ExoHad

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GIESSEN

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Helmholtz Forschungsakademie Hessen für FAIR

Paraty 2024

Exotic mesons with functional methods

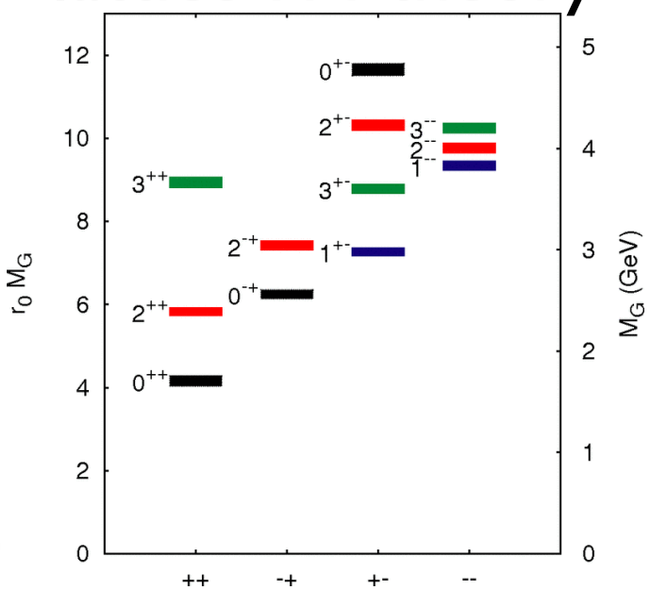
with Markus Huber, Gernot Eichmann
and Joshua Hoffer

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]
Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025
Hoffer, Eichmann, CF, 2409.05779

Glueballs

Theory:

lattice YM-theory

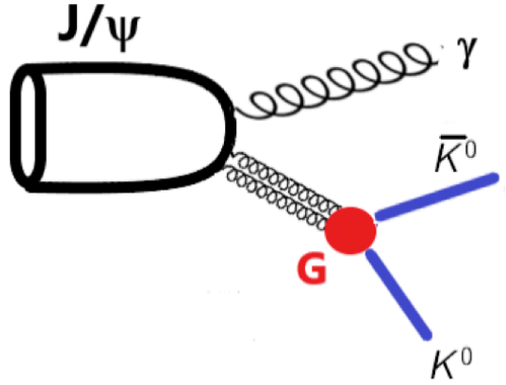


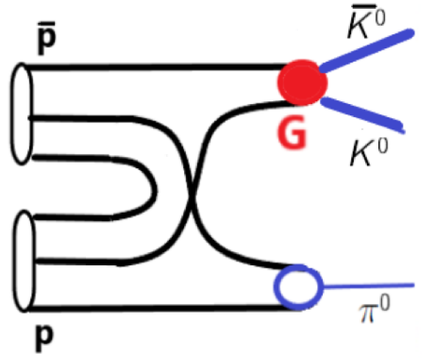
Morningstar and Peardon, PRD 60 (1999)

models

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} \begin{pmatrix} |n\bar{n}\rangle \\ |s\bar{s}\rangle \\ |gg\rangle \end{pmatrix}$$

Experiment:





Klempt, arXiv:2211.12901

$$M_{0^{++}} = 1865 \pm 25^{+10}_{-30}$$

Sarantsev et al., PLB 816 (2021) 136227

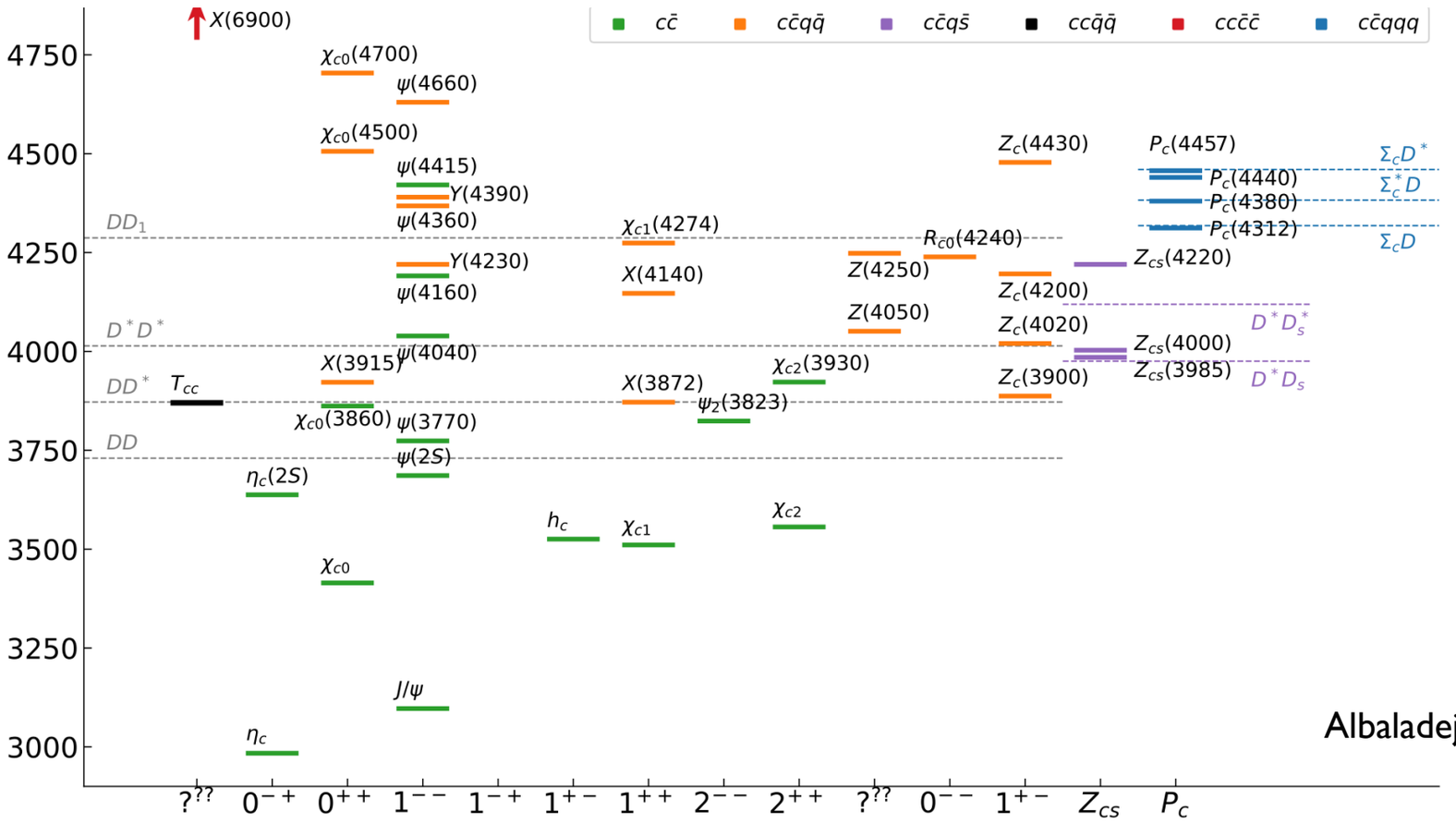
$$M_{0^{++}} \to f_0(1710)$$

Rodas et al. (JPAC), EPJC 82 (2022) 1, 80

$$M_{0^{-+}} = 2395 \pm 11(stat)^{+26}_{-94}(syst)$$

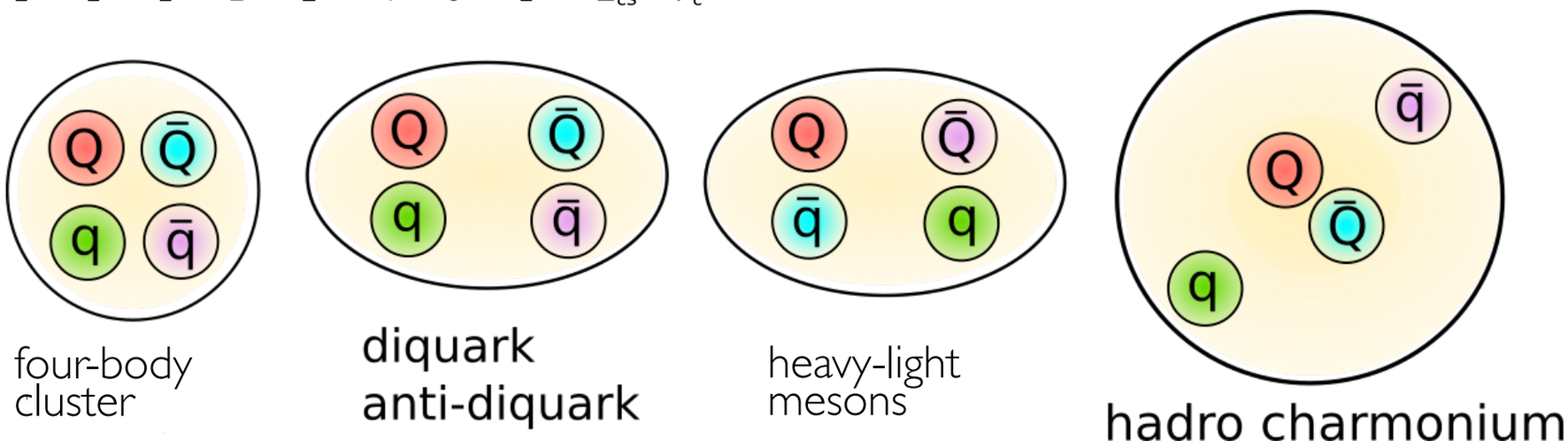
M.~Ablikim et al. [BESIII], PRL 132 (2024) no.18, 1819014

Exotic hadrons at Belle, BABAR, BES, LHCb,…



Albaladejo et al. [JPAC], PPNP 127 (2022), 103981

Four-quark states:



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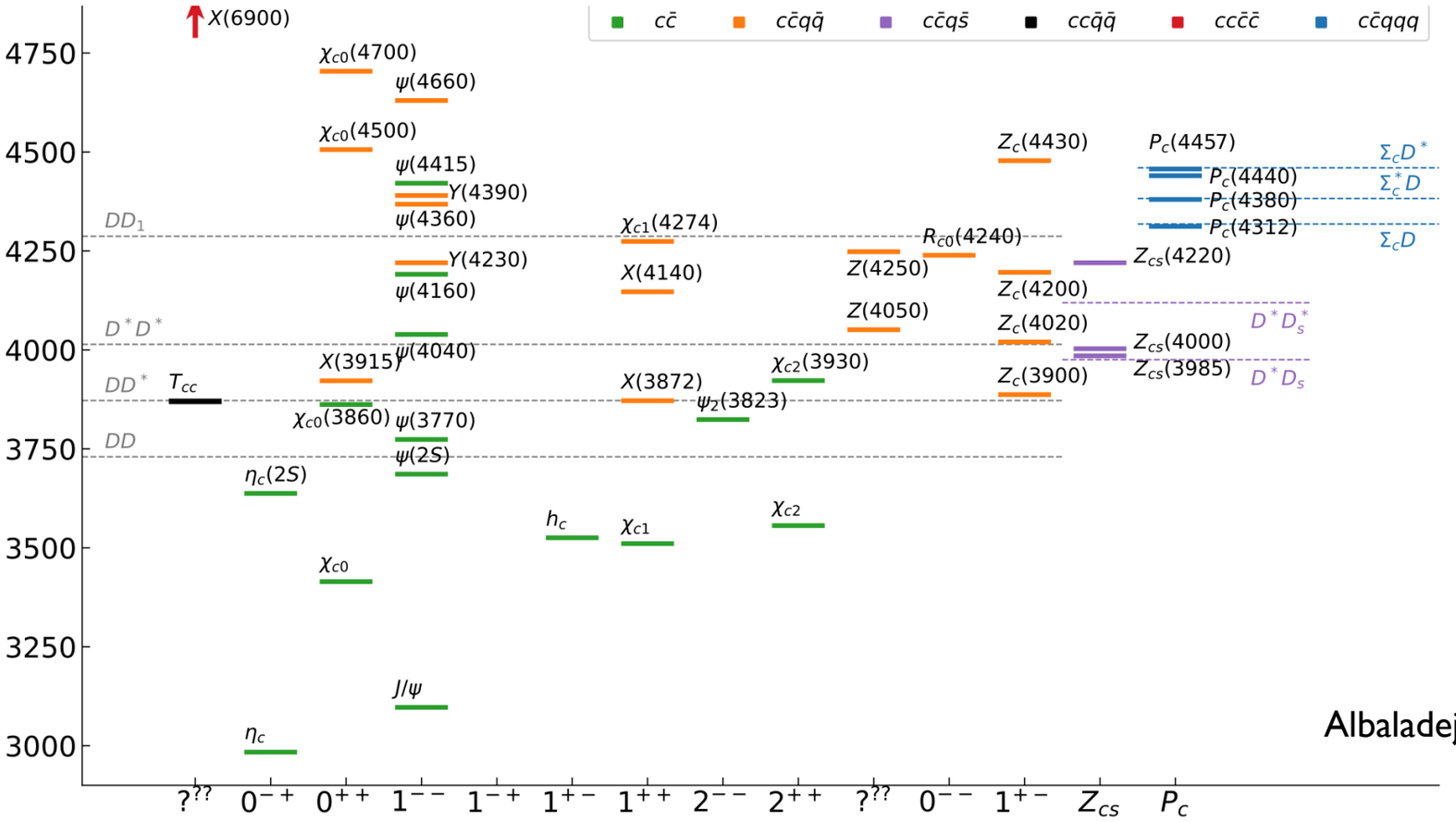

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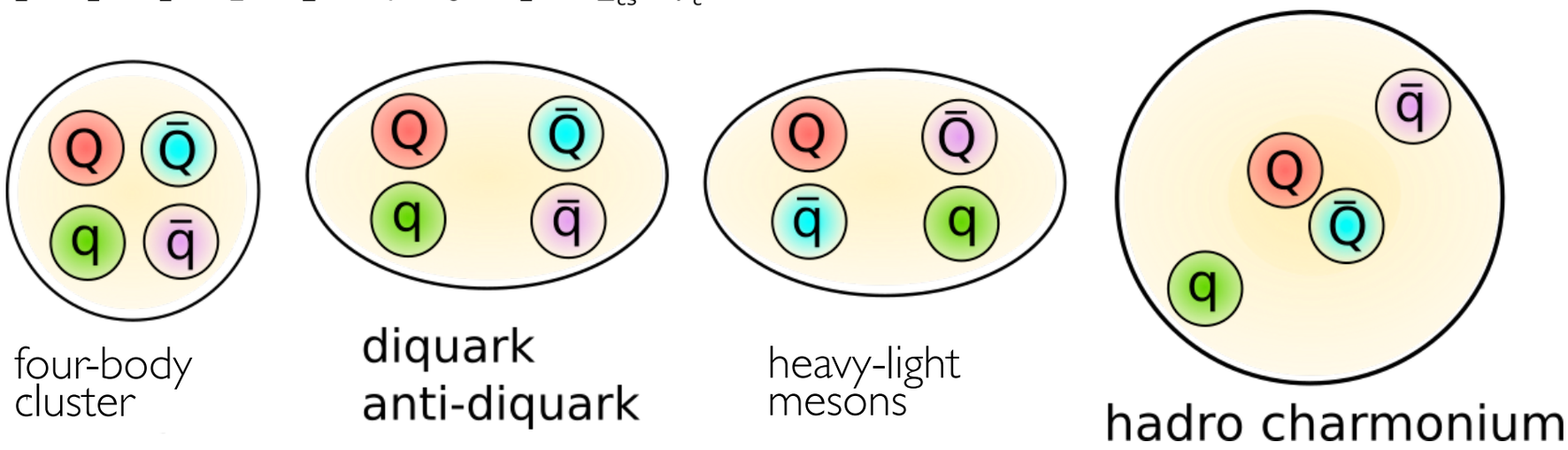
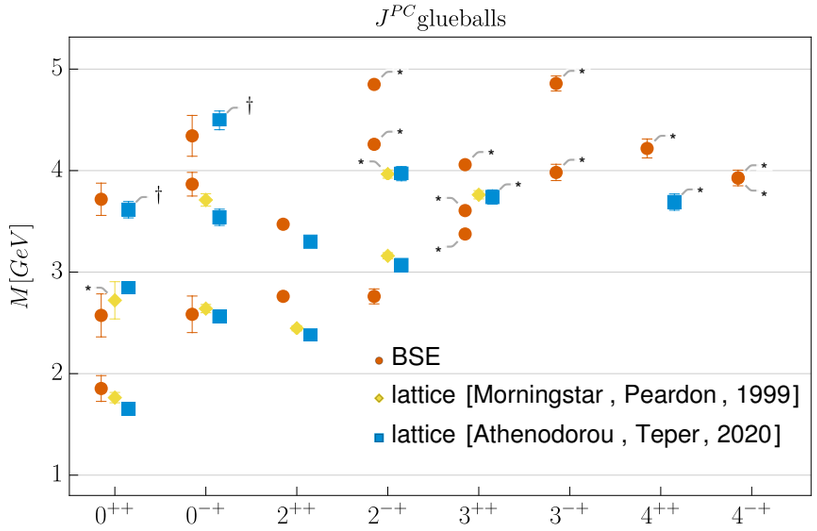


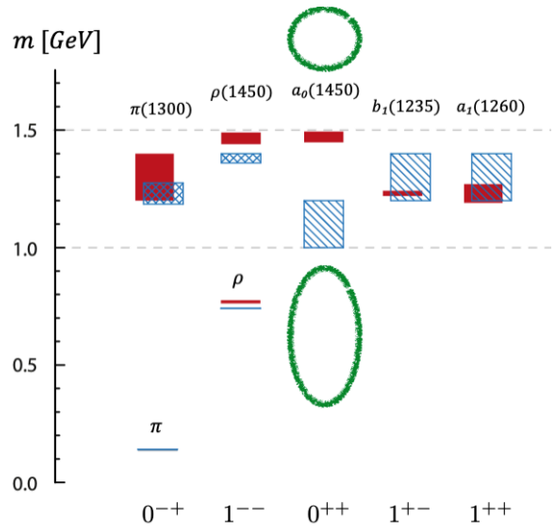
Related to details of underlying QCD forces

Overview

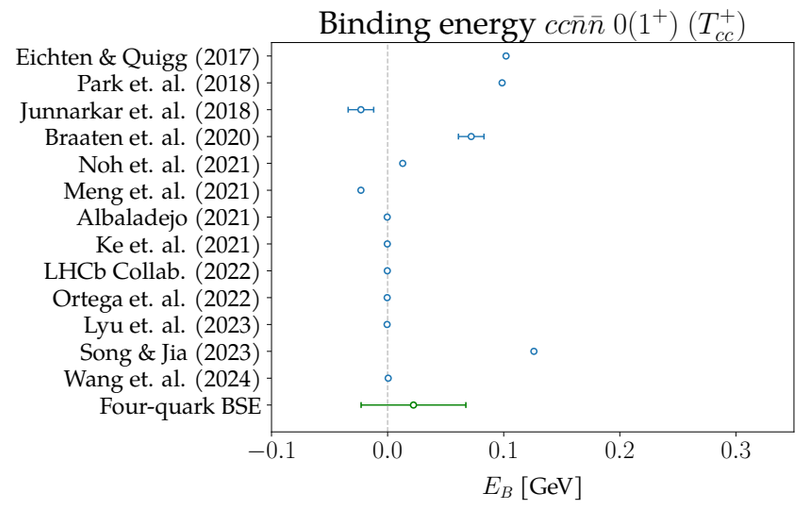
1. Glueballs: pure Yang-Mills



2. Conventional mesons (and baryons)



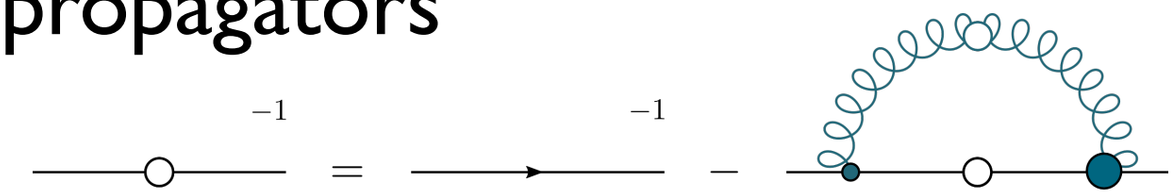
3. Four-quark states: hidden and open flavour



Dyson-Schwinger equations: QCD vs YM-Theory

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i \not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, EPJ C77 (2017) no.11, 733

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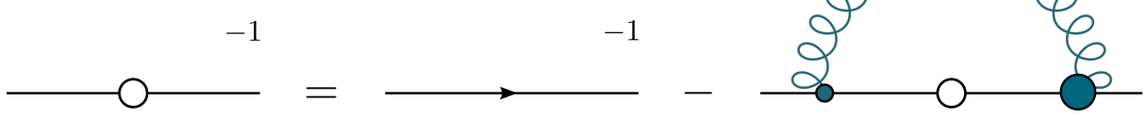

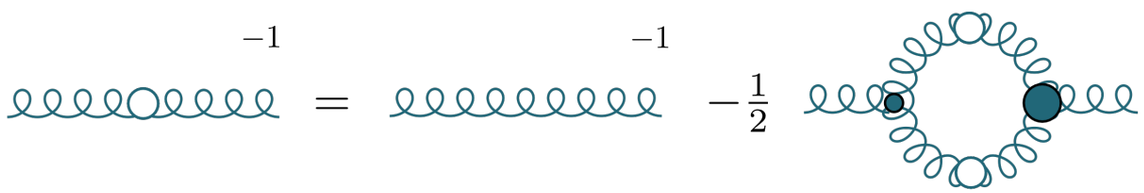

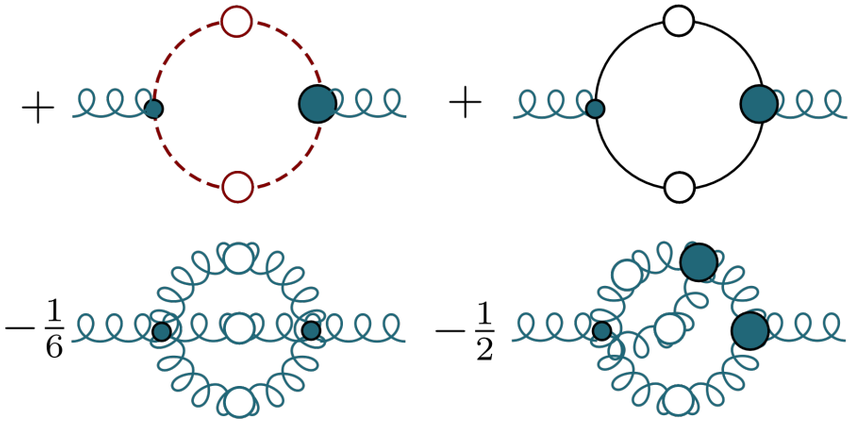

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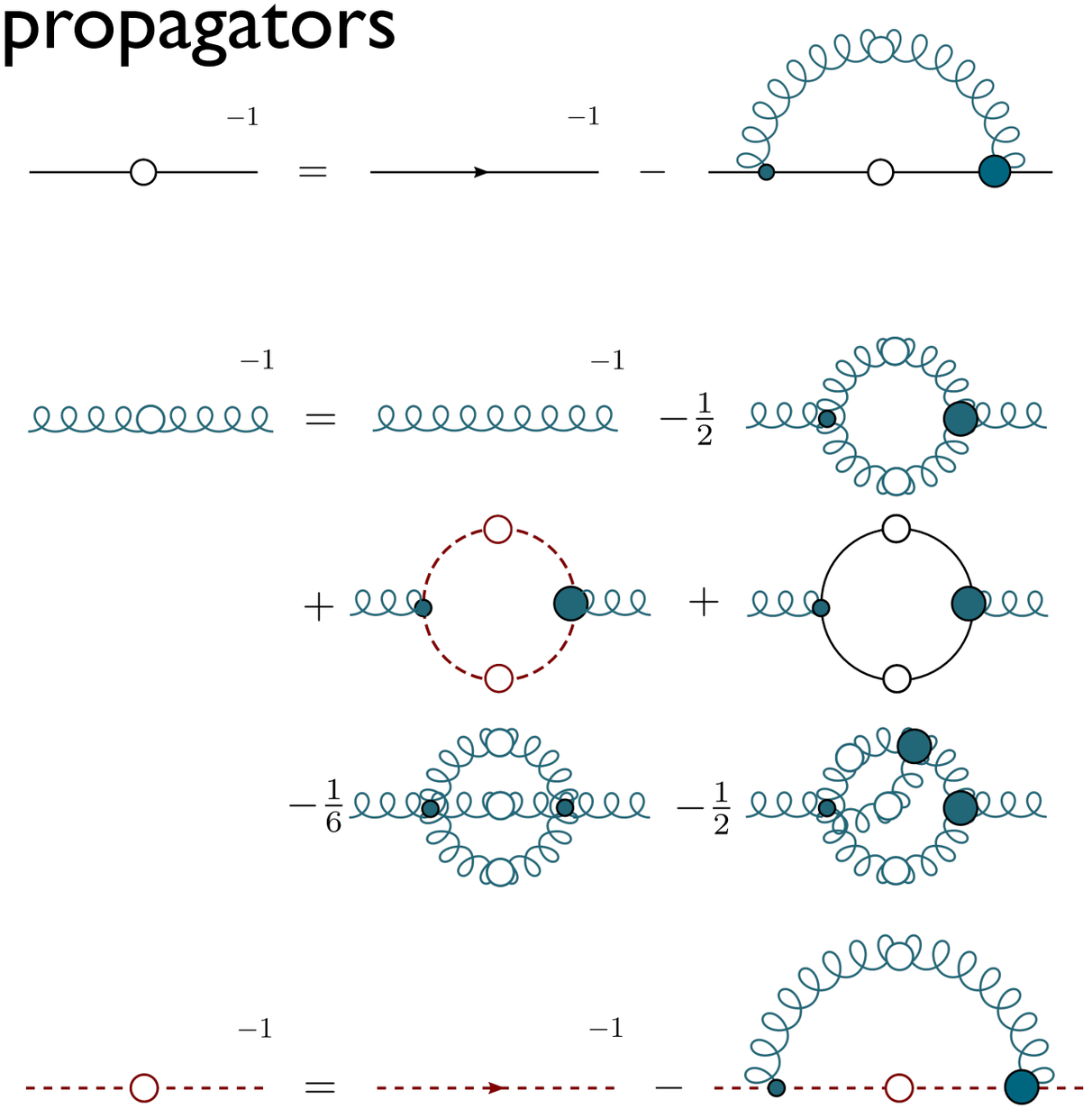
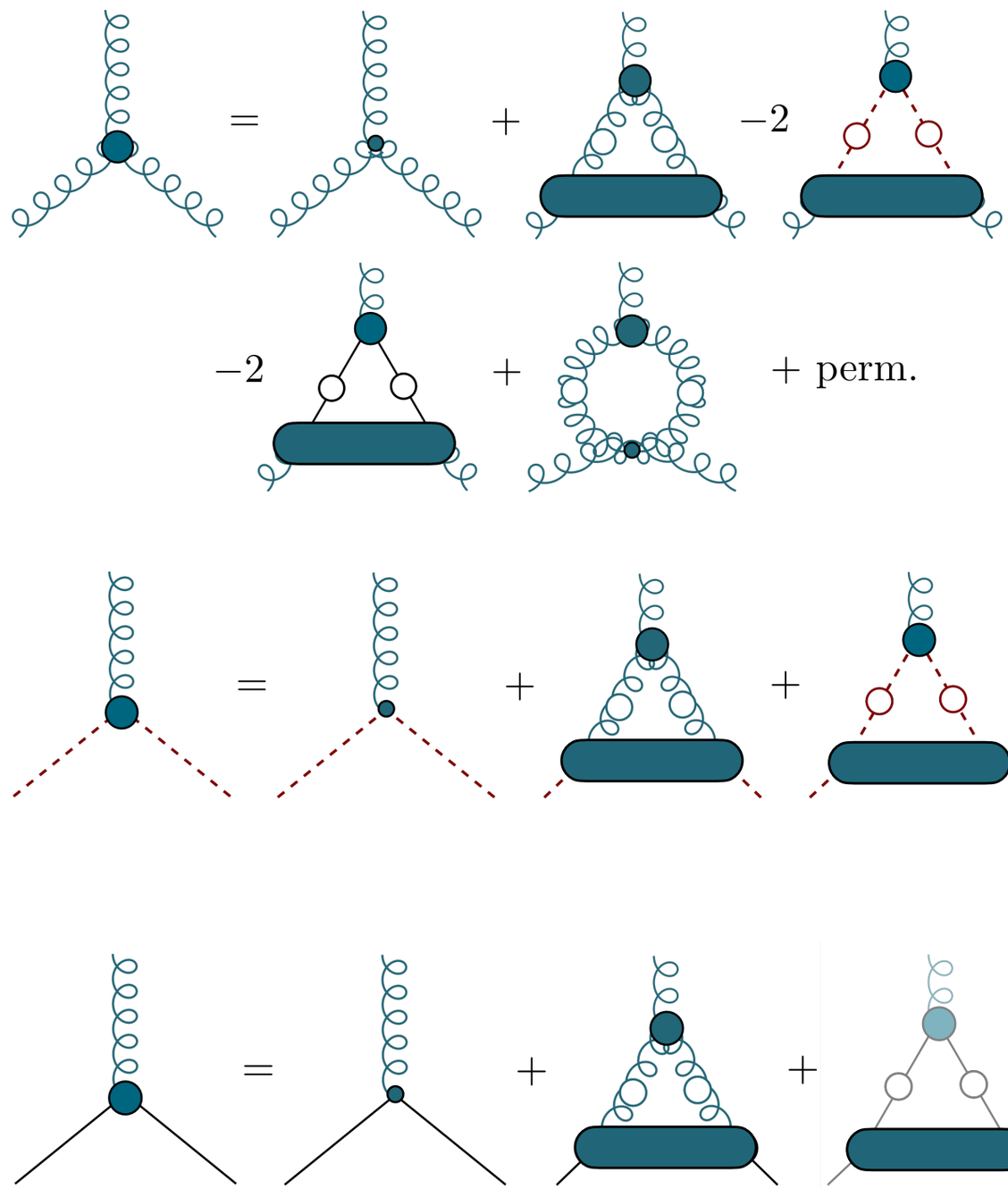


vertices



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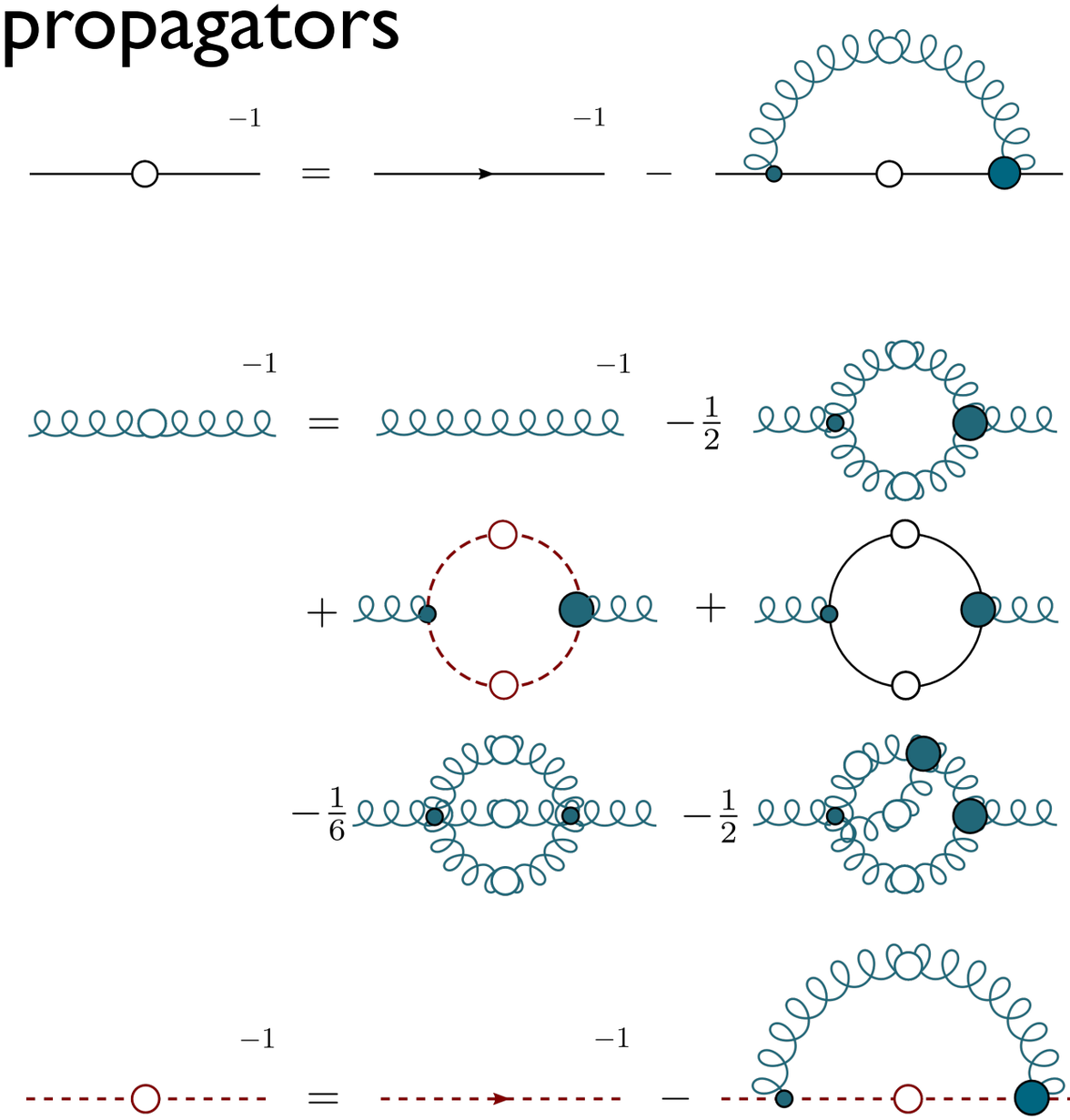

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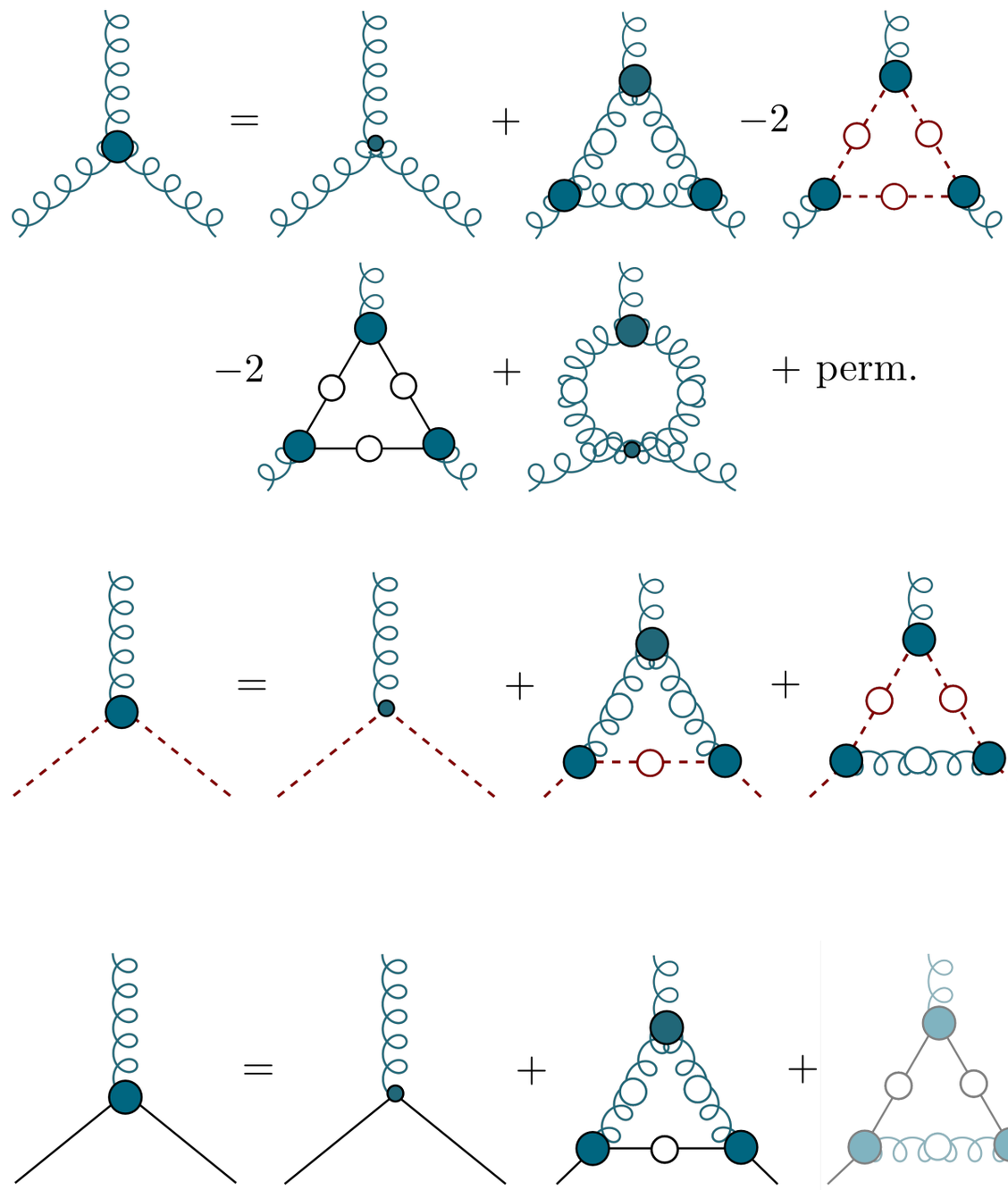

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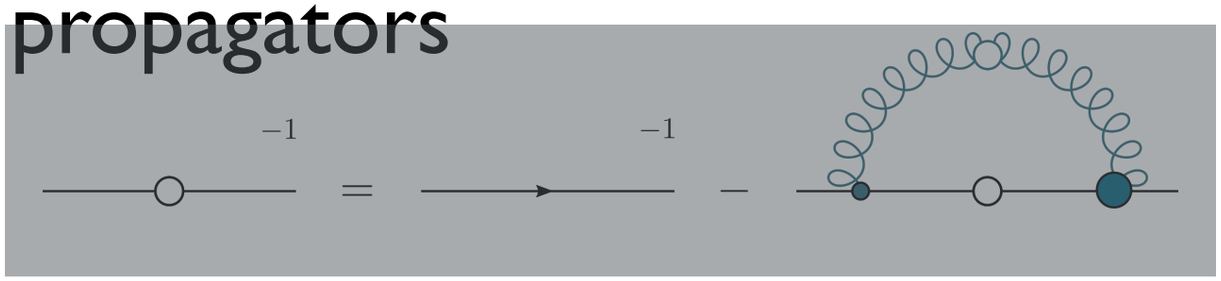

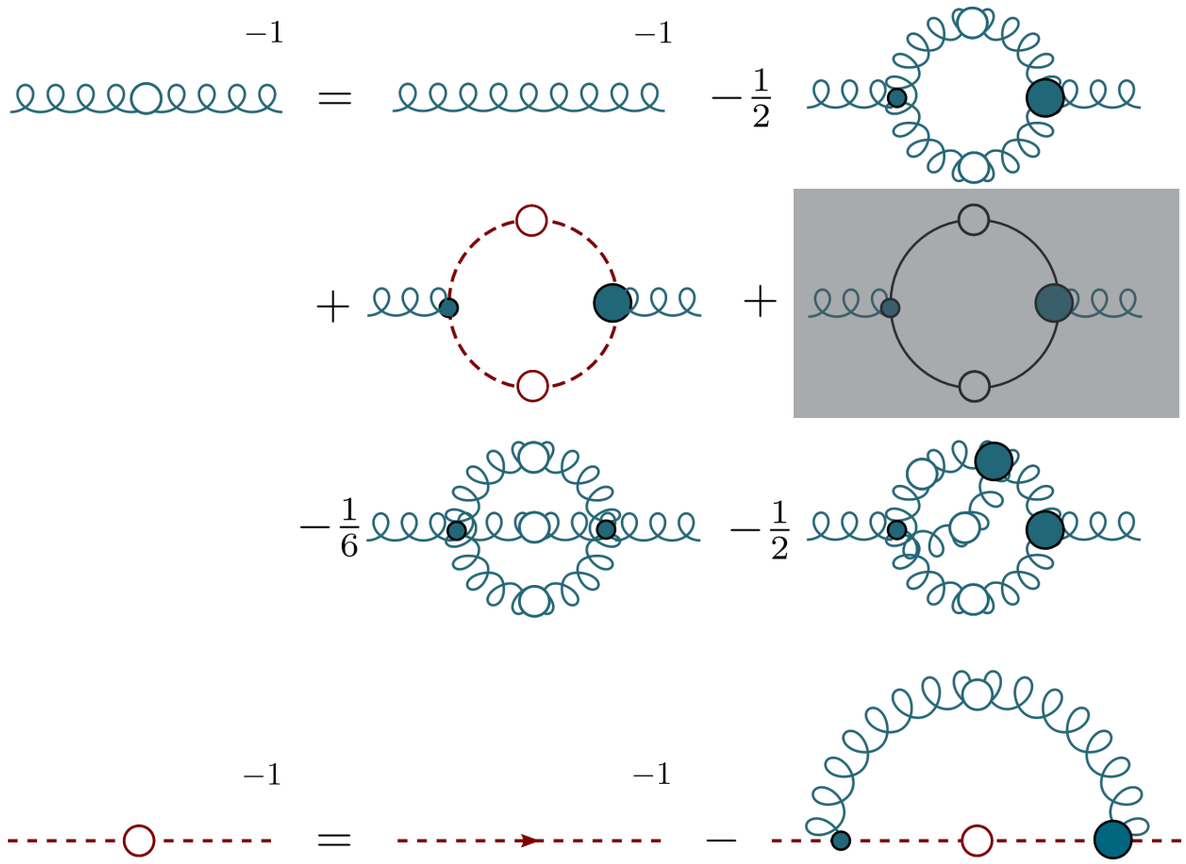

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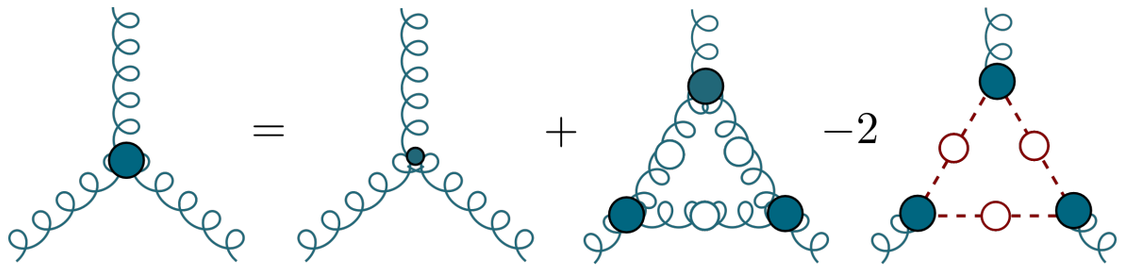

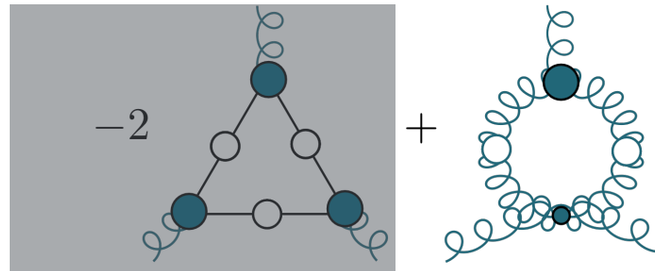

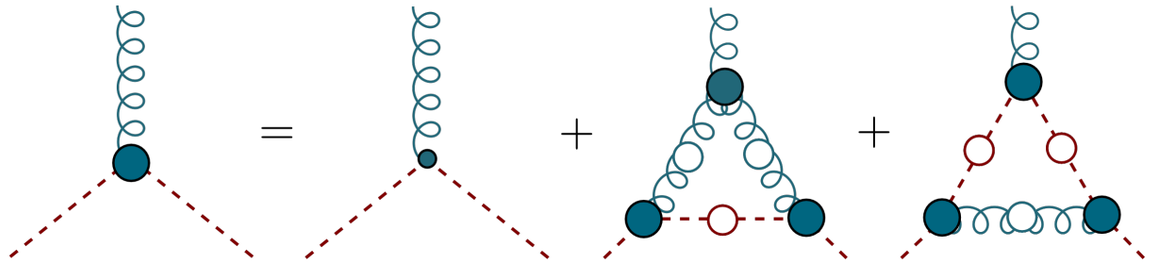

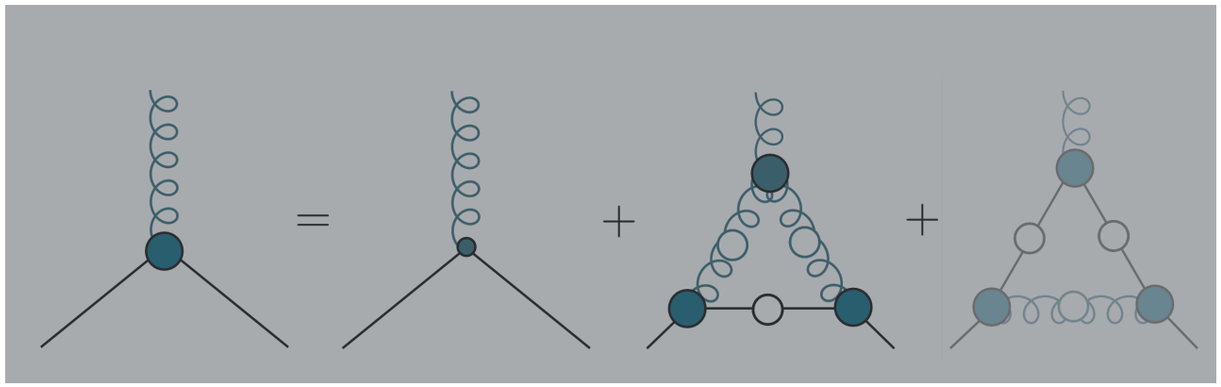

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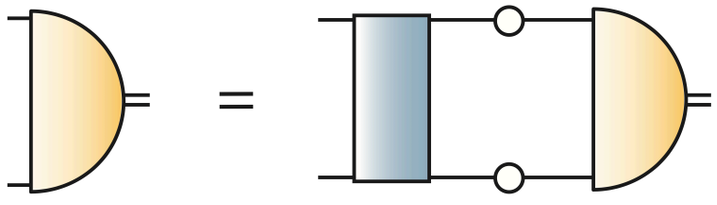
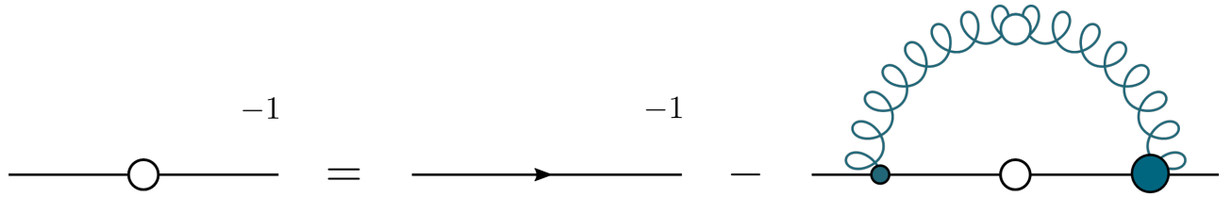
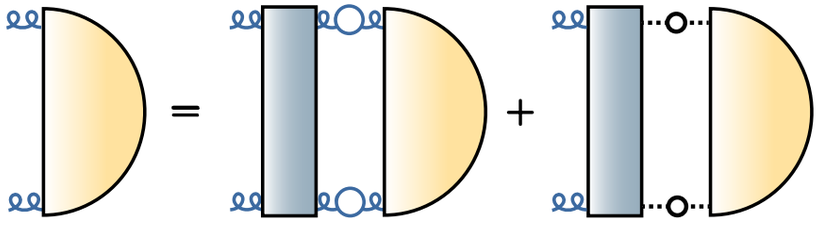
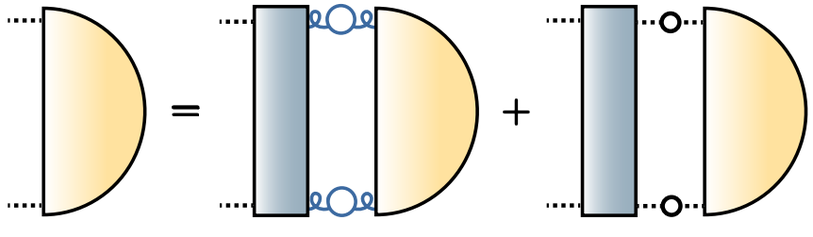
Bound states and Bethe-Salpeter equations

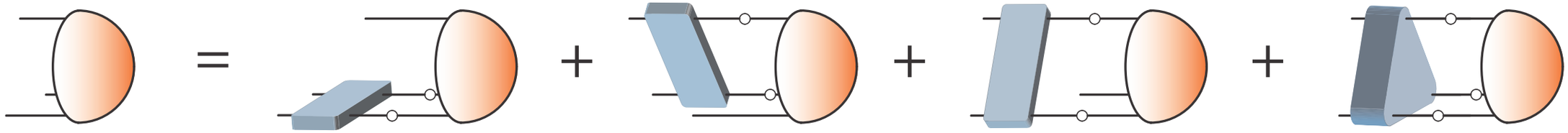
BSEs:

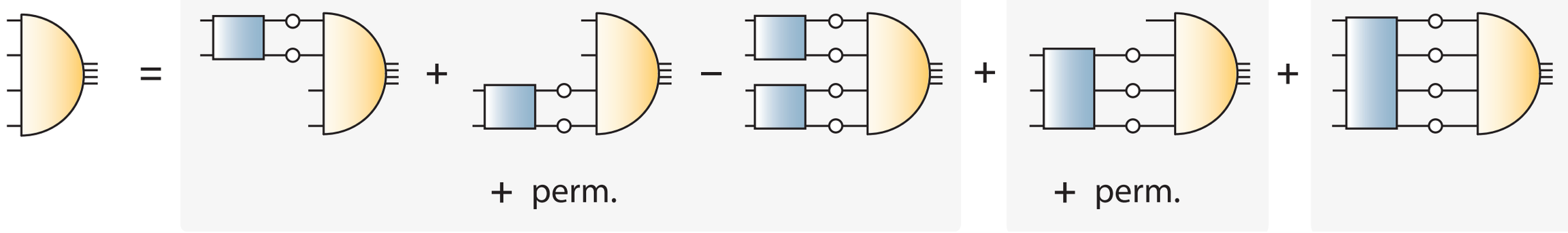








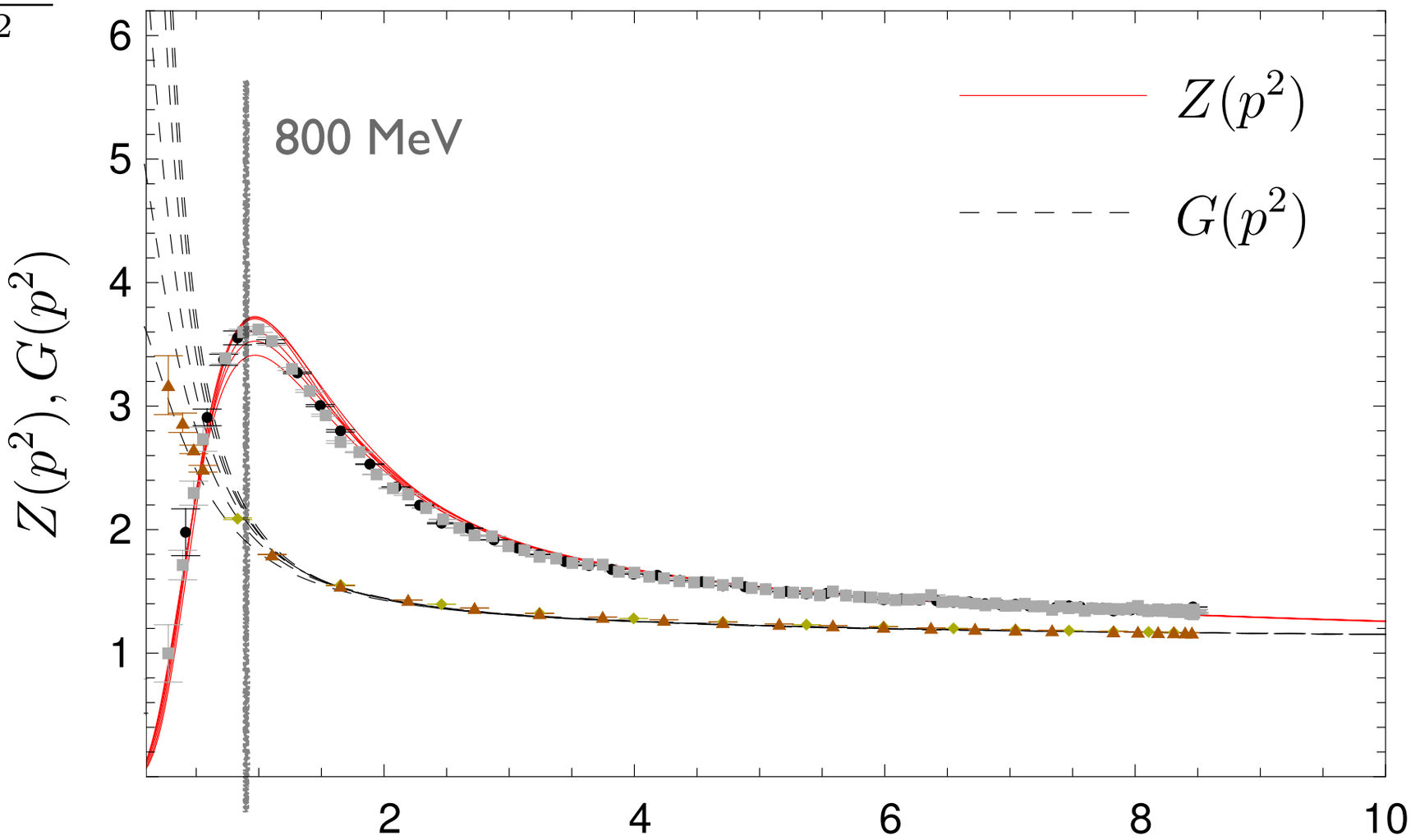




Eigenvalue equations: masses and wave functions

Landau gauge gluon propagator

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

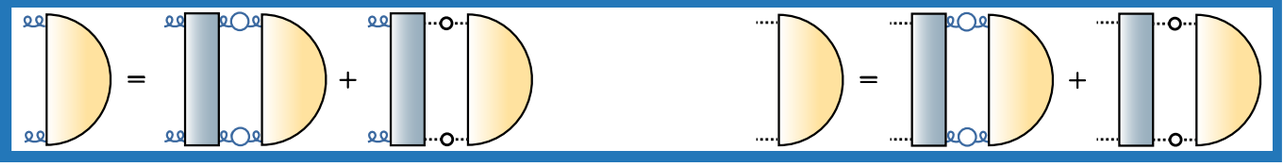


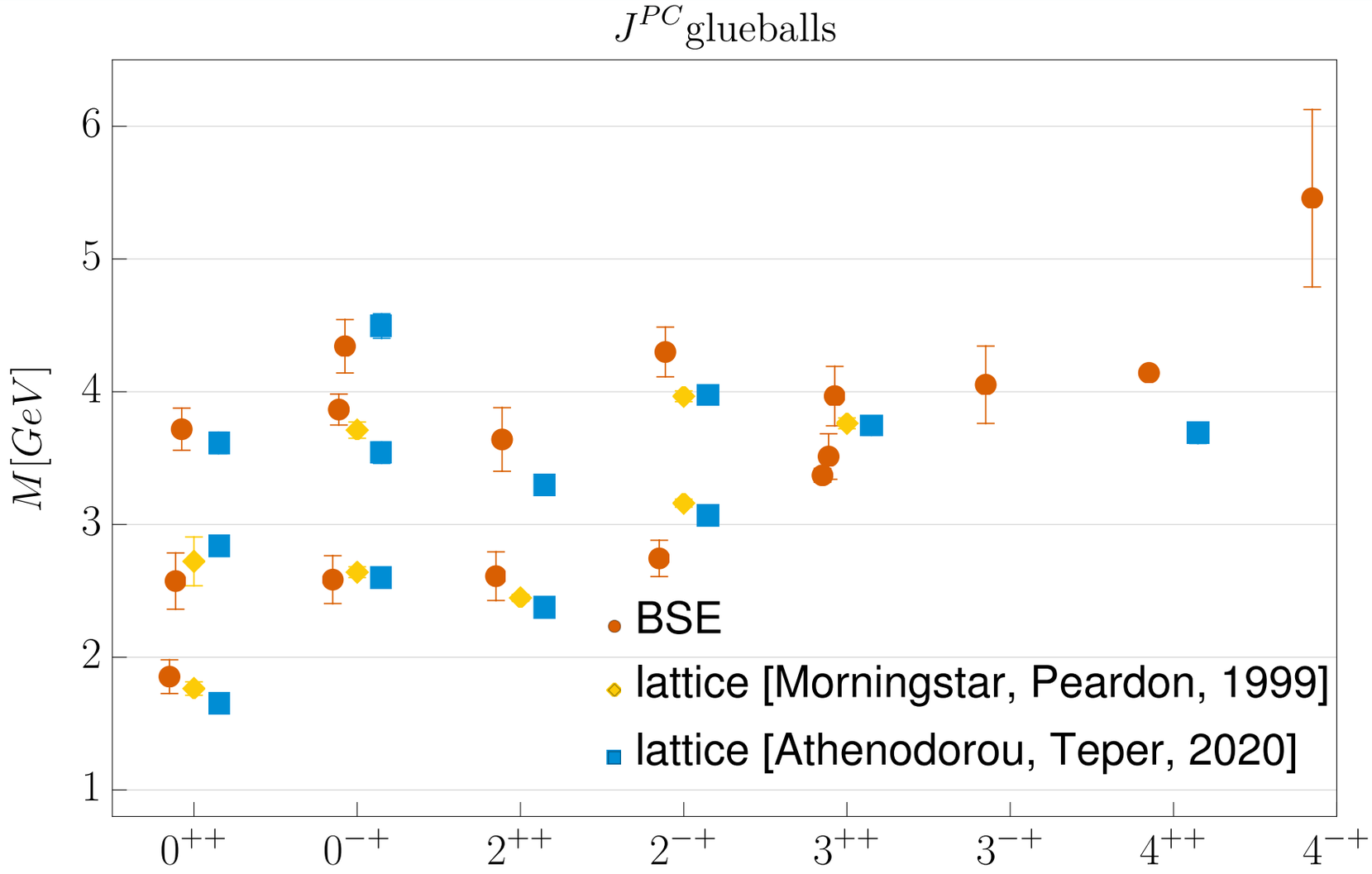
- fully dressed gluon appears massive

Cornwall PRD 26 (1982);
Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099;
CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

DSE: Huber, PRD 101 (2020) 114009, arXiv:2003.13703
Lattice: Sternbeck, Müller-Preussker, PLB 726 (2013)

Glueballs: results





- confirmation of results from lattice YM-theory
- predictions for some channels

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To do:
chart the mixing of glueballs with conventional meson states…

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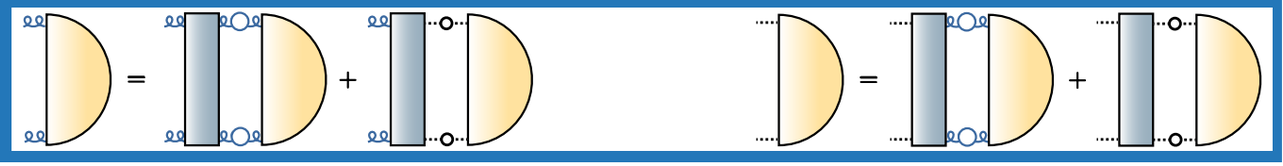

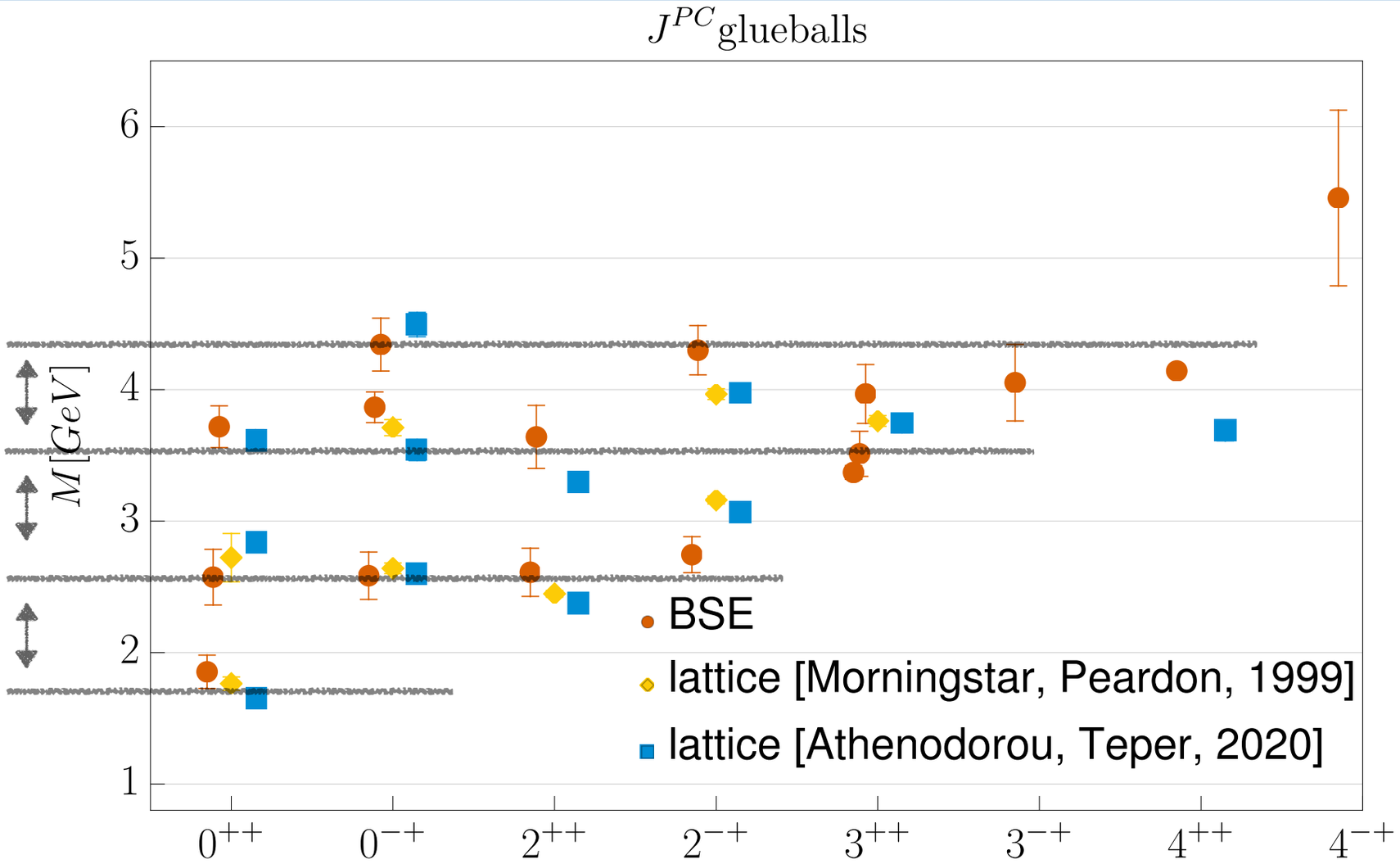

800 MeV

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Overview

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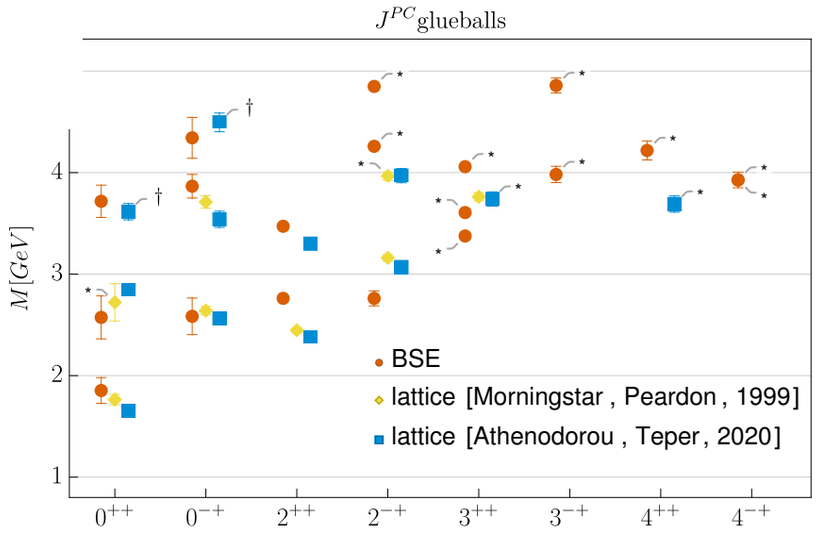

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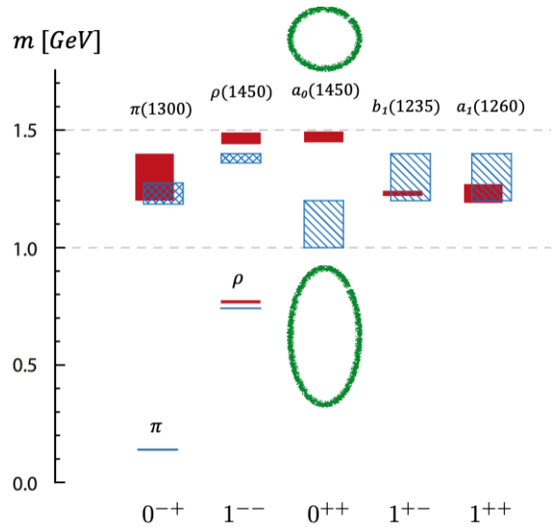

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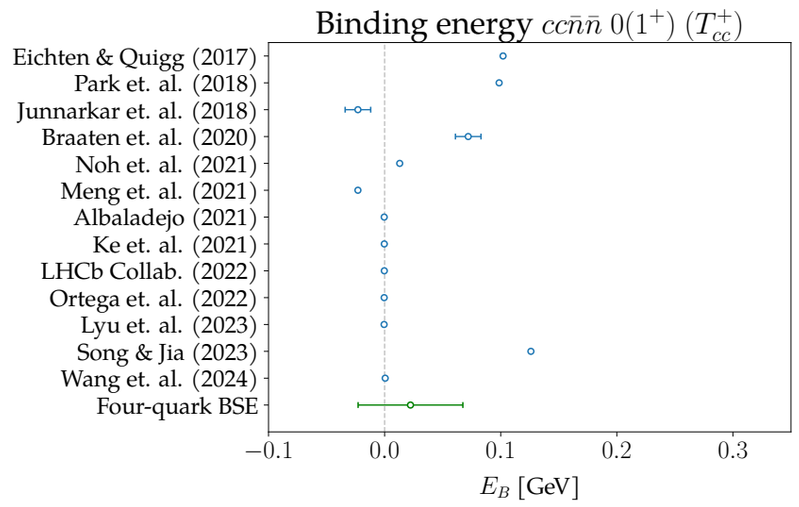

Dyson-Schwinger equations - "3PI vs RL"

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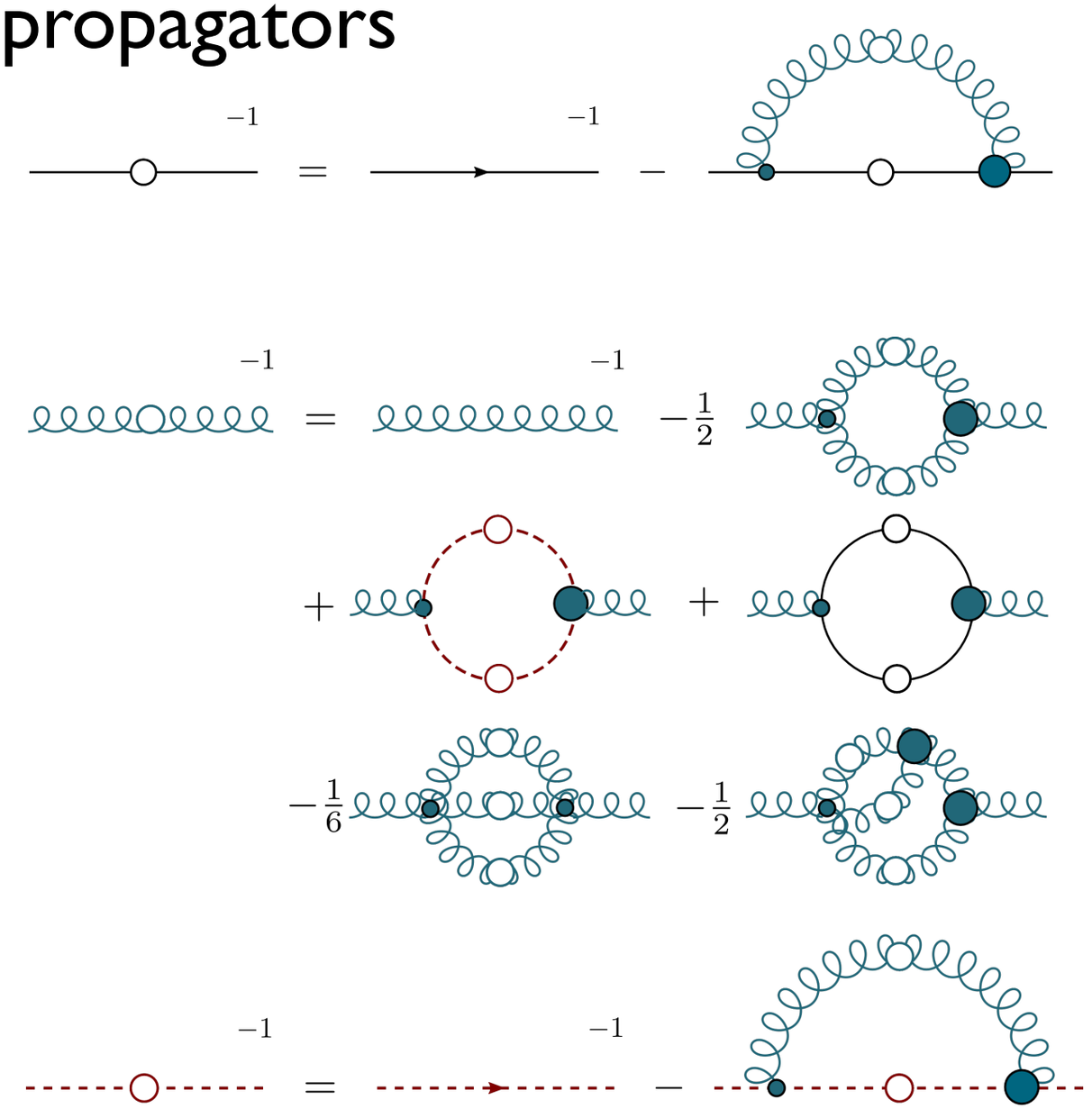
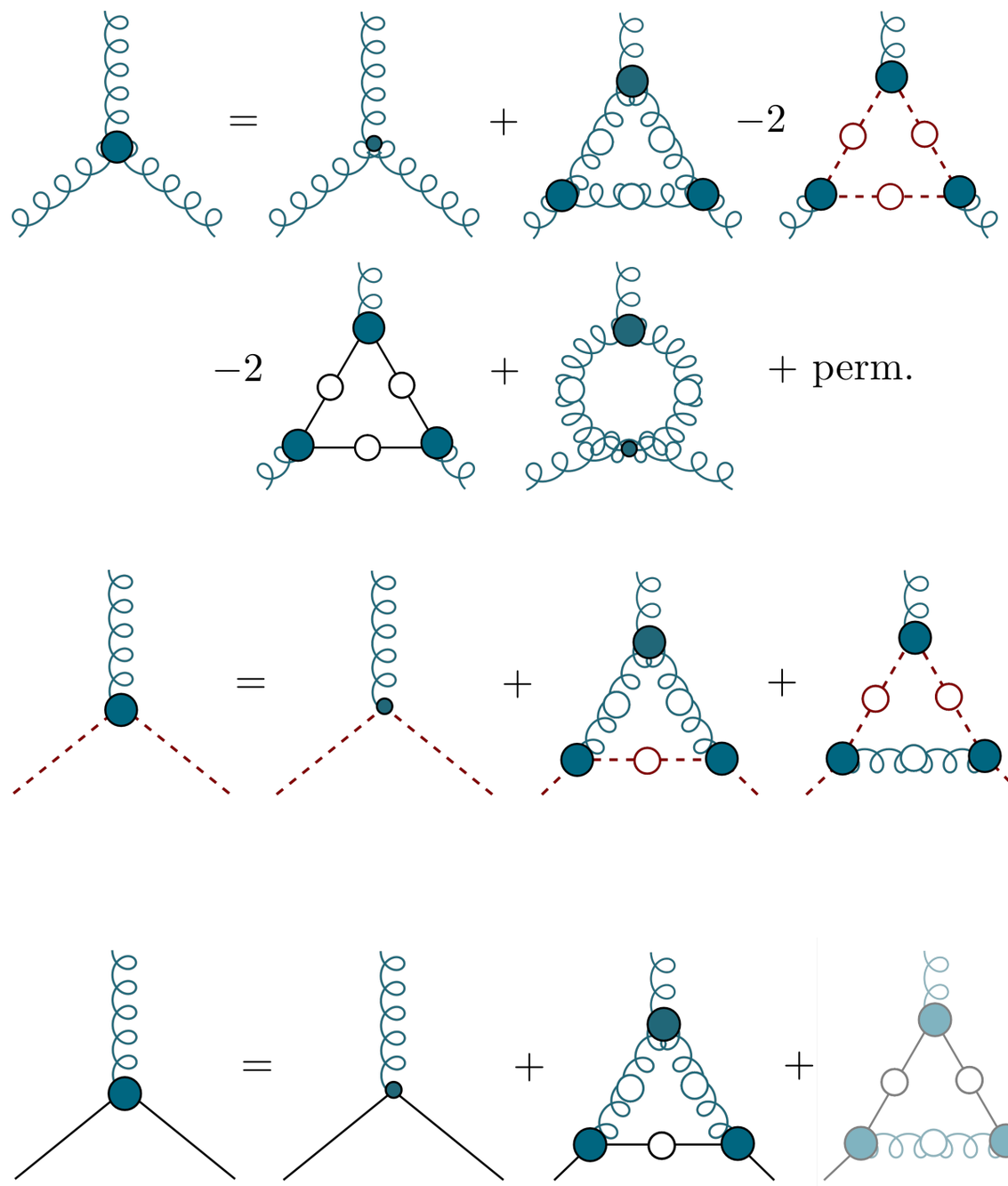
propagators



vertices



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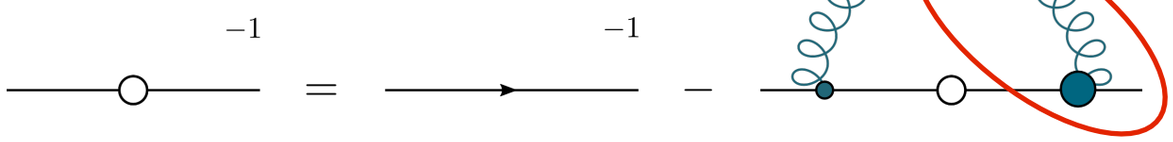

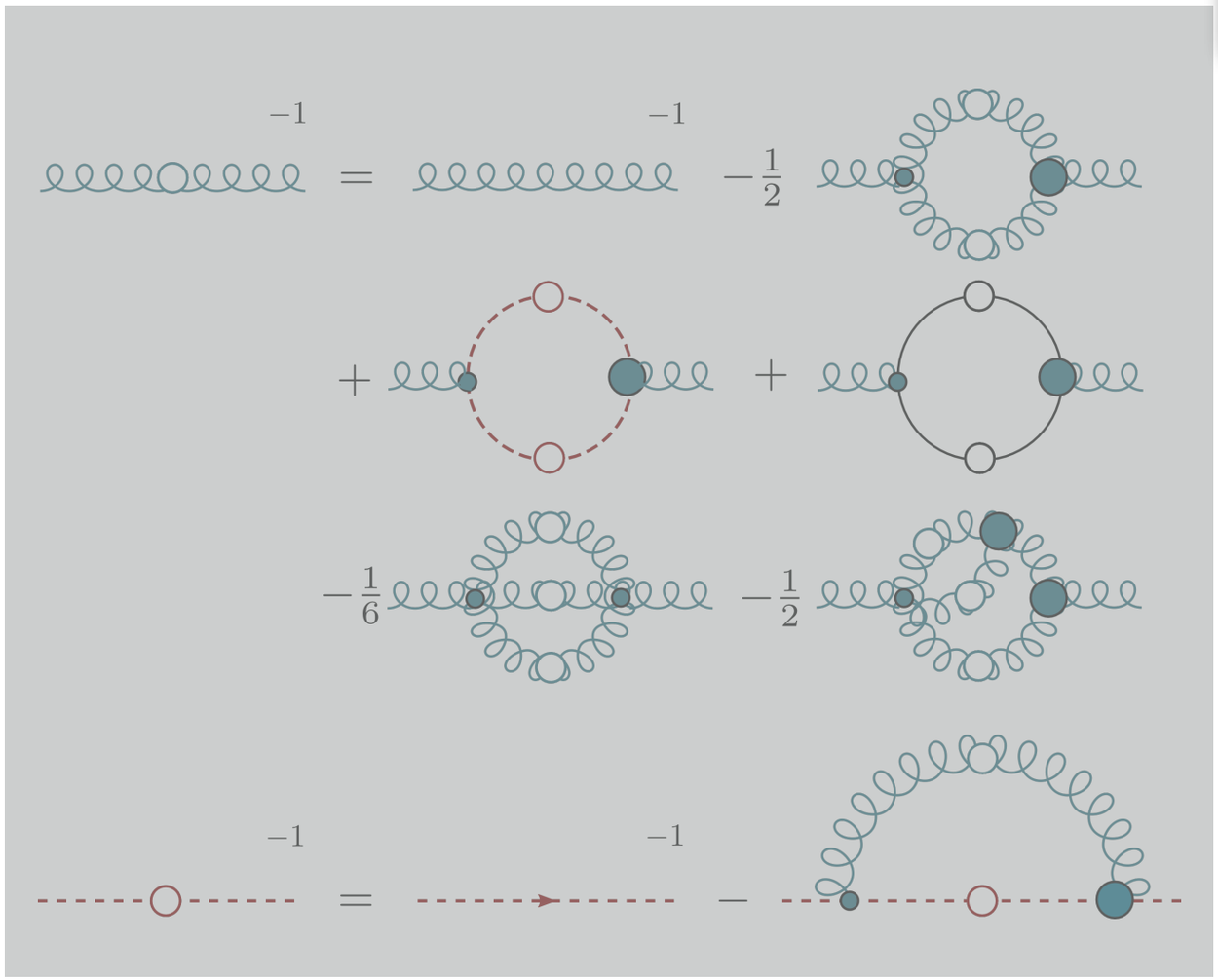

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Williams, CF, Heupel, PRD93 (2016) 034026
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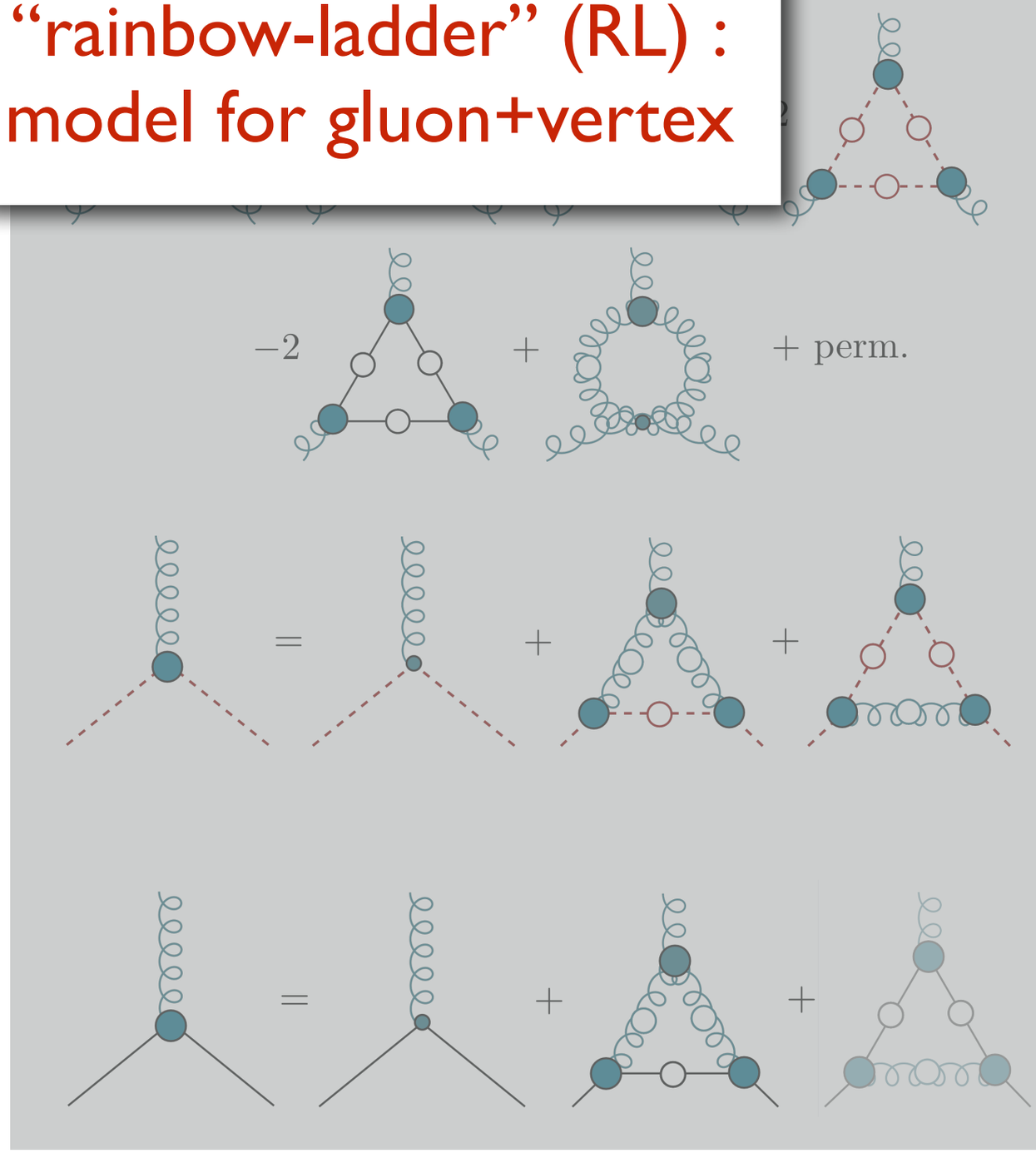
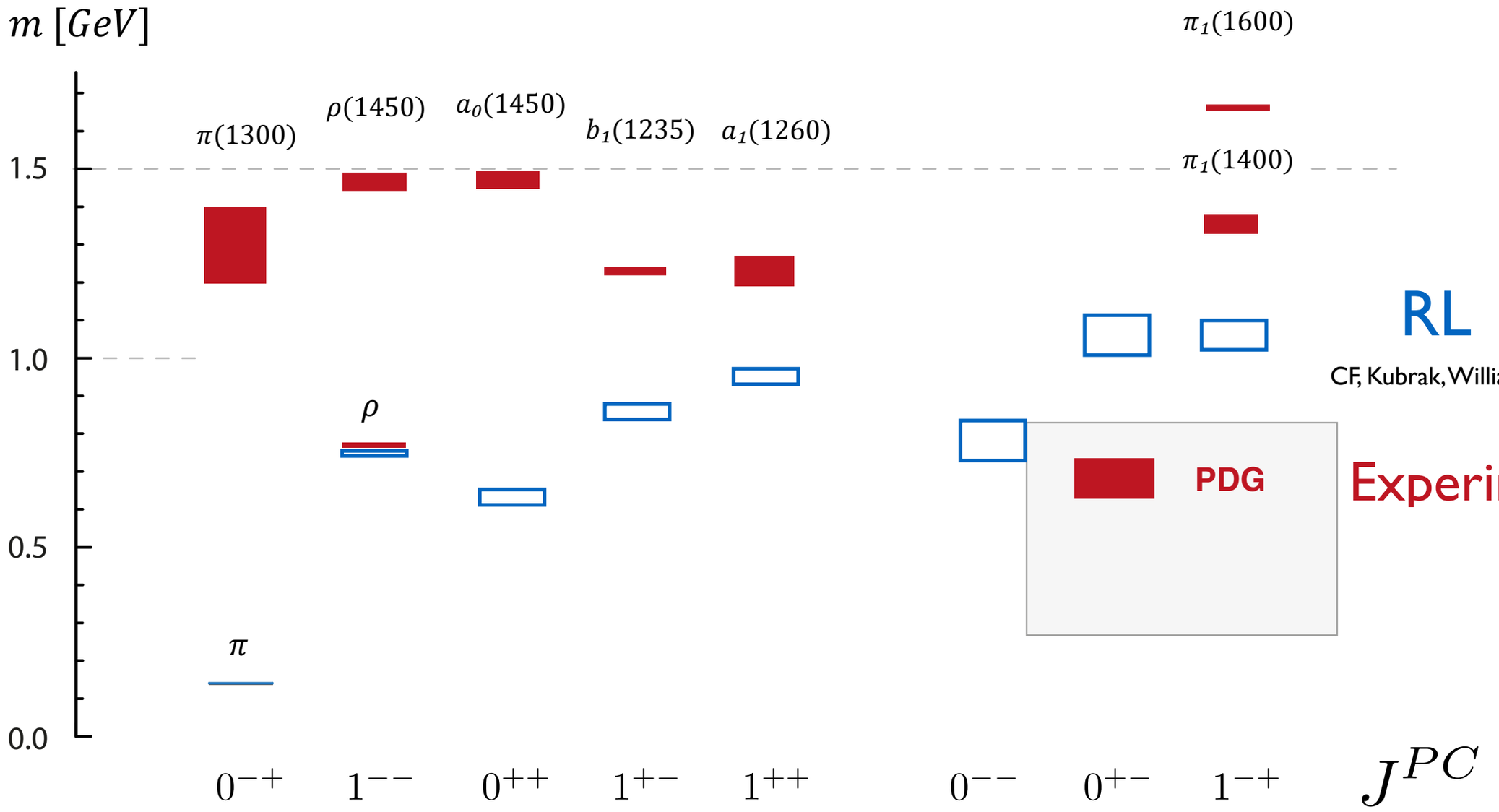
vertices

"rainbow-ladder" (RL) : model for gluon+vertex



Light meson spectrum - full 3PI-calculation



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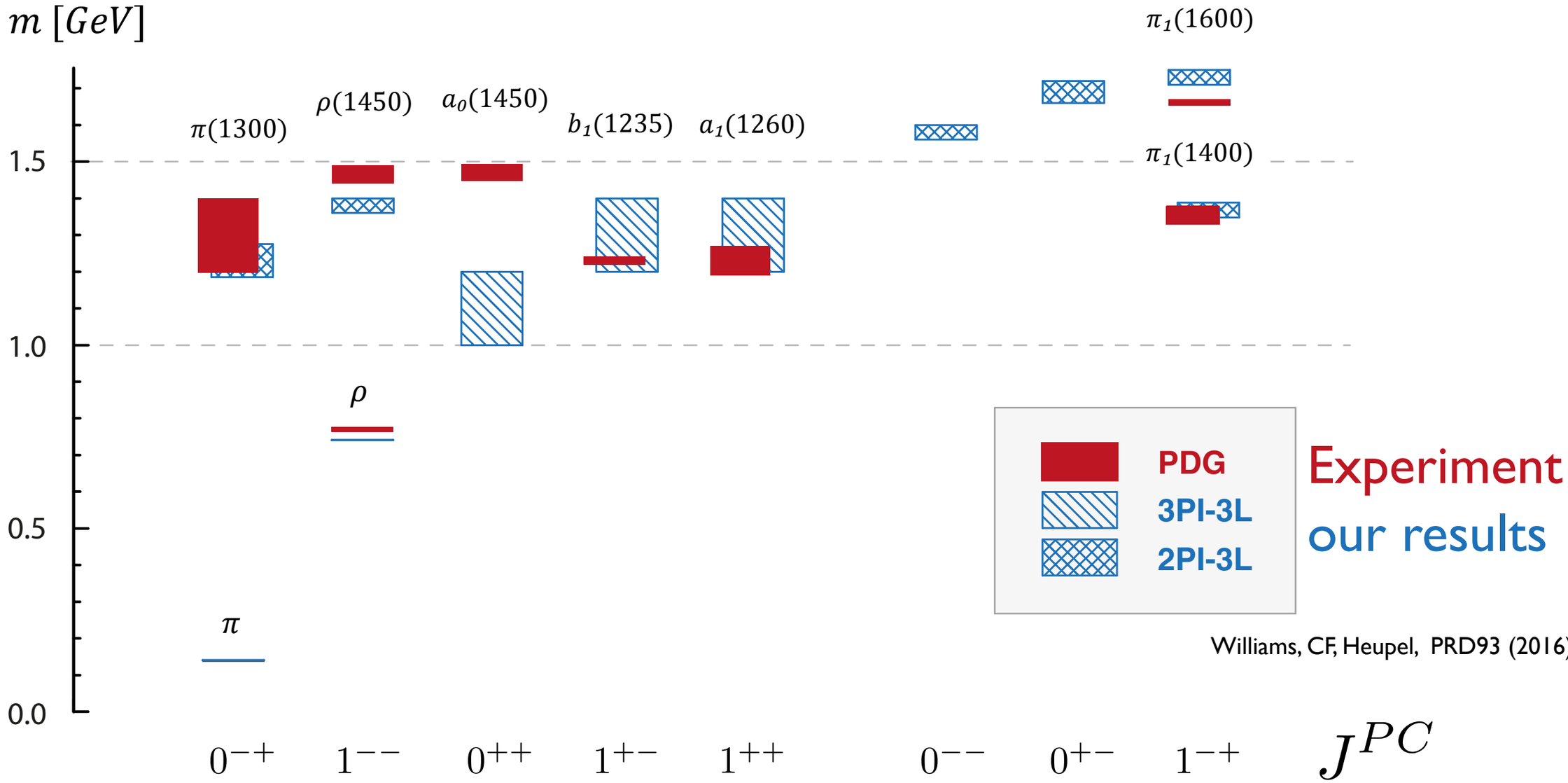
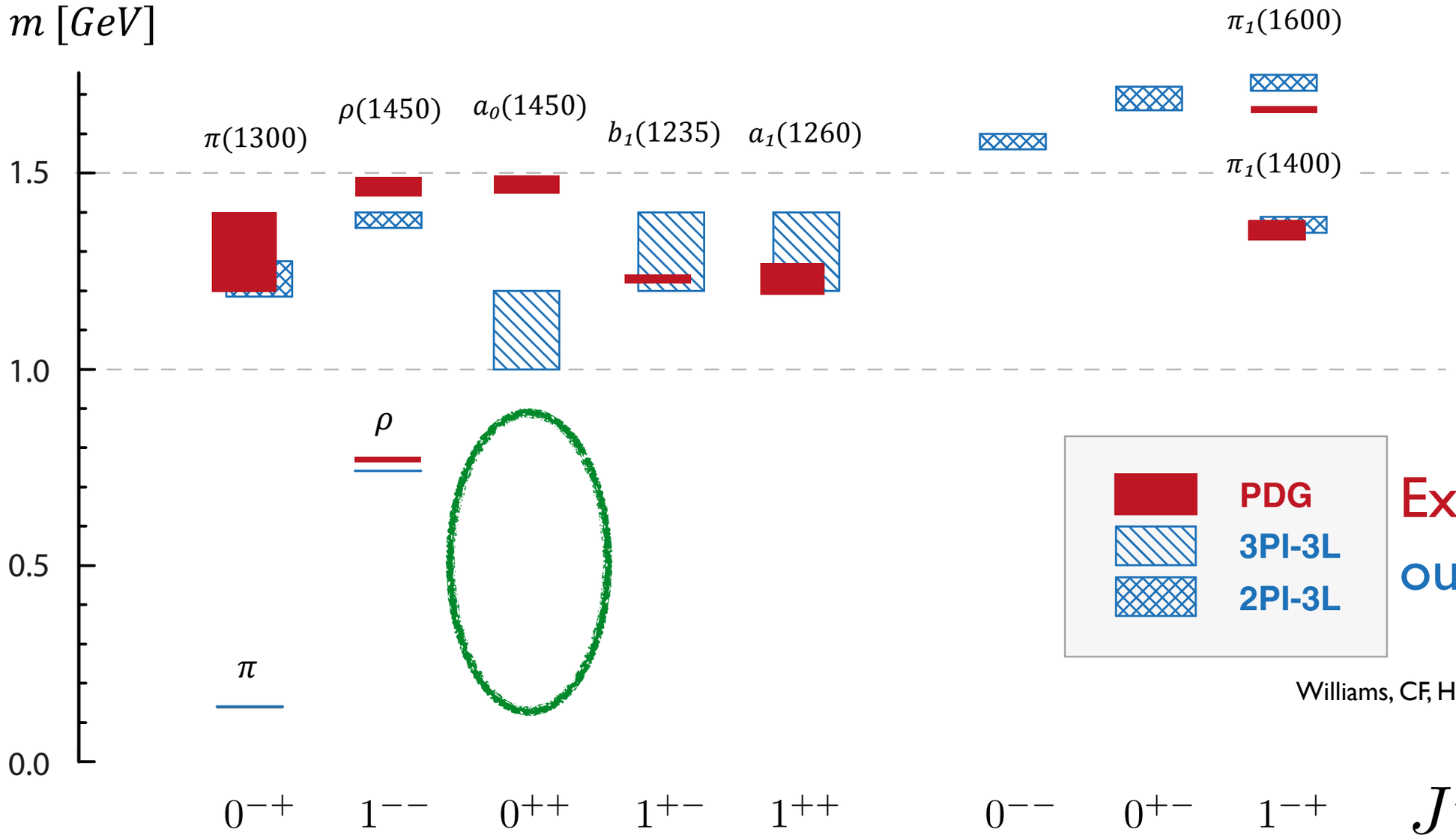


Experiment
our results

Williams, CF, Heupel, PRD93 (2016) 034026

- good agreement with experiment in most channels
- special channels:
  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
  scalar 0^{++} : complicated channel…

Light meson spectrum - full 3PI-calculation

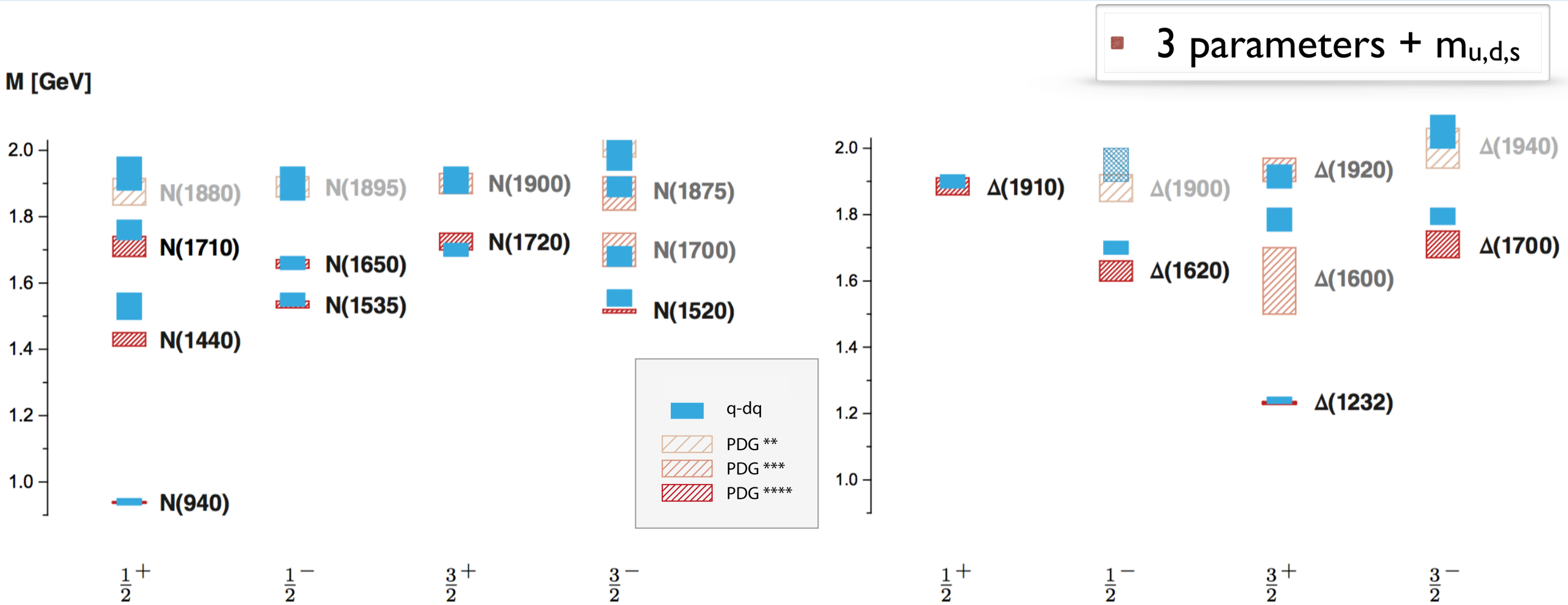


Williams, CF, Heupel, PRD93 (2016) 034026

- good agreement with experiment in most channels
- special channels:
  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
  scalar 0^{++} : complicated channel…

Light baryon spectrum: diquark-picture

- 3 parameters + $m_{u,d,s}$



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- strange baryons
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Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Qin, Roberts, Schmidt, Few Body Syst. 60 (2019) no.2, 26
Torcato, Arriaga, Eichmann and Pena, FBS 64 (2023) 45

Review on diquarks: Barabanov et al, PPNP 116 (2021), 103835

Light baryon spectrum: diquark-picture

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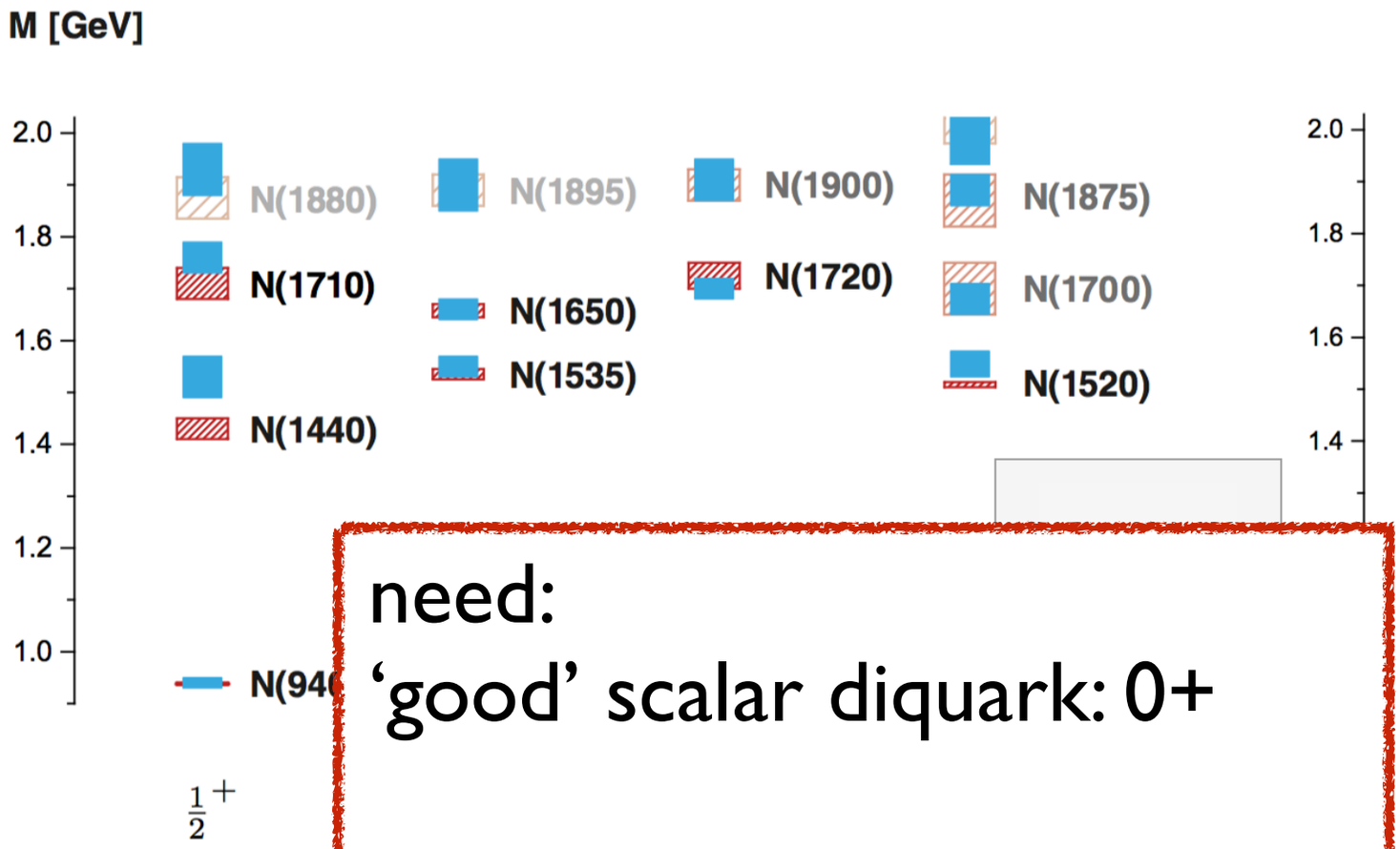
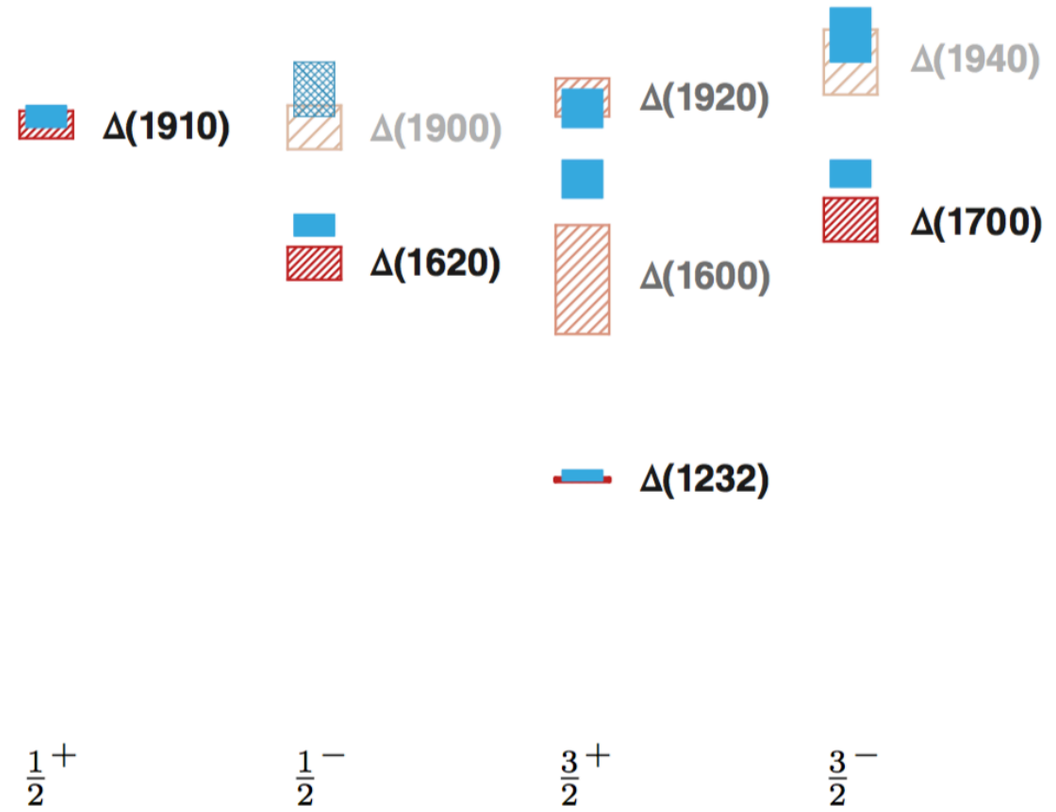


need:
‘good’ scalar diquark: 0+



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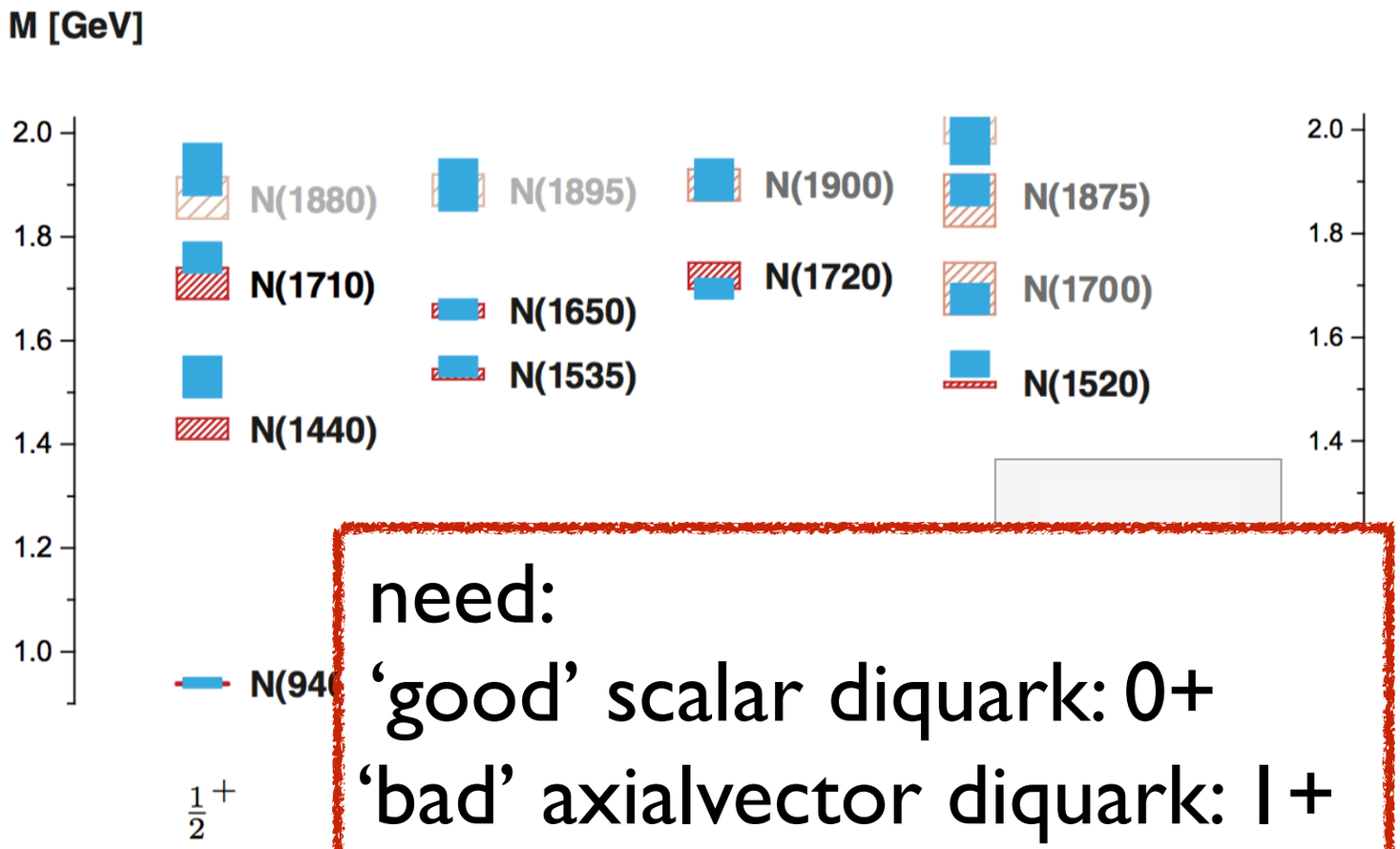
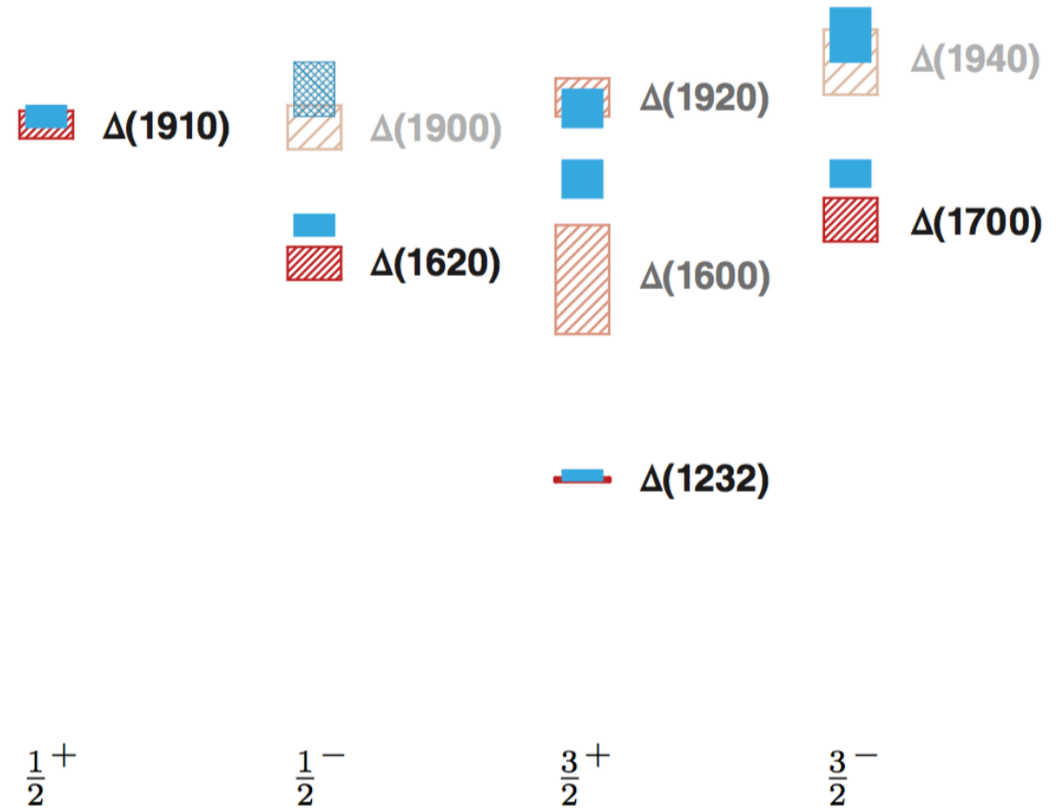


need:

'good' scalar diquark: 0+

'bad' axialvector diquark: 1+



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Light baryon spectrum: diquark-picture

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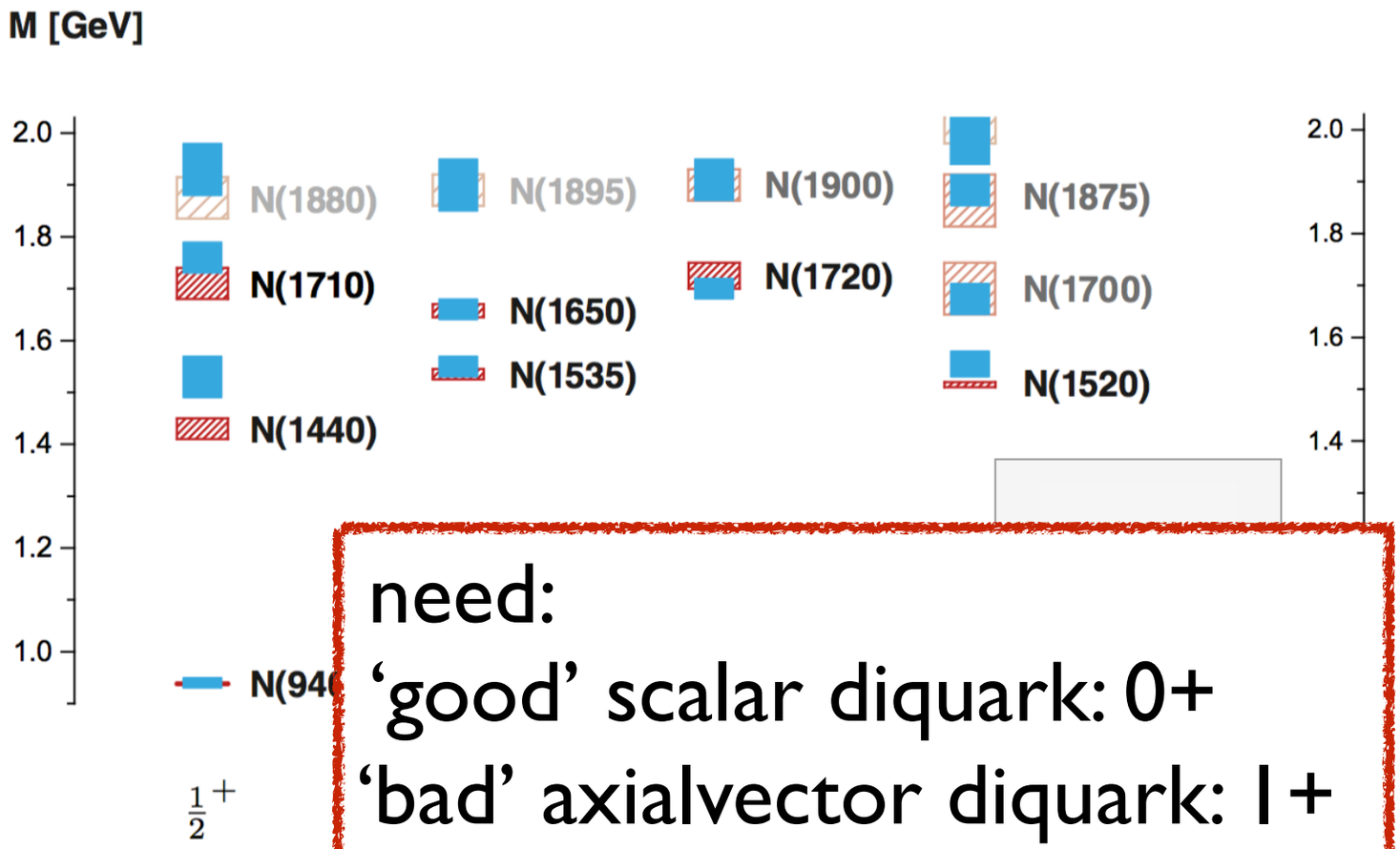
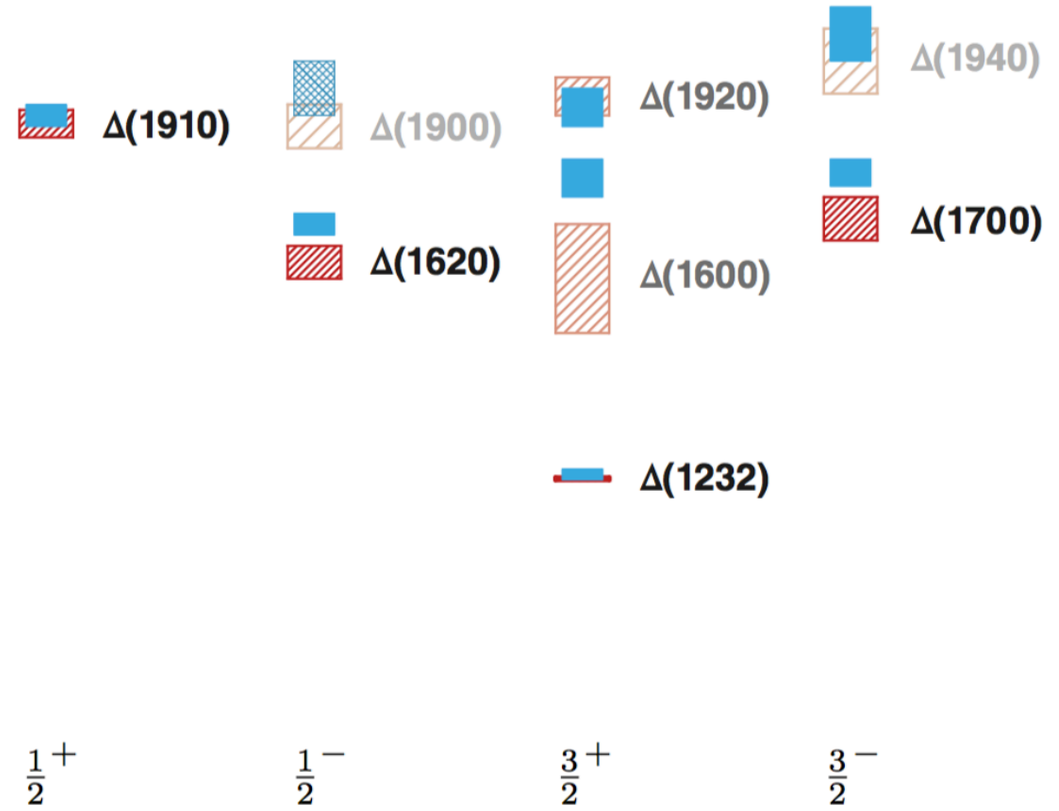


need:

'good' scalar diquark: 0+

'bad' axialvector diquark: 1+

'ugly' ps/v diquark



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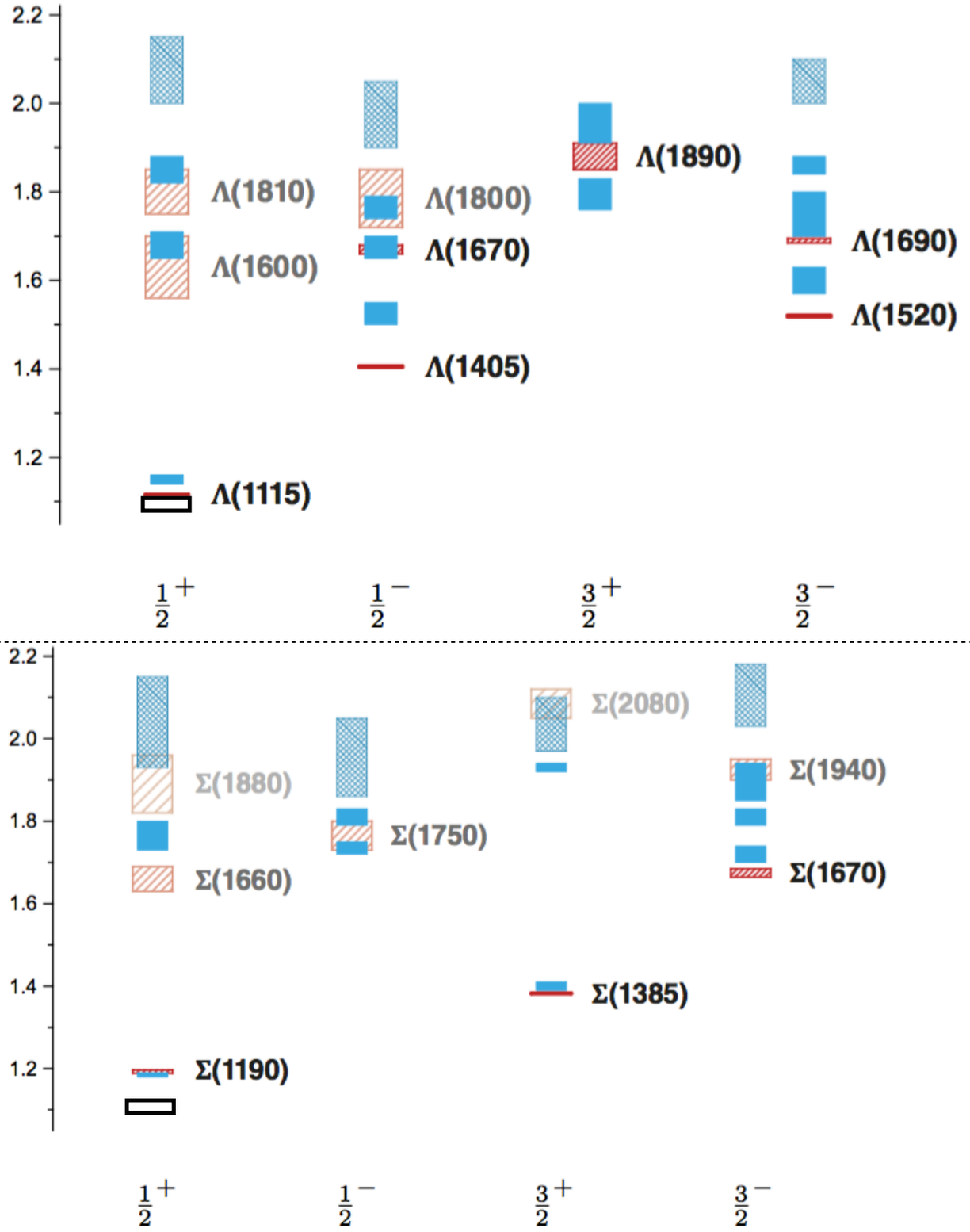
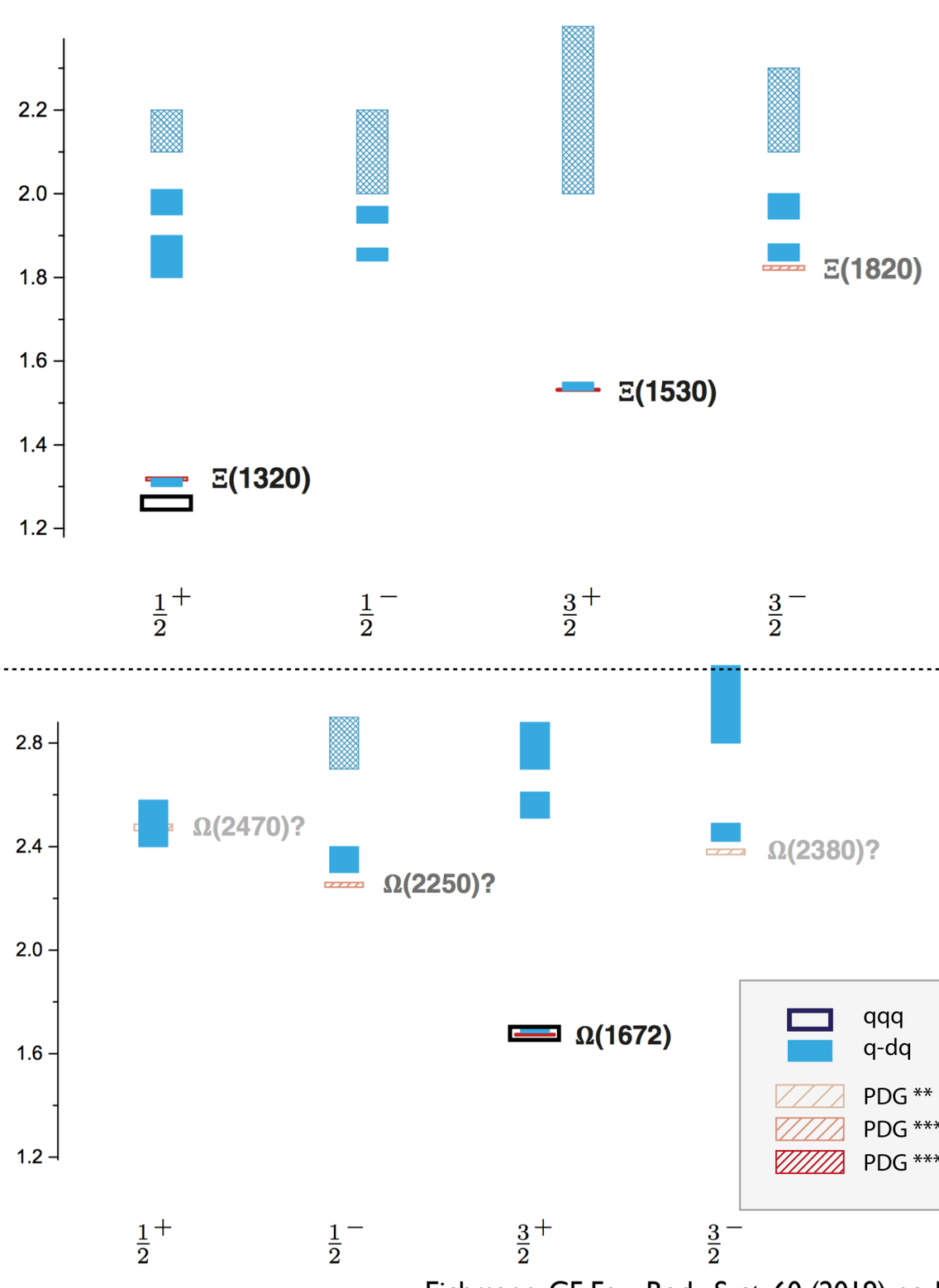
Strange baryon spectrum: DSE-RL (preliminary !)





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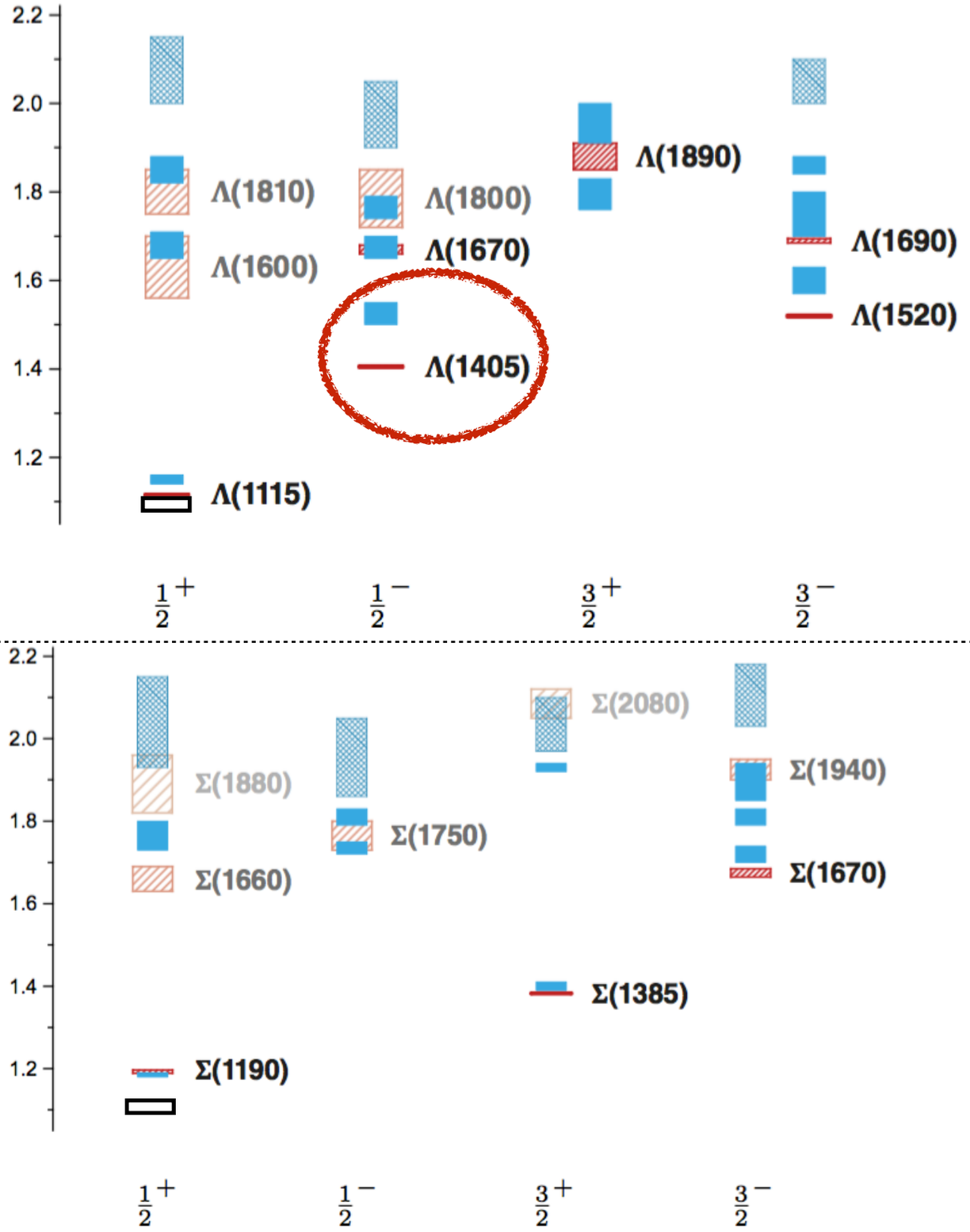

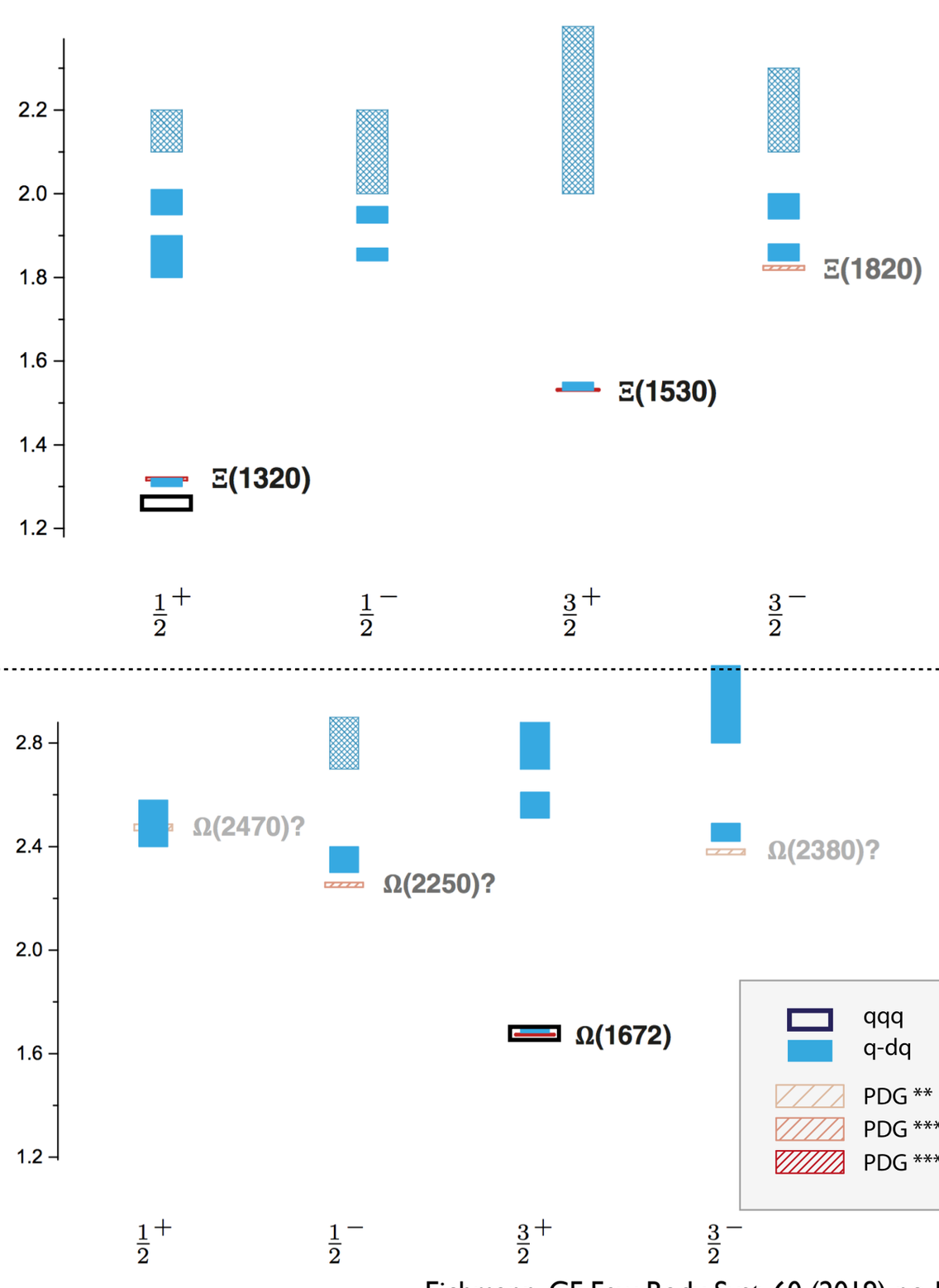

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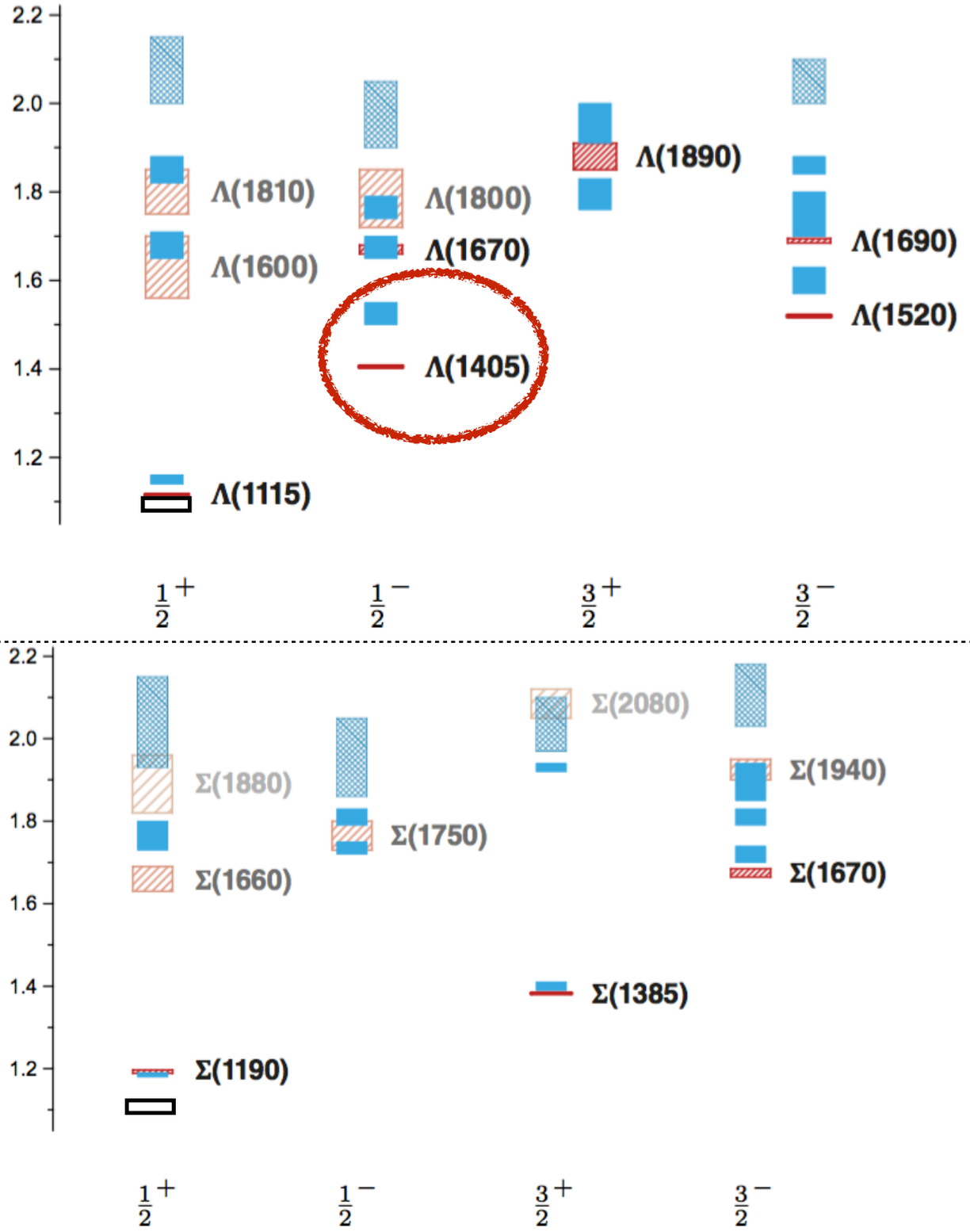

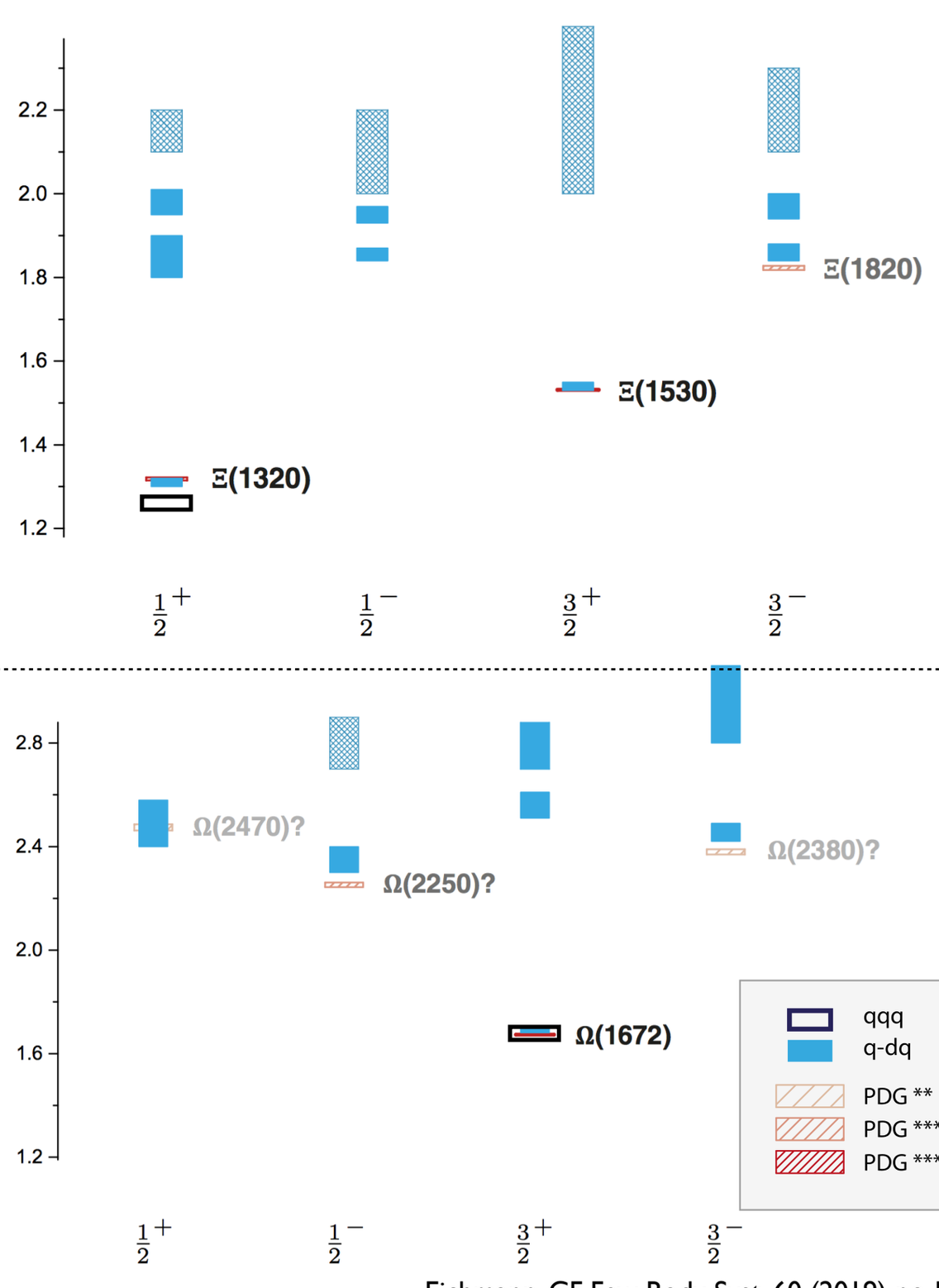

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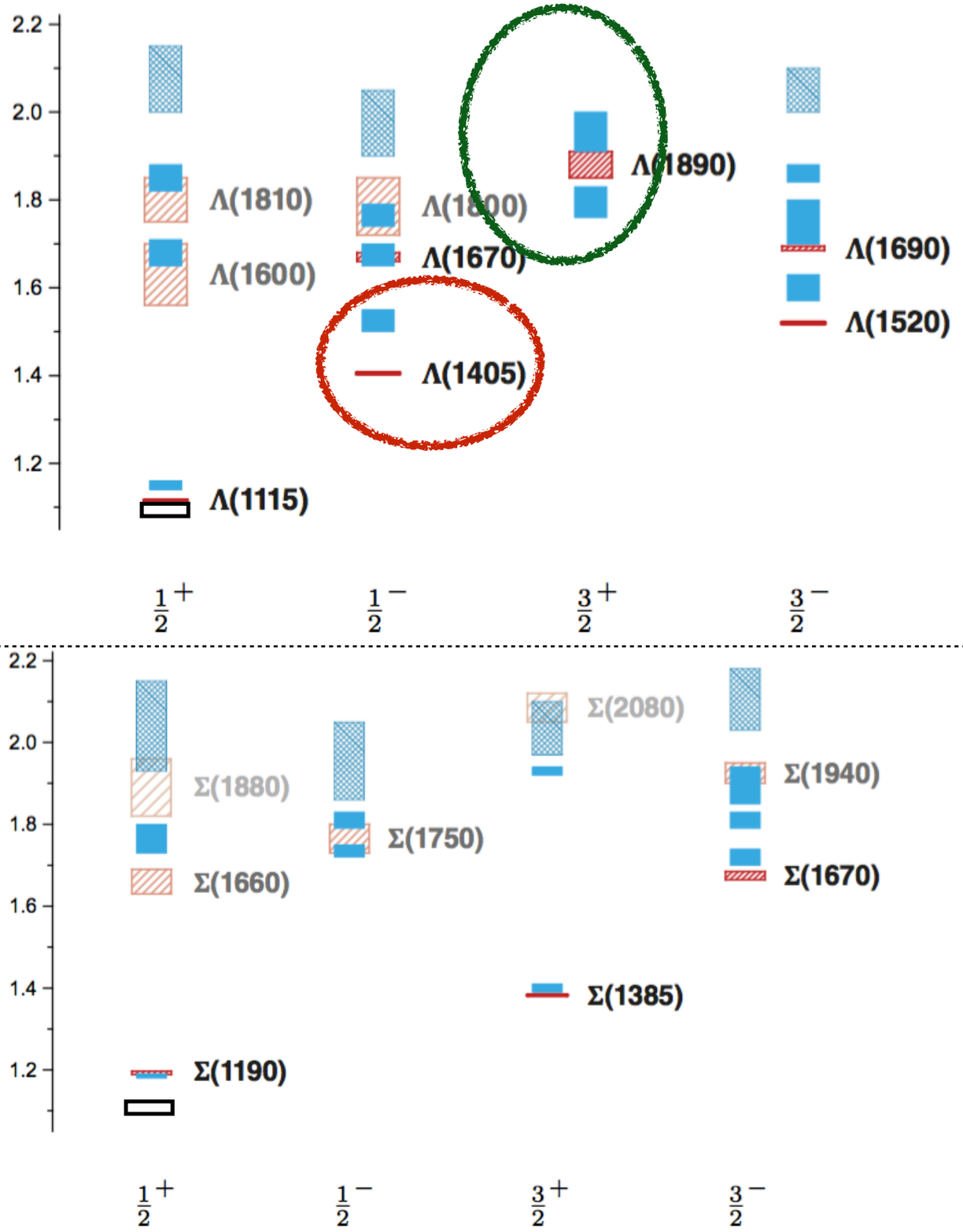

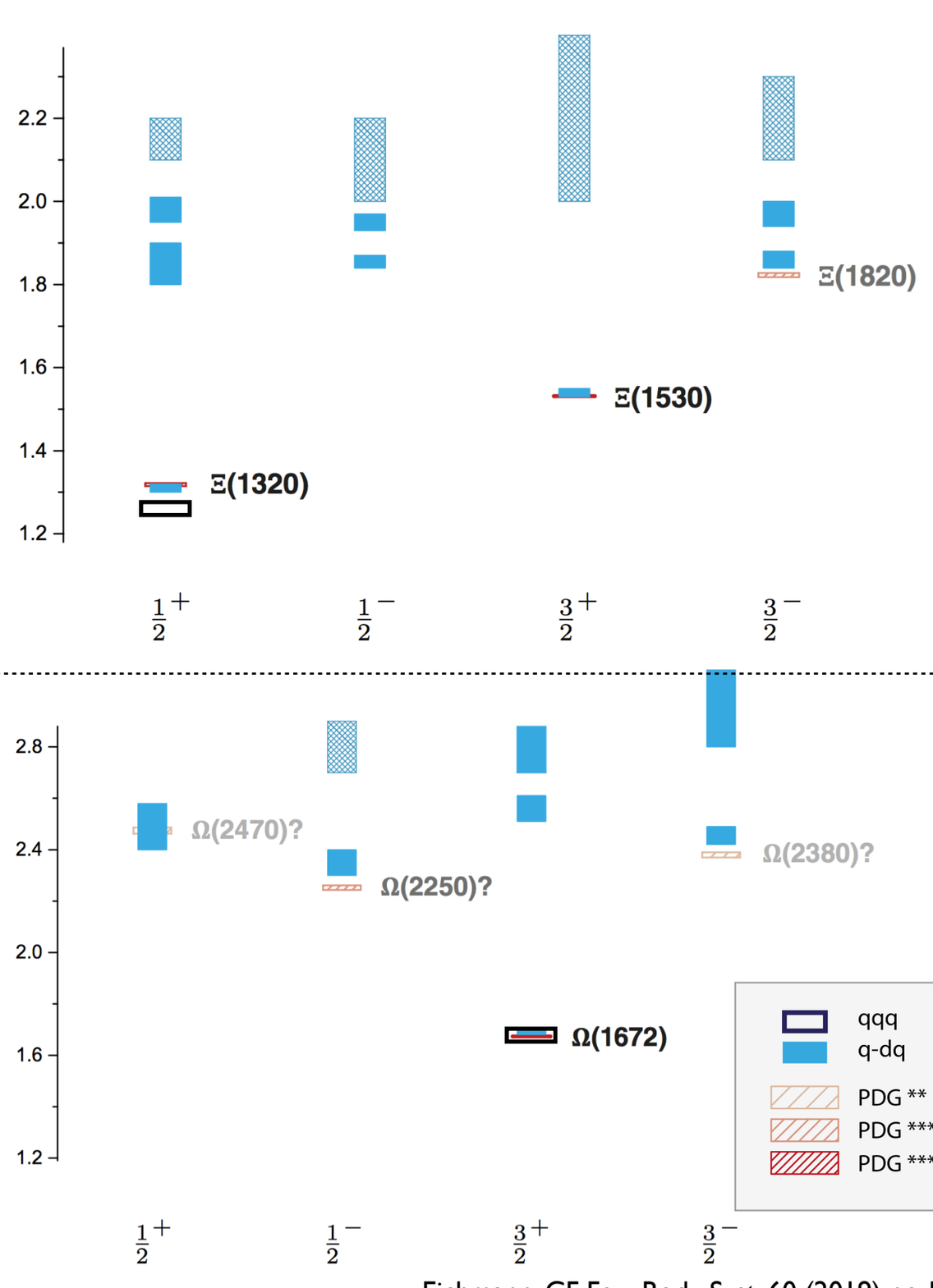

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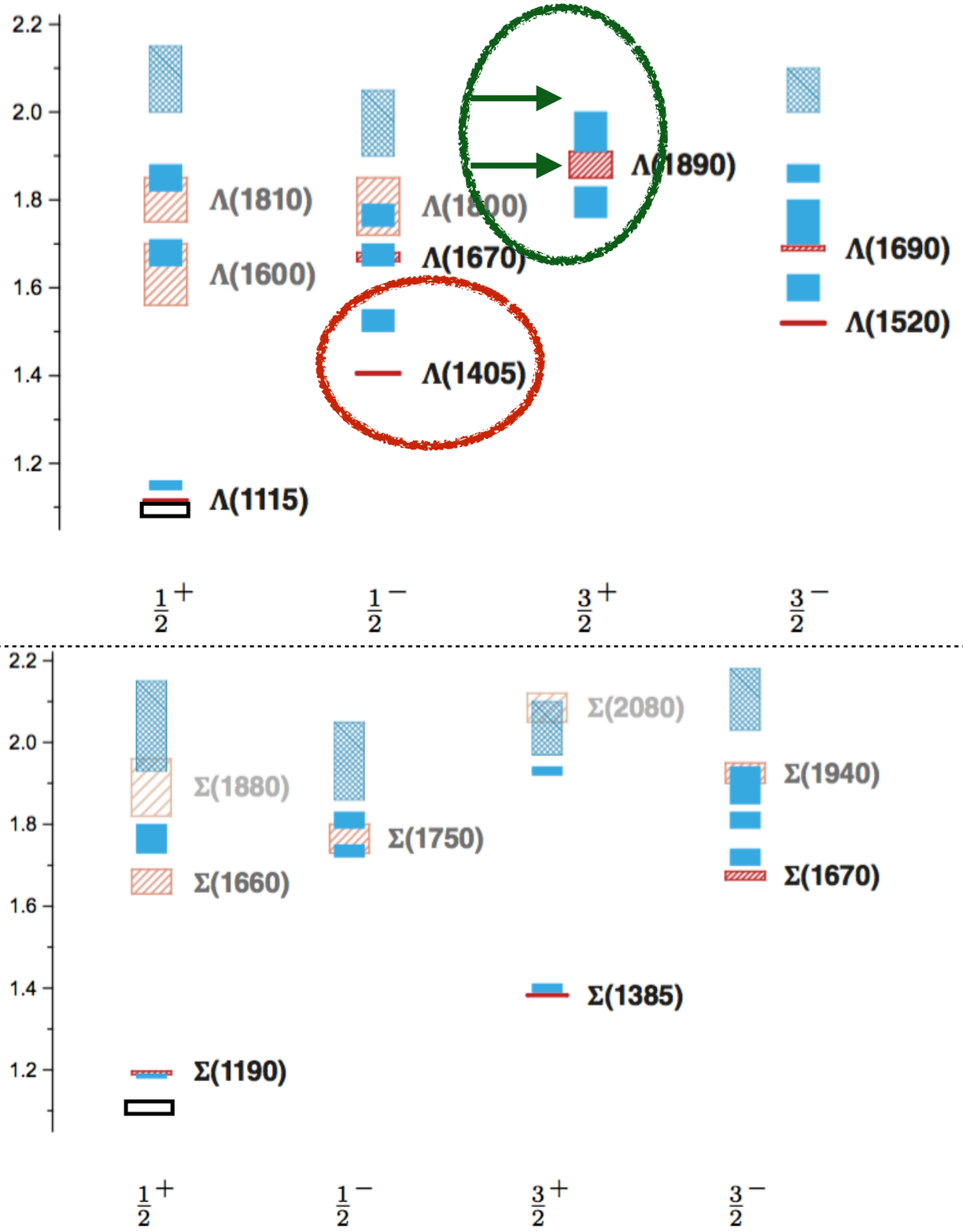

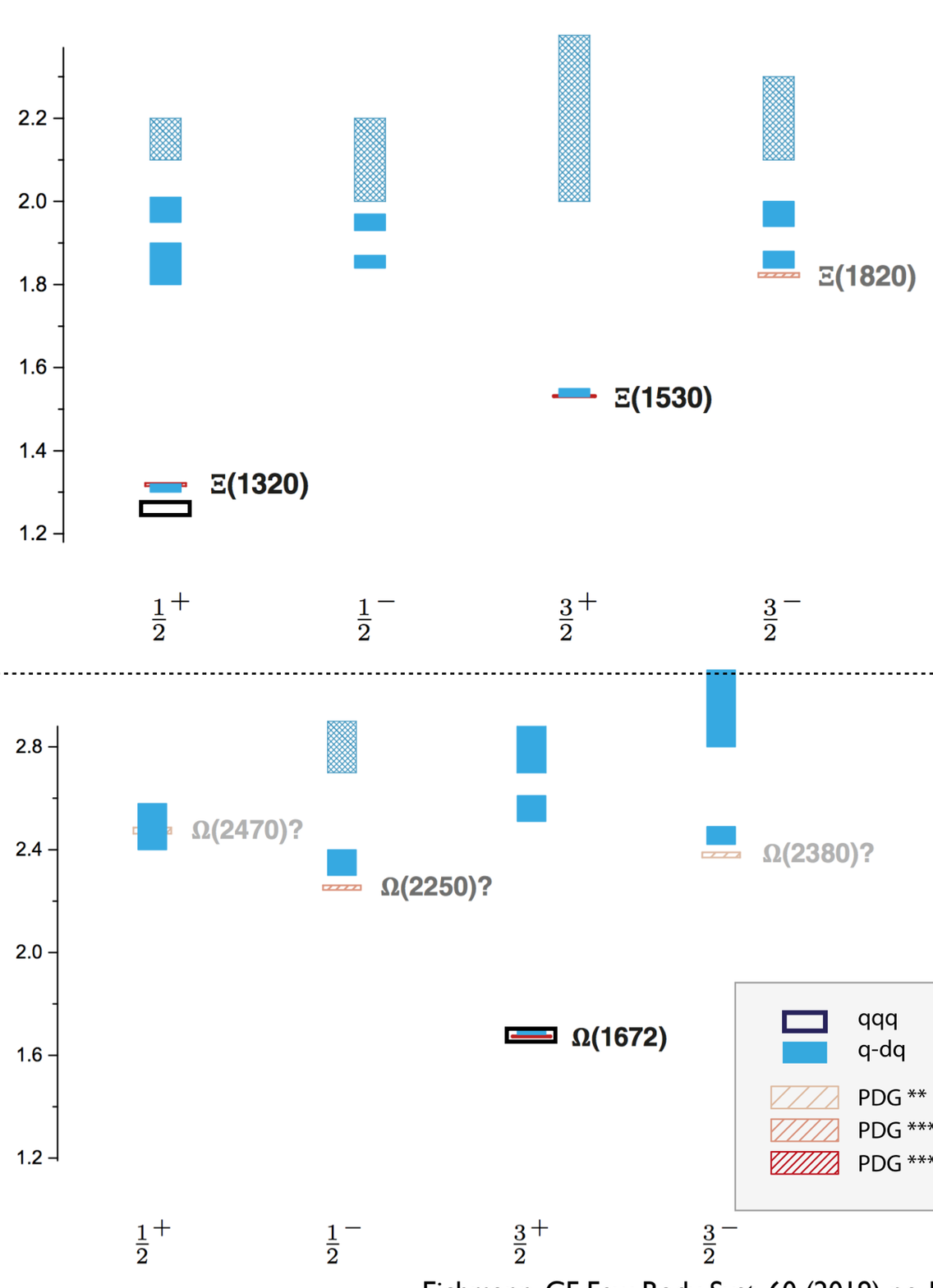

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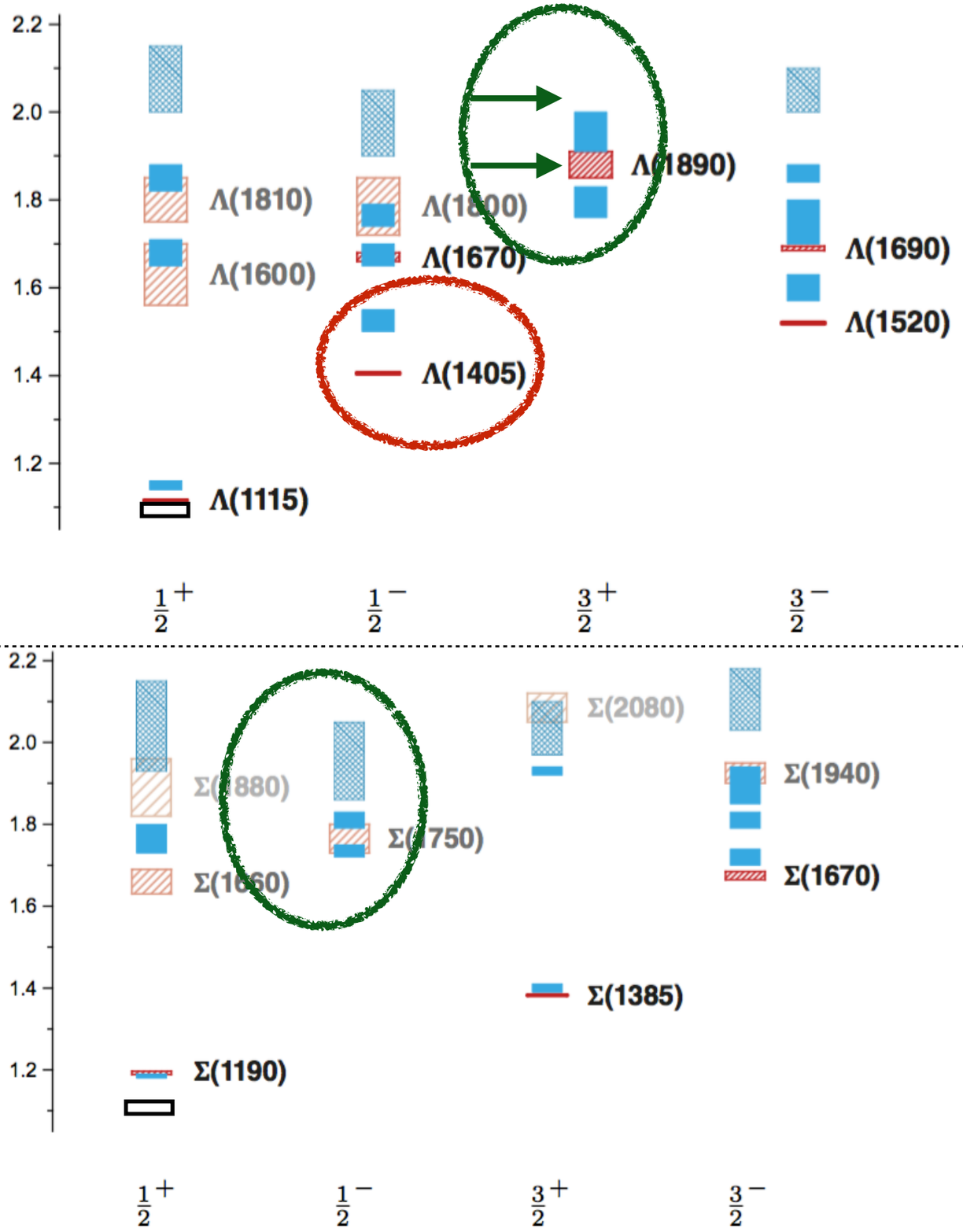

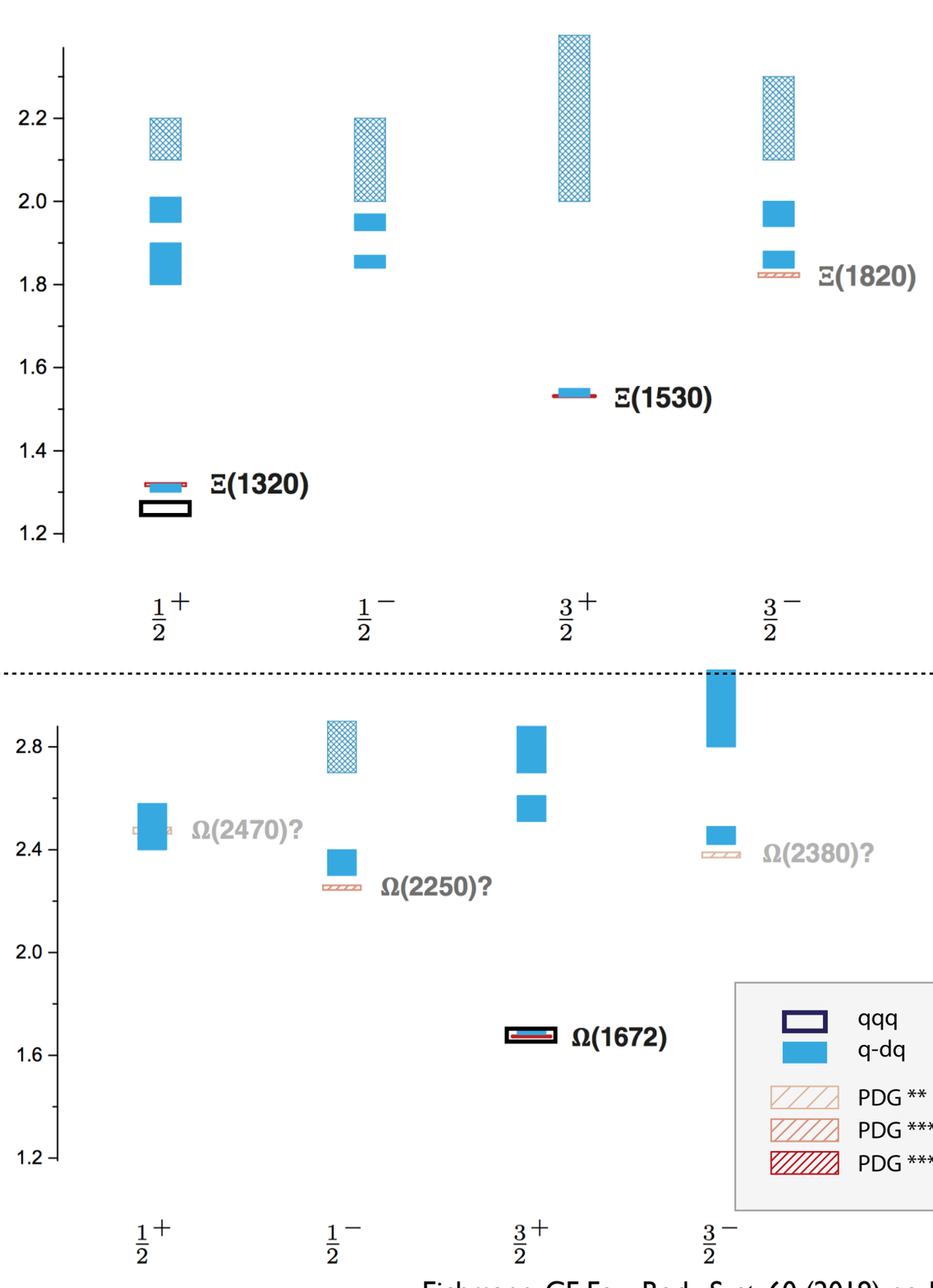

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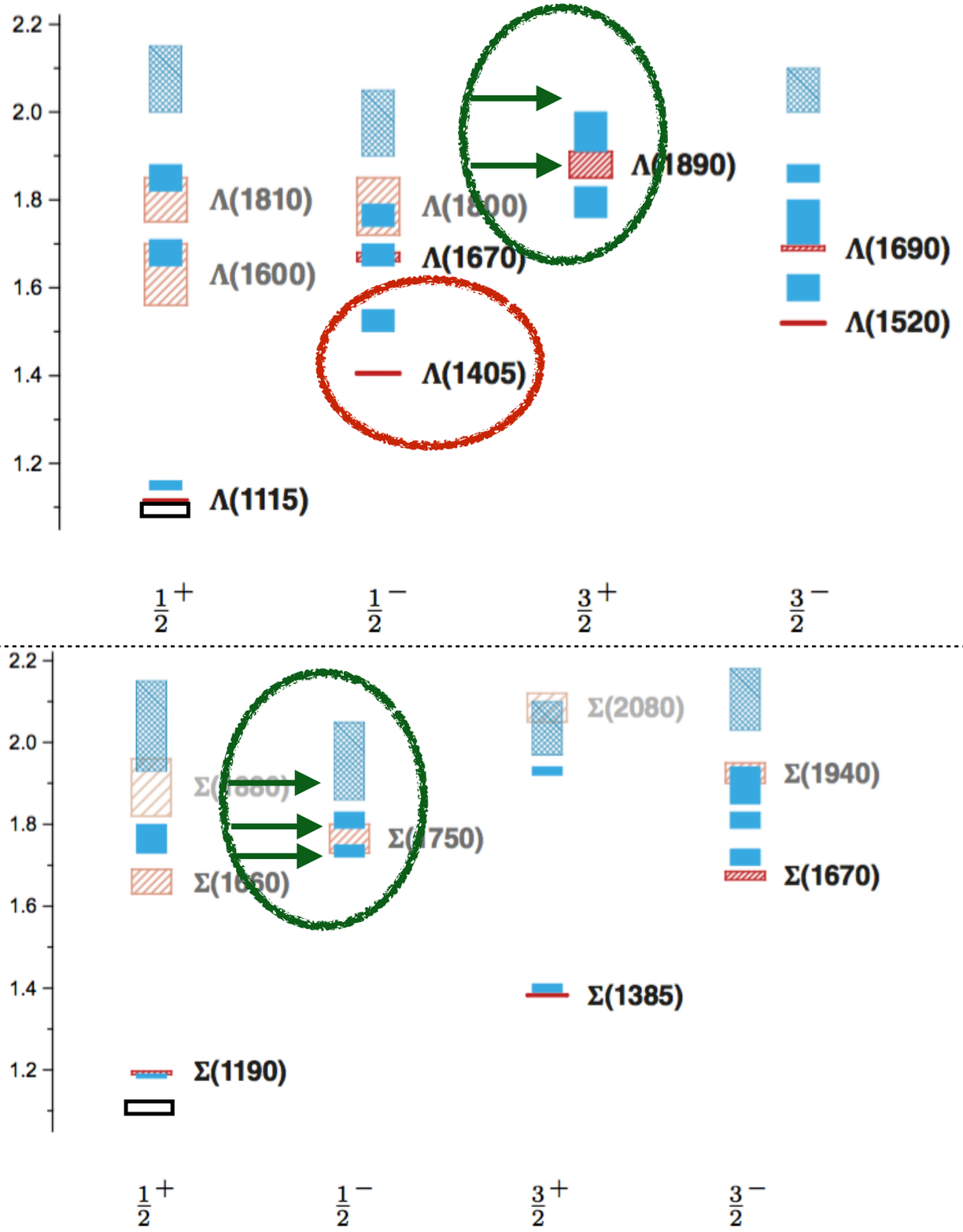

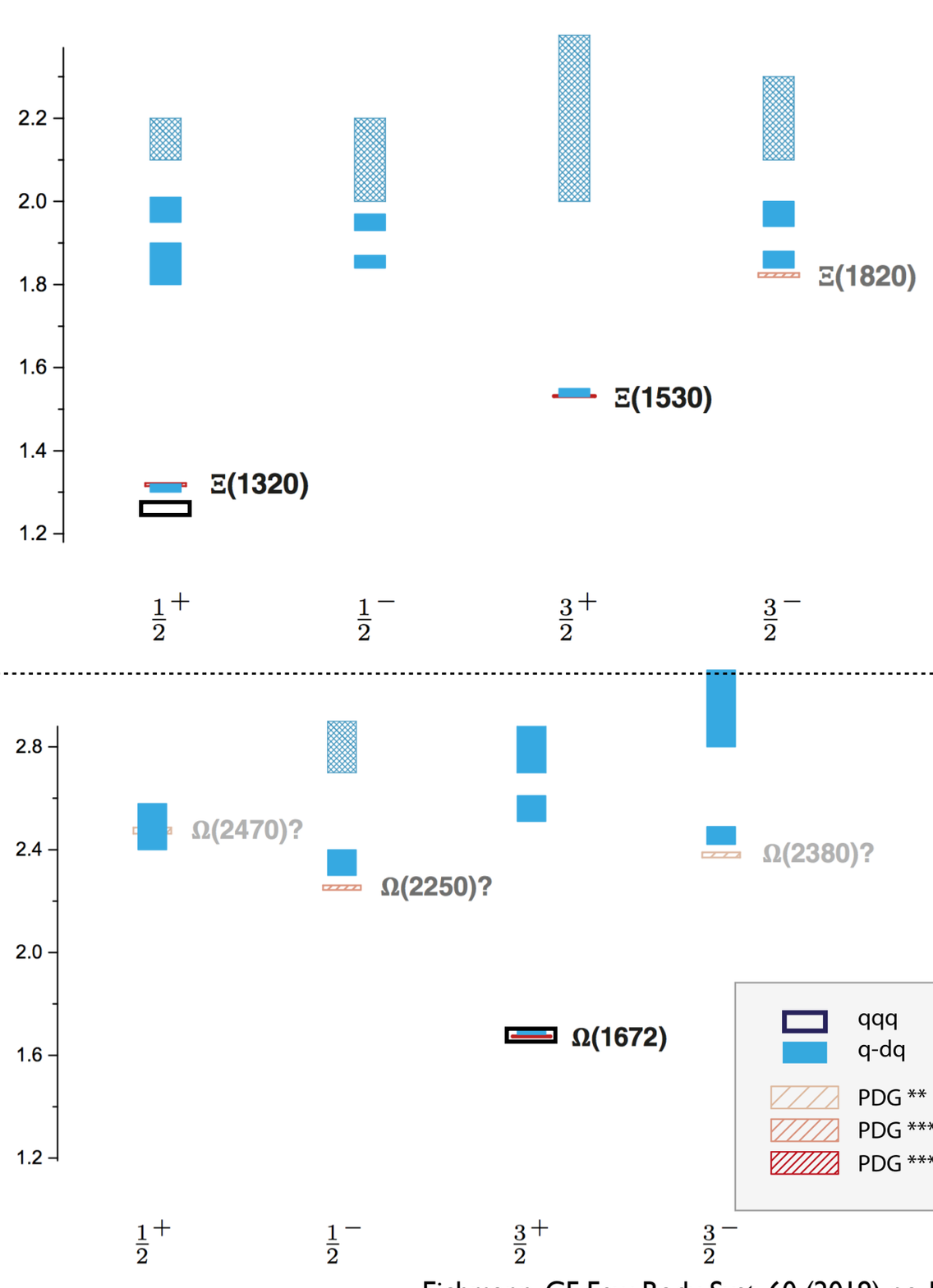

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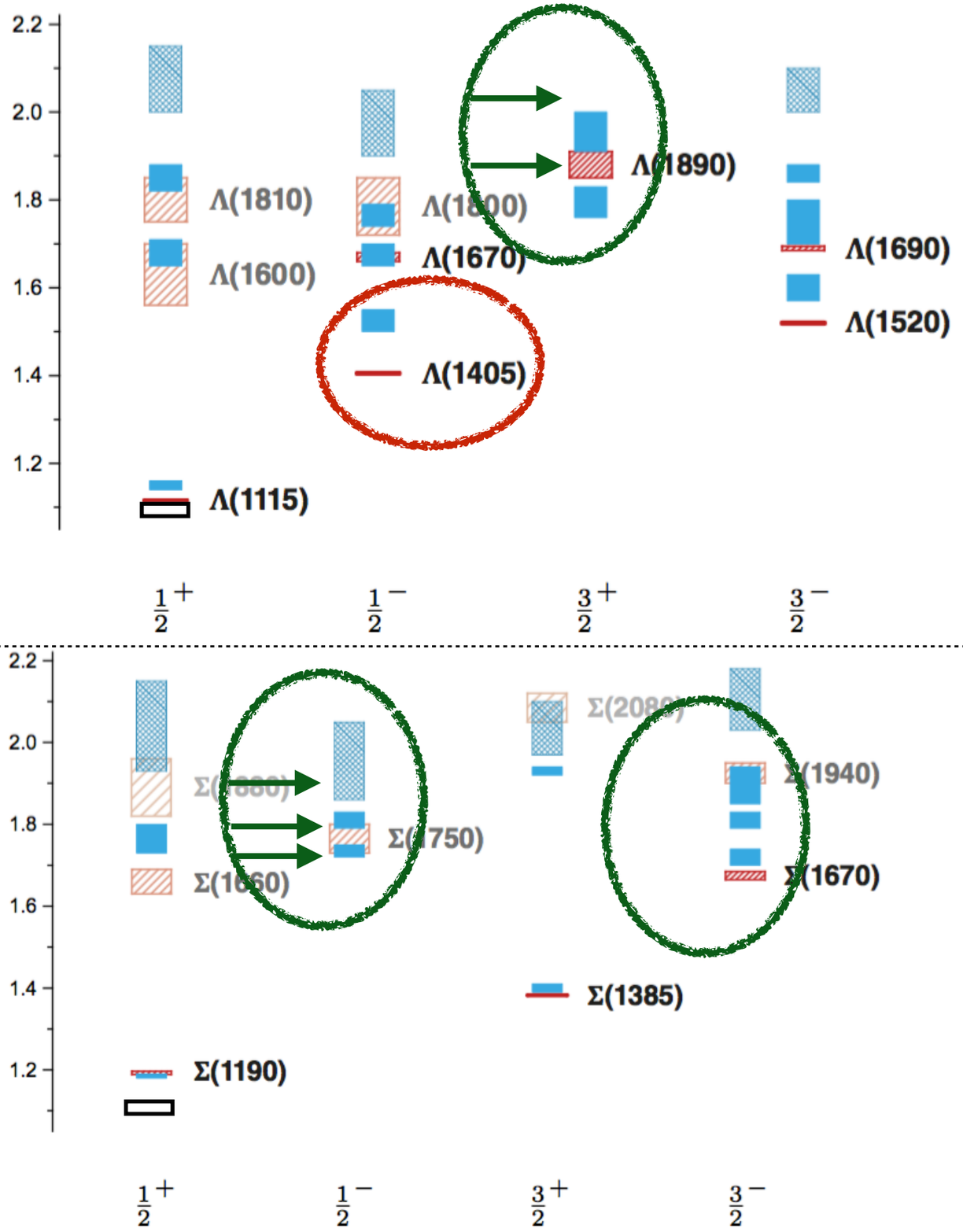

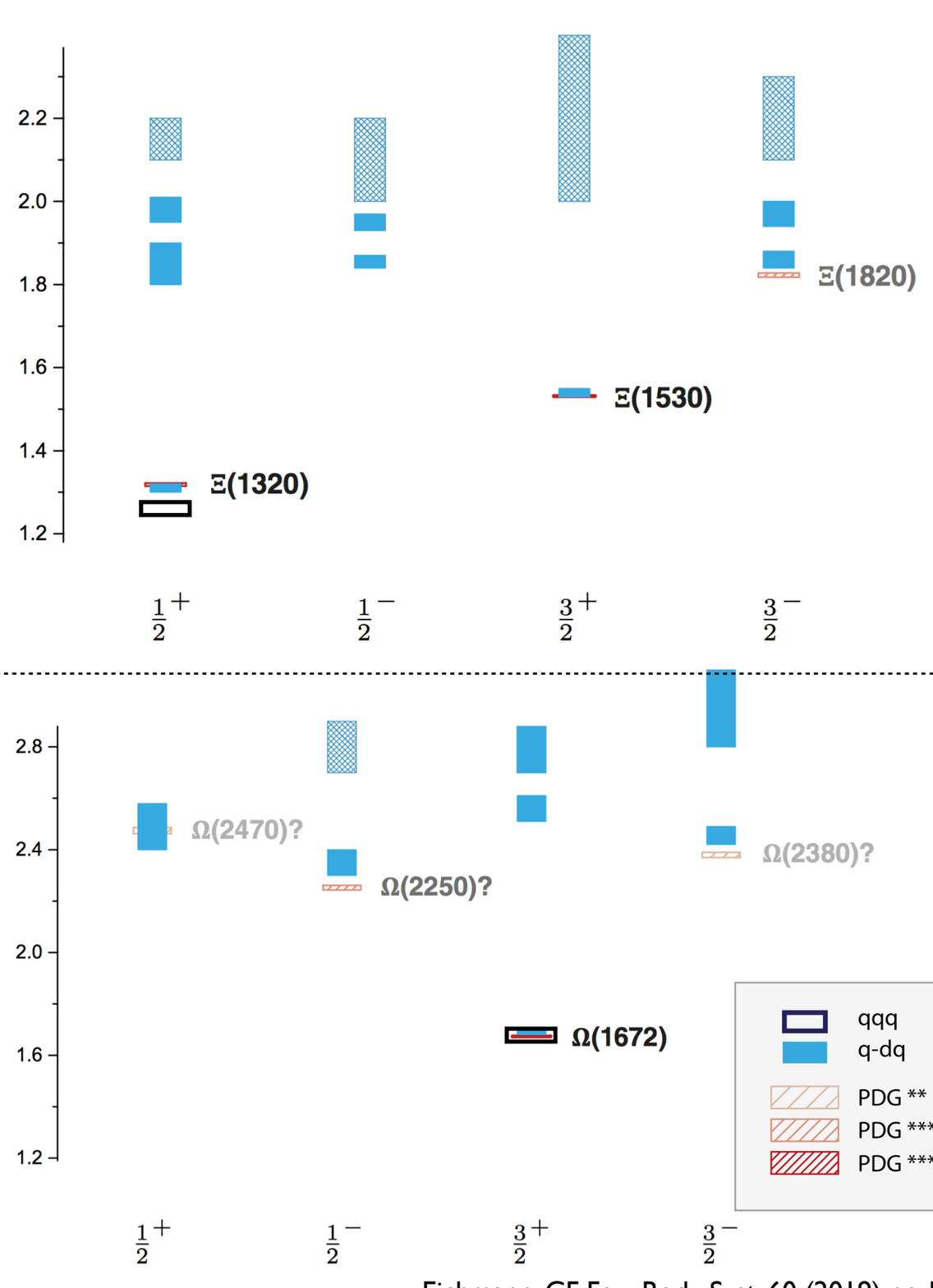

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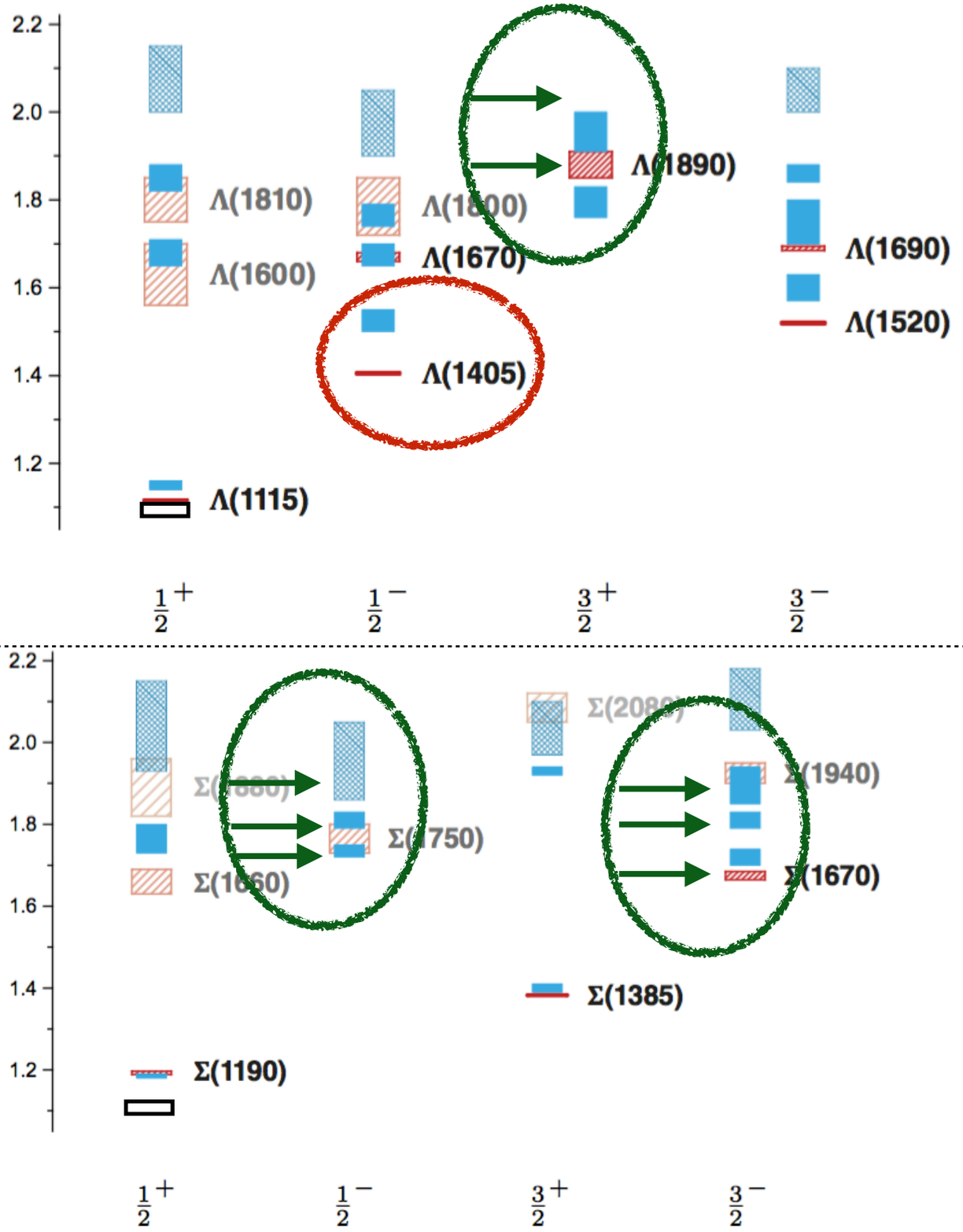

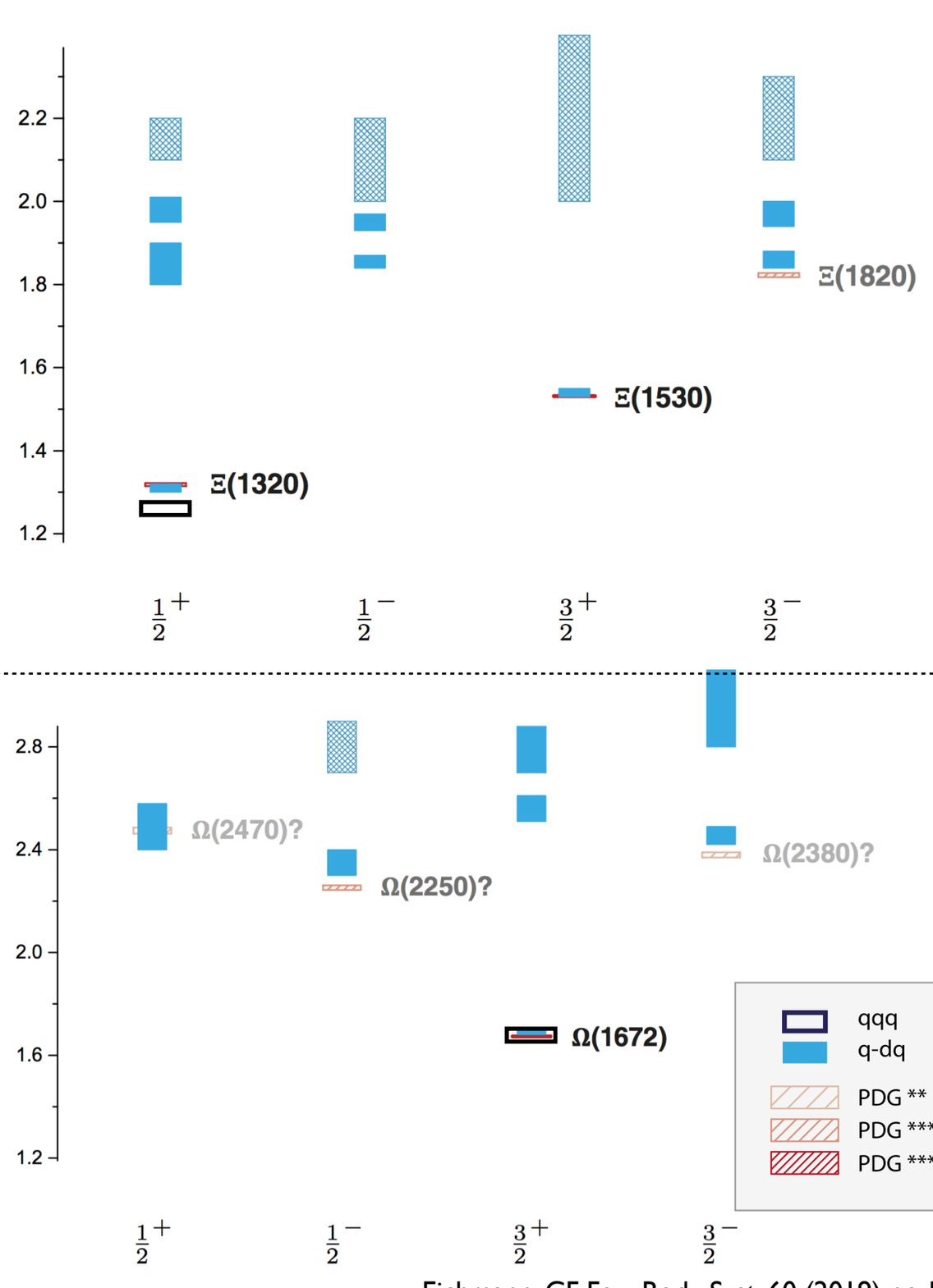

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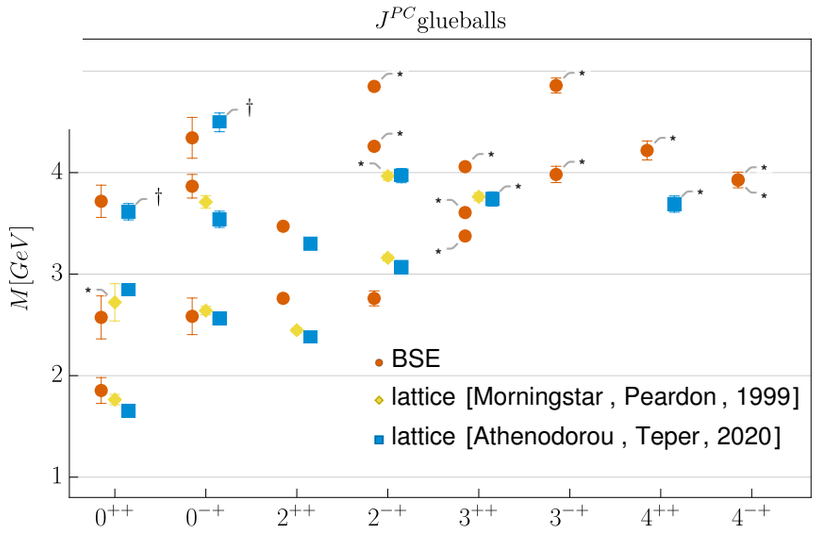
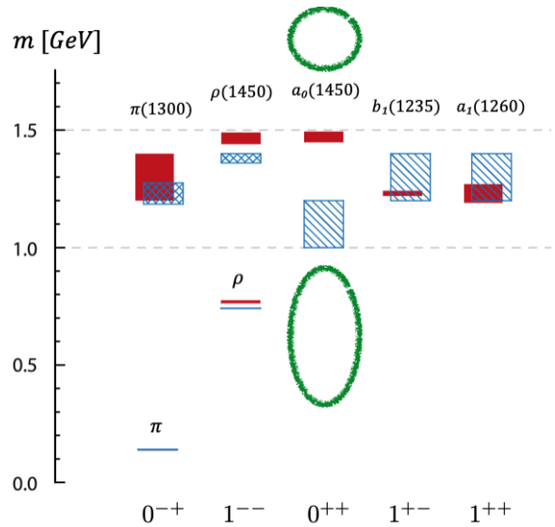
Overview

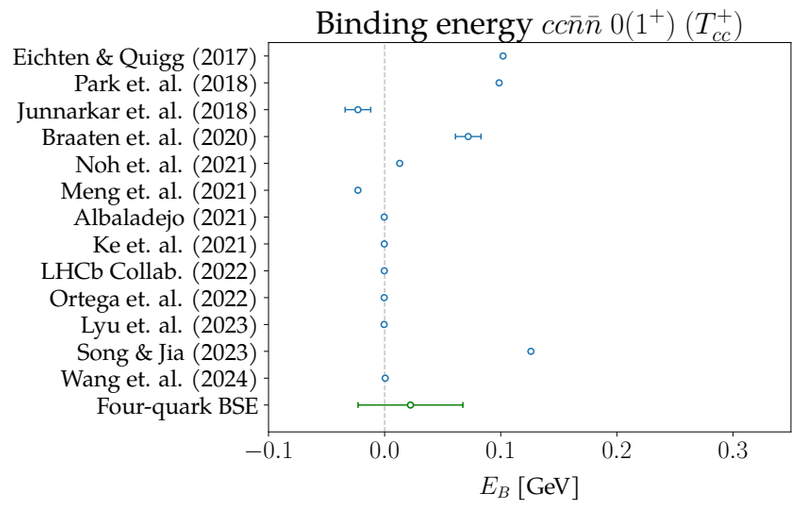
1. Glueballs: pure Yang-Mills



2. Conventional mesons (and baryons)

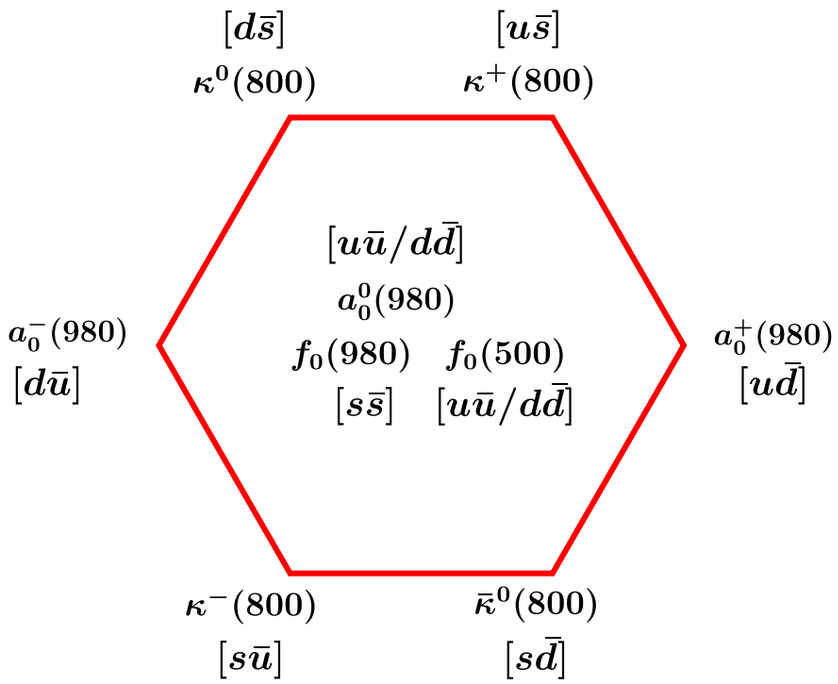


3. Four-quark states: hidden and open flavour



Light mesons with $\bar{q}\bar{q}qq$-content

Light scalar mesons:



wrong level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

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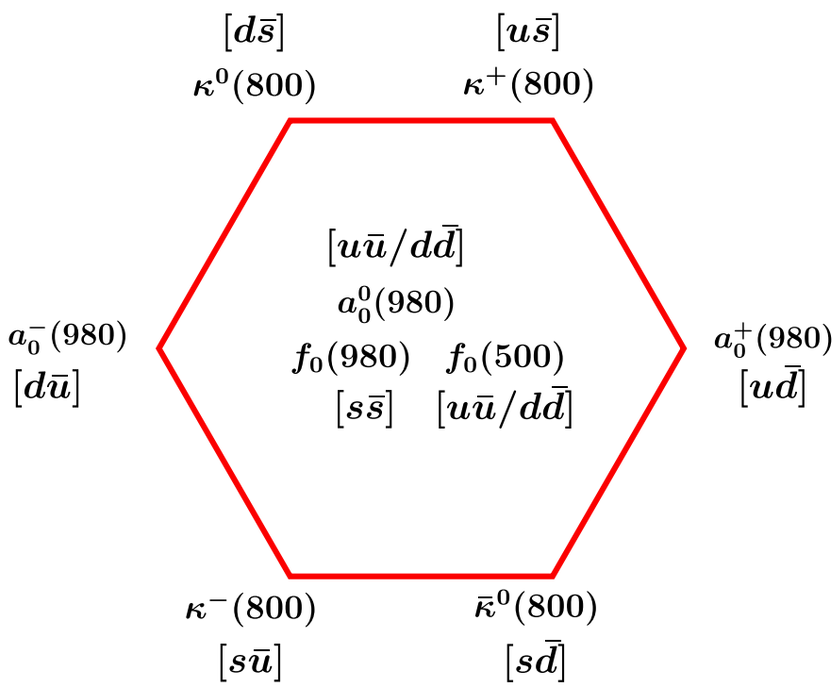

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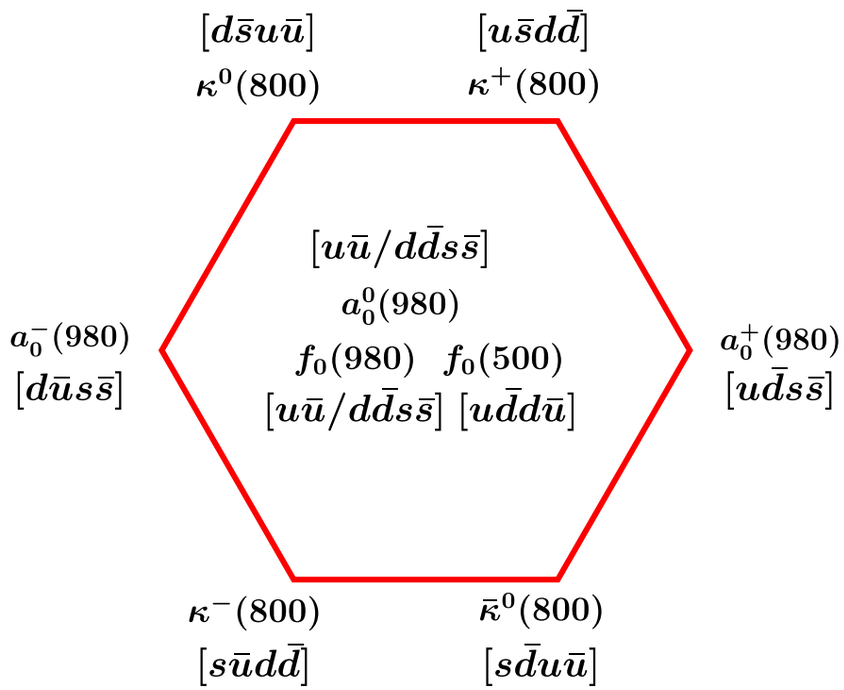


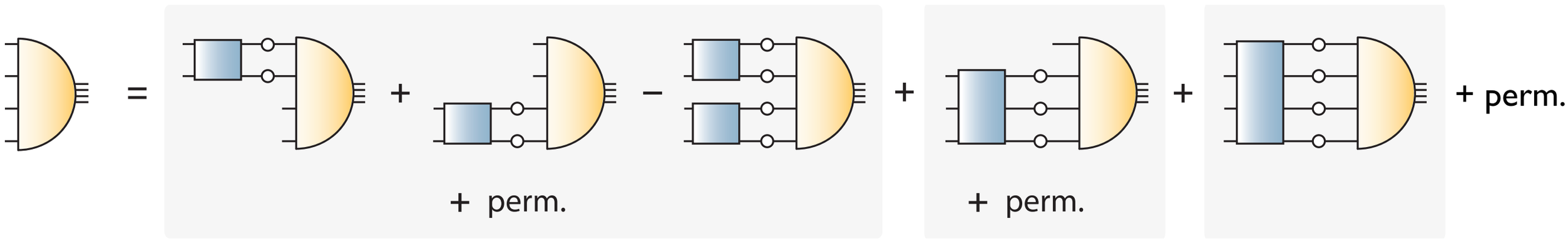
correct level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

Tetraquarks from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



Two-body interactions

Three- and four-body interactions

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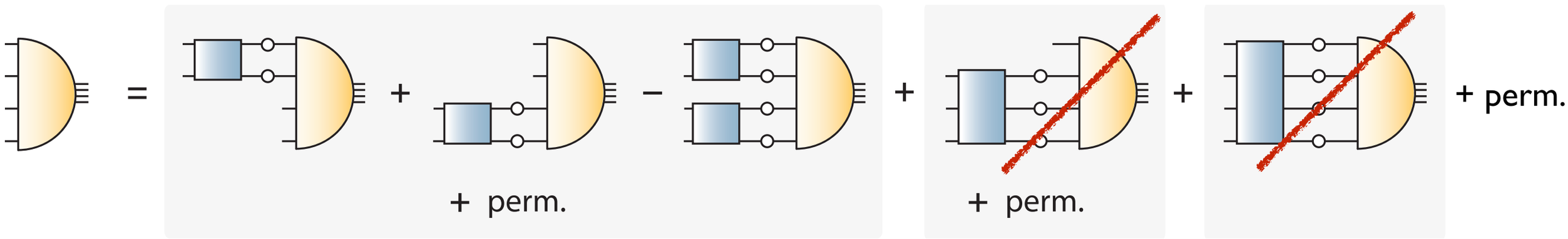

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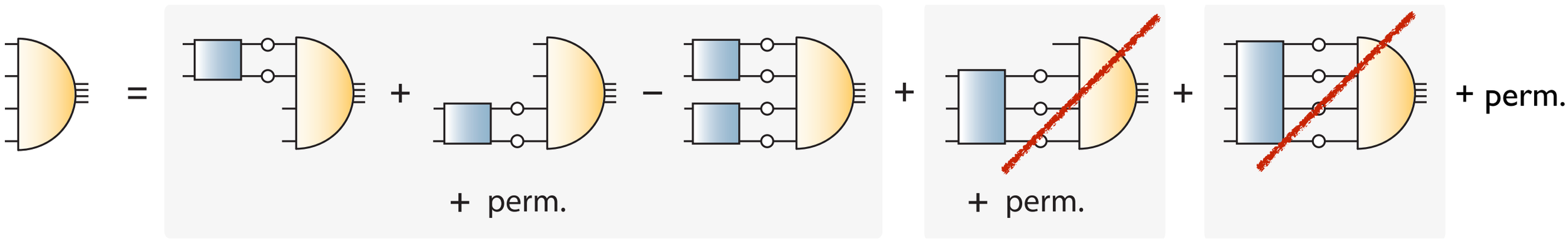

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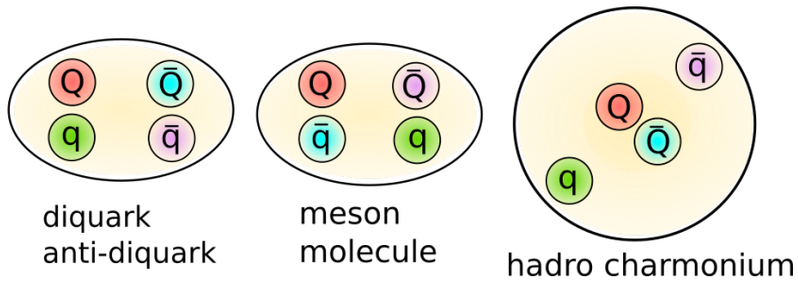

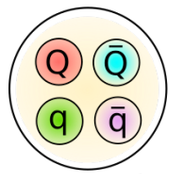

- Two-body interactions: allow for internal clustering
- use rainbow-ladder approximation…

Tetraquarks from the four-body equation

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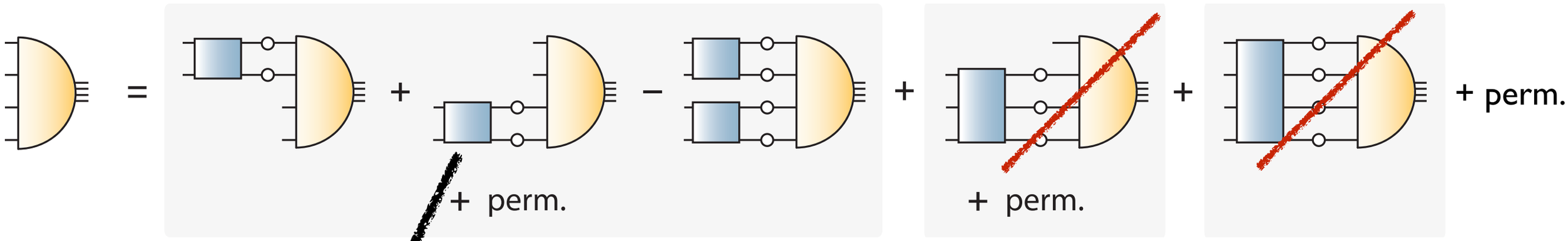

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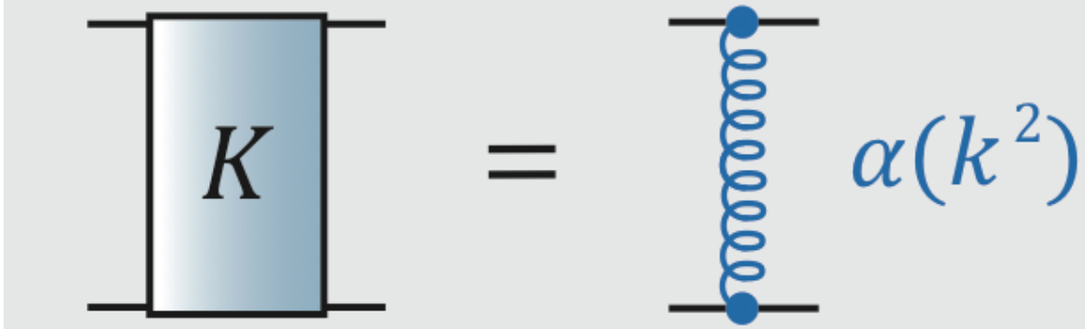
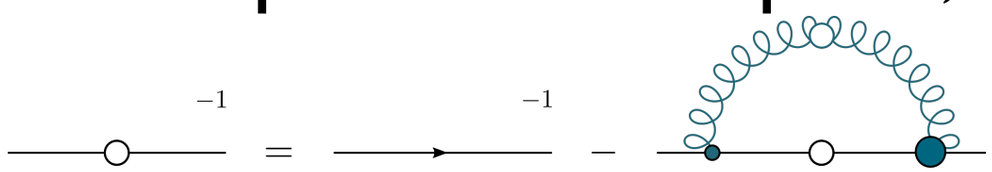


- Input: Non-perturbative quark, quark-gluon interaction



$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

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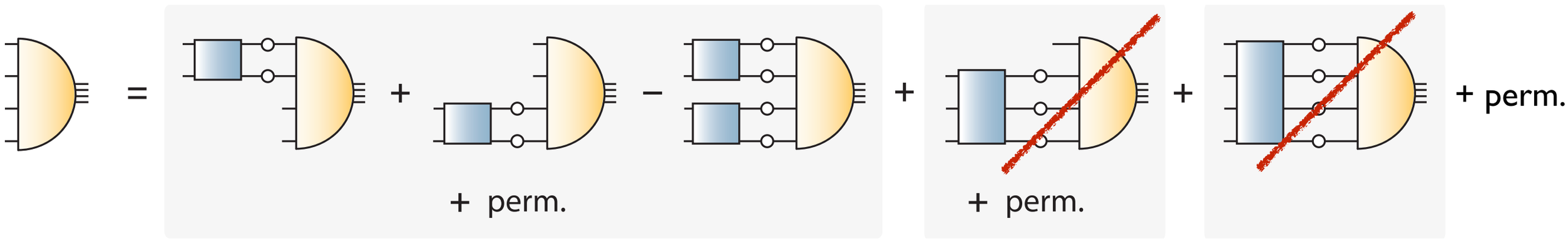


Two-body interactions

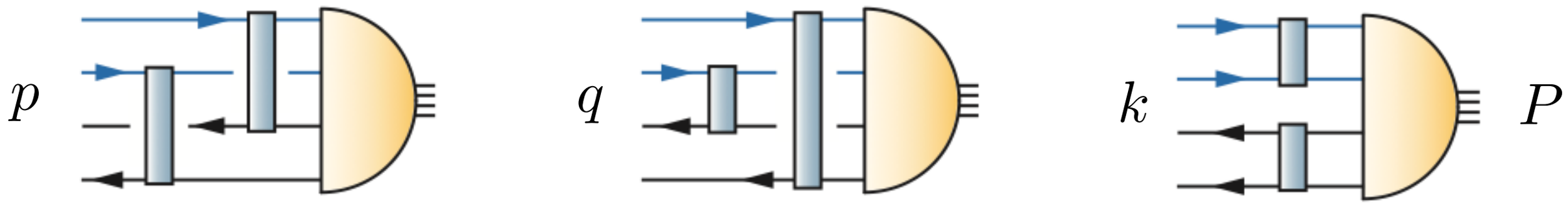
Three- and four-body interactions

$$f_0(500) : \pi\pi - \text{component dominates!}$$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

9 Lorentz scalars (built from P,p,q,k)

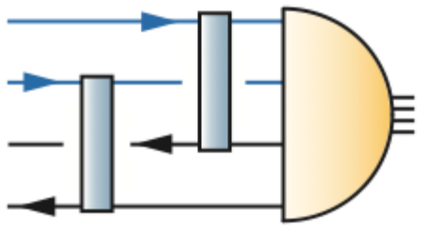
256 tensor structures (scalar)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

- reduce # tensor structures guided by physics

  light scalars: π-π and diquark-antidiquark

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, \cancel{s, a,} \ldots)$$

without twobody-clustering

0

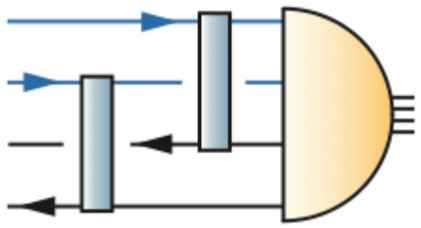
1200

$M_{\rm Tetra}[MeV]$

Bound state of four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, s, a, ...)$$

without twobody-clustering

0 — 300-400 — 1200 — $M_{\rm Tetra}[MeV]$

with π-clustering

Two-pion resonance

Bound state of

four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
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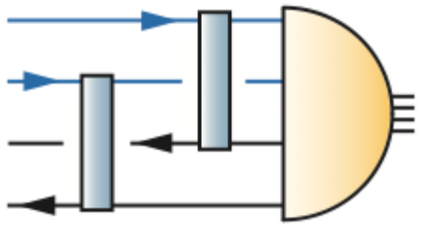
Bound state vs resonance: scalar four-quark states



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⟶ identify with $f_0(500)$ (‘σ-meson’)

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
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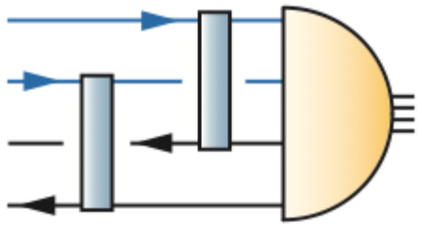

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Two-pion resonance

Bound state of

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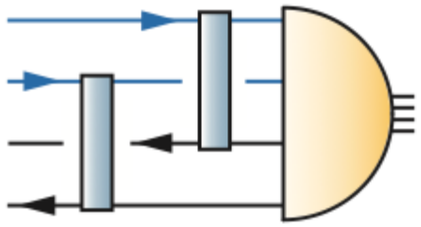
⟶ identify with $f_0(500)$ ('σ-meson')

with strange quarks: $m(a_0, f_0) \approx 1GeV$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



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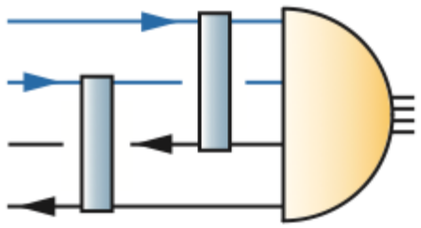
Meson-meson components dominate over diquarks !

Bound state vs resonance: scalar four-quark states



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0 — 300-400 — 1200 — $M_{\rm Tetra}[MeV]$

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Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

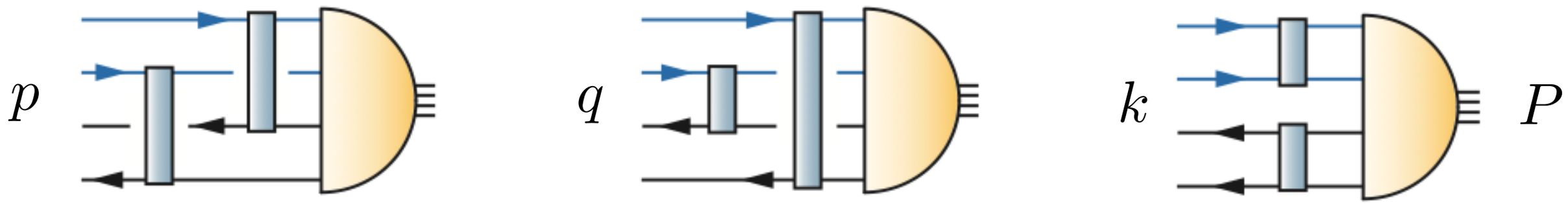
Meson-meson components dominate over diquarks !

Mixing with $q\bar{q}$: small effect

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014
Santowsky, CF, PRD 105 (2022) 4,313

Structure of the amplitude

Vector tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

9 Lorentz scalars (built from P,p,q,k)

768 tensor structures (vector)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

- reduce # tensor structures guided by physics

Structure of the amplitude

Vector

p

$\Gamma(P, p$

- red

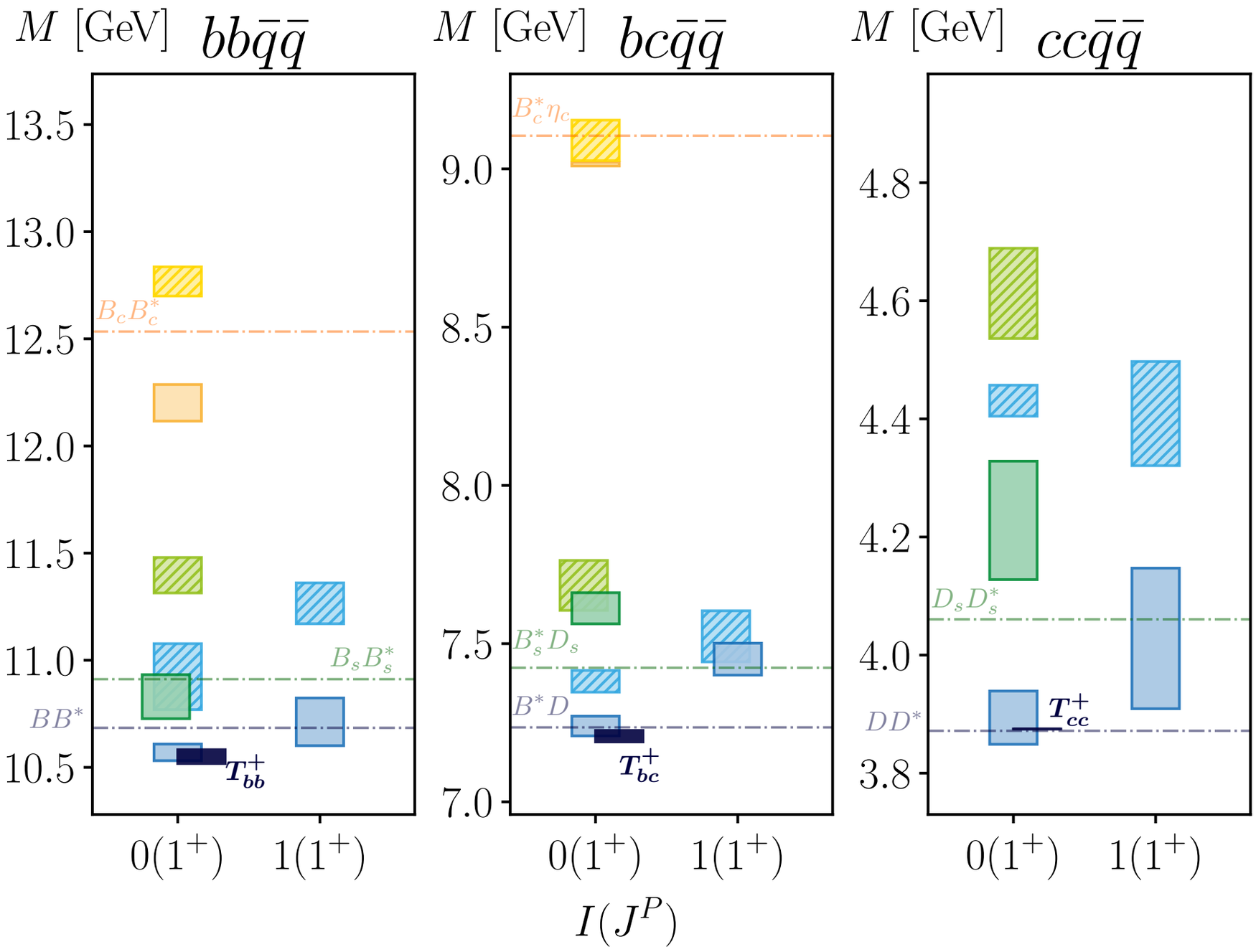
$n \in \{u, d\}$

$I(J^P)$	Physical components					
	$\mathbf{1}\otimes\mathbf{1}$		$\bar{\mathbf{3}}\otimes\mathbf{3}$	$\mathbf{8}\otimes\mathbf{8}$		$\mathbf{6}\otimes\bar{\mathbf{6}}$
	f_0	f_1	f_2	f_3	f_4	f_5
$0(1^+)$ $bb\bar{n}\bar{n}$	BB^*	B^*B^*	$A_{bb}S$	BB^*	B^*B^*	$S_{bb}A$
$bc\bar{n}\bar{n}$	BD^*	B^*D	$A_{bc}S$	BD^*	B^*D	$S_{bc}A$
$cc\bar{n}\bar{n}$	DD^*	D^*D^*	$A_{cc}S$	DD^*	D^*D^*	$S_{cc}A$
$bb\bar{s}\bar{s}$	$B_sB_s^*$	–	$A_{bb}A_{ss}$	$B_sB_s^*$	–	–
$bc\bar{s}\bar{s}$	$B_sD_s^*$	$B_s^*D_s$	$S_{bc}A_{ss}$	$B_sD_s^*$	$B_s^*D_s^*$	$A_{bc}S_{ss}$
$cc\bar{s}\bar{s}$	$D_sD_s^*$	–	$A_{cc}A_{ss}$	$D_sD_s^*$	–	–
$1(1^+)$ $bb\bar{q}\bar{q}$	BB^*	–	$A_{bb}A$	BB^*	–	–
$bc\bar{q}\bar{q}$	BD^*	B^*D	$S_{bc}A$	BD^*	B^*D^*	$A_{bc}S$
$cc\bar{q}\bar{q}$	DD^*	–	$A_{cc}A$	DD^*	–	–

P

$avor$

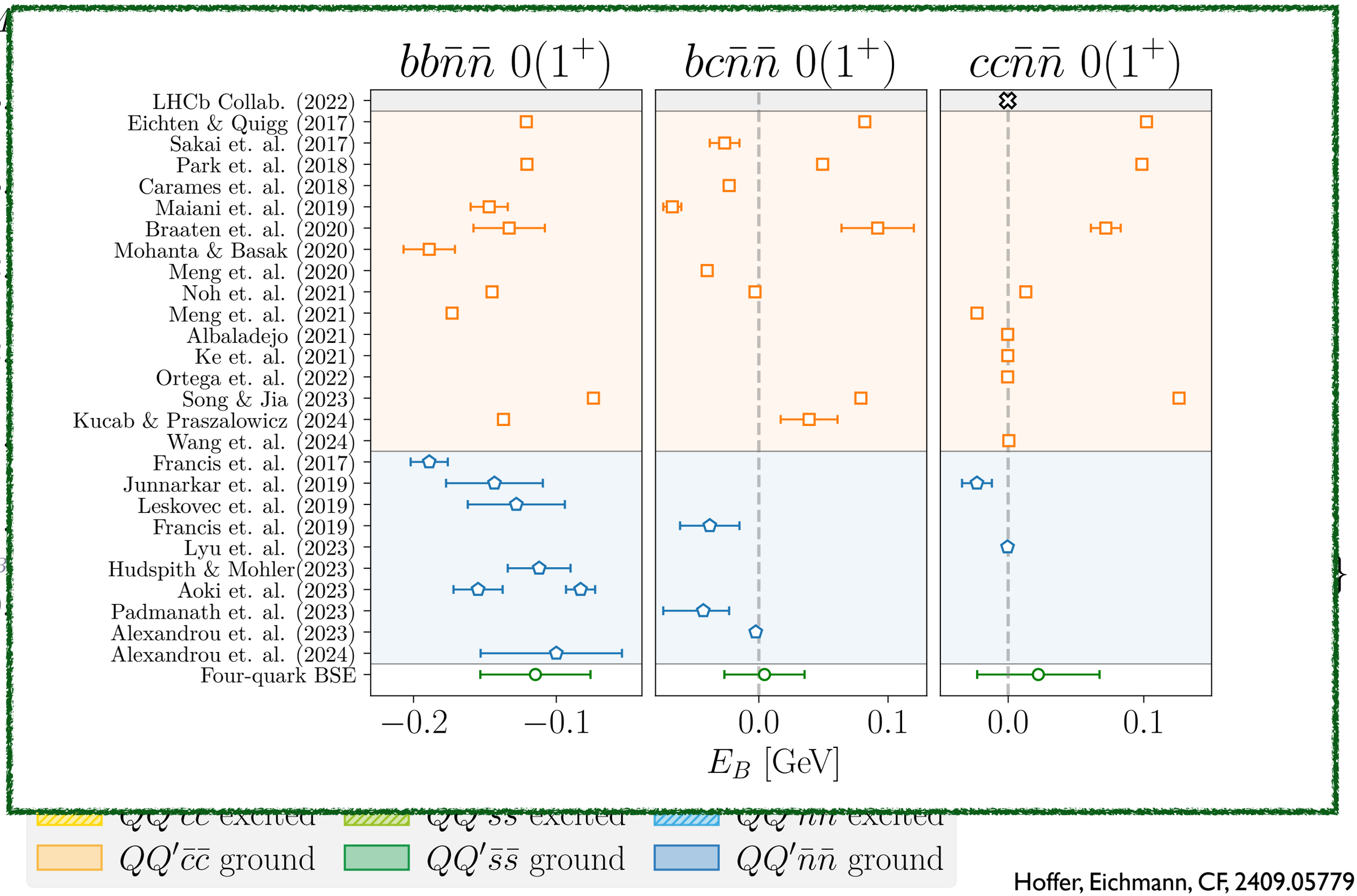
bb, bc and cc four-quark-states



$n \in \{u, d\}$

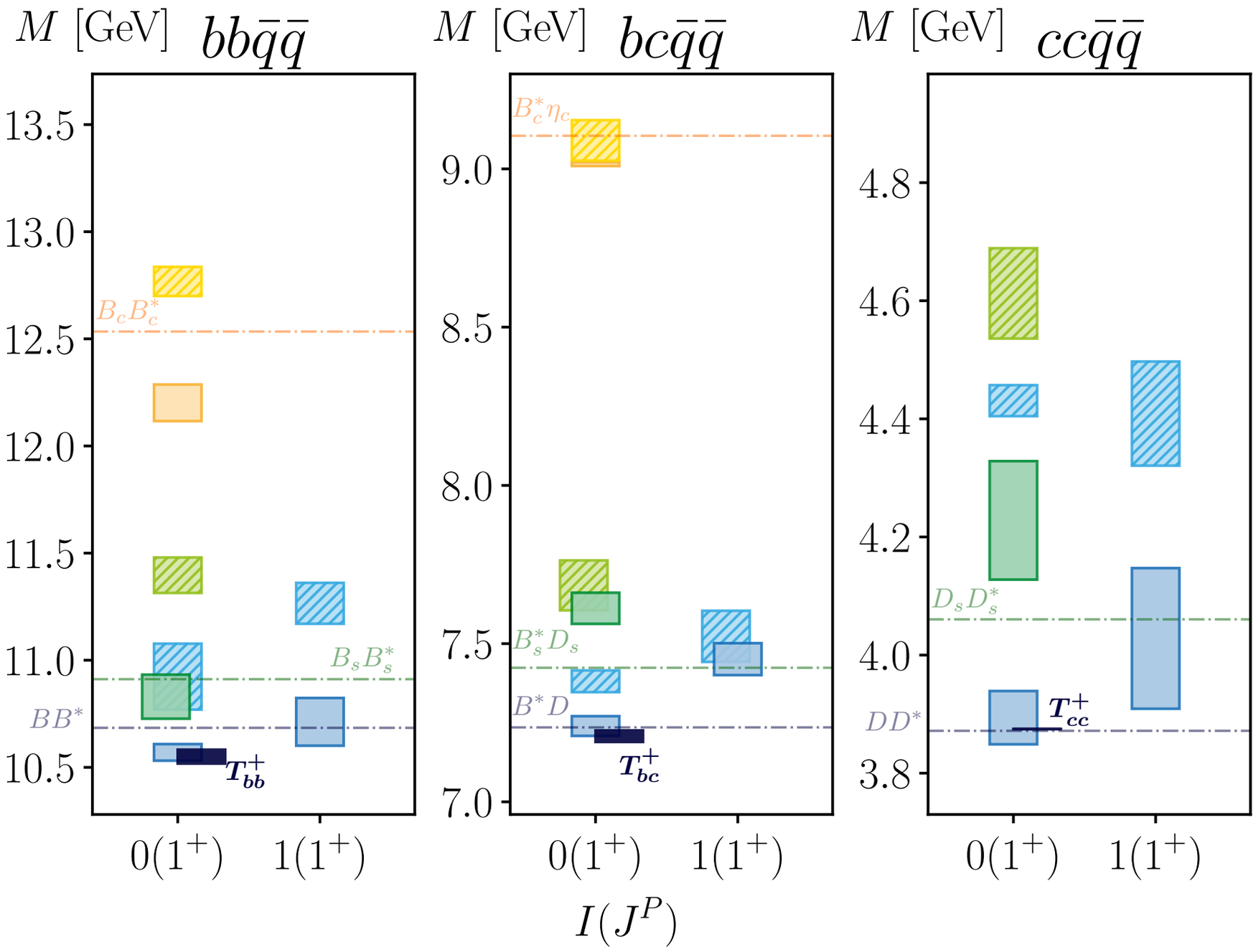
Hoffer, Eichmann, CF, 2409.05779

bb, bc and cc four-quark-states



Hoffer, Eichmann, CF, 2409.05779

bb, bc and cc four-quark-states



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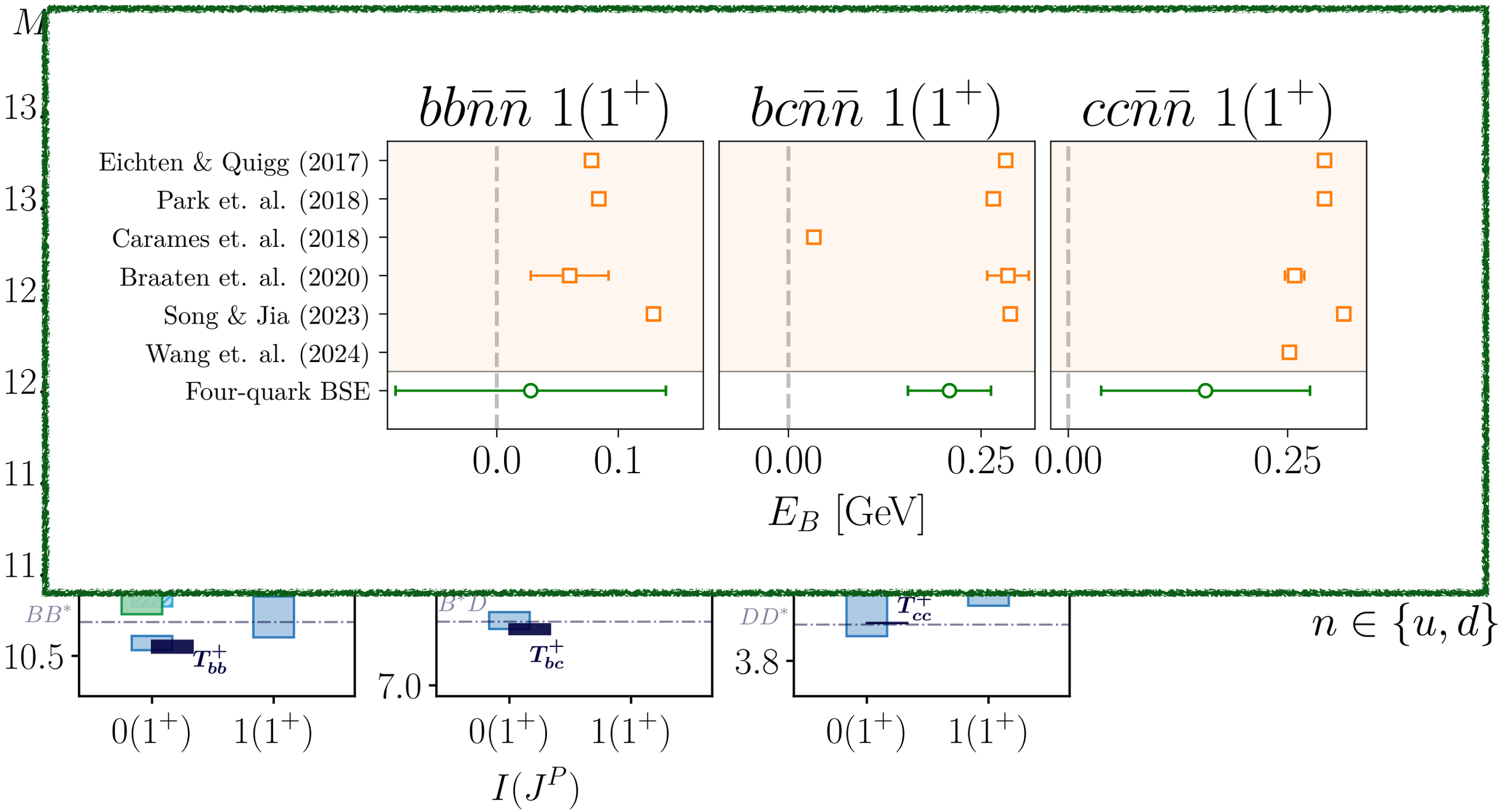
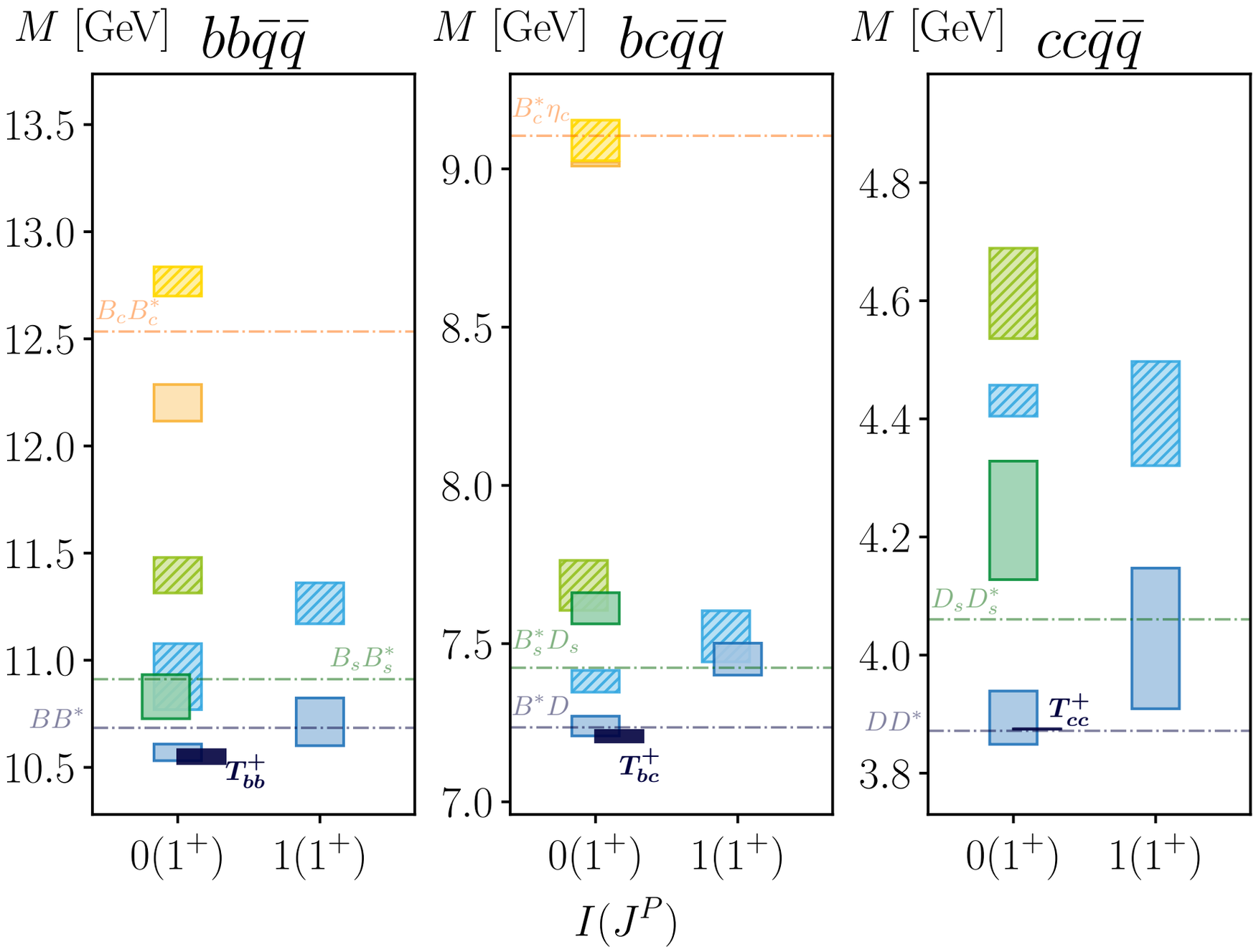
Hoffer, Eichmann, CF, 2409.05779

bb, bc and cc four-quark-states



Hoffer, Eichmann, CF, 2409.05779

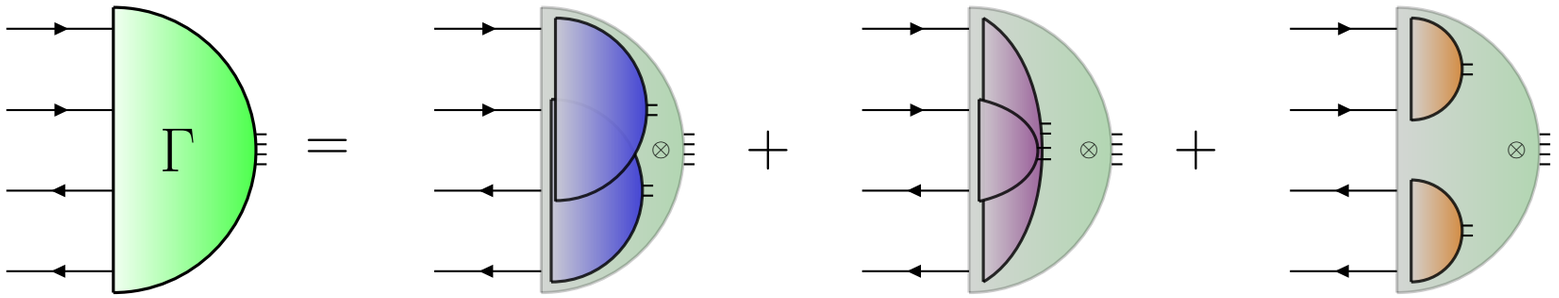
bb, bc and cc four-quark-states



$n \in \{u, d\}$

Hoffer, Eichmann, CF, 2409.05779

Identifying leading structures…

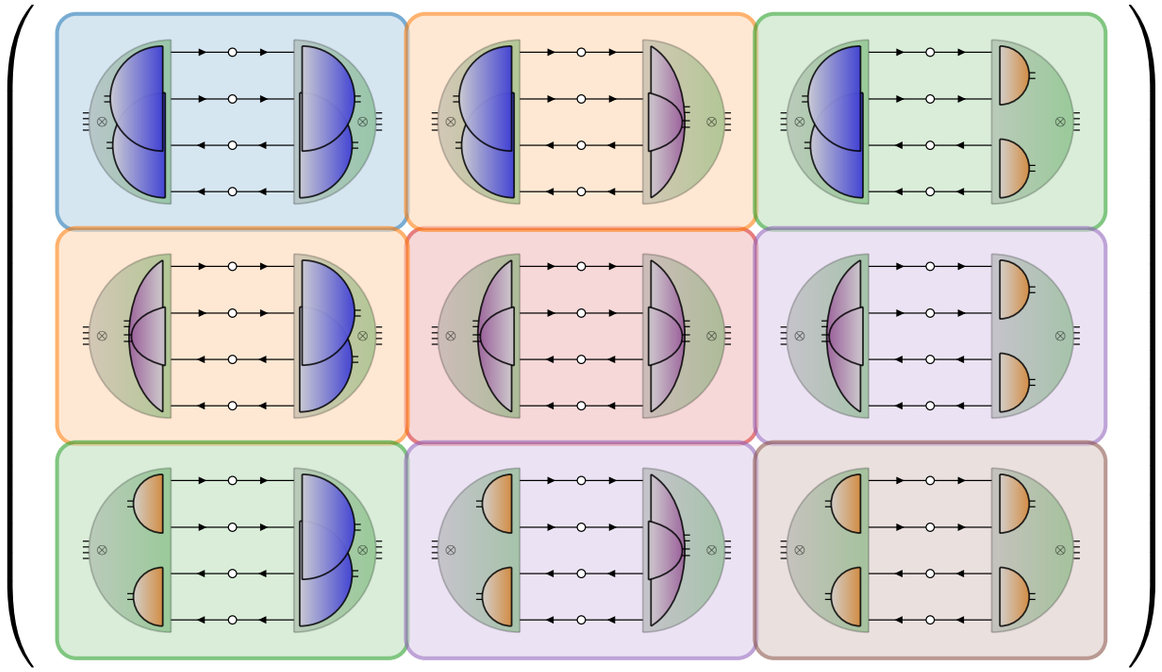


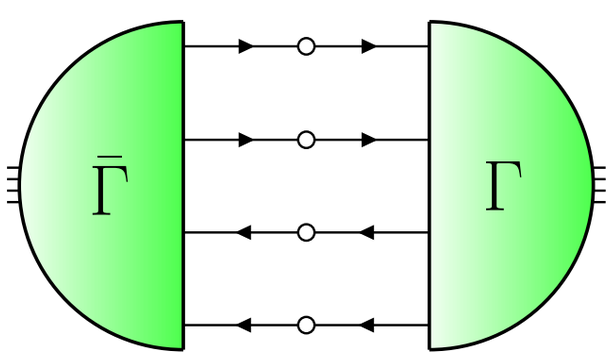
meson-meson

hadro-onium

diquark-antidiquark

- norm contributions

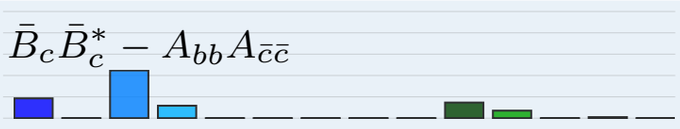




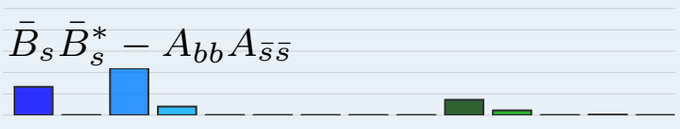
Internal structure

$n \in \{u, d\}$

$bb\bar{c}\bar{c}$

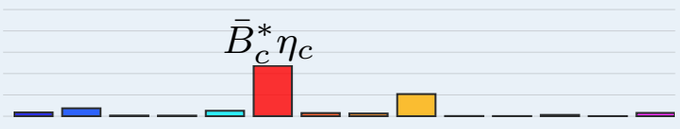


$bb\bar{s}\bar{s}$

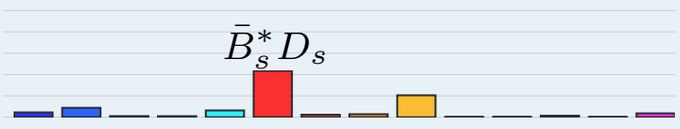


$bb\bar{n}\bar{n}$

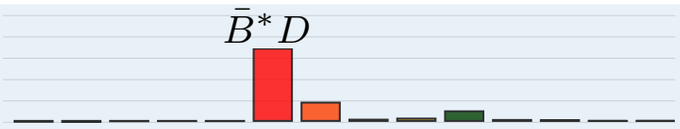
$bc\bar{c}\bar{c}$



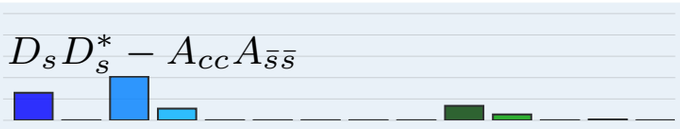
$bc\bar{s}\bar{s}$



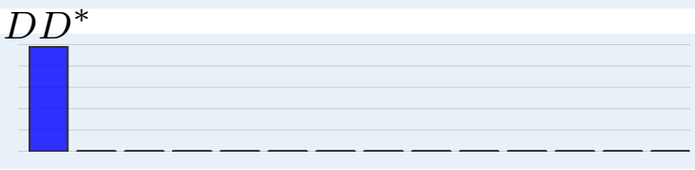
$bc\bar{n}\bar{n}$



$cc\bar{s}\bar{s}$



$cc\bar{n}\bar{n}$



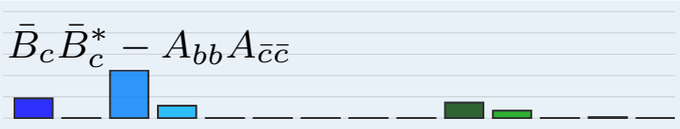
$0(1^+)$ $1(1^+)$

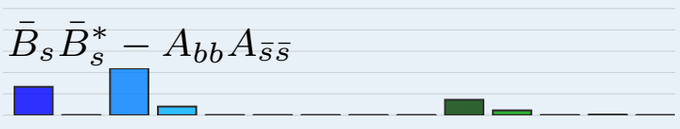
- decided dynamically !
- flavour and spin dependent…prediction for bc

Hoffer, Eichmann, CF, 2409.05779

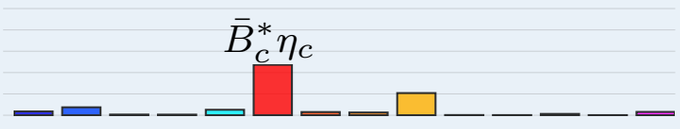
Internal structure

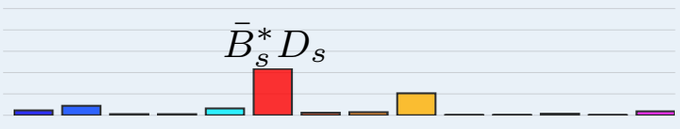
$n \in \{u, d\}$

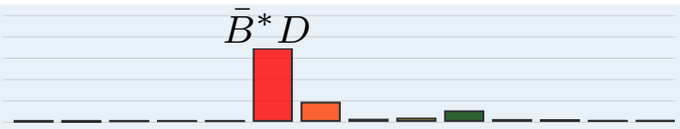
$bb\bar{c}\bar{c}$ 

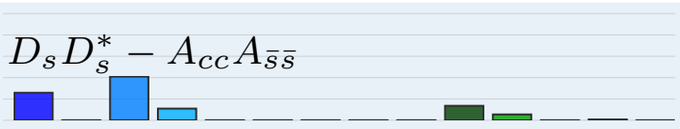
$bb\bar{s}\bar{s}$ 

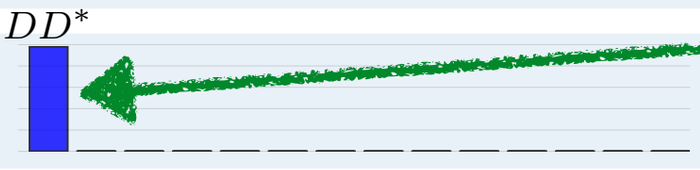
$bb\bar{n}\bar{n}$ 

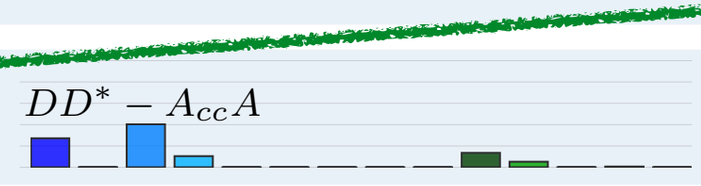
$bc\bar{c}\bar{c}$ 

$bc\bar{s}\bar{s}$ 

$bc\bar{n}\bar{n}$ 

$cc\bar{s}\bar{s}$ 

$cc\bar{n}\bar{n}$ 



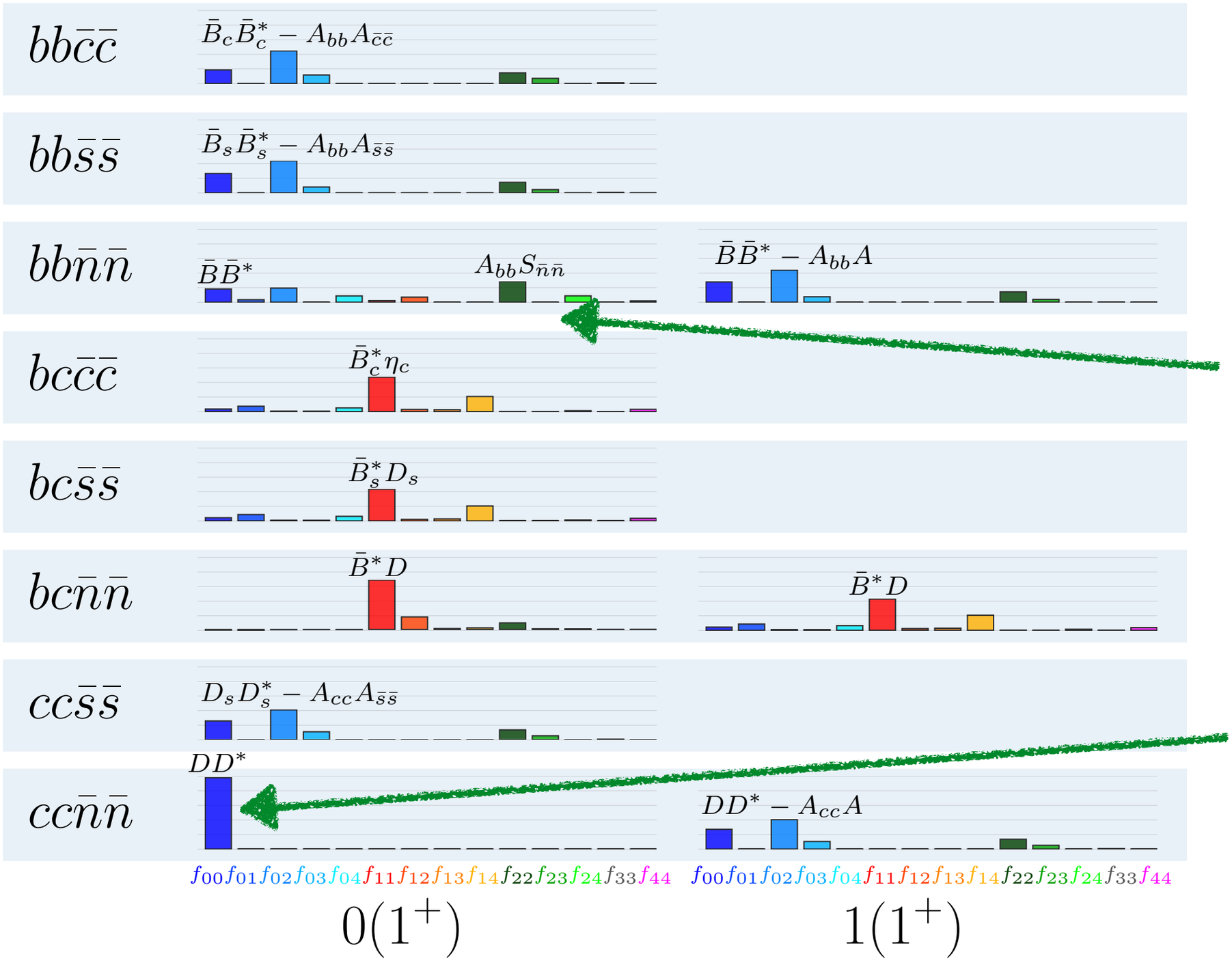
T_{cc}^+

- decided dynamically !
- flavour and spin dependent…prediction for bc

Hoffer, Eichmann, CF, 2409.05779

Internal structure

$n \in \{u, d\}$



also seen on the lattice

Bicudo et al, PRD D103 (2021)

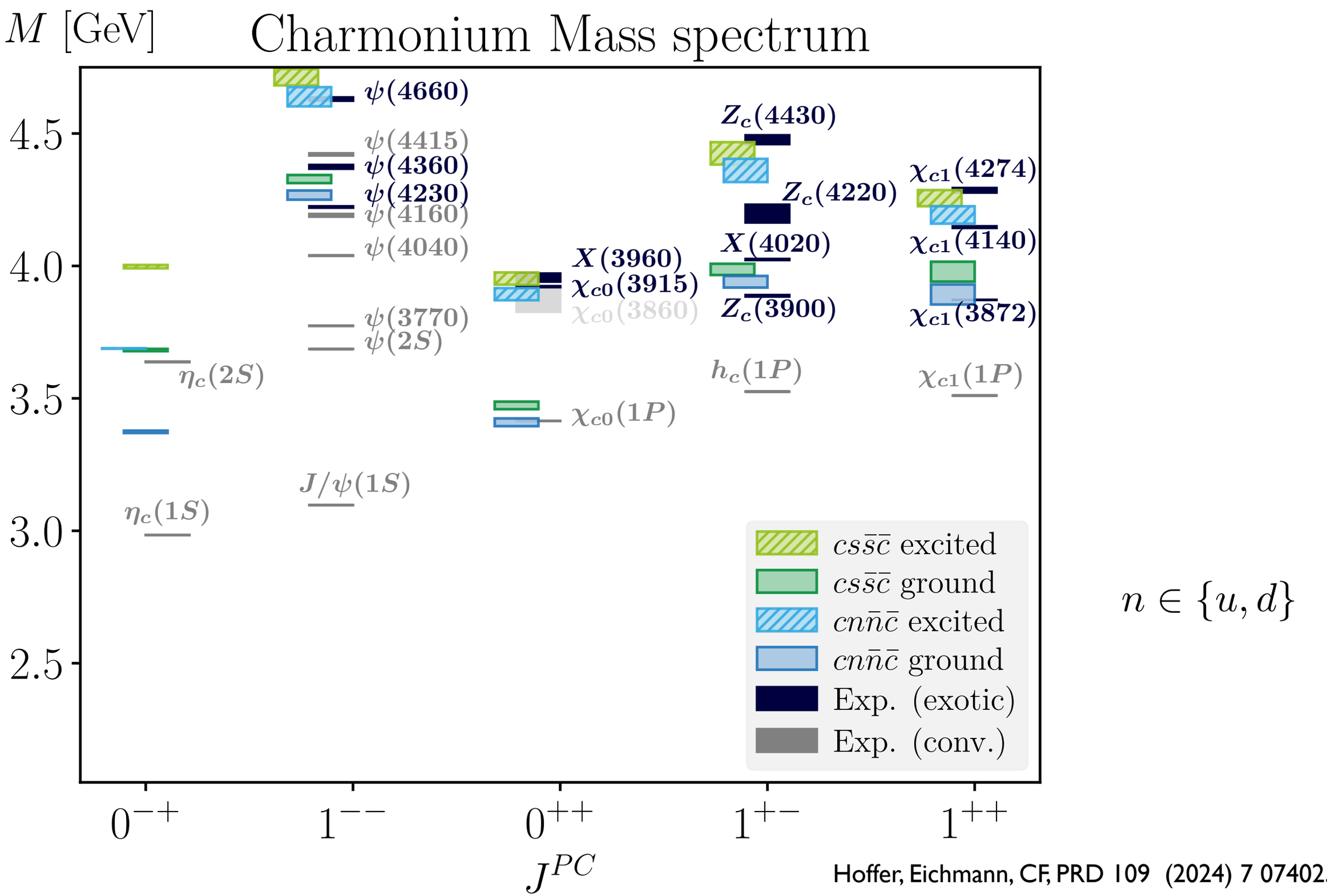
Review:
Francis, accepted for PPNP

T_{cc}^+

- decided dynamically !
- flavour and spin dependent…prediction for bc

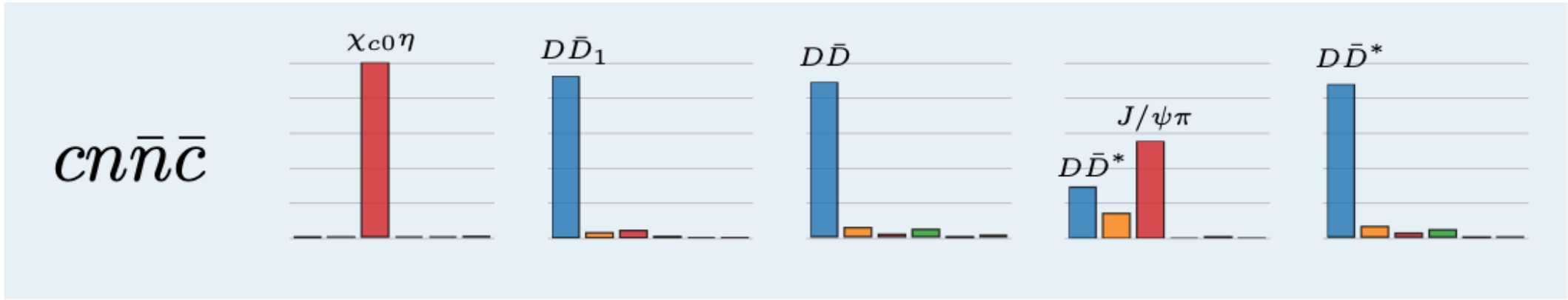
Hoffer, Eichmann, CF, 2409.05779

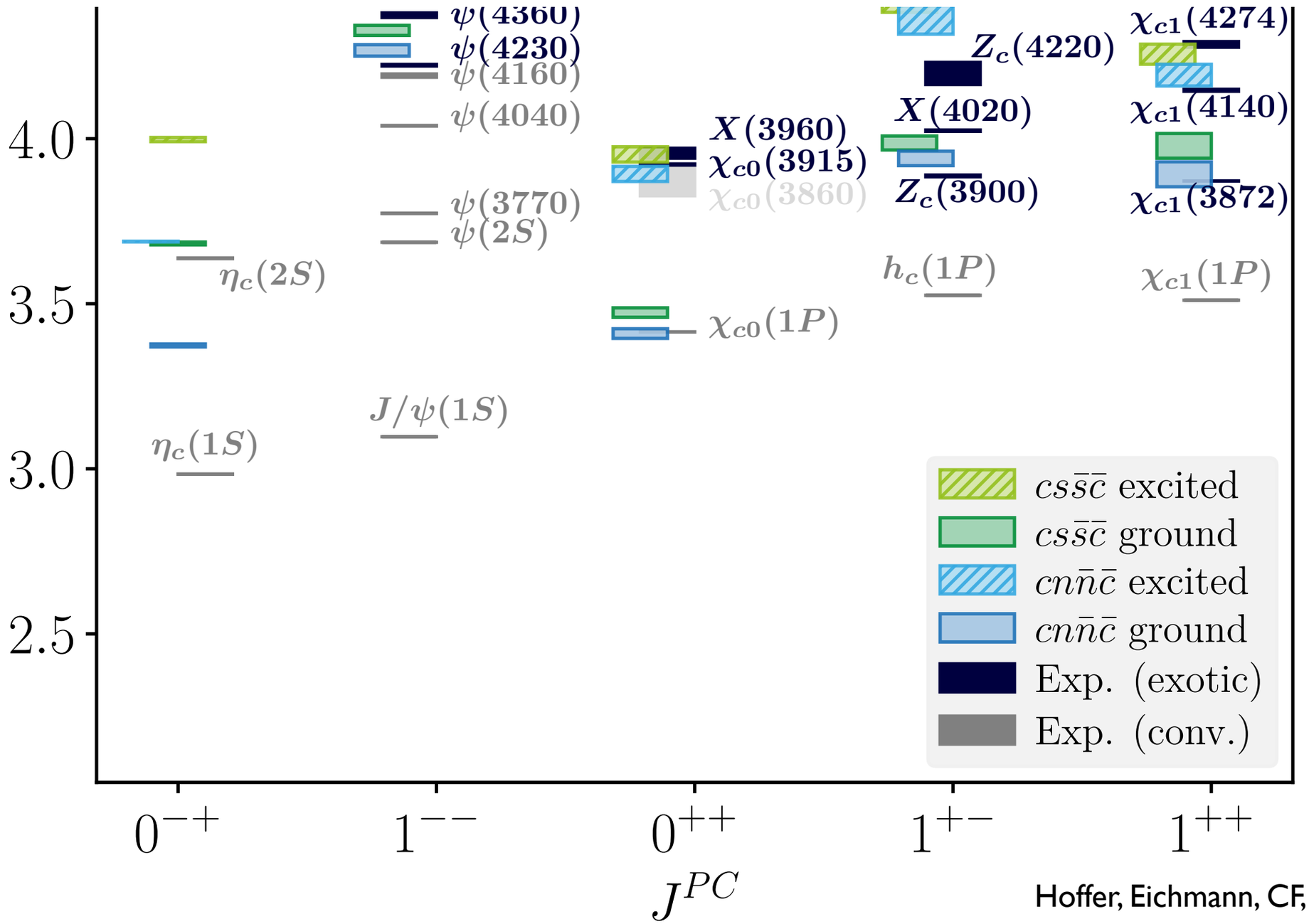
Hidden flavour four-quark states



Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025

- no repulsive color channels included yet…





$n \in \{u, d\}$

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025

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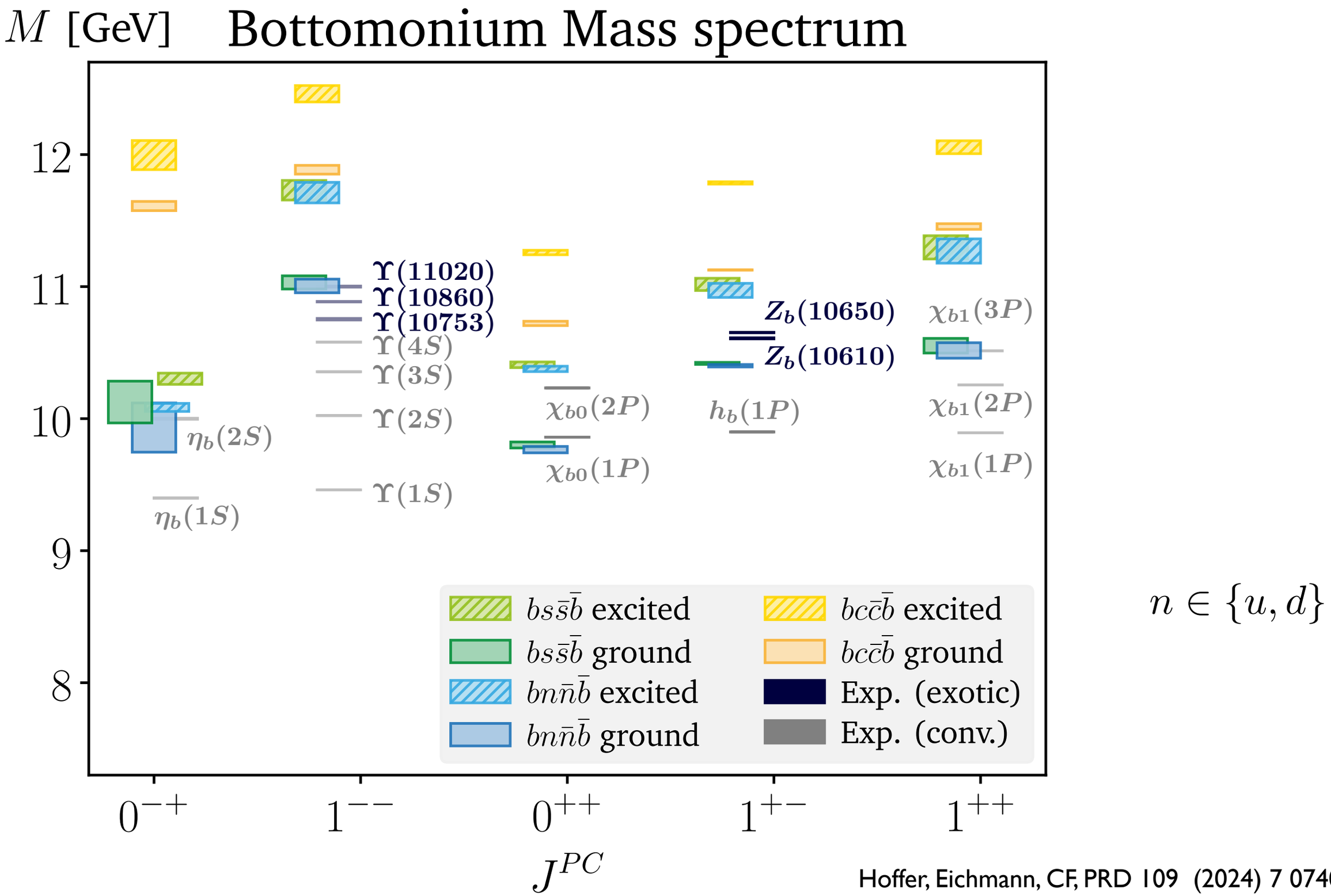
Hidden flavour four-quark states



$n \in \{u, d\}$

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025

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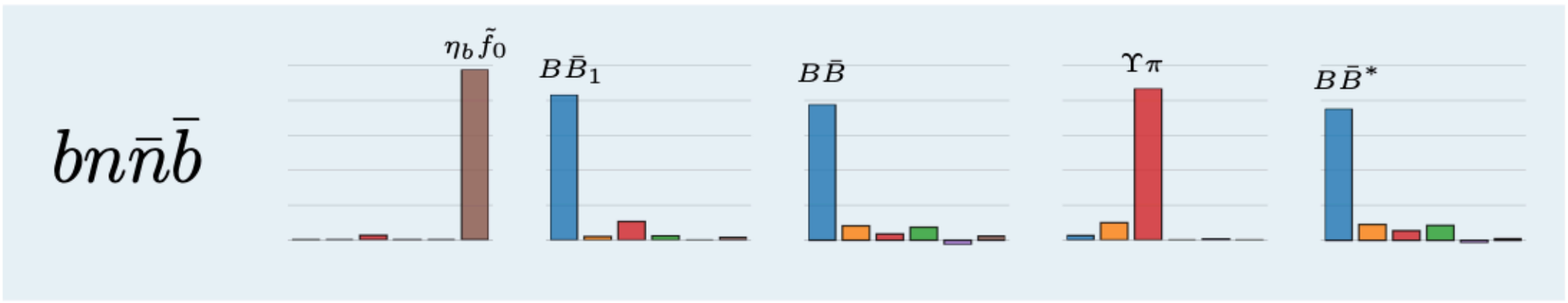
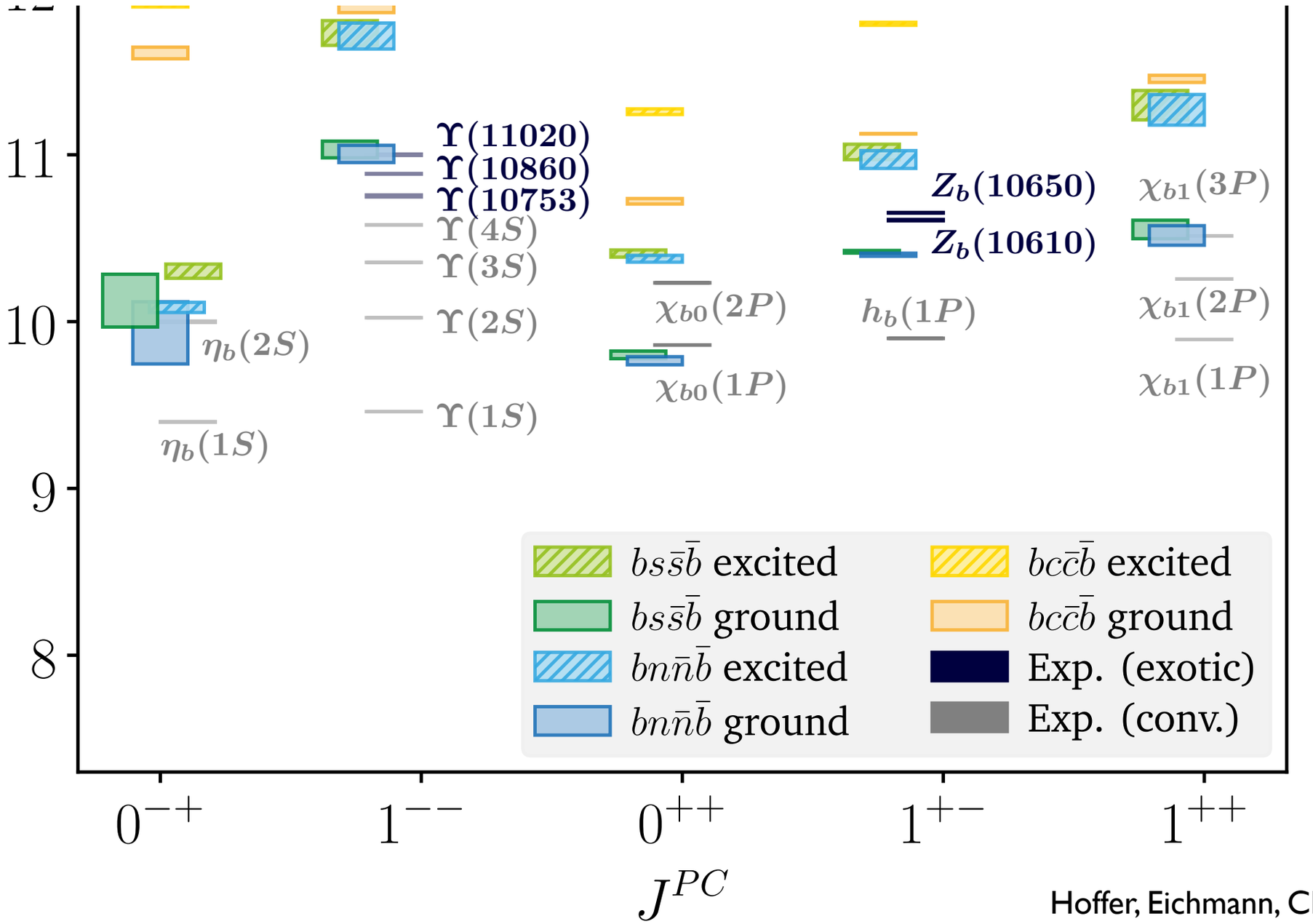




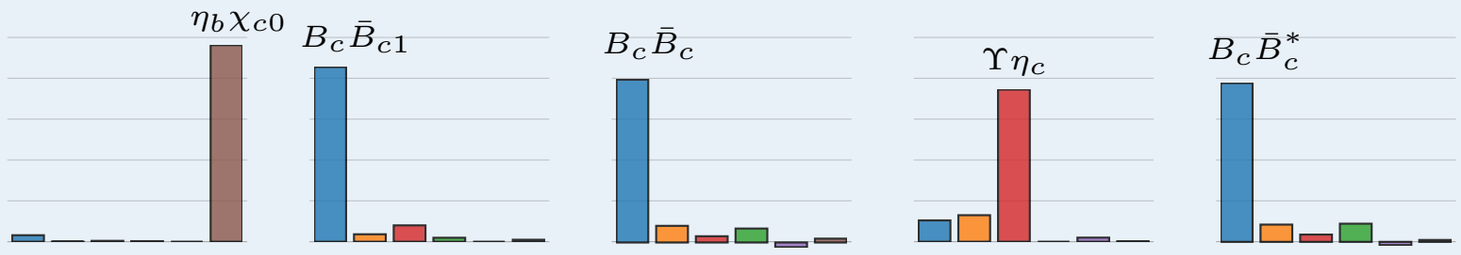
$n \in \{u, d\}$

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025

- no repulsive color channels included yet…

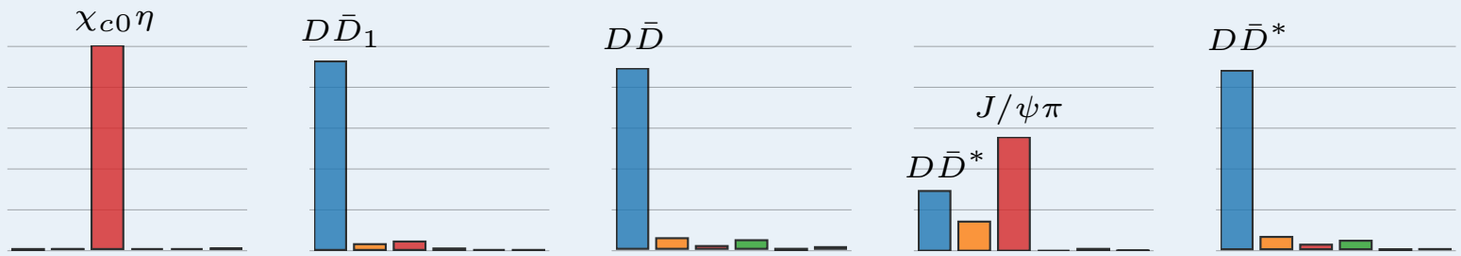
Internal structure

$bc\bar{c}\bar{b}$



$bn\bar{n}\bar{b}$

$cn\bar{n}\bar{c}$



$n \in \{u, d\}$

0^{-+} 1^{--} 0^{++} 1^{+-} 1^{++}

- decided dynamically !
- flavour and spin dependent…

Hoffer, Eichmann, CF, PRD 109 (2024) 7 074025

Summary

Internal dynamics very important !!

Glueballs:

- First quantitatively reliable results using very involved truncation

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

Hidden flavour four-quark states:

- Dynamical description of σ: π-π resonance

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Mixing with $q\bar{q}$ studied for light mesons

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014.

- Results for hidden charm and bottom (without repulsive channels)

Wallbott, Eichmann and CF, PRD100 (2019) 014033
Wallbott, Eichmann and CF, PRD102 (2020) 051501
Hoffer, Eichmann, CF, PRD 109 (2024) 074025

Open flavour four-quark states:

- Results for open charm and bottom (attractive and repulsive channels)

Hoffer, Eichmann, CF, 2409.05779

- Internal structure is flavour and spin dependent!
  - meson-meson is dominating/important

Mini-Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4 38, [2008.10240]

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INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS

46th **Course**

QCD under extreme conditions

- from heavy-ion collisions to the phase diagram

Erice-Sicily: September 16-22, 2025

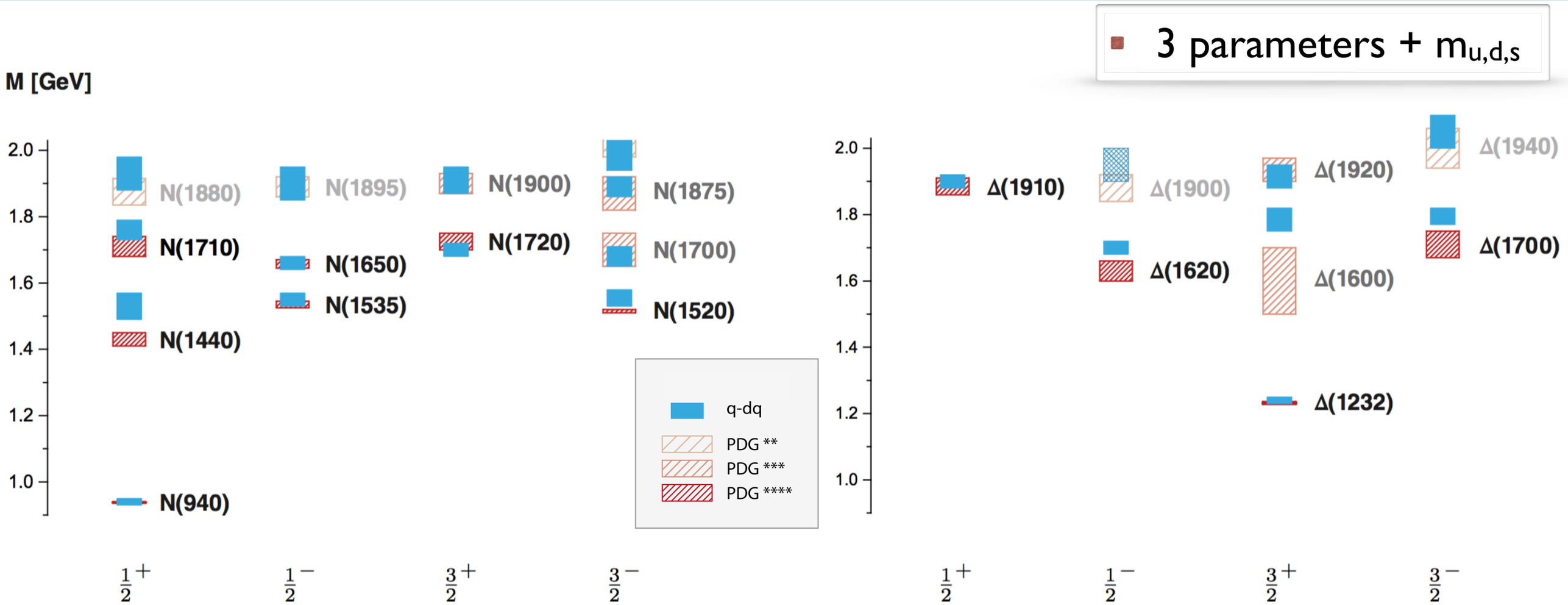
Directors of the school

Christian Fischer

Backup Slides

Light baryon spectrum: diquark-picture

- 3 parameters + $m_{u,d,s}$



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- strange baryons
- heavy baryons

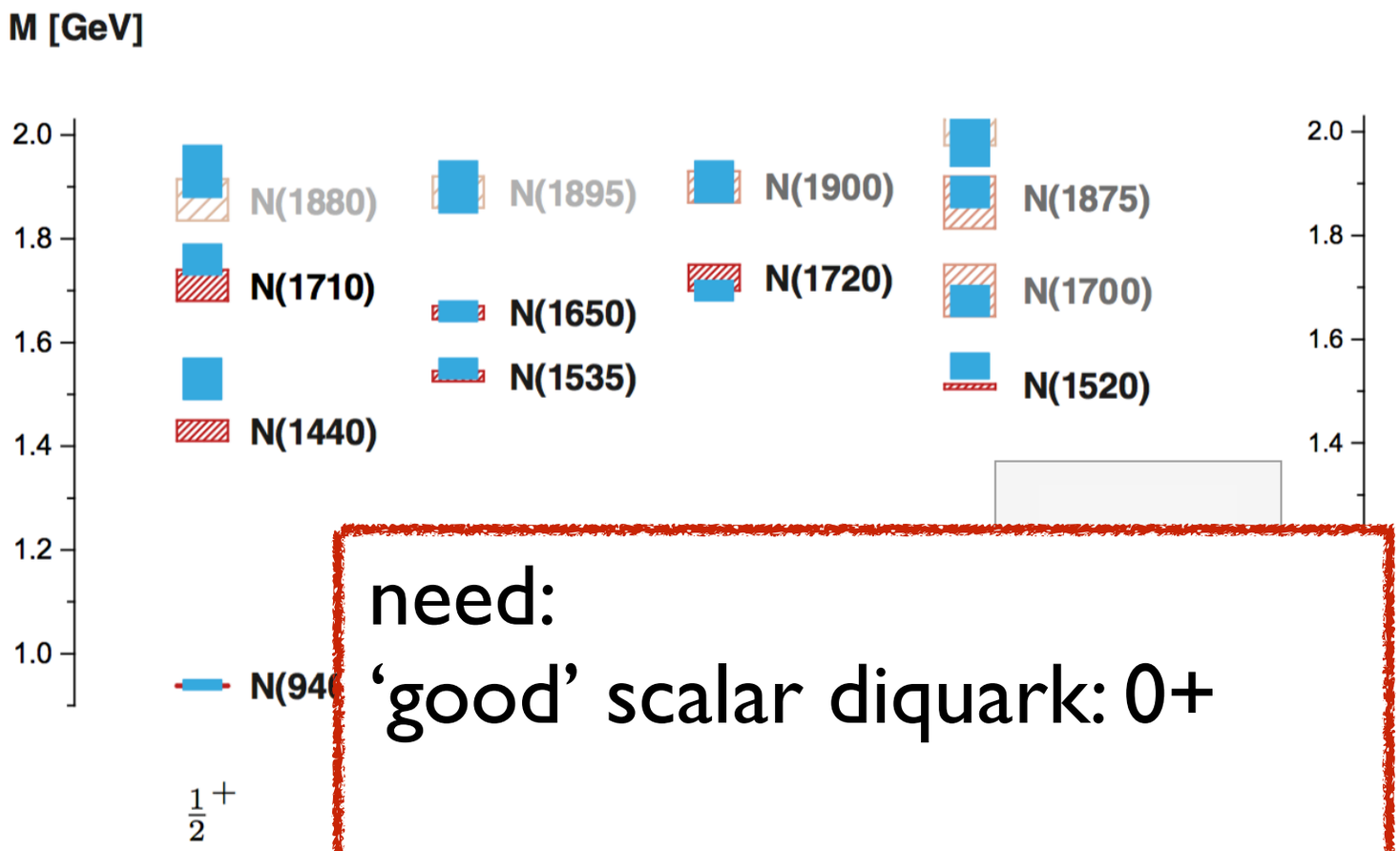
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Qin, Roberts, Schmidt, Few Body Syst. 60 (2019) no.2, 26
Torcato, Arriaga, Eichmann and Pena, FBS 64 (2023) 45

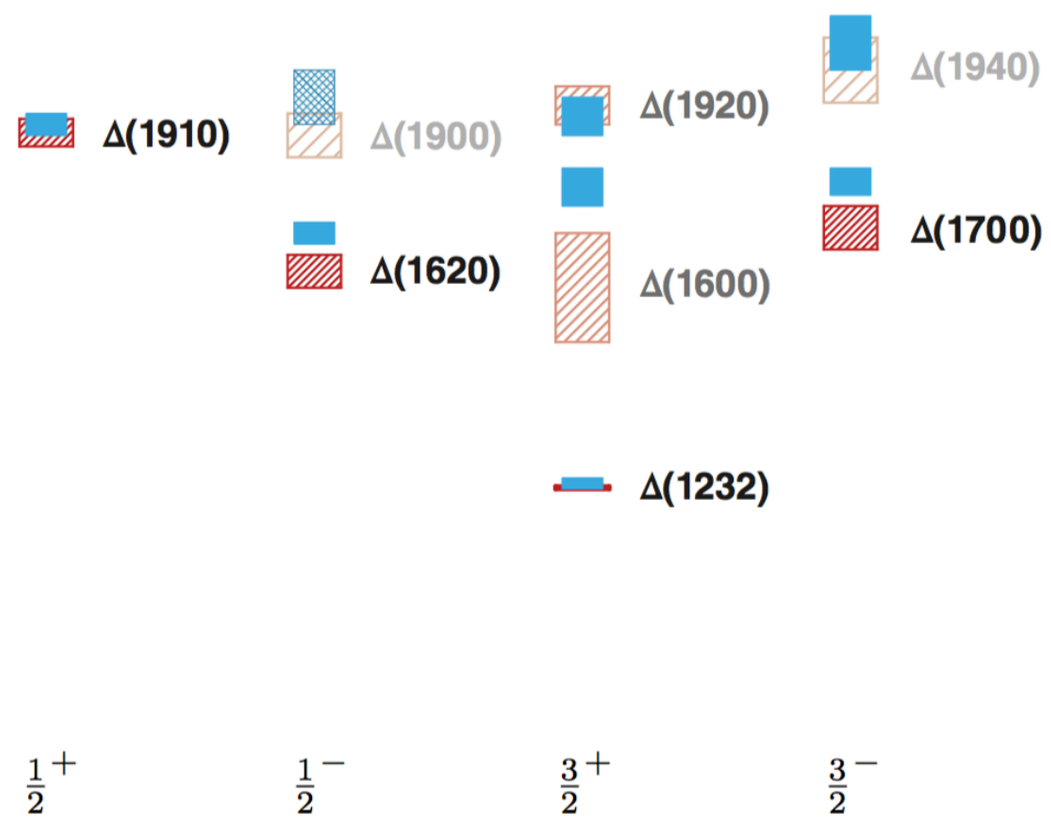
Review on diquarks: Barabanov et al, PPNP 116 (2021), 103835

Light baryon spectrum: diquark-picture

- 3 parameters + $m_{u,d,s}$



need:
‘good’ scalar diquark: 0+



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

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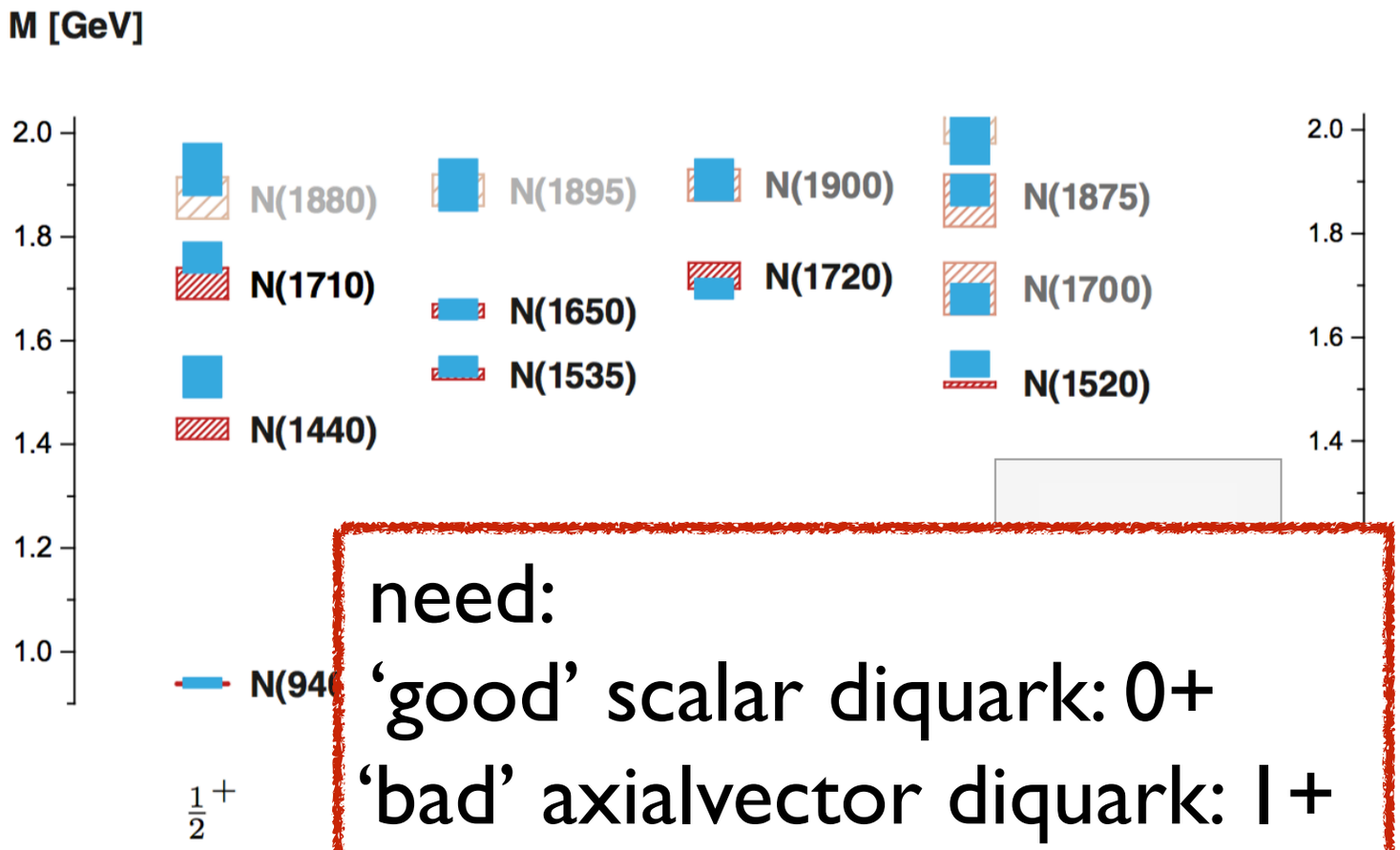

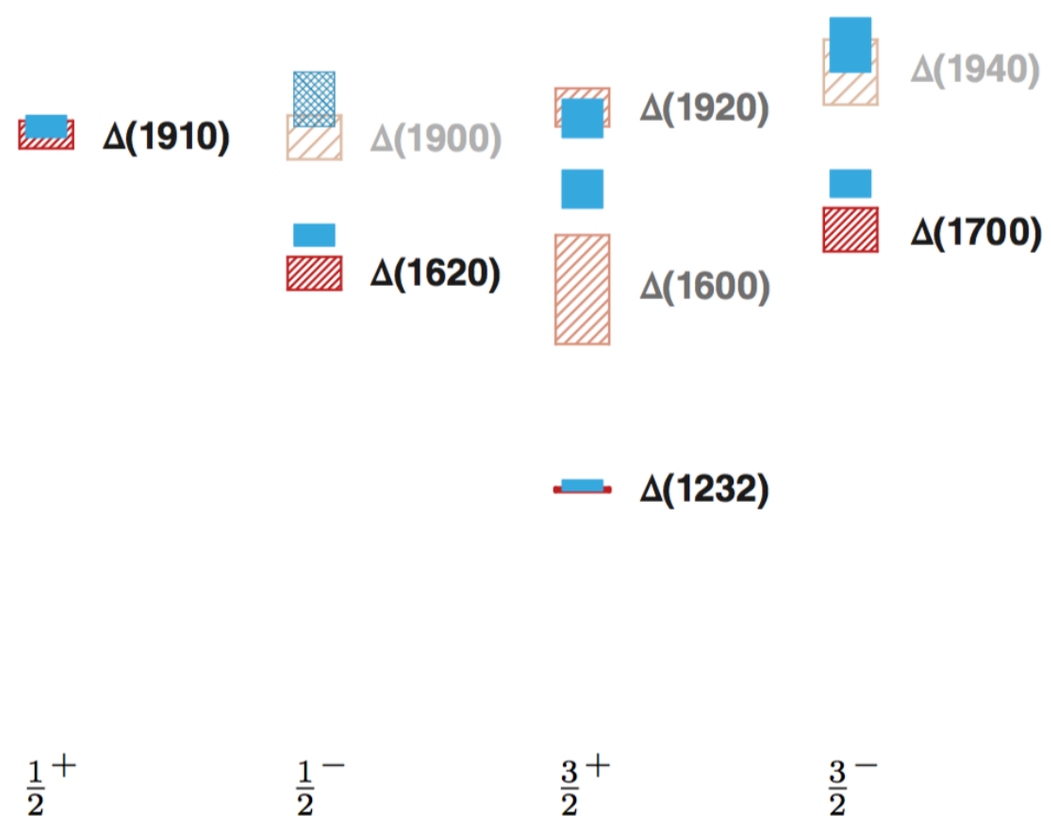

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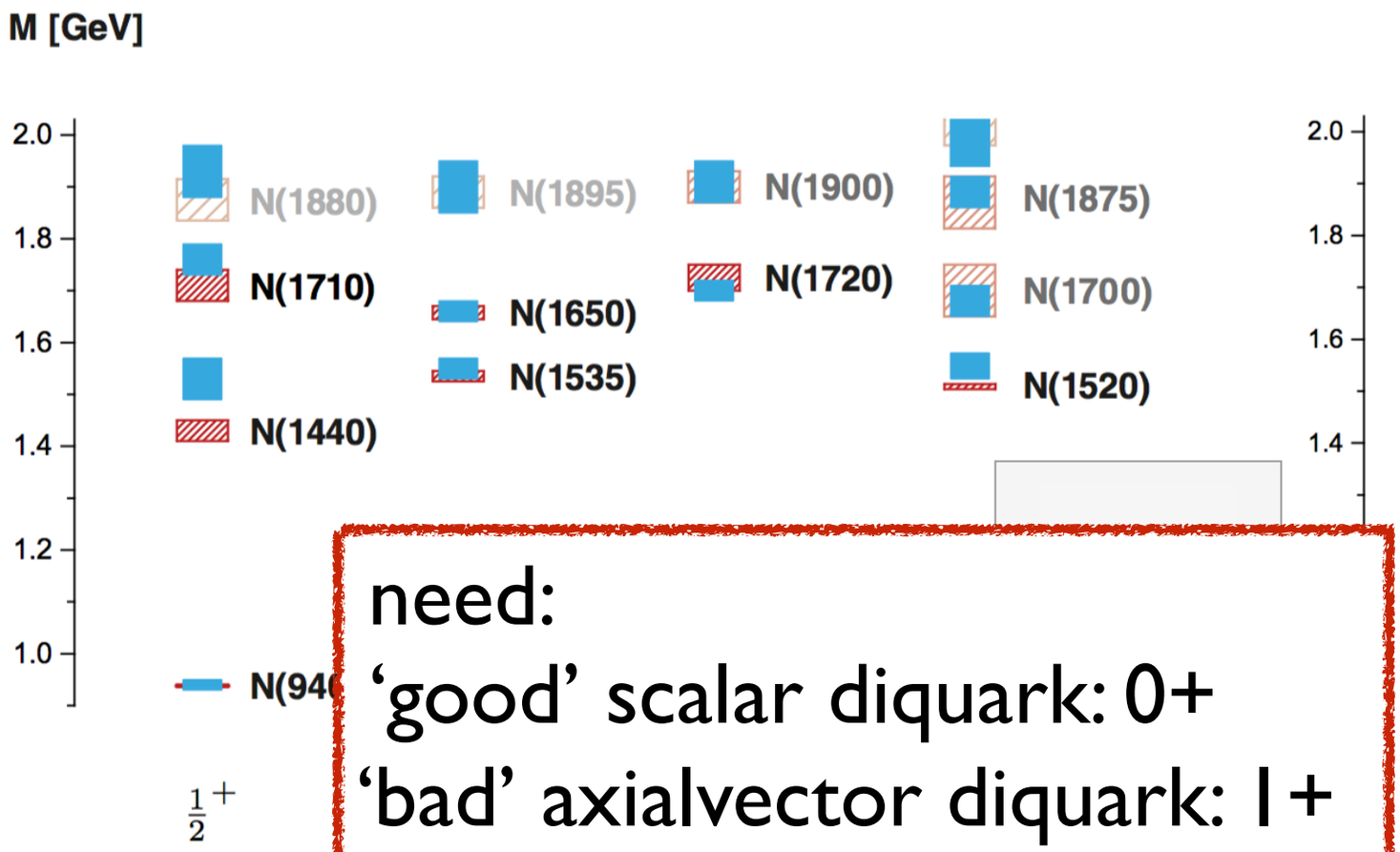

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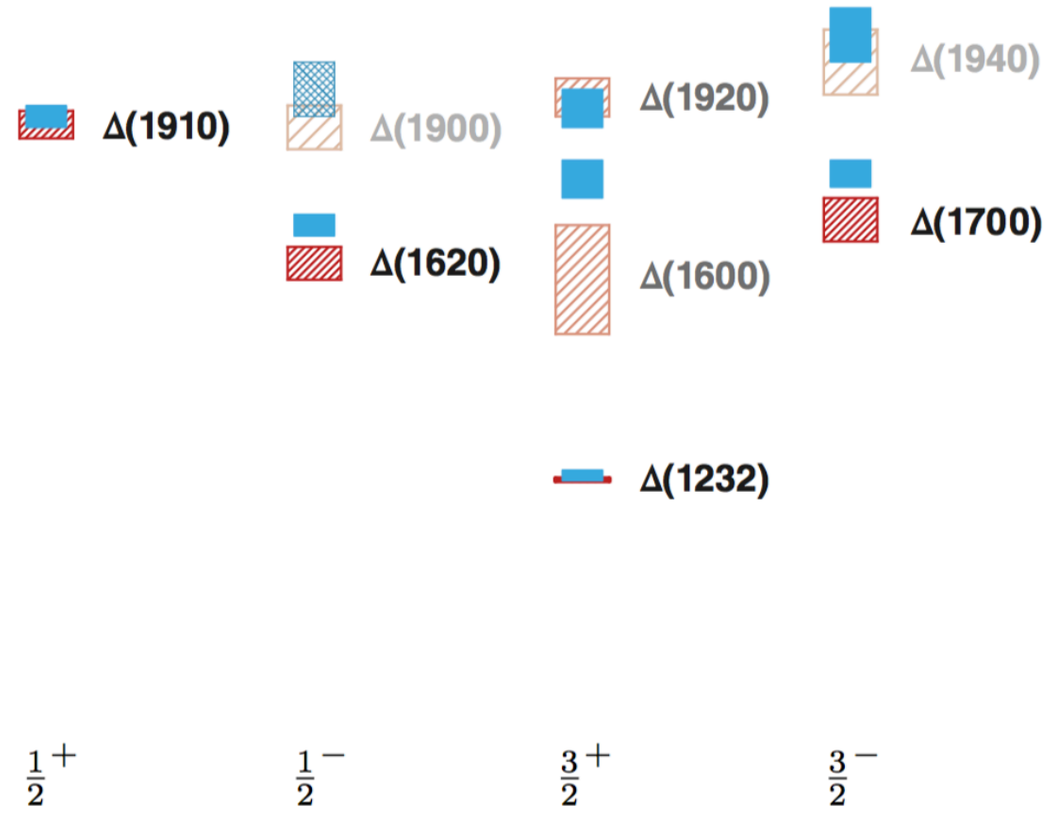
'ugly' ps/v diquark



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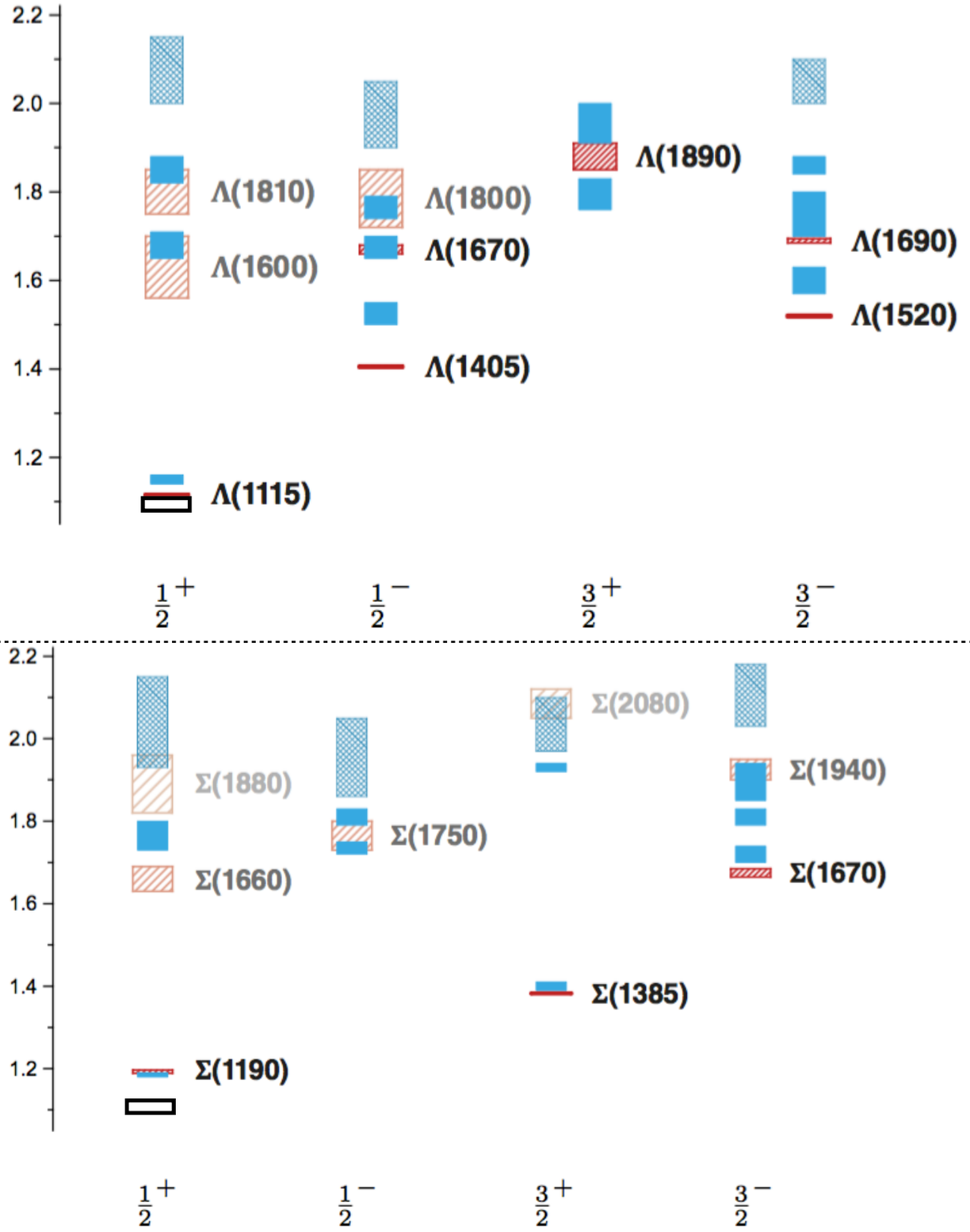
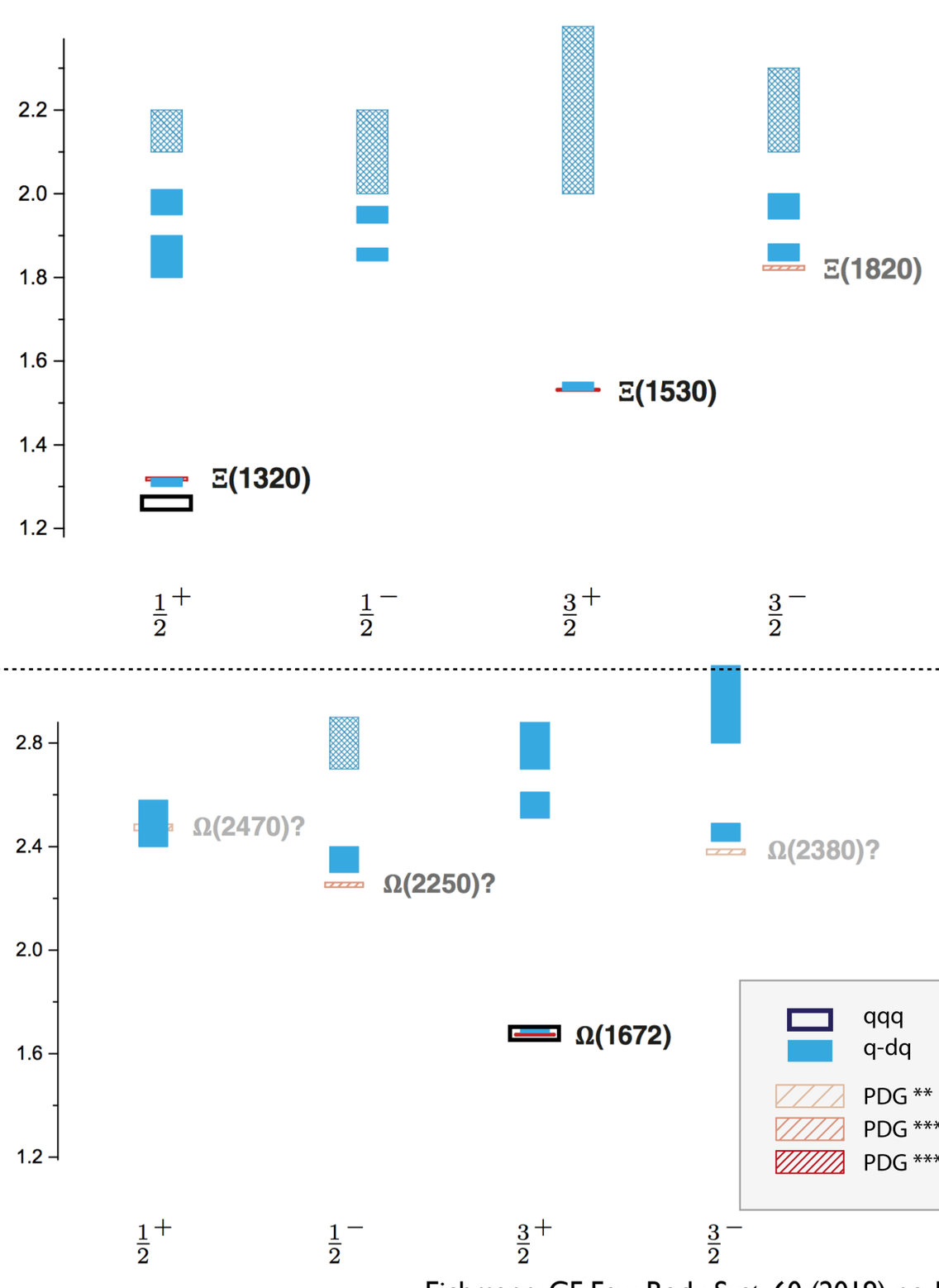
Strange baryon spectrum: DSE-RL (preliminary !)





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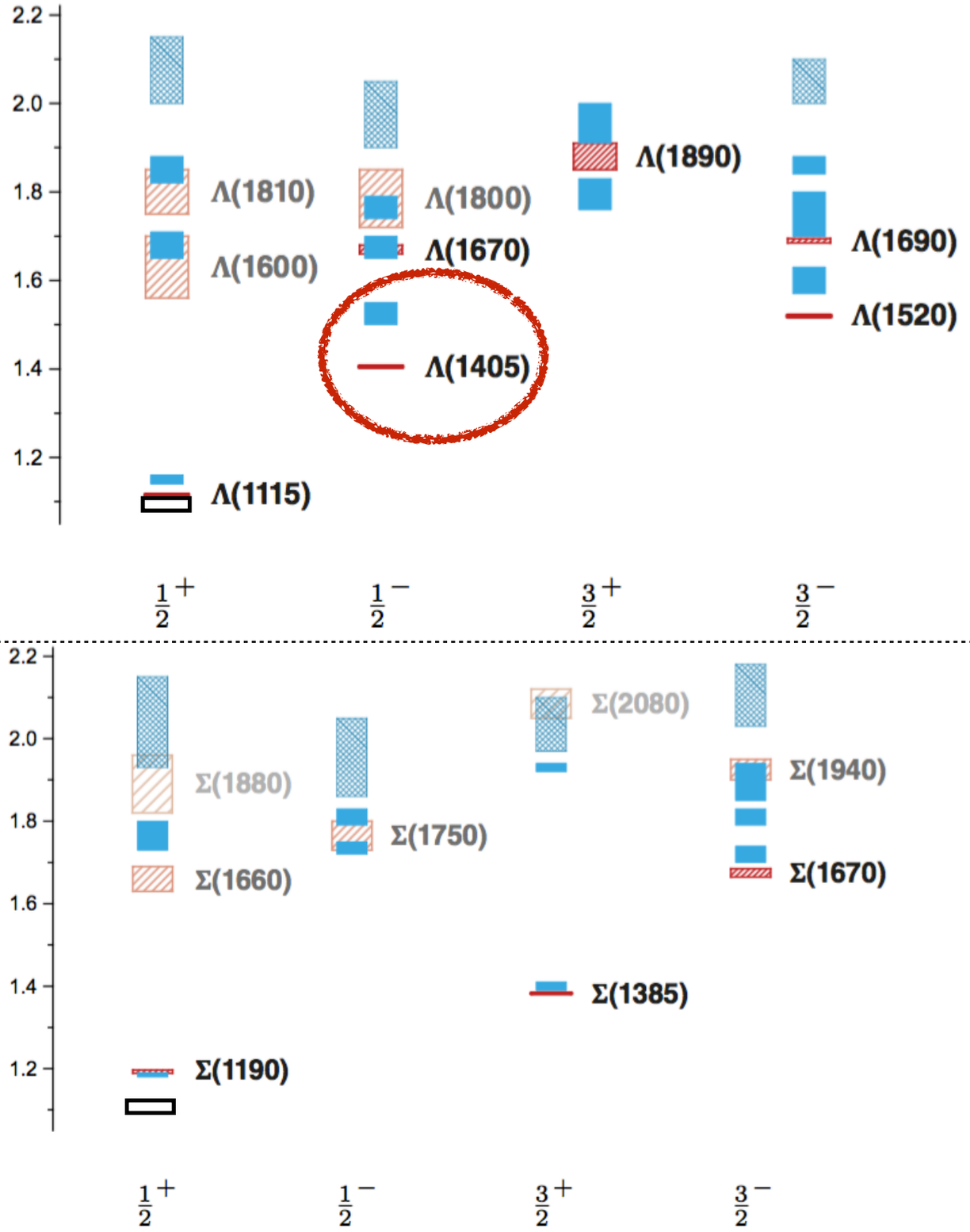

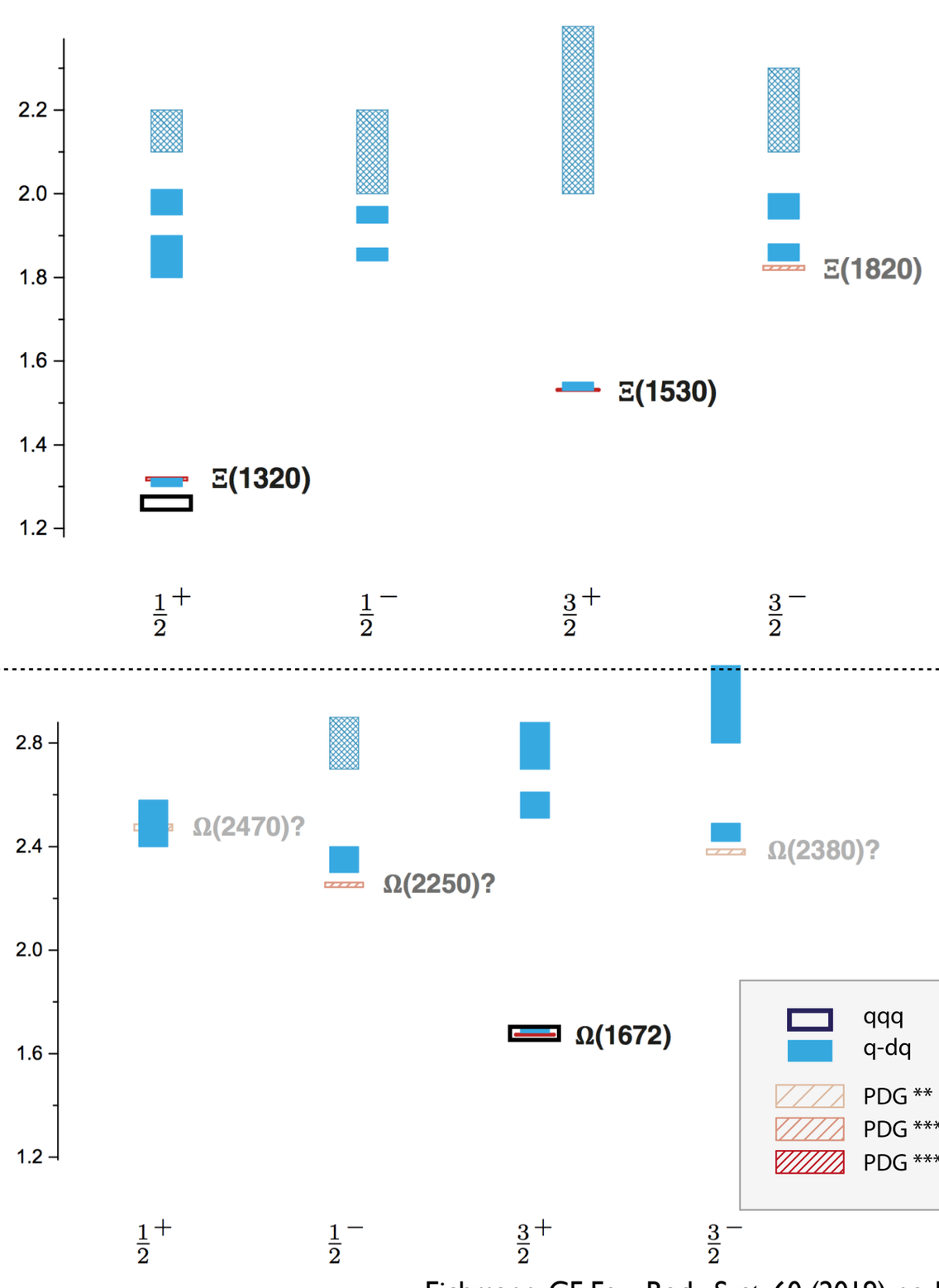

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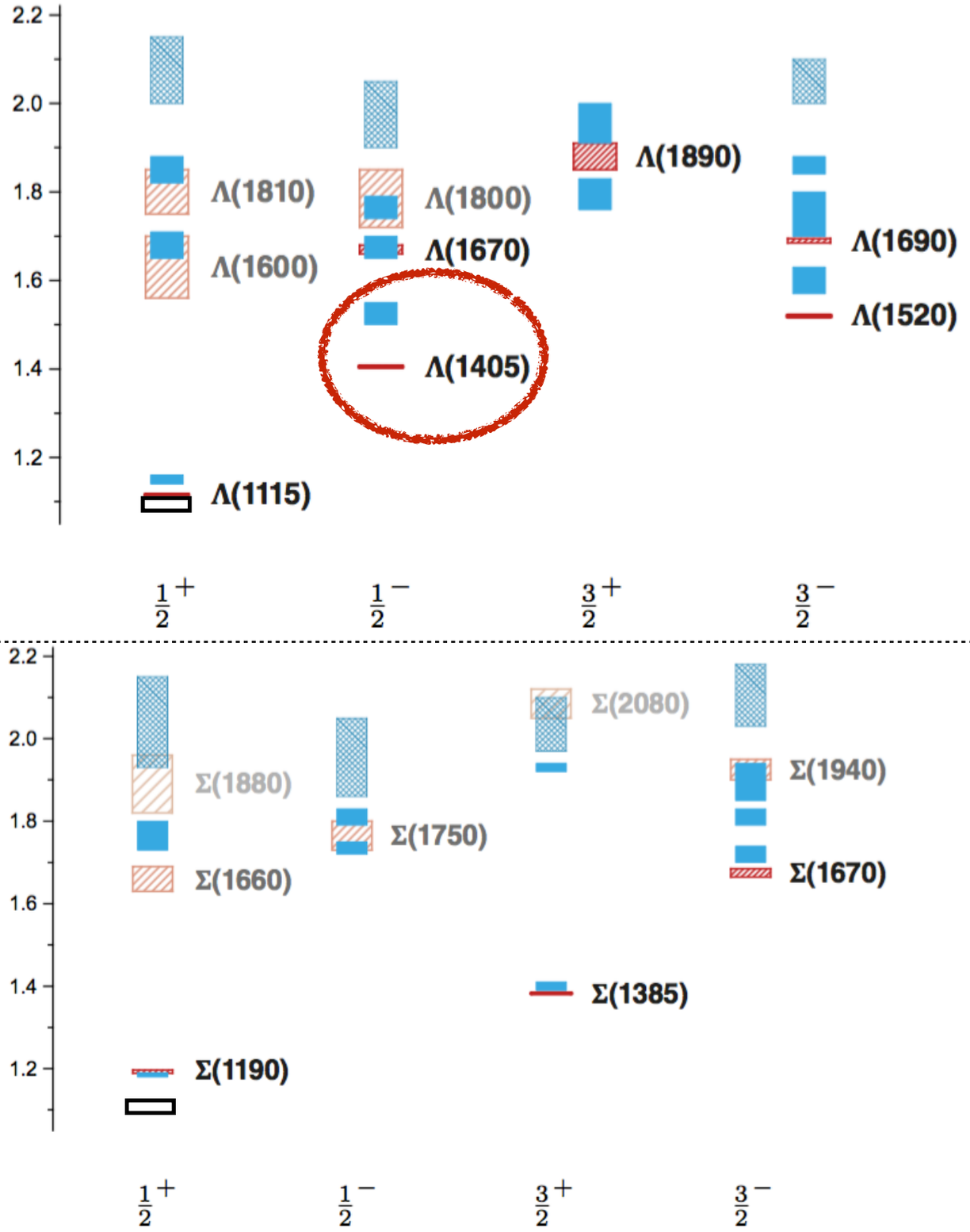

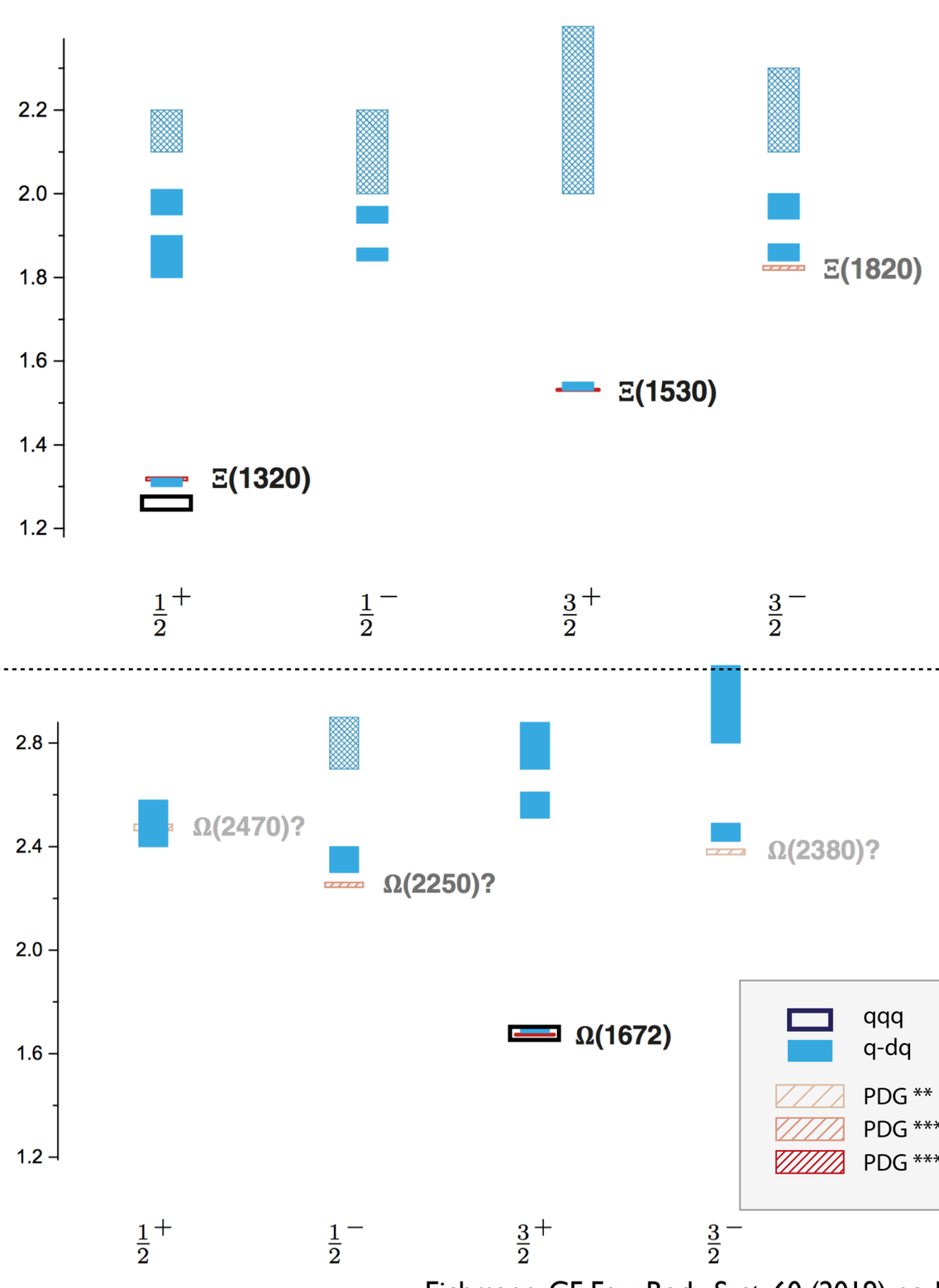

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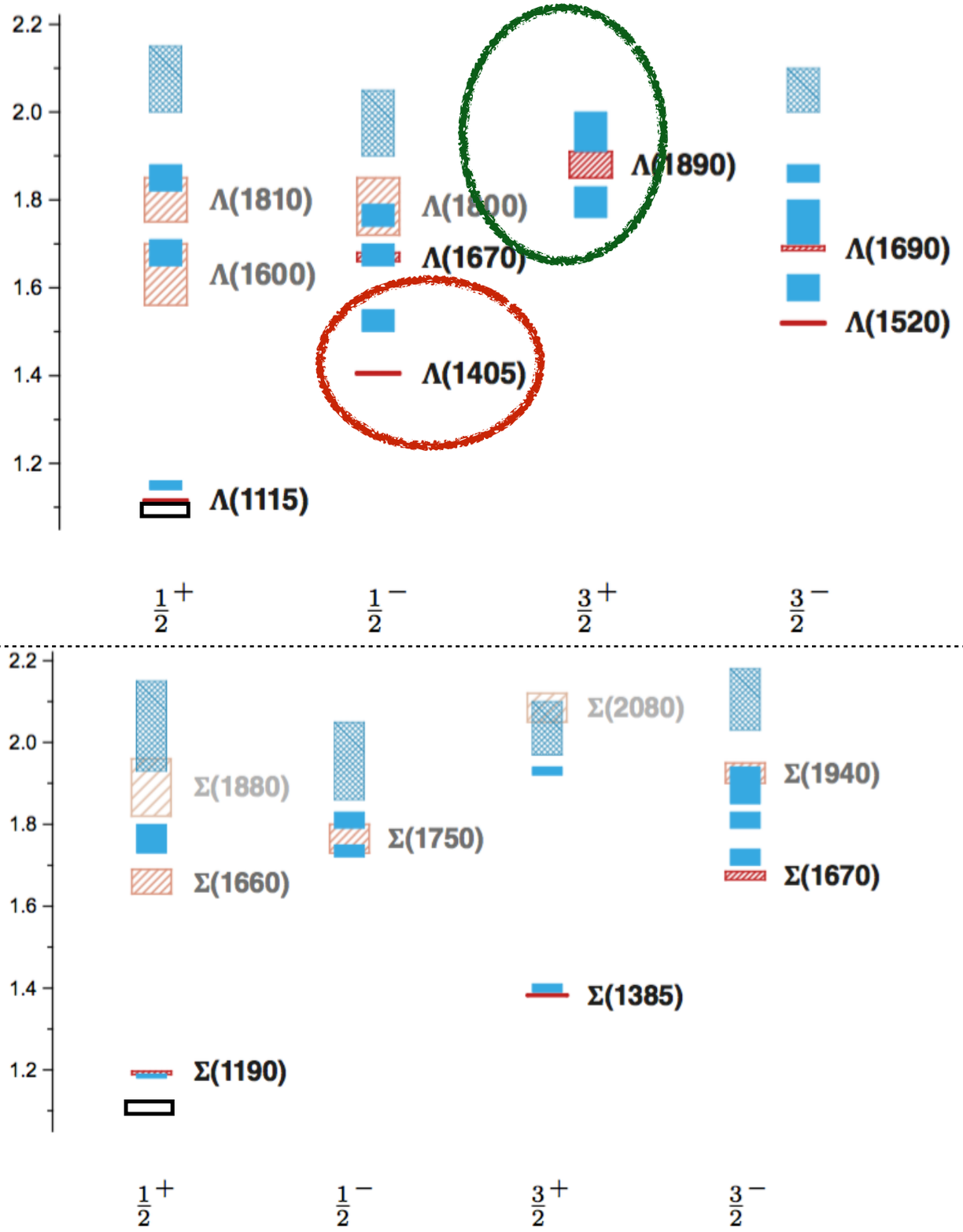

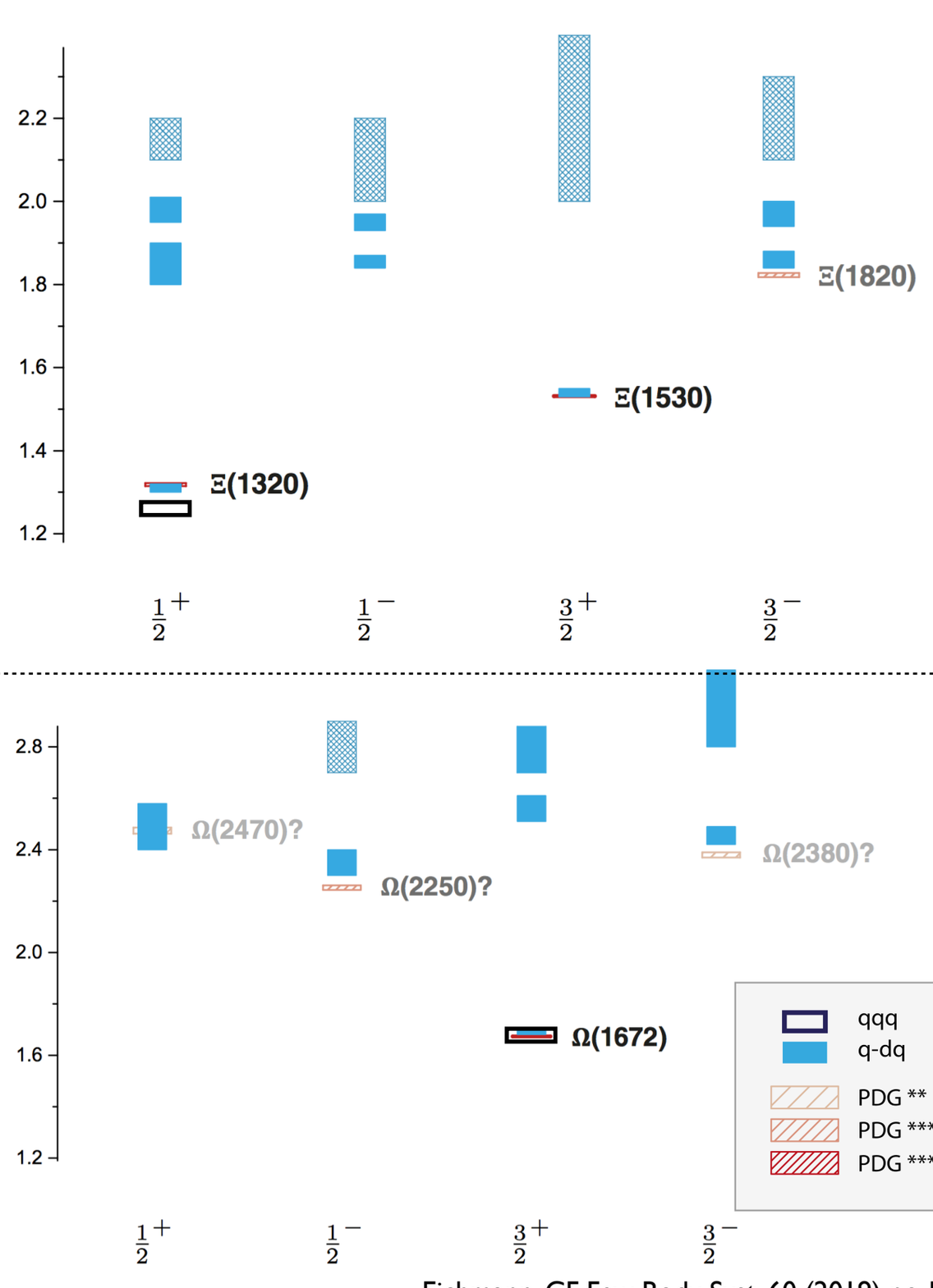

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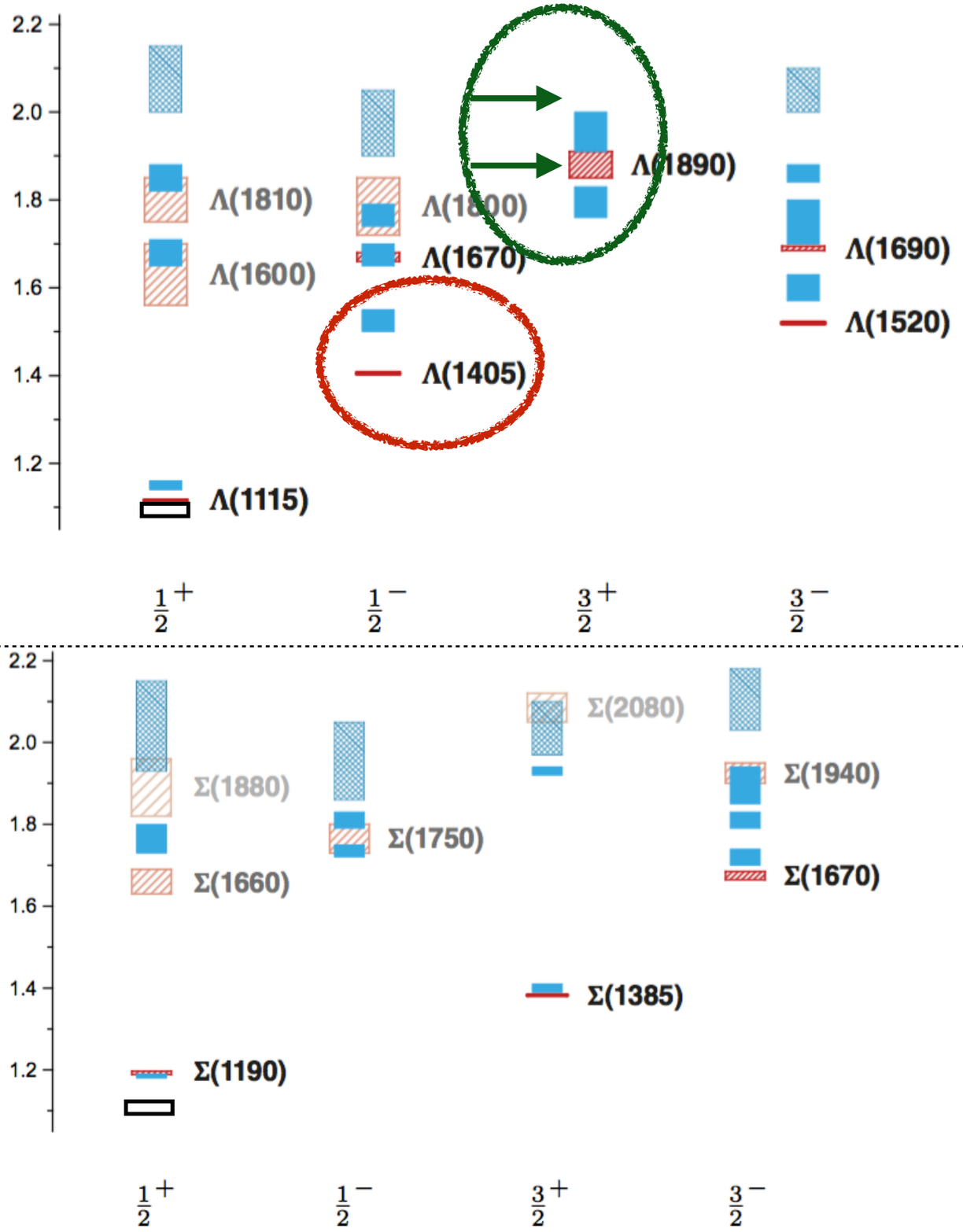

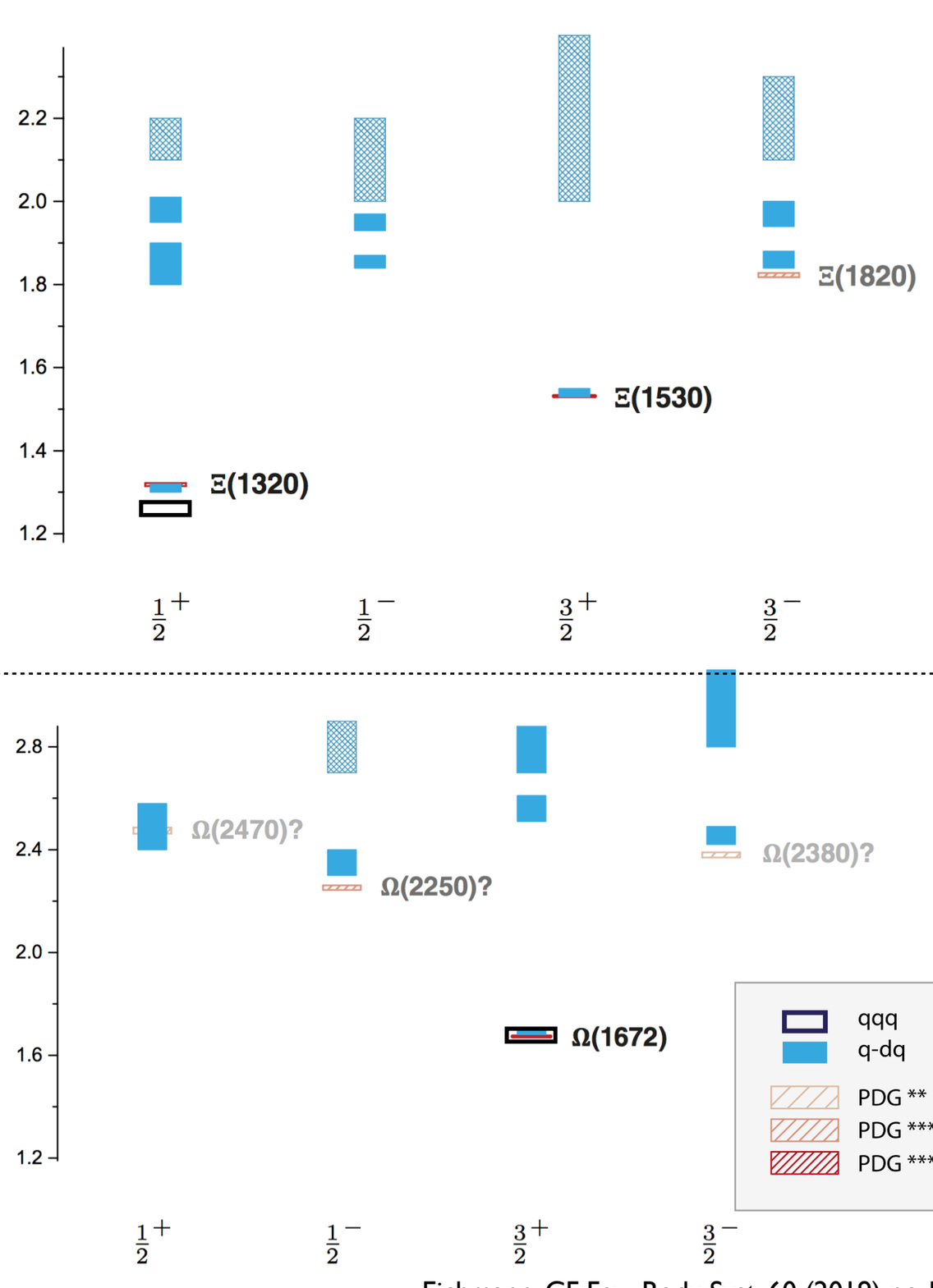

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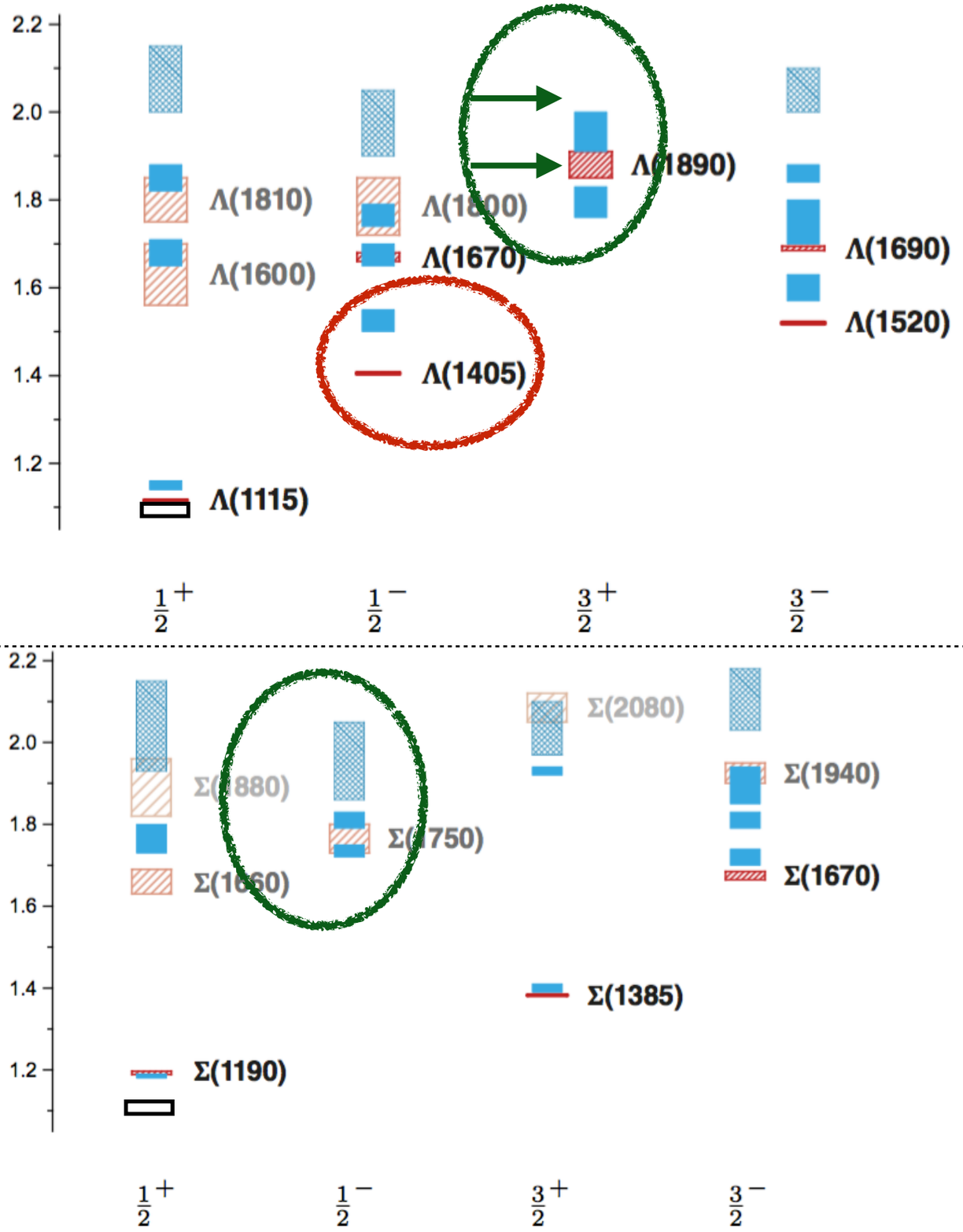

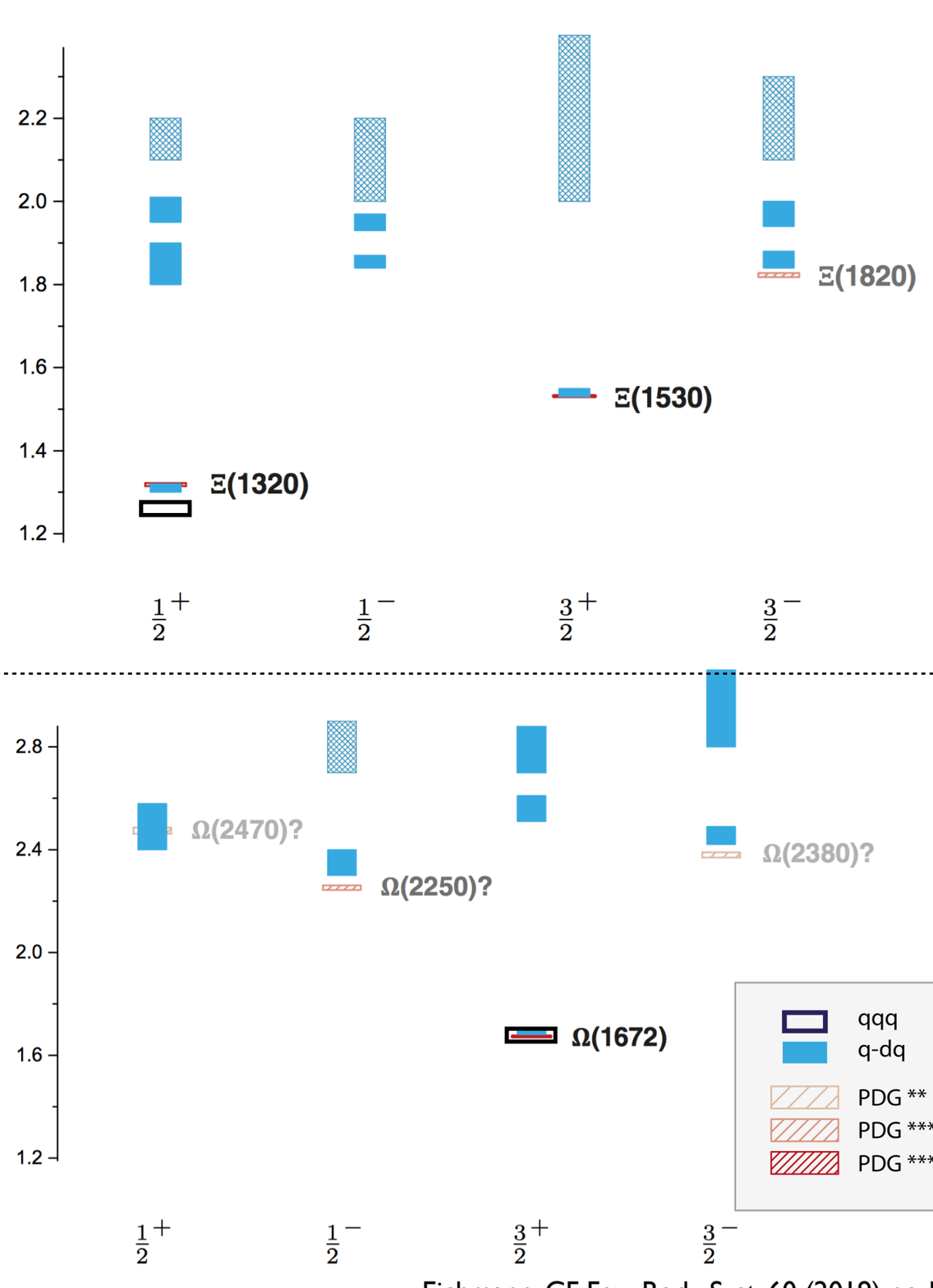

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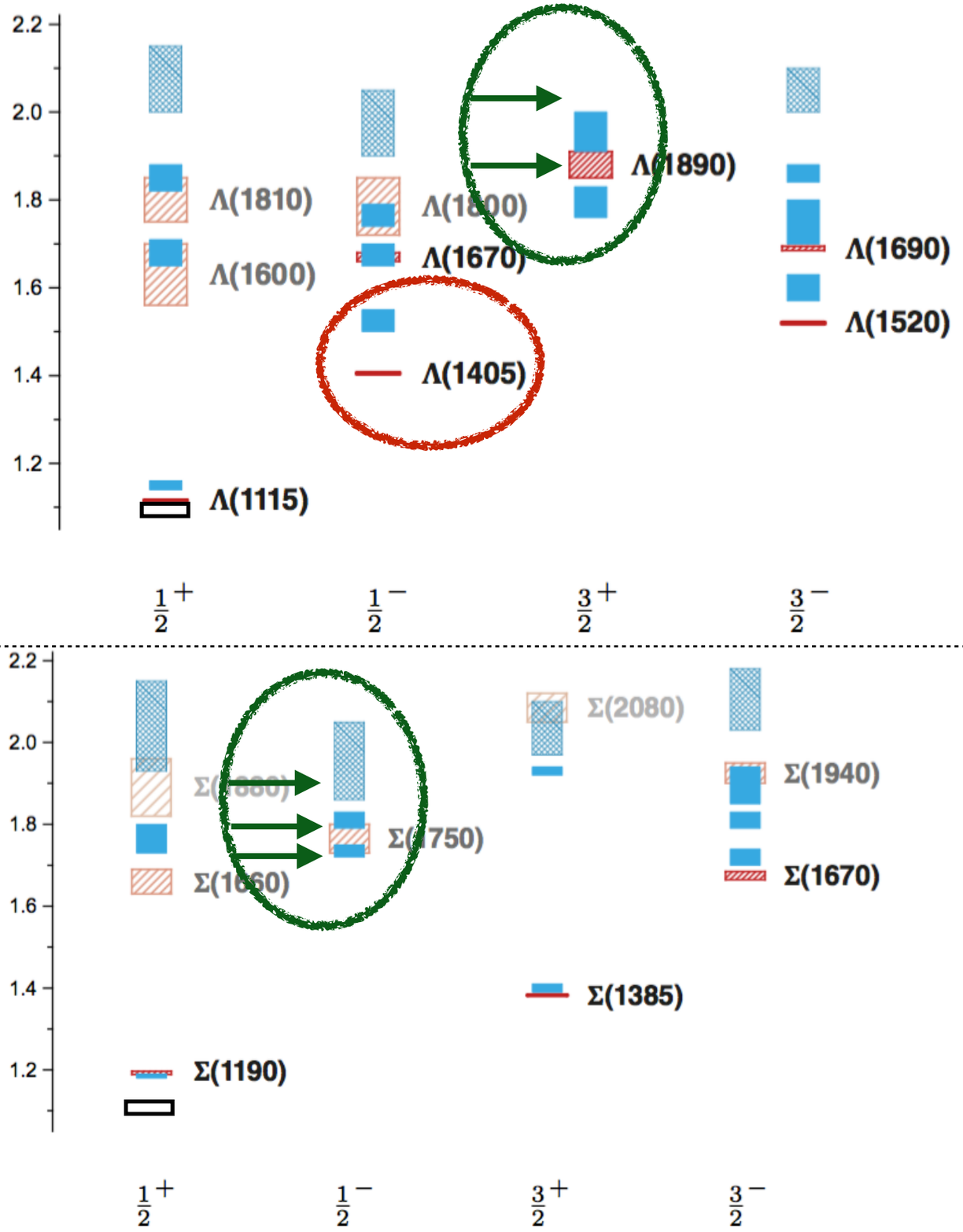

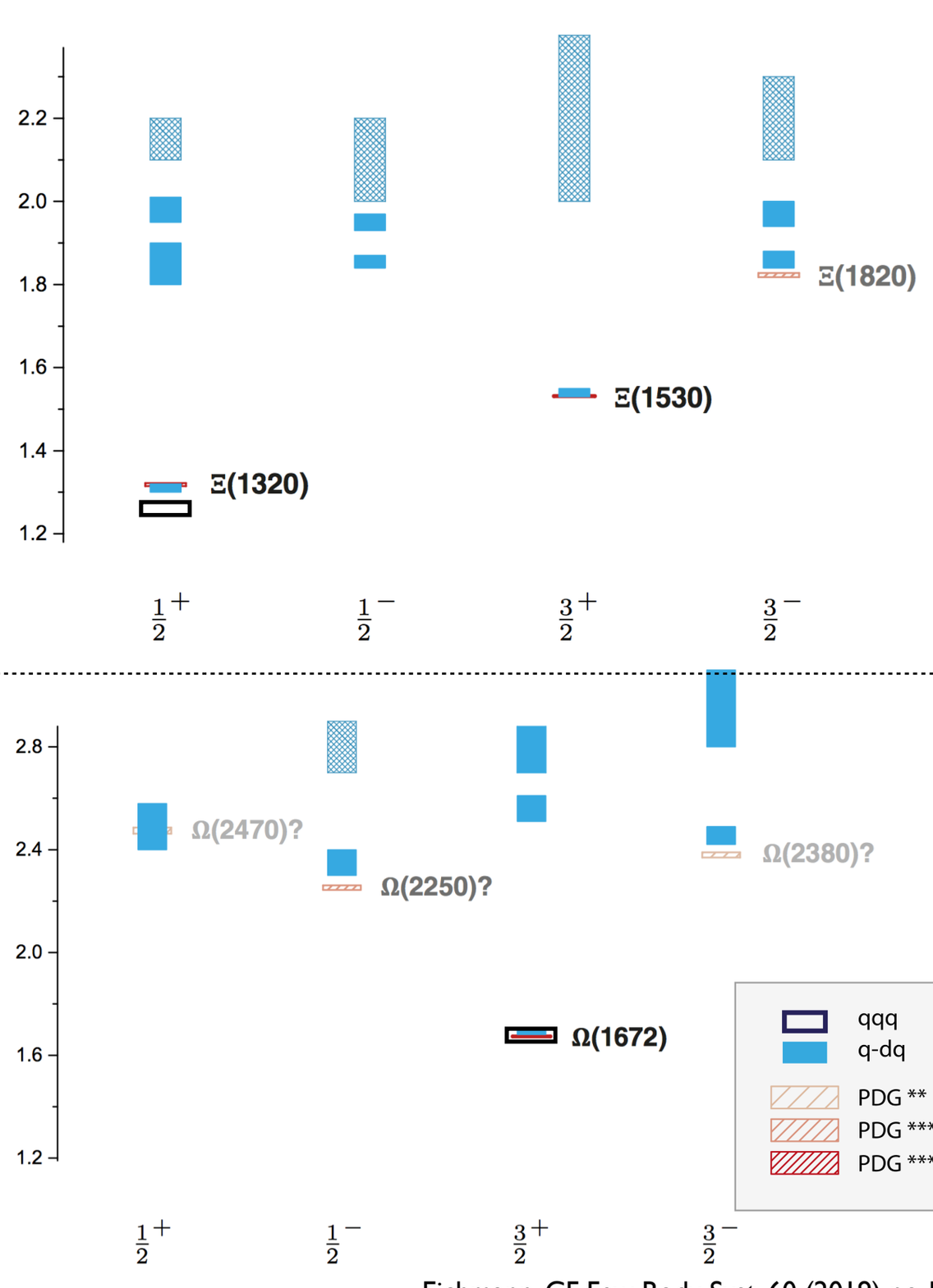

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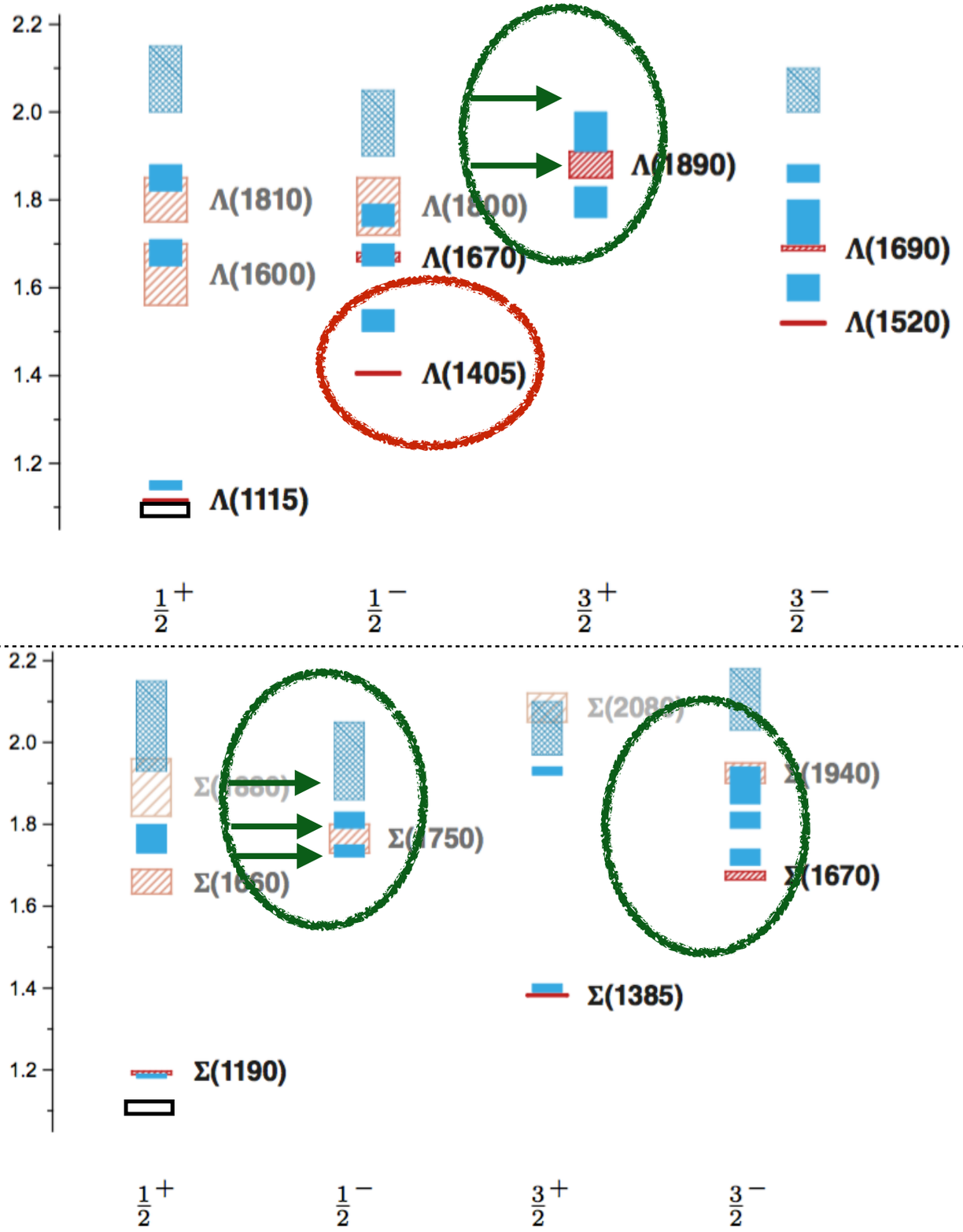

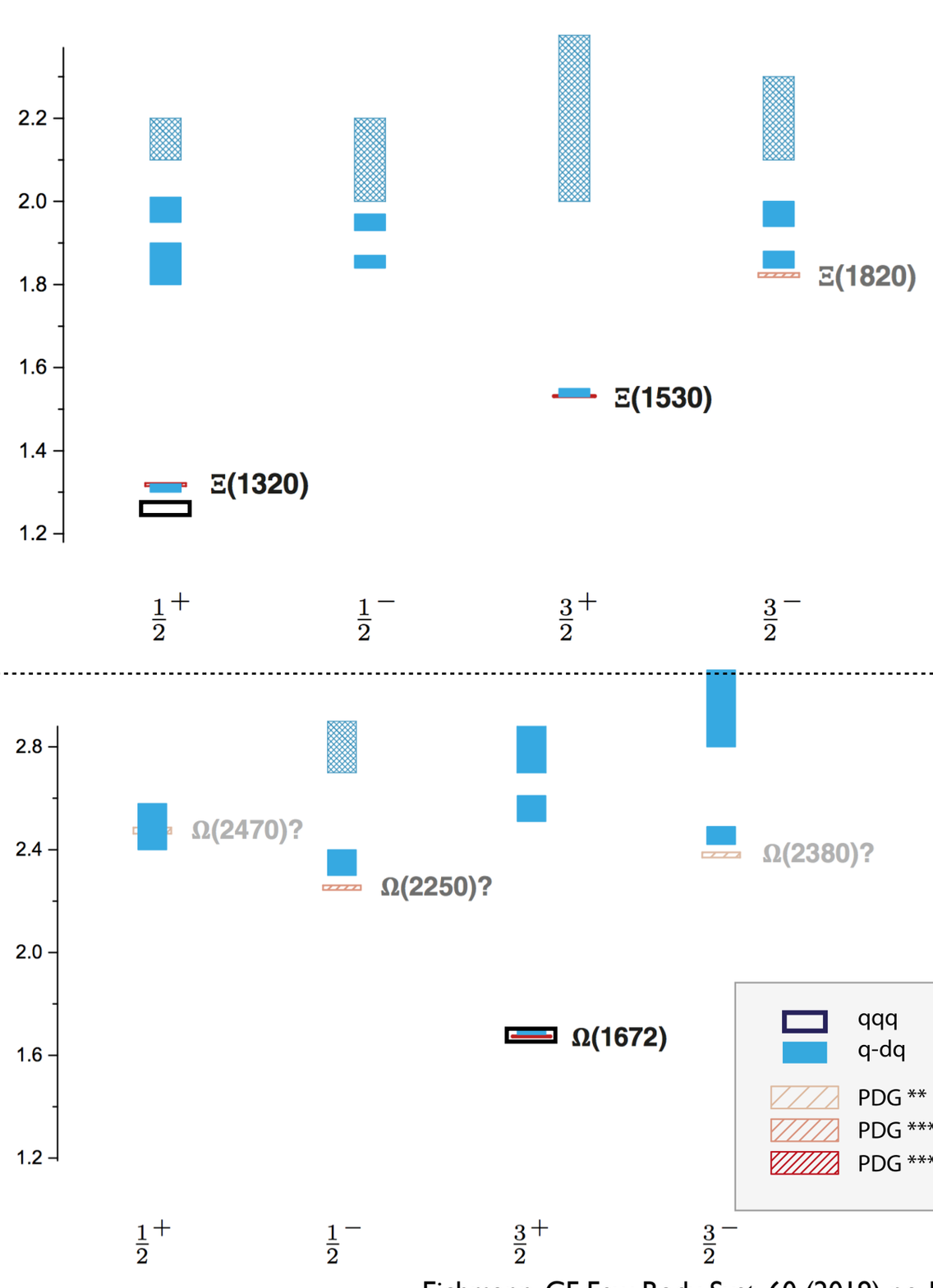

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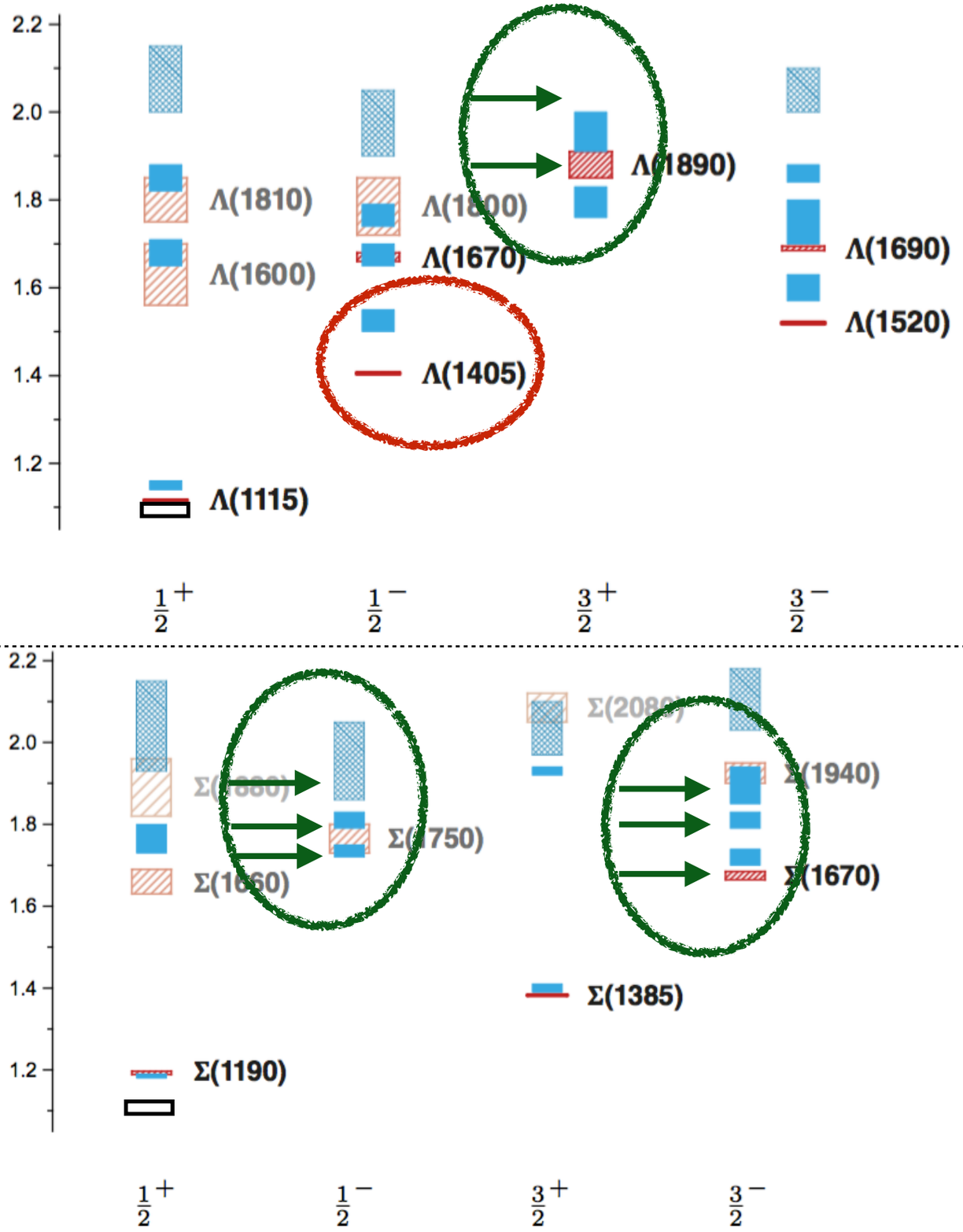

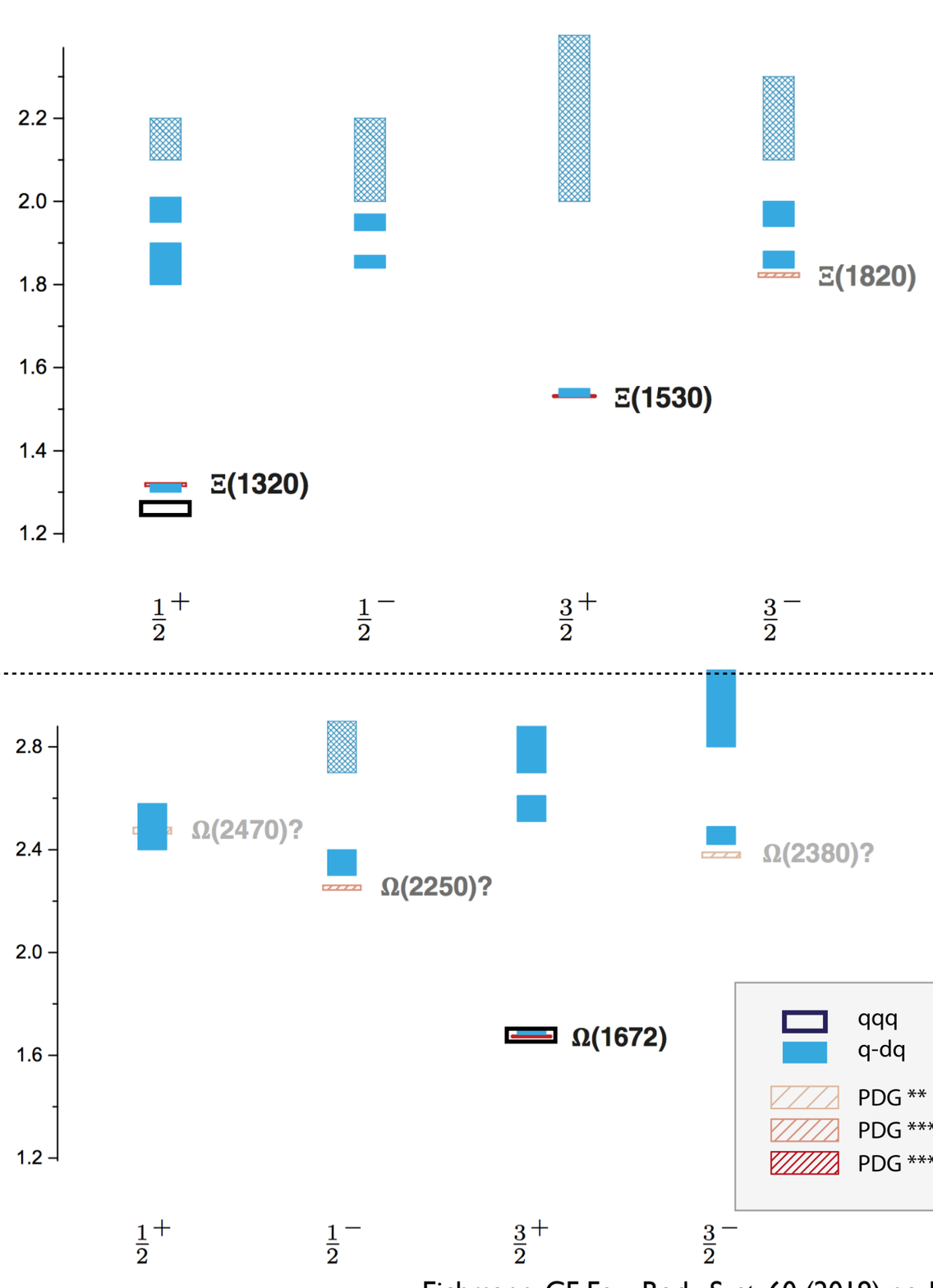

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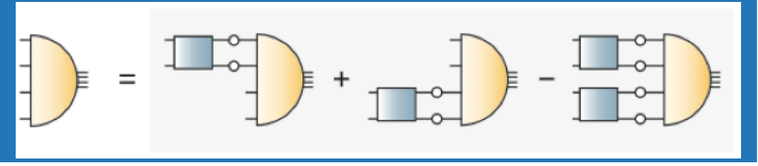
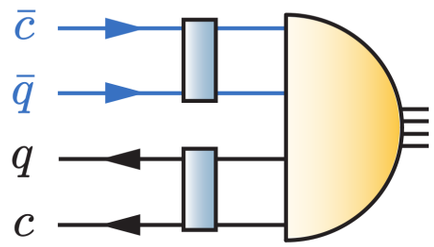
Four-body equation: permutations



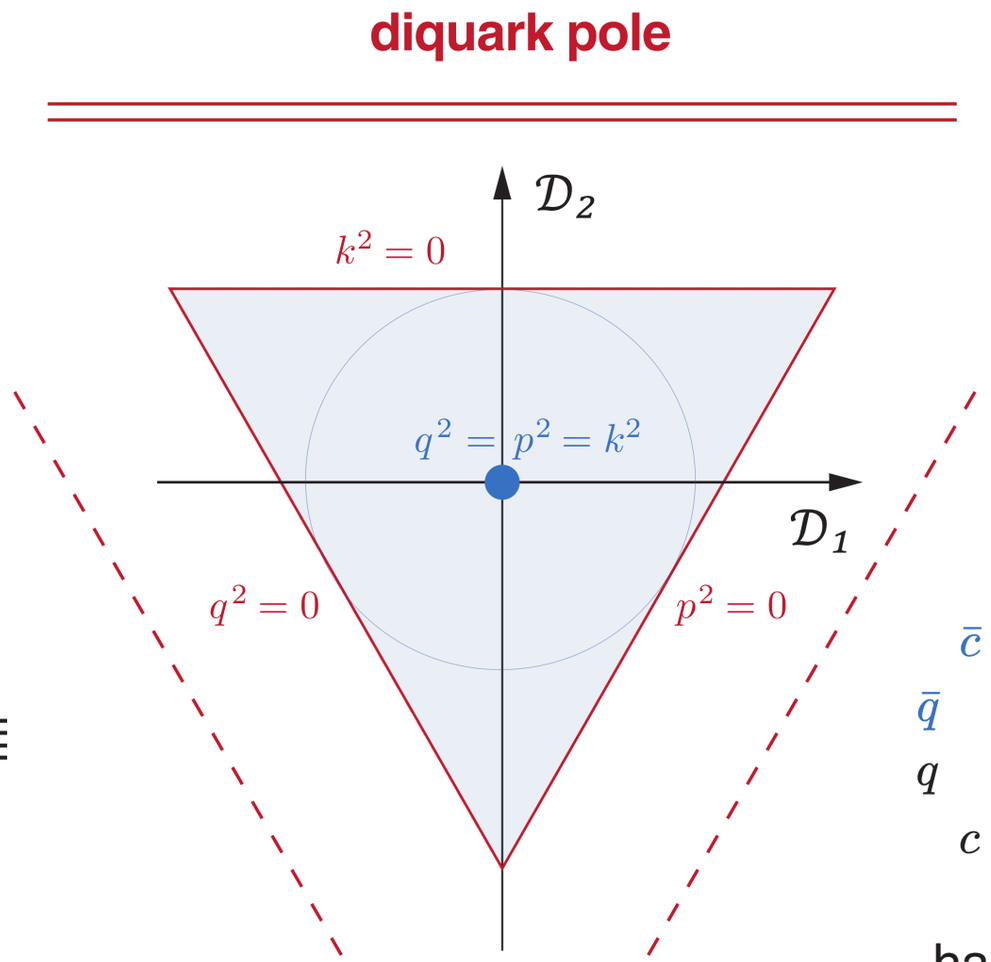
- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

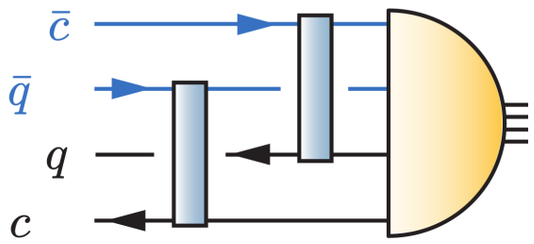
  $\mathcal{D}_2 \sim q^2 - p^2$

p, q, k : relative momenta

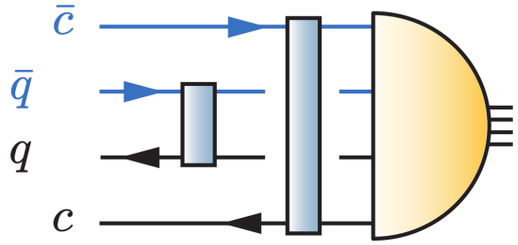


diquark-antidiquark





heavy-light mesons



hadrocharmonium

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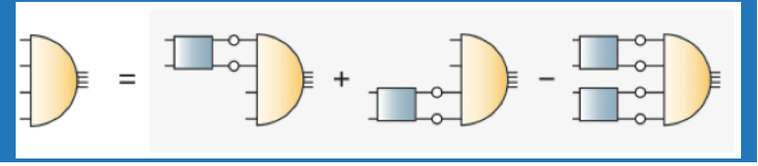

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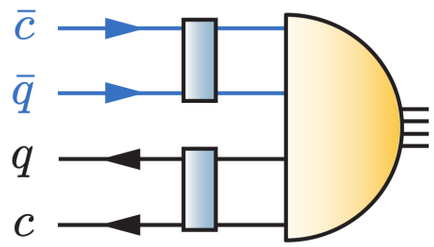
p, q, k : relative momenta

- model independent: heavy-light meson poles more important than diquark poles (color factor !)



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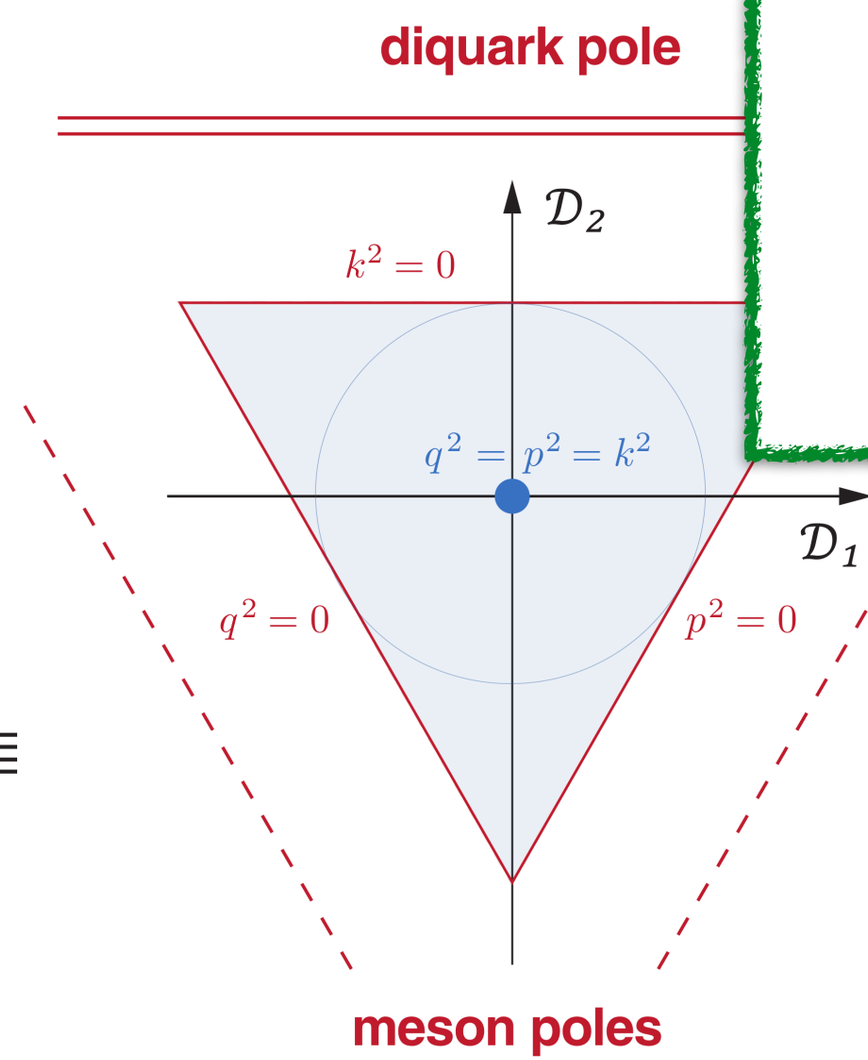

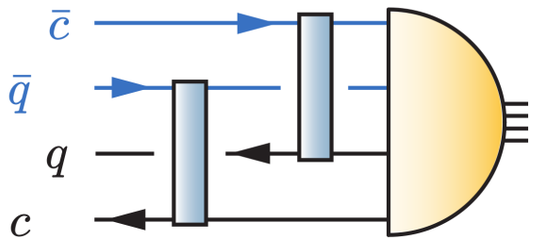

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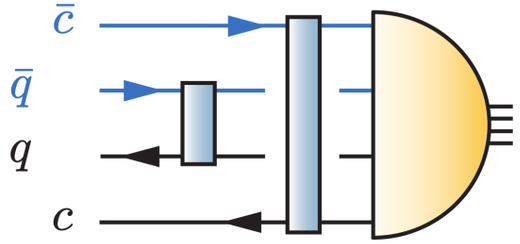
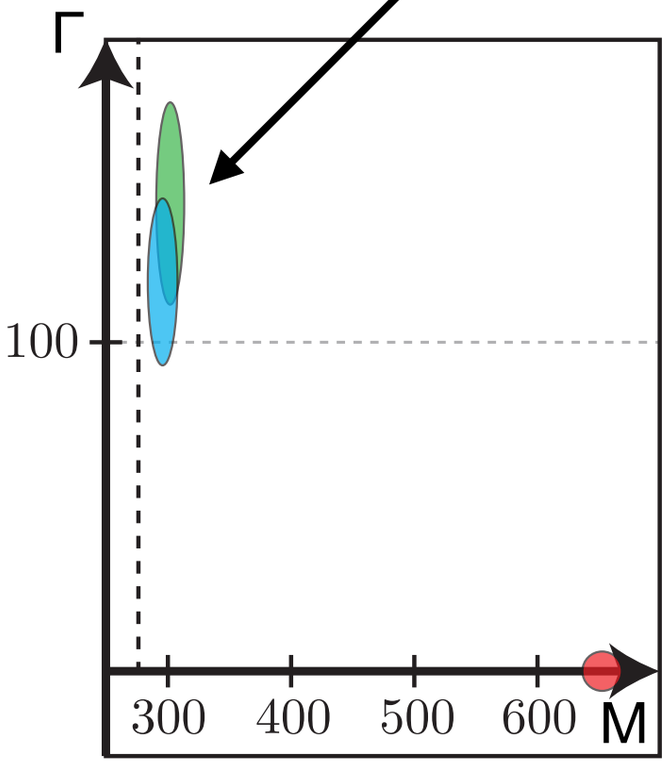


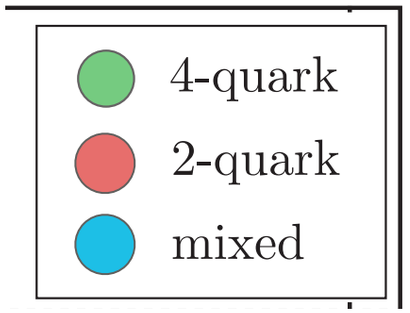
hadrocharmonium

Mass evolution of four-quark state: 0++

$$f_0(500) : \pi\pi - \text{component dominates!}$$



$$m_q = 3.8\,\mathrm{MeV}$$



Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

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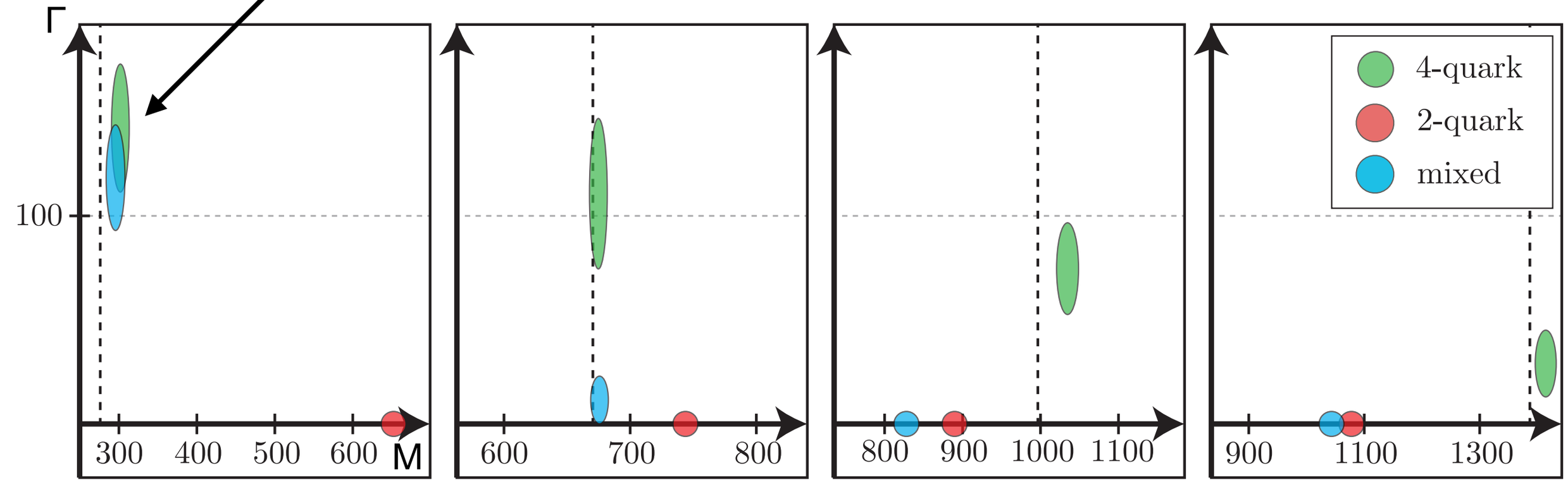

$m_q = 3.8\,\mathrm{MeV}$ $m_q = 20.0\,\mathrm{MeV}$ $m_q = 50.0\,\mathrm{MeV}$ $m_q = 85.5\,\mathrm{MeV}$

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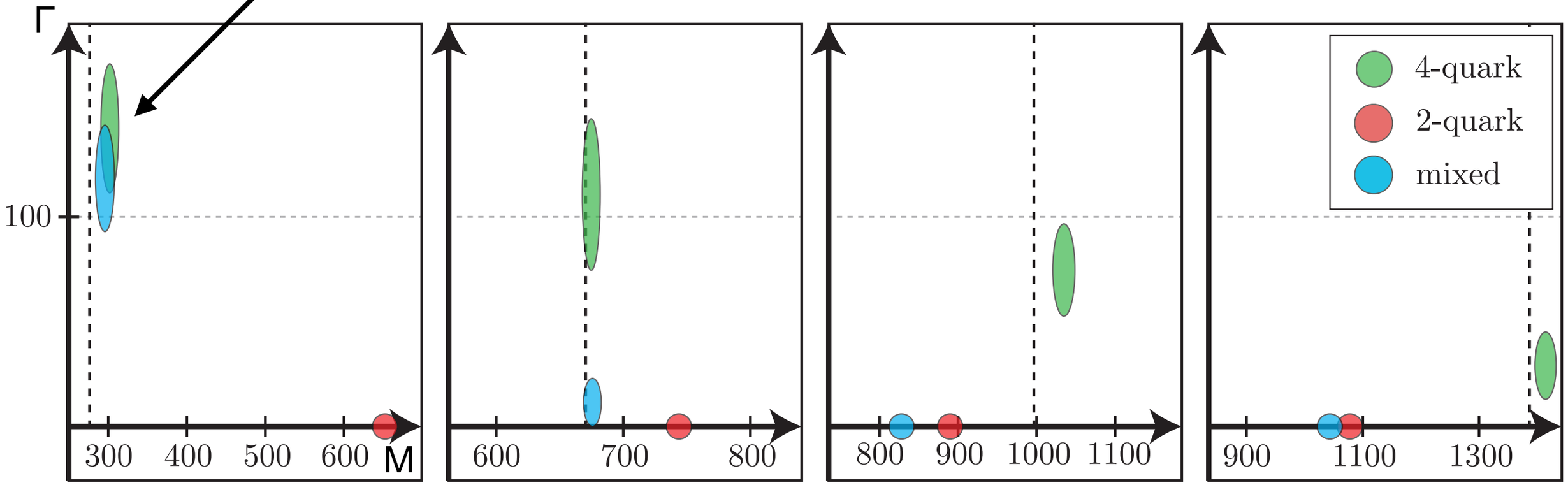

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- mixed state becomes qq-dominated for large m_q
- dynamical decision !

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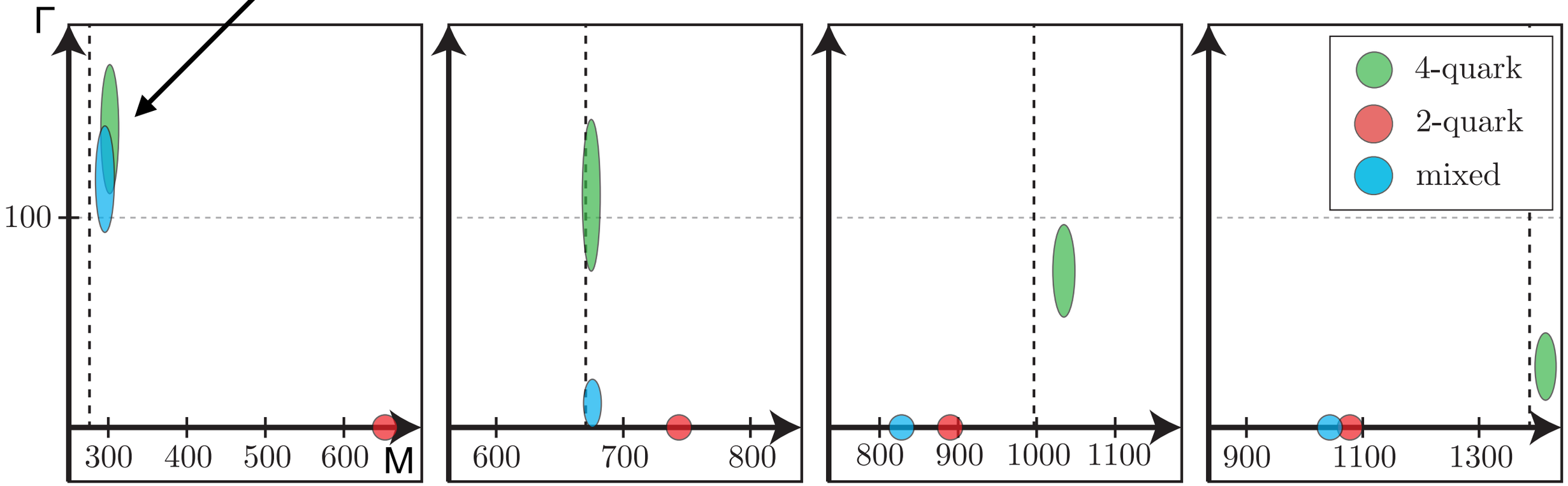

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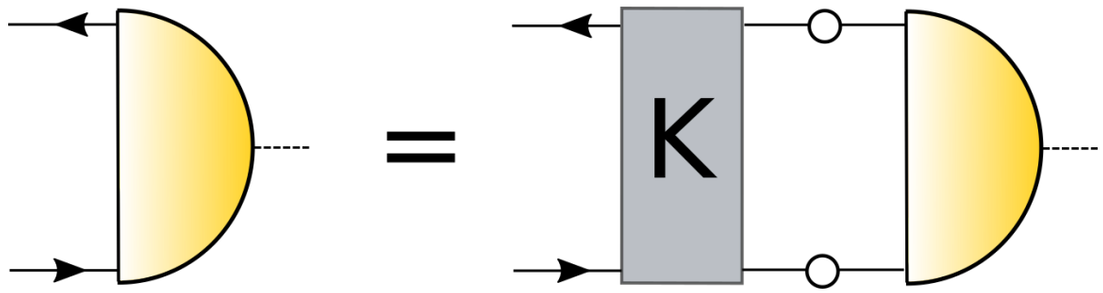
- mixed state becomes qq-dominated for large m_q
- dynamical decision !

⟶ consequences for ccqq, ccss, bbq, bbss, bbcc ?

work to be done!

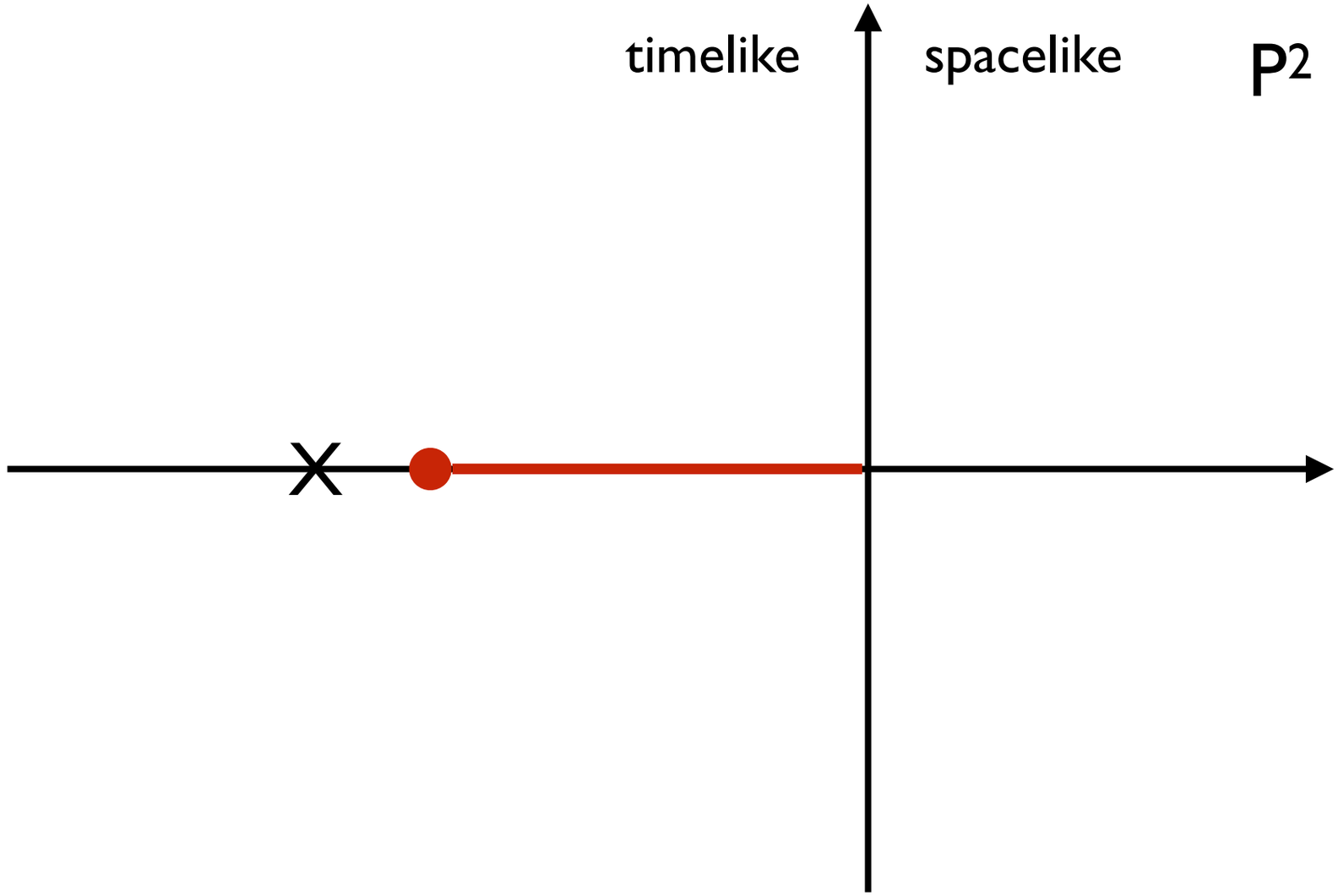
The complex P^2-plane



$\lambda(P^2) \circ BSA = kernel \circ BSA$

$\lambda(P^2) \stackrel{!}{=} 1$

generic situation



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

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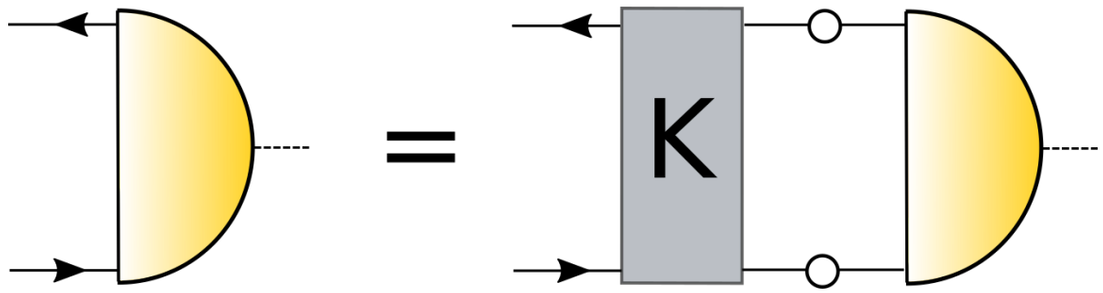

$\lambda(P^2) \circ \mathrm{BSA} = \mathrm{kernel} \circ \mathrm{BSA}$

$\lambda(P^2) \stackrel{!}{=} 1$

SPM

(see talk by Tripolt)

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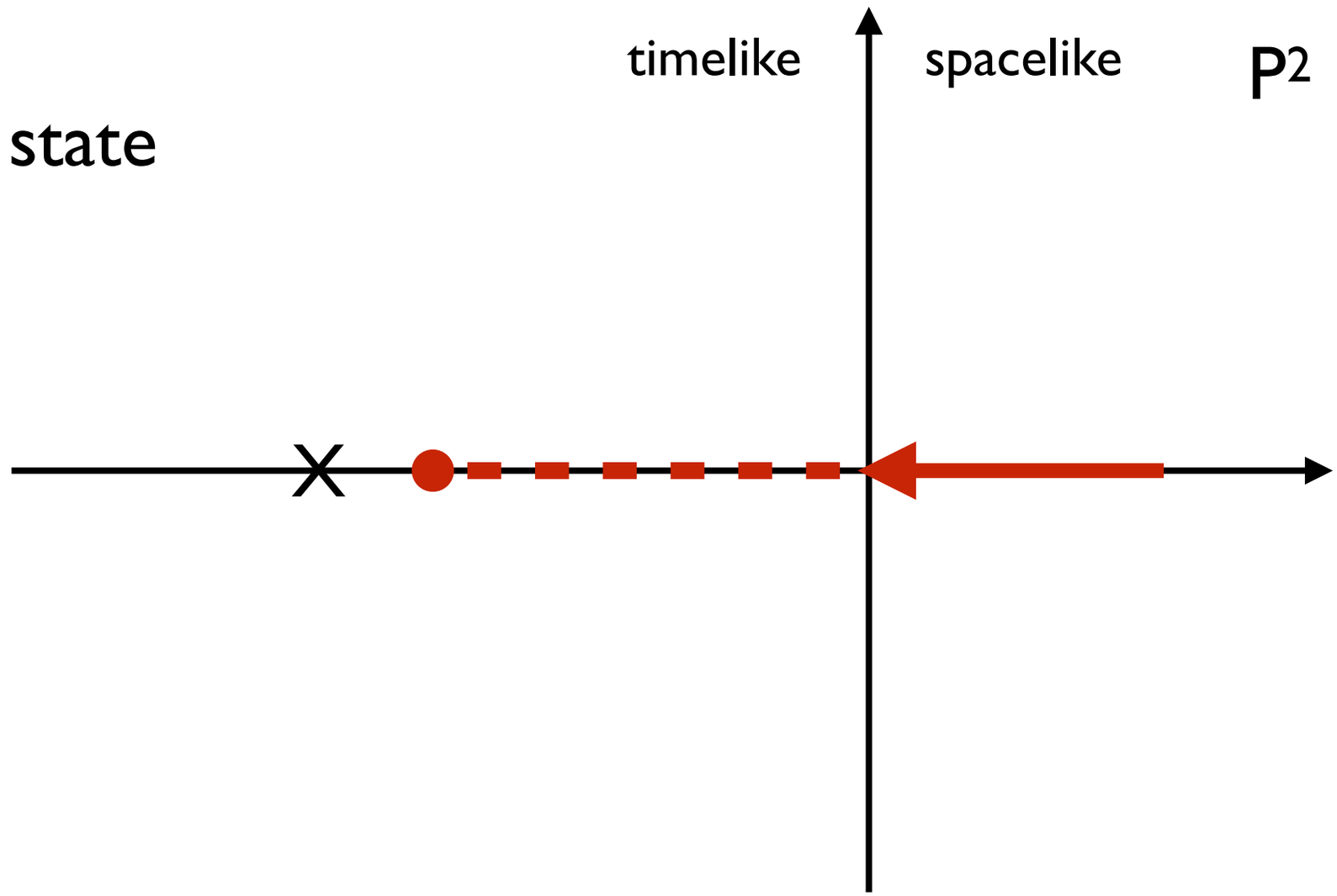
extrapolation to bound state



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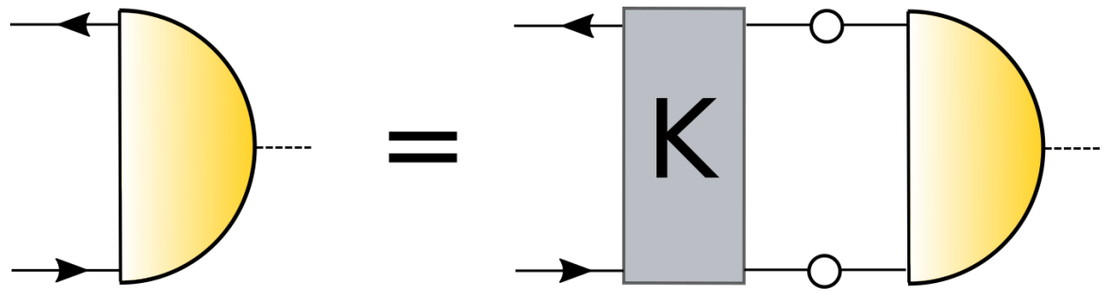
The complex P^2-plane



$\lambda(P^2) \circ \mathrm{BSA} = \mathrm{kernel} \circ \mathrm{BSA}$

$\lambda(P^2) \stackrel{!}{=} 1$

SPM

(see talk by Tripolt)

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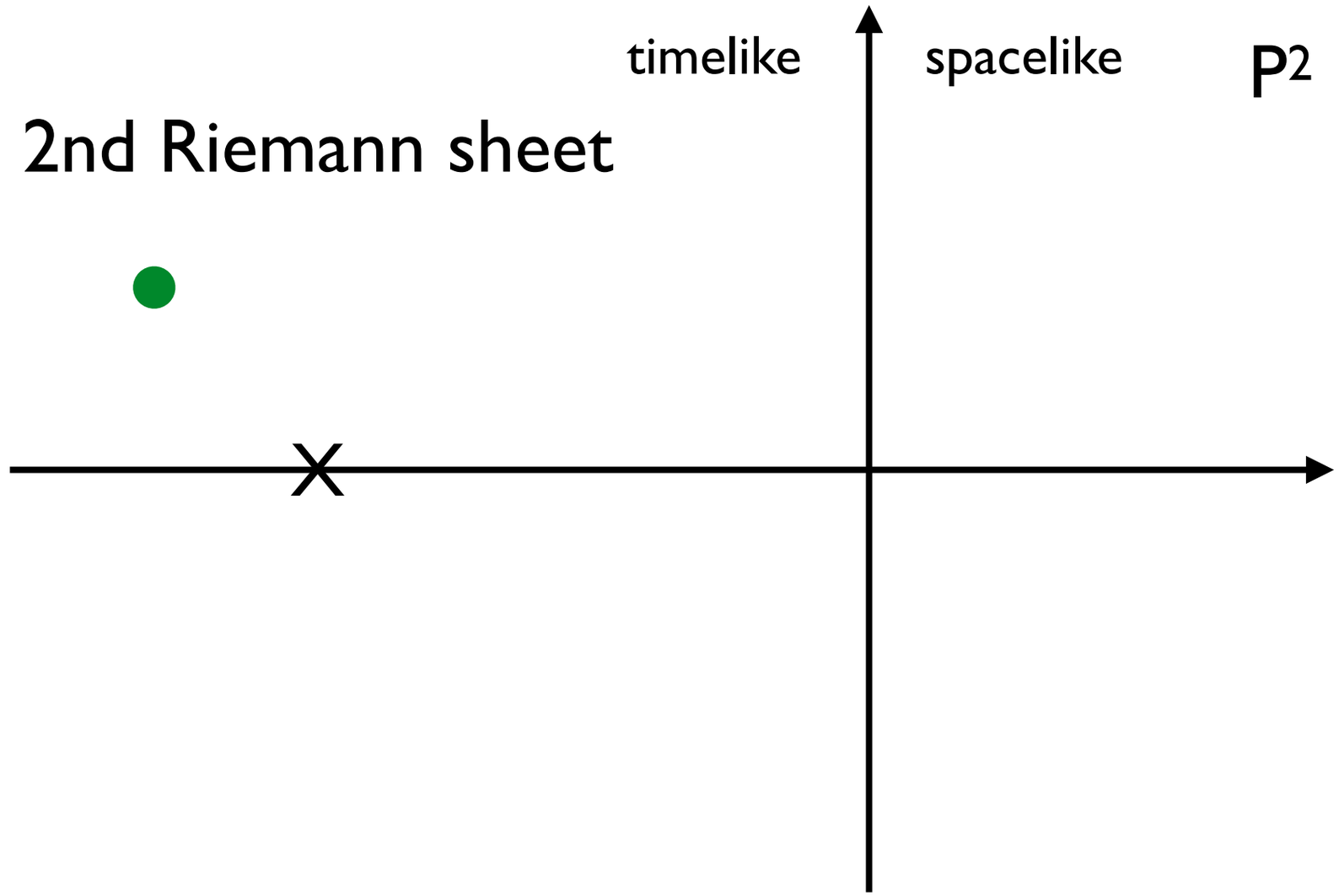
extrapolation to pole in 2nd Riemann sheet

$\rho \to \pi\pi$

$\sigma \to \pi\pi$



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

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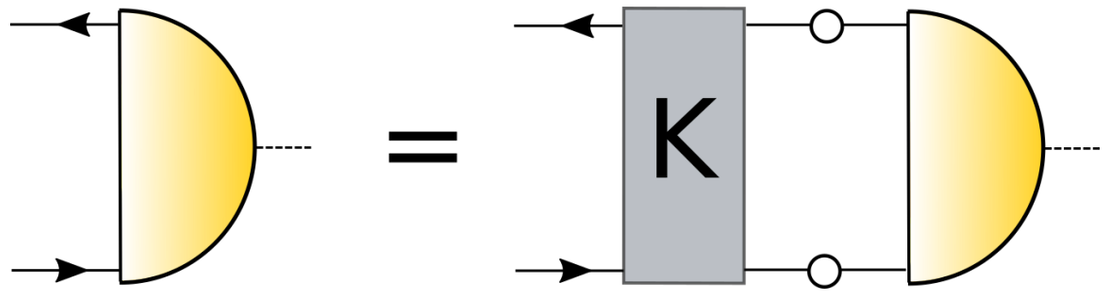

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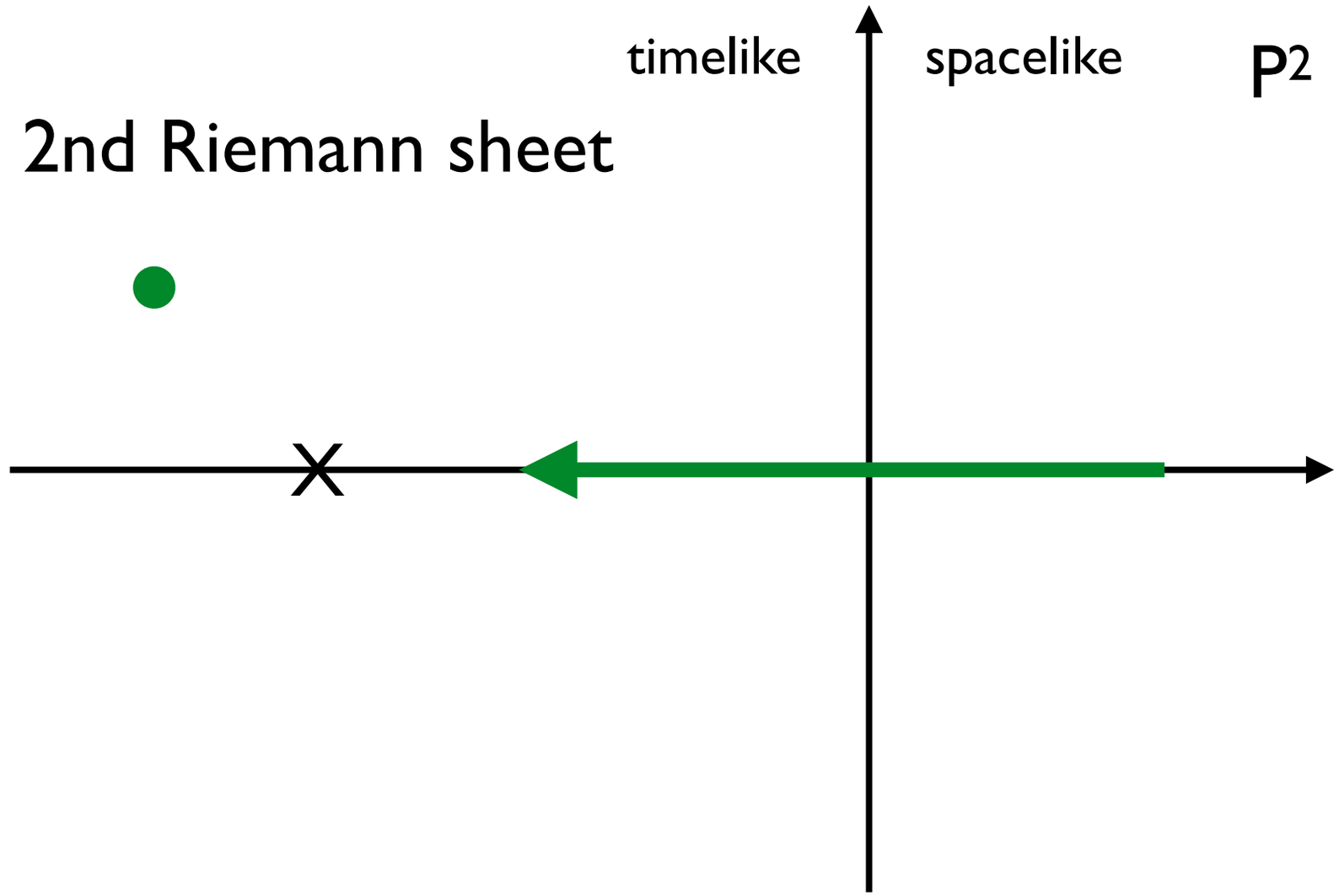

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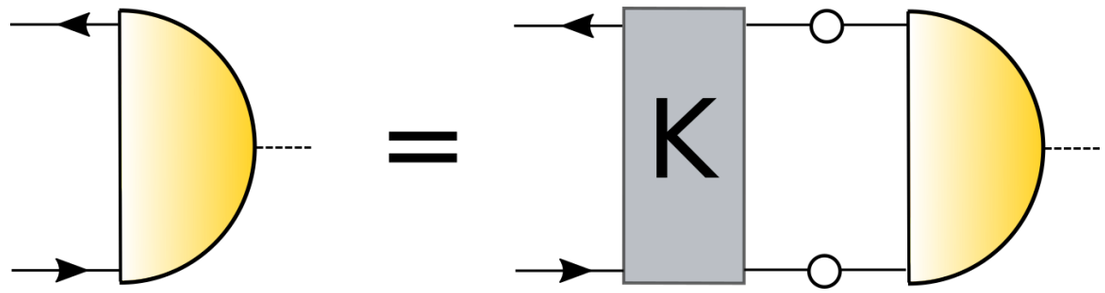

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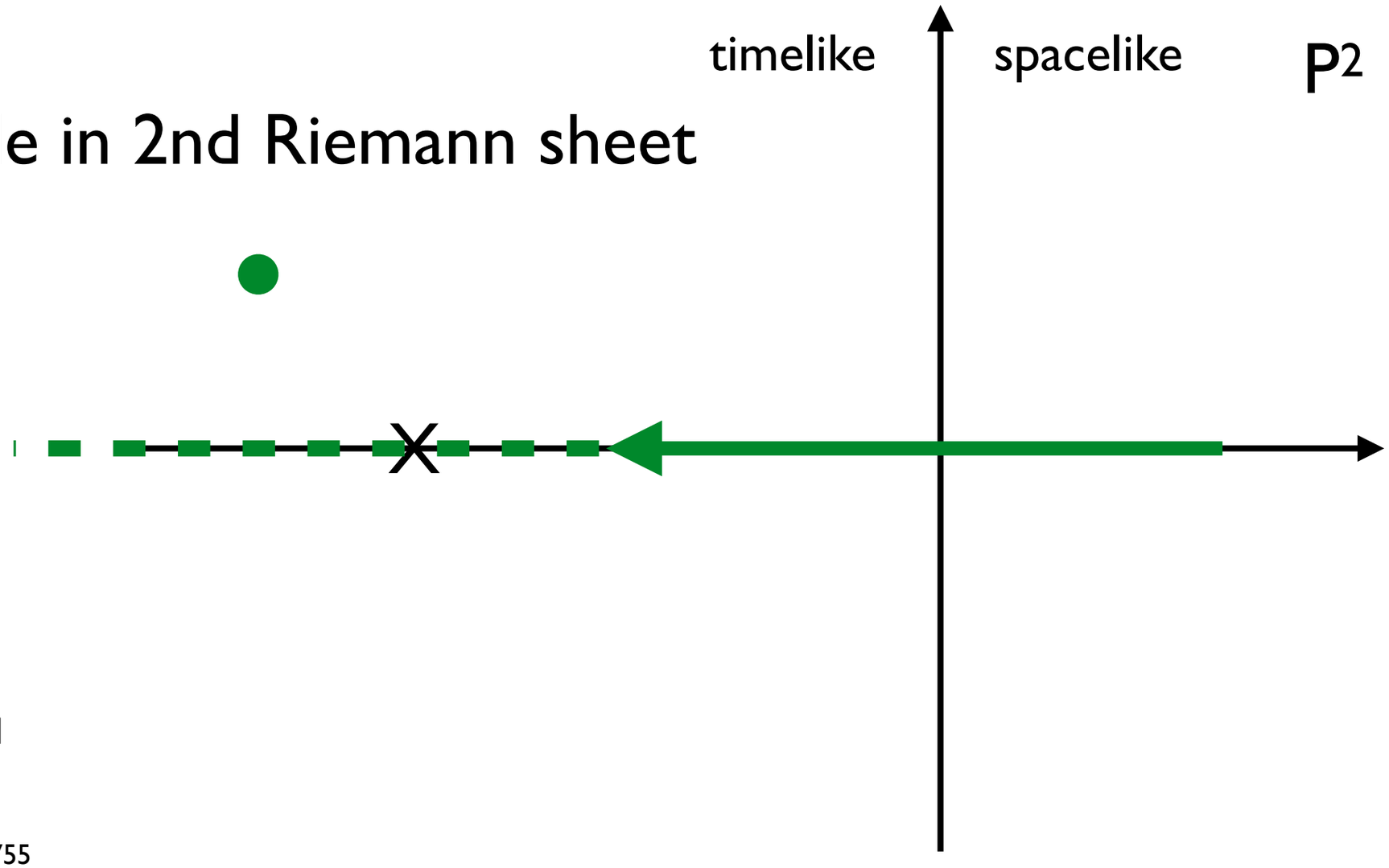

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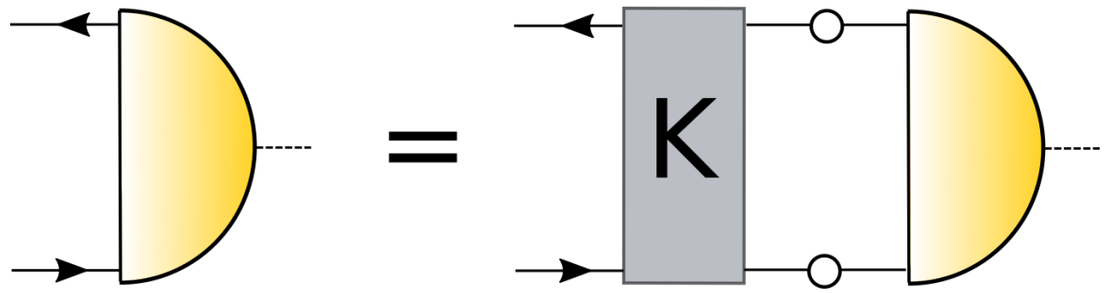

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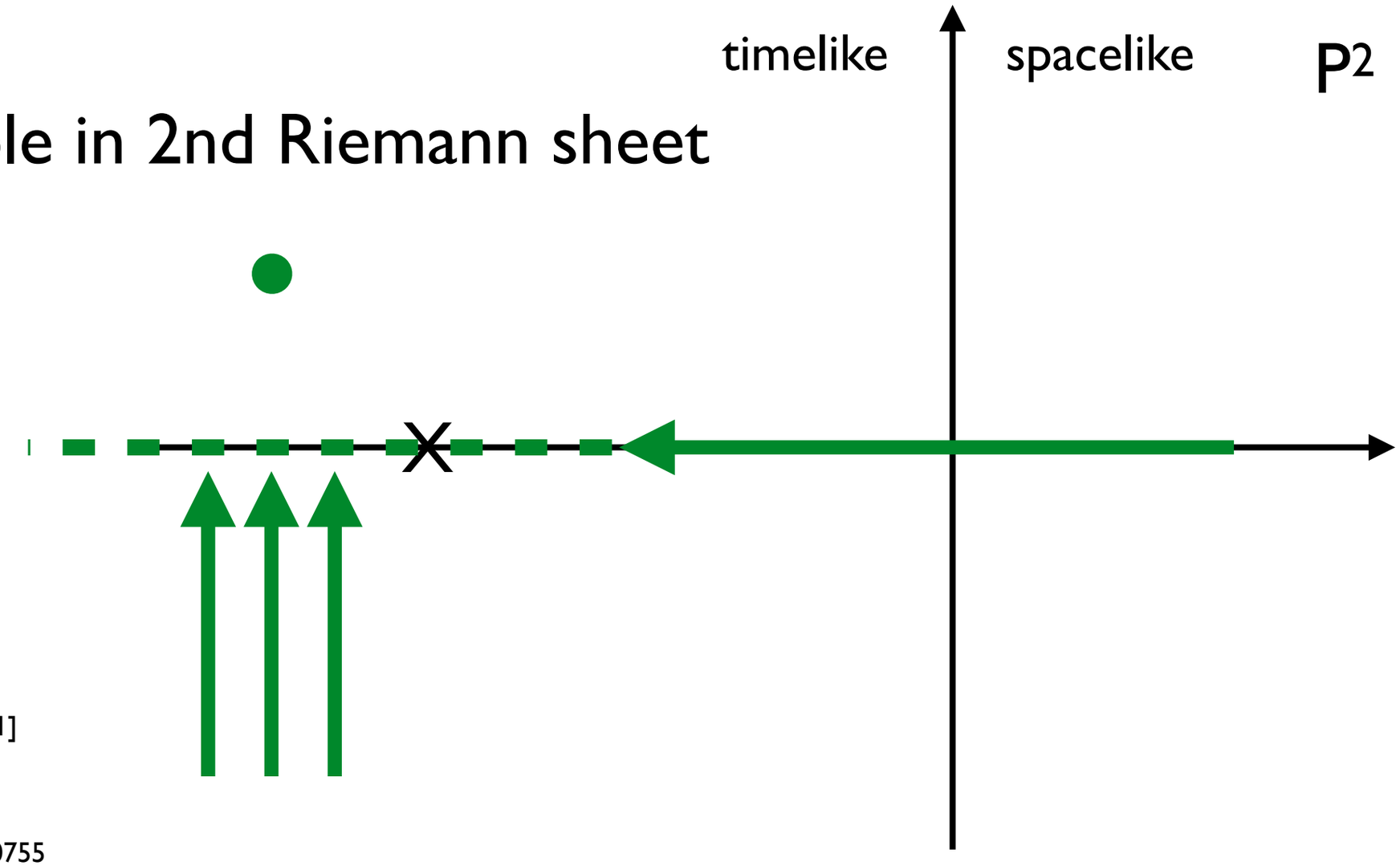

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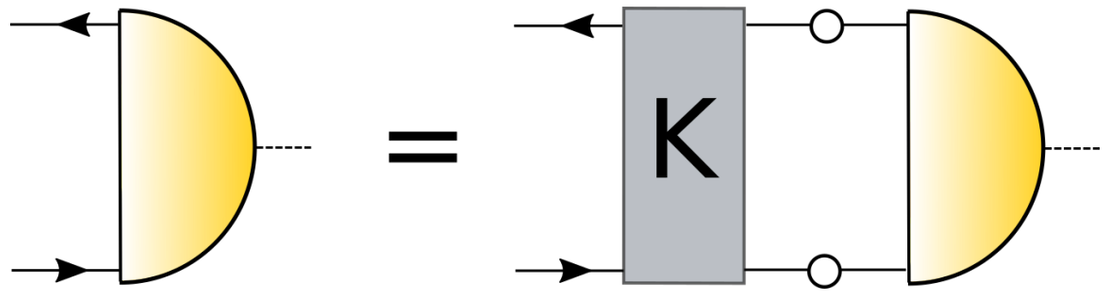

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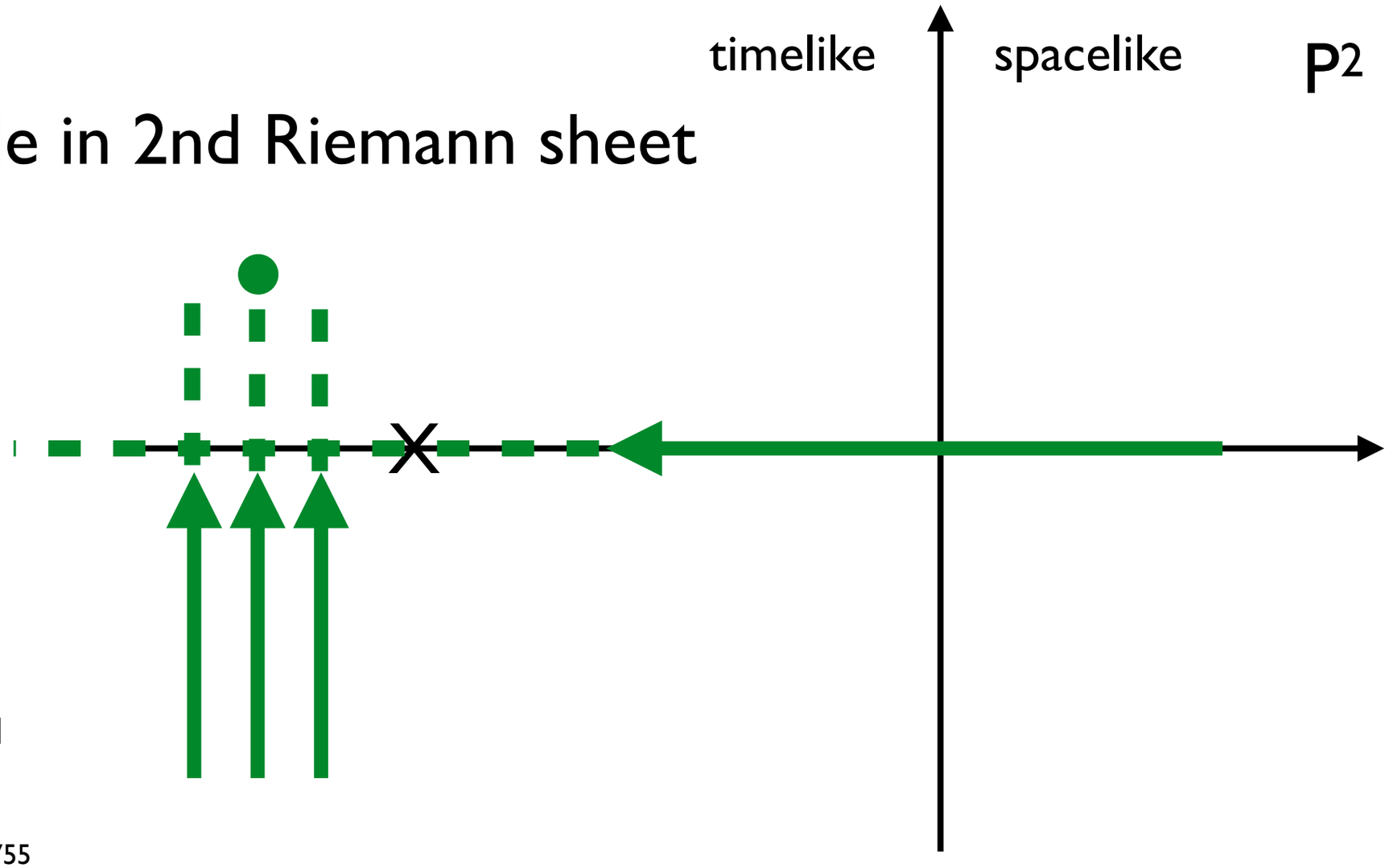

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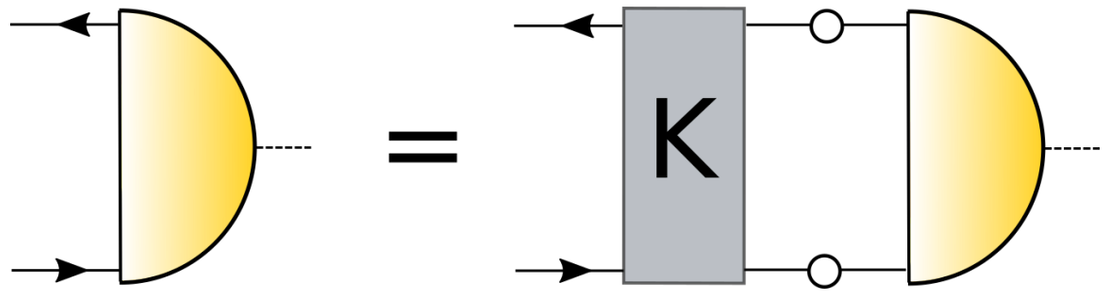

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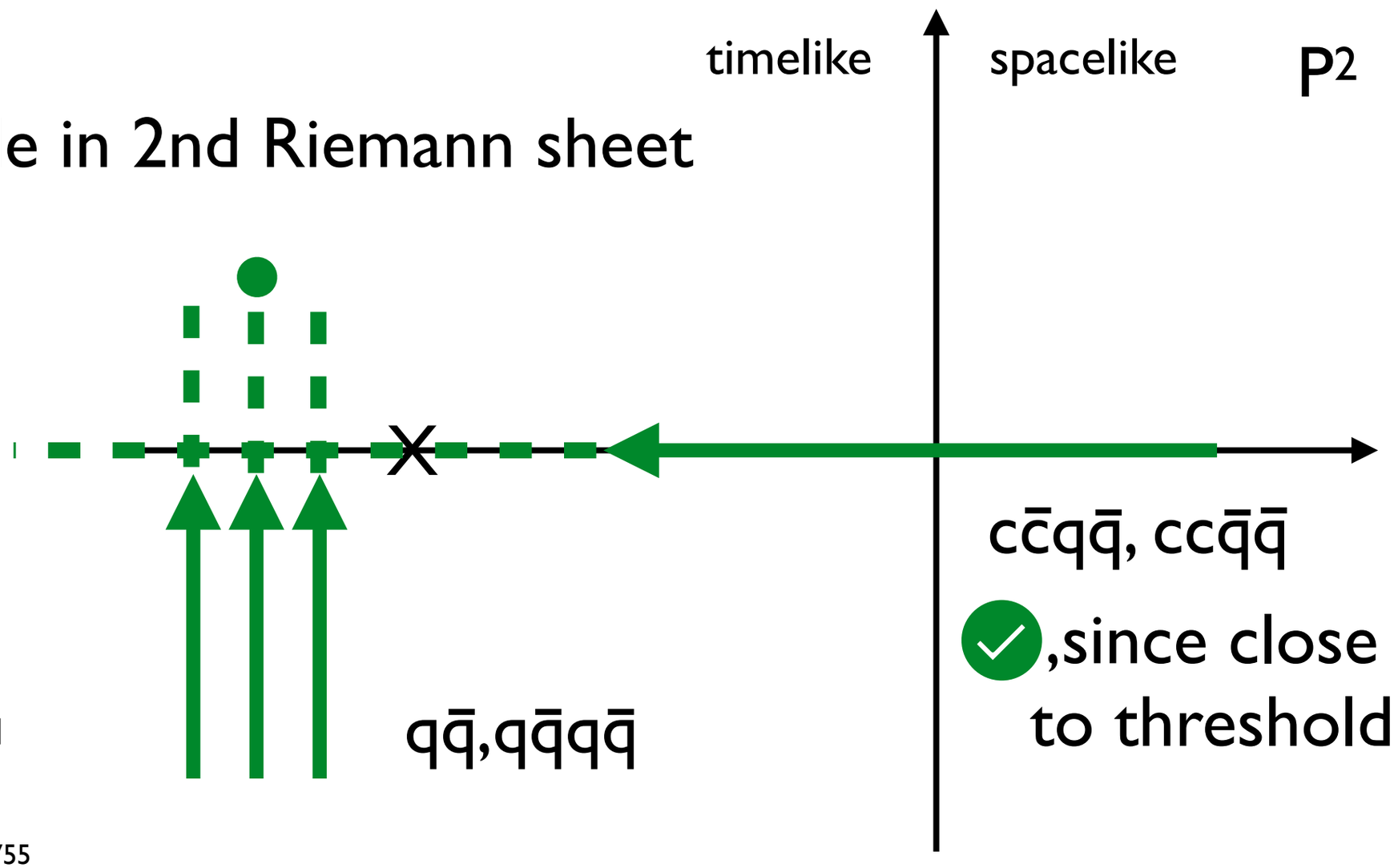

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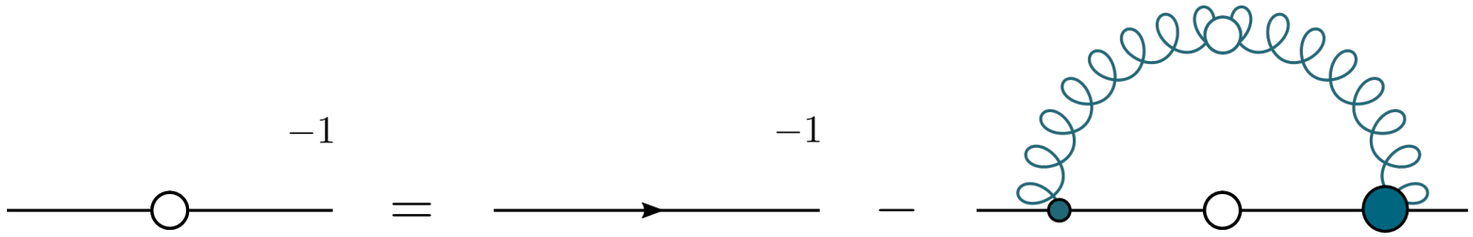
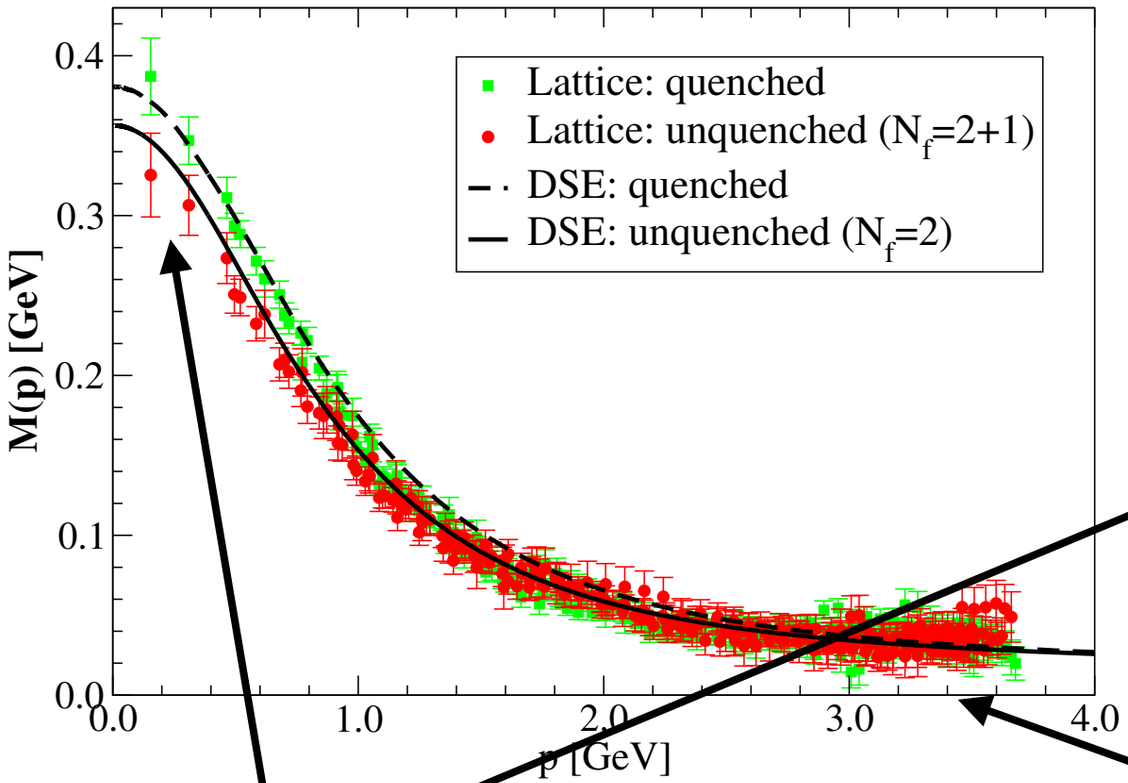
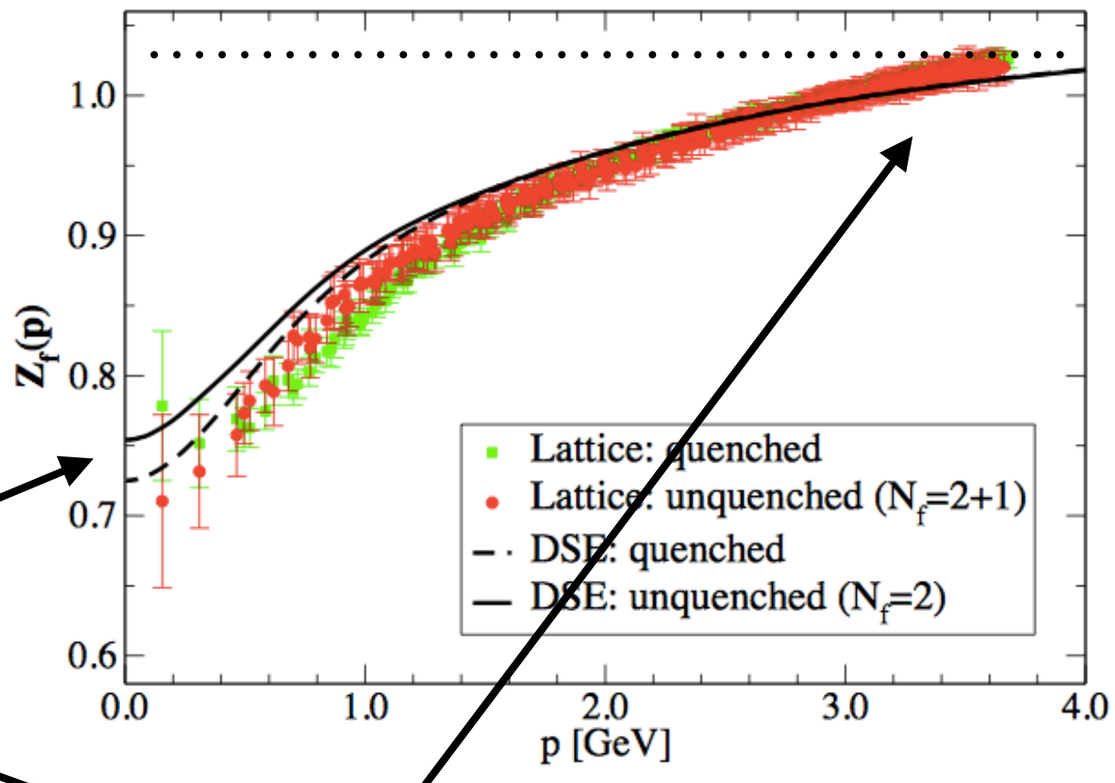
Quarks: mass from interaction



$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





'constituent quark':
large mass; very composite

'current quark':
- small mass; non-composite