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Precision nucleon charges and form factors from lattice QCD

Sungwoo Park, LLNL

Lattice QCD workshop on hadron and quark matter (LQCDW1),
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LLNL-PRES-XXXX
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PNDME Collaboration: Thirteen 2+1+1-flavor HISQ ensembles = clover-on-HISQ formulation

NME Collaboration: Thirteen 2+1-flavor clover ensembles = clover-on-clover formulation

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- Huey-Wen Lin (MSU)
- Balint Joo (ORNL)
- Frank Winter (JLab)

References

PNDME

- Charges: Gupta et al, PRD.98 (2018) 034503
- AFF: Gupta et al, PRD 96 (2017) 114503, Jang et al, PRL 124 (2020) 072002, Jang et al, arXiv:2305.11330

  VFF: Jang et al, PRD 100 (2020) 014507
- $\sigma_{\pi N}$ Gupta et al, PRL 127 (2021) 242002
- d_n from Θ-term Bhattacharya etal, PRD 103 (2021) 114507
- d_n from qEDM Gupta et al, PRD 98 (2018) 091501
- Moments of PDFs Mondal et al, PRD 102 (2020) 054512
- Proton spin: Lin et al, PRD 98 (2018) 094512

NME

- Charges, FF: Park et al, PRD 105 (2022) 054505
- Moments of PDFs Mondal et al, JHEP 04 (2021) 044

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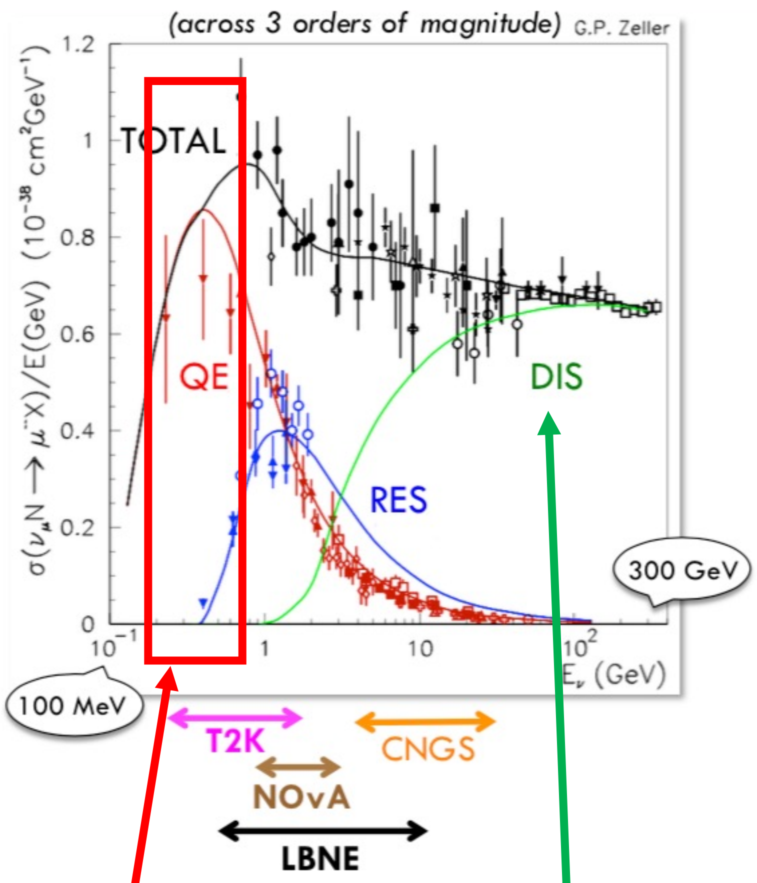
Introduction

Physics from **nucleon form factors and charges**

Methodology for calculation of **nucleon matrix elements** using **lattice QCD**

Lepton-nucleon scattering

- Nucleon charges and form factors give the strength of the interaction of external probes (electrons, neutrinos, ⋯) with nucleons and are critical inputs in experimental searches of physics beyond the standard model.
- High precision results for axial, electric and magnetic form factors versus Q^2 needed for determining **(quasi-) elastic cross-section** of (ν, e, μ) scattering off nuclei

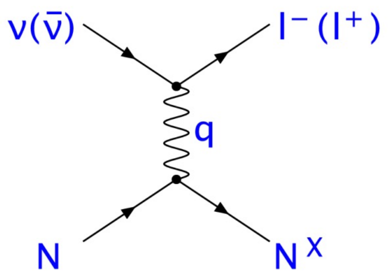


F_A = axial form factor
$\tilde{F}_P$ = induced pseudoscalar
$G_E = F_1 - \tau F_2$ Electric
$G_M = F_1 + F_2$ Magnetic
$\tau = Q^2/4M^2$
$M = M_n = M_p \approx 939$ MeV
$m = M_\pi$

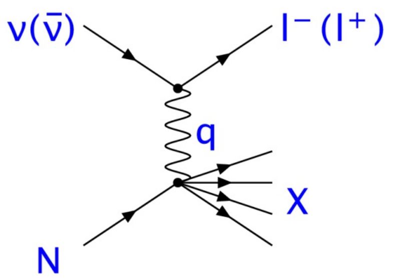
$$\frac{d\sigma}{dQ^2}\begin{pmatrix} \nu_l + n \to l^- + p \\ \bar{\nu}_l + p \to l^+ + n \end{pmatrix} = \frac{M^2 G_F{}^2 cos^2\theta_c}{8\pi E_\nu{}^2}\left\{A(Q^2) \pm B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right\},$$

$$\begin{aligned} A(Q^2) &= \frac{(m^2+Q^2)}{M^2}\Big[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \\ &\quad - \frac{m^2}{4M^2}\left((F_1+F_2)^2 + (F_A + 2F_P)^2 - 4\left(1+\frac{Q^2}{4M^2}\right)F_P^2\right)\Big], \\ B(Q^2) &= \frac{Q^2}{M^2}F_A(F_1+F_2), \\ C(Q^2) &= \frac{1}{4}(F_A^2 + F_1^2 + \tau F_2^2). \end{aligned}$$

- Quasi-elastic scattering (QE)



- Deep inelastic scattering (DIS)



Review:
A.Meyer et al., Annu. Rev. Nucl. Part. Sci. 2022. 72:1–30, 2201.01839

Physics from flavor diagonal nucleon charges

- $\boldsymbol{g_A^q} = \mathbf{\Delta q}$: Quark contributions to the nucleon spin

$$\frac{1}{2} = \sum_{u,d,s,\cdots} \left(\frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{\Delta q} + L_q \right) + J_g$$

X. Ji (1997),
L_q: orbital angular momentum of the quark
J_g: total angular momentum of the gluons

- $\boldsymbol{g_T^q}$: Quark EDM contributions to the neutron EDM d_n

nEDM collab. (2020)

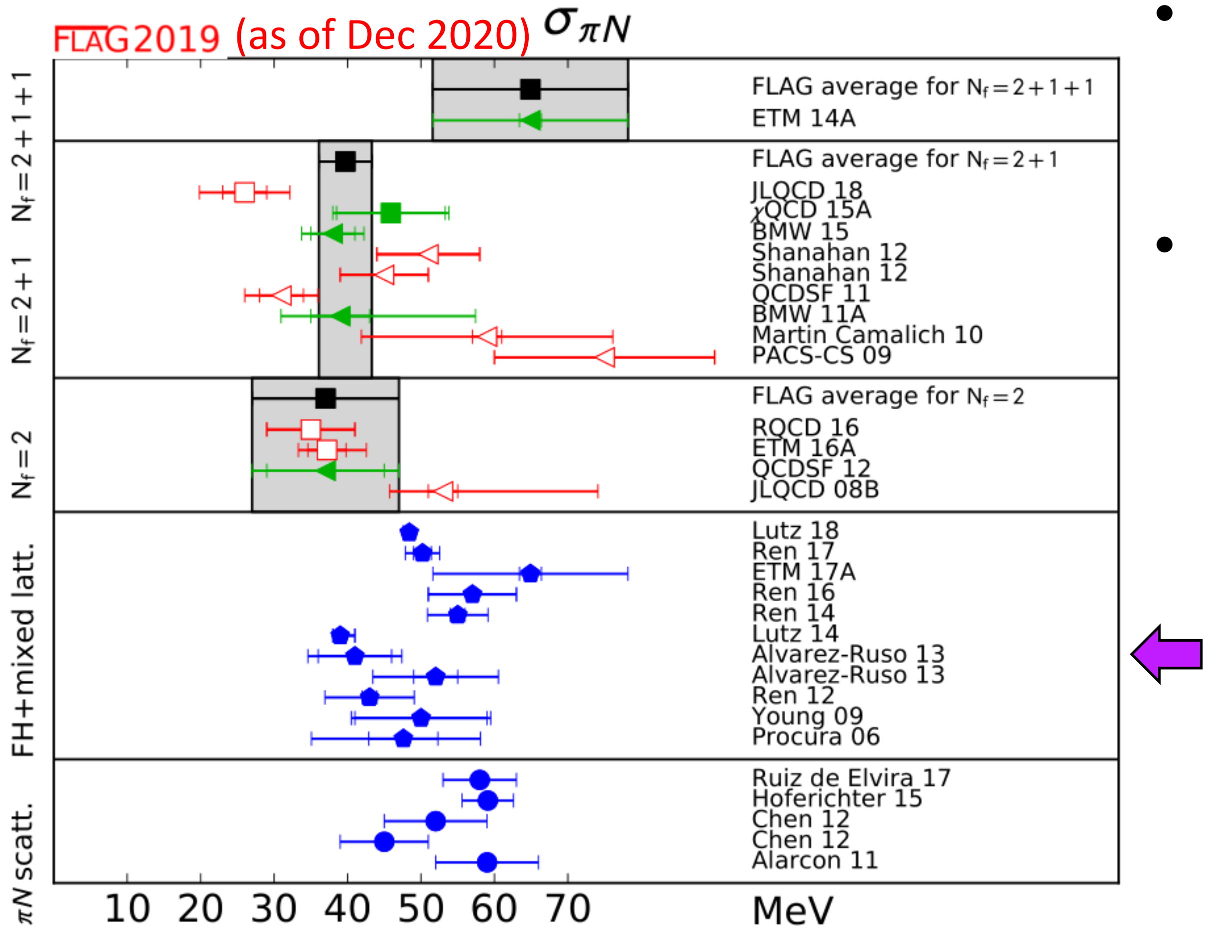
$$|d_n| = |d_u^\gamma \boldsymbol{g_T^u} + d_d^\gamma \boldsymbol{g_T^d} + d_s^\gamma \boldsymbol{g_T^s} + \cdots| \leq 1.8 \times 10^{-26}\ e\ \mathrm{cm}$$

- $\boldsymbol{g_S^q} = \frac{\boldsymbol{\partial M_N}}{\boldsymbol{\partial m_q}}$: Slope of the nucleon mass with respect to the quark mass

$\sigma_{\pi N} = m_l \boldsymbol{g_S^{u+d}}$: Quark contributions to the nucleon mass
$\sigma_s = m_s \boldsymbol{g_S^s}$

Pion-nucleon sigma term

- $\sigma_{\pi N} \equiv m_{ud} g_S^{u+d} \equiv m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle$, isospin limit

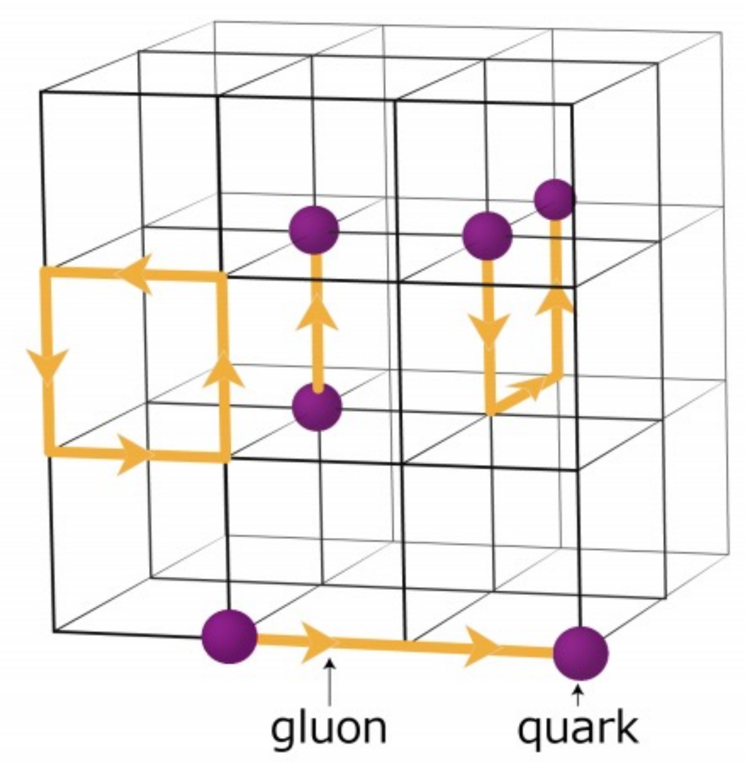


- Fundamental parameter of QCD that quantifies the amount of the nucleon mass generated by u&d quarks.
- g_S: important input in the search of BSM physics!

- Lattice calculation:
  Direct vs Feynman-Hellmann ($\frac{g_S^q}{Z_S} = \frac{\partial M_N}{\partial m_q}$)
- From phenomenology: connection to πN-scattering amplitude via Cheng-Dashen low-energy theorem

Lattice QCD

[Formulated by K. Wilson (1974). Numerical computation field opened by M. Creutz (1979)]



Lattice QCD is QCD defined on a 4-dimensional Euclidean space-time lattice

- Finite lattice spacing: (a)
- Quark fields $(q, \bar{q})$, Gauge fields (gluons): (U_μ)
- Perturbative & **Numerical (nonperturbative) calculations**

The simulation allows **ab initio** calculations of nonperturbative QCD interactions of quarks and gluons using the **Feynman path integral** formulation of QFT.

Major systematic errors coming from:

- Finite lattice spacing a (UV cut-off effect)
- Chiral fit to get value at physical pion mass
- Finite Volume
- Statistical errors
- **Excited state contaminations**
- Renormalization

Calculation of expectation values using importance sampling

Expection value of observables are calculated using the Feynman path integral approach. Starting point is the partition function:

$$Z = \int D[U]D[\bar{\psi},\psi]e^{-S_{\text{lattice}}} = \int D[U]e^{-S_{gluon}[U]}det(M_{\text{quark}})$$

- $S_{\text{lattice}} = S_{\text{gluon}} + \bar{\psi}M_{\text{quark}}\psi$: Lattice QCD action of gluons and quarks
- M_{quark} : Lattice Dirac operator, a matrix in (x, y, z, t)+color+spin space

An ensemble is generated using **Markov Chain Monte-Carlo with the probability distribution**

$$P(\{U\}) = \frac{1}{Z}e^{-S_{gluon}[U]}det(M_{\text{quark}})$$

so that the expectation value of an observable $\mathcal{O}$ is obtained as an ensemble average

$$\langle\mathcal{O}\rangle = \frac{1}{N_{\text{samp}}}\sum_{\{U\}}\mathcal{O}(\{U\})$$

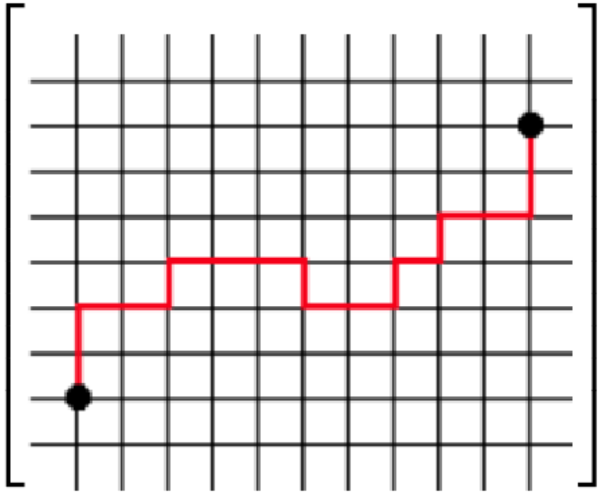
Calculation of $\Delta det(M_{\text{quark}})$ for the generation of ensemble, and M^{-1}_{quark} are the most expensive parts of the calculations.

Calculation of quark propagator

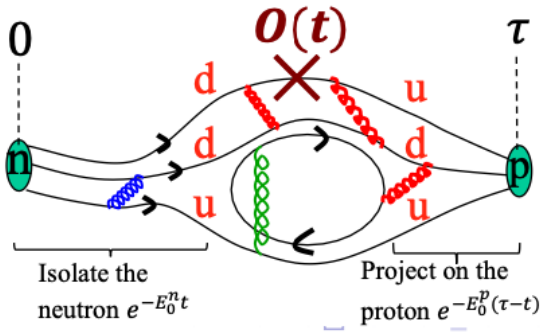
- M : Lattice Dirac operator, a matrix in (x, y, z, t)+color+spin space (dimension$\sim 10^9$)

The inverse of the Dirac operator $M^{-1}(y, x)$ is the quark propagator on a given background gauge configuration $\{U\}$.

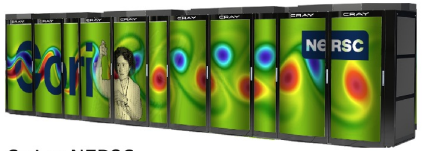
$M^{-1}(y, x) =$



This will be a building block of diagrams like



- To solve the linear equation, x = M · y where M is very large (size ~ $10^9 \times 10^9$, but sparse and mostly diagonal) matrix
- Multigrid method (state-of-the-art)



Cori at NERSC

Los Alamos NATIONAL LABORATORY

Jefferson Lab

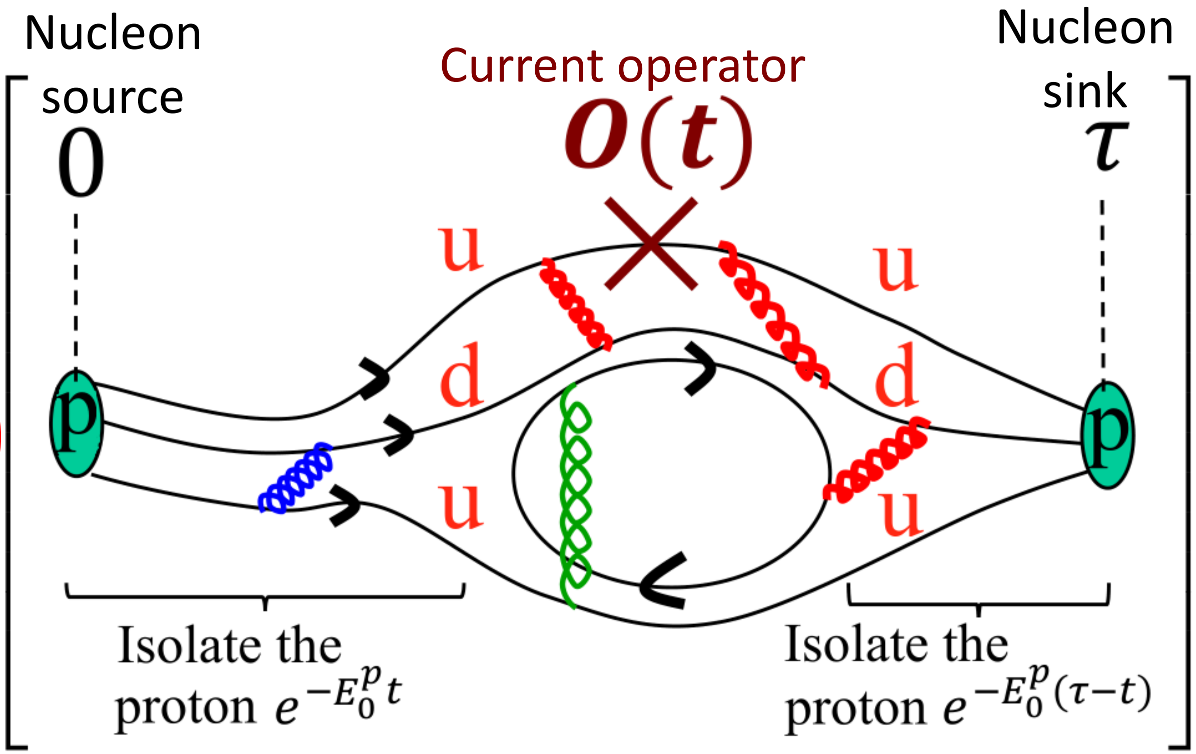
Summit at ORNL

Calculation of Nucleon Matrix Element

- Properties of nucleons ($\langle p|O|p\rangle$, Form Factor) are extracted from the **3-point correlation function** $C(t,\tau) \equiv \langle N^p(\tau) O(t) \bar{N}^p(0)\rangle$:

Average over the "Gauge ensemble" generated using Markov Chain Monte-Carlo

$$C(t,\tau) = \frac{1}{N_{\text{samp}}} \sum_{\{U\}}$$



$$Z = \int \mathcal{D}[U]\mathcal{D}[\bar{\psi},\psi] e^{-S_{\text{gluon}} - \bar{\psi} D_{\text{quark}} \psi} = \int \mathcal{D}[U] e^{-S_{gluon}[U]} det(D_{\text{quark}})$$

$\rightarrow$ **Monte-Carlo!**

$$P(\{U\}) = \frac{1}{Z} e^{-S_{gluon}[U]} det(D_{\text{quark}})$$

Clover fermions on 2+1-flavor Clover Ensembles

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	N_{conf}	N_{HP}	N_{LP}
a127m285	0.127	285	5.87	2002	8008	256256
a094m270	0.094	269	4.09	2469	7407	237024
a094m270L	0.094	269	6.15	4510	18040	577280
a093m220	0.093	216	4.95	2000	8000	256000
a093m220X	0.093	214	4.81	2005	8020	256640
a091m170	0.091	169	3.35	4012	16048	513536
a091m170L	0.091	170	5.01	3000	15000	480000
a073m270	0.073	272	4.81	4720	18800	604160
a072m220	0.072	223	5.10	2000	12000	192000
a071m170	0.071	166	4.28	2500	15000	240000
a070m130	0.070	127	4.37	980	5880	94080
a056m280	0.056	281	5.10	2700	16200	259200
a056m220	0.056	214	4.38	2049	12294	196704

Summit (GPU) at OLCF

- **13 gauge ensembles** generated by the Jlab/W&M/LANL/MIT collaborations
- $O(2-6\times10^5)$ measurements done, Truncated solver method with bias correction

$$C^{\text{imp}} = \sum_{i=1}^{N_{\text{LP}}} \frac{C_{\text{LP}}(\mathbf{x}_i^{\text{LP}})}{N_{\text{LP}}} + \sum_{i=1}^{N_{\text{HP}}} \left[\frac{C_{\text{HP}}(\mathbf{x}_i^{\text{HP}}) - C_{\text{LP}}(\mathbf{x}_i^{\text{HP}})}{N_{\text{HP}}} \right]$$

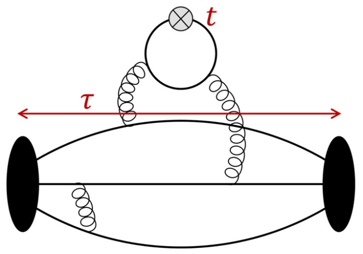
- Simulations are being done over a range of the three free (*unphysical*!) parameters $(a, M_\pi, M_\pi L)$
- Results obtained by extrapolation to the physical values $(\boldsymbol{a=0, M_\pi^{\text{Phys}}, M_\pi L=\infty})$

Disconnected on 2+1+1-flavor HISQ Ensembles

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	$N_{\rm conf}^{\rm conn}$	$N_{\rm conf}^{\rm disc}$ light/strange
a15m310	~0.15	320	3.93	1917	1917 / 1917
a12m310	~0.12	310	4.55	1013	1013 / 1013
a12m220	~0.12	228	4.38	744	958 / 870
a09m310	~0.09	313	4.51	2263	1017 / 1024
a09m220	~0.09	226	4.79	964	712 / 847
a09m130	~0.09	138	3.90	1290	1270 / 994
a06m310	~0.06	320	4.52	500	808 / 976
a06m220	~0.06	235	4.41	649	1001 / 1002

PNDME, PRD98, 034503 (2018)
: Statistics for connected diagrams

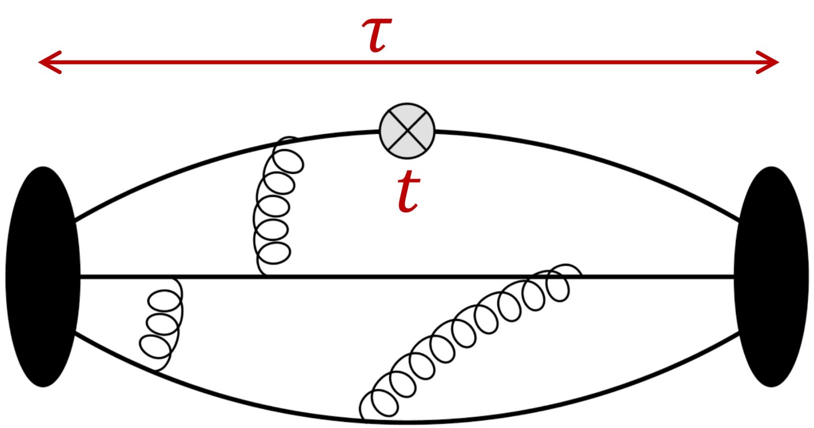
Analyzed for the disconnected diagrams

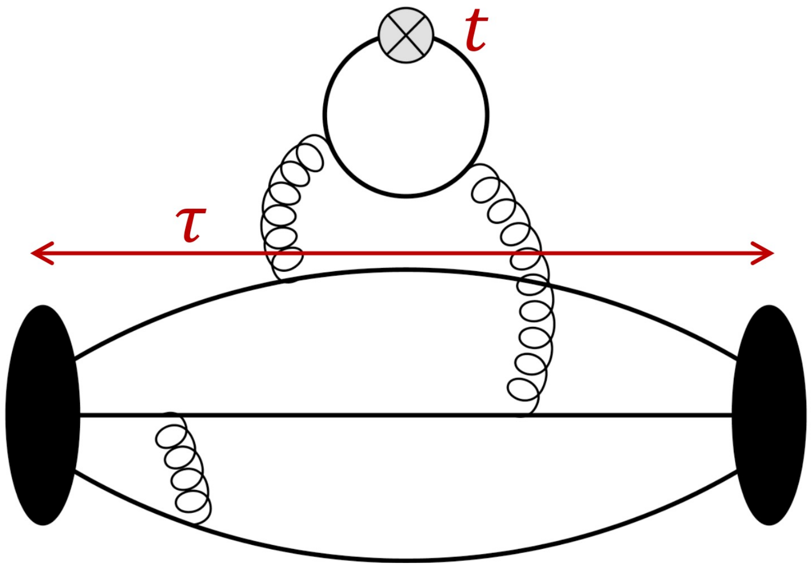


- Ensembles generated by MILC Collaboration
- 8 ensembles including one physical $M_\pi^{\rm phys}$ ensemble
- HYP smeared $N_f = 2+1+1$ MILC HISQ lattices,
- Clover fermion with a tree-level tadpole improved c_{SW}

Connected and disconnected diagrams

- Charges / Form factors are obtained from the nucleon ME $\langle N|\bar{q}\,\Gamma\,q|N\rangle$
- Require high precision measurements of quark bilinear operators within the nucleon state for both "connected" and "disconnected" 3-point correlation functions,





Calculated with covariant Gaussian source smearing, multiple source-sink separation $0.9 \lesssim \tau \lesssim 1.4$, accelerated with coherent sequential inversions and the truncated solver method with bias correction.
PNDME, PRD98, 034503 (2018)

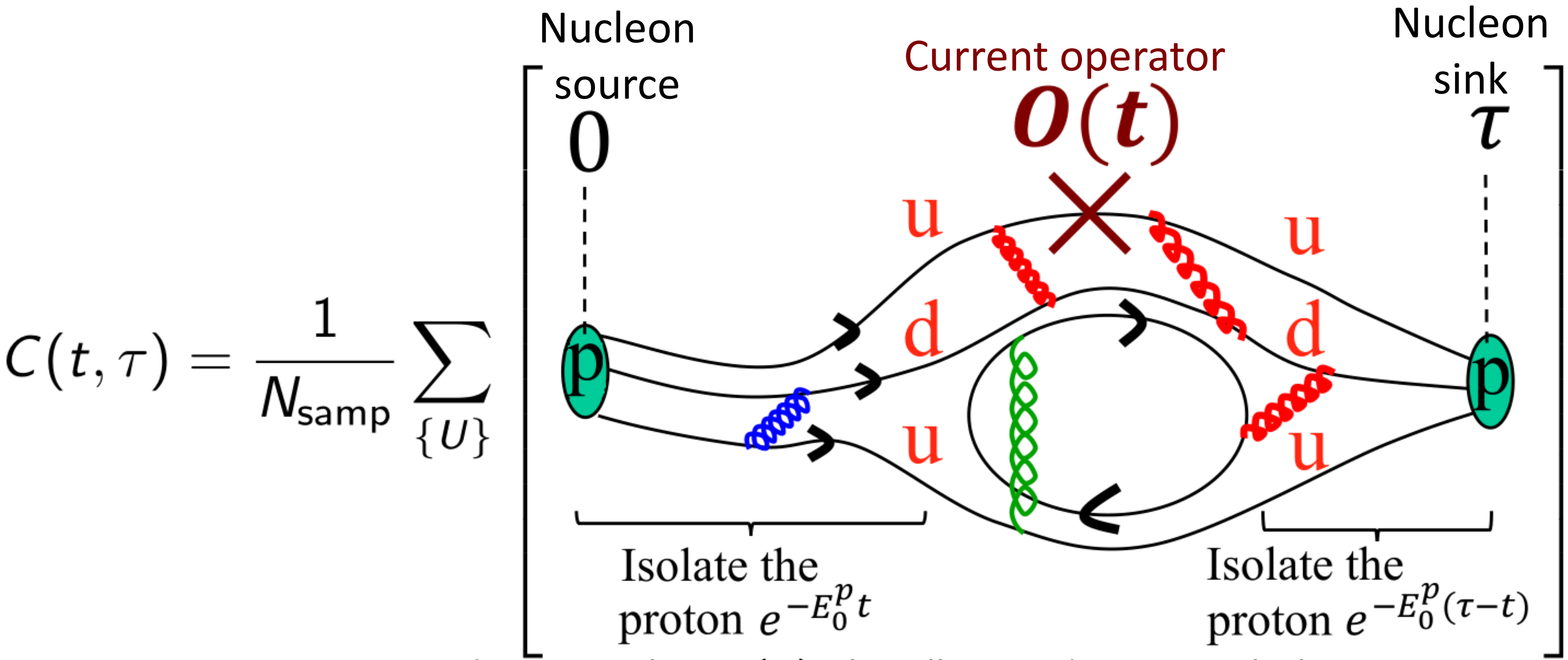
All-to-all quark propagator estimated by stochastic method using Z_4 random sources, accelerated with the truncated solver method with bias correction and hoping parameter expansion.
PNDME, PRD92, 094511 (2015)

Excited-state effect

Effect from $N\pi$ / $N\pi\pi$ multihadron excited states

Calculation of Nucleon Matrix Element

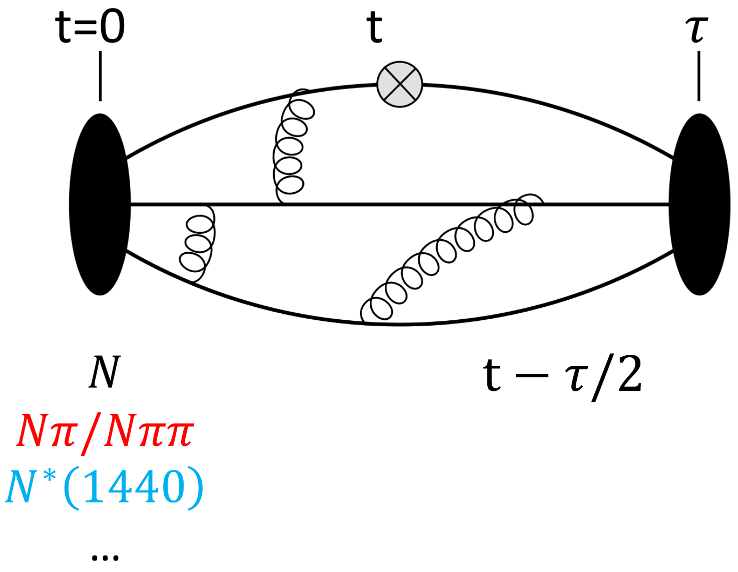
- Properties of nucleons ($\langle p|O|p\rangle$, Form Factor) are extracted from the **3-point correlation function** $C(t,\tau) \equiv \langle N^p(\tau) O(t) \bar{N}^p(0)\rangle$:



- Nucleon operator creates ground state nucleons (N) plus all excited states with the same quantum number, including $N\pi$, $N\pi\pi$, $N\rho$, $N^*(1440)$, $N^*(1710)$, $\cdots$.
- Nucleon signal/noise decays $\propto e^{-(E-1.5M_\pi)\tau}$ with Euclidean time τ.

Excited state contamination (ESC)

- Excited states (ES) that give significant contribution to a particular nucleon matrix element are not known a priori. → χPT is a very useful guide



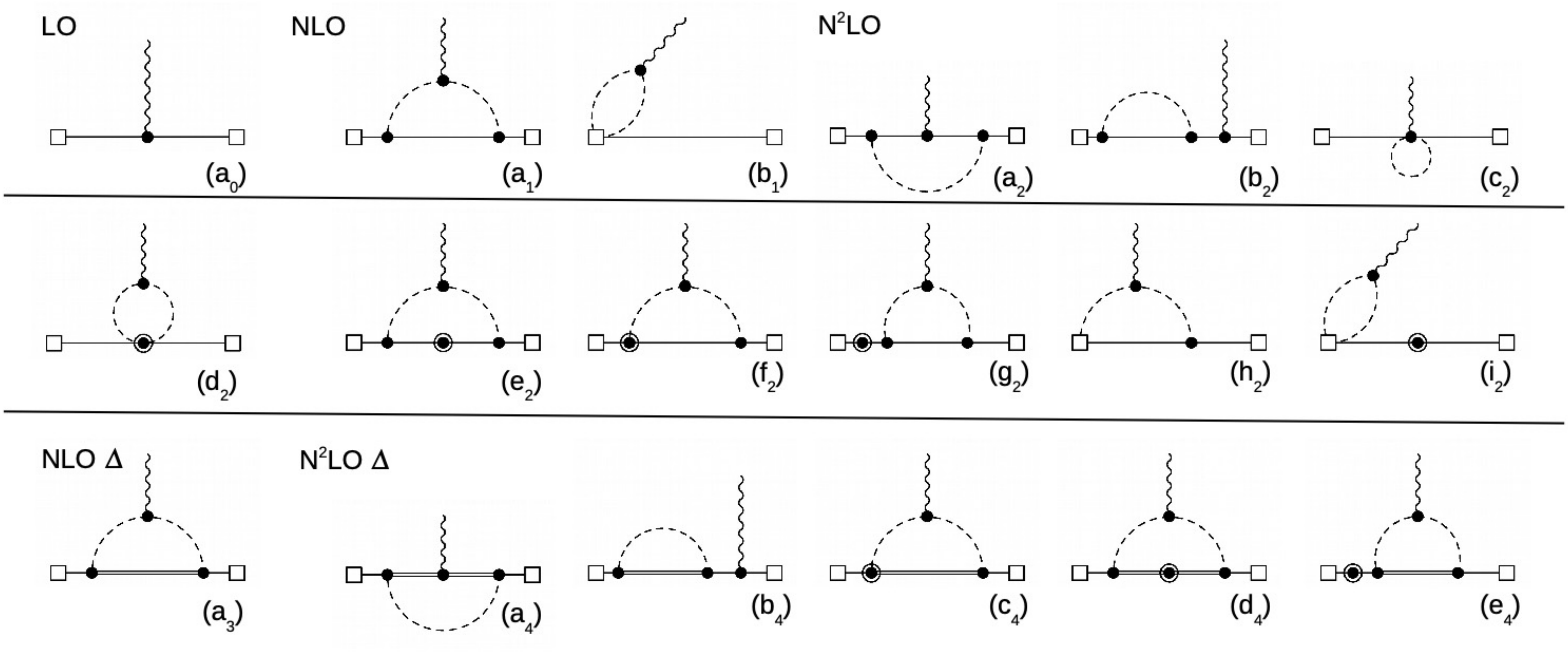
Spectral decomposition of
3-point correlation function $\boldsymbol{C(t,\tau) \equiv \langle N^p(\tau) O(t) \bar{N}^p(0) \rangle}$:

$$C^{3pt} = \langle 0|\mathcal{O}|0\rangle |A_0|^2 e^{-M_0\tau} \times \Big[1 + \frac{\langle 1|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle} \frac{|A_1|^2}{|A_0|^2} e^{-\Delta M_1 \tau} + \frac{\langle 2|\mathcal{O}|2\rangle}{\langle 0|\mathcal{O}|0\rangle} \frac{|A_2|^2}{|A_0|^2} e^{-(\Delta M_2 + \Delta M_1)\tau} + \frac{\langle 0|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle} \frac{|A_1|}{|A_0|} e^{-\Delta M_1 \frac{\tau}{2}} \times 2\cosh\left(\Delta M_1 (t - \frac{\tau}{2})\right) + (\cdots)$$

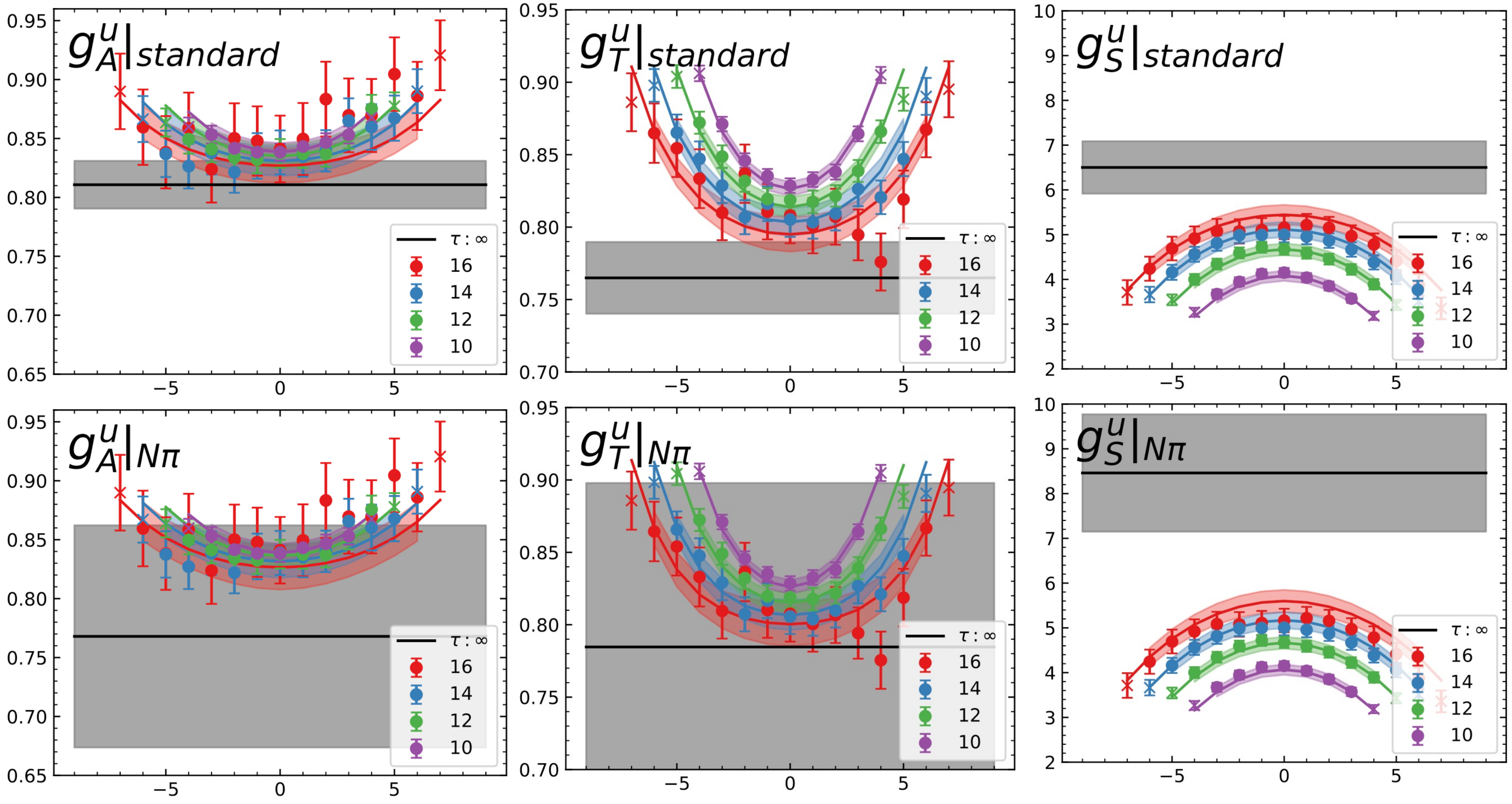
ESC!

Ground-state matrix element
→Nucleon charge or Form Factor

Corrections to the scalar charge in χPT



ESC from $\boldsymbol{N\pi}$ and $\boldsymbol{N\pi\pi}$



- We carry out two types of analyses:
  1. The "**standard**" fit to $C_{2\text{pt}}(\tau)$ uses wide priors for all the excited-state amplitudes, A_i, and masses, M_i, to stabilize the fits.
  2. The "$\boldsymbol{N\pi}$" fit in which a narrow prior is used for M_1 with the central value given by the non-interacting energy of the lowest allowed $N\pi$ or $N\pi\pi$ state on the lattice

- For g_Γ^S, the leading multi-hadron ES is expected to be ΣK → "standard" analysis

$a \approx 0.09fm$
$M_\pi \approx 135MeV$

SP et al., *PoS* LATTICE2022 118

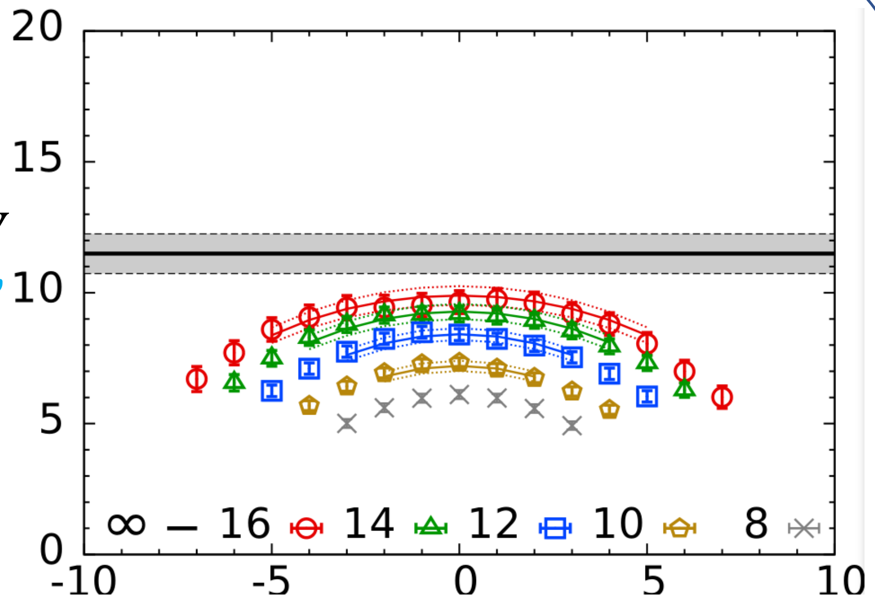
g_S^{u+d}: Excited state effect

$a \approx 0.09 fm$
$M_\pi \approx 135 MeV$

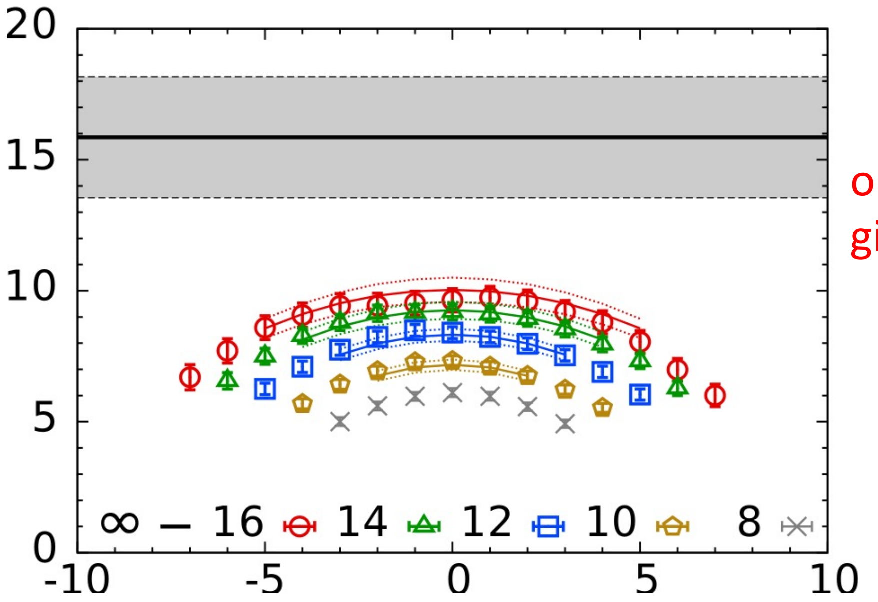
PNDME, PRL 127 (2021) 242002

$M_1 \approx 1.6\ GeV$

"Standard"



$$\sigma_{\pi N} = m_l g_S^{u+d} \sim 40 \text{ MeV}, \ \frac{\chi^2}{dof} = 1.1$$



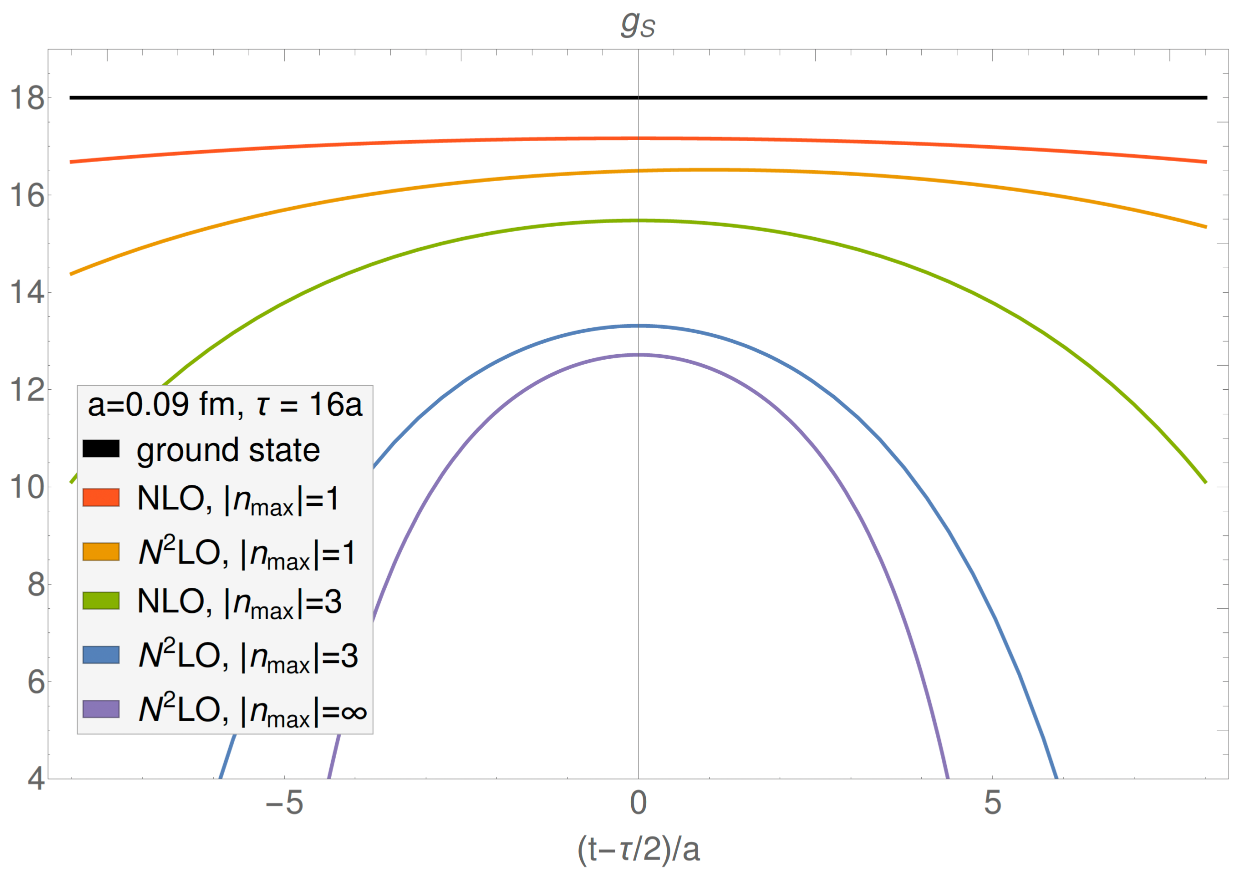
$N(1)\pi(-1)$
or, $N(0)\pi(0)\pi(0)$
gives $M_1 \approx 1.2\ GeV$

"$N\pi$"

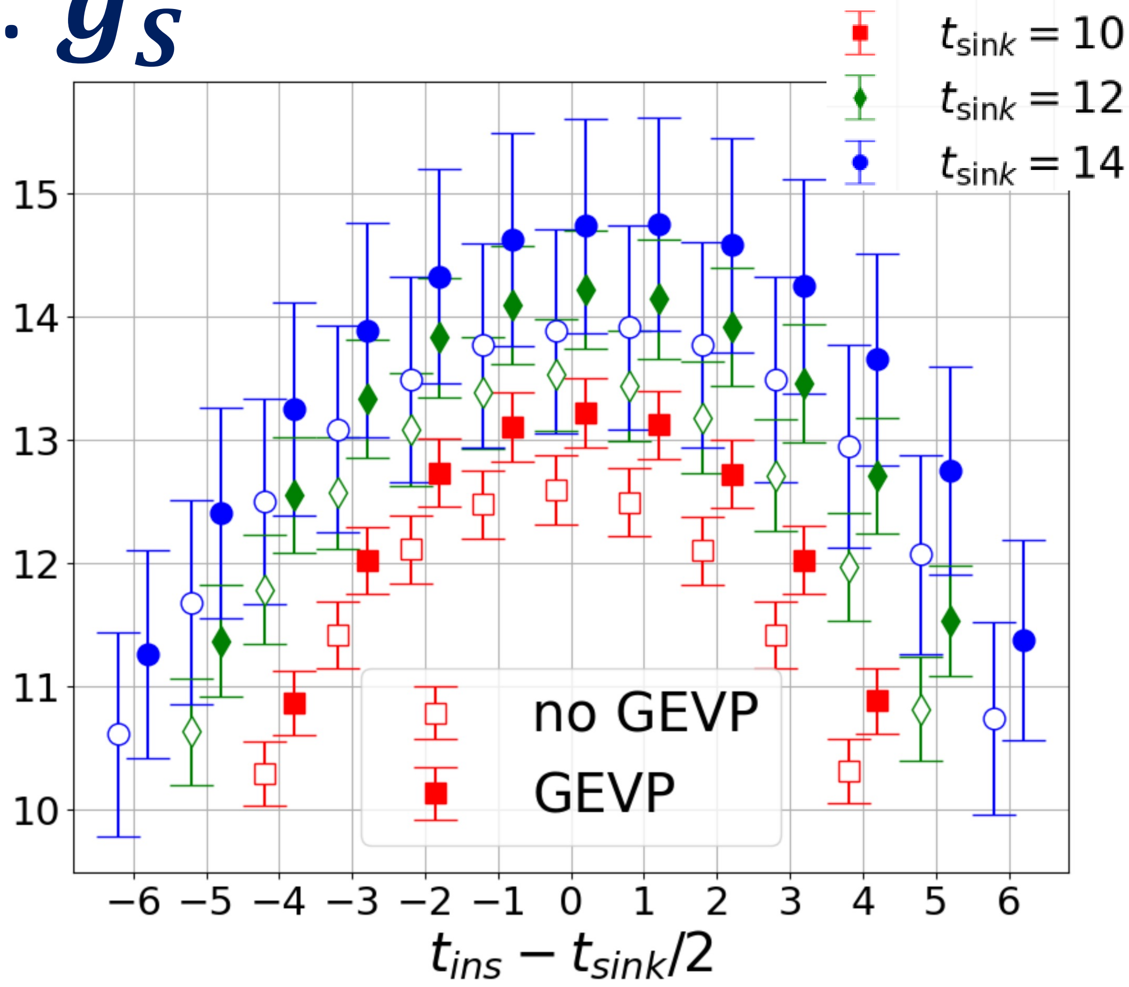
$$\sigma_{\pi N} = m_l g_S^{u+d} \sim 60 \text{ MeV}, \ \frac{\chi^2}{dof} = 1.2$$

- Scalar is sensitive to $N\pi$ state
- Output is close to the phenomenological determination

ESC from $N\pi$ and $N\pi\pi$: g_S^{u+d}



ChPT estimate of ESC
from $N(\boldsymbol{k})\pi(-\boldsymbol{k})$ and $N(\boldsymbol{0})\pi(\boldsymbol{k})\pi(-\boldsymbol{k})$ intermediate states
PNDME, PRL 127, 242002 (2021), Supplemental Material

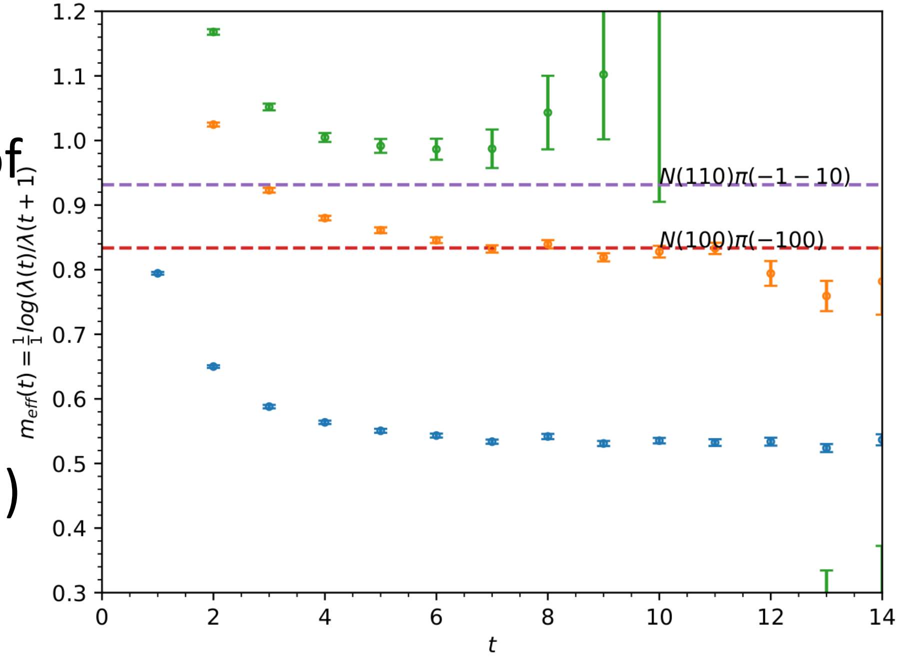


GEVP improvement with $\{N(\boldsymbol{0}),\ N(\boldsymbol{1})\pi(-\boldsymbol{1})\}$
Talk from Yan Li (ETMC), Lattice 2023

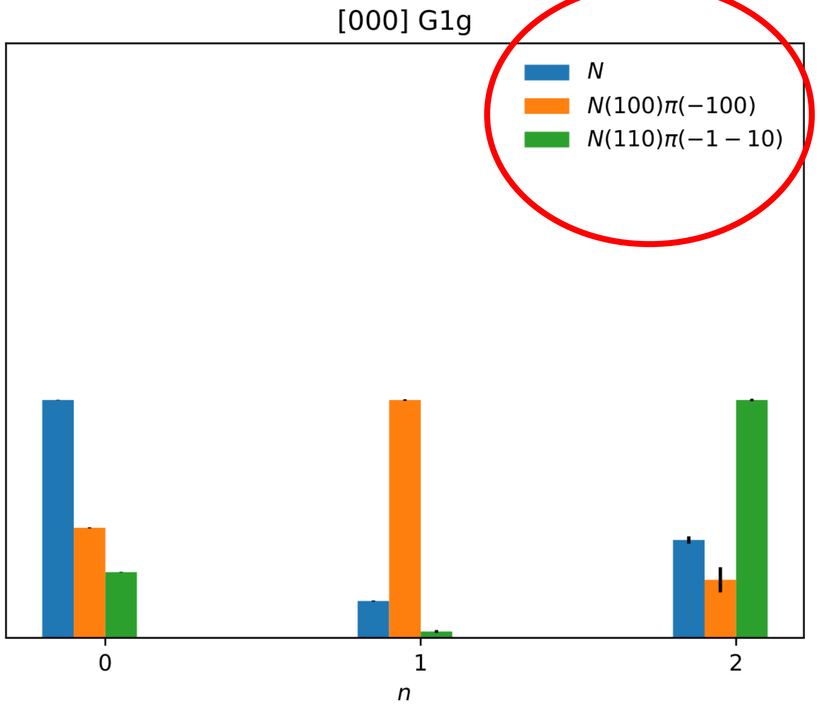
Analysis using multihadron operators
Generalized eigenvalue problem: $I = 1/2$, G_{1g}

$a \approx 0.094 fm$
$M_\pi \approx 358 MeV$

- Distillation with 64 eigenvectors
- redstar software generates complete list of graphs and makes contractions
- Solved GEVP for 3 $N, N\pi$ operators ($I = \frac{1}{2}, I_z = +\frac{1}{2}, J^P = \frac{1}{2}^+$)
- Study on the corresponding 3pt correlation functions are in progress!



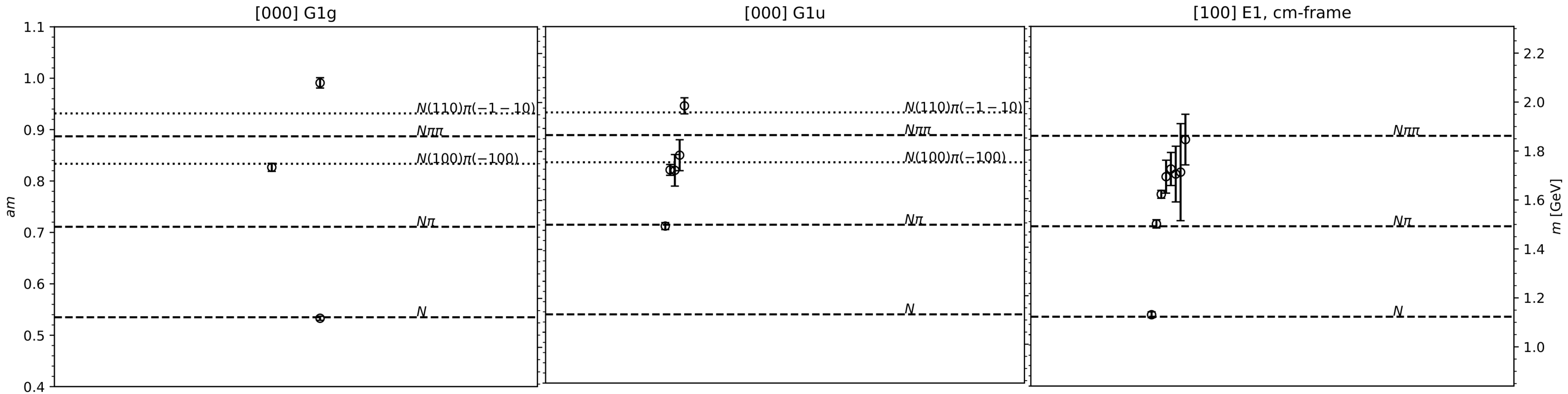
Effective mass from the eigenstates



Normalized overlap factor $Z_i^n = \langle n | O_i^+ | 0 \rangle$

$I = 1/2$ spectrum in the cm frame

$a \approx 0.094 fm$
$M_\pi \approx 358 MeV$



$N(0)\pi(0), N(1)\pi(-1),..$

$\left(N_M \otimes \left(\frac{1}{2}^+\right)^1_M \otimes D^{[1]}_{L=1,M}\right)^{\frac{1}{2}^-}$

$\left(N_M \otimes \left(\frac{3}{2}^+\right)^1_M \otimes D^{[1]}_{L=1,M}\right)^{\frac{1}{2}^-}$

$N(0)\pi(1), N(1)\pi(0),..$

$(N_M \otimes (\frac{1}{2}^+)^1_M \otimes D^{[0]}_{L=0})^{1/2[100]E_1}$

$(N_M \otimes (\frac{1}{2}^+)^1_M \otimes D^{[1]}_{L=1,M})^{1/2[100]E_1}$

$(N_M \otimes (\frac{1}{2}^+)^1_M \otimes D^{[1]}_{L=1,M})^{3/2[100]E_1}$

$(N_M \otimes (\frac{3}{2}^+)^1_S \otimes D^{[1]}_{L=1,M})^{1/2[100]E_1}$

$(N_M \otimes (\frac{3}{2}^+)^1_S \otimes D^{[1]}_{L=1,M})^{3/2[100]E_1}$,...

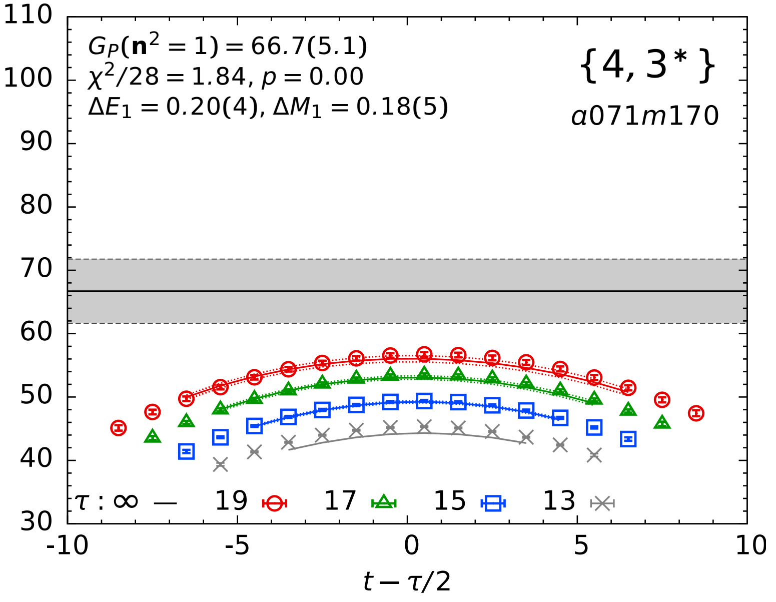
G_P^{u-d}: Excited state effect

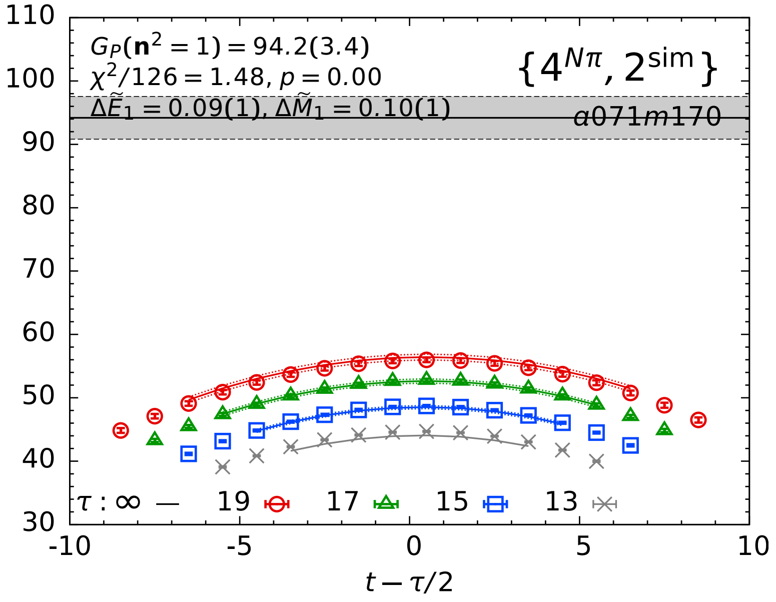
[NME (2021), PRD 105 054505]
$a \approx 0.071$ fm, $M_\pi \approx 170$ MeV
At $\vec{q} = \frac{2\pi}{L}(1,0,0)$

- Data displayed: 3-point/2-point ratio of correlation functions showing dependence on t, τ due to ES
- Gray band: $G_P^{u-d}(\vec{q})$ determined from the ES fit.

Standard 3-state fit to $\langle P \rangle$

$\Delta E_1 \sim \Delta M_1$
$\sim 0.5 GeV$
from 2pt analysis





Simultaneous 2-state to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ correlators

$E_1 \sim N(\vec{0})\pi(\vec{q})$
$M_1 \sim N(\vec{q})\pi(-\vec{q})$

$\Delta E_1 \sim \Delta M_1$
$\sim 0.25 GeV$

- χPT: $N\pi$ state coupling large in the axial current
- Output of a simultaneous fit *increases* the axial form factors by $G_A \sim 5\,\%$, $\tilde{G}_P \sim 35\,\%$, $G_P \sim 35\,\%$
- Satisfies PCAC relation!

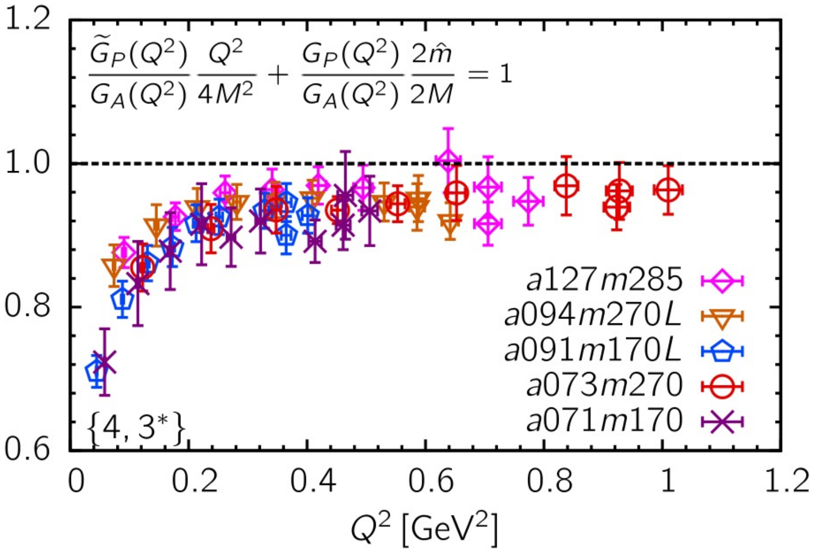
$$2\hat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

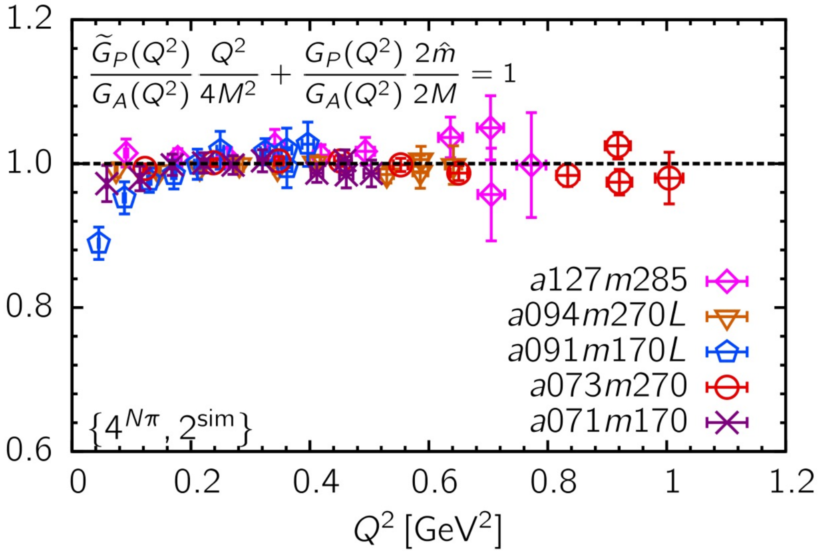
PCAC: Excited state effect

[NME (2021), PRD 105 054505]

Standard 3-state fit to Axials

$\Delta E_1 \sim \Delta M_1$
$\sim 0.5 GeV$





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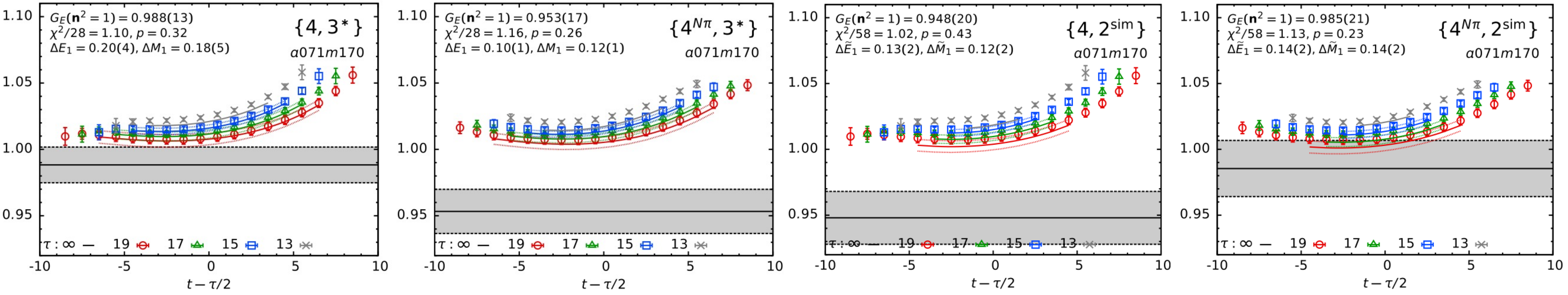
$$2\hat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

G_E^{u-d}: Excited state effect

[NME (2021), PRD 105 054505]
$a \approx 0.071$ fm, $M_\pi \approx 170$ MeV
At $\vec{q} = \frac{2\pi}{L}(1,0,0)$

- Over 4 different strategies to control the ES effect, $G_E^{u-d}(\vec{q})$ has $\approx 4\%$ variation
- At larger momentum transfer $\vec{q}$, the data and fit become less sensitive to ES



- Data displayed: 3-point/2-point ratio of correlation functions showing dependence on t, τ due to ES
- Gray band: $G_E^{u-d}(\vec{q})$ determined from the ES fit.

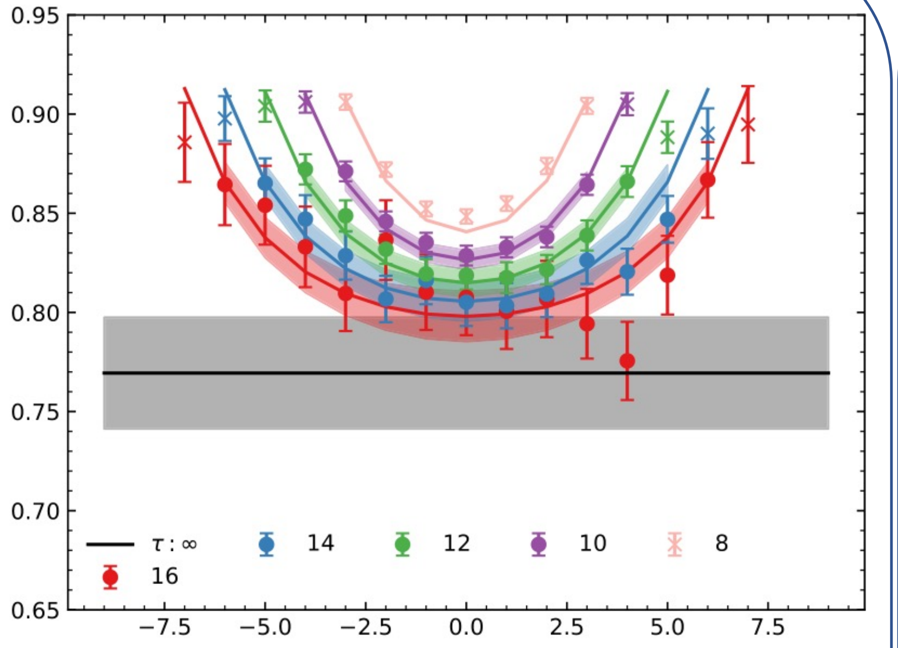
$a \approx 0.09 fm$
$M_\pi \approx 135 MeV$

g_T^u: Excited state effect

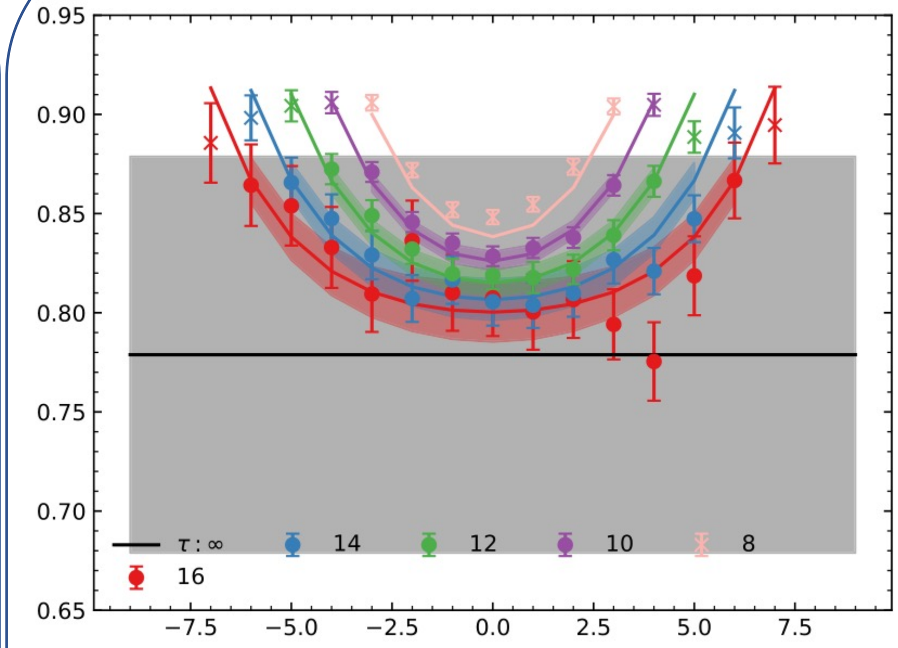
PNDME (2022) PRELIMINARY

$M_1 \approx 1.6\ GeV$

without $N\pi$



0.769(28), $\frac{\chi^2}{dof} = 1.1$



0.78(10), $\frac{\chi^2}{dof} = 1.1$

$N(1)\pi(-1)$
or, $N(0)\pi(0)\pi(0)$
gives $M_1 \approx 1.2\ GeV$

with $N\pi$

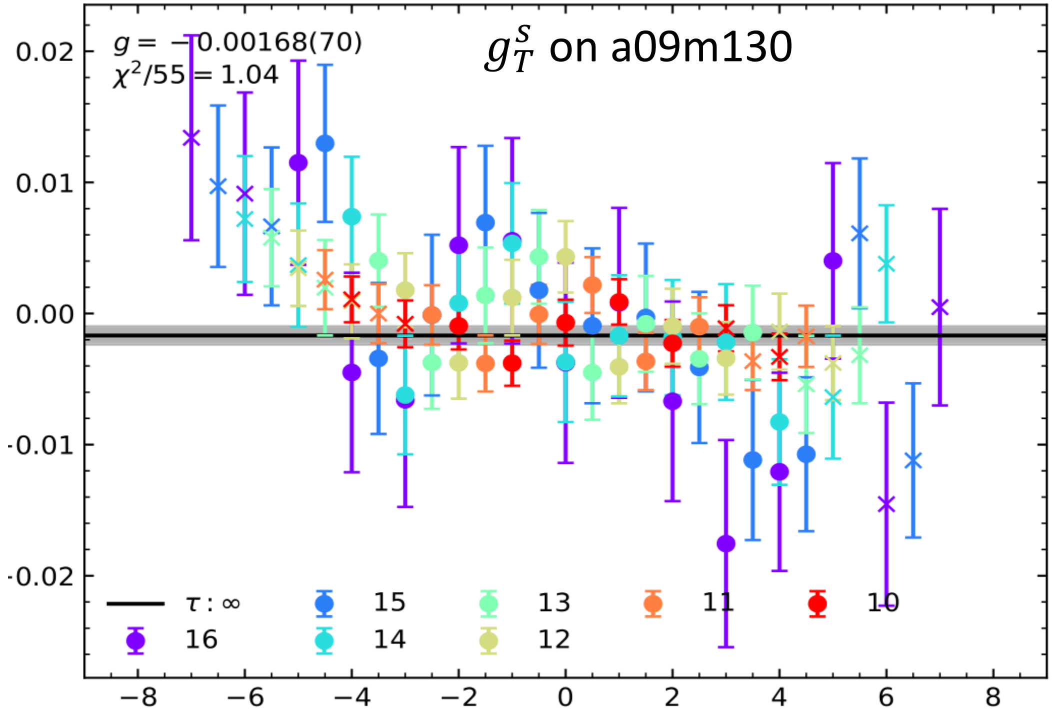
- Tensor is not sensitive to $N\pi$ state

$a \approx 0.09 fm$
$M_\pi \approx 135 MeV$

ESC in g_T^s is not resolved

PNDME (2022) PRELIMINARY

- For g_T^s, 3pt function doesn't show excited state effect
- Constant fit to 3pt/2pt ratio.



Results

Isovector axial, electric and magnetic form factors

(Q^2 dependence fit, chiral-continuum extrapolation)

Flavor diagonal axial, scalar, and tensor charges

Nonperturbative renormalization in RI-sMOM

- Regularization independent (symmetric) momentum subtracted scheme (RI-sMOM)

$$_{p'}\langle f|\mathcal{O}_R^{f'}|f\rangle_p = \delta_{ff'}.$$

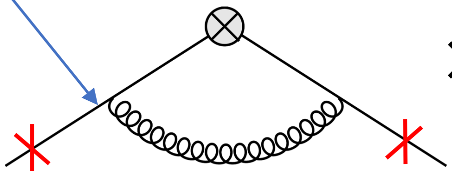
- For flavor diagonal charges, we explicitly evaluated the 3×3 flavor (u, d, s) mixing matrices in

$$g_\Gamma^f = \sum_{f'} Z_\Gamma^{ff'} g_\Gamma^{f'}|_{\mathrm{bare}}$$

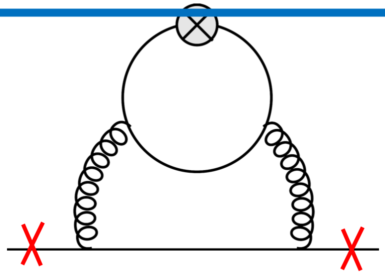
Landau gauge fixed quark propagators using momentum source with $p \propto (1,1,1,1)$

Projected amputated Green's function $\mathrm{Tr}[(..)\mathbb{P}] \equiv \Lambda_\Gamma^{\mathrm{PA}}$

$$(Z_\Gamma^{-1})^{ff'} = \sum_{f'} \frac{1}{Z_\psi^f} \mathrm{Tr}[($$



$$\times\delta^{ff'} -$$

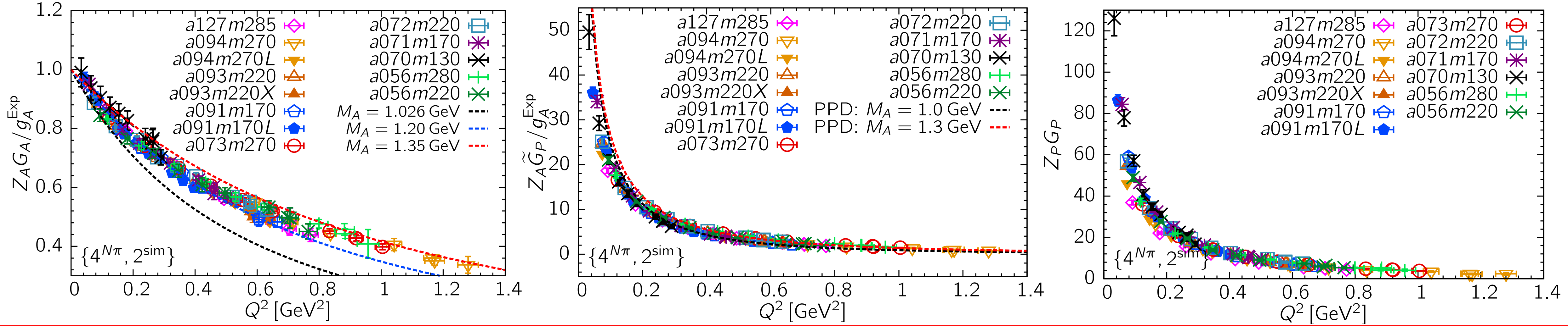


$$)\mathbb{P}(p', p)]$$

Nucleon Isovector Form Factors

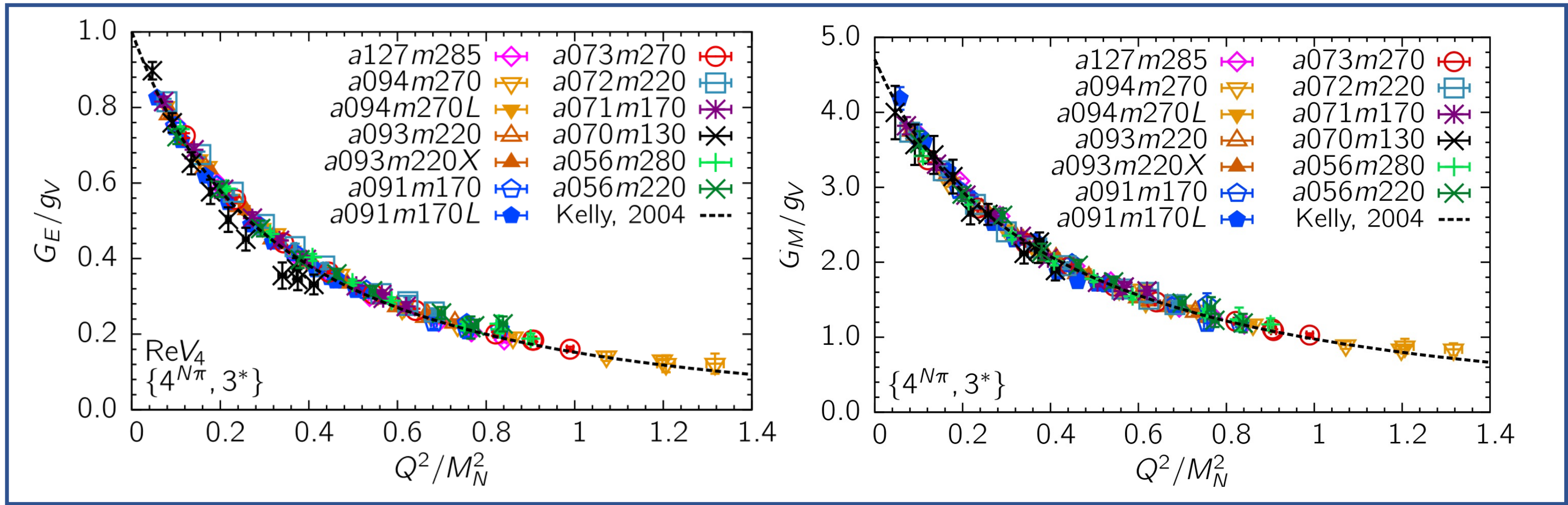
[NME collab., all preliminary]

- Clover fermion on $N_f = 2+1$ clover ensembles



Axial form factors

- $N\pi$ excited state needed to satisfy PCAC relation. Impact on FF is large

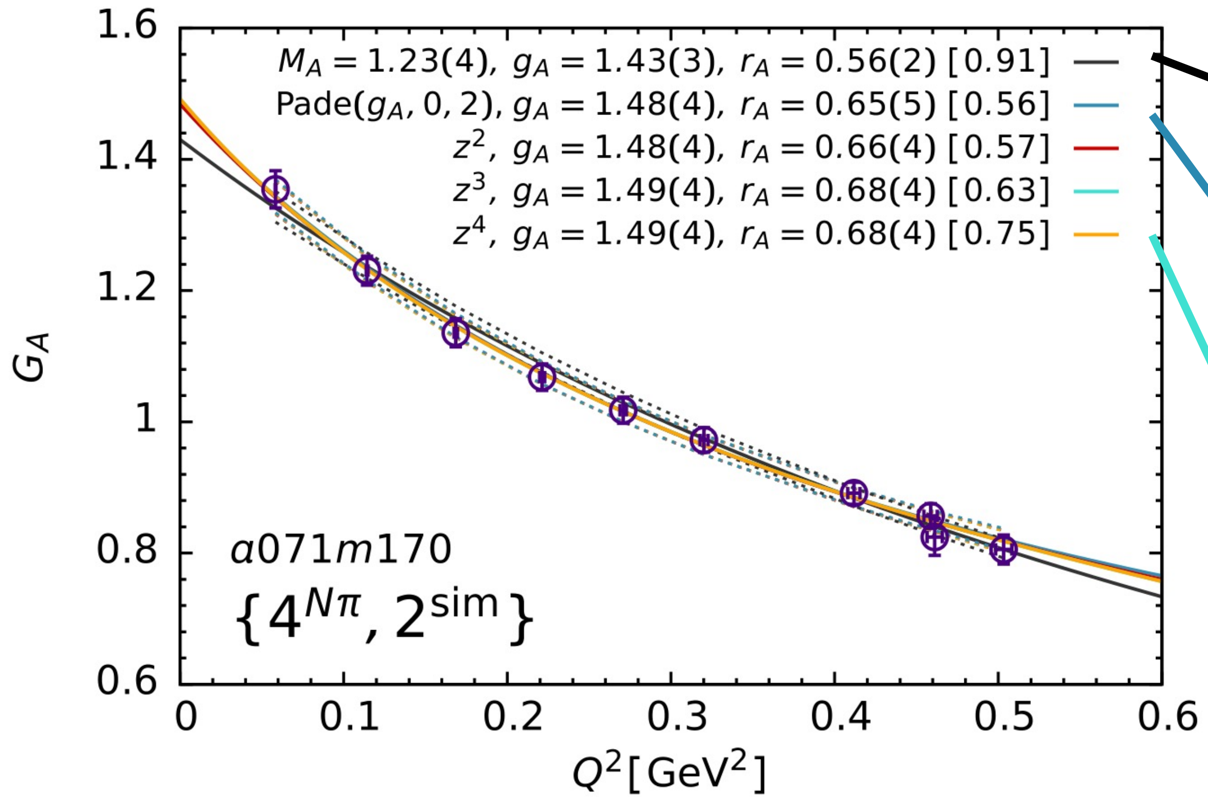


Electric & Magnetic form factors

- Less sensitive to the details of the excited states
- Good agreement with the Kelly curve

[J.J.Kelly, PRC 70, 068202 (2004)]

G_A^{u-d} : Examined Dipole, Pade and z-expansion fits

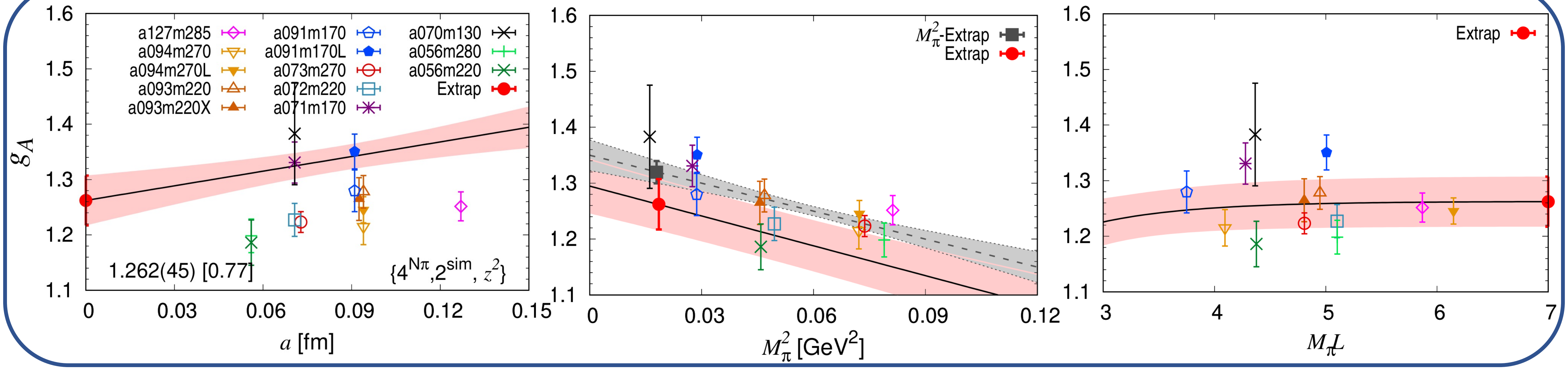


- Dipole: $\frac{G_A(0)}{(1+Q^2/M_E^2)}$
- Pade: $\frac{g}{1+b_1Q^2+b_2Q^4}$
- z^n-expansion: $\sum_{k=0}^{n} a_k z(Q^2)^k$

[NME (2022), PRD 105 054505]

g_A^{u-d}: chiral continuum extrapolation

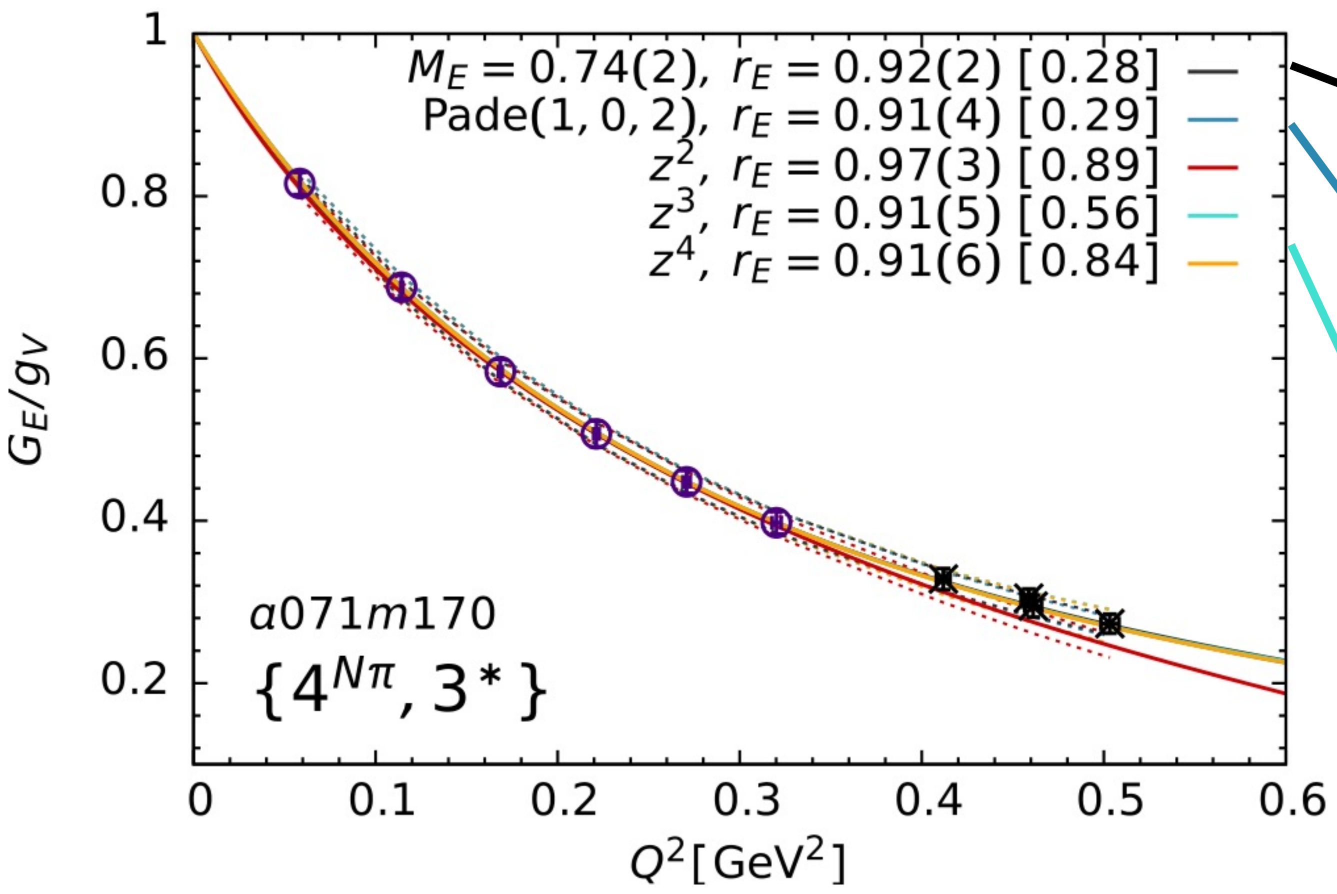
[NME (2023), preliminary]



- Axial charges obtained from the $Q^2 \to 0$ extrapolation to $G_A(Q^2)$

$$g(a, M_\pi, M_\pi L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 \frac{M_\pi^2 e^{-M_\pi L}}{\sqrt{M_\pi L}}$$

G_E^{u-d} : Examined Dipole, Pade and z-expansion fits



- Dipole: $\frac{G_E(0)/g_V}{(1+Q^2/M_E^2)}$
- Pade: $\frac{g}{1+b_1Q^2+b_2Q^4}$
- z^n-expansion: $\sum_{k=0}^{n} a_k z(Q^2)^k$

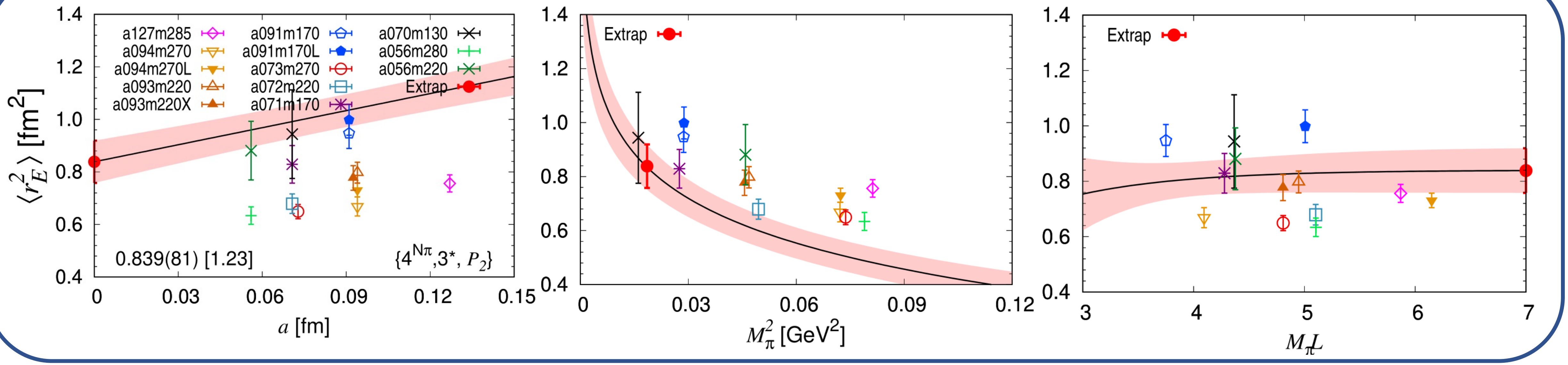
  Need z^3 to fit the data

[NME (2022), PRD 105 054505]

$\langle r_E^2 \rangle^{u-d}$: Chiral-Continuum-Finite Volume extrapolation

$$\langle r_E^2 \rangle^{u-d} = -6 \frac{d}{dQ^2} \left(\frac{G_{E,M}^{u-d}(Q^2)}{G_{E,M}^{u-d}(0)} \right) \Big|_{Q^2=0}$$

NME (2022), preliminary

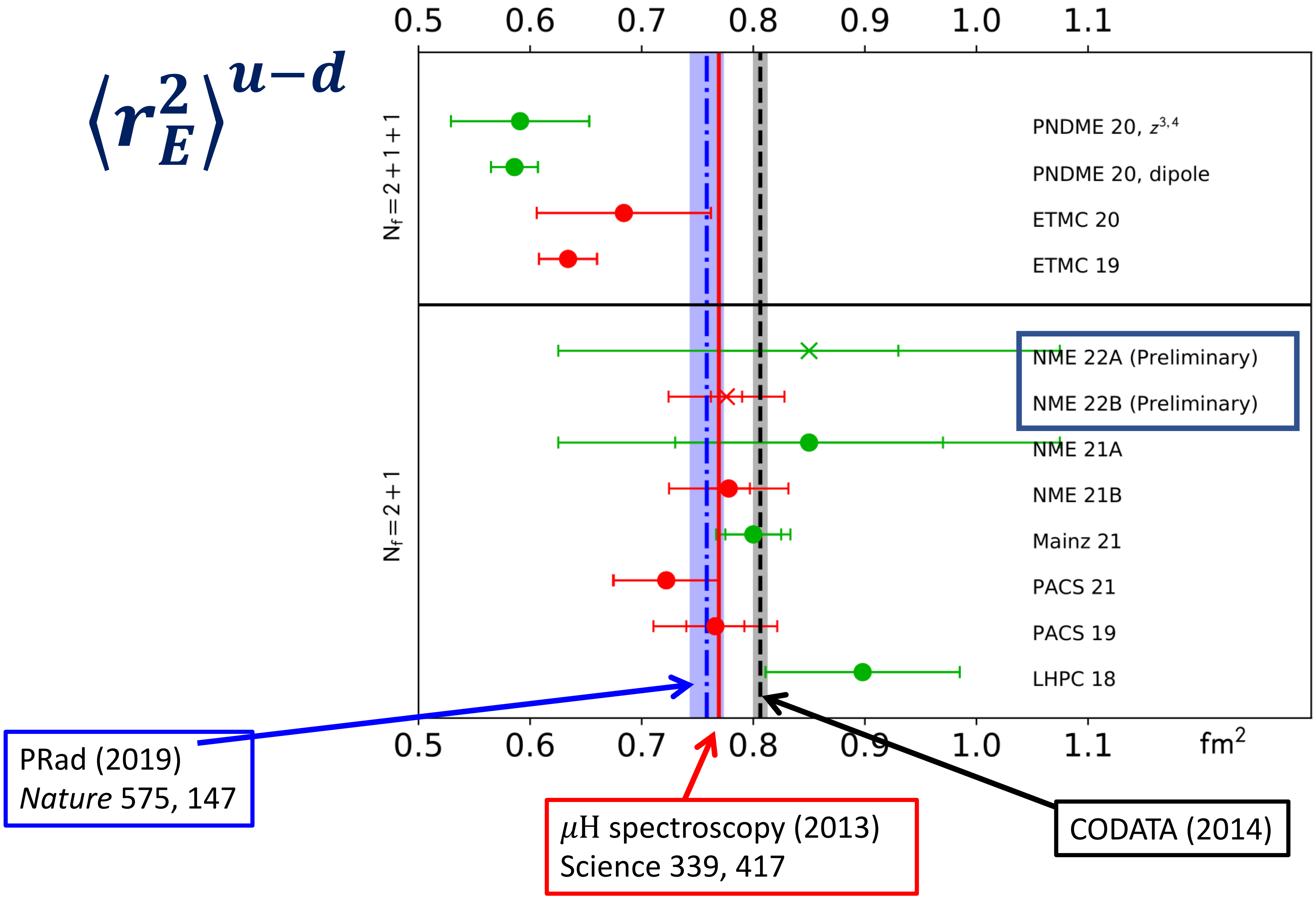


Leading corrections included in the fit ansatz

- $c_1 + c_2 a + c_3 \log \frac{M_\pi^2}{\lambda^2} + c_4 \log \frac{M_\pi^2}{\lambda^2} e^{-M_\pi L}$

Nucl.Phys.A635, 121 (1998)
Nucl.Phys.A679, 698 (2001)
Phys.Rev.D71, 034508 (2005)

$\langle r_E^2 \rangle^{u-d}$



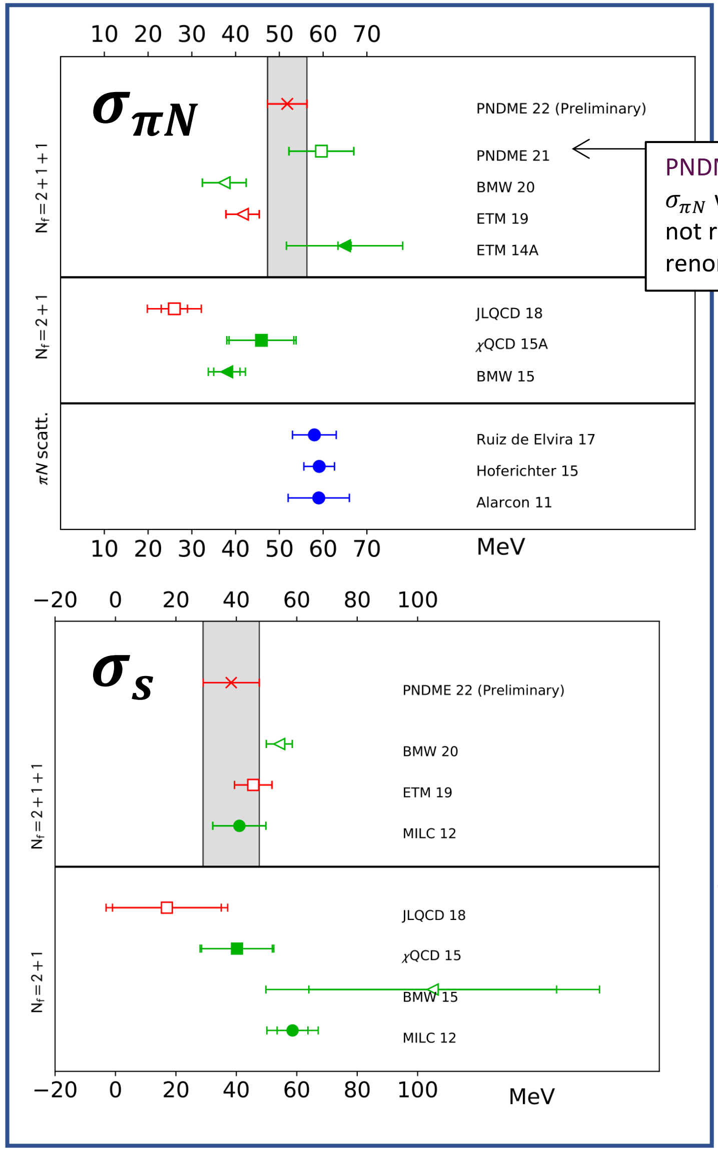
Large systematic uncertainty from the excited state effect

The slope (r_E) is turns out to be very sensitive to various ES fits even though we had a relatively small $G_E^{u-d}(\vec{q}) \approx 4\%$ variation at smallest $\vec{q}$

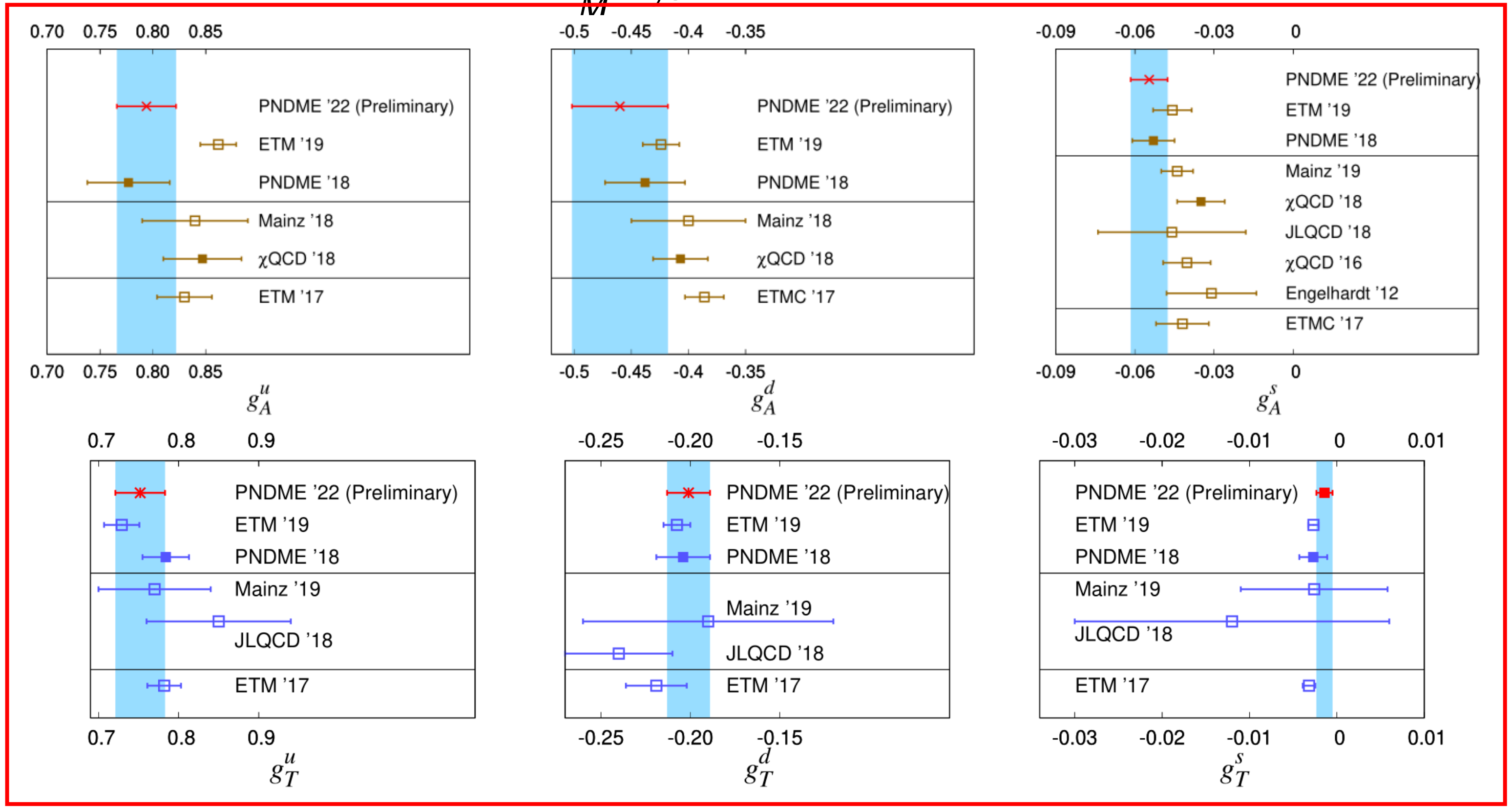
Nucleon Flavor Diagonal Charges
: Comparison with FLAG 2021 results

[PNDME collab., preliminary]

- Clover fermion on $N_f = 2+1+1$ HISQ ensembles
- Flavor mixing calculated nonperturbatively
- Chiral-Continuum extrapolation including a data at M^{Phys}



PNDME (2021)
$\sigma_{\pi N}$ which does not require renormalization



Nucleon sigma terms (Scalar charges)

- $\sigma_{N\pi}$: Excited-state effects are large and results very sensitive to $N\pi$ / $N\pi\pi$ states

Axial and Tensor charges

- Less sensitive to the details of the excited states

Summary

- Using ***lattice QCD***, we are calculating *nucleon isovector form factors and flavor diagonal charges* as part of a comprehensive analysis of nucleon structure
- Form factors presented as a function of Q^2 over $0.04 < Q^2 < 1\ \mathrm{GeV}^2$.
- We are investigating excited state effects
  - Contributions from $N\pi$ / $N\pi\pi$ multihadron excited states
    - Evidence of large ES for $\tilde{G}_P, G_P$, and $g_S^{u,d}$ ($\sigma^{\pi N}$).
  - Need higher statistics to resolve the ES at M_π^{Phys} and on finer lattices (smaller a)
  - Higher order ES fits are under investigation
  - Study with multihadron $N\pi$ operators is in progress

Acknowledgements

- We thank the MILC collaboration for providing the 2+1+1-flavor HISQ lattices.
- The calculations used the CHROMA software suite.
- We thank DOE for computer time allocations at NERSC and OLCF.
- We thank the USQCD collaboration for computer time
- Institutional Computing at LANL for computer time