Black Hole Binary Dynamics and Radiation from Classical and Quantum Gravitational Scattering

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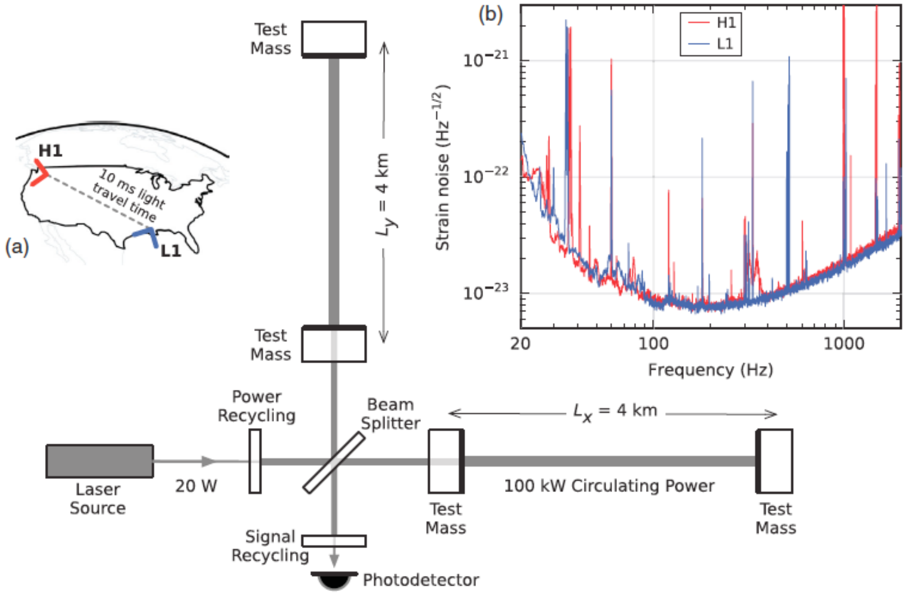
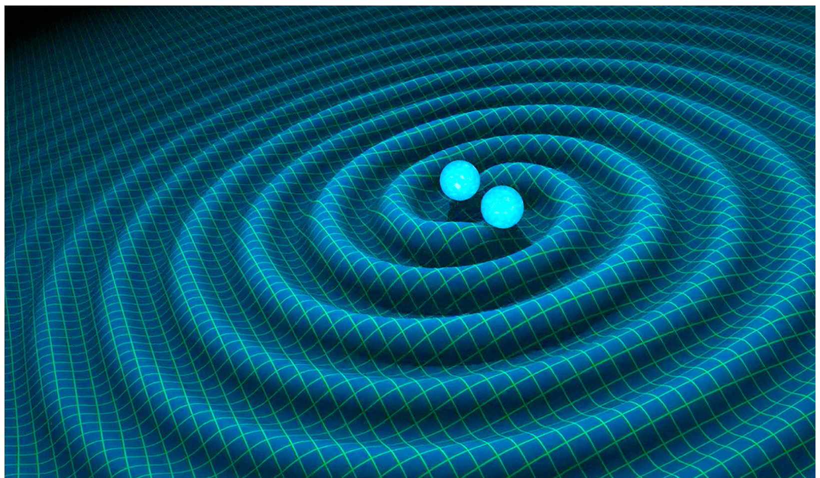
STARTING FROM 14 SEPT 2015: GRAVITATIONAL WAVE (GW) DETECTIONS BY TWO LIGO (+ VIRGO+KAGRA+...) GW DETECTORS

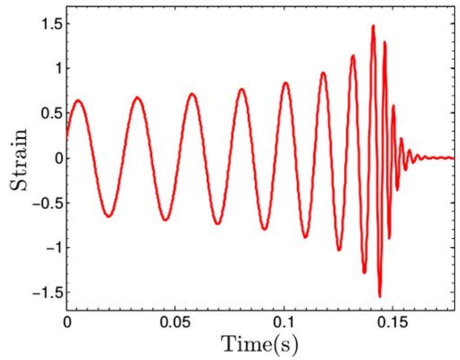


LIGO Hanford

LIGO Livingston

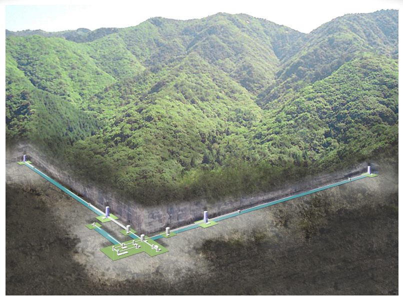
Virgo (IT)





waveform h(t)

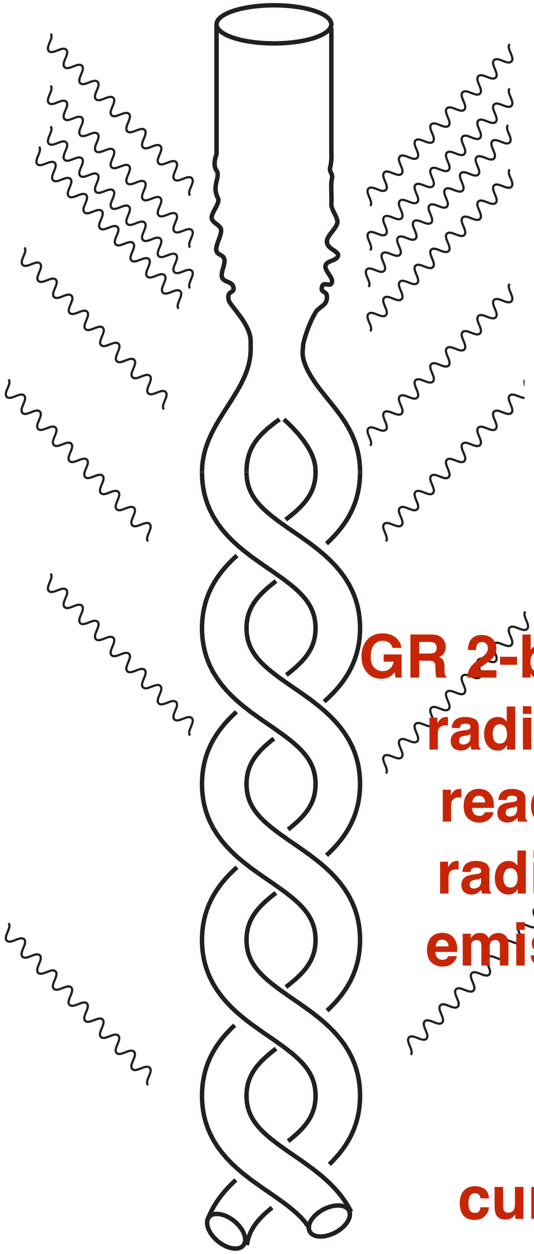
KAGRA



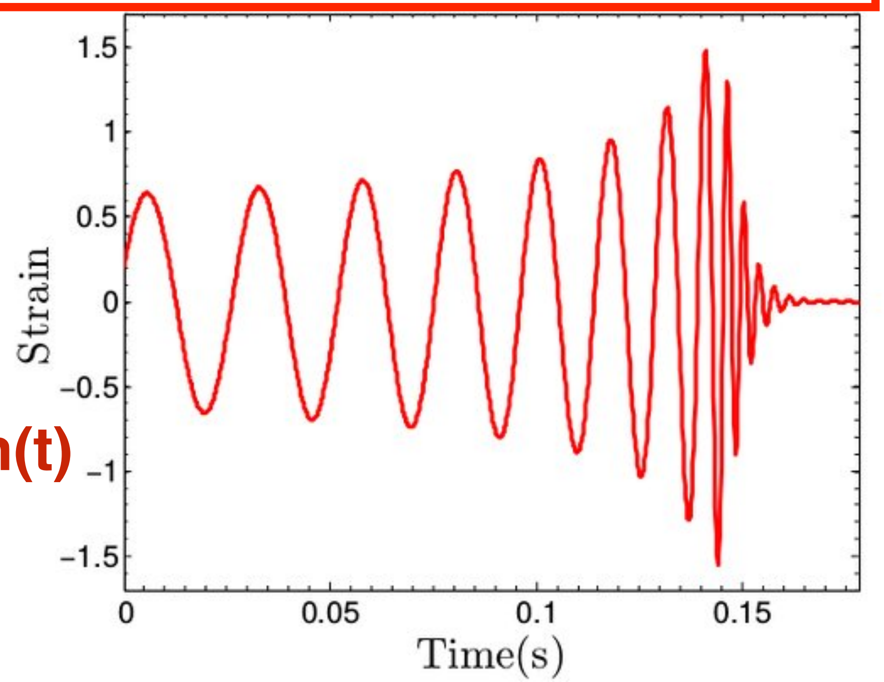
$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad\qquad R_{\mu\nu} = 0$$

$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$

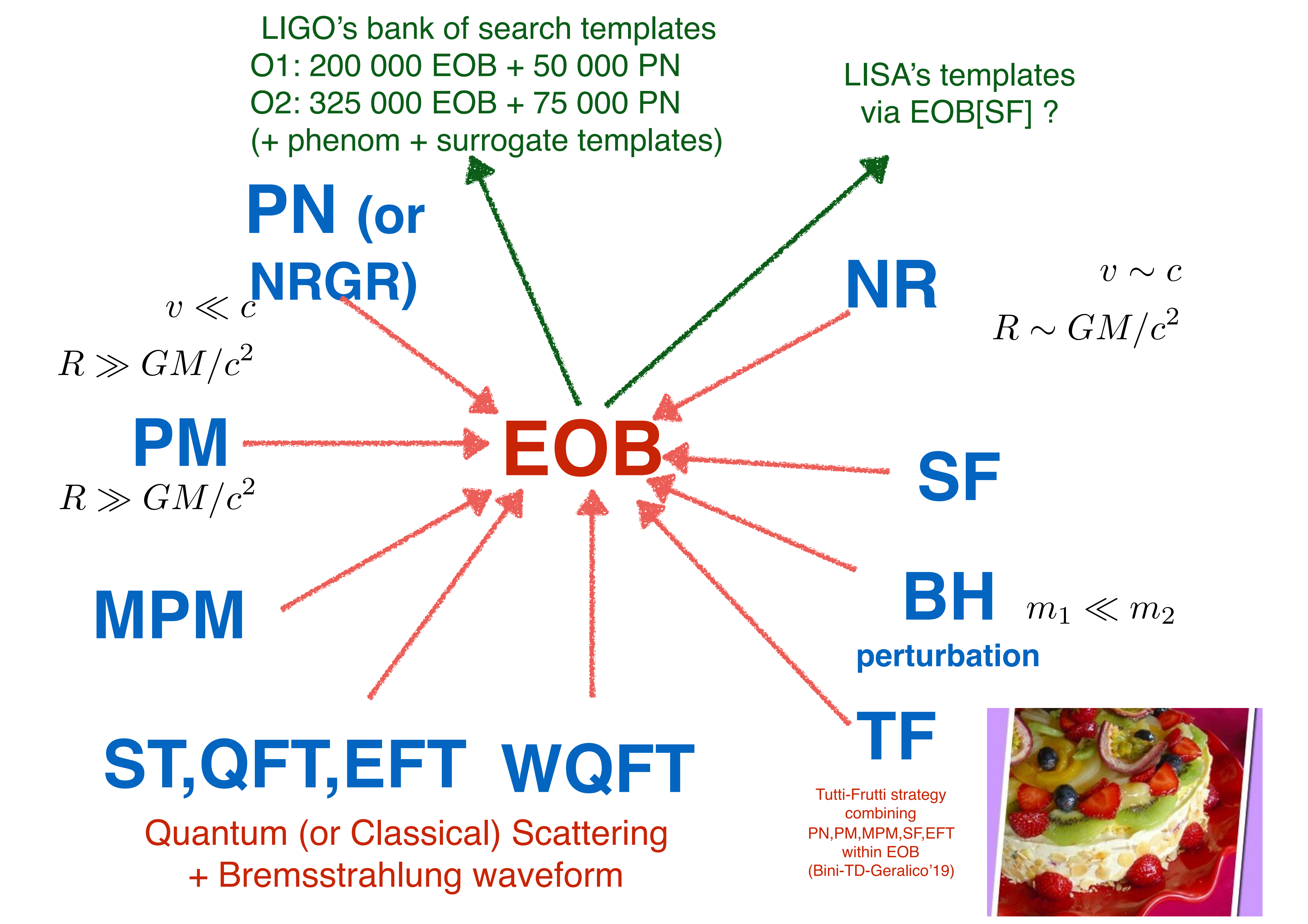
$$-g^{\mu\nu} g_{\alpha\beta,\,\mu\nu} + g^{\mu\nu} g^{\rho\sigma} \big(g_{\alpha\mu,\,\rho}\, g_{\beta\nu,\,\sigma} - g_{\alpha\mu,\,\rho}\, g_{\beta\sigma,\,\nu} + g_{\alpha\mu,\,\rho}\, g_{\nu\sigma,\,\beta} + g_{\beta\mu,\,\rho}\, g_{\nu\sigma,\,\alpha} - \frac{1}{2} g_{\mu\rho,\,\alpha}\, g_{\nu\sigma,\,\beta}\big) = 0$$



GR 2-body pb, radiation-reaction, radiation emission. → **waveform h(t)**



needed with ever-decreasing unfaithfulness: currently 10^-2 (0.1 rad); future 10^-4 (0.01 rad) or more



LIGO's bank of search templates
O1: 200 000 EOB + 50 000 PN
O2: 325 000 EOB + 75 000 PN
(+ phenom + surrogate templates)
LISA's templates via EOB[SF] ?
PN (or NRGR)
$v \ll c$
$R \gg GM/c^2$
NR
$v \sim c$
$R \sim GM/c^2$
PM
$R \gg GM/c^2$
EOB
SF
MPM
BH perturbation
$m_1 \ll m_2$
TF
Tutti-Frutti strategy combining PN,PM,MPM,SF,EFT within EOB (Bini-TD-Geralico'19)
ST,QFT,EFT
WQFT
Quantum (or Classical) Scattering + Bremsstrahlung waveform

Perturbative (PN) computation of 2-body Hamiltonian

$$\hat{H}^{\rm cm}_{\leq 4PN} = \hat{H}_N + \hat{H}_{1PN} + \hat{H}_{2PN} + \hat{H}_{3PN} + \hat{H}_{4PN}$$

DJS'14

$$\hat{H}_{\rm N}(\mathbf{r},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{r},$$

$$c^2\hat{H}_{\rm 1PN}(\mathbf{r},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\{(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\}\frac{1}{r} + \frac{1}{2r^2},$$

$$c^4\hat{H}_{\rm 2PN}(\mathbf{r},\mathbf{p}) = \frac{1}{16}(1-5\nu+5\nu^2)(\mathbf{p}^2)^3 + \frac{1}{8}\{(5-20\nu-3\nu^2)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\}\frac{1}{r} + \frac{1}{2}\{(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\}\frac{1}{r^2} - \frac{1}{4}(1+3\nu)\frac{1}{r^3},$$

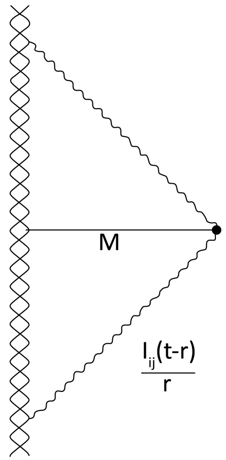
$$\begin{aligned}
c^6\hat{H}_{\rm 3PN}(\mathbf{r},\mathbf{p}) ={}& \frac{1}{128}(-5+35\nu-70\nu^2+35\nu^3)(\mathbf{p}^2)^4 + \frac{1}{16}\{(-7+42\nu-53\nu^2-5\nu^3)(\mathbf{p}^2)^3 \\
&+ (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\}\frac{1}{r} \\
&+ \left\{\frac{1}{16}(-27+136\nu+109\nu^2)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\right\}\frac{1}{r^2} \\
&+ \left\{\left(-\frac{25}{8} + \left(\frac{\pi^2}{64} - \frac{335}{48}\right)\nu - \frac{23\nu^2}{8}\right)\mathbf{p}^2 + \left(-\frac{85}{16} - \frac{3\pi^2}{64} - \frac{7\nu}{4}\right)\nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{r^3} \\
&+ \left\{\frac{1}{8} + \left(\frac{109}{12} - \frac{21}{32}\pi^2\right)\nu\right\}\frac{1}{r^4}.
\end{aligned}$$

$$H_{4\mathrm{PN}}[\mathbf{r},\mathbf{p}] = H^{\mathrm{loc}}_{4\mathrm{PN}}(\mathbf{r},\mathbf{p}) + H^{\mathrm{nonloc}}_{4\mathrm{PN}},$$

nonlocal

$$H^{\mathrm{nonloc}}_{4\mathrm{PN}}(t) = -\frac{1}{5}\frac{G^2 M}{c^8} I^{(3)}_{ij}(t) \times \mathrm{Pf}_{2r_{12}/c} \int_{-\infty}^{+\infty} \frac{dv}{|v|} I^{(3)}_{ij}(t+v),$$

time-nonlocality due to tails



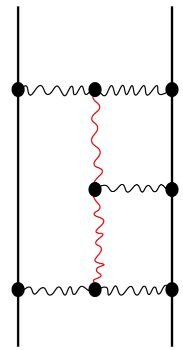
order G^4/c^8

local part

$$\begin{aligned}
c^8\frac{H^{\mathrm{loc}}_{4\mathrm{PN}}(\mathbf{r},\mathbf{p})}{\mu} &= \left(\frac{7}{256}-\frac{63}{256}\nu+\frac{189}{256}\nu^2-\frac{105}{128}\nu^3+\frac{63}{256}\nu^4\right)(\mathbf{p}^2)^5 \\
&+\Bigg\{\frac{45}{128}(\mathbf{p}^2)^4-\frac{45}{16}(\mathbf{p}^2)^4\nu+\left(\frac{423}{64}(\mathbf{p}^2)^4-\frac{3}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3-\frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2\right)\nu^2 \\
&+\left(-\frac{1013}{256}(\mathbf{p}^2)^4+\frac{23}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3+\frac{69}{128}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2-\frac{5}{64}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2+\frac{35}{256}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^3 \\
&+\left(-\frac{35}{128}(\mathbf{p}^2)^4-\frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3-\frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2-\frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2-\frac{35}{128}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^4\Bigg\}\frac{1}{r} \\
&+\Bigg\{\frac{13}{8}(\mathbf{p}^2)^3+\left(-\frac{791}{64}(\mathbf{p}^2)^3+\frac{49}{16}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2-\frac{889}{192}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2+\frac{369}{160}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu \\
&+\left(\frac{4857}{256}(\mathbf{p}^2)^3-\frac{545}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2+\frac{9475}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2-\frac{1151}{128}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^2 \\
&+\left(\frac{2335}{256}(\mathbf{p}^2)^3+\frac{1135}{256}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2-\frac{1649}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2+\frac{10353}{1280}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^3\Bigg\}\frac{1}{r^2} \\
&+\Bigg\{\frac{105}{32}(\mathbf{p}^2)^2+\left(\left(\frac{2749\pi^2}{8192}-\frac{589189}{19200}\right)(\mathbf{p}^2)^2+\left(\frac{63347}{1600}-\frac{1059\pi^2}{1024}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 \right.\\
&\left.+\left(\frac{375\pi^2}{8192}-\frac{23533}{1280}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu+\left(\left(\frac{18491\pi^2}{16384}-\frac{1189789}{28800}\right)(\mathbf{p}^2)^2 \right.\\
&\left.+\left(-\frac{127}{3}-\frac{4035\pi^2}{2048}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2+\left(\frac{57563}{1920}-\frac{38655\pi^2}{16384}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^2 \\
&+\left(-\frac{553}{128}(\mathbf{p}^2)^2-\frac{225}{64}(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2-\frac{381}{128}(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^3\Bigg\}\frac{1}{r^3} \\
&+\Bigg\{\frac{105}{32}\mathbf{p}^2+\left(\left(\frac{185761}{19200}-\frac{21837\pi^2}{8192}\right)\mathbf{p}^2+\left(\frac{3401779}{57600}-\frac{28691\pi^2}{24576}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu \\
&+\left(\left(\frac{672811}{19200}-\frac{158177\pi^2}{49152}\right)\mathbf{p}^2+\left(\frac{110099\pi^2}{49152}-\frac{21827}{3840}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu^2\Bigg\}\frac{1}{r^4} \\
&+\Bigg\{-\frac{1}{16}+\left(\frac{6237\pi^2}{1024}-\frac{169199}{2400}\right)\nu+\left(\frac{7403\pi^2}{3072}-\frac{1256}{45}\right)\nu^2\Bigg\}\frac{1}{r^5}.
\end{aligned} \tag{5.13}$$

recent progress in computing H^loc at G^4

(Dlapa+'24, Bini-TD'24)



Perturbative computation of GW flux from binary system

- lowest order : Einstein 1918 Peters-Mathews 63
- 1 + (v^2/c^2) : Wagoner-Will 76
- … + (v^3/c^3) : Blanchet-Damour 92, Wiseman 93
- … + (v^4/c^4) : Blanchet-Damour-Iyer Will-Wiseman 95
- … + (v^5/c^5) : Blanchet 96
- … + (v^6/c^6) : Blanchet-Damour-Esposito-Farèse-Iyer 2004
- … + (v^7/c^7) : Blanchet
- … + (v^8/c^8) + (v^9/c^9) : Blanchet et al 2023

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$x = \left(\frac{v}{c}\right)^2 = \left(\frac{G(m_1+m_2)\Omega}{c^3}\right)^{\frac{2}{3}} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{2}{3}}$$

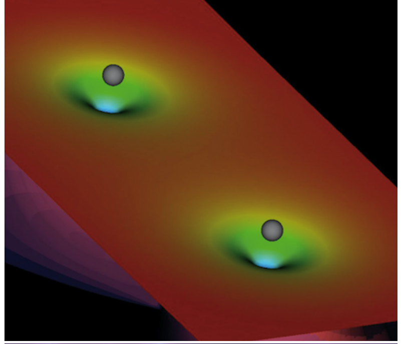
LO quadrupole radiation

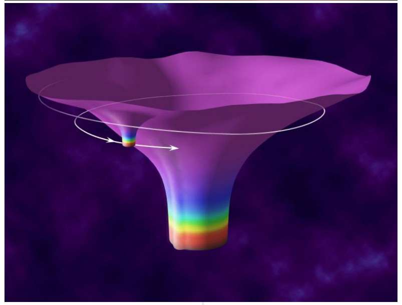
$$\begin{aligned}
\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \Bigg\{ & 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} \\
& + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_\mathrm{E} - \frac{856}{105}\ln(16\,x) + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right]x^3 \\
& + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} \\
& + \left[-\frac{323105549467}{3178375200} + \frac{232597}{4410}\gamma_\mathrm{E} - \frac{1369}{126}\pi^2 + \frac{39931}{294}\ln 2 - \frac{47385}{1568}\ln 3 + \frac{232597}{8820}\ln x \right. \\
& \quad + \left(-\frac{1452202403629}{1466942400} + \frac{41478}{245}\gamma_\mathrm{E} - \frac{267127}{4608}\pi^2 + \frac{479062}{2205}\ln 2 + \frac{47385}{392}\ln 3 + \frac{20739}{245}\ln x\right)\nu \\
& \quad \left. + \left(\frac{1607125}{6804} - \frac{3157}{384}\pi^2\right)\nu^2 + \frac{6875}{504}\nu^3 + \frac{5}{6}\nu^4\right]x^4 \\
& + \left[\frac{265978667519}{745113600} - \frac{6848}{105}\gamma_\mathrm{E} - \frac{3424}{105}\ln(16\,x) + \left(\frac{2062241}{22176} + \frac{41}{12}\pi^2\right)\nu \right. \\
& \quad \left. - \frac{133112905}{290304}\nu^2 - \frac{3719141}{38016}\nu^3\right]\pi x^{9/2} + \mathcal{O}(x^5)\Bigg\}. \qquad (4)
\end{aligned}$$

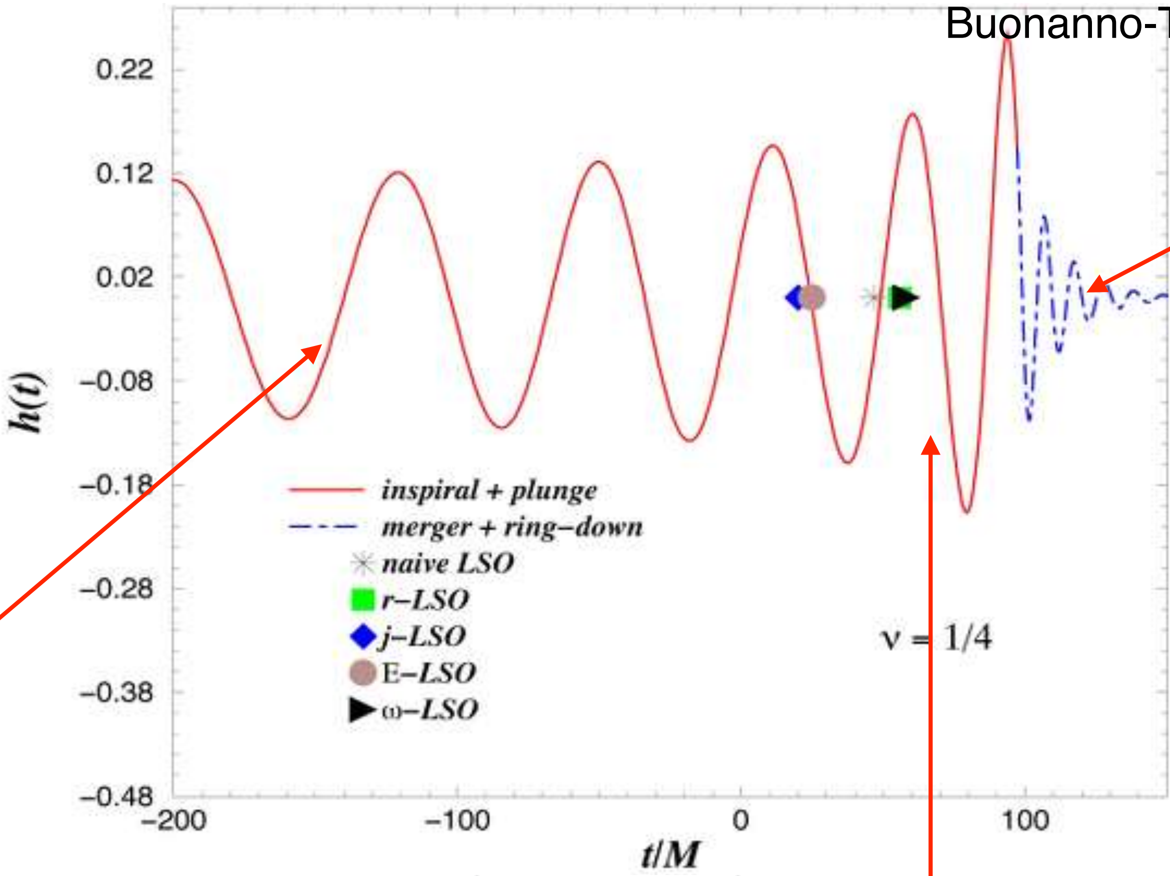
4PN

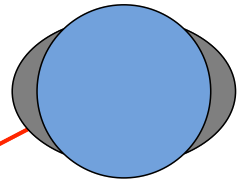
4.5PN

The Effective One-Body (EOB) approach to the GW signal emitted by the Merger of two Black Holes



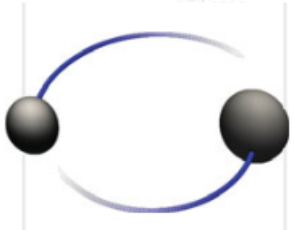






Ringdown (BBH):
« vibration modes » of final BH (QNM); perturbation of BHs à la Regge-Wheeler-Zerilli-Teukolsky +Vishveshwara

Inspiral:
perturbative computation of higher-order contributions to E=H and F (expansion in v^2/c^2 + tidal polarizability of NS)



Late inspiral, « plunge » and merger:
first estimated by the Effective One-Body method (AB-TD 2000) later confirmed and improved by using numerical simulations (Pretorius…2005)

From EOB vs NR to EOB-NR waveforms

Buonanno-Cook-Pretorius 2007

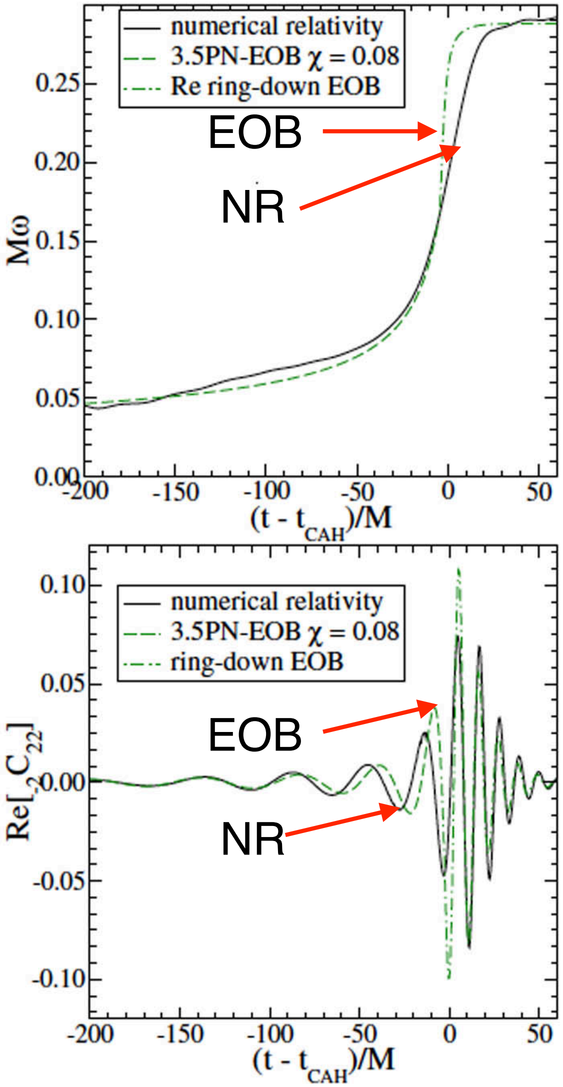
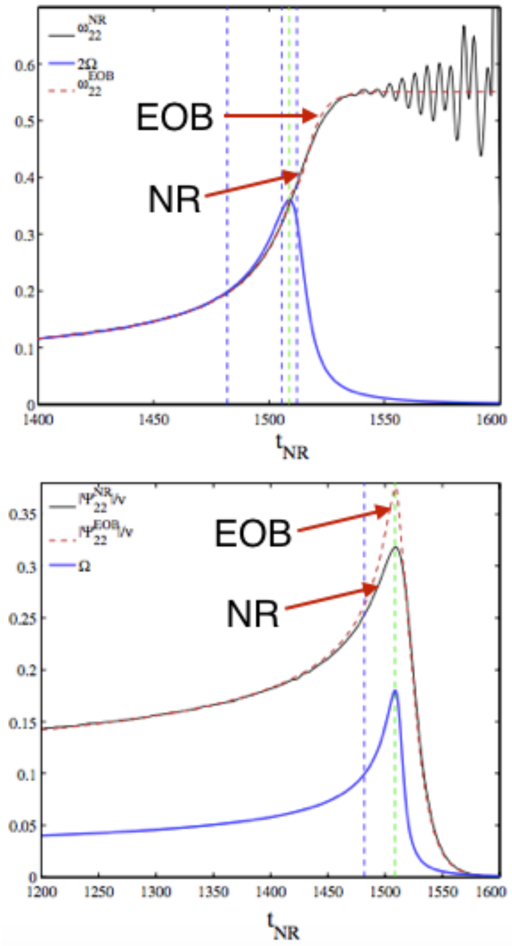
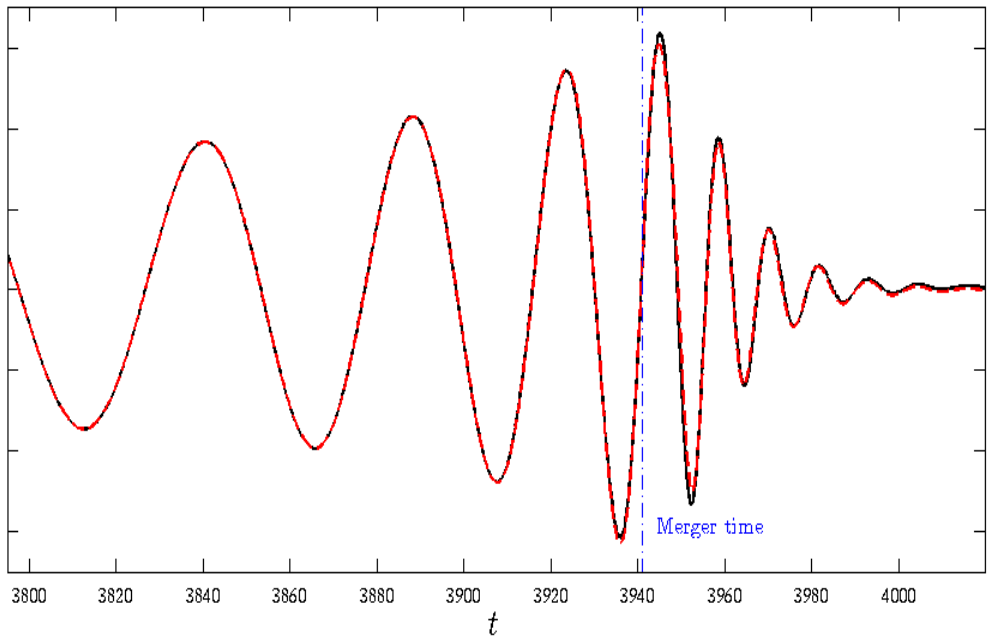


FIG. 21 (color online). We compare the NR and EOB frequency and Re[$_{-2}C_{22}$] waveforms throughout the entire inspiral–merger–ring-down evolution. The data refers to the $d = 16$ run.

TD-Nagar-Dorband-Pollney-Rezzolla 2008



EOB-NR vs NR



EOB-NR is obtained by tuning some yet unknown theoretical EOB parameter to a sample of NR simulations

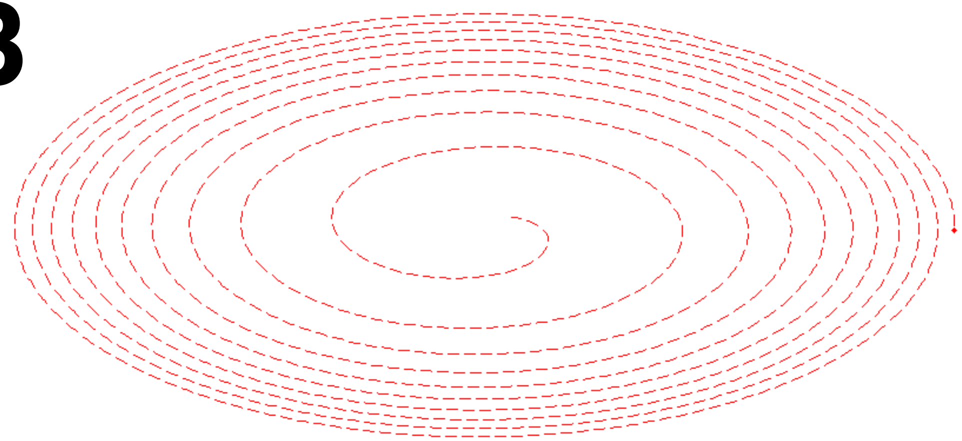
$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$

$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$

$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$

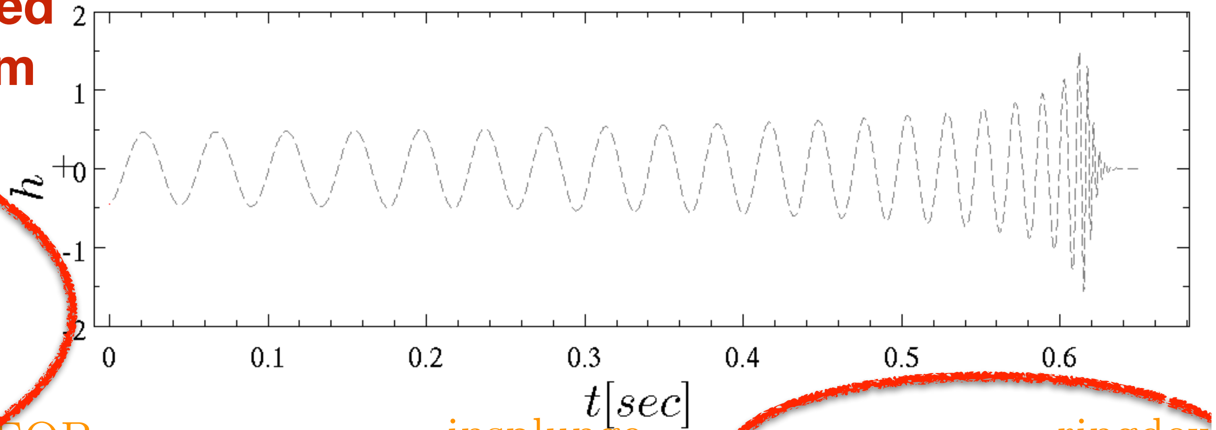
EOB



Hamiltonian: conservative dynamics

Rad Reac Force

Resummed waveform



$$h_{\ell m} \equiv h^{(N,\epsilon)}_{\ell m} \hat{h}^{(\epsilon)}_{\ell m} \hat{h}^{\rm NQC}_{\ell m}$$

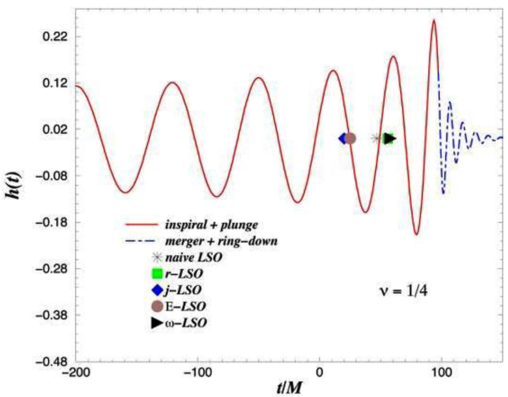
$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{\rm eff} T_{\ell m} e^{i\delta_{\ell m}} \rho^{\ell}_{\ell m}$$

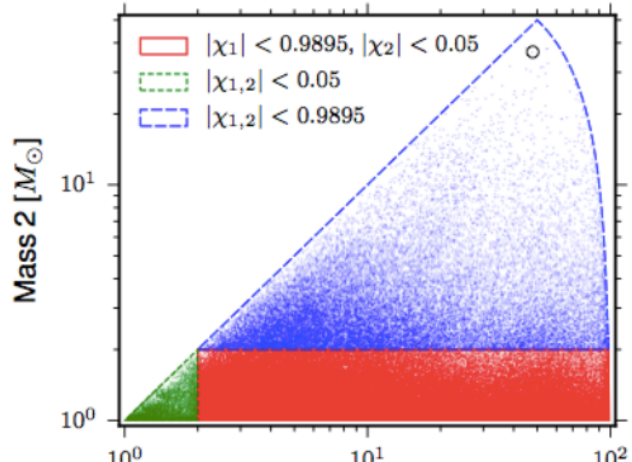
$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega} \sum_{\ell=2}^{\ell_{\max}} \sum_{m=1}^{\ell} (m\Omega)^2 |R h^{(\epsilon)}_{\ell m}|^2$$

$$h^{\rm EOB}_{\ell m} = \theta(t_m - t) h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m) h^{\rm ringdown}_{\ell m}$$

$$h^{\rm ringdown}_{\ell m}(t) = \sum_N C^+_N e^{-\sigma^+_N (t - t_m)}$$

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell + 1)} e^{\pi \hat{\hat{k}}} e^{2i\hat{\hat{k}} \log(2kr_0)},$$



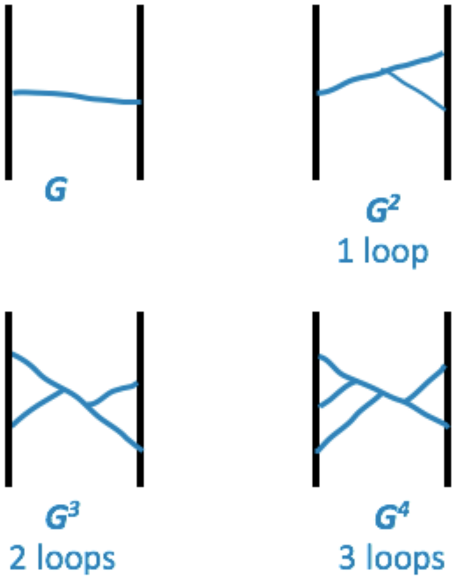


Complete waveforms for BBH coalescenceS:

Effective One-Body (EOB) approach: H + Rad-Reac Force

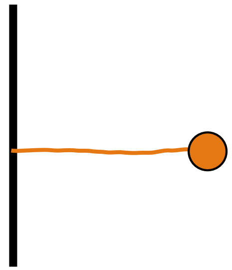
Historically rooted in QM: Brezin-Itzykson-ZinnJustin'70
eikonal scattering amplitude+ Wheeler's:`Think quantum mechanically'

Real 2-body system
(in the c.o.m. frame)



An effective particle of mass mu in some effective metric

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$



$$g_{\rm eff}^{\mu\nu}(X)$$

1:1 map

mass-shell constraint

$$0 = g_{\rm eff}^{\mu\nu}(X) P_\mu P_\nu + \mu^2 + Q(X,P)$$

Level correspondence
in the semi-classical limit:
Bohr-Sommerfeld ->
identification of
quantized action variables

$$J = \ell\hbar = \frac{1}{2\pi}\oint p_\varphi d\varphi$$

$$N = n\hbar = I_r + J$$

$$I_r = \frac{1}{2\pi}\oint p_r dr$$

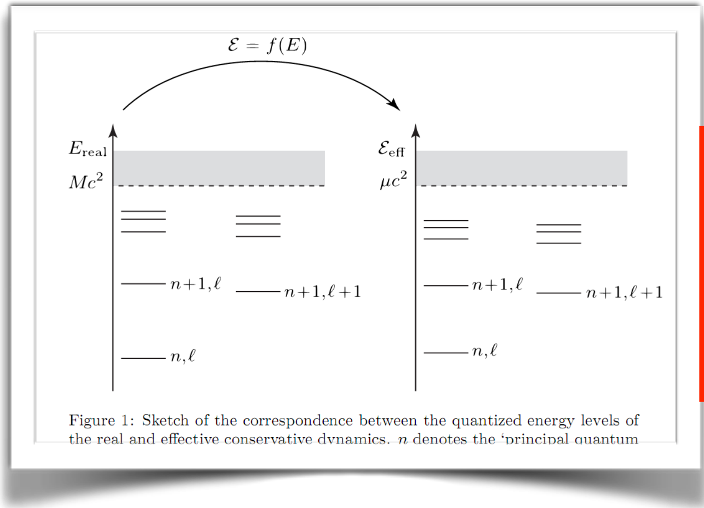


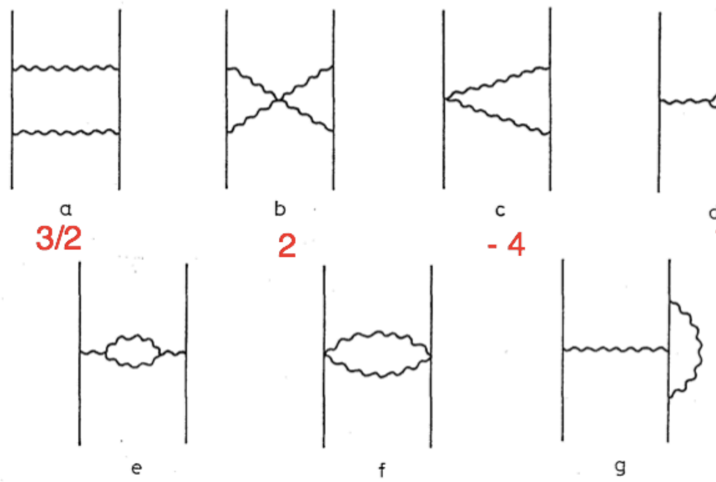
Figure 1: Sketch of the correspondence between the quantized energy levels of the real and effective conservative dynamics. n denotes the 'principal quantum

Crucial energy map

$$\mathcal{E}_{\rm eff} = \frac{(\mathcal{E}_{\rm real})^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}$$

as functions of I_r and I_phi=J

Quantum Scattering Amplitudes and 2-body Dynamics



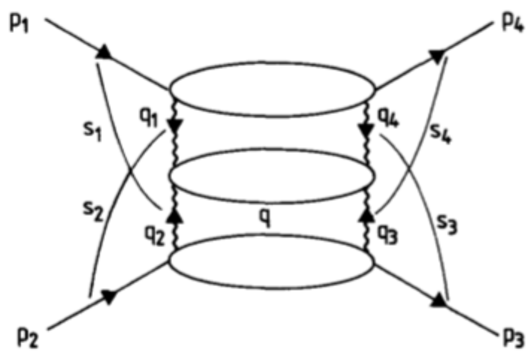


Fig. 3. The "H" diagram that provides the leading correction to the eikonal.

• Quantum Scattering Amplitudes —> Potential
one-graviton exchange :
Corinaldesi '56 '71,
Barker-Gupta-Haracz 66,
Barker-O'Connell 70, Hiida-Okamura72

Nonlinear: Iwasaki 71 [1PN],
Okamura-Ohta-Kimura-Hiida 73[2 PN]

Using modern amplitude techniques: Bjerrum-Bohr+..2003-

Amati-Ciafaloni-Veneziano 1987-2008

Ultra-High-Energy (s >> M_Planck^2)
Four-graviton Scattering at 2 loops

Eikonal phase \delta in D=4
with one- and two-loop corrections using the Regge-Gribov approach

confirmed by DiVecchia+'19

$$\delta = \frac{Gs}{\hbar}\left(\log\left(\frac{L_{IR}}{b}\right) + \frac{6\ell_s^2}{\pi b^2} + \frac{2G^2 s}{b^2}\left(1 + \frac{2i}{\pi}\log(\cdots)\right)\right)$$

Having so computed $\mathcal{E}$ and J one might then, for instance, compare the EOB prediction for the scattering angle $\theta(\mathcal{E}, J)$ (which follows from the EOB Hamiltonian) with GSF computations of θ for a sample of values of $\mathcal{E}$ and J. We see that, in principle, we have access here to one function of *two* real variables, which is ample information for determining the functions entering the EOB formalism.

Personally becoming aware of the ACV results in Parma 2008, plus discussions at IHES with Donoghue and Vanhove —> GSF and EOB (TD 2010): scattering and zero-binding zoom-whirl orbit (Barack et al'19)

Reviving the PM Two-Body Dynamics

(pioneered by Bertotti'56, Havas-Goldberg'62, Rosenblum'78, Westpfahl'79, Portilla'80, Bel et al.81)

using Classical and/or Quantum Two-Body Scattering

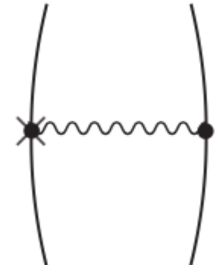
TD 2016, 2017:

Gravitational scattering, post-Minkowskian approximation, and effective-one-bodv theorv

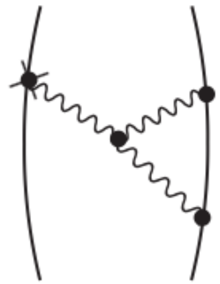
High-energy gravitational scattering and the general relativistic two-body problem

A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [Phys. Rev. D **94**, 104015 (2016)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the two-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian.

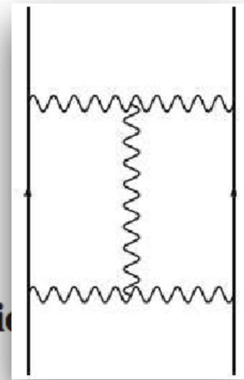
tree-level
G^1



one-loop
G^2



two-loop
G^3+G^4



Cheung-Rothstein-Solon 2018

From Scattering Amplitudes to Classical Potentials in the Post-Minkowskian Expansi

We combine tools from effective field theory and generalized unitarity to construct a map between on-shell scattering amplitudes and the classical potential for interacting spinless particles. For general relativity, we obtain analytic expressions for the classical potential of a binary black hole system at second order in the gravitational constant and all orders in velocity. Our results exactly match all known results up to fourth post-Newtonian order, and offer a simple check of future higher order calculations. By design, these methods should extend to higher orders in perturbation theory.

one-loop
G^2

Simple Map: Scattering angle <-> EOB dynamics

TD'16-18
Bini-TD-Geralico'20

$$\frac{1}{2}\chi = \Phi(E_{\rm real}, J; m_1, m_2, G)$$

$$0 = g_{\rm eff}^{\mu\nu} P_\mu P_\nu + \mu^2 + Q$$

$$\mathcal{E}_{\rm eff} = \frac{(\mathcal{E}_{\rm real})^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}$$

$$\frac{\mathcal{E}_{\rm eff}}{\mu} = \gamma = -\frac{p_1 \cdot p_2}{m_1 m_2}$$

$$\frac{1}{j} = \frac{G m_1 m_2}{J}$$

$$\chi(\gamma, j) = 2\frac{\chi_1(\gamma)}{j} + 2\frac{\chi_2(\gamma)}{j^2} + 2\frac{\chi_3(\gamma)}{j^3} + 2\frac{\chi_4(\gamma)}{j^4} + O\left[\frac{1}{j^5}\right]$$

$g_{\rm eff}^{\mu\nu}$ **= Schwarzschild metric M=m1+m2**

$$Q = \left(\frac{GM}{R}\right)^2 q_2(E) + \left(\frac{GM}{R}\right)^3 q_3(E) + O(G^4)$$

$$q_2 = -\frac{4}{\pi}(\chi_2 - \chi_2^{\rm Schw})$$

$$q_3 = \frac{4}{\pi}\frac{2\gamma^2 - 1}{\gamma^2 - 1}(\chi_2 - \chi_2^{\rm Schw}) - \frac{\chi_3 - \chi_3^{\rm Schw}}{\gamma^2 - 1}$$

Application to the ACV eikonal scattering phase (massless or ultra-relativistic scattering)

Amati-Ciafaloni-Veneziano'90+ Ciafaloni-Colferai'14+ Bern et al'20+ DiVecchia et al'20

$$\delta^{\text{eikonal}} = \frac{1}{\hbar}(\delta^{\text{R}} + i\delta^{\text{I}}) + \text{quantum corr.}$$

$$\frac{1}{2}\chi^{\text{eikonal}} = 2\frac{\gamma}{j} + \frac{16}{3}\frac{\gamma^3}{j^3} + \cdots$$

valid in the HE limit gamma-> infty

Using the chi—> Q dictionary this corresponds to the HE limits:

$$q_2^{\text{HE}} = \frac{15}{2}\gamma^2$$
$$q_3^{\text{HE}} = \gamma^2$$

i.e. an HE limit for the EOB mass-shell condition (TD'18)

$$0 = g_{\text{eff}}^{\mu\nu}(X)P_\mu P_\nu + \mu^2 + Q(X,P)$$

$$0 = g_{\text{Schw}}^{\mu\nu}P_\mu P_\nu + \left(\frac{15}{2}\left(\frac{GM}{R}\right)^2 + \left(\frac{GM}{R}\right)^3\right)P_0^2$$

Translating quantum scattering amplitudes into classical dynamical information (1)

The domain of validity of the Born-Feynman expansion

$$\mathcal{M}(s,t) = \mathcal{M}^{(\frac{G}{\hbar})}(s,t) + \mathcal{M}^{(\frac{G^2}{\hbar^2})}(s,t) + \cdots, \qquad \mathcal{M}^{(\frac{G}{\hbar})}(s,t) = 16\pi \frac{G}{\hbar} \frac{2(p_1 \cdot p_2)^2 - p_1^2 p_2^2}{-t}.$$

is

$$\frac{Gs}{\hbar v} \sim \frac{G E_1 E_2}{\hbar v} \ll 1$$

while the domain of validity of classical scattering is (Bohr 1948)

$$\frac{Gs}{\hbar v} \sim \frac{G E_1 E_2}{\hbar v} \gg 1$$

Amati-Ciafaloni-Veneziano faced this issue by assuming **eikonalization in b space**

$$\widetilde{\mathcal{A}}(s,b) = \int \frac{d^{D-2}q}{(2\pi)^{D-2}} \frac{\mathcal{A}(s,q^2)}{4pE} e^{-ib\cdot q}$$

$$1 + i\widetilde{\mathcal{A}}(s,b) = (1 + 2i\Delta(s,b))\, e^{2i\delta(s,b)}$$

classical phase

$$i\,\frac{\mathcal{A}(s,Q^2)}{4pE} = \int d^{D-2}b \left(e^{2i\delta(s,b)} - 1\right) e^{ib\cdot Q}$$

$$2\delta(s,b) = \frac{\Delta S_r(s,J)}{\hbar}$$

subtracted radial action of potential scattering

total classical momentum transfer:

$$Q^\mu = -\frac{\partial\, \mathrm{Re}\, 2\delta(s,b)}{\partial b^\mu}$$

Other approaches to extracting classical info: Bern et al., KMOC, Porto, Plefka, Damgaard,…

Translating quantum scattering amplitudes into classical dynamical information (2)

TD'17: EOB potential Q(R,E) or W(R,E)
Cheung-Rothstein-Solon'18, Bern et al'19
different EFT potential V(R,P^2) and methods for
taking the classical limit at the integrand level,
and extracting the « classical part » of the scattering amplitude

EOB

$$Q^E(u,\mathcal{E}_{\rm eff}) = u^2 q_2(\mathcal{E}_{\rm eff}) + u^3 q_3(\mathcal{E}_{\rm eff}) + u^4 q_4^E(\mathcal{E}_{\rm eff}) + O(G^5)$$

$$w(r,p_\infty) = \frac{w_1(\gamma)}{r} + \frac{w_2(\gamma)}{r^2} + \frac{w_3(\gamma)}{r^3} + \frac{w_4(\gamma)}{r^4} + \cdots$$

EFT

$$H(\mathbf{P},\mathbf{X}) = \sqrt{m_1^2+\mathbf{P}^2} + \sqrt{m_2^2+\mathbf{P}^2} + V(R,\mathbf{P}^2)$$

$$V(R,\mathbf{P}^2) = G\frac{c_1(\mathbf{P}^2)}{R} + G^2\frac{c_2(\mathbf{P}^2)}{R^2} + G^3\frac{c_3(\mathbf{P}^2)}{R^3} + \cdot$$

non-relativistic potential scattering !

$$-\widehat{\hbar}^2\Delta_{\mathbf{x}}\psi(\mathbf{x}) = \left[p_\infty^2 + \frac{w_1}{r} + \frac{w_2}{r^2} + \frac{w_3}{r^3} + O\left(\frac{1}{r^4}\right)\right]\psi(\mathbf{x})$$

$$\mathcal{M}^{QFT}_{\rm classical} = \frac{8\pi G s}{\hbar} f^{EOB} = \mathcal{M}^{EFT}$$

issue: extracting the « classical »
piece of the amplitude

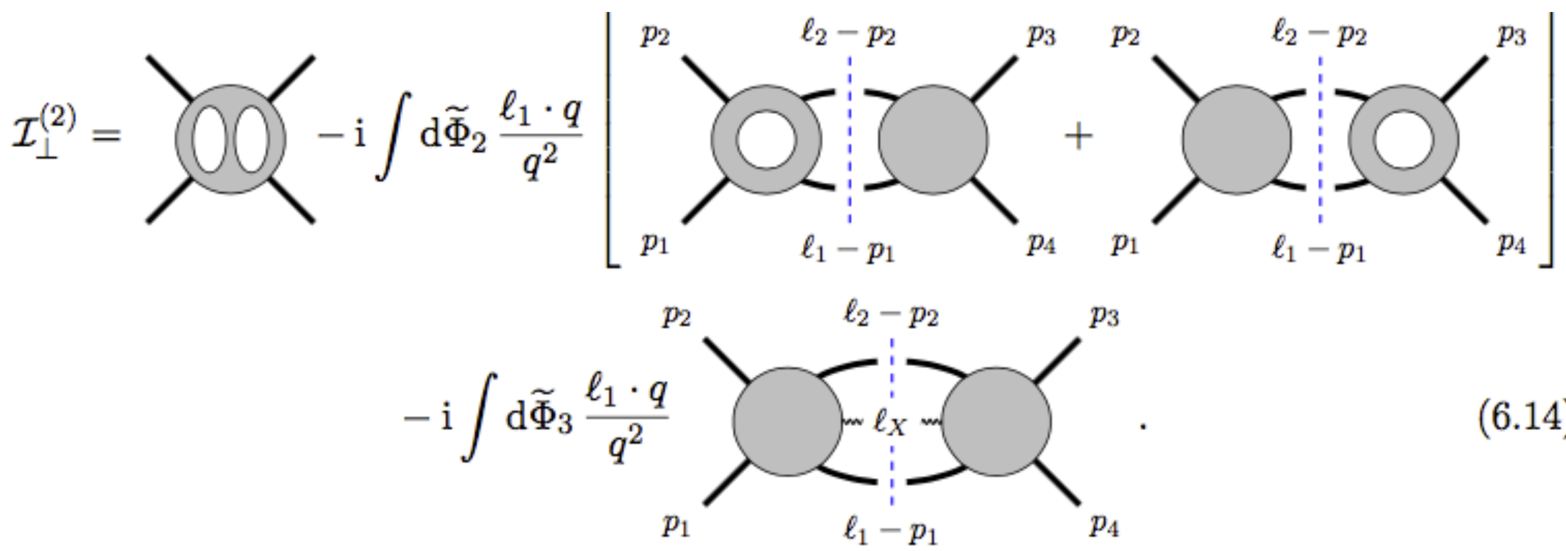
Translating quantum scattering amplitudes into classical dynamical information (3)

Kosower-Maybee-O'Connell'19 formalism **for any quasi-classical observable O**

$$\Delta O = \langle \mathrm{out}|\mathbb{O}|\mathrm{out}\rangle - \langle \mathrm{in}|\mathbb{O}|\mathrm{in}\rangle$$ with |out> = S |in> and S=1+i T

$$\Delta O = \langle \mathrm{in}|i[O,T]|\mathrm{in}\rangle + \langle \mathrm{in}|T^\dagger[O,T]|\mathrm{in}\rangle$$

Hermann-Parra-Martinez-Ruf-Zeng'21 making use of: generalized unitarity, reverse unitarity (for phase-space integrals), method of regions, integration by parts canonical differential eqs applied KMOC to O= p_1^mu and p_rad^mu



(6.14)

Scattering Amplitudes and the Conservative Hamiltonian for Binary Systems at Third Post-Minkowskian Order

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two-loop level G^3

We present the amplitude for classical scattering of gravitationally interacting massive scalars at third post-Minkowskian order. Our approach harnesses powerful tools from the modern amplitudes program such as generalized unitarity and the double-copy construction, which relates gravity integrands to simpler gauge-theory expressions. Adapting methods for integration and matching from effective field theory, we extract the conservative Hamiltonian for compact spinless binaries at third post-Minkowskian order. The resulting Hamiltonian is in complete agreement with corresponding terms in state-of-the-art expressions at fourth post-Newtonian order as well as the probe limit at all orders in velocity. We also derive the scattering angle at third post-Minkowskian order and find agreement with known results.

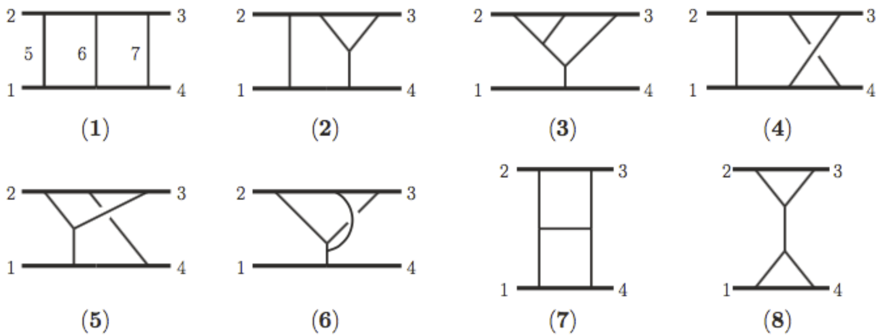
Double-copy Einstein=YM^2

rooted in StringThy

$$V^{\mu\nu} \propto \partial_z X^\mu \partial_{\bar{z}} X^\nu e^{ik.X}$$

KLT'86, BCJ'08

the eight 2-loop diagrams contributing to the O(G^3/r^3) classical potential



two-loop level

$$\mathcal{M}_3 = \frac{\pi G^3 \nu^2 m^4 \log \boldsymbol{q}^2}{6\gamma^2 \xi}\Big[3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 - \frac{48\nu(3+12\sigma^2-4\sigma^4)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} - \frac{18\nu\gamma(1-2\sigma^2)(1-5\sigma^2)}{(1+\gamma)(1+\sigma)}\Big] + \frac{8\pi^3 G^3 \nu^4 m^6}{\gamma^4 \xi}[3\gamma(1-2\sigma^2)(1-5\sigma^2)F_1 - 32 m^2\nu^2(1-2\sigma^2)^3 F_2], \qquad (8)$$

arcsinh

3PM computation (Bern-Cheung-Roiban-Shen-Solon-Zeng'19)

using a combination of techniques: generalized unitarity; BCJ double-copy; 2-loop amplitude of quasi-classical diagrams; **EFT transcription** (Cheung-Rothstein-Solon'18); **resummation of PN-expanded integrals for potential-gravitons**

$$\chi_3^{\text{cons}} = \chi_3^{\text{Schw}} - \frac{2\nu\sqrt{\gamma^2-1}}{h^2(\gamma,\nu)}\bar{C}^{\text{cons}}(\gamma)$$

$$q_3^{\text{cons}} = \frac{3}{2}\frac{(2\gamma^2-1)(5\gamma^2-1)}{\gamma^2-1}\left(\frac{1}{h(\gamma,\nu)}-1\right) + \frac{2\nu}{h^2(\gamma,\nu)}\bar{C}^{\text{cons}}(\gamma)$$

G^3 contrib. to H_EOB

$$\overline{C}^{\text{cons}}(\gamma) = \frac{2}{3}\gamma(14\gamma^2+25) + 2(4\gamma^4-12\gamma^2-3)\frac{\mathcal{A}(v)}{\sqrt{\gamma^2-1}}$$

$$h(\gamma,\nu) \equiv \frac{\sqrt{s}}{M} = \sqrt{1+2\nu(\gamma-1)}$$

$$\mathcal{A}(v) \equiv \text{arctanh}(v) = \frac{1}{2}\ln\frac{1+v}{1-v} = 2\,\text{arcsinh}\sqrt{\frac{\gamma-1}{2}}$$

puzzling HE limits when compared to ACV and Akcay et al'12

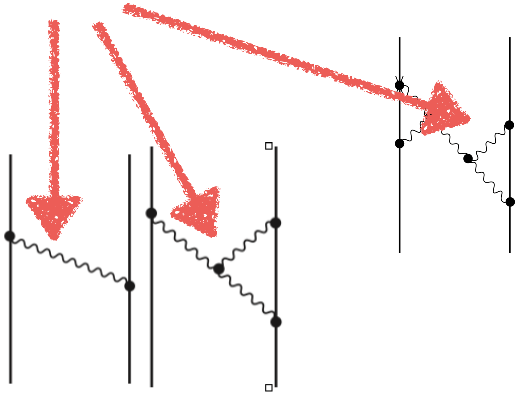
$$\frac{1}{2}\chi^{\text{cons}} = 2\frac{\gamma}{j} + (12 - 8\ln(2\gamma))\frac{\gamma^3}{j^3} + O(G^4)$$

$q_3^{\text{cons}} \approx +8\ln(2\gamma)\gamma^2$ instead of $q_3^{\text{ACV}} \approx +1\gamma^2$

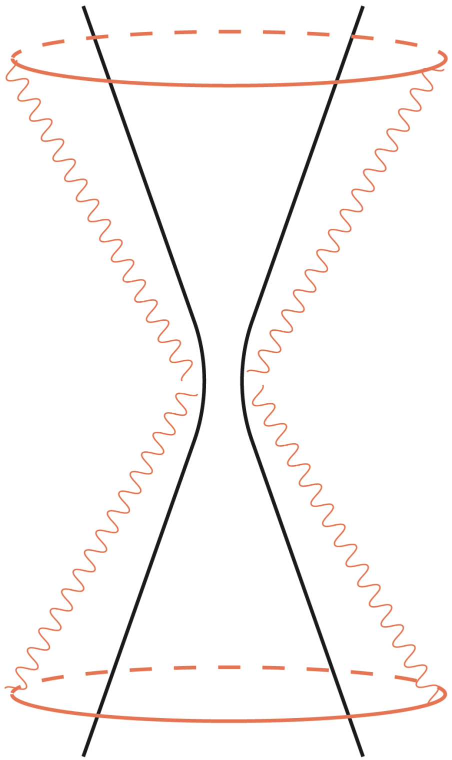
confirmations: 5PN (Bini-TD-Geralico'19); 6PN (Blümlein-Maier-Marquard-Schäfer'20, Bini-TD-Geralico'20); 3PM (Cheung-Solon'20, Kälin-Porto'20)

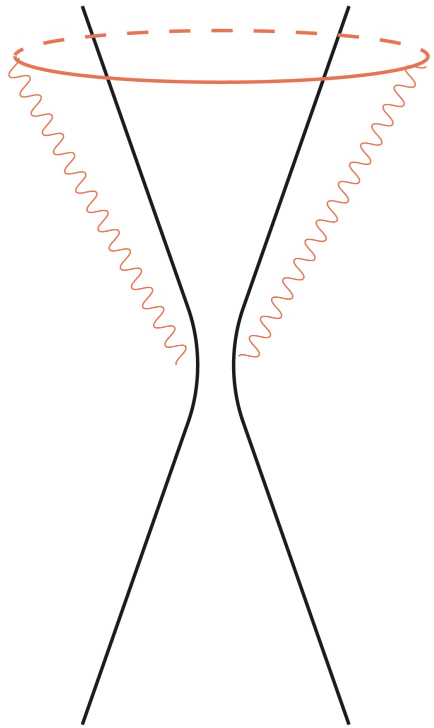
Conservative vs Radiation-reacted
Classical Gravitational Scattering

Fokker-Wheeler-Feynman conservative action using G_sym=1/2(G_ret+Gadv) =Re[G_F]=PV(1/p^2)



Subtleties arise at G^4 when iterating several PV(1/p^2)





Radiation-reaction effects enter scattering at G^3/c^5 (Bini-TD'12)

$$\frac{1}{2}\chi^{\rm rad} = +\frac{8G^3}{5c^5}\frac{m_1^3 m_2^3}{J^3}\nu v^2 + \cdots$$

chi^rad linked to radiated E and J

Radiation-reaction effects in scattering play a crucial role at **high-energy**

(DiVecchia-Heissenberg-Russo-Veneziano'20, TD'21, Hermann-Parra-Martinez-Ruf-Zeng'21,….)

they resolve the O(G^3) puzzle of the discrepancy between the HE limit of Amati-Ciafaloni-Veneziano'90(+ Ciafaloni-Colferai'14), and the G^3 result of Bern et al'19,20

Tutti-Frutti method

(**Bini-TD-Geralico** ’19,’20’21)

combines PN, MPM, EOB, Delaunay, Self-Force, mass-polynomiality of scattering angle

SIXTH POST-NEWTONIAN LOCAL-IN-TIME DYNAMIC

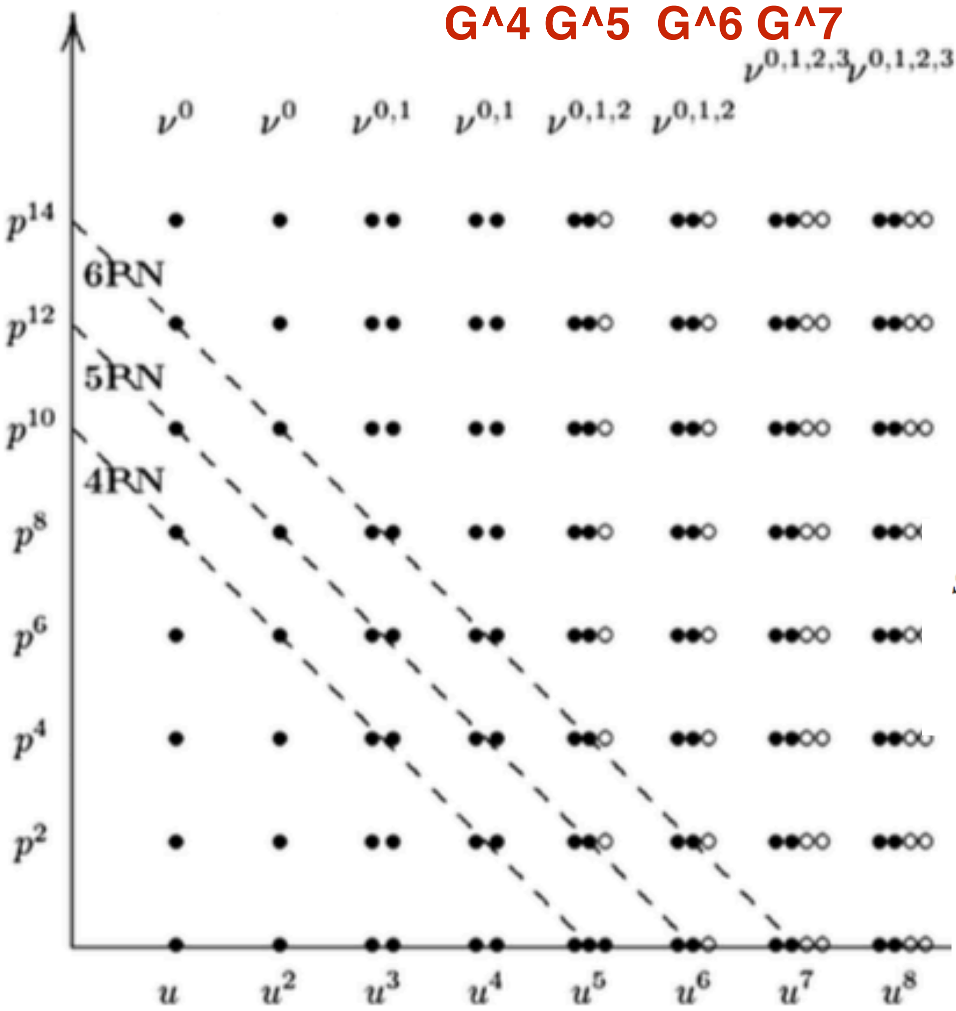


FIG. 1. Schematic representation of the irreducible information contained, at each post-Minkowskian level (keyed by a power of $u = GM/r$), in the local dynamics. Each vertical column of dots describes the post-Newtonian expansion (keyed by powers of p^2) of an energy-dependent function parametrizing the scattering angle. The various columns at a given post-Minkowskian level correspond to increasing powers of the symmetric mass-ratio ν. See text for details.

6PN conservative dynamics complete at 3PM and 4PM

$$S^{4+5\text{PN}}_{\text{nonloc}}[x_1(s_1), x_2(s_2)] = \frac{G^2\mathcal{M}}{c^3}\int dt \text{PF}_{2r^h_{12}(t)/c} \times \int \frac{dt'}{|t-t'|}\mathcal{F}^{\text{split}}_{1\text{PN}}(t,t').$$

$$\mathcal{F}^{\text{split}}_{1\text{PN}}(t,t') = \frac{G}{c^5}\left(\frac{1}{5}I^{(3)}_{ab}(t)I^{(3)}_{ab}(t') + \frac{1}{189c^2} + \frac{16}{45c^2}J^{(3)}_{ab}(t)J^{(3)}_{ab}(t')\right).$$

with both potential- and soft (radiationlike) gravitons

Scattering Amplitudes, the Tail Effect, and Conservative Binary Dynamics at $\mathcal{O}(G^4)$

Zvi Bern,[1] Julio Parra-Martinez,[2] Radu Roiban,[3] Michael S. Ruf,[1]
Chia-Hsien Shen,[4] Mikhail P. Solon,[1] and Mao Zeng[5]

three-loop level G^4

Conservative Dynamics of Binary Systems at Fourth Post-Minkowskian Order in the Large-eccentricity Expansion

Christoph Dlapa,[1] Gregor Kälin,[1] Zhengwen Liu,[1] and Rafael A. Porto[1]

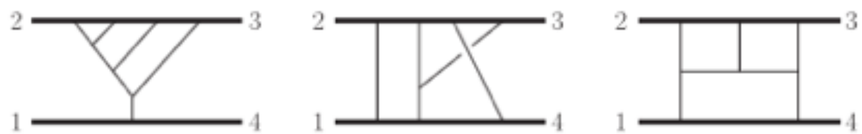
$$\mathcal{M}_4^{\rm cons} = G^4 M^7 \nu^2 |\boldsymbol{q}| \pi^2 \left[\mathcal{M}_4^{\rm p} + \nu\left(4\mathcal{M}_4^{\rm t}\log\left(\tfrac{p_\infty}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\rm rem}\right)\right] + \int_\ell \frac{\tilde{I}^4_{r,1}}{Z_1 Z_2 Z_3} + \int_\ell \frac{\tilde{I}^2_{r,1}\tilde{I}_{r,2}}{Z_1 Z_2} + \int_\ell \frac{\tilde{I}_{r,1}\tilde{I}_{r,3}}{Z_1} + \int_\ell \frac{\tilde{I}^2_{r,2}}{Z_1},$$

$$\mathcal{M}_4^{\rm p} = -\frac{35\left(1 - 18\sigma^2 + 33\sigma^4\right)}{8\left(\sigma^2 - 1\right)},$$

$$\mathcal{M}_4^{\rm t} = r_1 + r_2 \log\left(\tfrac{\sigma+1}{2}\right) + r_3 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}},$$

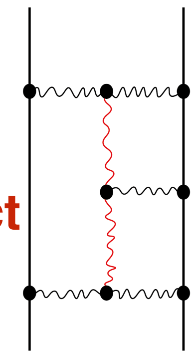
$$\mathcal{M}_4^{\pi^2} = r_4\pi^2 + r_5\,{\rm K}\left(\tfrac{\sigma-1}{\sigma+1}\right){\rm E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_6\,{\rm K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_7\,{\rm E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right),$$

$$\mathcal{M}_4^{\rm rem} = r_8 + r_9\log\left(\tfrac{\sigma+1}{2}\right) + r_{10}\tfrac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + r_{11}\log(\sigma) + r_{12}\log^2\left(\tfrac{\sigma+1}{2}\right) + r_{13}\tfrac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\log\left(\tfrac{\sigma+1}{2}\right) + r_{14}\tfrac{\operatorname{arccosh}^2(\sigma)}{\sigma^2-1}$$
$$+ r_{15}\,{\rm Li}_2\left(\tfrac{1-\sigma}{2}\right) + r_{16}\,{\rm Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) + r_{17}\tfrac{1}{\sqrt{\sigma^2-1}}\left[{\rm Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - {\rm Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right)\right].$$



(3)

three-loop



tail effect

$$\mathcal{M}_4^{\rm radgrav,f} = \frac{12044}{75}p_\infty^2 + \frac{212077}{3675}p_\infty^4 + \frac{115917979}{793800}p_\infty^6 - \frac{9823091209}{76839840}p_\infty^8 + \frac{115240251793703}{1038874636800}p_\infty^{10} - \frac{411188753665637}{4155498547200}p_\infty^{12} + \cdots, \quad (6)$$

whose first three terms match the sixth PN order result in Eq. (6.20) of Ref. [42].

matches the 6PN result of the Tutti Frutti approach (Bini-TD-Geralico'21)

New Subtleties in Radiative Contributions to Gravitational Scattering

X. NONLINEAR RADIATION-REACTION CONTRIBUTIONS TO SCATTERING

(Bini-TD-Geralico'21)

Effects linked to rad-reac^2

rad-reac force=O(G^2)

$$\epsilon_{rr} \equiv + \frac{4}{5} \frac{G^2 m_1 m_2}{c^5}.$$

$$\frac{d^2 \mathbf{x}_1}{dt^2} = -G m_2 \frac{\mathbf{x}_1 - \mathbf{x}_2}{r_{12}^3} - \epsilon_{rr} \frac{v_{12}^2}{r_{12}^3} [\mathbf{v}_{12} - 3(\mathbf{v}_{12} \cdot \mathbf{n}_{12}) \mathbf{n}_{12}],$$

rad-reac^2 =O(G^4)
contribution to scattering

$$\lim_{t \to +\infty} \epsilon_{rr}^2 \mathbf{v}_1^{(2)}(t) = -\frac{3\pi}{8} \epsilon_{rr}^2 \frac{v_0^3}{b^4} \hat{\mathbf{b}},$$

Classical (or « Quantum ») scattering worldline perturbation theory enhanced by using QFT integration methods

$$\frac{dx_a^\mu}{d\sigma_a} = g^{\mu\nu}(x_a)p_{a\nu},$$

$$\frac{dp_{a\mu}}{d\sigma_a} = -\frac{1}{2}\partial_\mu g^{\alpha\beta}(x_a)p_{a\alpha}p_{a\beta}.$$

$$R^{\mu\nu} - \tfrac{1}{2}Rg^{\mu\nu} = 8\pi G T^{\mu\nu}$$

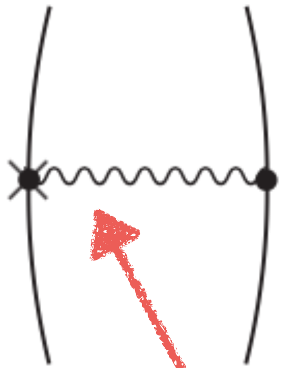
$$T^{\mu\nu}(x) = \sum \int d\sigma_a p_a^\mu p_a^\nu \frac{\delta^4(x - x_a(\sigma_a))}{\sqrt{g}}$$

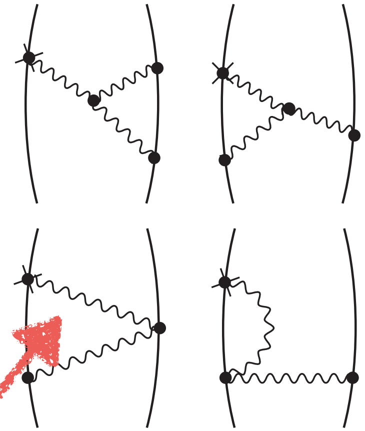
$$\Delta p_{a\mu} = \int_{-\infty}^{+\infty} d\sigma_a \frac{dp_{a\mu}}{d\sigma_a}$$

$$= -\frac{1}{2}\int_{-\infty}^{+\infty} d\sigma_a \partial_\mu g^{\alpha\beta}(x_a)p_{a\alpha}p_{a\beta}.$$

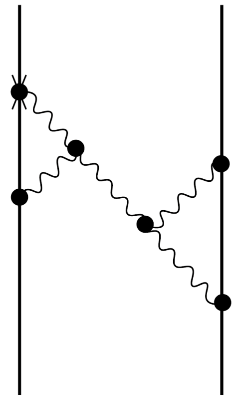
$$\Delta p_{1\mu} = 2G\int d\sigma_1 d\sigma_2 p_{1\alpha}p_{1\beta}$$

$$\times\, \partial_\mu \mathcal{P}^{\alpha\beta;\alpha'\beta'}(x_1(\sigma_1) - x_2(\sigma_2))p_{2\alpha'}p_{2\beta'}$$





with retarded propagator



$$\mathcal{P}^{\alpha\beta;\alpha'\beta'}(x-y) = \left(\eta^{\alpha\alpha'}\eta^{\beta\beta'} - \frac{1}{2}\eta^{\alpha\beta}\eta^{\alpha'\beta'}\right)\mathcal{G}(x-y)$$

with retarded propagator

Approach initiated long ago: Rosenblum'78 Westpfahl'79,'85 Portilla'80 Bel et al.'81 limited by the technical difficulty of computing the integrals beyond G^2, ie at G^2=2-loop.

Recently developed in two different flavors:
Dlapa-Kalin-Liu-Porto vs Jakobsen-Mogull-Plefka+…,…

Radiation Reaction and Gravitational Waves at Fourth Post-Minkowskian Order

Christoph Dlapa,[1] Gregor Kälin,[1] Zhengwen Liu,[2,1] Jakob Neef,[3,4] and Rafael A. Porto[1] (PRL 10 March 2023)

confirmed by recent QFT computation
Damgaard-Hansen-Plant'e-Vanhove'23

G^4 term

$$\Delta^{(n)} p_1^\mu = c_{1b}^{(n)} \frac{\hat{b}^\mu}{b^n} + \frac{1}{b^n} \sum_a c_{1\check{u}_a}^{(n)} \check{u}_a^\mu,$$

$$\begin{aligned}
\frac{c_{1b}^{(4)\,\mathrm{tot}}}{\pi} = & -\frac{3h_1 m_1 m_2 (m_1^3+m_2^3)}{64(\gamma^2-1)^{5/2}} + m_1^2 m_2^2 (m_1+m_2)\left[\frac{21 h_2 \mathrm{E}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{32(\gamma-1)\sqrt{\gamma^2-1}} + \frac{3h_3 \mathrm{K}^2\left(\frac{\gamma-1}{\gamma+1}\right)}{16(\gamma^2-1)^{3/2}} - \frac{3h_4 \mathrm{E}\left(\frac{\gamma-1}{\gamma+1}\right)\mathrm{K}\left(\frac{\gamma-1}{\gamma+1}\right)}{16(\gamma^2-1)^{3/2}} + \frac{\pi^2 h_5}{8\sqrt{\gamma^2-1}}\right. \\
& \left. + \frac{h_6 \log\left(\frac{\gamma-1}{2}\right)}{16(\gamma^2-1)^{3/2}} + \frac{3h_7 \mathrm{Li}_2\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\right)}{(\gamma-1)(\gamma+1)^2} - \frac{3h_7 \mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{4(\gamma-1)(\gamma+1)^2}\right] + m_1^3 m_2^2 \left[\frac{h_8}{48(\gamma^2-1)^3} + \frac{\sqrt{\gamma^2-1}\, h_9}{768(\gamma-1)^3\gamma^9(\gamma+1)^4} + \frac{h_{10}\log\left(\frac{\gamma+1}{2}\right)}{8(\gamma^2-1)^2}\right. \\
& - \frac{h_{11}\log\left(\frac{\gamma+1}{2}\right)}{32(\gamma^2-1)^{5/2}} + \frac{h_{12}\log(\gamma)}{16(\gamma^2-1)^{5/2}} - \frac{h_{13}\mathrm{arccosh}(\gamma)}{8(\gamma-1)(\gamma+1)^4} + \frac{h_{14}\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^{7/2}} - \frac{3h_{15}\log\left(\frac{\gamma+1}{2}\right)\log\left(\frac{\gamma-1}{\gamma+1}\right)}{8\sqrt{\gamma^2-1}} + \frac{3h_{16}\mathrm{arccosh}(\gamma)\log\left(\frac{\gamma-1}{\gamma+1}\right)}{16(\gamma^2-1)^2} \\
& \left. - \frac{3h_{17}\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{64\sqrt{\gamma^2-1}} - \frac{3}{32}\sqrt{\gamma^2-1}\, h_{18}\mathrm{Li}_2\left(\frac{1-\gamma}{\gamma+1}\right)\right] + m_1^2 m_2^3 \left[\frac{3h_{15}\log\left(\frac{2}{\gamma-1}\right)\log\left(\frac{\gamma+1}{2}\right)}{8\sqrt{\gamma^2-1}} + \frac{3h_{16}\log\left(\frac{\gamma-1}{2}\right)\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^2} + \frac{h_{19}}{48(\gamma^2-1)^3}\right. \\
& + \frac{h_{20}}{192\gamma^7(\gamma^2-1)^{5/2}} + \frac{h_{21}\log\left(\frac{\gamma+1}{2}\right)}{8(\gamma^2-1)^2} + \frac{h_{22}\log\left(\frac{\gamma+1}{2}\right)}{16(\gamma^2-1)^{3/2}} + \frac{h_{23}\log(\gamma)}{2(\gamma^2-1)^{3/2}} - \frac{h_{24}\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^3} + \frac{h_{25}\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^{7/2}} - \frac{3h_{26}\mathrm{arccosh}^2(\gamma)}{32(\gamma^2-1)^{7/2}} \\
& \left. + \frac{3h_{27}\log^2\left(\frac{\gamma+1}{2}\right)}{2\sqrt{\gamma^2-1}} + \frac{3h_{28}\log\left(\frac{\gamma+1}{2}\right)\mathrm{arccosh}(\gamma)}{16(\gamma^2-1)^2} + \frac{h_{29}\mathrm{Li}_2\left(\frac{1-\gamma}{\gamma+1}\right)}{4\sqrt{\gamma^2-1}} + \frac{3h_{30}\mathrm{Li}_2\left(\frac{\gamma-1}{\gamma+1}\right)}{8\sqrt{\gamma^2-1}}\right],
\end{aligned}$$

Its PN expansion agrees with Bini-TD-Geralico'23 notably for the **nu^2 =O(RR^2) contribution.**
Moreover, Bini-TD-Geralico'23 went beyond the linear-response formula by using balance+mass-polynomiality

$$\Delta p_{a\mu} = \Delta p_{a\mu}^{\rm cons} + \Delta p_{a\mu}^{\rm rr\,lin} + \Delta p_{a\mu}^{\rm rr\,nonlin}$$

$$\Delta p^{\rm rr}_{1\,\mu G^4} = \Delta p^{\rm rr\,lin-odd}_{1\,\mu G^4} + \frac{G^4}{b^4} m_1^3 m_2^2 p_x^{G^4}(\gamma)\hat{b}^\mu_{12},$$

$$\Delta p^{\rm rr}_{1\,\mu G^4} = \Delta p^{\rm rr\,lin-odd}_{1\,\mu G^4} + \frac{m_1}{m_2-m_1} P^{\rm rad}_{x\,G^4}\hat{b}^\mu_{12}.$$

relation between the rad-reac^2 term and P^rad_x

Strong-field scattering of two black holes: Numerical relativity meets post-Minkowskian gravity

Thibault Damour[1] and Piero Rettegno[2,3]

(PRD March 2023)

$$\chi_{n\mathrm{PM}}(\gamma, j) \equiv \sum_{i=1}^{n} 2\frac{\chi_i(\gamma)}{j^i}$$

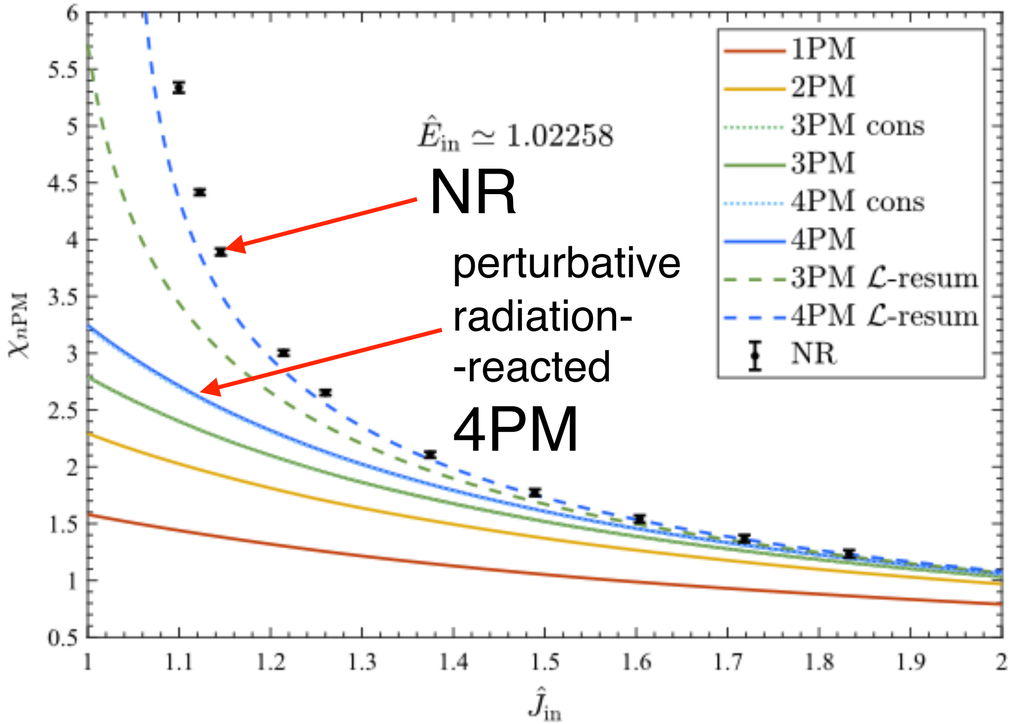
$$\mu^2 + g_{\mathrm{eff}}^{\mu\nu} P_\mu P_\nu + Q(X^\mu, P_\mu) = 0,$$

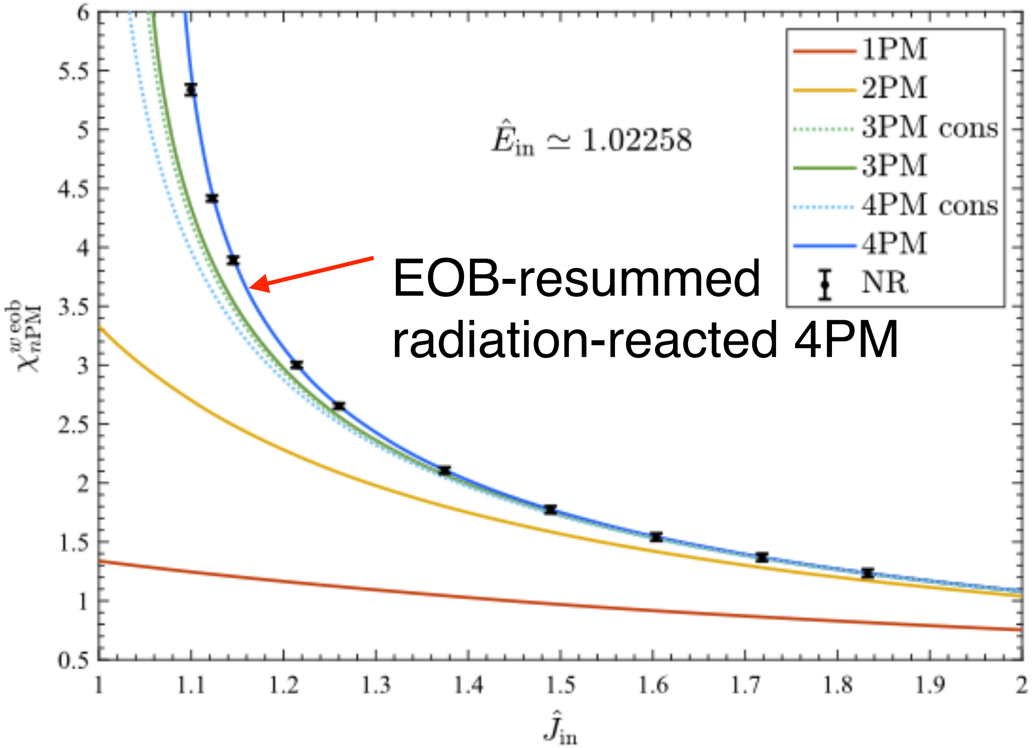
$$p_{\bar r}^2 + \frac{j^2}{\bar r^2} = p_\infty^2 + w(\bar r, \gamma).$$

$$w(\bar r, \gamma) = \frac{w_1(\gamma)}{\bar r} + \frac{w_2(\gamma)}{\bar r^2} + \frac{w_3(\gamma)}{\bar r^3} + \frac{w_4(\gamma)}{\bar r^4} + O\left[\frac{1}{\bar r^5}\right]$$

Newtonianlike EOB radial potential

$$\chi_{n\mathrm{PM}}^{w\,\mathrm{eob}}(\gamma, j) \equiv 2j \int_0^{\bar u_{\max}(\gamma, j)} \frac{d\bar u}{\sqrt{p_\infty^2 + w_{n\mathrm{PM}}(\bar u, \gamma) - j^2 \bar u^2}} - \pi.$$

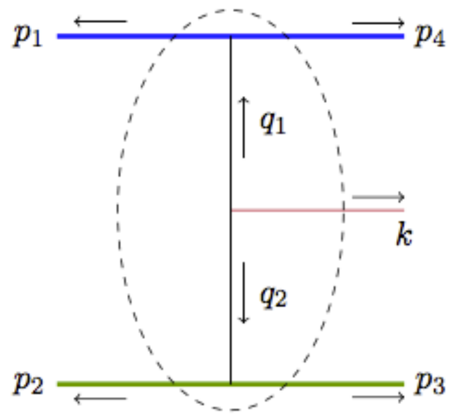




PM waveform computation $W(k^\mu) = \epsilon^\mu \epsilon^\nu h_{\mu\nu}(\omega, \theta, \phi)$

G^1=1PM (linearized,Einstein 1918) stationary $\propto \delta(\omega)$

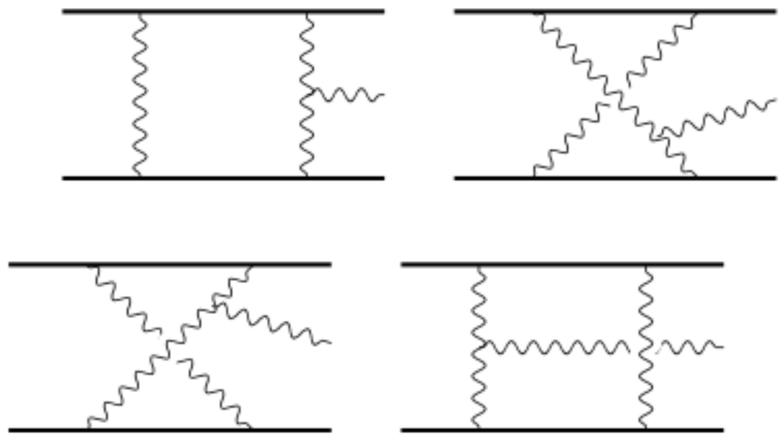
LO (tree level) waveform



G^2=2PM: classical time-domain W(t,n): Kovacs-Thorne 1977
quantum-based: yields W(k,p1,p2,p3,p4)=W(k,p1,p2,q1)
Johansson-Ochirov'15, GoldbergerRidgway'17 Luna-Nicholson-OConnellWhite'18
Mougiakakos-Riva-Vernizzi'21,Bautista-Siemonsen'22, De Angelis-Gonzo-Novichkov'23

Recent NLO (one-loop) waveform

G^3=3PM
Brandhuber+'23, Herderschee+'23, Georgoudis+'23, Bohnenblust+'24



5-point HEFT one-loop amplitude
—-> O(G^3) waveform via KMOC

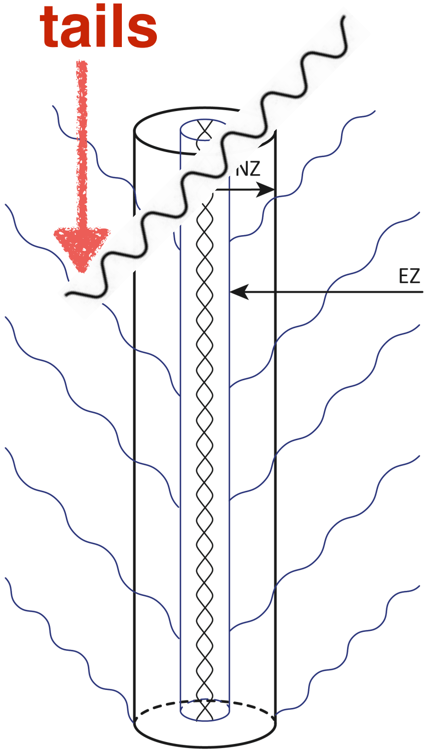
5-point amplitude

$$\begin{aligned}
&\mathscr{M}(\varepsilon, k, p_1, p_2, q_1, q_2) \\
&\quad \equiv i\langle p_3 p_4 | \hat{a}(k) \mathbb{T} | p_1 p_2 \rangle + \langle p_3 p_4 | \mathbb{T}^\dagger \hat{a}(k) \mathbb{T} | p_1 p_2 \rangle \\
&\quad = i\langle p_3 p_4 k | \mathbb{T} | p_1 p_2 \rangle + \langle p_3 p_4 | \mathbb{T}^\dagger \hat{a}(k) \mathbb{T} | p_1 p_2 \rangle ,
\end{aligned}$$

« cut term » important (Caron-Huot+'23)

Comparing one-loop amplitude to MPM waveform

(Bini-TD-Geralico'23)



algorithmic STF tensors encoding multipole moments (related to the source moments I_L,J_L)

$$g = \eta + G h_1 + G^2 h_2 + G^3 h_3 + ...,$$
$$\Box h_1 = 0,$$
$$\Box h_2 = \partial\partial h_1 h_1,$$
$$\Box h_3 = \partial\partial h_1 h_1 h_1 + \partial\partial h_1 h_2,$$
$$h_1 = \sum_\ell \partial_{i_1 i_2 \dots i_\ell}\left(\frac{M_{i_1 i_2 \dots i_\ell}(t-r/c)}{r}\right) + \partial\partial \dots \partial\left(\frac{\epsilon_{j_1 j_2 k} S_{k j_3 \dots j_\ell}(t-r/c)}{r}\right),$$
$$h_2 = FP_B \Box^{-1}_{\rm ret}\left(\left(\frac{r}{r_0}\right)^B \partial\partial h_1 h_1\right) + ...,$$
$$h_3 = FP_B \Box^{-1}_{\rm ret} \dots$$

radiative multipole moments (observable at infinity) U_L, V_L

$$r h_{ij}^{\rm TT} = \frac{4G}{c^2} P(n)_{ijab} \sum_{l=2}^{\infty} \frac{1}{c^l}\frac{1}{l!}\left(U_{abL-2} n_{L-2} - \frac{2l}{c(l+1)} n_{cL-2}\epsilon_{cd(a} V_{b)dL-2}\right)$$

$$\mathcal{M}^{\rm MPM}(k,b,u_1,u_2,m_1,m_2) = -i\frac{\kappa}{2}\epsilon^\mu\epsilon^\nu h^{\rm MPM}_{\mu\nu}(\omega,\theta,\phi) = -i\frac{\kappa}{2}\int dt e^{i\omega t}\epsilon^\mu\epsilon^\nu h^{\rm MPM}_{\mu\nu}(t,\theta,\phi)$$

$$\mathcal{M}^{\rm HEFT}(k,b,u_1,u_2,m_1,m_2) =$$
$$e^{i\frac{b_1+b_2}{2}\cdot k}\int \frac{d^D q}{(2\pi)^{D-2}}\delta\left(2p_1\cdot\left(q+\frac{k}{2}\right)\right)\delta\left(2p_2\cdot\left(-q+\frac{k}{2}\right)\right) e^{iq\cdot(b_1-b_2)}\mathcal{M}^{(1)}_{5,\rm HEFT}\left(q+\frac{k}{2},-q+\frac{k}{2};h\right)$$

Comparison one-loop amplitude vs MPM waveform

$$W(t,\theta,\phi) \sim \frac{1}{c^4}\left(G\,(\text{stationary}) + G^2(1+\frac{1}{c^1}+\frac{1}{c^2}+\frac{1}{c^3}+\cdots) + G^3(1+\frac{1}{c^1}+\frac{1}{c^2}+\frac{1}{c^3}+\cdots) + O(G^4)\right)$$

tree-level

one-loop

Aim: accuracy up to radiation-reaction effects: O(1/c^5) beyond LO quadrupole

$$U_{ij}(\omega) \sim \left(G(1+\frac{1}{c^2}+\frac{1}{c^4}) + G^2(1+\frac{1}{c^2}+\frac{1}{c^3}+\frac{1}{c^4}+\frac{1}{c^5}) + O(G^4)\right) + O(\frac{1}{c^6})$$

Newtonian G^2 **LO tail** **rad-reac plus similar effects**

$$U_{ij}^{\text{tail}}(t) = \frac{2G\mathcal{M}}{c^3}\int_0^\infty d\tau I_{ij}^{(4)}(t-\tau)\left(\ln\left(\frac{\tau}{2b_0}\right)+\frac{11}{12}\right)$$

Main results of the initial EFT-MPM comparison (Bini-TD-Geralico, 2023): mismatch at the Newtonian level, except if one refers the one-loop amp. to classical averaged momenta, rather than incoming momenta; then the terms linked to time-even PN corrections to multipoles agree but there are many mismatches at the G^2/c^5 level

Updated comparisons (Georgoudis et al.'23,'24, Bini et al. '24) lead to perfect agreement after taking into account three subtle effects:

(1) the bilinear-in-amplitude KMOC term generates the needed rotation
(2) IR divergences generate an additional (D-4)/(D-4) contribution
(3) zero-frequency gravitons contribute additional terms at h~G and h~G^3
(4) interesting links beween zero-freq gravitons and BMS frame (Veneziano-Vilkovisky)

Current Puzzles

high-energy limits?

G^3 energy loss too large

G^3 angular momentum loss too large (Manohar-Ridgway-Shen'22)

Rad-reacted G^4 scattering diverges (Porto..,Damgaard..)

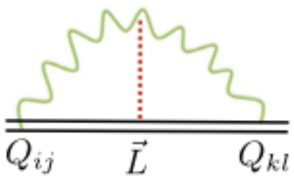
cf ACV motivation: BH formation in HE scattering

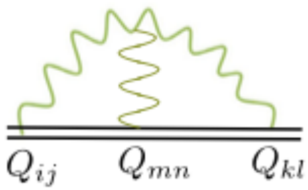
Subtleties in defining/computing angular momentum flux

(Ashtekar et al., Veneziano-Vilkovisky, Riva-Vernizzi,…)

low-energy discrepancy at 5PN between

Foffa-Sturani'19,21,22 Bluemlein et al'21 and Bini-TD-Geralico





$$\begin{aligned} S_{QQL} &= C_{QQL} G^2 \int dt I^{(4)}_{is} I^{(3)}_{js} \varepsilon_{ijk} L_k \\ S_{QQQ_1} &= C_{QQQ_1} G^2 \int dt I^{(4)}_{is} I^{(4)}_{js} I_{ij} \, , \\ S_{QQQ_2} &= C_{QQQ_2} G^2 \int dt I^{(3)}_{is} I^{(3)}_{js} I^{(2)}_{ij} \, . \end{aligned}$$

TF-constraint on 5PN O(nu^2)
EFT radiative terms

$$0 = \frac{2973}{350} - \frac{69}{2} C_{QQL} + \frac{253}{18} C_{QQQ_1} + \frac{85}{9} C_{QQQ_2}$$

not solved by recent in-in results **(Foffa-Sturani'22)**

Conclusions

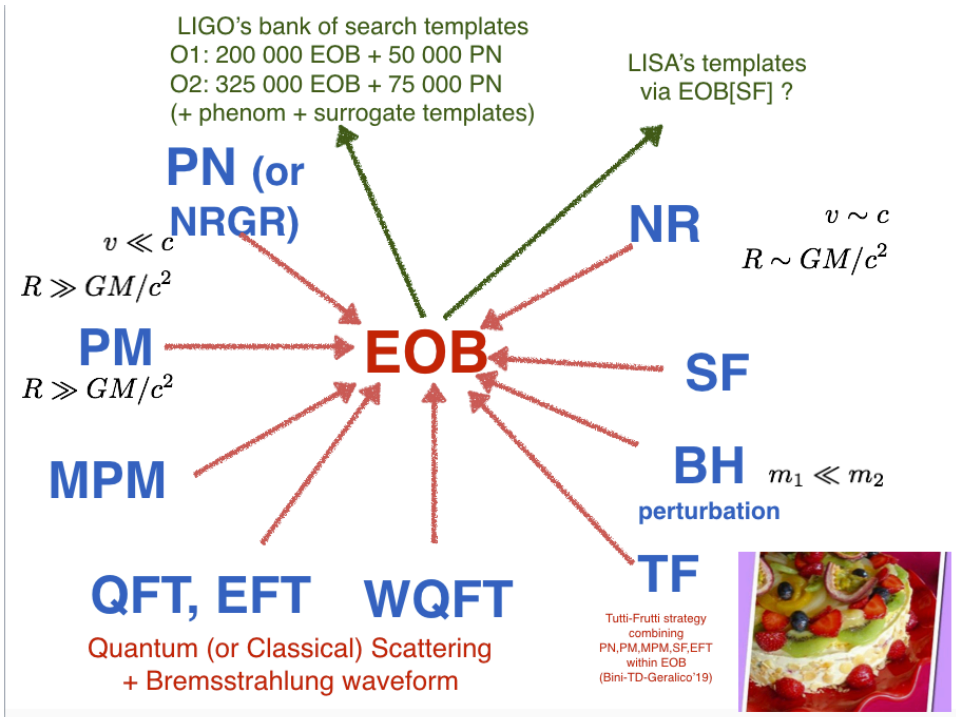
The recent synergy between various methods (time-honored and recent QFT-based ones) has led to many very interesting new vistas on the gravitational 2-body interaction.

Many impressive new results have been derived and more are in store, though one is close to reaching the limits of the new techniques

There remains puzzles to clarify

Though Numerical Relativity is and will remain very important and useful, analytical approaches will continue to play an important role.

Some improved avatar of the time-honored PN+MPM (+EFT) approach might remain most useful.



The flexible analytical nature of the EOB formalism makes it useful for incorporating new information in LIGO-useful form.