Noisy gates for simulating quantum computers

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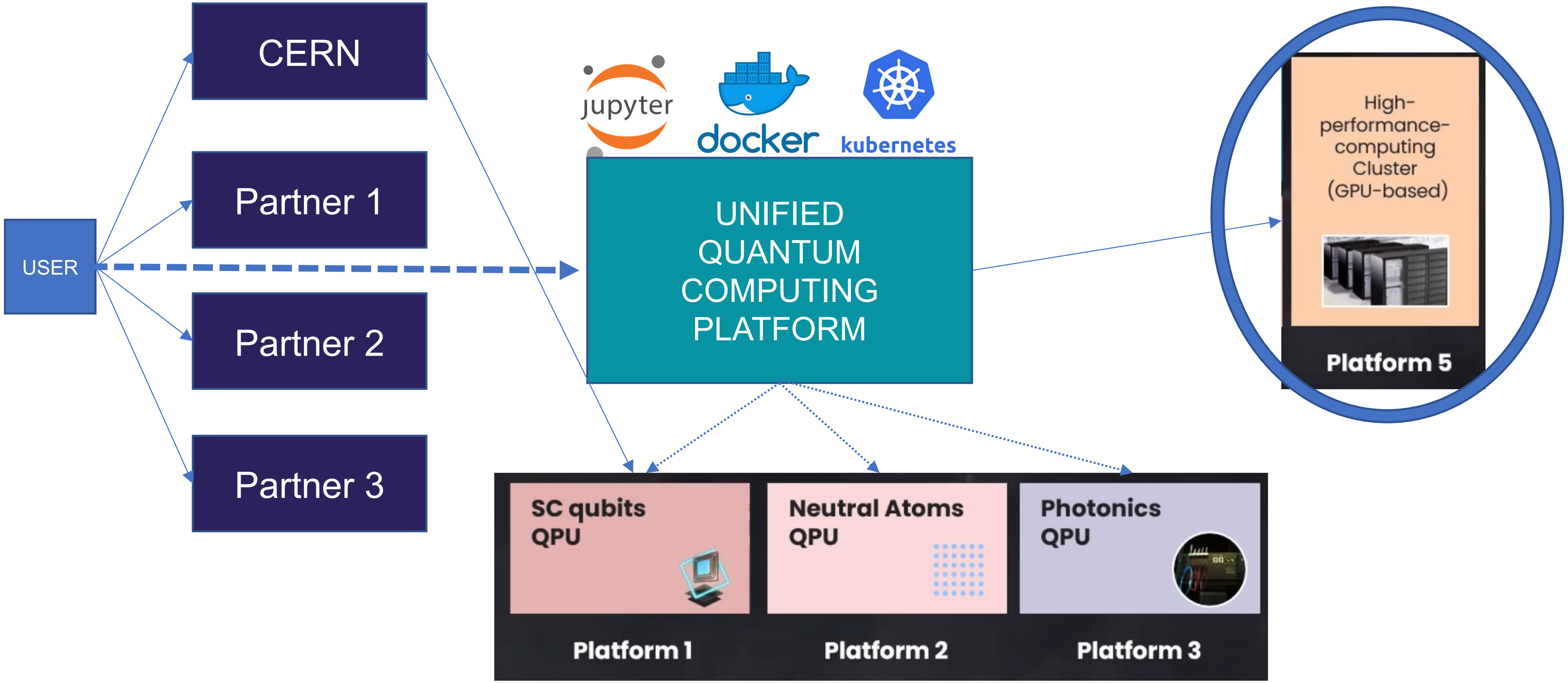
CERN Openlab Technical Workshop 2024

QUANTUM TECHNOLOGY INITIATIVE

Quantum Computing Platform



MOTIVATION

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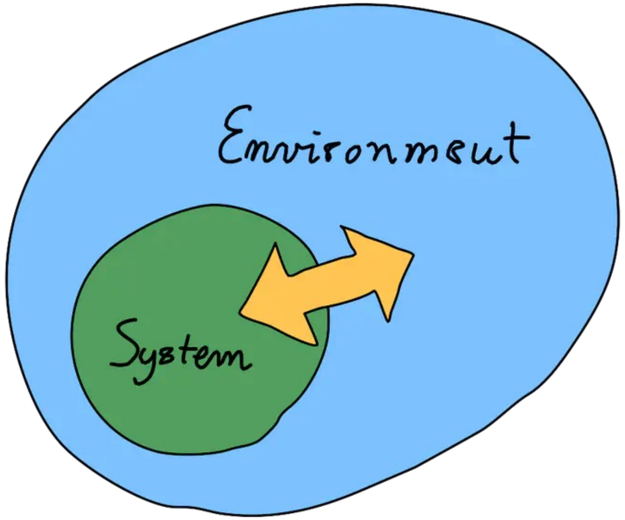
Study the noise

A **proper theoretical modelling** of the effect of the environment on a quantum systems allows to:



- Have a **physical understanding** of the sources of noise
- Suggest strategies to **mitigate errors**
- Perform **accurate simulations** to predict how the performances scale with the number of qubits/gates.

Georgopoulos, K., Emary, C., & Zuliani, P. (2021). Modeling and simulating the noisy behavior of near-term quantum computers. *Physical Review A*, *104*(6), 062432.
Sun, J., Yuan, X., Tsunoda, T., Vedral, V., Benjamin, S. C., & Endo, S. (2021). Mitigating realistic noise in practical noisy intermediate-scale quantum devices. *Physical Review Applied*, *15*(3), 034026.
Guerreschi, G. G., & Matsuura, A. Y. (2019). QAOA for Max-Cut requires hundreds of qubits for quantum speed-up. *Scientific reports*, *9*(1), 1-7.
Xue, C., Chen, Z. Y., Wu, Y. C., & Guo, G. P. (2021). Effects of quantum noise on quantum approximate optimization algorithm. *Chinese Physics Letters*, *38*(3), 030302.
Resch, S., & Karpuzcu, U. R. (2021). Benchmarking quantum computers and the impact of quantum noise. *ACM Computing Surveys (CSUR)*, *54*(7), 1-35.

NOISY GATES

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Open quantum systems

Breuer and Petruccione: *The Theory of Open Quantum Systems*, Oxford University Press (2002)

Theory of **open quantum systems**

Master Equation

$$|\psi\rangle \;\rightarrow\; \rho = |\psi\rangle\langle\psi|$$

State vector Density matrix

$$\frac{d}{dt}\rho_t = -\frac{i}{\hbar}[H_t, \rho_t] + \sum_k \gamma_k \left[L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_t\}\right]$$

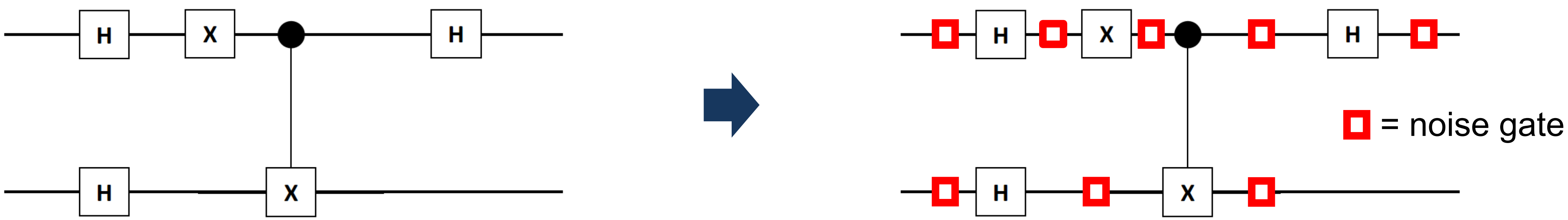
Internal evolution Effect of the environment

Issues to deal with:

- More complicated dynamics; how to model the environment efficiently
- With the density matrix, the problem scales quadratically with the size of the system.

Standard noise models

Noises (like gates) formally act instantly: Lindblad → Kraus $\rho \rightarrow \sum_i K_i \rho K_i^\dagger \qquad \sum_i K_i^\dagger K_i = 1$



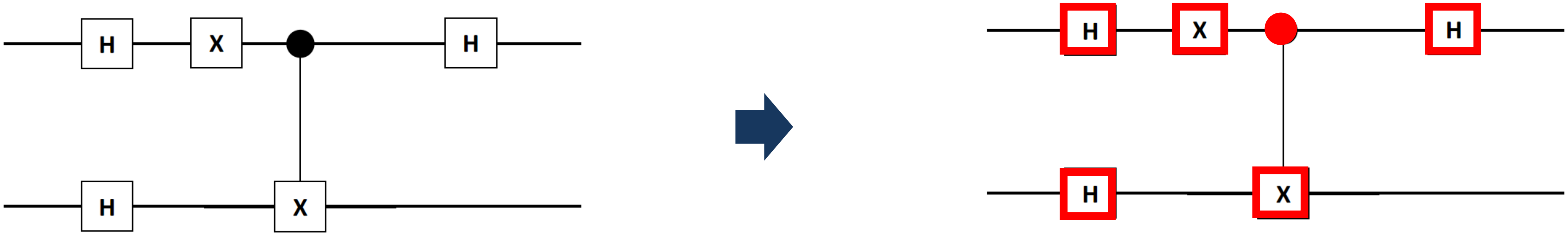
Standard noise simulation (e.g. in Qiskit)

- Gates and noise are formally **decoupled** (a sort of Trotterizzation), because time scales are small (IBM: gate time ~ 10^{-8} s, decoherence times ~ 10^{-4} s) ?
- Use the **quantum-jump-like approach** to replace the density matrix with (stochastic) state vector → stochastic dynamics ✓

Noisy Gates

G. Di Bartolomeo, M. Vischi, F. Cesa, R. Wixinger, M. Grossi, S. Donadi, A. Bassi. A novel approach to noisy gates for simulating quantum computers, *Phys. Rev. Research 5, 043210*

Our approach: provide a more accurate description of the noisy behaviour of a quantum computer



- Noises are **embedded** in the gate → more realistic picture ✓
- State vector (stochastic) description ✓

In SC: (SPAM) depolarizing +relaxation

From Lindblad to stochastic differential equations (SDE)

$$\frac{d}{dt}\rho_t = -\frac{i}{\hbar}[H_t, \rho_t] + \sum_k \gamma_k \left[L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_t\}\right] = \mathfrak{D}(\rho)$$

Gate Noise

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar}H_t dt + \sum_k \left(i\sqrt{\gamma_k}L_k dW_{k,t} - \frac{\gamma_k}{2}L_k^\dagger L_k dt\right)\right]|\psi_t\rangle$$

Stochastic evolution for the state vector (stochastic unravelling)

Formal equivalence: $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$

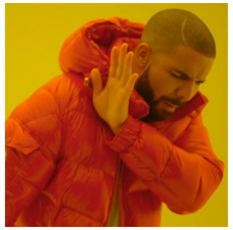
Comparison of the approximations

Standard approximation

$$\rho_t = U_{t,t_0} \mathrm{T}\left[e^{\gamma \int_{t_0}^{t} \mathrm{d}s \mathfrak{L}(s)}\right] \rho_{t_0} U_{t,t_0}^{\dagger}$$

$$\mathfrak{L}(t) \simeq \mathfrak{L}$$

$$\rho_t \simeq U_{t,t_0} e^{\gamma \mathfrak{L} \cdot (t-t_0)} \rho_{t_0} U_{t,t_0}^{\dagger}$$



Noisy gates

$$\rho_t = U_{t,t_0} \mathrm{T}\left[e^{\gamma \int_{t_0}^{t} \mathrm{d}s \mathfrak{L}(s)}\right] \rho_{t_0} U_{t,t_0}^{\dagger}$$

$$\mathrm{T}\left[e^{\gamma \int_{t_0}^{t} \mathrm{d}s \mathfrak{L}(s)}\right] \simeq 1 + \gamma \int_{t_0}^{t} \mathrm{d}s \mathfrak{L}(s)$$

$$\rho_t \simeq U_{t,t_0} \left(1 + \gamma \int_{t_0}^{t} \mathrm{d}s \mathfrak{L}(s)\right) \rho_{t_0} U_{t,t_0}^{\dagger}$$

IBMQ devices: main single qubit noises

Gates

Native gate set $\{RZ(\phi), \text{X}, \text{SX}, \text{CNOT}\}$

Cross resonance (CR) gate

θ : rotation angle

ϕ : phase, realizes virtual Z gates

$$H(\theta,\phi) = \frac{\theta\hbar}{2} R_{xy}(\phi)$$
$$R_{xy}(\phi) = \cos(\phi)X + \sin(\phi)Y$$

+

$$\mathrm{H}^{(1,2)}(\theta,\phi) = \frac{\hbar\theta}{2} \mathrm{Z}^{(1)} \otimes \mathrm{R}_{xy}^{(2)}$$
$$R_{xy}(\phi) = \cos(\phi)X + \sin(\phi)Y$$

Note: how to implement the pulse

Noises

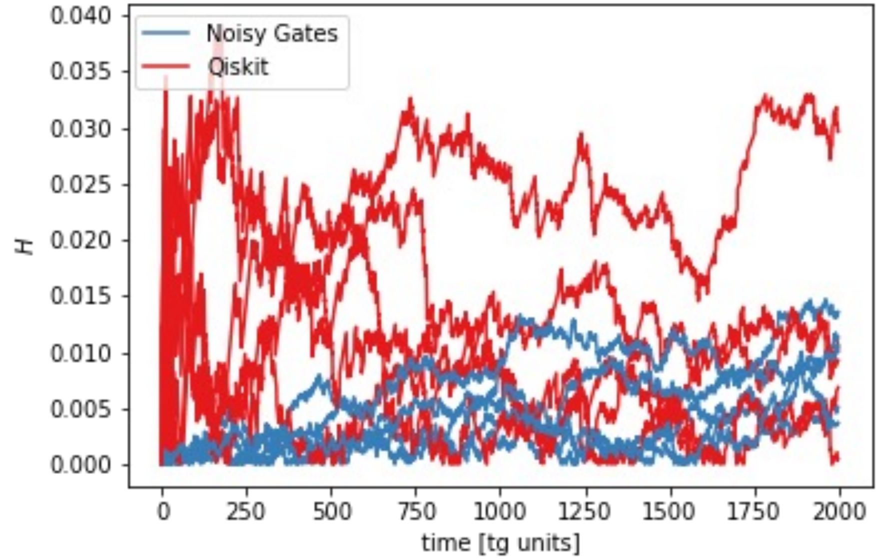
-Single qubit depolarization: γ_d

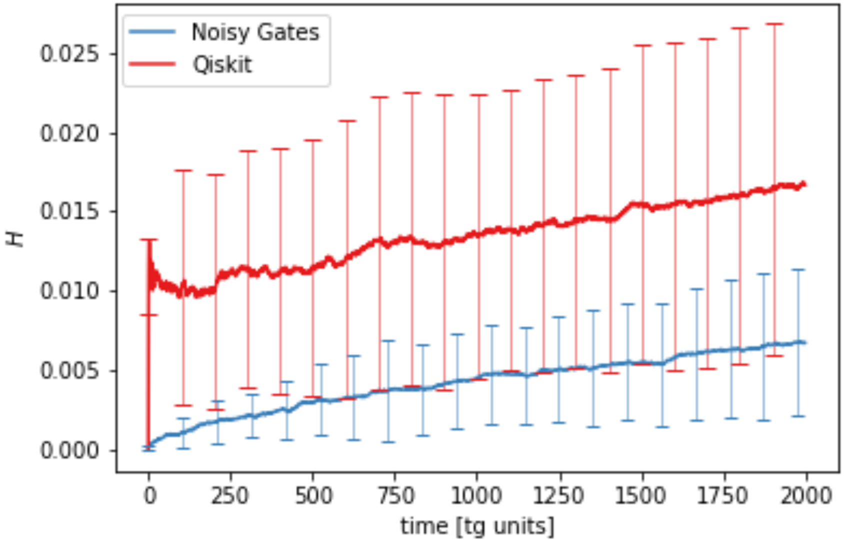
-Single qubit amplitude and phase damping: γ_1, γ_z

$$\mathrm{L}_1 = \sqrt{\frac{\lambda_1}{\lambda}}\sigma^-, \quad \mathrm{L}_2 = \sqrt{\frac{\lambda_2}{\lambda}}\sigma^+, \quad \mathrm{L}_3 = \sqrt{\frac{\lambda_3}{\lambda}}\mathrm{Z}$$
$$\lambda_1 = 2\gamma_d, \ \lambda_2 = 2\gamma_d + \gamma_1, \ \lambda_3 = \gamma_d + \gamma_z$$
$$\lambda = \lambda_1 + \lambda_2 + \lambda_3$$

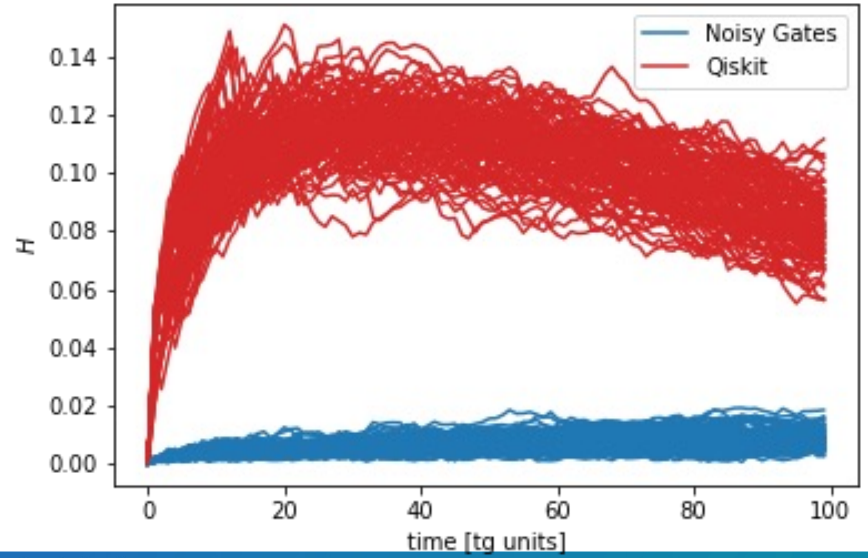
$$\lambda_k \sim 10^4 \mathrm{Hz}$$
$$t_g \sim 10^{-8} s$$
$$\epsilon = \sqrt{\lambda t_g} \ll 1$$

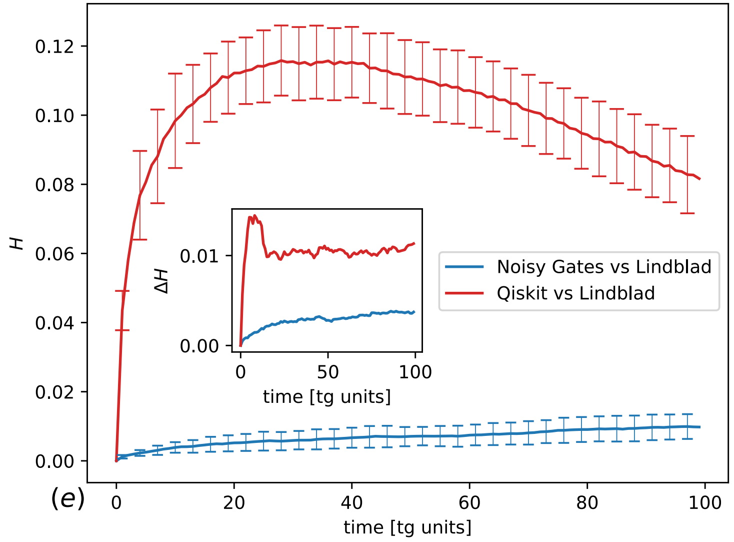
Simulation of the noisy X gate





Simulation of the noisy CR gate





Hellinger Distance

Lindblad and noisy gates

Lindblad and Qiskit simulator

$$H(P,Q) = \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{n} (\sqrt{p_i} - \sqrt{q_i})^2}$$

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quantum-gates 1.0.4

```
pip install quantum-gates
```

Latest version

Released: Mar 29, 2023

Quantum Noisy Gates Simulation with Python

Navigation

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Project description

Noisy Quantum Gates

University of Trieste | Bassi Group CERN | CERN openlab CERN | Open Source CERN | QTI

Implementation of the Noisy Quantum Gates model, which is soon to be published. It is a novel method to simulate the noisy behaviour of quantum devices by incorporating the noise directly in the gates, which become stochastic matrices.

Documentations

The documentation for Noisy Quantum Gates can be accessed on the website Read the Docs.

Run your own noisy simulation

We create a quantum circuit with Qiskit.

```
circ = QuantumCircuit(2,2)
circ.h(0)
circ.cx(0,1)
circ.barrier(range(2))
circ.measure(range(2),range(2))
circ.draw('mpl')
```

```
sim = MrAndersonSimulator(gates=standard_gates, CircuitClass=EfficientCircuit)

t_circ = transpile(
    circ,
    backend,
    scheduling_method='asap',
    initial_layout=qubits_layout,
    seed_transpiler=42
)

probs = sim.run(
    t_qiskit_circ=t_circ,
    qubits_layout=qubits_layout,
    psi0=np.array(run_config["psi0"]),
    shots=run_config["shots"],
    device_param=device_param_lookup,
    nqubit=2)

counts_ng = {format(i, 'b').zfill(2): probs[i] for i in range(0, 4)}
```

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https://quantum.cern/