Sharp Bounds on the Landscape

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For gravitational systems, there are non-obvious constraints on their low-energy effective theories.

How can we identify and derive them from the first principles?

I will present proof of concept.

It is often said that low-energy effective theory is a way to parameterize our ignorance.

It is important to make sure that we are not parameterizing an empty set.

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These constraints delineate the boundary between the **Landscape** and the **Swampland**.

Vafa: 0509212

In this talk, I will introduce:

- No arbitrary parameters (1949)

QUANTIFY ⇒ Distance Conjecture (2006)

$$m = \exp(-\alpha\,\phi + O(1))$$

- No global symmetry (1957)

QUANTIFY ⇒ Weak Gravity Conjecture (2006)

$$m^2 \leq \frac{d-2}{8\pi G_{\mathrm{N}}(d-3)} Q^2$$

Swampland conditions on low-energy effective theories of gravitational systems are motivated by

- string worldsheets,
- string compactifications,
- quantum black holes,
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In this talk, I will **prove** some of them for gravitational systems in anti-de Sitter space (AdS) using the **AdS/CFT correspondence as the first principles**.

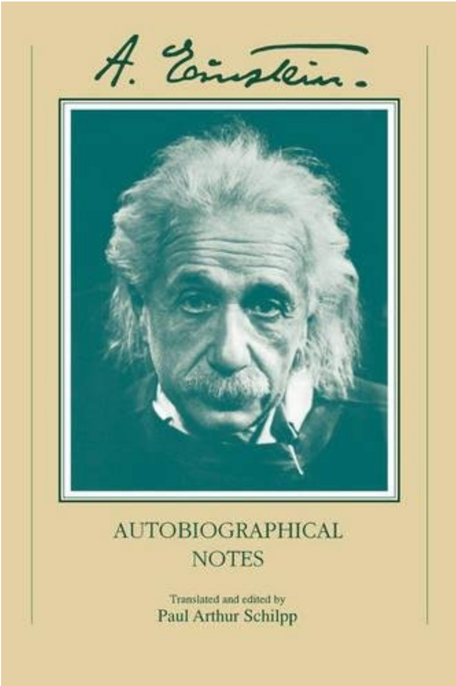
No arbitrary parameters

QUANTIFY ⇒

Distance Conjecture

$$m = \exp(-\alpha\,\phi + O(1))$$

Perhaps, the earliest Swampland condition:



Autobiographical Notes
by Albert Einstein, 1949

After explaining the notion of the natural units,

"…, then **only dimensionless constants could occur in the basic equations of physics**. Concerning such I would like to state a theorem which at present cannot be based upon anything more than upon a faith in the simplicity, *i.e.*, intelligibility, of nature: **there are no arbitrary constants of this kind** …"

A modern formulation (Distance Conjecture):

Vafa + H.O.:0605264

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Conjecture 0: Every parameter in quantum gravity is an expectation value of a dynamical field and can be varied by changing its expectation value.

Proof in AdS/CFT: If there is a continuous parameter in the AdS, there is a corresponding parameter in the dual CFT. In the CFT, each parameter is associated with an "exactly marginal" interaction term, which then corresponds to a massless scalar field in the AdS or a deformation of the boundary condition at the infinity. Therefore, **a continuous parameter in the AdS Lagrangian density must be an expectation value of a massless scalar field**.

A modern formulation (Distance Conjecture):

Vafa + H.O.:0605264

Conjecture 0: Every parameter in quantum gravity is an expectation value of a dynamical field and can be varied by changing its expectation value.

This by itself **does not provide a useful constraint** since what appear to be parameters in a low energy theory may be fixed by potentials in a more fundamental high energy theory.

For example, the Standard Model of Particle Physics has 18 parameters and the ΛCDM Model has 6 parameters, but they do not contain dynamical fields corresponding to these parameters.

A modern formulation (Distance Conjecture):

Vafa + H.O.:0605264

Conjecture 0: Every parameter in quantum gravity is an expectation value of a dynamical field and can be varied by changing its expectation value.

Conjecture 1: Choose any point p_0 in the moduli space $\mathcal{M}$. For any positive t, there is another point $p \in \mathcal{M}$ such that $d(p, p_0) > t$.

Conjecture 2: Compared to the theory at $p_0 \in \mathcal{M}$, the theory at p with $d(p, p_0) > t$ has an infinite tower of light particles starting with mass of the order of $e^{-\alpha t}$ for some $\alpha > 0$.

Distance Conjecture

Vafa + H.O.: 0605264

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- These conditions have been tested many examples in string theory, and **no counter-examples have been found**.
- They can impose **sharp constraints** on a low energy Lagrangian, especially **if α can be bounded**.

We **proved a part of Distance Conjecture** in $\mathrm{AdS}_3/\mathrm{CFT}_2$.

Wang + H.O.: 2405.00674

For any unitary 2d CFT, if there is a light state whose energy gap ΔE vanishes in some limit on the moduli space,

- The distance t to the limit is infinite.
- The approach to this limit is exponential,

$$\Delta E = \exp\big(-\alpha\, t + O(1)\big).$$

- The decay rate obeys the universal bounds,

$$c^{-1/2} \leq \alpha \leq 1.$$

The bounds are shape, and we have identified the necessary and sufficient conditions for saturation of the bounds.

ΔE can vanish only at infinite distance.

$$\Delta E = \exp(-\alpha\, t + O(1))$$

$$\frac{1}{\sqrt{c}} \leq \alpha \leq 1$$

In the AdS gravity units:

$$\Delta E = \exp(-\,\alpha_{\mathrm{AdS}}\,\phi + O(1))$$

$$\left(\frac{2}{3}\,L_{\mathrm{Planck}}\right)^{1/2} \leq \alpha_{\mathrm{AdS}} \leq (8\pi L_{\mathrm{AdS}})^{1/2}.$$

$L_{\mathrm{Planck}} = 8\pi G_N$

L_{AdS} : curvature radius

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The tower of light states **must emerge** when

$$\phi \gtrsim \left(\frac{2}{3}\,L_{\mathrm{Planck}}\right)^{-1/2}.$$

With supersymmetry:

$$\Delta E = \exp(-\,\alpha_{\mathrm{Ads}}\,\phi + O(1))$$

$$(L_{\mathrm{Planck}})^{1/2} \leq \alpha_{\mathrm{AdS}} \leq (8\pi L_{\mathrm{AdS}})^{1/2}.$$

The tower of light states **must emerge** when

$$\phi \gtrsim (L_{\mathrm{Planck}})^{-1/2}.$$

No global symmetry

QUANTIFY →

Weak Gravity Conjecture

$$m^2 \leq \frac{d-2}{8\pi G_{\mathrm{N}}(d-3)} Q^2$$

Perhaps, the second-earliest Swampland condition:

Misner, Wheeler, “Classical Physics as Geometry," Annals Phys. 2 (1957) 525.

ANNALS OF PHYSICS: **2**, 525–603 (1957)

Classical Physics as Geometry

Gravitation, Electromagnetism, Unquantized Charge, and Mass as Properties of Curved Empty Space*

CHARLES W. MISNER† AND JOHN A. WHEELER‡

Lorentz Institute, University of Leiden, Leiden, Netherlands, and Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

If classical physics be regarded as comprising gravitation, source free electromagnetism, unquantized charge, and unquantized mass of concentrations of electromagnetic field energy (geons), then classical physics can be described in terms of curved empty space, and nothing more. No changes are made in existing theory. The electromagnetic field is given by the "Maxwell square root" of the contracted curvature tensor of Ricci and Einstein. Maxwell's equations then reduce, as shown thirty years ago by Rainich, to a simple statement connecting the Ricci curvature and its rate of change. In contrast to unified field theories, one then secures from the standard theory of Maxwell and Einstein an "*already* unified field theory." This purely geometrical description of electromagnetism is traced out in detail. Charge receives a natural interpretation in terms of source-free electromagnetic fields that (1) are everywhere subject to Maxwell's equations for free space but (2) are trapped in the "worm holes" of a space with a multiply-connected topology. Electromagnetism in such a space receives a detailed description in terms of the existing beautiful and highly developed mathematics of topology and harmonic vector fields. Elementary particles and "real masses" are completely excluded from discussion as belonging to the world of quantum physics.

"I transmit but I do not create; I am sincerely fond of the ancient."—Confucius.

A modern formulation:

There is no global symmetry in quantum gravity.

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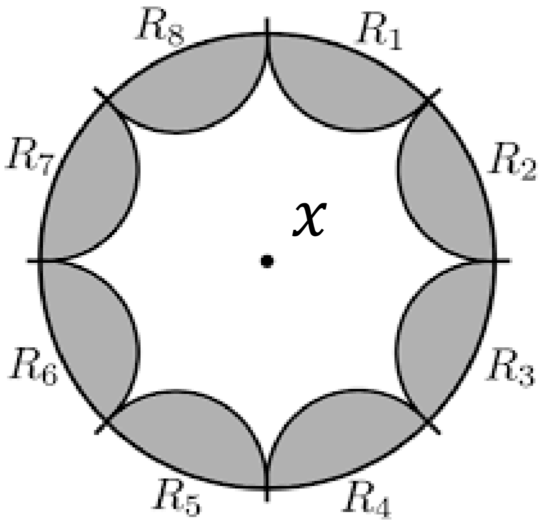
Harlow + H.O.: 1810.05338

Proof in AdS/CFT:

If a gravitational theory in AdS has global symmetry G, there must be a local operator in AdS that transforms faithfully into another local operator at the same point.



The Noether theorem implies that the symmetry generator in the CFT can be decomposed as,

$$U(g) = \prod_i U(g_i).$$

Such a symmetry operator would commute with the local operator at x in AdS.

Contradiction.

There is no global symmetry in quantum gravity.

This by itself **does not give a useful constraint** since a low energy theory may have accidental symmetry which is violated in high energy.

For example, the Standard Model of Particle Physics preserves the $(B - L)$ symmetry, but it can be violated or gauged in high energy.

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For example, the Standard Model of Particle Physics preserves the $(B - L)$ symmetry, but it can be violated or gauged in high energy.

This is analogous to the absence of non-dynamical parameters discussed earlier:
What seems like a non-dynamical parameter may be an expectation value of a dynamical field fixed by a potential.

Weak Gravity Conjecture

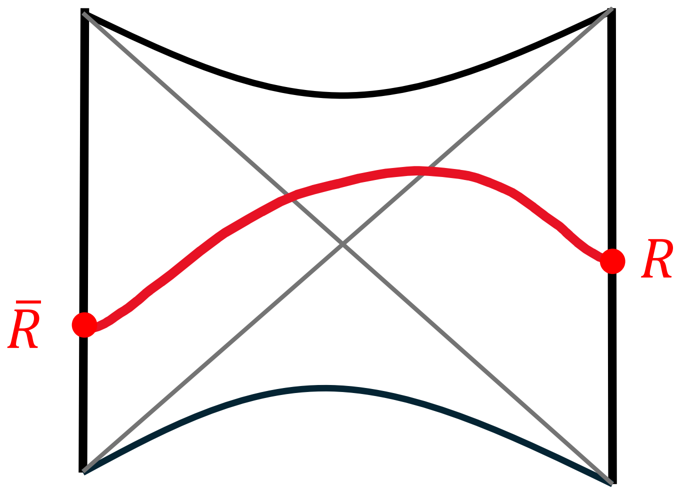
Arkani-Hamed, Motl, Nicolis, Vafa: 0601001

In any low energy theory described by the Einstein gravity, a Maxwell field, and a finite number of matter fields, if it has an UV completion as a consistent quantum theory,

there must be a particle with charge $\boldsymbol{Q}$ and mass $\boldsymbol{m}$ $\ll M_{\mathrm{Planck}}$ such that

$$m^2 \leq \frac{d-2}{8\pi G_{\mathrm{N}}(d-3)} Q^2.$$

Weak Gravity Conjecture: $m^2 \leq \frac{d-2}{8\pi G_{\mathrm{N}}(d-3)} Q^2$



In the **AdS Schwarzschild geometry**, the Wilson line in any representation R in a long-range gauge symmetry connecting the two asymptotic boundaries must be a sum of products of operators acting on the two Hilbert spaces.

Harlow: 1510.07911

In a model where a bulk $U(1)$ gauge field emerges in low energy, we can show

$$m^2 \lesssim \frac{1}{G_{\mathrm{N}}} Q^2,$$

Harlow: 1510.07911
Harlow + H.O.: 1810.05338

assuming that the Landau pole $< \Lambda_{\mathrm{QG}}$.

- No arbitrary parameters (1949)

QUANTIFY ⇒

Distance Conjecture (2006)

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QUANTIFY ⇒

Weak Gravity Conjecture (2006)

$$m^2 \leq \frac{d-2}{8\pi G_{\mathrm{N}}(d-3)}\,Q^2$$

How about scalar fields with potentials?

Constraints on scalar potentials:

$$S = \int dx^{d+1} \sqrt{g} M_p^{d-1} \left[R + G_{ij}(\phi) \partial\phi^i \partial\phi^j - V(\phi) + \cdots \right]$$

(Refined) de Sitter Conjecture.

Obied, Spodyneiko, Vafa + H. O.: 1806.0836

Palti, Shiu, Vafa + H. O.: 1810.05506

The potential $V(\phi)$ satisfies

$$|\nabla V(\phi)| \geq \frac{\gamma}{M_p} V(\phi) \quad \text{or} \quad \min\left[\nabla_i \nabla_j V\right] = \frac{\gamma'}{M_p} V(\phi) \,,$$

where γ and γ' are some $\mathrm{O}(1)$ constants.

It is desirable to bound γ and γ'.

In AdS, there are constraints on $V(\phi)$ that can be proven by the CFT consistency.

$$\mathcal{L} = \sqrt{g} M_p^{d-1} \left[R + G_{ij}(\phi) \partial \phi^i \partial \phi^j - V(\phi) + \cdots \right]$$

The **Penrose inequality** was motivated by the cosmic censorship hypothesis. In AdS, the inequality has been **proven by the CFT consistency**.

Horowitz, Engelhardt: 1903.00555

$$4\pi^{1/2} G_N M \geq \left(1 + \frac{A}{{L_{\mathrm{AdS}}}^2}\right) A^{1/2} \quad \text{in } \mathrm{AdS}_4.$$

M: mass

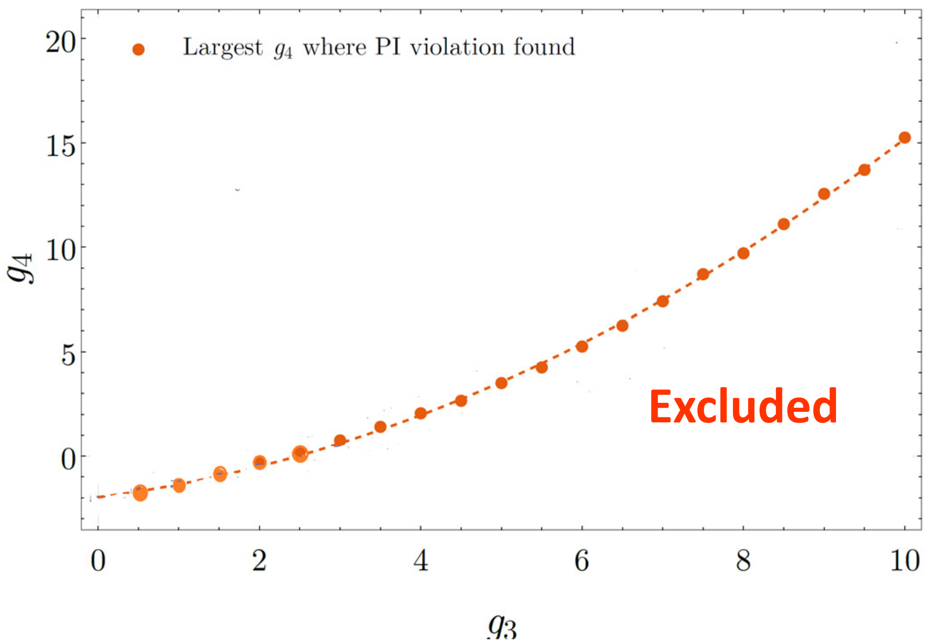
A : apparent horizon

$$\mathcal{L} = \sqrt{g} M_p^{d-1} \left[R + G_{ij}(\phi) \partial \phi^i \partial \phi^j - V(\phi) + \cdots \right]$$

There are scalar potentials that violate the **Penrose inequality** in AdS. They are therefore in the Swampland.

Åsmund Folkestad: 2209.00013

For example,



$$V(\phi) = -\left(\frac{3}{4L_{\mathrm{AdS}}}\right)^2 \phi^2 + g_3\phi^3 + g_4\phi^4$$

in AdS_4.

Summary

There are constraints on low-energy effective theories of gravitational systems that cannot be captured by the standard Wilsonian paradigm.

- No arbitrary parameters ⇒ QUANTIFY ⇒ Distance Conjecture
- No global symmetry ⇒ QUANTIFY ⇒ Weak Gravity Conjecture

In AdS, we can quantify and prove parts of these conjectures.

I hope to strengthen these results and generalize them for spacetimes with zero and positive vacuum energies.