Asymmetric Errors

A statistician's perspective

Joint work with: Roger Barlow and Igor Volobouev

PHYSTAT-Informal Reviews
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Thanks!

March Hare: *"See all the trouble you've started?"*

Alice: *"Really, I didn't think…"*

March Hare: *"But that's the point. If you don't think, you shouldn't talk."*

You made me think a lot!

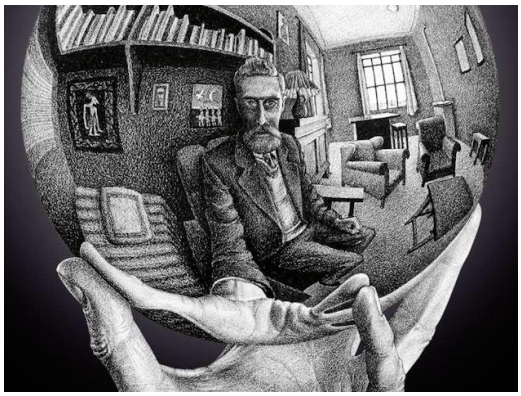
Lessons learnt

Alice: *"I give myself very good advice, but I very seldom follow it."*

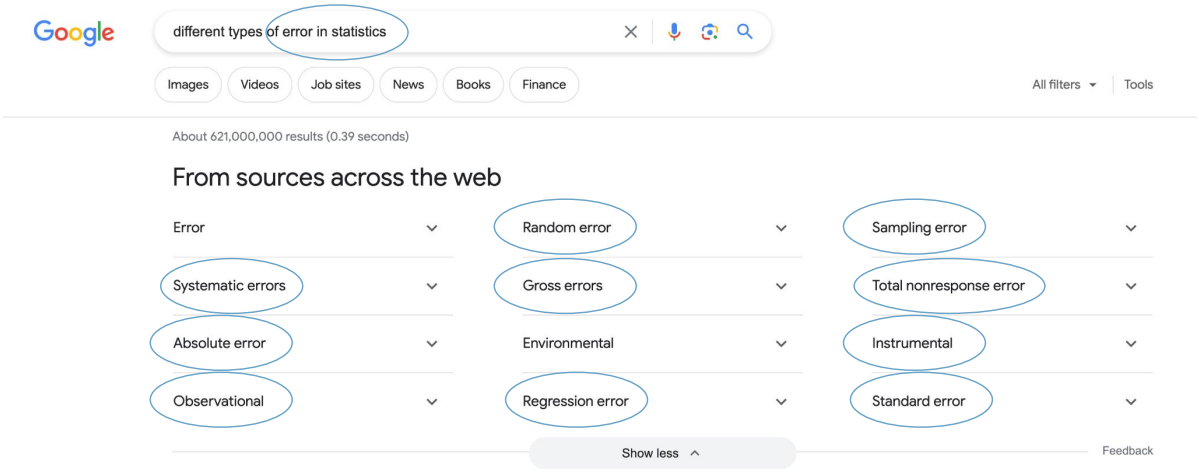
Re-look at things!

- They may look differently.
- You may discover new things.

Never take anything for granted!



What's an error?



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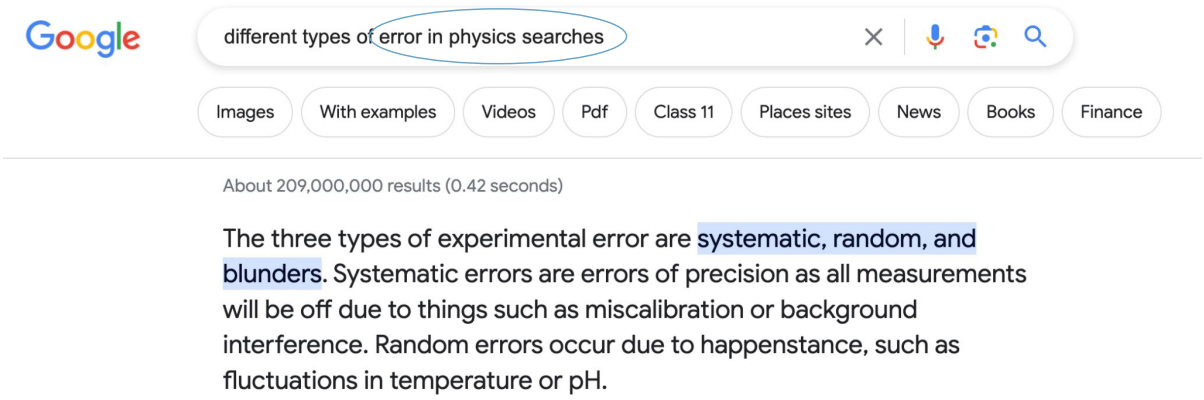


Ask a physicist!

"Systematic"

From an OPAT (One Parameter At a Time) error analysis when the response is not linear. Usually 3 points at $\pm\sigma$ but can be more detailed.

"Statistical"

From $\Delta \ln L = -\frac{1}{2}$ errors when the likelihood is not parabolic

BIAS

Tendency to produce results that differ in a systematic manner from the 'true' value.

VARIANCE

Scatter of values obtained from a data collection process.

What type of error?

$x \mp \sigma$

1 - α = 0.68

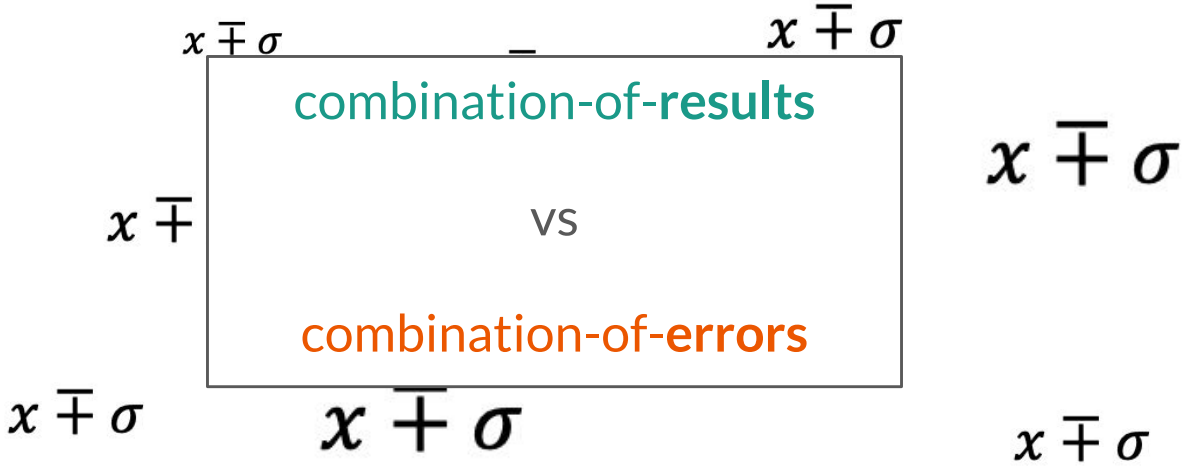
First option : (Wald type) **CONFIDENCE INTERVAL**

→ LIKs (statistical) → **confidence level**

Second option : **NUMERICAL** (sample/population) **SUMMARIES**

→ PDFs (systematic) → **probability**

What next?



Combination of results (LIKs)

Meta-analysis

Article Talk

From Wikipedia, the free encyclopedia

For the process in historical linguistics known as metanalysis, see Rebracketing.

Meta-analysis is the statistical combination of the results of multiple studies addressing a similar research question. An important part of this method involves computing an effect size across all of the studies; this involves extracting effect sizes and variance measures from various studies. Meta-analyses are integral in

Combination of errors (PDFs)

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Convolution of probability distributions

2 languages

Article Talk Read Edit View history Tools

From Wikipedia, the free encyclopedia

The **convolution/sum of probability distributions** arises in probability theory and statistics as the operation in terms of probability distributions that corresponds to the addition of independent random variables and, by extension, to forming linear combinations of random variables. The operation here is a special case of convolution in the context of probability distributions.

Yes, but …

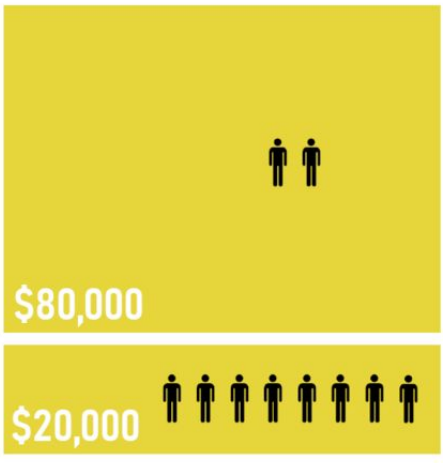
$$x \mp \sigma \longrightarrow x^{+\sigma^{+}}_{-\sigma^{-}}$$

Not much arround … Why?

My fault !

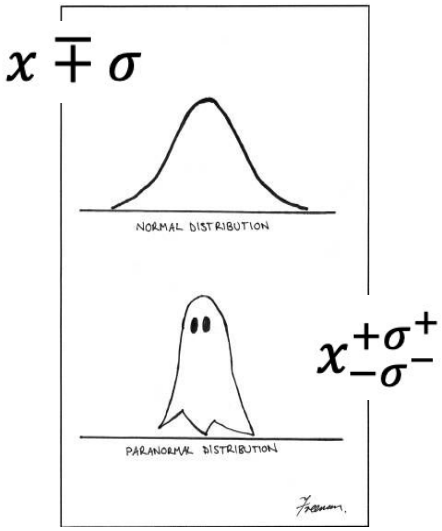
“personalized” Pareto rule :

95 - 5



Not much arround… Why?

1. Great faith in **Central Limit Theorem**
   - Wald type confidence intervals
2. Or then, distribution given
   - LR (& Co.)
3. …



Asymmetric Errors

Roger Barlow
The University of Huddersfield
Huddersfield, UK
Roger.Barlow@cern.ch

Alessandra Rosalba Brazzale
University of Padova
Padova, Italy

Igor Volobouev
Texas Tech University
Lubbock, Texas, USA

and others

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Abstract

We present a procedure for handling asymmetric errors. Many results in particle physics are presented as values with different positive and negative errors, and there is no consistent procedure for handling them. We consider the difference between errors quoted using likelihood and using pdfs, and the difference between the rms spread of a measurement and the 68% central confidence region. We provide a full analysis of the possibilities, and software tools to enable their use.

Combination of …

ERRORS

Identify asymmetric "surrogate" model

Convolution

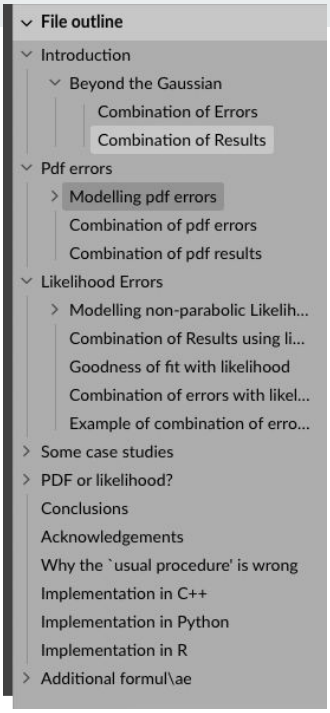
RESULTS

Identify non quadratic "surrogate" LIK

Combine LIKs

Of course …

- Combination of results with PDFs
  → Weighted sum
- Combination of errors with LIKs
  → Profiling
- …



“Surrogate” PDFs

skew Normal	log Normal	Edgeworth expansion
dimidiated Gaussian	distorted Gaussian	“railway” Gaussian
quantile variable width Gaussian	Johnson system	??

"It goes like this: / the fourth, the fifth, / the minor falls, and the major lifts» (L. Cohen)

“Surrogate” LIKs

ln(PDF)	generalised Poisson	broken parabola	logarithmic
linear sigma	linear sigma in log space	double cubic sigma in log space	linear variance
PDG method	**cubic**	constrained **quartic**	molded **quartic**
simple double **quartic**	molded double **quartic**	matched **quintic**	simple double **quintic**
molded double **quintic**	**quintic** sigma in log space	interpolated **seventh order**	??

Which LIK approximation?

- converge to quadratic LIK if asymmetry → 0
- interpretable
- "regular" model

hard boundary on / **maximum "practicable"** value for **asymmetry**

Submitted to the Statistical Science

Likelihood Asymptotics in Nonregular Settings: A Review with Emphasis on the Likelihood Ratio

Alessandra R. Brazzale * and Valentina Mameli †

Condition 1 The model functions defined by any two distinct values of θ are distinct almost surely. Moreover, all components of θ are identifiable.

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Condition 2 The support $\mathcal{Y}$ of $f(y;\theta)$ does not depend on θ.

Condition 3 The parameter space Θ is a compact subset of $\mathbb{R}^p$, for a fixed positive integer p, and the true value θ^0 of θ is an interior point of Θ.

Condition 4 The partial derivatives of the log-likelihood function $l(\theta;y)$ with respect to θ up to the order three exist in a neighbourhood of the true parameter value θ^0 almost surely. Furthermore, in such a neighbourhood, n^{-1} times the absolute value of the log-likelihood derivatives of order three are bounded above by a function of Y whose expectation is finite.

Condition 5 The first two Bartlett identities hold, so

$$E[u(\theta;Y)] = 0, \quad i(\theta) = \mathrm{Var}[u(\theta;Y)],$$

in addition to $0 < \mathrm{Var}[u(\theta;Y)] < \infty$.

Which PDF approximation?

closure under CONVOLUTION

Stable distribution

Article Talk

From Wikipedia, the free encyclopedia

Not to be confused with Stationary distribution.

In probability theory, a distribution is said to be **stable** if a linear combination of two independent random variables with this distribution has the same distribution, up to location and scale parameters. A random variable is said to be **stable** if its distribution is stable. The stable

at least approximate…
(work in progress)

… need suitable distance metric

Ongoings and open questions

1. **Beta testing**
2. **Model selection**
3. Correlated errors?
4. Combining LIKs and PDFs
5. What if we want to be Bayesian?
6. …

The floor to you!

Questions?