

The scale(s) of quantum gravity and integrable black holes

Roberto Casadio

D.I.F.A. "A. Righi"

Bologna University

INFN

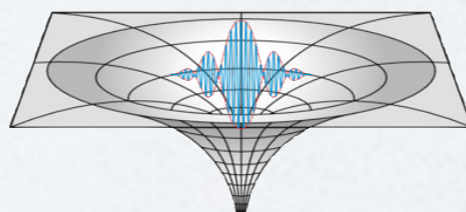
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Theory and Phenomenology
of Fundamental Interactions

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Preamble - Units and scales

Units (of measure) are chosen for practical reasons + physics is a practical (observational) science
= units reflect our understanding of nature

UNNATURAL



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CHAPTER 4. UNITS AND NATURAL SCALES

4.3.4 Planck units—completely natural units

Define $c \equiv \hbar \equiv G \equiv 1$, so that $m_P \equiv \ell_P \equiv T_P \equiv 1$. This system of units serves to keep both classical relativists and particle physicists happy. This system also serves to keep both classical relativists and particle physicists confused since it is essentially impossible to use dimensional analysis to check results for consistency while using this system. This system works in the first place because for suitable exponents (α, β, γ) and (a, b, c) almost all interesting physical quantities X are dimensionless multiples of

$$[X] \equiv m_P^\alpha \ell_P^\beta T_P^\gamma \equiv \hbar^a c^b G^c. \quad (4.20)$$

[Please, no niggling comments about thermodynamics (Boltzmann's constant), chemistry (moles), luminous intensity (candelas), etc.]

4.3 Units

4.3.1 Seminatural units

Most of this book will be written using “seminatural units”. I shall define $c \equiv 1$, so that $m_P = \sqrt{\hbar/G}$; $\ell_P \equiv T_P \equiv \sqrt{\hbar G}$. Equivalently $G \equiv \hbar/m_P^2 \equiv \ell_P^2/\hbar \equiv \ell_P/m_P$. For pedagogical purposes this seems to me to be the best compromise between constantly keeping track of explicit factors of the fundamental constants and quickly getting totally confused about the dimensionality of various physical quantities.

Other conventions in common use are as follows:

4.3.2 Geometrodynamic units

Define $c \equiv G \equiv 1$, so that $m_P \equiv \ell_P \equiv T_P \equiv \sqrt{\hbar}$. These units are commonly used by classical relativists but are guaranteed to drive particle physicists rapidly up the wall.

4.3.3 Quantum units

Define $c \equiv \hbar \equiv 1$, so that $m_P \equiv 1/\sqrt{G}$, while $\ell_P \equiv T_P \equiv \sqrt{G}$. These units are commonly used by particle physicists but are guaranteed to drive classical relativists rapidly up the wall.

Strong (quantum) gravity = high compactness $X \equiv GM/R \longrightarrow [G] = L/M$

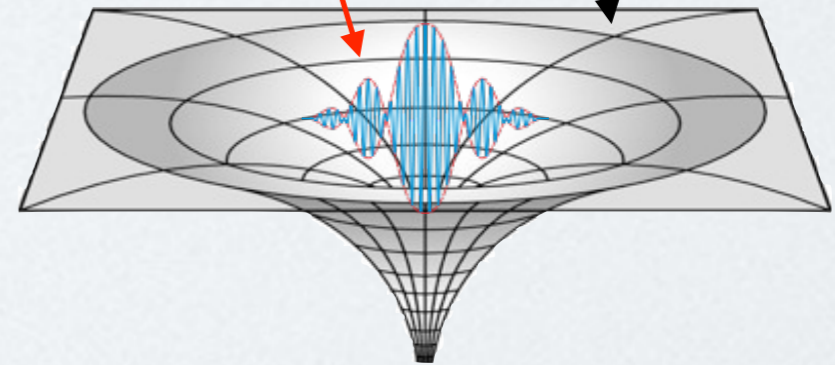
Preamble - QG and Planck scale

Planck scale:

$$m = m_p \equiv \sqrt{\frac{c \hbar}{G_N}} \sim 10^{-8} \text{ kg}$$

$$\lambda_C = \ell_p \equiv \sqrt{\frac{\hbar G_N}{c^3}} \sim 10^{-35} \text{ m}$$

$$\frac{\hbar}{m c} \equiv \lambda_C \sim R_H \equiv \frac{2 G_N m}{c^2}$$

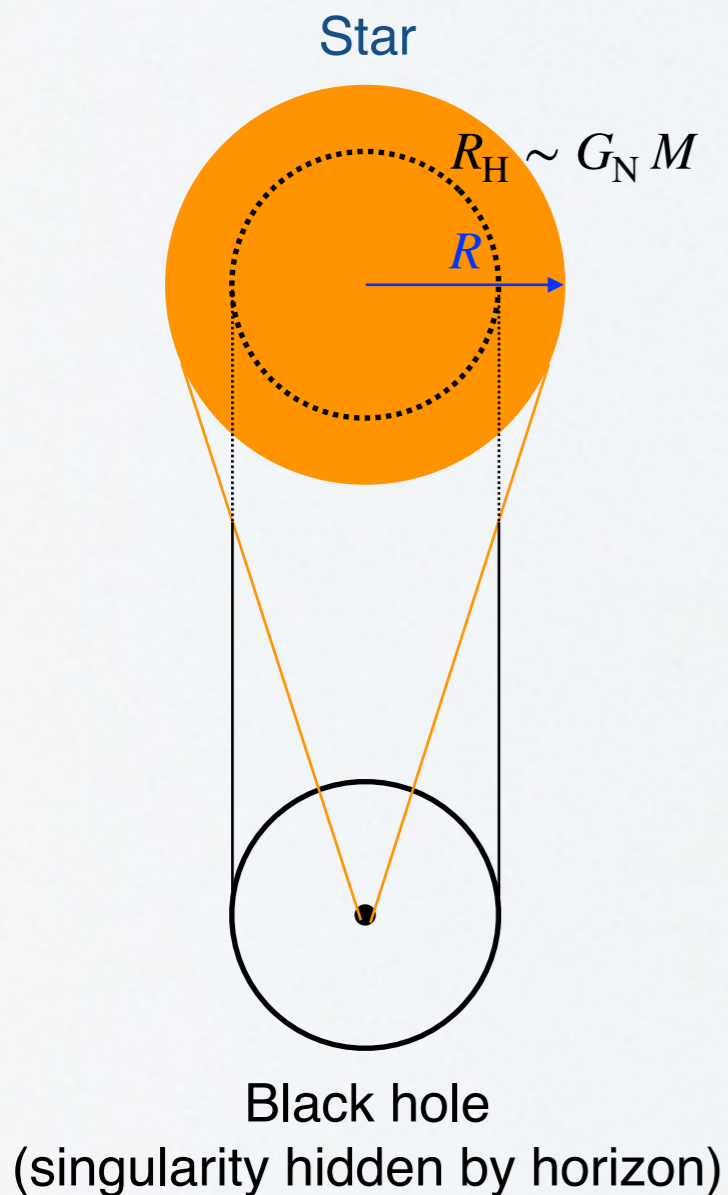


Is Quantum Gravity at the Planck scale ($\ell_p \sim m_p$) or compactness $\frac{G_N M}{R} = \frac{M \ell_p}{m_p R} \sim 1$?*

* Analogy: what is the scale of QED/SM? Electron Compton length \ll atom \ll BEC size \ll neutron star

Preamble - QG and gravitational collapse

- BH form from collapse - the classical view:



- $|\text{matter}\rangle \sim$ very large number of SM particles ($M_\odot \sim 10^{57}$ neutrons)
- $|\text{gravity}\rangle \sim$ very large number of gravitons* ($N_G \sim M_\odot^2 \sim 10^{76}$)



Reduce and simplify^{**}: the Lemaître-Oppenheimer-Snyder-Tolman model

* J.D. Bekenstein, PRD 7 (1973) 2333

** Rovelli, Thiemann,... Relational approach: "Quantise only what you measure"... But this is just what physics is about

1 - Quantum dust core

- Collapsing ball of dust [1]

$$ds^2 = - \left(1 - \frac{2 G_N M}{r} \right) dt^2 + \left(1 - \frac{2 G_N M}{r} \right)^{-1} dr^2 + r^2 d\Omega^2$$

$$\left(\frac{dR}{d\tau} \right)^2 + 1 - \frac{2 G_N M}{R} \simeq \frac{E^2}{\mu^2}$$

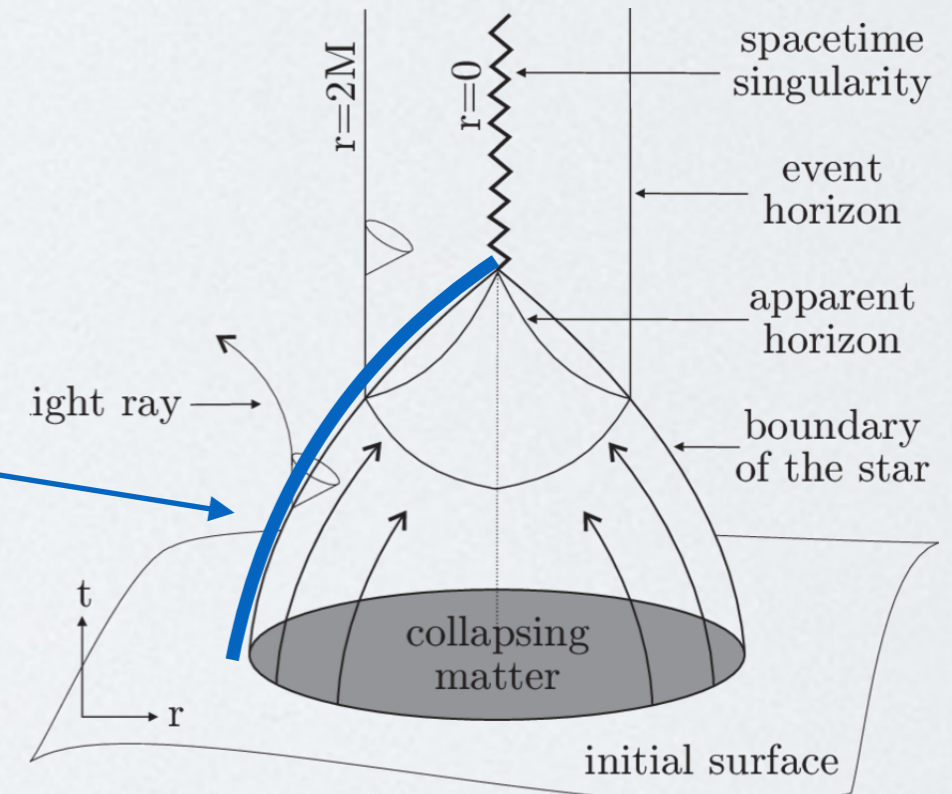
- Effective one-body Hamiltonian [2,3]:

$$H \equiv \frac{P^2}{2\mu} - \frac{G_N \mu M}{R} = \frac{\mu}{2} \left(\frac{E^2}{\mu^2} - 1 \right) \equiv \mathcal{E}$$

$$\hat{H} \Psi_n = \mathcal{E}_n \Psi_n$$

$$\frac{\mathcal{E}_n}{\mu} \simeq - \frac{G_N^2 \mu^3 M}{2 \hbar^2 n^2} \sim - \frac{\mu^3 M}{m_p^4 n^2}$$

$$\bar{R}_n \equiv \langle \Psi_n | \hat{R} | \Psi_n \rangle \simeq n^2 \ell_p \left(\frac{m_p^3}{\mu^2 M} \right)$$



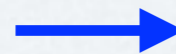
Newtonian physics:

$$n \geq 1$$



$$\bar{R}_1 \sim \ell_p \frac{m_p^3}{\mu^2 M} \sim 0$$

GR: $\frac{E_n^2}{\mu^2} = 1 + \frac{\mathcal{E}_n}{\mu} \geq 0$



$$n \geq N_M \sim \frac{\mu M}{m_p^2} \longrightarrow \bar{R}_{N_M} \sim G_N M$$

1 - Quantum dust core

- Collapsing ball of dust [2]

$$ds^2 = - \left(1 - \frac{2 G_N M}{r} \right) dt^2 + \left(1 - \frac{2 G_N M}{r} \right)^{-1} dr^2 + r^2 d\Omega^2$$

$$\left(\frac{dR_i}{d\tau} \right)^2 + 1 - \frac{2 G_N M_i}{R_i} \simeq \frac{E_i^2}{\mu^2}$$

- Dust particle Hamiltonians:

$$H_i \equiv \frac{P_i^2}{2\mu} - \frac{G_N \mu M_i}{R_i} = \frac{\mu}{2} \left(\frac{E_i^2}{\mu^2} - 1 \right) \equiv \mathcal{E}_i$$

- Schrödinger equation:

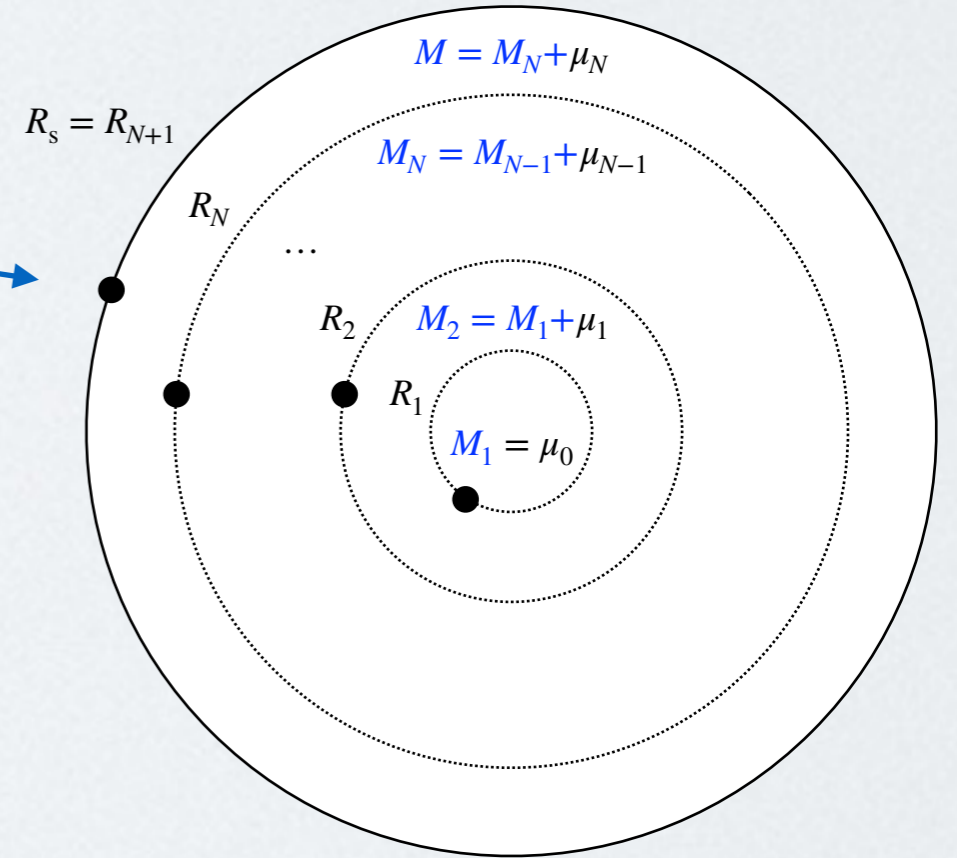
$$\hat{H}_i \Psi_{n_i} = \mathcal{E}_{n_i} \Psi_{n_i}$$

- Spectrum of bound states ($n_i \geq 1$):

$$\frac{\mathcal{E}_{n_i}}{\mu} \simeq - \frac{G_N^2 \mu^3 M_i}{2 \hbar^2 n_i^2} = - \frac{1}{2 n_i^2} \left(\frac{\mu M_i}{m_p^2} \right)^2 = \frac{1}{2} \left(\frac{E_i^2}{\mu^2} - 1 \right)$$

$$\bar{R}_{n_i} \equiv \langle \Psi_{n_i} | \hat{R}_i | \Psi_{n_i} \rangle \simeq n_i^2 \ell_p \left(\frac{m_p^3}{\mu^2 M_i} \right)$$

Newtonian spectrum



“GR” [2]

1 - Quantum dust core

• Allowed spectrum * [1,2]: $0 \leq \frac{E_i^2}{\mu^2} \simeq 1 - \frac{1}{n_i^2} \left(\frac{\mu M_i}{m_p^2} \right)^2 \longrightarrow n_i \geq N_i = \frac{\mu M_i}{m_p^2} \longrightarrow \bar{R}_{N_i} \simeq \frac{3}{2} G_N M_i$

• Layer thickness [1]: $\frac{\Delta \bar{R}_{n_i}}{\bar{R}_{n_i}} = \frac{\sqrt{n_i^2 + 2}}{3 n_i} \simeq \frac{1}{3} \longrightarrow \bar{R}_{N_{i+1}} \simeq \bar{R}_{N_i} + \Delta \bar{R}_{N_i} \simeq \frac{4}{3} \bar{R}_{N_i} \longrightarrow M_{i+1} \simeq \frac{4}{3} M_i$

• Bekenstein's area law: $N_G = \frac{M}{\mu} N_N \simeq \frac{M^2}{m_p^2}$

$R_s \simeq \frac{3}{2} G_N M = \frac{3}{4} R_H$

• Bounded compactness



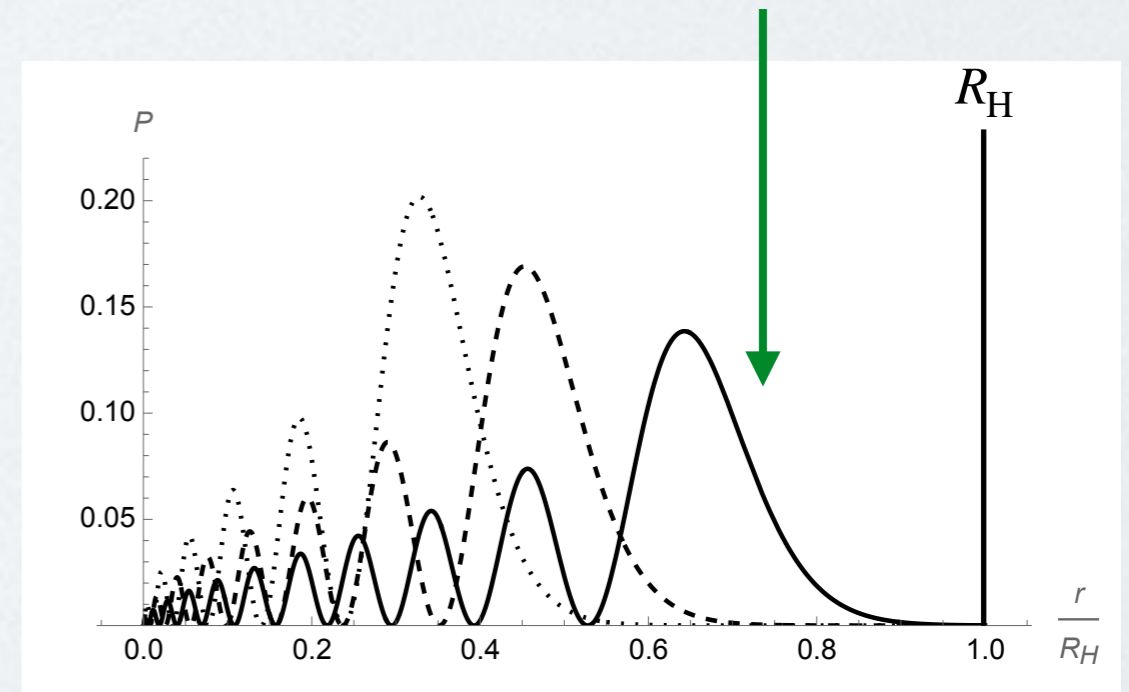
Semiclassical "bounce" (~ BH-to-WH transition)



non-linearity



* *Classicalization* ~ GUP in action



$\bar{R}_{n=1} \sim \ell_p \left(\frac{m_p^3}{\mu^2 M} \right) \sim \ell_p$

[1] R.C., *Quantum dust cores of black holes*, PLB 843 (2023) 138055 [arXiv:2304.06816]

[2] R.C., *A quantum bound for the compactness*, EPJC 82 (2022) 1 [arXiv:2103.14582]

1 - Quantum dust core

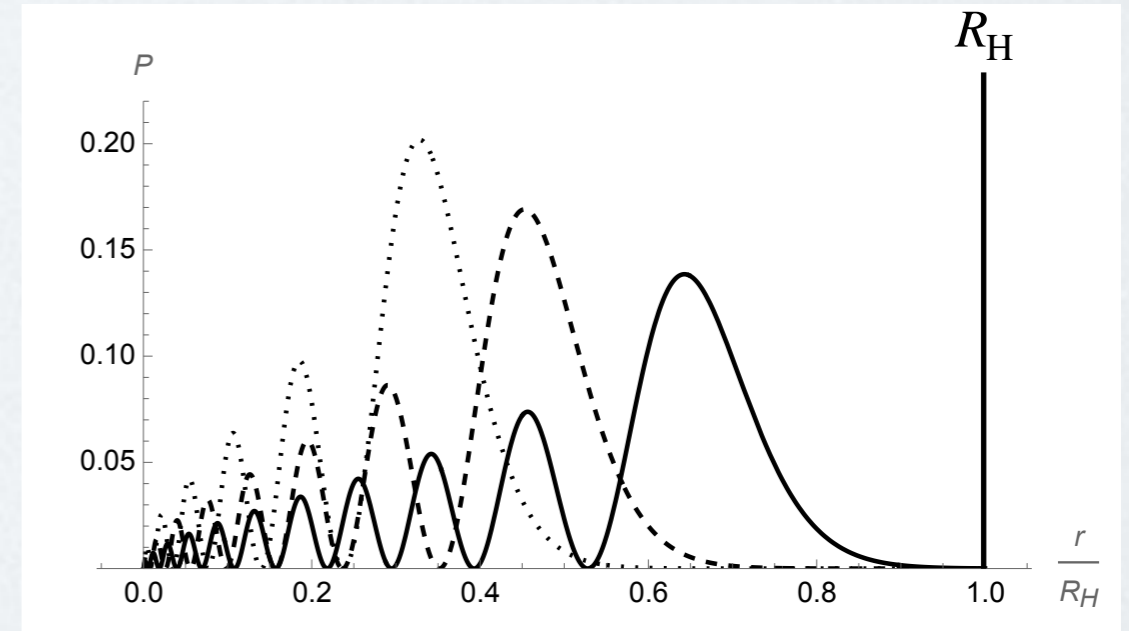
- Allowed spectrum * [1,2]: $0 \leq \frac{E_i^2}{\mu^2} \simeq 1 - \frac{1}{n_i^2} \left(\frac{\mu M_i}{m_p^2} \right)^2 \longrightarrow n_i \geq N_i = \frac{\mu M_i}{m_p^2} \longrightarrow \bar{R}_{N_i} \simeq \frac{3}{2} G_N M_i$

- Layer thickness [1]: $\frac{\Delta \bar{R}_{n_i}}{\bar{R}_{n_i}} = \frac{\sqrt{n_i^2 + 2}}{3 n_i} \simeq \frac{1}{3} \longrightarrow \bar{R}_{N_{i+1}} \simeq \bar{R}_{N_i} + \Delta \bar{R}_{N_i} \simeq \frac{4}{3} \bar{R}_{N_i} \longrightarrow M_{i+1} \simeq \frac{4}{3} M_i$

- Effective mass and energy density [1]: $m(r) \sim r$



- Integrable singularity: $\rho(r) \sim r^{-2}$

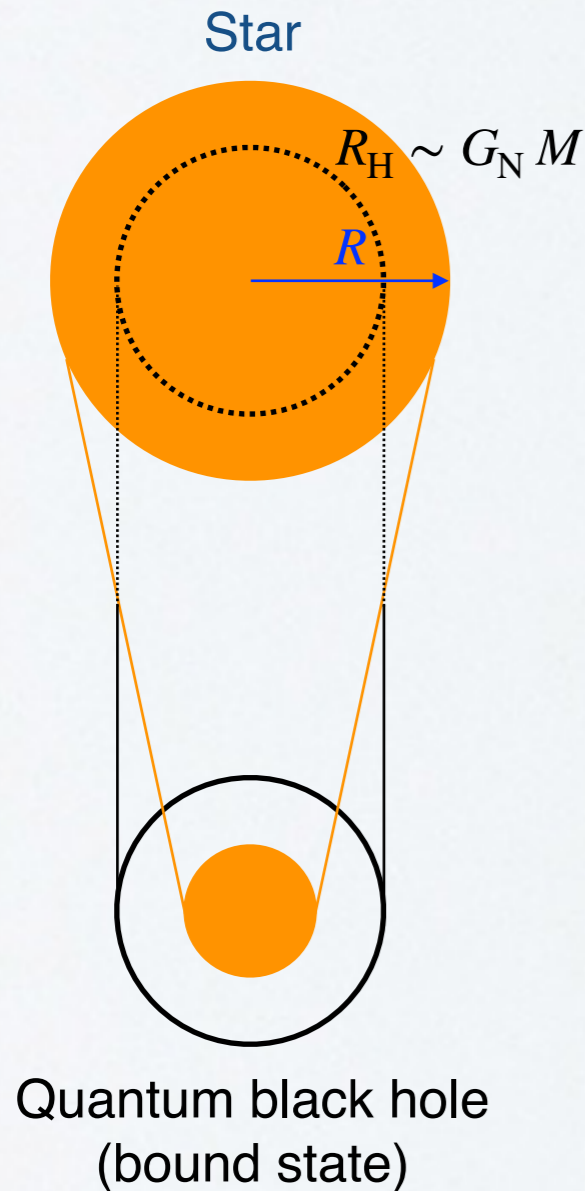


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Preamble - QG and gravitational collapse

- BH form from collapse - the quantum view:



- $|\text{matter}\rangle \sim$ very large number of SM particles ($M_{\odot} \sim 10^{57}$ neutrons)
- $|\text{gravity}\rangle \sim$ very large number of gravitons* ($N_G \sim M_{\odot}^2 \sim 10^{76}$)

- $|\text{gravity}\rangle$ always entangled with $|\text{matter}\rangle \iff$ “quantum hair” [1,2]

Dynamics

$$|\mathbf{g} \phi\rangle = \sum_{ij} C_{ij} |\mathbf{g}_i\rangle |\phi_j\rangle \quad \Rightarrow \quad \left(\sum_{ab} c_{ab} |\mathbf{g}_a\rangle |\phi_b\rangle \right) \left(\sum_{AB} c_{AB} |\mathbf{g}_A\rangle |\phi_B\rangle \right)$$

$\hat{H}^{\mu} |\mathbf{g} \phi\rangle = 0$ BH interior BH exterior

[1] R.C., *A quantum bound for the compactness*, EPJC 82 (2022) 1 [arXiv:2103.14582]

[2] R.C., *Quantum dust cores of black holes*, PLB 843 (2023) 138055 [arXiv:2304.06816]

2 - Coherent state for classical geometry

- SdS geometry [1]:

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f = 1 + 2V_{\text{SdS}} = 1 - \frac{2G_{\text{N}}M}{r} - \frac{\Lambda r^2}{3}$$

$$V_{\text{SdS}} = V_M + V_\Lambda = -\frac{G_{\text{N}}M}{r} - \frac{\Lambda r^2}{6}$$

- Horizons ($f = 0 \leftrightarrow 2V = -1$):

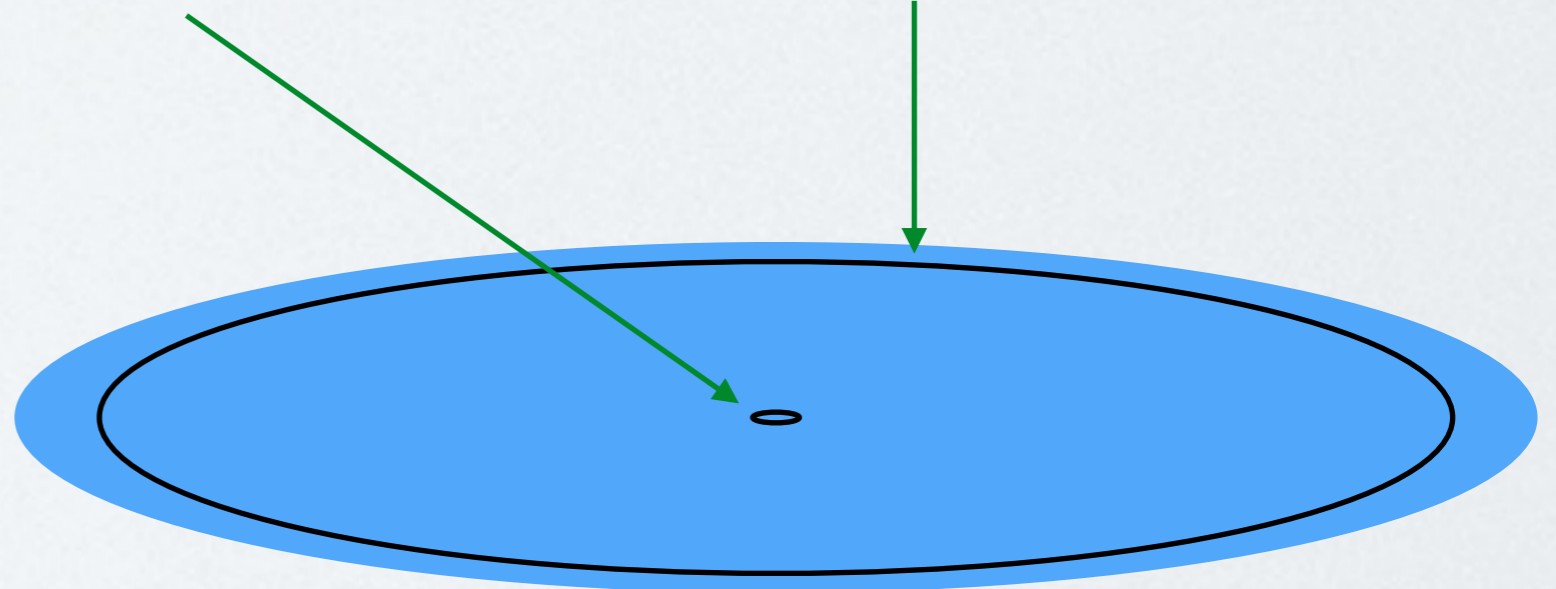
$$1/\sqrt{\Lambda} \gg G_{\text{N}}M$$

Localised source gravitational radius

$$R_{\text{H}} \simeq 2G_{\text{N}}M$$

Cosmological horizon

$$H^{-1} = L \simeq \sqrt{\frac{3}{\Lambda}}$$



2 - Coherent state for classical geometry

- “Effective” massless scalar field in Minkowski (~ true QG vacuum $\hat{a}_k | 0 \rangle = 0$ *):

$$\left\{ -\frac{\partial^2}{\partial t^2} + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \left[\frac{\partial}{\partial \theta} \left(\sin^2 \theta \frac{\partial}{\partial \theta} \right) + \frac{\partial^2}{\partial \varphi^2} \right] + \frac{\partial^2}{\partial y_1^2} + \dots + \frac{\partial^2}{\partial y_n^2} \right\} \Phi = 0$$

↑
ON

↑
OFF

* Metric ~ causal structure ~ gravity emerges from “excitations” along with matter (~ LQG, Regge calculus, etc...)

2 - Coherent state for classical geometry

- “Effective” massless scalar field in Minkowski (\sim true QG vacuum $\hat{a}_k | 0 \rangle = 0$ *):

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) \right] \Phi(t, r) = 0$$

$$u_k(t, r) = e^{-ikt} j_0(kr)$$

$$4\pi \int_0^\infty r^2 dr j_0(kr) j_0(pr) = \frac{2\pi^2}{k^2} \delta(k-p)$$

- Normal mode expansion of operators:

$$\hat{\Phi}(t, r) = \int_0^\infty \frac{k^2 dk}{2\pi^2} \sqrt{\frac{\hbar}{2k}} \left[\hat{a}_k u_k(t, r) + \hat{a}_k^\dagger u_k^*(t, r) \right]$$

$$\left[\hat{\Phi}(t, r), \hat{\Pi}(t, s) \right] = \frac{i\hbar}{4\pi r^2} \delta(r-s)$$

$$\hat{\Pi}(t, r) = i \int_0^\infty \frac{k^2 dk}{2\pi^2} \sqrt{\frac{\hbar k}{2}} \left[\hat{a}_k u_k(t, r) - \hat{a}_k^\dagger u_k^*(t, r) \right]$$

$$\left[\hat{a}_k, \hat{a}_p^\dagger \right] = \frac{2\pi^2}{k^2} \delta(k-p)$$

- Coherent state:

$$\hat{a}_k |g\rangle = g(k) e^{i\gamma_k(t)} |g\rangle$$

$$\langle g | \hat{\Phi}(t, r) | g \rangle = \int_0^\infty \frac{k^2 dk}{2\pi^2} \sqrt{\frac{2\ell_p m_p}{k}} g(k) \cos[\gamma_k(t) - kt] j_0(kr)$$

* Metric \sim causal structure \sim gravity emerges from “excitations” along with matter (\sim LQG, Regge calculus, etc...)

2 - Coherent state for classical geometry

- “Classical” coherent state:

$$\sqrt{\frac{\ell_p}{m_p}} \langle g | \hat{\Phi}(t, r) | g \rangle = V(r) = \int_0^\infty \frac{k^2 dk}{2\pi^2} \tilde{V}(k) j_0(kr)$$

Single mode occupation number: $g(k) = \sqrt{\frac{k}{2}} \frac{\tilde{V}(k)}{\ell_p}$

$$\gamma_k = k t$$

← virtual non-propagating modes *

- Total occupation number (**must be finite!**) ~ distance from vacuum:

$$|g\rangle = e^{-N_G/2} \exp \left\{ \int_0^\infty \frac{k^2 dk}{2\pi^2} g(k) \hat{a}_k^\dagger \right\} |0\rangle$$

$$N_G = \int_0^\infty \frac{k^2 dk}{2\pi^2} g^2(k) < \infty$$

$$\langle k \rangle = \int_0^\infty \frac{k^2 dk}{2\pi^2} k g^2(k) < \infty$$

* Potentials in QFT are generated by virtual / non propagating modes (same for spherical symmetry)

2 - Coherent state for classical geometry

- Localised source: $V_M = -\frac{G_N M}{r}$ $\tilde{V}_M = -4\pi G_N \frac{M}{k^2}$

- Mass scaling [1]:

Divergences
↓

$$N_M = 4 \frac{M^2}{m_p^2} \int_0^\infty \frac{dk}{k} \longrightarrow 4 \frac{M^2}{m_p^2} \int_{k_{\text{IR}}}^{k_{\text{UV}}} \frac{dk}{k} = 4 \frac{M^2}{m_p^2} \ln \left(\frac{k_{\text{UV}}}{k_{\text{IR}}} \right)$$

- Compton length scaling:

$$\langle k \rangle = 4 \frac{M^2}{m_p^2} \int_0^\infty dk \longrightarrow 4 \frac{M^2}{m_p^2} \int_{k_{\text{IR}}}^{k_{\text{UV}}} dk = 4 \frac{M^2}{m_p^2} (k_{\text{UV}} - k_{\text{IR}})$$

- Quantum core (BH = ground state): $k_{\text{UV}}^{-1} \simeq R_s \simeq G_N M$

* Cut-offs = existence condition for quantum state: $g(k < k_{\text{IR}}) = g(k > k_{\text{UV}}) \simeq 0!$

2 - Coherent state for classical geometry

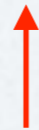
- Localised source: $V_M = -\frac{G_N M}{r}$

- Localised source in dS: $k_{UV}^{-1} \simeq R_s \simeq G_N M$

$$k_{IR}^{-1} \simeq L$$



$$V_{QM} \equiv \sqrt{\frac{\ell_p}{m_p}} \langle g | \hat{\Phi}(t, r) | g \rangle \simeq V_M(r)$$



(Observable?) quantum hair

$$V_{QM} = V_M(r) \left\{ 1 - \left[1 - \frac{2}{\pi} \text{Si} \left(\frac{r}{R_s} \right) \right] \right\}$$

$$\text{Si}(x) = \int_0^x j_0(z) dz$$

- Excited (coherent) states \iff deformations (Love numbers...)

3 - Quantum integrable black holes

- Corpuscular scaling laws:

$$N_M \sim \frac{M^2}{m_p^2} \ln \left(\frac{R_\infty}{R_s} \right)$$

$$\lambda_M \simeq \frac{N_M}{\langle k \rangle} \sim R_H \sim G_N M$$

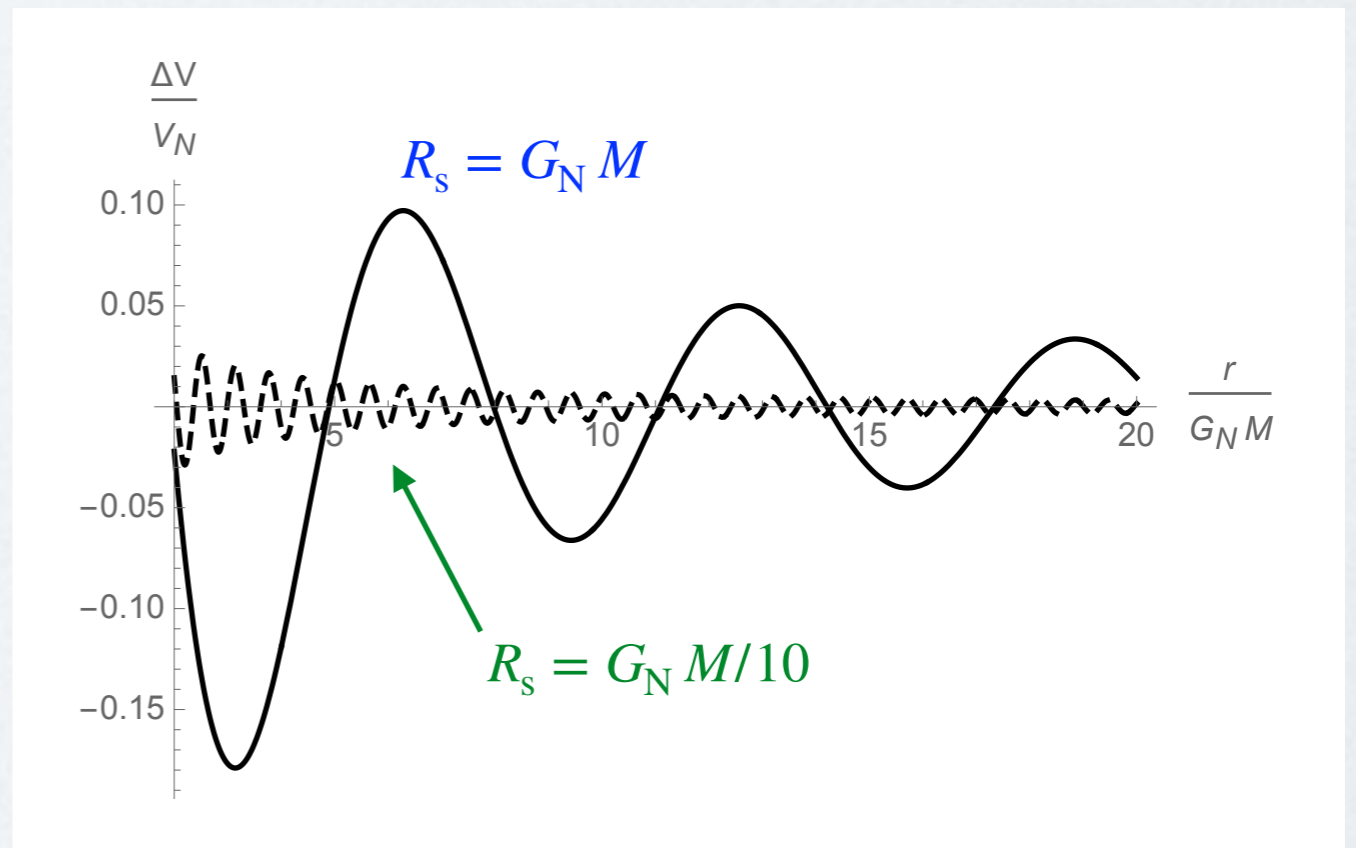
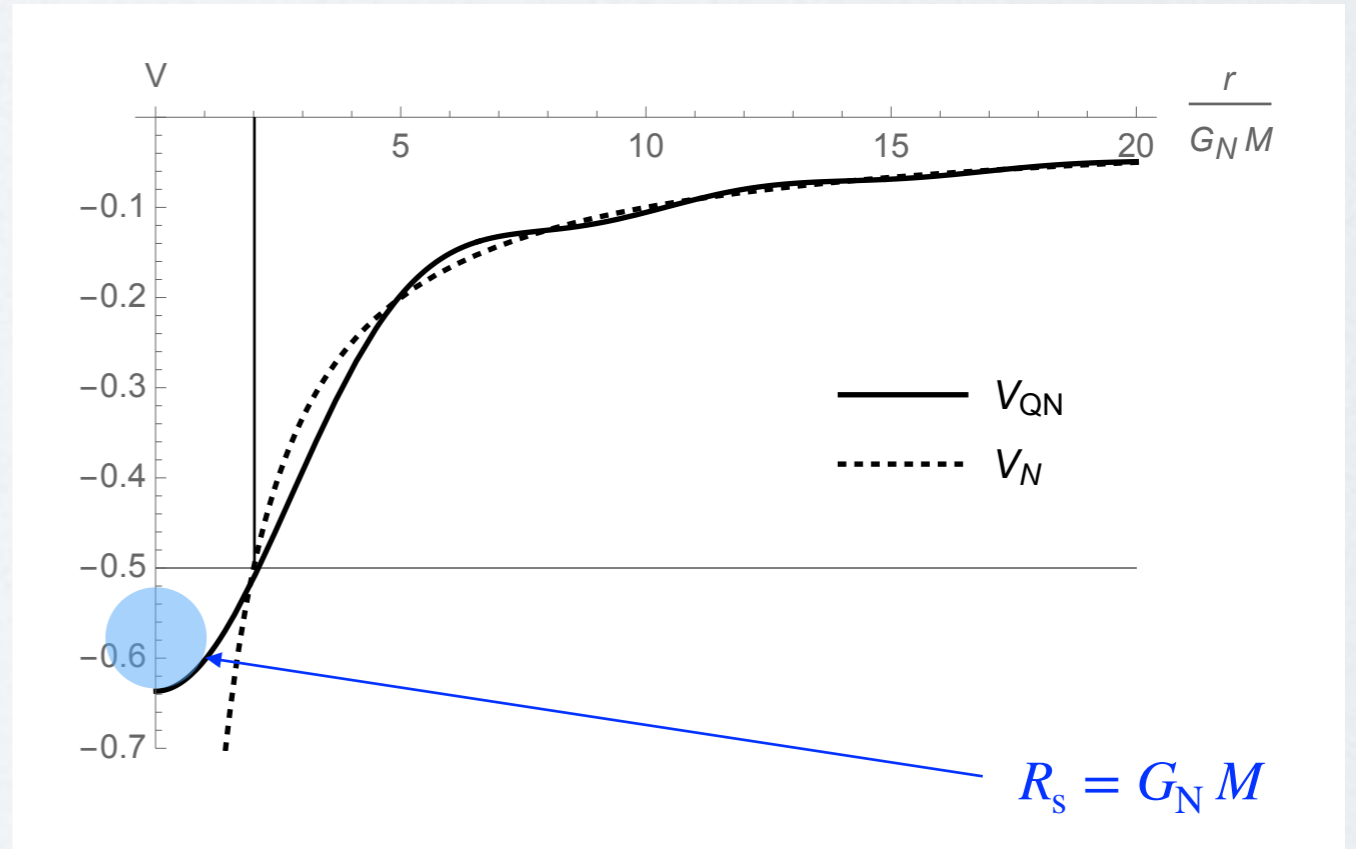
cutoffs \rightarrow proper $g(k) \sim$ time-dependent “quantum hair”

- Quantum metric [1]:

$$ds^2 \simeq - \left(1 + 2 V_{QM} \right) dt^2 + \frac{dr^2}{1 + 2 V_{QM}} + r^2 d\Omega^2$$

Integrable singularity without inner horizon

$$R^2 \sim R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \sim r^{-4}$$



3 - Quantum integrable black holes

- Spherical *integrable singularity* without inner horizon [1,2]

$$\rho_{\text{eff}} \sim |\Psi^2(r)| \sim r^{-2}$$

$$m(r) \sim \int_0^r \rho(x) x^2 dx < \infty$$

$$p_{\text{eff}}^r \sim -\rho_{\text{eff}} \sim -r^{-2}$$

$$p_{\text{eff}}^t \sim r^0$$

$$m(r) \sim r$$

$$\Delta = r_{\pm}^2 - 2r_{\pm}m(r_{\pm}) = 0$$

$$\Delta(r \sim 0) \leq 0$$

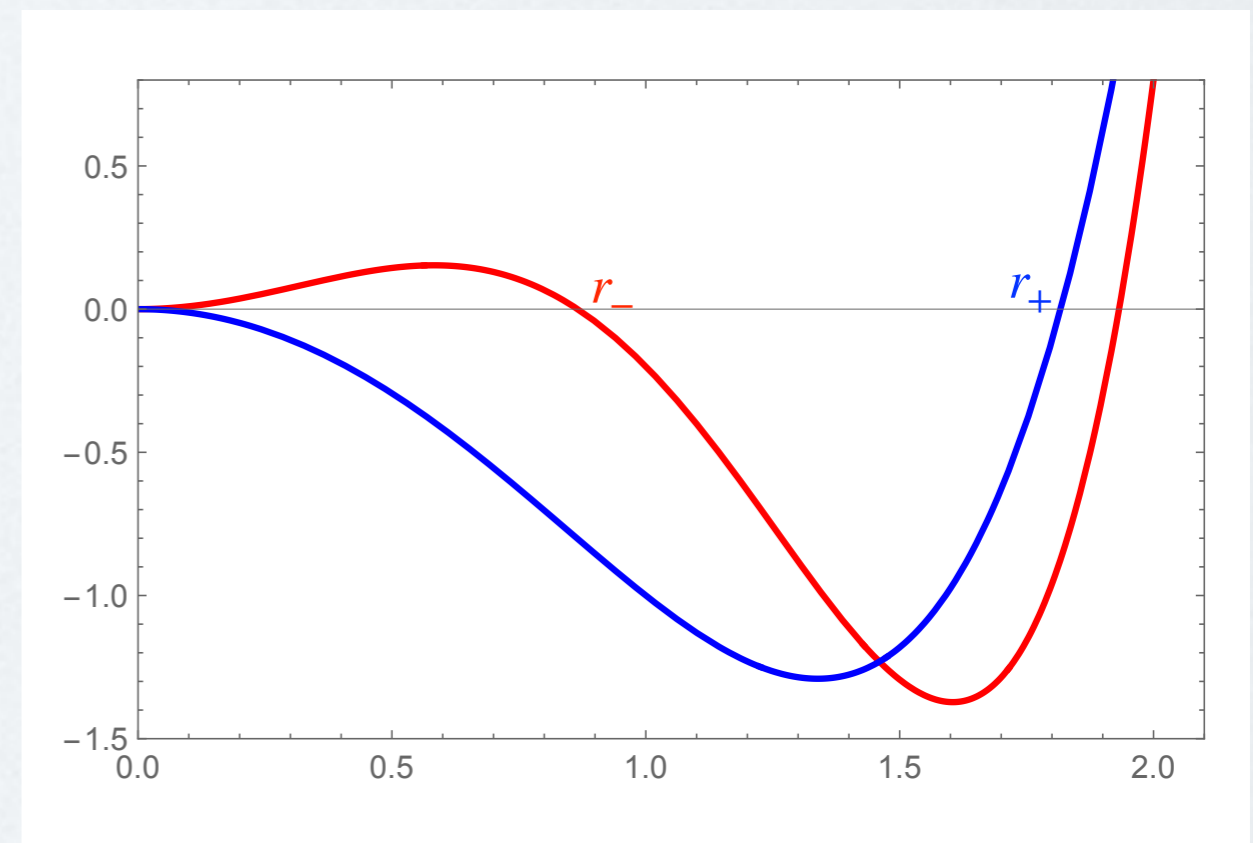
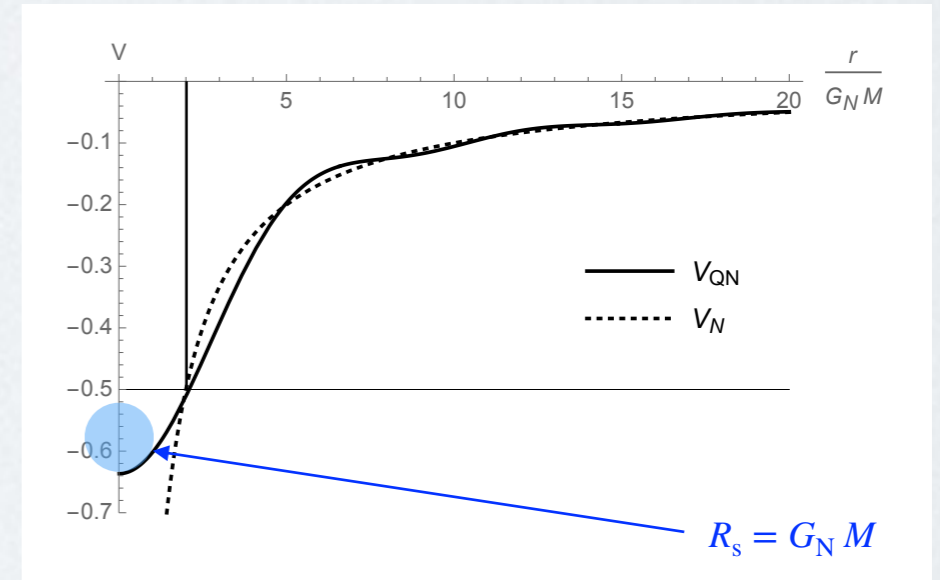


Regular (classical) black holes:

$$\rho \sim r^0 \implies m \sim r^3$$

$$\Delta = r_{\pm}^2 - 2r_{\pm}m(r_{\pm}) = 0$$

$$\Delta(r \sim 0) \sim r^2 \geq 0$$



[1] R.C., IJMPD 31 (2022) 2250128 [arXiv:2103.00183]

[6] R.C., A. Giusti, J. Ovalle, PRD 105 (2022) 124026 [arXiv:2203.03252]

Conclusions

- Black holes as (macroscopic) quantum states (*bound states* far from vacuum)
- Singularity is not resolved (integrable “fuzzy” geometry)
- Exterior quantum hair (from core size)
- No Cauchy horizon (also for electrically charged black holes)
- No Cauchy horizon for non-rigidly rotating black holes
- Effective cosmological DM
- Test the model*: perturbations \implies binary systems \implies GW
- Test the model**: geodesic motion \implies shadow

*R. Brustein, *Quantum Love numbers*, PRD 105 (2022) 024043 [arXiv:2008.02738]

**A. Urmanov, H. Chakrabarty, D. Malafarina, arXiv:2406.04813

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Thank you!

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**D. Malafarina et al, in preparation