Towards
two more orders of magnitude of precision
with
scattering amplitudes

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Supported by U.S. DEPARTMENT OF ENERGY | Office of Science

The detection of gravitational waves opened a new window on our Universe

- Probe aspects of dynamics in General Relativity in strong field regime
- Probe properties of black holes
- Probe/discriminate extensions of General Relativity
- Probe certain astrophysical environments, including dark matter
- Probe properties of (ultra-) dense nuclear matter
- Probe BH origin, formation mechanisms, population, etc
- …

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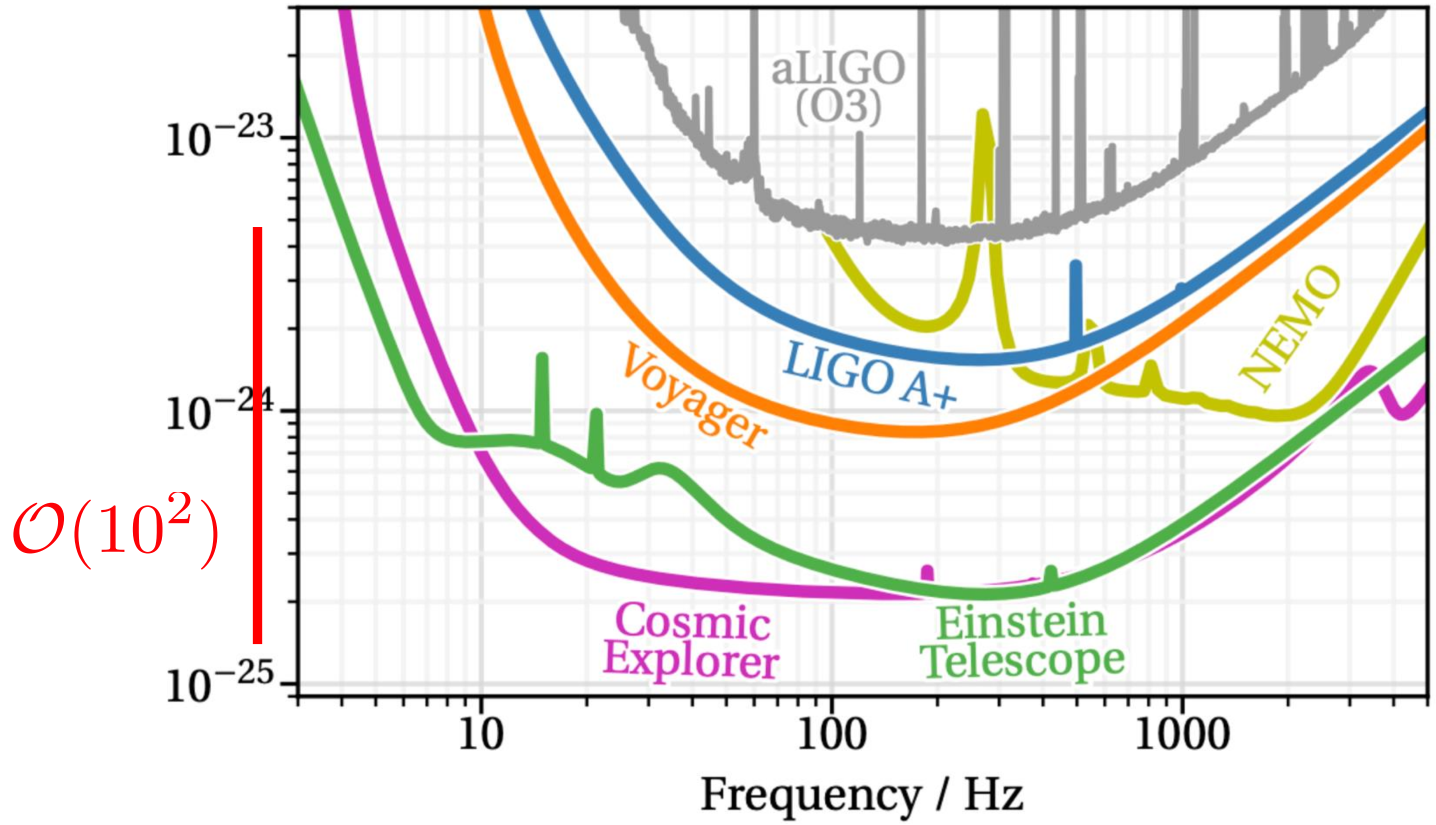
and gave new impetus towards new theoretical tools and structures

- Search for new symmetries
- Explore of the structure of perturbation theory
- Resummation of perturbation theory/nonperturbative methods
- Analytic continuations
- …

Future detectors will increase the sensitivity, observable sources and observable frequency

Advanced LIGO, Einstein Telescope, Cosmic Explorer



Space-based detectors:

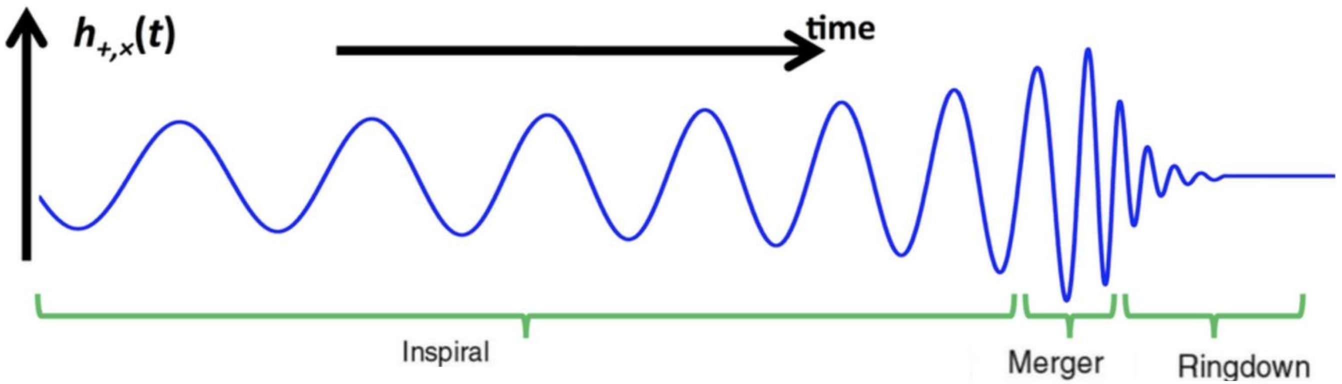
- Long-time signals
- Lower frequency
- Many events

Physics output = Precise Theoretical Predictions

Two more orders of magnitude of theoretical precision are needed

LISA Consortium
Waveform Working Group

Anatomy of an idealized binary merger



Favata/SXS/K.Thorne

- Numerical relativity – "the truth", but expensive and slow
- Post-Newtonian (weak field, nonrelativistic): $\boldsymbol{v}^2 \sim \frac{GM}{|\boldsymbol{r}|} \ll 1$
- Post-Minkowskian (weak-field, relativistic): $\frac{GM}{|\boldsymbol{r}|} \ll \boldsymbol{v}^2 \sim 1$
- Small mass ratio expansion (GSF) $\boldsymbol{v}^2 \sim GM/|\boldsymbol{r}| \sim 1$

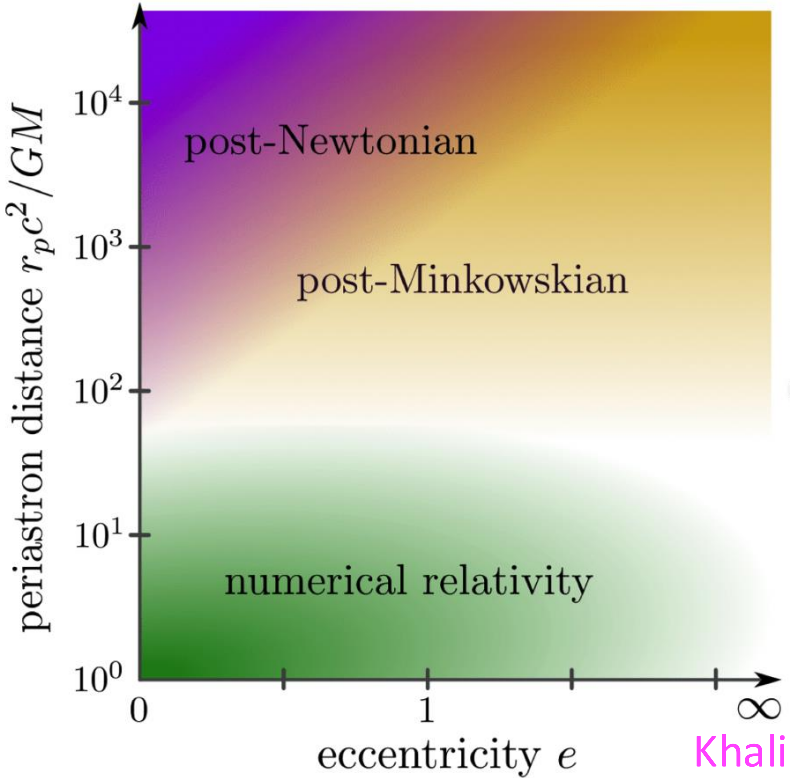
see Mikhail Solon's talk

- Ringdown: black hole perturbation theory

see Vitor Cardoso's talk

→ Effective one-body theory (EOB) consolidate available results

See Alessandra Buonanno's talk



Khalil, Buonanno,Steinhoff, Vines

Why worry about scattering and post-Minkowskian expansion:

- Motion on eccentrical orbits reaches high velocity at periastron

- Cleaner environment to sort out theoretical subtleties

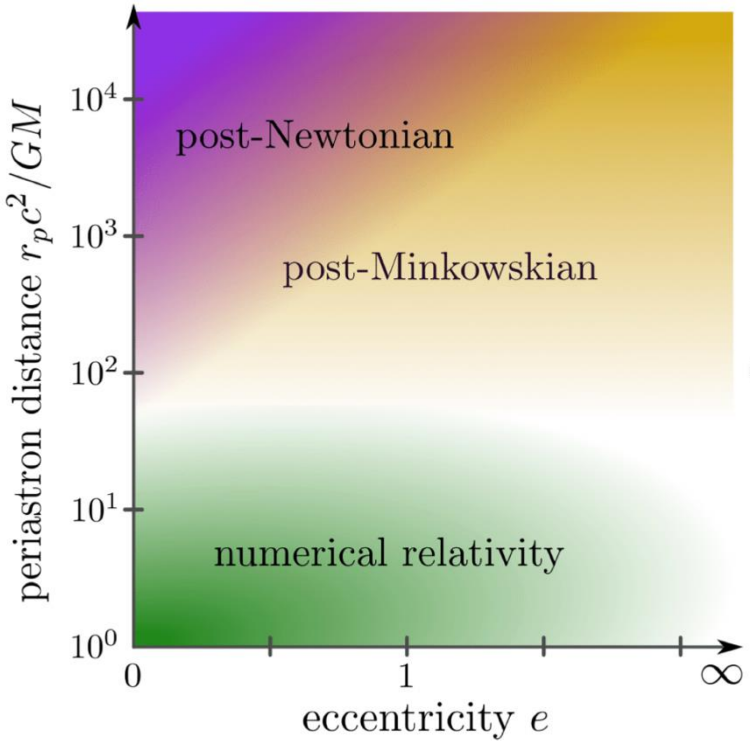
- Explore the structure of gravitational perturbation theory

  including symmetries, functional structures, etc

- Information for semi-analytic/semi-numerical approaches

  e.g. function basis required for fitting numerical data

- Aid with post-Newtonian calculations



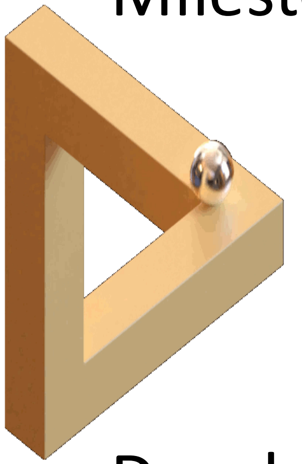
- Through resummation, contact between perturbation theory and strong-field gravity

  QCD-style resummation, EOB-style resummation, GSF-style resummation

- Analytic continuations can yield bound observables, including waveforms see Z. Zhou's talk + B2B

A Plan:

- Milestones from QFT-based approaches: spinless, spinning, radiation

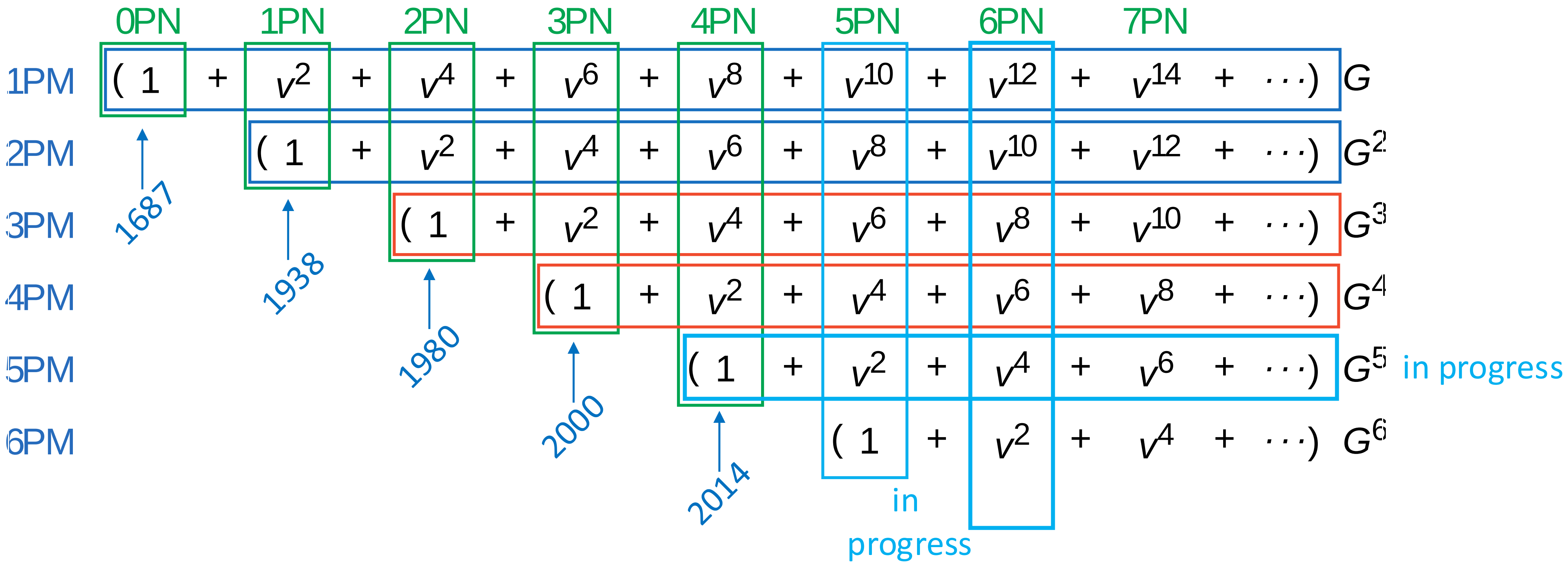


- Theoretical structures and connections

- Developing (computational) tools

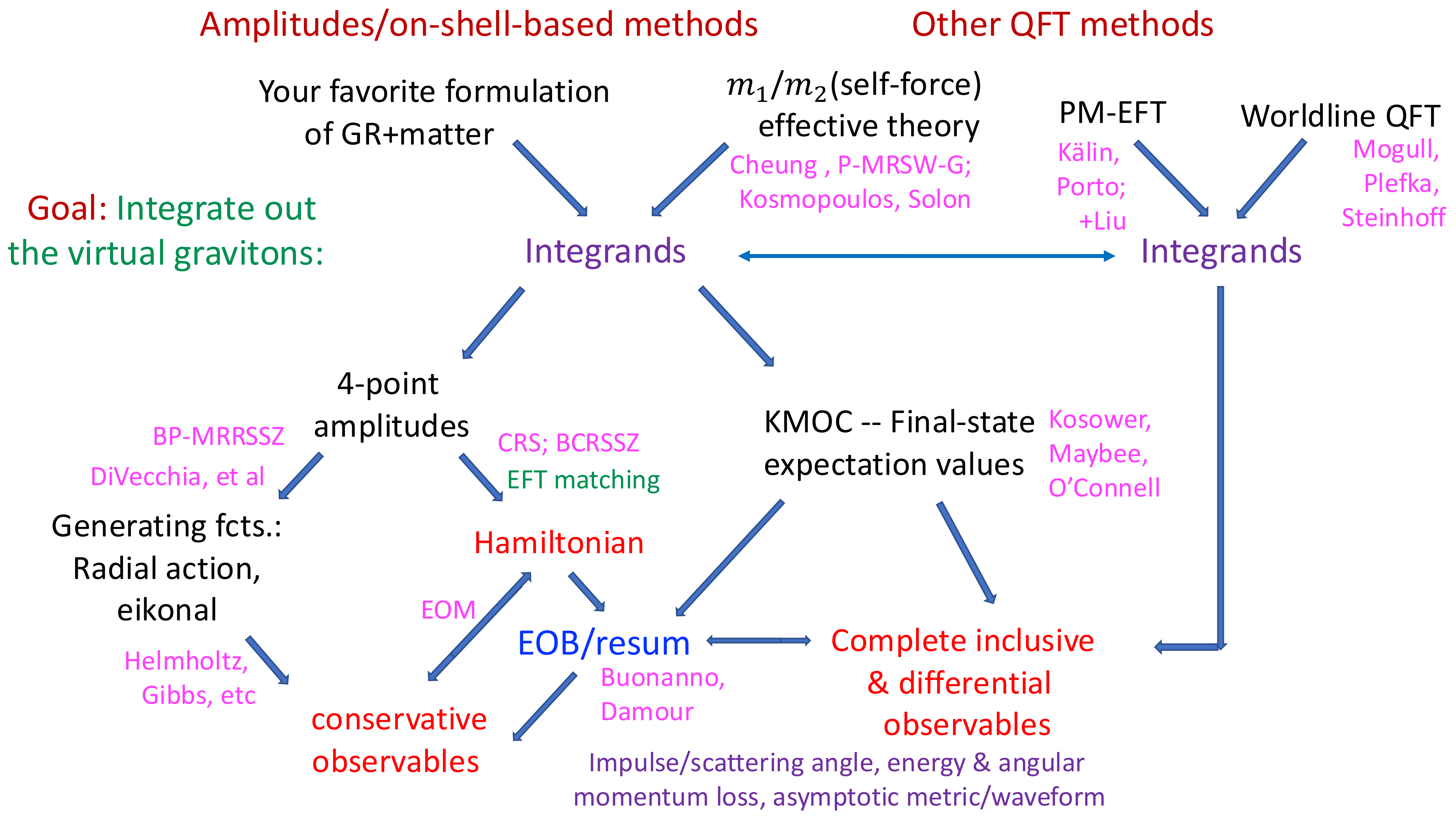
- Future

Extensive work in the spinless PN theory, using (mostly) traditional methods:



Ohta, Okamura, Kimura, Hiida, Jaranowski, Schäfer, Damour, Jaranowski, Blanchet, Faye, Porto, Rothstein, Iyer, Will, Wiseman, Poisson, Cutler, Finn, Flanagan, Deruelle, Thorne, Sathyaprakash, Bini, Geralico, Goldberger, Rothstein, Buonanno, Le Tiec, Marsat, Foffa, Sturani, Mastrolia, Sturm, Torres Bobadilla, Blümlein, Maier, Marquard, Mandal, Patil, *etc.*

+ MPM results



Amplitudes/on-shell-based methods
Other QFT methods
Your favorite formulation of GR+matter
m_1/m_2(self-force) effective theory
Cheung , P-MRSW-G; Kosmopoulos, Solon
PM-EFT
Kälin, Porto; +Liu
Worldline QFT
Mogull, Plefka, Steinhoff
Goal: Integrate out the virtual gravitons:
Integrands
Integrands
4-point amplitudes
BP-MRRSSZ
DiVecchia, et al
CRS; BCRSSZ
EFT matching
KMOC -- Final-state expectation values
Kosower, Maybee, O'Connell
Generating fcts.: Radial action, eikonal
Hamiltonian
EOM
EOB/resum
Buonanno, Damour
Complete inclusive & differential observables
Helmholtz, Gibbs, etc
conservative observables
Impulse/scattering angle, energy & angular momentum loss, asymptotic metric/waveform

Extensive perturbative QFT experience in gauge and gravity theories helps produce relativistic state of the art predictions

- Unitarity methods, recycles trees into loops — Bern, Dixon, Dunbar, Kosower
- Double-copy: gravity from gauge theory — Kawai, Lewellen, Tye; Bern, Carrasco, Johansson
- NRQCD/HEFT and EFT methods — Caswell, Lepage; Luke, Manohar, Rothstein; Golberger, Rothstein; Cheung, Rothstein, Solon
- Method of regions: integrate out virtual graviton modes — Beneke, Smirnov
- Reduction to master integrals/Integration-by-parts reduction — Chetyrkin, Tkachov; Laporta
- Method of differential equations for the evaluation of master integrals — Kotikov; Bern, Dixon, Kosower; Gehrmann, Remiddi; Henn, Smirnov
- Methods for evaluation of phase-space integrals — Kosower, Page

Classical limit, implemented as Bohr's correspondence principle, helps tremendously

classical limit = large charge limits

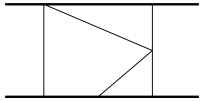
see also Giulia Isabella's talk

Some general features of PM perturbation theory

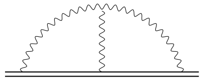
$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

	ν^0	ν^1	ν^2	Conservative	With radiation
1PM	0SF				
2PM	0SF				
3PM	0SF	1SF		Poor $E \to \infty$ limit	Fixed by radiation
4PM	0SF	1SF		Poor $E \to \infty$ limit	survives

Di Vecchia, Heissenberg, Russo, Veneziano; Damour



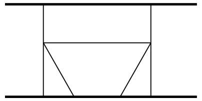
First appearance of nonlocal in time effects

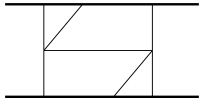


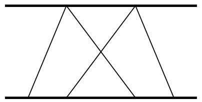
Elliptic integrals feature in observables

ν^2 Clean separation of conservative and dissipative parts

5PM 0SF 1SF 2SF







First appearance of more involved CY 3-fold integrals

Conservative and dissipative parts are difficult to separate

Breakdown of point-particle approximation/Compton amp.

Renormalization of tidal operators $RR(\partial\partial\phi)^2$; RG

6PM 0SF 1SF 2SF Mixing of binary evolution and tidal op's; Renormalization

⋮

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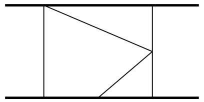

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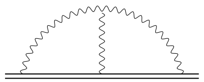

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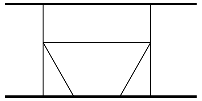

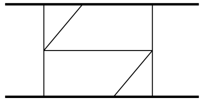

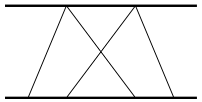

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6PM 0SF 1SF 2SF Mixing of binary evolution and tidal op's; Renormalization

corrections $ds^2 = -H dt^2 + H^{-1} dr^2 + r^2 d\Omega^2$

Resummation of geodesic motion in Schwarzschild/Kerr spacetime

See talks by Mikhail, Giulia, Julio, Zihan

Milestones

- Higher-order PM calculations
- Old and new puzzles
- Absorption
- Tidal & beyond GR
- Resummation
- Waveforms
- Spin
- EFT for self force

See Mikhail Solon's talk

Spinless PM scattering dynamics through G^4

$$\gamma = \frac{E}{m},\quad \xi = \frac{E_1 E_2}{E^2}$$
$$\nu = \frac{m_1 m_2}{m_1 + m_2}$$
$$m = m_1 + m_2$$
$$\sigma = p_1 \cdot p_2$$

Westphal; Damour

Conservative: Hamiltonian for hyperbolic motion: $H^{\text{hyp}} = E_1 + E_2 + \sum_{n=1}^{\infty} \frac{G^n}{r^n} c_n(\boldsymbol{p}^2)$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}\left(1 - 5\sigma^2\right) - \frac{4\nu\sigma\left(1 - 2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1 - 2\sigma^2\right)^2}{2\gamma^3\xi^2}\right]$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\left[\frac{1}{12}\left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3\right) - \frac{4\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right.$$
$$- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7 - 20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right)\left(1 - 2\sigma^2\right)}{4\gamma^3\xi^2}$$
$$\left. + \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\right]$$

Bern, Cheung, RR, Solon, Shen, Zeng

$$c_4^{\text{hyp}} = \frac{m^7\nu^2}{4\xi E^2}\left[\mathcal{M}_4^{\text{p}} + \nu\left(4\mathcal{M}_4^{\text{t}}\log\left(\tfrac{p_\infty}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\text{rem}}\right)\right] + \mathcal{D}^3\left[\frac{E^3\xi^3}{3}c_1^4\right] + \mathcal{D}^2\left[\left(\frac{E^3\xi^3}{\boldsymbol{p}^2} + \frac{E\xi(3\xi-1)}{2}\right)c_1^4 - 2E^2\xi^2 c_1^2 c_2\right]$$
$$+ \left(\mathcal{D} + \frac{1}{\boldsymbol{p}^2}\right)\left[E\xi(2c_1c_3 + c_2^2) + \left(\frac{4\xi-1}{4E} + \frac{2E^3\xi^3}{\boldsymbol{p}^4} + \frac{E\xi(3\xi-1)}{\boldsymbol{p}^2}\right)c_1^4 + \left((1-3\xi) - \frac{4E^2\xi^2}{\boldsymbol{p}^2}\right)c_1^2 c_2\right]$$

Bern, Parra-Martinez, RR, Ruf, Solon, Shen, Zeng

Parts of 4PM Hamiltonian from QFT techniques:

Bern, Parra-Martinez, RR, Ruf, Solon, Shen, Zeng

$$\mathcal{M}_4^{\mathrm{p}} = -\frac{35\left(1-18\sigma^2+33\sigma^4\right)}{8\left(\sigma^2-1\right)}$$ (p p p)

$$\mathcal{M}_4^{\mathrm{t}} = r_1 + r_2 \log\left(\frac{\sigma+1}{2}\right) + r_3 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}$$ (p r r)

$$\mathcal{M}_4^{\pi^2} = r_4\pi^2 + r_5\,\mathrm{K}\left(\tfrac{\sigma-1}{\sigma+1}\right)\mathrm{E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_6\,\mathrm{K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_7\,\mathrm{E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right)$$ (p p p)

$$\sigma = \frac{p_1\cdot p_2}{m_1 m_2} \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\mathcal{M}_4^{\mathrm{rem}} = r_8 + r_9 \log\left(\tfrac{\sigma+1}{2}\right) + r_{10}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + r_{11}\log(\sigma) + r_{12}\log^2\left(\tfrac{\sigma+1}{2}\right) + r_{13}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\log\left(\tfrac{\sigma+1}{2}\right) + r_{14}\frac{\operatorname{arccosh}^2(\sigma)}{\sigma^2-1}$$
$$+ r_{15}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{2}\right) + r_{16}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) + r_{17}\frac{1}{\sqrt{\sigma^2-1}}\left[\mathrm{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - \mathrm{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right)\right]$$ (p p p) + (p r r)

- Remarkably compact given that it is the result of higher-loop GR calculation
- Dissipative contributions and inclusive observables with radiation reaction — Manohar, Ridgeway, Shen; Dlapa, Kälin, Liu, Porto; Jakobsen, Mogull, Plefka, Sauer, Xu; Damgaard, Hansen, Planté, Vanhove
- Tail effect ⟶ difficulties with analytic continuation to bound motion: recent progress — Bini, Damour; Dlapa, Kälin, Liu, Porto
  Continue local + universal nonlocal; supply PN bound-orbit nonuniversal nonlocal
- Conservative part reproduces 6PN results in the overlap and through $\mathcal{O}(\nu)$ — Bini, Damour, Geralico
  Progress with resolving a $\mathcal{O}(\nu^2)$ puzzles, but some remains — Blümlein, Maier, Marquard, Schäfer

Original Puzzle:

PM conservative scattering angle

$$\chi \sim \nu^2 (1 \oplus \nu)$$

Damour; Antonelli, Buonanno, Steinhoff, van de Meent, Vines

EFT PN conservative angle

$$\chi \sim \nu^2 (1 \oplus \nu \oplus \underline{\nu^2})$$

Blümlein, Maier, Marquard, Schäfer; Foffa, Sturani

New calculations: EFT-inspired approach to conservative + radiative effective action

GR $\mathcal{O}(\nu^2)$	GR MPM $h_{ij}^{\rm TT}$; E, J fluxes	
✓	✗	Porto, Riva, Yang
✗	✓	Luz Alameda, Müller, Foffa, Sturani
✓	✓	QFT

So there is still a puzzle; suggested possible origins for the difference:

- Scattering frame or/and BMS frame mismatch
- Differences in the multipole definition (with trace or without trace, different coordinates)

Consensus: QFT-based results are correct

Higher post-Minkowskian order calculations

Amplitude/Worldline methods

Unitarity, double copy,
Feynman rules;
expansion in classical limit

Under control

Loop integrand

Integration by parts reduction
Differential equations

The bottle neck

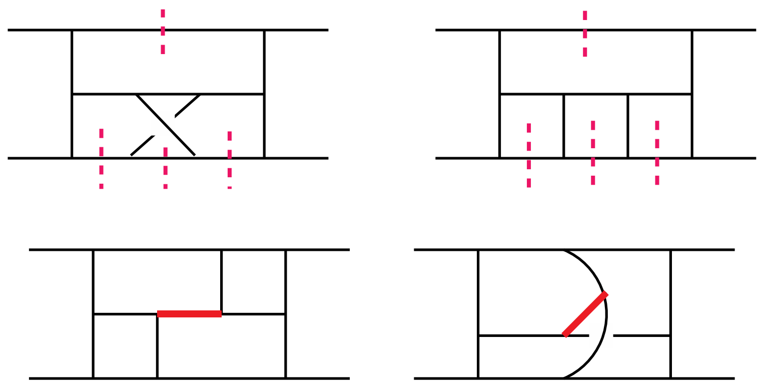
Integrated amplitude

EFT matching; Eikonal,
Amplitude-radial action relation
KMOC, 1-point fcts, etc

Under control

Hamiltonian/Observables

Hard and harder
5PM integral families



e.g. relevant 5PM integrals:
13 propagators, 9 ISP-s; up to
8/9 numerator factors and
4 doubled prop's

Bottle neck: higher-loop analytic integration, despite enormous advances over the last decade

IBP reduction & differential equations — Chetyrkin, Tkachov; Laporta; Beneke, Smirnov; Henn; Henn, Smirnov

IBP reduction requires care & tuning of public codes (FIRE, KIRA), and private codes

Bern, Herrmann, RR, Ruf, Smirnov, Smirnov, Zeng; Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovich

Attack in stages: embrace self-force organization + learn from simpler yet relevant theories

$$\mathcal{M}_{4,L} = Gm^4|\boldsymbol{q}|^{-2}(Gm|\boldsymbol{q}|)^L\nu^2\left[\mathcal{M}_{4,L}^{0\mathrm{SF}} + \nu\,\mathcal{M}_{4,L}^{1\mathrm{SF}} + \nu^2\,\mathcal{M}_{4,L}^{2\mathrm{SF}} + \ldots + \nu^{[L/2]}\,\mathcal{M}_{4,L}^{[L/2]SF}\right](\ln|\boldsymbol{q}|)^x$$

$$\nu = m_1 m_2/(m_1+m_2)^2$$

At 5PM, or *L* = 4 loops:

1. Warmup in QED, $\mathcal{O}(\nu)$ and $\mathcal{O}(\nu^2)$, potential — Bern, Herrmann, RR, Ruf, Smirnov, Smirnov, Zeng
2. $\mathcal{O}(\nu)$ conservative in general relativity — WQFT: Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovich
3. $\mathcal{N}$ = 8 supergravity — $\mathcal{O}(\nu)$ Bern, Herrmann, RR, Ruf, Smirnov, Smirnov, Zeng; $\mathcal{O}(\nu^2)$ in progress
4. $\mathcal{O}(\nu)$ radiative effects — WQFT: Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovich
5. GR $\mathcal{O}(\nu^2)$ — Difficult but looks doable

see Benjamin Sauer's talk

Example: Most important part of the $\mathcal{N}$ = 8 supergravity $\mathcal{O}(\nu)$ amplitude

Bern, Herrmann, RR, Ruf, Smirnov, Smirnov, Zeng

$$\mathcal{M}_{4,L=4}^{\mathcal{O}(\nu),\text{fin.}} = r_1 + r_2\,\text{F}_0 + r_3\,\text{F}_0^2 + r_4\,\text{F}_1 + r_5\,\text{F}_2$$

$$\text{F}_0 = \frac{2x}{1-x^2}\ln(x)$$

$$\text{F}_1 = \frac{2x}{1-x^2}\left[-\text{Li}_2(1-x) - \text{Li}_2(-x) - \ln(x)\ln(x+1) - \frac{1}{2}\zeta_2\right]$$

$$\text{F}_2 = \frac{2x}{1-x^2}\left[-\text{Li}_2(1-x) + \text{Li}_2(-x) - \frac{1}{2}\ln^2(x) + \ln(x)\ln(x+1) + \frac{1}{2}\zeta_2\right]$$

Conservative radiation contributions not included

- Extremely simple for a 4-loop amplitude, even with $\mathcal{N}$ = 8 supersymmetry
- Unexpected cancellations: master integrals include more complicated functions
- Simpler than the analogous 3-loop $\mathcal{N}$=8 amplitude; simplicity translates to observables
- Analogous very nice 1SF GR results, derived with WQFT methods, including conservative radiation exhibits similar features

Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovich

See Benjamin Sauer's talk

Are there similar simplifications at $\mathcal{O}(\nu^2)$? How to figure it out in advance and use them?

Example: Classical 1SF + 2SF QED potential amplitude

Bern, Herrmann, RR, Ruf, Smirnov, Smirnov, Zeng

$$\mathcal{M}_{4,L=4}^{\text{QED, finite}} = -(\alpha Q_1 Q_2)^5 |\boldsymbol{q}|^2 \ln |\boldsymbol{q}| \left[r_0^{(0)} + \sum_{k=1}^{12} \left(\nu\, r_k^{(1)} + \nu^2\, r_k^{(2)} \right) f_k \right] + \mathcal{O}(Q_1^n Q_2^{10-n})$$

$$\frac{1}{2}\left(x + \frac{1}{x}\right) = y \equiv \frac{\bar{p}_1 \cdot \bar{p}_2}{\bar{m}_1 \bar{m}_2}$$

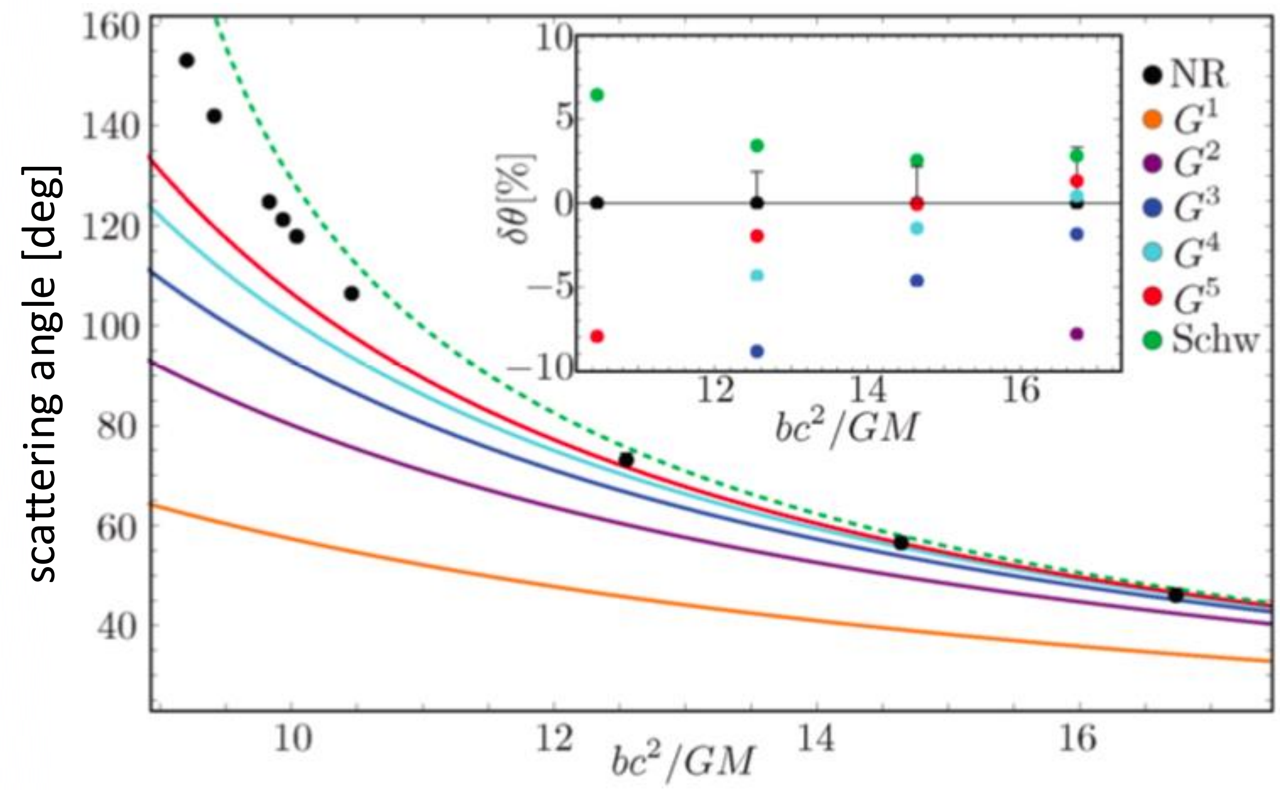
- $\mathcal{O}(\nu^2)$ amplitude is no more complicated than $\mathcal{O}(\nu)$
- Natural functions: cyclotomic polylogarithms $\mathbb{W} = \{x, x \pm 1, x^2 + 1, x^2 + x + 1, x^2 - x + 1\}$
- Unexpected simplifications -- solely in terms of logarithms and real Li_2 and Li_3
- No nonlocal-in-time parts (to any order in perturbation theory)
- Agrees with results obtained by transferring exact binary-charge solution of Schild to scattering configuration via EOB; higher orders available

Bini, Damour

We might expect that gravitational $\mathcal{O}(\nu^2)$ also exhibits some simplifications

Radiative contributions to the 0SF and 1SF scattering angle recently computed from WQFT

Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovich



- No EOB resummation
- Steady approach to numerical results
- Interesting to include resummation of geodesic motion e.g. as in SF scalar model Long, Whittall, Barack

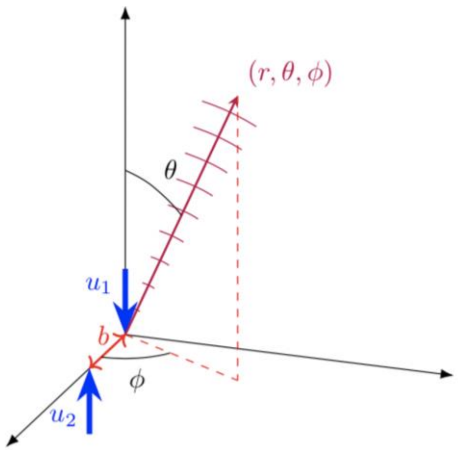
- Interesting to compare EOB-resumed 1SF 5PM & NR and study relative numerical importance of 2SF terms over parameter space

$$\chi^{w\ eob}_{\mathrm{nPM}}(\sigma, j) \equiv 2j \int_0^{\bar{u}_{\max}(\sigma,j)} \frac{d\bar{u}}{\sqrt{p_\infty^2 + w_{\mathrm{nPM}}(\bar{u},\sigma) - j^2\bar{u}^2}} - \pi$$

- Motivation to complete 5PM/2SF calculation and study improvement in precision of analytic calculation

Spinless scattering waveform:

$$W(T_R,\theta,\phi) = \frac{1}{4G}\lim_{r\to\infty} r\epsilon^{\mu\nu}_{++}(g_{\mu\nu}-\eta_{\mu\nu}) = \frac{1}{4G}(h_+^\infty - ih_\times^\infty)$$



Governed by: 5-point amplitudes and their cuts

KMOC/CGKOC

$$\langle h^{\rm TT}_{\mu\nu}\rangle = \langle\psi_{\rm out}|h^{\rm TT}_{\mu\nu}|\psi_{\rm out}\rangle - \langle\psi_{\rm in}|h^{\rm TT}_{\mu\nu}|\psi_{\rm in}\rangle$$

$$h^{\rm TT}_{\mu\nu}(x) = \sum_{h=\pm}\int \hat{\rm d}\Phi(k)\left[\varepsilon^{(hh)*}_{\mu\nu}(k)\, e^{-ik\cdot x}\,\hat{a}_{hh}(k) + \varepsilon^{(hh)}_{\mu\nu}(k)\, e^{+ik\cdot x}\,\hat{a}^\dagger_{hh}(k)\right]$$

graviton 1-point function in WQFT $\langle h_{\mu\nu}\rangle = \int DzDh\; h_{\mu\nu}e^{iS}$ Jakobsen, Mogull, Plefka, Steinhoff

- Next-to-leading order scattering waveform (+ differential fluxes)

Herderschee, RR, Teng; Elkhidir, O'Connell, Sergola, Vazquez-Holm; Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini; Georgoudis, Heissenberg, Vazquez-Holm; + Caron-Huot, Giroux, Hannesdottir, Mizera

- NLO waveform to quadratic order in spin see Lara Bohnenblust's talk Bohnenblust, Ita, Kraus, Schlenk

  NLO waveform memory to higher powers of spin Aoude, Haddad, Heissenberg, Helset

Summary of spinless comparison with GR results

Bini, Damour, De Angelis, Geralico, Herderschee, RR, Teng; Georgoudis, Heissenberg, Russo

- Analytic results agree with available GR calculations of Bini, Damour and Geralico (through 2.5PN)

$$W^{\text{KMOC}} \sim 1 + \frac{GM^2\nu}{p_\infty}\left(1 + p_\infty + \cdots + p_\infty^5 + \ldots\right) + \frac{GM^2\nu}{p_\infty}\frac{GM}{bp_\infty^2}\left(1 + p_\infty + \cdots + p_\infty^5 + \ldots\right)$$

$$M = m_1 + m_2 \qquad \nu = m_1 m_2/M^2 \qquad p_\infty = \sqrt{\sigma^2 - 1}$$

- Analytic results in the soft ω expansion agree with results of Sahoo and Sen

$$W^{\text{KMOC}} \sim \frac{\mathcal{A}}{\omega} + \mathcal{B}\ln\omega + \mathcal{C}\omega(\ln\omega)^2 + \mathcal{D}\omega\ln\omega + \ldots$$

- BMS frame is important: GR results are intrinsic frame ($t = -\infty$ metric: linearized Schwarzschild)

  Captured at NLO by vanishingly-soft (zero-energy) gravitons

$$\kappa\mathcal{F}[h_{\mu\nu}^{\text{ZFM}}, \omega] = \delta(\omega)\frac{2G}{r}\left[\frac{\Pi_{\mu\nu}^{ab}p_{1,a}p_{1,b}}{p_1\cdot n} + \frac{\Pi_{\mu\nu}^{ab}p_{2,a}p_{2,b}}{p_2\cdot n}\right] + \frac{G}{r}\,\omega\,\left[m_1 u_1\cdot n\log(u_1\cdot n)^2 + m_2 u_2\cdot n\log(u_1\cdot n)^2\right]\mathcal{F}[\mathcal{W}_{\mu\nu}^{\text{LO}}, \omega] + \ldots$$

Remarkable agreement with results of GR methods; motivation to proceed to higher orders

Summary and comparison with GR results

Bini, Damour, De Angelis, Geralico, Herderschee, RR, Teng
Georgoudis, Heissenberg, Russo

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$$M = m_1 + m_2$$
$$\nu = m_1 m_2/M^2$$
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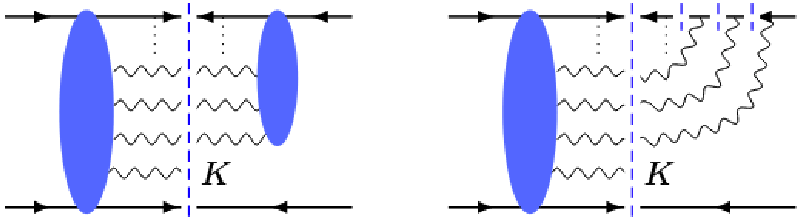
$$W^{\rm KMOC} \sim \frac{\mathcal{A}}{\omega} + \mathcal{B}\ln\omega + \mathcal{C}\omega(\ln\omega)^2 + \mathcal{D}\omega\ln\omega + \dots$$

- BMS frame is important: GR results are intrinsic frame ($t = -\infty$ metric: linearized Schwarzschild)
  Captured at NLO by vanishingly-soft (zero-energy) gravitons

- Resummation of vanishingly-soft modes ⟶ All-order of BMS frame contribs   Elkhidir, O'Connell, RR



+ contribution to angular momentum loss including contributions from nonlinear memory

+ reproduce classical supertranslation generators

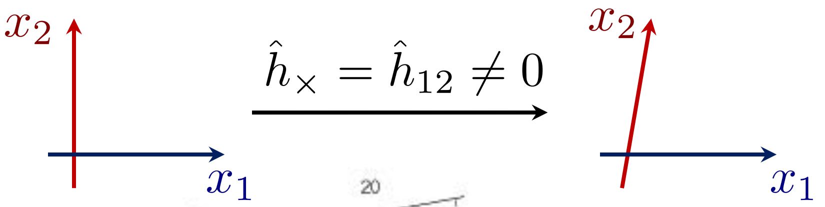
Remarkable agreement with results of GR methods motivates higher-order calculations

What would we "see" at a fancy gravitational wave detector

Herderschee, RR, Teng

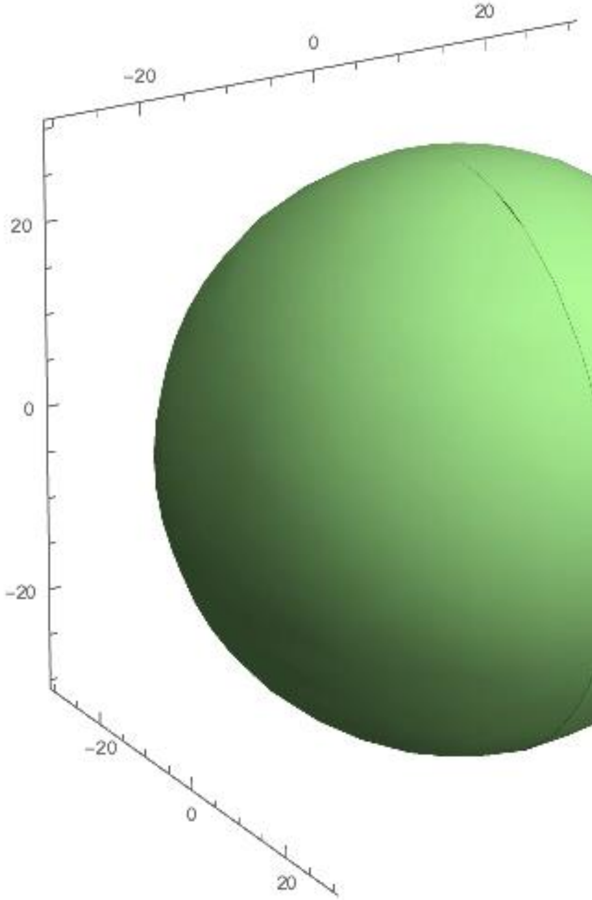
LO also by Jakobsen, Mogull, Plefka from WQFT

$$g_{\mu\nu}\Big|_{|\boldsymbol{x}|\to\infty} = \eta_{\mu\nu} + \frac{\kappa^2 \mathrm{M}}{8\pi|\boldsymbol{x}|}\left[\frac{\kappa^2\mathrm{M}}{\sqrt{-b^2}}\hat{h}^{(1)}_{\mu\nu} + \left(\frac{\kappa^2\mathrm{M}}{\sqrt{-b^2}}\right)^2 \hat{h}^{(2)}_{\mu\nu} + \dots\right]$$



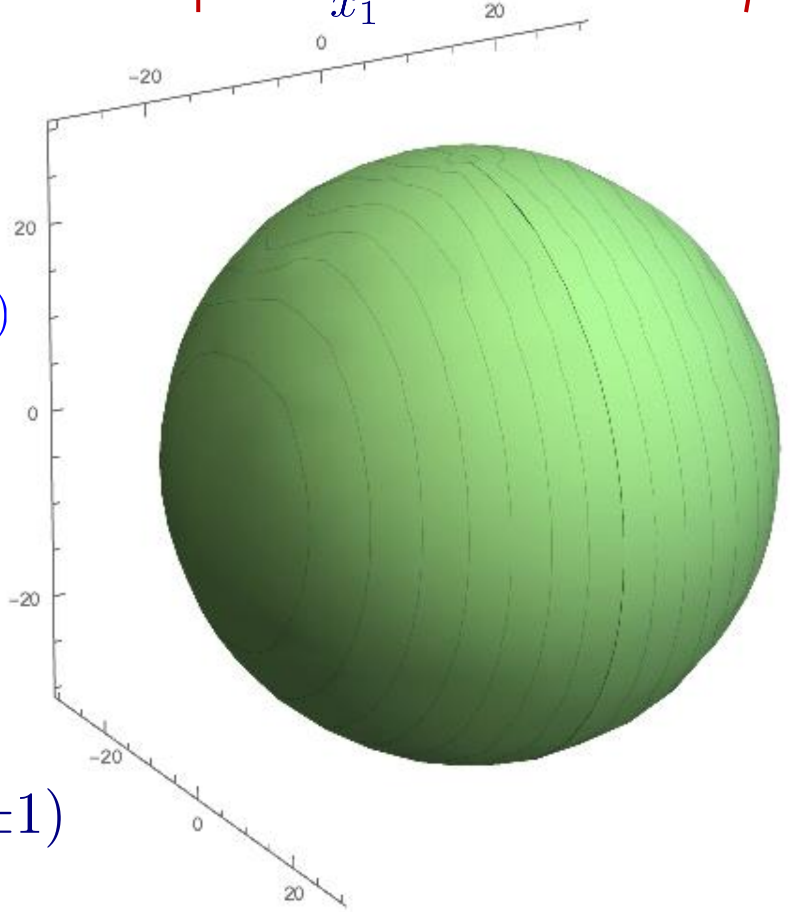
$\hat{h}^{(1)}_\times(t)$

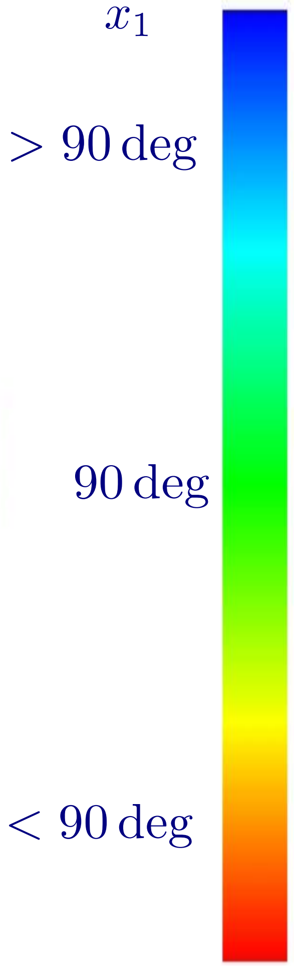
LO



$\hat{h}^{(2)}_\times(t)$

NLO





$$m_1 = m_2$$

$$u_{1,2} = \frac{1}{\sqrt{24}}(5, 0, 0, \pm 1)$$

Technically-challenging to obtain exact analytic expressions; recent progress

Brunello, De Angelis

Spinless scattering waveform at higher orders

- Conjecture for the universal part of waveform to all orders in *G* — Alessio, Di Vecchia ; + Heissenberg

$$\tilde{w}^{\mu\nu} = e^{2iGE\,\omega\ln\omega}\left[\frac{i}{\omega}a_0^{\mu\nu} - \sum_{n\geq 1}(-i\omega)^{n-1}(\ln\omega)^n\frac{a_n^{\mu\nu}}{n!} + \dots\right]$$

$$a_n^{\mu\nu} = (-)^n G^n\left[\frac{\sigma(2\sigma^2-3)}{(\sigma^2-1)^{3/2}}(p_1\cdot n + p_2\cdot n)^{n-1}B^{\mu\nu}(p_1,p_2) + (1,2)\leftrightarrow(3,4)\right]$$

$$B^{\mu\nu}(p_1,p_2) = \frac{p_2\cdot n}{p_1\cdot n}p_1^\mu p_1^\nu + \frac{p_1\cdot n}{p_2\cdot n}p_2^\mu p_2^\nu - (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu)$$

- Proven from GR perspective — Fucito, Morales, Russo
  - Universal part is fixed by the probe limit, found by solving the Teukolsky equation
  - Revealed a formal (?) connection w/ Liouville theory and $\mathcal{N}$ = 2 quiver gauge theories w/ $SU(2)^2$ gauge group via the AGT correspondence; also in — Bautista, Bonelli, Iossa, Tanzini, Zhou

- Useful comparison point for future NNLO $\mathcal{O}(G^4)$ calculations: — Bini, Damour, Geralico

  NNLO frequency space waveform at 2PN known from GR/MPM methods

All things spinning: more spins and more loops

Extensive work in the PN expansion

Hanson, Regge, Bailey, Israel, Yee, Bander, Tulczyjew, Damour, Buonanno, Levi, Steinhoff, Porto, Rothstein, Perodin, Khriplovich, Pomeranski, Antonelli, Faye, Hinderer, Kavanagh, Khalil, Mandal, Mastrolia, Patil, Vines, Kunst, Mougiakakos, Kim, Yin, Morales, Vieira, McLeod, von Hippel, Teng, *etc.*

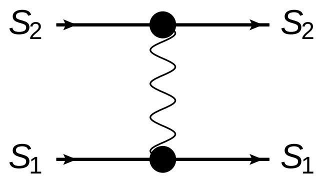
PN order	0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5
S^0	0PN	1PN		2PN		3PN		4PN		5PN		6PN	
S^1			LO		NLO		N2LO		N3LO		N4LO		
S^2				LO		NLO		N2LO		N3LO			
S^3							LO		NLO				
S^4								LO		NLO			
S^5											LO		NLO
S^6												LO	

(L+1)PM/loop order
tree
1-loop
2-loop
3-loop
4-loop
5-loop
6-loop
7-loop

Credit: J. Vines

Eight/Nine amplitudes-based approaches to higher-spin interactions

- 3-point "minimal" amplitude for arbitrary-spin particles — Arkani-Hamed, Huang, Huang
  - Naïve higher-point amplitudes with spurious poles at $S^{\geq 5}$; fixed — (2022) Chen, Chung, Huang, Kim
- Exponentiated soft factors — Guevara, Ochirov, Vines
  - Same higher-point spurious pole issue as AHH at $S^{\geq 5}$
- 4d EFT for arbitrary-spin particles in classical limit — Bern, Luna, RR, Shen, Zeng; + Teng; + Kosmopoulos, Scheopner, Vines; + Alaverdian
- Heavy-particle EFT-like theory — Aoude, Haddad, Helset
- Worldline QFT with spin, now capturing all powers of spin — Jakobsen, Mogull, Plefka, Steinhoff; Haddad, Jakobsen, Mogull, Plefka
- Fixed order in spin from fixed quantum spin — Spin ½ and 1: Holstein, Ross, Vaydia, Cachazo, Guevara, Bautista, Febres Cordero, Kraus, Lin, Ruf, Zeng; Damgaard, Haddad, Helset, … ; spin 5/2: Chiodaroli, Johansson, Pichini
  - $\text{spin-}k \longrightarrow S^{2k}$
- Chiral higher-spin gravity/massive higher-spin gauge symmetry — Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov
- Extract (Compton/Raman) amplitudes from Teukolsky equation — Bautista, Huang, J-W Kim; Bautista, Bonelli, Iossa, Tanzini, Zhou; Chen, Wang; Chen, Hsieh, Y-t Huang, J-W Kim
- Background field calculations — Kosmopoulos, Solon; Cheung, Parra-Martinez, Rothstein, Shah, Wilson-Gerow
  - see talks by Mikhail Solon & Julio Parra-Martinez



- Kerr gravitational form factor ⟶ 1PM amplitude:

Vines

$$\hat{T}^{\mu\nu}(a,q) = m\exp(ia*q)^{(\mu}{}_{\rho}u^{\nu)}u^{\rho} = m\cosh(a\cdot q)u^{\mu}u^{\nu} - \frac{i}{a\cdot q}\sinh(a\cdot q)\, q^{\rho}S(p)_{\rho}{}^{(\mu}u^{\nu)} \qquad u^{\mu} = p^{\mu}/m, \quad a^{\mu} = S^{\mu}/m$$

What principle determines higher order gravitational interactions of Kerr BHs?

- E.g. massive higher-spin symmetries + massless limit — Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov
- E.g. minimality — Arkani-Hamed, Huang, Huang
- Other symmetries? E.g spin-shift symmetry seems broken beyond S^4 and beyond probe limit — Bautista, Guevara, Kavanagh, Vines; Akpinar, Febres Cordero, Kraus, Ruf, Zeng

Postpone answering this question and aim to describe the physics of a general spinning object

- spin magnitude change
- horizon effects/absorption
- spin-related tidal effects
- ??

⟶ Construct a higher-spin EFT

Fixed-spin worldline description: Levi, Steinhoff; Perrodin; Porto, Rothstein

$$S_{\rm pp} = \int d\tau\Big[-m\sqrt{u^2} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + L_{\rm SI}[u^{\mu}, S_{\mu\nu}, g_{\mu\nu}]\Big] \qquad \Omega^{\mu\nu} = e^{\mu}_{A}\frac{de^{A\nu}}{d\tau}$$

$$L_{\rm SI} = \sum_{n=1}^{\infty}\frac{(-1)^{n+m}}{(2n)!}\frac{C_{ES^{2n}}}{m^{2n-1}}D_{\mu_{2n}}\cdots D_{\mu_3}\frac{E_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}}$$
$$+\sum_{n=1}^{\infty}\frac{(-1)^{n}}{(2n+1)!}\frac{C_{BS^{2n+1}}}{m^{2n}}D_{\mu_{2n+1}}\cdots D_{\mu_3}\frac{B_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}}S^{\mu_{2n+1}}$$

Extension to general compact objects using a 4d EFT valid in the classical limit mirroring the worldline action of Levi & Steinhoff plus more QFT-specific operators

Bern, Luna, RR, Shen, Zeng
Bern, Kosmopoulos, Luna, RR, Teng

$$\mathcal{L} = -R(e,\omega) + \frac{1}{2}g^{\mu\nu}\nabla(\omega)_\mu\phi_s\nabla(\omega)_\nu\phi_s - \frac{1}{2}m^2\phi_s\phi_s + \mathcal{L}_{\rm LS} + \mathcal{L}_H + \mathcal{L}_{R^2}$$

Equivalent classically and w/ covariant spin supplementary condition
Inequivalent q.m.

$$\mathcal{L}_{\rm LS} = \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n)!}\frac{C_{ES^{2n}}}{m^{2n}}\nabla(\omega)_{f_{2n}}\cdots\nabla(\omega)_{f_3}R_{f_1af_2b}\nabla(\omega)^a\phi_s\,\mathbb{S}^{(f_1}\ldots\mathbb{S}^{f_{2n})}\nabla(\omega)^b\phi_s$$
$$-\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{C_{BS^{2n}}}{m^{2n+1}}\nabla(\omega)_{f_{2n+1}}\cdots\nabla(\omega)_{f_3}\frac{1}{2}\epsilon_{ab(c|f_1}R^{ab}{}_{|d)\,f_2}\nabla(\omega)^c\phi_s\mathbb{S}^{(f_1}\ldots\mathbb{S}^{f_{2n+1})}\nabla(\omega)^d\phi_s$$

$$\mathcal{L}_H = -\sum_{n=1}^{\infty}\frac{(-1)^n(2n-1)}{(2n)!(2n+1)}\frac{H_{2n}}{m^{2n-2}}\nabla_{f_{2n}}\ldots\nabla_{f_3}R_{(a|f_1|b)f_2}\phi_sM^{a(f_1}M^{|b|f_2}\mathbb{S}^{f_3}\ldots\mathbb{S}^{f_{2n})}\phi_s$$
$$+\sum_{n=1}^{\infty}\frac{(-1)^n n}{(2n+1)!(n+1)}\frac{H_{2n+1}}{m^{2n-1}}\nabla_{f_{2n+1}}\ldots\nabla_{f_3}*R_{(a|f_1|b)f_2}\phi_sM^{a(f_1}M^{|b|f_2}\mathbb{S}^{f_3}\ldots\mathbb{S}^{f_{2n+1})}\phi_s$$

→ Puzzle: incompatibility w/ Teukolsky eq. at $\mathcal{O}(G^2S^5)$ b/c of non-decoupling of absorption

a solution: include operators describing absorption (more later)

Included in the action of Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov

$$\mathcal{L}_{\rm Kerr} = \sqrt{-g}\Big\{\frac{1}{2}\langle\nabla_\mu\Phi|\nabla^\mu\Phi\rangle - \frac{m^2}{2}\langle\Phi|\Phi\rangle - \frac{1}{4}\sum_{k=0}^{2s-2}\frac{2s-k-1}{m^{2k}}\langle\Phi|\Big\{\big(|\overleftarrow{\nabla}|\overrightarrow{\nabla}|\big)^{\odot k}\odot|R_-|\Big\}|\Phi\rangle\Big\} + \mathcal{O}(R^2)$$

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Bern, Luna, RR, Shen, Zeng

Bern, Kosmopoulos, Luna, RR, Teng

$$\mathcal{L} = -R(e,\omega) + \frac{1}{2} g^{\mu\nu} \nabla(\omega)_\mu \phi_s \nabla(\omega)_\nu \phi_s - \frac{1}{2} m^2 \phi_s \phi_s + \mathcal{L}_{\rm LS} + \mathcal{L}_H + \mathcal{L}_{R^2}$$

$$\mathcal{L}_{\rm LS} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n}} \nabla(\omega)_{f_{2n}} \cdots \nabla(\omega)_{f_3} R_{f_1 a f_2 b} \nabla(\omega)^a \phi_s \, \mathbb{S}^{(f_1} \ldots \mathbb{S}^{f_{2n})} \nabla(\omega)^b \phi_s$$
$$- \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n}}}{m^{2n+1}} \nabla(\omega)_{f_{2n+1}} \cdots \nabla(\omega)_{f_3} \frac{1}{2} \epsilon_{ab(c|f_1} R^{ab}{}_{|d)\, f_2} \nabla(\omega)^c \phi_s \mathbb{S}^{(f_1} \ldots \mathbb{S}^{f_{2n+1})} \nabla(\omega)^d \phi_s$$

Equivalent classically and w/ covariant spin supplementary condition

Inequivalent q.m.

$$\mathcal{L}_H = -\sum_{n=1}^{\infty} \frac{(-1)^n (2n-1)}{(2n)!(2n+1)} \frac{H_{2n}}{m^{2n-2}} \nabla_{f_{2n}} \ldots \nabla_{f_3} R_{(a|f_1|b) f_2} \phi_s M^{a(f_1} M^{|b| f_2} \mathbb{S}^{f_3} \ldots \mathbb{S}^{f_{2n})} \phi_s$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n n}{(2n+1)!(n+1)} \frac{H_{2n+1}}{m^{2n-1}} \nabla_{f_{2n+1}} \ldots \nabla_{f_3} * R_{(a|f_1|b) f_2} \phi_s M^{a(f_1} M^{|b| f_2} \mathbb{S}^{f_3} \ldots \mathbb{S}^{f_{2n+1})} \phi_s$$

→ Puzzle: What is the physical meaning of the extra Wilson coefficients?

- Related to conservative spin-magnitude change
- Acknowledge that the spin supplementary condition $p_\mu S^{\mu\nu}(p) = 0$ can be relaxed

from worldline: d'Ambrosi, Kumar, van Holten

Complementary descriptions of same physics: Initially in QED: Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines

EFT with one unconstrained field, no SSC, + analytic continuation ⟷ EFT with distinct fields of different spins SSC for each of them (more later)

$$\mathsf{S}^{\mu\nu} = \frac{1}{m}\epsilon^{\mu\nu\rho\sigma}p_\rho S_\sigma + \frac{i}{m}(p^\mu K^\nu - p^\nu K^\mu) \qquad \mathcal{E}_1 \cdot M^{\mu\nu} \cdot \mathcal{E}_2 = \mathsf{S}(p_1)^{\mu\nu}\mathcal{E}_1 \cdot \mathcal{E}_2 + \mathcal{O}(q^0)$$

Gravitational Lagrangian: reorganization of previous Lagrangian Alaverdian, Bern, Kosmopoulos, Luna, RR, Scheopner, Teng

$$L = L_0 + L_{\text{non-min}} \qquad L_0 = -\frac{1}{2}\phi_s(\nabla^2 + m^2)\phi_s + \frac{1}{8}R_{abcd}\phi_s M^{ab}M^{cd}\phi_s$$

E.g.

$$L_{\text{non-min}} = -\frac{C_2}{2m^2}R_{af_1bf_2}\nabla^a\phi_s\mathbb{S}^{(f_1}\mathbb{S}^{f_2)}\nabla^b\phi_s + \frac{D_2}{2m^2}R_{abcd}\nabla_i\phi_s\{M^{ai}M^{cd}\}\nabla^b\phi_s + \frac{E_2 - 2D_2}{2m^4}R_{abcd}\nabla^{(a}\nabla^{i)}\phi_s\{M^b{}_iM^d{}_j\}\nabla^{(c}\nabla^{j)}\phi_s$$

Same amplitudes as from previous Lagrangian; different EFT/Hamiltonian b/c of additional dof-s

For special values of Wilson coefficients -- $D_2 = E_2 = 0$ -- K vector decouples

⟶ recover SSC-satisfying theory ⟶ Method for fixed-spin computations w/o SSC

Verified explicitly through $\mathcal{O}(G^2S^4)$ + eikonal + H ... To appear Alaverdian, Bern, Kosmopoulos, Luna, RR, Scheopner, Teng

Worldline counterpart:

Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines
Alaverdian, Bern, Kosmopoulos, Luna, RR, Scheopner, Teng + to appear

$$L = -p_\mu \dot{z}^\mu + \frac{1}{2}\mathsf{S}^{\mu\nu}\Lambda_{A\mu}\frac{D\Lambda^A{}_\nu}{D\lambda} + \frac{\xi}{2}(p^2 - \mathrm{M}^2)$$

e.g. $\mathrm{M}^2 = m^2 + \frac{1-C_2}{4}R_{\hat{p}S\hat{p}S} + \frac{1-D_2}{2}R^\star_{\hat{p}S\hat{p}\mathrm{K}} + \frac{1-E_2}{4}R_{\hat{p}\mathrm{K}\hat{p}\mathrm{K}}$

- Same Compton amplitude as the 4D QFT calculation
- $K^\mu = 0$ preserved under time evolution for the special Wilson coefficients enforcing SSC

WQFT beyond $\mathcal{O}(S^2)$ recovered Kerr impulse through $\mathcal{O}(G^2S^4)$

Haddad, Jakobsen, Mogull, Plefka

Hamiltonian: - Poisson algebra of tangent space coordinates, momenta and spin tensor

- Imposes $\dot{Z} = \{H_T, Z\} \approx 0 \qquad H_T = eH + \zeta_\mu Z^\mu + a(S_{\mu\nu}S^{\mu\nu} - 2s^2)$

$Z^\mu = S^{\mu\nu}\hat{\pi}_\nu$

$\longrightarrow$ Determines $\zeta_\mu S^{\mu\nu}$ i.t.o. spin tensor, curvature, momenta, etc.

K is denoted by *Z* there

- Poisson algebra can be realized in terms of bosonic oscillators $S^{\mu\nu} \sim \bar{\alpha}^\mu\alpha^\nu$

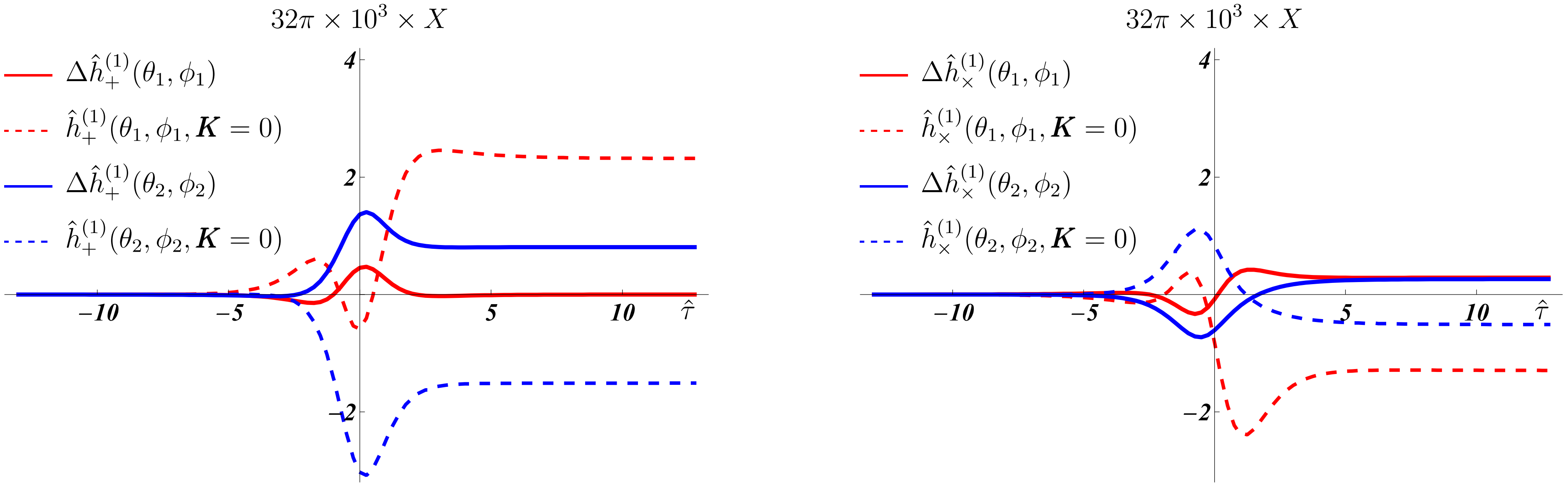
Lagrangian: - Ansatz inspired by Legendre transform of H_T w.r.t. α & w/ free coefficients

- $\Delta Z^\mu = 0$ fixes some, leaving one per power of spin; Z remains in action

$$S = -m\int \mathrm{d}\tau\left(\frac{1}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu - \mathrm{i}\eta_{ab}\bar{\alpha}^a\frac{\mathrm{D}\alpha^b}{\mathrm{d}\tau} - \mathcal{L}_{\mathrm{nm}}\right) \qquad \mathcal{L}_{\mathrm{nm}} = \sum_{n=1}^{\infty}(\nabla_{\mathfrak{a}})^{2n-2}\left(\frac{C_{ES^{2n}}}{(2n)!}\mathcal{E}_{\mathfrak{a}\mathfrak{a}} - \frac{\mathrm{i}C_{BS^{2n+1}}}{(2n+1)!}\nabla_{\mathfrak{a}}\mathcal{B}_{\mathfrak{a}\mathfrak{a}}\right) + \frac{C_2^{(Z)}}{2}B_{\mathfrak{a}\mathfrak{z}} - \frac{\nabla_{\mathfrak{z}}}{|\dot{x}|}\sum_{n=2}^{\infty}(\nabla_{\mathfrak{a}})^{2n-4}\left(\frac{C_{2n}^{(Z)}}{(2n)!}\nabla_{\mathfrak{a}}\mathcal{B}_{\mathfrak{a}\mathfrak{a}} + \frac{\mathrm{i}C_{2n-1}^{(Z)}}{(2n-1)!}\mathcal{E}_{\mathfrak{a}\mathfrak{a}}\right)$$

K affects observables – impulse, spin kick and waveform

Alaverdian, Bern, Kosmopoulos, Luna, RR, Scheopner, Teng; + to appear



⟶ K not degenerate with Wilson coefficients

Connection with existing worldline description of spin-induced multipole interactions

Poisson; Steinhoff; Porto, Goldberger, Li, Rothstein; Vines, Steinhoff;...

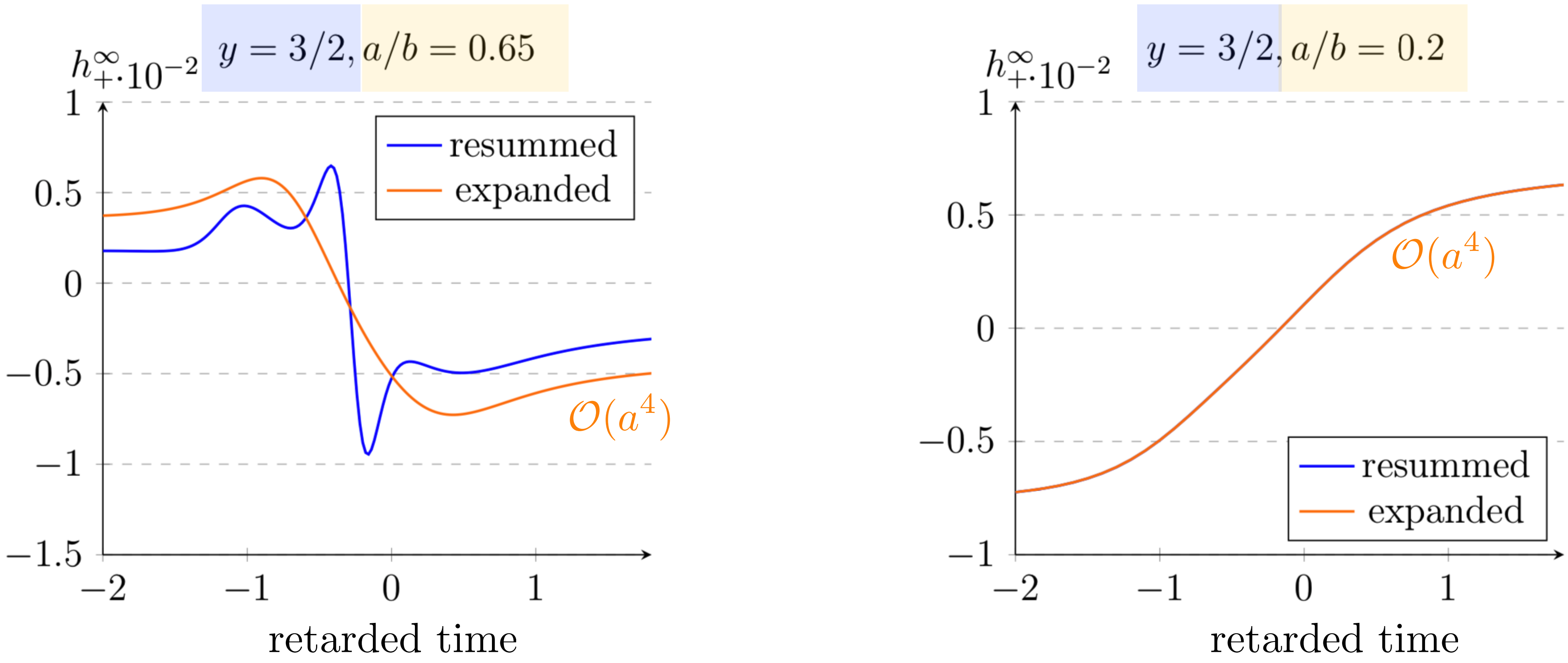
$$S_{\text{fin}} = \int d\lambda \left(Q_E^{ij} \frac{E_{ij}}{\sqrt{u^2}} + Q_B^{ij} \frac{B_{ij}}{\sqrt{u^2}} \right) + \dots \qquad Q_E^{ij} = \frac{C_2(r_s)}{2m} S^i S^j + \frac{E_2}{2m} K^i K^j \qquad Q_B^{ij} = \frac{D_2}{2m} S^{(i} K^{j)}$$

Like the spin, K describes massless internal d.o.f-s of (certain) massive (spinning) bodies
lower scale than that of the high energy modes governing spin-induced multipoles

All-order-in-spin waveforms at tree level:

Brandhuber, Brown, Chen, Gowdy, Travaglini;
De Angelis, Gonzo, Novichkov

Resummation of spin dependence can lead to sizeable effects, depending on impact parameter



Interesting to see the counterpart of this comparison for bound motion

State of the art spin-dependent results at higher orders

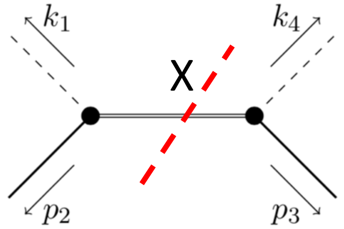
- Conservative $\mathcal{O}(G^3 S^2)$ Hamiltonian & radial action, EM & GR, including Casimir $S_\mu S^\mu$ contributions — Febres Cordero, Kraus, Lin, Ruf, Zeng; + Akpinar
  - Uses spin universality/fixed spin
  - Confirms $\mathcal{O}(S^2)$ WQFT results based on supersymmetric action of Jakobsen, Mogull
  - Spin-shift symmetry holds in the probe limit at this order
- NLO waveform quadratic in spin — see Lara Bohnenblust's talk — Bohnenblust, Ita, Kraus, Schlenk
- Incorporate spinning PM perturbative results into EOB — Buonanno, Jakobsen, Mogull; Buonanno, Mogull, Patil, Pompilli

  See Alessandra Buonanno's talk for import into EOB

Absorption with 4D QFT methods (see Goldberger, Li, Rothstein for a worldline perspective)

(Porto; Endlich, Penco for slow rotation)

- 4D QFT: include internal states with 2-point function of Kählen-Lehmann form — Jones, Ruf



$$\mathcal{M}^{(\psi)}\Big|_{\text{forward}} = -\kappa^2 \int_{m_2^2}^{\infty} \mathrm{d}\mu^2 \frac{\rho_0(\mu^2)}{m_2^2 + 2m_2\omega - \mu^2 + i0} \qquad \sigma_{\text{total}}(\omega) = \frac{\mathrm{Im}\mathcal{M}^{\text{forward}}(\omega)}{2m_2\omega}$$

 - Heavy X modes ⟶ Love numbers/ contact terms; Light modes ⟶ absorption

 - Some results: Schwarzschild mass always increases; agreement w/ energy loss for scalar model

- On-shell approach to absorption for Kerr — Chen, Hsieh, Y-t Huang, J-W Kim

 - Asymptotic states: coherent superpositions of spins; internal density of states

 - Three-point amplitudes fixed by matching against BHPT of Bautista, Guevara, Kavanagh, Vines

See also work by Bautista, Huang, J-W Kim using Compton amplitude from BHPT analysis for 2PM calculation

 - Interesting to compare with GR scattering observables with absorption

 - Some puzzles related to contributions to static Love numbers from density of states

- Absorption of mediators ⟶ Absorption of matter, i.e. dynamical capture or merger

See talk by Katsuki Aoki — Aoki, Cristofoli, Y-t Huang

Results on tidal effects & beyond GR from field theory methods

- R^3 deformations of general relativity — Brandhuber, Brown, Chen, Travaglini, Vives Matasan
  - Leading order impulse and spin kick, all-orders in spin
  - Leading order waveform to all orders in spin

- Extensive studies of two-body dynamics in Chern-Simons gravity with GR methods

  Goal: distinguish it from GR (e.g. using QN modes for spinning BHs Wagle, Li, Chen, Yunes)

  Action: $$\mathcal{L} = -\frac{1}{2}M_{\rm Pl}^2 R + \frac{1}{2}(\partial\varphi)^2 - \frac{l_{\rm dCS}^2}{M_{\rm Pl}^3}\,\varphi\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu}{}^{\gamma\delta}R_{\alpha\beta\gamma\delta}$$

  Kerr (but not Schwarzschild) metric receives corrections $\longrightarrow$ contrib. to scattering/inspiral

  - Eikonal to linear order in spin — Bhattacharyya, Ghosh, Ghosh, Pal
  - Leading order linear-in-spin waveform for scalar radiation — Falkowski, Marinellis

    also in scalar-Gauss-Bonnet theory, where Schwarzschild metric grows scalar hair

    + analytic continuation to bound-orbit observables in scalar-Gauss-Bonnet and dynamical Chern-Simons theories

Results on tidal effects, beyond GR from amplitudes

- Contributions leading-order tidal operators to 4PM impulse — Jakobsen, Mogull, Plefka, Sauer

$$\mathcal{L}_{\rm WL} \sim c_{E^2} \left(R_{\mu\rho\nu\sigma}\dot{x}^\rho \dot{x}^\sigma\right)^2 + c_{B^2} \left(R^*_{\mu\rho\nu\sigma}\dot{x}^\rho \dot{x}^\sigma\right)^2 = c_{E^2} \left(E_{\mu\nu}\right)^2 + c_{B^2} \left(B_{\mu\nu}\right)^2$$

Renormalization required; mixing with with higher tidal operators $(\dot{E}_{\mu\nu})^2$ and $(\dot{B}_{\mu\nu})^2$

renormalized coefficients, related to Love numbers, obtained from other consideration

Earlier discussion of renormalization in classical context:

Tidal operator renormalization in PN gravity — Goldberger, Rothstein; + Ross;… at 3PN: Mandal, Mastrolia, Silva, Patil, Steinhoff

Tidal operator renormalization in scalar model — BBHLP-MRSSTZ

Earlier renormalization in classical GR + matter: construction of Reissner-Nordström BH — Sardelis

- Results on vanishing static Love nrs. for BH from standard methods; symmetries args.

Chia; Hui, Joyce, Penco, Santoni, Solomon; Charalambous, Dubovski, Ivanov; Levi, Yin

Is it possible to build these symmetries into the QFT framework?

Structure or Curiosity:

Novel Theoretical Structures and Connections in the classical limit

- Cut = rotation, causality restauration, KMOC ⟷ causal response functions
- Various nonperturbative structures
- Structure in simpler theories
- New connections

- NLO Scattering waveform: Cut = $\chi/2$ rotation $\langle h^{\rm TT}_{\mu\nu}\rangle = i\langle\psi_{\rm in}|h^{\rm TT}_{\mu\nu}T|\psi_{\rm in}\rangle - i\langle\psi_{\rm in}|T^\dagger h^{\rm TT}_{\mu\nu}|\psi_{\rm in}\rangle + \langle\psi_{\rm in}|T^\dagger h^{\rm TT}_{\mu\nu}T|\psi_{\rm in}\rangle$

$$({\rm Conn.\ Amplitude} + {\rm Conn.\ Cut})(\phi) = ({\rm Conn.\ Amplitude})(\phi + \chi/2) + \delta\tau W^{\rm tree}$$

Bini, Damour, Geralico; Georgoudis, Heissenberg, Russo; Bini, Damour, De Angelis, Herderschee, Geralico, RR, Teng

Indication that amplitude & cut combine naturally at $\mathcal{O}(G_N^n)$ restoring causality in classical limit

Caron-Huot, Giroux, Hanesdottir, Mizera; Biswas, Parra-Martinez

KMOC observables ⟷ Causal response functions

- Amplitudes can be much simpler than integrals:

Bern, Herrmann, RR, Ruf, Smirnov, Smirnov, Zeng; Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovich

Nontrivial 4PM cancellations of complex polylogs; 5PM cancellations of elliptic ints, letters, etc

- Nonperturbative structures: (1) Resummation

- Resummation of zero-energy gravitons vs. BMS supertranslations — Elkhidir, O'Connell, RR

Other BMS supertranslations in on-shell framework? Extensions of BMS symmetry?

Other consequences of different supertranslation frames?

- Direct all-order resummation of amplitude contributions to Schwarzschild metric and of the geodesic equation — Mougiakakos, Vanhove

- Nonperturbative structures: (2) Exponentiation of amplitudes

- Canonical eikonal exponentiation of 4-point amplitudes with K vector $\quad i\mathcal{M} = e^{i\delta} - 1$

$$\Delta\mathcal{O} = e^{-i\delta\mathcal{D}}[\mathcal{O}, e^{i\delta\mathcal{D}}]\,,\ \delta\mathcal{D}g \equiv \delta g + \mathcal{D}_L(\delta, g)\,,\ \mathcal{D}_L\,(\delta, g) \equiv -\epsilon^{ijk} \sum_{a=1,2} S_a^k \frac{\partial\delta}{\partial S_a^i}\frac{\partial g}{\partial L^j} + K_a^k \frac{\partial\delta}{\partial K_a^i}\frac{\partial g}{\partial L^j}$$

Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines

- Covariant eikonal exponentiation with spin and K vector

Luna, Moynihan, O'Connell, Ross; Gatica

$$\Delta\mathcal{O} = \mathcal{O}(\Delta p) + \{\delta, \mathcal{O}\} + \frac{1}{2}\{\delta, \{\delta, \mathcal{O}\}\} + \ldots$$

$\mathcal{D}_L$ originates from the evaluation of lower order terms in shifted variables

- Operatorial exponentiation; Eikonal for higher-point amplitudes

Damgaard, Planté, Vanhove

Di Vecchia, Heissenberg, Russo, Veneziano

$$S = e^{i\hat{N}}\,, \quad S = e^{i\delta} e^{i\sqrt{-2i(\delta-\delta^\dagger)}C} e^{i\sqrt{-2i(\delta-\delta^\dagger)}C^\dagger}$$

- Eikonal techniques for absorptive processes

Aoude, Cristofoli, Elkhidir, Sergola

$$S|\psi\rangle = \begin{pmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{pmatrix} \begin{pmatrix} |\varphi_{1a}, \varphi_2\rangle \\ |\varphi_{1b}, \varphi_2\rangle \end{pmatrix} \qquad \begin{matrix} S_{aa} = \hat{\eta}_{aa} e^{i\hat{N}_{aa}} \\ S_{ba} = \hat{\eta}_{ba} e^{i\hat{N}_{ba}} \end{matrix}$$

extends exponential rep of Damgaard, Planté, Vanhove

- Search for structure in simpler theories:

QED

De la Cruz, Maybee, O'Connell, Ross; Bern, Gatica, Herrmann, Luna, Zeng; Saketh, Vines
Steinhoff, Buonanno; Bern, Herrmann, RR, Ruf, A&V Smirnov, Zeng; Brunello, De Angelis

A charged scalar model: SF + PM aim to extend both beyond respective validity regime

Barack, Bern, Herrmann, Long, Parra-Martinez, RR, Ruf, Shen, Solon, Teng, Zeng

Resummation accounting for: (1) known 4PM terms (2) near separatrix: $\chi \sim \ln(b - b_c(v))$

Long, Whittall, Barack

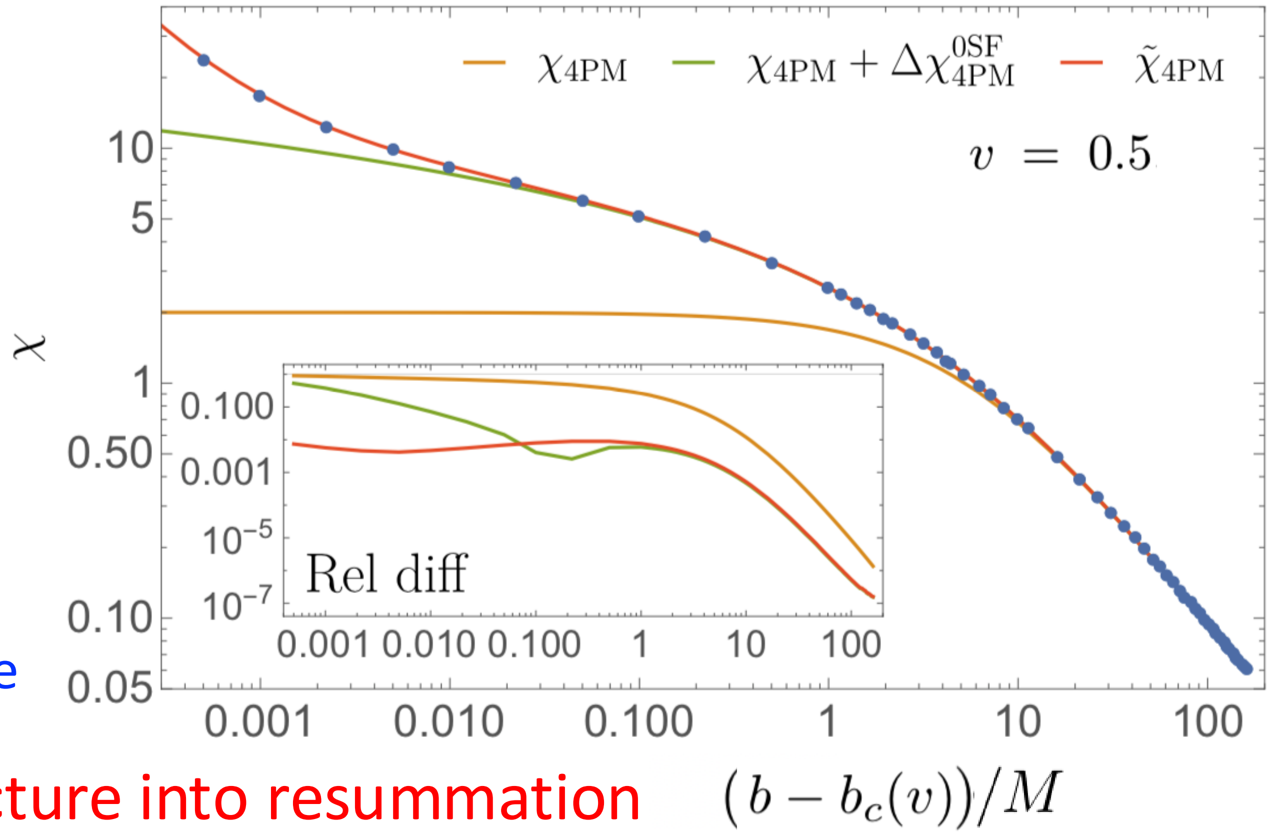
$$\tilde{\chi}(b,v) := \chi_{4\mathrm{PM}}(b,v) + \Delta\chi(b,v)$$

$$\Delta\chi(b,v) = A_0(v)\left[\log\left(1 - \frac{b_c(v)[1-\epsilon A_1(v)/A_0(v)]}{b}\right) + \sum_{k=1}^{4}\frac{1}{k}\left(\frac{b_c(v)[1-\epsilon A_1(v)/A_0(v)]}{b}\right)^k\right]$$

Best fit of numerical scalar 1SF calculations:

$$A_1(v) \approx 0.0222 - 0.0398v + 0.0199v^2$$

Improves the faithfulness of fixed-order PM expressions, uniformly across the parameter space



Illustrates power of building analytic structure into resummation

- Search for structure in simpler theories:

QED

De la Cruz, Maybee, O'Connell, Ross; Bern, Gatica, Herrmann, Luna, Zeng; Saketh, Vines
Steinhoff, Buonanno; Bern, Herrmann, RR, Ruf, A&V Smirnov, Zeng; Brunello, De Angelis

A charged scalar model: SF + PM aim to extend both beyond respective validity regime

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Interesting puzzle:

- Numerical analysis: static scalar Love number $c_1 = 0.31 \pm 0.38$ consistent w/ 0 within errors
- Analytic 1SF calculation: $c_1 = 1/6$ consistent with numerical analysis within error bars; used in fit — Bini, Geralico, Kavanagh, Pound, Usseglio
- Worldline analysis suggests $c_1 = 0$ — Ivanov, Li, Parra-Martinez, Z. Zhou

See talks by Julio Parra-Martinez and Giulia Isabella

- New connection from analysis of universal terms in waveform:

Radial solution to Teukolsky eq for ψ_4 $\xleftrightarrow{\text{solutions to Heun equation}}$ Five-point degenerate correlator in Louville theory $\xleftrightarrow{\text{AGT}}$ Instanton partition function in $\mathcal{N}$ = 2 $SU(2) \times SU(2)$ sYM theory

Consoli, Fucito, Morales, Poghossian; Fucito, Morales ; + Russo
Bautista, Bonelli, Iossa, Tanzini, Z. Zhou

Accident or deep structure?

“Necessity is the mother of invention”:

New special purpose & general purpose tools

- New/Improved Integration Methods
- New/Improved amplitudes and amplitudes methods
- New (proposals for) Computational Methods
- Synergy with traditional methods

- New/Improved Integration Methods:

  - Massive improvements to public IBP programs — Bern, Herrmann, RR, Ruf, Smirnov, Smirnov, Zeng; Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovich

    FIRE and KIRA + private codes ⟶ public

  - Systematic analysis of single-scale integrals and associated geometries appearing in the classical limit at 4 loops; first CY integrals — Frellesvig, Morales, Wilhelm; Klemm, Nega, Sauer, Plefka

    + intersection theory perspective; classical limit ⟶ increased efficiency — Teschke; Frellesvig, Teschke

  - Fourier-transforms of and Fourier-transform-like Feynman integrals

    - IBP-based approach to time-domain analytic scattering waveforms — Brunello, De Angelis; Brunello, Crisanti, Giroux, Mastrolia, Smith

    - Tensor-integral generating functions (PV alternative); spin-resumed obs. — Chen, J-W Kim, Wang

  - Construction of finite integral basis — De la Cruz, Kosower, Novichkov; Gambuti, Kosower, Novichkov, Tancredi

- New/Improved amplitudes and amplitudes methods
  - New Integrand construction: - Defines an integrand basis before IBP — Bern, Herrmann, RR, Ruf, Zeng
    - Streamlines cut merging: $\text{cut terms} = \text{amplitude terms}$
    - Double-copy to all loop orders

    See Zvi Bern's talk later today
  - New construction of eikonal: Magnus expansion — J-H Kim, J-W Kim, S. Kim, S. Lee
    - the log of a time-ordered exponential

    See Sangmin Lee's talk for details
  - New tree-level quantum Compton amplitudes for arbitrary spin — Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov
    - Includes insight from massive higher-spin field theory & massless limit
    - With free parameters; overlaps with solution of Teukolsky equation
    - Sewn into loops — Bohnenblust, Cangemi, Johansson, Pichini
  - New Compton amplitudes from classical scattering — Bautista, Guevara, Kavanagh, Vines; Bautista, Bonelli, Iossa, Tanzini, Zhou; Chen, Wang; Chen, Hsieh, Y-t Huang, J-W Kim; Bautista, Huang, J-W Kim
    - Including horizon/absorption effects
    - Sewn into loop amplitudes; w/ KMOC
  - New tree-level Compton amplitudes from string theory — Azevedo, Matamoros, Menezes; Cangemi, Pichini
  - Tree-level Compton amplitudes from bootstrap — Bjerrum-Bohr, Chen, Skowronek

- New (proposals for) Computational Methods:
  - Schwinger/Keldysh in 4d QFT as a means to enforce causality — Carron-Huot, Giroux, Hannesdottir, Mizera; Biswas, Parra-Martinez

    Equivalence with KMOC framework; manifest superclassical and acausal term cancellations
  - Model-agnostic KMOC-based approach to spinning observables — Gatica
  - K/Z and Wilson coefficients as a means to enforce the SSC — Bern, Kosmopoulos, Luna, RR, Scheopner, Teng, Vines; + Alaverdian; Haddad, Jakobsen, Mogull, Plefka
- Synergy with traditional methods & other fields:
  - Bound regime, analytic continuation

    - Bound inclusive observables from unbound inclusive observables: — Dlapa, Kälin, Liu, Porto

      B2B for local in time & direct bound regime PN calculation of rest
    - Proposal for bound-unbound connection for waveforms — Adamo, Gonzo, Ilderton

      See Zihan Zhou's talk at higher orders
  - PM theory meets EOB for wave emission and scattering — Buonanno, Jakobsen, Mogull; Buonanno, Mogull, Patil, Pompili
  - Study of simpler models (QED, scalar model) continue to yield insight

    - Love numbers, integration, resummation, absorption, etc — BHRRSSZ; Long, Wittall, Barack; Jones, Ruf; Bini, Geralico, Kavanagh, Pound, Usseglio; Ivanov, Li, Parra-Martinez, Z. Zhou

    - Potential input to phenomenology of ultraperipheral and diffractive scattering in QED

Outlook

Enormous progress towards the desired two orders of magnitude of precision; more is needed

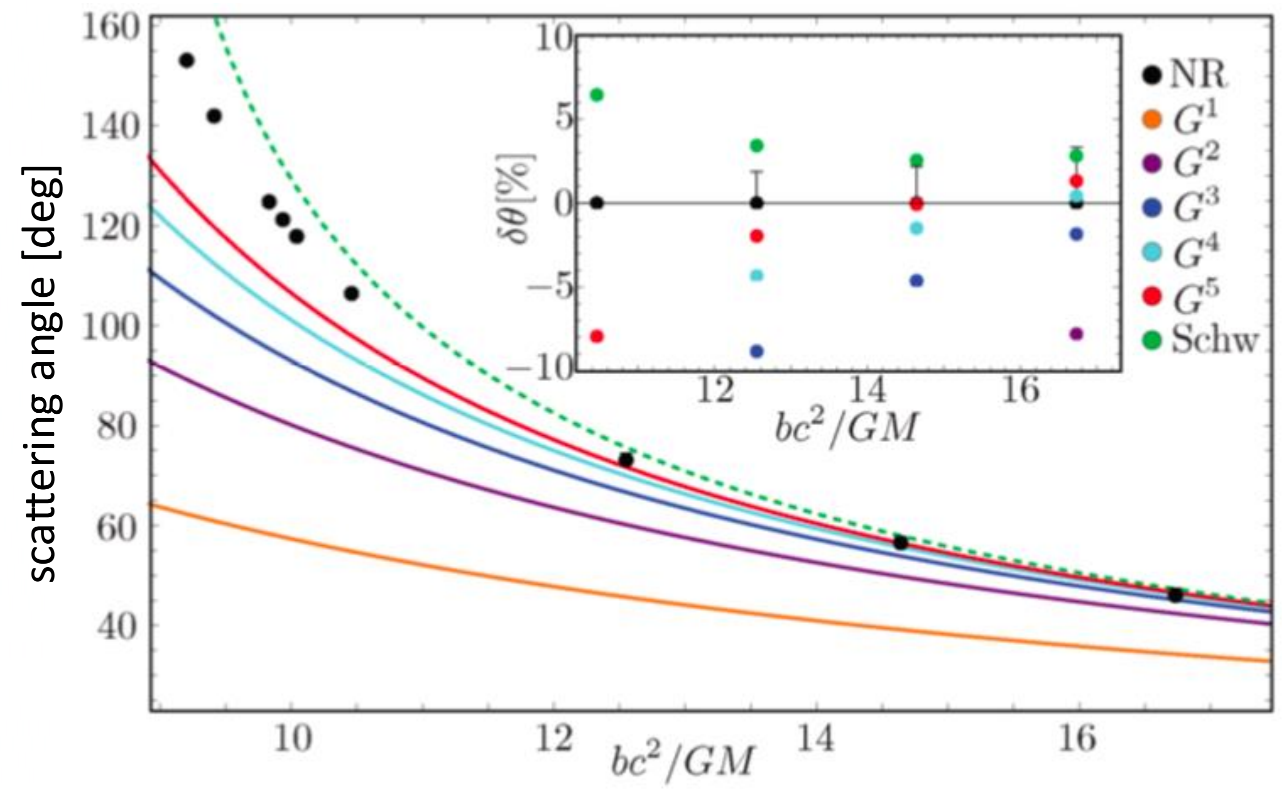
- Explicit higher-order spinning and spinless computations
- Systematic inclusion of tidal effects and absorption
- Answer basic questions about the description of spinning bodies
- New observables?
- Explore and use analytic structure of observables, bound-unbound analytic continuations
- Understand and use the structure of higher order perturbation theory
  - e.g. diagram resummation
  - e.g. via synergy between various approaches to the two-body problem
- Exploit feedback into amplitudes and other fields

Given the recent rate of progress, expect many more results,
both for gravitational-wave physics, and more generally for understanding gravity

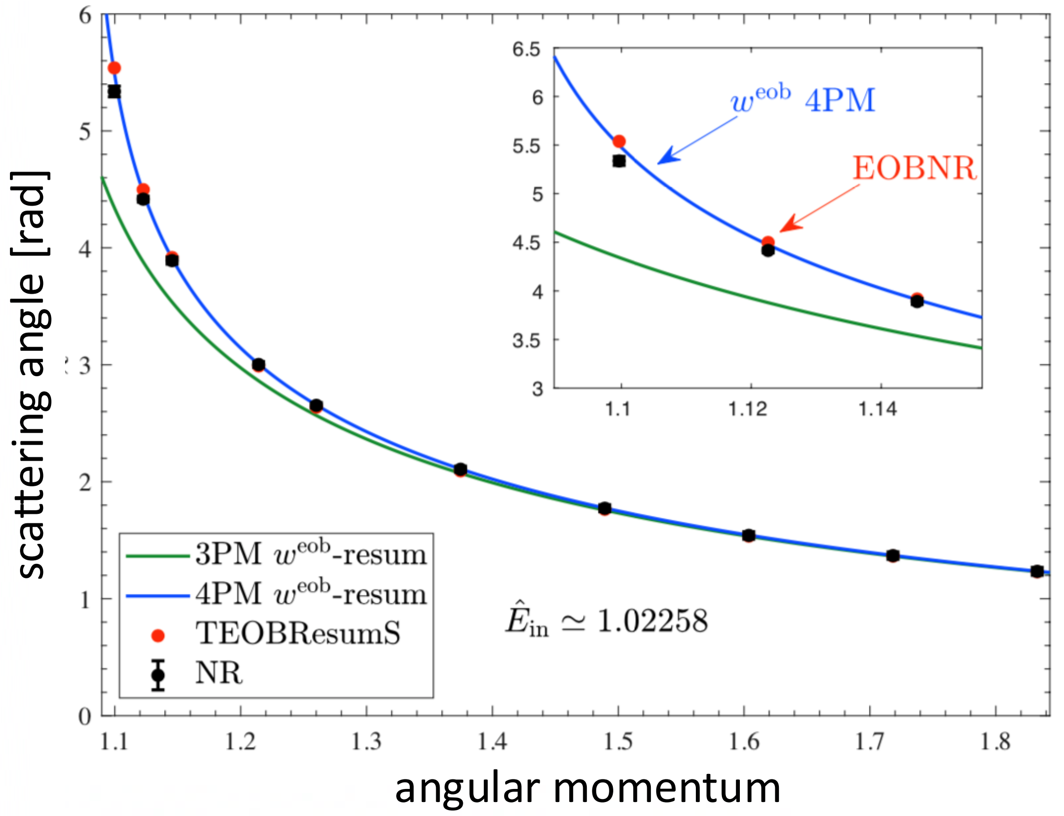
The Beginning

Radiative contributions to the 0SF and 1SF scattering angle recently computed from WQFT

Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovich



Damour, Rettegno



Interesting to compare EOB-resumed 1SF 5PM & NR and study relative numerical importance of 2SF terms over parameter space

$$\chi^{w\ eob}_{\rm nPM}(\sigma, j) \equiv 2j \int_0^{\bar{u}_{\max}(\sigma,j)} \frac{d\bar{u}}{\sqrt{p_\infty^2 + w_{\rm nPM}(\bar{u},\sigma) - j^2\bar{u}^2}} - \pi$$

PM STATE-OF-THE-ART

WQFT
[Driesse, Jakobsen, Sauer, Klemm, Mogull, Nega , Steinhoff, Usovitsch]

WEFT **Worldline effective theory**
[Källin,Porto,Dlapa,Cho,Liu,..]
[Riva,Vernizzi,Mougiakakos..]

HEFT **Heavy BH effective theory**
[Aoude,Haddad,Helset,Damgaard]
[Brandhuber,Travaglini,Chen]

Amps **Scattering amplitudes**
[Bern,Roiban,Shen,Parra-Martinez,Ruf,Zeng..]
[Bjerrum-Bohr,Damgaard,Plante,Vanhove,..]
[Di Vecchia,Veneziano,Heissenberg,Russo]
[Solon,Cheung,..][Huang,..]
[Guevera,Ochirov,Vines,...]
[Johansson,Pichini[Kosower,O'Connell,Maybee

order	deflection & spin kick					waveform			Integration complexity
	plain	spin-orbit	spin-spin	spin>2	tidal	plain	spin-orbit spin-spin	tidal	
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	WQFT WEFT Amps	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps	~ 1-loop
3PM cons	WQFT WEFT Amps HEFT	WQFT Amps	WQFT (Amps)		WQFT WEFT	Amps HEFT	Amps		
3PM diss	WQFT WEFT Amps HEFT	WQFT	WQFT		WQFT WEFT				~ 2-loop
4PM cons	WQFT WEFT Amps	WQFT			WQFT				
4PM diss	WQFT WEFT Amps	WQFT			WQFT				~ 3-loop
5PM-1SF cons	WQFT								~ 4-loop

r-r: Radiation-reaction (...) : partial results

Source: J. Plefka Talk at MIAPbP