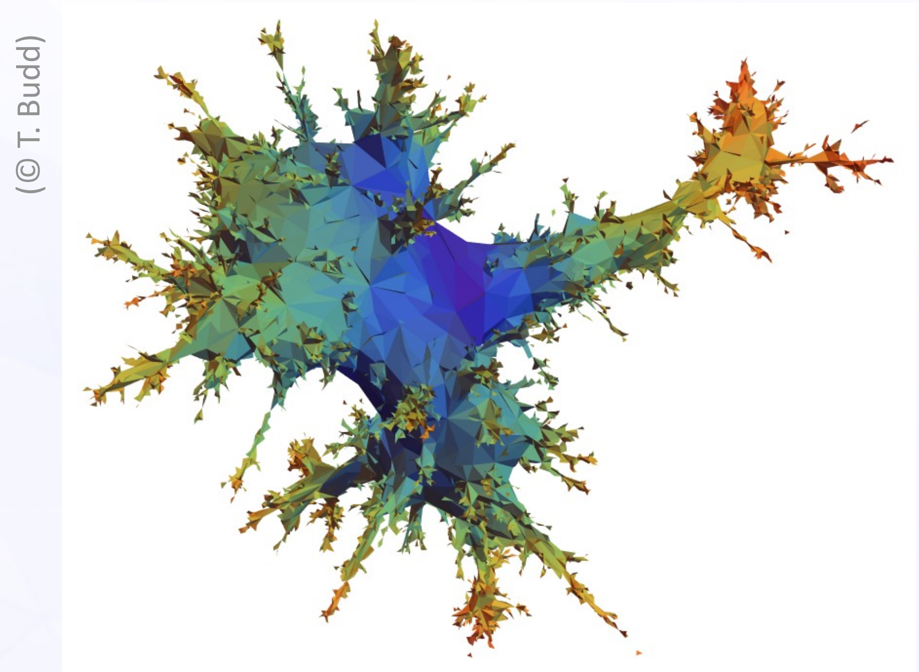
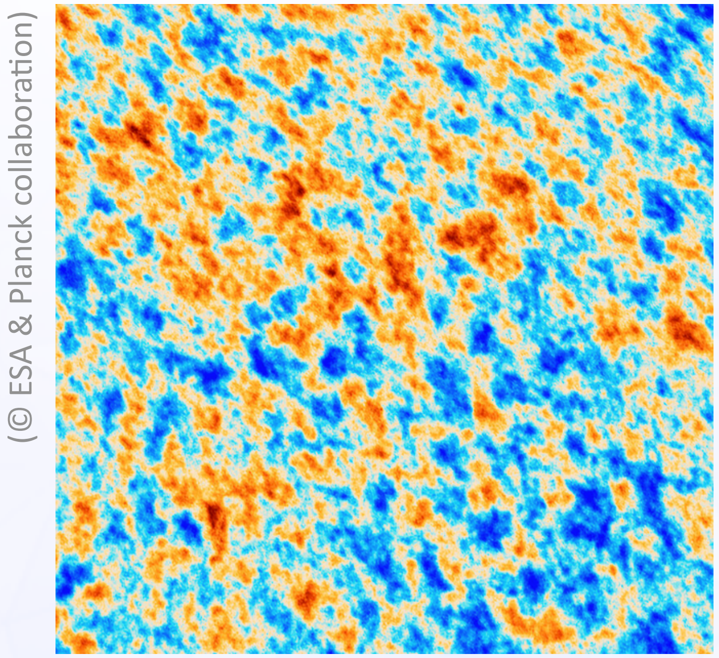



typical nonperturbative quantum space(time)




cosmic microwave background

# Mapping the Road to Cosmology from the Planckian End

**Renate Loll**
Radboud University (NL)

**Lemaître Conference** **Vatican Observatory, 17 June 2024**

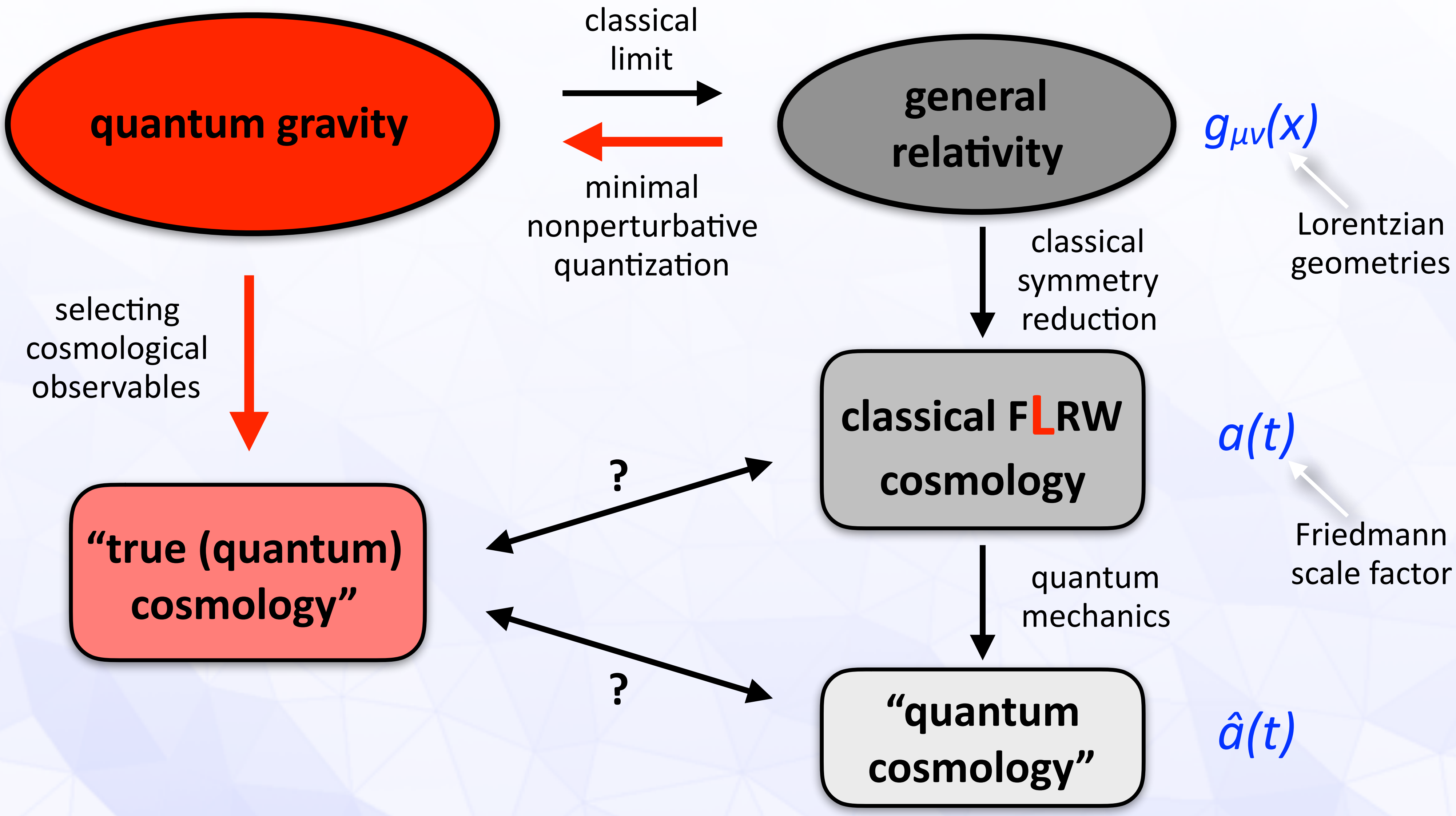
# Honouring Lemaître's legacy …

**… in the ongoing quest to understand the beginning of the universe** — to the extent this lies within the realm of physical theory.

Some of **Lemaître's favourite themes**:

- the universe expanded from a quantum state ("primeval atom")
- such a quantum beginning defies our intuition but is "not at all repugnant"; space and time may fail to have any meaning
- an early proponent of computers for cosmological calculations

***Today:*** **a roadmap to cosmology from a fundamental quantum description of gravity** (standard, first principles (QFT), no "exotics", nonperturbative evaluation relies crucially on *computer simulation*; ***Results:*** observables in a Planckian realm do *not* conform to classical expectation, but guide our intuition; emergent *de Sitter* properties)




FLRW spacetimes have no fundamental status; can we ***derive/verify*** assumptions about the early universe from the full quantum theory?

Theorists would like "spacetime as we know it" to ***emerge*** from their favourite quantum bits, but this is very nontrivial to realize concretely.

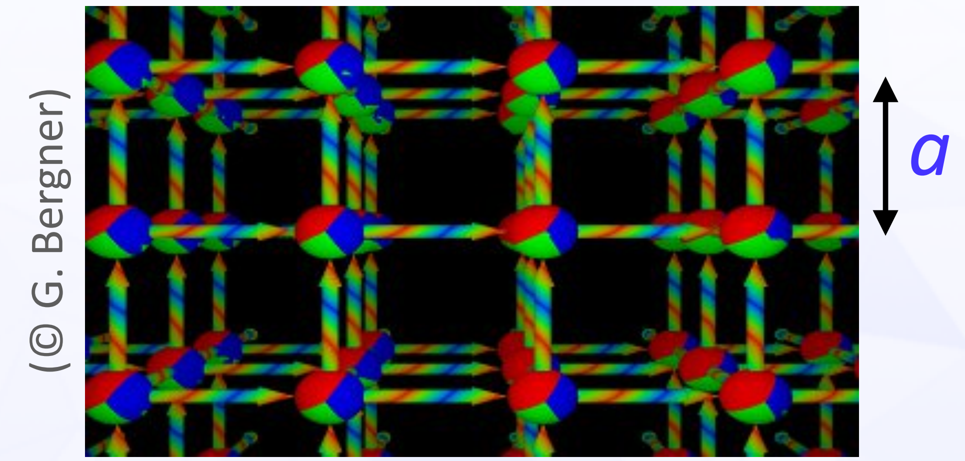
# Some key takeaways

- we have learned how to put 4D quantum gravity (the gravitational path integral) on the lattice and investigate it ***beyond perturbation theory***
- the best lattice methodology today is based on ***causal dynamical triangulations (CDT)***, and is coordinate-invariant (a gravitational analogue of Wilson's formulation of lattice QCD in terms of holonomies) **(Wilson, 1974)**
- it provides a ***unique window on unexplored Planckian physics*** and experimental data on the (often unexpected) behaviour of its observables
- it also provides a concrete blueprint for the ***emergence of quantum spacetime*** and a new perspective on what quantum gravity can deliver
- the new ***quantum Ricci curvature*** allows us to construct further observables that can inform our understanding of the ***very early universe***
- new mathematics ***"beyond $g_{\mu\nu}$"*** is required

**for a larger context, see R.L. et al.: "Quantum Gravity in 30 Questions", arXiv:2206.06762**

# Putting quantum gravity on the lattice, correctly

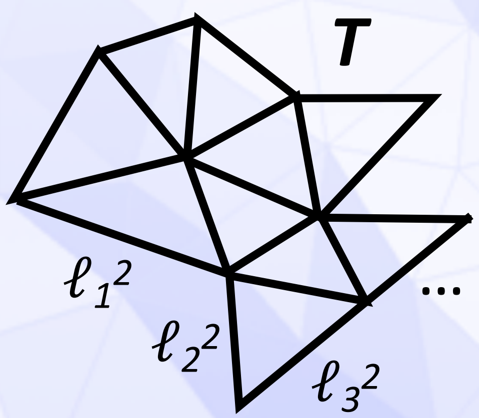
**Strategy**: lattice acts as a *regulator*, with UV cutoff $a$; search for a continuum limit near a 2nd-order phase transition as $a \to 0$, thus attaining ***universality*** (cf. stat. mech.)




- early work put gravity on a *fixed* 4D lattice, like in lattice QCD, but Monte Carlo simulations never found an interesting phase structure

→ broken diffeomorphism invariance! action unbounded! Euclidean!

- these issues are now ***resolved*** ( → slide)




- inspired by an old idea, "General Relativity without coordinates" (Regge, 1961): regularize smooth curved manifolds by triangulations, $(M, g_{\mu\nu}(x)) \to (T, \{\ell_i^2, i=1,\dots,n\})$

- use this to obtain a ***geometric regularization*** of the path integral

# Lattice quantum gravity à la CDT: the basics

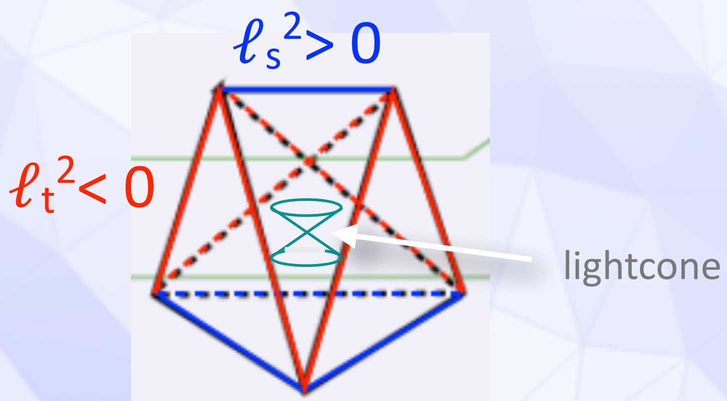
- We use CDT to implement Feynman's quantum superposition principle as a Lorentzian ***"sum over histories"***, i.e. spacetime geometries $g$.

$$Z = \int_{g \in \frac{Lor(M)}{Diff(M)}} \mathcal{D}g \; e^{iS_{\rm grav}[g]}$$

gravitational action

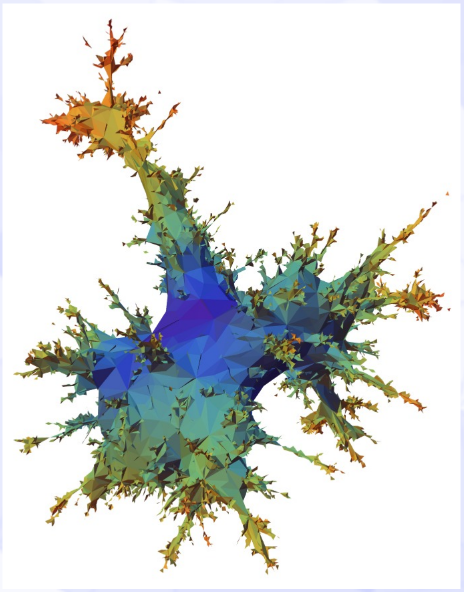
- piecewise flat, simplicial manifolds (aka triangulations *T*), defined in terms of their edge lengths $\{\ell_i\}$ and connectivity, provide a *regularized* version of the space of geometries, $\mathcal{G}(M) = Lor(M)/Diff(M)$

- *dynam. triangulation* = identical building blocks (two types in 4D CDT)



Minkowskian four-simplex, (4,1)-building block of 4D CDT, with time- and spacelike edges

- "typical" histories are highly nonclassical (higher-dimensional analogues of the nowhere differentiable 1D paths in the Wiener measure)

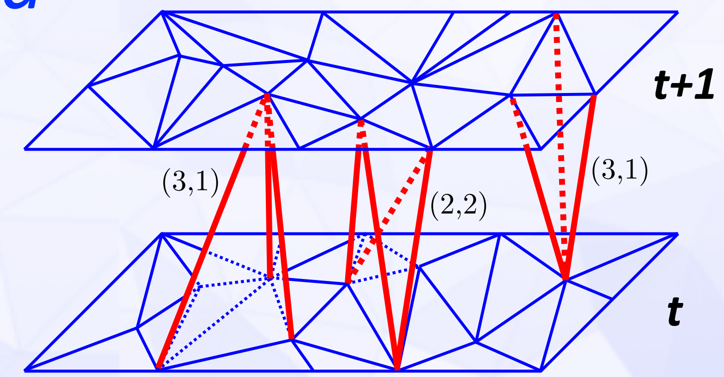

typical history, 2D path integral

[formal continuum path integral] [well-defined regularized path integral + cont. limit]

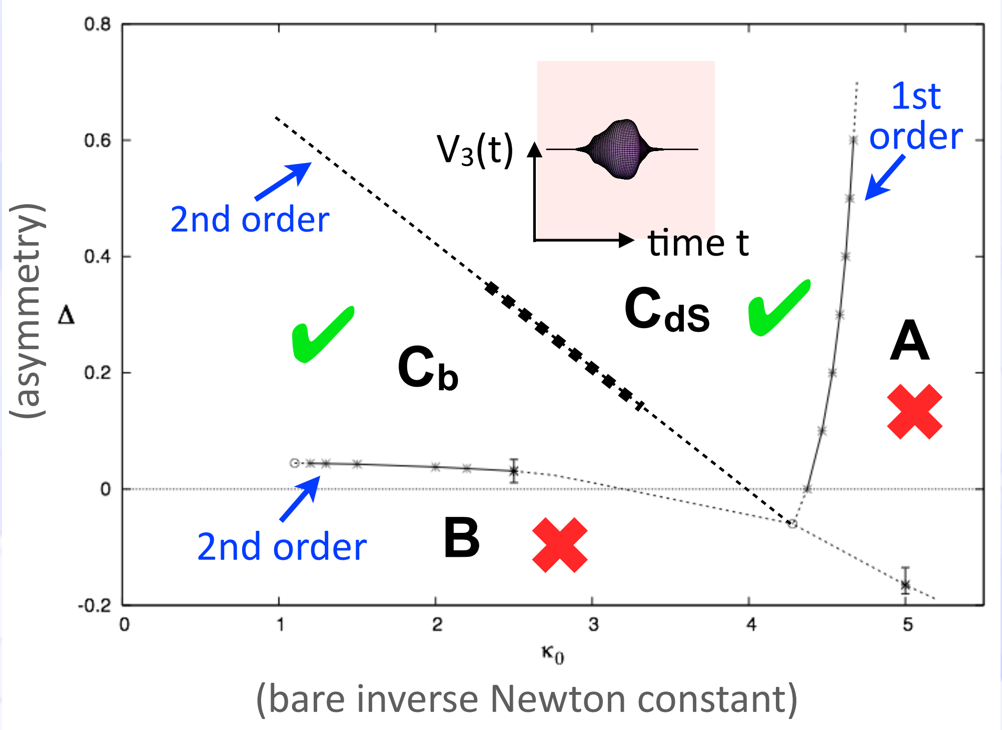
$$Z = \int_{\mathcal{G}(M)} \mathcal{D}g \, e^{iS[g]}, \quad S = \kappa^2 \int_M d^4x \sqrt{|g|}(R - 2\Lambda) \xrightarrow{\text{CDT}} Z = \lim_{a \to 0} \sum_{\substack{causal \\ triang.\,T}} \frac{1}{C(T)} e^{iS^{\text{Regge}}[T]}$$

# symmetries of T

- simple (bare) action: $S^{\text{Regge}}[T] = -(\kappa_0 + 6\Delta) N_0(T) + \kappa_4 N_4(T) + \Delta N_{41}(T)$
- limit $a \to 0$, $N \to \infty$, for finite physical volume $V = N a^4$
- path integral histories have a well-defined causal structure ($M = S^3 \times S^1$, no spatial topology changes!)




'stacked' structure of CDT configuration (3D)



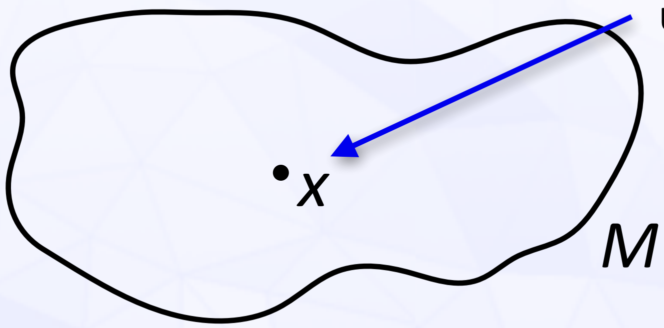

- $Z^{CDT}$ is amenable to Monte Carlo methods after analytic continuation
- CDT has a rich phase diagram with 2nd-order transition lines

J. Ambjørn, S, Jordan, J. Jurkiewicz, R.L. PRL 107 (2011) 211303

# Lattice quantum gravity à la CDT: results

- the physics of ***quantum spacetime*** is captured by diffeomorphism-invariant ***quantum observables*** $\hat{\mathcal{O}}$ :

$$\langle\hat{\mathcal{O}}\rangle = \frac{1}{Z}\int \mathcal{D}g\ \mathcal{O}[g]\, \mathrm{e}^{-S_{\mathrm{grav}}[g]}$$




- observables $\mathcal{O}$ typically are nonlocal integrals of scalars, like $\int_M d^4x\,\sqrt{g}\,R(x)$
- this also reflects the absence of meaningful reference frames @$\ell_{Pl}$
- "expectation management": many popular quantum gravity questions do ***not*** have a Planckian implementation and cannot be defined operationally in a background-independent, highly quantum-fluctuating setting (this is a feature)
- adjust expectations and exploit new opportunities (for cosmology)!

# Planckian surprises

The dimension of the microscopic building blocks does not fix the dimension of "quantum spacetime". This was discovered by ***computer experiments***, measuring the spectral and Hausdorff dimensions.

It can happen because of the nontrivial limit $a \to 0$ and in a region of coupling-constant space where "energy" and "entropy" compete,

$$Z = \int \underbrace{\mathcal{D}g}_{entropy} \; \underbrace{\mathrm{e}^{-S[g]}}_{energy} \longleftarrow \text{analytic cont. of } \mathrm{e}^{iS[g]}$$

The ***spectral dimension*** $D_S$ of a space is determined from a diffusion process. CDT was first to find a dimensional reduction $4 \to 2$ @$\ell_{Pl}$ J. Ambjørn, J. Jurkiewicz, R.L., PRL 95 (2005) 171301, a conjectured universal property of quantum gravity.

S. Carlip, CQG 34 (2017) 193001

➔ ***remarkable: nonperturbative quantum signature we can calculate!***

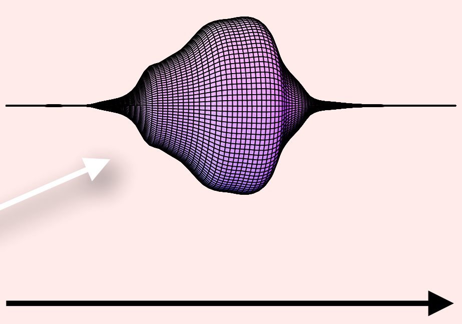
# Emergent classicality from pure "quantum foam"

The shape $\langle V_3(t) \rangle$ of the dynamically generated quantum universe (spatial volume as a function of proper time) matches that of a classical 4D de Sitter space, ***although no background geometry was ever put in***.

J. Ambjørn, A. Görlich, J. Jurkiewicz, R.L., PRL 100 (2008) 091304, PRD 78 (2008) 063544

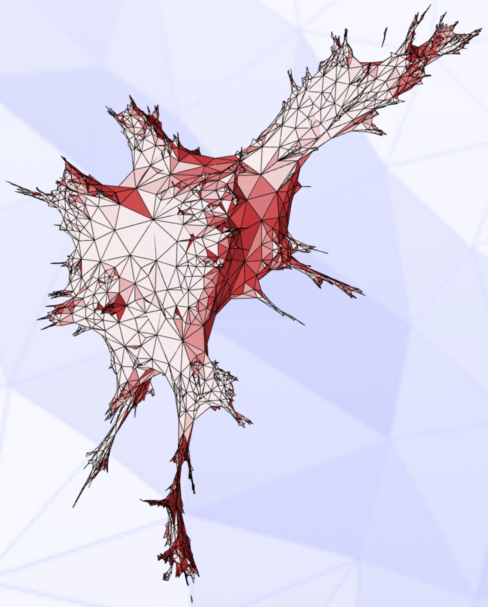
Remarkable, but from the global scale factor $a(t)$ alone we cannot conclude that this ***is*** a (Euclidean) de Sitter universe (≉symmetry-reduced qu. cosmology, no $g_{\mu\nu}$!).

*MC snapshot of the shape* $\langle V_3(t) \rangle$ *of the universe*




N.B. local fluctuations are not depicted here!

$R^{\kappa}{}_{\lambda\mu\nu}[g,\partial g,\partial^2 g,x) = ?$



We have recently constructed a renormalized notion of ***quantum Ricci curvature*** that is well defined in a Planckian regime. It yields a finite average curvature, also compatible with a de Sitter space!

N. Klitgaard & R.L., Eur. Phys. J. C80 (2020) 990,
cf. Y. Ollivier, J. Funct. Anal. 256 (2009) 810

# Relation to our actual universe

Lattice QG predicts a universe with $\Lambda^{ren} > 0$, which is 4D on large scales, and whose ***average shape and curvature*** are compatible with a ***de Sitter space***, matching our current understanding of the early universe.

These properties have been derived ***from first principles*** and measured in an observational window $\lesssim 20\ell_{Pl}$; we can also reverse-engineer an effective action for the Friedmann scale factor $a(t) \backsim V_3(t)^{1/3}$.

At what scales and how does gravity interact with ***matter***?

$$Z = \int_{\mathcal{G}(M)} \mathcal{D}g \int_{\Phi} \mathcal{D}\phi \; \mathrm{e}^{i(S_{\mathrm{grav}}[g] + S_{\mathrm{matter}}[g,\phi])}$$

Investigations of CDT coupled to matter fields have so far not found a significant impact on the geometry ⇒ "matter doesn't matter @$\ell_{Pl}$"?

***There is now a clear roadmap to connect to early-universe physics.***

# Zoom in on the emergent quantum universe …

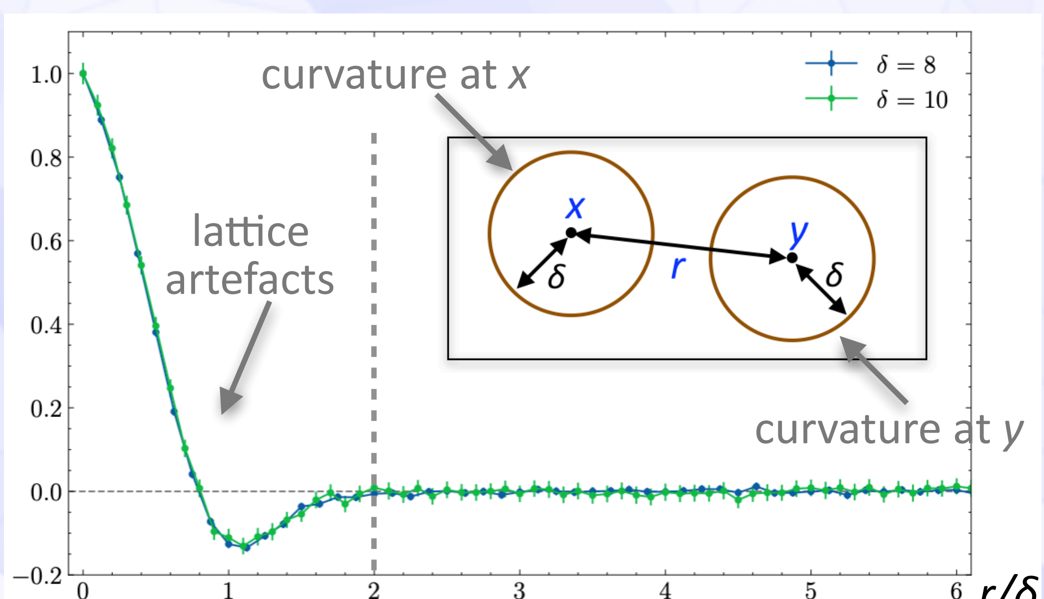
… by measuring more fine-grained (curvature) observables:

- classical de Sitter space is maximally symmetric; can homogeneity and isotropy of quantum spacetime “emerge”? we have constructed ***nonperturbative homogeneity measures*** **A. Silva, R.L., PRD 107 (2023) 086013**
- fluctuations/inhomogeneities are captured by ***diffeomorphism-invariant two-point functions*** of local scalars $\mathcal{O}$:

$$G_{\mathcal{O}}(r) = \frac{1}{Z}\int \mathcal{D}g\; e^{-S[g]} \int d^4x\sqrt{g(x)} \int d^4y\sqrt{g(y)}\; \mathcal{O}(x)\mathcal{O}(y)\,\delta(d_g(x,y)-r)$$

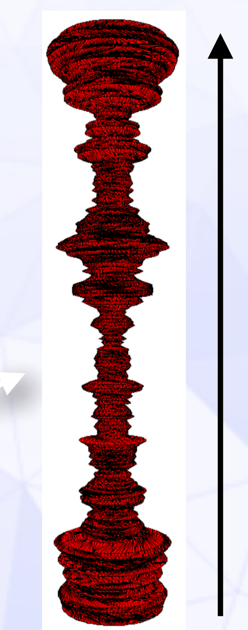
geod. distance

note operator dependence!




$\langle (K_q(x)-\bar{K}_q)(K_q(y)-\bar{K}_q)\rangle$ in 2D CDT on $T^2$, N=100k

typical configuration in 2D CDT



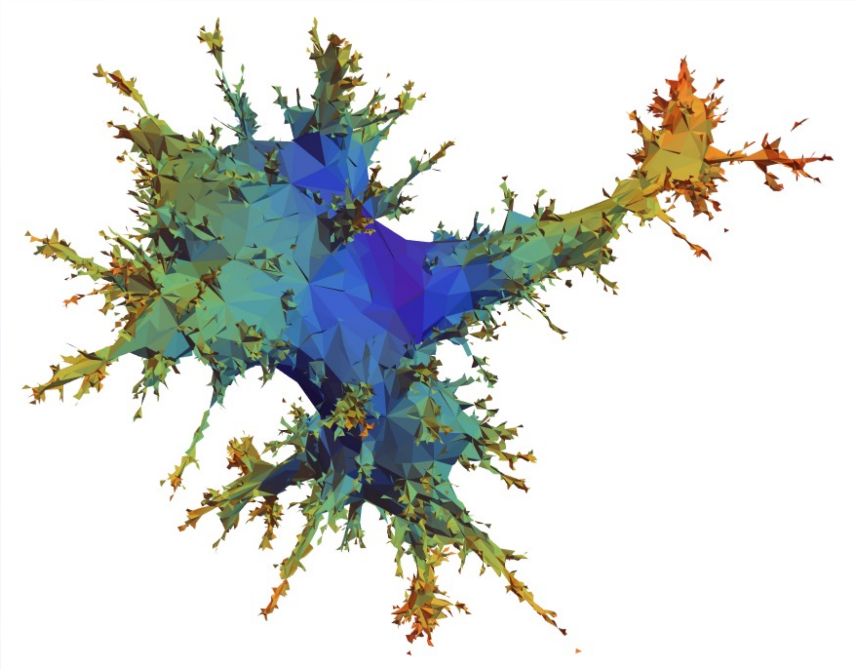

***curvature-curvature correlator*** in 2D CDT: no correlations for $r > 2\delta$

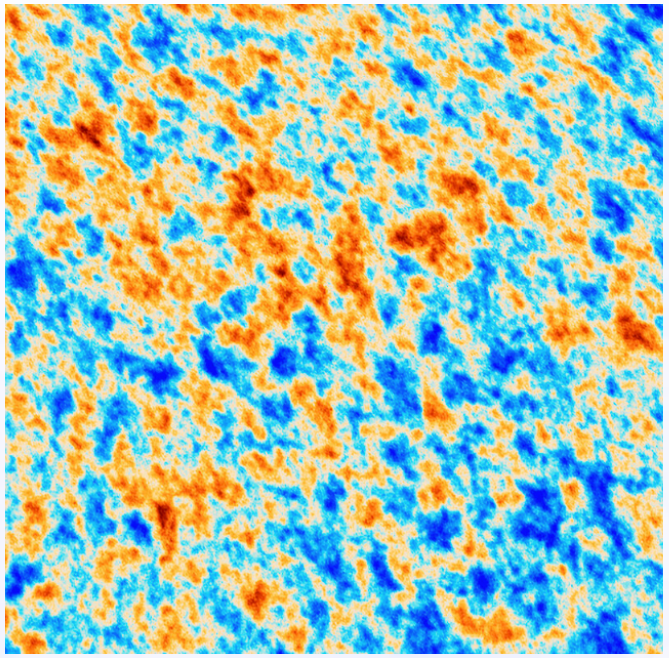
**J. van der Duin, R.L., arXiv:2404.17556**

# Summary and outlook

- ***computer-assisted theory construction:*** new, data-driven insights; lattice quantum gravity is a method, not an 'approach'; may be the ***primary gateway*** to the strongly coupled quantum regime @$\ell_{Pl}$
- ***what fundamental quantum gravity can deliver:*** compute and compare *observables:* universal properties & dynamical mechanisms (pitfalls!), new quantum signatures, what does (not) 'emerge'
- ***there is something rather than nothing!*** — rich dynamics, related to new mathematics of beyond-Riemannian and random geometry
- ***path to early-universe phenomenology:*** emergence of symmetry (homogeneity, isotropy, …) and structure (correlators); tests of standard assumptions in cosmology; hopefully, new predictions!


**Reviews of CDT lattice quantum gravity:**
**J. Ambjørn, A. Görlich, J. Jurkiewicz, R.L., Phys. Rep. 519 (2012) 127, arXiv:1203.3591**
**R.L., Class. Quant. Grav. 37 (2020) 013002, arXiv:1905.08669**
**J. Ambjørn, R.L., Encyclopedia of Mathematical Physics, arXiv:2401.09399**



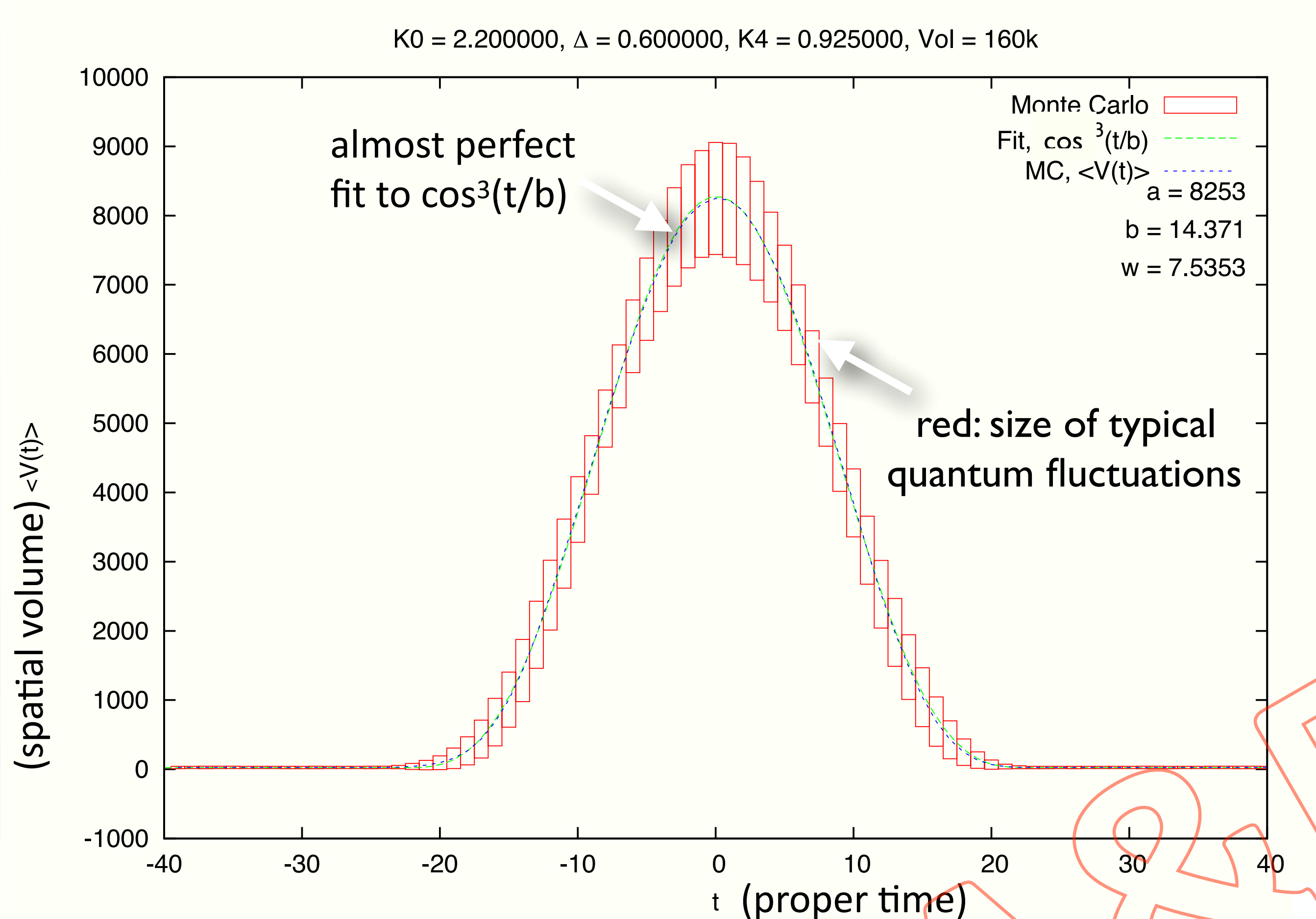


# Thank you!

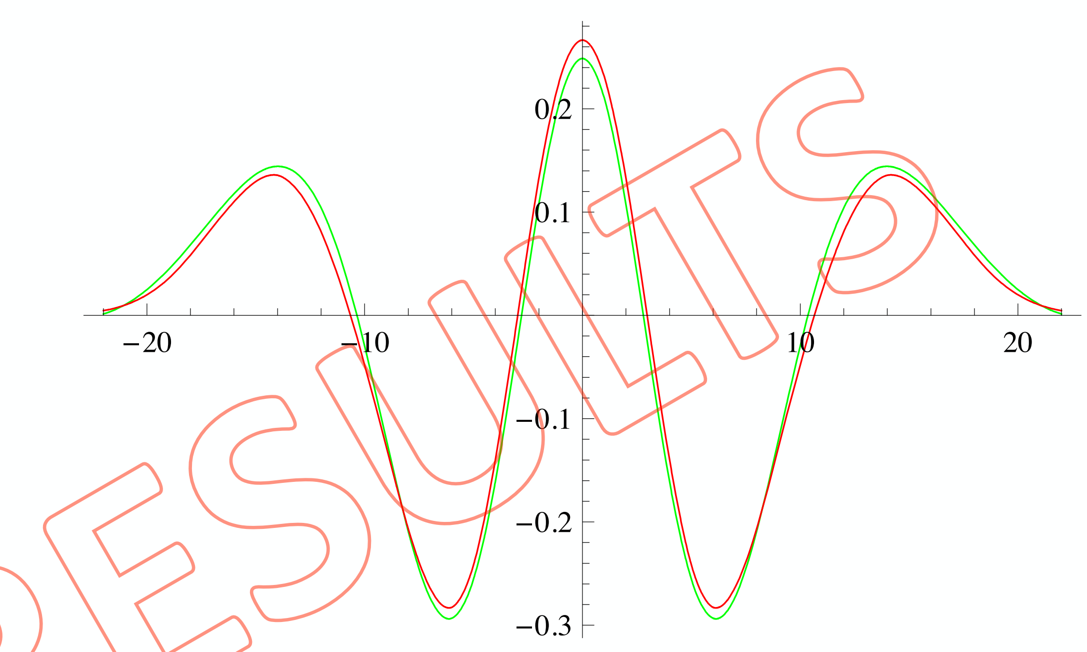
# Troubles of the path integral (PI), resolved

- cannot evaluate a complex PI, do Euclidean *∫Dg exp(-$S^{eu}$)* instead
  - ☑ CDT has a well-defined analytic continuation ("Wick rotation")
- the conformal divergence renders the Euclidean PI unstable
  - ☑ CDT has a compensating contribution from the measure ("entropy")
- difficult to renormalize compatible with diffeomorphism symmetry
  - ☑ CDT has no residual gauge symmetries, has a geometric cutoff $a$
- PI is highly divergent, no unique renormalization;
  - ☑ numerical evidence of exponential bound on # of configurations
- unclear what happens to perturbative UV divergences
  - ☑ first explicit match with an asymptotic safety analysis; CDT is an "effective theory" near @$\ell_{Pl}$ , agnostic of nature of UV completion
- cannot do any computations, since the PI is not Gaussian
  - ☑ CDT amenable to Monte Carlo simulations; get quantitative results
- unclear whether the PI leads to a unitary theory
  - ☑ CDT is reflection-positive w.r.t. discrete "proper time"

## The universe is de Sitter-shaped

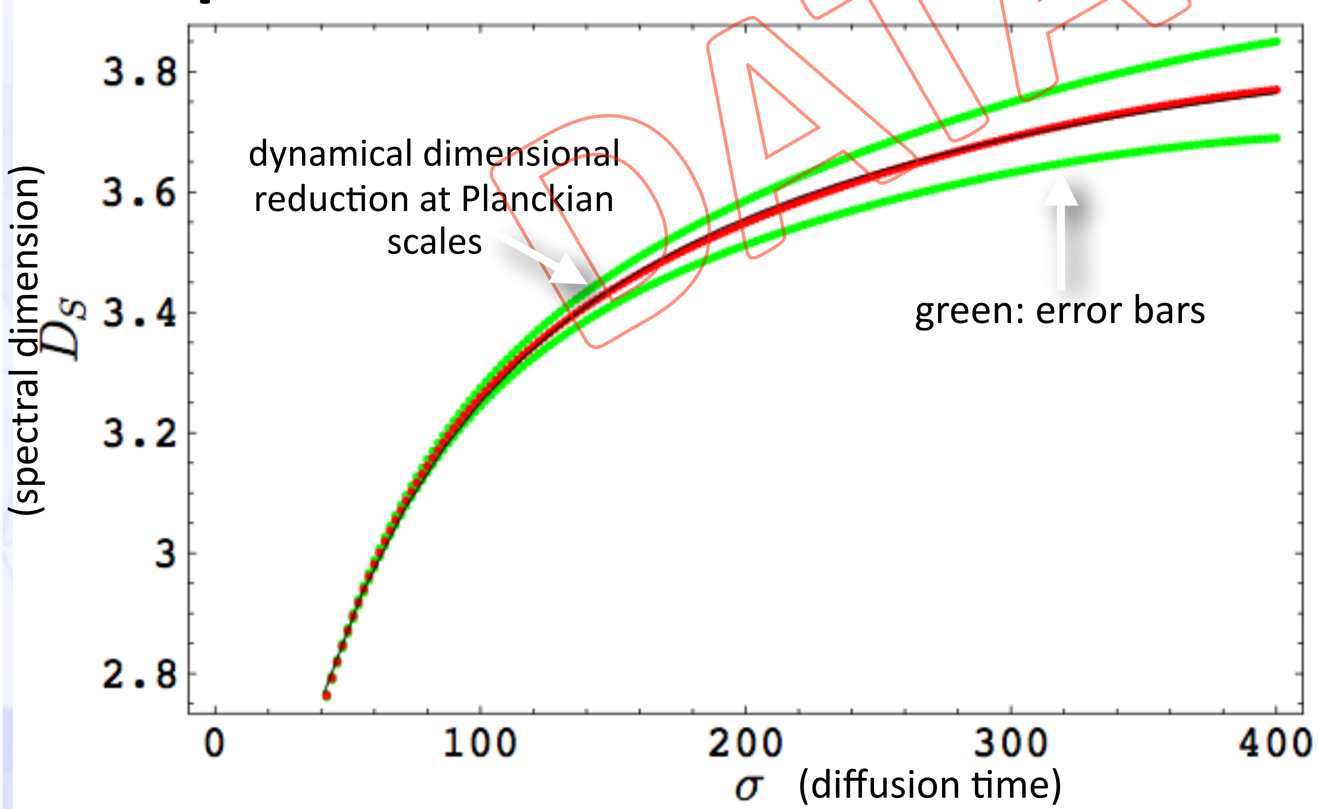



## Volume fluctuations around de Sitter

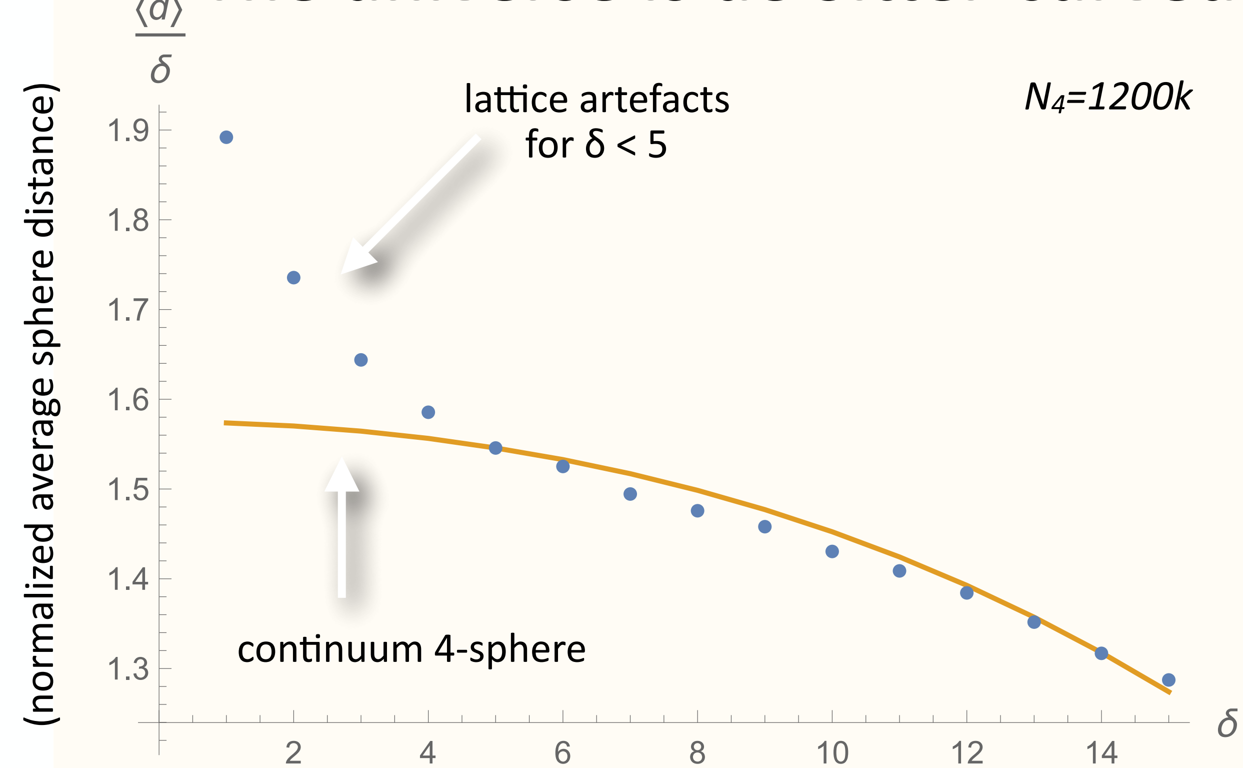



(low-lying eigenmode matches with semiclassical expectation)

## Spectral dimension of the universe




## The universe is de Sitter-curved




(quantum Ricci curvature of the dynamically generated dS universe)