Classical Gravity Knows About Quantum Mechanics

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I. General Relativity knows about its own quantum states

Two ways to determine the entropy of a system in a thermal state:

(1) In your lab, measure how much energy you have to add to raise the temperature. Integrating

$$dS = \frac{dE}{T} \ .$$

yields $S(T)$.

(2) If you know the microscopic structure of the system (for example, a monatomic ideal gas, a particular crystal, etc.), determine the spectrum and compute the partition function. Let's review how this works.

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Consider the unnormalized density operator

$$e^{-\beta H} = \sum_{k=0}^{\infty} e^{-\beta E_k} \left| k \right\rangle \left\langle k \right| \ .$$

The trace of this operator is the thermal partition function,

$$Z(\beta) \equiv \mathrm{Tr}\, e^{-\beta H} = \sum_k e^{-\beta E_k} \ .$$

Notice that this trace can be evaluated by performing a path integral around a Euclidean-time circle of length β. The action is $I = \beta H$.

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To get a normalized density operator, we need to divide by the partition function:

$$\rho(\beta) = \frac{e^{-\beta H}}{Z(\beta)} \ .$$

The entropy of the state is:

$$S \equiv -\mathrm{Tr}\, \rho \log \rho = (1 - \beta \frac{d}{d\beta}) \log Z(\beta) \ .$$

I. General Relativity knows about its own quantum states

For a black hole, we have neither experimental access, nor do we know its microscopic structure.

Nevertheless, we have tremendous confidence that the entropy of any black hole is given by

$$S = \frac{A}{4G\hbar} \ ,$$

where G is Newton's constant and A is the area of the event horizon of the black hole. (I have set $c = k_B = 1$.)

How can we possibly know this?

I. General Relativity knows about its own quantum states

Hawking (1974) computed the state of the quantum fields in the presence of a black hole, assuming no new matter is added. He found that the outgoing modes far from the black hole are in a thermal state, with temperature $\hbar/(8\pi GE)$, where E is the mass of the black hole. The most outrageous aspect of this result is not *that* black holes radiate, but the claim that they radiate thermally with a particular temperature. This is like computing the radiation spectrum of an unknown substance given only its thermal energy. Indeed, Hawking's calculation determines the entropy of a black hole, via the first law:

$$S = \frac{A}{4G\hbar} \ .$$

(See also Bekenstein 1972.)

I. General Relativity knows about its own quantum states

Gibbons and Hawking (1977) reproduced this result in a new, suggestive way, starting with two radical assumptions:

1. Quantum gravity is a conventional quantum-mechanical theory that can be associated to the boundary of the spacetime.

2. The boundary path integral can be evaluated by integrating over geometries that “fill in” the boundary conditions.

These assumptions were not spelled out by Gibbons and Hawking. But they are, in hindsight, how we can make sense of their calculation.

I. General Relativity knows about its own quantum states

The first assumption implies that we should compute $Z(\beta)$ of an unknown quantum theory associated with the boundary of spacetime.

$$Z(\beta) = \mathrm{Tr}\, e^{-I} \,,$$

where $I = \beta H$ is the action of the unknown theory.

The second assumption instructs us to perform this “impossible” task by evaluating a gravitational path integral with boundary conditions given by a large sphere $\times$ Euclidean-time circle of length β, and I given by the Einstein-Hilbert action.

I. General Relativity knows about its own quantum states

One finds that the gravitational path integral is dominated not by flat space ($B^3 \times S^1$), but by a cigar-shaped geometry (topologically, $B^2 \times S^2$) that corresponds to the analytic continuation of a black hole solution. More precisely, the dominant geometry is the Schwarzschild black hole that has Hawking temperature $1/\beta$.

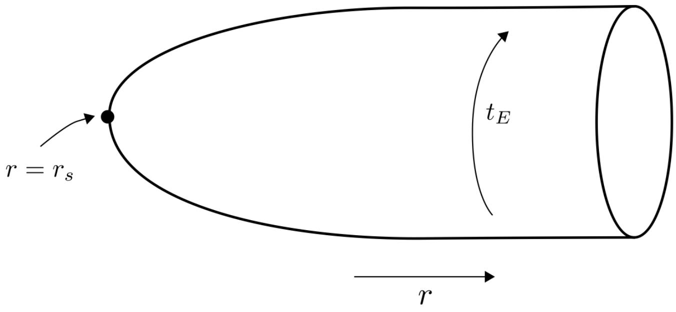


Figure: Almheiri et al. 2020

I. General Relativity knows about its own quantum states

Computing the Einstein-Hilbert action of this geometry, one thus finds

$$Z(\beta) \sim e^{-I(\beta)} .$$

The entropy turns out to be

$$S = (1 - \beta \frac{d}{d\beta}) \log Z(\beta) = \frac{A}{4G\hbar} ,$$

where A is the area of the event horizon of the black hole.

I. General Relativity knows about its own quantum states

Holography, broadly understood, is the fact that in the presence of gravity information is associated with the boundary of regions.

Here, the unknown quantum theory lives on the boundary, and the path integral is carried out over geometries matching the boundary. A nontrivial partition function is obtained in the saddlepoint approximation, from classical solutions of General Relativity, with no knowledge of the quantum spectrum.

$\rightarrow$ Jacobson's talk this afternoon
$\rightarrow$ Witten's talk on Friday

I. General Relativity knows about its own quantum states

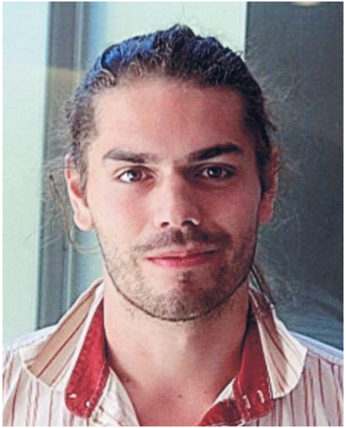
Recent addendum:

In general, e^S is not an integer, because the thermal state is really a weighted ensemble of infinitely many pure states. Moreover, there are subleading corrections to $S = A/4G\hbar$ in the gravitational path integral.

However, for certain supersymmetric black holes one expects a degenerate ground state Hilbert space of dimension approximately given by $\exp(A/4G\hbar)$. In this case the dimension should of course be an integer. One of the greatest triumphs of string theory was the use of string dualities to compute this integer in some special cases (Strominger, Vafa 1996).

I. General Relativity knows about its own quantum states

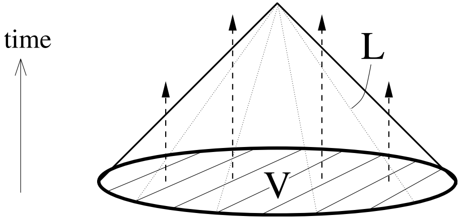
Iliesiu, Murthy, and Turiaci (2020) were able to reproduce this result using only the gravitational path integral. This required its exact evaluation using supersymmetric localization techniques. Gravity produces many non-integer terms that coincide with the Hardy-Ramanujan-Rademacher expansion developed in analytic number theory, and which thus sum up to an integer.



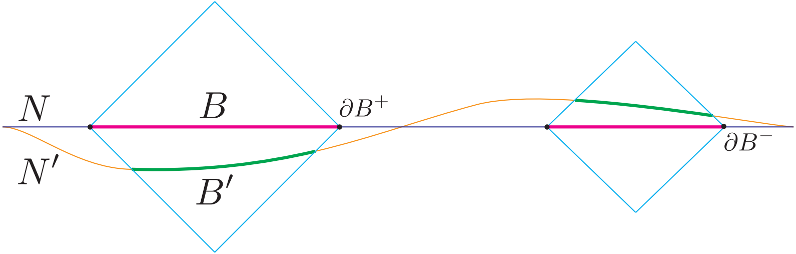
II. General Relativity knows about the quantum states of matter

Covariant Entropy Bound (RB 1999):

$$S_{\text{matter}}(L) \leq \frac{A}{4G\hbar} .$$



Today we understand this to be an implication of the Quantum Focussing Conjecture (RB, Fisher, Leichenauer, Wall 2015), a semiclassical generalization of the classical GR result that classical matter focusses lightrays (e.g., bending of light by the sun).



II. General Relativity knows about the quantum states of matter

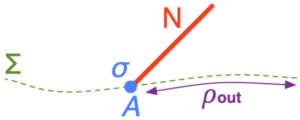
These conjectures have highly nontrivial implications for Quantum Field Theory without gravity, which can be proven (laboriously, in some cases):

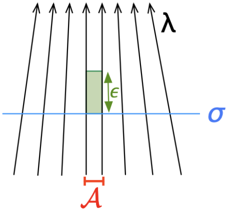
- Bekenstein bound (Bekenstein 1981; Casini 2008):

$$S \leq \frac{2\pi}{\hbar} \int_{z>0} dx\, dy\, dz\, z\, \langle T_{tt} \rangle \approx \frac{\pi E \Delta z}{\hbar}$$

- Quantum Null Energy Condition (RB, Fisher, Koeller, Leichenauer, Wall 2015; Ceyhan, Faulkner 2018):

$$\langle T_{kk} \rangle \geq \frac{\hbar}{2\pi} S''$$





III. General Relativity knows that information is ~~lost~~ preserved

Hawking (1976) found that information about the initial quantum state is lost when a black hole forms and fully evaporates:

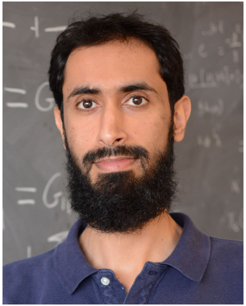
$$\rho = e^{-\beta H} \ .$$

The entropy $S(\rho)$ of the Hawking radiation grows monotonically.

III. General Relativity knows that information is ~~lost~~ preserved

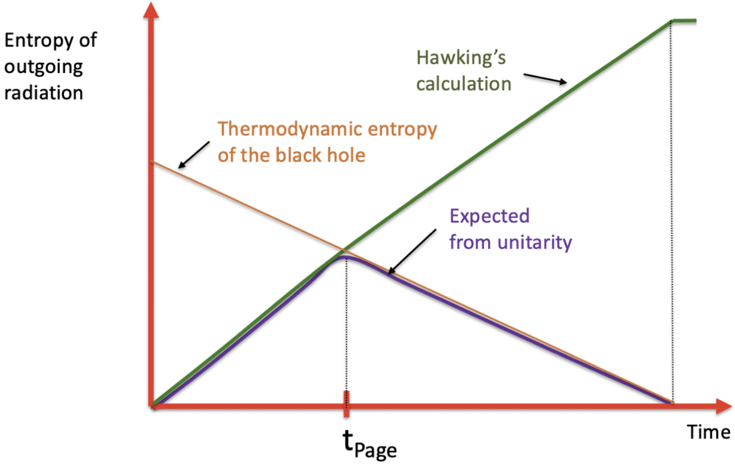
One can also use the gravitational path integral to compute the entropy of the radiation directly, without first computing ρ.

Penington 2019; Almheiri, Engelhardt, Marolf, Maxfield 2019



III. General Relativity knows that information is ~~lost~~ preserved

One finds a phase transition at the “Page time”, when the coarse-grained black hole and radiation entropies are equal. The entropy will first increase as predicted by Hawking. But after the Page time, the entropy decrease back to zero.



This is exactly how the entropy of the radiation should behave, if the black hole returns all information.

(Page 1993)

Figure:
Almheiri et al. 2020

III. General Relativity knows that information is ~~lost~~ preserved

How does this work?

The von Neumann entropy can be written as an analytic continuation of the $n = 2, 3, 4, \ldots$ Renyi entropies:

$$S = \lim_{n\to 1} S_n \ ; \ S_n = (1-n)^{-1} \log \mathrm{Tr}\, \rho^n \ .$$

$\mathrm{Tr}\, \rho^n$ can be computed from a gravitational path integral with n times replicated boundary conditions (Lewkowycz, Maldacena 2013).

III. General Relativity knows that information is ~~lost~~ preserved

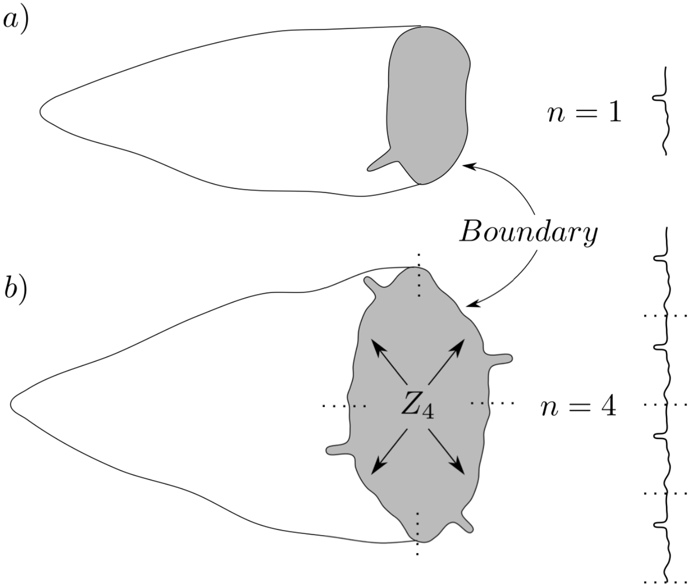


Figure: Lewkowycz, Maldacena 2013

III. General Relativity knows that information is ~~lost~~ preserved

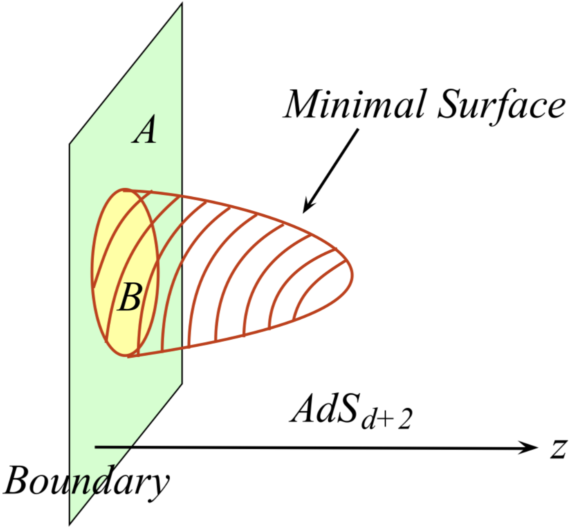
The analytic continuation of Renyi entropies to $n = 1$ can be performed geometrically. Find a minimal surface homologous to a given boundary region, in the original spacetime. Its area gives the (fine-grained) von Neumann entropy of the unknown theory in one step (Lewkowycz, Maldacena 2013).

In some cases the fundamental theory is known: AdS/CFT Maldacena (1997). Then the gravitational predictions for the entropy can be checked directly. In fact, this is how the “RT” prescription was first discovered (Ryu, Takayanagi 2006; Hubeny et al. 2007; Engelhardt, Wall 2014).

III. General Relativity knows that information is ~~lost~~ preserved

The RT prescription implies that spacetime induces a fundamental quantum state as if it were a quantum error correcting code. Not all of the boundary is needed to reconstruct the gravitating spacetime. The homology region between the minimal surface and any boundary portion is precisely the bulk region that can be described (Wall 2014). The reconstructible region is called the entanglement wedge of the boundary region.

Figure credit: Nishioka et al. 2009



III. General Relativity knows that information is ~~lost~~ preserved

The RT or Quantum Extremal Surface prescription arises from the Gravitational Path Integral and is unrelated to AdS/CFT.

It can be used to compute, for example, the fine-grained entropy of the Hawking radiation emitted by a black hole, for example in asymptotically flat spacetime.

III. General Relativity knows that information is ~~lost~~ preserved

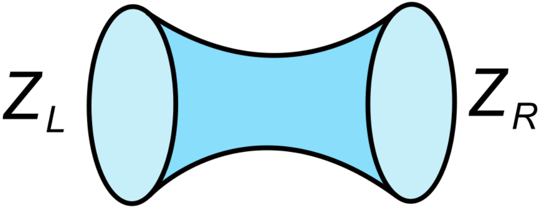
Both Hawking's calculation of the state of the radiation, and the recent direct calculation of its entropy, used the gravitational path integral. This apparent contradiction may offer an interesting hint: suppose that the gravitational path integral computes some kind of average. Then it is consistent that $\overline{S(\rho)} \neq S(\overline{\rho})$.

RB, Tomasevic 1999;
RB, Wildenhain 2000

III. General Relativity knows that information is ~~lost~~ preserved

In general, it is not known what (if anything) gravity is averaging over. However, certain 2-dimensional toy models of gravity were previously shown to be dual to an ensemble average of quantum mechanical theories such as the Sachdev-Ye-Kitaev model, or random matrix models. In the SYK model, the average is over couplings; more generally, over matrices drawn from an ensemble. See e.g. Saad, Shenker, Stanford (2018, 2019).

III. General Relativity knows that information is ~~lost~~ preserved



Another example: Euclidean wormholes connect two unrelated boundaries, so that

$$\overline{Z_L(\beta_L)}\ \overline{Z_R(\beta_R)} \neq \overline{Z_{\text{both}}(\beta_L, \beta_R)}$$

Without averaging, this would be a contradiction!

Figure credit:
Saad, Shenker, Yao 2021

IV. Gravity as Single-Shot Quantum Communication

In quantum (and classical) communication theory, one distinguishes between single-shot and asymptotic tasks.

Single-shot means (roughly) that you would like to send someone a message once, with success probability $1 - \epsilon$. The minimum capacity of the communication channel required for this task is set by smallest number of messages whose total probability exceed $1 - \epsilon$. This is controlled by a quantity called the smooth max entropy. It has a counterpart called the smooth min entropy, related to the capacity required for ϵ probability of success.

Asymptotic means that you intend to send $n \to \infty$ messages simultaneously, all drawn from the same ensemble. The capacity per message can be smaller than in the single-shot setting because of compression. In the quantum case, it can also be useful to entangle the messages. The capacity required is now set by the von Neumann entropy, which is always less than the max entropy.

IV. Gravity as Single-Shot Quantum Communication

A yet more obscure task is known as quantum state merging: Alice and Bob share a quantum state that is entangled with a third system. Alice would like to send her share of this entanglement to Bob, so that only Bob is entangled with the third system. Again this task comes in one-shot and asymptotic variants. The required capacities are quantified by the smooth conditional max- and min-entropies (Renner, Wolf 2004); and in the asymptotic case, simply by the conditional von Neumann entropy.

IV. Gravity as Single-Shot Quantum Communication

It turns out that the size of the entanglement wedge is correctly determined only if we treat the bulk-to-boundary isometry as a one-shot communication task (Akers, Penington 2020; Akers, Levine, Penington, Wildenhain 2024).

This implies that in general there are two entanglement wedges, $e_{\max} \subset e_{\min}$. $e_{\max}$ is the largest region that can be completely reconstructed from boundary data. $e_{\min}$ is the smallest region such that nothing at all can be reconstructed in its complement.

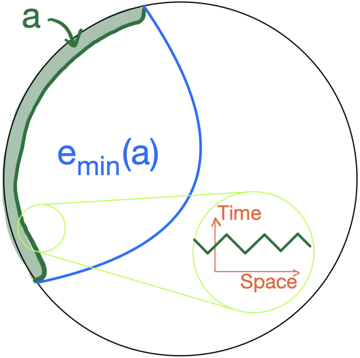
IV. Gravity as Single-Shot Quantum Communication

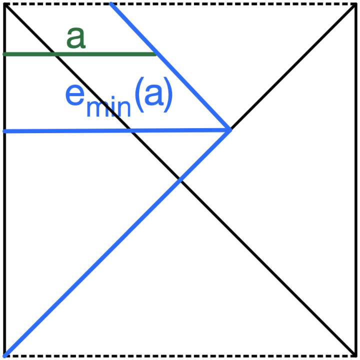
When reconstructing from a boundary region, $e_{\mathrm{max}} = e_{\mathrm{min}}$ unless the bulk matter is in an "incompressible" quantum state. Such states are rarely considered.

Recently, RB, Penington (2022, 2023) generalized the notion of entanglement wedges to arbitrary spacetimes, not just AdS. Our construction suggests that it is possible to reconstruct from gravitating (bulk) regions, in the sense that the full algebra of operators generated by the semiclassical operators in a region can access a larger region. Our main long term goal was to generalize quantum gravity beyond AdS/CFT, to our own universe. But we encountered a big surprise right away.

IV. Gravity as Single-Shot Quantum Communication

A consistent prescription for bulk-bulk entanglement wedges required us to distinguish between $e_{\max}$ and $e_{\min}$ already at the level of classical geometry!





Thus, sophisticated concepts from quantum communication theory are baked deep into the classical structure of space and time.

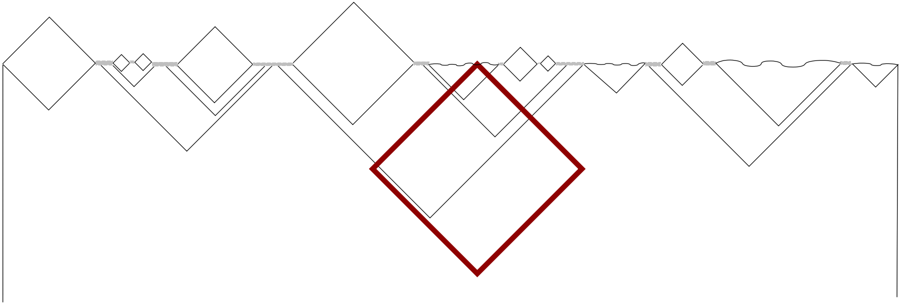
V. Some Implications For Cosmology

Spacetime organizes quantum information.

Everything I talked about is true in semiclassical gravity, for arbitrary spacetimes including cosmological solutions. Not limited to AdS.

The Page curve calculation demonstrates that semiclassical gravity is a far more rigorous tool than many had anticipated. Solve $G_{ab} = 8\pi G \langle T_{ab} \rangle$ to all orders in $cG\hbar$, where c is the number of matter fields. The quantum fields evolve unitarily.

V. Some Implications For Cosmology



Unitary evolution of quantum fields is incompatible with the global description of eternal inflation, where vacuum decay is assumed to happen stochastically at specific times and locations. Indeed, a local description is sufficient for understanding how vacua like ours can be produced (RB 2006). This is important for solving the cosmological constant problem via a large vacuum landscape (RB, Polchinski 2000).

V. Some Implications For Cosmology

Singularities: Original singularity theorems were based on assumptions that are known to be false in Nature, either classically (SEC) or at the level of QFT (NEC).

Robust singularity theorems can be proven by assuming the Generalized Second Law (Wall 2011) or the Quantum Focussing Conjecture (RB, Shahbazi-Moghaddam 2022, 2023).

VI. Summary

The workings, ultimate power, and limitations of the gravitational path integral remain mysterious.

There are indications that at least in some settings it must be given an interpretation involving ensemble averaging.

At present, gravity is oracular – you have to ask nicely, and come up with clever strategies for extracting its wisdom.

Yet, the fact that General Relativity gives us any information about quantum theory at all (let alone highly nontrivial discoveries like the QNEC) is utterly remarkable.