Ligo is quantum

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Lemaitre Vatican Museum
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Lemaitre

The first to realize that Schw Horizon was regular

Very strange coords. Part was Painelevi-Gulstrand.
First to show they were coord. trans. of Schwarz. (Suspected by Einstein in '23). Second part was proper time σ along

$$\tau = t + 2\left(\sqrt{r} + \ln(\frac{\sqrt{r}-1}{\sqrt{r}+1})\right)$$

const to match to dust universe.

$$ds^2 = d\tau^2 - \frac{d\sigma^2}{r} - r^2(d\theta^2 + \sin(\theta)^2 d\phi^2)$$

$$r = \left(\frac{3}{2}(\tau - \sigma)\right)^{\frac{2}{3}}$$

World Atom (I hate it because it gives entirely wrong image of birth of universe) The whole universe as a quantum system.

Is there a limit to QM? Is Macroscopic different from Microscopic-- classical vs quantum?

Belief vs experiment. How do we show that that there is no such difference?

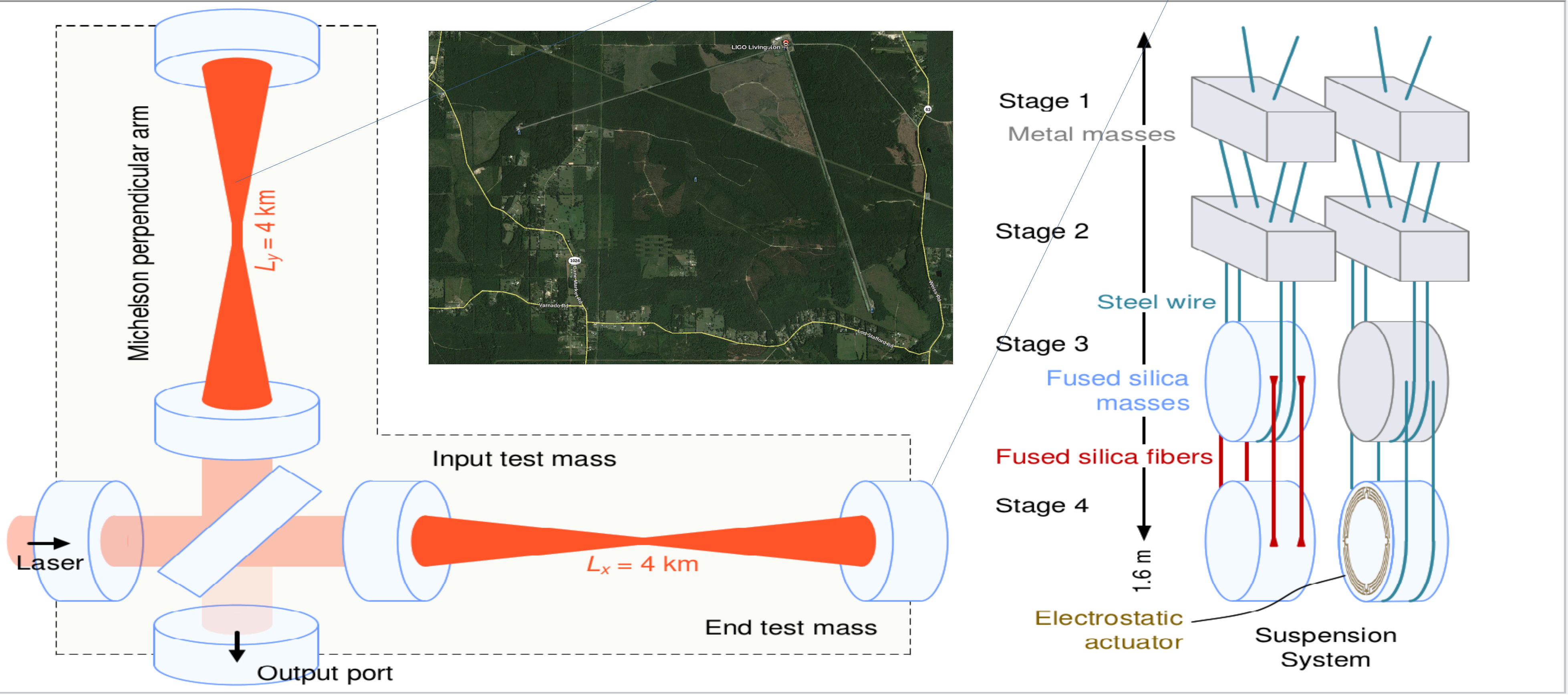
Many have argued the opposite. Bohm, Leggett, Penrose, Oppenheim,...

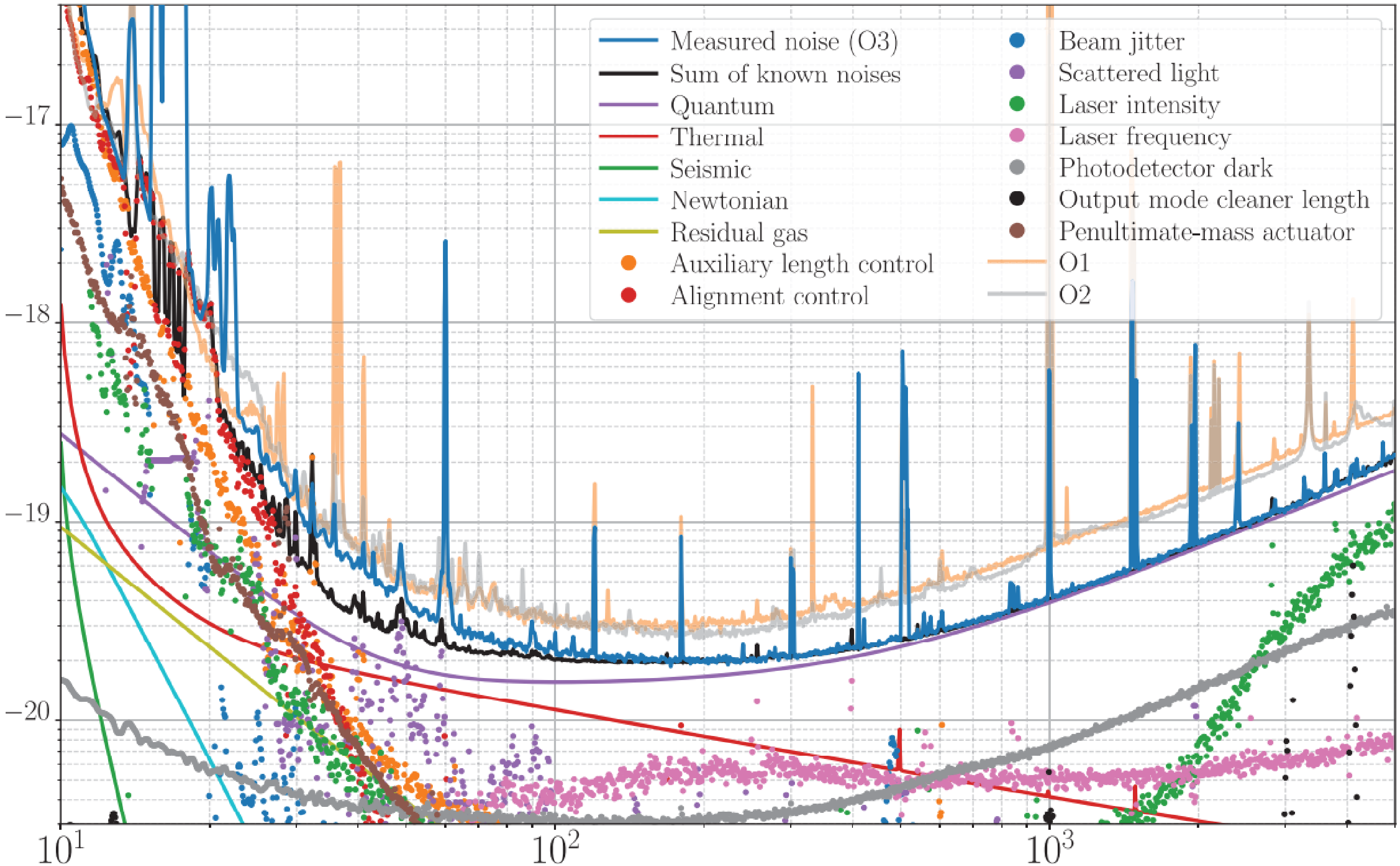
Since invention of QM, there has been conflict as to whether there are limits to Quantum Mechanics?

Not helped by Bohr’s discussion of measurement in which measurement made by coupling a quantum system to a macroscopic classical system, where “Measurement” has a well accepted meaning (values exist even if not measured Measurement simply reveal those pre-existing values.)

Is there a limit to the validity of Quantum Mechanics?
Many otherwise very good physicists have taken the position that there is.

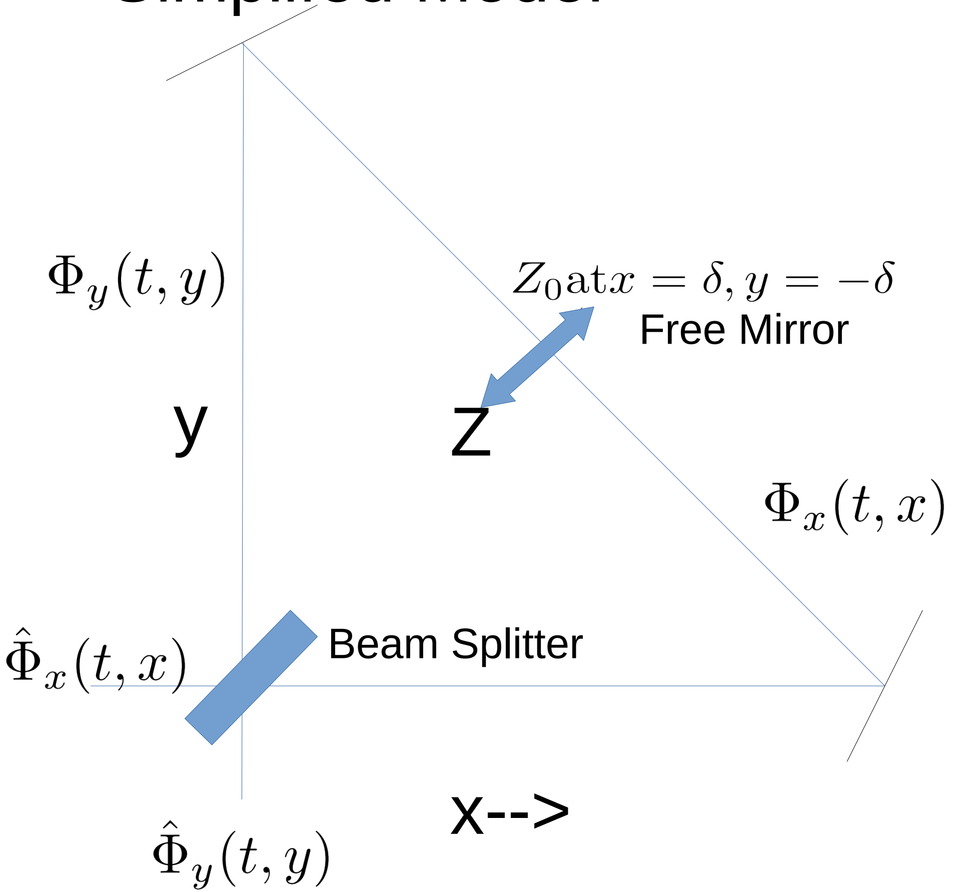
LIGO (Laser Interferometer gravity wave observatory)
Huge system-- 4 km long, masses of mirrors 40Kg
Energy of laser beam: ~1MW (10^{-11} kg/s)




Measured noise (O3)
Sum of known noises
Quantum
Thermal
Seismic
Newtonian
Residual gas
Auxiliary length control
Alignment control
Beam jitter
Scattered light
Laser intensity
Laser frequency
Photodetector dark
Output mode cleaner length
Penultimate-mass actuator
O1
O2
−17
−18
−19
−20
10^1
10^2
10^3

Quantum noise is dominant noise source.
Does not mean that mirror CM are quantum objects.

Simplifed Model



$$\Phi_x(t,x) = \Phi_{xI}(t-x)\,\overbrace{-\Phi_{xI}(t+x-2Z(t+x..))}^{\Phi_{xO}(t+x)}$$

$$\Phi_y(t,y) = \Phi_{yI}(t-y)\,\overbrace{-\Phi_{yI}(t+y+2Z(t+y..))}^{\Phi_{yO}(t+y)}$$

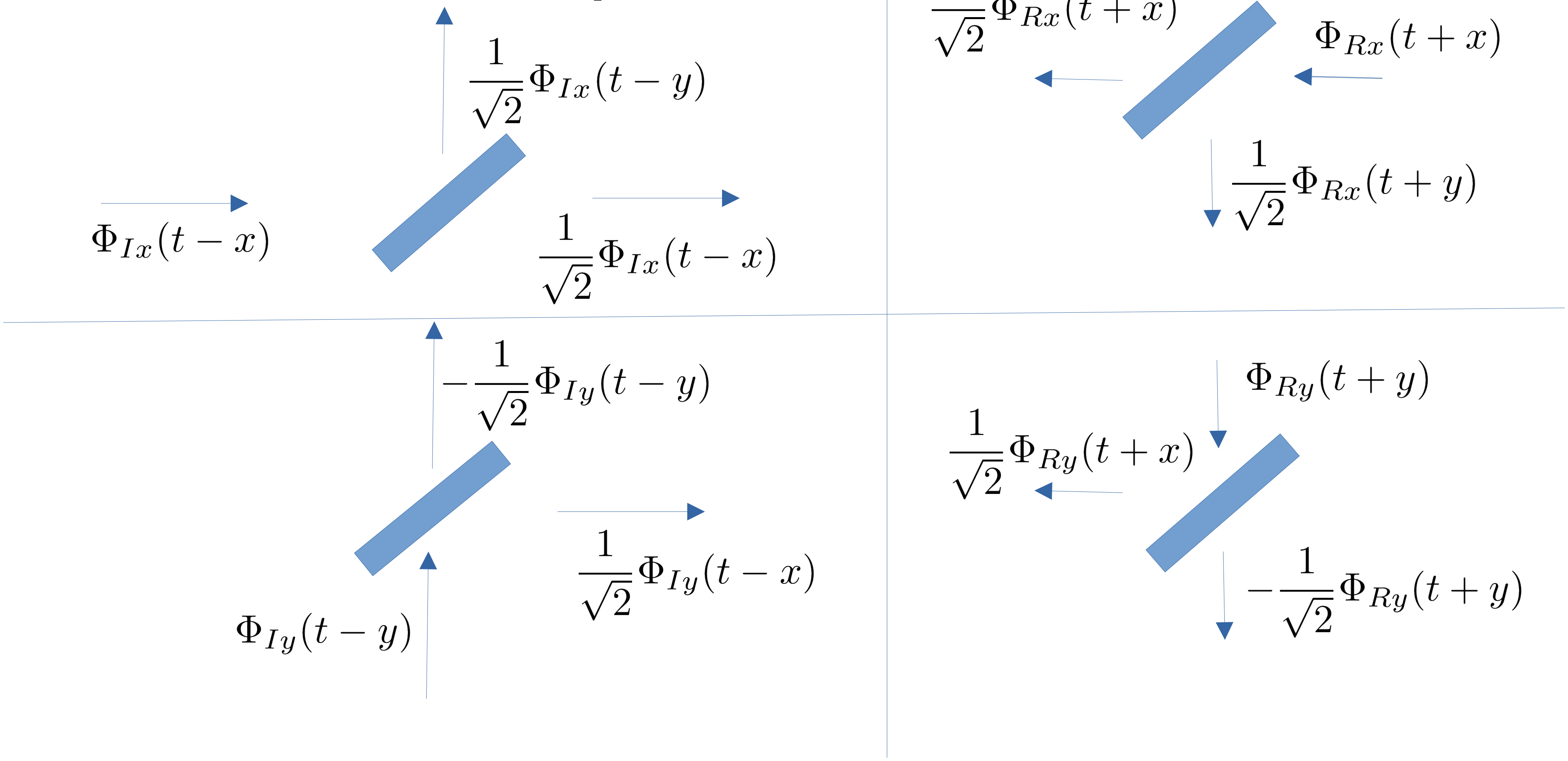
$$\Phi_{xO}(t-x) = \frac{1}{\sqrt{2}}(\hat{\Phi}_{xI}(t-x)+\hat{\Phi}_{yI}(t-x))$$

$$\Phi_{yO}(t-y) = \frac{1}{\sqrt{2}}(\hat{\Phi}_{yO}(t-y)-\hat{\Phi}_{xO}(t-y))$$

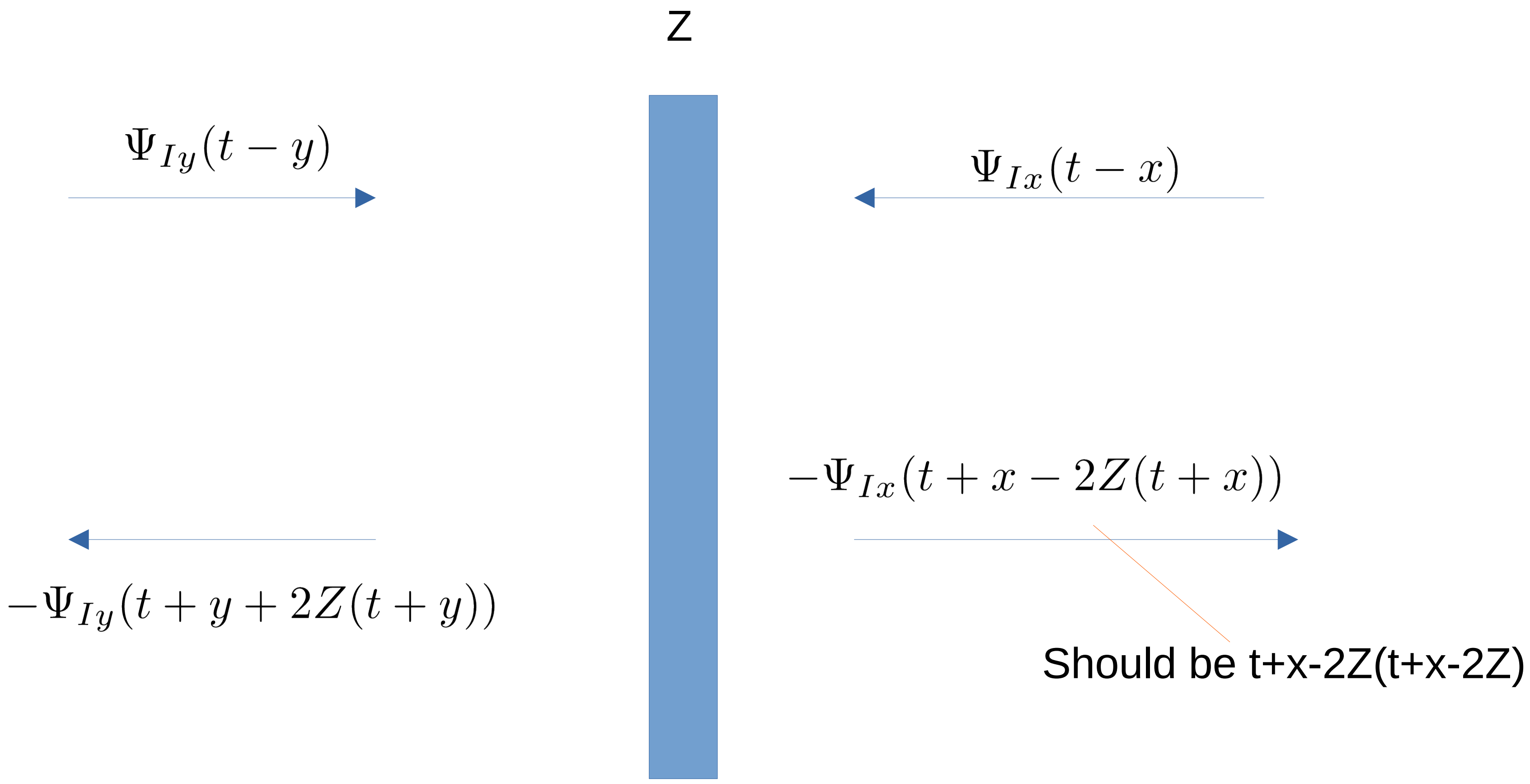
$$\Phi_{xO}(t-x) = \frac{1}{\sqrt{2}}(\hat{\Phi}_{xI}(t-x)+\hat{\Phi}_{yI}(t-x))$$

$$\Phi_{yO}(t-y) = \frac{1}{\sqrt{2}}(\hat{\Phi}_{yO}(t-y)-\hat{\Phi}_{xO}(t-y))$$

Beam Splitter



Mirror



At mirror Ψ is zero. Z occurs in phase of reflected beam.

Motion of Mirror

$$M\partial_t^2 Z = A(P_x - P_y)|_{x=L+Z}$$

A is the beam area

P is radiation pressure of beam on mirror.

$$P_x = \frac{1}{4}\left((\partial_t \Psi)^2 + (\partial_x \Psi)^2\right)$$

$$\text{so } P_x = \frac{1}{4}(\partial_x \Psi_x(t-x))^2|_{x=L+2Z(t-L)}$$

and similarly for P_y

Detection

At detection, Light falls onto a Photodiode
Photodiode detects number of photons in the beam

Photocurrent in diode measures the flux of photons

$$J_y = i[\Phi^-_{Oy}(t+y)\partial_y\Phi^+_{Oy}(t+y) - (\partial_y\Phi^-_{Oy}(t+y))\Phi^+_{Oy}(t-y)]$$

"Coherent State"

Incoming EM field

$$\Phi_{Ix}(t-x) = e^{-i\omega(t-x)} \int_{-\Lambda}^{\Lambda} \left(\frac{\alpha}{\sqrt{4\pi\omega}} \delta(\nu) + \mathbf{A}_\nu \frac{e^{-i\nu(t-x)}}{\sqrt{4\pi(\omega+\nu)}} \right) d\nu + \mathbf{HC}$$

$$\Phi_{Iy}(t-y) = e^{-i\omega(t-y)} \int_{-\Lambda}^{\Lambda} \left(\mathbf{B}_\nu \frac{e^{-i\nu(t-x)}}{\sqrt{4\pi(\omega+\nu)}} \right) d\nu + \mathbf{HC}$$

$\mathbf{A}_\nu, \ \ \mathbf{B}_\nu$ are annihilation operators at frequency $\omega + \nu$

Assume $\alpha >> A, \ \ B$

$\Lambda << \omega$

$$\Psi_{Ix}(t-x) = \frac{1}{\sqrt{2}}(\Phi_{Ix}(t-x-L) + \Phi_{Iy}(t-x+L))$$

$$\Psi_{Iy}(t-y) = \frac{1}{\sqrt{2}}(\Phi_{Ix}(t-x+L) - \Phi_{Iy}(t-y+L)))$$

and

$$\Phi_{Rx} = \frac{1}{\sqrt{2}}(-\Phi_{Ix}(t+x-2L-2Z(t+x-L)) - \Phi_{Iy}(t+x-2L-Z(t+x-L)))$$

$$\Phi_{Ry} = \frac{1}{\sqrt{2}}(-\Phi_{I}x(t+y-2L+2Z(t+y-L)) + \Phi_{Iy}(t+y-2L+2Z(t+y-L)))$$

Now choose $e^{-i\omega 2L} = 1 \qquad |\omega Z_0| << 1$

Then $\alpha e^{-i(\omega(t+x-2Z(t+x)))} \approx \alpha[e^{-i(\omega(t+x))}(1-(2i\omega Z(t+x)))$

$$\alpha e^{-i(\omega(t+y+2Z(t+y)))} \approx \alpha[e^{-i(\omega(t+y))}(1+(2i\omega Z(t+y))$$

Output out the y (B) port.

$$e^{-i\omega(t+y)}\left(4i\frac{\alpha\sin(\omega Z_0)}{\sqrt{4\pi\omega}}\omega Z(t+y)+\int_{-\Delta}^{\Delta}\frac{B_\nu}{\sqrt{4\pi(\omega+\nu)}}e^{-i\nu(t+y)}d\nu\right)$$

The quantum noise comes from the B operators (coming into the dark port (ie the one which is NOT where the laser comes in). (Caves 1980). The noise from the laser port is suppressed by $\tan(\omega Z_0)$

The B in that formula comes from the direct reflection of the incoming beams from the mirror.

However there is an additional source of noise. Namely due to the Radiation pressure on the mirrors. The radiation pressure on the mirrors due to and A are symmetric on the mirror, and thus exert the same pressure on the x side as on the y side and cancel in their effect on the mirror.

However the radiation pressure due to the B noise is, to lowest order, of opposite sign on the two sides of the mirror. They do not cancel.

The force on the mirror is thus given by

$$\mathbf{F} = \frac{\mathcal{A}}{\pi}\omega\alpha\int(B_\nu + B^\dagger_{-\nu})e^{-i\nu(t)}d\nu$$

The equation of motion of the mirror (I will approximate it as a free particle, rather than an oscillator, since the restoring force is small and the resonant frequency is much less than the frequencies of interest (about 3Hz rather than 40-1000 Hz)

$$M\partial_t^2 Z = \mathbf{F} = \frac{\mathcal{A}}{\pi}\omega\alpha\int(B_\nu + B^\dagger_{-\nu})e^{-i\nu(t)}d\nu$$

$$Z(\nu) = -\frac{\mathcal{A}}{\pi M\nu^2}\omega\alpha(B_\nu + B^\dagger_{-\nu})$$

The motion of the mirror due to the quantum nature of the light dies off with frequency . It also dies off with the mass of the mirror, and increases with the amplitude of the laser light on the mirror.

One thus, at a given frequency, one has two sources of quantum noise due to the quantum fluctuations coming in to the dark port. One is the direct reflection of the quantum light from the mirror, and one is due to the motion of the mirror due to the radiaiton pressure on the mirror. These also have different phases -- $B+B^\dagger \text{ and } i(B-B^\dagger).$

and different frequency dependencies.(ν)

How does the detector detect photons as a function of time?

Hard to model. Glauber argued that it measured some average of the flux

$$J_y = i[\Phi^-_{Oy}(t+y)\partial_y\Phi^+_{Oy}(t+y) - (\partial_y\Phi^-_{Oy}(t+y))\Phi^+_{Oy}(t-y)]$$

Ie the normal ordered symplectic flux of the field
(The +,- superscripts refer to the negative and positive freq
of the EM field. $e^{-i(\omega+\nu)t}$ and $e^{i(\omega+\nu)t}$)
there will be two terms-- the one prop to the

$$\alpha^2 sin(\omega Z_0)^2$$

the other prop. to terms linear in B

That other term will have the form of

$$D_\nu = [\rho B_\nu + i\frac{\sigma}{\nu^2}(B_\nu + B^\dagger_{-\nu})]e^{-i\nu(t+y)}$$

where rho and sigma are complex constants which depend on the amplitude of the incoming flux, the Mass of the mirror, the freq of the light, etc. For Ligo, the two terms have equal magnitude around 50 Hz.

Ie the outgoing flux will that of a two mode squeezed state.

Two mode sqeezed states

$$D = \rho C + \sigma \tilde{C}^\dagger$$
$$\tilde{D} = \tilde{\rho}\tilde{C} + \tilde{\sigma} C^\dagger$$

commutation relations $$|\rho|^2 - |\sigma|^2 = |\tilde{\rho}|^2 - |\tilde{\sigma}|^2 = 1$$
$$\frac{\sigma}{\rho} = \frac{\tilde{\sigma}}{\tilde{\rho}}$$

Amplifier $$C|c\rangle = c|c\rangle; \qquad \tilde{C}|c\rangle = 0$$
$$\langle c|D|c\rangle = \rho c$$
$$\langle c|D^2 - \rho^2 c^2|c\rangle = \rho^2 + \sigma^2$$

Ligo acts as an amplifier of noise.

Radiation noise and shot noise are approx equal in the 50-100 Hz band.
Note that they simply add to each other. If instead of feeding in a vacuum state into the B port , one feeds in the vacuum for D (Ie a two mode squeezed state

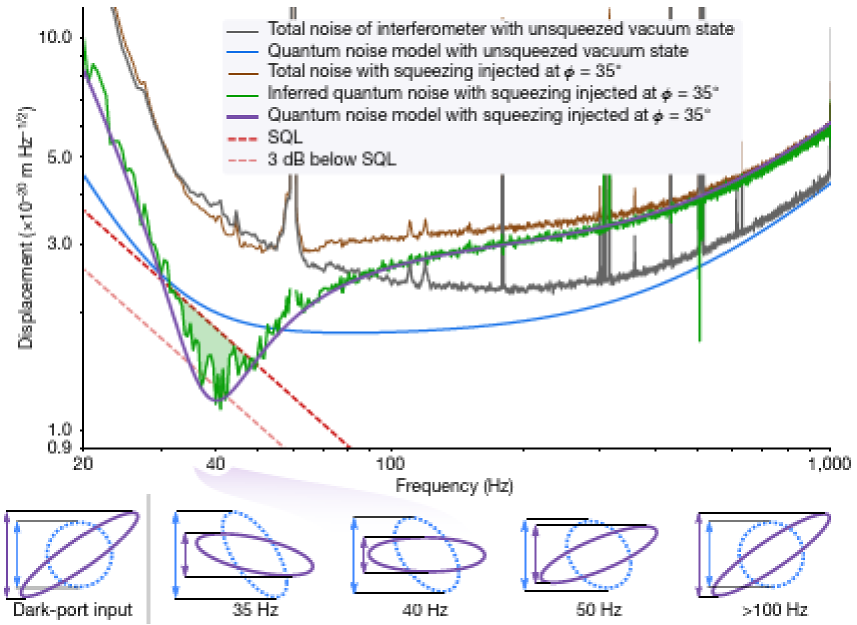
$$(\rho B_\nu + \sigma B^\dagger_{-\nu})|0\rangle_B = 0$$

one will reduce the noise to vacuum noise.
One can reduce it even further for a single phase by doing a further single mode squeezing. Eg for single mode

$$[\cosh(r)(\rho B_{nu} + \sigma B^\dagger_{-\nu}) + \sinh(r)(\rho B^\dagger_{nu} + \sigma B_{-\nu})]|0\rangle_S = 0$$

One can make the noise arb. small for a given phase

Ie, one can make the shot noise and the rad pressue noise interfere destructively with each other.



Ligo test about 2 years ago now. (theory 1979-1983)

Quantum Interference.

What if the mirrors were classical?
The shot noise would not change since it is due to purely specular deflection by the mass. It does not probe any quantum dynamics of the mirror.

However, the radiation pressure noise would change. It is due to the response of the mirror to the quantum nature of the light.

How does a classical system respond to a quantum system?

Semiclassical: Mirror responds to expectation-value of the Quantum force.
But we have seen that to linear order, the quantum force is proportional to the Operators B and its conjugate.
Their expectation value is 0. Ie, under this theory, if the mirror were classic, there would be no radiation pressure noise.
Already disproven 20 years ago. It has never been a serious contender for classicality.

Stochastic Gravity.-- Take into account statistical nature of QM

Jonathan Oppenheim has been exploring classical/quantum interaction (Gravity classical, matter quantum).

Quantum system exerts forces which are classical but stochastic

Quantum effect is, when measuring the response of a classical system, like a stochastic noise.
Response of mirror would be

$$\frac{1}{\nu^2}(B_\nu + B^\dagger_{-\nu}) \to (b_\nu + b^*_{-\nu})$$

b_ν has the same prob distribution as B_ν

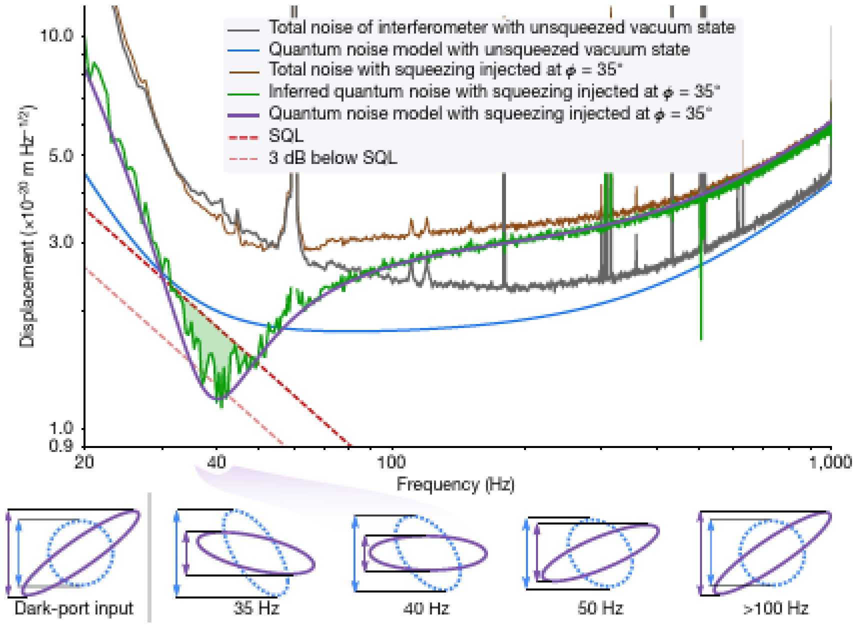
(zero mean, Gaussian distr, with same variance as the quantum operator)

However a quantum operator and a classical one cannot interfer (cannot cancel) $B_\nu + b_\nu$ has same distribution as

$B_\nu - b_\nu$

while single mode squeezing can reduce the single phase uncertainty of a quantum operator, so one can use single mode squeezing to reduce the shot noise or the rad pressure noise, it cannot reduce the sum of them.

LIGO is Quantum. 40kg mirrors behave as quantum systems.



2021 Squeezing of input state (Ie, Feeding in noise --2mode squeezed state – into the “dark port” of interferometer reduces the noise in the output of the interferometer, as I predicted in 1979-83)

This has now been implimented in both Hanford and Livingston and is part of current runs. About 3dB (2) improvement to noise.

The current LIGO depends crucially on the mirrors –40Kg of silicon di-oxide-- being quantum objects.

Quantum mechanics is the physics of the small.
Well 40Kg is not small.
However the motion of that mirror IS small.

$$\Delta Z \approx 10^{-16}\mathrm{m}$$

Is that which makes it quantum?

Room for creating a theory which is quantum for small dist., but classical for large is shrinking.

Note that if your theory acts in the same way as a quantum theory for short times, Maybe it could still pass the ligo test. But it is hard.

Ligo IS Quantum!