QCD meets Gravity, Taipei, December 2024

Semiclassical Gravitational Radiation

Donal O'Connell
Edinburgh, IPhT CEA-Saclay

Funded by the European Union
European Research Council
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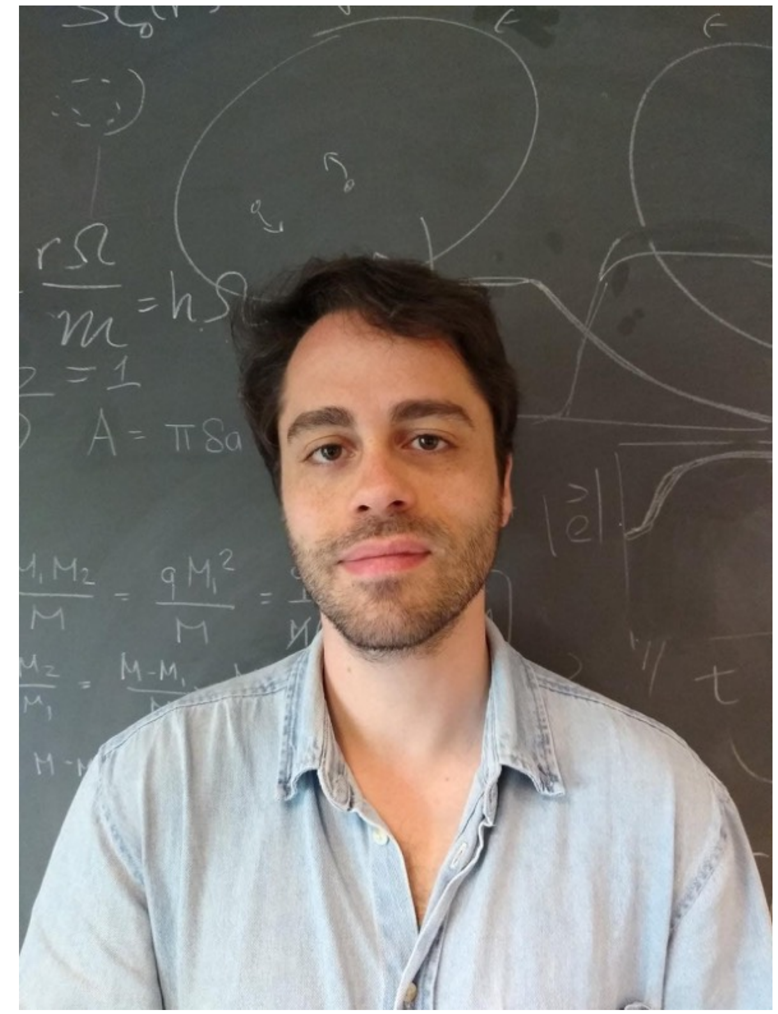
With

Rafael Aoude
Edinburgh



Matteo Sergola
IPhT

Motivation

Amplitudes / GW programme: great success

→ Radu’s talk

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Amplitudes are quantum

Use our methods to compute semiclassical observables?

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Amplitudes are quantum

Use our methods to compute semiclassical observables?

Yes: Hawking radiation is an example

Outline

This talk: Hawking's computation in amplitudes language

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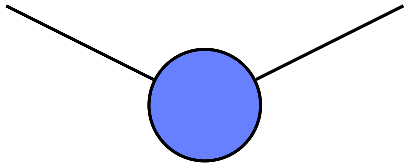
1. Hawking's scattering process



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This talk: Hawking's computation in amplitudes language

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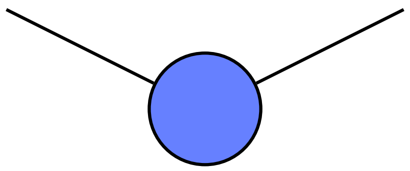
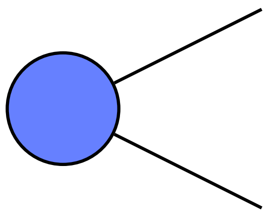


2. Pair production by crossing



Outline

Throughout: dynamical background metric "Vaidya"

$$g^{\mu\nu} = \eta^{\mu\nu} + \frac{2GM\Theta(t+r)}{r} k^\mu k^\nu$$

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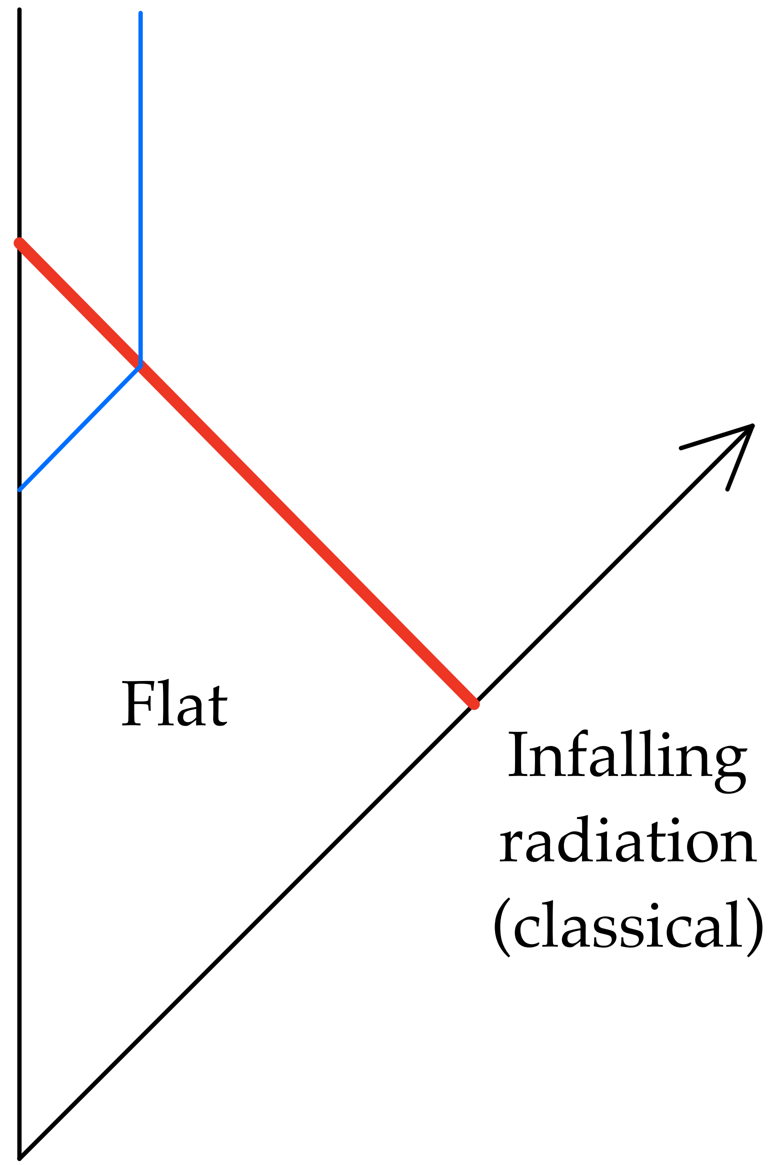

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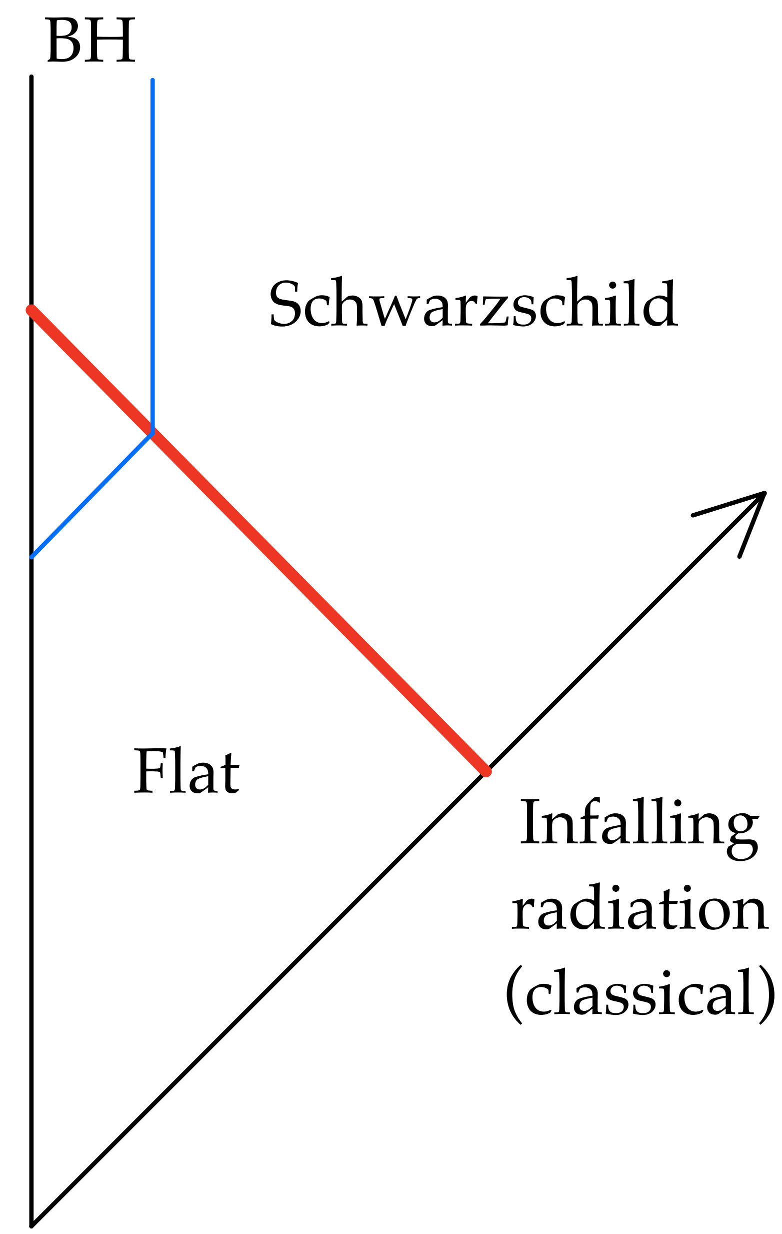
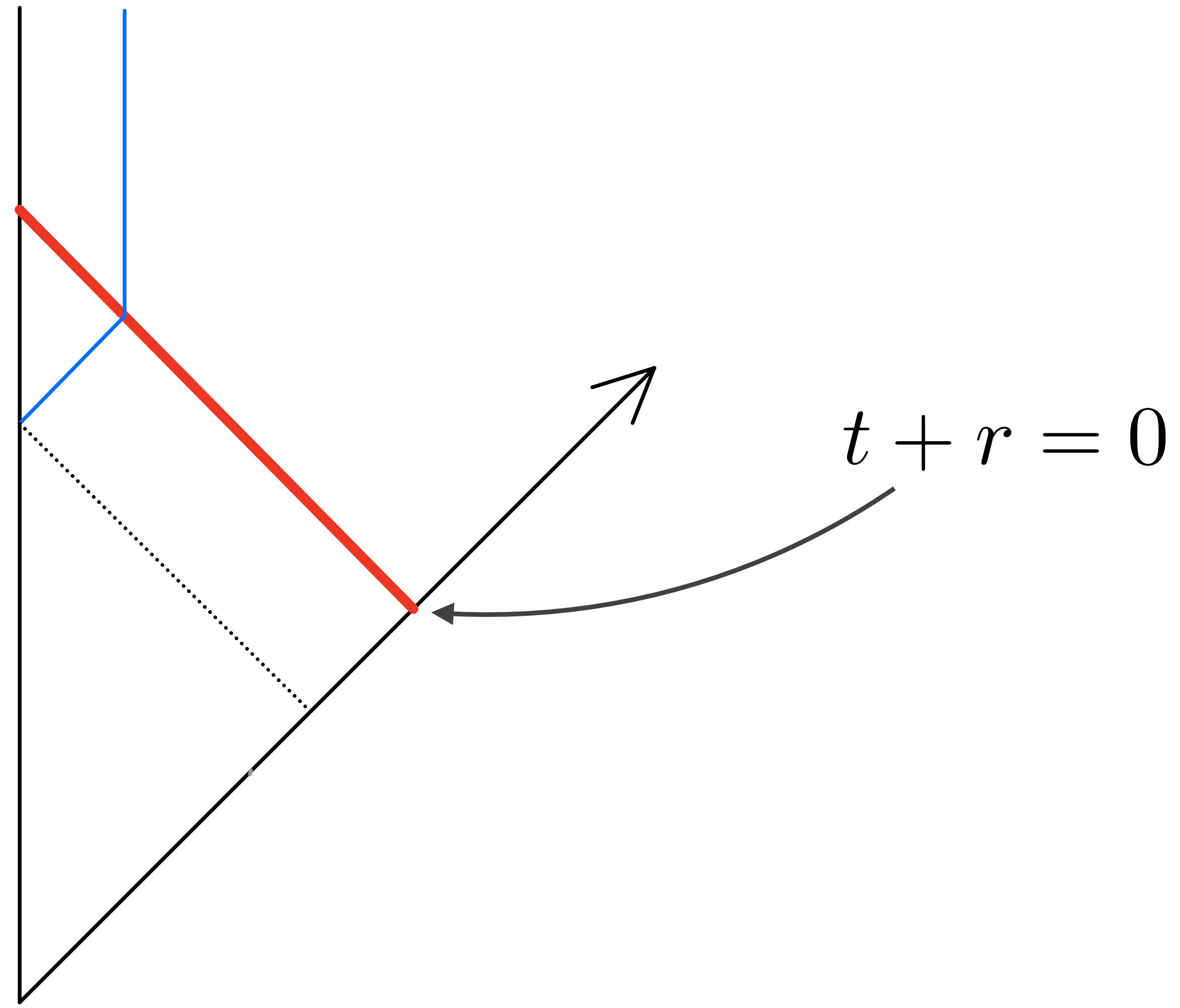


Hawking Scattering

Scatter spherical particle through background



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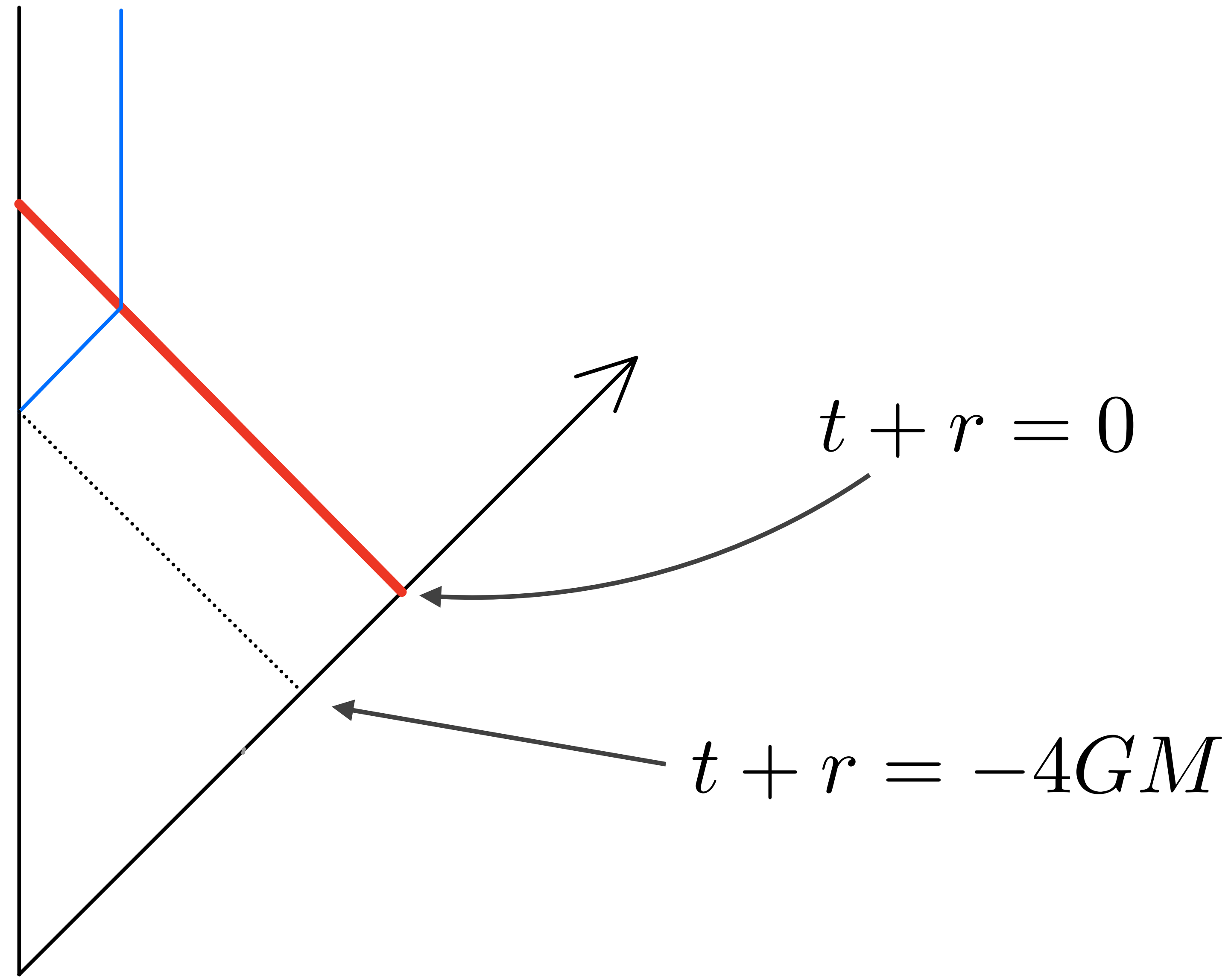

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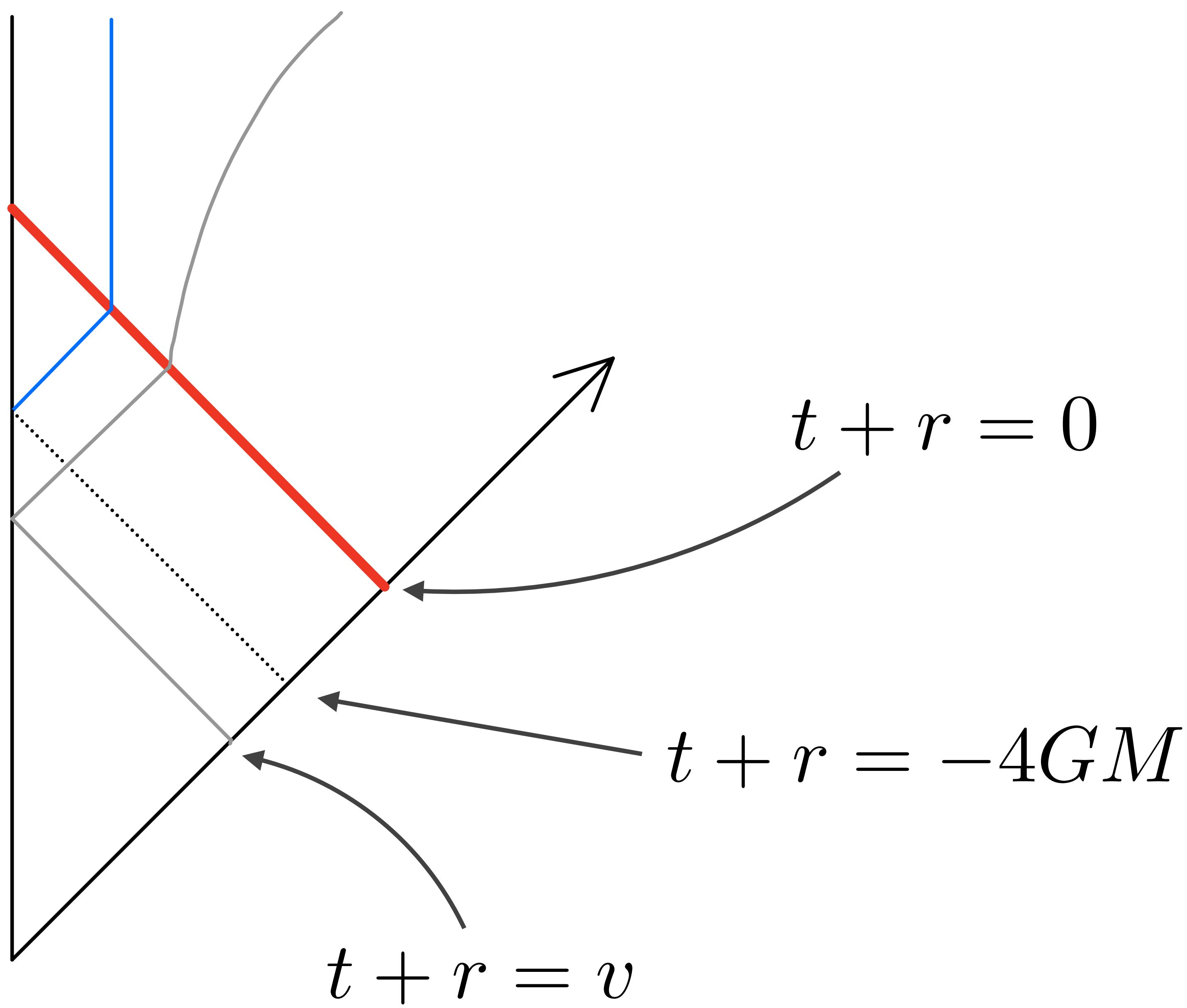

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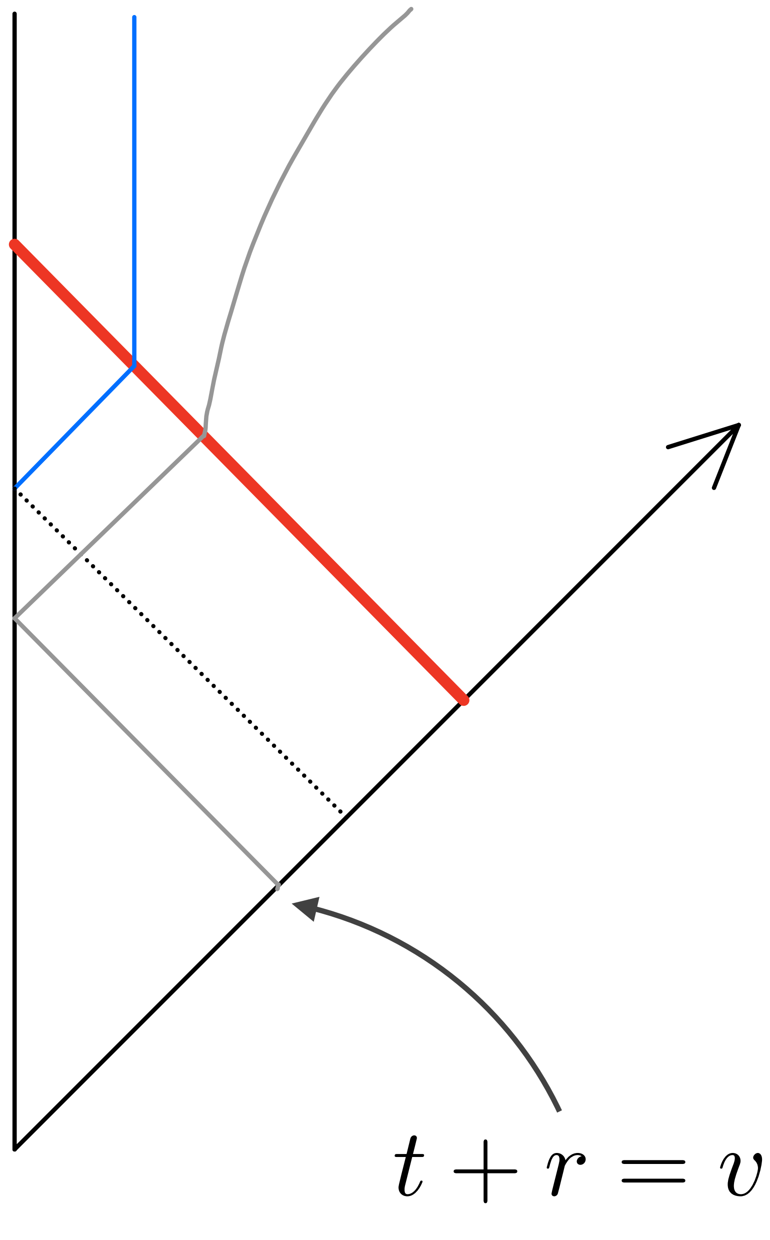

Perturbative parameter

$$\frac{v}{4GM}$$

Hawking Scattering

Scatter spherical particle through background

$$|\psi\rangle = \int \mathrm{d}\Phi(p)\,\varphi(p)\,|p\rangle\, = \int \mathrm{d}\Phi(p) \int \mathrm{d}v\,\varphi(v)e^{iEv}\,|p\rangle$$

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Massless, quantum

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Spherical symmetry

$$E = |\mathbf{p}|$$

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$$= \int \mathrm{d}\Phi(p) \int \mathrm{d}v\, \varphi(v) e^{ip\cdot b(v)}\, |p\rangle$$

Spherical symmetry

$E = |\mathbf{p}|$

Time-like "impact parameter"

Hawking Scattering

Quantum particle: hard

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Simplify: high energy "geometric optics" $\sim e^{iS(x)/\eta}$

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Analogous to $\hbar$

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→ Giulia's talk

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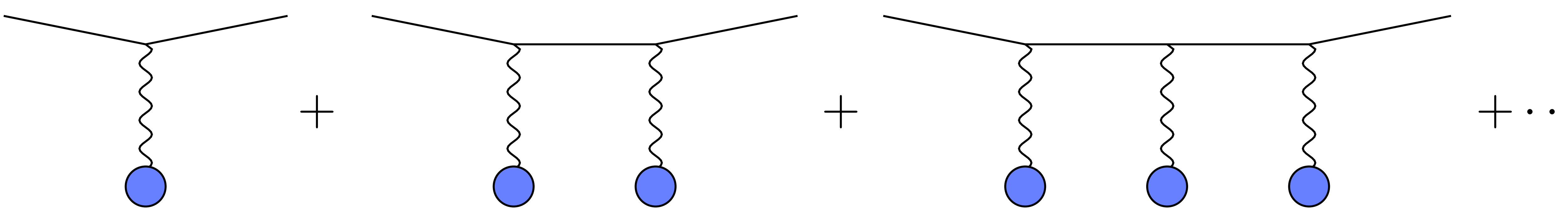
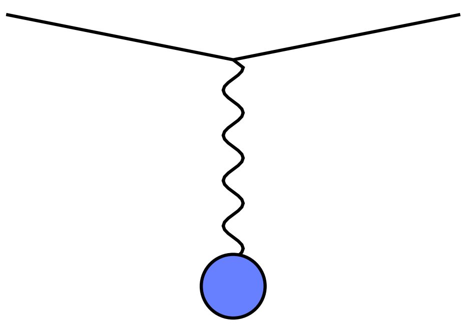
$\mathcal{A}_0^{(-1)} \sim \frac{1}{\eta}$ $\qquad$ $\mathcal{A}_1^{(-2)} \sim \frac{1}{\eta^2}$ $\qquad$ $\mathcal{A}_2^{(-3)} \sim \frac{1}{\eta^3}$



Hawking Scattering

Tree



$$S_{\text{int}} = \frac{1}{2}\int \mathrm{d}^4x\, h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

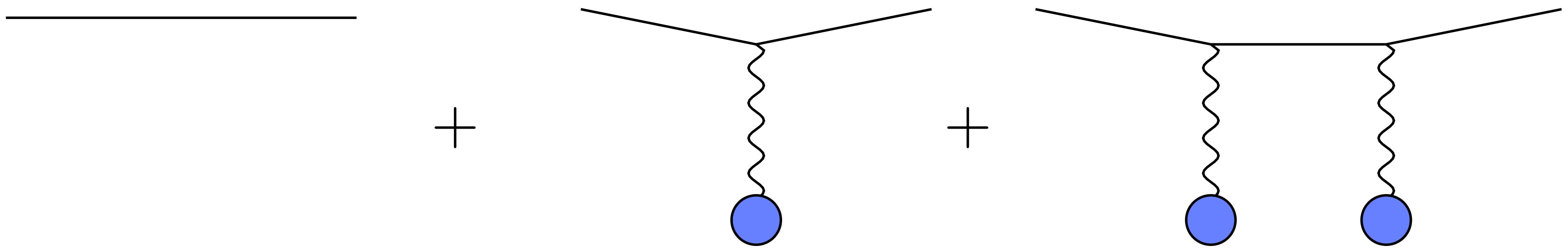
Compute using usual methods

$$\langle p'|S-1|\psi\rangle = \int \mathrm{d}v\,\varphi(v)e^{ip'\cdot b(v)}\big(-4GME'\log(-v/\mu)\big)$$

Hawking Scattering

Iterates, higher loops exponentiate

$$\langle p' | S | \psi \rangle =$$



$$= \int \mathrm{d}v \, \varphi(v) e^{ip' \cdot b(v)} \exp\big(-4iGME' \log(-v/\mu)\big)$$

Hawking Scattering

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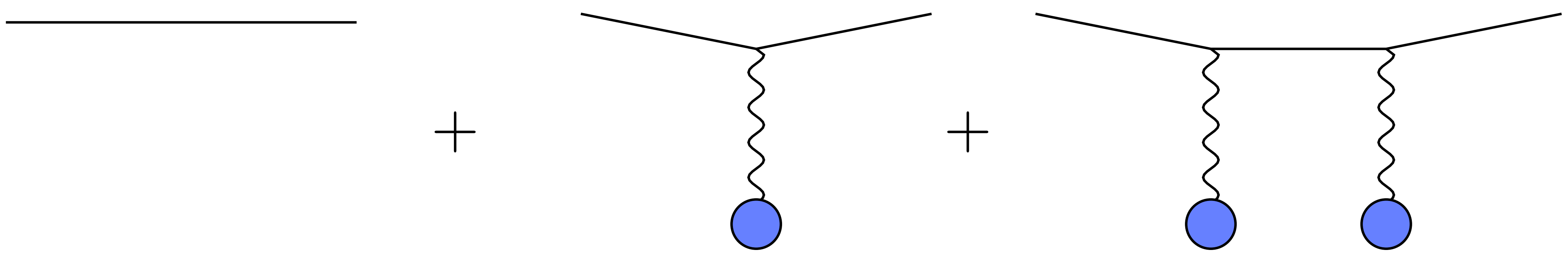
$\langle p'|S|\psi\rangle =$ 

$$= \int \mathrm{d}v\, \varphi(v) e^{ip'\cdot b(v)} \exp\big(-4iGME' \log(-v/\mu)\big)$$

Spherical energy eigenstate $|\psi\rangle = \frac{1}{4\pi}\int \mathrm{d}\Omega\, |E_0, E_0\mathbf{n}\rangle$

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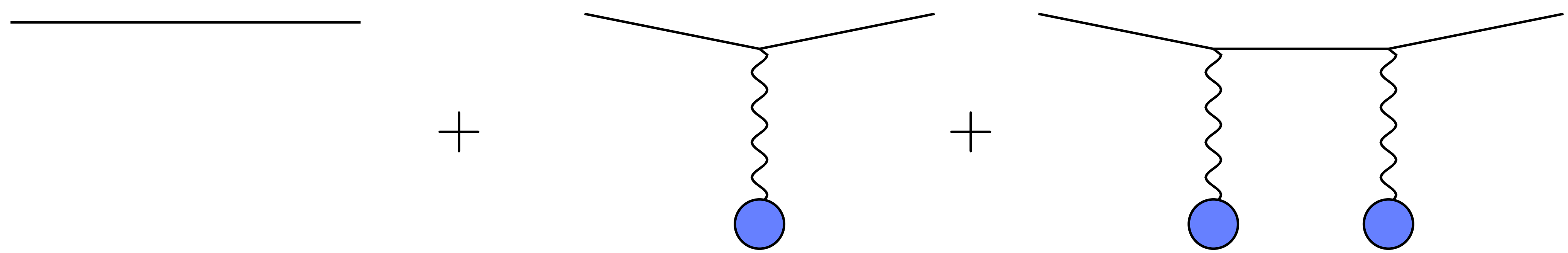

$$\mathcal{A} = \mathcal{N} \int \mathrm{d}v\, e^{i(E'-E_0)} e^{-4iGM\,E'\,\log(-v/\mu)}$$

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Hawking Scattering

Not fully inclusive

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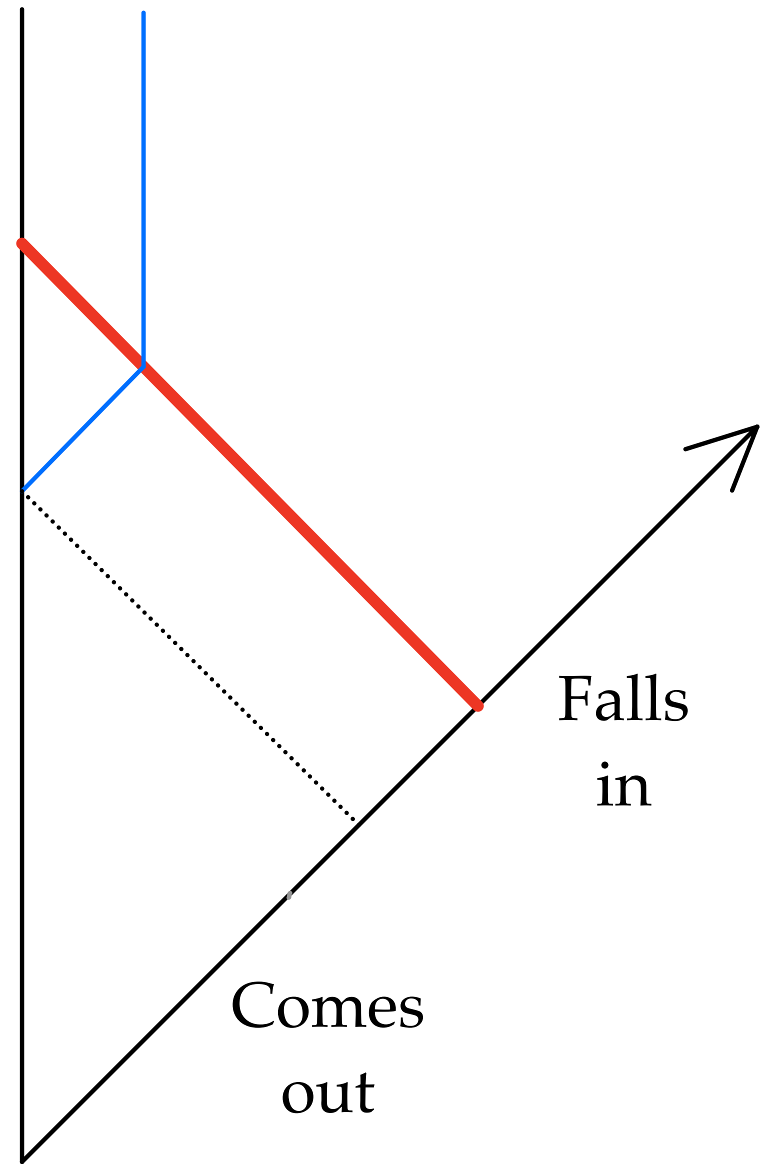

Hawking Scattering

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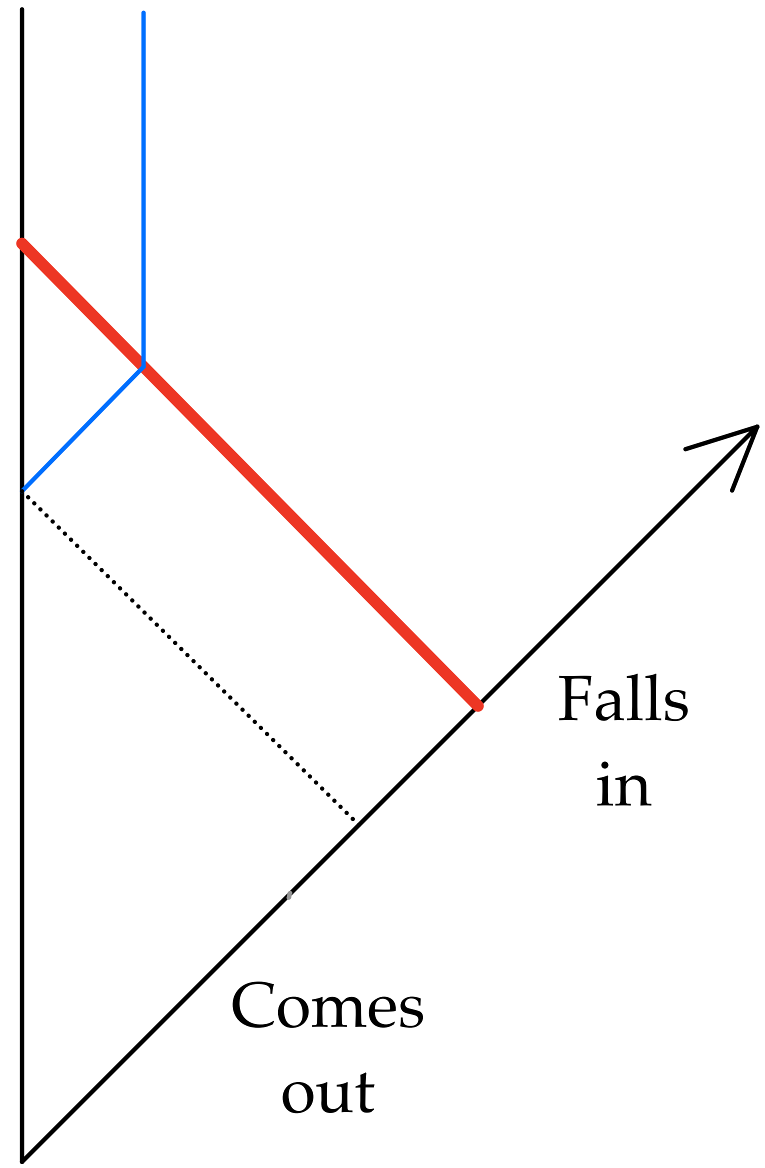
Horizon

$$\mathcal{A} = \mathcal{N} \int_{-\infty}^{0} \mathrm{d}v\, e^{i(E'-E_0)} e^{-4iGM\,E' \log(-v/\mu)}$$



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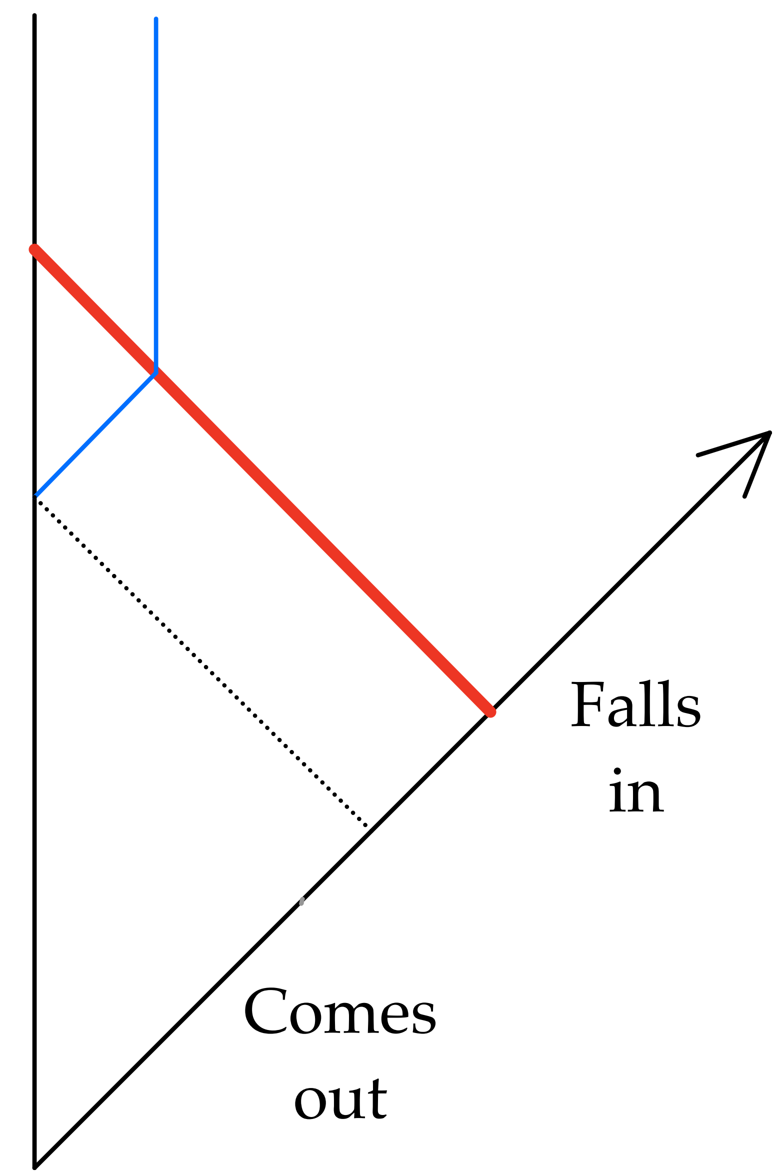
$$\boxed{\mathcal{A} = \mathcal{N} \int_{-\infty}^{0} \mathrm{d}v \, e^{i(E' - E_0)} e^{-4iGM\, E' \log(-v/\mu)}}$$

Hawking's amplitude



Pair Production

Simply cross in-state to out-state

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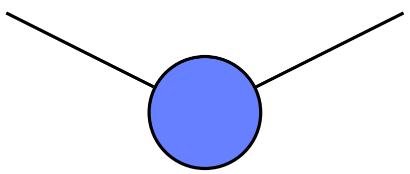

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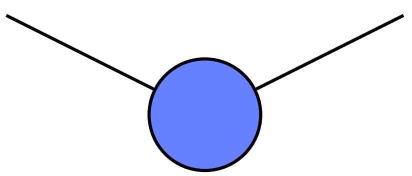
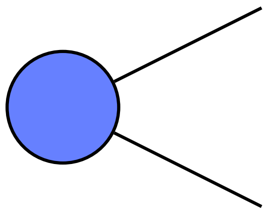


Crossing

$$\mathcal{B} = \mathcal{N} \int_{-\infty}^{0} \mathrm{d}v \, e^{i(E' + E_0)} e^{-4iGM\,E' \log(-v/\mu)}$$

$$\sim e^{-2\pi GME} \Gamma(1 - 4iGME)$$



Pair Production

Count outgoing states

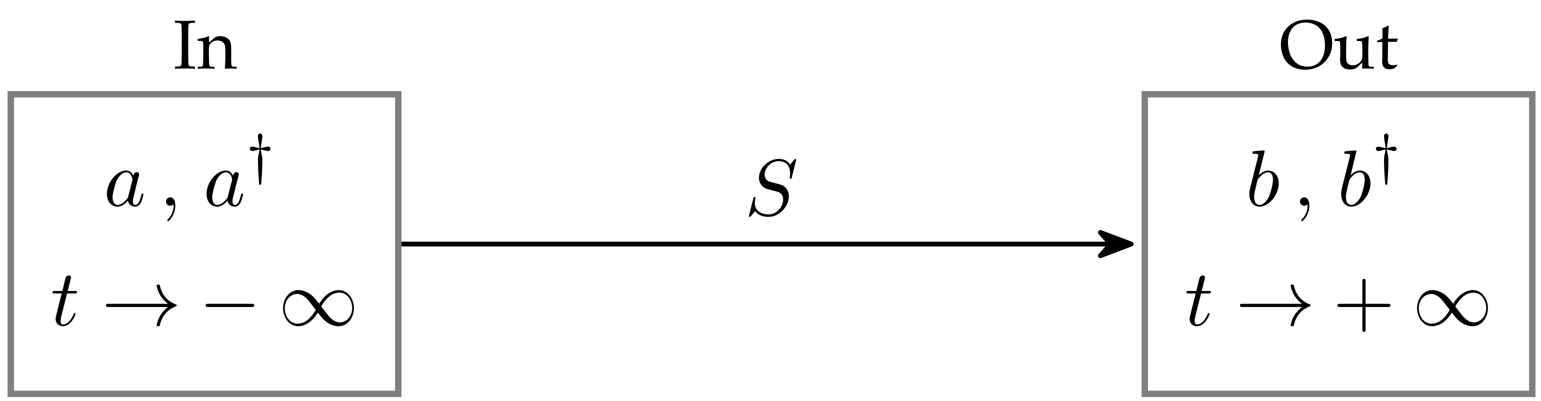
$$|\mathcal{B}|^2 = (\text{factor}) \frac{1}{e^{8\pi G M E'} - 1}$$

Thermal distribution

$$T = \frac{1}{8\pi G M}$$

Pair Production

Deeper understanding: Bogoliubov transformations



$$b(p) = S^\dagger a(p) S$$

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Exact field eom $\partial^2\phi + \partial_\mu \, h^{\mu\nu}(x)\partial_\nu\phi = 0$

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Past solutions $P(x,p)$

$$P(x,p) \xrightarrow[t\to-\infty]{} e^{-ix\cdot p}$$

$$\phi(x) = \int \mathrm{d}\Phi(p)\left[P(x,p)a(p) + \text{h.c.}\right]$$

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$$P(x,p) \xrightarrow[t\to-\infty]{} e^{-ix\cdot p}$$

Future solutions $F(x,p)$

$$F(x,p) \xrightarrow[t\to\infty]{} e^{-ix\cdot p}$$

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$$\phi(x) = \int \mathrm{d}\Phi(p)\left[P(x,p)a(p) + \text{h.c.}\right] \;=\; \phi(x) = \int \mathrm{d}\Phi(p)\left[F(x,p)b(p) + \text{h.c.}\right]$$

Pair Production

$$P \sim \Sigma\left(AF + B\bar{F}\right)$$

F is a basis

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$$b(k) = S^\dagger a(k) S = \int \mathrm{d}\Phi(p) \big(A(k,p) a(p) + B(k,p) a^\dagger(p) \big)$$

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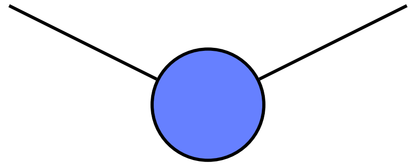
Bogoliubov coefficients = (generalised) amplitudes

Vaidya vacuum

$$A(k,p) = \langle\Omega|S^\dagger a(k) S a^\dagger(p)|\Omega\rangle$$



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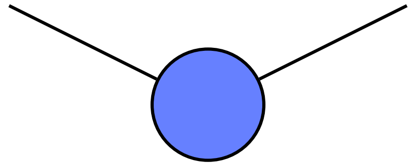

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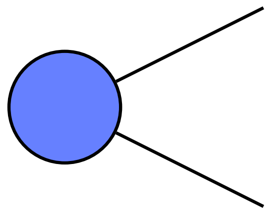

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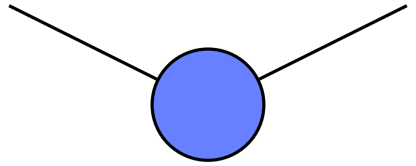

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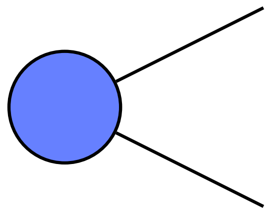

Perturbative expansion: Lippmann-Schwinger series

→ Giulia's talk

Observable

Count number of particles in future

$$\langle\Omega|S^\dagger a^\dagger(p)a(p)S|\Omega\rangle = \int \mathrm{d}\Phi(k)\,\bar{B}(p,k)B(p,k)$$

Differential number operator

$$\mathrm{d}n = |B(k,p)|^2 \mathrm{d}\Phi(k,p)$$

$$|\mathcal{B}|^2 = (\text{factor})\frac{1}{e^{8\pi GME'}-1}$$

Higher Orders

$$\exp\left(i\frac{S}{\eta}\right) = \exp\left(i\frac{S^{(0)} + S^{(1)} + \cdots}{\eta}\right)$$

Also computed $S^{(1)}$

$$S^{(0)} + S^{(1)} = -4GME\log\big(-v/\mu\big) - \frac{16G^2M^2E}{v}$$

$$\simeq -4GME\log\big(-(v+4GM)/\mu\big)$$

Higher Orders

Analog of PM expansion

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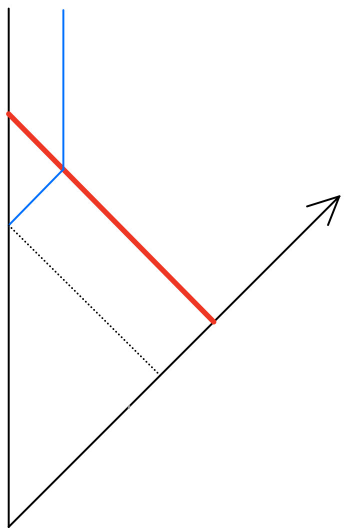

Finite radius of horizon

Higher Orders

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Finite size effect

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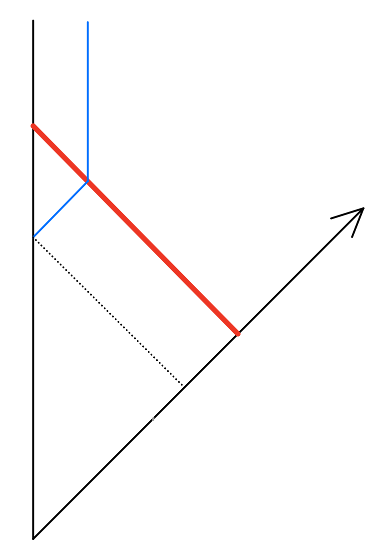

Finite radius of horizon

Outlook

We're tooled up on this problem

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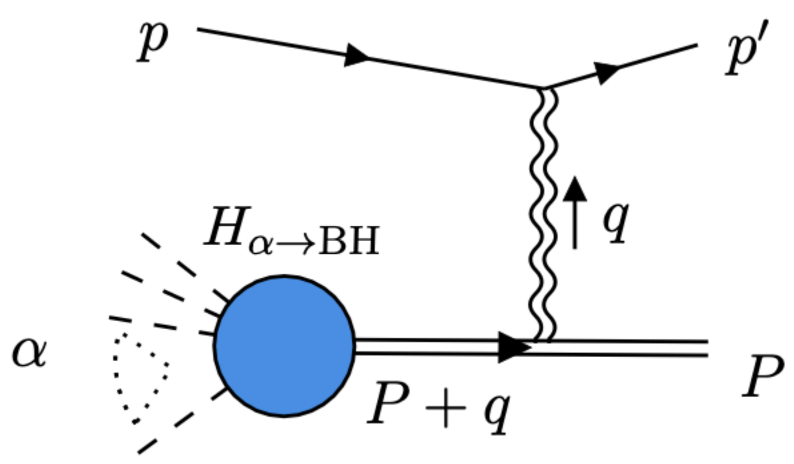

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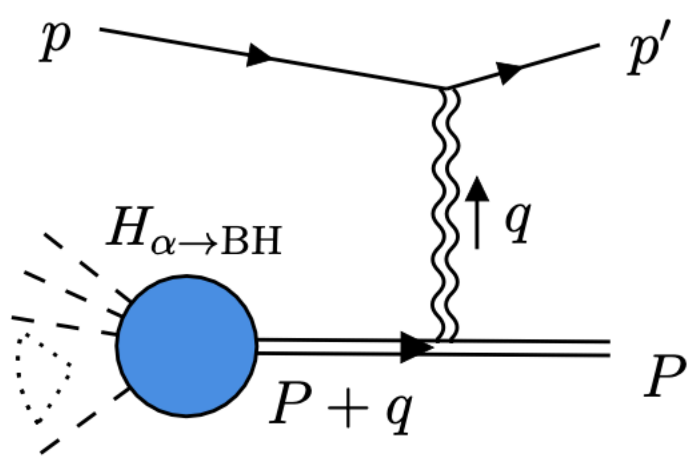

Infalling radiation:
coherent state

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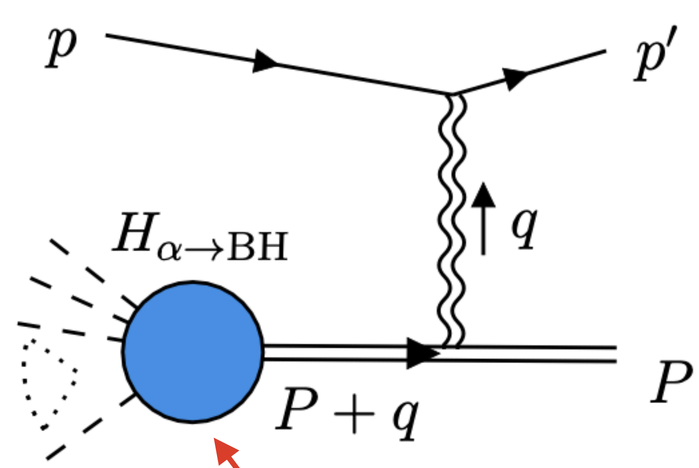

Infalling radiation:
coherent state

Matter to BH:
form factor?

Outlook

We’re tooled up on this problem

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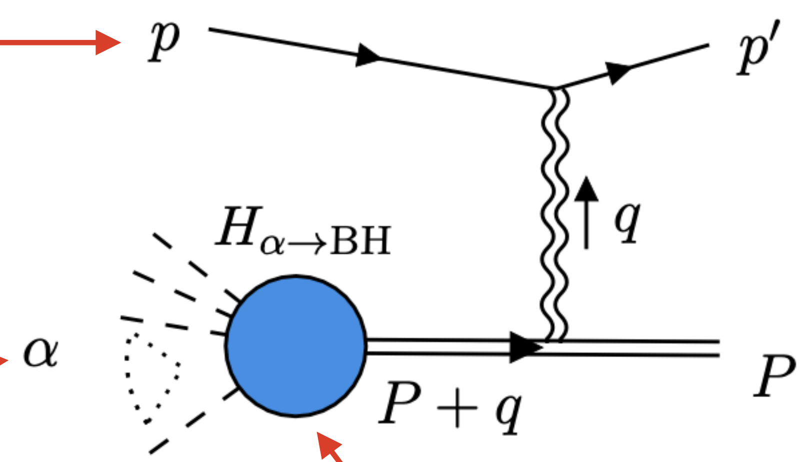
Pair production: crossing
One pair enough
(Squeezing)



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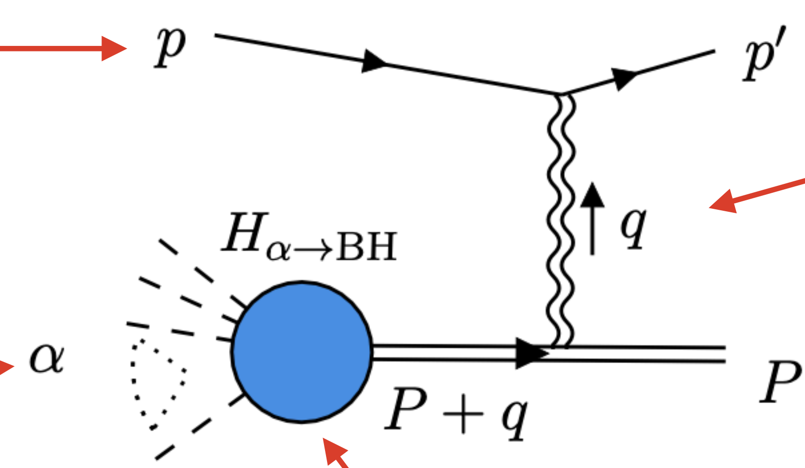
One pair enough
(Squeezing)



Single graviton exchange: enough

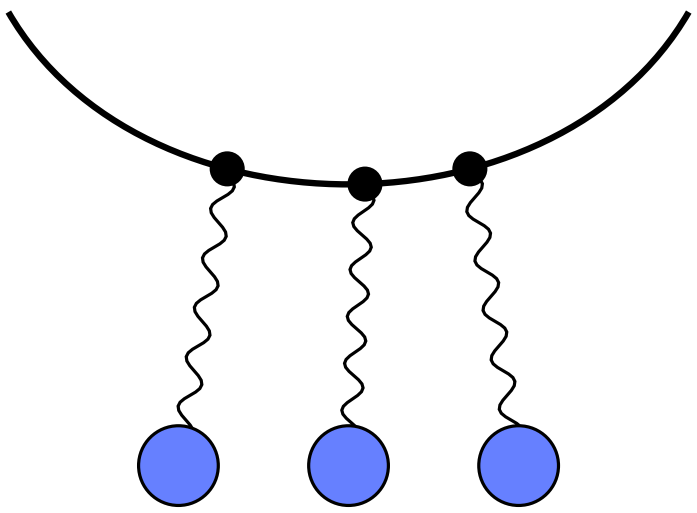
Infalling radiation: coherent state

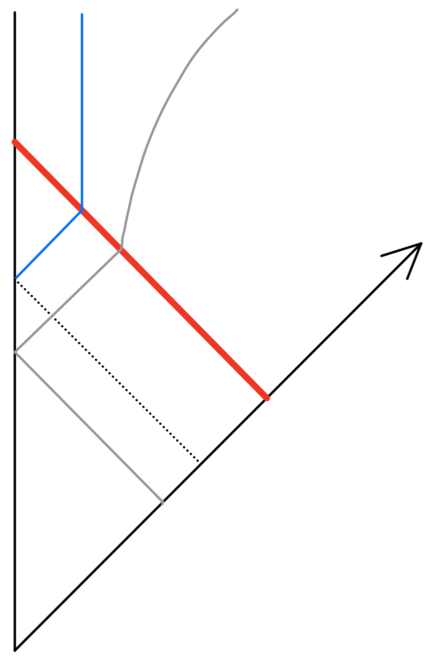
Matter to BH: form factor?

Outlook

We're tooled up on this problem

- Remove background?
- Non-local pair production?

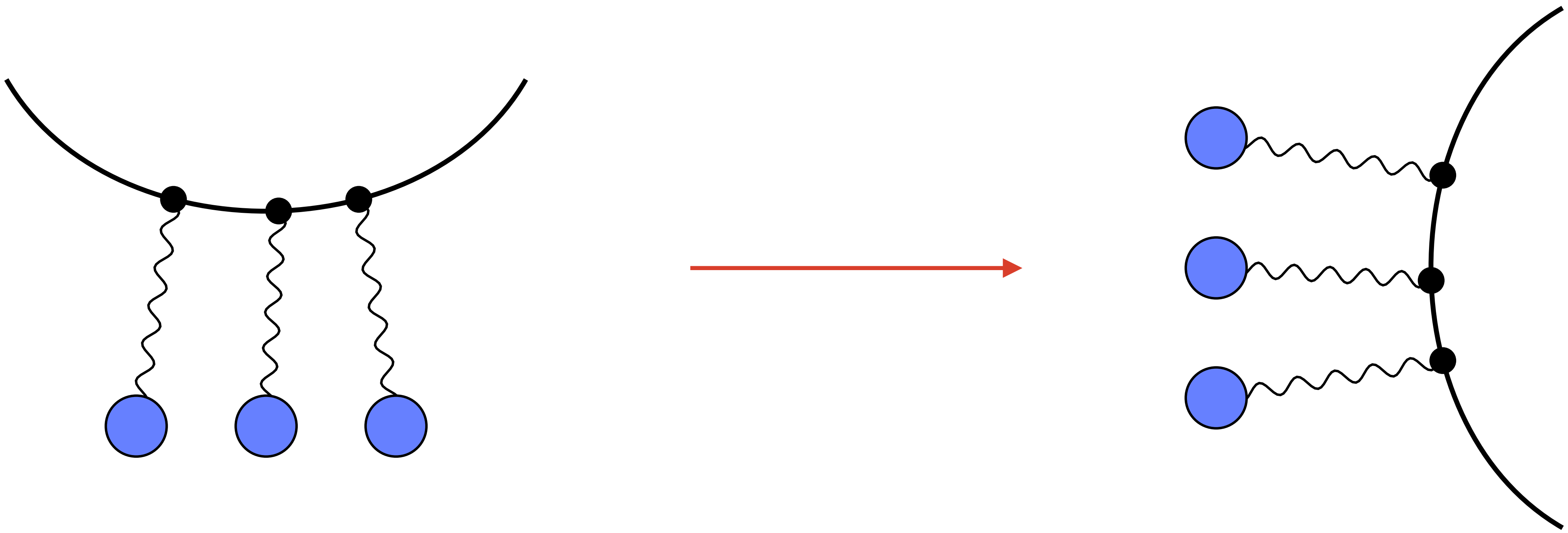




Outlook

We're tooled up on this problem

- Remove background?
- Non-local pair production?



Outlook

We're tooled up on this problem

- Remove background?
- Non-local pair production - tunneling? *Parikh & Wilczek*
- Exponentiate with more care: stationary phase
- Radiation reaction?
- Page curve?