Muon beam for neutrino CP Violation: Connecting energy and neutrino frontiers

Alim Ruzi
Physics Department, Peking University

This talk is about:

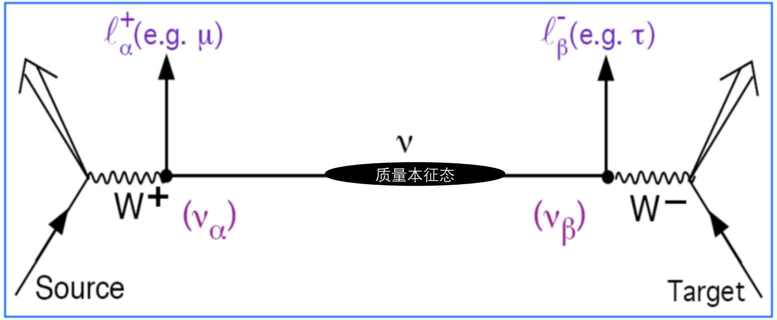
◆ **Neutrino oscillation**

  - Theory and experimental status
  - Current status of CP violating phase: $\boldsymbol{\delta_{\mathrm{CP}}}$
  - New muon source: e^+e^- collision
  - Simulation results of the new experiment

◆ **Muon beams for dark Matter detection**

Neutrino oscillation：a quantum phenomenon

- **Oscillation: spontaneous periodic change from one neutrino flavor to another, a direct result of neutrino mixing with mass eigenstates, and is a quantum phenomenon. In a neutrino oscillation experiment, the neutrino beam is produced and detected via the weak Charged-Current (CC) interaction。**



- **Neutrino state of flavor $\alpha = e, \mu, \tau$ produced in a weak interaction can be written as superposition of mass eigenstates：**

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle$$

Neutrino Mixing Matrix or PMNS matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U^\dagger U = 1, \quad \sum_i U_{\alpha i} U^*_{\beta i} = \delta_{\alpha\beta}, \quad \sum_i U_{\alpha i} U^*_{\alpha j} = \delta_{ij}$$

Neutrino Oscillation probability

- **The corresponding transition amplitude for flavor α to β can be obtained with the old-fashioned way as**

$$A(\nu_\alpha \to \nu_\beta) = \langle \nu_\beta | \nu_\alpha(t,L)\rangle = \sum_{i,j} U^*_{\alpha i} U_{\beta j} e^{-iE_j t + i p_j L} \langle \nu_j | \nu_i \rangle = \sum_j U^*_{\alpha j} U_{\beta j} e^{-iE_j t + i p_j L}$$

$$\boxed{E_i = \sqrt{m_i^2 + p_i^2} \simeq p_i + \frac{m_i^2}{2p_i} \simeq E + \frac{m_i^2}{2E}}$$

- Highly relativistic: $\vec{p} \gg m, \quad p = E$

- **The oscillation probability (for 3 flavor) is than given as**

$$P(\nu_\alpha \to \nu_\beta) = |A(\nu_\alpha \to \nu_\beta)|^2 = \sum_{i,j} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} e^{-i(E_i - E_j)t}$$

$$= \delta_{\alpha\beta} - 4Re \sum_{j>i} \left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right] \sin^2(X_{ij})$$

$$\boxed{+ 2 \sum_{j>i} Im \left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right] \sin 2X_{ij}}$$

$$X_{ij} = \frac{(m_i^2 - m_j^2)L}{4E} = 1.267 \frac{\Delta m^2_{ij}}{\mathrm{eV}^2} \frac{L}{\mathrm{Km}} \frac{\mathrm{GeV}}{E}$$

$$2 \sum_{i<j} Im \left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right] \sin(2X_{ij})$$

$$= \pm 8J \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

(+) for (e→μ), (μ→τ), (τ→e), otherwise (-)

- ***Jarlskog* factor**

$$J = \cos\theta_{12} \sin\theta_{12} \cos^2\theta_{13} \sin\theta_{13} \cos\theta_{23} \sin\theta_{23} \sin(\delta_{\mathrm{CP}})$$

Jarlskog invariant PRL. 58, 1698 (1987)

CP violation in neutrino oscillation

- The oscillation probability for $\bar{\nu}_\alpha \to \bar{\nu}_\beta$ is obtained through a CP transformation on the corresponding *wave functions* of ν_α, ν_β, or simply by taking $U \to U^*$, which only changes the sign of the Imaginary part in $P(\nu_\alpha \to \nu_\beta)$.

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4Re\sum_{j>i}\left[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right]\sin^2(X_{ij}) \pm 8J\sin\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin\left(\frac{\Delta m^2_{32}L}{4E}\right)\sin\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

CP transformed

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \delta_{\alpha\beta} - 4Re\sum_{j>i}\left[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right]\sin^2(X_{ij}) \mp 8J\sin\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin\left(\frac{\Delta m^2_{32}L}{4E}\right)\sin\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

$$\boxed{J = \cos\theta_{12}\sin\theta_{12}\cos^2\theta_{13}\sin\theta_{13}\cos\theta_{23}\sin\theta_{23}\sin(\delta_{\mathrm{CP}})}$$

$$\mathbf{\Delta P}(\nu_\alpha \to \nu_\beta) = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \pm 16\mathbf{J}\sin\left(\frac{\mathbf{\Delta m^2_{21}L}}{\mathbf{4E}}\right)\sin\left(\frac{\mathbf{\Delta m^2_{32}L}}{\mathbf{4E}}\right)\sin\left(\frac{\mathbf{\Delta m^2_{31}L}}{\mathbf{4E}}\right) \neq \mathbf{0}$$

If $\delta_{CP} \neq 0$

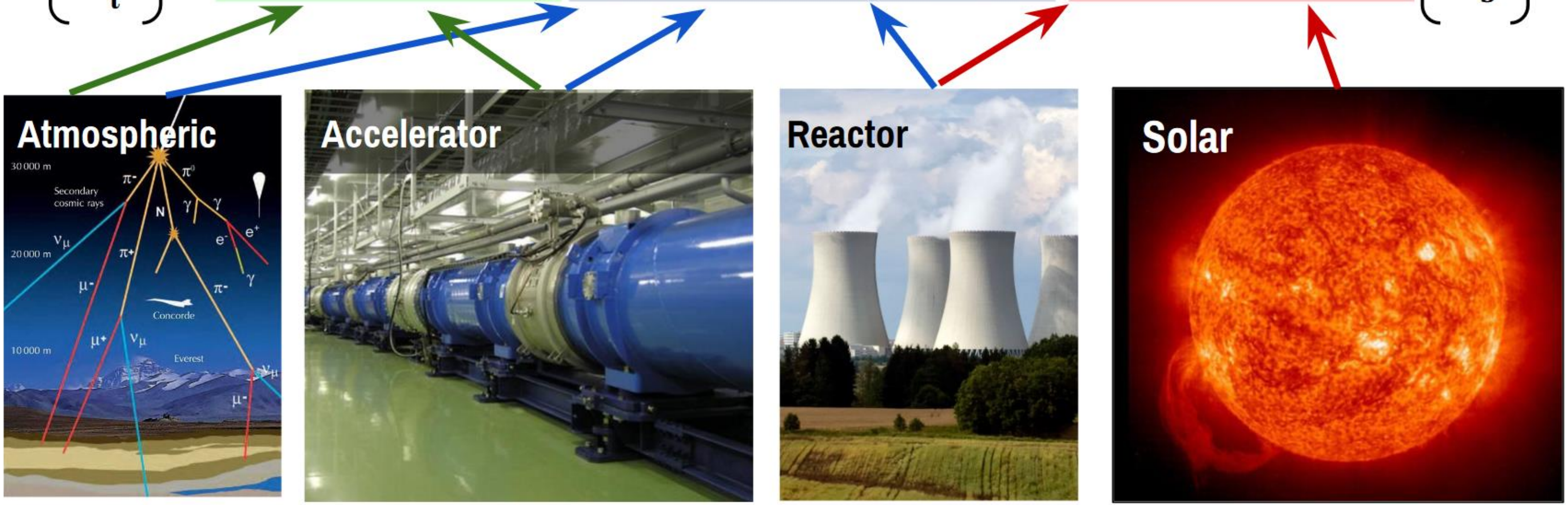
$$\Delta P \neq 0$$

CP transformation is violated!

Neutrino Sources and Mixing Parameters

$c_{ij} = \cos\theta_{ij}$, and $s_{ij} = \sin\theta_{ij}$.

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{bmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{bmatrix}\begin{bmatrix}c_{13}&0&s_{13}e^{-i\delta}\\0&1&0\\-s_{13}e^{i\delta}&0&c_{13}\end{bmatrix}\begin{bmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{bmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

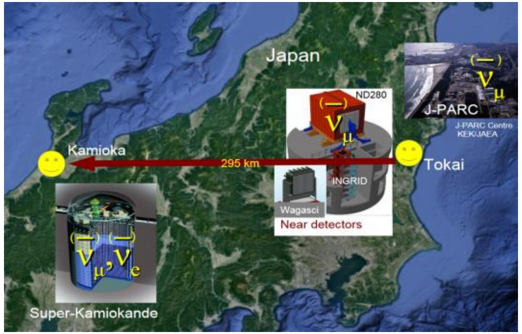


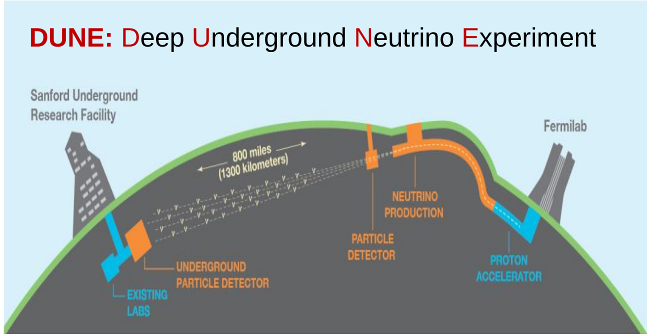
Neutrino Experiments and Oscillation parameters

◆ **Parameters to be determined**

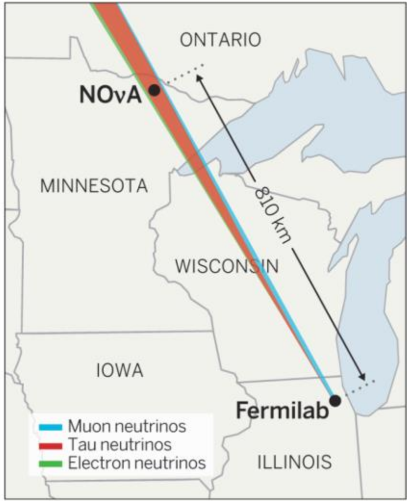
1. Three mixing angles: $\boldsymbol{\theta_{13}, \theta_{12}, \theta_{23} \neq 0}$
1. Two mass differences: $\boldsymbol{\Delta m^2_{12}, \Delta m^2_{13}, \Delta m^2_{23}}$
2. One Dirac phase :$\boldsymbol{\delta_{cp}}$

T2K




DUNE: Deep Underground Neutrino Experiment

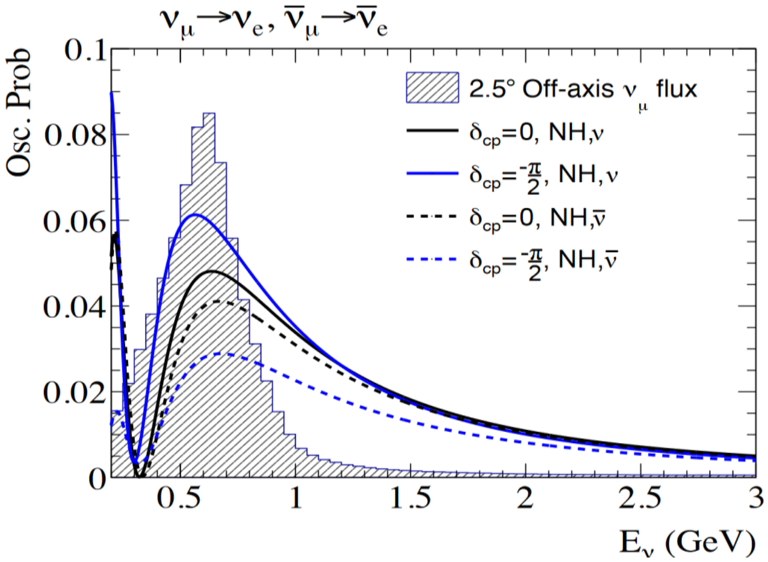
K. ENGMAN/SCIENCE 345, 6204

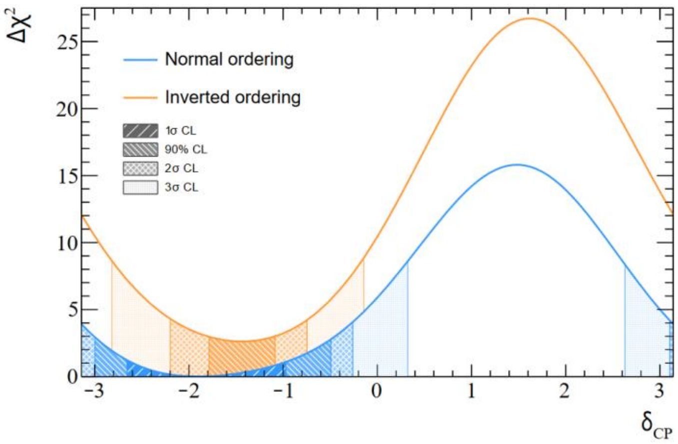


NuFit results:

JHEP 09 (2020) 178,
JHEP 01 (2019) 106
JHEP 01 (2017) 087
JHEP 09 (2015) 200
JHEP 11 (2014) 052

Probing CP phase: *T2K Experiment*

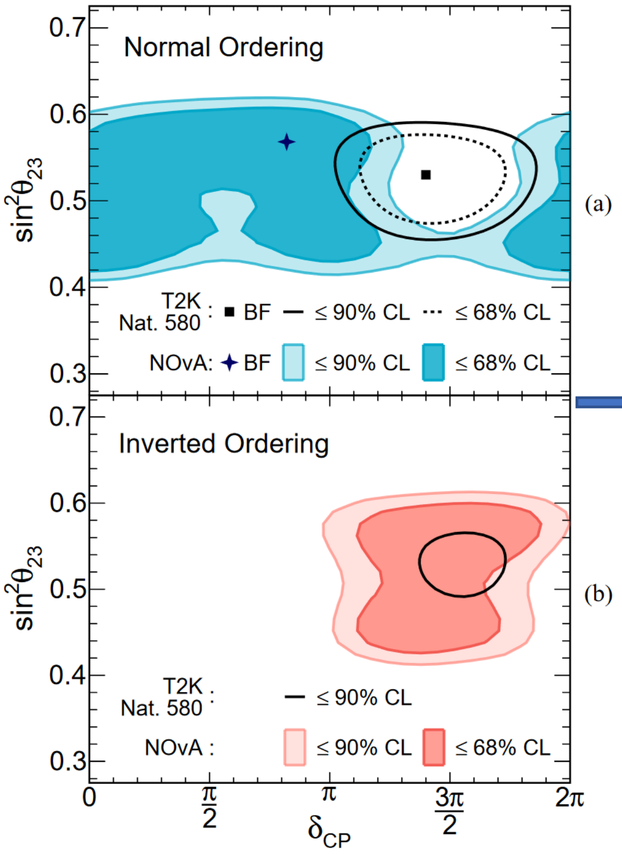




Eliminates $\boldsymbol{\delta_{CP} = \frac{\pi}{2}}$ at 3 sigma level

T2K Collaboration
Eur.Phys.J.C 83 (2023) 9, 782

Probing CP phase: *NOvA Experiment*



Phys. Rev. D 106, 032004 (2022)

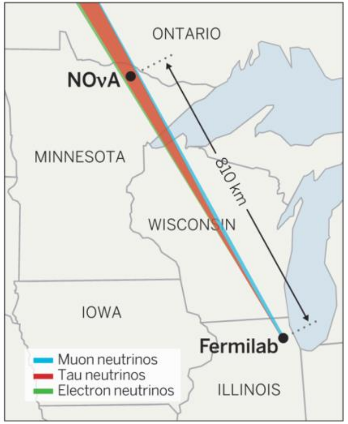
FIG. 6. The 68% and 90% confidence level contours in $\sin^2\theta_{23}$ vs. $\delta_{\rm CP}$ in the (a) normal mass ordering and (b) inverted mass ordering [95]. The cross denotes the NOvA best-fit point and colored areas depict the 90% and 68% FC corrected allowed regions for NOvA. Overlaid black solid-line and dashed-line contours depict allowed regions reported by T2K [91][3].

Non-standard (**NC&CC**) interactions of the neutrinos

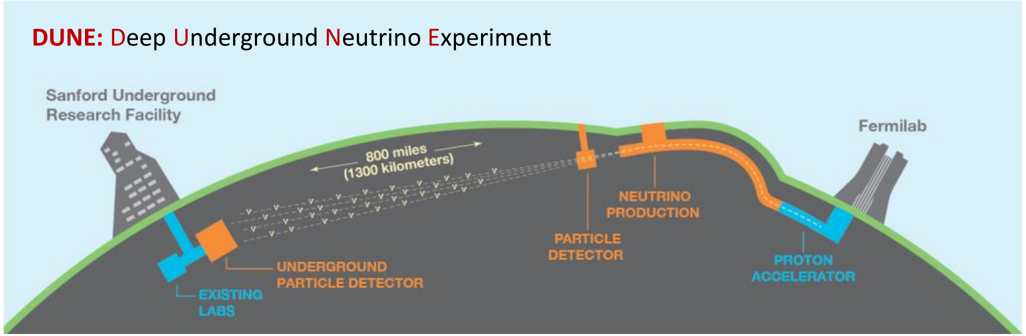
$$\mathcal{L}_{\rm NC-NSI} = -2\sqrt{2}G_F\varepsilon^{fC}_{\alpha\beta}(\overline{\nu_\alpha}\gamma^\mu P_L\nu_\beta)(\bar{f}\gamma_\mu P_C f)$$

$$\mathscr{L}_{\rm CC\text{-}NSI} = -2\sqrt{2}G_F\sum_{f,f',\alpha,\beta,P}\epsilon^{f,f',P}_{\alpha\beta}[\bar{\nu}_\beta\gamma^\mu P_L l_\alpha][\bar{f}\gamma_\mu P f']$$

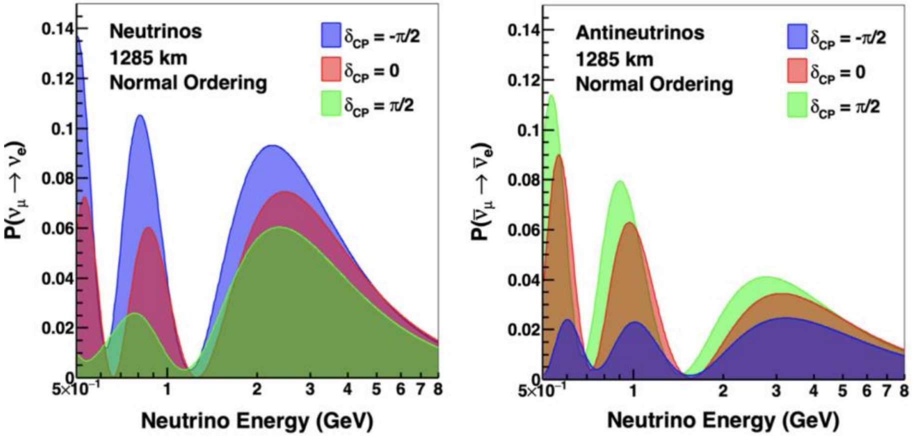
arXiv:2401.02901, Daya Bay

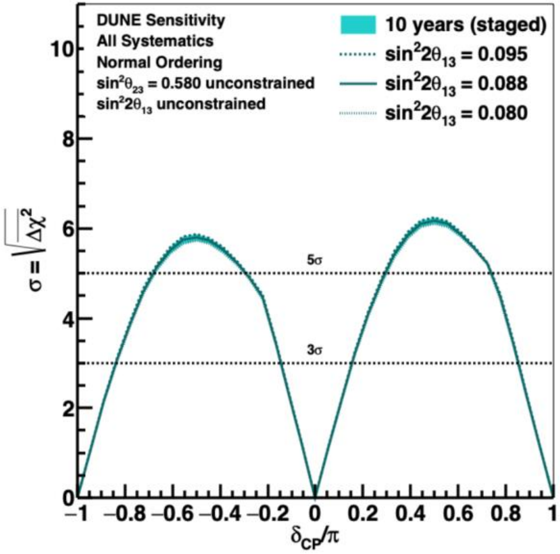


Probing CP phase: DUNE simulation



Under construction, to be completed in 2029 and start data taking in 2035 to 2040

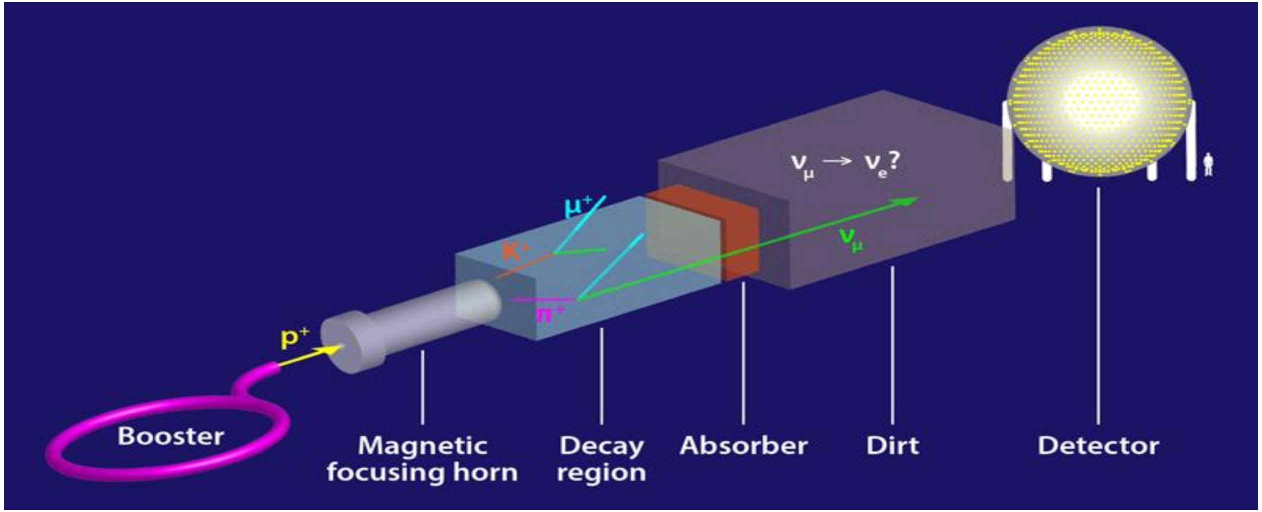




EPJC. 80 (2020) 10, 978

Accelerator neutrinos for Oscillation experiments

- **Conventional muon sources: accelerated proton-on-target**



Limitations

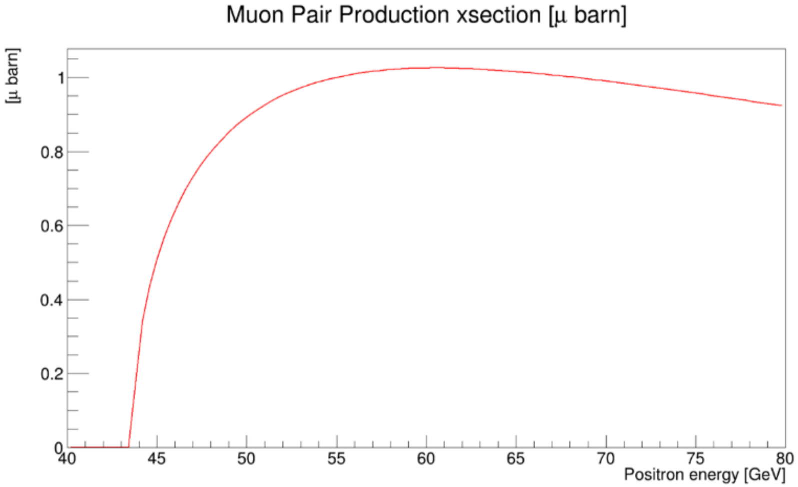
- Lower neutrino flux
- Limited neutrino energy spectrum
- Background contamination

APS Physics 11 (2018) 122

Positron driven muon sources for Anything!

Low **EM**ittance **M**uon **A**ccelerator (LEMMA)

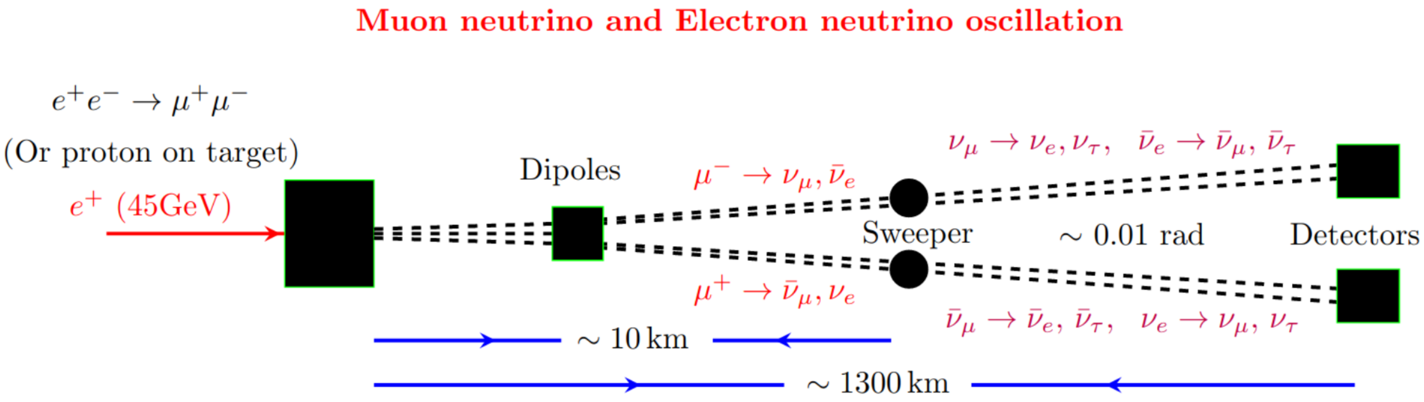
D. Alesini *et al*
arXiv:1905.05747



Positron driven muon sources for neutrino oscillation

arXiv:2301.02493

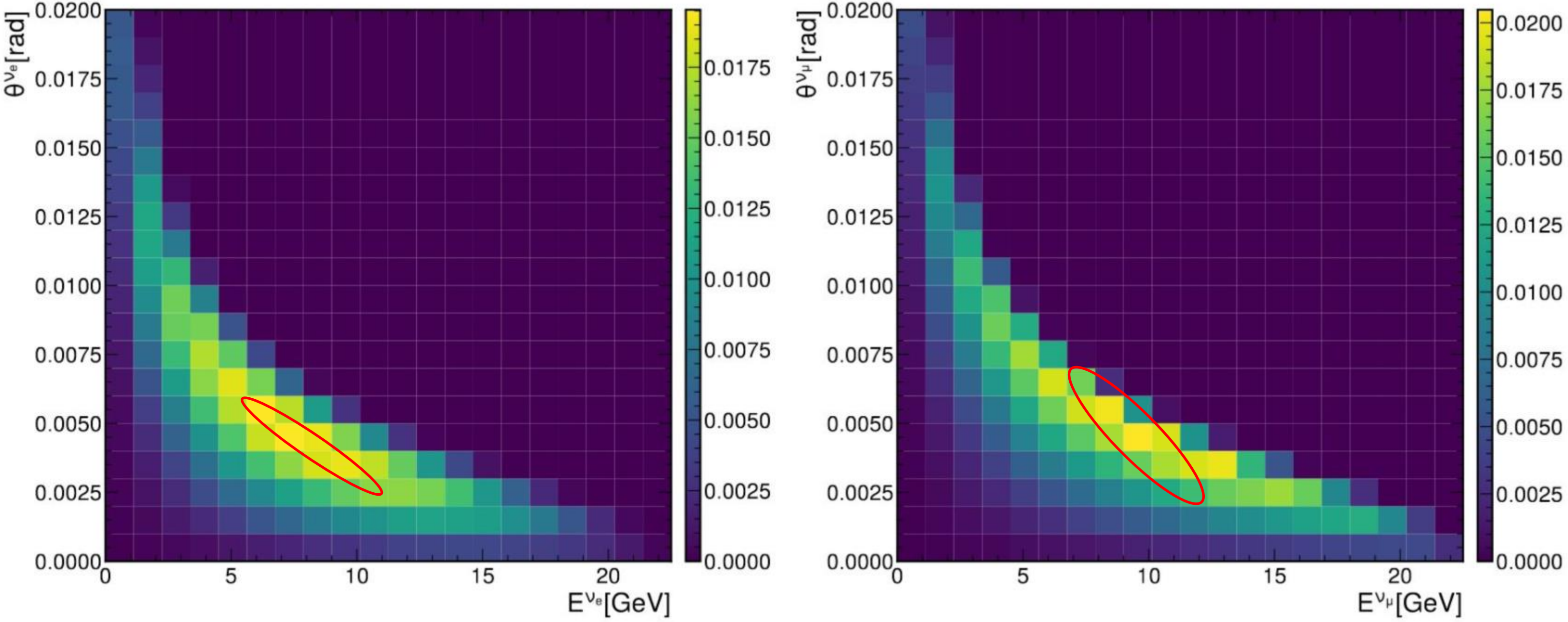
A. Ruzi & Qiang Li, et al



Muon neutrino and Electron neutrino oscillation

- **Collimated and manipulable** muon beams, which lead to a larger acceptance of neutrino sources in the far detector side.
- **Symmetric μ+ and μ– beams,** and thus symmetric neutrino and antineutrino sources, ideally useful for measuring neutrino CP violation.

Neutrino profile



5~10 GeV energy range > tau threshold

Oscillation probability in Matter

Series expansion of oscillation probability: ***JHEP*** **04 (2004) 078**

$$P_{\alpha\beta} = P_{\alpha\beta}(\Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{13}, \theta_{23}, \delta_{\rm CP}; E, L, V(x)), \quad \alpha, \beta = e, \mu, \tau$$

$$H \simeq \frac{1}{2E} U \,\mathrm{diag}(0, \Delta m^2_{21}, \Delta m^2_{31}) U^\dagger + \mathrm{diag}(V, 0, 0) \,.$$

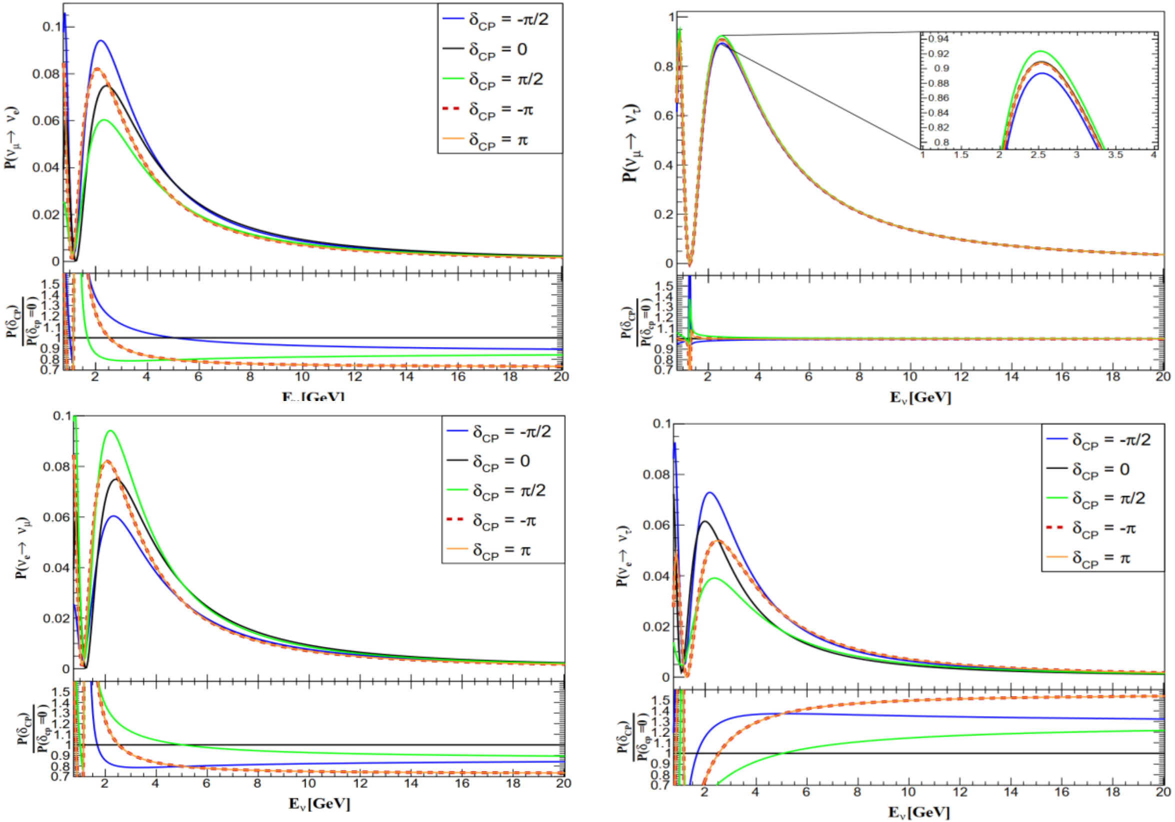
$$V(x) \simeq 7.56 \times 10^{-14} \left(\frac{\rho(x)}{\mathrm{g/cm^3}} \right) Y_e(x) \ \ \mathrm{eV}$$

Experimental Parameters	**Values**
Stored Muons	1×10^{20}
E_μ[GeV]	22.5 GeV
Run time	5 years
Matter density	$2.8\ g/cm^3$
Base line length	1300 Km
Target mass (Detector)	40 Kt Liquid Argon

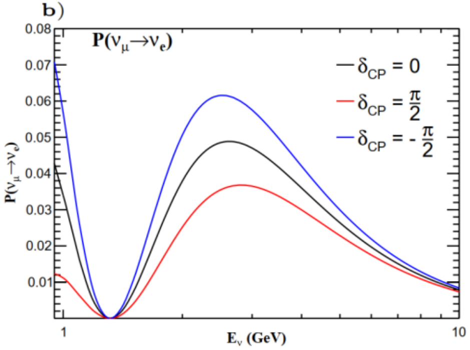
$$Y_e(x) = 0.5$$

$Y_e(x)$ is the number of electrons per nucleon. For the matter of the Earth.

Matter effects on the Oscillation probability

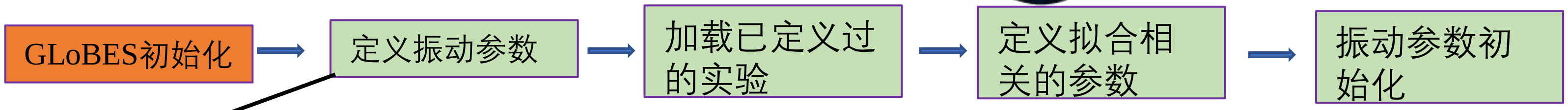


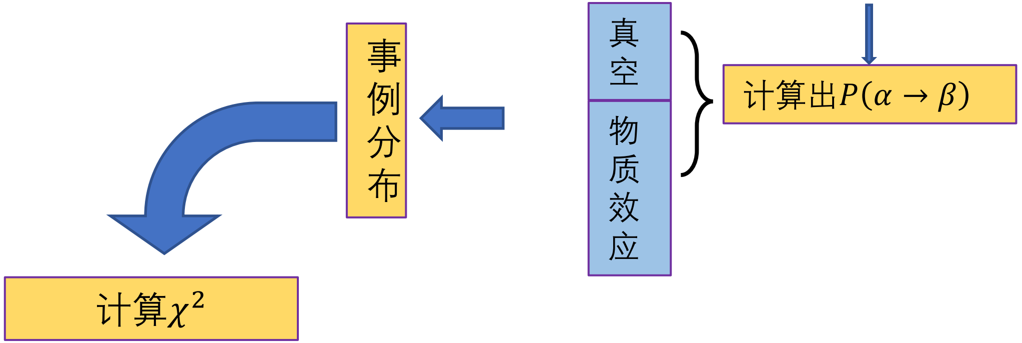
Oscillations in Vacuum

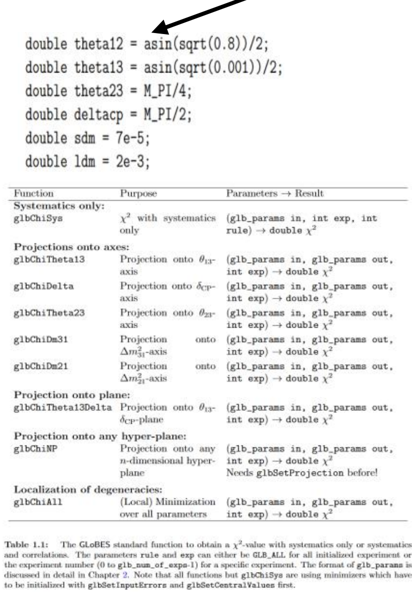


General Long Baseline Experiment Simulation

GLoBES
General Long Baseline Experiment Simulator



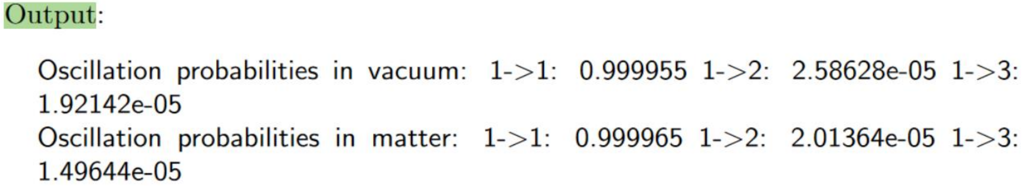




```
double theta12 = asin(sqrt(0.8))/2;
double theta13 = asin(sqrt(0.001))/2;
double theta23 = M_PI/4;
double deltacp = M_PI/2;
double sdm = 7e-5;
double ldm = 2e-3;
```

Function	Purpose	Parameters → Result
Systematics only:		
`glbChiSys`	χ^2 with systematics only	`(glb_params in, int exp, int rule)` → `double` χ^2
Projections onto axes:		
`glbChiTheta13`	Projection onto θ_{13}-axis	`(glb_params in, glb_params out, int exp)` → `double` χ^2
`glbChiDelta`	Projection onto δ_{CP}-axis	`(glb_params in, glb_params out, int exp)` → `double` χ^2
`glbChiTheta23`	Projection onto θ_{23}-axis	`(glb_params in, glb_params out, int exp)` → `double` χ^2
`glbChiDm31`	Projection onto Δm^2_{31}-axis	`(glb_params in, glb_params out, int exp)` → `double` χ^2
`glbChiDm21`	Projection onto Δm^2_{21}-axis	`(glb_params in, glb_params out, int exp)` → `double` χ^2
Projection onto plane:		
`glbChiTheta13Delta`	Projection onto θ_{13}-δ_{CP}-plane	`(glb_params in, glb_params out, int exp)` → `double` χ^2
Projection onto any hyper-plane:		
`glbChiNP`	Projection onto any n-dimensional hyper-plane	`(glb_params in, glb_params out, int exp)` → `double` χ^2 Needs `glbSetProjection` before!
Localization of degeneracies:		
`glbChiAll`	(Local) Minimization over all parameters	`(glb_params in, glb_params out, int exp)` → `double` χ^2

Table 1.1: The GLoBES standard function to obtain a χ^2-value with systematics only or systematics and correlations. The parameters `rule` and `exp` can either be `GLB_ALL` for all initialized experiment or the experiment number (0 to `glb_num_of_exps`-1) for a specific experiment. The format of `glb_params` is discussed in detail in Chapter 2. Note that all functions but `glbChiSys` are using minimizers which have to be initialized with `glbSetInputErrors` and `glbSetCentralValues` first.



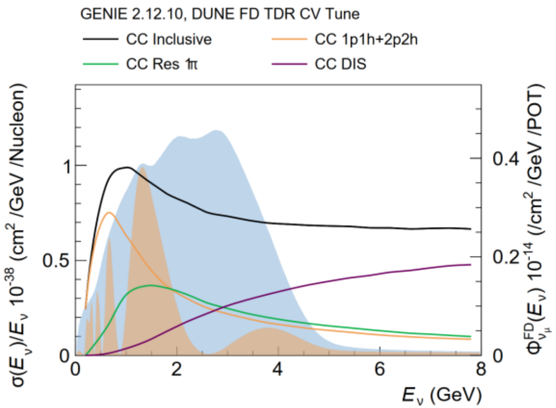
```
Output:

Oscillation probabilities in vacuum: 1->1: 0.999955 1->2: 2.58628e-05 1->3:
1.92142e-05
Oscillation probabilities in matter: 1->1: 0.999965 1->2: 2.01364e-05 1->3:
1.49644e-05
```

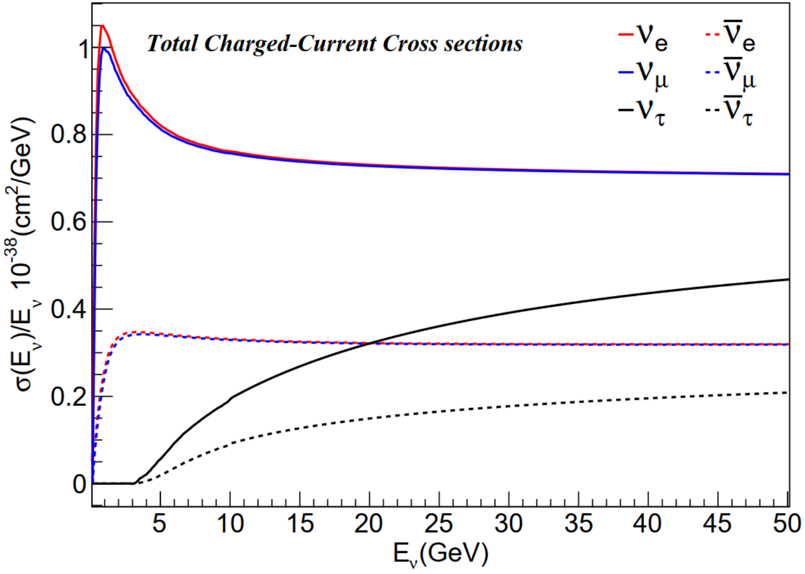
Neutrino CC interactions inside detector

Detection of neutrino is really challenging for most detectors. The Charge-Current process helps us detect neutrinos on the detector side!

Cross sections are true results obtained from GENIE simulation: *an event generator for neutrino nucleon interactions*



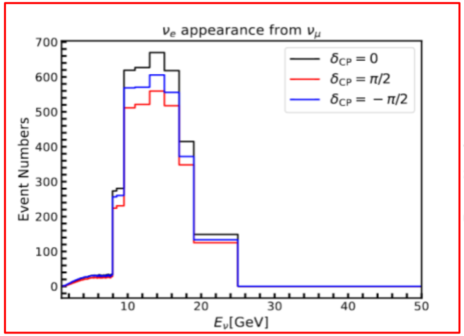
DUNE Collaboration • B. Abi (Oxford U.) et al.

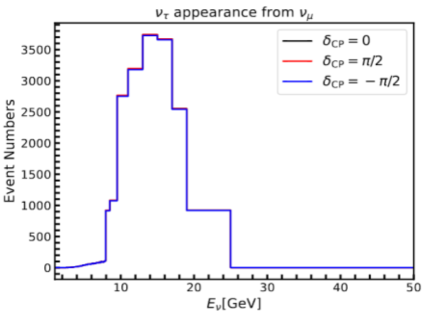


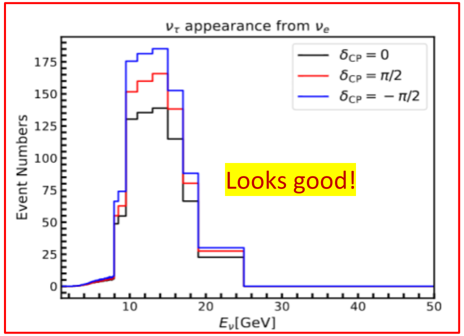
Event spectrum

- **Positron source:** positron bunch density $\mathbf{10^{12}}$**/bunch** with crossing frequency as $\mathbf{10^{5}/sec}$, which means $10^{17}/sec\ e^{+}$ on target. Eventually, we have **muon production rates** as $\frac{dN_\mu}{dt} \sim 10^{12}$/**sec or** $\mathbf{10^{19}}$**/year.**

n(μ) = 1.e20, L = 1300 Km, Detector Mass = 4万吨液氩, 运行5年







Looks good!

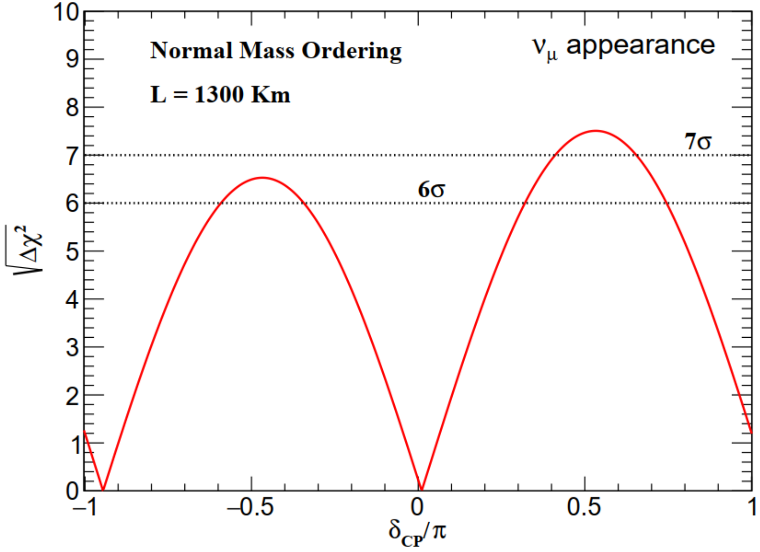
- $\boldsymbol{\nu_\mu \rightarrow \nu_e}$: the basic channel used by many neutrino oscillation experiment and shows fairly good sensitivity on $\boldsymbol{\delta_{CP}}$ here
- $\boldsymbol{\nu_\mu \rightarrow \nu_\tau}$: gives the largest tau neutrino events, but poor sensitivity on $\boldsymbol{\delta_{CP}}$
- $\boldsymbol{\nu_e \rightarrow \nu_\tau}$: gives fairly good sensitivity too!

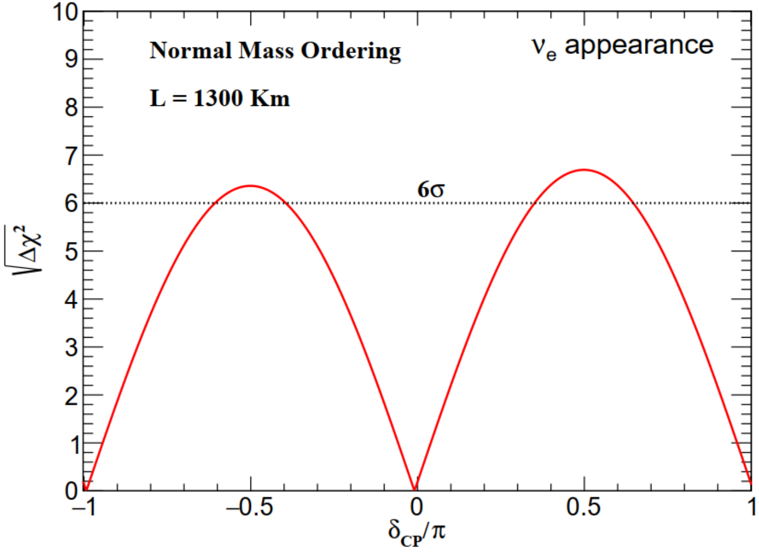
Sensitivity on δ_{CP}

$$\chi^2 = \frac{(N^{\rm true} - N^{\rm test})^2}{N^{\rm true}}$$

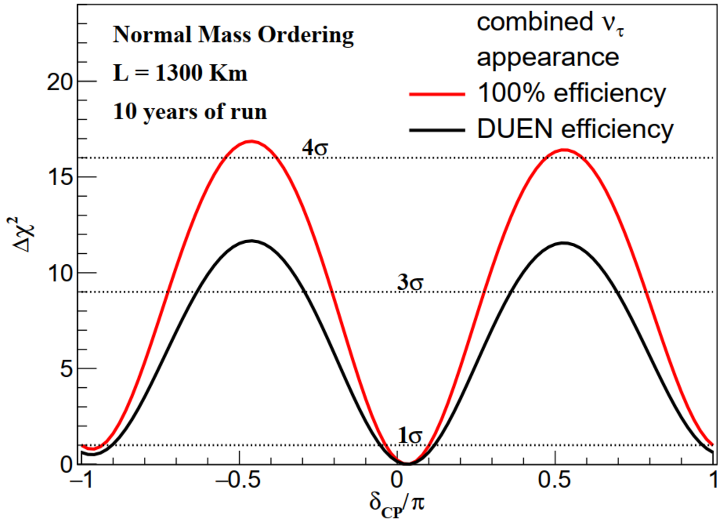
N^{true}: Events produced using $\delta_{CP} = \frac{\pi}{2}$.

N^{test}: Events simulated using $\delta^{CP} = 0\ or\ \pi$





Significance (2)



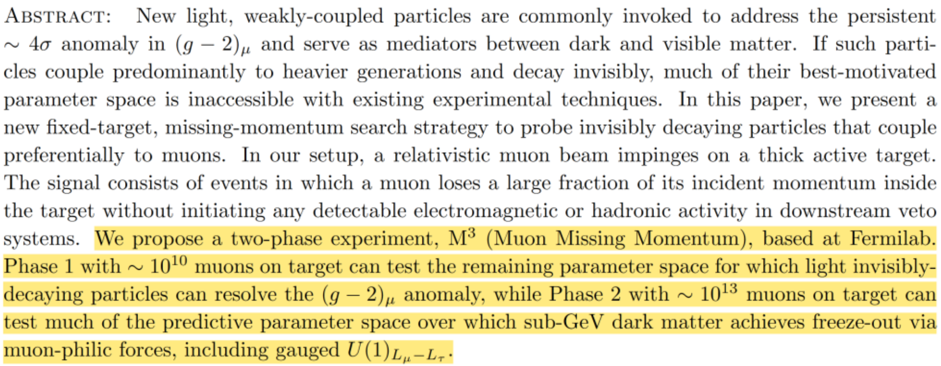
Now formally accepted by Nature Communications Physics
orcid.org/0000-0002-9569-8231

Extra remark on Muon beams for Dark Matter detection

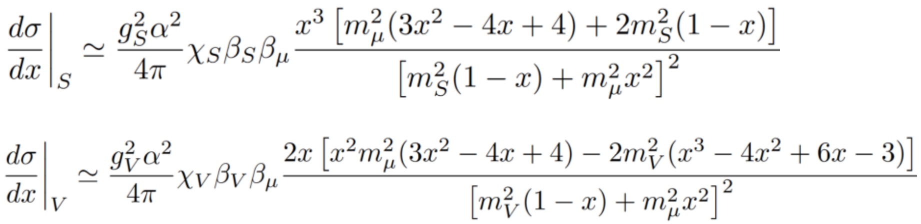
Detecting Muon-Philic Dark Matter

Direct Dark Matter search using muon beams: M^3 experiment

M^3: a new muon missing momentum experiment to probe $(g-2)_\mu$ and dark matter at Fermilab ------- *JHEP 09 (2018), 153*



Abstract: New light, weakly-coupled particles are commonly invoked to address the persistent $\sim 4\sigma$ anomaly in $(g-2)_\mu$ and serve as mediators between dark and visible matter. If such particles couple predominantly to heavier generations and decay invisibly, much of their best-motivated parameter space is inaccessible with existing experimental techniques. In this paper, we present a new fixed-target, missing-momentum search strategy to probe invisibly decaying particles that couple preferentially to muons. In our setup, a relativistic muon beam impinges on a thick active target. The signal consists of events in which a muon loses a large fraction of its incident momentum inside the target without initiating any detectable electromagnetic or hadronic activity in downstream veto systems. We propose a two-phase experiment, M^3 (Muon Missing Momentum), based at Fermilab. Phase 1 with $\sim 10^{10}$ muons on target can test the remaining parameter space for which light invisibly-decaying particles can resolve the $(g-2)_\mu$ anomaly, while Phase 2 with $\sim 10^{13}$ muons on target can test much of the predictive parameter space over which sub-GeV dark matter achieves freeze-out via muon-philic forces, including gauged $U(1)_{L_\mu - L_\tau}$.

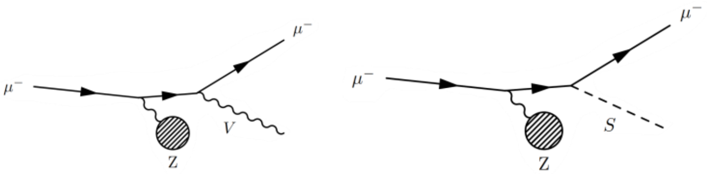


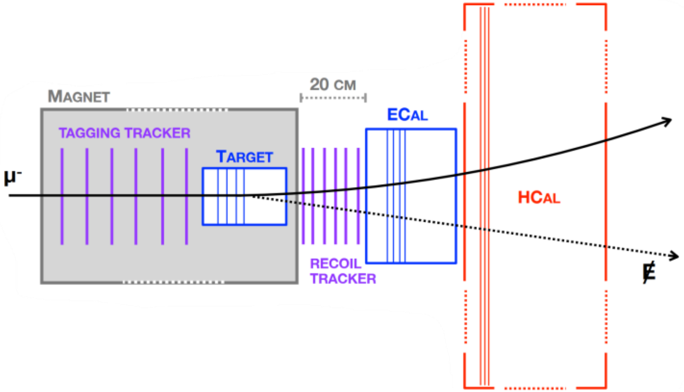
$$\left.\frac{d\sigma}{dx}\right|_S \simeq \frac{g_S^2\alpha^2}{4\pi}\chi_S\beta_S\beta_\mu \frac{x^3\left[m_\mu^2(3x^2-4x+4)+2m_S^2(1-x)\right]}{\left[m_S^2(1-x)+m_\mu^2x^2\right]^2}$$

$$\left.\frac{d\sigma}{dx}\right|_V \simeq \frac{g_V^2\alpha^2}{4\pi}\chi_V\beta_V\beta_\mu \frac{2x\left[x^2m_\mu^2(3x^2-4x+4)-2m_V^2(x^3-4x^2+6x-3)\right]}{\left[m_V^2(1-x)+m_\mu^2x^2\right]^2}$$

Vector-like

Scalar-like





Free muon beams for detecting Dark Matter

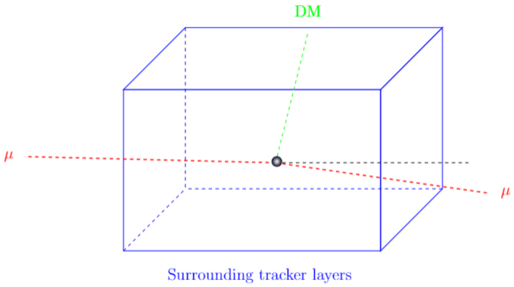
Int.J.Mod.Phys.A 38 (2023) 29n30, 2350154

- New proposal using atmospheric muons or accelerator muons
  - Highly energetic muons from cosmic rays are collided by surrounding dark matter particles that preferentially interacts with muons. Muons obtains some recoil energy dependently on DM mass as below,

$$E_{\rm recoil}^{\rm max} = \frac{(2 \times \mathrm{M_D} \times v)^2}{\mathrm{M}_\mu}$$

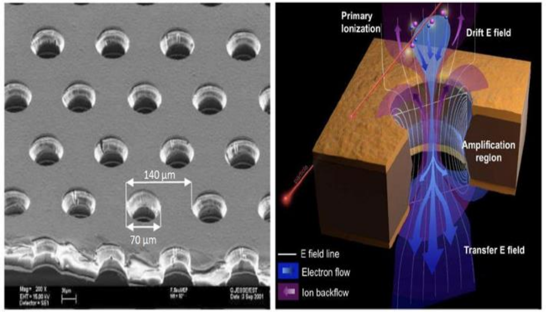
  - v = 300 km/s, velocity of DM near earth.
  - $\mathrm{M_D}$ is mass of the DM particle

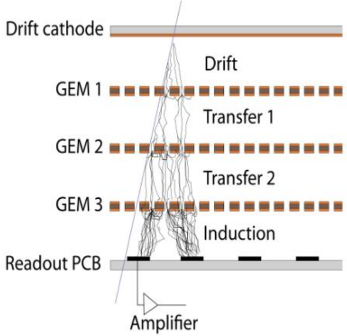
The maximum velocity for muon can be 0.1-10 km/s for M_D\sim 1-10 MeV, then the maximum shift of muons from the original beam axis will be 10-100 microns for a cubic device with a length of 1 meter



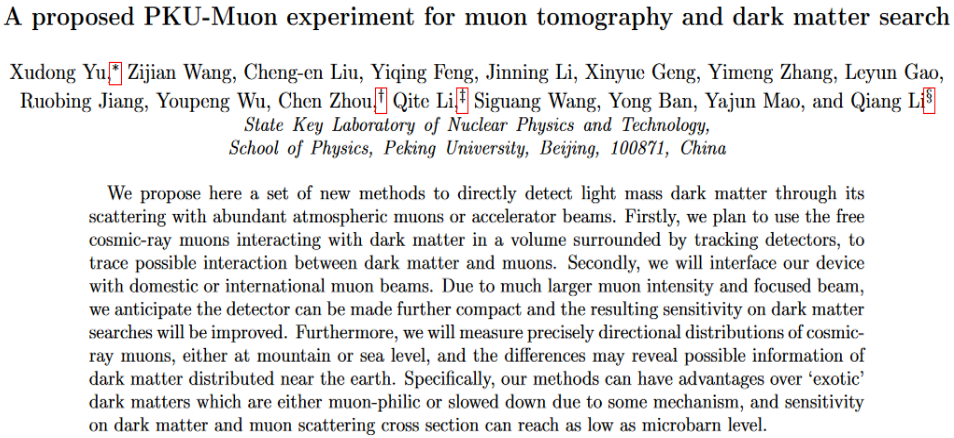
- DM flux can be estimated to $\mathbf{10^{10}\,cm^{-2}\,s^{-1}}$for $\mathbf{M_D \sim 1\ MeV}$, then the scattering rate will be $\boldsymbol{\Phi_{DM} \times \sigma_D \times N_\mu}$, the muon number inside the detector can be estimated to be 1000-10000, based on muon flux at sea level.

- Within one year, the sensitivity on cross section $\boldsymbol{\sigma_D \sim 10^{-21}\,\mathrm{cm}^2}$





Free muon beams for detecting Dark Matter



A proposed PKU-Muon experiment for muon tomography and dark matter search

Xudong Yu,[*] Zijian Wang, Cheng-en Liu, Yiqing Feng, Jinning Li, Xinyue Geng, Yimeng Zhang, Leyun Gao,
Ruobing Jiang, Youpeng Wu, Chen Zhou,[†] Qite Li,[‡] Siguang Wang, Yong Ban, Yajun Mao, and Qiang Li[§]
State Key Laboratory of Nuclear Physics and Technology,
School of Physics, Peking University, Beijing, 100871, China

We propose here a set of new methods to directly detect light mass dark matter through its scattering with abundant atmospheric muons or accelerator beams. Firstly, we plan to use the free cosmic-ray muons interacting with dark matter in a volume surrounded by tracking detectors, to trace possible interaction between dark matter and muons. Secondly, we will interface our device with domestic or international muon beams. Due to much larger muon intensity and focused beam, we anticipate the detector can be made further compact and the resulting sensitivity on dark matter searches will be improved. Furthermore, we will measure precisely directional distributions of cosmic-ray muons, either at mountain or sea level, and the differences may reveal possible information of dark matter distributed near the earth. Specifically, our methods can have advantages over 'exotic' dark matters which are either muon-philic or slowed down due to some mechanism, and sensitivity on dark matter and muon scattering cross section can reach as low as microbarn level.

arXiv:2402.13483

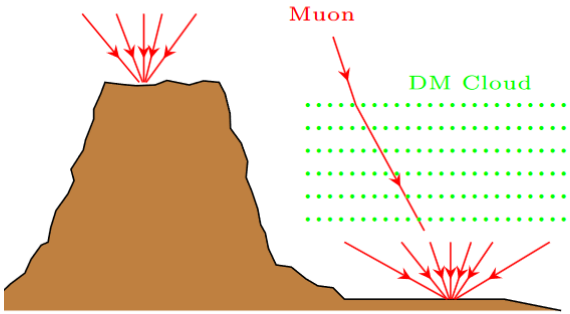
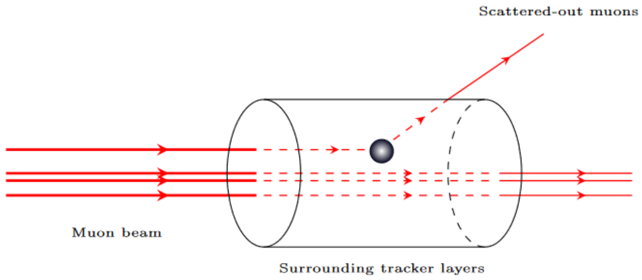


FIG. 3. Illustration of an experiment to detect muon-philic DM through precisely measuring directional distributions of cosmic-ray muons, either at mountain or sea level.



Summary

- Neutrino oscillation is one of the observed physical phenomenon beyond Standard Model, still contains undiscovered physics.
- CP violation in neutrino oscillation still demands compelling data from super-beam experiments.
- LEMMA approach may provide better Muon sources in the super-beam experiments, **HyperK and DUNE.**
- Muons may also enable us to discover light-mass dark matter particle that interacts with muons.

Thanks a lot for your attention!

Back Ups

To do list for future work

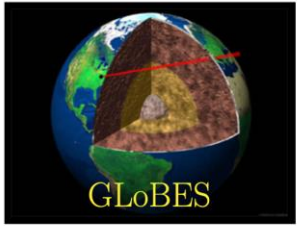
Neutrino Physics

Theory and pheno

- **Standard and non-standard oscillation (*goal of SK, HK and DUNE*)**
  - ✓ Sensitivity on CP phase (being worked out)
  - ✓ Modification of PMNS-matrix
  - ✓ Search for sterile neutrino and do sensitivity check on the new mixing parameters
  - ✓ Neutrino Global fit (precision measurements of mixing parameters and Δm^2)
- **Neutrino mass problem (Hard)**
  - ✓ Origin of neutrino mass (EFT approach: Weinberg operator)
  - ✓ Solving mass ordering problem (matter effects can help)

Software

- **GLoBES: neutrino oscillation simulator**
  - ✓ Simulation of neutrino experiment (Nuclear, accelerator and atmospheric neutrino)
  - ✓ χ^2 analysis: projections on $\theta_{ij}, \delta_{CP}, \Delta m^2_{ij}$
- **Genie: Neutrino event generator**
  - ✓ Cross section calculation
  - ✓ Detector simulation
  - ✓ Neutrino-target experiment, νN DIS
  - ✓ Elastic and DIS Dark matter-Nucleon cross section and event generation

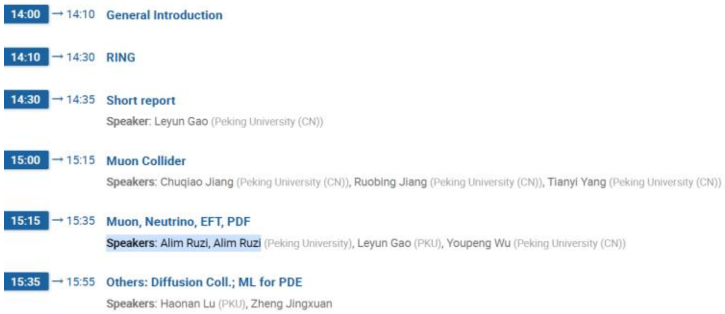


Version from May 5, 2020 for GLoBES 3.2.18

Main activities in Current group

- **Particle physics (Theory)**
- **Paper writing**
- **Guiding new students (An undergraduate student)**

Discussion on Future collider and
Neutrino physics study



14:00 → 14:10 **General Introduction**

14:10 → 14:30 **RING**

14:30 → 14:35 **Short report**
Speaker: Leyun Gao (Peking University (CN))

15:00 → 15:15 **Muon Collider**
Speakers: Chuqiao Jiang (Peking University (CN)), Ruobing Jiang (Peking University (CN)), Tianyi Yang (Peking University (CN))

15:15 → 15:35 **Muon, Neutrino, EFT, PDF**
Speakers: Alim Ruzi, Alim Ruzi (Peking University), Leyun Gao (PKU), Youpeng Wu (Peking University (CN))

15:35 → 15:55 **Others: Diffusion Coll.; ML for PDE**
Speakers: Haonan Lu (PKU), Zheng Jingxuan

PMNS matrix

- The PMNS matrix is usually expressed by 3 rotation matrices and three complex phases:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta_{23} & sin\theta_{23} \\ 0 & -sin\theta_{23} & cos\theta_{23} \end{pmatrix} \begin{pmatrix} cos\theta_{13} & 0 & sin\theta_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -sin\theta_{13}e^{i\delta_{CP}} & 0 & cos\theta_{13} \end{pmatrix} \begin{pmatrix} cos\theta_{12} & sin\theta_{12} & 0 \\ -sin\theta_{12} & cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & e^{i\gamma} \end{pmatrix}$$

Quantifies the CP violation effect in the neutrino oscillation because $\boldsymbol{\theta_{13} \neq 0}$

Two flavor oscillation

Majorana phase, can ignored (for now)

- Ignoring the Majorana phases, we find that, when multiplied out, the PMNS matrix becomes

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

$$\boldsymbol{c_{ij} = \cos\theta_{ij},\ \ s_{ij} = \sin\theta_{ij}}$$

PhysRevLett.51.1945

Neutrino oscillation in vacuum

$$P(\nu_\mu \to \nu_\tau) \simeq \sin^2(2\theta_{23})\cos^4(\theta_{13})\sin^2\left(1.27\frac{\Delta m^2_{32}L}{E_\nu}\right) \pm 1.27\Delta m^2_{21}\frac{L}{E_\nu}\sin^2\left(1.27\frac{\Delta m^2_{32}L}{E_\nu}\right)\times 8J_{\mathrm{CP}},$$

$$P(\nu_\mu \to \nu_e) \simeq \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right) \mp 1.27\Delta m^2_{21}\frac{L}{E_\nu}\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right)\times 8J_{\mathrm{CP}},$$

$$P(\nu_e \to \nu_\tau) \simeq \sin^2(2\theta_{13})\cos^2(\theta_{23})\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right) \mp 1.27\Delta m^2_{21}\frac{L}{E_\nu}\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right)\times 8J_{\mathrm{CP}},$$

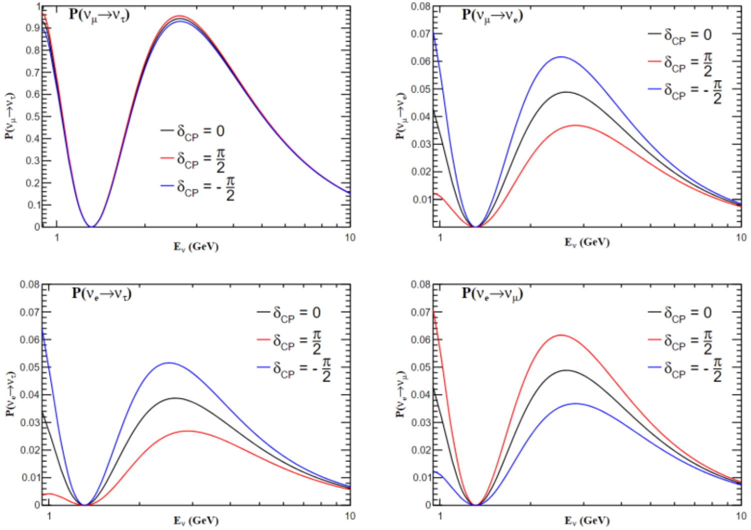
$$P(\nu_e \to \nu_\mu) \simeq \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right) \pm 1.27\Delta m^2_{21}\frac{L}{E_\nu}\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right)\times 8J_{\mathrm{CP}},$$

$$P(\nu_\mu \to \nu_\tau) = 0.2916 \pm 0.0026\sin\delta_{\mathrm{CP}}\ (0.5093 \pm 0.0048\sin\delta_{\mathrm{CP}}),$$

$$P(\nu_\mu \to \nu_e) = 0.0151 \mp 0.0026\sin\delta_{\mathrm{CP}}\ (0.0264 \mp 0.0048\sin\delta_{\mathrm{CP}}),$$

$$P(\nu_e \to \nu_\mu) = 0.0151 \pm 0.0026\sin\delta_{\mathrm{CP}}\ (0.0264 \pm 0.0048\sin\delta_{\mathrm{CP}}),$$

$$P(\nu_e \to \nu_\tau) = 0.0119 \mp 0.0026\sin\delta_{\mathrm{CP}}\ (0.0209 \mp 0.0048\sin\delta_{\mathrm{CP}}).$$



Neutrino oscillation in matter

$$\begin{aligned}
P_{\mu\tau} = {} & \sin^2 2\theta_{23}\, \sin^2\Delta - \alpha\, c_{12}^2\, \sin^2 2\theta_{23}\, \Delta\, \sin 2\Delta + \alpha^2\, c_{12}^4\, \sin^2 2\theta_{23}\, \Delta^2\, \cos 2\Delta \\
& - \frac{1}{2A}\alpha^2\, \sin^2 2\theta_{12}\, \sin^2 2\theta_{23} \left(\sin\Delta\, \frac{\sin A\Delta}{A}\, \cos(A-1)\Delta - \frac{\Delta}{2}\, \sin 2\Delta\right) \\
& + \frac{2}{A-1}\, s_{13}^2\, \sin^2 2\theta_{23} \left(\sin\Delta\, \cos A\Delta\, \frac{\sin(A-1)\Delta}{A-1} - \frac{A}{2}\Delta\, \sin 2\Delta\right) \\
& + 2\,\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \sin\delta_{\mathrm{CP}}\, \sin\Delta\, \frac{\sin A\Delta}{A}\, \frac{\sin(A-1)\Delta}{A-1} \\
& - \frac{2}{A-1}\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \cos 2\theta_{23}\, \cos\delta_{\mathrm{CP}}\, \sin\Delta \left(A\sin\Delta - \frac{\sin A\Delta}{A}\, \cos(A-1)\Delta\right)
\end{aligned}$$

$$\begin{aligned}
P_{e\mu} = {} & \alpha^2\, \sin^2 2\theta_{12}\, c_{23}^2\, \frac{\sin^2 A\Delta}{A^2} + 4\, s_{13}^2\, s_{23}^2\, \frac{\sin^2(A-1)\Delta}{(A-1)^2} \\
& + 2\,\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23} \cos(\Delta - \delta_{\mathrm{CP}})\, \frac{\sin A\Delta}{A}\, \frac{\sin(A-1)\Delta}{A-1}
\end{aligned}$$

$$\begin{aligned}
P_{e\tau} = {} & \alpha^2\, \sin^2 2\theta_{12}\, s_{23}^2\, \frac{\sin^2 A\Delta}{A^2} + 4\, s_{13}^2\, c_{23}^2\, \frac{\sin^2(A-1)\Delta}{(A-1)^2} \\
& - 2\,\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23} \cos(\Delta - \delta_{\mathrm{CP}})\, \frac{\sin A\Delta}{A}\, \frac{\sin(A-1)\Delta}{A-1}
\end{aligned}$$

Akhmedov， Johansson， Lindner， J. High Energy Phys. 2004-05-05

Event simulation in GLoBES

$$n_i^c = N/L^2 \int_{E_i-\Delta E_i/2}^{E_i+\Delta E_i/2} dE' \int_0^{\infty} dE \; \Phi^c(E)\; P^c(E)\; \sigma^c(E)\; R^c(E,E')\, \epsilon^c(E')$$

- N: renormalization factor.
- L : baseline length.
- E: energy of incoming neutrino.
- E': reconstructed energy.
- Φ^c: incoming neutrino flux in specific channel.
- $P^c(E)$: oscillation probability.
- $\sigma(E)$: cross section of neutrino-nucleus interaction inside detector.
- $R^c(E,E')$: Energy resolution function.
- $\epsilon^c(E')$: Post smearing efficiency , or energy efficiency.

$$R^c(E,E') = \frac{1}{\sigma(E)\sqrt{2\pi}}\, e^{-\frac{(E-E')^2}{2\sigma^2(E)}}$$

$$\sigma(E) = \alpha \cdot E + \beta \cdot \sqrt{E} + \gamma$$