ARCHIVES GEORGES LEMAÎTRE

UCLouvain, Louvain-la-Neuve

The Origin(s) and Meaning(s) of the Primeval Atom Hypothesis: Quantum Physics meets Lemaître's Cosmology

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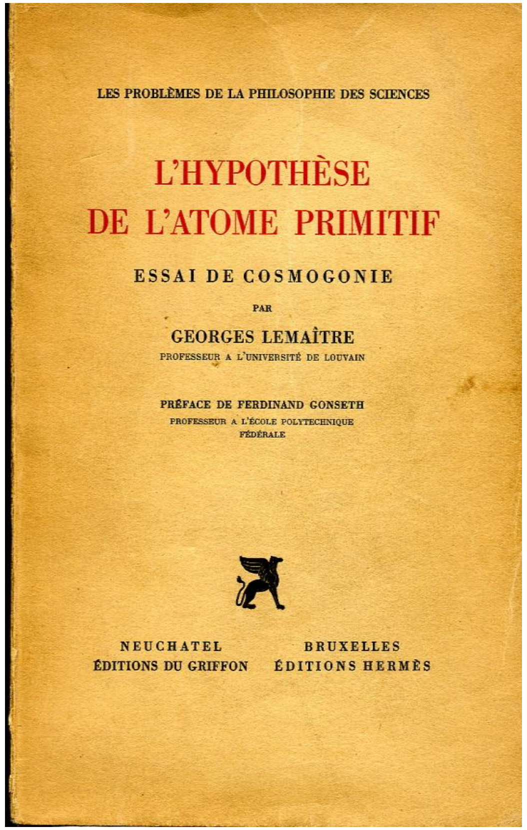

LES PROBLÈMES DE LA PHILOSOPHIE DES SCIENCES

L'HYPOTHÈSE DE L'ATOME PRIMITIF

ESSAI DE COSMOGONIE

PAR

GEORGES LEMAÎTRE

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PRÉFACE DE FERDINAND GONSETH

PROFESSEUR A L'ÉCOLE POLYTECHNIQUE FÉDÉRALE

NEUCHATEL ÉDITIONS DU GRIFFON

BRUXELLES ÉDITIONS HERMÈS

Caricature made by Xavier de Callataÿ (student of Lemaître) copyright A. de Callataÿ

Primeval Atom Hypothesis as you see here was a crucial node in the intellectual network of Lemaître.

I want to answer two questions:

1) From where did this Hypothesis come? **2 sources**!

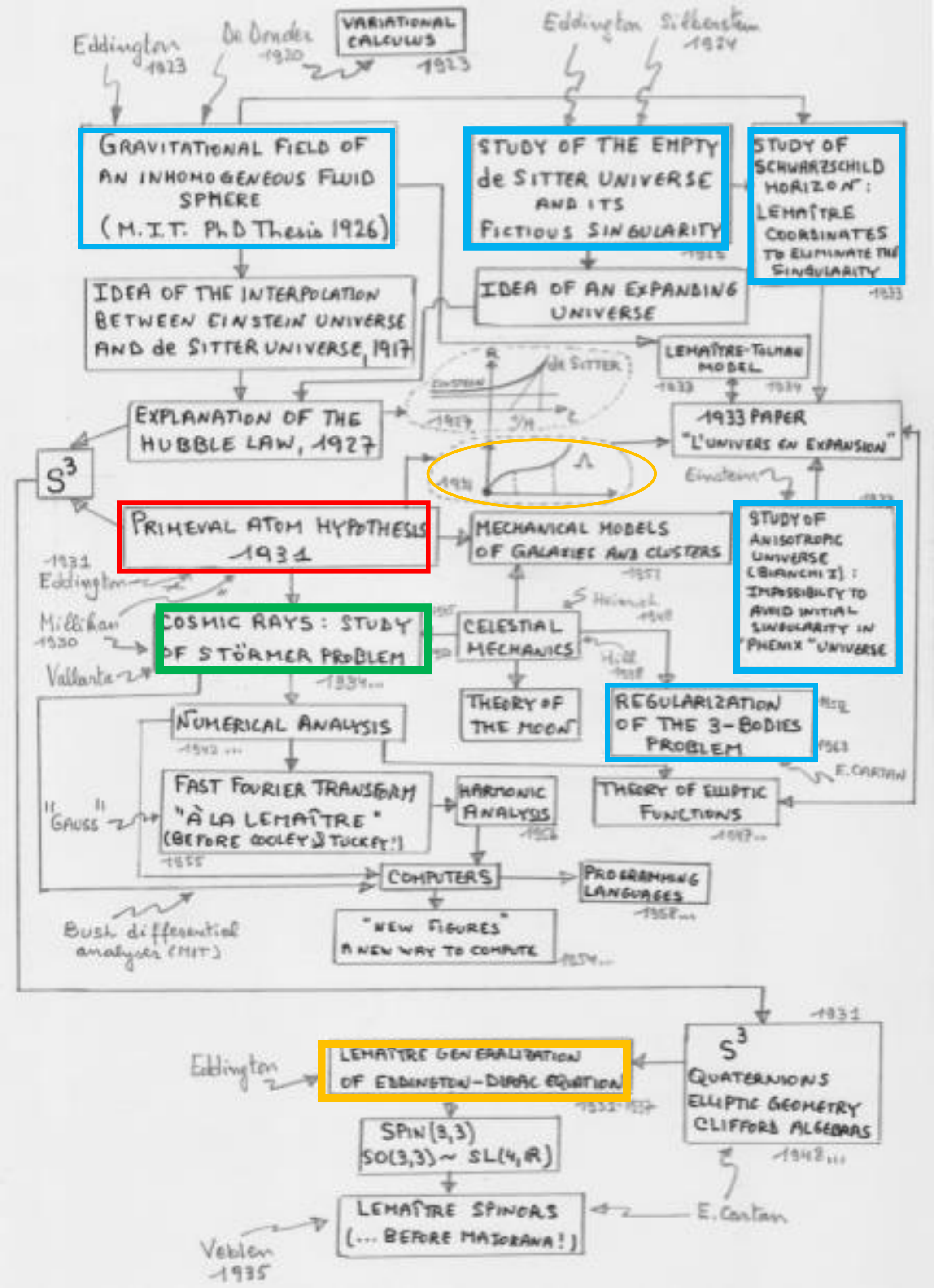
2) What is the nature of the Primeval Atom? An ambiguous quantum image! **2 interpretations**!

The **Primeval Atom Hypothesis** is connected to three main subjects of Lemaître's works:

1) The study of **Singularities** is one of the major theme of Lemaître's work (not only in Cosmology but also in Celestial Mechanics and…)

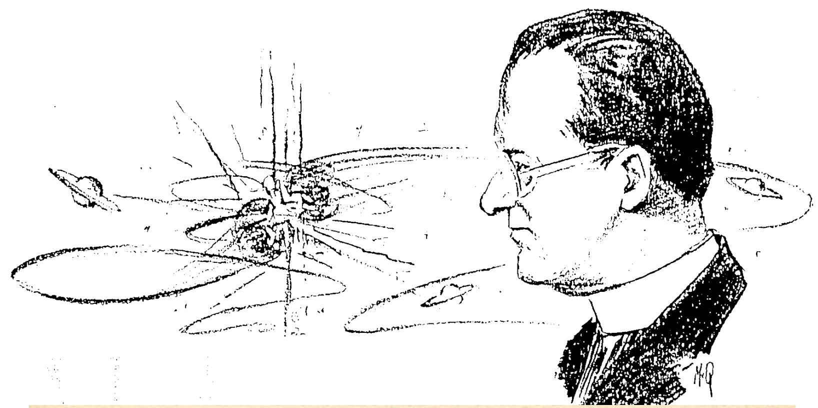
2) The study of **quantum theory**… à la Eddington!

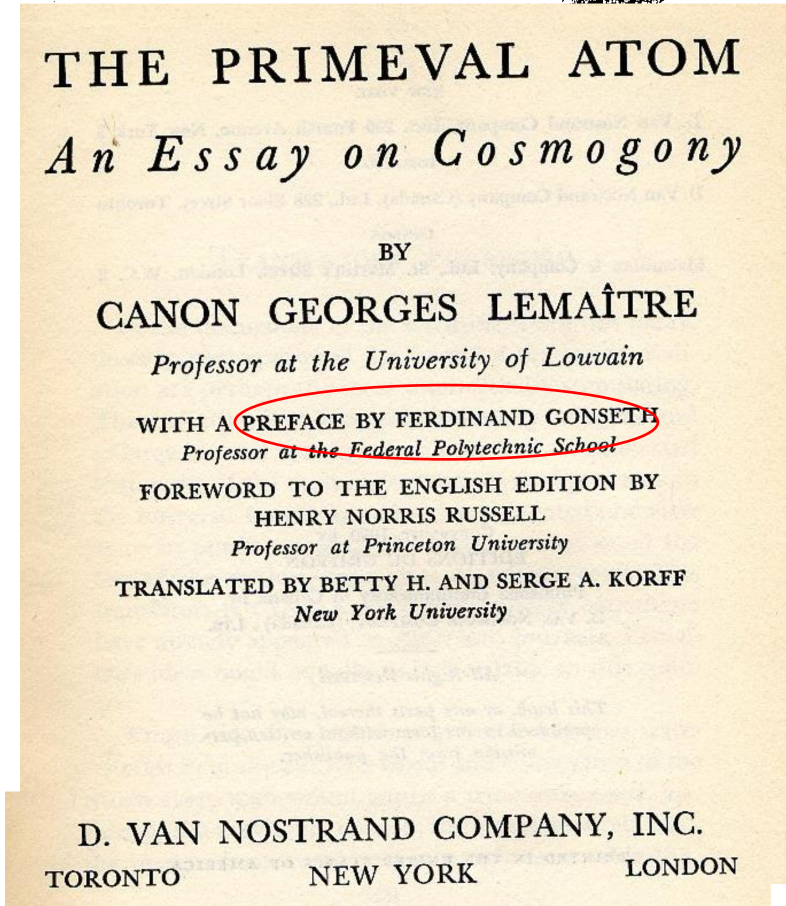
3) The study of **Cosmic rays**



The Universe and Abbé Lemaître

What is the historical origin of the Primeval Atom Hypothesis (1931-1945-46)?





THE PRIMEVAL ATOM

An Essay on Cosmogony

BY

CANON GEORGES LEMAÎTRE

Professor at the University of Louvain

WITH A PREFACE BY FERDINAND GONSETH
Professor at the Federal Polytechnic School

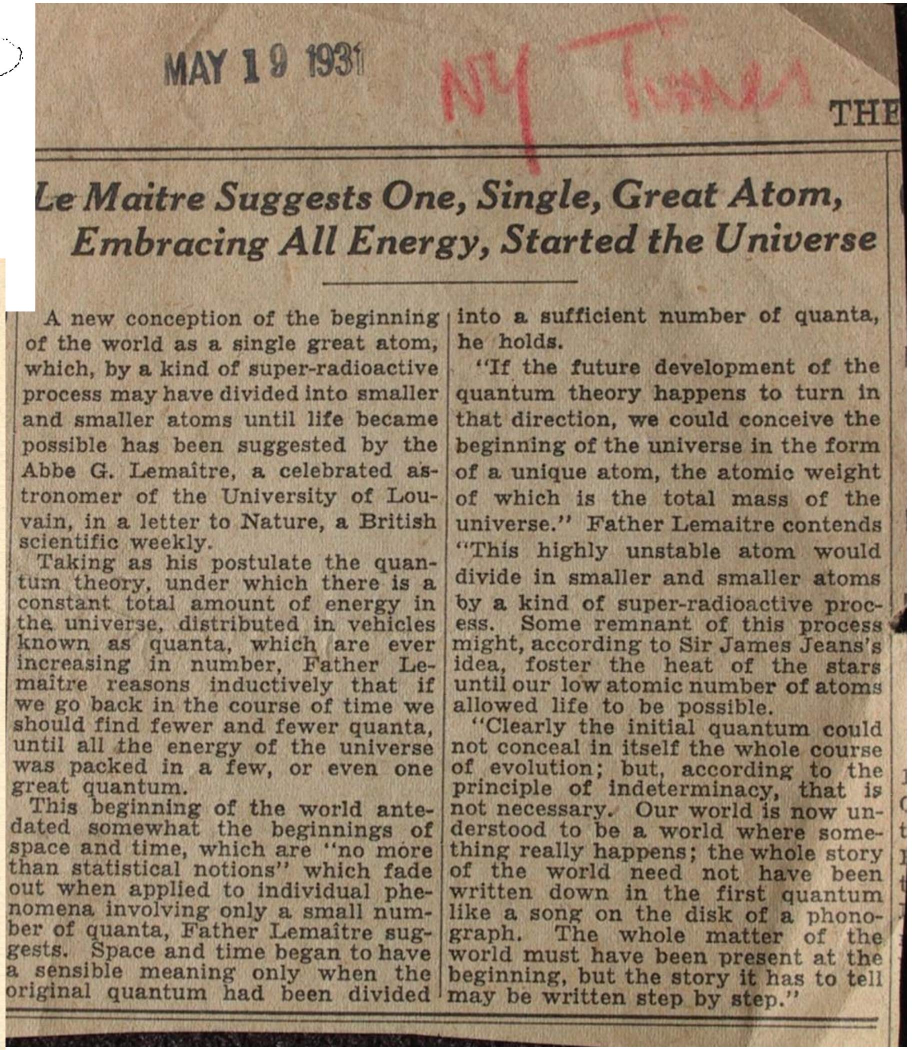
FOREWORD TO THE ENGLISH EDITION BY
HENRY NORRIS RUSSELL
Professor at Princeton University

TRANSLATED BY BETTY H. AND SERGE A. KORFF
New York University

D. VAN NOSTRAND COMPANY, INC.
TORONTO NEW YORK LONDON

1945-1946…

1931



MAY 19 1931 NY Times

THE

Le Maitre Suggests One, Single, Great Atom, Embracing All Energy, Started the Universe

A new conception of the beginning of the world as a single great atom, which, by a kind of super-radioactive process may have divided into smaller and smaller atoms until life became possible has been suggested by the Abbe G. Lemaitre, a celebrated astronomer of the University of Louvain, in a letter to Nature, a British scientific weekly.

Taking as his postulate the quantum theory, under which there is a constant total amount of energy in the universe, distributed in vehicles known as quanta, which are ever increasing in number, Father Lemaitre reasons inductively that if we go back in the course of time we should find fewer and fewer quanta, until all the energy of the universe was packed in a few, or even one great quantum.

This beginning of the world antedated somewhat the beginnings of space and time, which are "no more than statistical notions" which fade out when applied to individual phenomena involving only a small number of quanta, Father Lemaitre suggests. Space and time began to have a sensible meaning only when the original quantum had been divided into a sufficient number of quanta, he holds.

"If the future development of the quantum theory happens to turn in that direction, we could conceive the beginning of the universe in the form of a unique atom, the atomic weight of which is the total mass of the universe." Father Lemaitre contends "This highly unstable atom would divide in smaller and smaller atoms by a kind of super-radioactive process. Some remnant of this process might, according to Sir James Jeans's idea, foster the heat of the stars until our low atomic number of atoms allowed life to be possible.

"Clearly the initial quantum could not conceal in itself the whole course of evolution; but, according to the principle of indeterminacy, that is not necessary. Our world is now understood to be a world where something really happens; the whole story of the world need not have been written down in the first quantum like a song on the disk of a phonograph. The whole matter of the world must have been present at the beginning, but the story it has to tell may be written step by step."

ARCHIVES GEORGES LEMAÎTRE

1 The unknown source of the Primeval Atom Hypothesis

At the beginning was a quantum of radiation…giving rise to the matter in the expanding universe

Context:

in 1930 Lemaître was working on the nature of Cosmic Rays a major subject in all his life! He had contacts with Auguste Piccard, Millikan, …

The question of the origin of matter leads Lemaître to the idea that this origin is (quanta of) radiation

Matter is coming out of the radiation, out of « the light »: At the beginning was (quanta of) radiation… this is the first stimulation leading to the idea of the Primeval Atom…

1930 : One year before stating the Primeval Atom Hypothesis Lemaître proposed the idea that all the matter in the universe is coming from radiation… but of course nearly nobody read this marginal text published in French…

> **« One could admit that the light was the original state of the matter and that all the matter condensed in stars was formed by the process proposed by Millikan »**
>
> « On pourrait admettre que la lumière a été l'état originel de la matière et que toute la matière condensée en étoiles s'est formée par le processus proposé par Millikan ».
>
> (G. Lemaître, «L'hypothèse de Millikan-Cameron dans un univers de rayon variable» in *Comptes rendus du Congrès national des sciences organisé par la fédération belge des sociétés scientifiques. Bruxelles, 29 Juin-2juillet 1930, Bruxelles, Fédération belge des sociétés Scientifiques, 1930, p. 180)*

At the beginning was (quanta of) radiation… but from where is this idea coming? From his interest in cosmic rays and from his study of a strange idea of Millikan about the nature of Cosmis Rays…

MILLIKAN – CAMERON HYPOTHESIS (1928) : in a STATIC UNIVERSE

(concerning an attempt to characterize the nature of cosmic rays and to avoid the 2d Principle))

1 ELECTROMAGNETIC RAYS

2 FREE PROTONS AND ELECTRONS

3 PROTONS BOUNDED IN NUCLEI (→ mass defect = 1 photon)

4 MATTER (stars,…) « COSMIC RAYS »

Cfr Helge Kragh, *Matter and Spirit in the Universe*, Imperial College Press, 2004, pp. 88-92

LEMAÎTRE HYPOTHESIS (1930) : in an EXPANDING UNIVERSE

(He kept only the idea that radiation produces matter)

Lemaître studied Quantum physics at the MIT (1924-25)

1 « **LIGHT** » = « INITIAL STATE » **Quanta of radiation**

2 FREE PROTONS AND ELECTRONS

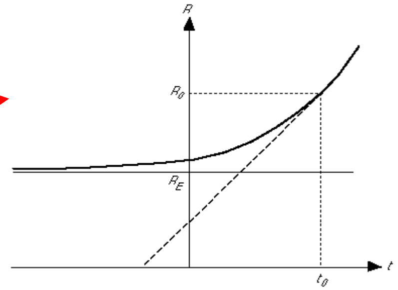
3 PROTONS BOUNDED IN NUCLEI (→ mass defect = 1 photon)

4 MATTER « COSMIC RAYS »

« Toute la matière de l'univers »

From where come the idea of a primeval Quantum?

In 1927 Lemaître had already expressed the idea that Radiation was the primeval state of the Universe…



At the end of his 1927 famous paper: something strange!

> "It remains to find the cause of the expansion of the universe. We have seen that the pressure of radiation does work during the expansion. This seems to suggest that the expansion has been set up by the radiation itself. In a static universe, light emitted by matter travels round space, comes back to its starting point and accumulates indefinitely. It seems that this may be the origin of the velocity of expansion R′/R which Einstein assumed to be zero and which in our interpretation is observed as the radial velocity of extragalactic nebulæ."

G. Lemaître, *A Homogeneous Universe of Constant Mass and Increasing Radius accounting for the Radial Velocity of Extra–Galactic Nebulæ* (translated by J.-P. Luminet)

Radiation → Matter → local pressure change (« stagnation ») → unstability → expansion…

The Unstability of Einstein universe and the effect of pressure change (« L'Univers en expansion », p. 18)
Cfr Lemaître's 1933 study of Inhomogeneous universes (Tolman-Lemaître universe; cfr Krasinski)
"L'univers en expansion" (séance du 3 mai 1933, 1re section), *Annales de la Société Scientifique de Bruxelles*, 1933, n°2, pp. 51-85.

Later a talk at an important meeting of the *British association for the advancement of science* : 29 September 1931 sheds some light on the meaning of this conclusion of the 1927 paper… and of the Primeval Atom…

« **A World full of radiation begins to expand as soon as radiation can turn into matter…** »
« …un monde plein de radiation commence à se dilater dès que la radiation peut se transformer en matière » (p. 16)

« **A complete revision of our cosmogonic hypothesis is necessary…We need a fireworks theory of evolution** »
« Une révision complète de nos hypothèses cosmogoniques est nécessaire…Nous avons besoin d'une théorie de l'évolution en « feu d'artifice » (p. 17)

At the beginning were quanta of radiation …. was a quantum of radiation…and the universe began to expand due to the radiation-matter transformation process : **This is the first root of the Primeval Atom Hypothesis**

(today small density of matter => the condensation process is recent => age of the universe several billions of years)

"The evolution of the universe : Contributions to a British association discussion on the evolution of the universe" (communications de J. Jeans, G. Lemaître, W. de Sitter, A.Eddington, R.A.Millikan, E.A.Milne, J.C. Smuts, E.W. Barnes et O. Lodge ; discussion tenue le 29 septembre 1931 dans la section des sciences mathématiques et physiques du colloque du centenaire de la *British association for the advancement of science*), *Nature : Supplément,* t. CXXVIII, 24 octobre 1931, n°3234, pp. 699-722 (communication de G.Lemaître, pp. 704-706).
"The evolution of the universe : Discussion", in *British association for the advancement of science : Report of the centenary meeting. London, 1931, september* 23-30, London, Office of the British Association, 1932, pp. 573-610 (talk of Lemaître, pp. 605- 610).
"Discussion" (traduction de l'intervention de G.Lemaître) in *Discussion sur l'évolution de l'univers* (traduction et avant-propos de P.Couderc), Paris, Gauthier-Villars, 1933, pp. 15-22.

2. The same year 1931, just before stating the Primeval Atom Hypothesis, Lemaître was working on Quantum theory

1) Heisenberg indeterminacy principle in electromagnetism (29 January 1931)

2) Eddington fundamental theory and Dirac-Eddington equation (22 April 1931)

This lead him to formulate his cosmological ideas about the origin of matter (and of the cosmic rays) using quantum notions… From radiation to Quanta and to a unique Quantum…

Lemaître's work on Heisenberg relations

G. Lemaître, "L'indétermination de la loi de Coulomb" (séance du **29 janvier 1931**, 2e section), *Annales de la Société Scientifique de Bruxelles,* série B : *Sciences Physiques et Naturelles,* 1re partie : Comptes rendus des Séances, t. LI, 1931, pp. 12-16.

L'INDÉTERMINATION DE LA LOI DE COULOMB

Note de M. l'Abbé G. LEMAITRE

Heisenberg [3] a montré que le principe d'indétermination du mouvement des particules entraîne une indétermination du champ électromagnétique.

[3] Die physikalischen Prinzipien der Quantentheorie, page 36 sq.

$$\Delta E_x \Delta H_z = \frac{hc}{l^4 T} = \frac{h}{l^3 t},$$

$$\Delta \int \sqrt{F_{\mu\nu} F^{\nu\sigma} F_{\sigma\tau} F^{\tau\mu}} \sqrt{-g}\, dx\, dy\, dz\, dt > 4\pi h$$

G. Lemaître, "The uncertainty of the electromagnetic field of a particle", *The Physical Review,* 2e série, t. XLIII, 15 janvier 1933, n°2, p. 148.

One month before the publication in *Nature* Lemaître publish a paper on Dirac equation (April 1931)

Extrait des *Annales de la Société scientifique de Bruxelles*.
Tome LI, série B, première partie, *Comptes rendus des séances*, p. 83.
Session du 22 avril 1931. Deuxième Section.

SUR L'INTERPRÉTATION D'EDDINGTON DE L'ÉQUATION DE DIRAC

Note de M. l'Abbé G. LEMAÎTRE

1. INTRODUCTION.

L'équation de Dirac et l'invariance de cette équation pour une transformation orthogonale (transformation de Lorentz) a donné lieu à une suite de travaux (1) qui ont mis progressivement en valeur la symétrie profonde de ces équations.

L'équation de Dirac peut s'écrire

$$H\psi = 0 \qquad (1)$$

où ψ représente un ensemble de quatre fonctions $\psi^1, \psi^2, \psi^3, \psi^4$ et H une matrice de quatre lignes et de quatre colonnes. Elle est donc équivalente aux quatre équations

$$\sum_{\nu=1}^{4} H_{\mu\nu}\,\psi^{\nu} = 0 \qquad (\mu = 1, 2, 3, 4) \qquad (2)$$

La matrice H s'exprime linéairement au moyen des moments p_1, p_2, p_3 et p_0 et de quatre matrices $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ par la relation

$$H = i\gamma_1 p_1 + i\gamma_2 p_2 + i\gamma_3 p_3 + \gamma_4 p_0 + mc \qquad (3)$$

les matrices γ doivent vérifier les relations

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu} \qquad (\mu, \nu = 1, 2, 3, 4) \qquad (4)$$

où $\delta_{\mu\nu}$ est le symbole de Kronecker égal à un ou à zéro suivant que μ est égal à ν ou en diffère.

Dirac a montré que les relations (4) sont vérifiées pour certaines matrices dont il donne l'expression explicite, soit $\bar{\gamma}_\mu$, et que toute solution de ces relations peut s'obtenir à partir de cette solution particulière par une transformation canonique, c'est-à-dire par une relation de la forme

$$\gamma_\mu = k\,\bar{\gamma}_\mu\,k' \qquad (5)$$

où k et k' sont deux matrices inverses, c.-à-d. telles que

$$k k' = k' k = 1. \qquad (6)$$

(1) P. DIRAC. — *Proc. R. S. L.*, vol. 117, p. 610.
» *ibid.* vol. 118, p. 351.
C. DARWIN. *ibid.* vol. 118, p. 654.
SIR A. EDDINGTON. *ibid.* vol. 121, p. 524.
» *ibid.* vol. 122, p. 358.
» *ibid.* vol. 126, p. 696.

DIRAC 1928 : $H\Psi = 0$

$$H = i\gamma^1 p_1 + i\gamma^2 p_2 + i\gamma^3 p_3 + \gamma^0 p_0 + mc$$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\delta^{\mu\nu}$$

+++ -

SO(3,1)

EDDINGTON 1928 :

$$\gamma^1 = B_3A_1 \quad \gamma^2 = B_3A_2 \quad \gamma^3 = B_3A_3 \quad \gamma^0 = iB_3B_1 \text{ ou } iB_3B_2$$

$$\boxed{\begin{array}{lll} A_1^2 = A_2^2 = 1 & B_1^2 = B_2^2 = 1 & A_iB_j = B_jA_i \\ A_1A_2 = -A_2A_1 & B_1B_2 = -B_2B_1 & \\ A_1A_2 = iA_3 & B_1B_2 = iB_3 & \end{array}}$$

$$-iB_3 \cdot H\psi = (A_1p_1 + A_2p_2 + A_3p_3 + B_1p_0 - imcB_3)\psi = 0$$

+++ - -

SO(3,2)=Sp(4,R)

LEMAÎTRE 1931 :

$$T\psi = (A_1p_1 + A_2p_2 + A_3p_3 + B_1q_1 + \underbrace{B_2q_2 + B_3q_3}\,)\psi = 0$$

$$\begin{cases} \psi \longmapsto \psi' = K\psi \\ T \longmapsto T' = (K^{-1})^{t}\,T\,K^{-1} \end{cases} \qquad \begin{array}{ll} K = \exp(A_iA_j\theta) \quad \exp(B_iB_j\theta) \quad i \neq j & 6 \\ K = \exp(A_iB_j\theta) & 9 \end{array}$$

+++ - - -

6+9 =15

$$\det(T) = p_1^2 + p_2^2 + p_3^2 - q_1^2 - q_2^2 - q_3^2$$

SO(3,3) = SL(4,R)

$Cl(3,2) = R(4)+R(4) = Cl_{even}(3,3)$
which contains Spin(3,3)
Majorana Spinor before Majorana (1937)

COMMENTARII
Vol. II - N. 11
pag. 1-20

PONTIFICIA ACADEMIA SCIENTIARVM

THE SCIENTIFIC WORK OF GEORGES LEMAITRE

P. A. M. DIRAC
Pontifical Academician

SVMMARIVM — Agitur de publico praeconio quod Auctor — cum Sessio Plenaria Academiae haberetur — die 25 Aprilis 1968 recitavit.

At the time when LEMAÎTRE was beginning his career, science and philosophy had been overwhelmed by the principle of relativity, which had swept away long-cherished modes of thought and provided an entirely new outlook to dominate man's view of nature. LEMAÎTRE entered into the new discipline and soon began making contributions to it.

One of the main points of discussion at that time was simultaneity, which had lost its absolute significance. LEMAÎTRE considered a rigid solid, moving generally in curved space-time, and examined the problem of introducing a standard of simultaneity inside the solid, referring to its motion [1]. Two neighbouring points of space-time are defined to be simultaneous if the line joining them is orthogonal to the world-

Paper presented on April 25th, 1968, during the Plenary Session of the Pontifical Academy of Sciences.

His contribution is described at length by Dirac in his paper on Lemaître' scientific legacy.

transformations orthogonales à 3 + 1 dimensions de p_1, p_2, p_3, p_0.
Enfin Eddington a montré comment les matrices γ peuvent être choisies (de plusieurs manières) parmi quinze matrices : six matrices antisymétriques et neuf matrices symétriques. Ces quinze matrices forment avec la matrice unité un groupe susceptible de seize « rotations » qui joue un rôle essentiel dans les développements ultérieurs par lesquels il essaie de déterminer l'équation quantique pour deux électrons et de justifier la relation numérique

$$\frac{hc}{2\pi e^2} = 137. \qquad (7)$$

Cfr C .W . Kilmister, *Eddington's Search for a Fundamental Theory*, Cambridge University Press, 1994.

Lemaître was invited by O. Veblen in 1935 to Princeton to give lectures on Spinors…

G. Lemaître and the Eddington Spinor theory 22 avril 1931…

The hope of describing of the universe using Eddington fundamental theory

Lemaître kep in mind several (strange) aspects of this theory: number of elementary particles…
…and minimal volume of the universe…

18. EDDINGTON'S QUANTUM THEORY.

The next question which remains open is to understand how the total mass of the universe is so adjusted to the cosmical constant so that the condition for an Einstein Universe is nearly fulfilled, the small difference being in favour of the everexpanding type of motion.

Relativity theory is unable to give any answer to this question. There is no doubt that fundamental questions about the cosmical constant depend on quantum theory.

The adaptation of quantum theory to the exigency of the relativity postulate is yet in a very unsatisfactory stage, some approximate solutions have been presented but non converging integrals appears anywhere.

There is no doubt that the presence of infinities in physical theory is a ~~marked~~ clear indication of errors, at least partial ones.

There has been ~~at least one~~ a theory which do not play with infinities and which has lead his author, sir Arthur Eddington, to most astonishing relations between physical constants

Nevertheless it is the first serious attempt to give a theoretical basis to the idea of Mach that the mass of the individual parti-cles is determined by the presence of all the other particles in the universe, and is so able to predict from the observed mass of the electron what must be the total mass of the universe. The result of such deduction is in perfect agreement with the results explained above.

In his book of "Relativity Theory of Protons and Electrons" Sir Arthur Eddington obtain some results which would give the solution of the question raised at the beginning of this section.

Investigating the conditions for an Einstein Universe from the point of view of his quantum theory, ~~Eddington~~ he finds that an Einstein universe (formed of hydrogen) could only be possible if the temperature is zero.

Therefore as the temperature of the universe is not zero the energy (or mass) of the universe would be somewhat greater than the value required for equilibrium and the constant α of section 7 would be but slightly greater than one, the universe being therefore, as required by the foregoing discussion, of the everexpanding type of motion with a long period of slow motion through the equilibrium configuration, giving full opportunity for the local instability due to fluctuations of density to come into play by producing nebulae and stars.

The ratio p/ρ at the time of equilibrium is a measure of the temperature and would ~~be~~ enable us to estimate the total age of the universe. Taking the value one over one thousand from the evidence from cosmic rays we find an age counted from the zero value of the radius of ten thousand million years.

We hope that further discussion and refinement of Eddington's theory will not upset this fine result and even that progress along his line of thought will ~~make possible to~~ permit to deduce all the essential features of the universe without introducing any arbitrary assumption about the mass and the velocity of expansion.

An important text intented to be (but never) published in the *Catholic Encyclopedia of Japan* (before 1940):
The expanding universe : Lemaîtres unknown manuscript (introduction by O. Godart and M. Heller), Tucson (Arizona), Pachart Publishing House, 1985, 50 pages, History of astronomy series, 2.

Lemaître retained his interest in in spinor theory throughout his life… here: January 1956

ARCHIVES GEORGES LEMAÎTRE

Janvier 1956 11

Les spineurs et la physique quantique.

Depuis Dirac 1928 les ψ de la mécanique ondulatoire ne sont plus des scalaires, mais appartiennent à un espace vectoriel qui est distinct de l'espace ou espace-temps physique et après avoir été appelé "wave-vector" par Eddington est finalement désigné comme vecteur de l'espace de spin ou plus simplement spineur. Les ψ et l'autre spineur φ (souvent conjugué du premier) sont multipliés par des opérateurs α, dits observables qui sont représentables par des matrices opérant sur les ψ ou les φ. Alternativement les ψ et les φ étant considérés (c'est à dire transformés comme) des contrevariants et des covariants, les α sont des tenseurs mixtes. La valeur moyenne d'un observable s'obtient par contraction des indices (ou en langage de matrices en prenant la trace (spur) de la matrice). La probabilité de passage d'un état $\psi\varphi$ à un état $\psi'\varphi'$ par produit contracté $\varphi'\alpha\psi$ ou $\varphi\alpha\psi'$.

Ce nouvel aspect de la mécanique quantique se combine naturellement avec l'espace de Hilbert déjà utilisé par la mécanique quantique et en particulier les opérateurs doivent y avoir un caractère hermitien. C'est dire particulièrement pour les spineurs, qu'a été développé (van der Waerden) le mécanisme des tenseurs hermitiens, ou quantités hybrides (Schouten) p. 51 auxquelles depuis quelque temps on attache plutôt le nom de Kähler (Schouten p. 397).

Plus récemment 1/ A S Eddington 1928 - Relativity Theory of protons and electrons 1936 Fundamental Theory 1947 2/ E Cartan - Théorie des spineurs 1938
3/ C C Chevalley - The algebraic Theory of Spinors ont séparé cet aspect hermitien en développant la théorie d'abord indépendamment très dans le corps des complexes (Cartan) puis dans un corps quelconque (Chevalley) quitte à introduire ensuite sous forme de condition de réalité (Eddington) les conditions hermitiennes.

On évite ainsi l'introduction du formalisme hilbertien qui était obscur dans les premiers travaux des physiciens, quitte bien entendu à traiter les questions (essentielles au point de vue physique) à le faire à titre d'application.

En particulier les deux sortes de spineurs φ et ψ ne sont pas nécessairement supposés conjugués l'un de l'autre.

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Preface

In so extensive a work I cannot expect that serious mistakes have been entirely avoided. But now that the theory can be viewed as a whole, I think the reader will be convinced that there is a practicable way of progress along the lines I have attempted. I hope therefore that he will see in the imperfections of this book an opportunity for developing, not an excuse for dismissing, the subject which it sets forth.

Prof. G. Lemaître and Prof. G. F. J. Temple have kindly read the book in proof. Their interest and criticism has encouraged me in the development of the theory, and I now owe them a further debt for many helpful suggestions.

A. S. E.

CAMBRIDGE,
June 1936.

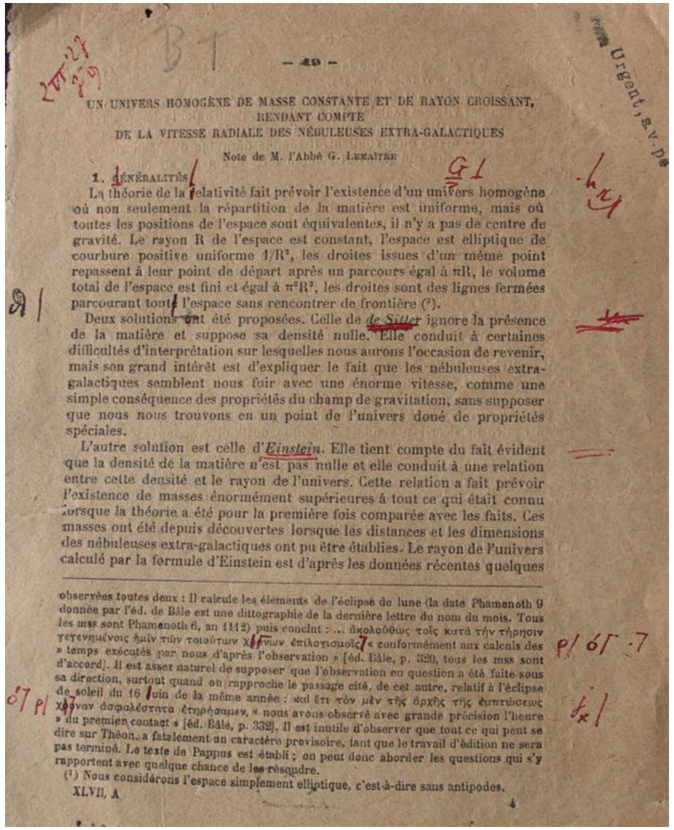
Lemaître corrected the proofs of Eddington's « Relativistic theory of protons and electrons » (Cambridge, 1936).

3 The well-known source of the Primeval Atom Hypothesis:

A reaction to Eddington's philosophical reluctance to consider

a beginning of the present order of the universe

12 The Primeval Atom Hypothesis as a scientific answer to Eddington's philosophical reluctance…



— 49 —

UN UNIVERS HOMOGÈNE DE MASSE CONSTANTE ET DE RAYON CROISSANT, RENDANT COMPTE DE LA VITESSE RADIALE DES NÉBULEUSES EXTRA-GALACTIQUES

Note de M. l'Abbé G. Lemaître

1. Généralités.

La théorie de la relativité fait prévoir l'existence d'un univers homogène où non seulement la répartition de la matière est uniforme, mais où toutes les positions de l'espace sont équivalentes, il n'y a pas de centre de gravité. Le rayon R de l'espace est constant, l'espace est elliptique de courbure positive uniforme $1/R^2$, les droites issues d'un même point repassent à leur point de départ après un parcours égal à πR, le volume total de l'espace est fini et égal à $\pi^2 R^3$, les droites sont des lignes fermées parcourant tout l'espace sans rencontrer de frontière (¹).

Deux solutions ont été proposées. Celle de *de Sitter* ignore la présence de la matière et suppose sa densité nulle. Elle conduit à certaines difficultés d'interprétation sur lesquelles nous aurons l'occasion de revenir, mais son grand intérêt est d'expliquer le fait que les nébuleuses extra-galactiques semblent nous fuir avec une énorme vitesse, comme une simple conséquence des propriétés du champ de gravitation, sans supposer que nous nous trouvons en un point de l'univers doué de propriétés spéciales.

L'autre solution est celle d'*Einstein*. Elle tient compte du fait évident que la densité de la matière n'est pas nulle et elle conduit à une relation entre cette densité et le rayon de l'univers. Cette relation a fait prévoir l'existence de masses énormément supérieures à tout ce qui était connu lorsque la théorie a été pour la première fois comparée avec les faits. Ces masses ont été depuis découvertes lorsque les distances et les dimensions des nébuleuses extra-galactiques ont pu être établies. Le rayon de l'univers calculé par la formule d'Einstein est d'après les données récentes quelques

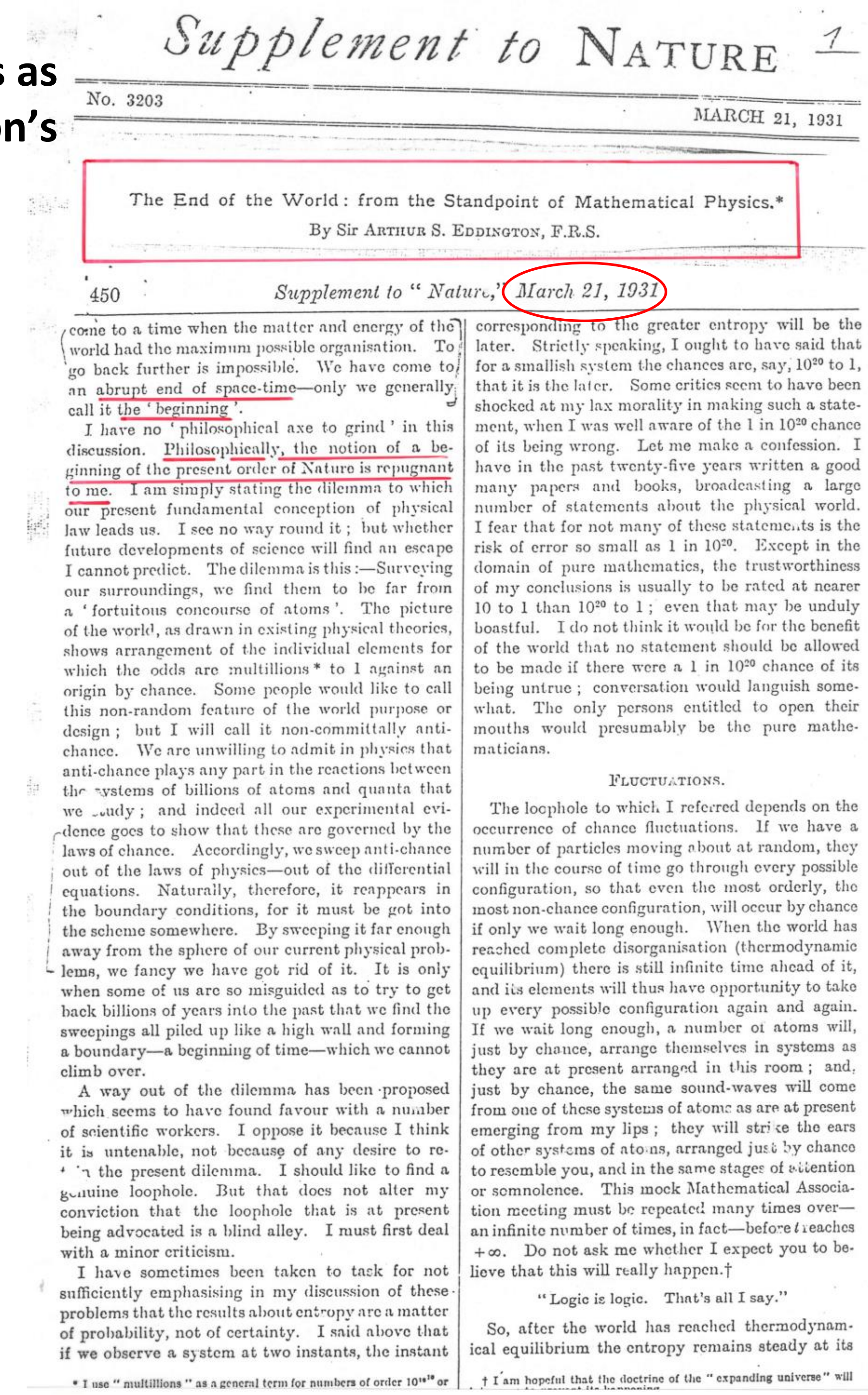
observées toutes deux : Il calcule les éléments de l'éclipse de lune (la date Phamenoth 9 donnée par l'éd. de Bâle est une dittographie de la dernière lettre du nom du mois. Tous les mss sont Phamenoth 6, an 1112) puis conclut : … ἀκολούθως τοῖς κατὰ τὴν τήρησιν γεγενημένοις ἡμῖν τῶν τοιούτων χρόνων ἐπιλογισμοῖς « conformément aux calculs des » temps exécutés par nous d'après l'observation » [éd. Bâle, p. 320, tous les mss sont d'accord]. Il est assez naturel de supposer que l'observation en question a été faite sous sa direction, surtout quand on rapproche le passage cité, de cet autre, relatif à l'éclipse de soleil du 16 juin de la même année : καὶ ἔτι τὸν μὲν τῆς ἀρχῆς τῆς ἐμπτώσεως χρόνον ἀσφαλέστατα ἐτηρήσαμεν, « nous avons observé avec grande précision l'heure » du premier contact » [éd. Bâle, p. 332]. Il est inutile d'observer que tout ce qui peut se dire sur Théon, a fatalement un caractère provisoire, tant que le travail d'édition ne sera pas terminé. Le texte de Pappus est établi ; on peut donc aborder les questions qui s'y rapportent avec quelque chance de les résoudre.

(¹) Nous considérons l'espace simplement elliptique, c'est-à-dire sans antipodes.

XLVII, A

ARCHIVES GEORGES LEMAÎTRE

1927 Lemaître seminal paper



Supplement to NATURE 1

No. 3203 MARCH 21, 1931

The End of the World: from the Standpoint of Mathematical Physics.*

By Sir Arthur S. Eddington, F.R.S.

450 Supplement to "Nature," March 21, 1931

come to a time when the matter and energy of the world had the maximum possible organisation. To go back further is impossible. We have come to an abrupt end of space-time—only we generally call it the 'beginning'.

I have no 'philosophical axe to grind' in this discussion. Philosophically, the notion of a beginning of the present order of Nature is repugnant to me. I am simply stating the dilemma to which our present fundamental conception of physical law leads us. I see no way round it; but whether future developments of science will find an escape I cannot predict. The dilemma is this:—Surveying our surroundings, we find them to be far from a 'fortuitous concourse of atoms'. The picture of the world, as drawn in existing physical theories, shows arrangement of the individual elements for which the odds are multillions* to 1 against an origin by chance. Some people would like to call this non-random feature of the world purpose or design; but I will call it non-committally anti-chance. We are unwilling to admit in physics that anti-chance plays any part in the reactions between the systems of billions of atoms and quanta that we study; and indeed all our experimental evidence goes to show that these are governed by the laws of chance. Accordingly, we sweep anti-chance out of the laws of physics—out of the differential equations. Naturally, therefore, it reappears in the boundary conditions, for it must be got into the scheme somewhere. By sweeping it far enough away from the sphere of our current physical problems, we fancy we have got rid of it. It is only when some of us are so misguided as to try to get back billions of years into the past that we find the sweepings all piled up like a high wall and forming a boundary—a beginning of time—which we cannot climb over.

A way out of the dilemma has been proposed which seems to have found favour with a number of scientific workers. I oppose it because I think it is untenable, not because of any desire to retain the present dilemma. I should like to find a genuine loophole. But that does not alter my conviction that the loophole that is at present being advocated is a blind alley. I must first deal with a minor criticism.

I have sometimes been taken to task for not sufficiently emphasising in my discussion of these problems that the results about entropy are a matter of probability, not of certainty. I said above that if we observe a system at two instants, the instant corresponding to the greater entropy will be the later. Strictly speaking, I ought to have said that for a smallish system the chances are, say, 10^{20} to 1, that it is the later. Some critics seem to have been shocked at my lax morality in making such a statement, when I was well aware of the 1 in 10^{20} chance of its being wrong. Let me make a confession. I have in the past twenty-five years written a good many papers and books, broadcasting a large number of statements about the physical world. I fear that for not many of these statements is the risk of error so small as 1 in 10^{20}. Except in the domain of pure mathematics, the trustworthiness of my conclusions is usually to be rated at nearer 10 to 1 than 10^{20} to 1; even that may be unduly boastful. I do not think it would be for the benefit of the world that no statement should be allowed to be made if there were a 1 in 10^{20} chance of its being untrue; conversation would languish somewhat. The only persons entitled to open their mouths would presumably be the pure mathematicians.

FLUCTUATIONS.

The loophole to which I referred depends on the occurrence of chance fluctuations. If we have a number of particles moving about at random, they will in the course of time go through every possible configuration, so that even the most orderly, the most non-chance configuration, will occur by chance if only we wait long enough. When the world has reached complete disorganisation (thermodynamic equilibrium) there is still infinite time ahead of it, and its elements will thus have opportunity to take up every possible configuration again and again. If we wait long enough, a number of atoms will, just by chance, arrange themselves in systems as they are at present arranged in this room; and, just by chance, the same sound-waves will come from one of these systems of atoms as are at present emerging from my lips; they will strike the ears of other systems of atoms, arranged just by chance to resemble you, and in the same stages of attention or somnolence. This mock Mathematical Association meeting must be repeated many times over—an infinite number of times, in fact—before t reaches $+\infty$. Do not ask me whether I expect you to believe that this will really happen.†

"Logic is logic. That's all I say."

So, after the world has reached thermodynamical equilibrium the entropy remains steady at its

* I use "multillions" as a general term for numbers of order 10^{10^n} or

† I am hopeful that the doctrine of the "expanding universe" will

In 1930 Eddington has accepted Lemaître's expanding universe… but…

« Philosophically, the notion of a beginning of the present order of Nature is repugnant to me » (Eddington, March 1931)

The strongest motivation leading to Primeval Atom Hypothesis: A Quantum theory answer to Eddington's reluctance to give physical meaning to the beginning of the universe

Nature
London
Date 9 MAY 1931

The Beginning of the World from the Point of View of Quantum Theory.

SIR ARTHUR EDDINGTON[1] states that, philosophically, the notion of a beginning of the present order of Nature is repugnant to him. I would rather be inclined to think that the present state of quantum theory suggests a beginning of the world very different from the present order of Nature. Thermodynamical principles from the point of view of quantum theory may be stated as follows: (1) Energy of constant total amount is distributed in discrete quanta. (2) The number of distinct quanta is ever increasing. If we go back in the course of time we must find fewer and fewer quanta, until we find all the energy of the universe packed in a few or even in a unique quantum.

Now, in atomic processes, the notions of space and time are no more than statistical notions; they fade out when applied to individual phenomena involving but a small number of quanta. If the world has begun with a single quantum, the notions of space and time would altogether fail to have any meaning at the beginning; they would only begin to have a sensible meaning when the original quantum had been divided into a sufficient number of quanta. If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time. I think that such a beginning of the world is far enough from the present order of Nature to be not at all repugnant.

It may be difficult to follow up the idea in detail as we are not yet able to count the quantum packets in every case. For example, it may be that an atomic nucleus must be counted as a unique quantum, the atomic number acting as a kind of quantum number. If the future development of quantum theory happens to turn in that direction, we could conceive the beginning of the universe in the form of a unique atom, the atomic weight of which is the total mass of the universe. This highly unstable atom would divide in smaller and smaller atoms by a kind of super-radioactive process. Some remnant of this process might, according to Sir James Jeans's idea, foster the heat of the stars until our low atomic number atoms allowed life to be possible.

Clearly the initial quantum could not conceal in itself the whole course of evolution; but, according to the principle of indeterminacy, that is not necessary. Our world is now understood to be a world where something really happens; the whole story of the world need not have been written down in the first quantum like a song on the disc of a phonograph. The whole matter of the world must have been present at the beginning, but the story it has to tell may be written step by step.

G. LEMAÎTRE.

40 rue de Namur,
Louvain.

[1] NATURE, Mar. 21, p. 447.

« **Quantum theory** suggests a beginning of the world very different from the present order of Nature »

« I think that such a beginning of the world is far enough from the present order of Nature to be not at all repugnant… » why?...

« Space and time are no more than statistical notions… »

« If the world has begun with a single **quantum**, the notion of space and time would altogether fail to have any meaning at the beginning »

« A unique **quantum** »

« An unstable Atom »

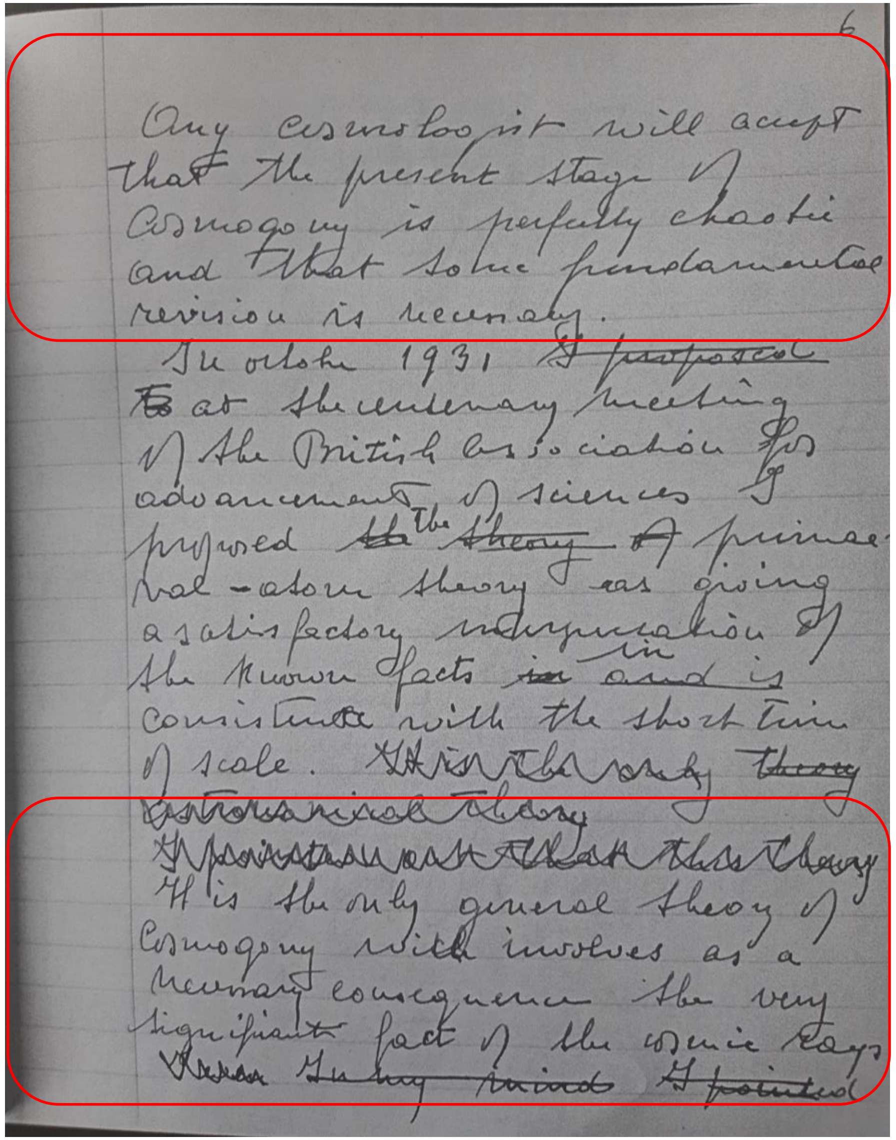
« Some remnant… »

The College in Leuven where Lemaître wrote his 1931 paper

Quantum indeterminacy principle: « The whole story of the world need not have been written down in the first **quantum** »

One important result…to explain the origin of Cosmis rays

The real humility of Lemaître…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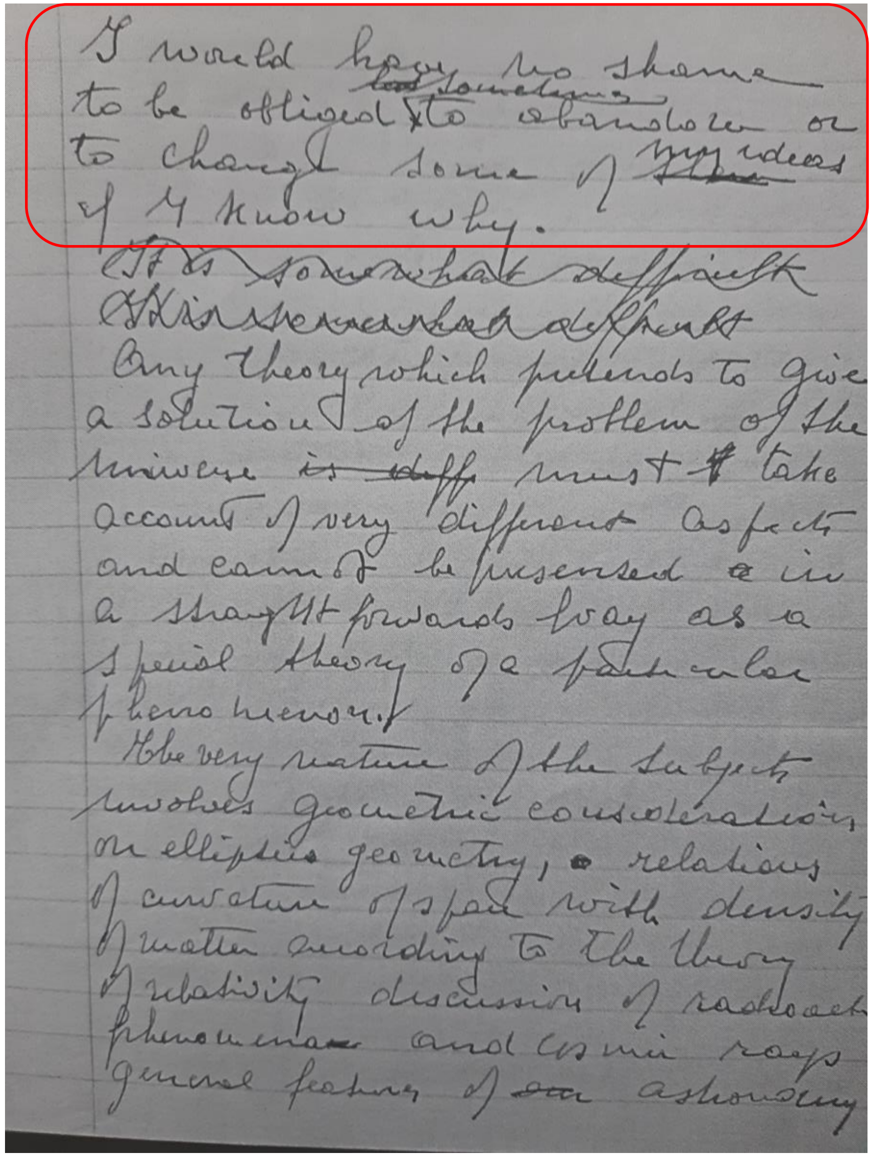
Any cosmologist will accept that the present stage of cosmogony is perfectly chaotic and that some fundamental revision is necessary.

In october 1931 ~~I proposed~~ ~~to~~ at the centenary meeting of the British association for advancement of sciences I proposed ~~the~~ the ~~theory of~~ primeval-atom theory as giving a satisfactory interpretation of the known facts ~~in~~ and is consistent with the short time of scale. ~~It is the only theory~~ ~~astronomical theory~~ ~~I pointed out that this theory~~

It is the only general theory of cosmogony with involves as a necessary consequence the very significant fact of the cosmic rays ~~I pointed~~

Notebook for an unpublished book

ARCHIVES GEORGES LEMAÎTRE


I would have no shame sometimes to be obliged to abandon or to change some of my ideas if I know why.

~~It is somewhat difficult~~ ~~It is somewhat difficult~~

Any theory which pretends to give a solution of the problem of the universe ~~is diff.~~ must take account of very different aspects and cannot be presented in a straightforwards way as a special theory of a particular phenomenon.

The very nature of the subject involves geometric considerations, on elliptic geometry, relations of curvature of space with density of matter according to the theory of relativity discussion of radioactive phenomena and cosmic rays general features of ~~our~~ astonishing

« It is the only general theory of cosmogony wich involves as a consequence the very significant fact of the cosmic rays »

« I would have no shame to be obliged to sometimes abandon or to change my ideas if I know why… »

4 The ambiguity of the Primeval Atom Hypothesis and its first meaning

Is Primeval Atom a physical state inside « space-time-matter » (atom, giant nucleus,…)
or
is it a state out of space-time-matter framework from which space-time-matter is emerging?

(what is its status? What would be a Physics without space-time-matter…?)

Lemaître is probably the first one (maybe with Eddington) to adress the question of a (mathematical) description of a Primeval State giving rise to space and time

One of the interpretation of the Primeval Atom suggested by Lemaître: a state which generates space, time and matter (whati is its status?): how to conceive the emergence of Space-time… The origin of geometry…

From Logic?
From algebra?
From topology?

How to give meaning to physics without space-time?

PONTIFICIA ACADEMIA SCIENTIARVM

ACTA
Vol. XII – N. 6
pag. 25-40

L'HYPOTHÉSE DE L'ATOME PRIMITIF (*)

GEORGES LEMAITRE
Académicien Pontifical

40 PONTIFICIA ACADEMIA SCIENTIARVM

la fragmentation initiale; l'instant où naissait l'espace avec un rayon partant de zéro, l'instant où naissait la multiplicité dans la matière.

Cette origine nous apparaît, dans l'espace temps comme un fond qui défie notre imagination et notre raison en leur opposant une barrière qu'elles ne peuvent franchir. L'espace-temps nous apparaît, semblable à une coupe conique. On progresse vers le futur en suivant les génératrices du cône vers le bord extérieur du verre. On fait le tour de l'espace en parcourant un cercle normalement aux génératrices. Lorsqu'on remonte par la pensée le cours du temps, on s'approche du fond de la coupe, on s'approche de cette instant unique, qui n'avait pas d'hier parce qu'hier, il n'y avait pas d'espace.

Commencement naturel du monde, origine pour laquelle la pensée ne peut concevoir une pré-existence, puisque c'est l'espace même qui commence et que nous ne pouvons rien concevoir sans espace. Le temps semble pouvoir être prolongé à volonté vers le passé comme vers l'avenir. Mais l'espace peut commencer, et le temps ne peut exister sans espace, on pourrait donc dire, que l'espace étrangle le temps, et empèche de l'étendre au delà du fond de l'espace-temps.

Mais cette origine est aussi le commencement de la multiplicité. C'est un instant où la matière est un seul atome, un instant où les notions statistiques qui supposent la multiplicité ne trouvent pas d'emploi. On peut se demander si dans ces conditions la notion même d'espace ne s'évanouit pas à la limite et n'acquière que progressivement un sens à fur à mesure que la fragmentation s'achève que les êtres se multiplient.

Devons-nous nous plaindre que nos notions les plus familiaires s'évanouissent lorsqu'elles s'approchent du terme ultime qu'elles ne doivent pas dépasser? Je ne le pense pas.

En terminant je ne puis mieux faire que de rappeler le mot de René Descartes par lequel je commençais et qui s'applique sans doute aussi à l'atome primitif *Mundus est fabula*.

« The beginning of the world happened a little before the beginning of space and time » : **space and time are statistical notions**…

into a sufficient number of quanta. If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time. I think that such a beginning of the world is far enough from the present order of Nature to be not at all repugnant.

Nature 1931

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These figures may be ~~largely~~ considerably increased if the cosmic rays have been produced at a time when the density of matter was much higher than it is now, twice greater, but the energy of the rays was also it has been reduced by the expansion by the same phenomenon which produces the redshift of light.

17. THE BEGINNING OF SPACE.

Matter is now expanding at an accelerated rate. The expansion will become so rapid that light emitted now by stars and nebulae will not be able to travel more than a few degrees of angular distance.

A few thousand millions years ago it had passed slowly through equilibrium and the diffuse original matter had taken opportunity of the unstability of the equilibrium to arrange itself into nebulae and clusters of nebulae. The collapsing matter in the nebulae had undergone ~~more~~ elastic collisions and had been agglomerated into stars.

At an earlier stage during the first rapid expansion of the everexpanding universe matter was engaged into super-radioactive activity and had emitted, the cosmic rays.

What happened before that?

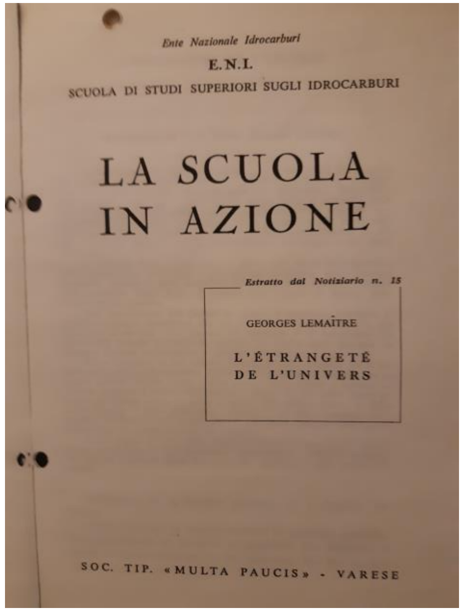
Before that we have to face the zero value of the radius. We have discussed how far it had to be taken as strictly zero, and we have seen that it means a very trifling quantity, let us say a few hours of light.

We may speak of this event as of a beginning. I do not say a creation. Physically it is a beginning in that sense that if something has happened before, it has no observable influence on the behaviour of our universe, as any feature of matter before this beginning has been completely lost by the extreme contraction at the theoretical zero. Any pre-existence of the universe has a metaphysical character. Physically every thing happens as if the theoretical zero was really a beginning. The question if it was really a beginning or rather a creation: something starting from nothing, is a philosophical question which cannot be settled by physical or astronomical considerations.

It is a very happy circumstance that relativity provides for a natural beginning. Otherwise we would not know when to stop our investigation of the remote past of our universe.

« The Expanding Universe »

In *L'étrangeté de l'Univers* Lemaître emphasizes the role of Spinor Theory Eddington and Cartan.



Elie Cartan

Lemaître students discussed with J. TITS

Jacques Tits

L'évolution des idées géométriques ne fait sans doute que commencer.

En 1913, Elie Cartan a découvert ce qu'on appelle maintenant les spineurs. Il a découvert qu'à l'espace euclidien à un nombre quelconque de dimensions est associé un espace d'une nature différente : l'espace de spin.

Pour des espaces d'un petit nombre de dimensions, l'espace de spin a aussi une dimension petite. En fait, dans l'espace à quatre dimensions, l'espace physique et l'espace de spin ont un même nombre de dimensions.

Mais, si on est amené à envisager des espaces d'un plus grand nombre de dimensions, l'espace de spin correspondant acquiert un nombre énorme de dimensions.

Pour un espace à cent dimensions, c'est par milliers de milliards que se comptent les dimensions de l'espace de spin.

Quelques années après la découverte de Cartan, Dirac a retrouvé et donné son nom aux spineurs de l'espace à quatre dimensions dans sa célèbre théorie de l'électron relativiste.

Il a montré que les ondes de de Broglie n'étaient pas, comme on le pensait, des scalaires, mais qu'elles étaient des vecteurs de l'espace de spin : des spineurs.

C'est l'équation de Dirac qui a été le point de départ des recherches d'Eddington. Eddington était convaincu que l'équation de Dirac n'était qu'un commencement. Il a entrepris l'étude des spineurs de l'espace à huit dimensions. Le nombre qu'il a proposé comme étant le nombre total des protons existant dans l'univers est essentiellement le nombre de dimensions de l'espace de spin d'un espace à 512 dimensions.

L'étrangeté de la description actuelle du monde atomique ne provient-elle pas de ce que nous nous efforçons de loger dans l'espace étriqué de notre intuition sensible un univers d'un nombre notablement plus grand de dimensions.

L'univers n'est-il pas un spineur d'un espace à quelques centaines de dimensions?

Chi lo sa?

Qui peut dire jusqu'où nous conduira la Science dans la découverte de l'étrangeté du monde?

A fundamental algebraic theory able was important for Lemaître

Je ne veux pas développer davantage en ce moment l'hypothèse de l'atome primitif. Mon sujet est l'étrangeté du monde. Je veux me borner à montrer comment l'étrangeté de l'atome primitif aide à comprendre comment l'univers a pu commencer d'une façon naturelle.

Je préfère pour ma part ce commencement naturel à la chiquenaude initiale dont parlait déjà Pascal avant Laplace et par laquelle Dieu serait intervenu dans le domaine des causes secondes pour mettre le monde en mouvement.

Je préfère penser au Dieu caché d'Isaïe : « Deus absconditus, salvator », au Dieu suprême et inaccessible : « Personne n'a connu Dieu », dit Saint Jean, au Dieu caché même dans le commencement du monde.

Où s'arrêtera la physique dans sa découverte de l'étrangeté? Sans doute est-il vain de faire des pronostics, mais comment pourrait-on s'en empêcher?

L'évolution des idées géométriques ne fait sans doute que commencer.

En 1913, Elie Cartan a découvert ce qu'on appelle maintenant les spineurs. Il a découvert qu'à l'espace euclidien à un nombre quelconque de dimensions est associé un espace d'une nature différente : l'espace de spin.

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Quelques années après la découverte de Cartan, Dirac a retrouvé et donné son nom aux spineurs de l'espace à quatre dimensions dans sa célèbre théorie de l'électron relativiste.

In 1960 Lemaître is still fascinated by spinor theory

From 1929 Lemaître became interested in topology of the universe: “La grandeur de l'espace” (conférence faite le 31 janvier 1929 à l'Assemblée générale de la Société scientifique de Bruxelles), *Revue des Questions Scientifiques,* 48e année, t. XCV (4e série, t. XV), 20 mars 1929, pp. 189-216.

At the beginning was topology!

Topologies of the Primeval Atom Universe

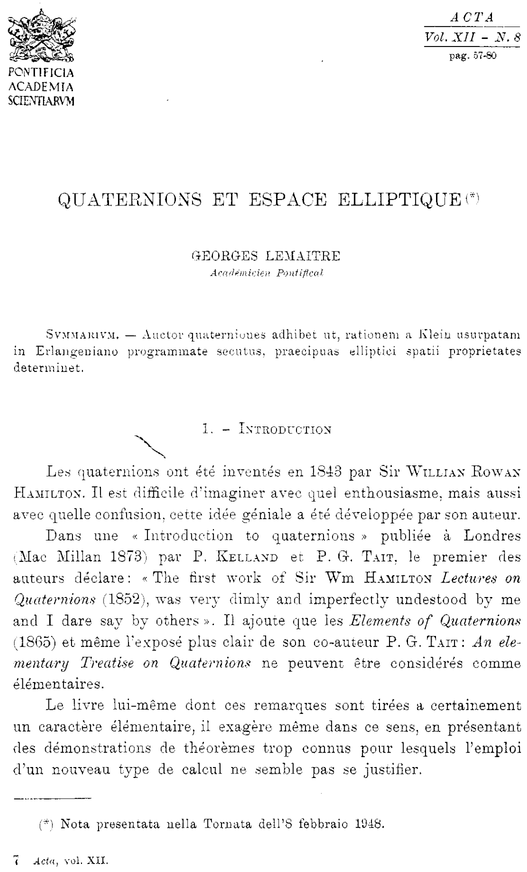
Elliptic spaces:

S^3 = SU(2)
Projection:
$S^3 \rightarrow S^3/S^1 = S^2$
(Hopf fibration, 1931)

Or

S^3/Z_2 = SU(2)/Z_2 = SO(3)
Projection:
$S^3/Z_2 \rightarrow S^3/S^1 \times Z_2 = S^2/Z_2$
real projective plane

Lemaître wondered if this Topological difference can be physically relevant?



PONTIFICIA ACADEMIA SCIENTIARVM

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QUATERNIONS ET ESPACE ELLIPTIQUE (*)

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SVMMARIVM. — Auctor quaterniones adhibet ut, rationem a Klein usurpatam in Erlangeniano programmate secutus, praecipuas elliptici spatii proprietates determinet.

1. – INTRODUCTION

Les quaternions ont été inventés en 1843 par Sir WILLIAN ROWAN HAMILTON. Il est difficile d'imaginer avec quel enthousiasme, mais aussi avec quelle confusion, cette idée géniale a été développée par son auteur.

Dans une « Introduction to quaternions » publiée à Londres (Mac Millan 1873) par P. KELLAND et P. G. TAIT, le premier des auteurs déclare: « The first work of Sir Wm HAMILTON *Lectures on Quaternions* (1852), was very dimly and imperfectly undestood by me and I dare say by others ». Il ajoute que les *Elements of Quaternions* (1865) et même l'exposé plus clair de son co-auteur P. G. TAIT: *An elementary Treatise on Quaternions* ne peuvent être considérés comme élémentaires.

Le livre lui-même dont ces remarques sont tirées a certainement un caractère élémentaire, il exagère même dans ce sens, en présentant des démonstrations de théorèmes trop connus pour lesquels l'emploi d'un nouveau type de calcul ne semble pas se justifier.

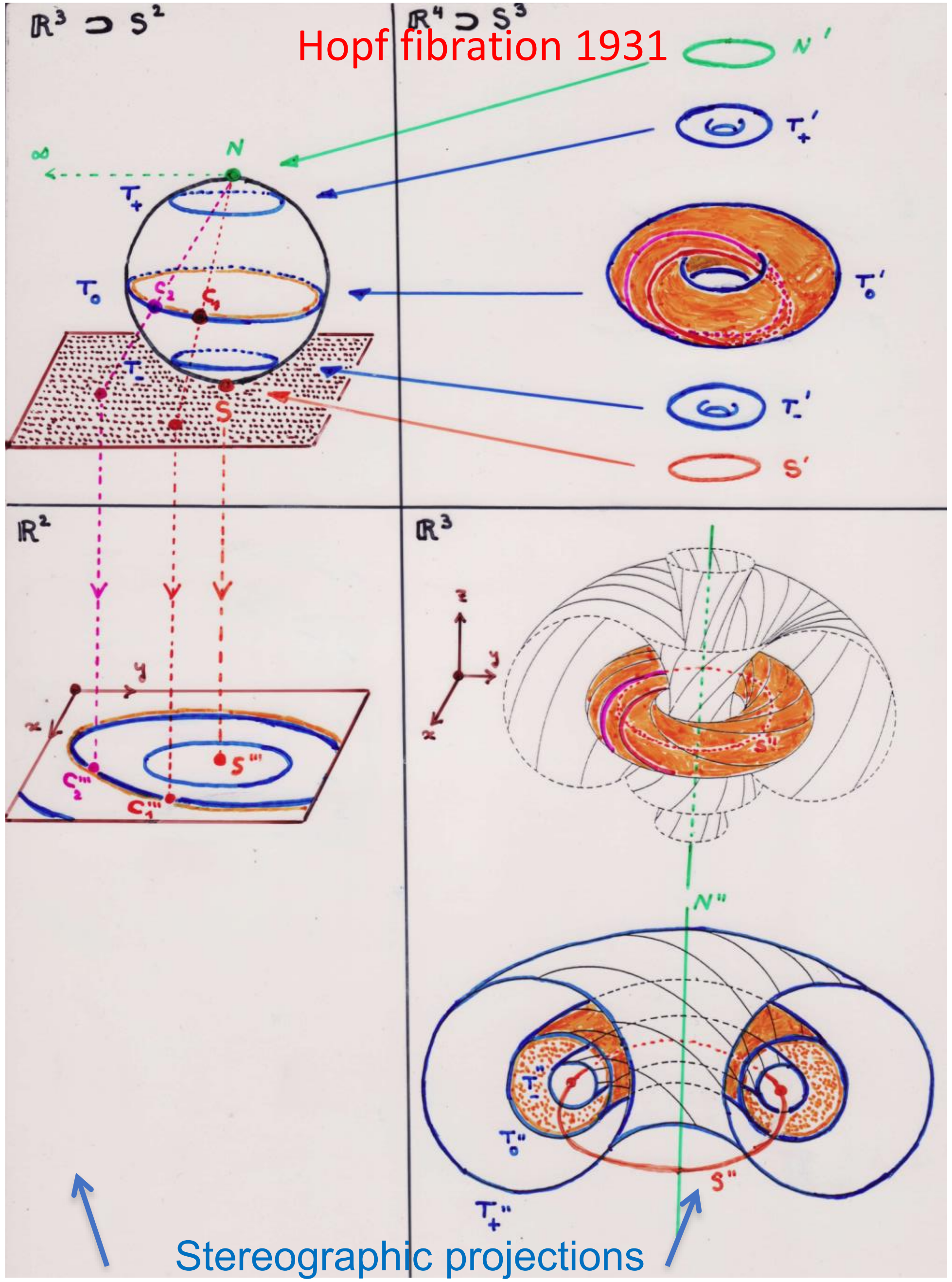
(*) Nota presentata nella Tornata dell'8 febbraio 1948.

7 *Acta*, vol. XII.

Quaternions of unit norm

$Q = q^0 + q^1\, i + q^2\, j + q^3\, k \qquad i.j = - j.i = k$

$QQ^* = 1 = (q^0)^2 + (q^1)^2 + (q^2)^2 + (q^3)^2$: **S^3**



Usual Sphere: S^2 Spherical Universe : S^3

According to Lemaître:

« it maybe difficult to follow the idea (of conceiving the beginning of the world before the beginning of space-time) »

We have thus first to come back to the idea of a unique (physical) atom…

« La cosmogonie est de la physique atomique à grande échelle… » (1931)

« Cosmogony is atomic physics on a large scale »(*)

(*) "The evolution of the universe : Discussion", in *British association for the advancement of science : Report of the centenary meeting. London, 1931, september* 23-30, London, Office of the British Association, 1932, pp. 573-610 (talk of Lemaître, pp. 605- 610).

5 The ambiguity of the status of the Primeval Atom and its Second interpretation

The Primeval Atom is a physical reality: a « Colossal Atomic Nucleus »?

Quantum forces stop the contraction (if any) before the singularity:

> When the distances between the atomic nuclei and the electrons become of the order of 10^{-12} cm, the non-Maxwellian forces which prevent the mutual interpenetration of elementary particles must become predominant and are without doubt capable of stopping the contraction. The universe would then be comparable to a colossal atomic nucleus. If the contraction is stopped, the process should continue in the opposite direction.

"The Expanding Universe" (English translation by M.A.H.Mac Callum), *General Relativity and Gravitation*, t. 29, 1997, n°5, p. 678.

« A l'origine toute la masse de l'univers existait sous forme d'un atome unique;
le rayon de l'univers quoique non strictement nul, était relativement très petit.
Tout l'univers résulte de la désintégration de cet atome primitif » 1931

« **The matter has to find, though, a way of avoiding the vanishing of its volume** » (in « L'expansion de l'univers », 1933; 1997 , p.678)

« Il faut pourtant bien que la matière trouve un moyen d'éviter l'évanouissement de son volume » (1933, p. 36)

The matter has to find, though, a way of avoiding the vanishing of its volume.

As long as the matter is made up of stars, this is manifestly impossible.

When it is condensed into a single mass, it is clear that it must have acquired a high temperature much greater than the critical temperature of liquids and that nothing prevents it attaining a degree of concentration comparable to the interior of the companion of Sirius.

Even for a degenerate gas it seems that nothing could oppose the concentration, since the available energy M/R is unbounded.

When the distances between the atomic nuclei and the electrons become of the order of 10^{-12} cm, the non-Maxwellian forces which prevent the mutual interpenetration of elementary particles must become predominant and are without doubt capable of stopping the contraction. The universe would then be comparable to a colossal atomic nucleus. If the contraction is stopped, the process should continue in the opposite direction.

Adopting, following Eddington, 10^{78} as the number of protons in existence, we have, as the order of magnitude of the radius of the universe when reduced to its atomic state

$$10^{(78/3)-12} = 10^{14} \text{ cm} ,$$

"The Expanding Universe" (English translation by M.A.H.Mac Callum), *General Relativity and Gravitation*, t. 29, 1997, n°5, pp.641-680 (présentation par A.Krasinski, "Editor's Note : The Expanding Universe", *General Relativity and Gravitation*, t. 29, 1997, n°5, pp.637-640).

Still a reference to Eddington's Fundamental Theory

which is about ten times the distance to the Sun.

We thus conclude that only the subatomic nuclear forces seem capable of stopping the contraction of the universe, when the radius of the universe is reduced to the dimensions of the solar system.

For the cosmological point of view, the zero of space must thus be treated as a beginning, in the sense that every astronomical structure with an earlier existence would have been completely destroyed there.

The epoch of this beginning, or, if one likes, of this recommencement, certainly dates from before the formation of the Earth's crust and the organization of the solar system, that is as a strict minimum from the study of radioactive rocks

$$1.6 \times 10^9 \text{ years.}$$

Comparing this value with Hubble's ratio

$$\frac{r}{v} = 1.8 \times 10^9 \text{ years,}$$

we conclude that all solutions in which the expansion speed has always been faster than it is now are excluded.

In particular, for Einstein's cycloidal universe (12.4) or the solution (10.10) for small R/R_0, one has

$$t = \frac{2}{3}\frac{r}{v} = 1.2 \times 10^9 \text{ years.}$$

We must thus exclude the solutions where the radius is less than the equilibrium radius and in particular the quasi-periodic solutions.

For a purely aesthetic point of view, one may perhaps regret this. Those solutions where the universe expands and contracts successively while periodically reducing itself to an atomic mass of the dimensions of the solar system, have an indisputable poetic charm and make one think of the phoenix of legend.

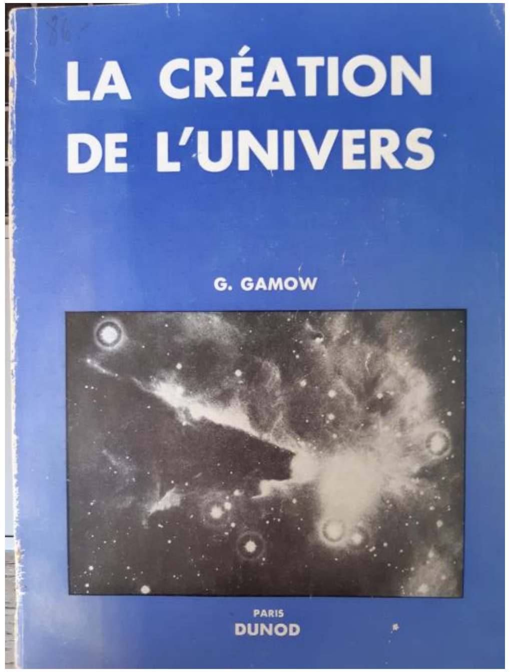
"L'univers en expansion" (séance du 3 mai 1933, 1re section), *Annales de la Société Scientifique de Bruxelles*, série A : Sciences Mathématiques, t. LIII, 1933, n°2, pp. 51-85.

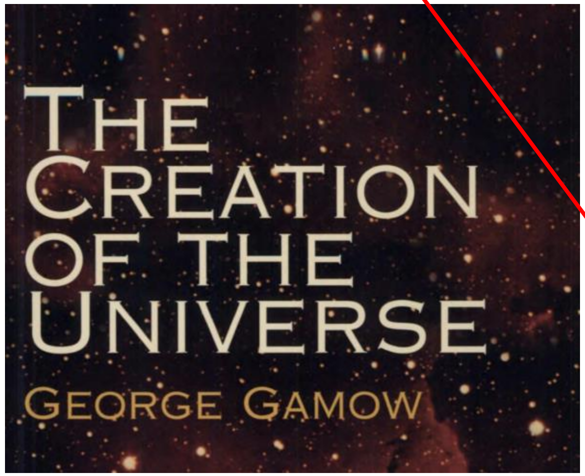
Lemaître had also a completely different idea. You cannot approach singularity due to physical a « Primeval Nucleus » and thus a physical state (described by Gamov)!

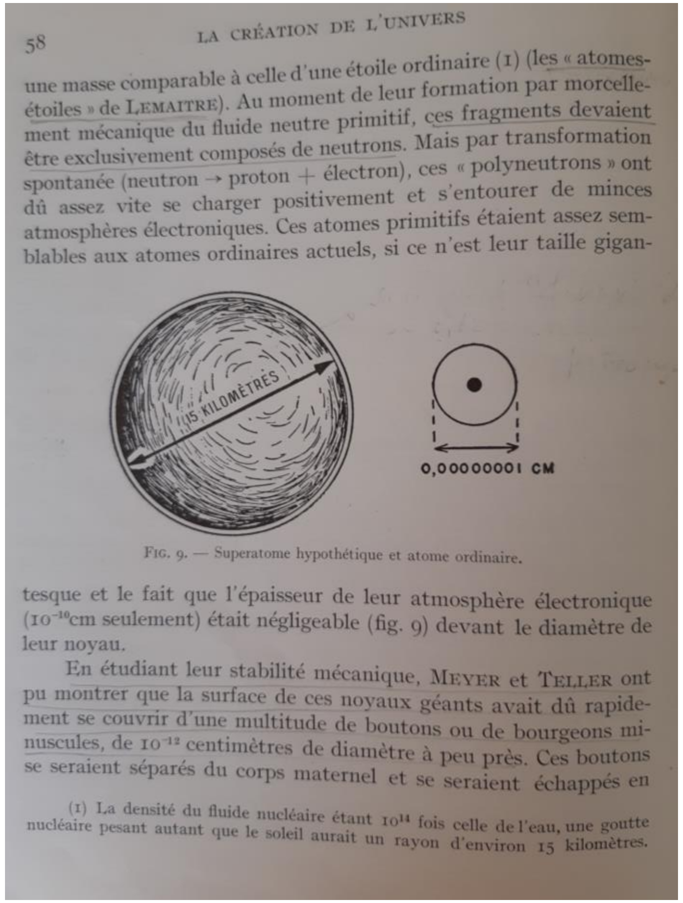
Quantum forces prevent the universe to go to the singularity (Eddington computation!): you reach an enormous physical atom which is not the Primeval Atom but as Gamov said a « Primeval Nucleus »!

Primeval Atom → « **initial atom** » a « giant nucleus » Physical state

The quantum nature of this « primeval nucleus » (Gamow polyneutron hypothesis) and its consequences explored by Maria Meyer and Edward Teller in order to understand nuclosynthesis







58 LA CRÉATION DE L'UNIVERS

une masse comparable à celle d'une étoile ordinaire (1) (les « atomes-étoiles » de LEMAITRE). Au moment de leur formation par morcellement mécanique du fluide neutre primitif, ces fragments devaient être exclusivement composés de neutrons. Mais par transformation spontanée (neutron → proton + électron), ces « polyneutrons » ont dû assez vite se charger positivement et s'entourer de minces atmosphères électroniques. Ces atomes primitifs étaient assez semblables aux atomes ordinaires actuels, si ce n'est leur taille gigantesque et le fait que l'épaisseur de leur atmosphère électronique (10^{-10}cm seulement) était négligeable (fig. 9) devant le diamètre de leur noyau.

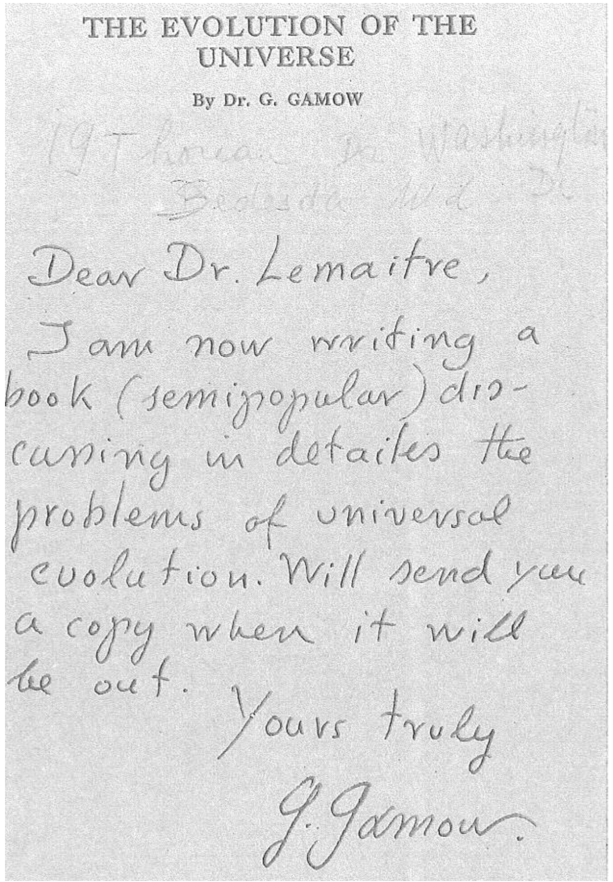
15 KILOMÈTRES

0,00000001 CM

FIG. 9. — Superatome hypothétique et atome ordinaire.

En étudiant leur stabilité mécanique, MEYER et TELLER ont pu montrer que la surface de ces noyaux géants avait dû rapidement se couvrir d'une multitude de boutons ou de bourgeons minuscules, de 10^{-12} centimètres de diamètre à peu près. Ces boutons se seraient séparés du corps maternel et se seraient échappés en

(1) La densité du fluide nucléaire étant 10^{14} fois celle de l'eau, une goutte nucléaire pesant autant que le soleil aurait un rayon d'environ 15 kilomètres.



THE EVOLUTION OF THE UNIVERSE

By Dr. G. GAMOW

Dear Dr. Lemaitre,

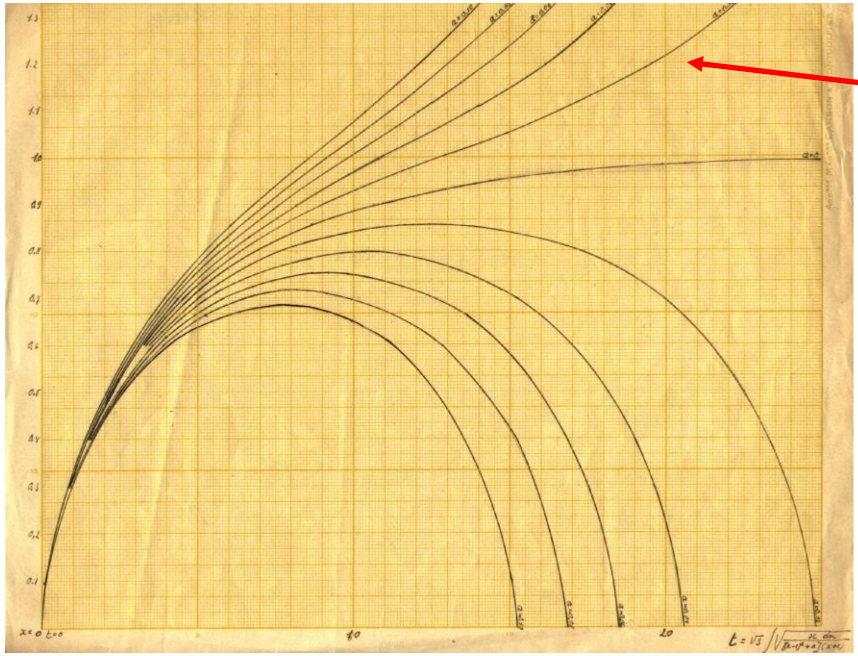
I am now writing a book (semipopular) discussing in detailes the problems of universal evolution. Will send you a copy when it will be out.

Yours truly

G. Gamow

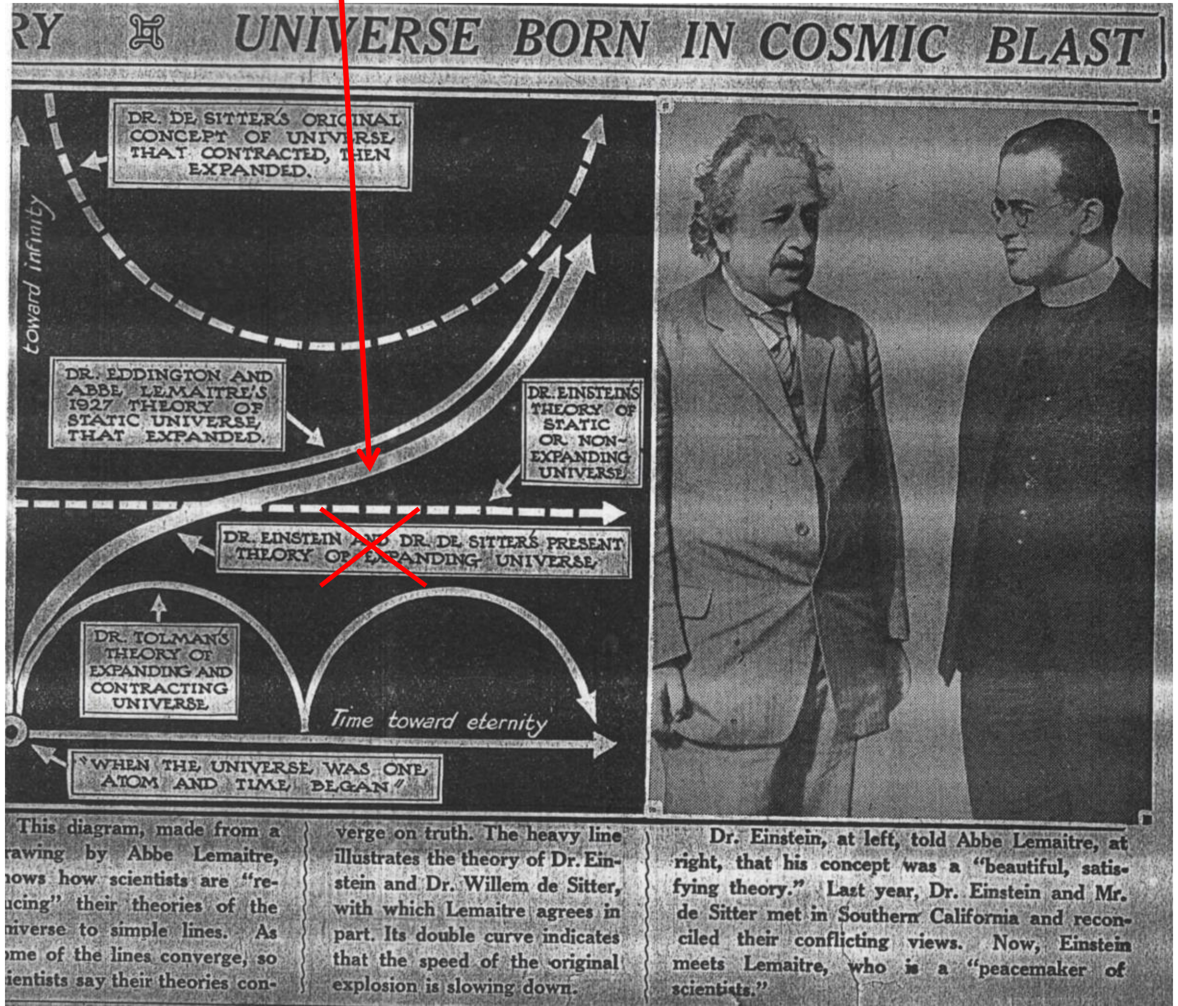
Lemaître had contact with Gamow be he never studied the results of Meyer and Teller!

6 But the beginning could also related to r=0. What is the link between singularity in Friedmann-Lemaître universes and the Primeval Atom Hypothesis?



In 1931 Lemaître chose a (spherical S^3/Z_2) model with an initial singularity (and positive value of the cosmological constant)

« When the universe was an atom and time began »



This diagram, made from a rawing by Abbe Lemaitre, hows how scientists are "re-ucing" their theories of the niverse to simple lines. As ome of the lines converge, so ientists say their theories converge on truth. The heavy line illustrates the theory of Dr. Einstein and Dr. Willem de Sitter, with which Lemaitre agrees in part. Its double curve indicates that the speed of the original explosion is slowing down.

Dr. Einstein, at left, told Abbe Lemaitre, at right, that his concept was a "beautiful, satisfying theory." Last year, Dr. Einstein and Mr. de Sitter met in Southern California and reconciled their conflicting views. Now, Einstein meets Lemaitre, who is a "peacemaker of scientists."

For the cosmological point of view, the zero of space must thus be treated as a beginning, in the sense that every astronomical structure with an earlier existence would have been completely destroyed there.

"The Expanding Universe" (English translation by M.A.H.Mac Callum), *General Relativity and Gravitation*, t. 29, 1997, n°5, p. 679

Some singularities are inescapable : Lemaître was really open to find a way to eliminate inital singularitiues if any and if possible!

Ilf the singularity is inescapable then Primeval Atom Hypothesis gives a quantum interpretation In terms of natural beginning.

Singularities and the way to escape them are one of the major themes of Lemaître’s work

Primeval Atom Hypothesis: a quantum way to interpret an inescapable initial singularity

« It is an inaccessible ground of space-time. Such a picture finds a natural geometrical support in the point-singularity which arises in Friedmann's theory. The radius of space can start from zero. Such singular event which arises when space has a zero-volume is a bottom of space-time which terminates every line of space-time. I do not pretend that such a singularity is inescapable in Friedman's theory, but I simply point out it fits with the quantum outlook as a natural beginning of multiplicity and of space-time. »

(Congrès Solvay 1958, *La structure et l'évolution de l'univers*, Bruxelles, Stoops, 1958, p.7)

Lemaître admitted that one could escape singularities

His aim is only to suggest that if it is inescapable it could corresponds adequately to what he described in his Primeval Atom Hypothesis as a natural beginning

-natural beginning which is the…
-beginning of the multiplicity which corresponds to an
-inaccessible beginning… because it is out of space-time!

McCrea, Oort, Lemaitre, Gorter, Pauli, Bragg, Oppenheimer, Moller, Shapley, Heckmann; Klein, Morgan, Hoyle, Kukaskin, Ambarzumian, van de Hulst, Fierz, Sandage, Baade, Schatzman, Wheeler, Bondi, Gold, Zanstra, Rosenfeld, Ledoux, Lovell, Geneniau

Various studies on singularities fictious or not

Avoiding singularities was a constant theme in Lemaître's carreer…

In Harvard College Observatory (1925): how to remove the Singularity in de Sitter metric?

We may sum up the above discussion in the following way: de Sitter's coördinates introduced a spurious inhomogeneity of the field which is not simply the mathematical appearance of center of an origin of coördinates, but really attributes distinct absolute properties to a center.

We tried to remove the difficulty by introducing other coördinates and were led to a homogeneous field; but first the field is not static and secondly, the space has no curvature. The first point may probably be accepted. Eddington writes on this subject: "It is sometimes urged against de Sitter's world that it becomes non-statical as soon as any matter is inserted in it. But this property is perhaps rather in favor of de Sitter's theory than against it."[2] Our treatment evidences this non-statical character of de Sitter's world which gives a possible interpretation of the mean receding motion of spiral nebulae.

The second point, on the contrary, seems completely inadmissible. We are led back to the euclidean space and to the impossibility of filling up an infinite space with matter which cannot but be finite. De Sitter's solution has to be abandoned, not because it is non-static, but because it does not give a finite space without introducing an impossible boundary.

[2]Eddington. The Mathematical Theory of Relativity, p. 161, Cambridge, 1923.

March, 1925.

NOTE ON DE SITTER'S UNIVERSE

By G. LEMAITRE[1]

The equations of the element of interval of a four-dimensional universe of constant positive curvature have been given by de Sitter in the form

$$ds^2 = R^2[-d\chi^2 - \sin^2\chi(d\Theta^2 + \sin^2\Theta d\phi^2) + \cos^2\chi d\tau^2], \quad (1)$$

$$ds^2 = R^2[-e^{-2T}(dx^2+dy^2+dz^2)+dT^2] \quad (4)$$

for geodesics parallel to the future, or

$$ds^2 = R^2[-e^{2T}(dx^2+dy^2+dz^2)+dT^2] \quad (5)$$

if parallel in the past direction.

This choice of coördinates is free from the objection of introducing a spurious assymmetry in space and time.

Which leads to the intuition of a non-static universe →

If we introduce de Sitter's coördinates, taking as center the light-source M, the observer M_0 will be at a distance

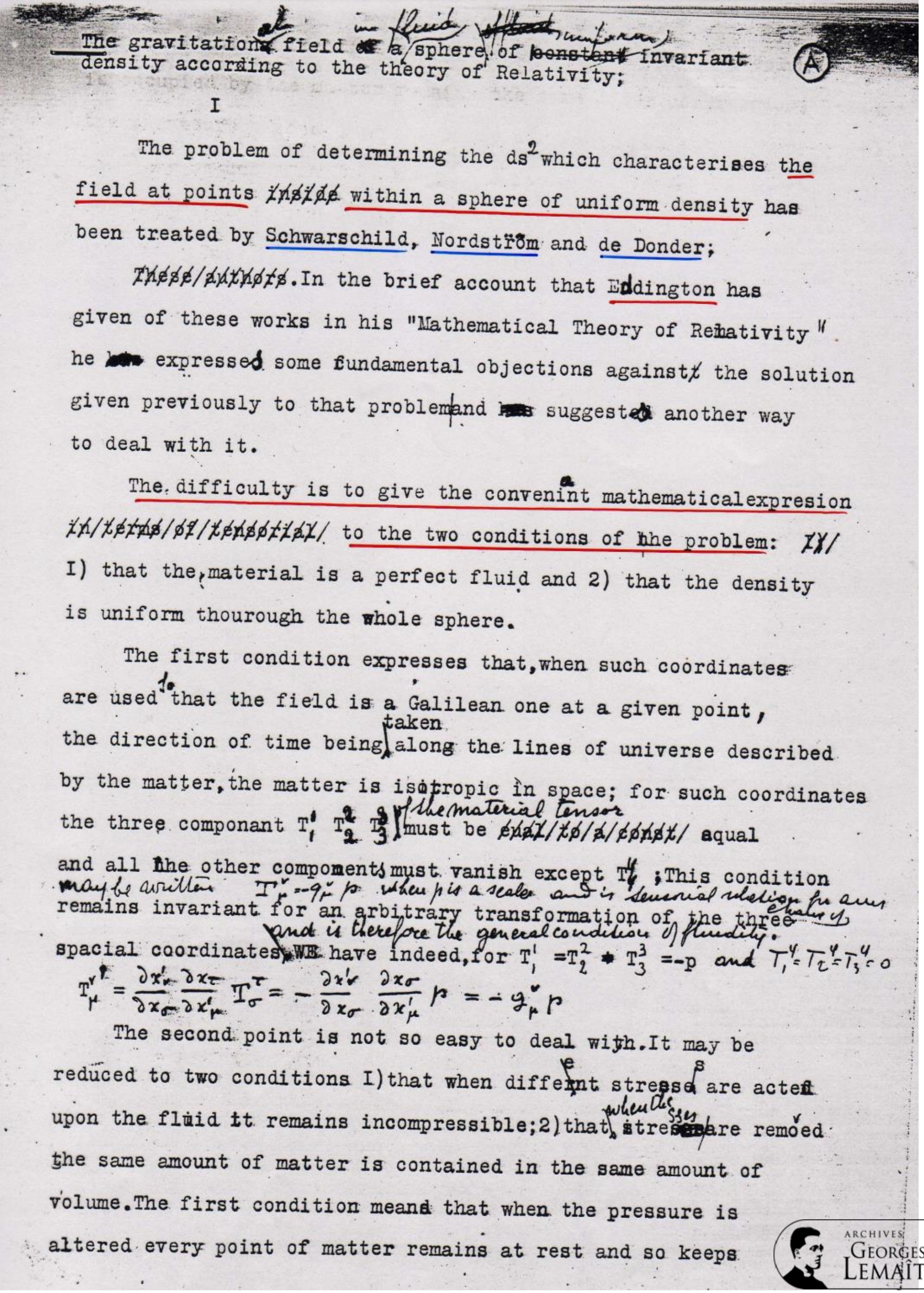
$$\sin\chi = \frac{r}{t_0};$$

we shall have,

$$\frac{d\lambda}{\lambda} = -\frac{r}{t_0} = -\sin\chi. \quad (9)$$

78. Note on de Sitter's universe. G. LEMAITRE, Massachusetts Institute of Technology (introduced by Paul Heymans).—De Sitter's solution of the relativistic equations of the gravitational field in the absence of matter introduces a spurious inhomogeneity which is not simply the mathematical appearance of center of an origin of coordinates, but really attributes distinct absolute properties to a particular point. A new form of de Sitter's solution is given in order to remove this difficulty. New coordinates are introduced and, with the corresponding separation of space and time, the field is found homogeneous but non-statical. Furthermore the geometry is euclidean. The singularity at de Sitter's horizon disappears. The Doppler effect has the numerical value given by Silberstein, but no way is found of introducing his double sign without spoiling the homogeneity of the field. The result that a homogeneous solution of de Sitter's world is non-statical and euclidean is rather against the physical significance of de Sitter's universe. However, the non-statical character of this universe has been advocated as an advantage by Eddington, but euclidean geometry is a very unsatisfactory feature of any conception of the whole universe.

"Note on de Sitter's universe", *Journal of Mathematics and Physics,* t. IV, mai 1925, n°3, pp. 188-192.


The gravitational field ~~of~~ at a fluid sphere of ~~constant~~ uniform invariant density according to the theory of Relativity;

I

The problem of determining the ds^2 which characterises the field at points ~~inside~~ within a sphere of uniform density has been treated by Schwarschild, Nordström and de Donder;

~~These authors~~. In the brief account that Eddington has given of these works in his "Mathematical Theory of Relativity" he ~~has~~ expressed some fundamental objections against the solution given previously to that problem and ~~has~~ suggested another way to deal with it.

The difficulty is to give the convenient mathematical expresion ~~in terms of tensors~~ to the two conditions of the problem: ~~I)~~ I) that the material is a perfect fluid and 2) that the density is uniform thourough the whole sphere.

The first condition expresses that, when such coordinates are used that the field is a Galilean one at a given point, the direction of time being taken along the lines of universe described by the matter, the matter is isotropic in space; for such coordinates the three componant T_1^1 T_2^2 T_3^3 of the material tensor must be ~~equal to a const~~ aqual and all the other components must vanish except T_4^4; This condition may be written $T^\nu_\mu = -g^\nu_\mu p$ when p is a scaler and is tensorial relation for any choice of coordinates remains invariant for an arbitrary transformation of the three spacial coordinates, and is therefore the general condition of fluidity. We have indeed, for $T_1^1 = T_2^2 = T_3^3 = -p$ and $T_1^4 = T_2^4 = T_3^4 = 0$

$$T'^{\nu}_{\mu} = \frac{\partial x'_\nu}{\partial x_\sigma}\frac{\partial x_\tau}{\partial x'_\mu} T^\tau_\sigma = -\frac{\partial x'_\nu}{\partial x_\sigma}\frac{\partial x_\sigma}{\partial x'_\mu} p = -g^\nu_\mu p$$

The second point is not so easy to deal with. It may be reduced to two conditions I) that when different stresses are acted upon the fluid it remains incompressible; 2) that when the stresses are removed the same amount of matter is contained in the same amount of volume. The first condition means that when the pressure is altered every point of matter remains at rest and so keeps

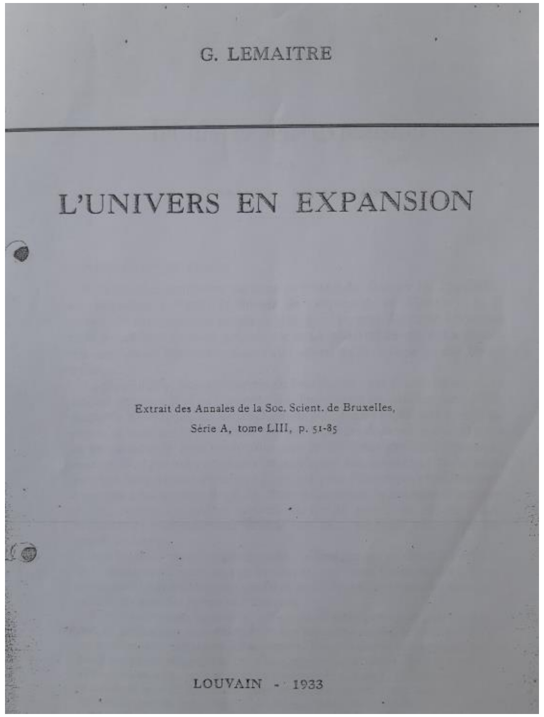
ARCHIVES GEORGES LEMAÎTRE

In his MIT PhD Thesis: study of the singularity in Schwarzschild (interior and exterior) metrics

The PhD Thesis of Lemaître at the M.I.T. 1925 « The gravitational field in a fluid sphere of uniform invariant density according to the theory of Relativity »

All these results are summarizes in a very important 1933 paper:

"L'univers en expansion" (séance du 3 mai 1933, 1re section), *Annales de la Société Scientifique de Bruxelles,* série A : Sciences Mathématiques, t. LIII, 1933, n°2, pp. 51-85.



« **The matter has to find, though, a way of avoiding the vanishing of its volume** » (1997, p.678)

"The Expanding Universe" (English translation by M.A.H.Mac Callum), *General Relativity and Gravitation*, t. 29, 1997, n°5, pp.641-680 (présentation par A.Krasinski, "Editor's Note : The Expanding Universe", *General Relativity and Gravitation*, t. 29, 1997, n°5, pp.637-640).

MIT PhD Thesis 1925-1926 : Lemaître is working on two singular situations… which will be published in 1933!

$$c^2(t,\chi)dt^2 - a^2(t,\chi)d\chi^2 - r^2(t,\chi)(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$[T_\mu^\nu] = \begin{bmatrix} \rho & & & \\ & -p & & \\ & & -\tau & \\ & & & -\tau \end{bmatrix}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa T_{\mu\nu}$$

$$4\pi r^2 \rho \frac{\partial r}{\partial \chi} = \frac{\partial m}{\partial \chi}$$

$$4\pi r^2 p \frac{\partial r}{\partial t} = -\frac{\partial m}{\partial t}$$

$$\frac{1}{c^2}\left(\frac{\partial r}{\partial t}\right)^2 = -\left(1 - \frac{1}{a^2}\left(\frac{\partial r}{\partial \chi}\right)^2\right) + \frac{2m}{r} + \frac{\Lambda r^2}{3}$$

$$\frac{\partial p}{\partial \chi} + \frac{2}{r}\frac{\partial r}{\partial \chi}(p - \tau) + \frac{1}{c}\frac{\partial c}{\partial \chi}(p + \rho) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{2}{r}\frac{\partial r}{\partial t}(\rho + \tau) + \frac{1}{a}\frac{\partial a}{\partial t}(\rho + p) = 0$$

$$V = \pi^2 R^3(t)$$

$$dU = \delta Q - p dV$$

$$\delta Q = 0, U = \rho V$$

Lemaître-Tolman inhomogeneous model

$$\frac{\partial r}{\partial t} = 0 \; ; r = a\sin\chi$$

Schwarzschild (interior)

Cfr J. Eisenstaedt

$$ds^2 = -a^2\left[d\chi^2 + \sin^2\chi(d\theta^2 \sin^2\theta d\varphi^2)\right] + \left[f_1(t) - f_2(t)\cos\chi\right]^2 dt^2$$

$$p = \frac{\rho f_2(t)\cos\chi}{f_1(t) - f_2(t)\cos\chi}$$

$f_1(t) = 0 \; ; \; \rho = 0$ (de Sitter)

$f_2(t) = 0 \; ; \; \kappa\rho = 2\lambda$ (Einstein)

Schwarzschild (exteriror)

and the new form of the field is written without singularity

$$ds^2 = -2m\frac{d\chi^2}{r} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + dt^2, \qquad (11.12)$$

where

$$r = \left[\tfrac{3}{2}\sqrt{2m}\,(t - \chi)\right]^{2/3}. \qquad (11.13)$$

The singularity of the Schwarzschild field is thus a fictitious singularity, analogous to that which appears at the horizon of the centre in the original form of the de Sitter universe.

Eddington 1924; Synge 1950-Fronsdal 1959 (Eddington-Lemaître coordinates map only the half of space-time) ; Kruskal 1960

$$a = \frac{1}{f(\chi)}\frac{\partial r}{\partial \chi}$$

$$\left(\frac{\dot r}{r}\right)^2 + \left(\frac{1 - f^2}{r^2}\right) + 2\frac{\dot r'}{r'}\frac{\dot r}{r} - 2\frac{ff'}{rr'} = \Lambda + \kappa\rho$$

$$\left(\frac{\dot r}{r}\right)^2 + 2\frac{\ddot r}{r} + \left(\frac{1 - f^2}{r^2}\right) = \Lambda - \kappa p$$

$$\dot r^2 = -(1 - f^2) + 2\frac{m(t,\chi)}{r} + \frac{\Lambda r^2}{3}$$

$$\frac{\partial \tau}{\partial \varphi} = \frac{\partial \tau}{\partial \theta} = 0$$

$$p = \tau,$$

$$(\tau - p)\frac{\partial r}{\partial \chi} = \frac{r}{2}\frac{\partial p}{\partial \chi}$$

$$\dot\rho + \left(\frac{\dot r'}{r'}\right)(\rho + p) + 2\left(\frac{\dot r}{r}\right)(\rho + \tau)$$

FLRW

$$r = R(t).E(\chi)$$

$$E(\chi) = \sin\chi, 1, \sinh\chi \;,\; f(\chi) = \cos\chi, 1, \cosh\chi$$

$$m = M\sin^3\chi \qquad m = M \qquad m = M\sinh^3\chi$$

An answer to Einstein objection: Lemaître is still working on singularities!

Lemaître showed that one cannot avoid singularity using Bianchi 1 metric
(an example of anisoptropy):bye bye « Phoenix Universes »? (Expanding Universe, p. 27).

- 27 -

8. PHOENIX UNIVERSES.

In the collapsing type of motion, the radius starts from zero and comes back to zero. Mathematically it could be possible to consider, that later on the radius starts again from zero and repeats over and again the same oscillations from zero to a maximum. This is particularly clear for the limiting case, the cycloidal universe, in which we would imagine that the full cycloids, with its infinite number of branches could be used.

This would give a kind of periodical universe which will pass over and over through the same periods of expansion and contraction.

The difficulty of this theory, comes from the vanishing of the radius, and a number of investigations have been made to see if by suitable modifications of the very idealised conditions postulated by Friedmann, it would not be possible to modify the conditions so that this zero would not be strictly a zero but would mean a very small value, then the radius would oscillate between a maximum and a very small minimum.

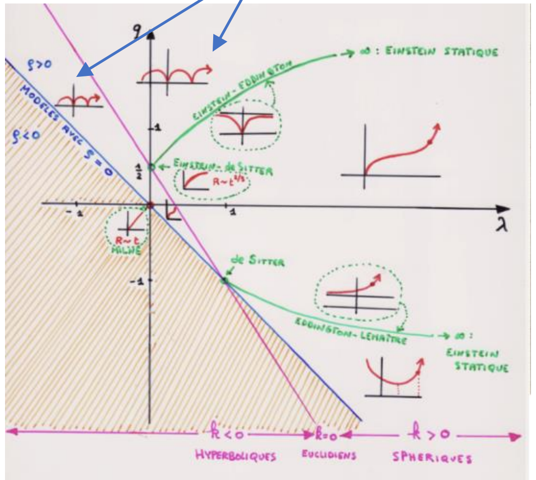
At first side it seems, that such thing is a very simple matter. We are free to consider any law of variation we like for the radius of the universe, and then to use the formulae to compare the density and the pressure.

But in order to accept such a solution as more than a mathematical exercice, we must find values of ρ and p acceptable from a physical point of view. For astronomical applications we have to consider an idealised gas formed of stars, nebulae and light. For such a gas density and also pressure must essentially be positive quantities.

Investigations by Tolman have shown that with such restrictions it is impossible to replace the zero value of the radius by a small non vanishing quantity. The effect of any positive pressure is even to precipitate the contraction.

Another line of attack has been followed by the author of this article, according to a suggestion privately made by Einstein. It was to investigate if the precise vanishing of the radius was not due to the assumption of perfect homogeneity of the repartition of matter. The suggestion was to investigate the behaviour of an universe defined by ~~equal to~~

$$ds^2 = -b_1^2 dx_1^2 - b_2^2 dx_2^2 - b_3^2 dx_3^2 + b_4^2 dx_4^2$$



12. THE VANISHING OF SPACE

The radius of space may pass through zero. We intend to discuss this passage, and to examine in particular if there is a way of interpreting this zero value of the radius physically as simply representing a small quantity and, in this case, of fixing its order of magnitude.

For the study of the zero point, we may neglect the cosmological constant; setting

$$\frac{Km}{c^2} = a, \tag{12.1}$$

we then have

$$\frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 = -1 + \frac{2a}{R}. \tag{12.2}$$

Introducing the angular distance U crossed by light during the time t,

$$dU = \frac{cdt}{R}, \tag{12.3}$$

we easily find Einstein's cycloidal universe

$$R = a(1 - \cos U),$$
$$ct = a(U - \sin U). \tag{12.4}$$

When U varies from 0 to π, R returns to its initial zero value, and light just has time to go round the simply elliptic space.

The question is to know if there is a way to smooth out the cusp of the cycloid.

One can ask first of all, if one would not obtain this result if one took into account the effect of the pressure which need not necessarily be negligible. It is easy to see, going back to the equation (3.7), that the

pressure only reinforces the gravitational action. Besides, the question has been treated in detail by Tolman.[9]

It is more important to examine the effect of a lack of isotropy in the distribution of tensions.

We intend to examine, following an idea which Einstein communicated to us, a universe defined by

$$\boxed{ds^2 = -b_1^2 dx_1^2 - b_2^2 dx_2^2 - b_3^2 dx_3^2 + dx_4^2} \tag{12.5}$$

where b_1, b_2 and b_3 are functions of $x_4 = t$.

Such a universe is naturally inadmissible from many points of view, but it has the interest of introducing a marked and largely arbitrary anisotropy.

We can easily calculate the matter tensor by the formulae of Section 1.

We have, for k and i different from 4, by (1.9),

$$\beta_{ik} = \frac{b_i' b_k'}{b_i b_k} \tag{12.6}$$

the primes denoting derivatives with respect to t, and

$$\beta_{i4} = \frac{b_i''}{b_i}. \tag{12.7}$$

The components $T_{\mu\nu}$ $(\mu \neq \nu)$ vanish.

$$\sqrt{-g} = b_1 b_2 b_3 = R^3 \tag{12.8}$$

measures the volume occupied by a specified part of the matter. Here R is no longer the radius of the universe, since the space is Euclidean, but the volume of space tends to zero if R tends to zero.

We have

$$\frac{3R'}{R} = \frac{b_1'}{b_1} + \frac{b_2'}{b_2} + \frac{b_3'}{b_3}$$

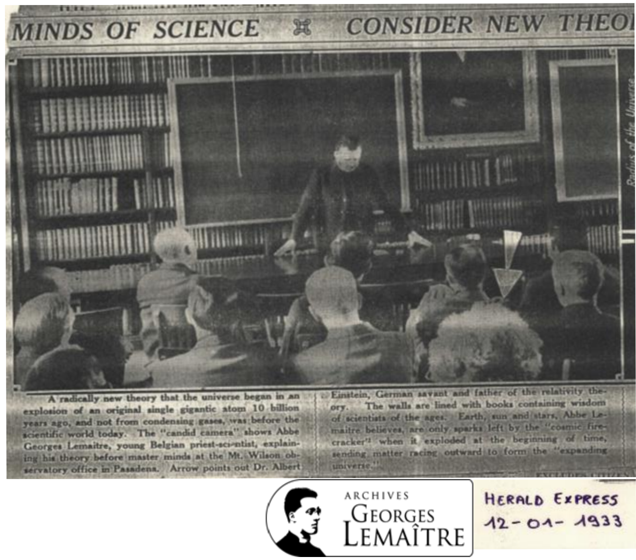
and

$$3\left(\frac{R''}{R} - \frac{R'^2}{R^2}\right) = \frac{b_1''}{b_1} + \frac{b_2''}{b_2} + \frac{b_3''}{b_3} - \frac{b_1'^2}{b_1^2} - \frac{b_2'^2}{b_2^2} - \frac{b_3'^2}{b_3^2}.$$

[9] This reference is: Tolman, Richard C., and Ward, M. (1932). "On the Behavior of Non-Static Models of the Universe when the Cosmological Constant is Omitted." Physical Review 39, 835-843 — Transl.

« The Expanding Universe, p. 27 »

One of the Lemaître's 1933 Lectures in Pasadena



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7 Primeval Atom Hypothesis is really a scientific notion: it can be falsified

The existence of a « pre-big Bang » era has to be tested

For example: the theoretical possibility of the Phoenix Universes…(Lemaître liked it!).

But according to Lemaître: they do not correspond to observations…

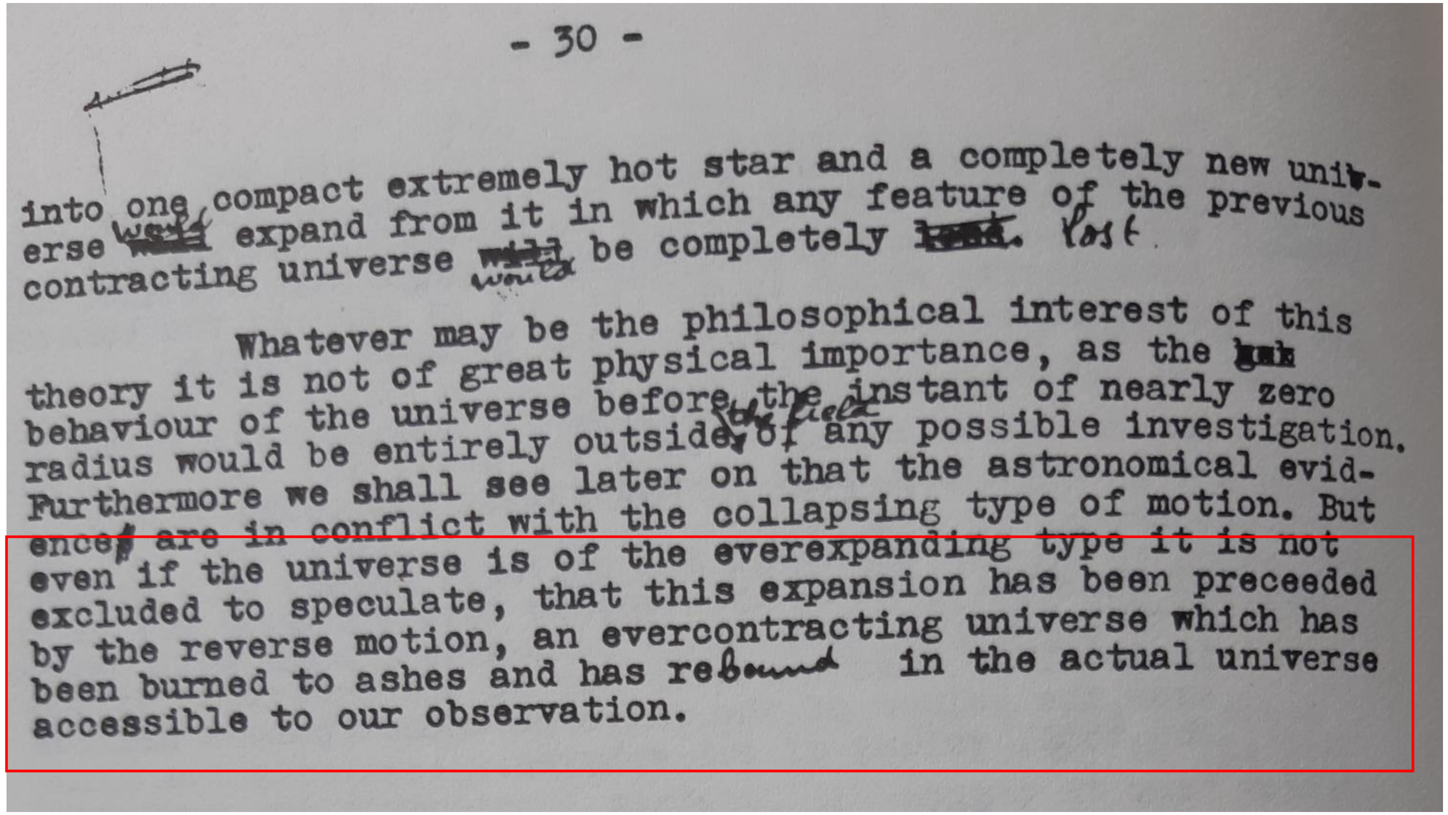
« We must exclude the solutions where the radius is less than the equilibrium radius and in particular the quasi-periodic solutions. **For a purely aesthetic point of view, one may perhaps regret this**. Those solutions where the universe expands and contracts successively while periodically reducing itself to an atomic mass of the dimensions of the solar system, have **an indisputable poetic charm** and make one think of the Phoenix of legend. » (p. 679)

"The Expanding Universe" (English translation by M.A.H.Mac Callum), *General Relativity and Gravitation*, t. 29, 1997, n°5, pp.641-680 (présentation par A.Krasinski, "Editor's Note : The Expanding Universe", *General Relativity and Gravitation*, t. 29, 1997, n°5, pp.637-640).

Remark : Lemaître does not exclude a pre-Big Bang era. He said that esthetically he would have prefered that case!!!

The initial singularity which cannot be avoided is not necessarily an absolute beginning

« It is not excluded to speculate, that this expansion has been preceeded by the reverse motion, an evercontracting universe which has been burned to ashes and has rebund in the actual universe… »



- 30 -

into one compact extremely hot star and a completely new universe ~~were~~ expand from it in which any feature of the previous contracting universe ~~will~~ would be completely lost.

Whatever may be the philosophical interest of this theory it is not of great physical importance, as the behaviour of the universe before the instant of nearly zero radius would be entirely outside the field of any possible investigation. Furthermore we shall see later on that the astronomical evidences are in conflict with the collapsing type of motion. But even if the universe is of the everexpanding type it is not excluded to speculate, that this expansion has been preceeded by the reverse motion, an evercontracting universe which has been burned to ashes and has rebound in the actual universe accessible to our observation.

« The Expanding Universe, p. 30 »

But Lemaître excluded that possibility due to 3 reasons:

- 1Thermodynamical reason (S is not so high today)
- 2 Quantum physics reason (forces prevent the collapse)

It is sometimes really possible to go through the singularity but they are objections… Phoenix universes have observable properties which can be refutable…

It is of some interest to consider what would have happened if we imagine as a preuniverse a universe perfectly similar to our own but going in the reverse direction, a contracting universe, collapsing to the zero, being burned into a small sphere and rebounding. This universe will have the same general features as our universe, it will even form stars and nebulae, but some essential features would be quite different.

First of all it would contain less uranium and thorium, about one tenth, and much more lead as a product of radioactive disintegration. But the chief difference would be that there will be no cosmic rays. There will be some indications that it is not formed of fresh matter.

There is a nice way to obtain a universe formed of fresh matter, I must admit that it has a very hypothetical character but I think that it is the duty of the theoretist to face the most absolute form of his line of thought even when this would lead to uncertain extrapolation from the known facts. I mean the primeval atom hypothesis.

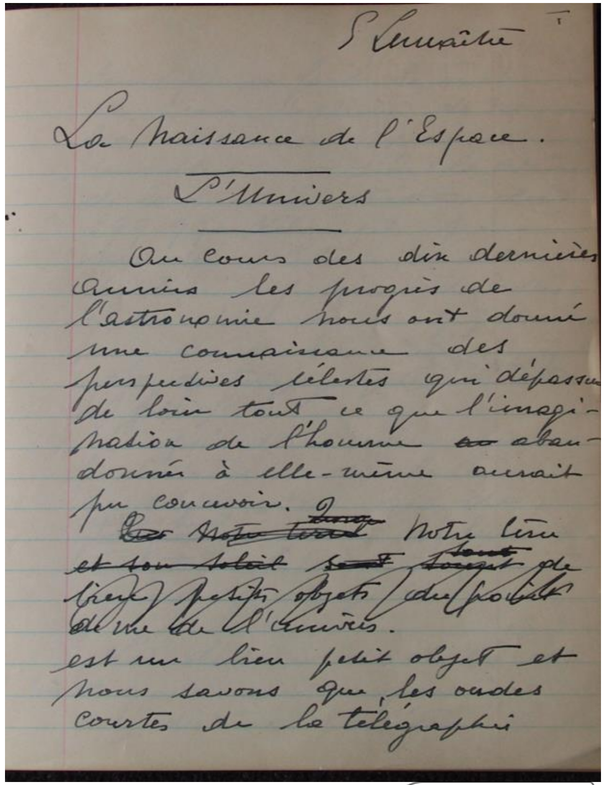
The universe would have started as an atom with atomic weight equal to the total mass of the universe. The radioactive disintegration of the primeval atom and the developments of space from zero would be two simultaneous processes. The atom would break into two parts, equal or unequal, each piece would break again until after some two hundred and fifty

No cosmic rays!

The universe today (and its « fresh » matter) is not compatible with the existence of a contracting preuniverse

(« The Expanding Universe », p. 48)

After 1931 Lemaître explains one of his main motivations…to propose Primeval Atom Hypothesis… and the necessity to express the ideas in a strong way!



G. Lemaître

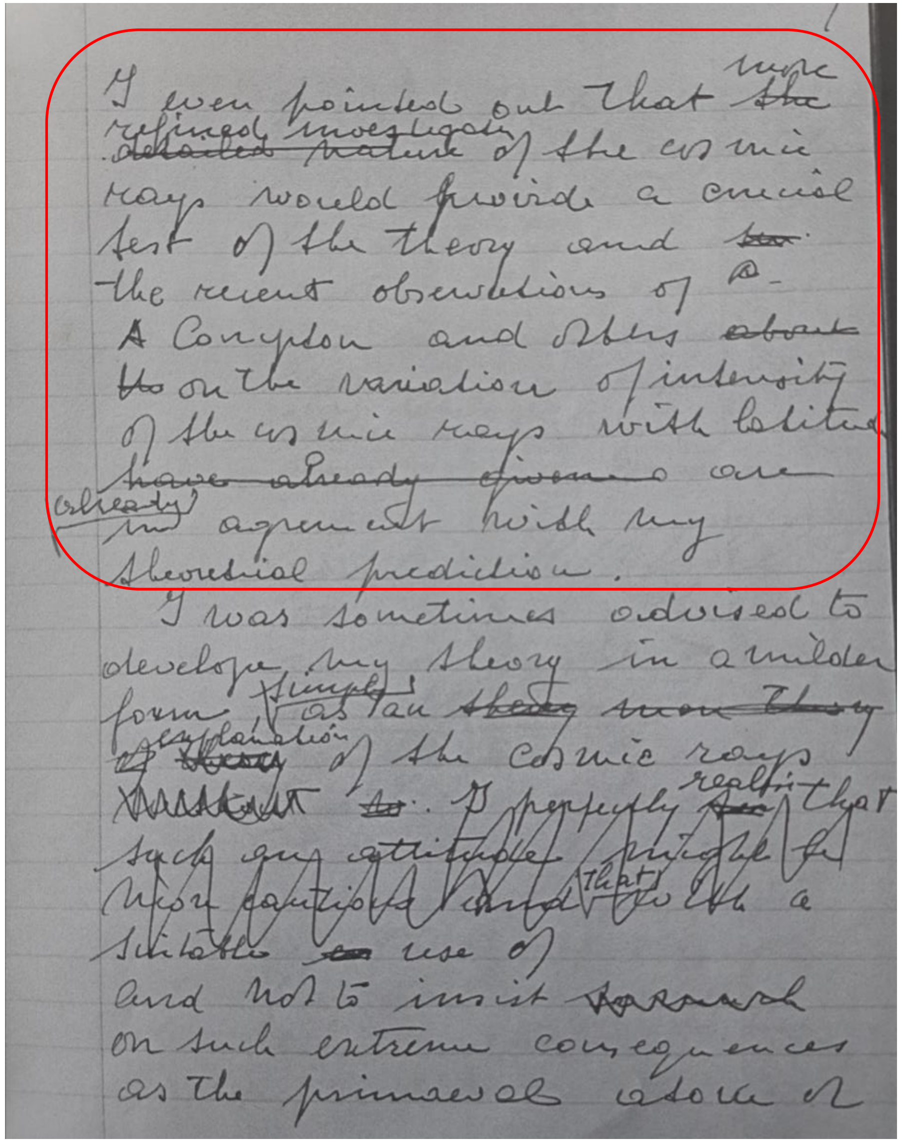
La naissance de l'Espace.

L'Univers

Au cours des dix dernières années les progrès de l'astronomie nous ont donné une connaissance des perspectives célestes qui dépassent de loin tout ce que l'imagination de l'homme ~~ou~~ abandonné à elle-même aurait pu concevoir. ~~Le~~ ~~Notre terre~~ Notre terre ~~et son soleil sont~~ [illegible] est un bien petit objet et nous savons que les ondes courtes de la télégraphie

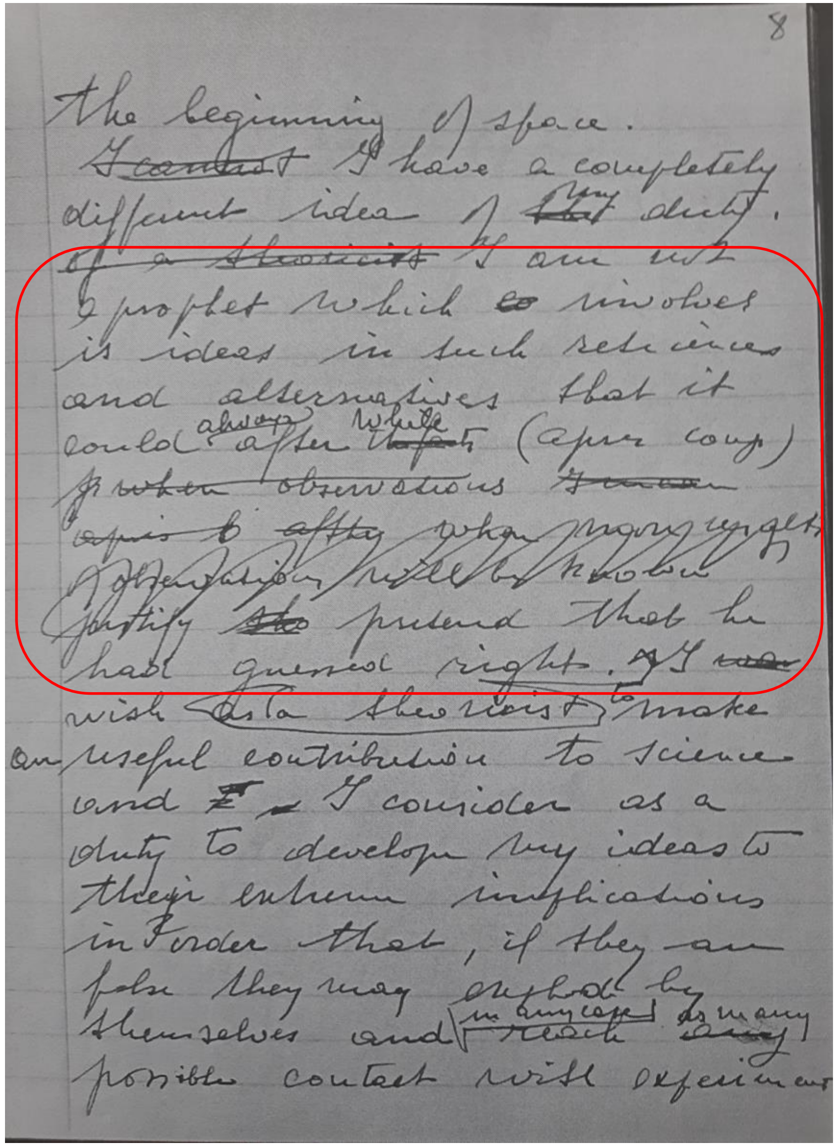
Archives Georges Lemaître

The crucial test of the Primeval Atom Hypothesis is the existence of Ultra-High-Energy Cosmis Rays!



I even pointed out that more ~~the~~ refined investigation ~~detailed nature~~ of the cosmic rays would provide a crucial test of the theory and ~~the~~ the recent observations of A. Compton and others ~~about~~ ~~the~~ on the variation of intensity of the cosmic rays with latitude ~~have already given~~ are already in agreement with my theoretical prediction.

I was sometimes advised to develop my theory in a milder form [illegible] of the cosmic rays [illegible]. I perfectly realise that such an attitude [illegible] would be more cautious [illegible] suitable use of and not to insist on such extreme consequences as the primaeval atom a

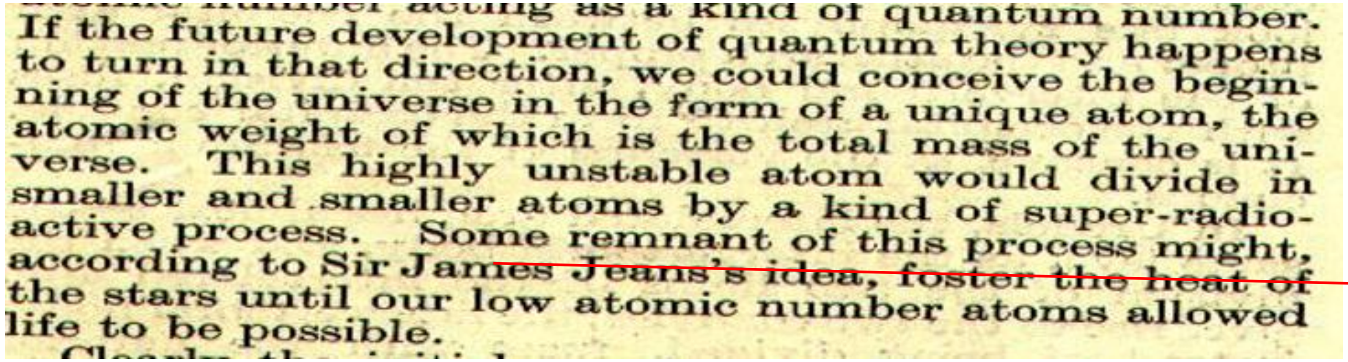


8

The beginning of space.

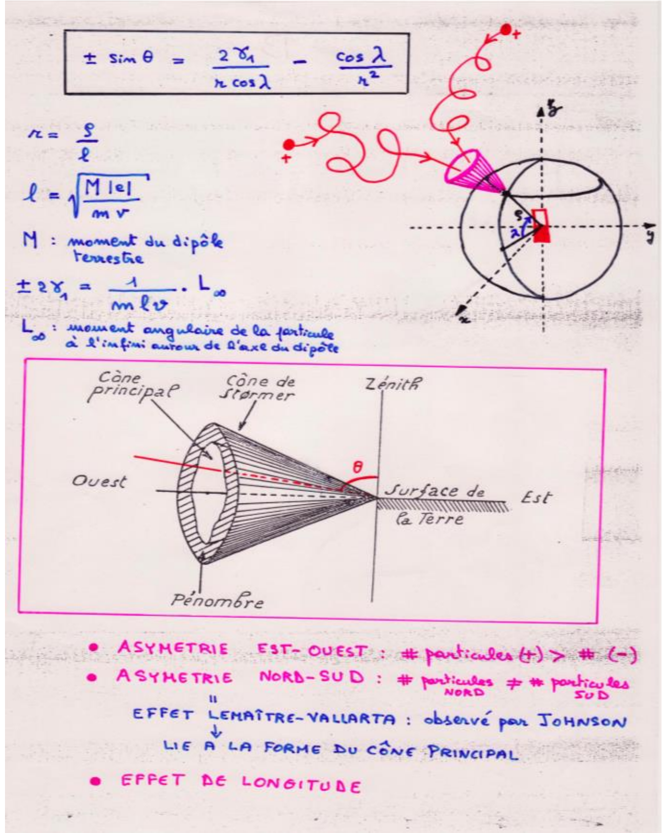
~~I cannot~~ I have a completely different idea of my duty. ~~of a theorist~~ I am not a prophet which involves his ideas in such reticences and alternatives that it could always after (après coup) when observations [illegible] will be known justify ~~the~~ pretend that he had guessed right. As I wish as a theorist to make an useful contribution to science and I consider as a duty to develop my ideas to their extreme implications in order that, if they are false they may exploded by themselves and in any case reach as many possible contact with experiment

What is remaining of the first moments of the Universe (Ultra-High Energy Cosmic rays): the beautiful mistake!

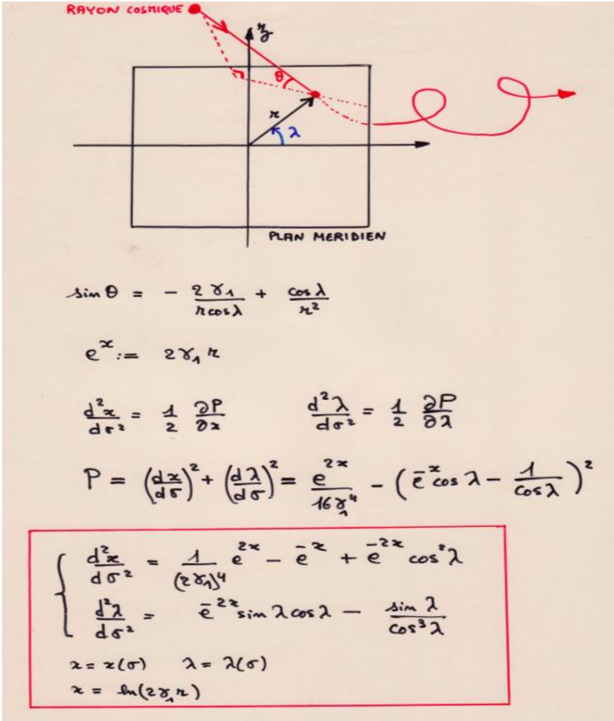

If the future development of quantum theory happens to turn in that direction, we could conceive the beginning of the universe in the form of a unique atom, the atomic weight of which is the total mass of the universe. This highly unstable atom would divide in smaller and smaller atoms by a kind of super-radio-active process. Some remnant of this process might, ~~according to Sir James Jeans's idea, foster the heat of~~ the stars until our low atomic number atoms allowed life to be possible.

Nature 1931

According to Lemaître: the disintegration products of the Primeval Atom give rise to the elements and to the cosmic rays. He computed many orbits of such charged particle using original numerical methods and computer (like the Bush machine at the MIT and his own computer)



How to compute the orbits of Cosmic rays in the magnetic field of the Earth?




Vannevar Bush computer, MIT


ARCHIVES GEORGES LEMAÎTRE

Lemaître computer, 1958

The Störmer Problem

— 7 —

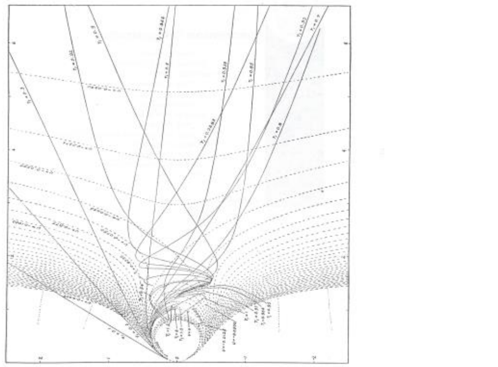


FIG. 1. — Modèle mécanique de C. Störmer. (*Terrestrial Magnetism and Atmospheric Electricity*. Vol. 37, 1932, 375).

Les lignes pointillées sont les lignes de niveau de la surface U (désignée par — U dans les inscriptions sur la figure).

Le col est au point de coordonnées (2,0), la cuvette est à sa gauche, les deux vallées en forme de cornes aboutissent au dipôle qui se trouve à l'origine. Le fond des vallées est la ligne U = 0.

Les courbes en trait plein sont des trajectoires différentes de celles que nous discutons dans le texte. Elles correspondent à une valeur de $\sigma = 1/16\ \gamma_1^4$. Elles aboutissent au dipôle par la vallée supérieure.

La trajectoire $v_1 = 0{,}7$ partant du bas à droite de la figure est très voisine de celles que nous discutons.

On the Allowed Cone of Cosmic Radiation

G. LEMAITRE AND M. S. VALLARTA, *University of Louvain and Massachusetts Institute of Technology*

(Received July 15, 1936)

Further results of an extensive study of trajectories asymptotic to a known family of unstable periodic orbits in the earth's dipolar magnetic field, carried out by means of Bush's differential analyzer, are presented in this paper. A detailed discussion is given of our methods of determining asymptotic trajectories by means of the differential analyzer and by numerical integration of a whole family at a time; comparison of the results obtained shows the absence of systematic errors of any consequence in the mechanical integrations and exhibits the precision attained with the differential analyzer. The families of asymptotic trajectories are then analyzed systematically in order to determine the main cones for latitudes up to 30°. This leads to the theory of the azimuthal effect and a study of the region in the vicinity of the zenith.

In a preceding paper[1] to which reference should be made for a complete statement of the problem treated here and of our methods of attack, we gave results obtained from the analysis of some three hundred asymptotic trajectories to a known family of unstable periodic orbits found by means of Bush's differential analyzer[2] and discussed fully the sections of the main allowed cones of cosmic radiation by the meridian plane, which in turn led us to the theory of the north-south asymmetry. Those results were presented at the time with reservations as far as a critical examination of their precision was concerned. We have now been able to complete the calculations announced in our preceding paper, to which we shall return below, with the result that suspected systematic errors are so small that they can well be neglected. Certain other points which were then summarily sketched will now be developed in detail; in particular we shall present here a full discussion of our method of determination of the asymptotic trajectories by means of the differential analyzer and a fairly complete determination of the main allowed cones for geomagnetic latitudes up to 30°. The last two sections will be devoted to the analysis of the azimuthal effect and to a study of the region in the vicinity of the zenith.

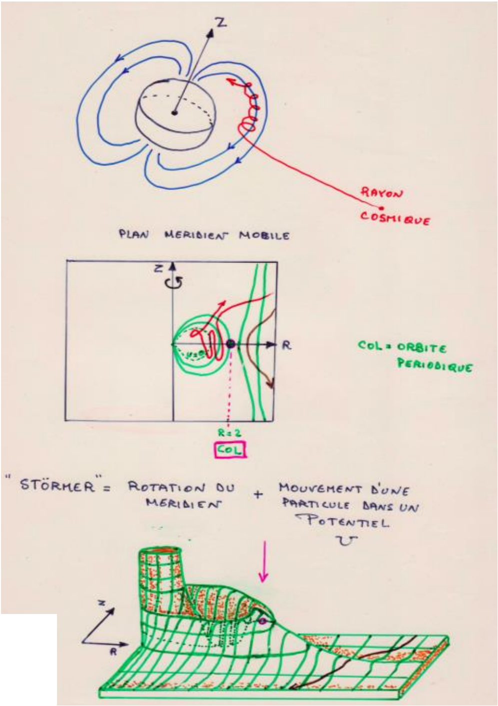
1. The Determination of Asymptotic Trajectories by Means of Bush's Differential Analyzer

The differential equations of motion to be integrated are (reference 1, Eqs. (4), (5))

$$d^2x/d\sigma^2=(1/(2\gamma_1)^4)e^{2x}-e^{-x}+e^{-2x}\cos^2\lambda,$$
$$d^2\lambda/d\sigma^2=e^{-2x}\sin\lambda\cos\lambda-(\sin\lambda/\cos^3\lambda). \quad (1)$$

Our problem is to find the solutions of these differential equations corresponding to trajectories asymptotic to a known family of unstable periodic orbits.

[1] G. Lemaitre and M. S. Vallarta, Phys. Rev. **49**, 719 (1936).
[2] V. Bush, J. Frank. Inst. **212**, 447 (1931).



Lemaître and M.S. Vallarta

MAY 15, 1936 PHYSICAL REVIEW VOLUME 49
Printed in U. S. A.

On the Geomagnetic Analysis of Cosmic Radiation

G. LEMAITRE AND M. S. VALLARTA,* *University of Louvain and Massachusetts Institute of Technology*

(Received March 30, 1936)

The results of an extensive study of trajectories asymptotic to a certain family of periodic orbits in the earth's magnetic field, carried out by means of Bush's differential analyzer, are presented in this paper. The theory of the region of full light, or main cone, is fully discussed. Attention is then restricted to the section of the main cone in the plane of the geomagnetic meridian and it is shown that the north-south asymmetry furnishes the most direct approach to the analysis of the energy spectrum in a wide region, independently of the particles' sign. Further it is shown that the general shape and the minimum of the north-south asymmetry discovered by Johnson in the course of his Mexican experiments are fully accounted for by the action of the earth's field.

1. Introduction

Recent experiments by the method of multiple coincidences carried out by Johnson[1] in Mexico (geomagnetic latitude 29°N) and by Clay[2] in Java (geomagnetic latitude 18°S) have shown that in the northern geomagnetic hemisphere the intensity of cosmic radiation in the geomagnetic meridian is, for equal zenith angles, greater from the south than from the north; conversely in the southern hemisphere it is greater from the north than from the south.[3] That this is a consequence of the action of the earth's magnetic field on the motion of charged particles was pointed out by the present authors as early as 1932.[4] Shortly afterwards Bouckaert[5] was able to calculate this north-south asymmetry for geomagnetic latitudes up to 20° and moderate zenith angles. Considerable difficulties stand in the way of extending these calculations to higher latitudes and larger zenith angles. The use of Bush's differential analyzer,[11] which was made available to us to carry out the investigation reported here, has made it possible to include in the present analysis latitudes as high as 40° and, in most cases, zenith angles as far as the horizon. Thus with the completion of these researches the problem of determining the allowed main cone, or region of full light, for the latitudes mentioned is essentially solved.

The chief results of our present research are, first, that the north-south asymmetry, already discussed in a previous paper,[6] depends for each latitude on a narrow band of energy. Therefore, as further shown in the sequel, the experimental study of this asymmetry provides a direct workable method for the analysis of the spectral distribution of corpuscular cosmic radiation, independent of the latter's sign. Second, we show that the minimum of the north-south asymmetry found by Johnson at a geomagnetic latitude 29°N and at about 45° zenith angle, as well as the general features of his experimental results, are fully accounted for by the action of the earth's field. In fact the theory predicts a slight reversal of the sign of the asymmetry for zenith angles between 45° and 55° which depends on the existence of a very narrow energy band. This reversal begins at latitudes around 20° for for zenith angles about 70° and eventually broadens out so as to include the whole region between the zenith and 40° at latitudes around 35°. Contrary to our earlier[6] suggestion it is not necessary to invoke atmospheric absorption in order to account for this minimum.

The results we wish to present here must be considered as preliminary insofar as a critical examination of their precision is concerned. There is in fact ground for suspecting that our method of finding asymptotic trajectories leads to slightly too large a value for the aperture of

* At present Visiting Professor of the C.R.B. Educational Foundation at the University of Louvain.
[1] T. H. Johnson, Phys. Rev. **47**, 91 (1935).
[2] J. Clay, Physica **8**, 867 (1935).
[3] A northern excess in Eritrea (geomag. lat. 11.5°N) at a zenith angle of 45°, slightly larger than the rather large experimental error, has been reported by B. Rossi, Ricerca Scient. **5**, 583 (1934).
[4] G. Lemaitre and M. S. Vallarta, Phys. Rev. **43**, 87 (1933).
[5] L. Bouckaert, Ann. de la Soc. Sci. de Bruxelles, **A54**, 174 (1934).
[6] G. Lemaitre, M. S. Vallarta and L. Bouckaert, Phys. Rev. **47**, 434 (1935). Full references bearing on our application of Liouville's theorem to the present problem are given in the footnote on p. 435.

The fruitfull mistake: Van Allen Belt theory!

Whatever the status of the primeval atom it can be considered as a fruitful image giving rise to many interesting Theoretical and observational consequences

The Primeval atom lead for example Lemaître to propose the idea of the existence of a fossil radition…

What can be the epistemological status of the Primeval Atom Hypothesis?

- **Primeval Atom wa never described mathematically : No equations…but full of intuitions…!**

- **Primeval Atom is a powerfull image belonging to what Philosophers of Science called the context of discovery…**

- **A kind of regulating idea able to generate theoretical or experimental notions, concepts, …**

What Lemaître's story shows is that « generative concepts » and « image » are maybe very important and necessary in the process of ceating new mathematical concept and new observational test in cosmology…

Primeval atom was a crucial yet ambiguous image (but this is its force!) in Lemaître's work, helping him to direct his thought in a field where nobody knew at that time where to go and how?

Primeval Atom Hypothesis as you see here was a crucial node in the intellectual network of Lemaître.

I have tried to answer two questions:

1) **From where did it come?** 2 sources!

   Millikan-Cameron (**Cosmic rays**) /reaction to Eddington

2) **What is the nature of the Primeval Atom?** An **ambiguous** quantum image! 3 interpetations!

   A state from which space-time-matter is emerging

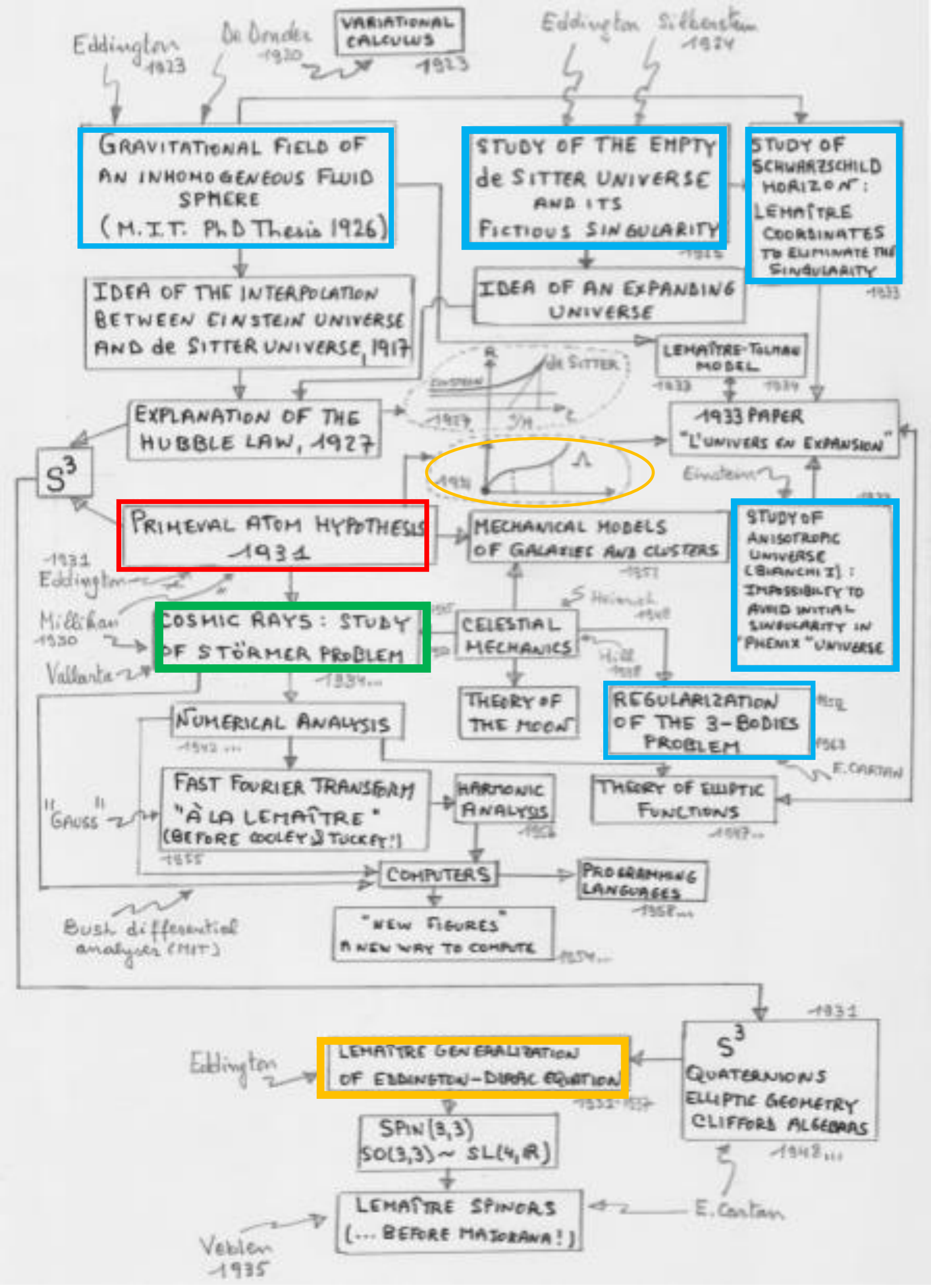
   An initial nucleus (//Gamov-Mayer-Teller polyneutron)

   A way to give meaning to inescapable r=0.

The **Primeval Atom Hypothesis** is connected to three main subjects of Lemaître's works:

1) The study of **Singularities** is one of the major theme of Lemaître's work (not only in cosmology but also in Celestial Mechanics and…) : **Primeval Atom Hypothesis** gives meaning to an initial inescapable singularity (natural beginning)
2) The study of **quantum theory**… à la Eddington! Algebra gives a way to describe the state from wich space-time physics is emerging
3) The study of **Cosmic rays :** a consequence of the **Primeval Atom Hypothesis**

This lead us maybe to reflect on the role of « images » in generating fruitful concepts…



Regularization of binary shocks in3-Body Problem:

From 1950 and 1964: Lemaître was still working on SINGULARITIES but in Classical Mechanics

Extrait du *Bulletin de l'Académie royale de Belgique* (Classe des Sciences)
Séance du 7 juin 1952

MÉCANIQUE CÉLESTE

Coordonnées symétriques dans le problème des trois corps,

par G. LEMAÎTRE,
Membre de la classe.

Résumé. — Le problème des trois corps dans le cas plan et pour des masses égales est équivalent au mouvement d'un point sous l'action d'une fonction des forces et d'un potentiel vecteur.
On montre comment dans le cas général l'asymétrie introduite par l'inégalité des masses modifie l'hamiltonien.

1. — PROBLÈME PLAN A MASSES ÉGALES.

Considérons tout d'abord le cas particulier du problème des trois corps où les masses sont égales et où les vitesses initiales sont telles que les trois corps restent dans un même plan.

Au lieu des trois distances r_i ($i = 1, 2, 3$), introduisons trois coordonnées r, σ et χ de telle façon que les coordonnées cartésiennes des trois corps par rapport à des axes rectangulaires convenables soient

$$(1) \qquad \begin{aligned} x_i &= r\sqrt{\frac{2}{3}}\cos\sigma\cos\left(\chi + \frac{2\pi i}{3}\right) \\ y_i &= r\sqrt{\frac{2}{3}}\sin\sigma\sin\left(\chi + \frac{2\pi i}{3}\right) \end{aligned}$$

Ceci est possible, car de (1) on déduit aisément les distances mutuelles des corps et on trouve

$$(2) \qquad r_i = r\sqrt{1 - \cos 2\sigma\cos\left(2\chi - \frac{2\pi i}{3}\right)}$$

Georges Lemaître in Berkeley: at the *Space Science Laboratory* (NASA Grants!)

Space Sciences Laboratory
University of California
Berkeley 4, California

THE THREE BODY PROBLEM

by

Prof. C. G. Lemaitre

Technical Report on
NsG 243-62 and NsG 122-61

Series 4
Issue 49

This work was jointly supported by NASA Grants NsG 243-62 and NsG 122-61.

August 20, 1963

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