Addressing the so called quantum/classical “ divide” in gravitational contexts, and its implications in cosmology.

Daniel Sudarsky, ICN-UNAM
Collaborations with: B. Juárez, T. Miramontes , B Kay, T. Maudlin, E Okón, S. Landau, M. Benetti, A. Perez, G. León, G. Bengochea, R.L. Lechuga and various other collaborators in precursor works.

Universidad Nacional Autónoma de México

JB INSTITUTE

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A word about methodology:

We approach the exploration of the GR/ QT regime in a top - bottom approach.

Usual bottom -up approach: postulates a fundamental theory (S.T. , LQG, Causal sets, CDT, etc.) and attempts to connect to regimes of interest of the "world out there" : Cosmology, Black Holes, etc.

The top - bottom approach, pushes existing, well tested and developed theories, to address open issues that seem to lie beyond their domain. Possible modifications can serve as clues about the nature of the more fundamental theory.

The idea is to push GR together with QFT (i.e. semi-classical gravity) while considering certain foundational difficulties.

Quantum uncertainties or indefiniteness are often referred to as quantum fluctuations. This terminology promotes confusion with stochastic fluctuations, which in turn, can refer to either, small changes in a system occurring in time, or to variations in localized aspects of an extended system, or among individuals within an actual ensemble.

They occur in various places in our cosmological ideas and theories:
i) The emergence of the Seeds of cosmic structure during inflation.
ii) The generation of primordial gravity waves during inflation.
iii) A fundamental problem afflicting many inflationary models : Eternal Inflation (if time permits).

Quantum uncertainties might justifiably be taken as measures of a *"stochastic "* kind of variations, in connection with an appropriate measurement, bringing in the “*M* problem".

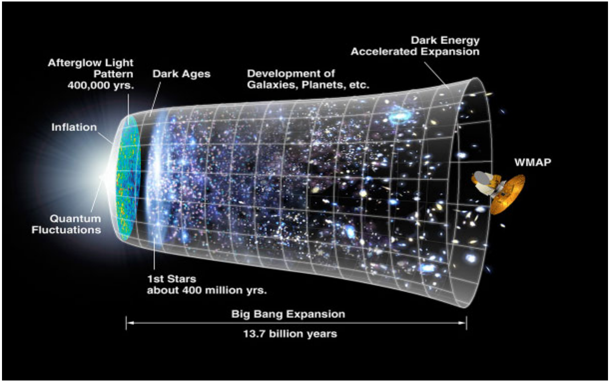
We note that the cosmological context is such that no satisfactory candidate of observer or measuring device can be identified.

Simply overlooking the conceptual issues and adopting a "practical posture" where one thinks that a quantum system just “jumps around" within the corresponding uncertainty range, is untenable on various grounds.

In fact, such (“shut up and compute") practical posture would bring us into serious conflict with actual experiments:

Example: searches for a non-vanishing EDM of the Neutron, where the quantum uncertainties of order $\sim 10^{-14} e - cm$ would, if viewed in that way, be incompatible with attainment of the current bounds, $d_n < 10^{-26} e - cm$ (obtained via a weak measurement).

COSMIC INFLATION Contemporary cosmology includes inflation as one of its most attractive components.



Early stage of accelerated (close to de-Sitter) expansion, that is turned off after at least $N = 60$ or so e-folds.

The (simplest version of) theory is described by the action

$$S = \int d^4x\sqrt{-g}[R - 1/2\nabla^a\phi\nabla_a\phi + V(\phi)]. \qquad (1)$$

The inflationary behavior is the result of a scalar field whose potential acts as a slowly varying cosmological constant $V(\phi_0(t))$.

The resulting dynamics drives the universe into an extremely flat homogeneous and isotropic stage where all matter content (and specially all defects remnant from earlier phase transitions) are exponentially diluted (and solves other “naturalness problems").

Its biggest success: the account for emergence of the seeds of cosmic structure as a result of “quantum fluctuations" with the correct spectrum.

However, at the theoretical/conceptual level, the account is not satisfactory.

The starting point of the analysis is a cosmological space-time (in a specific gauge)

$ds^2 = a^2(\eta)\{-(1+2\Psi)d\eta^2 + [(1-2\Psi)\delta_{ij} + h_{ij}]dx^i dx^j\}$

with matter represented by an inflaton field written as $\phi = \phi_0(\eta) + \delta\phi$ with $\delta\phi, \Psi, ..., \delta h_{ij}$ “small perturbations" containing the spatial dependencies, which might or might not be present.

The background (a, ϕ_0) is treated classically and assumed to be dominated by the inflaton potential (slow roll regime), (so ϕ_0 changes slowly, and a is approximately $a(\eta) = \frac{-1}{\eta H_I}$. .

[Set $a = 1$ “today" and inflation to take place: η in $(-\mathcal{T}, \eta_0), \eta_0 < 0$].

The “perturbations": $\delta\phi, \Psi, ..., \delta h_{ij}$, are treated quantum mechanically & assumed to be characterized by a (BD or adiabatic) “ vacuum state" $|0\rangle$.

[As noted, Inflation dilutes all preexisting features and drives all space dependent fields towards their vacuum states.]

The state of the quantum field is *"also characterized"* by the so called “quantum fluctuations" or “uncertainties".

Here, we face an instance of the kind of confusion discussed before:

In the usual treatments, those quantum indeterminacies are **unjustifiably** identified as the primordial inhomogeneities which eventually evolved into all the structure in our Universe: galaxies, stars, planets, etc... and left their traces in the CMB.

However, note that, according to the inflationary picture: The Universe was H&I, (both in the parts described in “classical", and “quantum" terms) as a result of inflation (up to e^{-N}).

The “present" situation (with galaxies, stars, planets, and us) is not.

How does this happen if the dynamics of the closed system does not break those symmetries.? This is an instance of M problem!.

A useful way to frame the M problem is given by Maudlin's Trilema:

The following 3 premises can not be held simultaneously in a self consistent manner. [Tim Maudlin (*Topoi* **14**, 1995)].

i) **The characterization of a system by its wave function is complete.** Its negation leads, for instance, to hidden variable theories (like Bohmian Mech)**.**

ii) **The evolution of the wave function is always according to Schrödinger's equation.** Its negation leads, for instance, to **spontaneous collapse theories.**

iii)**The results of experiments lead to definite results.** Its negation leads, for instance, to Many World/ Minds Interpretations, Consistent Histories approach, etc.

Collapse theories

Unify the U and R evolution processes. Large amount of previous work GRW, Continuous Spontaneous Localization (CSL P. Pearle). Relativistic versions (R . Tumulka, D Bedinham,..), L. Diosi & R. Penrose considered tying them to gravity.

Example (CSL). The theory is defined by two equations:

i) A modified Schrödinger equation, whose solution is:

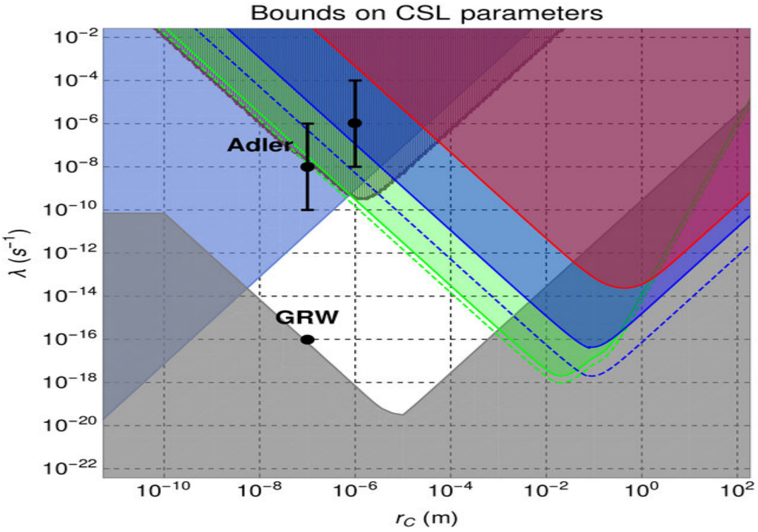
$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\lambda}[w(t') - 2\lambda\hat{A}]^2 \right]} |\psi, 0\rangle. \qquad (2)$$

($\hat{\mathcal{T}}$ is the time-ordering operator). $w(t)$ is a random classical function of time, of white noise type, whose probability is given by the second equation, ii) the Probability Rule:

$$PDw(t) \equiv {}_w\langle\psi, t|\psi, t\rangle_w \prod_{t_i=0}^{t} \frac{dw(t_i)}{\sqrt{2\pi\lambda/dt}}. \qquad (3)$$

For non-relativistc QM : $\hat{A} = \hat{\vec{X}}$ (smeared with scale $r_c \sim 10^{-5} cm$).

The theories are being tested. Example CSL



With λ small enough to avoid conflict with tests of QM and big enough to result in rapid localization of “macroscopic objects". GRW suggested range: $\lambda \sim 10^{-17} sec^{-1}$. (Likely depends on particle mass).

These can account for the breakdown of symmetries, so consider incorporating those into “inflationary cosmology".

They involve modified space-time evolution of quantum systems, so we need space-time in order to use them. Space-time is thus treated classically. The scalar field (as all matter) is treated using QFT in curved space-time.

We are driven to use a semiclassical gravity formalism. Many arguments have been put forward against this option, including a famous ‘experiment" discussed in Page and Gleiker (PRL ,1981) involving precisely the exploration of the gravitational field (or the space-time metric) associated with massive sphere in a quantum superposition of two localizations

.

Their analysis concludes that:
a) If there are no collapses of the quantum state, the theory conflicts with experimental evidence.
b) If there are collapses of the quantum state the theory is inconsistent.
The last statement is based on the observation that the kind of collapse required implies $\nabla^a < T_{ab} > \neq 0$ while the other side of EE automatically satisfies $\nabla^a G_{ab} = 0$.

On the other hand, in ["On the status of conservation laws in physics: Implications for semiclassical gravity", T. Maudlin, E. Okón, D.S., Studies in History and Philosophy of Science, 69, 67 (2020).] we argued that all reasonable paths to deal with the M problem [i) ii) & iii)] lead very same problem.

Semi-classical gravity (together with any resolution of the *M* problem), **cannot** be considered as a fundamental characterization of the interface of geometrical and quantum aspects of nature.

We view semi-classical gravity together with suitably adapted spontaneous collapse theories as an approximate description.

Hydrodynamics analogy: The N-S eqs. work fine in a broad set of circumstances but they are clearly not fundamental (and breakdowns occur).

Take it valid in regions with no collapse events, but acknowledge that departures would occur when spontaneous collapses are involved.

To make things more explicit and precise, we will use the following definition characterizing the situations in which semi-classical GR is valid:

Incorporate collapse to GR. At the formal level, we rely on the notion of *Semi-classical Self-consistent Configuration* (SSC).

DEFINITION: The set $g_{\mu\nu}(x), \hat{\varphi}(x), \hat{\pi}(x), \mathcal{H}, |\xi\rangle \mathrm{in} \mathcal{H}$ represents a SSC iff $\hat{\varphi}(x)$, $\hat{\pi}(x)$ and $\mathcal{H}$ corresponds to QFT in CS over the space-time with metric $g_{\mu\nu}(x)$, and MOREOVER the state $|\xi\rangle$ in $\mathcal{H}$ is such that:

$$G_{\mu\nu}[g(x)] = 8\pi G \langle\xi|\hat{T}_{\mu\nu}[g(x), \hat{\varphi}(x), \hat{\pi}(x)]|\xi\rangle^{(Ren)}.$$

Involves self reference (is a GR version of the Schrödinger-Newton system).

Incorporating spontaneous Collapse:

Involves a change in the quantum state, which requires a change in the space-time metric, what in turn requires a change in the Hilbert space to which the state can belongs. It should not be looked as *jumps in states* but *jumps* of the form:

$$....SSC1.... \rightarrow ....SSC2.... \tag{4}$$

A scheme is needed to interpolate between, or join SSC's.

Matching conditions: for space-time and states in the Hilbert space. Involves delicate issues. (renormalization of the EMT, well posedness of the Initial value formulation of Semi-classical gravity, etc).

Furthermore, we note that an extension of any collapse theory from the Non Rel QM, many particle setting to the Relativistic QFT one is highly nontrivial.

Applying the approach to Inflation:
The zero mode of the field ($\hat{\phi}_0$) is taken to start in highly excited (and sharply peaked) state, while the space dependent modes are in the vacuum (BD or adiabatic) state $|0\rangle$.

The quantum state of the scalar field and the space-time metric satisfy Einstein's semi-classical eq.

$$G_{\mu\nu} = 8\pi G \langle\xi|\hat{T}_{\mu\nu}|\xi\rangle.$$

under those conditions one obtains essentially the standard behavior for the background.

$a(\eta) = \frac{-1}{\eta H_I}$ and slow roll for $\langle\hat{\phi}_0\rangle$ in $(-\mathcal{T}, \eta_0), \eta_0 < 0$.

Concentrate next on the $\vec{k} \neq 0$ modes.

[We have studied the case of the individual collapse of a single mode using the SSC formalism and a natural gluing recipe. More recently a generic collapse for general situations.]

Here, we will rely on a *practical procedure* which gives equivalent results as the more rigorous formalism.
In the vacuum, the operators $\delta\hat{\phi}_k$ $\hat{\pi}_k$ are characterized by gaussian wave functions centered on 0 with uncertainties $\Delta\delta\phi_k$ and $\Delta\pi_k$, and $\Psi(\eta, x) = 0, h_{ij}(\eta, x) = 0$.

The collapse modifies the quantum state, and the expectation values of $\delta\hat{\phi_k}(\eta)$ and $\hat{\pi}_k(\eta)$.

Assume the collapse occurs mode by mode and is described by an adapted version of collapse theories.

Our universe would correspond to one specific realization of the stochastic functions (one for each $\vec{k}$).

First, consider the scalar perturbations $\Psi(\eta, x)$. The Fourier decomposition of the semi classical Einstein's Equations give:

$$-k^2\Psi(\eta, \vec{k}) = \frac{4\pi G\langle\phi_0'(\eta)\rangle}{a}\langle\hat{\pi}(\vec{k}, \eta)\rangle \qquad (5)$$

With reasonable choices in the details of the collapse theory, agreement with observations can be achieved:

In CSL version: Collapse in the field operator or the momentum conjugate operators with $\lambda = \tilde{\lambda} k^{\pm 1}$ fixed by dimensional considerations (or collapse in the operators $(-\nabla^2)^{-1/2}\hat{\pi}(\vec{x})$ or $(-\nabla^2)^{1/2}\hat{\phi}(\vec{x})$) . Why is this the right thing?.

The resulting prediction for the power spectrum is:

$$P_S(k) \sim (1/k^3)(1/\epsilon)(V/M_{Pl}^4)\tilde{\lambda}\mathcal{T} \qquad (6)$$

Taking GUT scale for the inflation potential, and standard values for the slow-roll, leads to agreement with observation for: $\tilde{\lambda} \sim 10^{-5} MpC^{-1} \approx 10^{-19} sec^{-1}$.

Not very different from GRW suggestion ! .

Other studies making different choices obtain rather different results !

TENSOR MODES

Similarly, the equation of motion for the tensor perturbations is:

$$(\partial_0^2 - \nabla^2)h_{ij} + 2(\dot{a}/a)\dot{h}_{ij} = 16\pi G \langle(\partial_i\delta\phi)(\partial_j\delta\phi)\rangle_{Ren}^{tr-tr} \quad (7)$$

$tr - tr$ stands for the transverse trace-less part of the expression (retaining only dominant terms).

Note that it is quadratic in the collapsing quantities !!

Passing to a Fourier decomposition, we solve the eq.

$$\ddot{\tilde{h}}_{ij}(\vec{k},\eta) + 2(\dot{a}/a)\dot{\tilde{h}}_{ij}(\vec{k},\eta) + k^2\tilde{h}_{ij}(\vec{k},\eta) = S_{ij}(\vec{k},\eta), \quad (8)$$

with zero initial data, and source term:

$$S_{ij}(\vec{k},\eta) = 16\pi G \int \frac{d^3x}{\sqrt{(2\pi)^3}} e^{i\vec{k}\vec{x}} \langle(\partial_i\delta\phi)(\partial_j\delta\phi)\rangle_{Ren}^{tr-tr}(\eta,\vec{x}). \quad (9)$$

The result is formally divergent. However, we must introduce a cut-off *(the scale of diffusion (Silk) dumping with* $p_{UV} \approx 0.078 MpC^{-1}$ *)*.

The prediction for the power spectrum of tensor perturbations is:

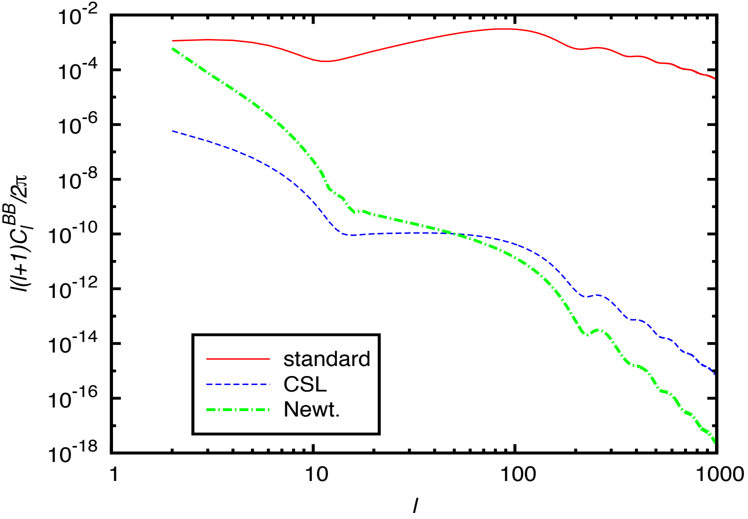
$$P_h(k) \sim (1/k^3)(V/M_{Pl}^4)^2(\tilde{\lambda}^2 \mathcal{T}^4 p_{UV}^5/k^3) \quad (10)$$

($\mathcal{T}$ the conformal time at the start of inflation taken for standard inflationary parameters as 10^4 MpC), while the power spectrum for the scalar perturbations is

$$P_S(k) \sim (1/k^3)(1/\epsilon)(V/M_{Pl}^4)\tilde{\lambda}\mathcal{T} \quad (11)$$

That is a very different relation between them than the "standard one" . Tensor modes are not expected at the level they are being looked for!!
PRD **96**, 101301(R) (2017); *PRD* **98** 023512 (2018)

We also considered a simpler collapse model, and again obtained **reduced tensor mode amplitude** but with a different shape.



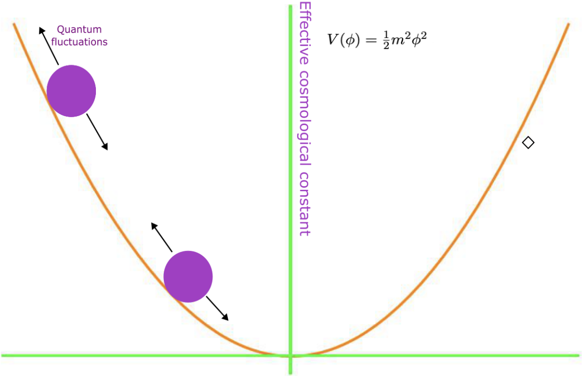
The Eternal Inflation Problem.

One of Inflation's most serious problem: the propensity of simple models to lead to a condition where inflation is extremely unlikely to ever end.

The argument: Inflation is driven by the the inflaton's zero mode $\phi_0(\eta)$, which sets into a “slow roll condition". In a characteristic time, its displacement $\Delta^{Class}\phi$ slowly decreases effective cosmological constant. However, it is argued the inflaton is also subject to quantum fluctuations $\Delta^{Quant}\phi$ and one must determine whether or not the latter dominates over the classical displacement.

In the latter case, in some regions, there is an increase of the inflaton's potential, and thus, of Λ_{eff}, and the opposite in other regions. The former regions grow faster and thus at slightly later times they will represent a larger portion of the universe. In time, the regions where the fluctuations were mostly upward would represent the overwhelming portion of the universe so we are likely to find ourselves in one such region. The expectation is that inflation will never end.

The Eternal Inflation Problem



Classical vs. quantum " displacements" of the field.

This argument raises some serious “concerns" :

1) It once more conflates quantum uncertainties with stochastic fluctuations. The quantum fluctuations by themselves do not indicate something is randomly changing in space or in time.

2) Moreover, as we’ve noted, inflation is driven by the inflaton’s zero mode, which is, by definition, homogeneous and isotropic, so nothing pertaining to it could be taken as indicating that something happens in some regions and something else occurs in others.

So, as it stands, there seems to be no solid argument for **eternal inflation.**

However, as we have seen, we should supplement the standard story in order to have a sensible account for the formation of structure, and when that is provided by a spontaneous collapse theory, we then do have actual stochastic fluctuations in the evolution of the inflaton field.

Thus, the issue must be faced anew, but from a rather different perspective: Compare the classical displacement $\Delta^{Class}\phi$ in a characteristic time, with the corresponding stochastic mean displacement in the same time (due to the collapses) .

Taking the implementation where the field is taken as the collapse operator, with $\lambda = \tilde{\lambda}k$, the zero mode is NOT subject to collapses, and so it seems we would not face the EIP!.

However, we must confront the fact that there are modes, with such a long wavelength that they would be *effectively acting* as the zero mode: the modes with wavelength larger that the particle (causal) horizon i.e. modes with $k < k^*$.

We must compare $(\Delta^{Stoch}\phi)^2 = \int_0^{k^*} d^3k \overline{< \delta_k\phi >^2}$ with $(\Delta^{Class}\phi)^2$ for all times during the inflationary period.

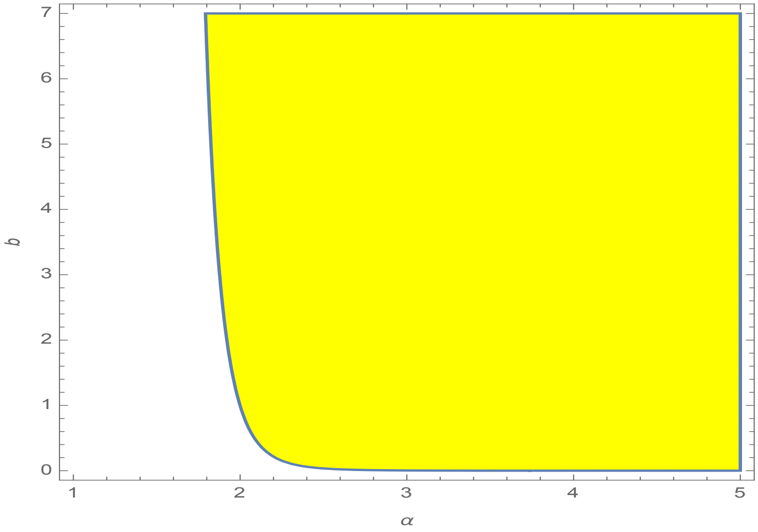
We considered a slightly generalized form for the k dependence of the collapse rate

$$\lambda = \tilde{\lambda} k \frac{k^{\alpha+1}}{(b+k)^{\alpha}} \quad (12)$$

which must reduce to $\lambda \sim k$ for the modes that are visible in the CMB (and BAO) : $10^{-3} Mpc^{-1} < k < 10^{2} Mpc^{-1}$.

[" Eternal inflation and collapse theories", R.L. Lechuga, D. Sudarsky, JCAP, Vol 01, 038 (2024). arXiv:2308.01383.]

The region where the Eternal Inflation Problem is avoided is marked in yellow:



b is in units of $10^{-5} Mpc^{-1}$.

The approach also offers paths to address other outstanding problems:

The BH information loss puzzle. [“The Black Hole Information Paradox and the Collapse of the Wave Function" *FoP* **45**, 461 (2015). “ Non-Paradoxical Loss of Information in Black Hole Evaporation in Collapse Theories" *PRD* **91**, 124009 (2015); (2016); “Black Holes, Information Loss and the Measurement Problem", *FoP* **47**, 120 (2017);“Losing stuff down a black hole", *FoP* **48**, 411-428, (2018),...]

The Problem of time In Canonical approaches to Quantum Gravity [“Benefits of Objective Collapse Models for Cosmology and Quantum Gravity" *FoP* **44**, 114 (2014).]

However, there is much work till to be done, as we are probably just at the beginning of the required exploration.

THANKS