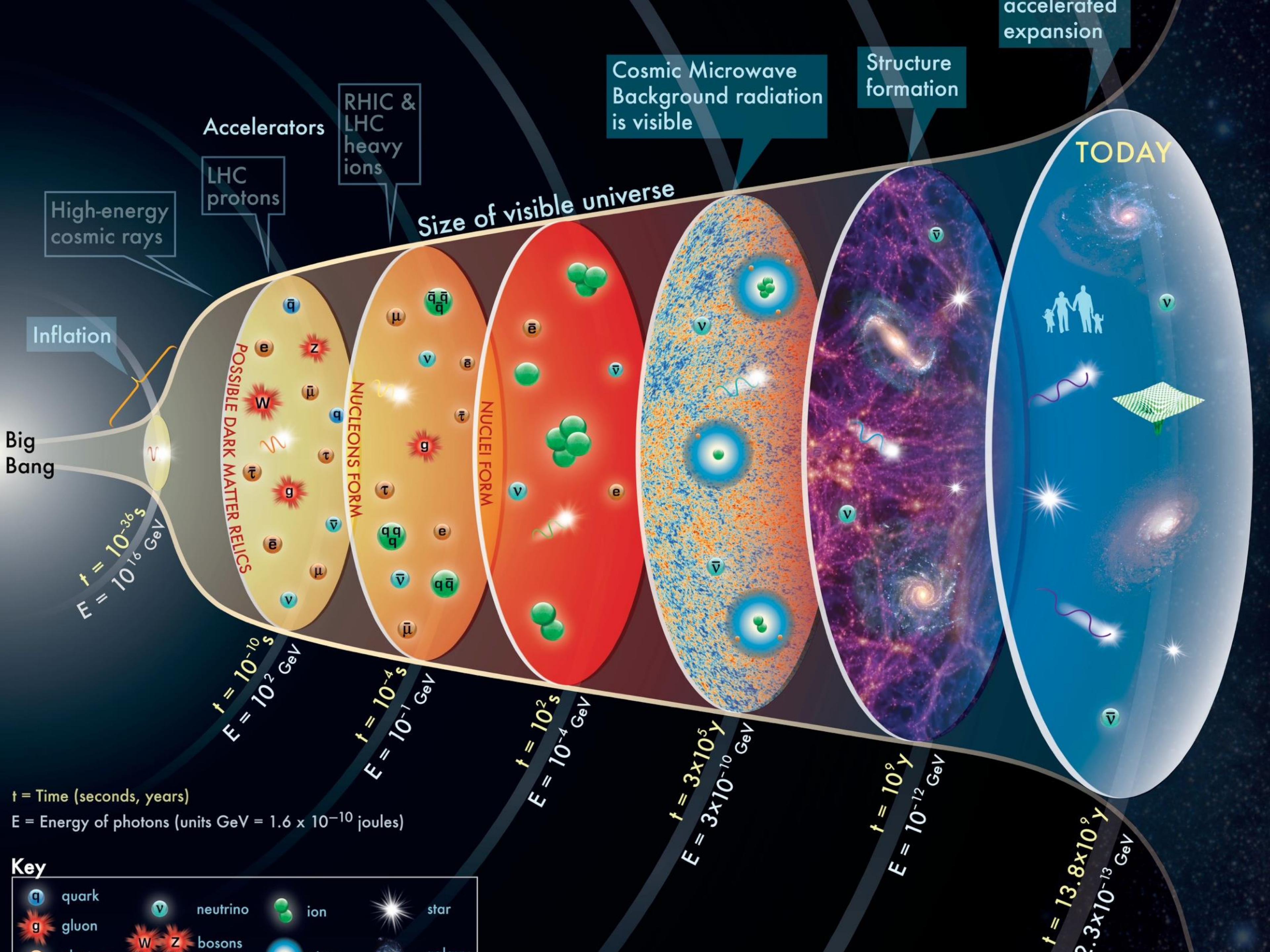
Present Status of Inflationary Cosmology

Andrei Linde

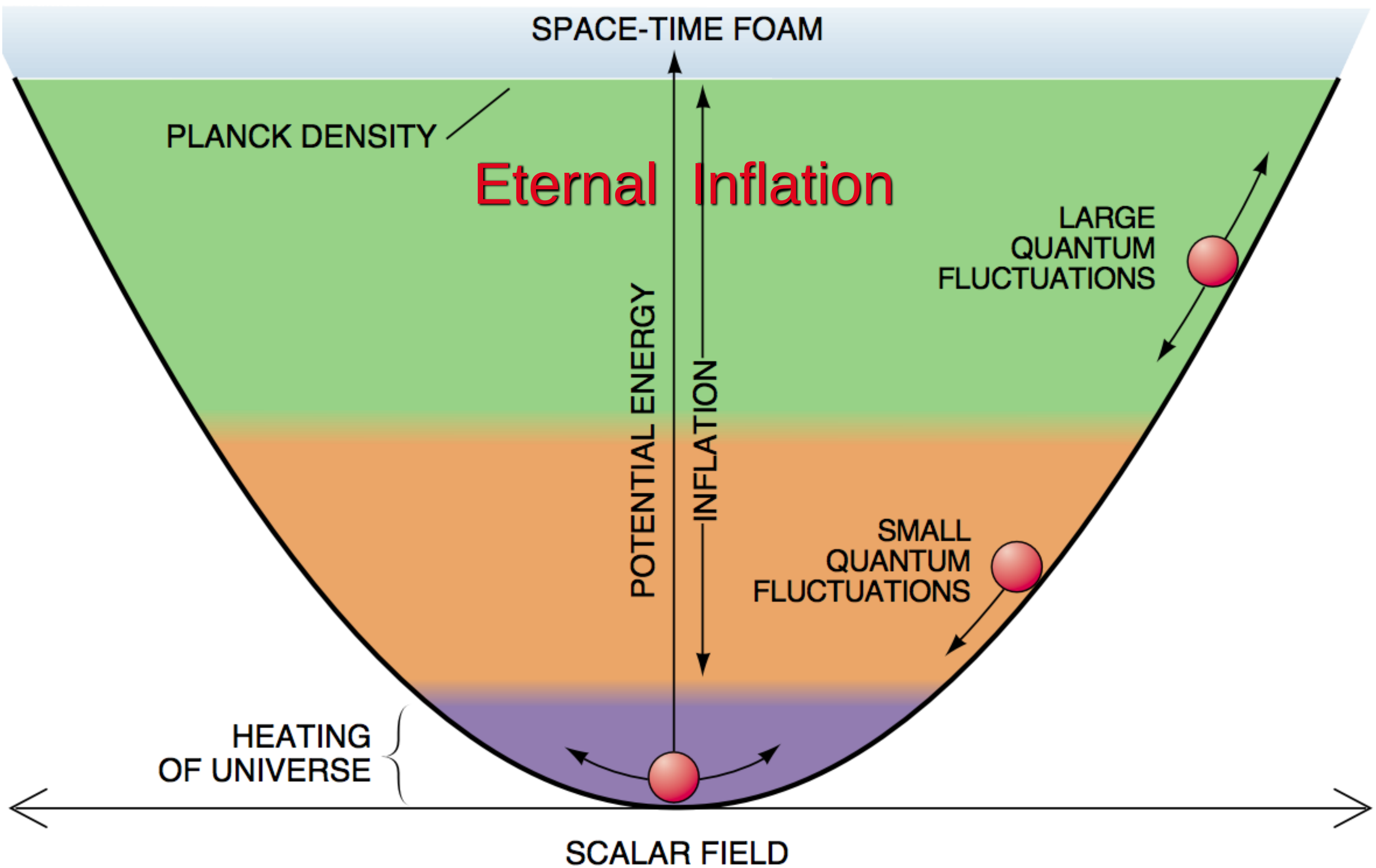
Lemaitre Conference 2024


accelerated expansion
Cosmic Microwave Background radiation is visible
Structure formation
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
neutrino
bosons
ion
star

The simplest inflationary model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$$



Consider a tiny universe of a smallest possible size 10^{-33} cm at the Planck density. If the potential energy of the scalar field in this domain was greater than its kinetic and gradient energy, it starts growing fast.

Within 10^{-42} s the universe becomes homogeneous and completely dominated by the potential energy of the scalar field.

Equation for the scalar field

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$$

Einstein's equation

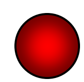
$$H = \frac{m\phi}{\sqrt{6}}$$

The solution shows that the universe grows approximately exponentially. At the end of inflation, the universe grows up by a factor

$$e^{\phi_0^2/4}$$

Here ϕ_0 is the initial value of the field.

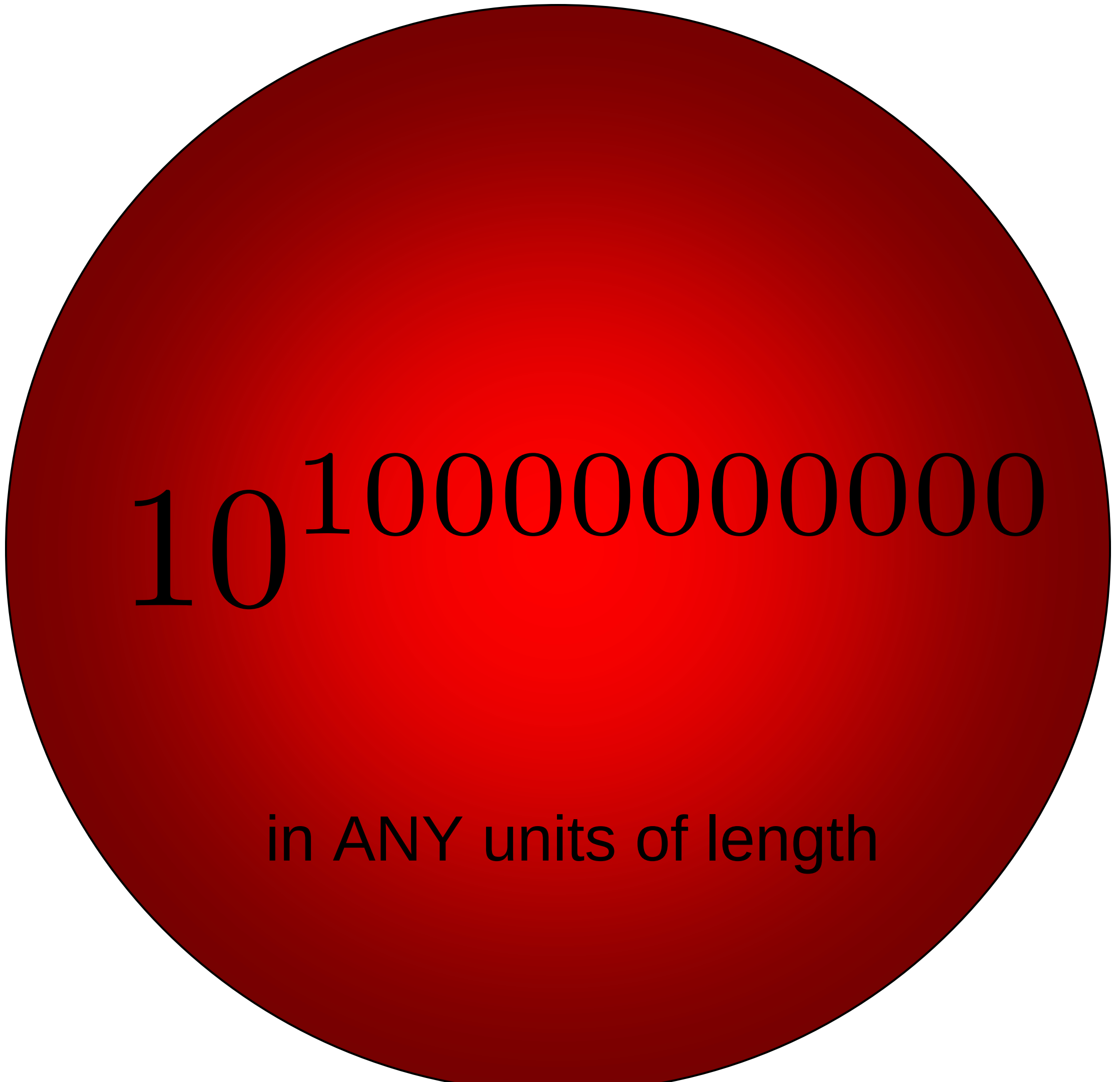
A newborn universe could be as small as 10^{-33} cm and as light as 10^{-5} g (it could be born from nothing at all…)



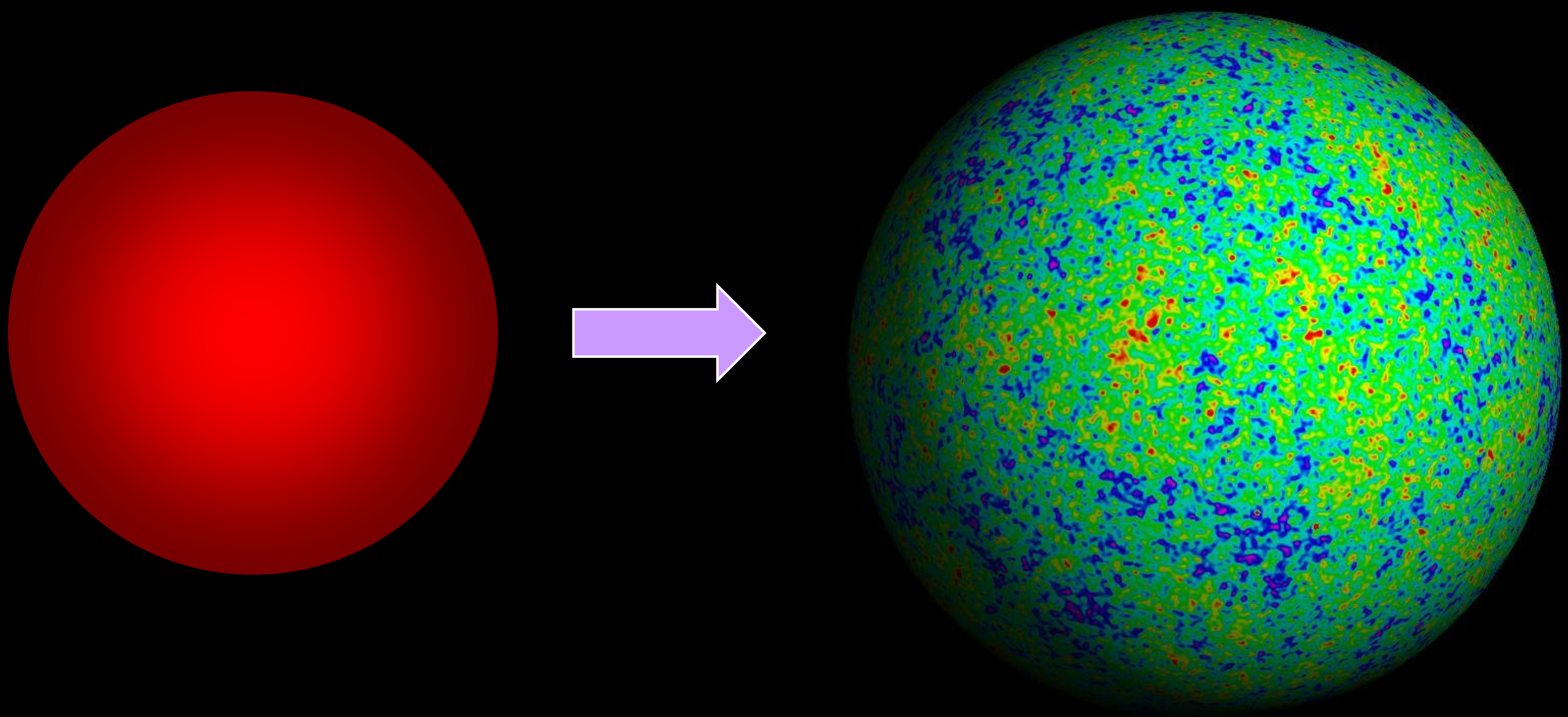
$$l \sim 10^{-33} \text{ cm}$$

$$m \sim 10^{-5} \text{ g}$$

Inflationary universe 10^{-35} seconds old



The universe after inflation becomes huge and almost absolutely uniform, but quantum fluctuations make it slightly non-uniform. This leads to formation of galaxies and tiny perturbations of the temperature of the universe



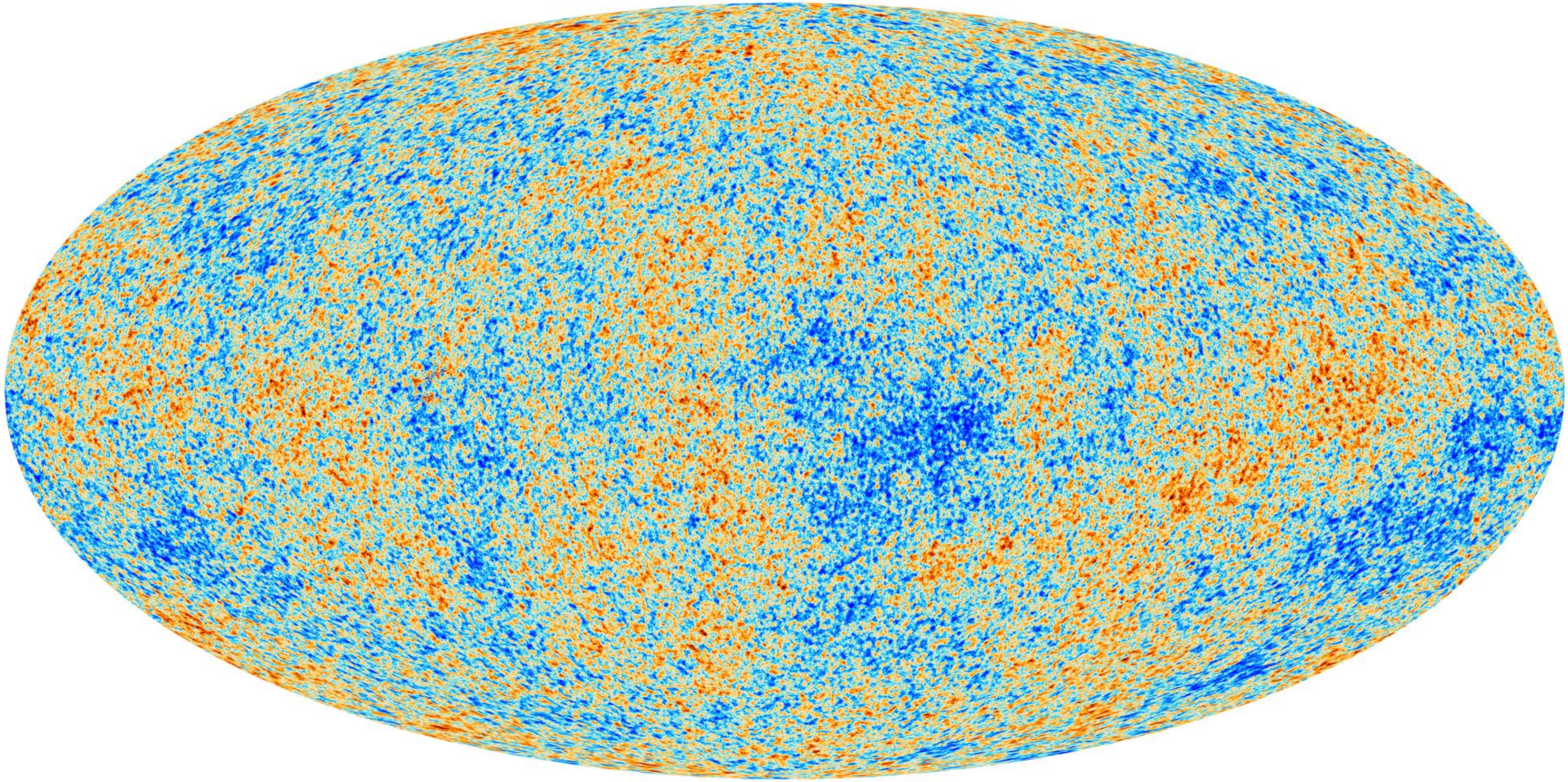
Origin of structure:

In this theory, original inhomogeneities are stretched away, but new ones are produced from **quantum fluctuations** amplified during the exponential growth of the universe.

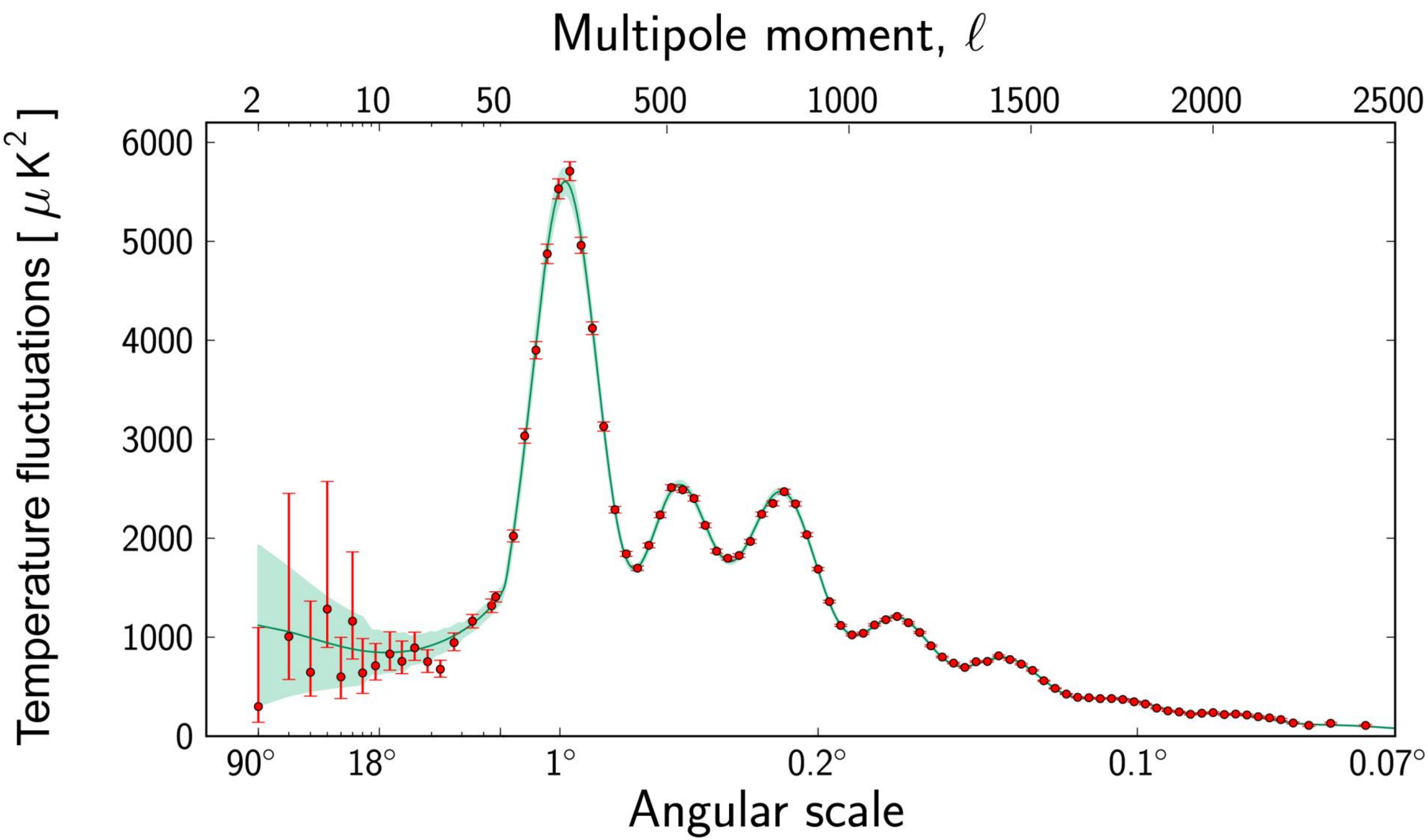
Galaxies are children of quantum fluctuations produced in the first 10^{-35} seconds after the birth of the universe.

Planck satellite: Perturbations of temperature

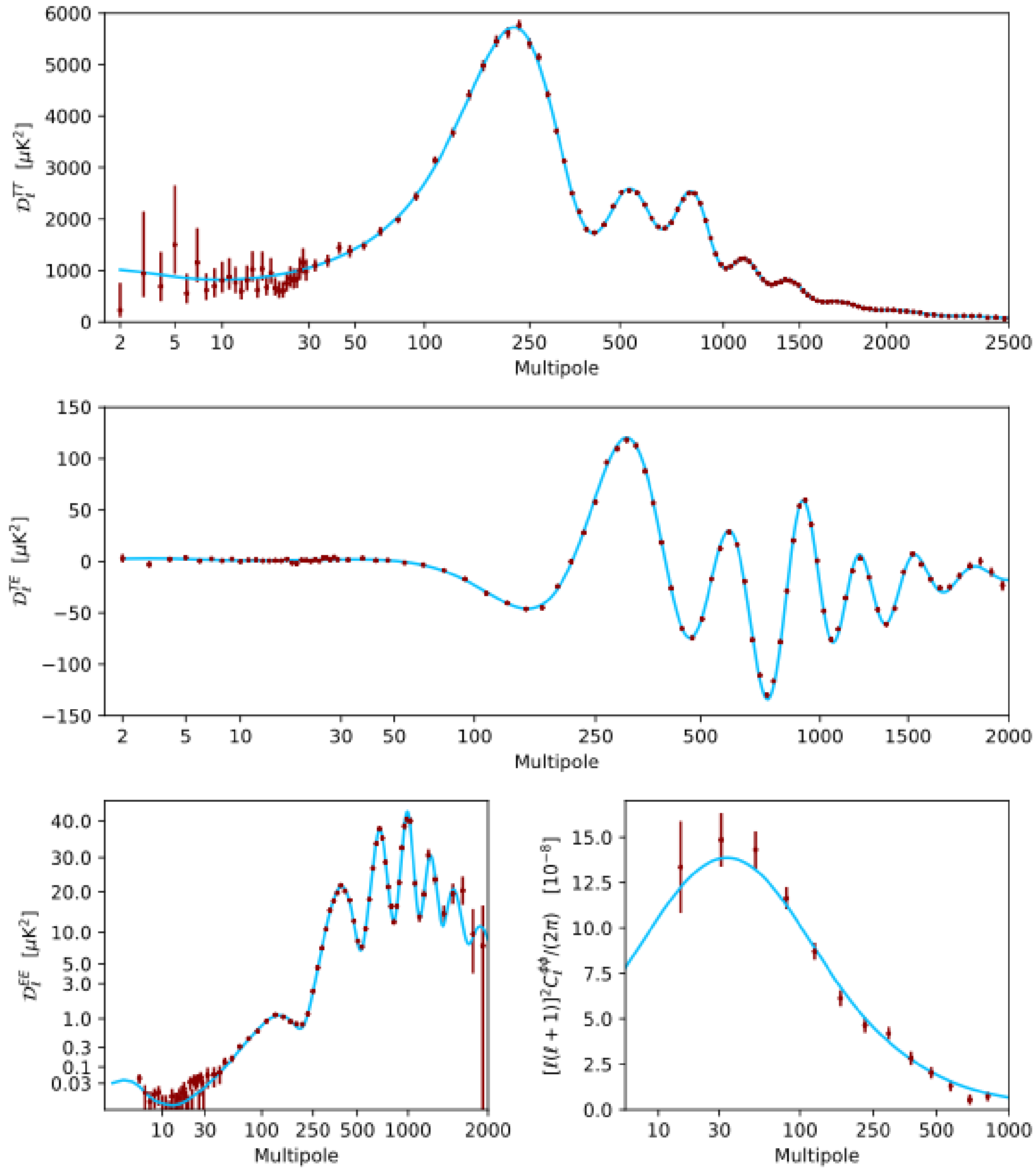
This is an image of **quantum fluctuations produced by inflation 10^{-35} seconds after the Big Bang**. These tiny fluctuations were **stretched by inflation** to incredibly large size, and now we can observe them **using all sky as a giant photographic plate**



Planck satellite: Perturbations of temperature (red dots) and predictions of inflationary theory (green line)



Planck 2018



6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell^{TT}$ [μK^2]
2 5 10 30 50 100 250 500 1000 1500 2000 2500
Multipole
150
100
50
0
−50
−100
−150
$\mathcal{D}_\ell^{TE}$ [μK^2]
2 5 10 30 50 100 250 500 1000 1500 2000
Multipole
40.0
30.0
20.0
10.0
5.0
3.0
1.0
0.3
0.1
0.03
$\mathcal{D}_\ell^{EE}$ [μK^2]
10 30 100 250 500 1000 2000
Multipole
15.0
12.5
10.0
7.5
5.0
2.5
0.0
$[\ell(\ell+1)]^2 C_\ell^{\phi\phi}/(2\pi)$ [10^{-8}]
10 30 100 250 500 1000
Multipole

Inflation and Planck 2018

$$\Omega = 1.009 \pm 0.0018$$

Planck + SPT + BAO

Universe is flat with accuracy about 10^{-2}

$$n_s = 0.965 \pm 0.004$$

Spectrum of perturbations is nearly flat

According to Planck 2018, non-inflationary HZ spectrum with $n_s = 1$ is ruled out at a better than 6σ level, just as predicted in 1981 by Mukhanov and Chibisov. (This is an important prediction of inflation, similar to asymptotic freedom in QCD.)

$$f_{\mathrm{NL}}^{\mathrm{local}} = 0.91 \pm 5$$

Agrees with predictions of the simplest inflationary models with accuracy $O(10^{-4})$.

An impressive success of inflationary theory

Can we test inflation even better ?

B-modes: a special polarization pattern which can be produced by gravitational waves generated during inflation. A discovery of the gravitational waves of this type could provide a strong additional evidence in favor of inflation.

A.A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. 30 (1979) 719
V.A. Rubakov, M.V. Sazhin, A.V. Veryaskin, Phys.Lett.B 115 (1982)

BICEP/Keck, LiteBIRD and other experiments

A non-discovery of B-modes is fine too: many models predict gravitational waves with a tiny amplitude.

A discovery of inflationary gravitational waves is **NOT** required for proving inflation, but it would be **a great gift indeed**, and not only for inflation, but for investigation of quantum gravity and processes at energies many orders above LHC.

Testing predictions of inflation

1) **The universe is flat, $\Omega = 1$**. (In the mid-90's, the consensus was that $\Omega = 0.3$, until the discovery of dark energy confirming inflation.)

2) The observable part of the universe is **uniform** (homogeneous).

3) It is **isotropic**. In particular, it does not rotate. (Back in the 80's we did not know that it is uniform and isotropic at such an incredible level.)

4) Perturbations produced by inflation are **adiabatic**

5) Unlike perturbations produced by cosmic strings, inflationary perturbations lead to many **peaks in the spectrum**

6) The large angle TE anti-correlation (WMAP, Planck) is a distinctive signature of **superhorizon fluctuations** (Spergel, Zaldarriaga 1997), ruling out many alternative possibilities

7) Inflationary perturbations should have a **nearly flat (but not exactly flat) spectrum**. A small deviation from flatness is one of the distinguishing features of inflation. It is as significant for inflationary theory as the asymptotic freedom for the theory of strong interactions

8) **Inflation produces scalar perturbations** and **tensor perturbations** with nearly flat spectrum, and it **does not produce vector perturbations**.

9) In the early 80's it could seem that inflation is ruled out because scalar perturbations are not observed at the expected level 10^{-3} required for galaxy formation. Thanks to dark matter, smaller perturbations are sufficient, and they were **found by COBE**.

10) Scalar perturbations are **Gaussian**. In non-inflationary models, the parameter f_{NL}^{local} describing the level of local non-Gaussianity can be as large as 10^4, but it is predicted to be O(1) in all single-field inflationary models. **Confirmed by Planck.** Prior to the Planck2013 data release, there were rumors that $f_{NL}^{local} \gg O(1)$, which would rule out **all** single field inflationary models

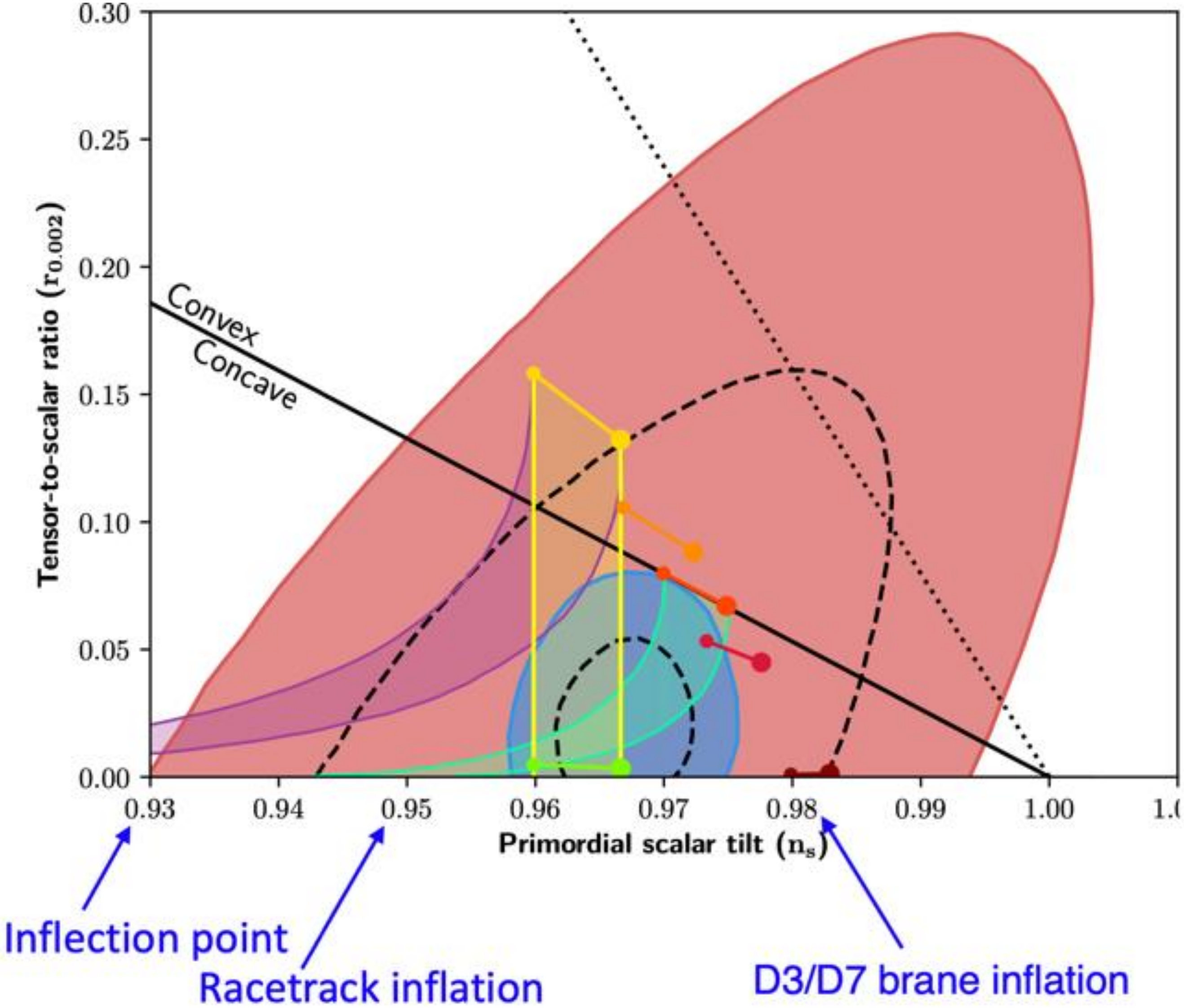
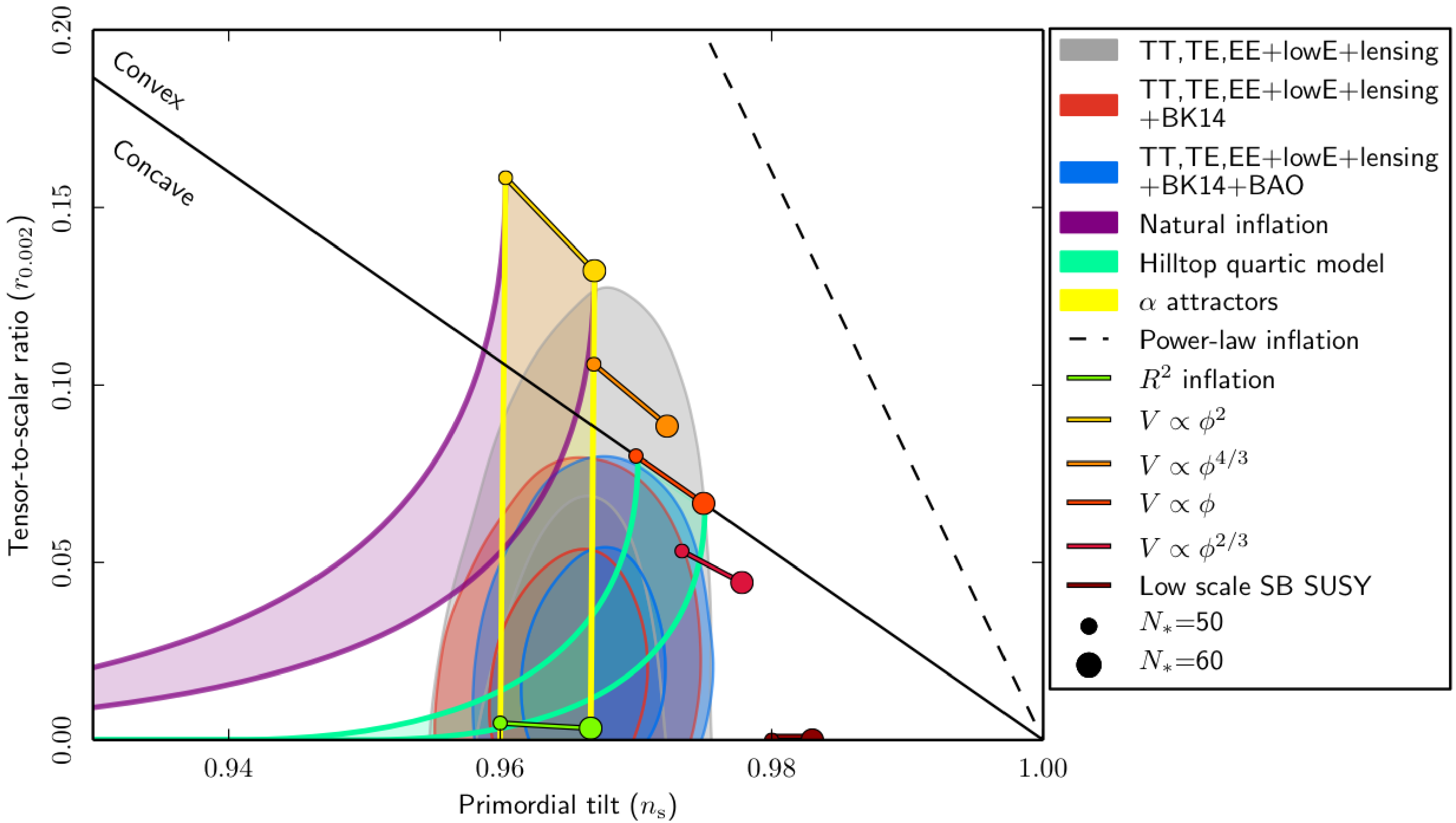


FIG. 5. Many favorite string inflation models from a decade ago, with very low r, are now ruled out by precision data on n_s. 7-year WMAP results [37] are in red, Planck 2018 results [12] are in blue.

Planck 2018



Tensor-to-scalar ratio ($r_{0.002}$)
0.20
0.15
0.10
0.05
0.00
0.94
0.96
0.98
1.00
Primordial tilt (n_s)
Convex
Concave
TT,TE,EE+lowE+lensing
TT,TE,EE+lowE+lensing +BK14
TT,TE,EE+lowE+lensing +BK14+BAO
Natural inflation
Hilltop quartic model
α attractors
Power-law inflation
R^2 inflation
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
Low scale SB SUSY
N_*=50
N_*=60

One can fit all Planck data by a polynomial, with inflation starting at the Planck density

$$V = \frac{m^2\phi^2}{2}\left(1 - a\phi + b\phi^2\right)$$

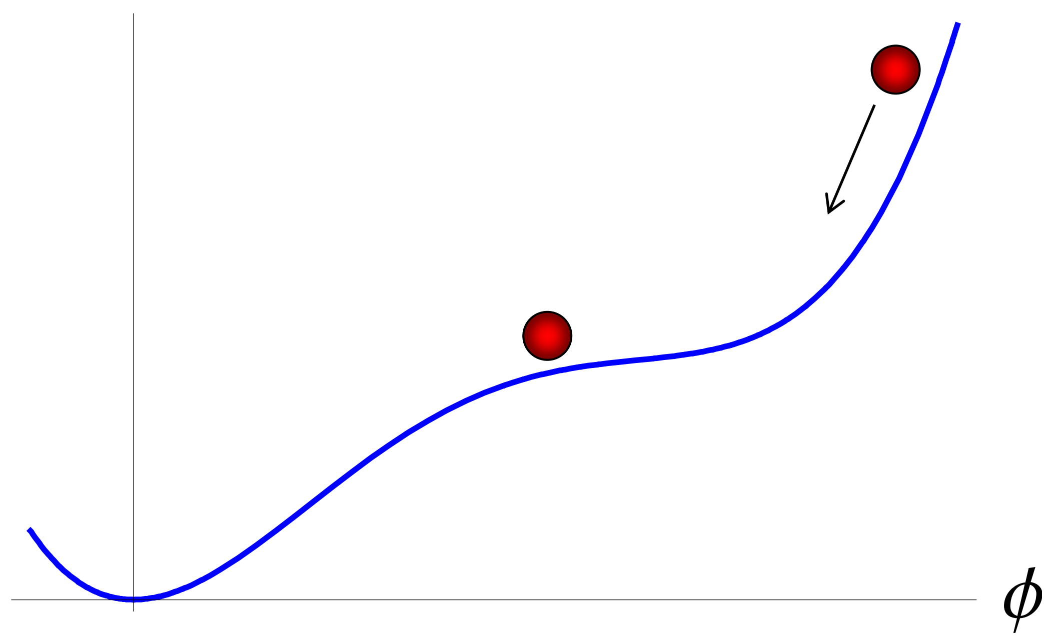
Destri, de Vega, Sanchez, 2007
Nakayama, Takahashi and Yanagida, 2013
Kallosh, AL, Westphal 2014
Kallosh, AL, Roest, Yamada 1705.09247

3 observables: A_s, n_s, r

3 parameters: m, a, b

Example: m =10^{-5}, a = 0.12, b=0.29

No problem with initial conditions



A simple polynomial superpotential with 3 parameters can describe the full range of all possible values of A_s, n_s and r, all the way to r = 0 and n_s = 1

Kallosh, AL, Westphal 2014

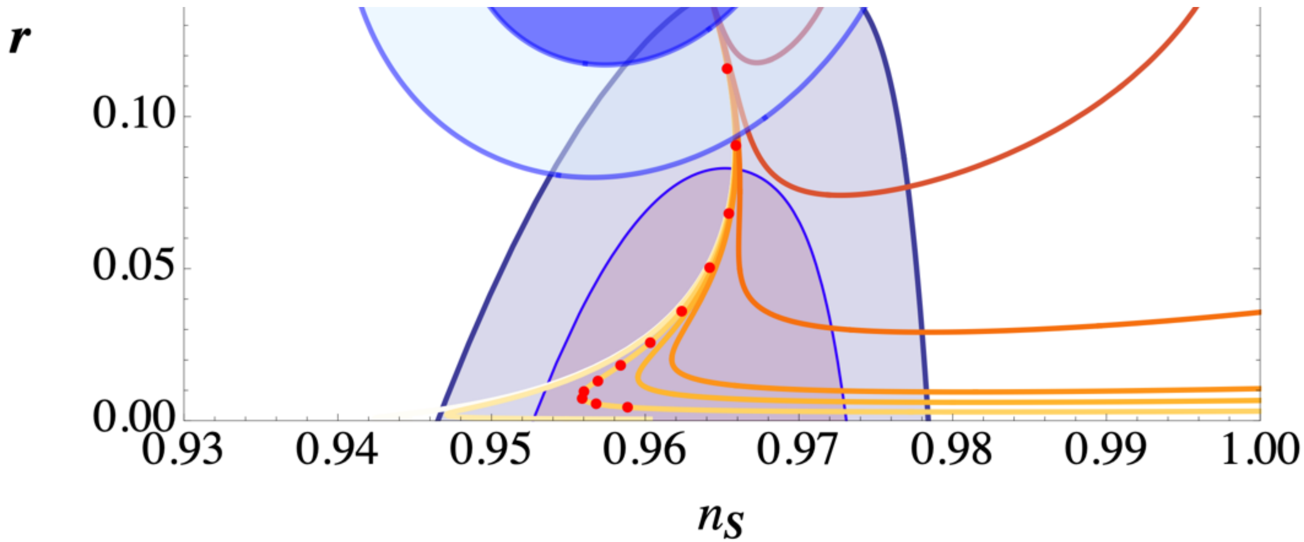


FIG. 3. Predictions for $n_s(a)$ and $r(a)$ in at 55 e-folds the model with $V(\phi) = \frac{m^2\phi^2}{2}\left(1 - a\phi + a^2 b\,\phi^2)\right)^2$ for various values of $b = 0.334\ldots5$. All curves have a running from 0.001 to 0.2. The red ($b = 0.34$) and green ($b = 5$) balls correspond to $a = 0.01\ldots0.13$

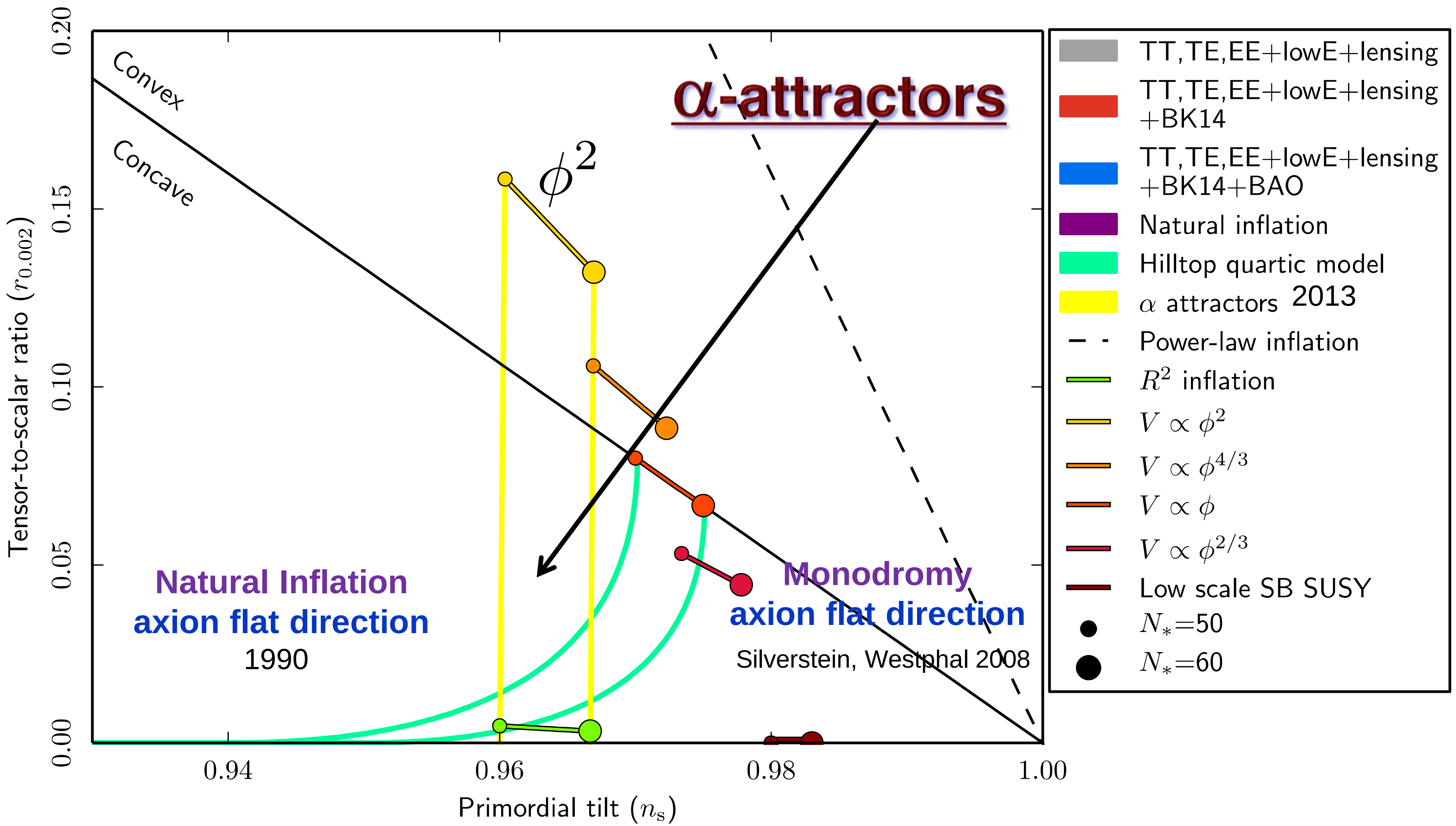
List of models favored by Planck2018

Inflationary model	Potential $V(\phi)$	Parameter range	$\Delta\chi^2$	$\ln B$
$R + R^2/(6M^2)$	$\Lambda^4\left(1 - e^{-\sqrt{2/3}\,\phi/M_{\rm Pl}}\right)^2$	...	...	...
Power-law potential	$\lambda M_{\rm Pl}^{10/3}\phi^{2/3}$	...	2.8	−2.6
Power-law potential	$\lambda M_{\rm Pl}^{3}\phi$	...	2.5	−1.9
Power-law potential	$\lambda M_{\rm Pl}^{8/3}\phi^{4/3}$	...	10.4	−4.5
Power-law potential	$\lambda M_{\rm Pl}^{2}\phi^{2}$	...	22.3	−7.1
Power-law potential	$\lambda M_{\rm Pl}\phi^{3}$	...	40.9	−19.2
Power-law potential	$\lambda\phi^{4}$	...	89.1	−33.3
Non-minimal coupling	$\lambda^4\phi^4 + \xi\phi^2 R/2$	$-4 < \log_{10}\xi < 4$	3.1	−1.6
Natural inflation	$\Lambda^4\left[1 + \cos(\phi/f)\right]$	$0.3 < \log_{10}(f/M_{\rm Pl}) < 2.5$	9.4	−4.2
Hilltop quadratic model	$\Lambda^4\left(1 - \phi^2/\mu_2^2 + \ldots\right)$	$0.3 < \log_{10}(\mu_2/M_{\rm Pl}) < 4.85$	1.7	−2.0
Hilltop quartic model	$\Lambda^4\left(1 - \phi^4/\mu_4^4 + \ldots\right)$	$-2 < \log_{10}(\mu_4/M_{\rm Pl}) < 2$	−0.3	−1.4
D-brane inflation ($p = 2$)	$\Lambda^4\left(1 - \mu_{D2}^2/\phi^p + \ldots\right)$	$-6 < \log_{10}(\mu_{D2}/M_{\rm Pl}) < 0.3$	−2.3	1.6
D-brane inflation ($p = 4$)	$\Lambda^4\left(1 - \mu_{D4}^4/\phi^p + \ldots\right)$	$-6 < \log_{10}(\mu_{D4}/M_{\rm Pl}) < 0.3$	−2.2	0.8
Potential with exponential tails	$\Lambda^4\left[1 - \exp(-q\phi/M_{\rm Pl}) + \ldots\right]$	$-3 < \log_{10} q < 3$	−0.5	−1.0
Spontaneously broken SUSY	$\Lambda^4\left[1 + \alpha_h\log(\phi/M_{\rm Pl}) + \ldots\right]$	$-2.5 < \log_{10}\alpha_h < 1$	9.0	−5.0
E-model ($n = 1$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\,\phi\left(\sqrt{3\alpha_1^{\rm E}}M_{\rm Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_1^{\rm E} < 4$	0.2	−1.0
E-model ($n = 2$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\,\phi\left(\sqrt{3\alpha_2^{\rm E}}M_{\rm Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_2^{\rm E} < 4$	−0.1	0.7
T-model ($m = 1$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_1^{\rm T}}M_{\rm Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_1^{\rm T} < 4$	−0.1	0.1
T-model ($m = 2$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_2^{\rm T}}M_{\rm Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_2^{\rm T} < 4$	−0.4	0.1

Planck 2018: Not all theories fit the data

α-attractors, Starobinsky, Higgs, fiber inflation, D-brane inflation

saxion flat direction: **plateau potentials**



Subsequent developments:

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Hilltop quartic model	$\Lambda^4\left(1 - \phi^4/\mu_4^4 + \dots\right)$	$-2 < \log_{10}(\mu_4/M_{Pl}) < 2$	−0.3	−1.4
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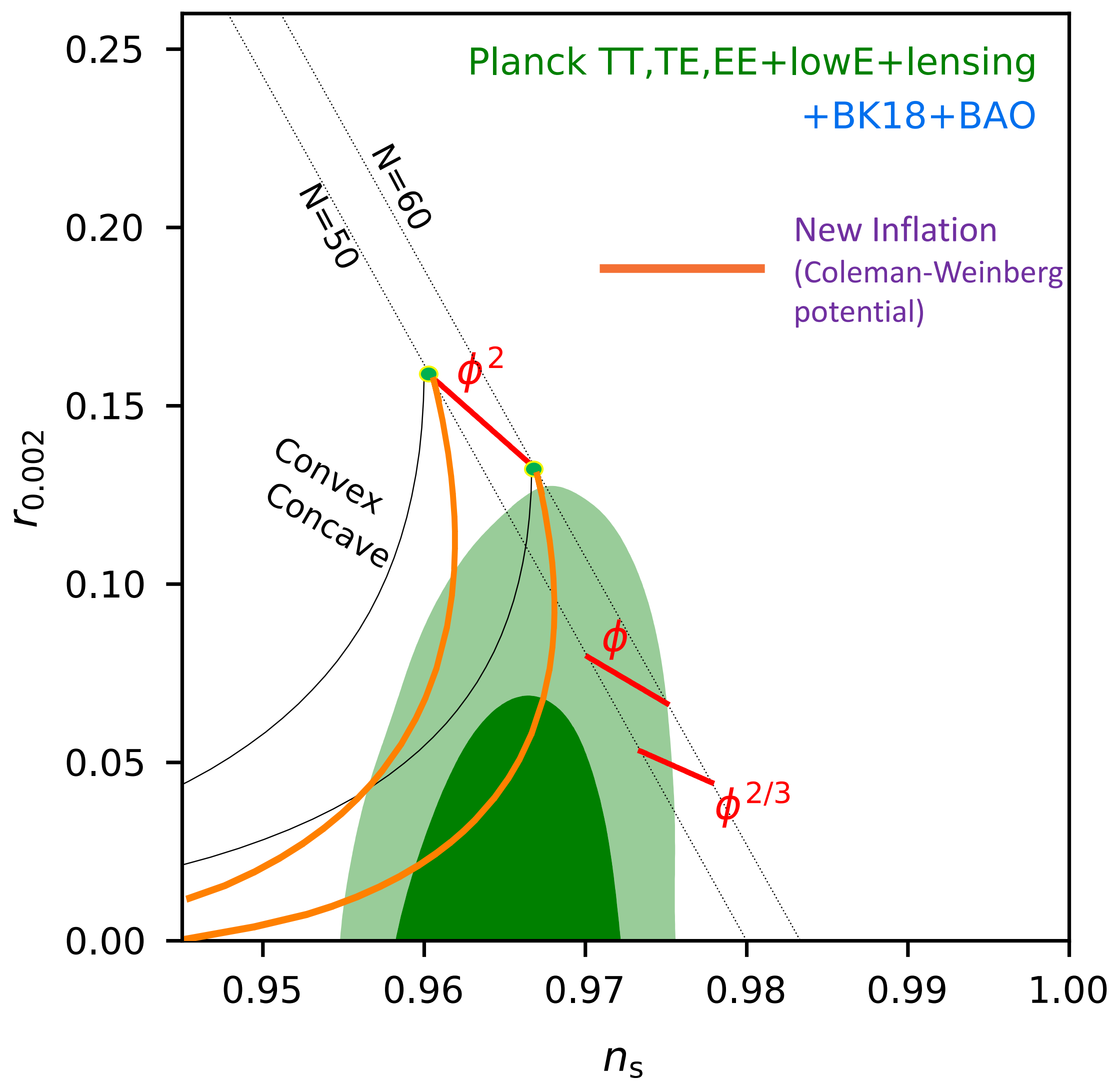
Spontaneously broken SUSY	$\Lambda^4\left[1 + \alpha_h \log(\phi/M_{Pl}) + \dots\right]$	$-2.5 < \log_{10}\alpha_h < 1$	9.0	−5.0
E-model ($n = 1$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\,\phi\left(\sqrt{3\alpha_1^E}\,M_{Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_1^E < 4$	0.2	−1.0
E-model ($n = 2$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\,\phi\left(\sqrt{3\alpha_2^E}\,M_{Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_2^E < 4$	−0.1	0.7
T-model ($m = 1$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_1^T}\,M_{Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_1^T < 4$	−0.1	0.1
T-model ($m = 2$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_2^T}\,M_{Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_2^T < 4$	−0.4	0.1

Natural inflation, new inflation, and monomial potentials are disfavored by Planck + BICEP/Keck 2021



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Updated constraints on amplitude and tilt of the tensor primordial spectrum

Giacomo Galloni,[1,2,*] Nicola Bartolo,[3,4,5] Sabino Matarrese,[3,4,5,6]
Marina Migliaccio,[1,2] Angelo Ricciardone,[3,4] and Nicola Vittorio[1,2]

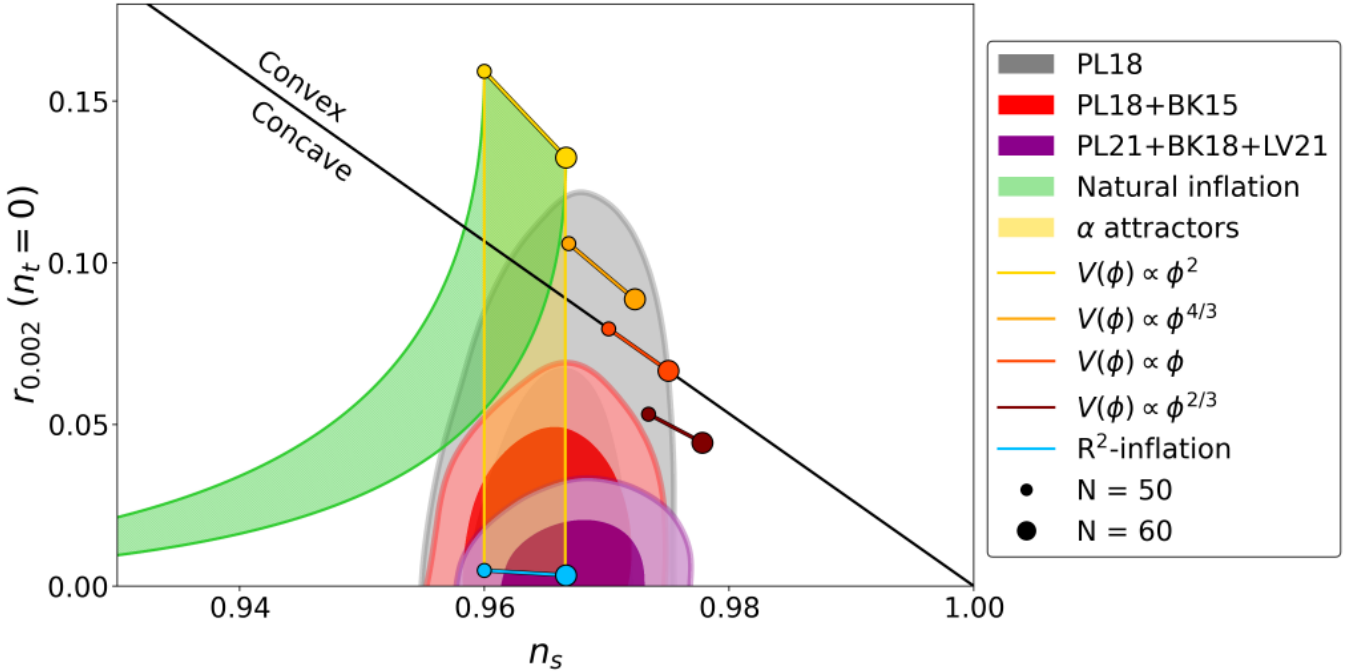

FIG. 12. 2D 68 and 95% CL intervals in the $(r_{0.002}, n_s)$-plane for PL18 (publicly available MCMC chains[a]), PL18+BK15 and PL21+BK18+LV21. $r_{0.002}$ is obtained from our chains assuming $n_t = 0$. For more details on the various inflationary models, see [13].

Constraining Cosmological Physics with DESI BAO Observations

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Instituto de Física Corpuscular (CSIC-Universitat de València), E-46980 Paterna, Spain

2σ constraint

$r_{0.05} = 0.017^{+0.020}_{-0.017}$

2404.06796

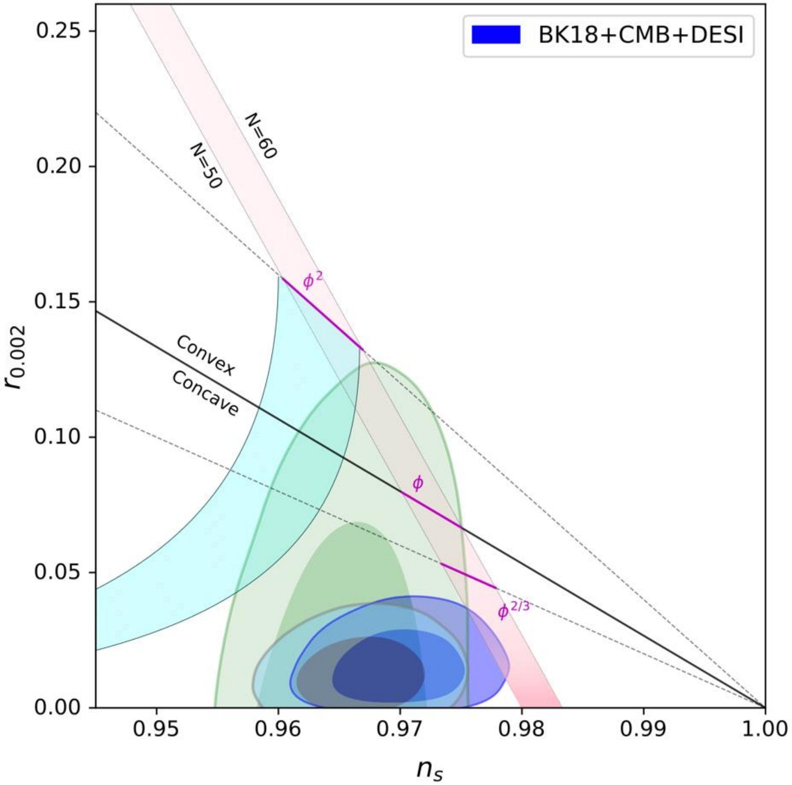


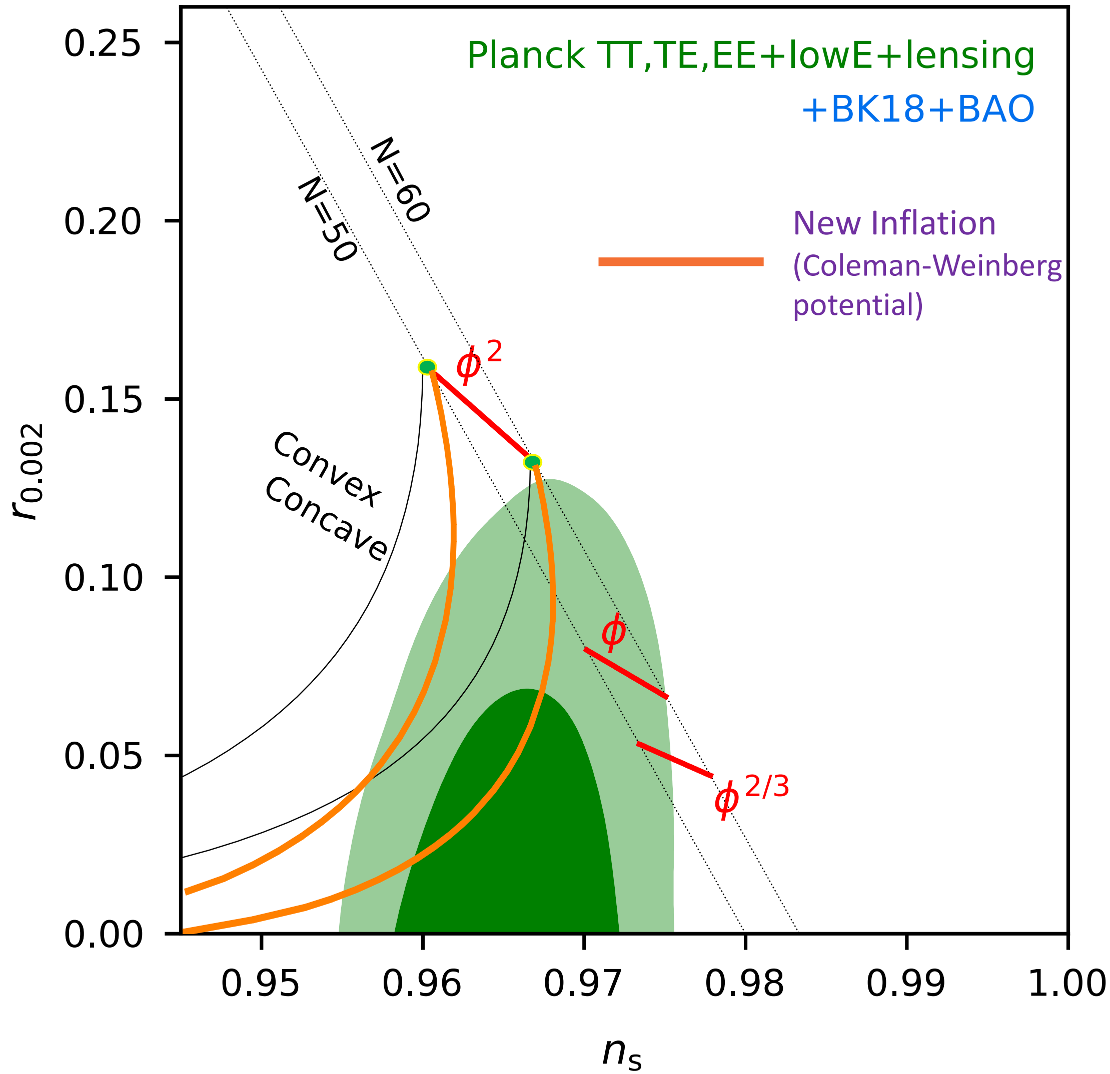
FIG. 1: The two-dimensional marginalized posterior distributions of the parameter pair (n_s, $r_{0.002}$) from CMB (green), BK18+CMB+SDSS (grey) and BK18+CMB+DESI (blue) observations in the $\Lambda+r$ scenario, compared to the theoretical predictions of selected inflationary models. Here ϕ denotes the inflationary scalar field and $N \equiv \ln a$ is the e-folding number.

How do we park H_0 ?

George Efstathiou, April 2024

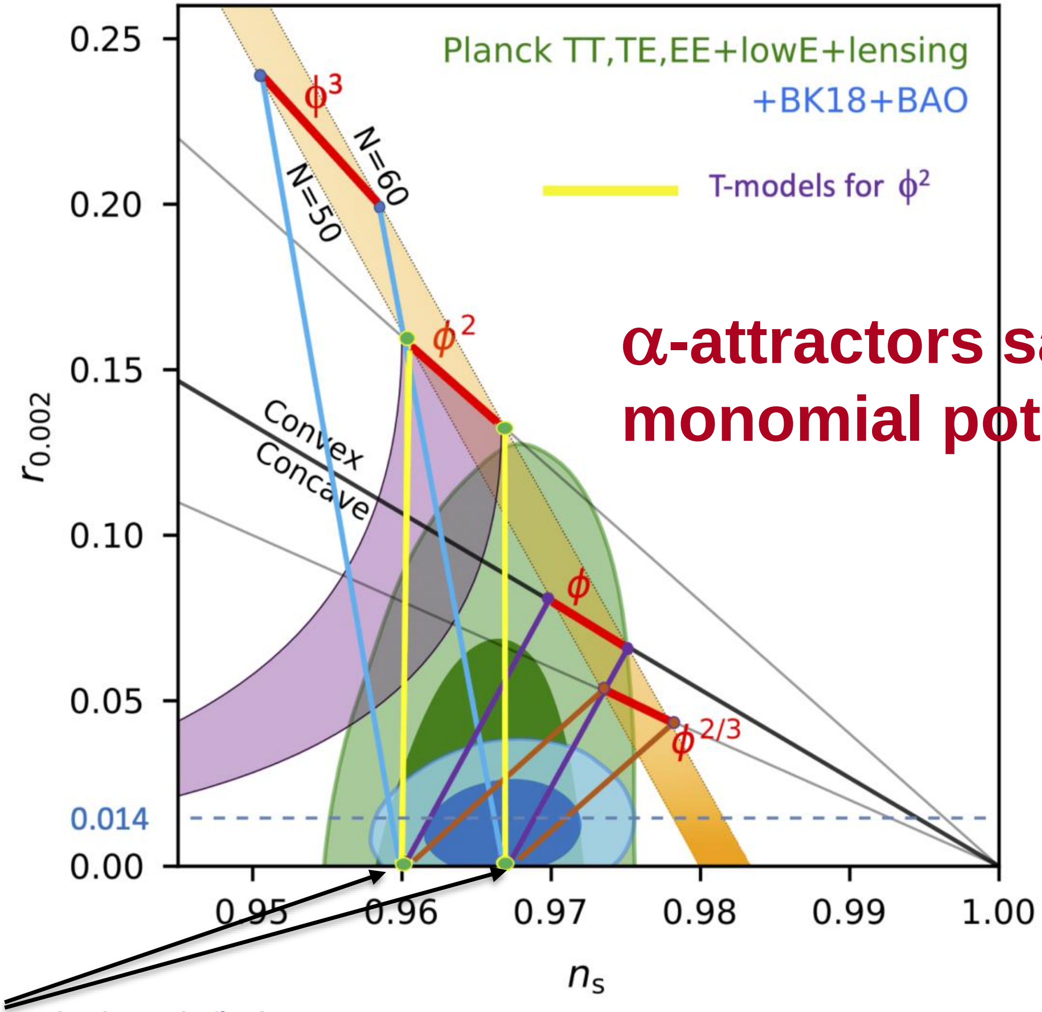
Planck2018 – BICEP/Keck2021 constraints

How do we "park" inflation?



Parking Inflation:

Predictions for n_s in α-attractor models at $\alpha < O(1)$ practically do not depend on the choice of the potential



α-attractors saving monomial potentials

Starobinsky model and Higgs inflation

α-attractors

Kallosh, AL, Roest 2013

To match observations, the simplest chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

should be modified:

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\underline{\frac{\phi^2}{6\alpha}})^2} - \frac{1}{2}m^2\phi^2$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$

The potential becomes

$$V = 3\alpha\, m^2 \tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

This model (**α-attractor T-model**) is consistent with observational data for m ~ 10^{-5} and **<u>any</u>** value of α smaller than O(5).

What is the meaning of α-attractors?

More generally:

$$\frac{L}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial_\mu \varphi)^2}{2\left(1 - \frac{\varphi^2}{6\alpha}\right)^2} - V(\varphi)$$

In canonical variables

$$\frac{L}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2} - V\left(\sqrt{6\alpha}\,\tanh\frac{\phi}{\sqrt{6\alpha}}\right)$$

Asymptotically at large values of the field

$$V(\phi) = V_0 - 2\sqrt{6\alpha}\, V_0'\, e^{-\sqrt{\frac{2}{3\alpha}}\phi}$$

Here $V_0' = \partial_\varphi V|_{\varphi=\sqrt{6\alpha}}$ This factor can be absorbed in the redefinition (shift) of the field. **Therefore, at small α, values of n_s and r depend only on V_0 and α**, not on the shape of V(ϕ).

$$n_s = 1 - \frac{2}{N_e}\,, \qquad r = \frac{12\alpha}{N_e^2}$$

E-models of α-attractors

Kallosh, AL, Roest 2014

Start with the model

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{3\alpha}{4}\frac{(\partial\rho)^2}{\rho^2} - V(\rho)$$

Switch to canonical variables

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{1}{2}(\partial\varphi)^2 - V(e^{-\sqrt{\frac{2}{3\alpha}}\varphi}).$$

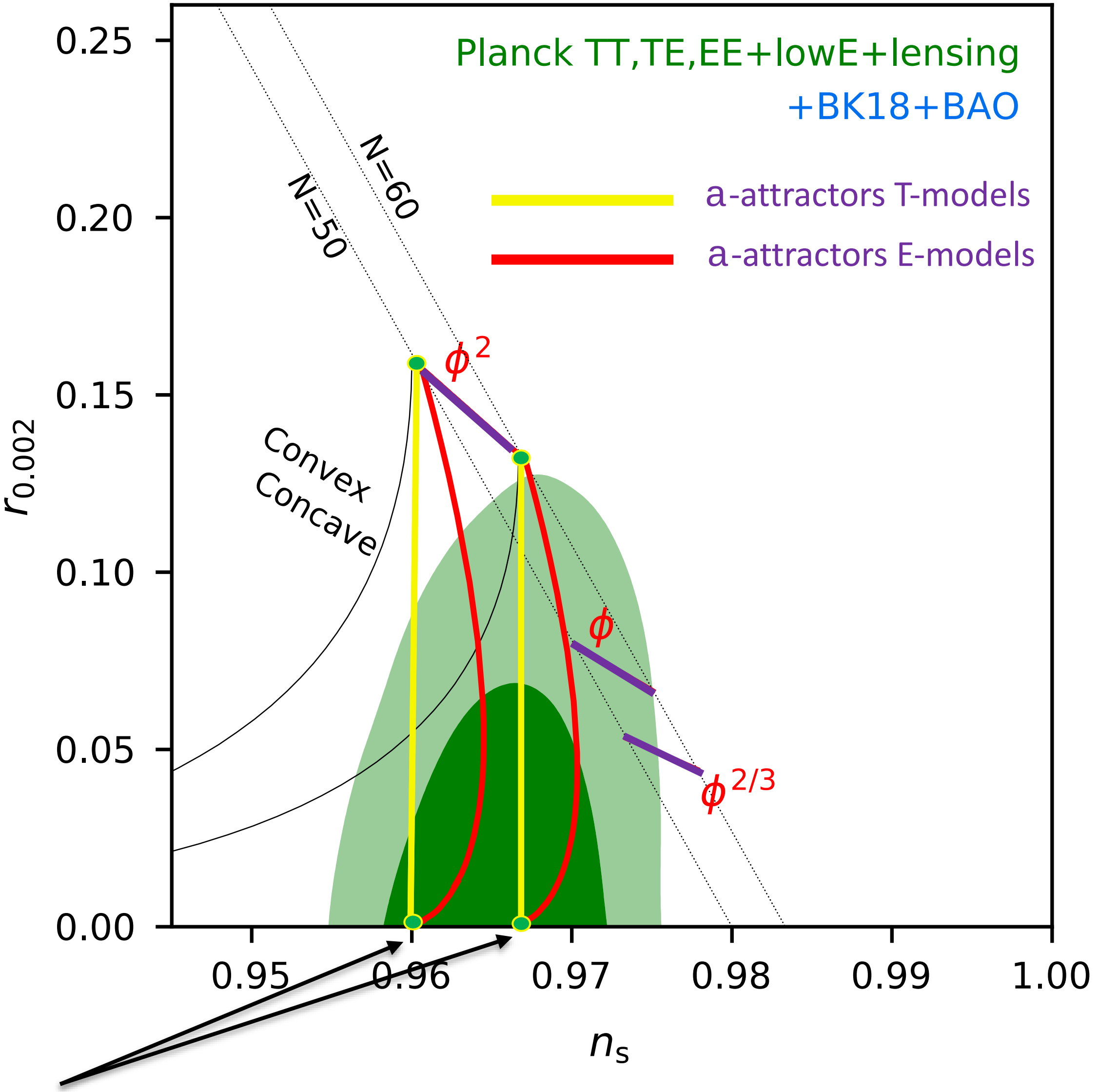
In particular, for $V(\rho) = V_0(1-\rho)^2$ the potential becomes

$$V = V_0\Big(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\Big)^2$$

This model (E-model) coincides with the Starobinsky model for α =1. In general case these models predict

$$n_s = 1 - \frac{2}{N_e}\,, \qquad r = \frac{12\alpha}{N_e^2}$$

Planck2018 – BICEP/Keck2021 constraints



Starobinsky model and Higgs inflation

Inflation in supergravity

Main problem:

$$V(\phi) = e^{K}\left(K^{-1}_{\Phi\bar{\Phi}}\,|D_\Phi W|^2 - 3|W|^2\right)$$

Canonical Kahler potential is $K = \Phi\bar{\Phi}$

Therefore, the potential blows up at large $|\Phi|$, and slow-roll inflation is impossible:

$$V \sim e^{|\Phi|^2}$$

Too steep, no inflation…

A general solution

Kallosh, A.L. 2010, Kallosh, A.L., Rube, 2010

$$W = X\, f(\Phi)$$

Superpotential must be a real holomorphic function. The Kahler potential is any function of the type

$$\mathcal{K}((\Phi - \bar{\Phi})^2,\, X\bar{X})$$

The potential as a function of the real part of Φ at X = 0 is

$$V = |f(\Phi)|^2$$

FUNCTIONAL FREEDOM in choosing inflationary potential

This method and its generalizations are especially powerful if X is a nilpotent field, $X^2=0$.

Antoniadis, Dudas, Ferrara, Sagniotti 2014
Ferrara, Kallosh, A.L. 2014

Model-building Paradise

Kallosh, A.L, Roest, Yamada 1705.09247;
Gunaydin, Kallosh, A.L, Yamada 2008.01494,
Kallosh, A.L, Wrase, Yamada 2108.08491,
2108.08492

Consider a theory with a Kahler potential

$$K(T,\overline{T}) = K_0(T,\overline{T}) + \frac{F_X^2}{F_X^2 + V_{\rm infl}(T,\overline{T})} X\overline{X}$$

and superpotential

$$W = (W_0 + F_X X)\, e^{-\kappa(T)/2}$$

Here X is a nilpotent field, and

$$\kappa(T) \equiv K_0(T,\overline{T})|_{\overline{T}\to T}$$

Then the potential along the direction $T = \overline{T} = t$ is given by

$$V_{\rm total}(T) = \Lambda + V_{\rm infl}(T,\overline{T})|_{T=\overline{T}=t}$$

and the cosmological constant is

$$\Lambda = F_X^2 - 3W_0^2$$

Example: single-field α-attractor

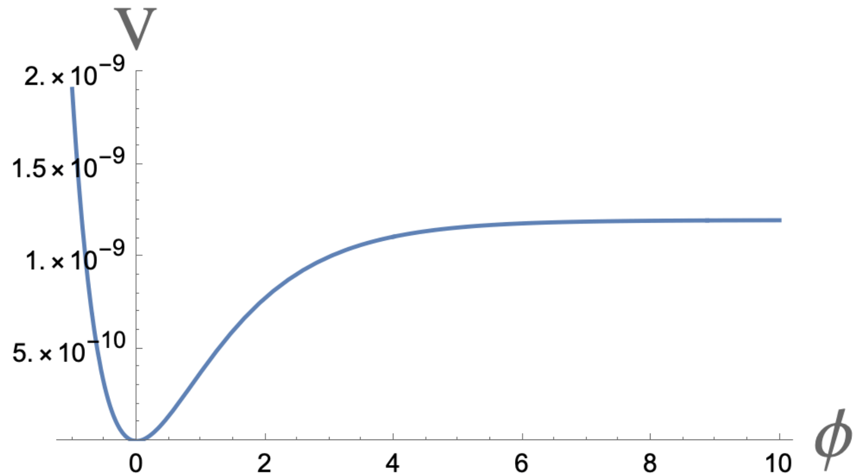
$$K(T,\overline{T}) = -3\alpha \log(T+\overline{T}) + \frac{F_X^2}{F_X^2 + V_{\rm infl}(T,\overline{T})} X\overline{X}$$

$$W(T) = (W_0 + F_X X)\sqrt{2T}$$

$$V_{\rm infl} = m^2(1-T)(1-\overline{T})$$

In canonical variables, along the real T flat direction one has the α-attractor potential

$$V_{\rm total}(\phi) = \Lambda + m^2\left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\phi}\right)^2$$



Towards sequestered inflation

The same results remain true in the theory with many moduli $\mathrm{T_i}$ if we add to the superpotential any function $W^{(I)}(T_i)$ such that

$$W^{(I)}(T_i) = 0 \ , \qquad \partial_{T_j} W^{(I)}(T_i) = 0$$

along the direction $T_i = \overline{T}_i = t_i$

In the absence of the nilpotent field X, this theory would describe supersymmetric Minkowski flat directions, but in our construction the potential along the flat (inflaton) directions is given by

$$V_{\rm total}(T_i) = \Lambda + V_{\rm infl}(T_i, \overline{T}_i)|_{T_i = \overline{T}_i = t_i}$$

Importantly, this potential does NOT depend on the value of the superpotential $W^{(I)}(T_i)$ outside of the flat inflaton directions.

This allows to disentangle, sequester, dynamics of inflation from the large energy scale encoded in $W^{(I)}(T_i)$.

IIB string theory: STU model

Kallosh, A.L, Wrase, Yamada 2108.08492

$$
\begin{aligned}
W =& e_0 + \mathrm{i}\sum_{I=1}^{3} e_I U_I - \sum_{I=1}^{3} q_I \frac{U_1U_2U_3}{U_I} + \mathrm{i} m U_1U_2U_3 \\
&+ S\left[\mathrm{i}h_0 - \sum_{I=1}^{3} a_I U_I + \sum_{I=1}^{3} \bar{a}_I \frac{U_1U_2U_3}{U_I} - \bar{h}_0 U_1U_2U_3\right] \\
&+ \sum_{I=1}^{3} T_I\left[-\mathrm{i}h_I - \sum_{J=1}^{3} U_J b_{JI} + \sum_{J=1}^{3} \mathrm{i}\bar{b}_{JI}\frac{U_1U_2U_3}{U_J} + \bar{h}_I U_1U_2U_3\right] - S\sum_{I=1}^{3} f_I T_I \\
&+ \sum_{I,J=1}^{3} \mathrm{i} g_{JI} S U_J T_I + \sum_{I,J=1}^{3} \bar{g}_{JI} S T_I \frac{U_1U_2U_3}{U_J} - \mathrm{i} S U_1U_2U_3 \sum_{I=1}^{3} \bar{f}_I T_I \,.
\end{aligned}
$$

Superpotential due to Aldazabal, Camara, Font and Ibanez, 2006

Tadpole cancellation: Bianchi Identities in 10D supergravity with local sources

$$
N_{\mathrm{D3}} = 16 - \frac{1}{2}\left[m h_0 - e_0\bar{h}_0 + \sum_{I=1}^{3}(q_I a_I + e_I \bar{a}_I)\right].
$$

$$
N_{\mathrm{NS7}_I} = \frac{1}{2}\left[h_0\bar{f}_I - \bar{h}_0 f_I - \sum_{J=1}^{3}(\bar{a}_J g_{JI} - a_J\bar{g}_{JI})\right]
$$

$$
N_{\mathrm{I7}_I} = -\frac{1}{2}\left[e_0\bar{f}_I - m f_I + \sum_{J=1}^{3}(q_J g_{JI} + e_J\bar{g}_{JI})\right]
$$

$$
\begin{aligned}
\bar{h}_0 h_J + \bar{a}_I b_{IJ} + \bar{a}_J \bar{b}_{KJ} - a_K \bar{b}_{KJ} + m f_J - q_I g_{IJ} - q_J g_{JJ} - e_K \bar{g}_{KJ} &= 0, \\
h_0 \bar{h}_J + a_I \bar{b}_{IJ} + a_J \bar{b}_{JJ} - \bar{a}_K b_{KJ} - e_0 \bar{f}_J - e_I \bar{g}_{IJ} - e_J \bar{g}_{JJ} - q_K g_{KJ} &= 0, \\
\bar{h}_0 b_{KJ} + \bar{a}_I \bar{b}_{JJ} + \bar{a}_J \bar{b}_{IJ} - a_K \bar{h}_J + m g_{KJ} - q_I \bar{g}_{JJ} - q_J \bar{g}_{IJ} - e_K \bar{f}_J &= 0, \\
h_0 \bar{b}_{KJ} + a_I b_{JJ} + a_J b_{IJ} - \bar{a}_K h_J - e_0 \bar{g}_{KJ} - e_I g_{JJ} - e_J g_{IJ} - q_K f_J &= 0.
\end{aligned}
$$

$$
\begin{aligned}
-g_{II} g_{JK} + \bar{g}_{KI} f_K + f_I \bar{g}_{KK} - g_{JI} g_{IK} &= 0, \\
-\bar{g}_{II} \bar{g}_{JK} + g_{KI} \bar{f}_K + \bar{f}_I g_{KK} - \bar{g}_{JI} \bar{g}_{IK} &= 0, \\
-g_{II} \bar{g}_{IJ} + \bar{g}_{JI} g_{JJ} + f_I \bar{f}_J - g_{KI} \bar{g}_{KJ} &= 0, \\
\bar{g}_{II} g_{IJ} - g_{JI} \bar{g}_{JJ} + f_I \bar{f}_J - g_{KI} \bar{g}_{KJ} &= 0.
\end{aligned}
$$

$$
\begin{aligned}
-b_{II} b_{JK} + \bar{b}_{KI} h_K + h_I \bar{b}_{KK} - b_{JI} b_{IK} &= 0, \\
-\bar{b}_{II} \bar{b}_{JK} + b_{KI} \bar{h}_K + \bar{h}_I b_{KK} - \bar{b}_{JI} \bar{b}_{IK} &= 0, \\
-b_{II} \bar{b}_{IJ} + \bar{b}_{JI} b_{JJ} + h_I \bar{h}_J - b_{KI} \bar{b}_{KJ} &= 0, \\
\bar{b}_{II} b_{IJ} - b_{JI} \bar{b}_{JJ} + h_I \bar{h}_J - b_{KI} \bar{b}_{KJ} &= 0.
\end{aligned}
$$

$$
\begin{aligned}
b_{KK} \bar{g}_{KJ} - h_K \bar{f}_J - \bar{b}_{JK} g_{JJ} + b_{IK} \bar{g}_{IJ} + g_{KK} \bar{b}_{KJ} - f_K \bar{h}_J - \bar{g}_{JK} b_{JJ} + g_{IK} \bar{b}_{IJ} &= 0, \\
b_{KK} g_{IJ} - h_K \bar{g}_{JJ} - \bar{b}_{JK} f_J + b_{IK} g_{KJ} + g_{KK} b_{IJ} - f_K \bar{b}_{JJ} - \bar{g}_{JK} h_J + g_{IK} b_{KJ} &= 0, \\
\bar{b}_{KK} \bar{g}_{IJ} - \bar{h}_K g_{JJ} - b_{JK} \bar{f}_J + \bar{b}_{IK} \bar{g}_{KJ} + \bar{g}_{KK} \bar{b}_{IJ} - \bar{f}_K b_{JJ} - g_{JK} \bar{h}_J + \bar{g}_{IK} \bar{b}_{KJ} &= 0, \\
\bar{b}_{KK} g_{KJ} - \bar{h}_K f_J - b_{JK} \bar{g}_{JJ} + \bar{b}_{IK} g_{IJ} + \bar{g}_{KK} b_{KJ} - \bar{f}_K h_J - g_{JK} \bar{b}_{JJ} + \bar{g}_{IK} b_{IJ} &= 0.
\end{aligned}
$$

Type IIB string theory and sequestered inflation

Kallosh, A.L, Roest, Yamada 2108.08491, 2108.08492

Seven chiral superfields (S, T_I, U_I) where $I = 1, 2, 3$

Example of a flux superpotential satisfying tadpole cancellation conditions with supersymmetric Minkowski flat directions

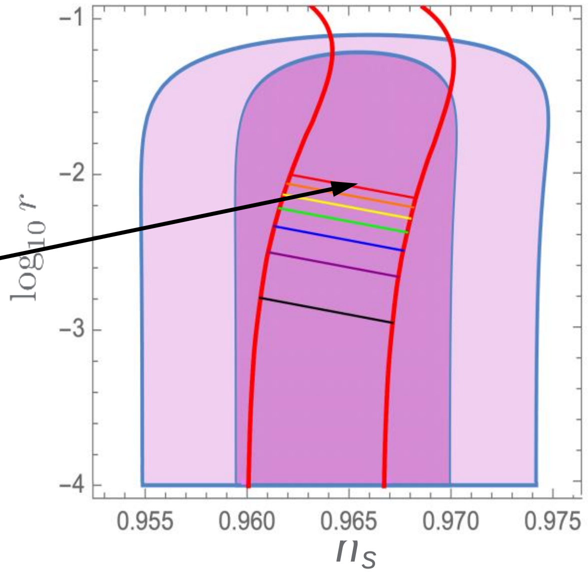
$$W = (S - U_1)(T_1 - U_2) + (S - U_2)(T_2 - U_3) + (S - U_3)(T_3 - U_1)$$

1 flat direction $S = T_1 = T_2 = T_3 = U_1 = U_2 = U_3$

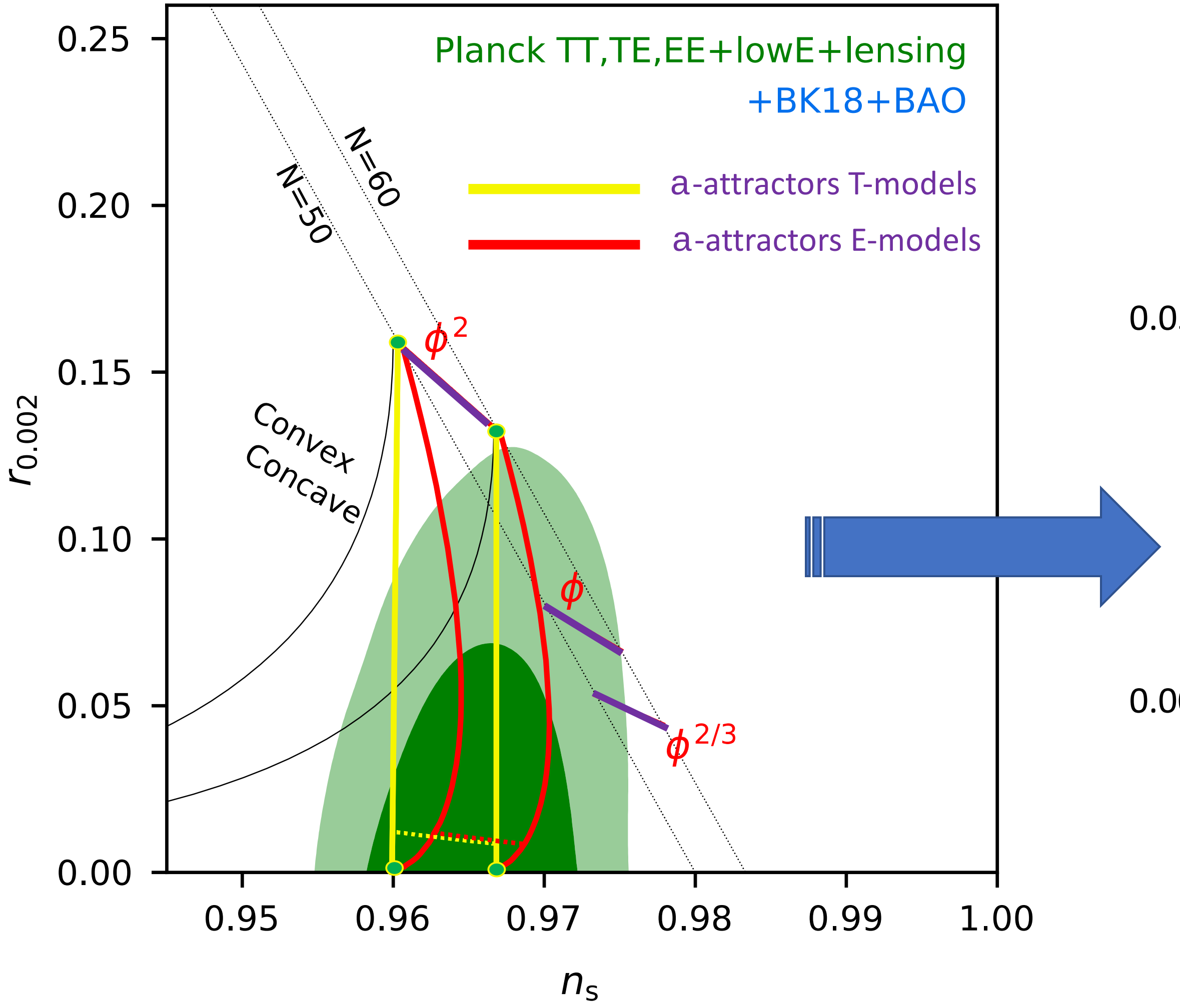
After uplifting of this flat direction and transformation to canonical variables, one finds α-attractor inflationary potential with $3\alpha = 7$ and $r = 10^{-2}$

$$V_{\rm total}(\phi) = \Lambda + m^2 \left(1 - e^{-\sqrt{\frac{2}{7}}\phi}\right)^2$$

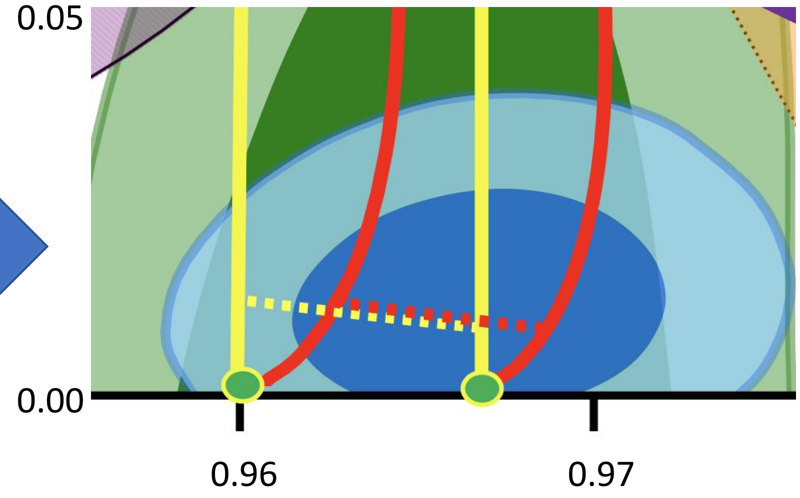
Describes the upper nearly horizontal line in this figure



BICEP/Keck2021 do not claim a discovery of the gravitational waves. The error bars of their result $r_{0.05} = 0.014^{+0.010}_{-0.011}$ are too large, $\sigma(r) = 0.009$. However, it is quite intriguing that the yellow and red dashed lines, which show the predictions of the largest option $\alpha = 7/3$, go straight through the center of the dark blue ellipse favored by Planck/BICEP/Keck data.



BICEP/Keck hope to reach σ(r) = 0.003 within 5 years.



Can we use the same method to "park" inflation AND dark energy?

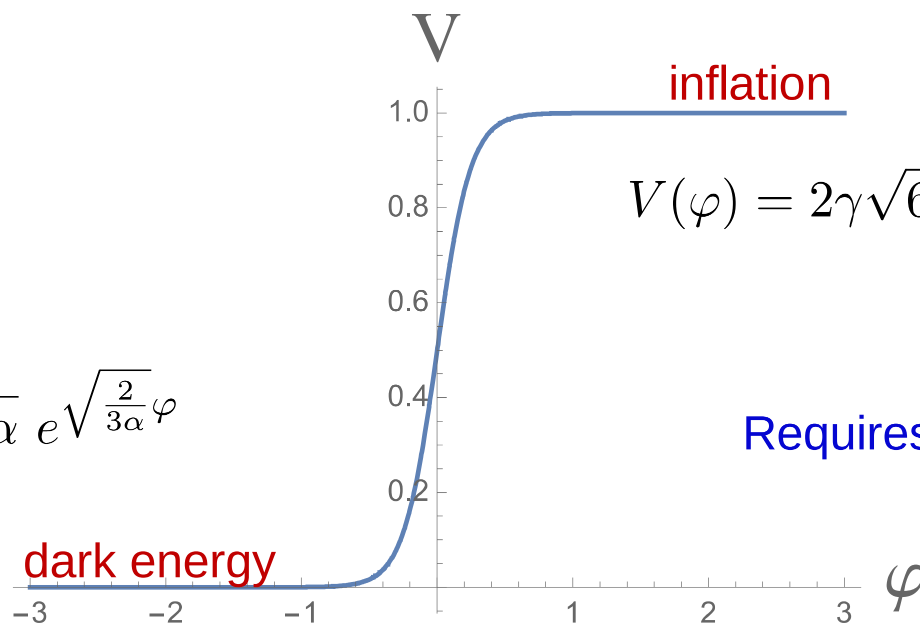
Akrami, Kallosh, AL, Vardanyan 2017

Consider the simplest linear potential

$$V(\phi) = \gamma\phi + \Lambda$$

The corresponding α-attractor potential in canonical variables is

$$V(\varphi) = \gamma\sqrt{6\alpha}(\tanh\frac{\varphi}{\sqrt{6\alpha}} + 1) + \Lambda$$



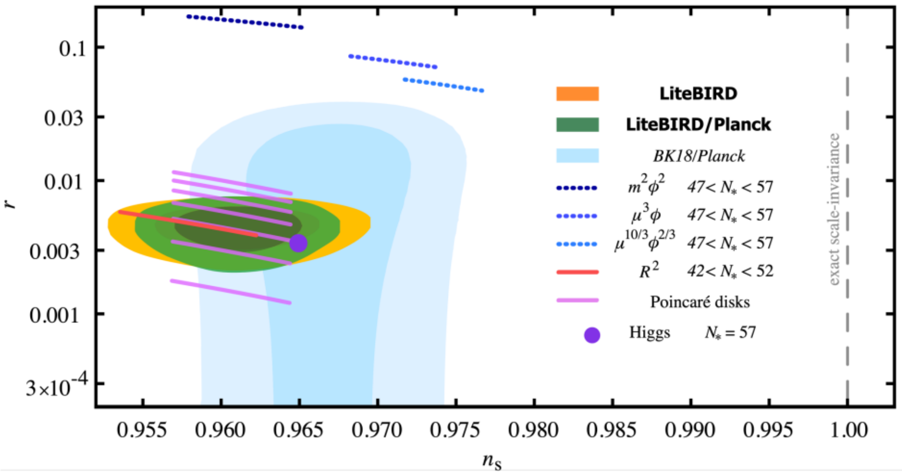
$$V(\varphi) = 2\gamma\sqrt{6\alpha}(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi})$$

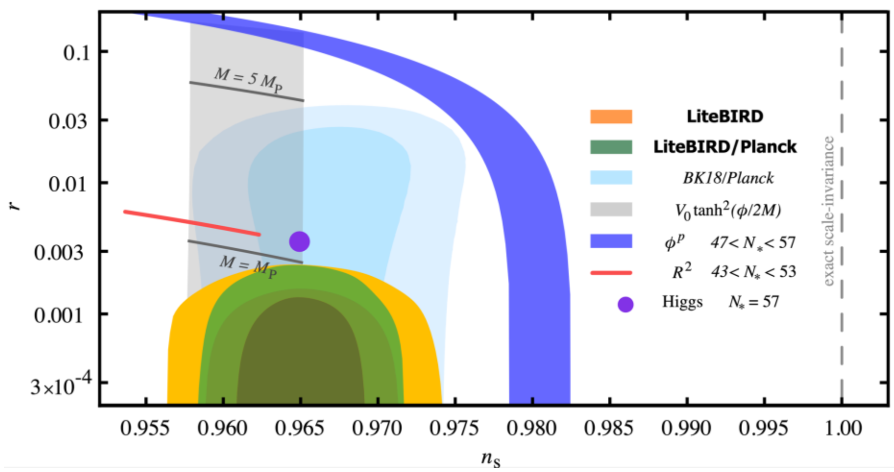
$$V(\varphi) = \Lambda + 2\gamma\sqrt{6\alpha}\ e^{\sqrt{\frac{2}{3\alpha}}\varphi}$$

Requires $\alpha \sim 10^{-2}$

LiteBIRD

Probing Cosmic Inflation with the LiteBIRD Cosmic Microwave Background Polarization Survey
2202.02773

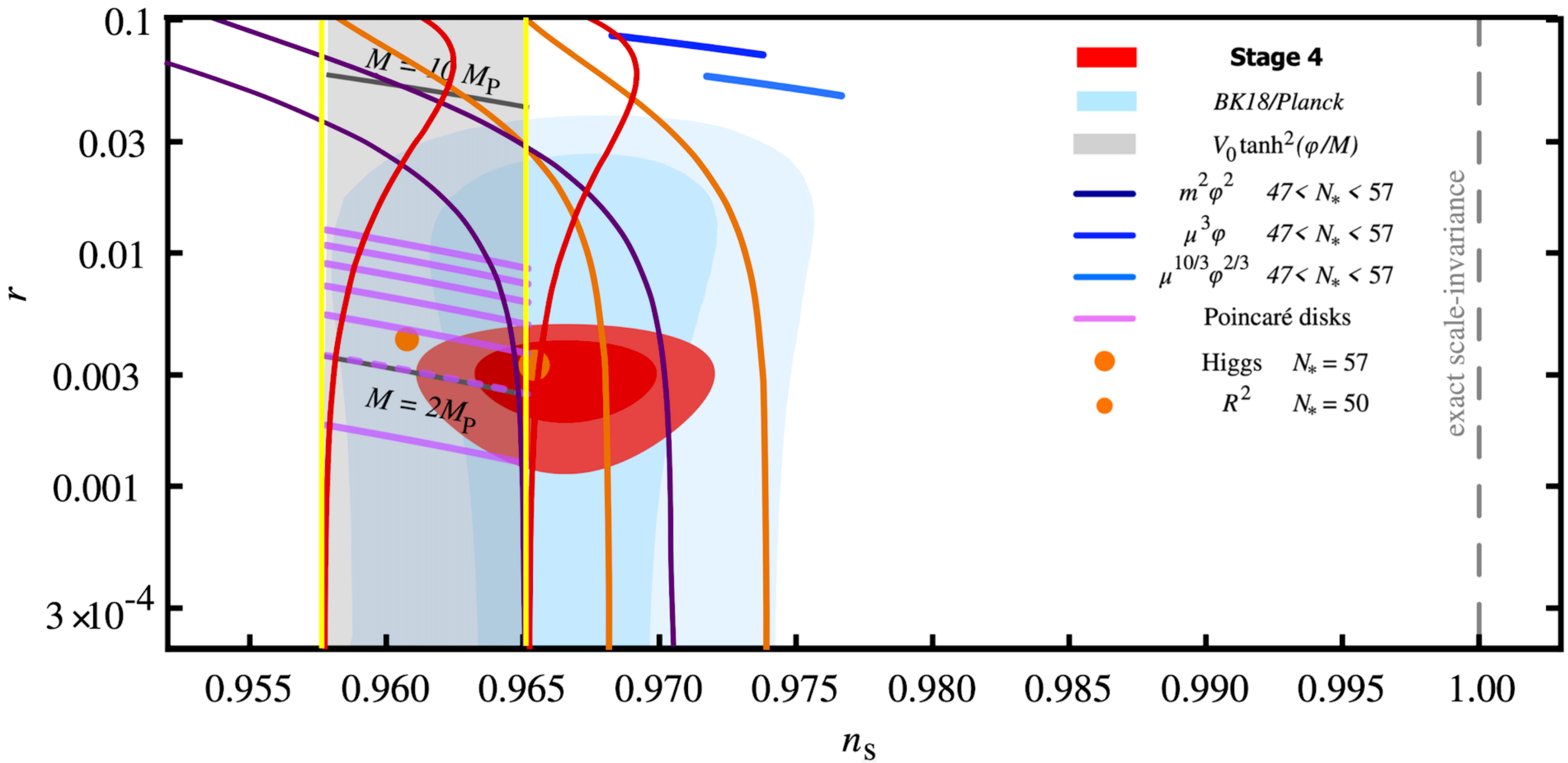




Are there any good models for the right side of the blue area favored by BICEP/Keck ??

CMB-S4

Snowmass2021 Cosmic Frontier: CMB Measurements White Paper



The gray area shows predictions of T-models. The two red lines show predictions of E-models. The purple and orange lines correspond to the polynomial α-attractors $\frac{\varphi^4}{\varphi^4+\mu^4}$ and $\frac{\varphi^2}{\varphi^2+\mu^2}$. These models completely cover the dark blue area favored by Planck/BICEP/Keck

Some authors trying to address the H0 problem complain that they do not know any inflationary model that could describe a broad range of n_s

Inflationary potential as seen from different angles

Giare, Pan, Yang, Di Valentino, De Haro, Melchiorri 2305.15378

"Despite significant efforts to explore various inflationary scenarios, no single model emerges as a comprehensive solution"

A simple polynomial superpotential with 3 parameters can describe the full range of all possible values of A_s, n_s and r, all the way to r = 0 and n_s = 1

Kallosh, AL, Westphal 2014

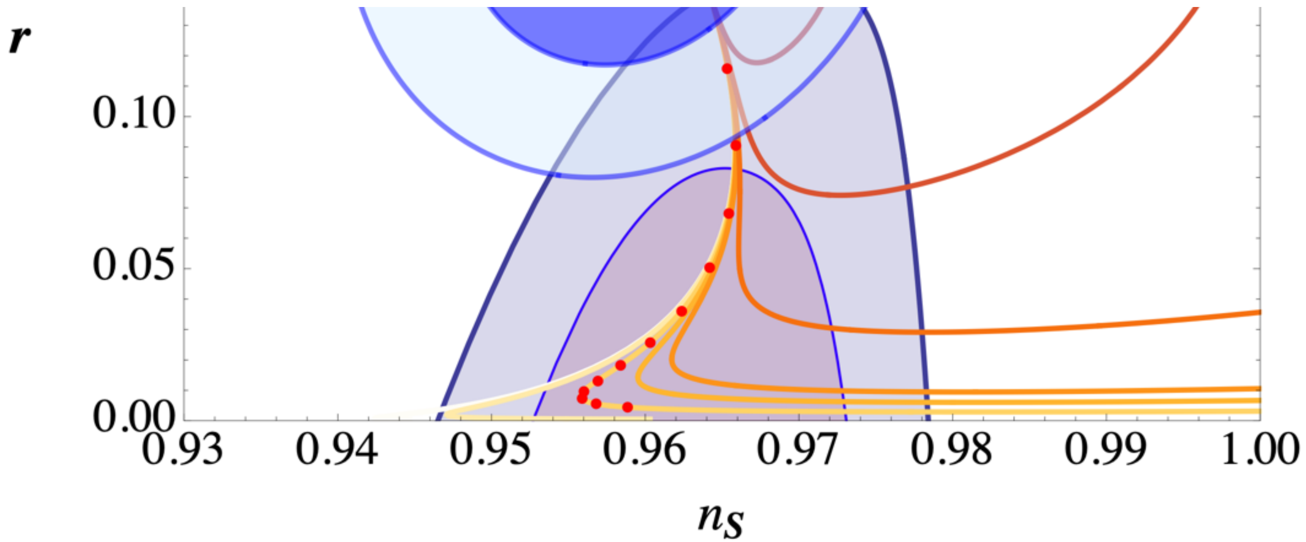
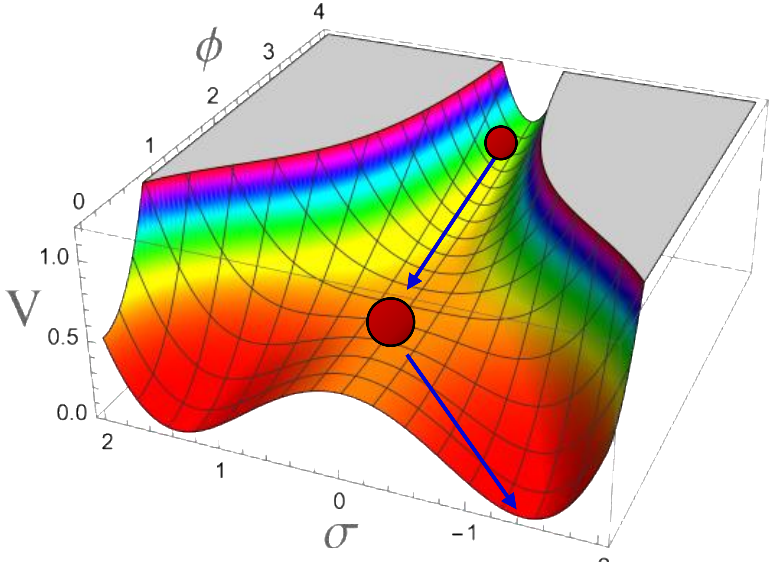


FIG. 3. Predictions for $n_s(a)$ and $r(a)$ in at 55 e-folds the model with $V(\phi) = \frac{m^2\phi^2}{2}\left(1 - a\phi + a^2 b\,\phi^2)\right)^2$ for various values of $b = 0.334\ldots 5$. All curves have a running from 0.001 to 0.2. The red ($b = 0.34$) and green ($b = 5$) balls correspond to $a = 0.01\ldots 0.13$

A possible comprehensive solution: Hybrid α-attractors

There is a special class of inflationary models where n_s = 1 is an attractor point: Hybrid Inflation

AL 1991, 1994



$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2$$

For σ = o, it is just the quadratic potential uplifted by $M^4/4\lambda$

$$n_s = 1 - 3\left(\frac{V'}{V}\right)^2 + 2\frac{V''}{V}$$

By increasing the uplifting term $V_{\rm uplift} = M^4/4\lambda$ one can increase V without changing any derivatives of V. Therefore, in the large uplift limit, for generic V we have an **attractor prediction n_s = 1**.

Hybrid α-attractors: Two attractor regimes

$$\frac{L}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial\varphi)^2}{2\left(1-\frac{\varphi^2}{6\alpha}\right)^2} - \frac{(\partial\sigma)^2}{2\left(1-\frac{\sigma^2}{6\beta}\right)^2} - V(\sigma,\varphi)$$

$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2-\lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2$$

Just as in all α-attractors, we have a universal large N attractor prediction

$$n_s = 1 - \frac{2}{N_e}\ , \qquad r = \frac{12\alpha}{N_e^2}$$

However, if uplift is very large, the last 60 e-foldings occur at small ϕ.
Then for N ~ 60 one has the standard hybrid inflation prediction $n_s = 1$.

Thus, by changing V_{uplift} one can obtain any value of n_s in the range between the two attractor predictions

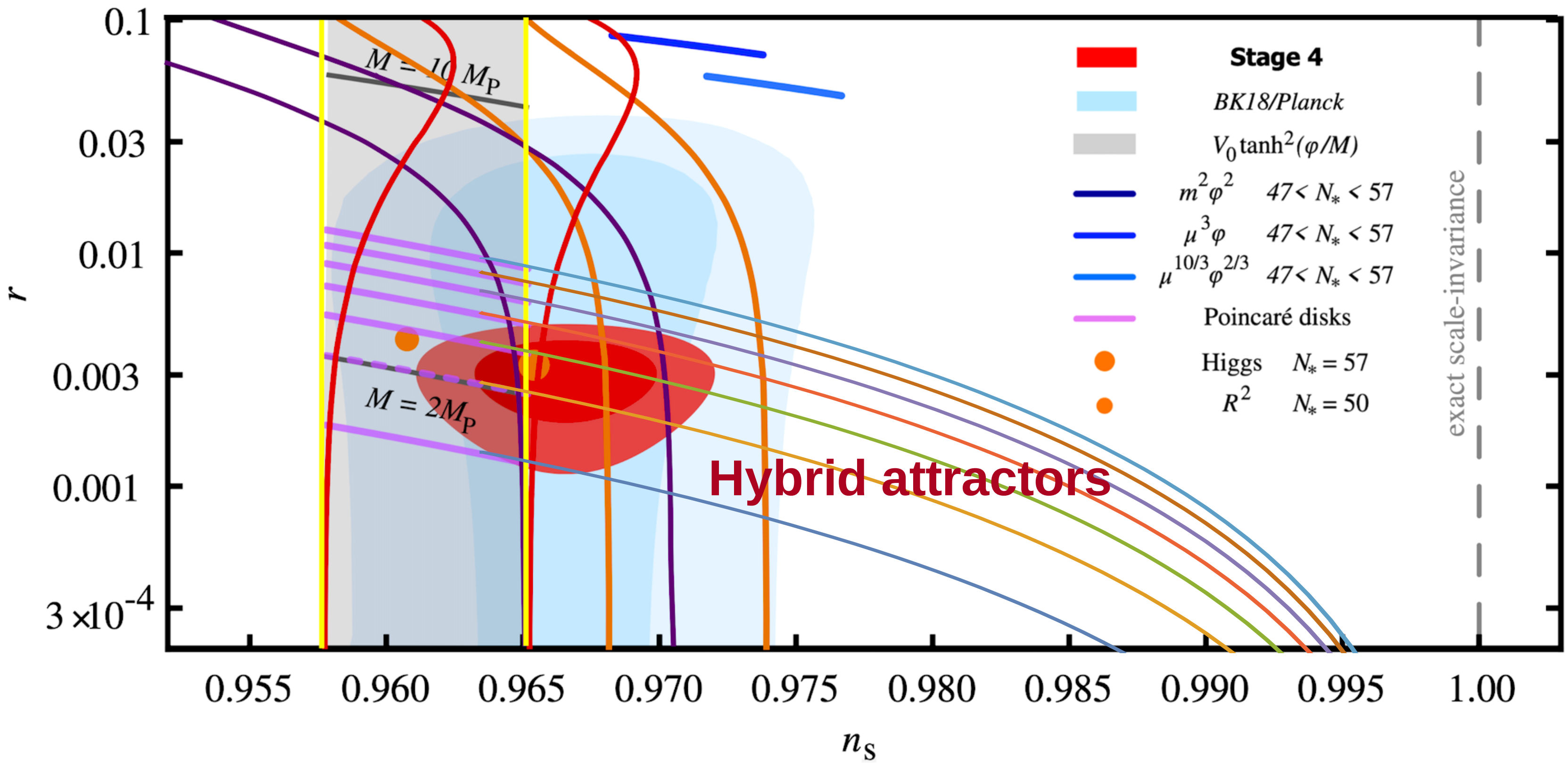
$$1 - \frac{2}{N_e} < n_s < 1$$

Kallosh, AL 2204.02425

Hybrid attractors are more complicated than the simplest α-attractors. However, if H_0 problem is real, this flexibility may be desirable.

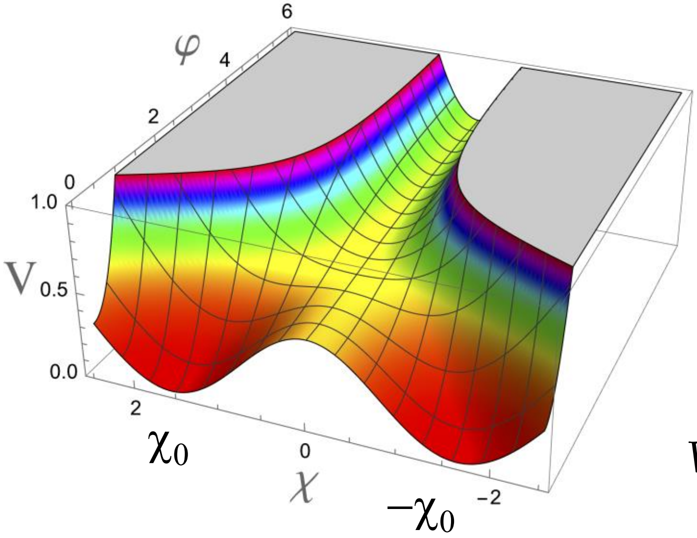
CMB-S4

Snowmass2021 Cosmic Frontier: CMB Measurements White Paper



The gray area shows predictions of T-models. The two red lines show predictions of E-models. The purple and orange lines correspond to the polynomial α-attractors $\frac{\varphi^4}{\varphi^4+\mu^4}$ and $\frac{\varphi^2}{\varphi^2+\mu^2}$. These models completely cover the dark blue area favored by Planck/BICEP/Keck

Hybrid α attractors: New possibilities



Original hybrid inflation model:

$$V(\chi,\phi) = M^2 \left[\frac{(\chi^2-\chi_0^2)^2}{4\chi_0^2} + \frac{\tilde{m}^2}{2}\phi^2 + \frac{\tilde{g}^2}{2}\phi^2\chi^2 + d\,\chi \right]$$

It's α-attractor generalization:

$$V(\chi,\varphi) = M^2 \left[\frac{(\chi^2-\chi_0^2)^2}{4\chi_0^2} + 3\alpha(\tilde{m}^2 + \tilde{g}^2\chi^2)\tanh^2\frac{\varphi}{\sqrt{6\alpha}} + d\,\chi \right]$$

The results that we discussed so far are valid for hybrid inflation where the amplitude of spontaneous symmetry breaking is sub-Planckian, $\chi_0 << 1$.

In the opposite case $\chi_0 > 1$ the tachyonic mass of the field χ along the ridge $\chi = 0$ is smaller than the Hubble constant, which leads to **eternal inflation** with **the amplitude of perturbations O(1)**. It is very easy to produce large PBH and even **eternally inflating topological defects** in this scenario.

Garcia-Bellido, A.L., Wands 1996,
Randall, Soljacic, Guth, 1996,
Braglia, A.L., Kallosh, Finelli, 2211.14262

Conclusions:

1. Many predictions of inflationary theory have been tested and confirmed by observations during the last 40 years.

2. Some inflationary models, such as the Starobinsky model, the Higgs inflation, and a broad class of α-attractors can describe all available CMB inflation-related data by a single parameter.

3. Predictions of α-attractors are stable with respect to modifications of the potential. These models can describe any small value of r, all the way down to $r = 0$.

4. BICEP/Keck results are moving close to the range necessary for testing tensor modes in these models. LiteBIRD would move us much further.

5. There is a significant progress in implementing inflationary models in supergravity.

6. Hybrid α-attractors can describe copious production of PBH, while remaining consistent with the Planck/BICEP/Keck data.

Backup slides about the multiverse

Here comes the multiverse

Pessimist:

If each part of the multiverse is huge, we will never see other parts, so it is **impossible to prove** that we live in the multiverse.

Optimist:

If each part of the multiverse is huge, we will never see other parts, so it is **impossible to disprove** that we live in the multiverse.

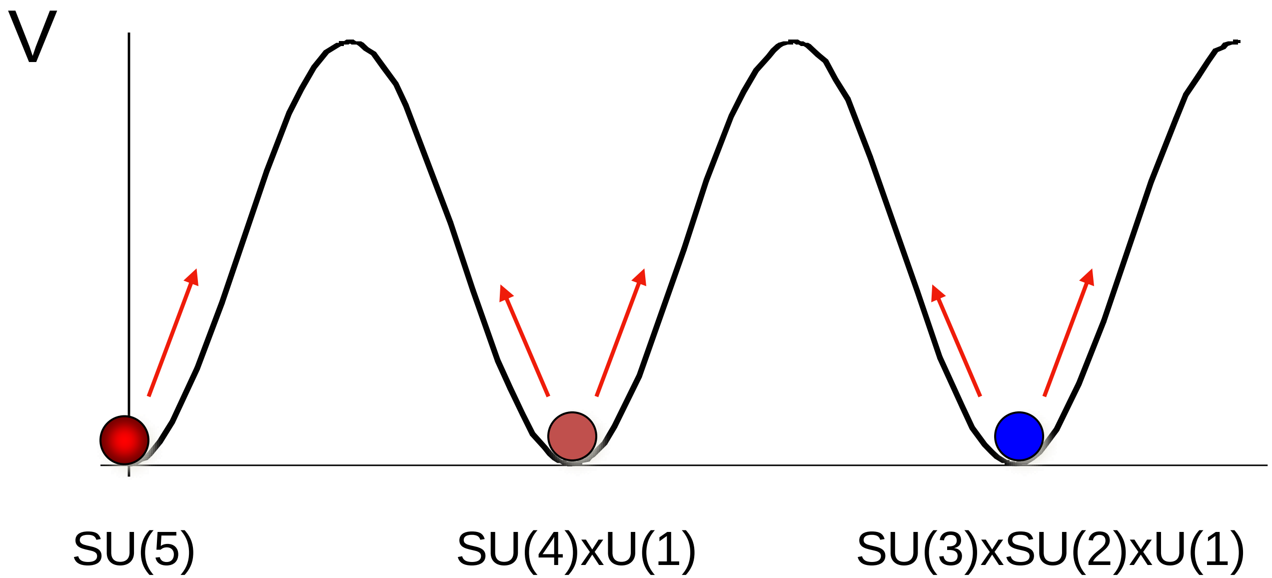
I'd rather be an optimist and a fool than a pessimist and right. Albert Einstein

This scenario is **more general** (otherwise one would need to explain why all colors but one are forbidden). Therefore, the theory of the **multi**verse, rather than the theory of the **uni**verse, is the basic theory.

Moreover, even if one begins with a single-colored universe, quantum fluctuations make it multi-colored.

Example: SUSY landscape

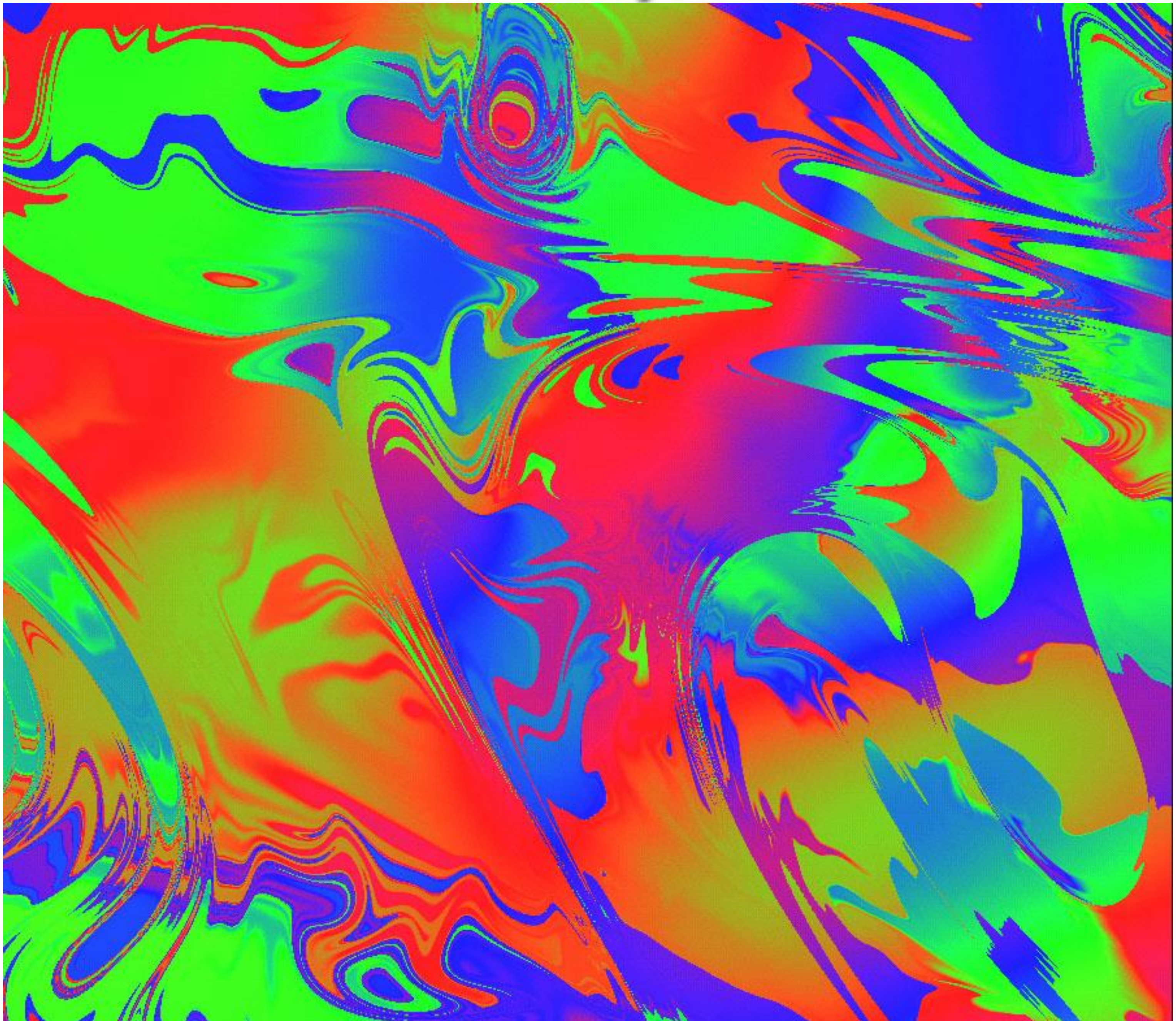
Supersymmetric SU(5)



Weinberg 1982: Supersymmetry forbids tunneling from SU(5) to SU(3)xSU(2)XU(1). This implied that we cannot break SU(5) symmetry.

A.L. 1983: Inflation solves this problem. Inflationary fluctuations bring us to each of the three minima. Inflation makes each of the parts of the universe exponentially large. We can live only in the SU(3)xSU(2)xU(1) minimum.

Kandinsky Universe

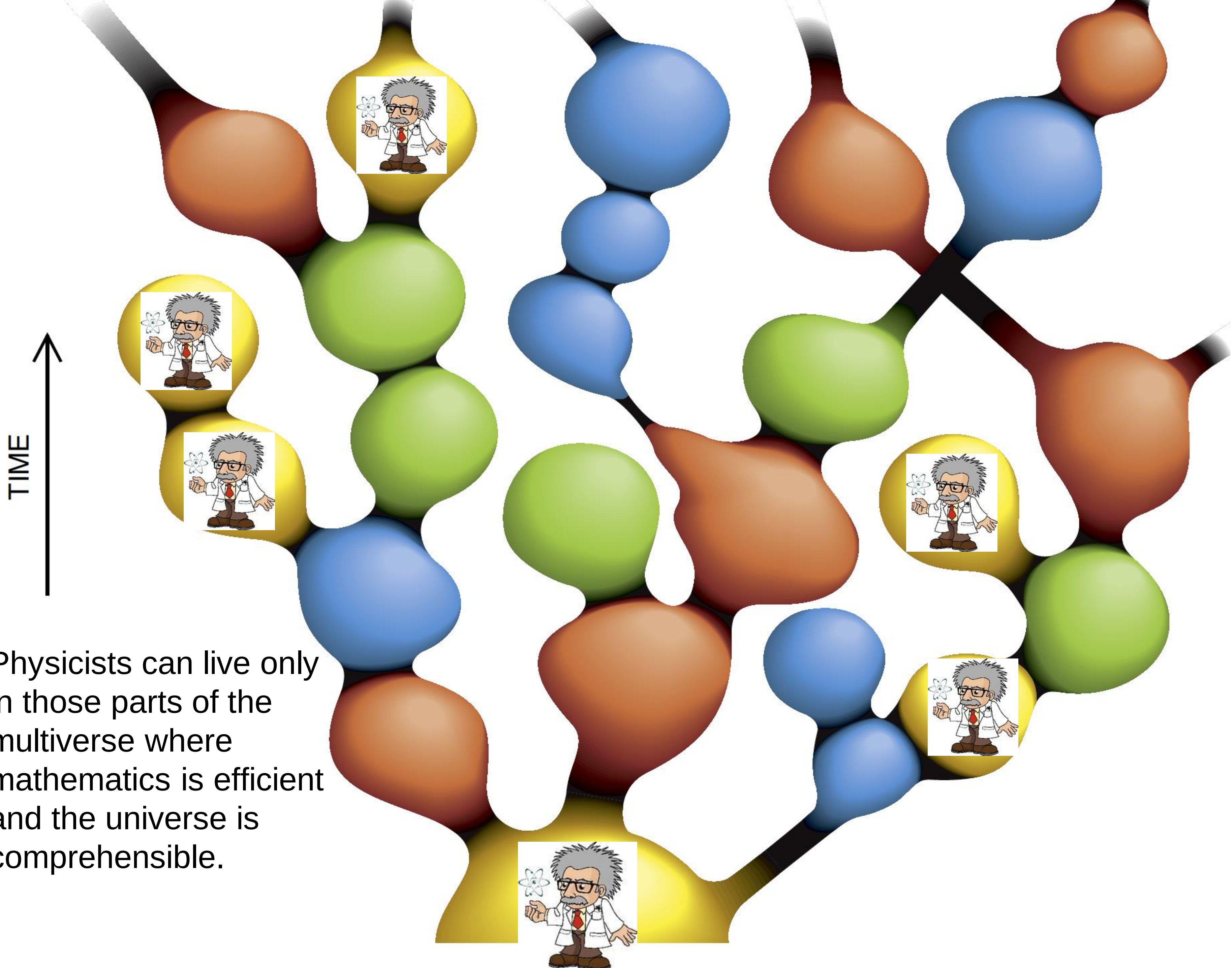


Can we test the multiverse theory ?

This theory provides the only known explanation of numerous experimental results (extremely small vacuum energy, strange masses of many elementary particles). **In this sense, it was already tested many times.**

“When you have eliminated the impossible, whatever remains, however improbable, must be the truth.”

Sherlock Holmes



Physicists can live only in those parts of the multiverse where mathematics is efficient and the universe is comprehensible.