

Investigating the multiplicities and femtoscopic correlation functions of heavy-flavor and exotic hadrons

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Hadron spectroscopy and the new unexpected resonances

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Paraty



- A brief overview on the state-of-the-art of exotic hadron spectroscopy
- Discussion about the underlying structure and the most promising approaches
- Summary of some of our recent contributions (focus on femtoscopy)



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1 Motivation

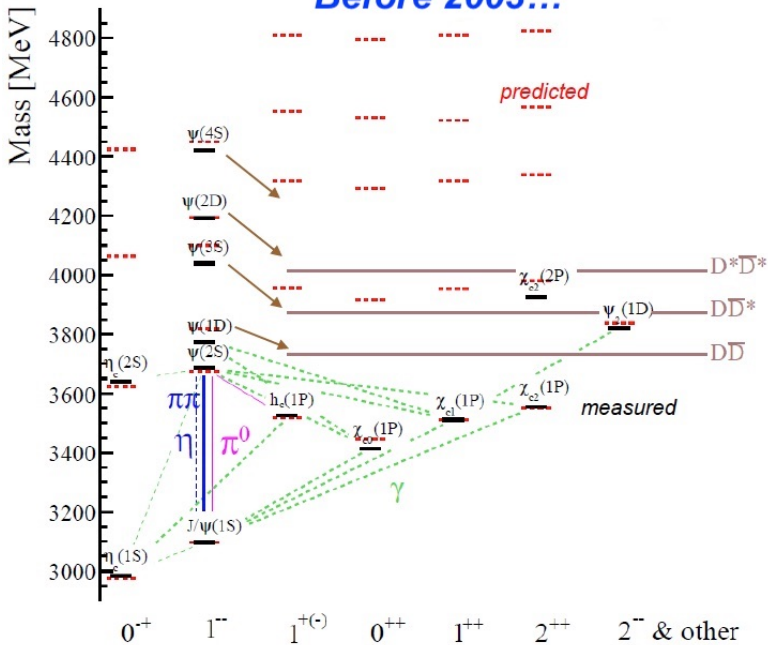
2 Our contributions

3 Femtoscopy

4 Summary

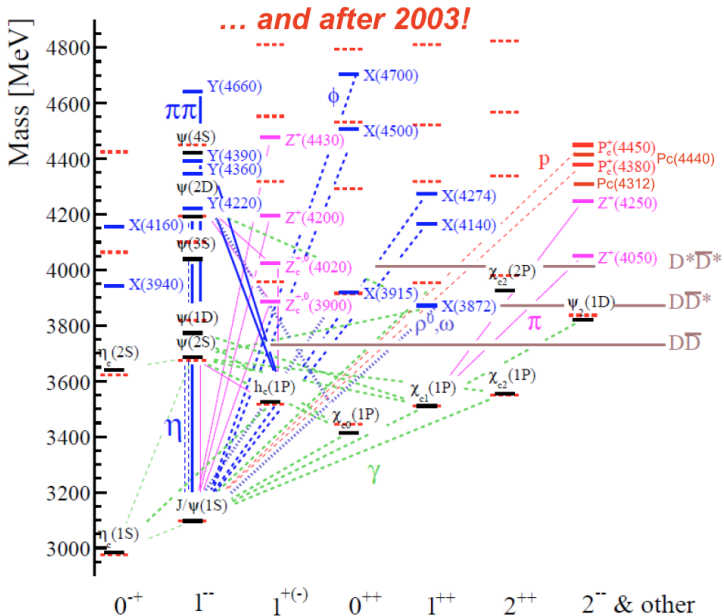


Before 2003...



Figures from Olsen, Skwarnicki, Zieminska



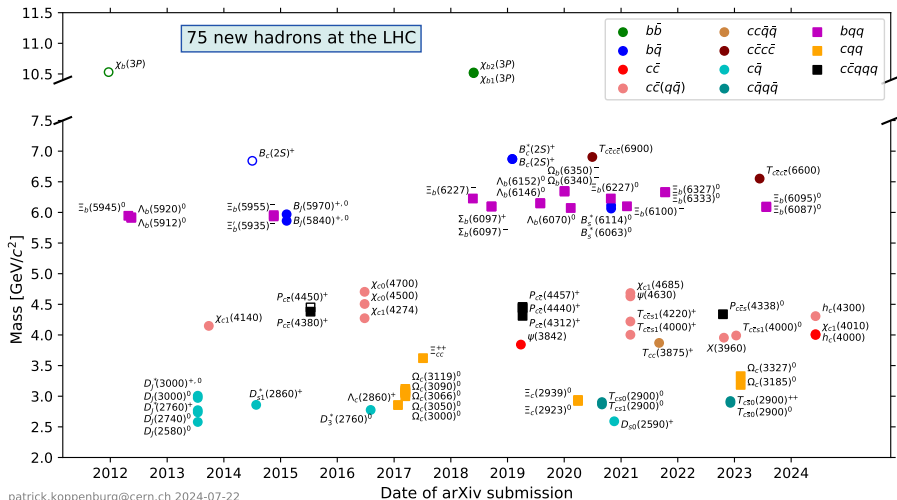


(Adapted from Skwarnicki, 2018)



The heavy exotics collection

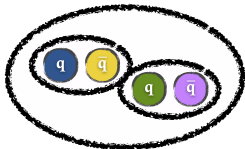
- Since 2003 [$X(3872)$]: about fifty candidates observed!



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Interpretations for composition and binding mechanisms?

- Hadron Molecules



- Hybrids



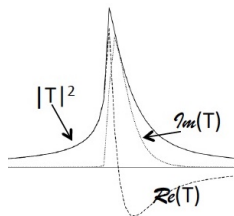
- Glueballs



- Tetraquarks



- Cusp effects (TS's)



- ...



Theoretical perspective

A compelling and unified understanding has not yet emerged

- No single theoretical framework explains the exotics collection
- Candidates: different interpretations (hadron molecule, diquark-antidiquark, kinematical effects, ...)
- (m, Γ) can be explained by different models or even superposition of them
- Necessity of more studies, more observables to distinguish their internal structure



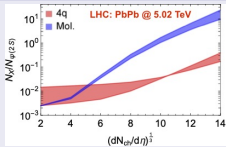
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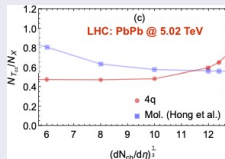
Strategy 1 \Rightarrow Exotics in Heavy-Ion Collisions

$X(3872) [(cq\bar{c}\bar{q}); 0(1^{++})]$



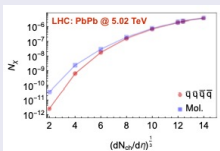
[PRD 90, 114023 (2014); 105, 116029 (2022); 110, 014011 (2024); PTEP 2016, 103B01 (2016), PLB 761, 303 (2016); EPJC (2022); ...] (Navarra's Talk)

$T_{cc}^+(3875) [(cc\bar{q}\bar{q}); 0(1^+)]$



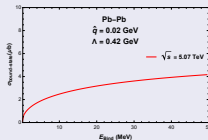
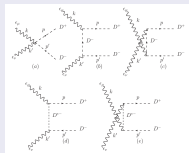
[EPJC 82, 296 (2022); PRD 105, 116029 (2022); NPB 985, 115994 (2022)]

$\chi_{c1}(4274) [(cs\bar{c}\bar{s}); 0^+(1^{++})]$



[PRD 108, 096028 (2023); PRD 109, 014041 (2024)]

$X(3700)^- [(c\bar{c}q\bar{q})0^+(0^{++})]$

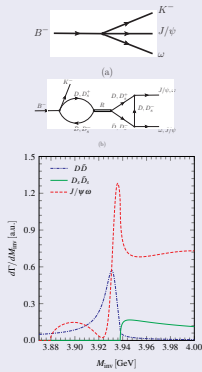


[PRD 110, 034037 (2024)] (Sobrinho's Talk)

[Collaboration USP-UNIFESP-UFBA: Navarra, Nielsen, Torres, Kamchandani, LMA, Vieira, Britto, Magalhães ...]

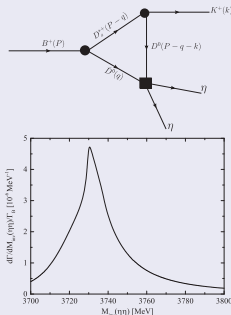
Strategy 2 → Exotics in hadron decays

$X(3930, 3960)$ in $B \rightarrow KJ/\psi\omega$



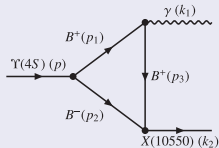
[Collaboration Valencia-UFBA
(Abreu, Albaladejo, Feijoo, Oset,
Nieves); EPJC 83, 309 (2023)]

$D\bar{D}(3720)$ in $B^+ \rightarrow K^+\eta\eta$



[Collaboration
Valencia-Beihang-UFBA (Brandão,
Song, Abreu, Oset); PRD 108,
054004 (2023)] (**Brandão's talk**)

$B\bar{B}(10550)$ in $\Upsilon(4S) \rightarrow \gamma X(10550)$



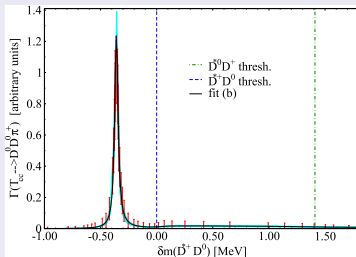
E_B [MeV]	g_{XBB} [GeV]	$\Gamma_{\Upsilon(4S) \rightarrow \gamma X(10550)}$ [keV]	$B_{\Upsilon(4S) \rightarrow \gamma X(10550)}$
5	28.50	1.55	7.55×10^{-5}
10	33.96	2.59	1.25×10^{-4}
25	47.94	6.90	3.37×10^{-4}
50	67.87	19.87	9.70×10^{-4}
75	85.85	42.34	2.06×10^{-3}
100	102.73	76.79	3.75×10^{-3}

$$\Gamma_{\Upsilon(4S) \rightarrow \gamma X(10550)} \sim 0.5 - 192 \text{ keV}$$

[Collaboration UFRB-UFBA (Britto,
Abreu); PRD 110, 056008 (2024)]

Strategy 3 → Compositeness of exotic states

Weinberg criterion + Bethe-Salpeter form. + fit

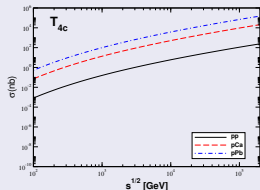
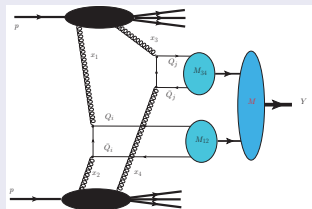


	a_i [fm]	$r_{0,i}$ [fm]
$i = 1 (D^{*+} D^0)$	$(7.60 \pm 0.14) - i (1.73 \pm 0.09)$	-2.94 ± 0.04
$i = 2 (D^{*0} D^+)$	$(1.99 \pm 0.07) - i (1.25 \pm 0.23)$	$(0.11 \pm 0.17) - i (2.74 \pm 0.22)$

B [KeV]	Γ [KeV]	g_1 [MeV]	g_2 [MeV]	P_1	P_2
360 ± 2	38 ± 1	3875 ± 51	-4077 ± 72	0.697 ± 0.017	0.301 ± 0.009

[Collaboration Valencia-Huzhou-UFBA: Dai, LMA, Feijoo, Oset, EPJC 83, 983 (2023)]

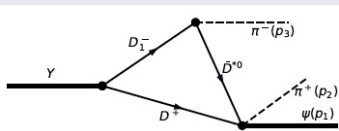
Strategy 4 → T_{4Q} through DPS



[Coll. UNIFESP-UFPEL-UFBA: LMA,

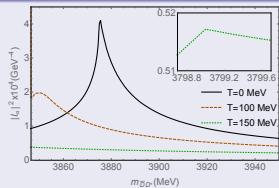
Cerqueira, Carvalho, Gonçalves; EPJC 84, 470 (2024)] (Carvalho's talk)

Strategy 5 → exotic states as kinematical effects

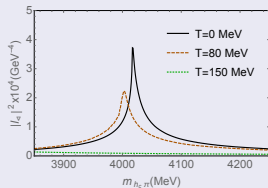


- $Z_c(3900)$: triangle singularity or new hadron?
- Can HICs help to discern the correct interpretation?

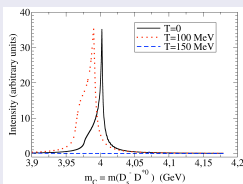
$Z_c(3900) [(cq\bar{c}\bar{q}); 1(1^-)]$



$Z_c(4020) [(cq\bar{c}\bar{q}); 1(?^?)]$



$Z_{cs}(3985)^- [(cs\bar{c}\bar{u})\frac{1}{2}(1^+)]$



- Singularity disappears at temperatures just below T_H
- Medium: spectroscopic filter to distinguish actual hadrons from TSs

[Collaboration U.Complutense Madrid-UFBA (F. Llanes-Estrada, ...); EPJ C 81, 430 (2021); PoS EPS-HEP2021 (2022) 278]; Nucl.Part.Phys.Proc. 318, 32 (2022)]

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Strategy 6 → Femtoscopy

Generalized coupled-channel CF for a specific channel i

$$\begin{aligned} C_i(k) &= \frac{N_i(\vec{k}_1, \vec{k}_2)}{N(\vec{k}_1)N(\vec{k}_2)} \simeq \int d^3\vec{r} S_{12}(\vec{r}) |\Psi_i(\vec{r}, \vec{k})|^2 \\ &= 1 + 4\pi \int_0^\infty dr r^2 S_{12}(\vec{r}) \left(\sum_j w_j |j_0(kr)\delta_{ji} + T_{ji}(\sqrt{s}) \tilde{G}_j(r; s)|^2 - j_0^2(kr) \right), \end{aligned}$$

\vec{k} : relative momentum;

w_j : weight of the observed channel j (common choice: $w_j = 1$);

$E = \sqrt{s}$: the CM energy;

T_{ji} : elements of the scattering matrix encoding the meson–meson interactions;

$$\tilde{G}_j(r; s) = \int_{|\vec{q}| < \Lambda} \frac{d^3q}{(2\pi)^3} \frac{\omega_1^{(j)} + \omega_2^{(j)}}{2\omega_1^{(j)}\omega_2^{(j)}} \frac{j_0(qr)}{s - (\omega_1^{(j)} + \omega_2^{(j)})^2 + i\epsilon},$$

$\omega_a^{(j)} \equiv \omega_a^{(j)}(k) = \sqrt{k^2 + m_a^2}$; $\Lambda = 700$ MeV;

$S_{12}(\vec{r})$: source function,

$$S_{12}(\vec{r}) = \frac{1}{(4\pi)^{\frac{3}{2}} R^3} \exp\left(-\frac{r^2}{4R^2}\right),$$

R : source size parameter (larger R : larger system size $pp \rightarrow pA \rightarrow AA$ collisions)



Lednicky-Lyuboshits (LL) approximation (asymptotic $\Psi(r \rightarrow \infty)$)

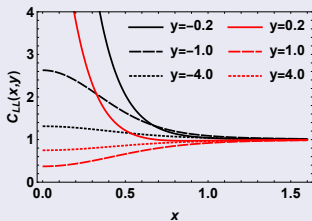
Using

$$-\frac{T}{8\pi\sqrt{s}} = f(k) \equiv \frac{1}{k \cot \delta(k) - ik} = \frac{R}{-R/a - ikR},$$

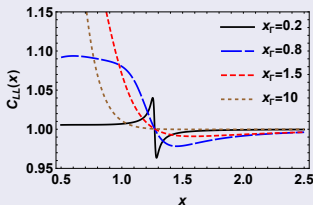
then

$$C_{LL}(x, y) = 1 + \frac{1}{x^2 + y^2} \left[\frac{1}{2} - \frac{2y}{\sqrt{\pi}} F_2(2x) - x F_3(2x) \right],$$

$x = kR$; $y = R/a$ (a : scattering length); $F_2(z) = \int dt \frac{e^{t^2 - z^2}}{z}$, $F_3(z) = \frac{1 - e^{-z^2}}{z}$



- $a < 0$: attractive interaction
- $a > 0$: with a bound state



- Resonance at $k_R(\delta = \pi/2)$:

$$C_{LL}(x_R) = 1 + \frac{e^{-4x_R^2}}{2x_R^2}$$

$$x_r = \sqrt{\mu \Gamma R^2} \quad (\mu: \text{reduced mass})$$

Dependence on $R, a \rightarrow$ bound or quasi-bound state; resonance

Predictions for the T_{bb}^+ state

$B^{*+}B^0; B^{*0}B^+$ interactions

Bethe-Salpeter formalism:

$$T = [1 - VG]^{-1} V ,$$

V : interaction potential
(Use of local hidden gauge approach);

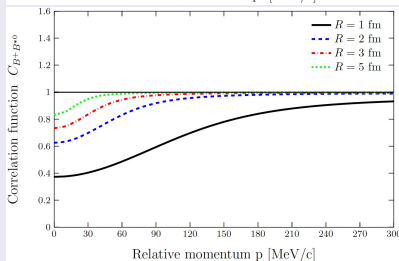
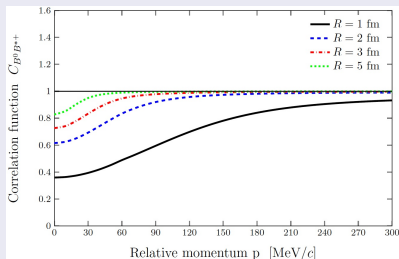
G : loop function ($q_{max} = 420$ MeV).

$I = 0$ state:

$$|BB, I = 0\rangle = -\frac{1}{\sqrt{2}}(B^{*+}B^0 - B^{*0}B^+).$$

[Collaboration Valencia-Huzhou-UFBA
(Dai, LMA, Feijoo, Molina, Oset);
PRD **109**, 016014 (2024)]

$C_{B^{*+}B^0}$ and $C_{B^{*0}B^+}$



T_{bb}^+ as a molecular state \rightarrow the only interpretation from these CF's?

Inverse Problem

- Extraction of relevant observables from the CFs
- Assumption: isospin symmetry, $\langle I = 0 | V | I = 0 \rangle = 1 \rightarrow V_{11} = V_{22}$
- No a priori choice: freedom is left for nonmolecular components
- Contribution from nonmolecular states: energy-dependent terms,

$$V_{11} = V'_{11} + \frac{\alpha}{m_V^2} (s - s_0)$$

$$V_{12} = V'_{12} + \frac{\beta}{m_V^2} (s - s_0)$$

Fit to the synthetic data

- Parameter space:
 $\{q_{max}, V'_{11}, V'_{12}, \alpha, \beta, R\}$
- CFs are used to produce synthetic data
- Fit for the CFs ($R^{(input)} = 1$ fm):

$$q_{max} = 445 \pm 29 \text{ MeV},$$

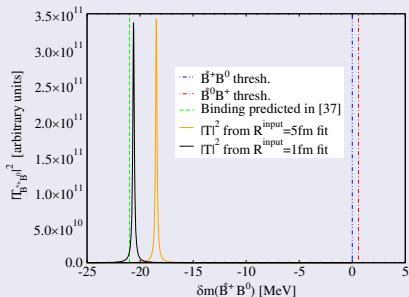
$$V'_{11} = 70 \pm 360,$$

$$V'_{12} = 3463 \pm 1272,$$

$$\alpha = -170 \pm 336,$$

$$\beta = 290 \pm 346,$$

$$R = 0.98 \pm 0.02 \text{ fm}$$



- From CFs: Bound state with $l = 0!$
- $E_b = 20.62$ MeV ($R^{(input)} = 1$ fm)
- $E_b = 18.48$ MeV ($R^{(input)} = 5$ fm)
- In Dai et al. PRD 105, **074017** (2022): $E_b = 21$ MeV

R^{input} (fm)	a_1 (fm)	$r_{0,1}$ (fm)	a_2 (fm)	$r_{0,2}$ (fm)
1	0.85 ± 0.18	-0.11 ± 0.51	$(0.81 \pm 0.13) - i(0.03 \pm 0.03)$	$(0.43 \pm 0.11) - i(0.38 \pm 0.29)$
5	0.85 ± 0.19	-0.92 ± 1.78	$(0.77 \pm 0.13) - i(0.05 \pm 0.06)$	$(0.26 \pm 0.40) - i(0.87 \pm 1.13)$

R^{input} (fm)	g_1 (MeV)	g_2 (MeV)	P_1	P_2	Z
1	33039 ± 14744	-32031 ± 17367	0.44 ± 0.06	0.43 ± 0.05	0.13 ± 0.11
5	30970 ± 19666	-31181 ± 19718	0.41 ± 0.11	0.39 ± 0.11	0.19 ± 0.22

- Molecular probability: $P_1 + P_2 = 0.87 \pm 0.11$ (compatible with 1)
- Nonmolecular probability: $Z = 1 - (P_1 + P_2) = 0.13 \pm 0.11$
- A clear molecular nature for the T_{bb}^+ state!

Can CFs shed light on the nature of the D_1 states?

PDG: evidence of two D_1 states with $J^P = 1^+$

$D_1(2420)$: $M = 2422.1 \pm 0.6$ MeV, $\Gamma = 31.3 \pm 1.9$ MeV.

$D_1(2430)$: $M = 2412 \pm 9$ MeV, $\Gamma = 314 \pm 29$ MeV.

Description of M 's and $R = \Gamma_{D_1(2420)}/\Gamma_{D_1(2430)} \sim 10$ from the same dynamics: controversies (see Kamchandani et al. PRD **110**, 036008 (2024))

Our purpose [Collaboration USP-UNIFESP-UFBA: Navarra, Torres, Kamchandani, LMA]

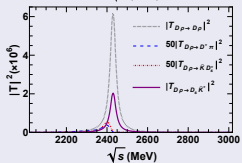
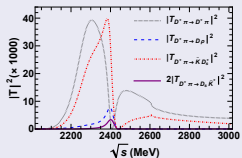
- Model: Meson-meson coupled channel + Bare quark-model pole (Interplay of quark-hadron degrees of freedom)
- Investigation if CFs can be useful
Focus: channels $D^{*+ (0)} \pi^{0 (+)}$, dominated by strong interactions



Model A

$$V_{QM} = -\frac{6000^2}{s - 2440^2}$$

(Godfrey and Isgur, PRD 32, 189 (1985))



- $D^* \pi$: $M \sim 2304$ MeV, $\Gamma \sim 160$ MeV

(Lower limit for $D_1(2430)$ from Babar (2006))

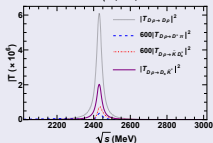
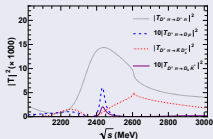
- $a_{D^* \pi}^{(1/2)} = -0.20$ fm

(In accordance with lattice results for $a_{D\pi}^{(1/2)}$ (Liu et al. PRD 87, 014508 (2013)))

Model B

$$V_{QM} = \frac{10000^2}{s - 2370^2}$$

(g_{QM}, M_{QM} considered as free parameters)

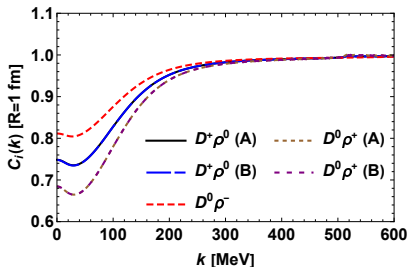
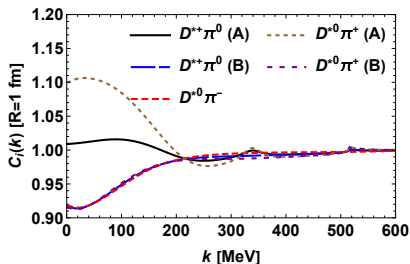


- $D^* \pi$: $M \sim 2436$ MeV, $\Gamma \sim 311$ MeV

(Agreement with LHCb and Belle data)

- $a_{D^* \pi}^{(1/2)} = 0.1$ fm

(In accordance with recent Alice results for $a_{D^* \pi}^{(1/2)}$ (e-Print: 2401.13541 [nucl-ex]))



- Model A: $C_{D^{*0}\pi^+}(k=0)|_{R=1\text{fm}} > 1$ (attractive character)
- Model B: $C_{D^{*0}\pi^+}(k=0)|_{R=1\text{fm}} < 1$ ($a_{D^{*0}\pi^+}^{(1/2)} = 0.1 \text{ fm} < 2.3R$)
- Both models: $C_{D^{*0}\pi^+}(k > 0)$ reflects the behavior of $T_{D^{*0}\pi^+}$
- Dip in $C_{\rho^+}(k=0)$: influence of the narrow state in $T_{D\rho, D\rho}^{(1/2)}$ below the $D\rho$ threshold
- $D^{*0}\pi^+$ and $D^0\rho^+$: more appropriate to test both models

[ϕN CF: LMA, Gubler, Khemchandani, Torres, Hosaka, arXiv:2409.05170]



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Summary

- Hadron Spectrum: richer than what we expected
- New particle zoo near $D^{(*)}\bar{D}^*$, $B^{(*)}\bar{B}^*$ thresholds: not $(\bar{q}q, qq\bar{q})$

General description of exotic states?

- It remains a great challenge!!!
- More experimental and theoretical investigations are necessary to shed light on their dynamics

Thank You!!!

Financial support:

