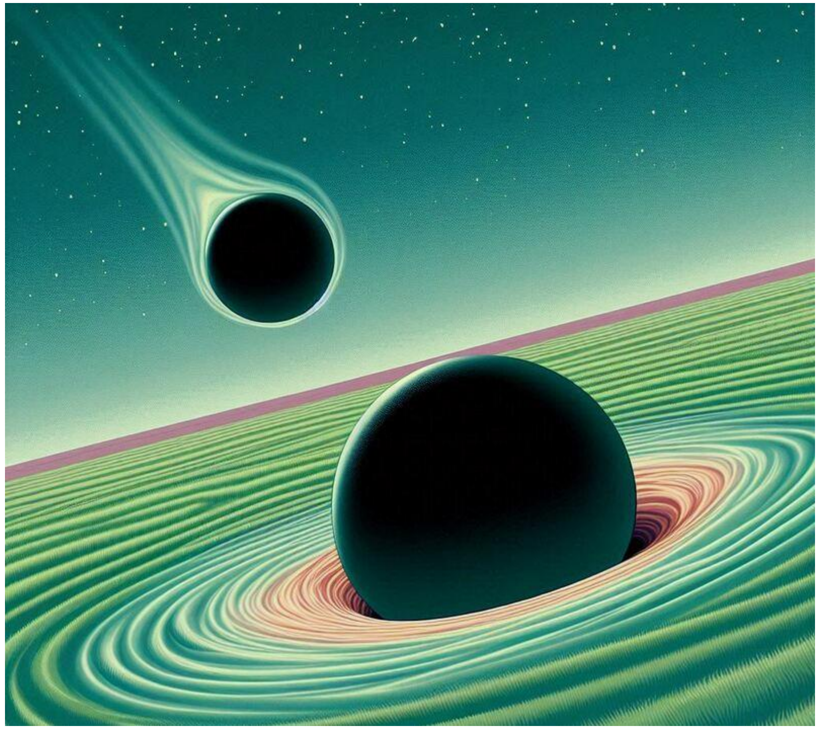
3PM Scattering Waveforms Including Spin Corrections



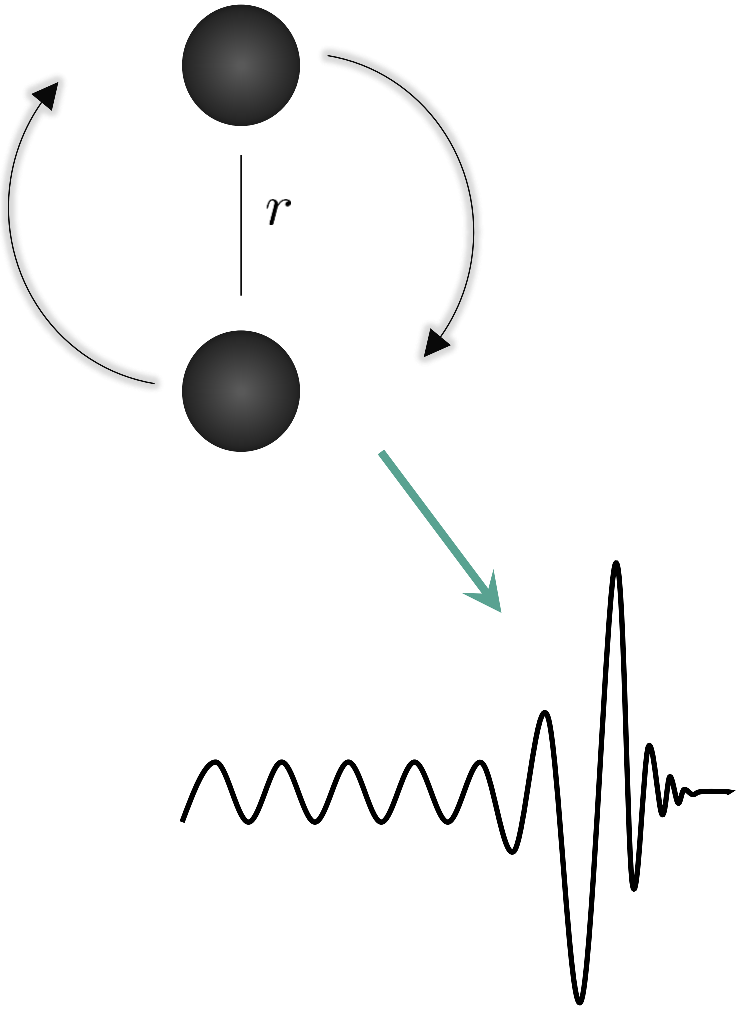
Based on 2312.14859 and ongoing work with H. Ita, M. Kraus and J. Schlenk

QCD Meets Gravity 2024, National Taiwan University
Lara Bohnenblust
12. December 2024

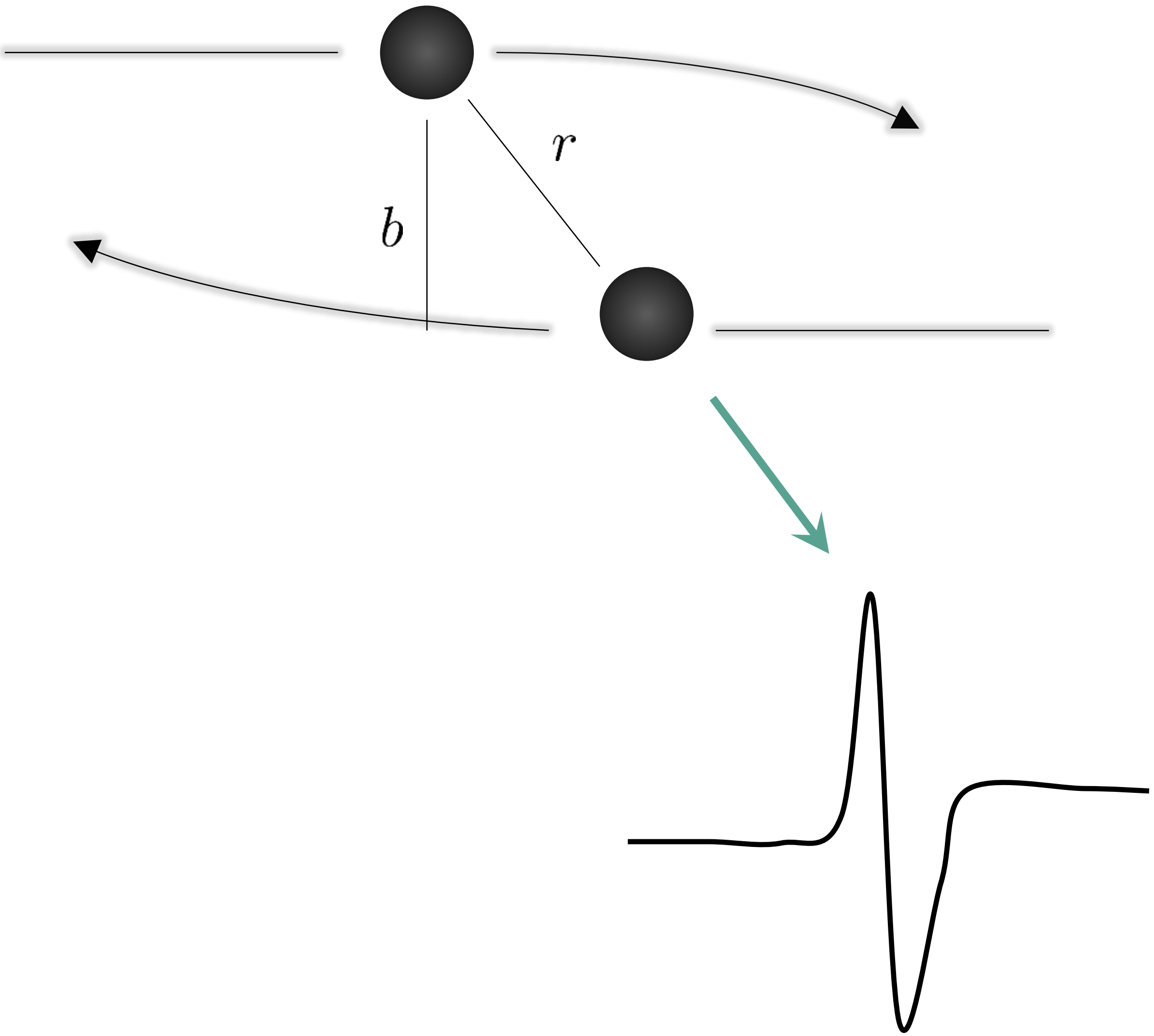
University of Zurich^{UZH}

Waveform Modelling

Bound System $v^2 \sim \frac{Gm}{r} \ll 1$



Unbound System $\frac{Gm}{r} \ll 1$



Waveform Modelling

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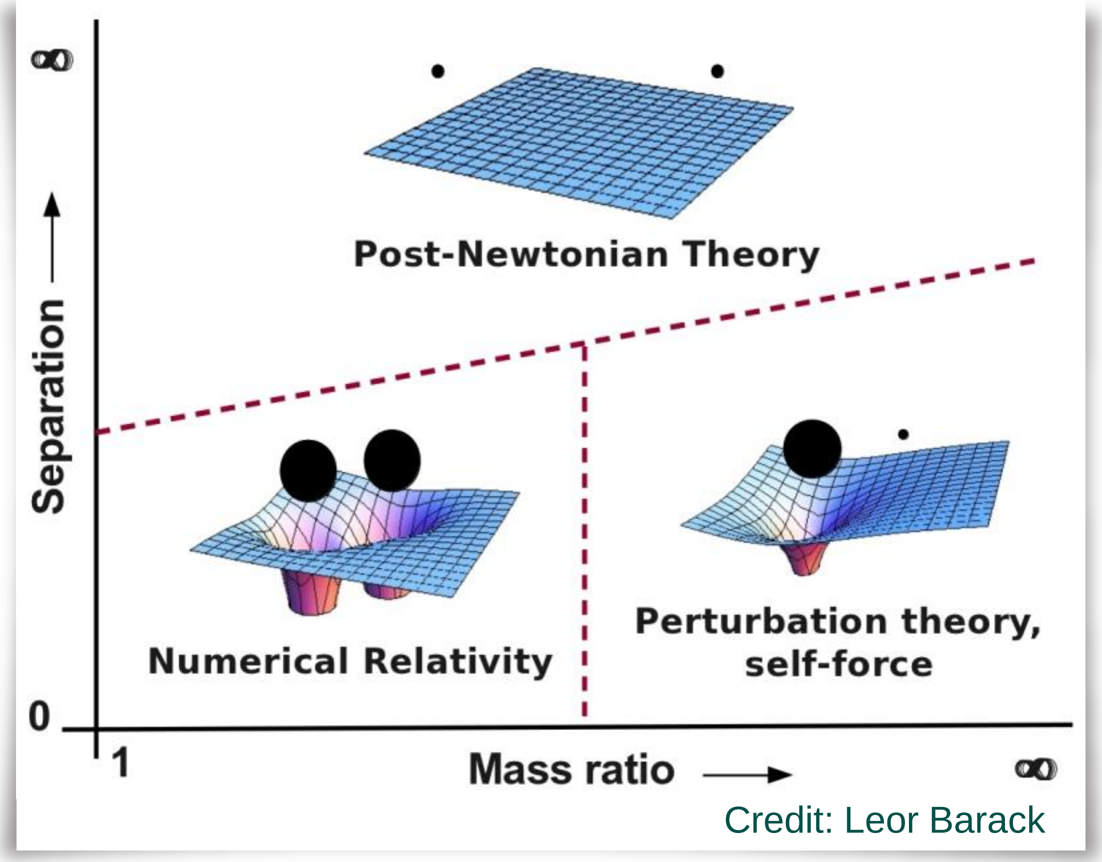
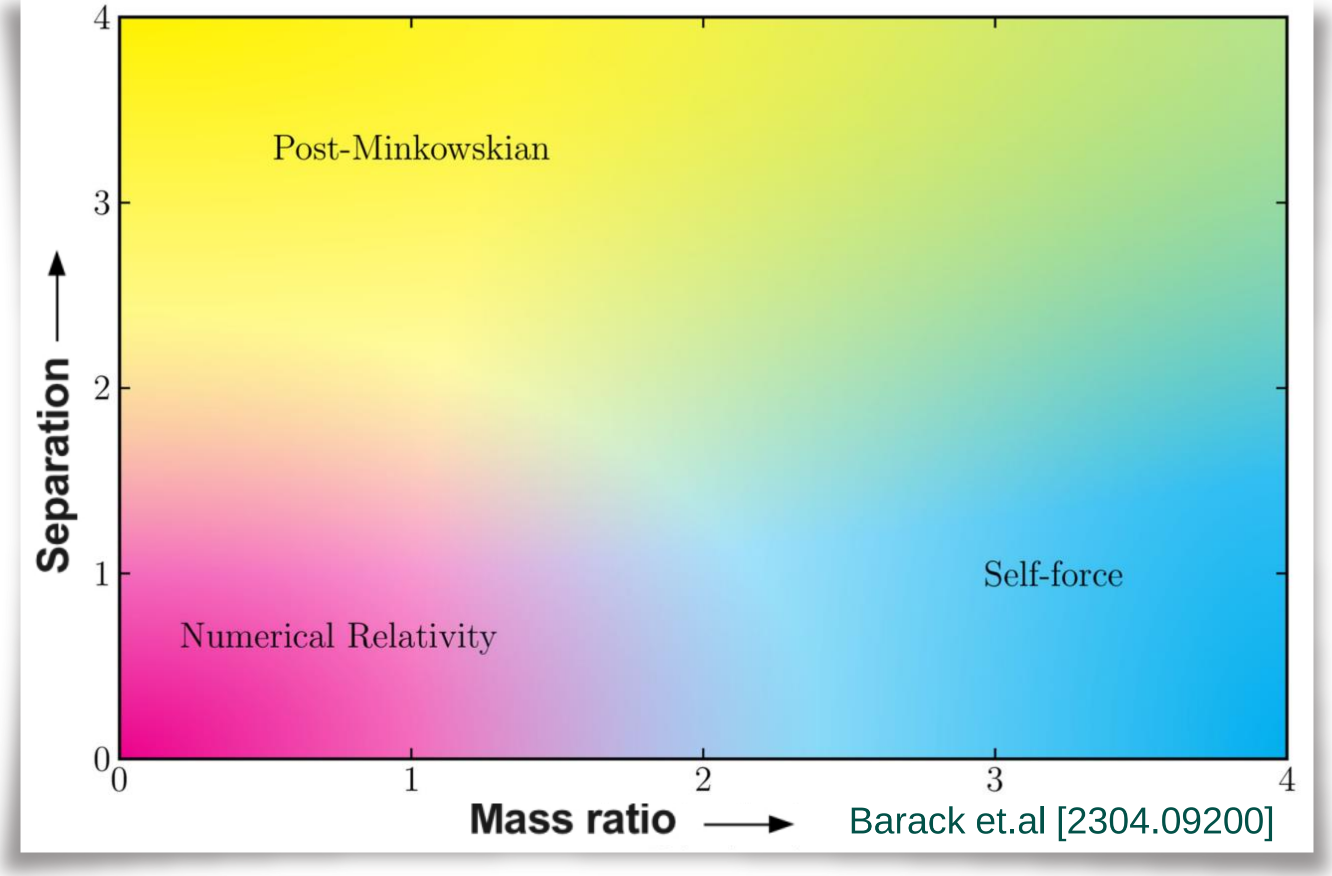


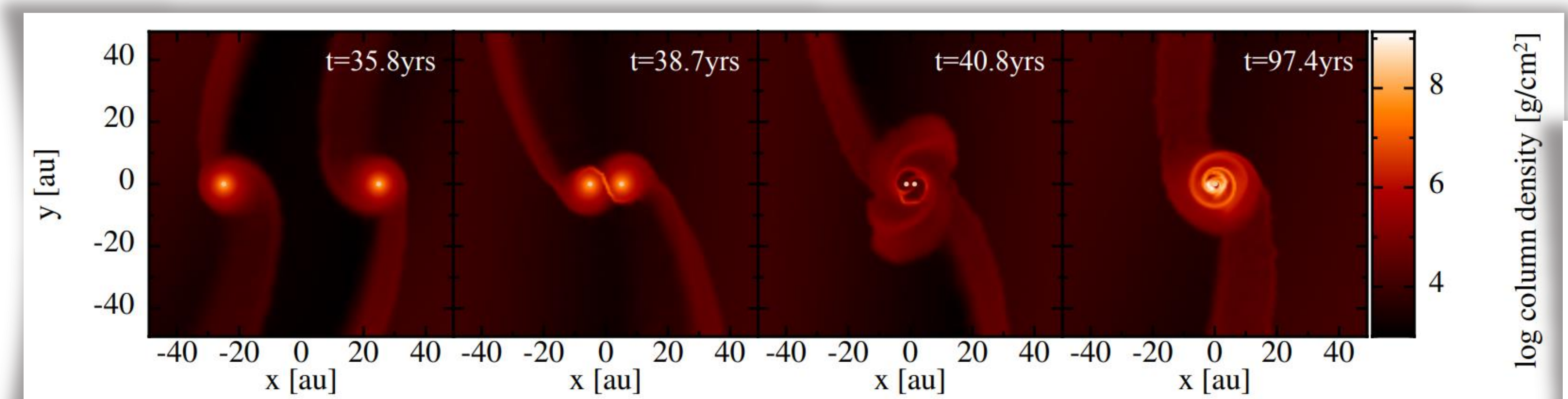
Credit: Leor Barack

Unbound System $\frac{Gm}{r} \ll 1$

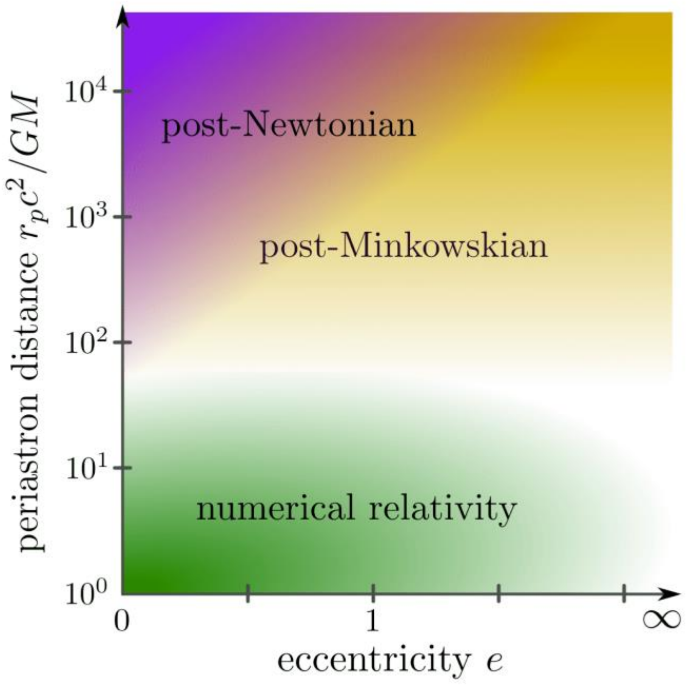


Barack et.al [2304.09200]

Motivation



Rowan, Boekholt, Kocsis, Haiman [2212.06133]

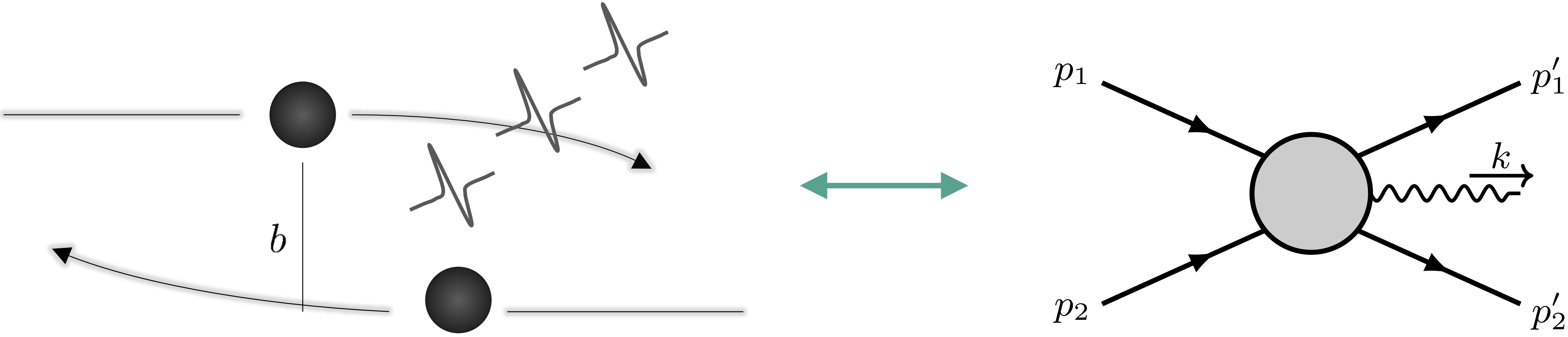


Khalil, Buonanno, Steinhoff, Vines [2204.05047]

- Precision and frequency ranges of future detectors
- Galactic nuclei harbor binaries on highly eccentric orbits and dynamical captures, as shown in simulations
- Analytic continuation from unbound to bound systems Kälin, Porto [1910.03008]; Adamo, Gonzo, Ilderton [2402.00124]; Bern, Cheung, Roiban, Shen, Solon [1908.01493]
- Input for effective-one-body (EOB) formalism Damour [1609.00354]; Antonelli, Buonanno, Steinhoff, van de Meent, Vines [1901.07102]
- Analytic, fast waveform templates Brunello, De Angelis [2403.08009]

PM Waveforms

$$h(t) \sim \sum_i c_i(p_1, p_2, k, b) \cdot \left(\frac{Gm}{b}\right)^i$$

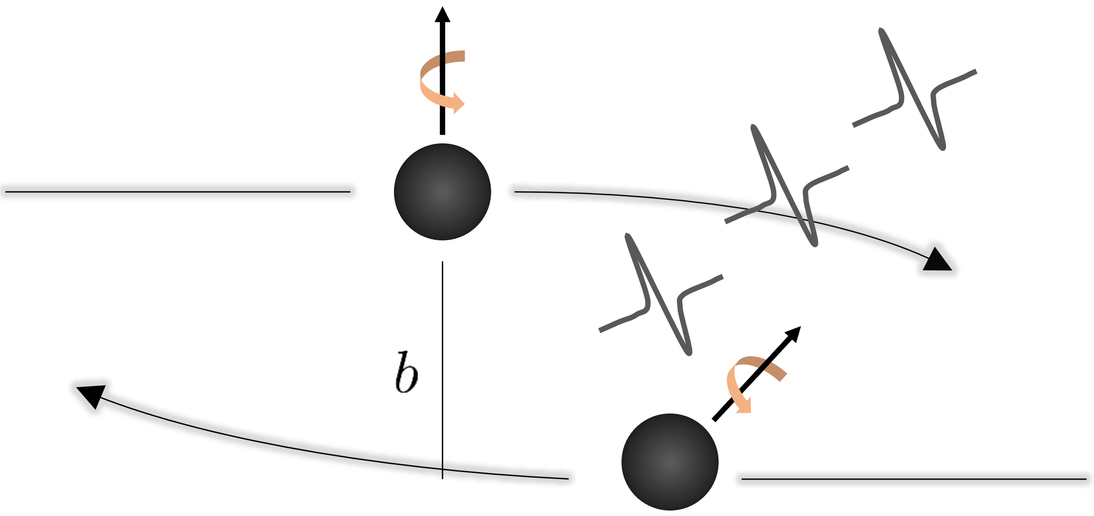


Scattering encounter

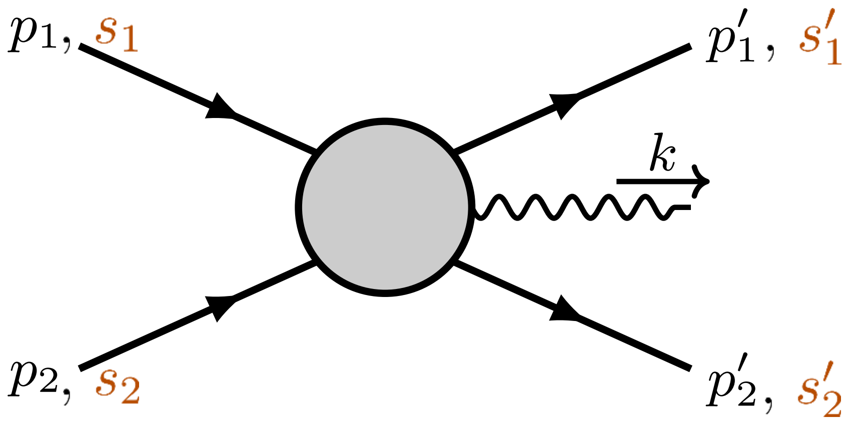
Scattering amplitude

PM Waveforms

$$h(t) \sim \sum_{ijk} c_{ijk}(p_1, p_2, k, b) \cdot \left(\frac{Gm}{b}\right)^i \left(\frac{S_1}{m_1 b}\right)^j \left(\frac{S_2}{m_2 b}\right)^k$$



Scattering encounter



Scattering amplitude

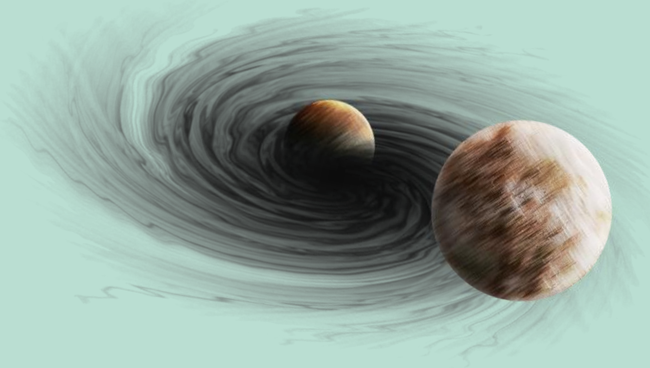
PM Waveforms

$$h(t) \sim \sum_{ijk} c_{ijk}(p_1, p_2, k, b) \cdot \left(\frac{Gm}{b}\right)^i \left(\frac{S_1}{m_1 b}\right)^j \left(\frac{S_2}{m_2 b}\right)^k$$

	S^0	S^1	S^2	S^3	S^4	S^∞ (conjectured)
Tree (G^2)	Kovacs & Thorne '78 Jakobsen, Mogull, Plefka, Steinhoff '21	Jakobsen, Mogull, Plefka, Steinhoff '21		De Angelis, Gonzo, Novichkov '23		Brandhuber, Brown, Chen, Gowdy, Travaglini; Aoude, Haddad, Heissenberg, Helset '23
1-loop (G^3)	Brandhuber, Brown, Chen, De Angelis, Gowdy; Herderschee, Roiban, Teng; Georgoudis, Heissenberg, Vazquez-Holm; **LB**, Ita Kraus, Schlenk '23	**LB**, Ita, Kraus, Schlenk '23	In progress (this talk)			

Extracting Waveforms

- Scattering Amplitudes + KMOC
- Worldline Formalism



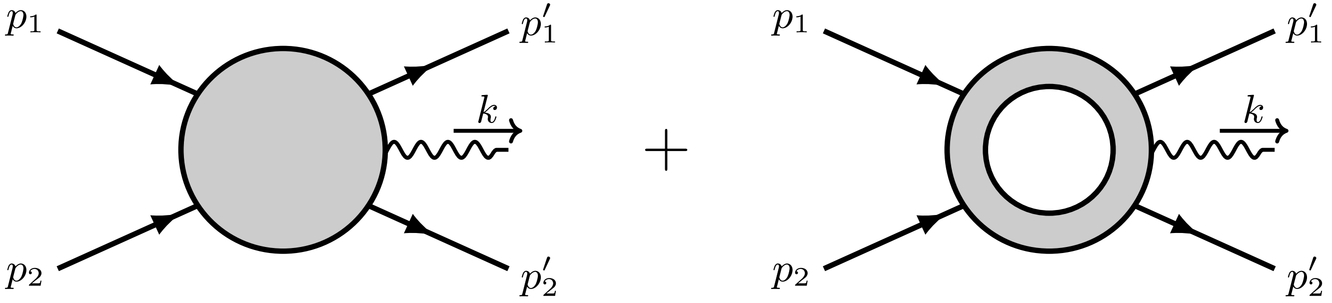
Observable-based Formalism (KMOC)

Kosower, Maybee, O'Connell [1811.10950]
Cristofoli, Gonzo, Kosower, O'Connell [2107.10193]

$$h^{(\eta)} = \lim_{r\to\infty} r\,\mathrm{FT}\left[i\langle p_1', p_2'|a_{(\eta)}(k)T|p_1, p_2\rangle + \langle p_1', p_2'|T^\dagger a_{(\eta)}(k)T|p_1, p_2\rangle + \mathrm{c.c.}\right]$$

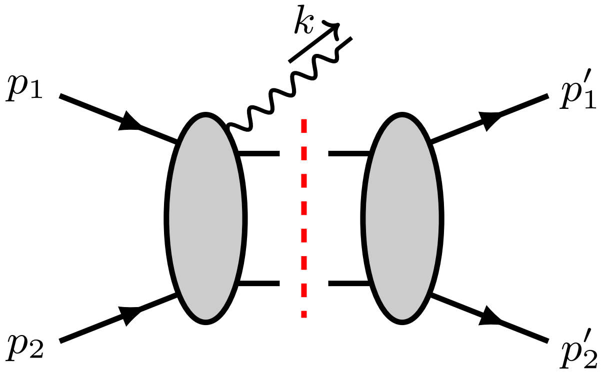
Up to order G^3 :

$$i\langle p_1'p_2'|a_{(\eta)}(k)T|p_1p_2\rangle =$$



$$= \mathcal{M}^{(\eta)}_{\mathrm{amp}}$$

$$\langle p_1'p_2'|T^\dagger a_{(\eta)}(k)T|p_1p_2\rangle =$$

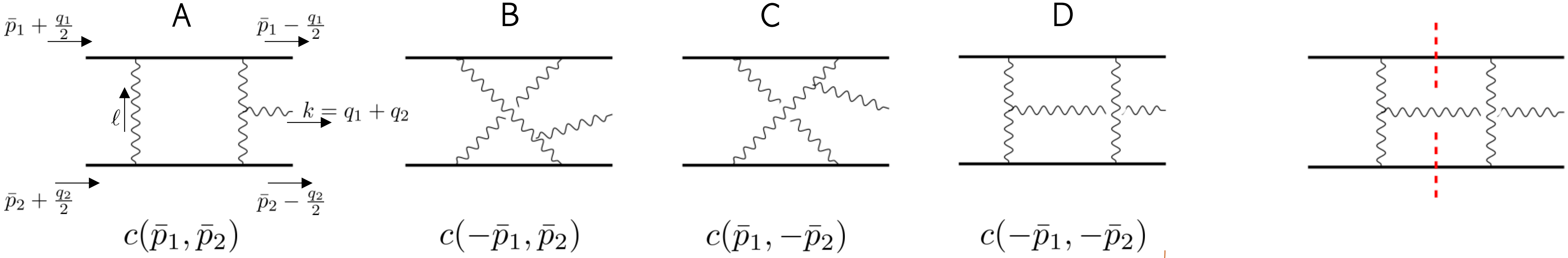


$$= \mathcal{M}^{(\eta)}_{\mathrm{cut}}$$

$$h^{(\eta)}(\tau) \sim \int_0^\infty d\omega e^{-i\omega\tau}\mathrm{FT}\Big[\Big(\mathcal{M}^{(\eta)}_{\mathrm{amp}} + \mathcal{M}^{(\eta)}_{\mathrm{cut}} + \mathrm{c.c.}\Big)\ \delta^{(D)}(q_1+q_2+k)\Big]$$

KMOC – Integrals

Caron-Huot, Giroux, Hannesdottir, Mizera
[2308.02125]



Eikonal Limit:

$$\sim \frac{1}{[2\bar{p}_1 \cdot \ell \pm i\epsilon][\bar{p}_2 \cdot \ell \pm i\epsilon)} \cdot \frac{1}{[\ell^2 + i\epsilon][(\ell + q_1)^2 + i\epsilon][\ell - q_2)^2 + i\epsilon]}$$

Weighted Sum

$$\sim \tilde{G}_{\text{ret}}(v_1 \cdot \ell)\,\delta(v_2 \cdot \ell)\tilde{G}_F(\ell)\tilde{G}_F(\ell + q_1)\tilde{G}_F(\ell - q_2)$$

Hyper-classical terms show up in the amplitude but get subtracted by the cut!

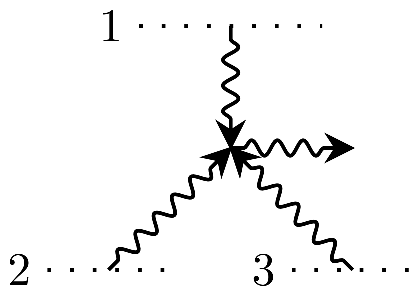
Worldline Formalism – Observables

(In-In formalism: see E. Pajer's talk)

Jakobsen, Mogull, Plefka, Sauer [2207.00569]
Kälin, Neef, Porto [2207.00580]

3-body waveform

$$h_{\mu\nu}(x) = k^2 \langle h_{\mu\nu} \rangle = \quad k^2 \times$$



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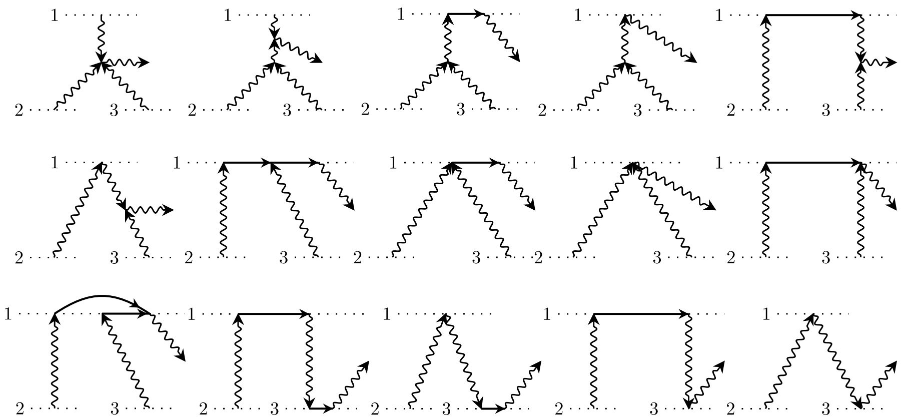

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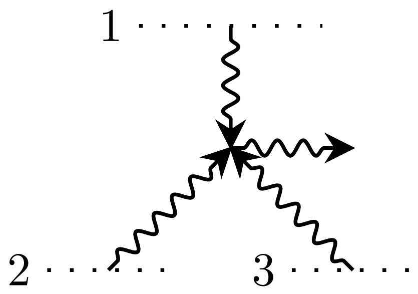

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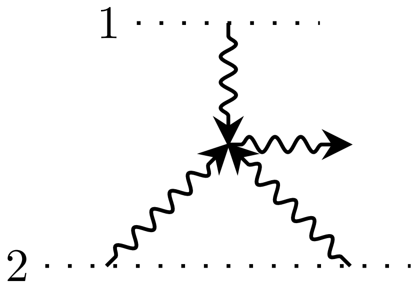
2-body waveform

$$h_{\mu\nu}(x) = k^2 \langle h_{\mu\nu} \rangle = \quad k^2 \times$$



Identify two of the black holes to get the NLO correction of the binary scattering!

Worldline Formalism – Observables

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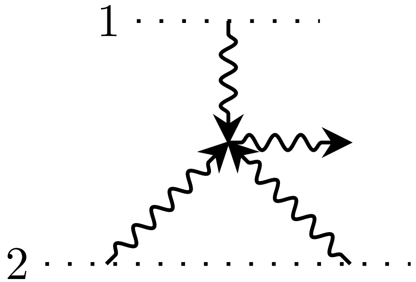

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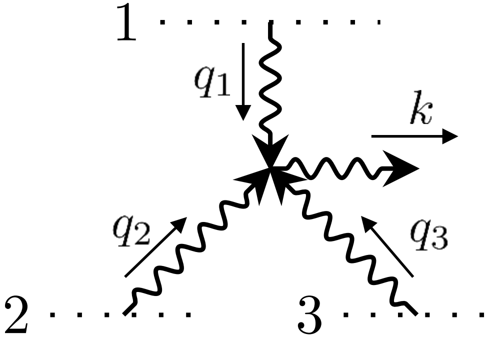
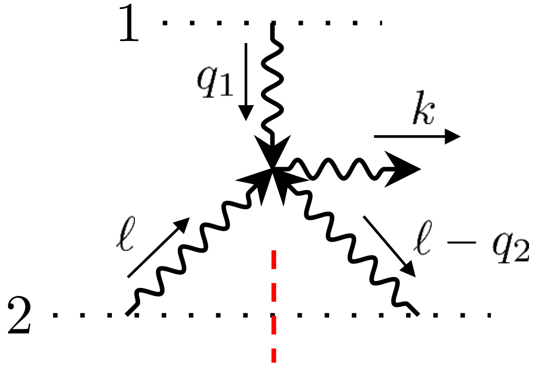


$$\xrightarrow{v_2 \equiv v_3}$$

$$q_2 \to \ell$$

$$q_3 \to q_2 - \ell$$

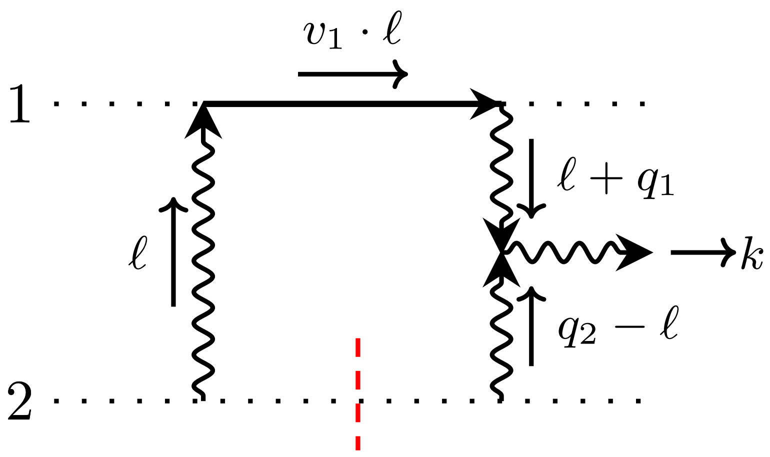


$$\sim \delta(v_1 \cdot q_1)\delta(v_2 \cdot q_2)\delta(v_3 \cdot q_3)\delta^d(k - q_1 - q_2 - q_3)$$

$$\sim \delta(v_2 \cdot \ell)\delta(v_1 \cdot q_1)\delta(v_2 \cdot q_2)\delta^d(k - q_1 - q_2)$$

This gives rise to Feynman-like integrals!

Worldline Formalism – Integrals



$$\sim \delta(v_2 \cdot \ell)\tilde{G}_{\mathrm{ret}}(v_1 \cdot \ell)\, \tilde{G}_{\mathrm{ret}}(\ell)\tilde{G}_{\mathrm{ret}}(\ell + q_1)\tilde{G}_{\mathrm{ret}}(q_2 - \ell)$$

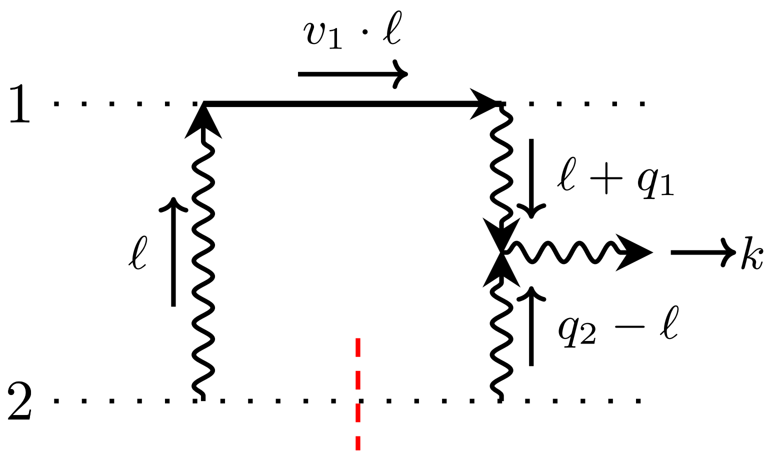
Feynman propagator: $\tilde{G}_{\mathrm{F}}(\ell) = \frac{-i}{\ell^2 + i\epsilon}$

Retarded propagator:
$$\tilde{G}_{\mathrm{ret}}(\ell) = \frac{-i}{(\ell_0 + i\tilde{\epsilon})^2 - \boldsymbol{\ell}^2} = \frac{-i}{\ell^2 + \mathrm{sign}(\ell_0) i\epsilon}$$

Momentum and energy conservation: $\delta(v_2 \cdot \ell)\delta(v_2 \cdot q_2)\delta^d(k - q_1 - q_2) \implies$

$$\boxed{\begin{aligned} \ell^0 &= q_2^0 - \ell^0 = 0 \\ \ell^0 + q_1^0 &= k^0 > 0 \end{aligned}}$$

Worldline Formalism – Integrals



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$$= \frac{-i}{\ell^2 + \mathrm{sign}(\ell_0)i\epsilon}$$

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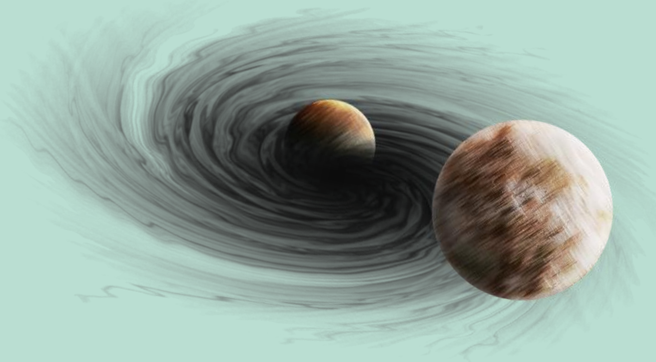
We can use Feynman propagators instead of retarded graviton propagators!

Computing Amplitudes

- Five-point one-loop
- Higher-point trees



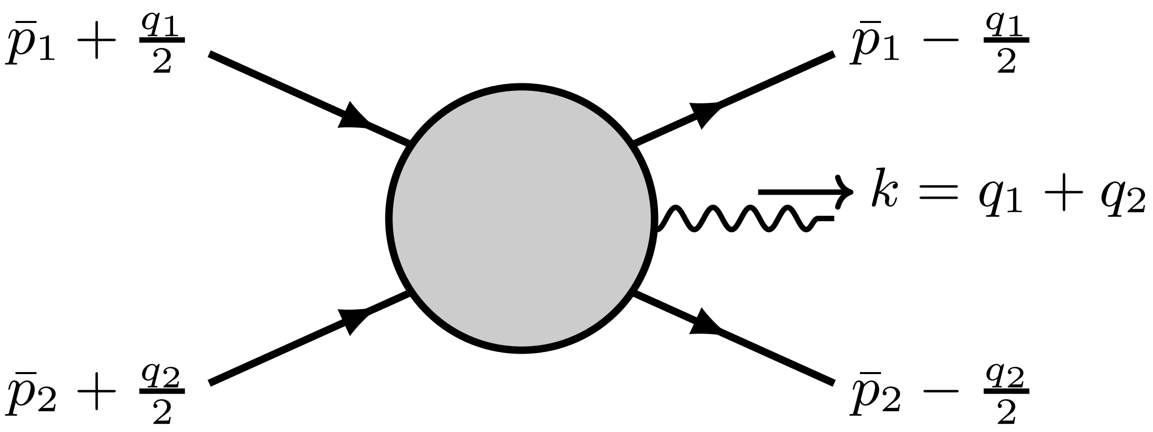
QFT and Kinematics

Minimally-coupled Proca field theory

$$\mathcal{L}_{(V,m)} = -\frac{1}{4}\sqrt{|g|}\Big[g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - 2m^2 g^{\mu\nu}V_\mu V_\nu\Big]\,, \quad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

Black-hole scattering

$$\mathcal{L}_{s^2} = \mathcal{L}_{\mathrm{EH}} + \mathcal{L}_{(V_1,m_1)} + \mathcal{L}_{(V_2,m_2)}$$



Multi-scale problem!

$$u_i = \bar{p}_i/\bar{m}_i\,,$$

$$\bar{m}_i^2 = \bar{p}_i^2 = p_i^2 - \frac{q_i^2}{4}\,,$$

$$y = u_1 \cdot u_2\,,$$

$$\omega_1 = u_1 \cdot q_2\,,$$

$$\omega_2 = u_2 \cdot q_1\,,$$

$$q_1^2\,, \quad q_2^2$$

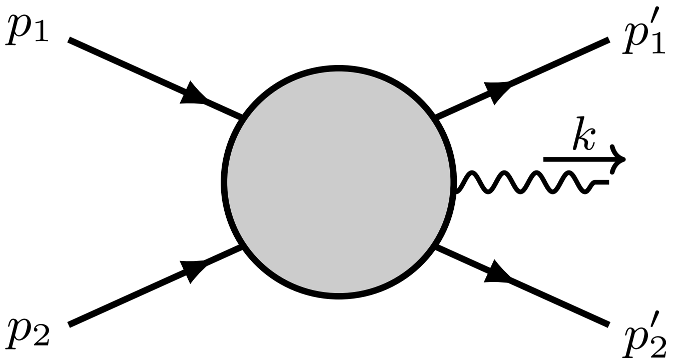
Classical limit $\bar{m}_i \to \infty$

Classical Spin

Boost

$$\Lambda^{\mu}{}_{\nu}(p_1,\bar{p}_1')\,\bar{\varepsilon}^{\nu}_{v}(\bar{p}_1') = \varepsilon^{\mu}_{v}(p_1)$$

Maybee, O'Connell, Vines [1906.09260]
Akpinar, Febres Cordero, Kraus, Ruf, Zeng [2407.19005]



$$\bar{\varepsilon}^{\mu}_{v'}(p_1') = \Lambda^{\mu}{}_{\nu}(p_1',\bar{p}_1')\,\bar{\varepsilon}^{\nu}_{v'}(\bar{p}_1')\,,$$

$$\bar{p}_1' \equiv \frac{m_1}{\bar{m}_1}\bar{p}_1$$

Pauli-Lubanski operator

e.g. Sexl, Urbantke, 1976

Chung, Huang, Kim, Lee [1812.08752]
Cangemi, Pichini [2207.03947]

Clebsch-Gordan decomposition

$$\bar{\varepsilon}^{\mu}_{v'}(p)\varepsilon^{\nu}_{v}(p) = \underbrace{\frac{1}{3}\bar{\varepsilon}_{v'}\cdot\mathbb{P}\cdot\varepsilon_{v}\left(\eta^{\mu\nu}-\frac{p^{\mu}p^{\nu}}{m^2}\right)}_{\text{scalar}} \underbrace{-\frac{i}{2m}\epsilon^{\mu\nu\rho\sigma}p_{\rho}\bar{\varepsilon}_{v'}\cdot\mathbb{S}_{\sigma}\cdot\varepsilon_{v}}_{\text{antisymmetric}} \underbrace{+\bar{\varepsilon}_{v'}\cdot\mathbb{S}^{\{\mu}\mathbb{S}^{\nu\}}\cdot\varepsilon_{v}}_{\text{symmetric-traceless}}$$

Classical spin replacement

$$\frac{\bar{\varepsilon}_{v}\cdot\mathbb{S}^{\mu}\cdot\varepsilon_{v}}{\bar{\varepsilon}_{v}\cdot\varepsilon_{v}} \to S^{\mu}$$

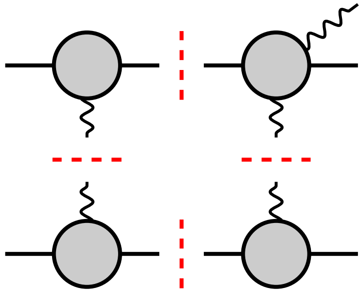
Numerical Unitarity

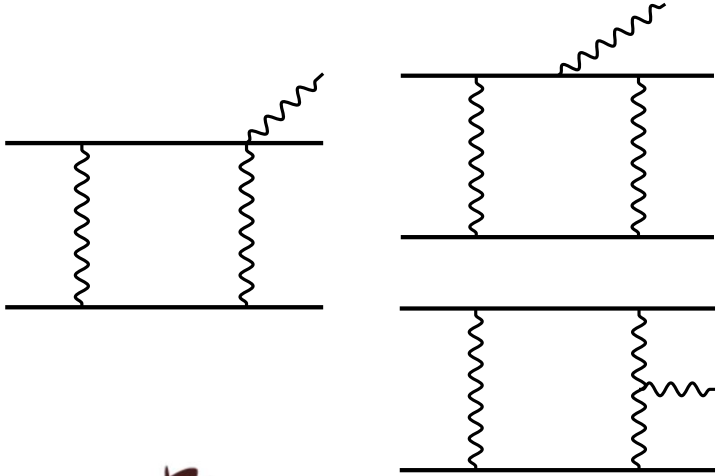
Integrand equation

$$\sum_{\substack{\text{virtual}\\ \text{states}}} \prod_{i\in T_\Gamma} M_i^{\text{tree}}(\ell^\Gamma) = \sum_{\Gamma'\geq\Gamma} \sum_{j\in M_{\Gamma'}\cup S_{\Gamma'}} c_{\Gamma',j}(\epsilon)\,\mathcal{I}_{\Gamma',j}(\ell^\Gamma).$$

Tree amplitudes corresponding to vertices of diagram Γ

Master and surface term integrands





Compute finite-field arithmetic numerical values for coefficients.

→ Analytically reconstruct

$$c^n_\Gamma(\epsilon, D_s, \bar{m}) = \bar{m}^{p^n_\Gamma} \sum_{k=-2}^{1} \sum_{l=0}^{\infty} c^n_{\Gamma,kl}(\epsilon)(D_s-2)^k \frac{1}{\bar{m}^l}\,.$$

Caravel

Abreu et al. [2009.11957]

Trees – WQFT

WQFT action

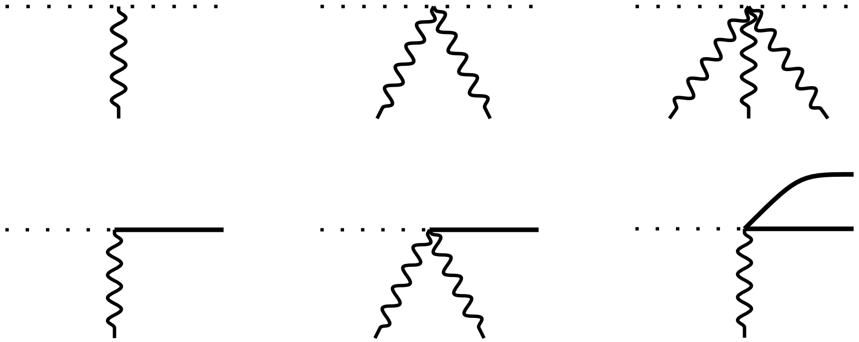
Jakobsen, Mogull, Plefka, Steinhoff [2109.04465]

(see talk by B. Sauer)

$$S_i = -m_i \int d\tau \Big[\frac{1}{2} g_{\mu\nu}\dot{x}_i^\mu \dot{x}_i^\nu + i\bar{\psi}_{i,a}\frac{D\psi_i^a}{D\tau} + \frac{1}{2}R_{abcd}\bar{\psi}_i^a\psi_i^b\bar{\psi}_i^c\psi_i^d\Big],$$

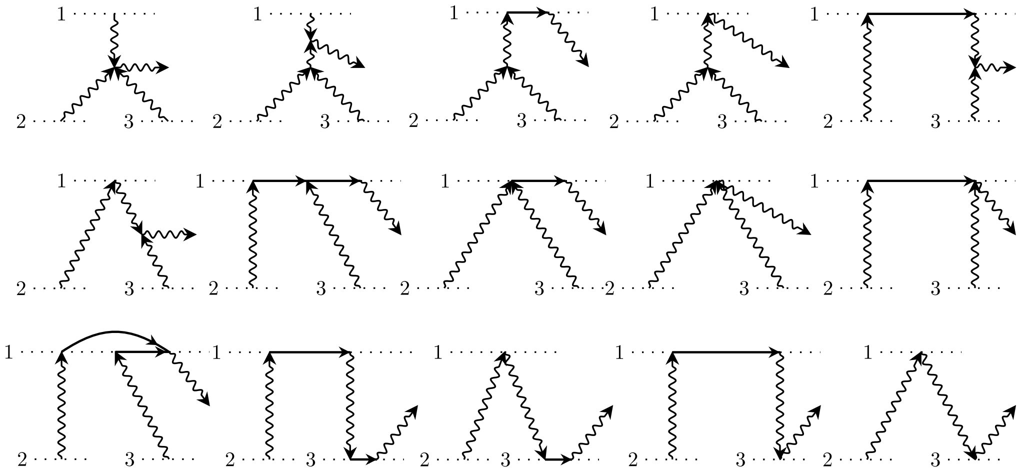
$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu\tau + z_i^\mu(\tau)\,, \qquad \psi_i^a(\tau) = \Psi_i^a + \phi_i^a(\tau)\,, \quad \mathcal{S}_i^{ab} = -2i\bar{\Psi}_i^{[a}\Psi_i^{b]}\,, \qquad S = S_{\rm EH} + S_1 + S_2.$$

Feynman rules



+ graviton interactions

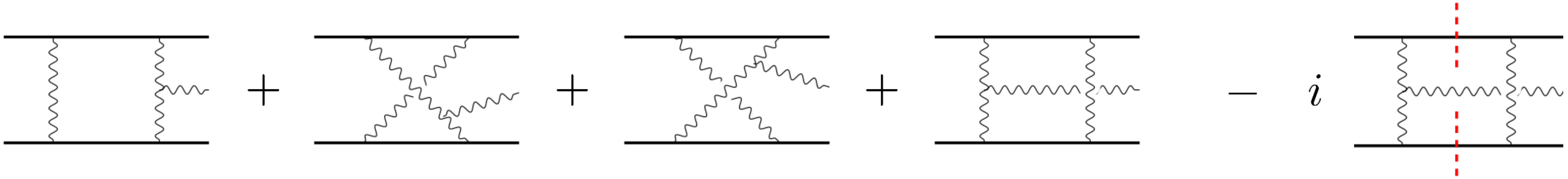
3-body trees



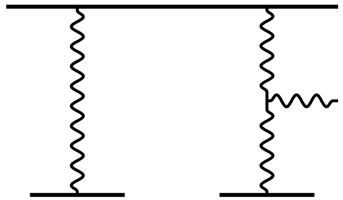
Trees – QFT

Equivalently: Compute seven-point trees for the 3-body scattering in QFT

Instead of



compute



This avoids hyper-classical terms!

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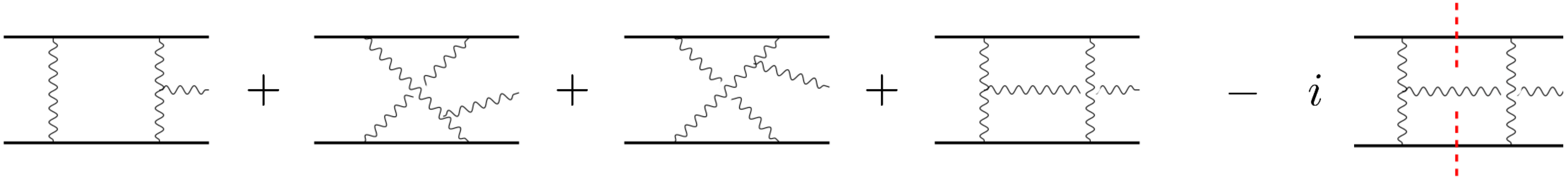

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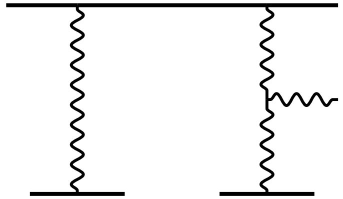


$$\mathcal{A} = \sum_n \mathcal{A}^n F^n\,,$$

$$\mathcal{A}^n = \frac{\sum_k c_k^n\,\bar{m}^k}{\sum_l c_l^n\,\bar{m}^l}$$

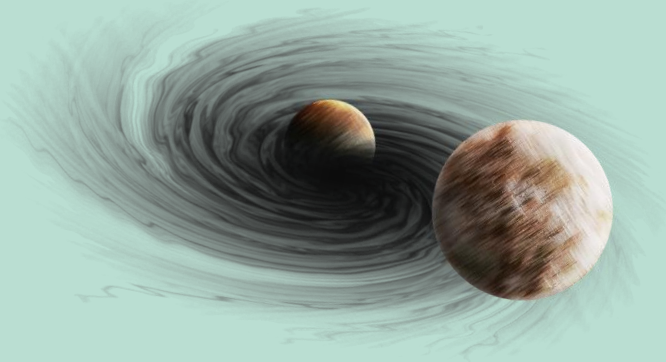
This avoids hyper-classical terms!

Caravel

Abreu et al. [2009.11957]

Future application: Check high-loop integrands in WQFT using Caravel!

Results & Validation

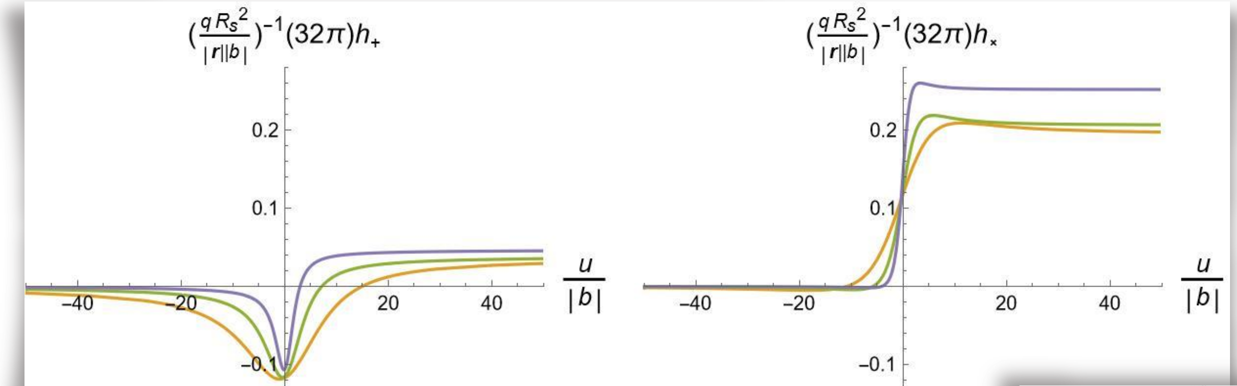


Results

LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

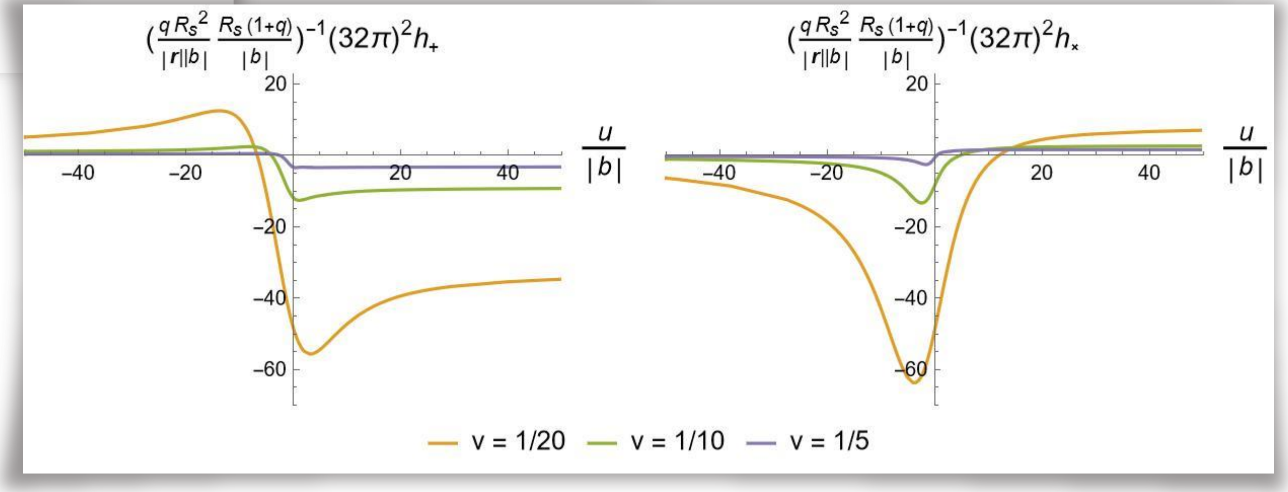
$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

S^0, S^1 and S^2



Scalar $\mathcal{M}^{\text{tree}}$

Scalar $\mathcal{M}^{\text{finite}}$



Validation

LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

$$\mathcal{M} = \mathcal{M}^{\mathrm{tree}} + \mathcal{M}^{\mathrm{IR}} + \mathcal{M}^{\mathrm{UV}} + \mathcal{M}^{\mathrm{tail}} + \mathcal{M}^{\mathrm{finite}}$$

	Other groups	MPM	Gauge invariance	IR	UV	Caravel
S^0						
S^1						
S^2						

✓ Validated

✗ Not checked

404 Results not available

🧑‍🔬 Work in progress

Validation

LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

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S^0						
S^1						
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✗ Not checked

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🧑‍🔬 Work in progress

Brandhuber, Brown, Chen, De Angelis, Gowdy [2303.06111]
Herderschee, Roiban, Teng [2303.06112]
Georgoudis, Heissenberg, Vazquez-Holm [2312.14710]

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	Other groups	MPM	Gauge invariance	IR	UV	Caravel
S^0	✓					
S^1	404					
S^2	404					

✓ Validated
✗ Not checked
404 Results not available
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S^0	✓					
S^1	404					
S^2	404					

✓ Validated

✗ Not checked

404 Results not available

👩‍🔬 Work in progress

Multipolar post-Minkowski (MPM)

Matching multipoles using the post-Newtonian expansion.

Bini, Damour, De Angelis, Geralico, Herderschee, Roiban, Teng [2402.06604]
Georgoudis, Heissenberg, Russo [2402.06361]

Validation

LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

	Other groups	MPM	Gauge invariance	IR	UV	Caravel
S^0	✓	✓				
S^1	404	✗				
S^2	404	✗				

- ✓ Validated
- ✗ Not checked
- 404 Results not available
- 🧑‍🔬 Work in progress

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	Other groups	MPM	Gauge invariance	IR	UV	Caravel
S^0	✓	✓				
S^1	404	✗				
S^2	404	✗				

- ✓ Validated
- ✗ Not checked
- 404 Results not available
- 🧑‍🔬 Work in progress

$$k_\mu h^{\mu\nu} = 0$$

Validation

LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

	Other groups	MPM	Gauge invariance	IR	UV	Caravel
S^0	✓	✓	✓			
S^1	404	✗	✓			
S^2	404	✗	✓			

- ✓ Validated
- ✗ Not checked
- 404 Results not available
- 🧑‍🔬 Work in progress

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Validation

LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

	Other groups	MPM	Gauge invariance	IR	UV	Caravel
S^0	✓	✓	✓			
S^1	404	✗	✓			
S^2	404	✗	✓			

- ✓ Validated
- ✗ Not checked
- 404 Results not available
- 🧑‍🔬 Work in progress

$$\mathcal{M}^{\text{IR}} = \left[\frac{1}{\epsilon} - \log\left(\frac{\mu_{\text{IR}}^2}{\mu^2}\right)\right] \mathcal{W}_S \, \mathcal{M}^{\text{tree}}_{D=4}$$

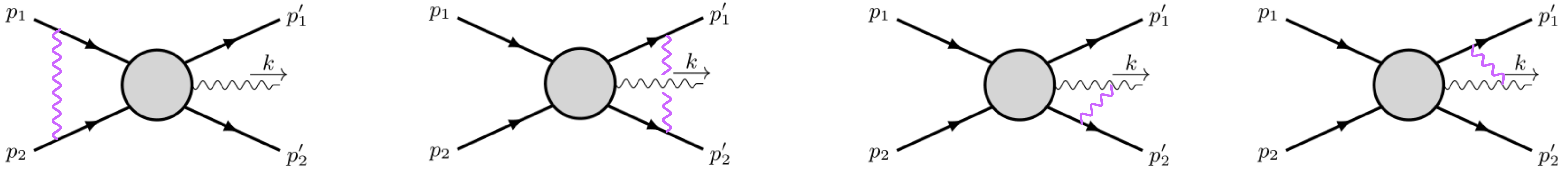
Weinberg soft factor: $\mathcal{W}_S = iG(\bar{m}_1\omega_1 + \bar{m}_2\omega_2)\left(1 + \frac{y(2y^2-3)}{2(y^2-1)^{3/2}}\right)$

Caron-Huot, Giroux, Hannesdottir, Mizera [2308.02125]

Weinberg for Observables

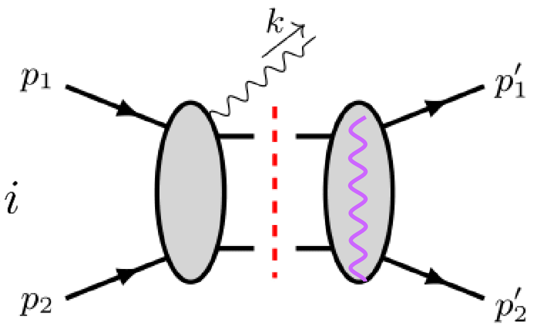
Weinberg soft theorem

Weinberg, 1965



$$W_S = W_{12} + W_{1'2'} + W_{1'k} + W_{2'k}$$

Subtract cut $-i$



$$\Longrightarrow \quad \mathrm{Cut}[W_{1'2'}] = 2\,\mathrm{Im}[W_{1'2'}]$$

$$\mathcal{W}_S \; = \; W_S - 2\,i\mathrm{Im}[W_{1'2'}]$$

Validation

LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

$$\mathcal{M} = \mathcal{M}^{\rm tree} + \mathcal{M}^{\rm IR} + \mathcal{M}^{\rm UV} + \mathcal{M}^{\rm tail} + \mathcal{M}^{\rm finite}$$

	Other groups	MPM	Gauge invariance	IR	UV	Caravel
S^0	✓	✓	✓			
S^1	404	✗	✓			
S^2	404	✗	✓			

✓ Validated

✗ Not checked

404 Results not available

👩‍🔬 Work in progress

$$\mathcal{M}^{\rm IR} = \left[\frac{1}{\epsilon} - \log\left(\frac{\mu_{\rm IR}^2}{\mu^2}\right)\right] \mathcal{W}_S\, \mathcal{M}^{\rm tree}_{D=4}$$

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Caron-Huot, Giroux, Hannesdottir, Mizera [2308.02125]

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LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

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S^0	✓	✓	✓	✓		
S^1	404	✗	✓	✓		
S^2	404	✗	✓	🧑‍🔬		

✓ Validated

✗ Not checked

404 Results not available

🧑‍🔬 Work in progress

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S^0	✓	✓	✓	✓		
S^1	404	✗	✓	✓		
S^2	404	✗	✓	👩‍🔬		

✓ Validated

✗ Not checked

404 Results not available

👩‍🔬 Work in progress

Contact term that integrates to $\delta(|b|)$ after the FT and does not contribute to the far-field waveform!

Validation

LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

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S^0	✓	✓	✓	✓	✓	
S^1	404	✗	✓	✓	✓	
S^2	404	✗	✓	👩‍🔬	👩‍🔬	

✓ Validated

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404 Results not available

👩‍🔬 Work in progress

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S^2	404	✗	✓	🧑‍🔬	🧑‍🔬	

✓ Validated

✗ Not checked

404 Results not available

🧑‍🔬 Work in progress

Caravel Abreu et al. [2009.11957]

Validation

LB, Ita, Kraus, Schlenk [2312.14859] and ongoing work

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S^0	✓	✓	✓	✓	✓	✓
S^1	404	✗	✓	✓	✓	✓
S^2	404	✗	✓	👩‍🔬	👩‍🔬	👩‍🔬

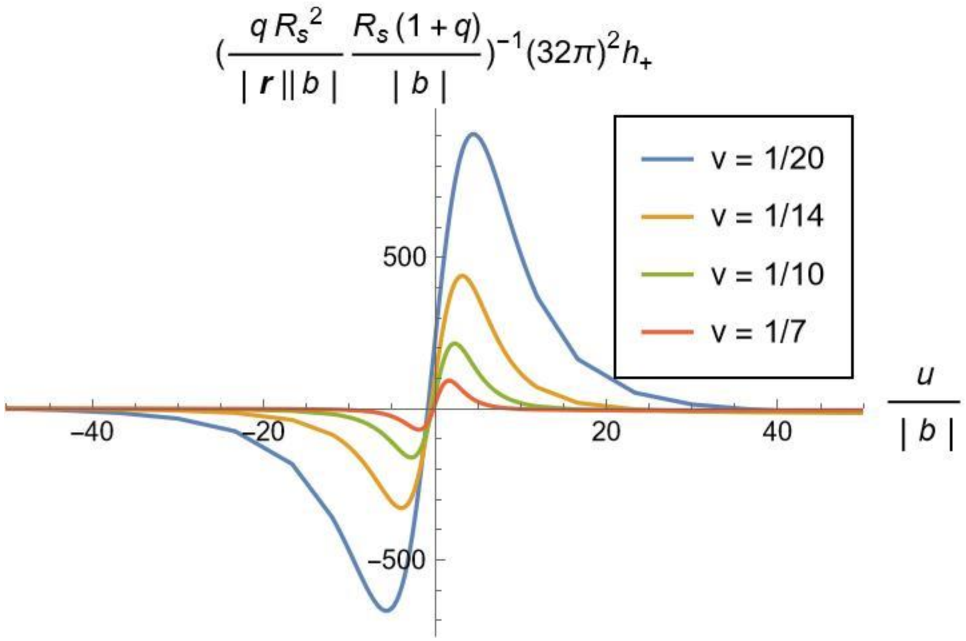
✓	Validated
✗	Not checked
404	Results not available
👩‍🔬	Work in progress

Abreu et al. [2009.11957]

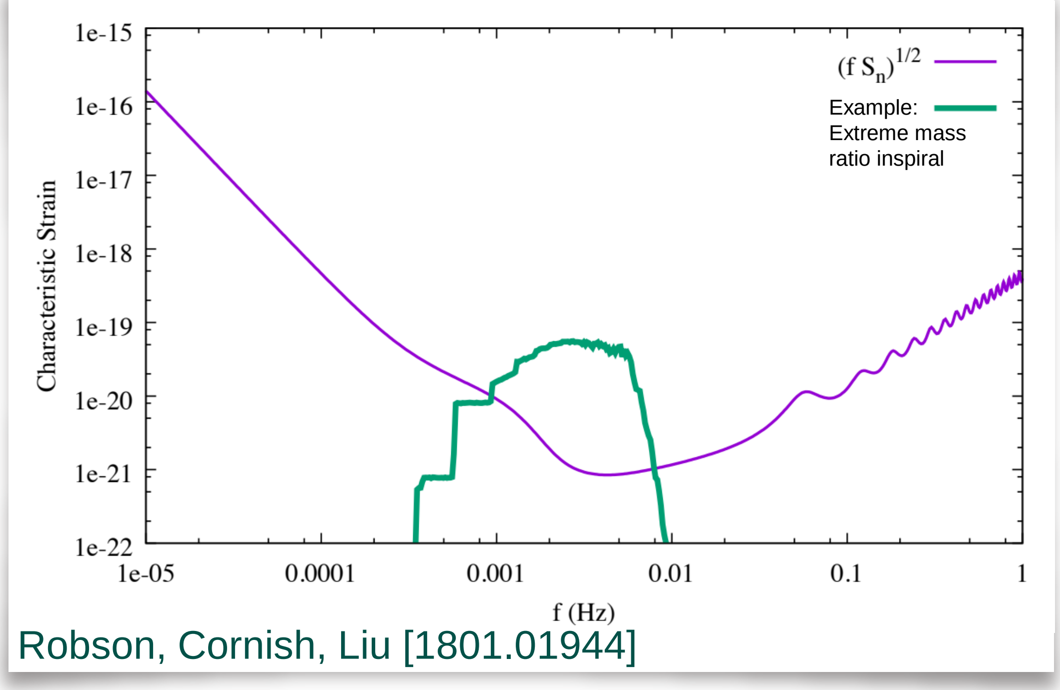
Relevance for Detections

LB, Ita, Kraus, Schlenk [2312.14859]

Scalar NLO correction:



LISA sensitivity curve



Robson, Cornish, Liu [1801.01944]

Detectability example

$$|b| = 1000\, r_S^1\,,$$

$$m_1 = 5\cdot 10^4\, M_\odot, m_2 = 50\, M_\odot\,, \qquad T \sim 10^4\,\mathrm{s}\,,$$

$$|\boldsymbol{r}| \approx 10^3\,\mathrm{kpc}, \text{ distance to Andromeda,} \qquad f \sim 10^{-4}\,\mathrm{Hz}\,,$$

$$h_{\mathrm{LO}} \sim 10^{-13},$$
$$h_{\mathrm{NLO-corr.}} \sim 10^{-15}$$

Relevance for Detections

LB, Ita, Kraus, Schlenk [2312.14859]

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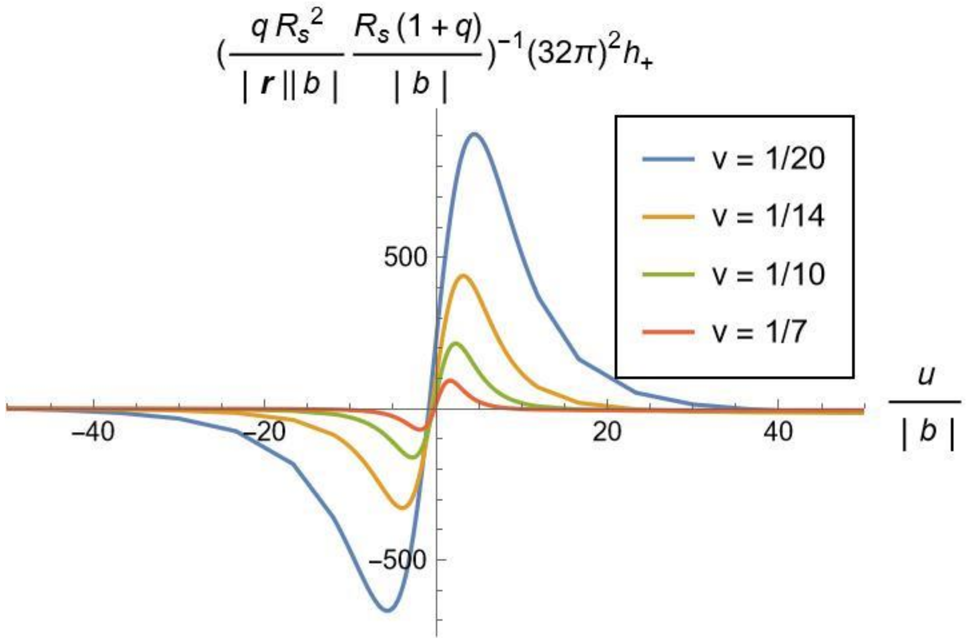
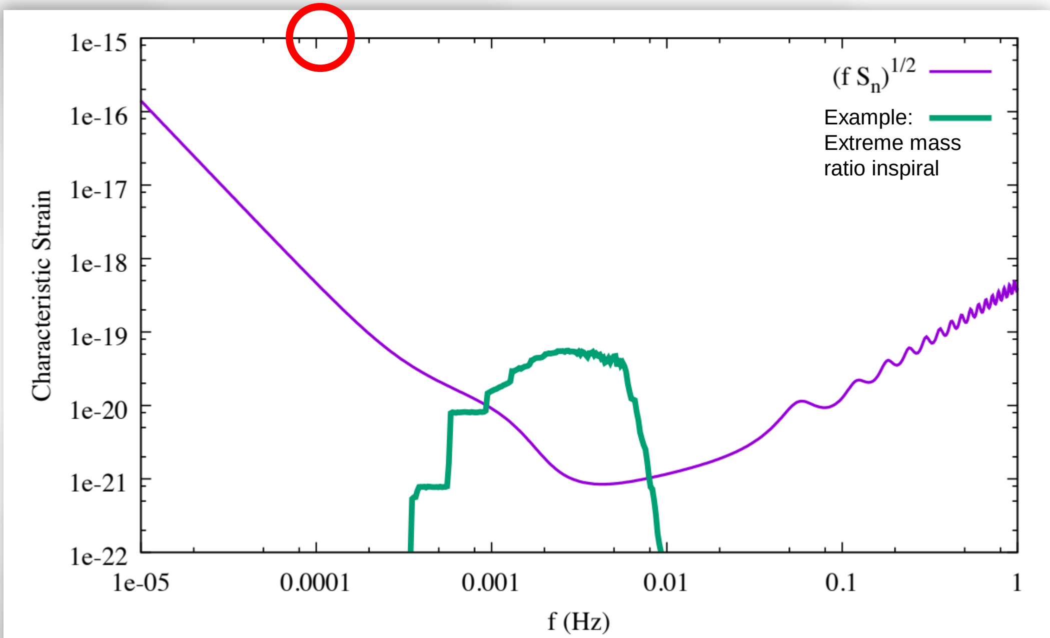


LISA sensitivity curve



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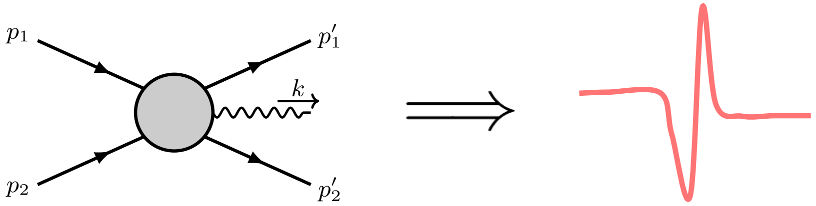
Summary

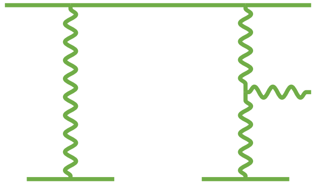
In this talk



- Extracting waveforms from field theory and WQFT
- Field theory integrals can be used for WQFT

$$\int d^d\ell \cdots$$

- Use trees in the classical limit to avoid hyper-classical terms



- Validation of results including S^2 ✓

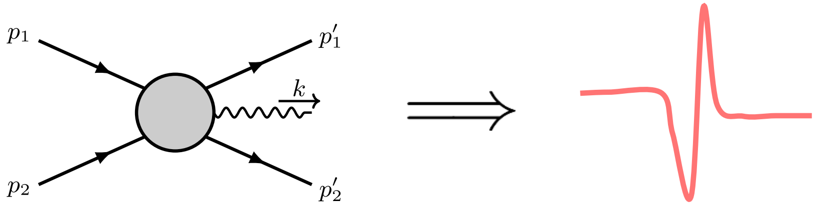
Caravel

Outlook

- Comparison of spin-corrected waveforms to MPM formalism
- Additions: Fourier transformation, non-constant spin, higher-loop order, finite-size effects …
- Mapping to bound system

Summary

In this talk



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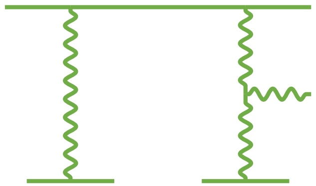

- Use trees in the classical limit to avoid hyper-classical terms
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Outlook

Thanks!

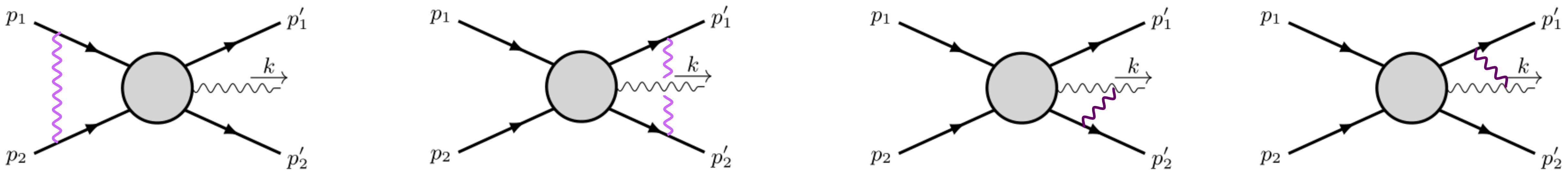
- Comparison of spin-corrected waveforms to MPM formalism
- Additions: Fourier transformation, non-constant spin, higher loop order, finite-size effects …
- Mapping to bound system

Backup Slides

Weinberg for Observables

Weinberg soft theorem

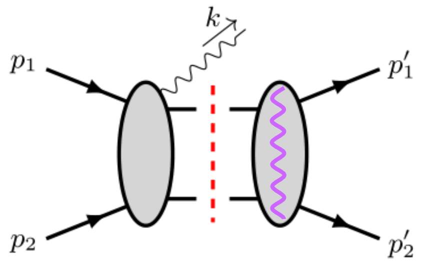
Weinberg, 1965



$$\mathrm{Re}\Big[W_S\Big] = 0$$

$$\mathrm{Im}\Big[W_S\Big] = \frac{1}{4}\sum_{\substack{i,j=1\\ i\neq j}}^{n} c_{ij}\,\mathrm{Im}[f_{ij}] = -\frac{\pi}{2}\left(2c_{12} + 2c_{1'2'} + c_{1'k} + c_{2'k}\right), \qquad \mathrm{Im}[f_{ij}] = \begin{cases} -\pi\Theta[(p_i\cdot p_j)]\,, & m_{i/j} = 0\,, \\ -2\pi\Theta[(p_i\cdot p_j)], & \text{else}\,. \end{cases}$$

Subtract cut: $-i$



$$\Longrightarrow \qquad \mathrm{Cut}[f_{1'2'}] = 2\,\mathrm{Im}[f_{1'2'}]$$

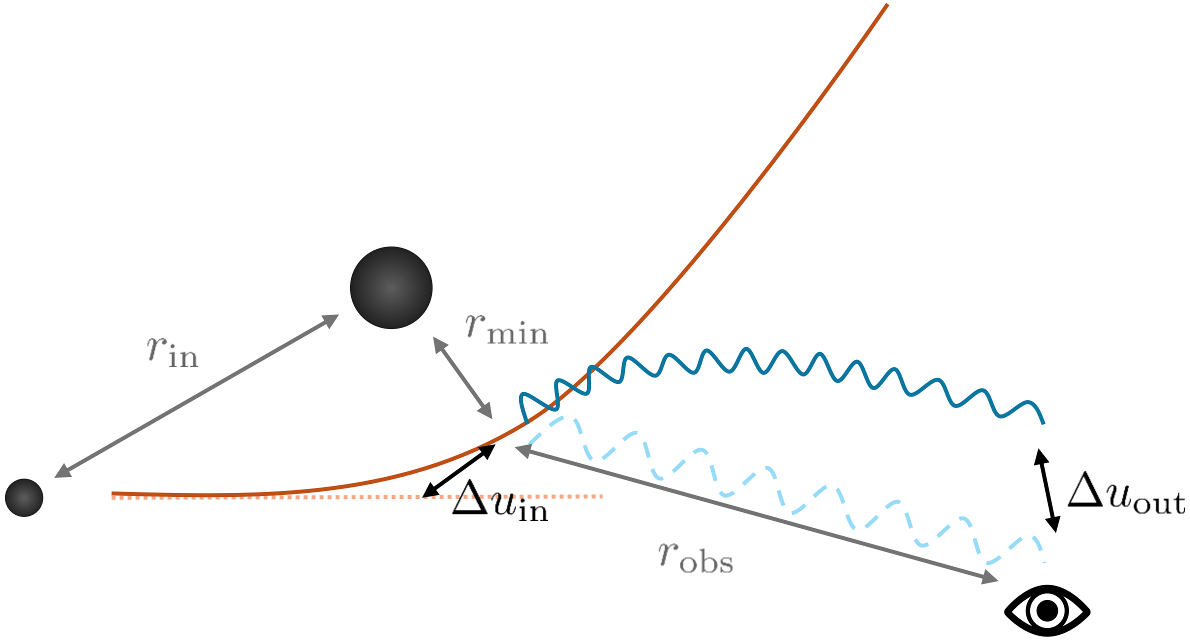
$$\mathrm{Im}\Big[\mathcal{W}_S\Big] = \mathrm{Im}\Big[W_S\Big] - \mathrm{Cut}_{1'2'}(W_S) = -\frac{\pi}{2}\left(2c_{12} - 2c_{1'2'} + c_{1'k} + c_{2'k}\right).$$

Waveform: IR Divergence

Exponentiation

$$e^{-i\omega u}\mathcal{M} = e^{-i\omega\left[u-\left(\frac{1}{\epsilon}-\log\frac{\mu_{\rm IR}^2}{\mu^2}\right)\frac{\mathcal{W}_S}{i\omega}\right]}\left(\mathcal{M}^{\rm tree}-\mathcal{W}_S\mathcal{M}^{\rm tree}_{\epsilon}+\mathcal{M}^{\rm UV}+\mathcal{M}^{\rm tail}+\mathcal{M}^{\rm finite}\right)+\mathcal{O}(G^3)$$

Classical interpretation



Shapiro delay

$$\Delta u_{\rm out} = \frac{2G}{\omega}(\bar{m}_1\omega_1+\bar{m}_2\omega_2)\log\left(\frac{r_{\rm obs}}{r_{\rm min}}\right)$$

Deflection of incoming trajectory

$$\Delta u_{\rm in} = \frac{2G}{\omega}(\bar{m}_1\omega_1+\bar{m}_2\omega_2)\frac{y(2y^2-3)}{2(y^2-1)^{3/2}}\log\left(\frac{r_{\rm in}}{r_{\rm min}}\right)$$

Cut contribution

Caron-Huot, Giroux, Hannesdottir, Mizera [2308.02125]

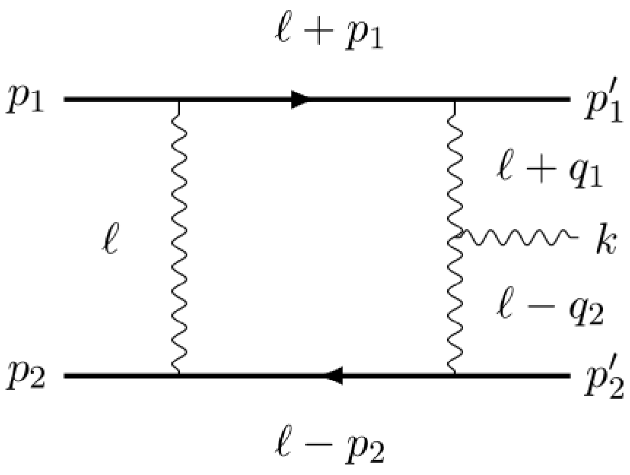
Waveform: UV Divergence

LB, Ita, Kraus, Schlenk [2312.14859]

$$\mathcal{M}^{\rm UV} = -\left(\frac{\kappa}{2}\right)^5 \frac{i}{8\pi\epsilon}(\bar{m}_1\omega_1+\bar{m}_2\omega_2)\frac{\bar{m}_1^2\bar{m}_2^2(\omega_1^2+\omega_2^2+y\omega_1\omega_2)(1-2y^2)^2}{\omega_1\omega_2^3(y^2-1)^{3/2}}\ F_1^{(2h)}$$

Contact term that integrates to $\delta(|b|)$ in the FT and does not contribute to the far-field waveform!

Where does the UV pole come from?



$$\Longrightarrow$$

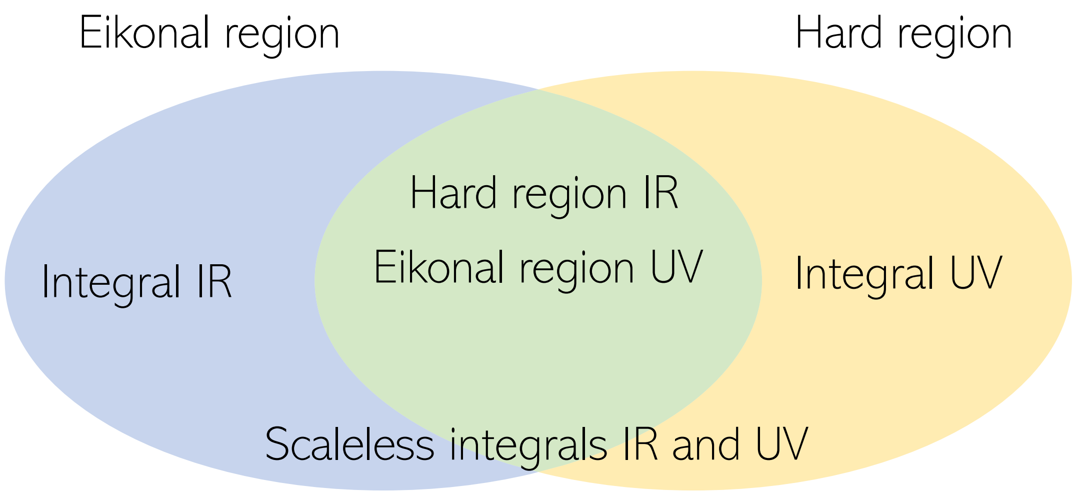
$$G^{\rm eik} \sim e^{\epsilon\gamma_E}\int \frac{d^D\ell}{i\pi^{D/2}}\frac{1}{\ell^2[2\ell\cdot p_1](\ell+q_1)^2(\ell-q_2)^2[-2\ell\cdot p_2]}\,, \qquad \ell^\mu \ll \bar{m}_i$$

$$G^{\rm hard} \sim e^{\epsilon\gamma_E}\int \frac{d^D\ell}{i\pi^{D/2}}\frac{1}{[\ell^2]^3[(\ell+p_1)^2-m_1^2][(\ell-p_2)^2+m_2^2]}\,, \qquad \ell^\mu \sim \bar{m}_i$$

Contributions polynomial in q_i

Waveform: UV Divergence

LB, Ita, Kraus, Schlenk [2312.14859]



Eikonal region

Hard region

Integral IR

Hard region IR

Eikonal region UV

Integral UV

Scaleless integrals IR and UV

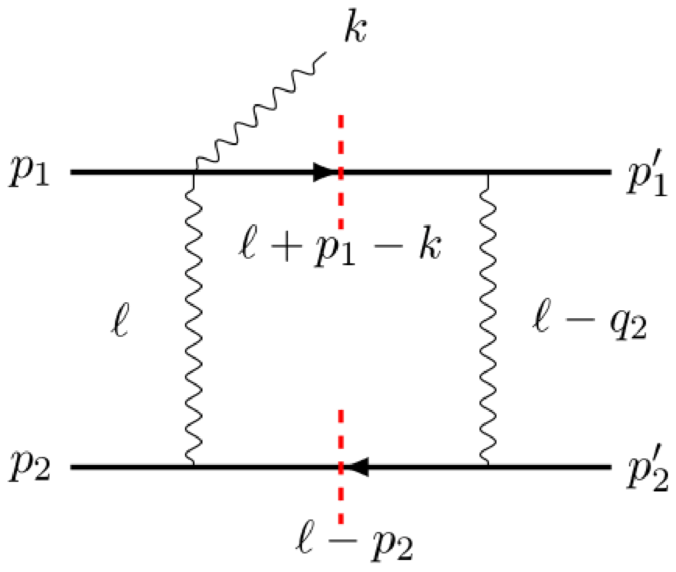
Full quantum amplitude

$$\mathcal{I}_{\text{box}} = \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{\ell^2\,[(\ell+p_1-k)^2-m_1^2](\ell-q_2)^2[(\ell-p_2)^2-m_2^2]}$$

Eikonal expansion of second massive propagator

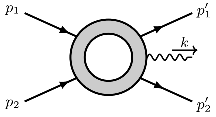
$$\frac{1}{(\ell-p_2)^2-m_2^2+i\epsilon} = \underbrace{\frac{1}{-2\bar{m}_2 u_2\cdot\ell+i\epsilon}}_{\sim\,\ell^{-1}} - \underbrace{\frac{\ell\cdot(\ell-q_2)}{[-2\bar{m}_2 u_2\cdot\ell+i\epsilon]^2}}_{\sim\,\ell^{0}} + \dots$$

$$\ell \to \infty$$

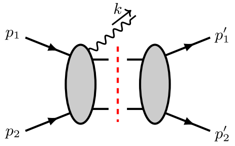


Amplitude and Cut

Master-integral decomposition



Amplitude



Cut

$$M = \sum_\Gamma c_\Gamma(\epsilon)\, \mathcal{I}_\Gamma \,, \qquad C = \sum_\Gamma c_\Gamma(\epsilon)\, \mathrm{Cut}_{1'2'}\left[\mathcal{I}_\Gamma\right]$$

Combine to new master integrals

$$\mathcal{M} = M - C = \sum_\Gamma c_\Gamma(\epsilon)\, \mathcal{I}_\Gamma^{\mathrm{exp}} \,,$$

$$\mathcal{I}_D^{\mathrm{exp}} \equiv \left[\mathcal{I}_D\right]^*$$

Integrals: Caron-Huot, Giroux, Hannesdottier, Mizera [2308.02125]
Brandhuber, Brown, Chen, De Angelis, Gowdy [2303.06111]
Herderschee, Roiban, Teng [2303.06112]
Georgoudis, Heissenberg, Vazquez-Holm [2303.07006]
LB, Ita, Kraus, Schlenk [2312.14859]

Rational Reconstruction

LB, Ita, Kraus, Schlenk [2312.14859]

Form-factor decomposition

$$\mathcal{M}^{(2h)} = \sum_{i=1,2} \mathcal{M}^i F_i^{(2h)}$$

with

$$F_1^{(2h)} = (\varepsilon_{(h)} \cdot u_2)\,(\varepsilon_{(h)} \cdot u_2)\,,$$
$$F_2^{(2h)} = (\varepsilon_{(h)} \cdot u_2)\,(\varepsilon_{(h)} \cdot q_1)\,.$$

$$c_\Gamma^{(2h)} = c_\Gamma^1 F_1^{(2h)} + c_\Gamma^2 F_2^{(2h)}$$

Rational ansatz

$$c_\Gamma^n(\epsilon, D_s, \bar{m}) = \bar{m}^{p_\Gamma^n} \sum_{k=-2}^{1} \sum_{l=0}^{\infty} c_{\Gamma,kl}^n(\epsilon)(D_s - 2)^k \frac{1}{\bar{m}^l}\,.$$

$$c_{\Gamma,kl}^n = \frac{\mathcal{N}_{\Gamma,kl}^n(\omega_1, \omega_2, q_1^2, q_2^2, \bar{m}_1, \bar{m}_2, y, \epsilon)}{\prod_j w_j^{q_{klj}^n}}$$

Polynomial in invariants

Alphabet of integral functions