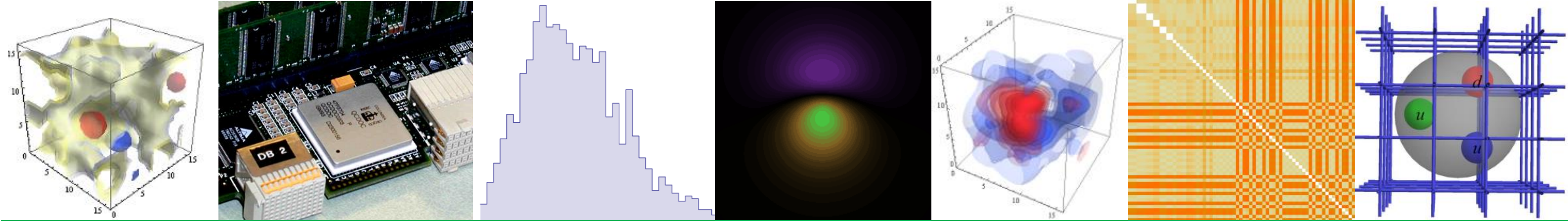


Mapping the Parton Distributions of Pion and Nucleon

Huey-Wen Lin

CTEQ

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

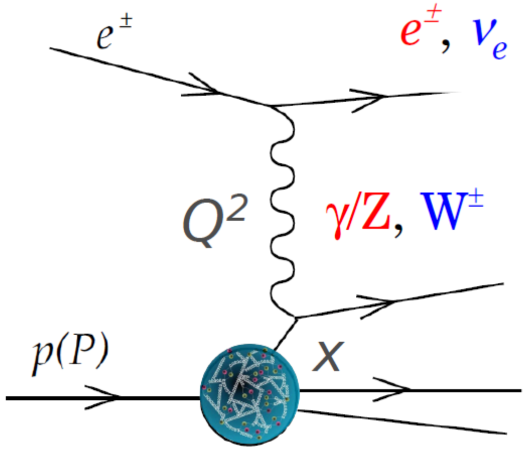
NSF

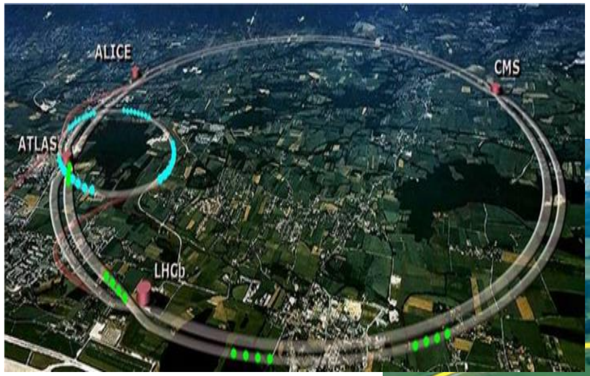
U.S. DEPARTMENT OF ENERGY

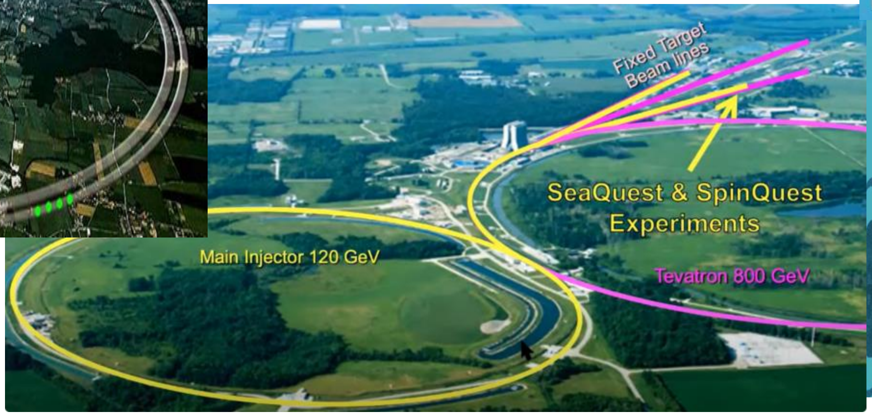
RESEARCH CORPORATION for SCIENCE ADVANCEMENT

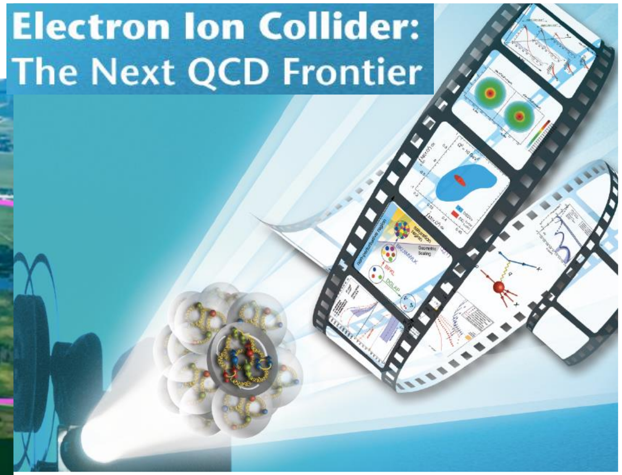
Parton Distribution Functions

- § PDFs provide the probability for a parton to carry a fraction *x* of the proton momentum
- § PDFs are intrinsic property of nucleons
  - Process-independent
  - Many ongoing/planned experiments (LHC, FNAL, BNL, JLab, J-PARC, COMPASS, EIC, LHeC, EIcC…)









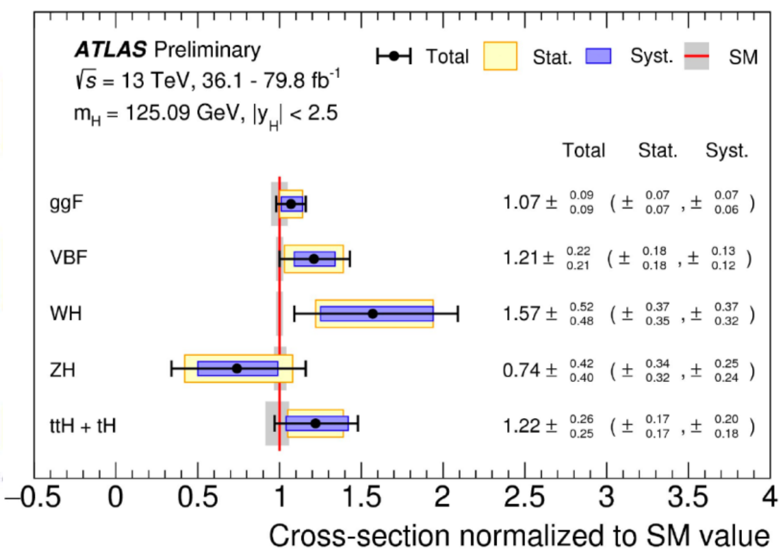
Why Study PDFs?

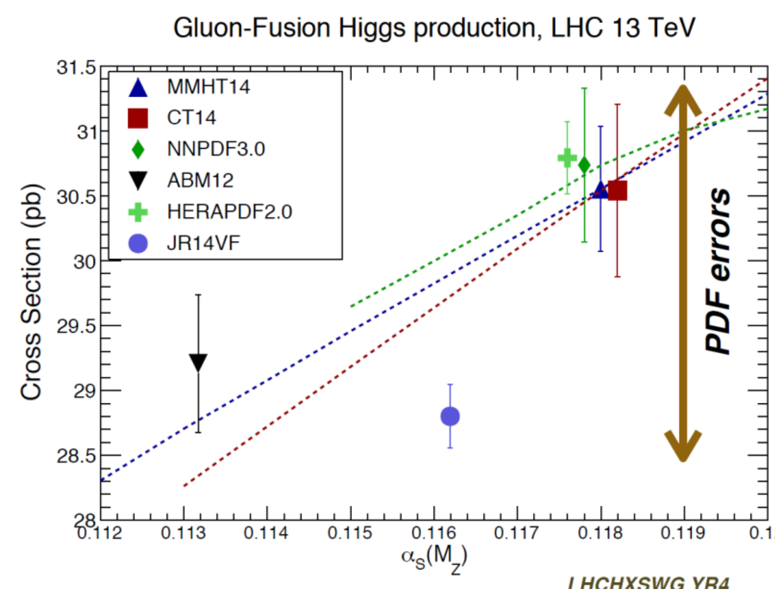
§ Crucial for precision theoretical predictions

- Important inputs to discern new physics at LHC, finding new elementary particles beyond the Standard Model
- Predict signal and background events at ultra high-energy neutrino detectors (IceCube, ANITA, etc.)

§ LHC example

- PDF uncertainty a dominant theory error in Higgs production cross sections





Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

- Assumptions imposed

  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

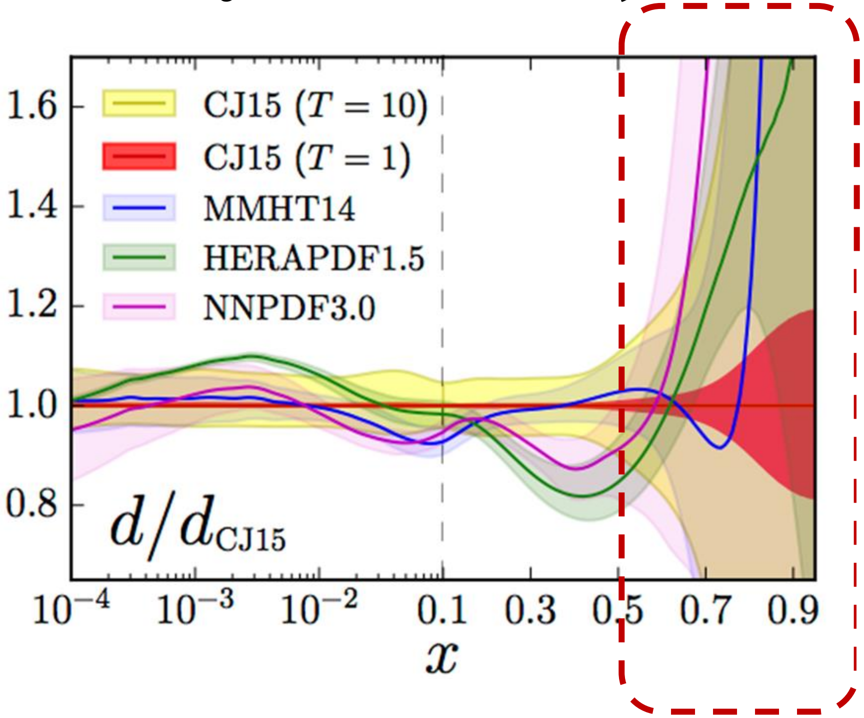
$$s = \bar{s} = \kappa\left(\bar{u} + \bar{d}\right)$$

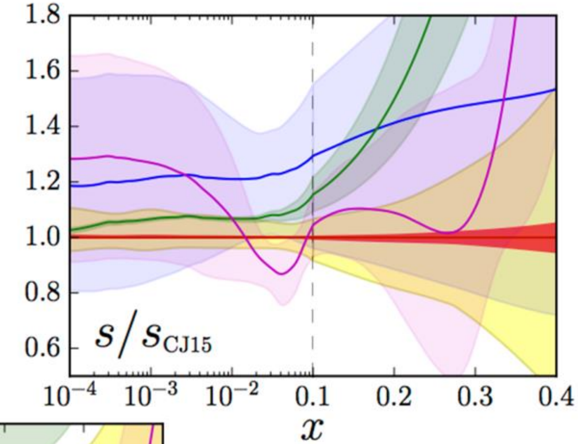
Global Analysis

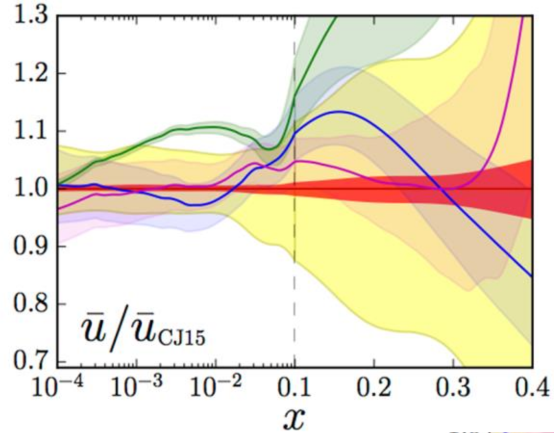
§ Discrepancies appear when data is scarce

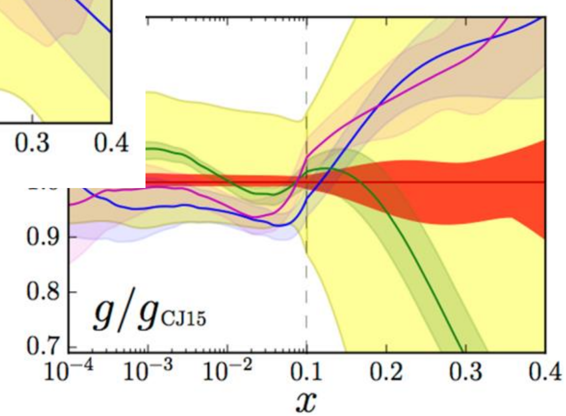
§ Many groups have tackled the analysis

- CTEQ, MSTW, ABM, JR, NNPDF, etc.







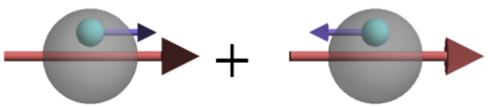


CTEQ-JLAB
https://www.jlab.org/theory/cj/

PDFs on the Lattice

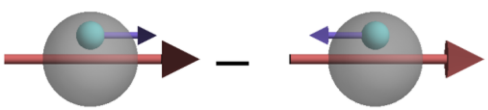
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

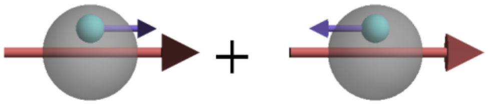
spin-dependent
transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

Moments of PDFs

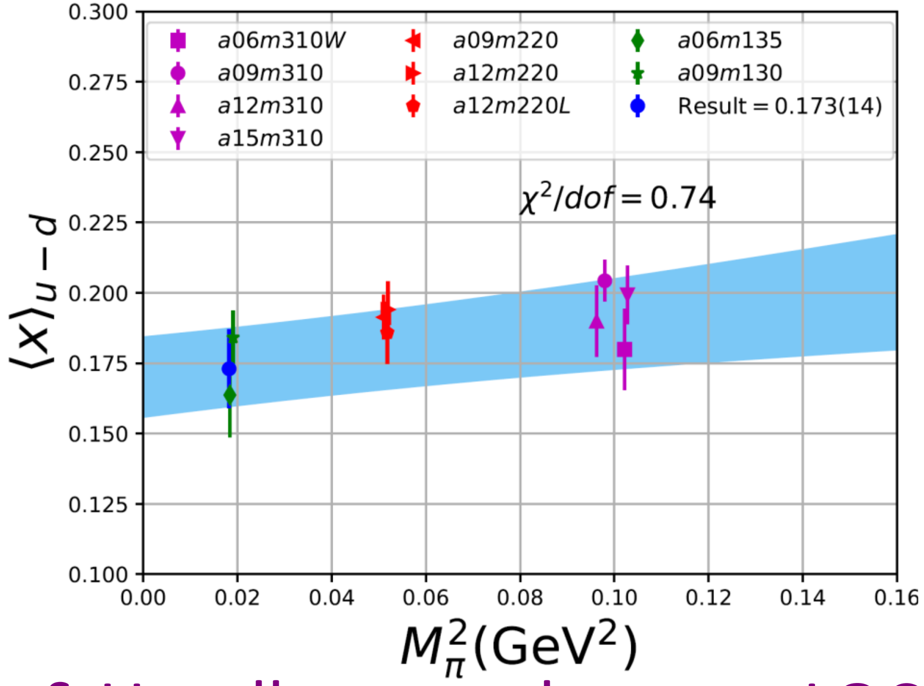
§ First moments are most commonly done 

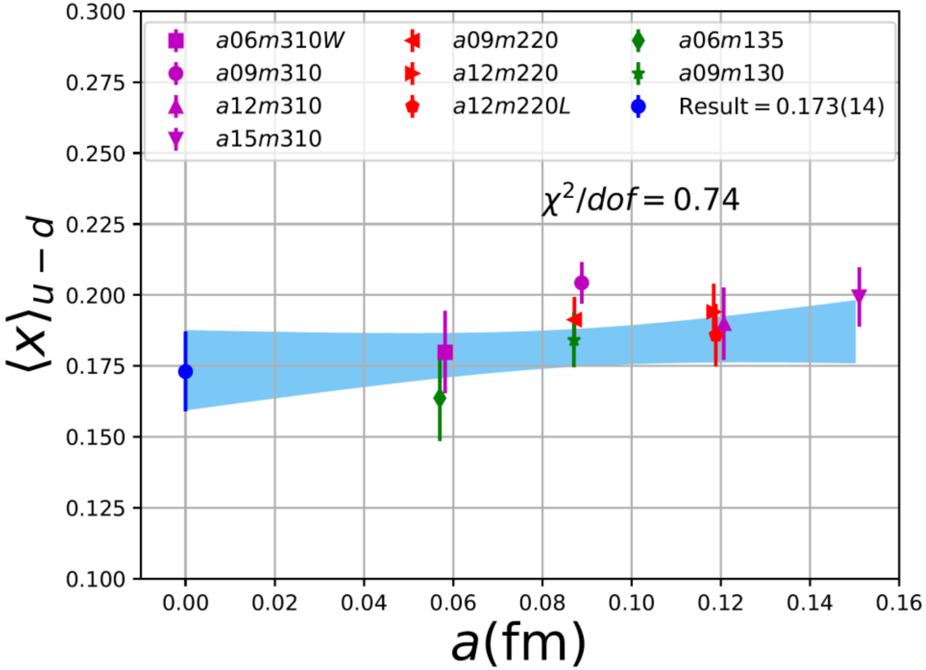
§ State-of-the art example

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

➢ Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779





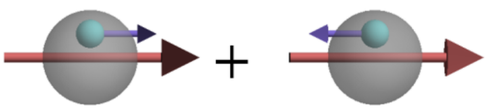
§ Usually more than one LQCD calculation

➢ Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

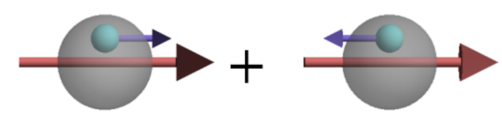
Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES	Value	Global Fit
$\langle x\rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.171(18)	0.161(18)
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★	0.173(14)(07)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★	$0.180(25)(^{+14}_{-6})$	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.151(28)(29)	
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★	0.195(07)(15)	
$\langle x\rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.359(30)	0.353(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.307(30)(18)	
$\langle x\rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.188(19)	0.192(6)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.160(27)(40)	
$\langle x\rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.052(12)	0.037(3)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.051(26)(5)	
$\langle x\rangle_g$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.427(92)	0.411(8)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.482(69)(48)	
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■	0.47(4)(11)	

** No quenching effects are seen.

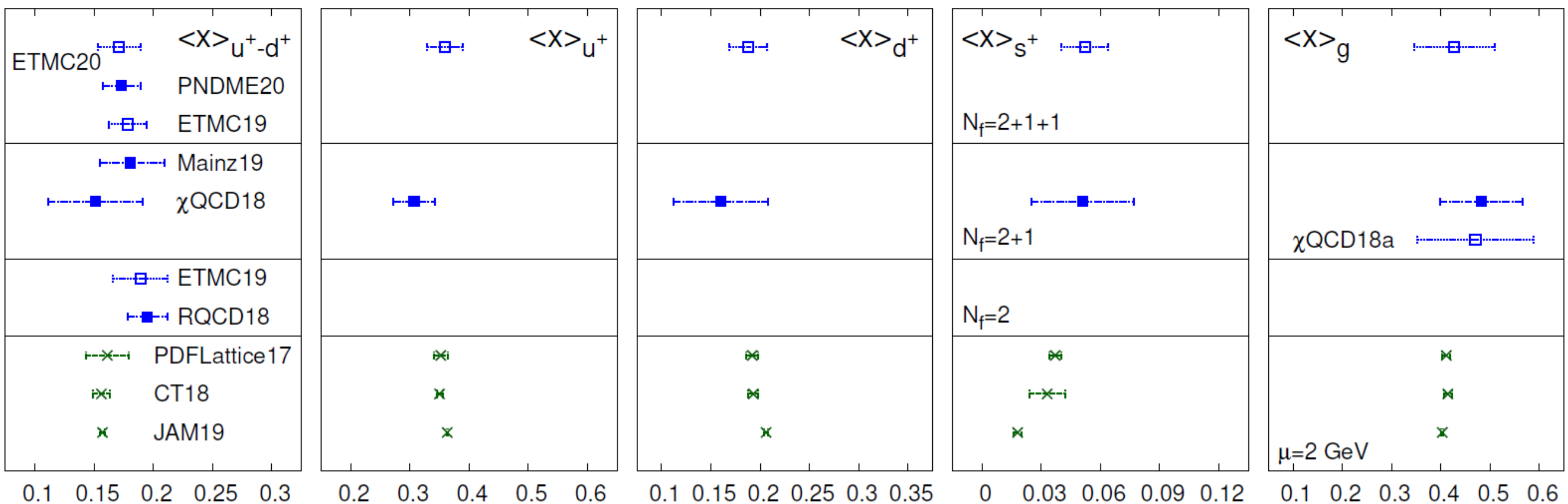
Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



LatticePDF Report, 1711.07916, 2006.08636



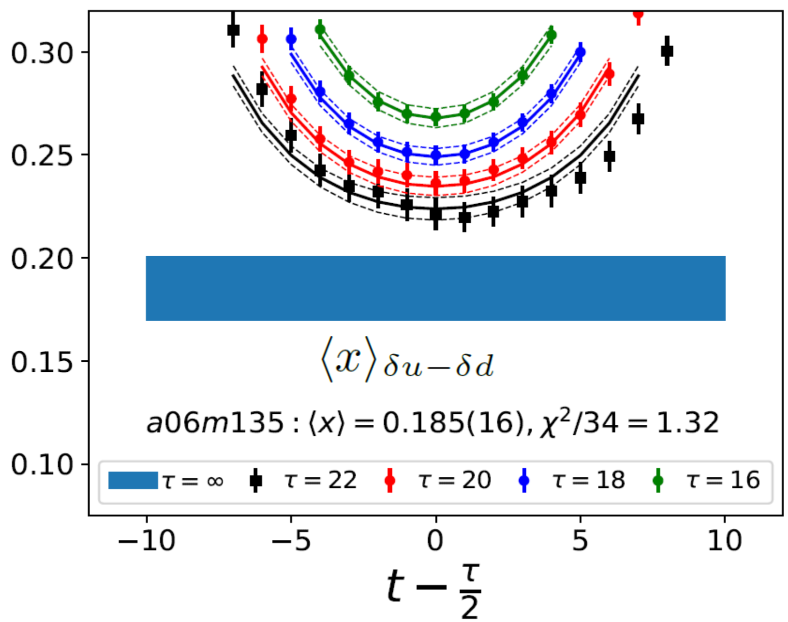
Moments of PDFs

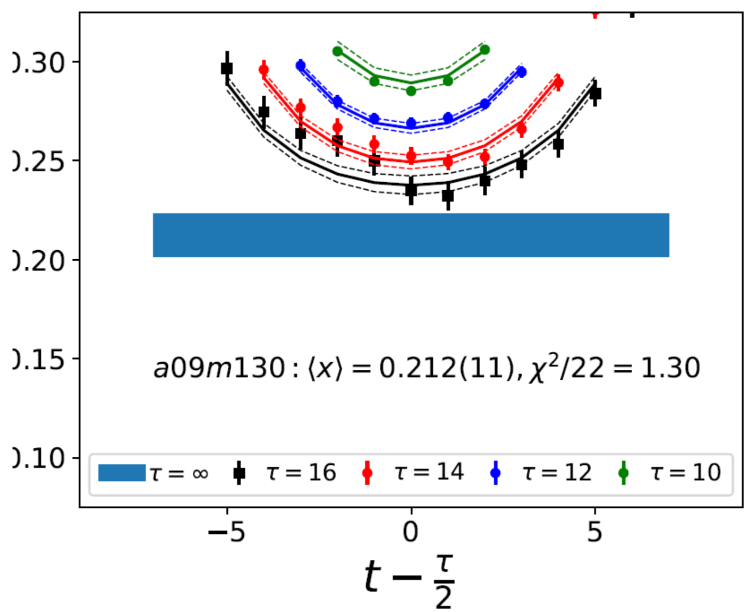
- Transversity first moments are most commonly done
- State-of-the art example
  - 2 physical pion mass ensembles
  - Extrapolate to the physical limit

$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$

Santanu Mondal et al (PNDME collaboration), 2005.13779

Plot by Santanu Mondal





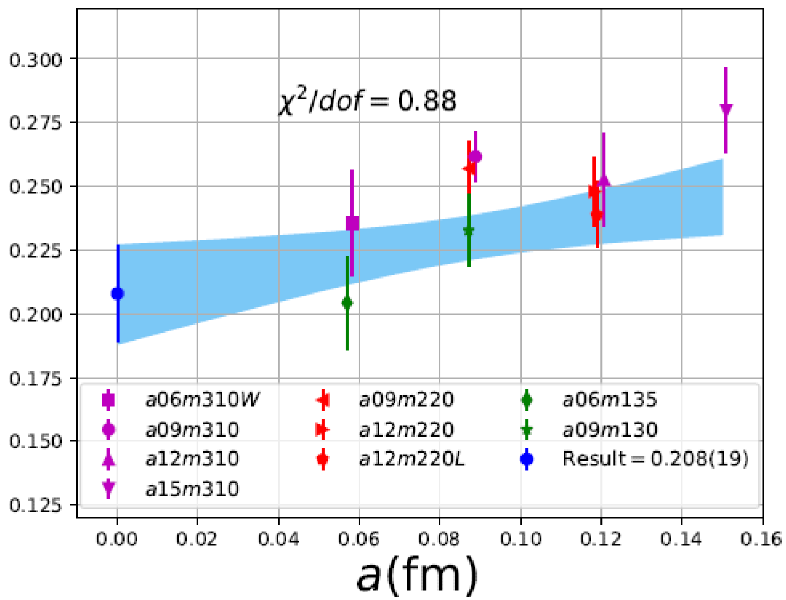
Moments of Transversity

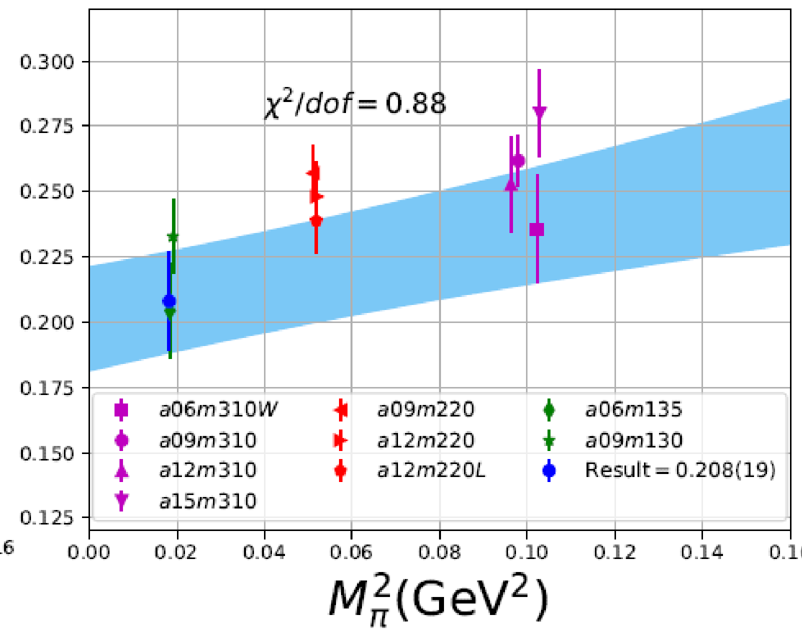
- § Transversity first moments are most commonly done
- § State-of-the art example
  - 2 physical pion mass ensembles
  - Extrapolate to the physical limit

$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$

Santanu Mondal et al (PNDME collaboration), 2005.13779

Plot by Santanu Mondal

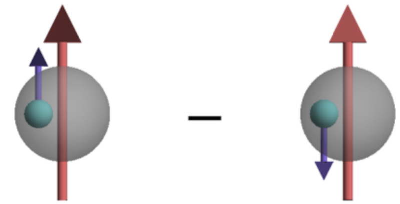




Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	0.10 — 1.1
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	-0.14 — 0.91
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	-0.97 — 0.47
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	N/A
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

Moments of PDFs

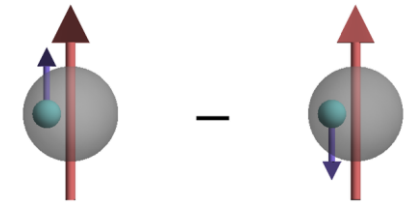
§ PDG-like rating system or average

§ LatticePDF Workshop

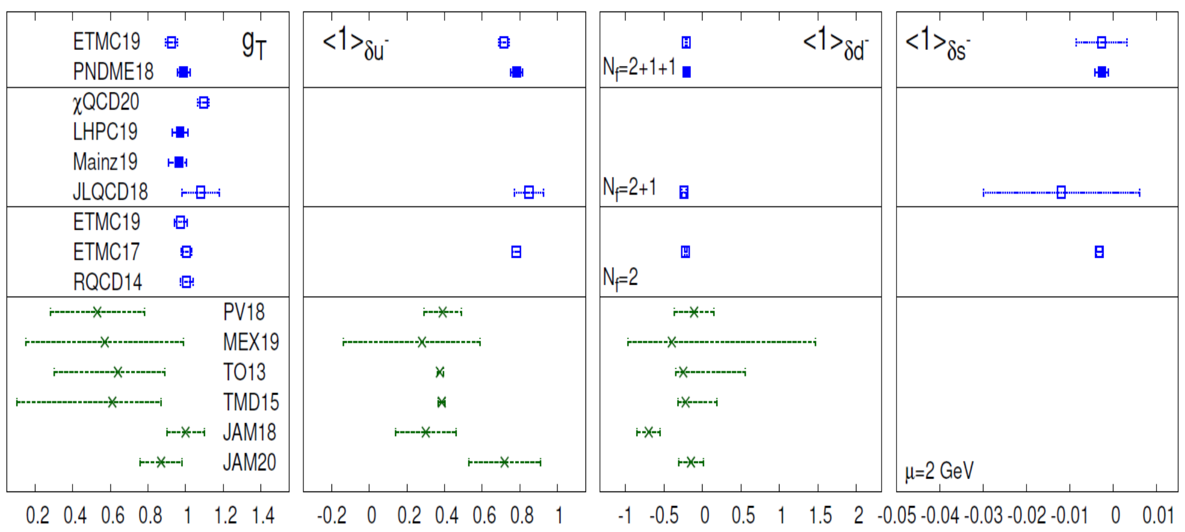
- Lattice representatives came together and devised a rating system

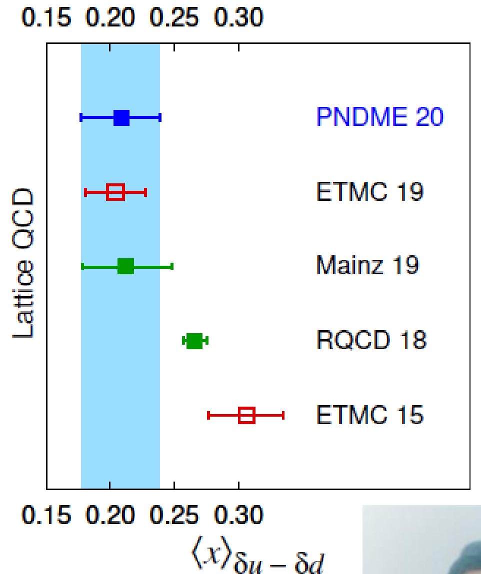
§ Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636



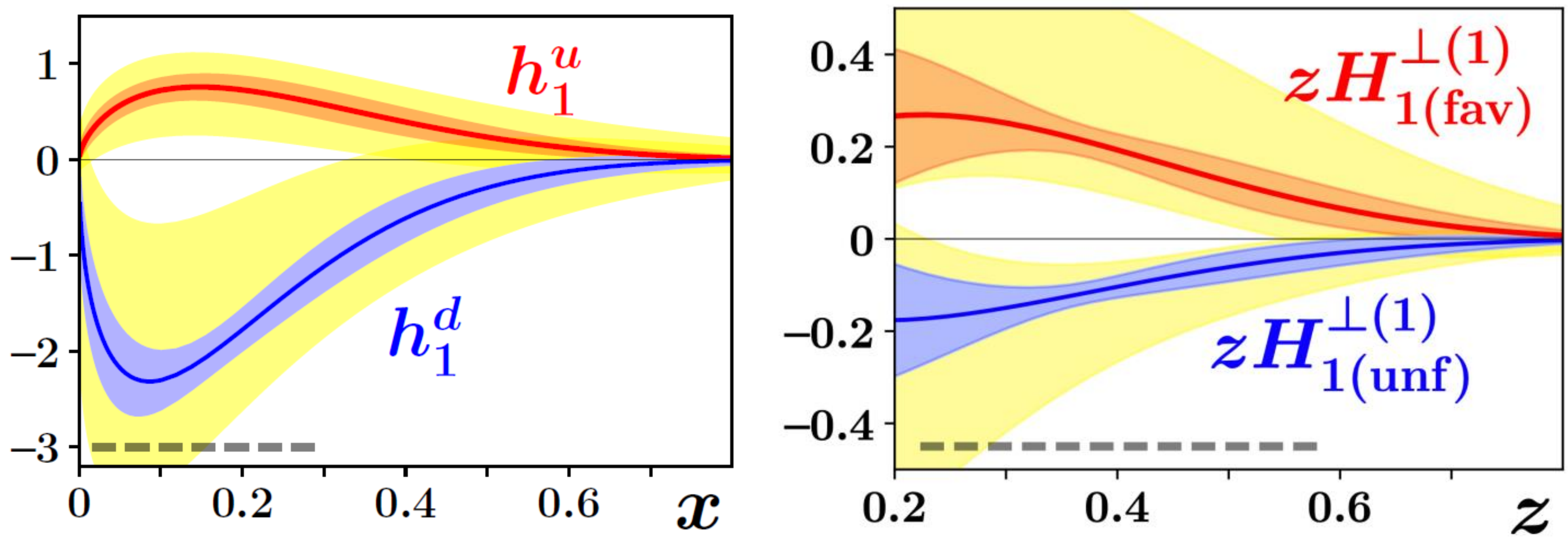


S. Mondal et al 2005.13779

From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets

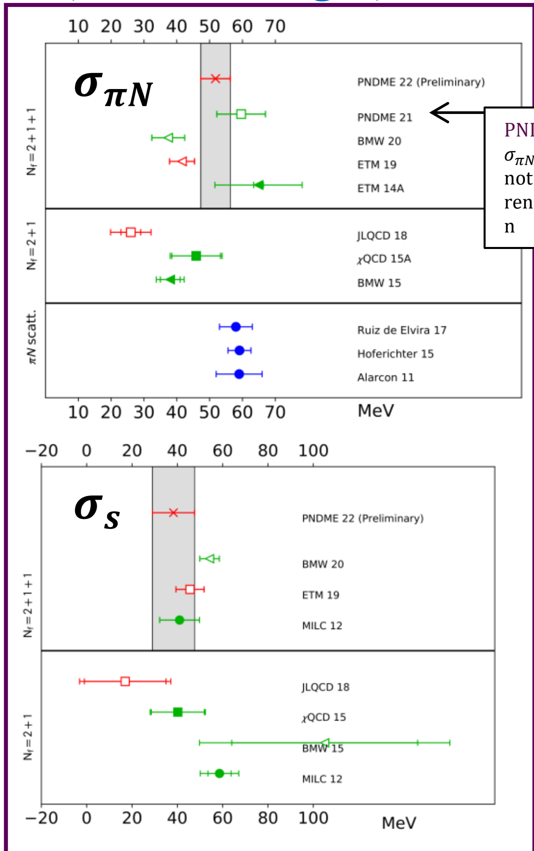


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Nucleon Flavor Diagonal Charges

Comparison with FLAG 2021 results

Nucleon sigma terms (Scalar charges)

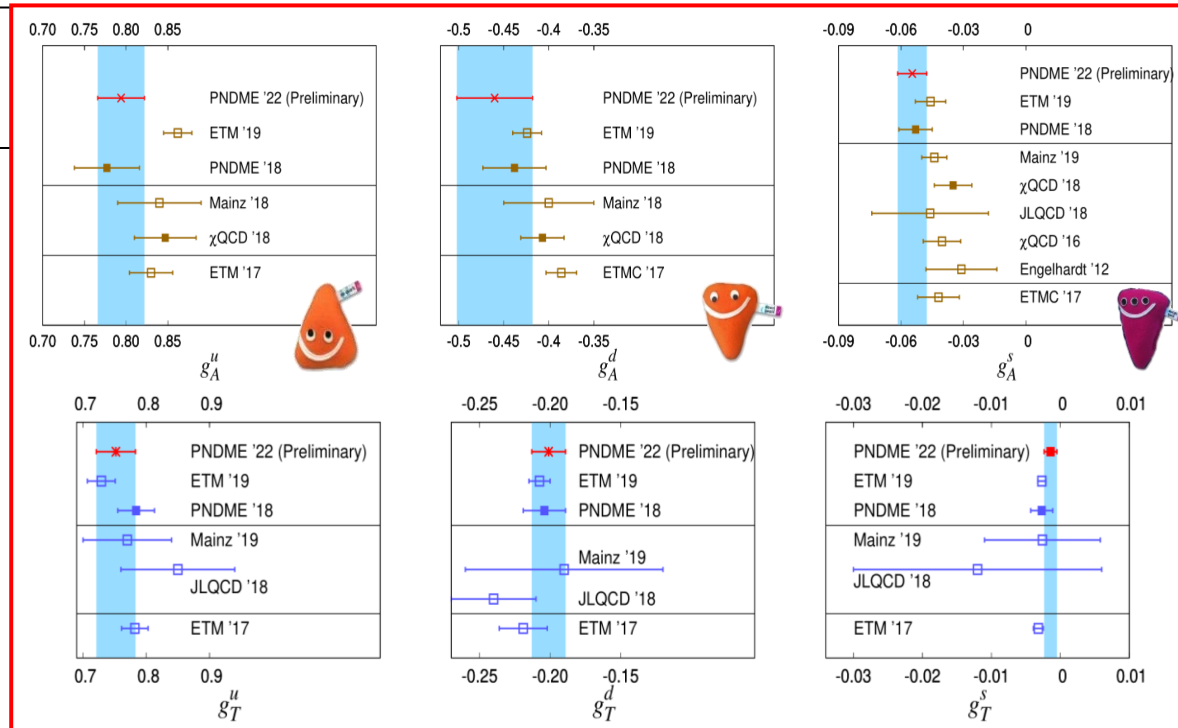


PNDME (2021) $\sigma_{\pi N}$ which does not require renormalizatio n

[PNDME, Lattice 2022 update, **preliminary**]

- Clover fermion on $N_f = 2+1+1$ HISQ ensembles
- Flavor mixing calculated nonperturbatively
- **Chiral-Continuum extrapolation** including a data at M_π^{Phys}

Axial and Tensor charges



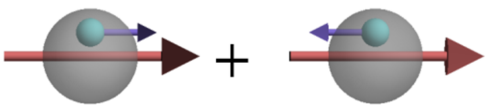
Plots by Sungwoo Park

Check out the latest updates in Sungwoo's talk next

PDFs on the Lattice

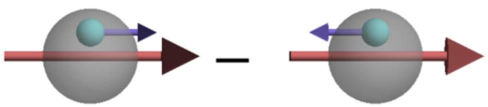
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



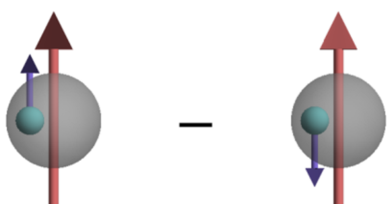
spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



spin-dependent
transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

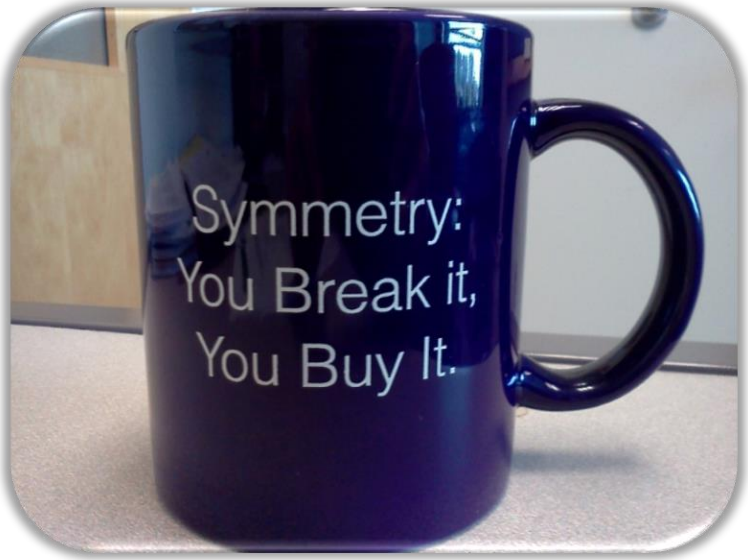
§ True distribution can only be recovered with all moments

PDFs on the Lattice

- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - Novel proposals to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Hard to separate valence contrib. from sea

W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501

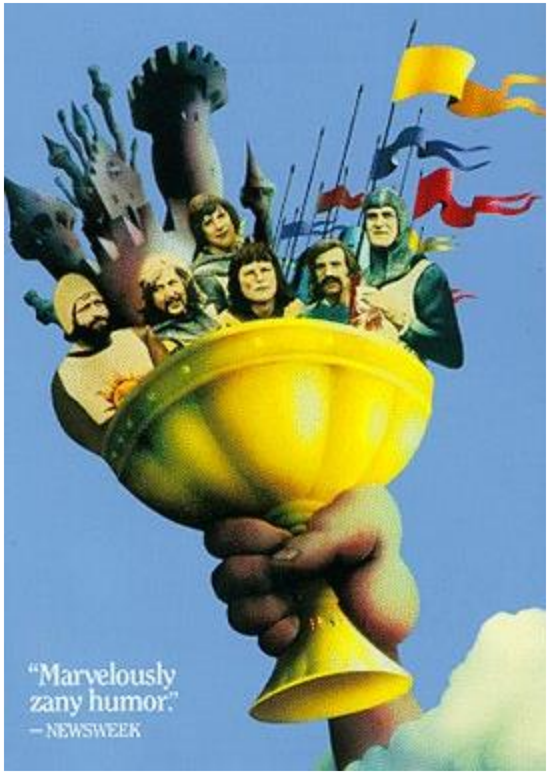
Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505



Check out David's talk in the afternoon on HOPE collaboration's latest work

Beyond Traditional Moments?

- Longstanding obstacle!
- Holy grail of structure calculations
- Applies to many structure quantities:
  - Generalized parton distributions (GPDs)
  - Transverse-momentum distributions (TMD)
  - Meson distribution amplitudes…
  - Wigner distribution



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

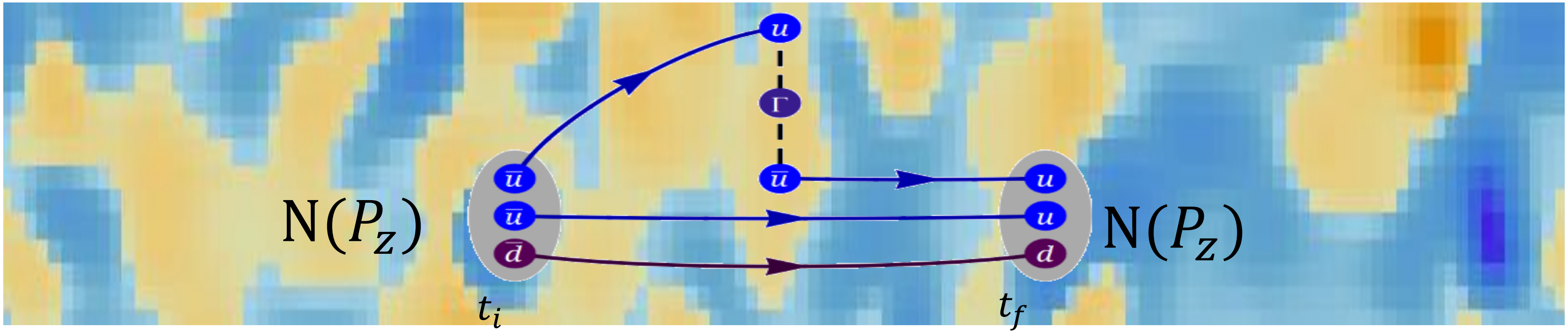
They carry secret plans to build the most powerful

Bjorken-x Dependent Hadron Structure

Biased selected results, highlighting work done by MSU students/postdocs

Lattice Parton Method

- Large-momentum effective theory (LaMET)/quasi-PDF
  (X. Ji, 2013; See 2004.03543 for review)



- Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

- Recover true distribution (take Pz → ∞ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{M_N^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P_z)^2}\right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Lattice Parton Method

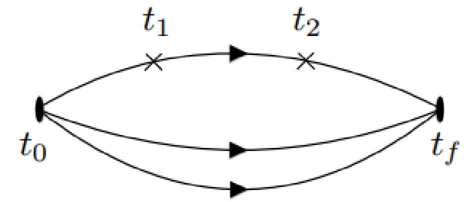
§ Short-distance factorization (SDF)

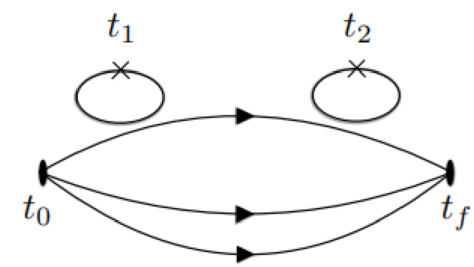
- **pseudo-PDF** method (A. Radyushkin, 2017)
- Hadronic tensor currents

  (Liu et al., hep-ph/9806491, … 1603.07352)
- Lattice cross-section method (LCS)

  (Y Ma and J. Qiu, 2014, 2017)
- Euclidean correlation functions

  (RQCD, 1709.04325)
- Compton amplitude approach (QCDSF, 1703.01153)



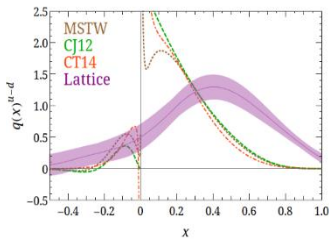


Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

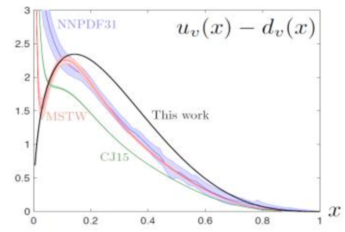
Lattice Parton Calculations

§ Rapid developments!

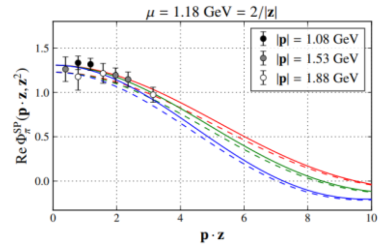
First unpol. PDF lattice calculation (2014)



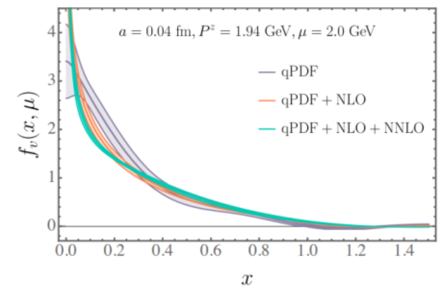
First lattice pseudo-PDFs (2017)



Euclidean correlation functions (2018)

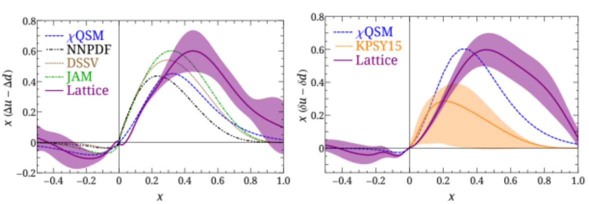


1st NNLO PDF (2022)

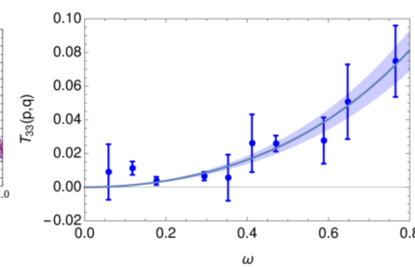


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

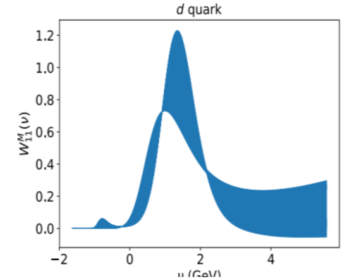
Pol. PDFs and mass corrections (2016)



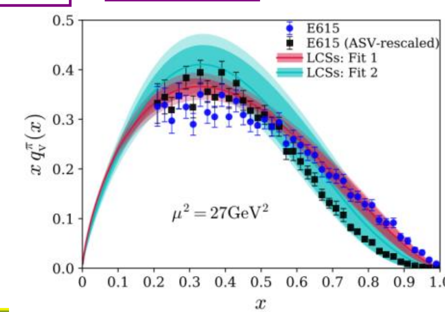
Compton amplitude (2017)



Hadronic tensor (2019)

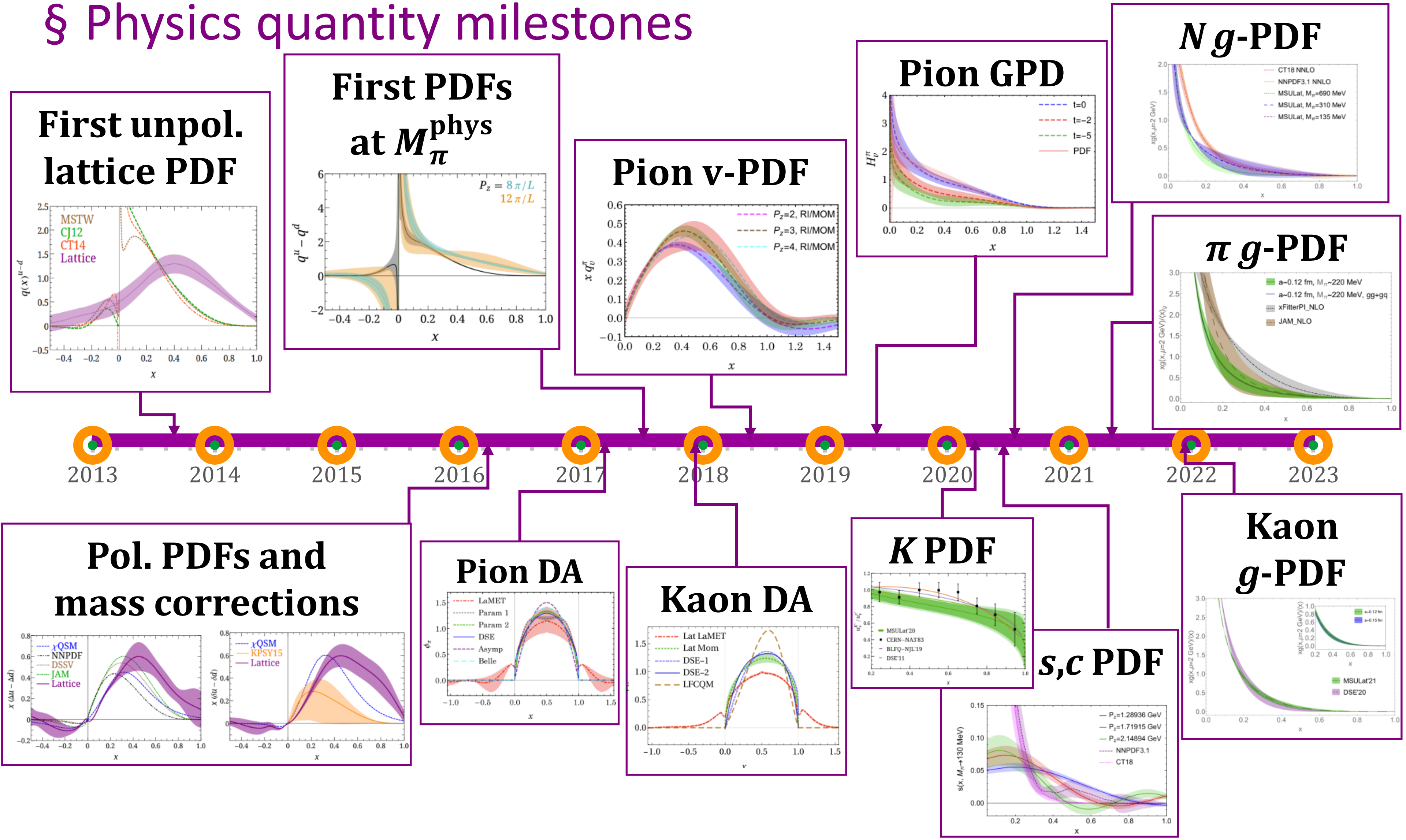


LCS (2020)



Lattice Parton Calculations

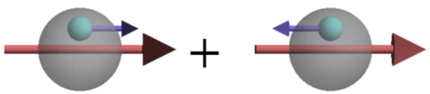
§ Physics quantity milestones

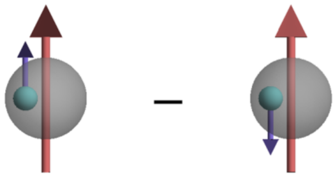


Lattice Example Results

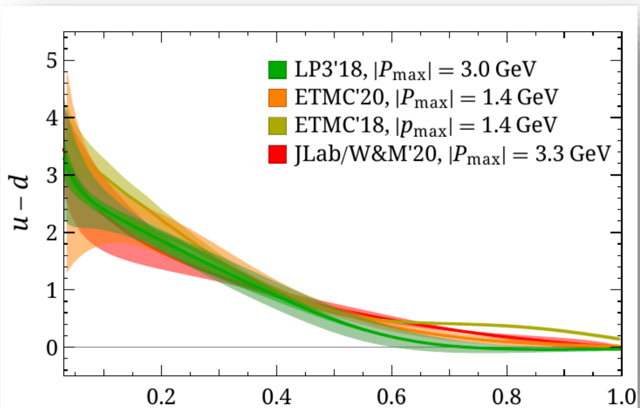
§ Summary of physical pion mass PDFs results

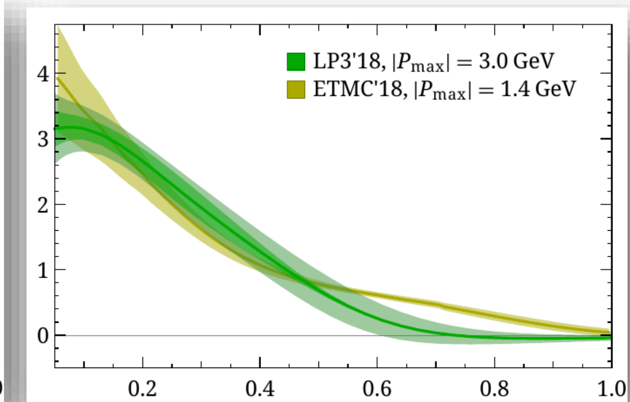
unpolarized	longitudinally polarized	transversely polarized

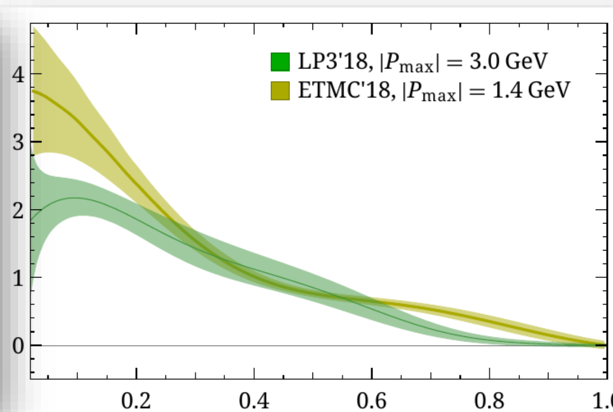


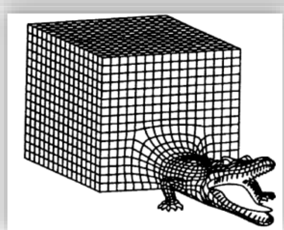


$u(x) - d(x)$ $\Delta u(x) - \Delta d(x)$ $\delta u(x) - \delta d(x)$









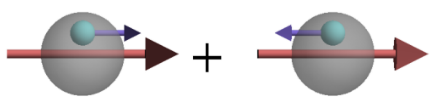
Finite volume,
Discretization,
...

2006.08636 (PDFLattice2019)

Lattice Example Results

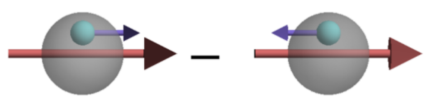
§ Summary of physical pion mass PDFs results

unpolarized



$u(x) - d(x)$

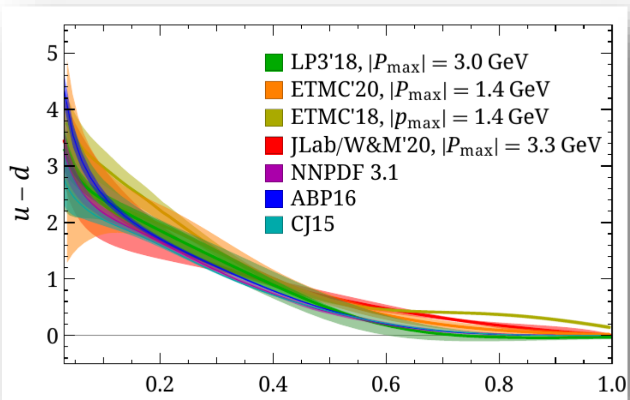
longitudinally polarized

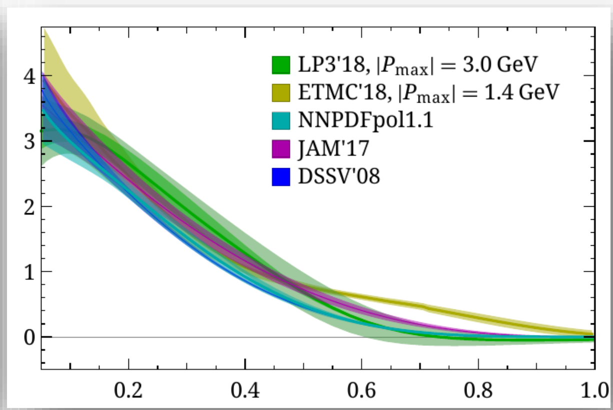


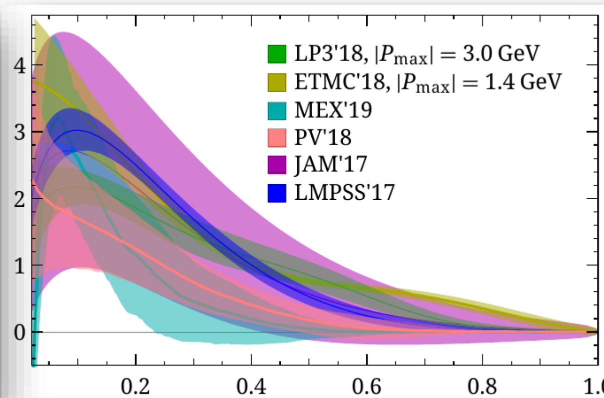
$\Delta u(x) - \Delta d(x)$

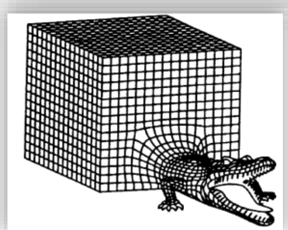
transversely polarized

$\delta u(x) - \delta d(x)$







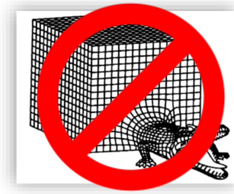


Finite volume,
Discretization,
…

2006.08636 (PDFLattice2019)

First Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

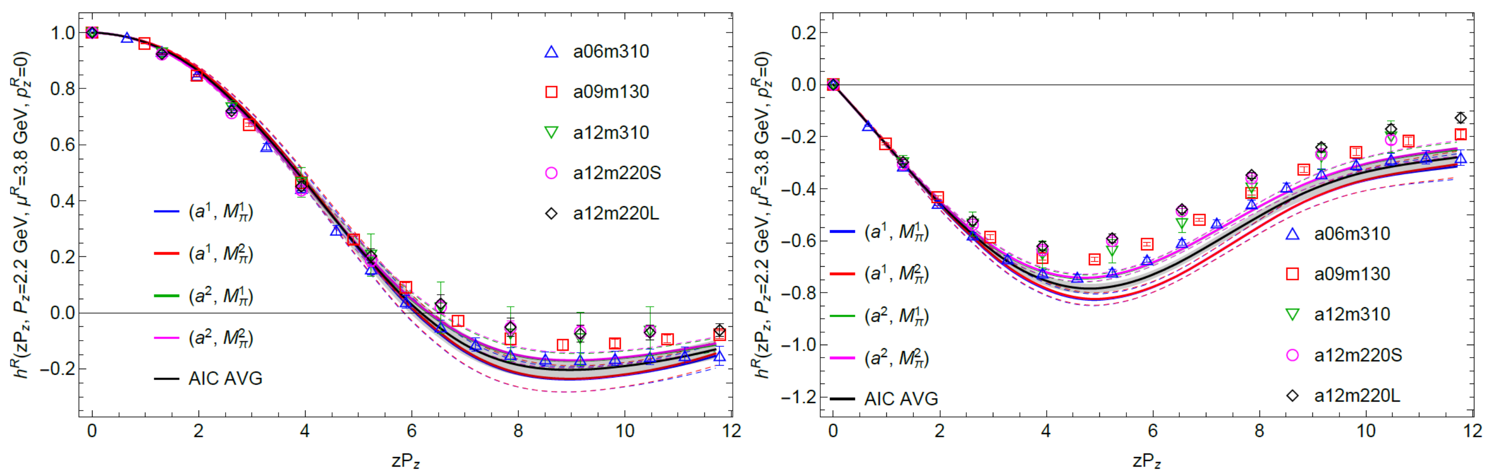


- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm,
  - $M_\pi \in \{135, 220, 310\}$-MeV pion,
  - $M_\pi L \in \{3.3, 5.5\}$.
  - $P_z \approx 2$ GeV

2011.14971, HL et al (MSULat)

- Naïve extrapolation to physical-continuum limit

Quantities that can be calculated on the lattice



First Continuum PDF

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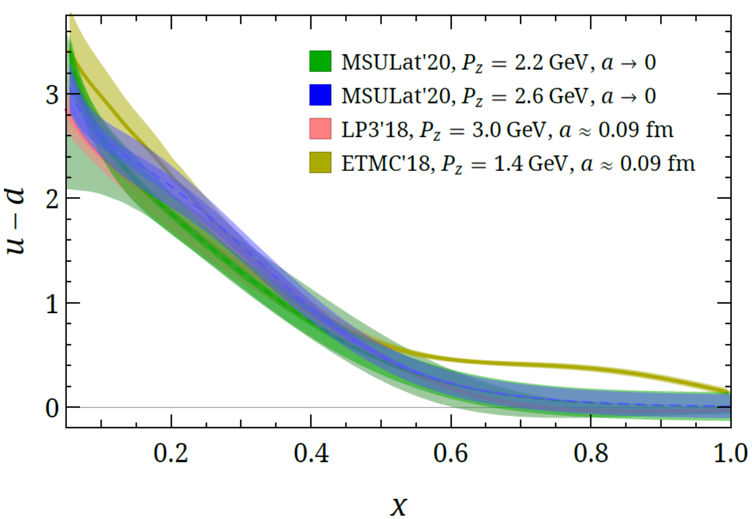
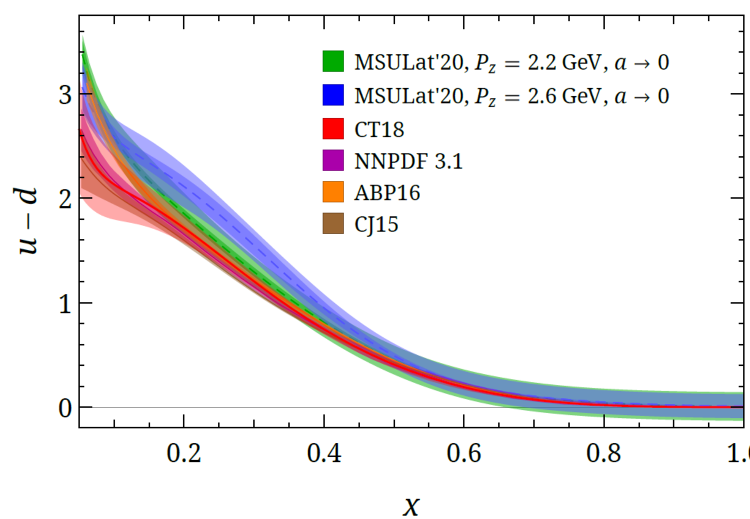
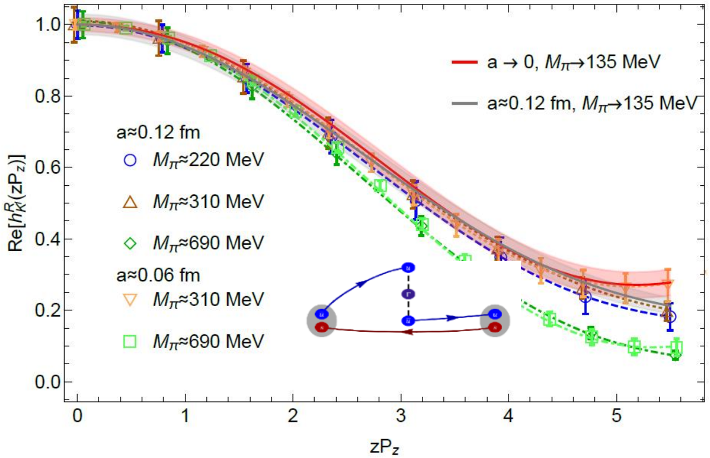
Wanted PDFs, GPDs, etc…

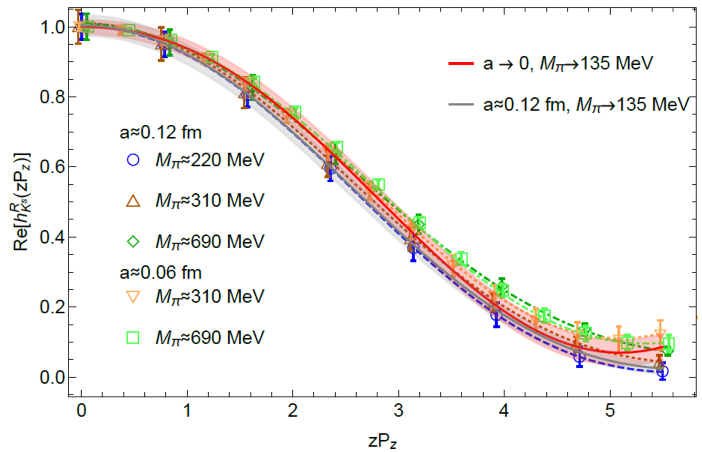




Meson Valence-quark PDFs

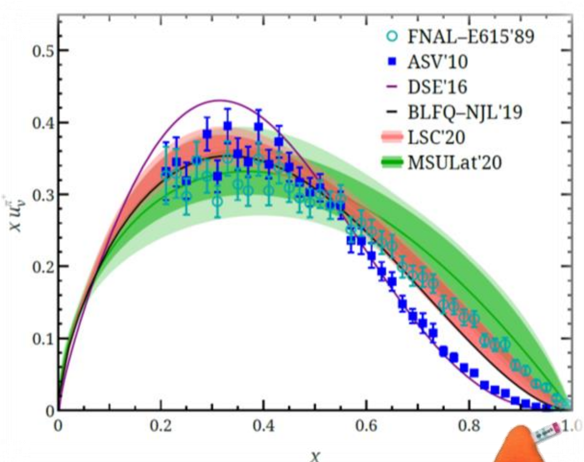
§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

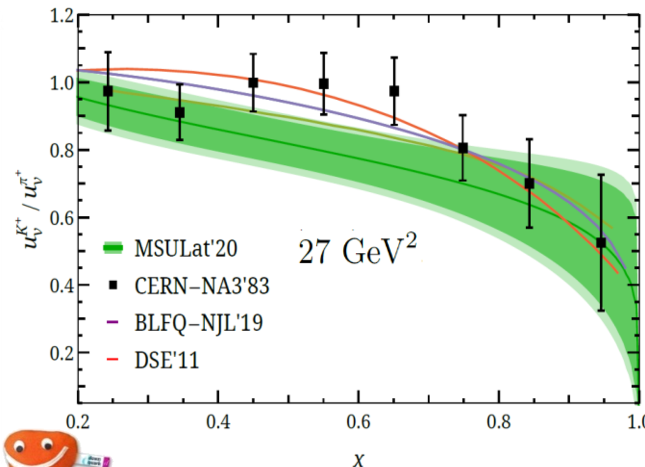


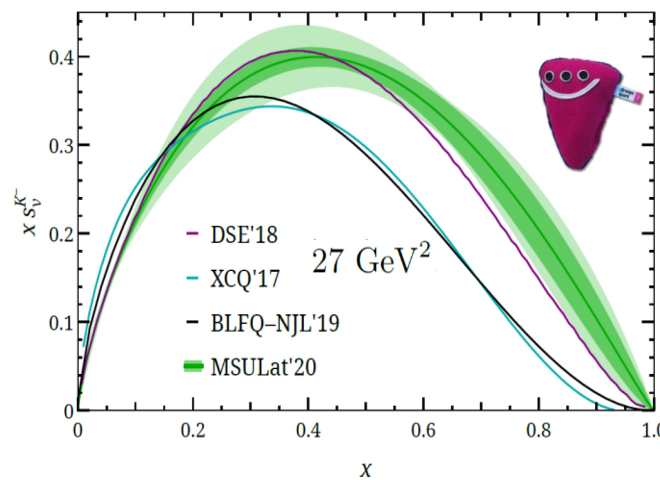


Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

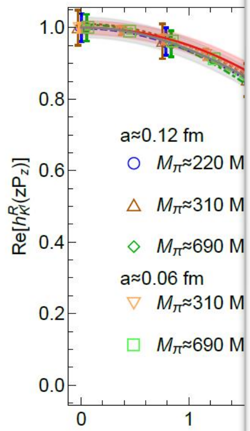




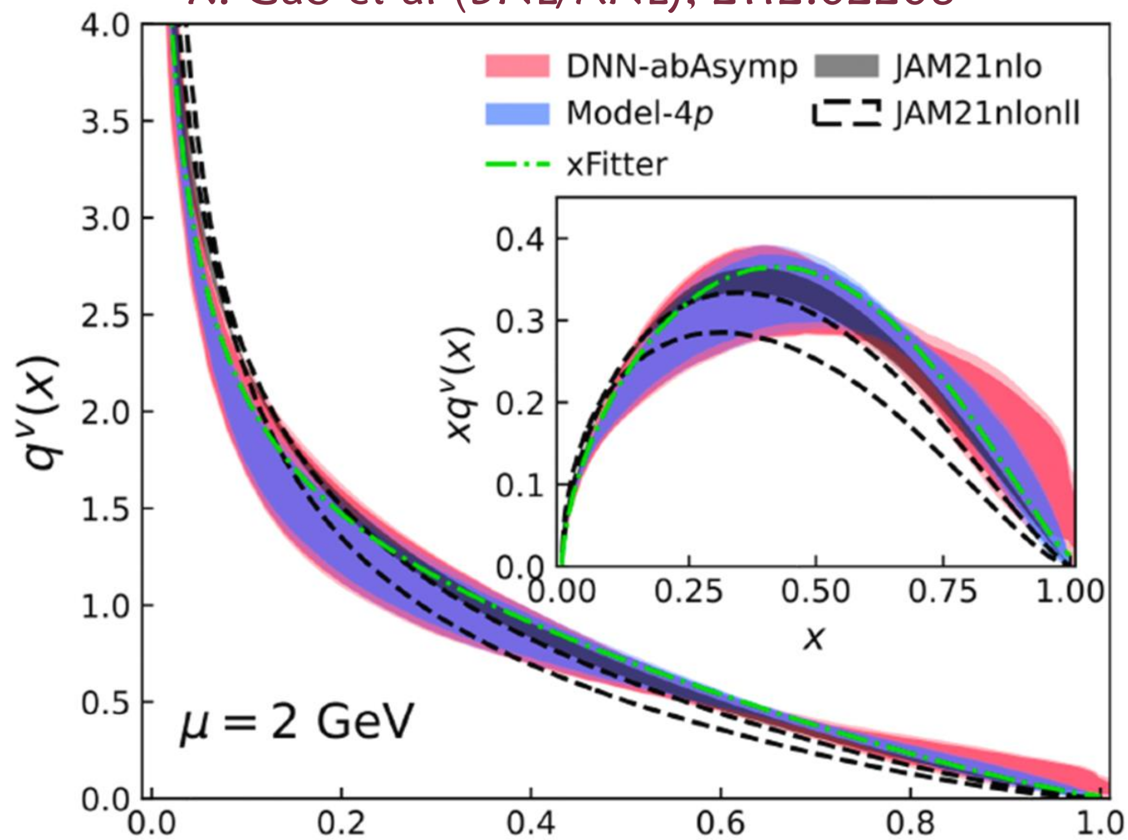


Meson Valence-quark PDFs

§ Pion/Ka … um limit



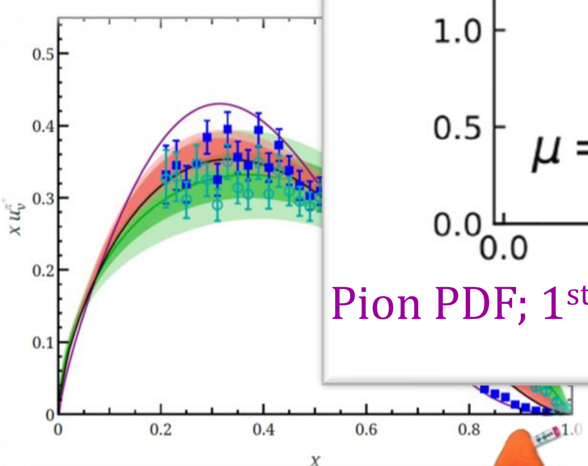
X. Gao et al (BNL/ANL), 2112.02208

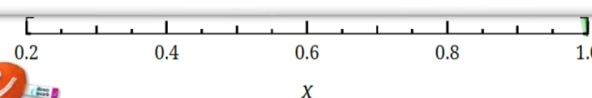


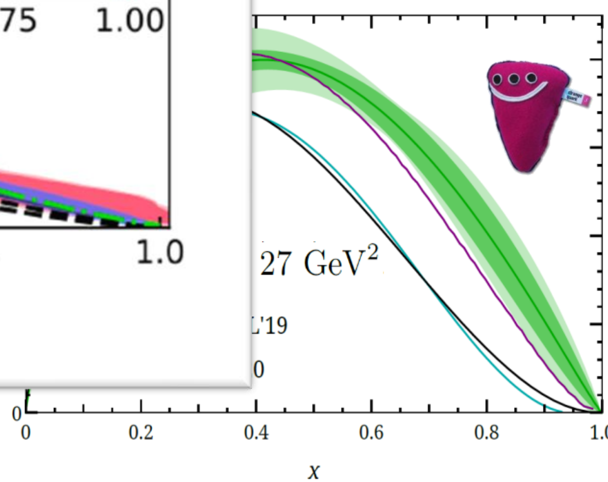
Pion PDF; 1st NNLO matching

Quantities that can be calculated on the lattice

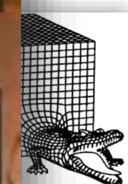
Wanted PDFs, GPDs, etc…







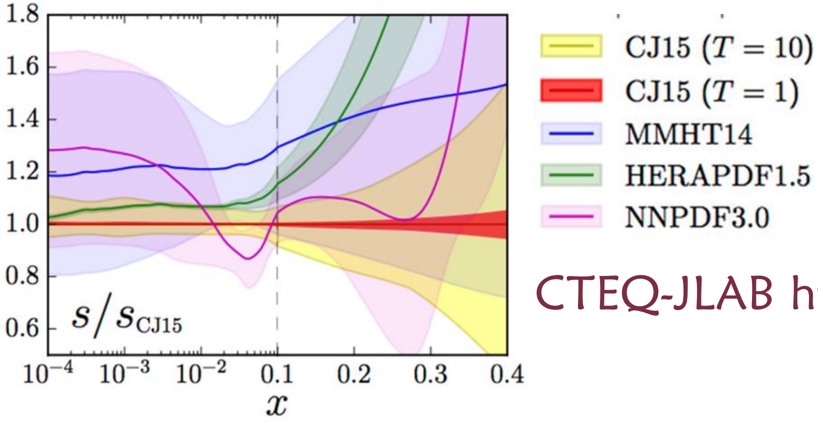
Str DFs



esults

First Lattice Strange PDF

§ Large uncertainties in global PDFs



- Assumptions imposed due to lack of precision data

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

CTEQ-JLAB https://www.jlab.org/theory/cj/

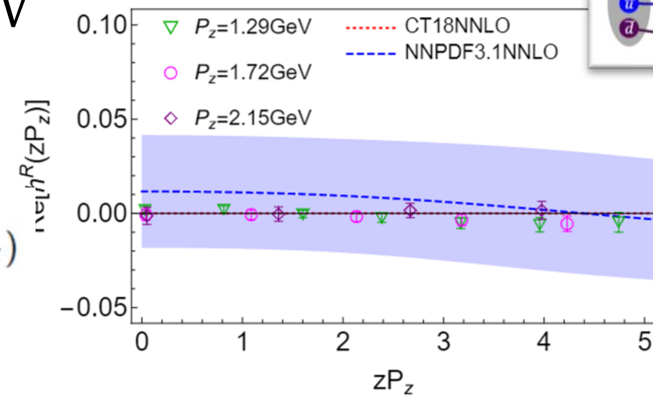
First Lattice Strange PDF

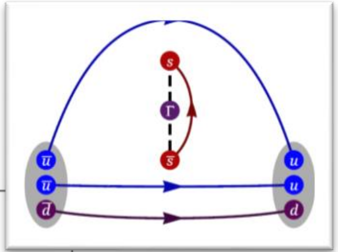
§ Results by MSULat/quasi-PDF method

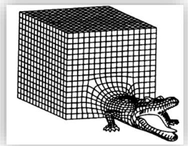
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

2005.01124, R. Zhang et al (MSULat)

$$\mathrm{Re}[h(z)] \propto \int dx\, (s(x) - \bar{s}(x)) \cos(xzP_z)$$







Quantities that can be calculated on the lattice

First Lattice Strange PDF

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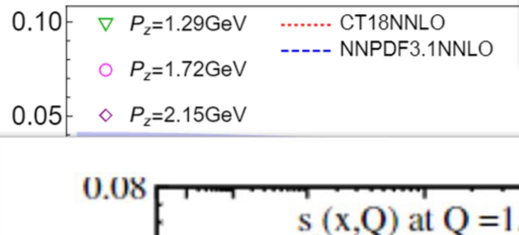
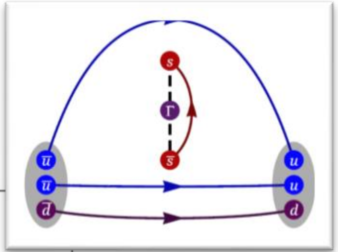
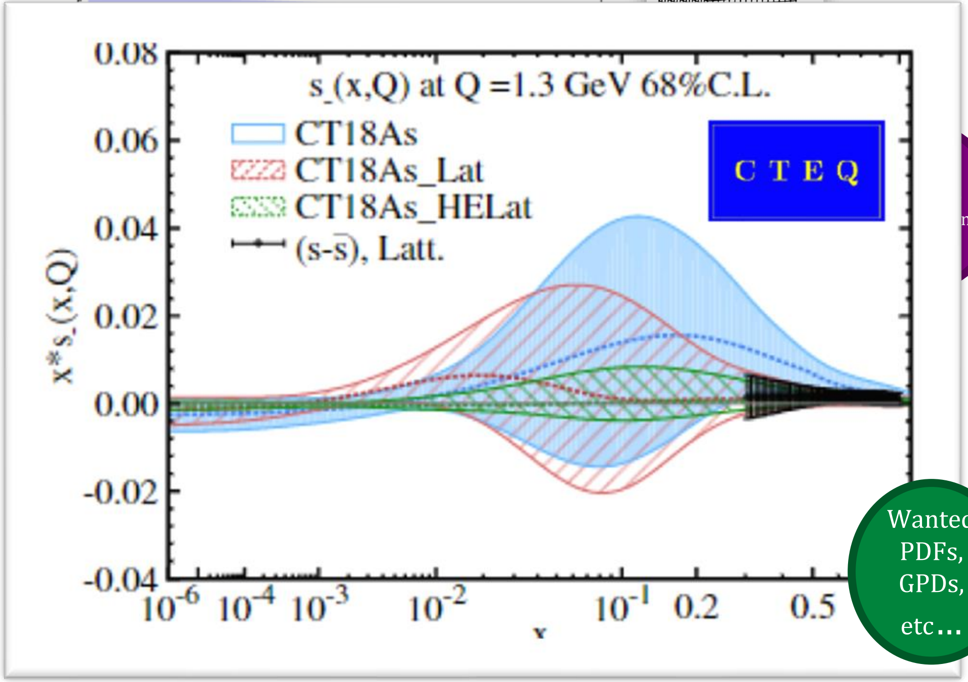




§ From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2 P_z^2}\right)$$

T. Hou, HL, M. Yan, C. Yuan, 2204.07944



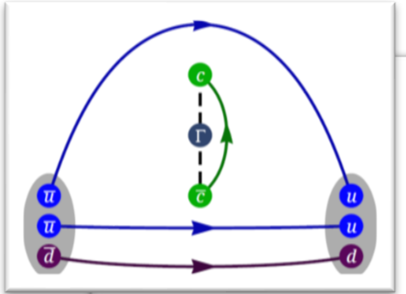
Wanted PDFs, GPDs, etc…

§ The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

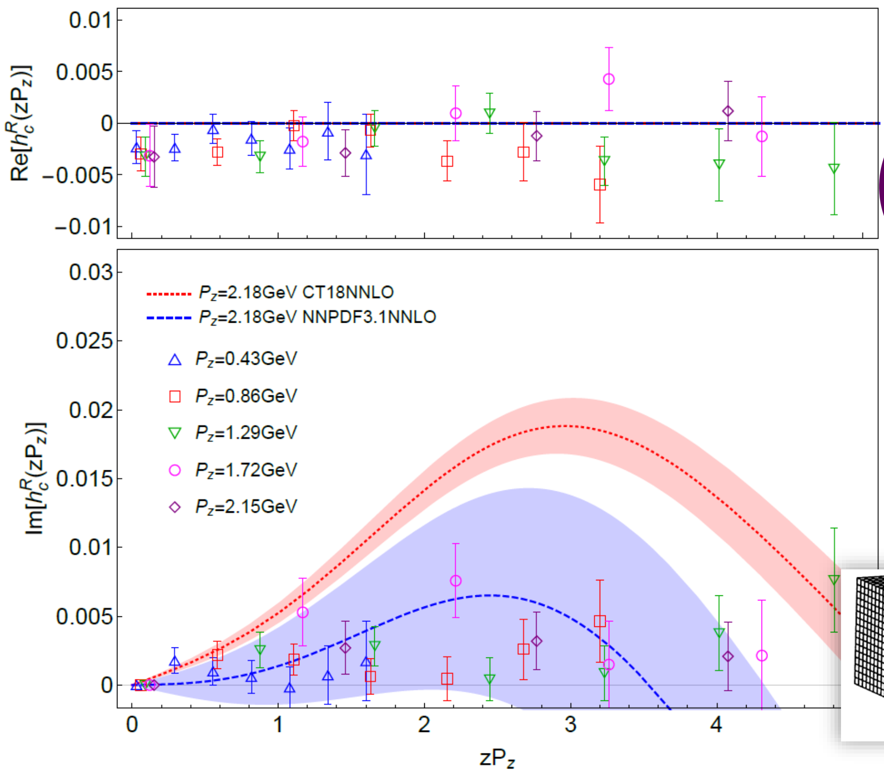
First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

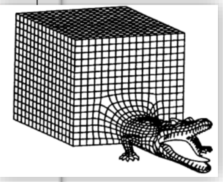
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Quantities that can be calculated on the lattice



First Lattice Charm PDF

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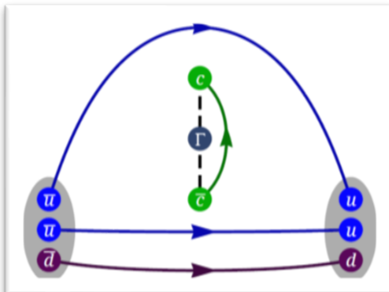
- Results by MSULat/quasi-PDF method
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

2005.01124, R. Zhang et al (MSULat)



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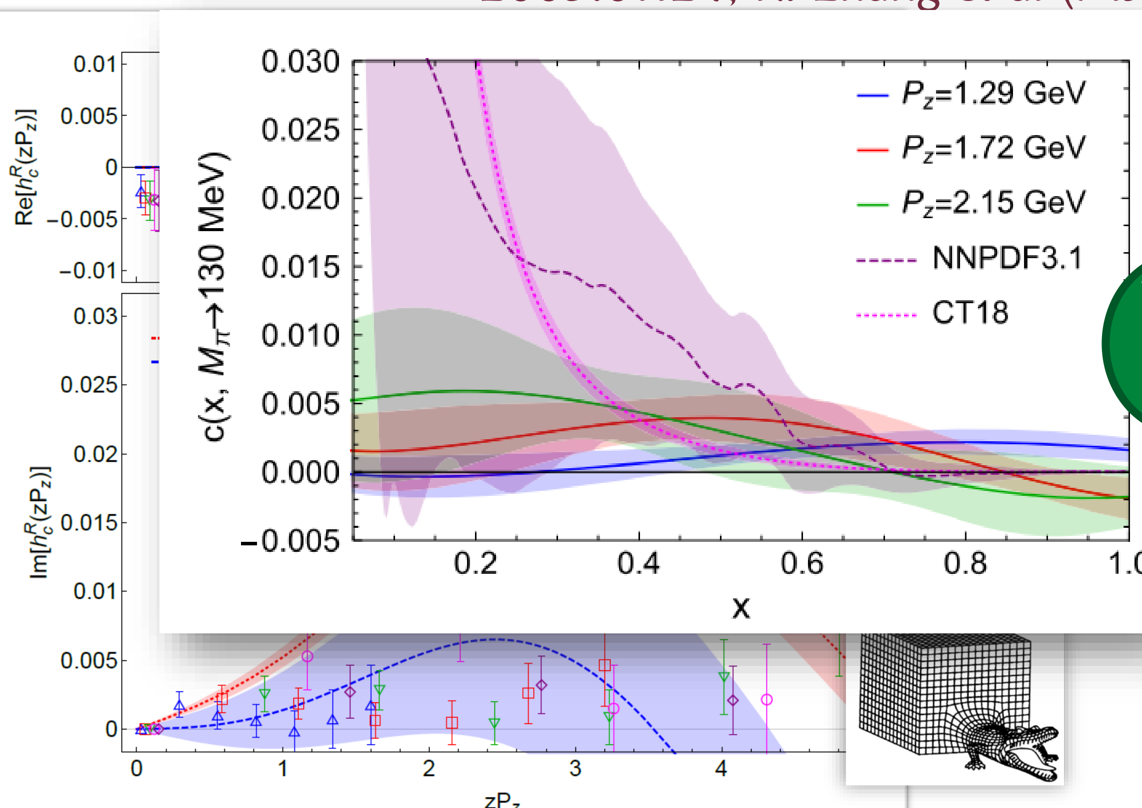
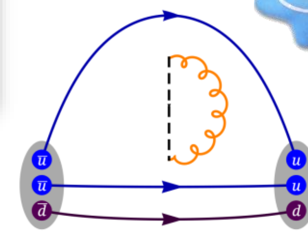


Wanted PDFs, GPDs, etc…

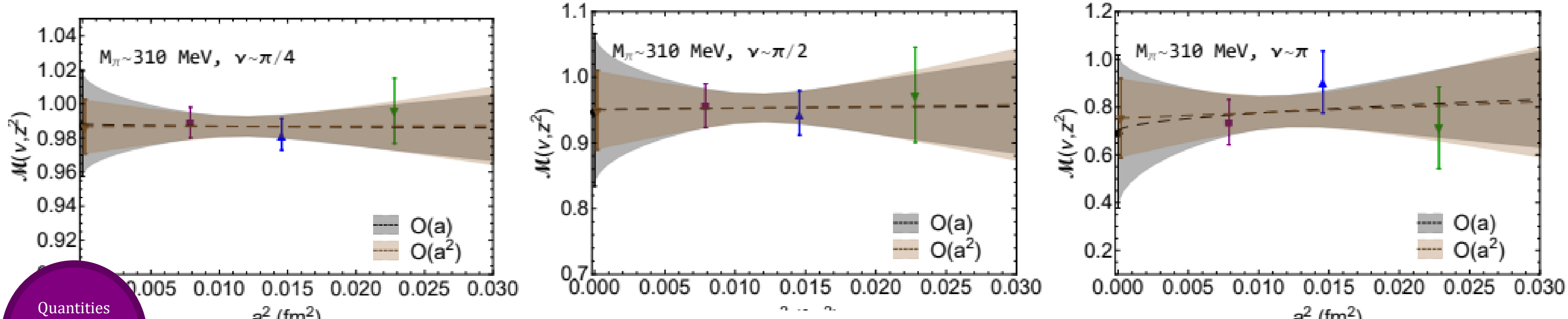
Gluon PDF in Nucleon

§ Continuum Gluon PDF w/ pseudo-PDF

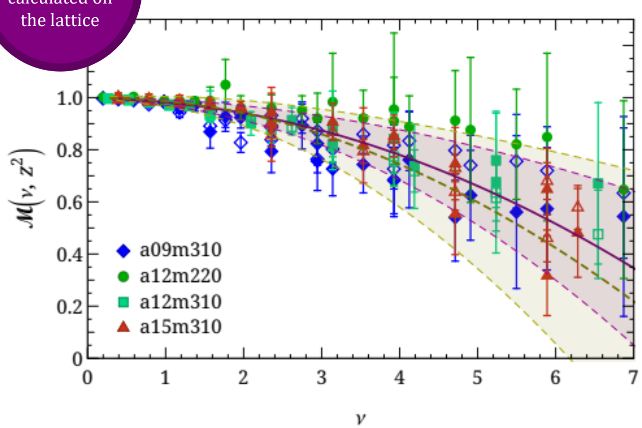


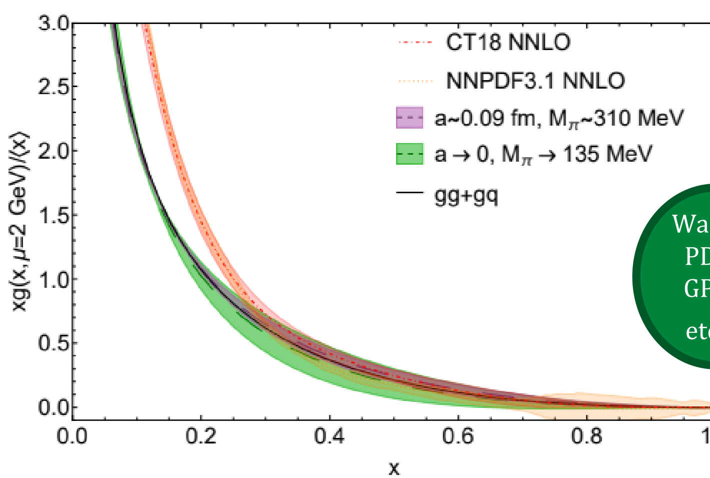
- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

  arXiv:2210.09985



Quantities that can be calculated on the lattice

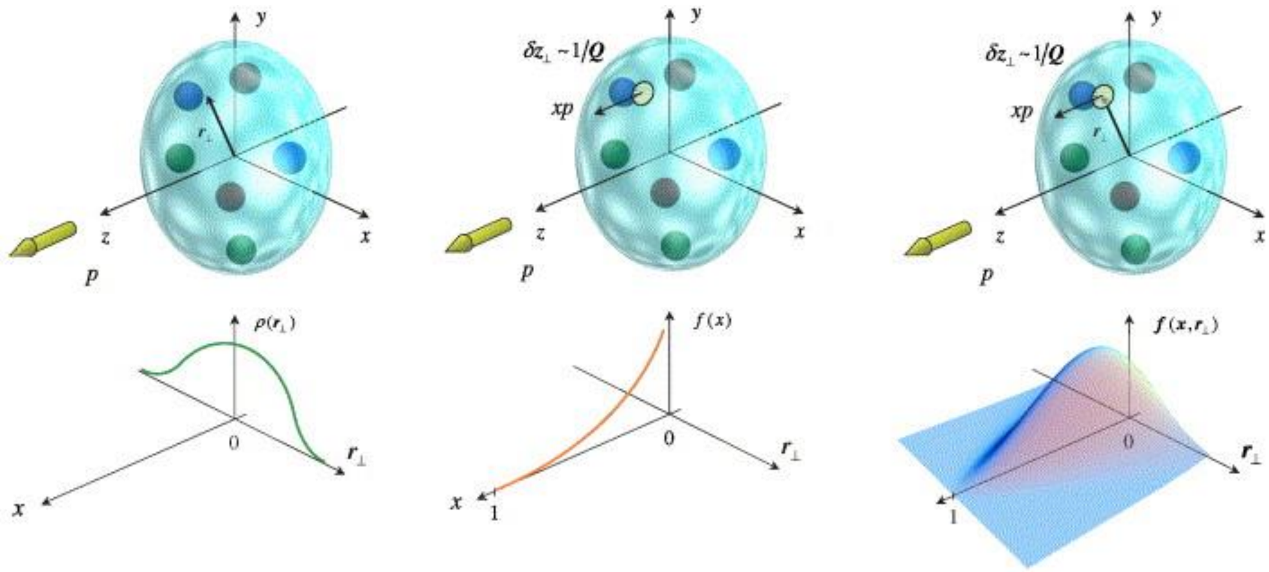




Wanted PDFs, GPDs, etc…

G: Bill Good

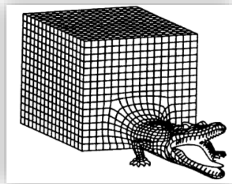
Bjorken-x Dependent GPDs



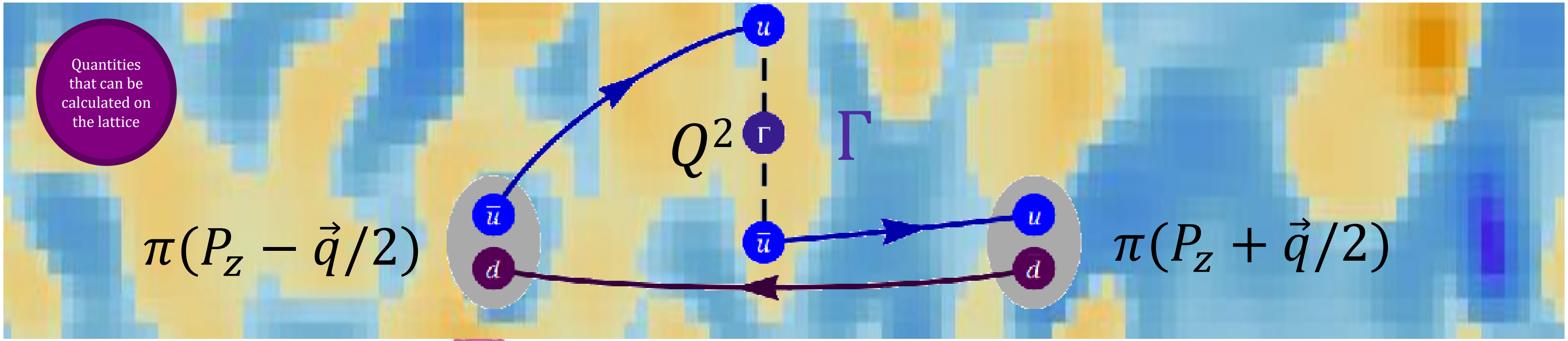
Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

Generalized Parton Distributions

Single-ensemble result



finite-volume,
discretization,
heavy quark mass,
...



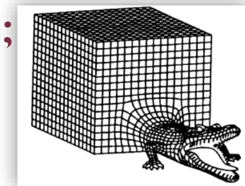
First Lattice GPDs

§ First glimpse into pion GPD using Quasi-PDF/LaMET

- Lattice details: clover/HISQ, **0.12fm**, **310-MeV** pion mass

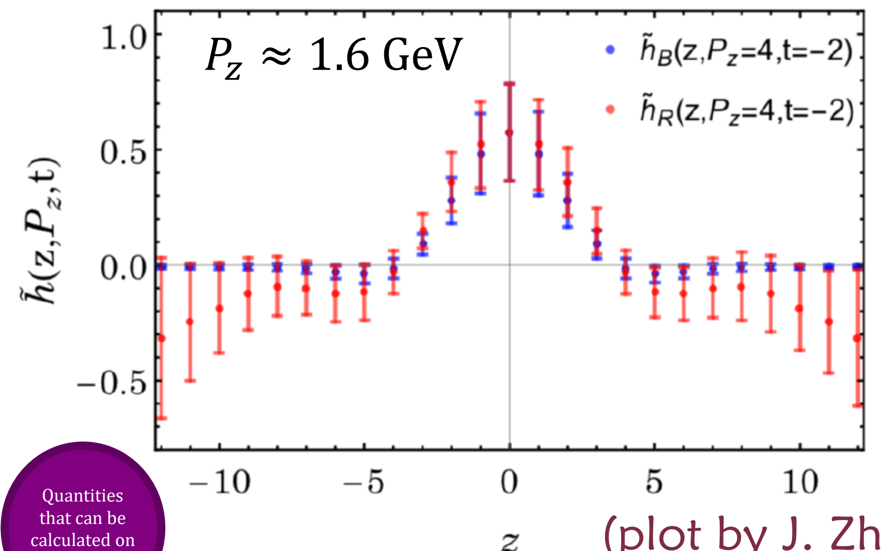
$P_z \approx 1.3, 1.6$ GeV

MILC, Phys. Rev. D, 82 (2010), 074501; Phys. Rev. D, 87 (2013), 0545056



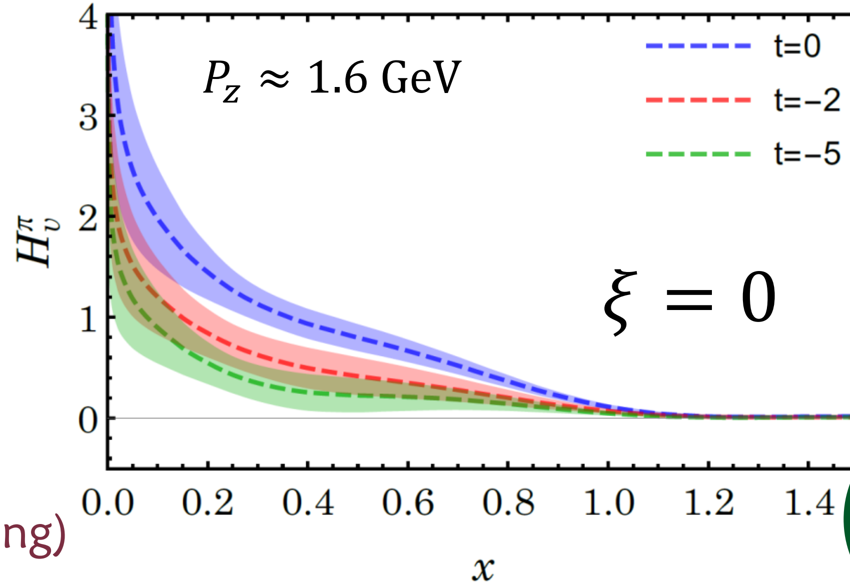
J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



Quantities that can be calculated on the lattice

(plot by J. Zhang)

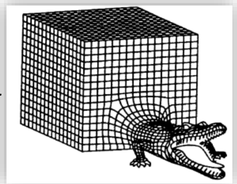


Wanted PDFs, GPDs, etc…

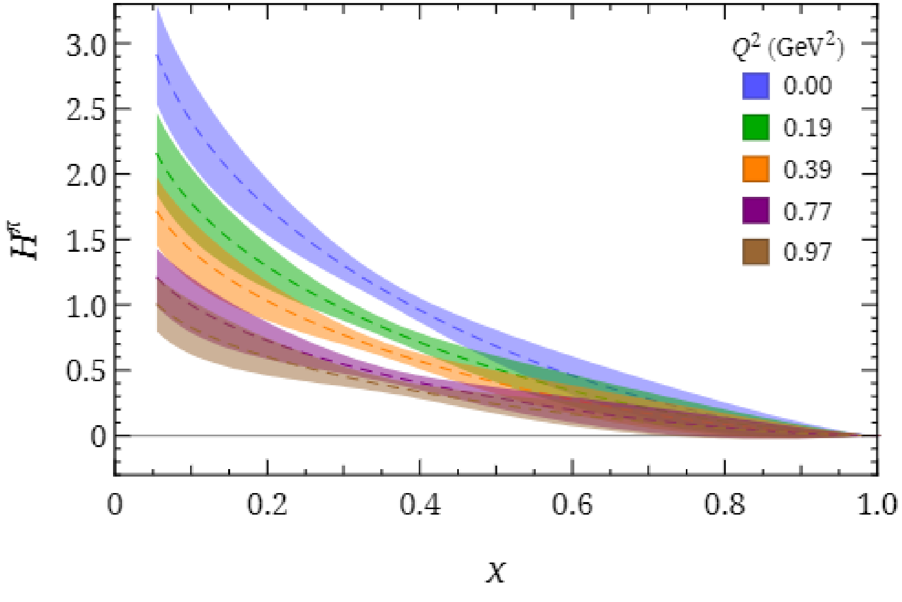
Valence-Quark Pion GPD

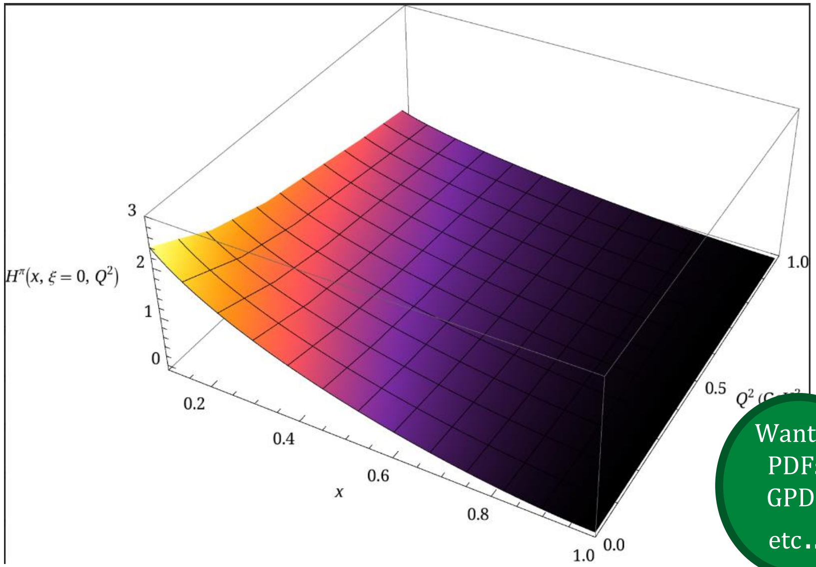
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results



finite-volume, discretization,





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HL (MSULat), Phys. Lett. B 846 (2023) 138181

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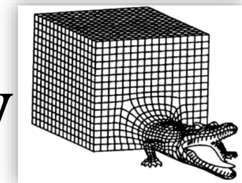

finite-volume,
discretization,

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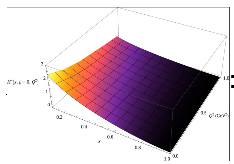
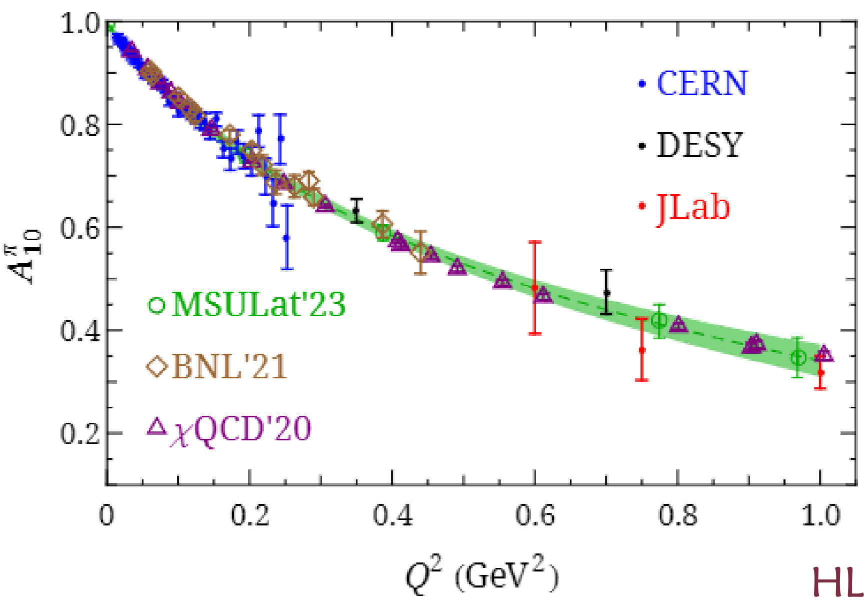
$\int_{-1}^{+1} dx\, x^{n-1}$  $= A^{\pi}_{ni}(t)$



HL (MSULat), Phys. Lett. B 846 (2023) 138181

Pion Tomography

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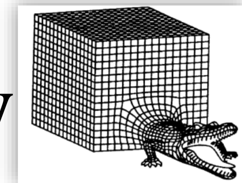

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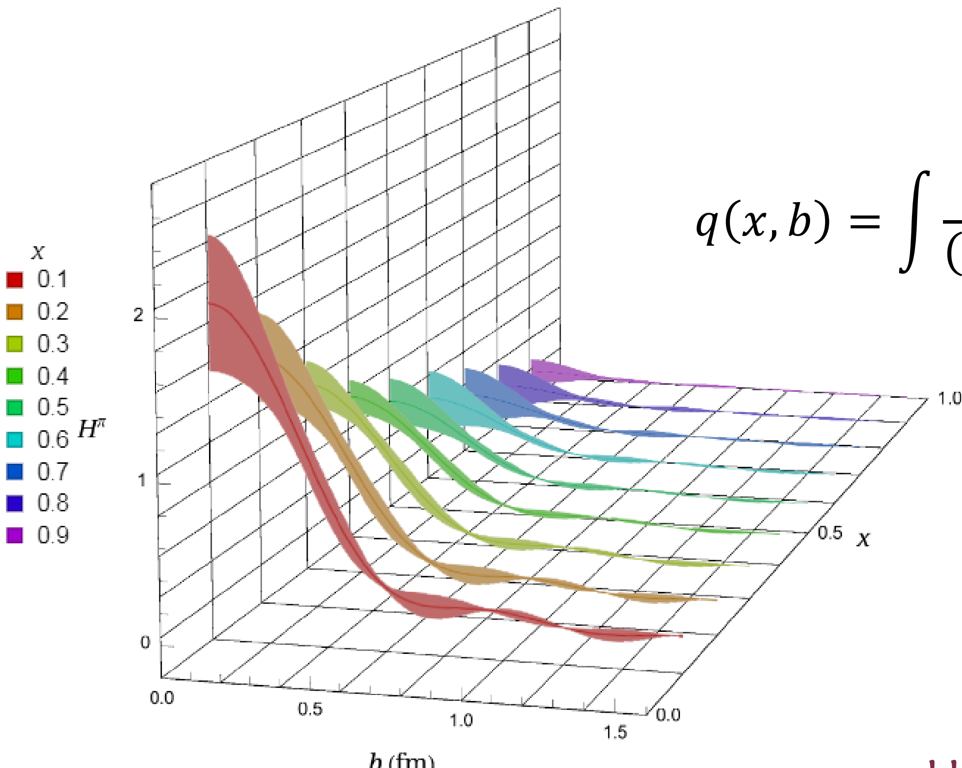
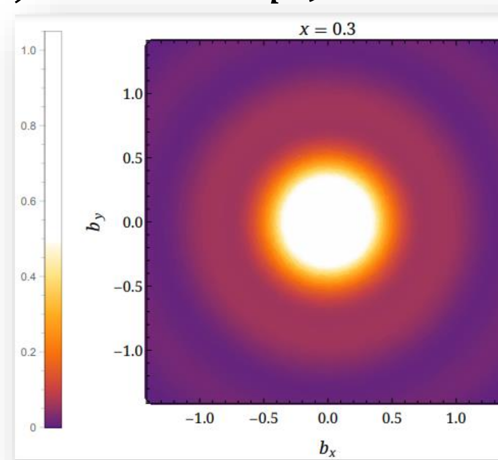


$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x,\xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$

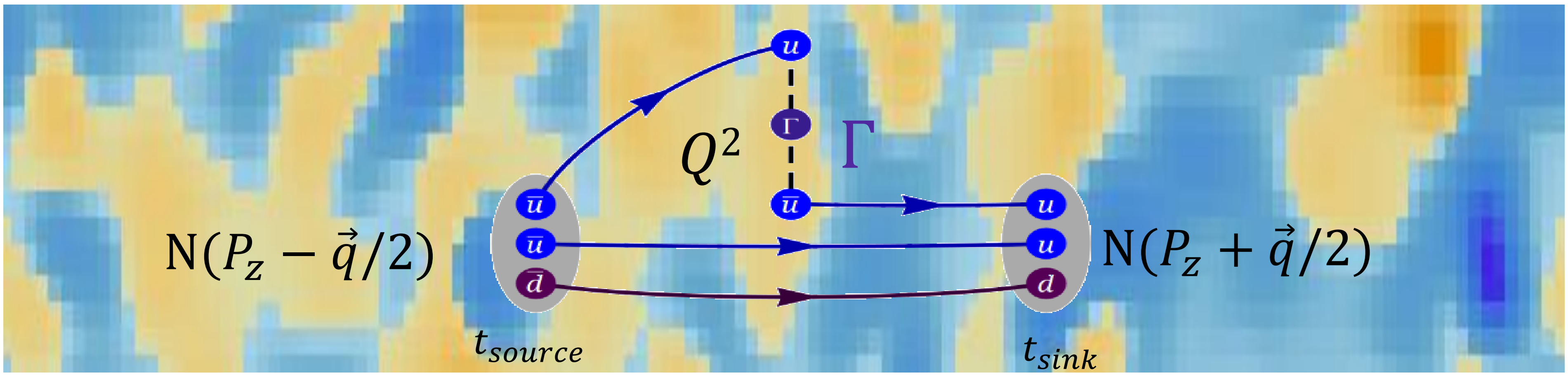


HL (MSULat), Phys. Lett. B 846 (2023) 138181

2020: Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(H(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + E(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

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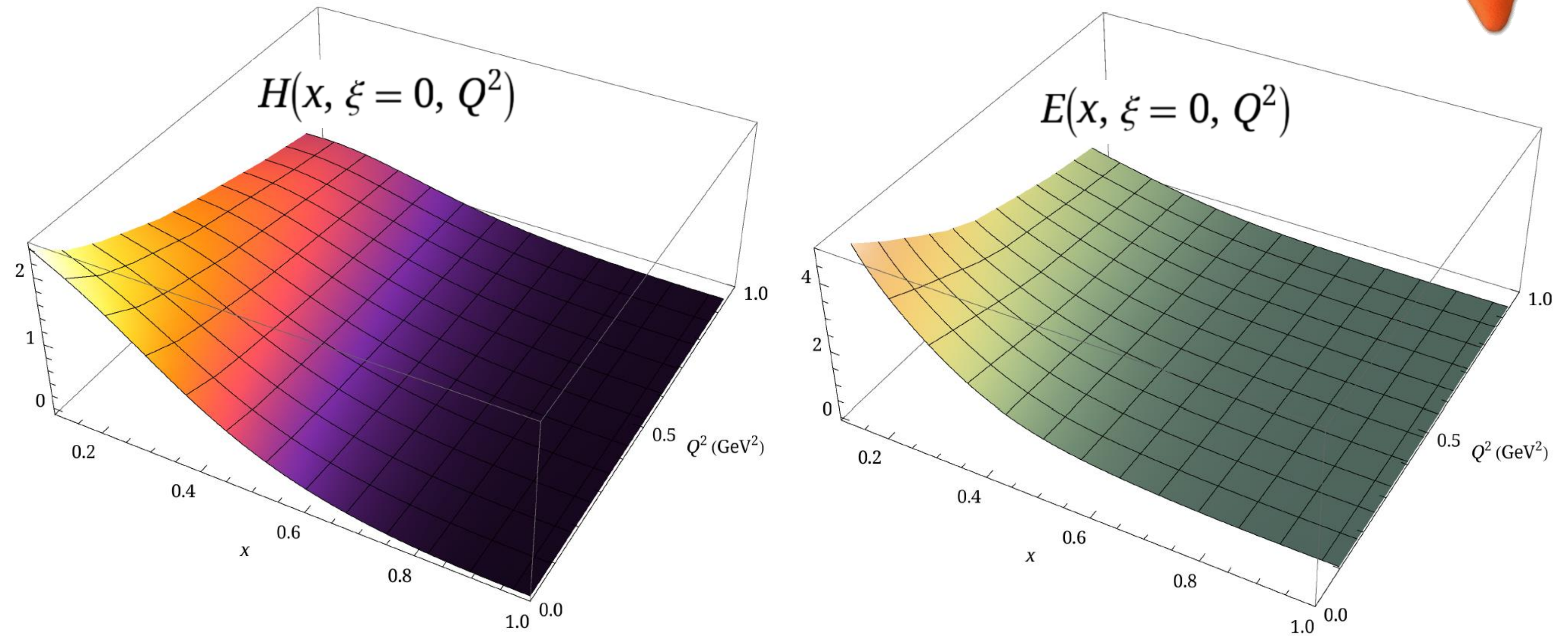

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

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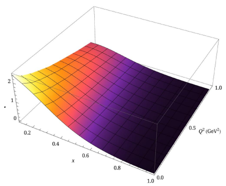
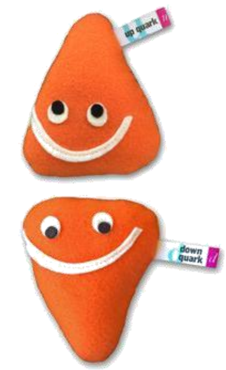
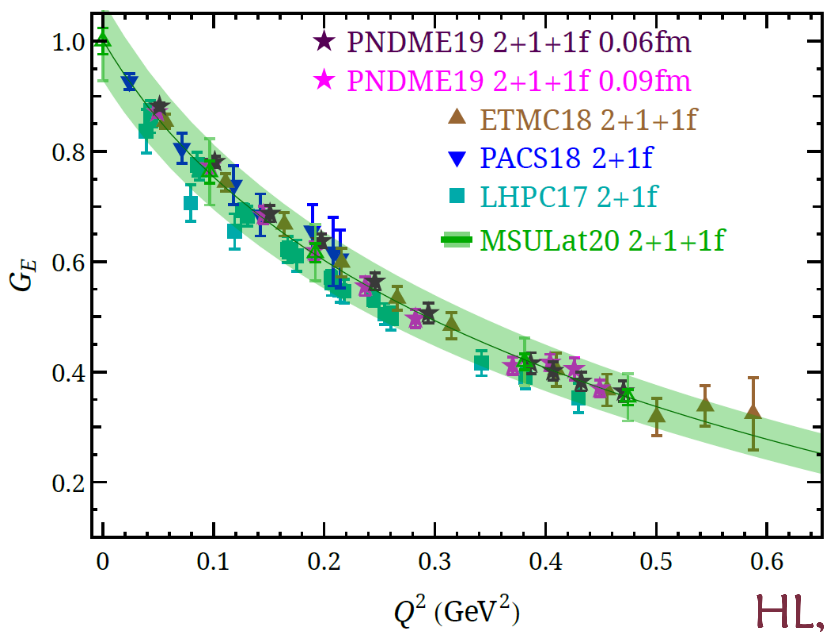
$$\int_{-1}^{+1} dx\, x^{n-1}$$



$$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A_{ni}^q(t) + (-2\xi)^n C_{n0}^q(t)\Big|_{n\ \text{even}}$$



$n = 1$



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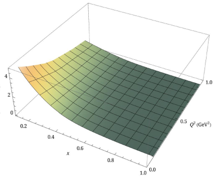
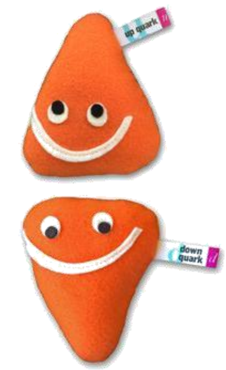
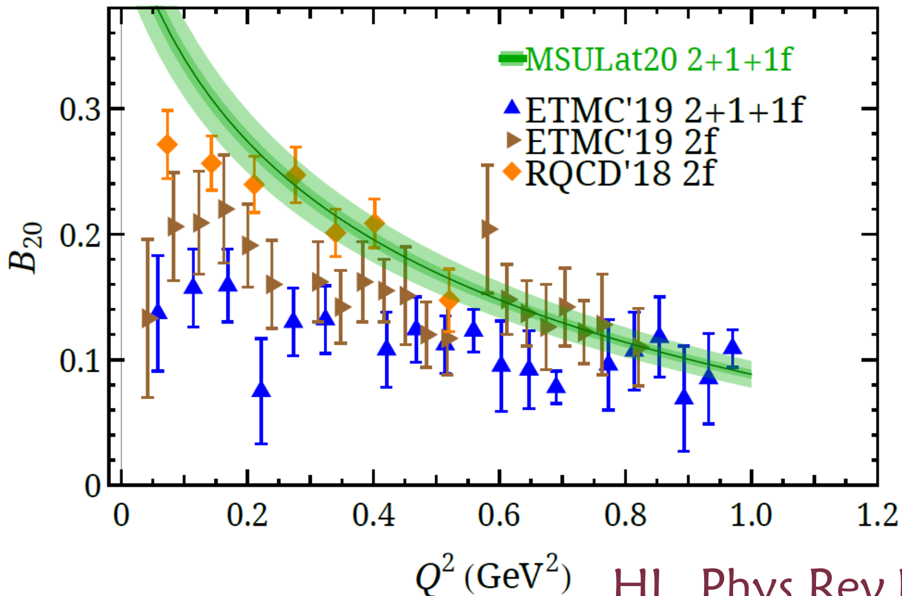


$$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B^q_{ni}(t) - (-2\xi)^n C^q_{n0}(t) \Big|_{n \text{ even}}$$



$n = 2$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

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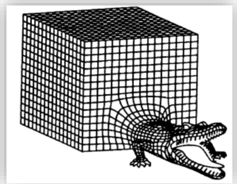

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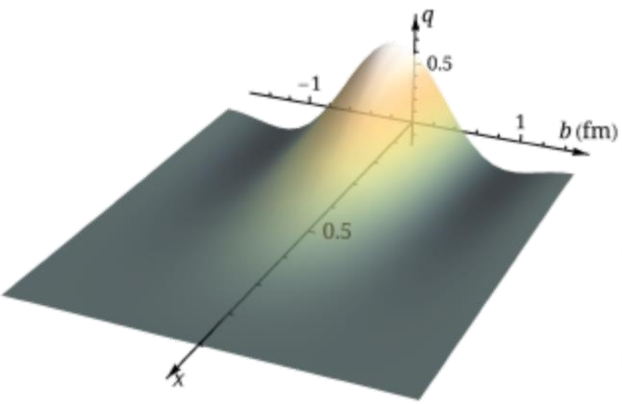
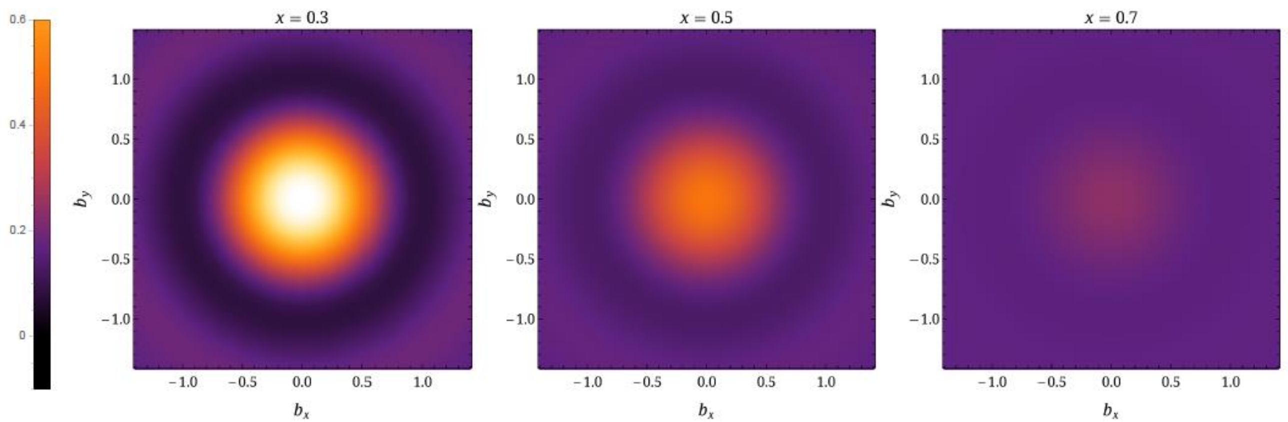




HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Caveats

§ Systematics in our earlier quasi-PDF calculation

- Renormalization: non-perturbative RI/MOM renormalization
  - State of the art: hybrid-ratio renormalization

X. Ji et. al. NPB 964, 115311 (2021)

- Next-leading order (NLO) matching only
  - State of the art: NNLO matching kernel available

X. Gao, PRL 128, 142003 (2022)

- Did not teat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al. PLB 844, 138081 (2023)

- For the rest of this presentation, we will focus on the uncertainties from the above (rather than typical lattice-calculation precision or systematics)

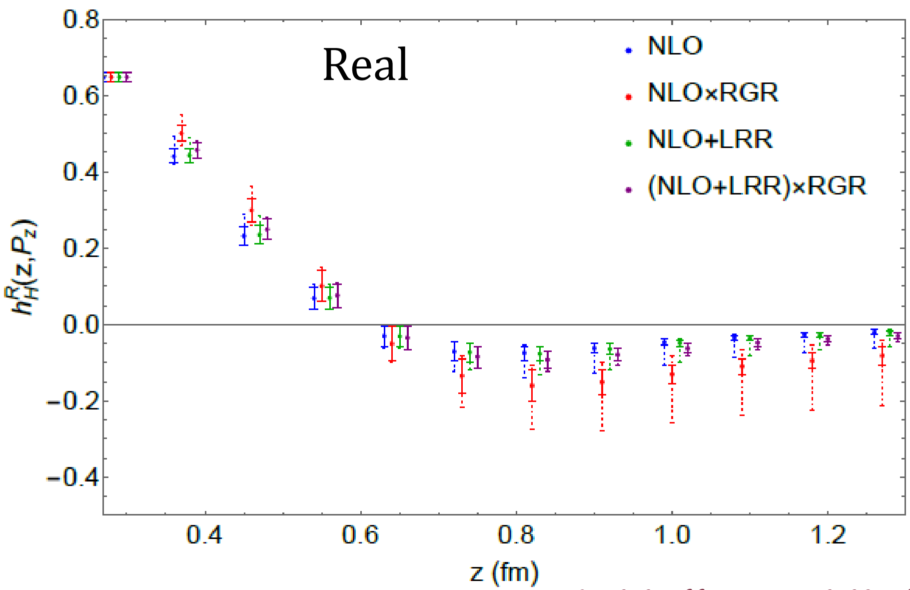
Forward-Limit Case: PDF

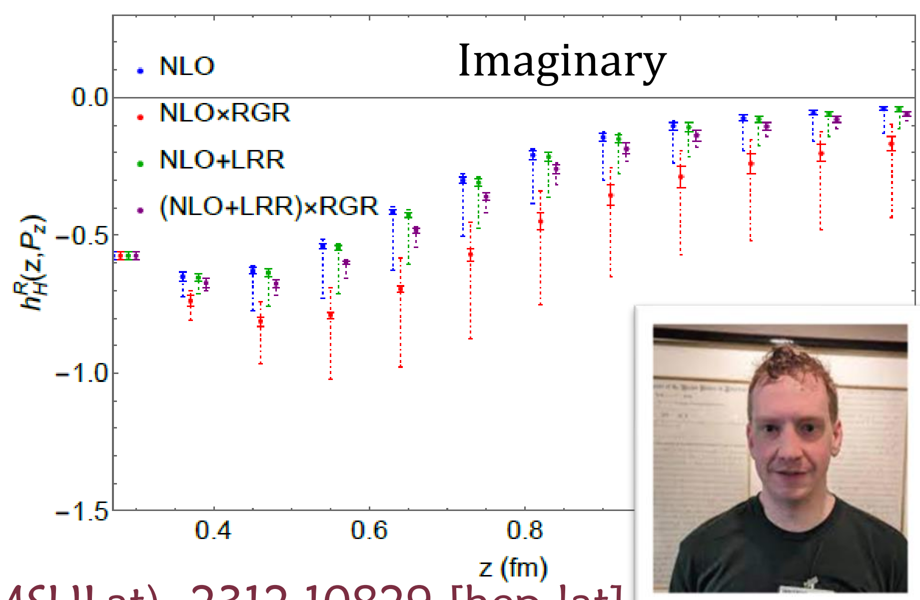
§ NLO hybrid-ratio renormalized matrix elements

$$h^R(z,P_z) = \begin{cases} N\dfrac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z < z_s \\ Ne^{(\delta m+m_0)(z-z_s)}\dfrac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z \geq z_s \end{cases}$$

Remove the linear divergence & renormalon ambiguity at large distances

- Vary the scale within $[0.75, 1.5]$: $\approx 15\%$ variation $\alpha_s(\mu = 2.0\text{ GeV})$
- Systematic errors shown below:



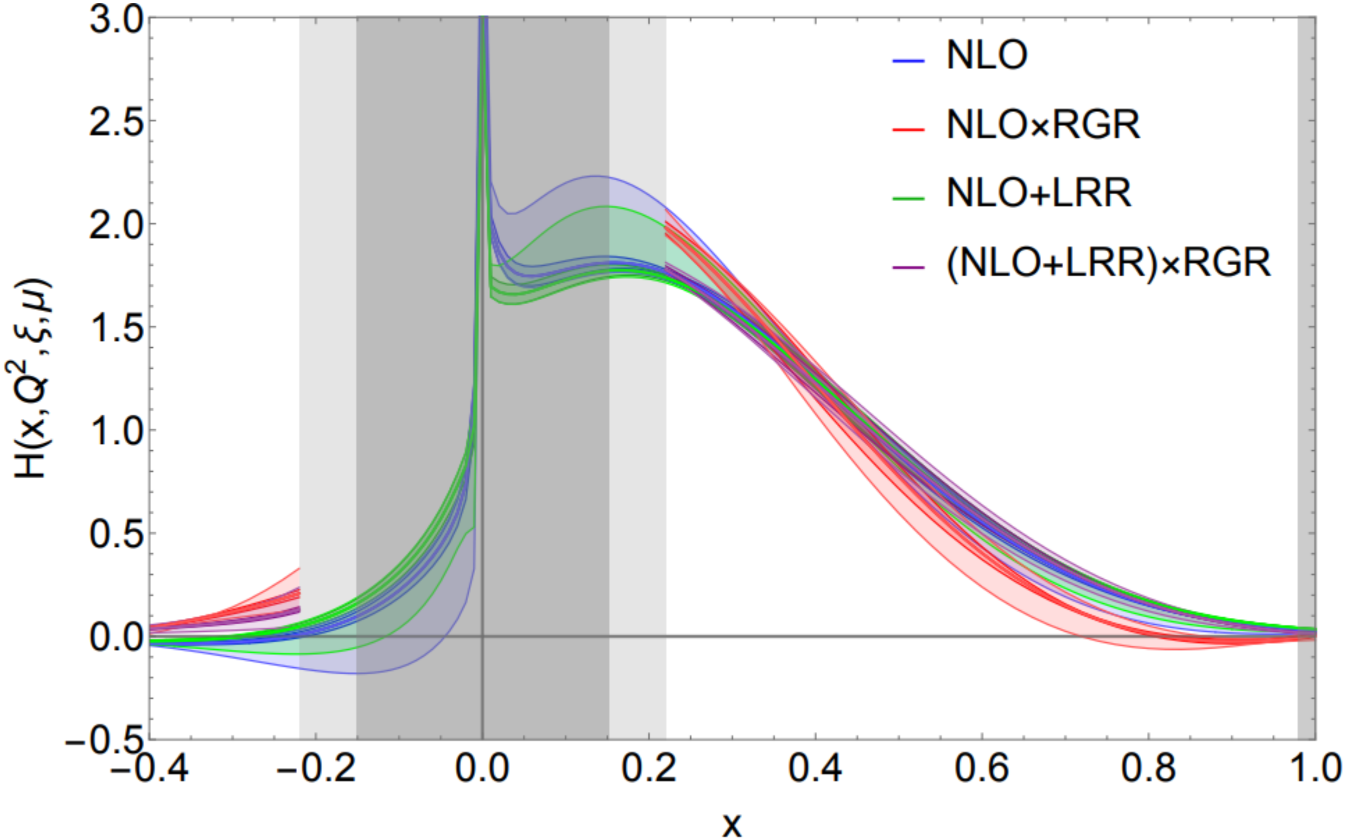


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Forward Limit Case: PDF

§ NLO isovector nucleon $H(\xi = 0, Q^2 = 0, x)$

- RGR process: DGLAP equation breaks down for $|x| \lesssim 0.2$ with $\mu = c' \times 2xP_z$



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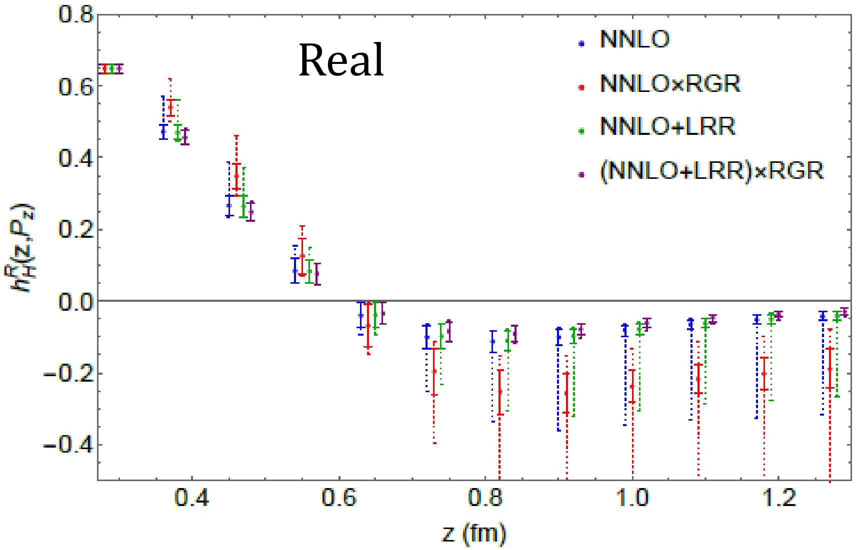

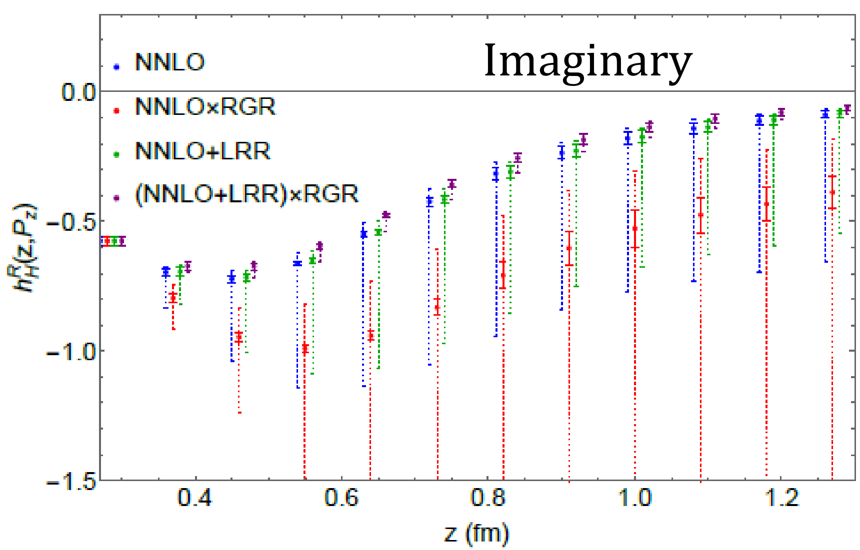

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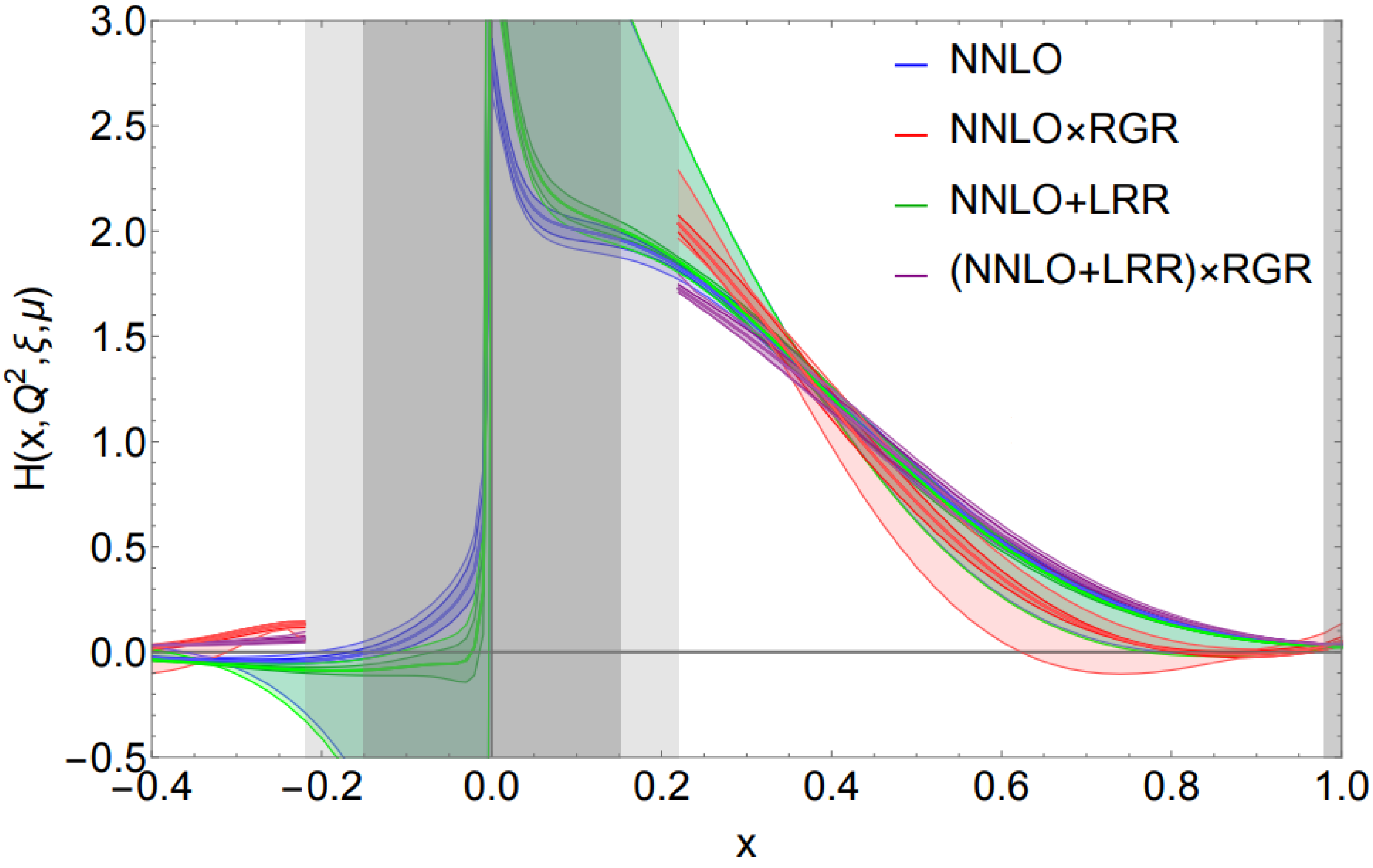
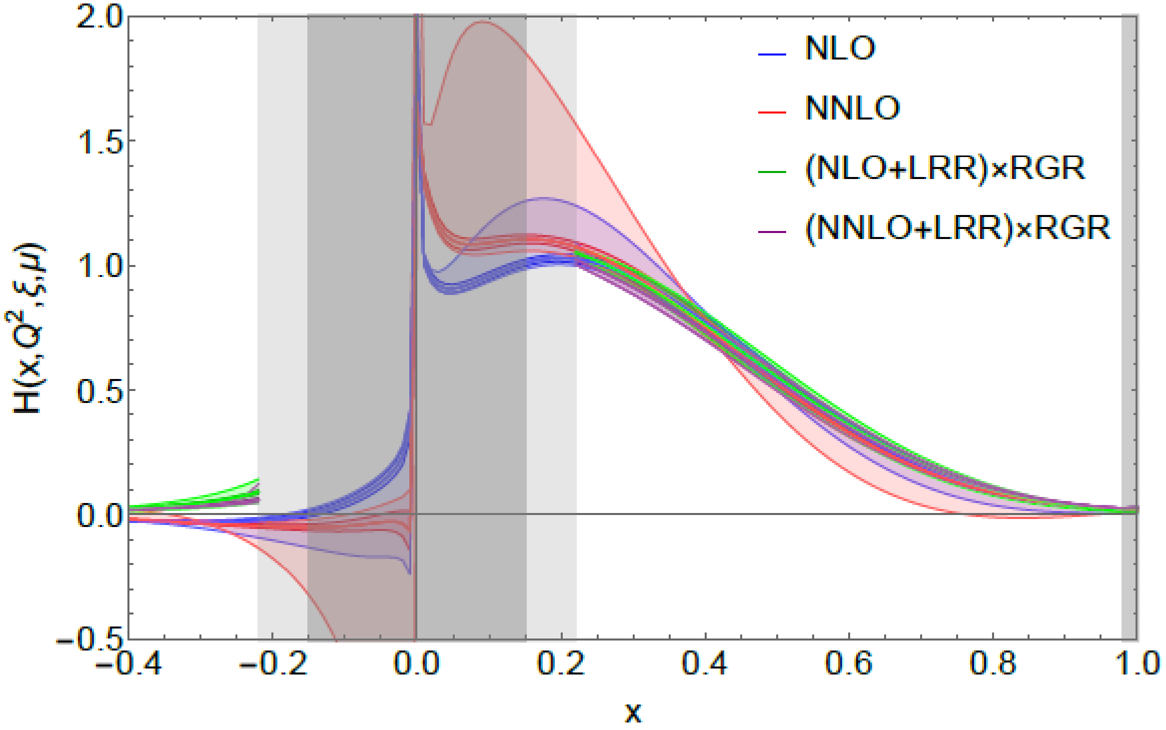


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

$\xi=0$, $Q^2=0.39$ GeV^2 GPDs

§ Repeat the procedure for nonzero transfer momentum



J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

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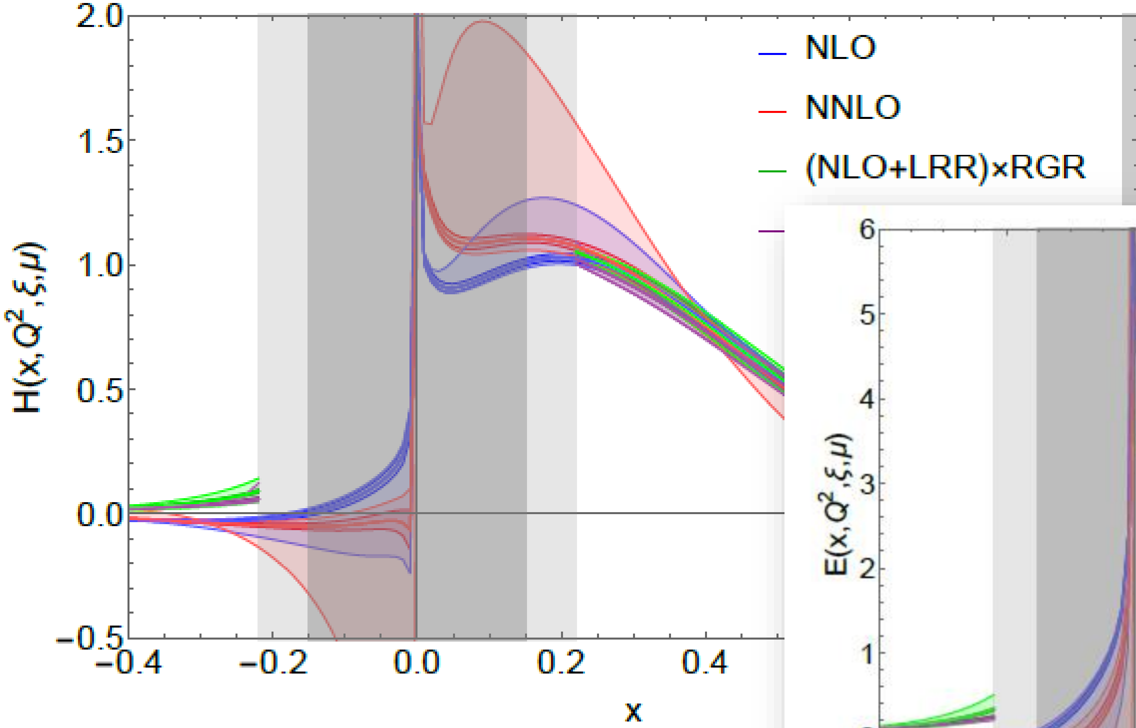

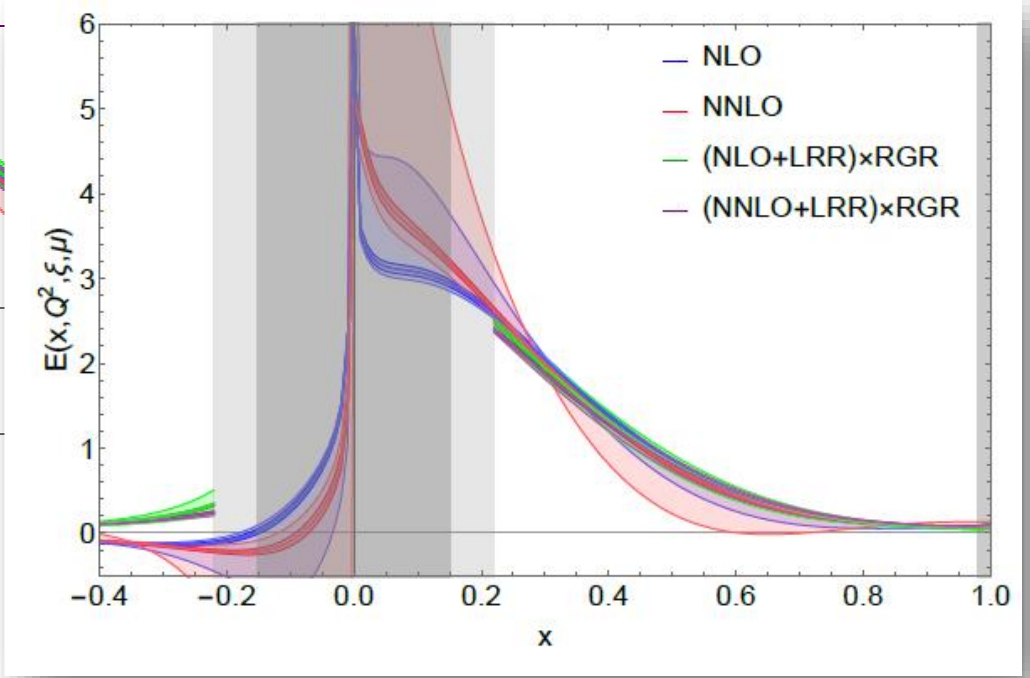

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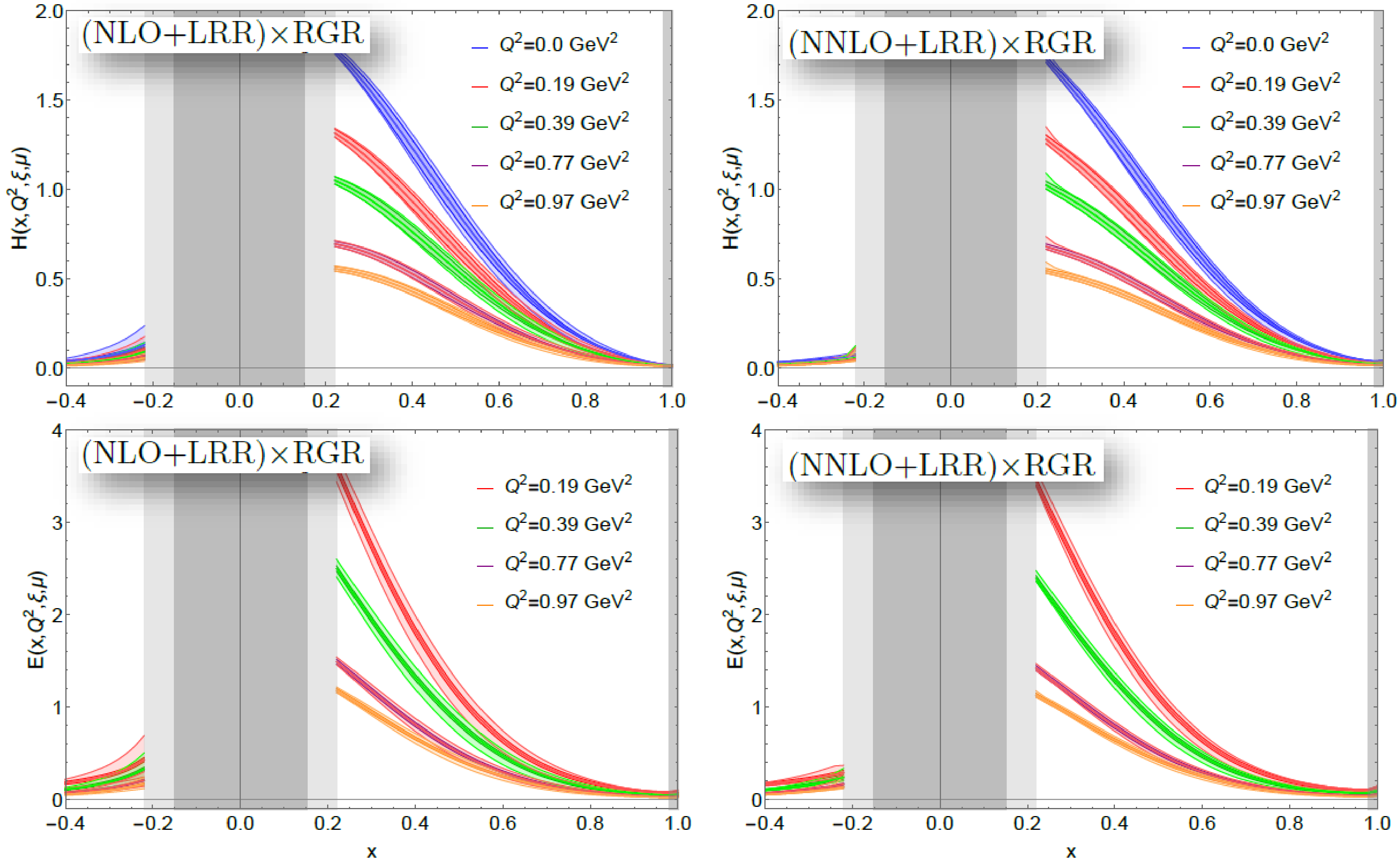


ξ≠0 GPDs

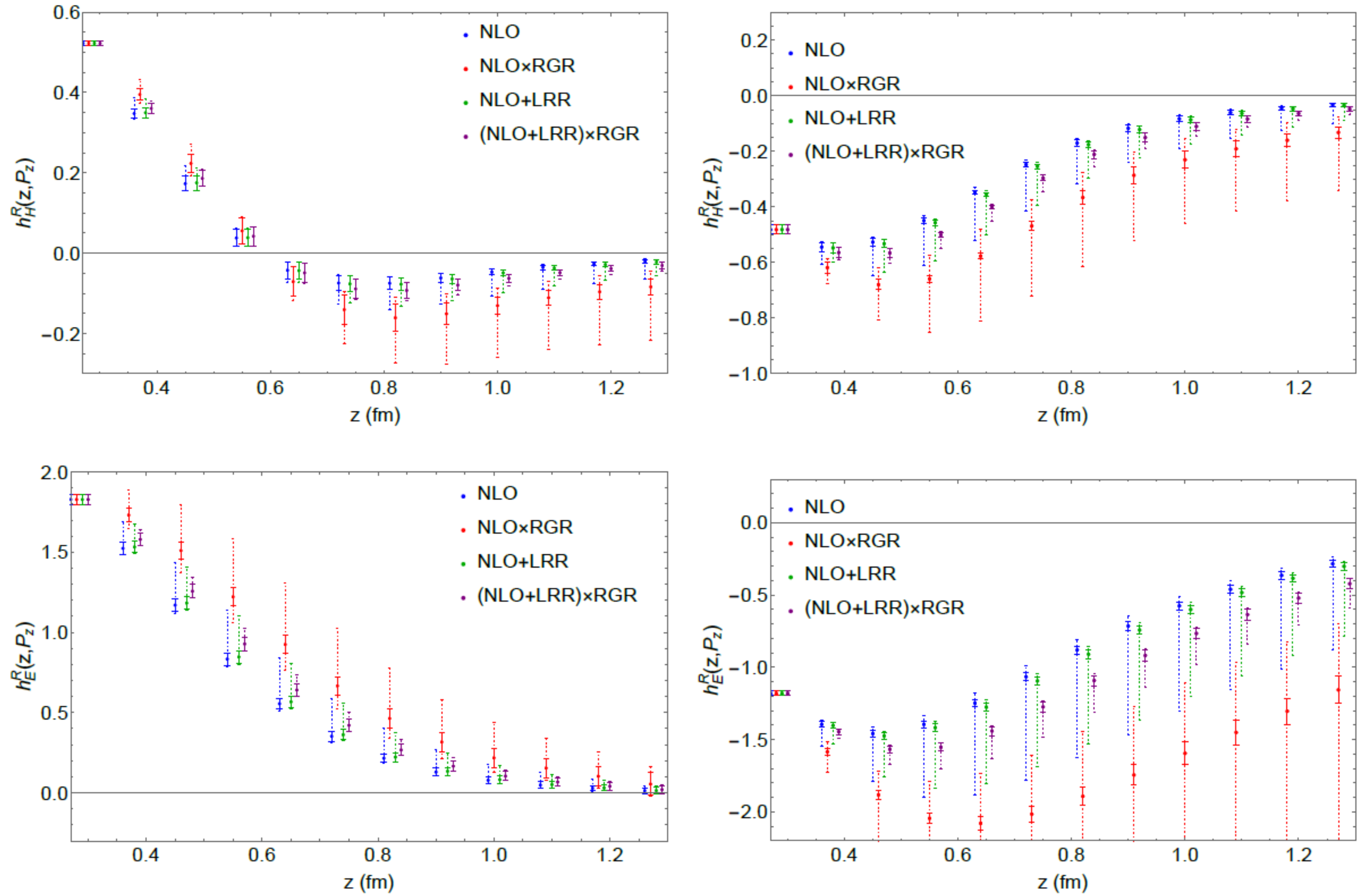
§ Only the NLO matching kernel is available

$$\mathcal{K}(x,y,\mu,\xi,P_z) = \delta(x-y) + \frac{\alpha_s C_F}{4\pi}\Bigg[\left(\frac{|\xi+x|}{2\xi(\xi+y)}+\frac{|\xi+x|}{(\xi+y)(y-x)}\right)\left(\ln\left(\frac{4y^2(\xi+x)^2P_z^2}{\mu^2}\right)-1\right) + \left(\frac{|\xi-x|}{2\xi(\xi-y)}+\frac{|\xi-x|}{(\xi-y)(x-y)}\right)\left(\ln\left(\frac{4y^2(\xi-x)^2P_z^2}{\mu^2}\right)-1\right) + \left(\left(\frac{\xi+x}{\xi+y}+\frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|}-\frac{|x-y|}{\xi^2-y^2}\right)\left(\ln\left(\frac{4y^2(x-y)^2P_z^2}{\mu^2}\right)-1\right)\Bigg]$$

F. Yoa et al, JHEP 11(2023) 021

$\xi \neq 0$ GPDs

§ NLO $\xi = 0.1$, $Q^2 = 0.23\ \mathrm{GeV}^2$



J. Holligan, HL (MSULat), in preparation

ξ≠0 GPDs

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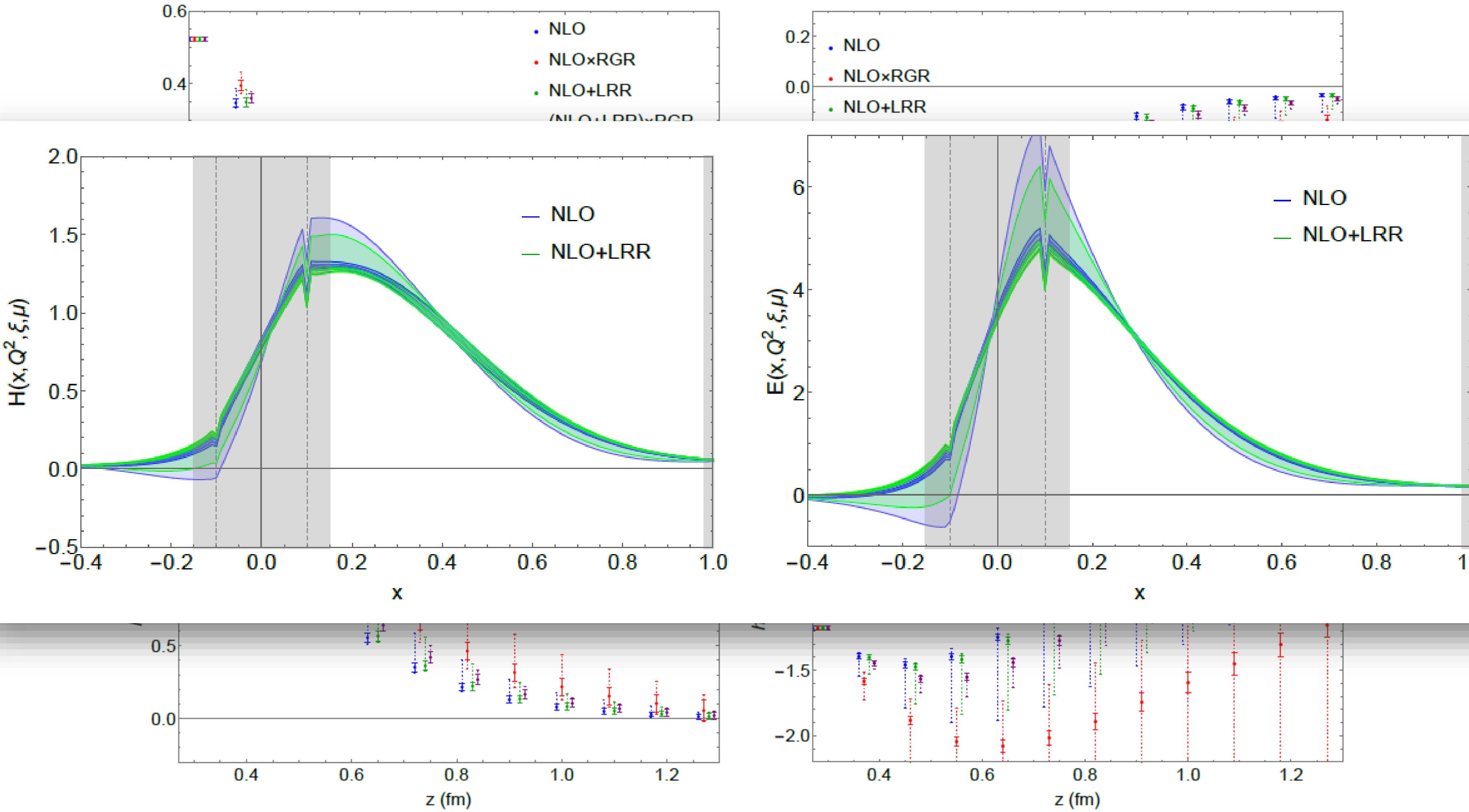


J. Holligan, HL (MSULat),2312.10829 [hep-lat]

Challenges

§ Large momentum is essential

- With sufficient statistics nucleons may reach 5 GeV

§ Renormalization of linear divergence

- Wilson-line ops have linear divergences that must be subtracted

§ Methods for signal-to-noise improvement

- Gluonic observables, new ideas for large momentum

§ Inverse problems PDF extraction in SDF

- Remove the model/preconditioner-choice dependence

§ Reaching long-range correlations in LaMET

- For small-*x* physics, new methods for calculating longer-range correlations must be developed

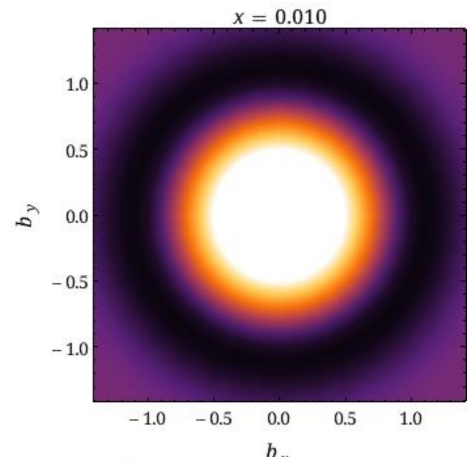
Whitepaper: Lattice QCD Calculations of Parton Physics, 2202.07193

Summary and Outlook

§ Exciting era using LQCD to study pion and nucleon structure

- Well-studied systematics → precision structures

§ Overcoming longstanding obstacles



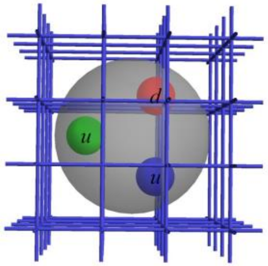
- Bjorken-*x* dependence of parton distributions are widely studied with LaMET and its variants

§ Precision and progress are limited on resources

- Challenges = new opportunities quantities

§ In the future

Theory Input → **Global Analysis of PDFs/GPDs** ← **Exp't Input** +



Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources

The work of HL is sponsored by NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 & RCSA Cottrell Scholar Award

Students Wanted

LGT4HEP website: https://lgt4hep.github.io/

Backup Slides

Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

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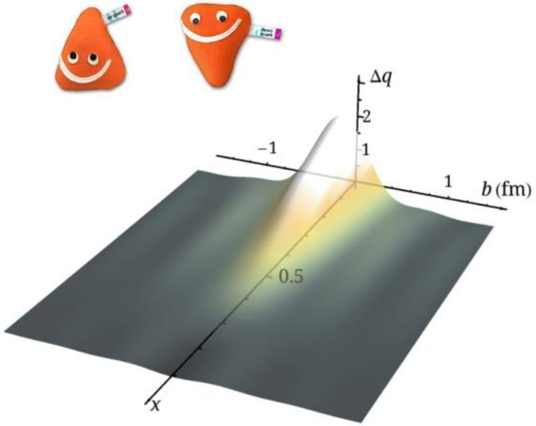
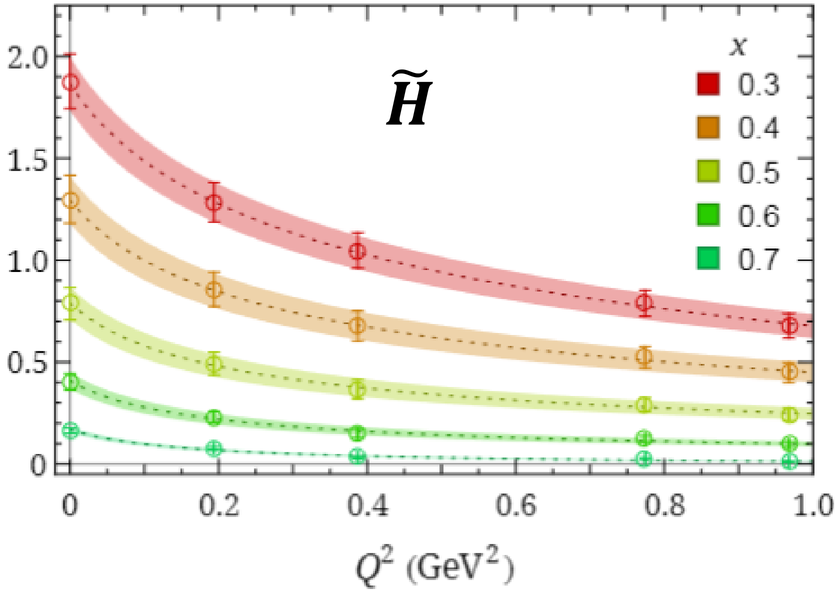
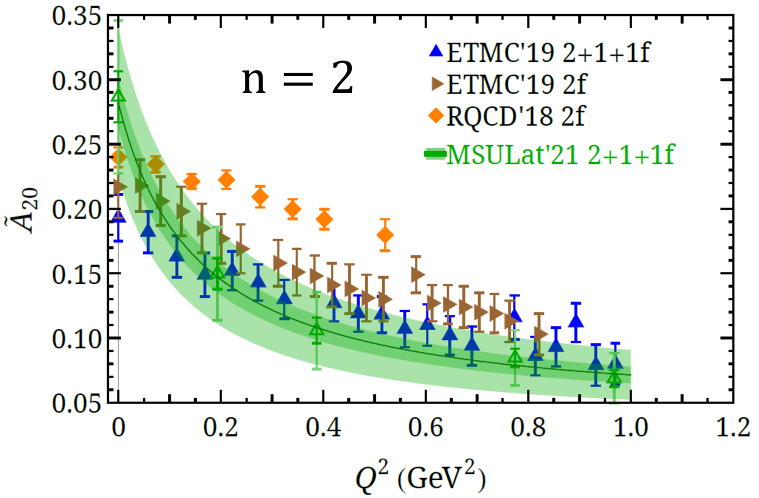
HL (MSULat), Phys.Lett.B 824 (2022) 136821

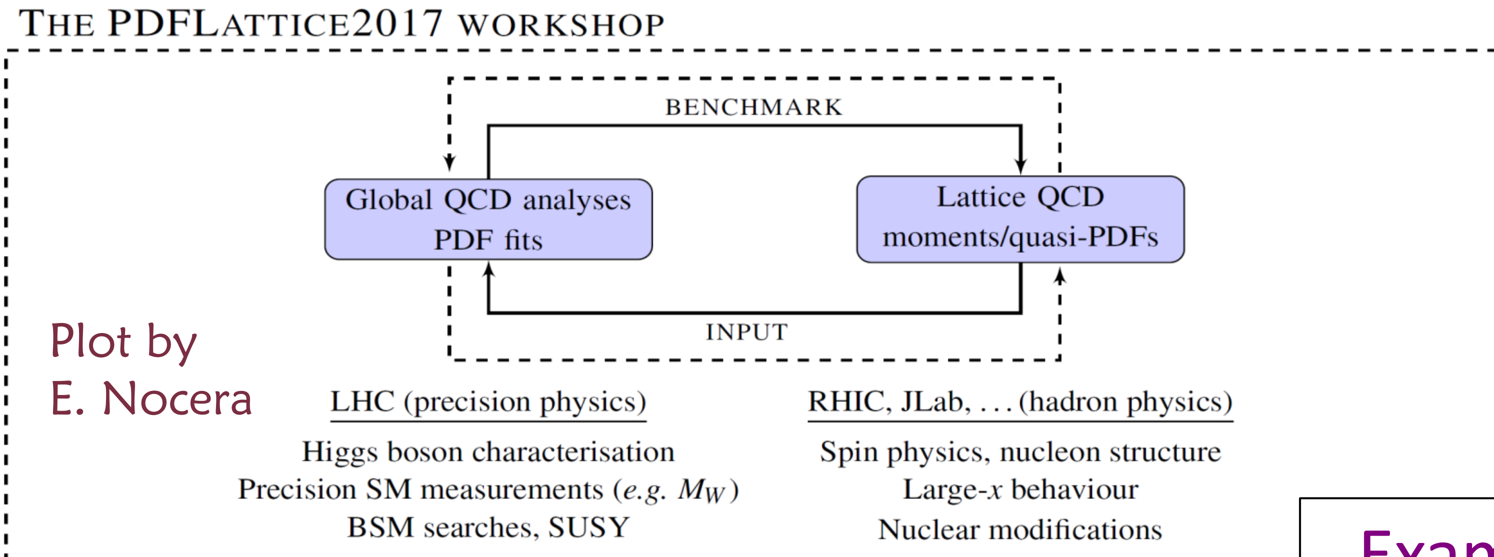




- Take the integral to form moments



How Can Lattice Help?



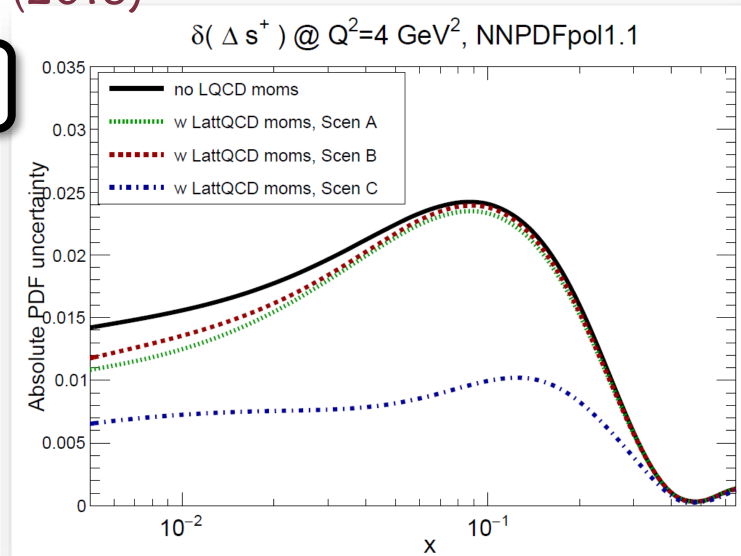
Plot by
E. Nocera

Whitepaper , Progress in Par. and Nuc. Phy. 100, 106 (2018)

Example study

A: 70% B: 50% C: 20%

§ Is there one quantity for which LQCD can achieve a precision at which it can make a significant difference?



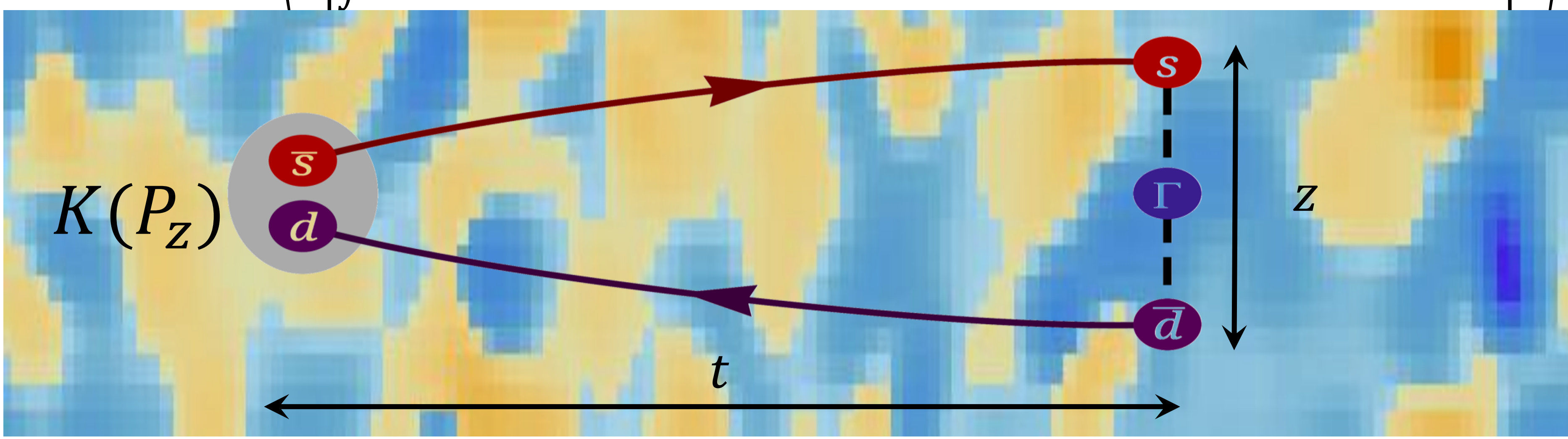
Application on Inverse Problem

Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

R. Zhang, C. Honkala, et al. (MSULat), 2005.13955

Example: Pion/Kaon Distribution Amplitude

$$C_M^{\mathrm{DA}}(z,P,t) = \left\langle 0 \left| \int d^3y\, e^{i\vec{P}\cdot\vec{y}} \bar{\psi}_1(\vec{y},t)\gamma_z\gamma_5 U(\vec{y},\vec{y}+z\,\hat{z})\psi_2(\vec{y}+z\,\hat{z},t)\bar{\psi}_2(0,0)\gamma_5\psi_1(0,0) \right| 0 \right\rangle$$



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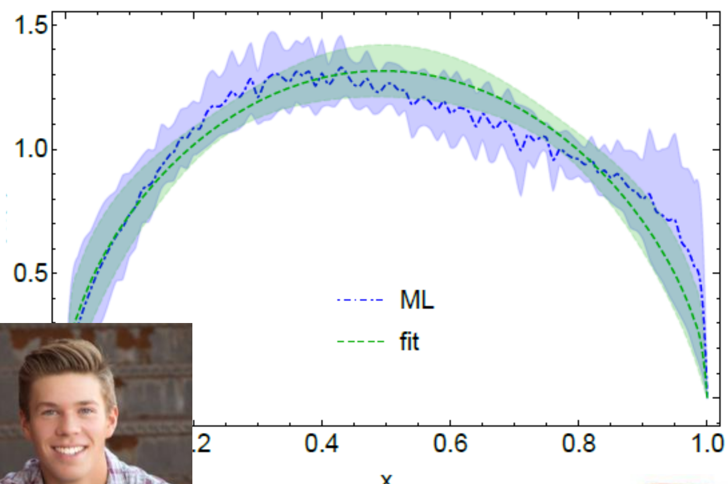
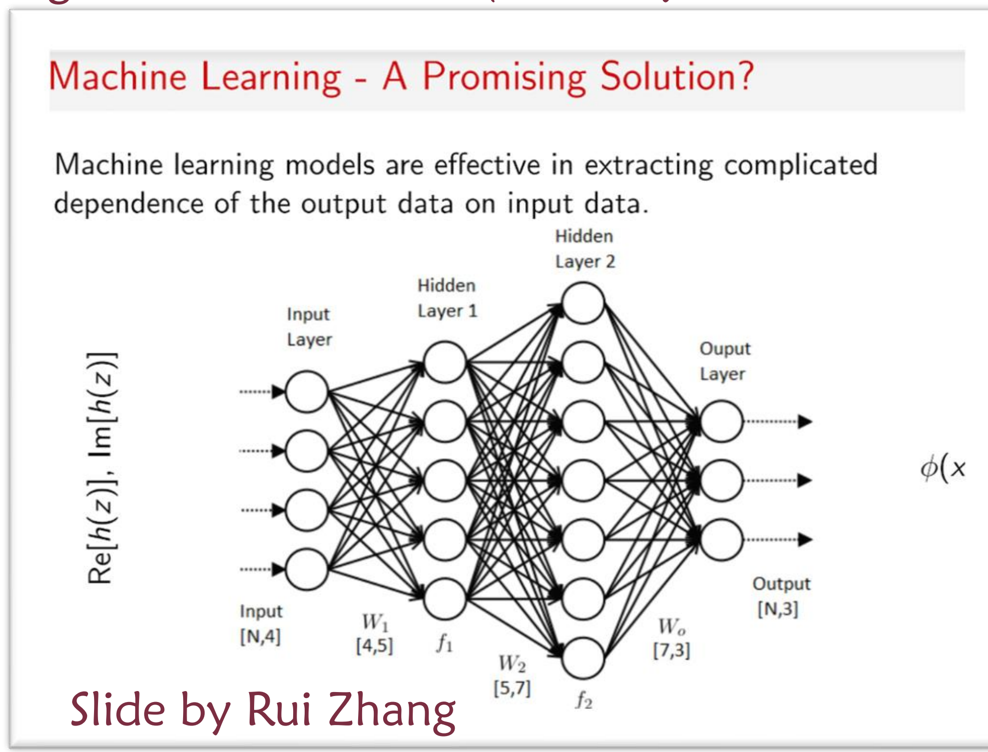
R. Zhang, C. Honkala, et al. (MSULat), 2005.13955

Pion Distribution Amplitude



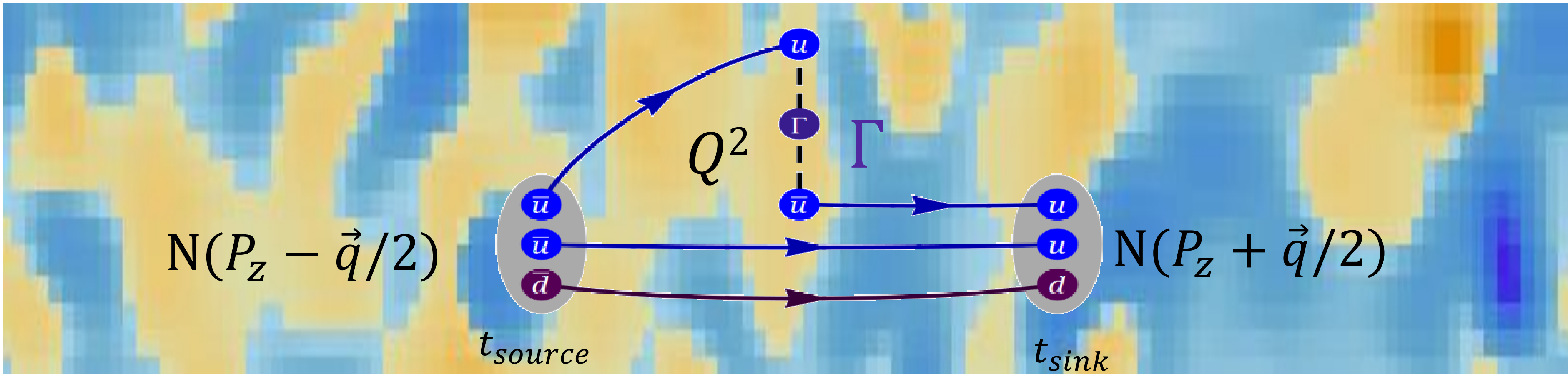
UG: Carson Honkala



Slide by Rui Zhang

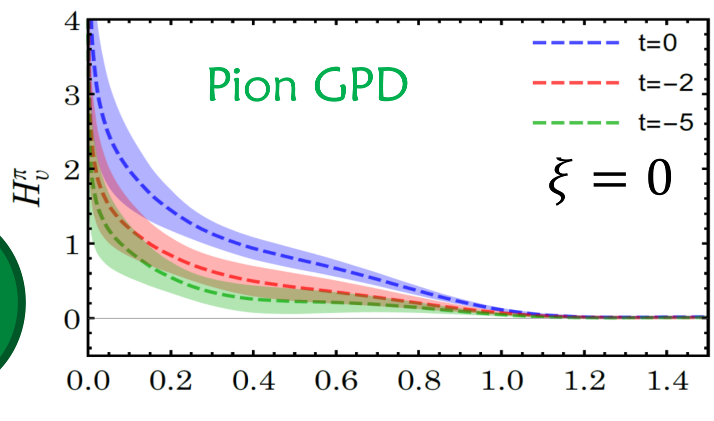
Generalized Parton Distributions

§ On the lattice, one needs to calculate the following

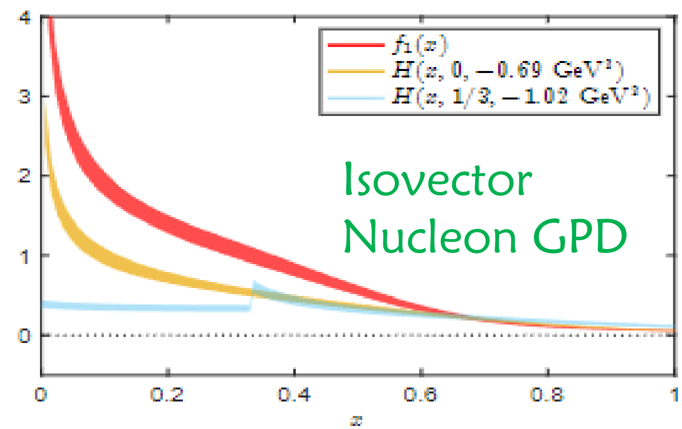


§ Heavy pion-mass results

Wanted PDFs, GPDs, etc…



J. Chen, HL, J. Zhang, 1904.12376



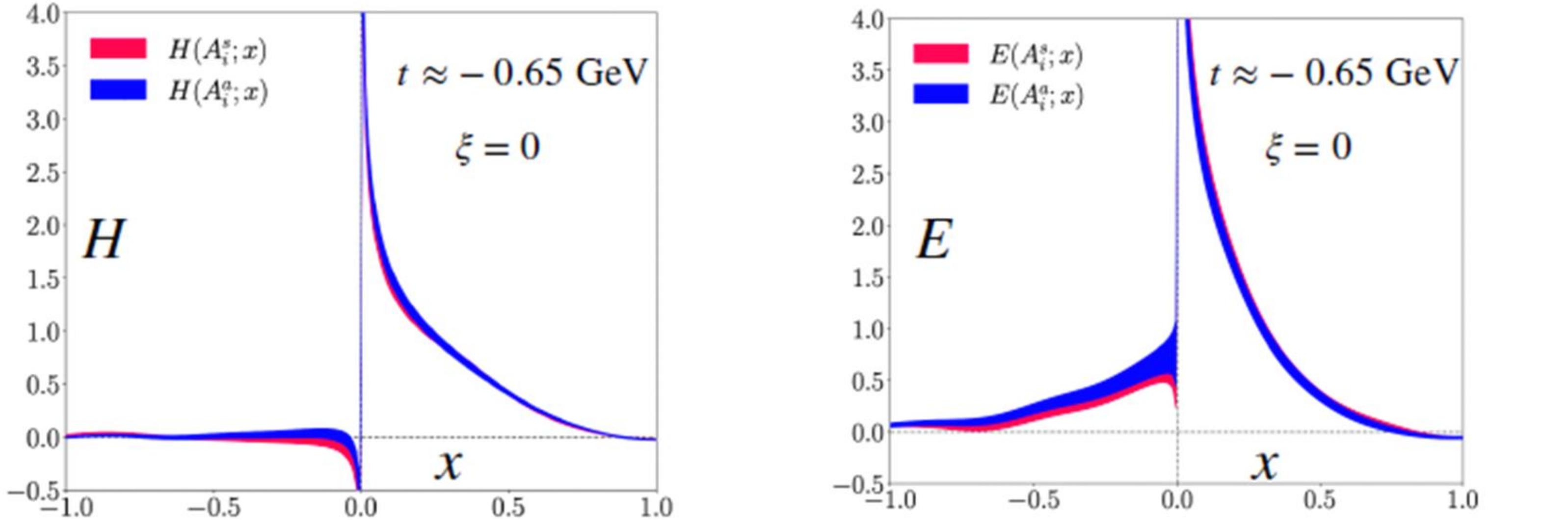
C. Alexandrou et al, 2008.10573

Ge

§ Or

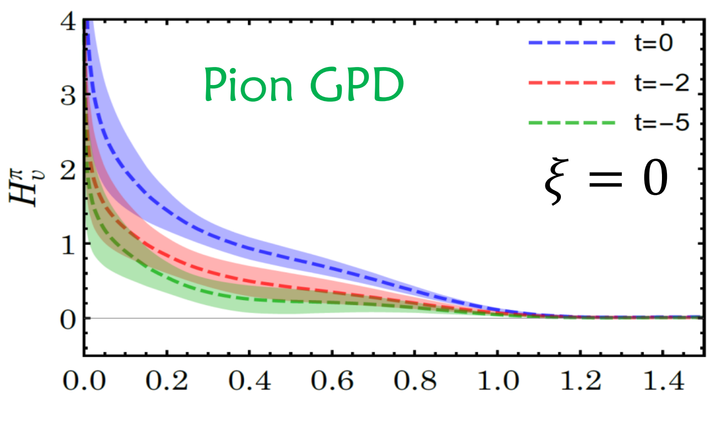
See Th

Shohini Bhattacharya *et al.*, arXiv:2209.05373 [hep-lat]

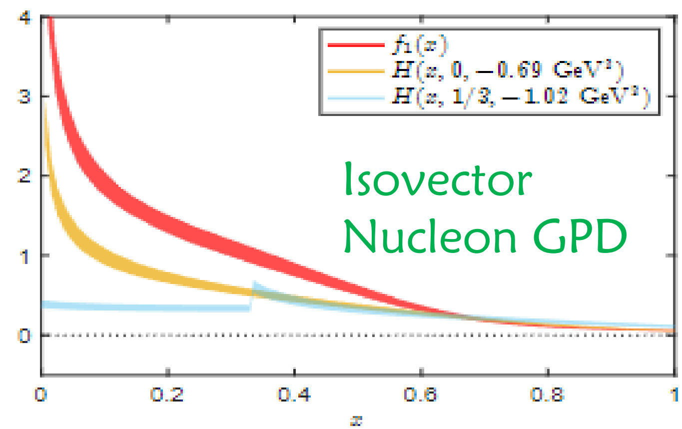


260-MeV results for asymmetric setup for GPD calculations to save computing time

§ Heavy pion-mass results



J. Chen, HL, J. Zhang, 1904.12376



C. Alexandrou et al, 2008.10573

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+ q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}\,(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}\,(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p)\right]$$

§ Helicity GPD ($\widetilde{H}$) using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right]$$

Lattice Progress & Challenges

- § Exploratory study on charm and gluon PDFs
- § Many approaches are moving to the NNLO level
  - Expect to see more improved lattice calculations
- § Beyond the standard twist-2 collinear PDFs
  - Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
  - Transverse-momentum– dependent distributions (TMDs)
    - Collins-Soper kernel, soft function and wavefunctions
  - Twist-3 PDFs and GPDs

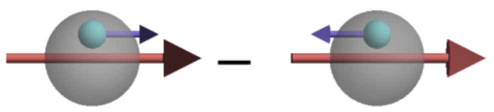
For more details and references, refer to 2202.07193

- § Challenges ahead for precision PDFs
  - Need large boost mom., better signal-to-noise, inverse problems in PDF extraction in SDF, more computational resources, etc.

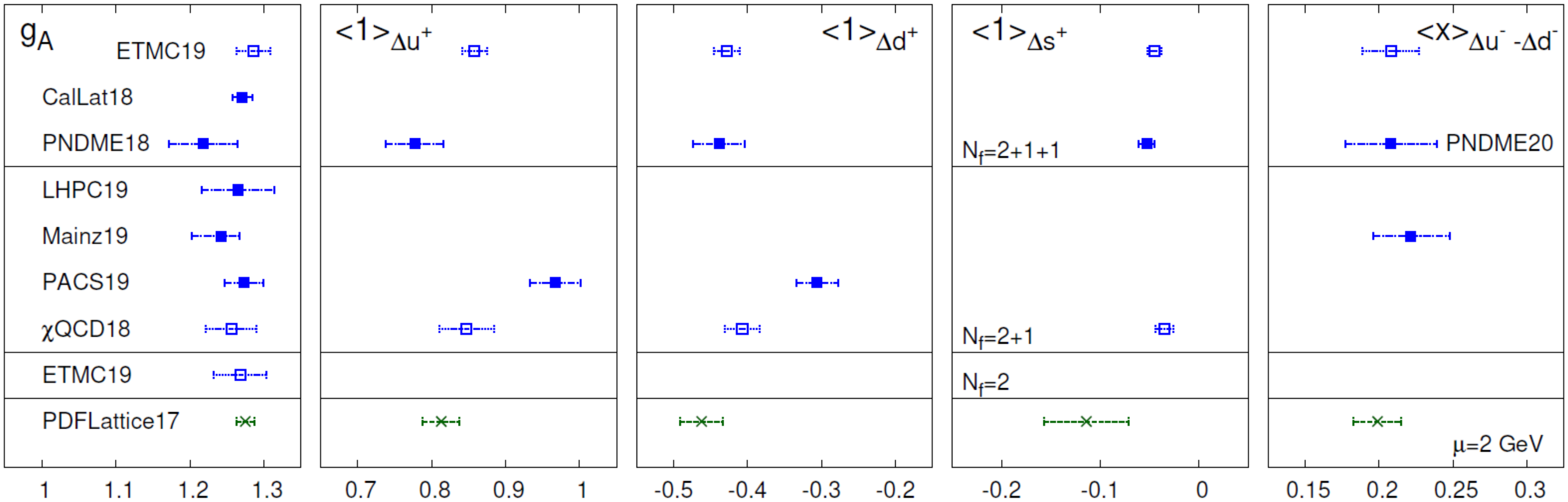
Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



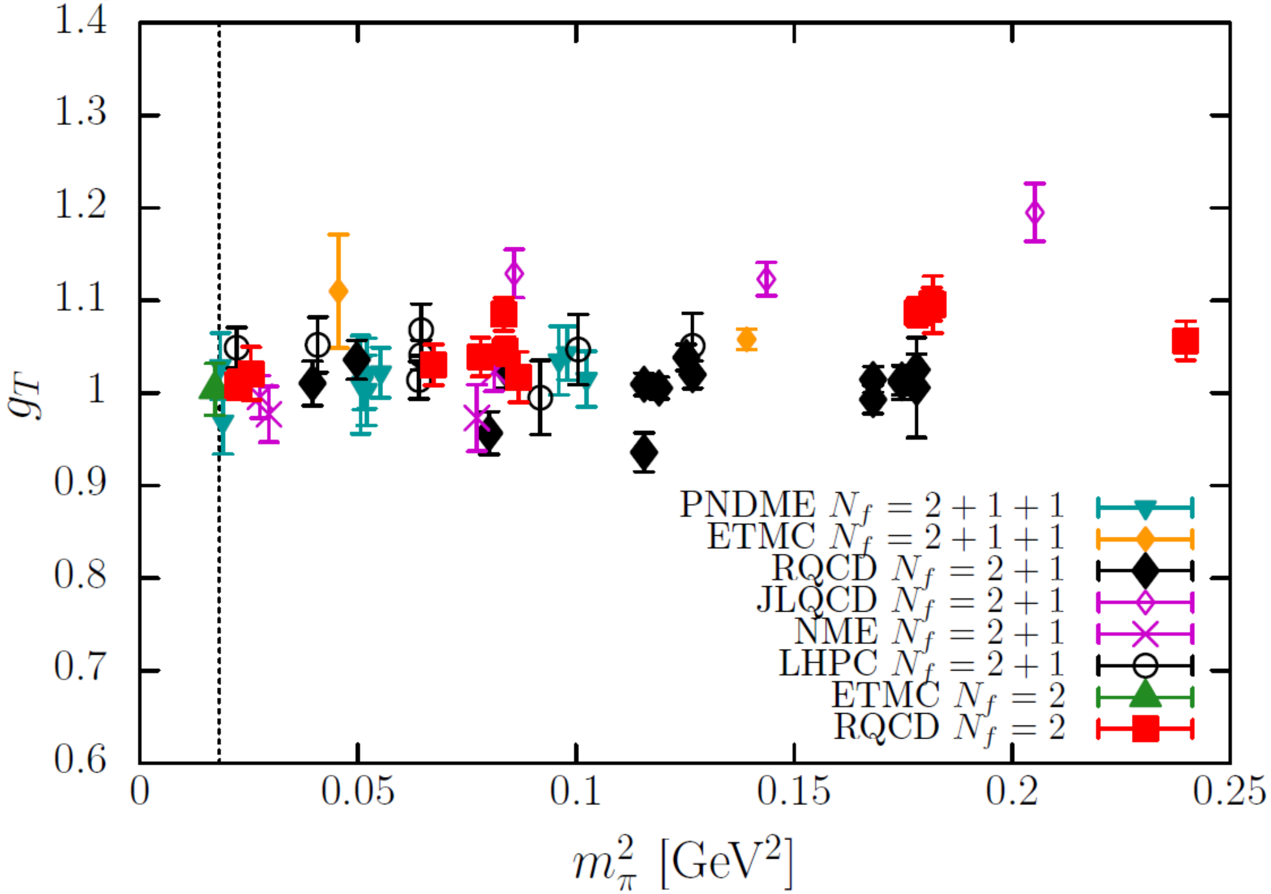
LatticePDF Report, 1711.07916,2006.08636



Precision Nucleon Couplings

§ Usually more than one LQCD calculation

- For example, tensor charge
- Lattice results should agree in the continuum limit



Plot by Sara Collins @ Lattice 2016

Precision Nucleon Couplings

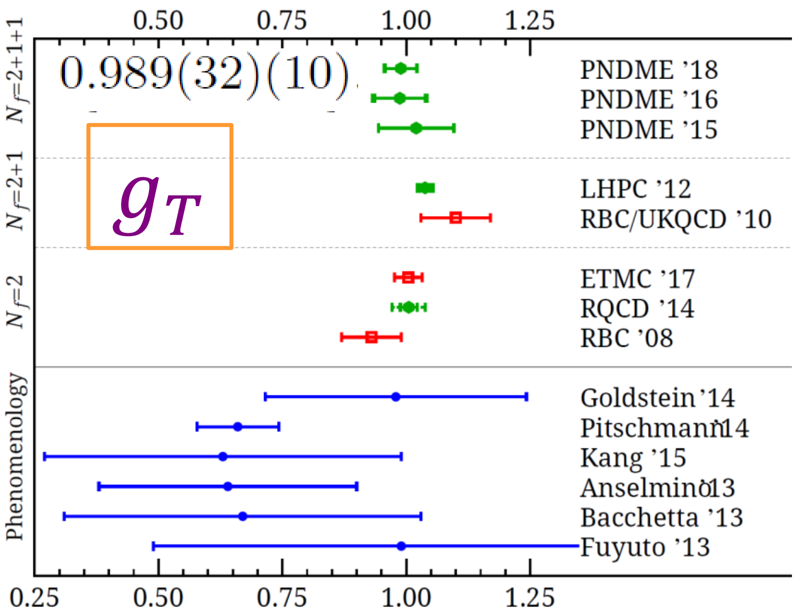
FLAG rating system

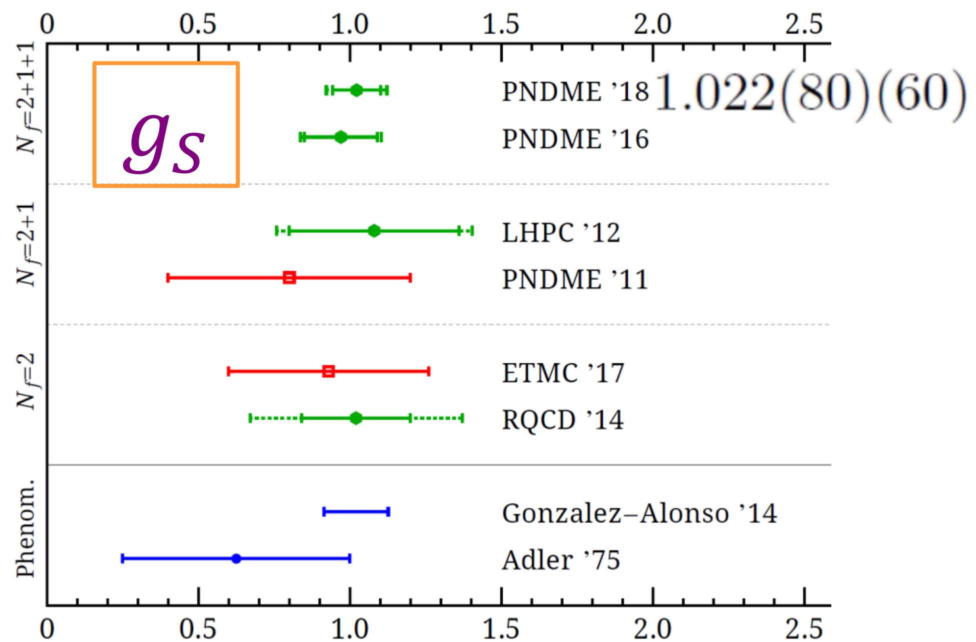
PNDME, 1506.06411; 1606.07049

New: excited-state rating

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76)[a]
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3)[b]
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20)[c]
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7)[d]
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29))[e]
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46)[f]
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6)[g]

PNDME, 1806.09006

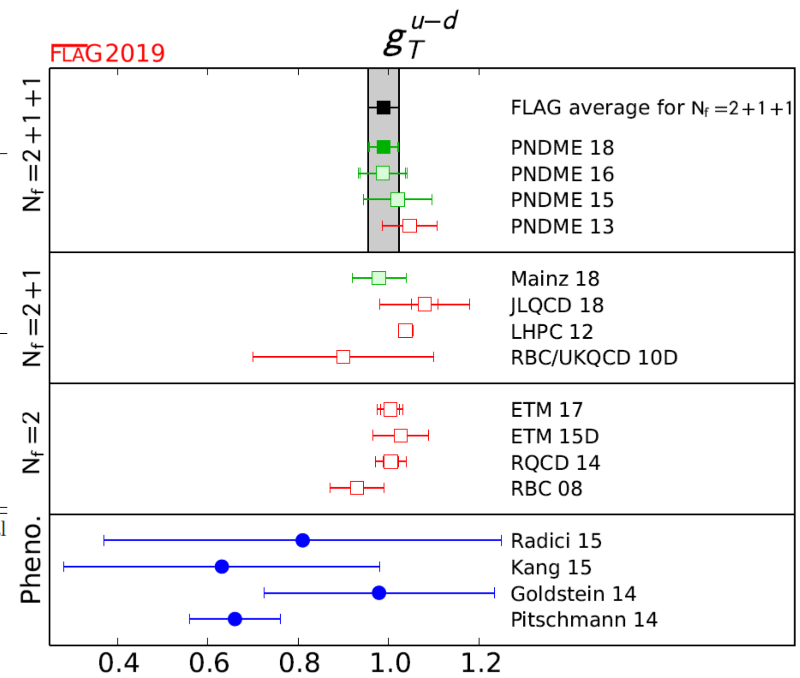




§ Finally adopted by FLAG! https://arxiv.org/pdf/1902.08191.pdf

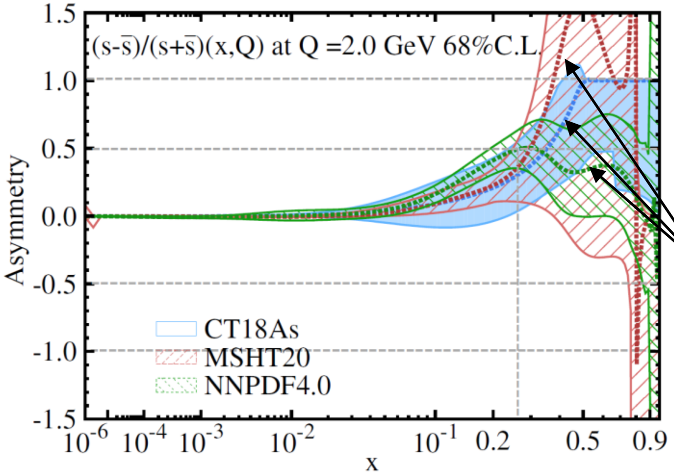
Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	g_T^{u-d}
PNDME 18	[84]	2+1+1	A	★‡	★	★	★	★	0.989(32)(10)
PNDME 16	[830]	2+1+1	A	○‡	★	★	★	★	0.987(51)(20)
PNDME 15	[828, 829]	2+1+1	A	○‡	★	★	★	★	1.020(76)
PNDME 13	[827]	2+1+1	A	■‡	■	★	★	★	1.047(61)
Mainz 18	[915]	2+1	C	★	○	★	★	★	0.979(60)
JLQCD 18	[839]	2+1	A	■	○	○	★	★	1.08(3)(3)(9)
LHPC 12	[920]	2+1	A	■‡	★	★	★	★	1.038(11)(12)
RBC/UKQCD 10D	[834]	2+1	A	■	■	○	★	■	0.9(2)
ETM 17	[826]	2	A	■	○	○	★	★	1.004(21)(2)(19)
ETM 15D	[822]	2	A	■	○	○	★	★	1.027(62)
RQCD 14	[819]	2	A	○	★	★	★	■	1.005(17)(29)
RBC 08	[918]	2	A	■	■	■	★	■	0.93(6)

‡ The rating takes into account that the action is not fully O(a) improved by requiring an additional lattice spacing.



Lattice Impact on Strange PDF

- lattice QCD can constrain PDFs (polarized, meson, TMDs, GPDs,…) that are difficult to access in experiments
- Example: the strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure (left), but can be predicted in lattice QCD (right)



differences reflect the pulls of LHC and other experiments

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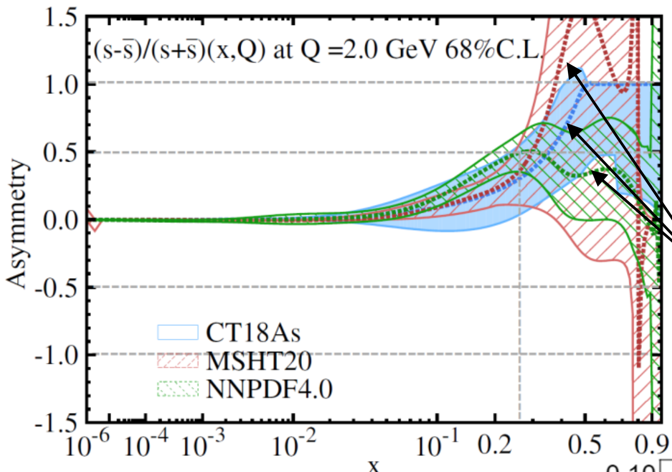
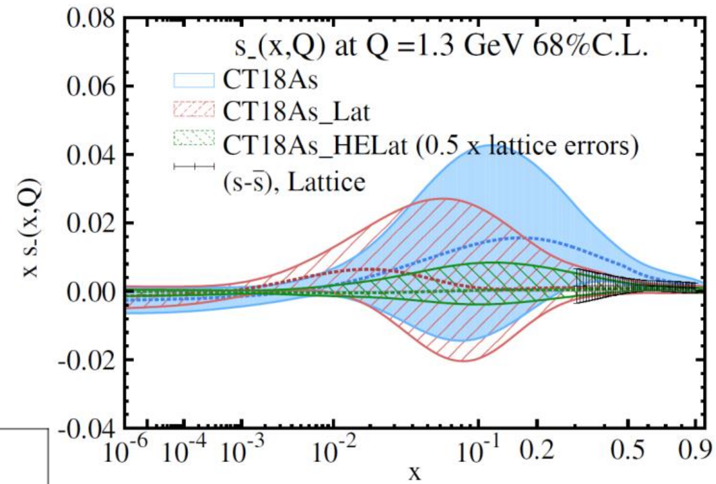
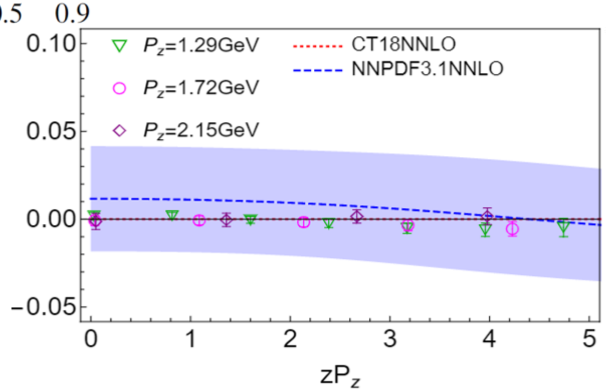


differences reflect the pulls of LHC and other experiments



$$\mathrm{Re}[h(z)] \propto \int dx \left(s(x) - \bar{s}(x)\right) \cos(xzP_z)$$



T. Hou, HL, M. Yan, C.-P. Yuan, 2204.07944