A Background Independent Algebra in Quantum Gravity

Edward Witten

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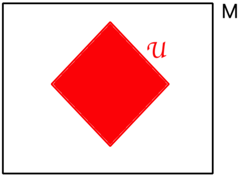

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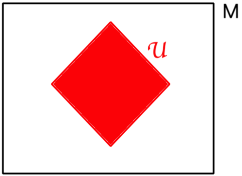

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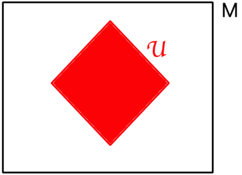

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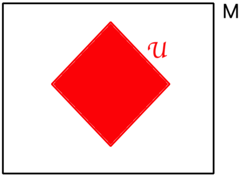

There are problems with this in the presence of gravity: With spacetime fluctuating, it is hard to explain what we mean by the region $\mathcal{U}$? but anyway why do we want to define an algebra unless it is the algebra of observables available to someone who lives in the spacetime?

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The algebra will depend on the laws of nature, but it is required to be *universal* and *background independent*, meaning that is is defined once and for all without any knowledge of the specific spacetime in which the observer is living.

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Penington and EW (2023), Kolchmeyer (2023)

In a general diamond-like region

Jensen, Sorce, and Speranza (2023)

Some recent developments

C. H. Chen and Penington (2024)

Kudler-Flam, Leutheusser, and Satishchandran (2024).

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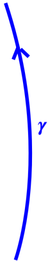

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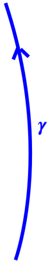

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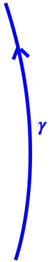

The worldline is parametrized by proper time τ. As in classic work of Unruh (1976), the observer measures along γ, for example, a scalar field ϕ, or the electromagnetic field $F_{\mu\nu}$, or the Riemann tensor $R_{\mu\nu\alpha\beta}$, as well as their covariant derivatives in normal directions.

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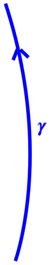

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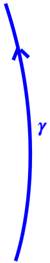

So $\phi(\tau)$ isn't by itself a meaningful observable: we need to introduce the observer's degrees of freedom and define τ relative to the observer's clock.

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where $H_{\rm bulk}$ is (any) gravitational constraint operator that generates a shift of τ along the worldline. An operator that commutes with $\widehat{H}$ is invariant under a spacetime diffeomorphism that moves the observer worldline forward in time, together with a time translation of the observer's system.

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$$[H_{\mathrm{bulk}}, \phi(\tau)] = -\mathrm{i}\dot{\phi}(\tau),$$

we need

$$[q, \phi(\tau)] = \mathrm{i}\dot{\phi}(\tau),$$

which we can achieve by just setting

$$\tau = p$$

or more generally

$$\tau = p + s$$

for a constant s.

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In addition to these operators (with ϕ possibly replaced by any local field along the worldline such as the electromagnetic field or the Riemann tensor) there is one more obvious operator that commutes with $\widehat{H}$, namely q itself. So we define an algebra $\mathcal{A}_{\rm obs}$ that is generated by the $\widehat{\phi}_s$ as well as q.

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To get background independence, we have to think of $\mathcal{A}_{\mathrm{obs}}$ as an operator product algebra, rather than an algebra of Hilbert space operators. The algebras for different M's are inequivalent representations of the same underlying operator product algebra.

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$$\phi(\tau)\phi(\tau') \sim C(\tau - \tau' - \mathrm{i}\epsilon)^{-2\Delta} + \cdots .$$

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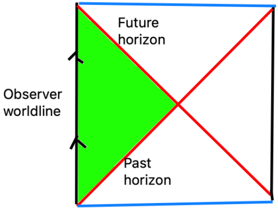

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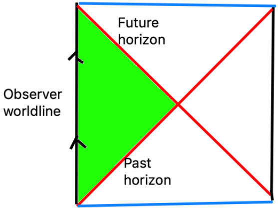

The green region is called a static patch, because it is invariant under a particular de Sitter generator H that advances the proper time of the observer.

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(1) Time translation symmetry:

$$\langle\Psi_{\mathrm{dS}}|\phi(\tau+s)\phi'(\tau'+s)|\Psi_{\mathrm{dS}}\rangle = \langle\Psi_{\mathrm{dS}}|\phi(\tau)\phi'(\tau')|\Psi_{\mathrm{dS}}\rangle.$$

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(2) The KMS condition, which says roughly:

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(A precise statement involves holomorphy of the correlation function in a strip in the complex plane.)

Including gravity and the observer, we define a special state in which the observer energy has a thermal distribution at the de Sitter temperature

$$\Psi_{\rm max} = \Psi_{\rm dS} e^{-\beta_{\rm dS} q/2} \sqrt{\beta_{\rm dS}},$$

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(1′) We still have time-translation symmetry

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(2′) The KMS condition simplifies:

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Condition (2′) tells us that if, for any $\mathbf{a} \in \mathcal{A}_{\rm obs}$, we define

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then the function Tr does have the algebraic property of a trace:

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This function has the property that $\mathrm{Tr}\,\mathbf{a}^\dagger\mathbf{a} > 0$ for all $\mathbf{a} \neq 0$, meaning in particular that it is "nondegenerate." Note that if $\Psi_{\rm max}$ is normalized then

$$\mathrm{Tr}\,1 = 1.$$

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Rather as in ordinary quantum mechanics, we expect ρ to be a positive element $\rho \in \mathcal{A}_{\rm obs}$ with $\mathrm{Tr}\,\rho = 1$. For example, let us find the density matrix of the state $\Psi_{\rm max}$.

The definition of the trace makes it clear that the density matrix of the state $\Psi_{\max}$ is $\sigma_{\max} = 1$, since to satisfy

$$\langle \Psi_{\max} | \mathbf{a} | \Psi_{\max} \rangle = \operatorname{Tr} \mathbf{a} \sigma_{\max} \equiv \langle \Psi_{\max} | \mathbf{a} \sigma_{\max} | \Psi_{\max} \rangle,$$

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we set

$$\sigma_{\mathrm{max}} = 1.$$

This means that Ψ_{max} is "maximally mixed," similar to a maximally mixed state in ordinary quantum mechanics whose density matrix is a multiple of the identity.

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If we want *all* states to have density matrices, we need to take a useful further step.

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In ordinary quantum mechanics, a maximally mixed state has a density matrix that is a multiple of the identity, and it has the maximum possible von Neumann entropy. The analog here is Ψ_{max}, with density matrix $\sigma_{\mathrm{max}} = 1$. It is clear that

$$S(\sigma_{\mathrm{max}}) = -\mathrm{Tr}\,1 \log 1 = 0,$$

and by imitating an argument that in ordinary quantum mechanics proves that a maximally mixed state has maximum possible entropy, one can prove that every other density matrix $\rho \neq 1$ has strictly smaller entropy:

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$$f'(0) = 0, \quad f''(t) < 0 \;\; \text{for} \;\; 0 \leq t \leq 1.$$

The first statement is almost immediate, and to prove the second, one uses $\log M = \int_0^\infty \mathrm{d}s \left(\frac{1}{s} - \frac{1}{s+M}\right)$, which leads to

$$f''(t) = -\int_0^\infty \mathrm{d}s\, \mathrm{Tr}\, \frac{1}{s+\rho_t}(1-\rho)\frac{1}{s+\rho_t}(1-\rho) = -\int_0^\infty \mathrm{d}s\, \mathrm{Tr}\, B^2 < 0,$$

where B is the self-adjoint operator $B = \left(\frac{1}{s+\rho_t}\right)^{1/2}(1-\rho)\left(\frac{1}{s+\rho_t}\right)^{1/2}$. Since $f'(0) = 0$, $f''(t) < 0$, we get $f(1) < 0$ so

$$S(\rho) < 0.$$

Thus, the system consisting of an observer in a static patch in de Sitter space has a state of maximum entropy

$$\Psi_{\mathrm{max}} = \Psi_{\mathrm{dS}} e^{-\beta_{\mathrm{dS}} q/2} \sqrt{\beta_{\mathrm{dS}}},$$

consisting of empty de Sitter space with a thermal distribution of the observer energy.

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consisting of empty de Sitter space with a thermal distribution of the observer energy. Why did this happen?

The original argument that empty de Sitter space has maximum entropy is due to Bousso (2000), who argued that this must be true, based on the Second Law of Thermodynamics, because the static patch is empty in the far future:

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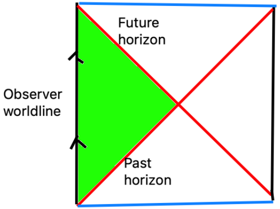

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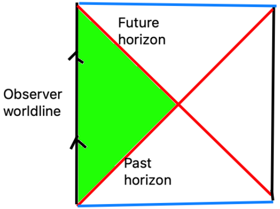

In the present context, we've defined the static patch by the presence of the observer, so by definition the observer doesn't leave the static patch even in the far future. But we can expect that in the far future the static patch will be empty except for the presence of the observer, and that the observer will be in thermal equilibrium with the bulk quantum fields, and that is what we see in the state $\Psi_{\max}$.

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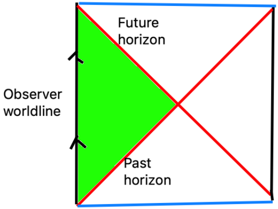

In the present context, we've defined the static patch by the presence of the observer, so by definition the observer doesn't leave the static patch even in the far future. But we can expect that in the far future the static patch will be empty except for the presence of the observer, and that the observer will be in thermal equilibrium with the bulk quantum fields, and that is what we see in the state $\Psi_{\rm max}$. So the maximum entropy state that we found is the one suggested by Bousso's argument.

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$$S(\rho) = -S(\rho|\sigma).$$

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* Though the definitions make sense regardless, I want to remark that the presentation that I've given seems most natural for an observer who because of black hole or cosmological horizons cannot see the whole universe.