Simulation Study of Muon Beam CT and Integration of MuDirac and Geant4 for Muonic X-ray Generation

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Experimental Muon Source (EMuS)

- EMuS was a research proposal for a muon facility at the China Spallation Neutron Source (CSNS) under the *NSFC Fund for Research on National Major Research Instruments* (2016–2021).

Research Article | Full Access

EMuS – A Pulsed Muon Facility for Multidisciplinary Research

Jing-Yu Tang, Ye Yuan, Bang-Jiao Ye, Zi-An Zhu, Zhi-Long Hou, Nikolaos Vassilopoulos, Jian Tang, Yu Chen, Yu-Kai Chen, Chang-Dong Deng, Jing-Yu Dong, Rui-Rui Fan, Chong-Chao He ... **See all authors**

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- Two schemes have been designed:

1. Phase-I or the simplified scheme →
   - Surface muon for μSR applications
   - Negative cloud muon for muonic X-ray analysis

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2. Phase-II or the baseline scheme →
   - Surface and decay muons for μSR
   - High-momentum decay muon for imaging
   - Negative decay muon for depth-selective muonic X-ray analysis

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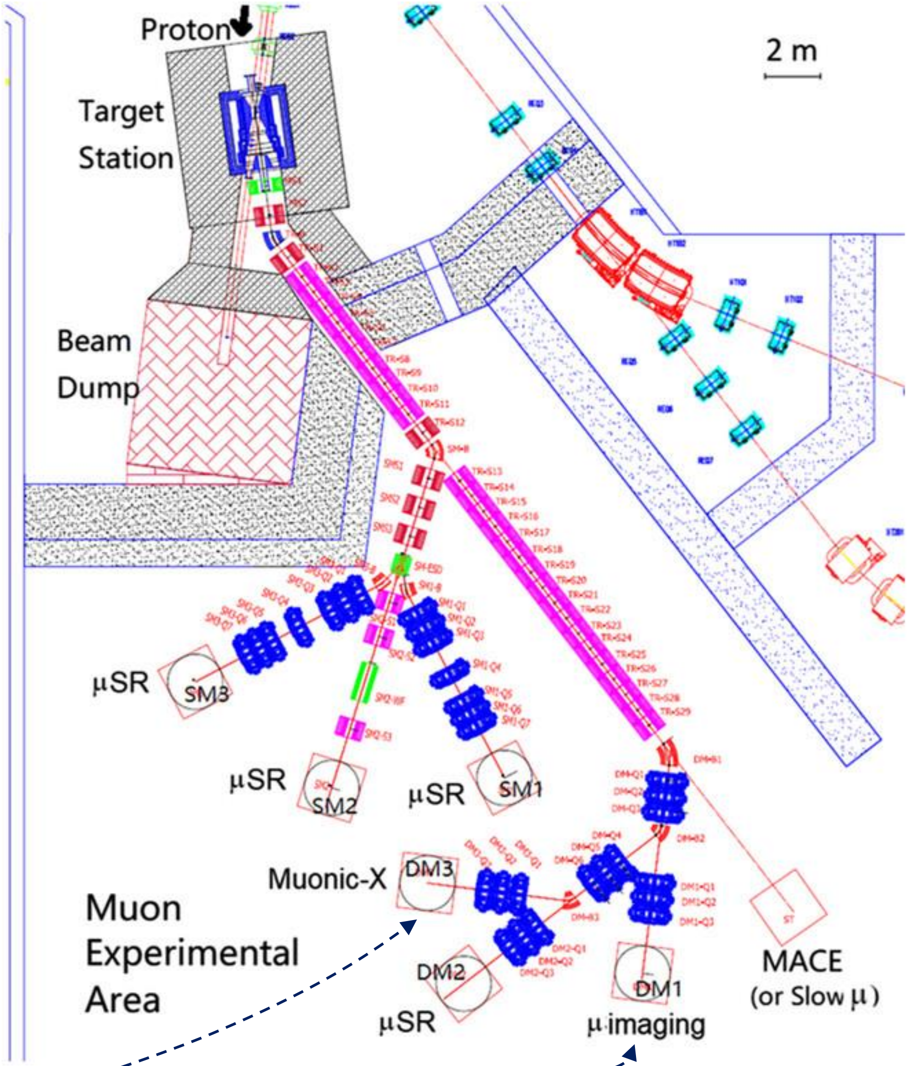
EMuS Baseline Scheme

- CSNS accelerator facility provides a proton beam with a kinetic energy of 1.6 GeV, and a repetition rate of 25 Hz. It currently has a beam power of 140 kW, which can be upgraded to 500 kW at Phase-II. EMuS will take about 5% or 25 kW of the total beam power (500 kW) from the CSNS-II accelerator complex.
- The high-momentum muon beams for imaging operate at a repetition rate of 2.5 Hz, with a momentum of up to 450 MeV/c and a beam intensity of 7.9×10^{8}.

End station	Momentum [MeV c^{-1}]	FWHM spot size [mm]	Intensity [10^6 μ s^{-1}]	Polar-z	$\Delta p/p$ [FWHM]
DM1	450	108 | 101	790	−0.760	9.7%
DM2	45	29.1 | 28.6	0.25	0.839	6.5%
	150	30.5 | 31.2	21	0.761	8.0%
DM3	45	31.2 | 25.8	0.24	−0.858	9.1%
	150	30.3 | 30.6	10	−0.681	9.1%



EMuS baseline scheme layout

- DM1 endstation: high-momentum muon beam imaging
- DM3 endstation: muonic X-ray analysis

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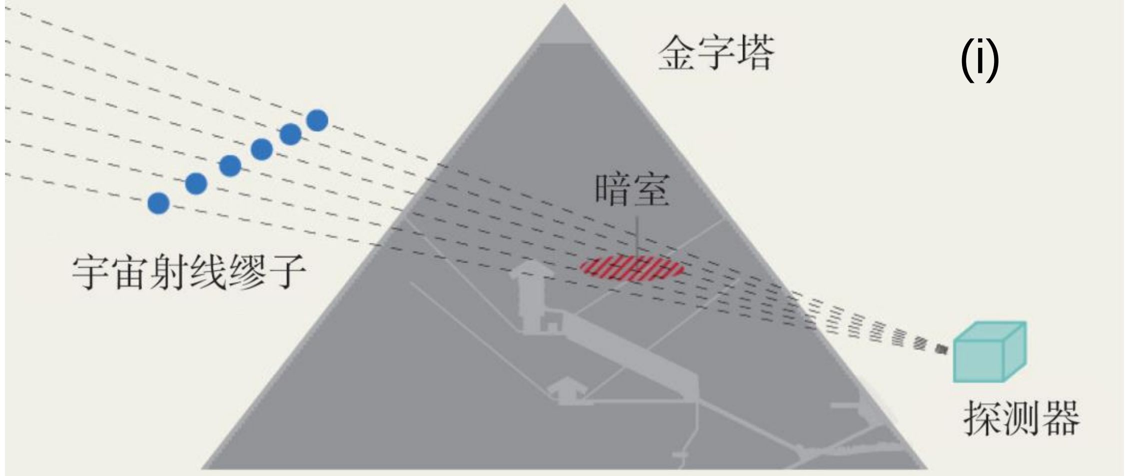
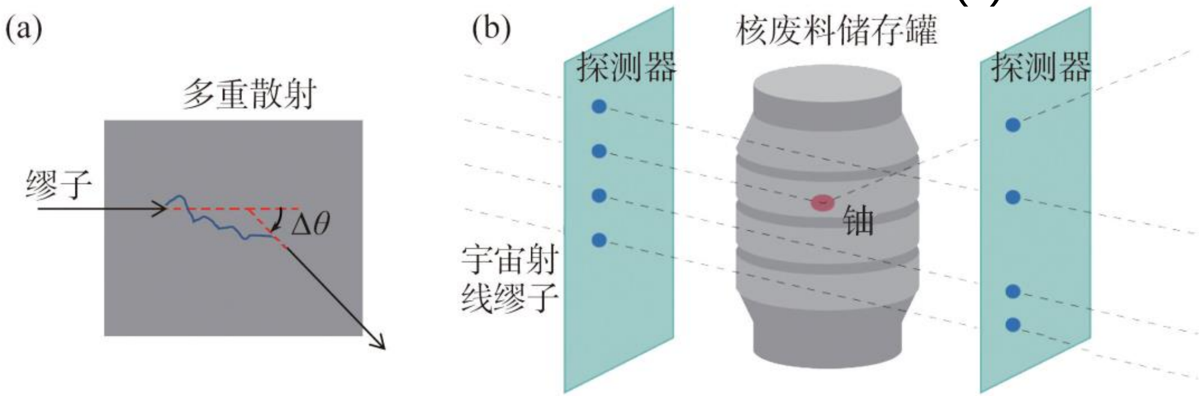
Cosmic-ray Muon Imaging Methods

- Cosmic-ray muons
  - Average energy ~4 GeV
  - Up to 100 TeV
  - ~1/(cm^2·s) at sea level
- Cosmic-ray muons have been used to image large objects
  - Transmission and absorption imaging
  - Muon scattering imaging(Muon tomography)



(ii)



Cosmic-ray muon imaging methods:
(i) transmission and absorption
(ii) scattering

Muon Beam CT

- Accelerator-based muon beam
  - ✓ **Controllable momentum**
  - ✓ **High intensity**
- Penetrating power of 450 MeV/c (360 MeV) muons
  - Could be used for imaging macroscopic samples
- The baseline scheme of EMuS provides possibility of imaging using high-momentum muon beam

End station	Momentum [MeV c^{-1}]	FWHM spot size [mm]	Intensity [10^6 μ s^{-1}]	Polar-z	$\Delta p/p$ [FWHM]
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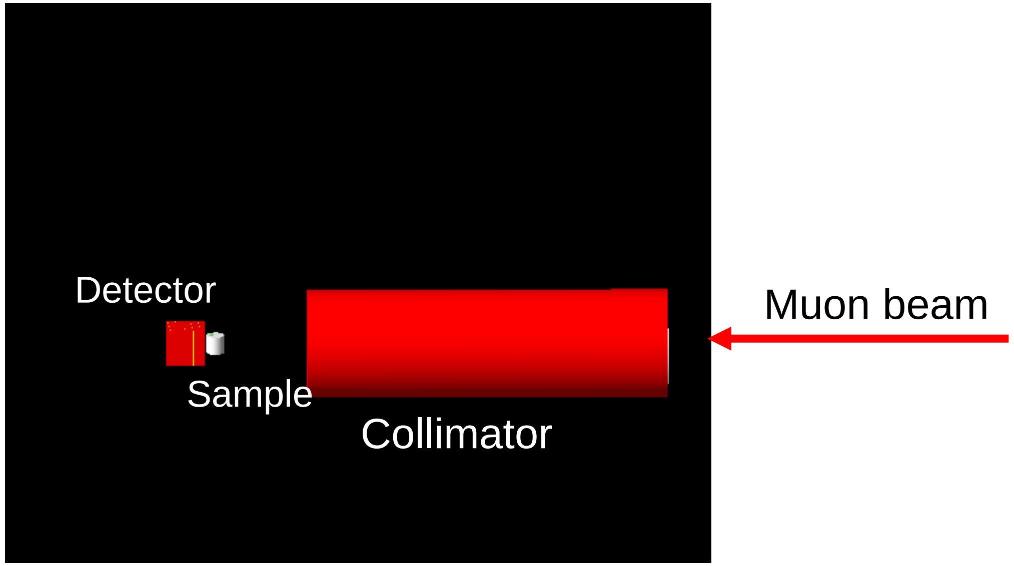
	150	30.3 | 30.6	10	−0.681	9.1%

Material	**Range [mm]**
Aluminum	718
Iron	271
Copper	248
Lead	238
Tungsten	137

Corresponding to ranges of 590 MeV proton

Simulation Study of Muon Beam CT

1. Projection Value Calculation → 2. Detector Design → 3. Collimator Design → 4. Result



Geant4 simulation

Projection Value

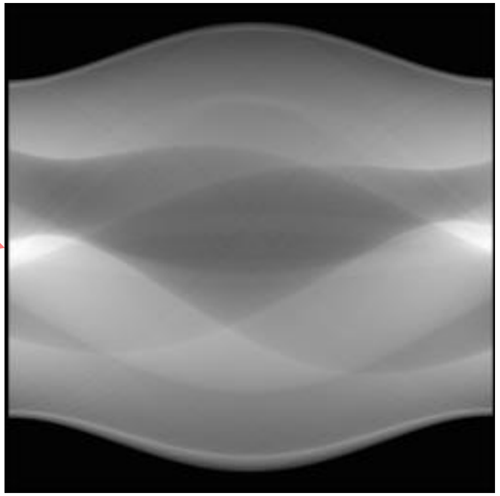
- As a tomographic imaging technique, CT images cannot be directly obtained from the detector data. Instead, the data is used to calculate projection values (PVs).
- Then, the calculated PVs are used as inputs for the reconstruction algorithm, which is used to produce the final image.

- Common CT reconstruction algorithms
  - Algebraic Reconstruction Technique (ART, this work)
  - Filtered Back Projection (FBP)
  - Maximum Likelihood Expectation Maximization (MLEM)

- The absorption of X-rays follows the Lambert-Beer law

$$I = I_0 e^{-u(x,y)\cdot l},$$

where u is the linear attenuation coefficient, I_0 the incident light intensity, and I the transmitted light intensity.



- For X-ray CT, the PV p is

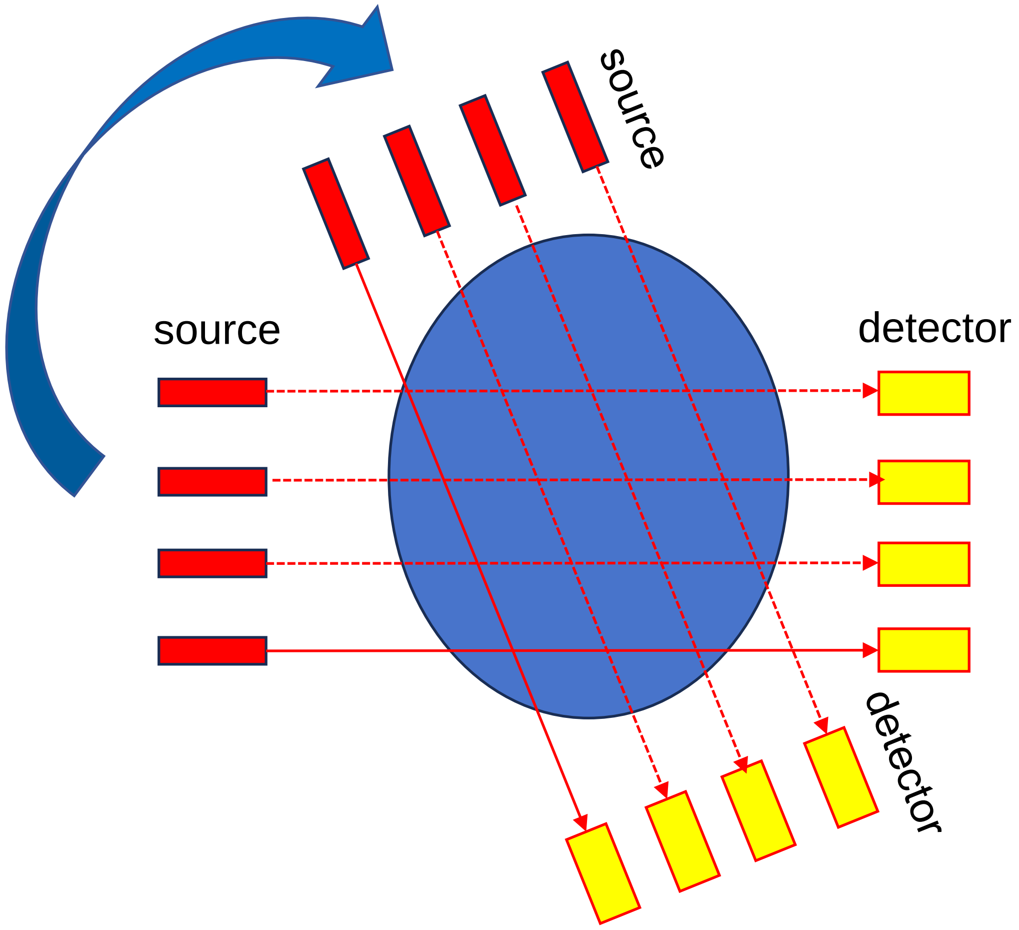
$$p = \ln \frac{I_0}{I} = \int_l u(x,y)\,\mathrm{d}l.$$

- When mono-energetic X-rays pass through a homogeneous medium, the PV is proportional to the distance traveled by the X-rays:

$$p = \ln \frac{I_0}{I} = u(x,y) \cdot l.$$

This forms the basis of CT reconstruction algorithm.

- In early CT scans, the source and detector move at an angle theta to cover the entire cross-section.
- The detector and collimator rotate multiple times to collect data from multiple angles.
- Nowadays, X-ray CT employs conical beams and detectors with larger areas to achieve multi-layer imaging at once.



- Proton CT (pCT) uses the energy loss of protons for imaging.
  - pCT can assist in proton therapy.
  - Proton beams usually from a cyclotron.
  - The projection value of pCT is:

$$\frac{dE}{dx} = -K\frac{Z}{A}\frac{1}{\beta^2(E)}\left[\ln\left(\frac{2m_e c^2\beta^2(E)}{I(r)(1-\beta^2(E))}\right) - \beta^2(E)\right],$$

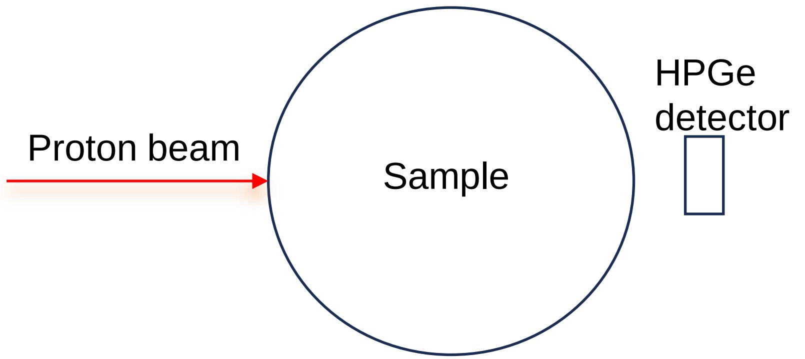
$$S = \frac{K}{\beta^2(E)}\left[\ln\left(\frac{2m_e c^2\beta^2(E)\gamma^2(E)}{I(r)}\right) - \beta^2(E)\right],$$

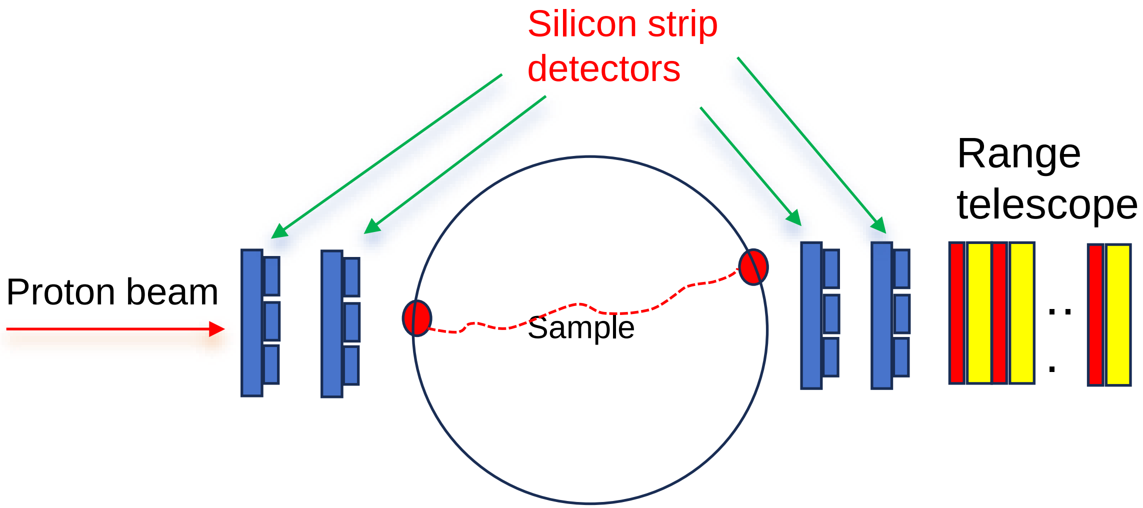
$$p = \int_{E_{in}}^{E_{out}} \frac{dE}{S(I_{water}, E)}.$$

Current proton CT uses silicon strip detectors (SSD) and silicon pixel detectors (SPD) to improve imaging efficiency.

- SSD: measure the entry and exit positions and angles of protons
- SPD: measure the energy of exiting protons
- With these measurements, trajectories of protons in the sample can be reconstructed using interpolation algorithms.

In early proton CT, high-purity germanium detectors were used to measure the energy of outgoing protons.





https://doi.org/10.1088/0031-9155/26/6/001
https://doi.org/10.1016/j.ejmp.2018.10.020

- The materials of samples for the μCT are not limited to those similar to human tissues.

✗ Proton: nucleon-nucleon interaction

✓ Muon: mainly electromagnetic interaction

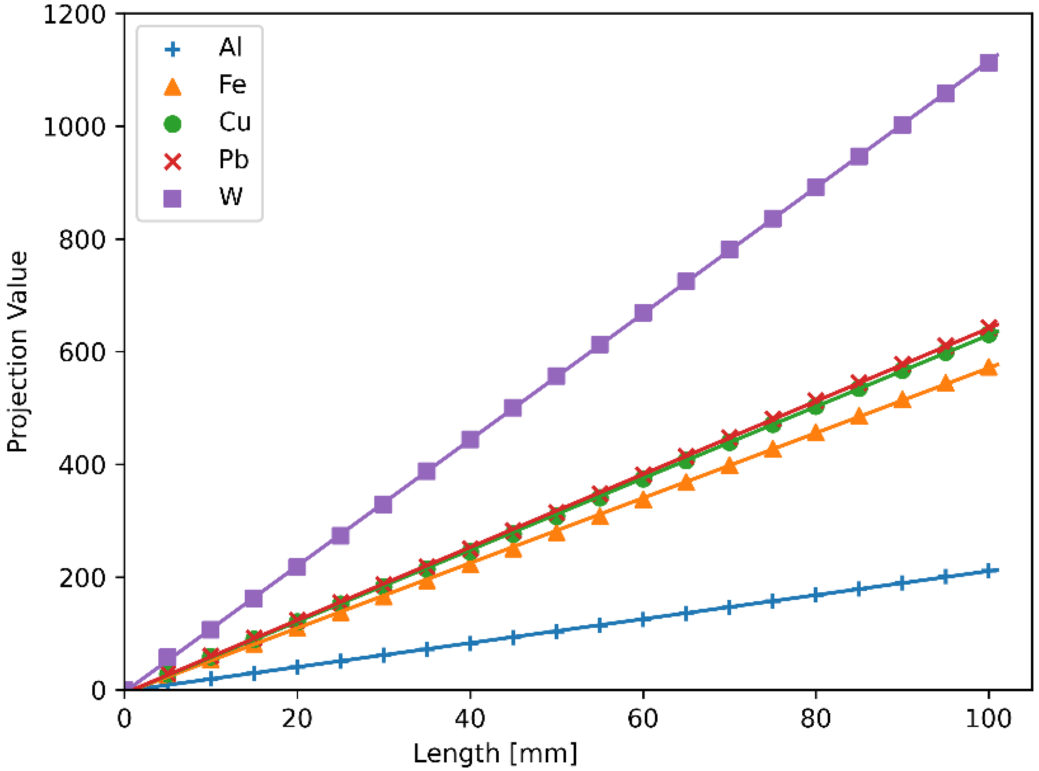
- Bethe-Bloch formula with density effect correction

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{2m_e c^2\beta^2\gamma^2 T_{max}}{I^2}\right) - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right],$$

- The projection value of μCT is

$$p = K\frac{Z}{A}l = -\int_{E_0}^{E}\frac{\mathrm{d}E}{\frac{1}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{2m_e c^2\beta^2\gamma^2 T_{max}}{I^2}\right) - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]}.$$

- Simulation
  - ➢ Aluminum、Iron、Copper、Lead、Tungsten
  - ➢ 1~100 mm



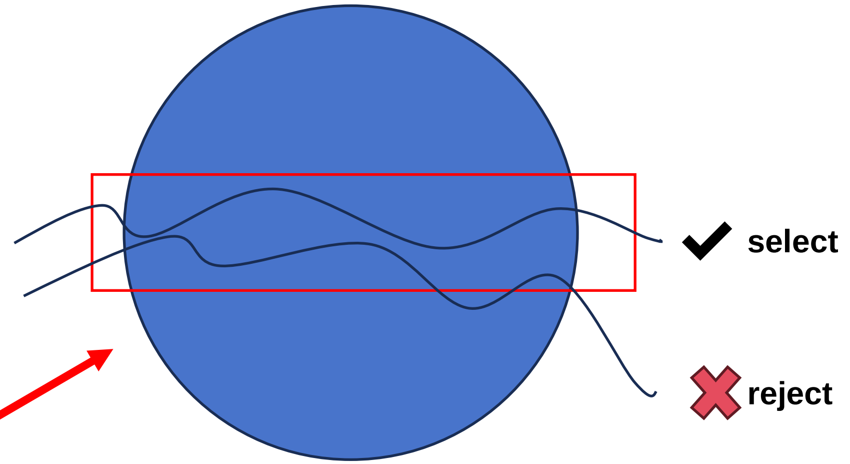
Relationship between the projection value and the thickness of the material which the muon passes through ➡ **Good linearity**

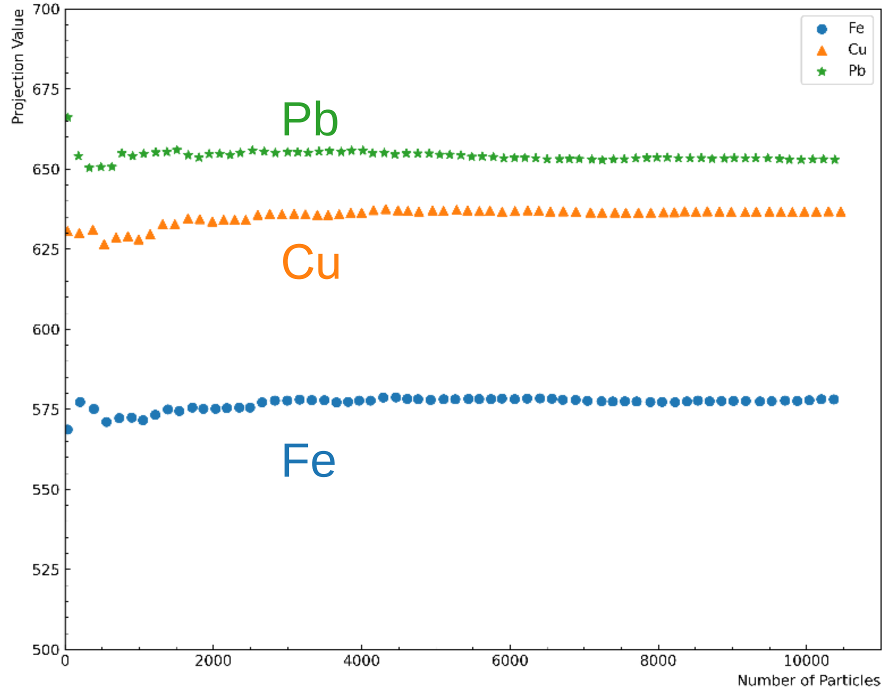
Muon Selection for CT

- As charged particles, muons undergo multiple Coulomb scatterings (MCS) when passing through objects. This brings challenges in accurately estimating the voxels through which the muons have passed, especially for the pulsed muon beam.
- To mitigate the effects of MCS and improve the quality of CT, only muons with incident and exit positions within a limited, small range are selected and labeled as “good”.
- The final projection value is the average of the projection values of all “good” muons:

$$\overline{p} = \frac{\sum_{i=1}^{N} p_i}{N}.$$

- How many “good” muons are needed to obtain a stable projection value?
  - Test samples: 100 mm thick iron, copper, and lead
  - Result: with >3000 muons, projection values tend to stabilize

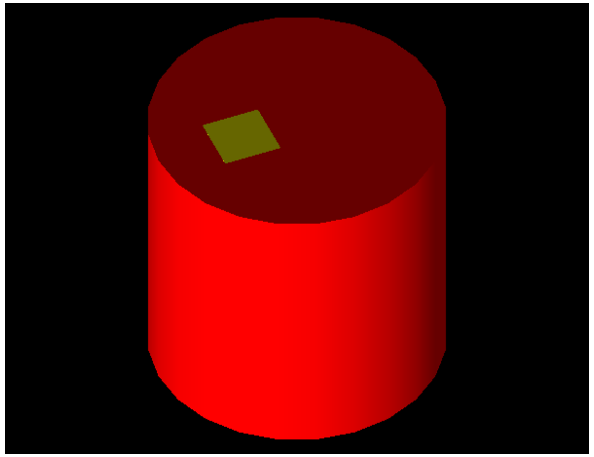




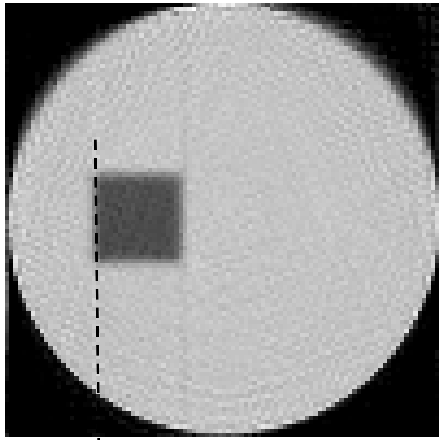
Relationship between the projection values of different materials and the number of “good” muons

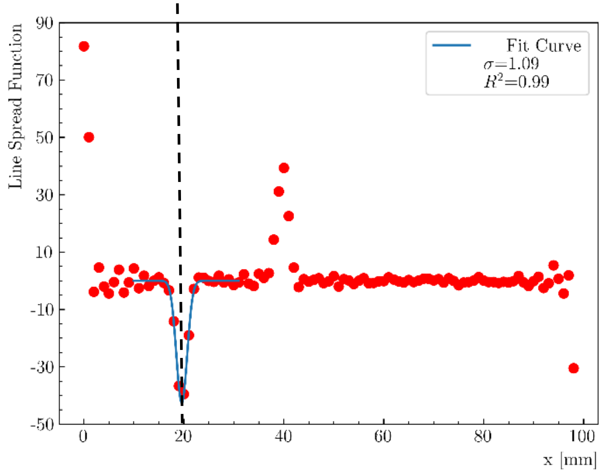
Projection Value Calculation

- The calculation of projection value was verified with Geant4 simulation.
- Simulation settings:
  - Small incident beam spot
  - Virtual detector
  - Sample: aluminum inside iron
  - **Iron**: red cylinder (diameter 100 mm, height 100 mm)
  - **Aluminum**: yellow cuboid (20*100*20 mm^3)



Reconstructed image





Fit to the Line Spread Function of the aluminum edge ➡ Resolution ~1.1 mm (σ)

Detector and Track Reconstruction

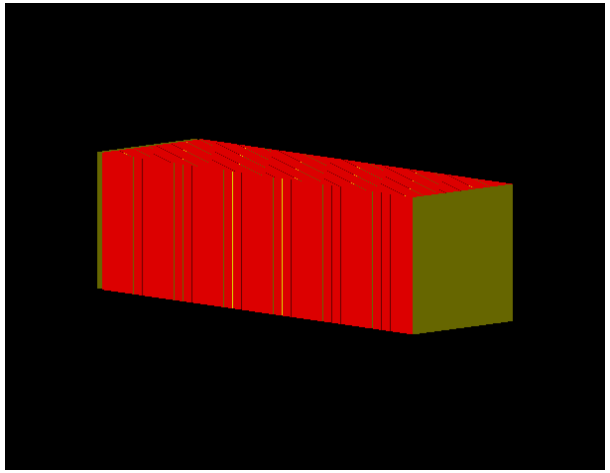
- Detector: **range telescope**
  - Silicon pixel detectors + Degraders
- Track reconstruction: **inward search tree**
  - Initially proposed in ALICE experiment at LHC
  - Now widely used in proton CT to reconstruct energy
  - Fixed threshold:

$$S_{\max} = 3\sigma_\theta$$

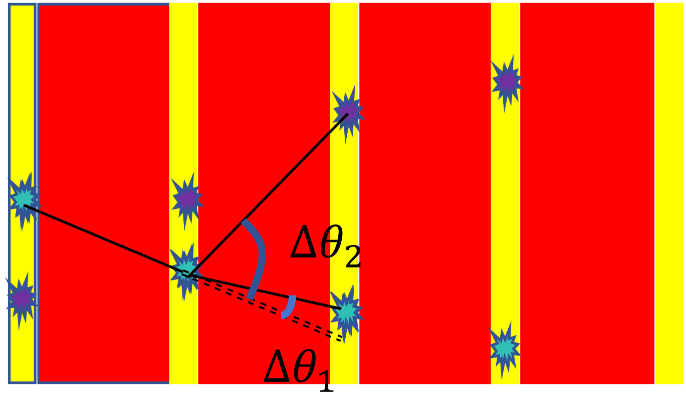
$$= 3\sqrt{2}\left[\int_0^x \left(\frac{14.1\ \mathrm{MeV}}{pv(x')}\right)^2 \frac{1}{X_0}\right]^{\frac{1}{2}} \left(1 + \frac{1}{9}\log_{10}\frac{x}{X_0}\right),$$

  - Score:

$$S = \sqrt{\sum_{i=1}^{n} (\Delta\theta_i)^2},$$



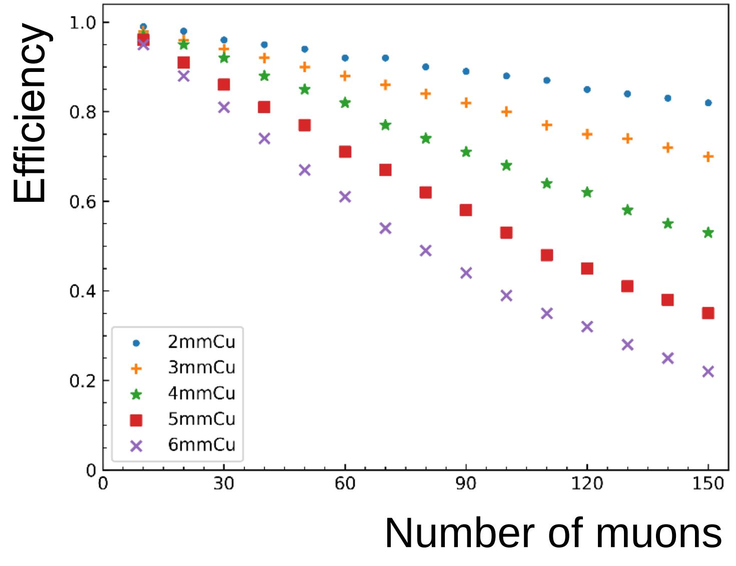
Range telescope model built with Geant4
- Yellow: silicon pixel detector
- Red: degrader



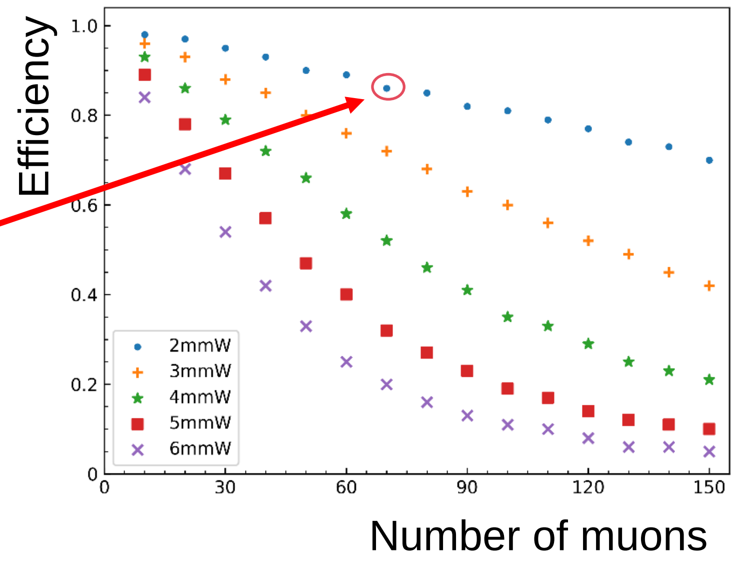
Schematic of track reconstruction algorithm

Design of Degrader

- Factors affecting track reconstruction efficiency:
  - number of incident muons
  - material and thickness of the energy degrader
- Track reconstruction efficiency $r = \frac{T_{\mathrm{S}}}{T_{\mathrm{N}}}$
  - T_{S}: the number of tracks in which the EventID of the seed signal is identical to the EventID of the signal on the first layer
  - T_{N}: the total number of tracks simulated
- Test materials and thickness: Cu and W, 2–6 mm
- Simulation results indicate that tungsten of 2 mm thick and ~70 muons incident each time is a good working point.



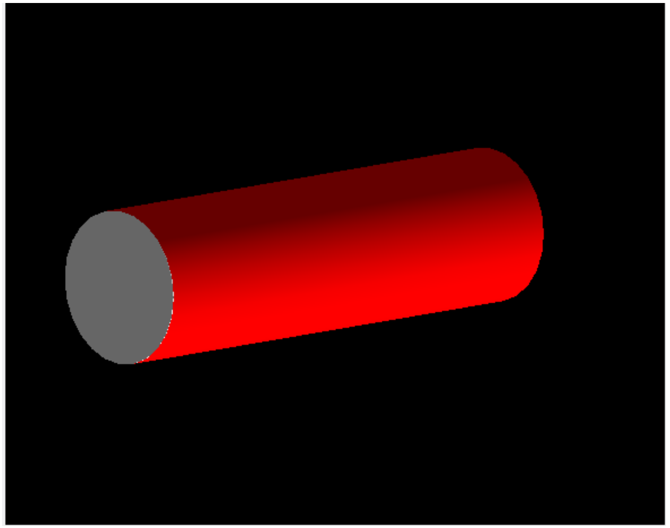
Copper



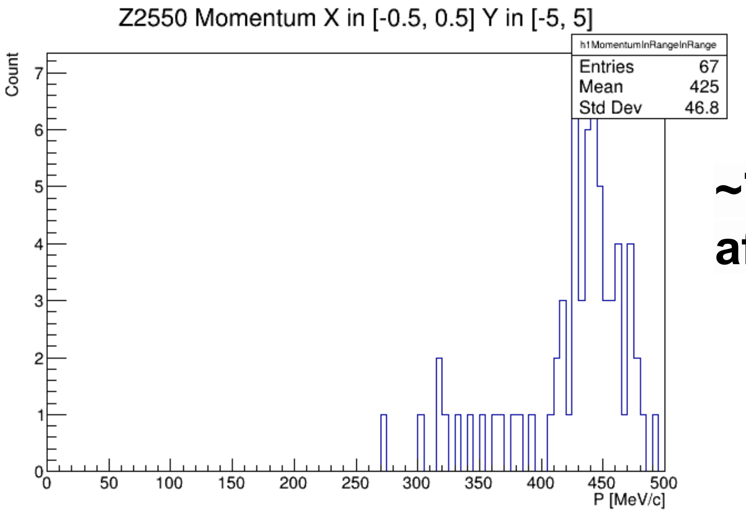
Tungsten

Design of Collimator

- Main design goals of the collimator
  - To collimate a small beam spot for CT: |beamX| < 0.5 mm, |beamY| < 5 mm
  - To control the number of muons for CT: around 70 muons per pulse
- Two components
  1. **Beam spreader (gray)**
     Φ 300 mm, 2 mm thickness
  2. **Beam collimator (red)**
     Φ 300 mm, 2000 mm length
- Beam intensity for muon CT can be tuned by varying the collimator design.



Design of the collimator with Geant4 simulation



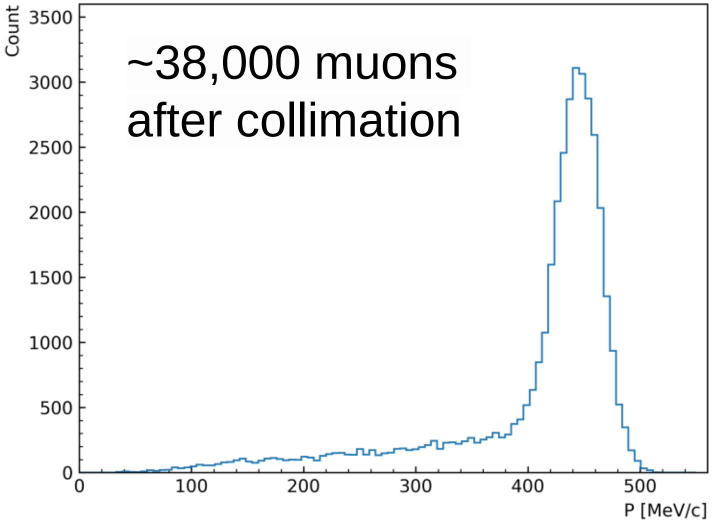
~70 muons remaining after collimation

Momentum of muons after collimation

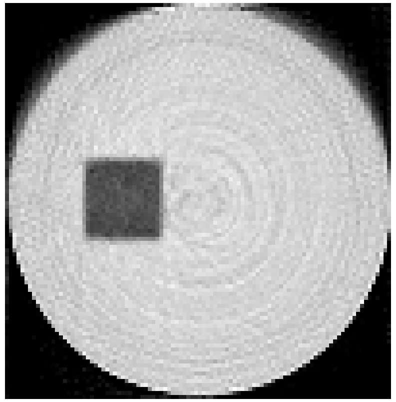
Simulation Result

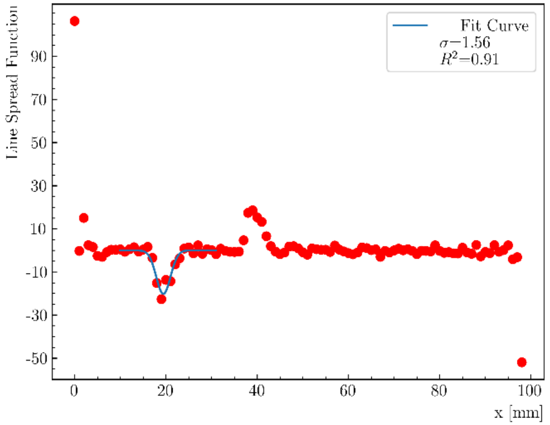
Full simulation of muon beam CT was conducted, including:

1. EMuS DM1 beam (~10^8 muons simulated)
2. Collimator
3. Sample
4. Range telescope



Momentum distribution of muons after passing through the collimator





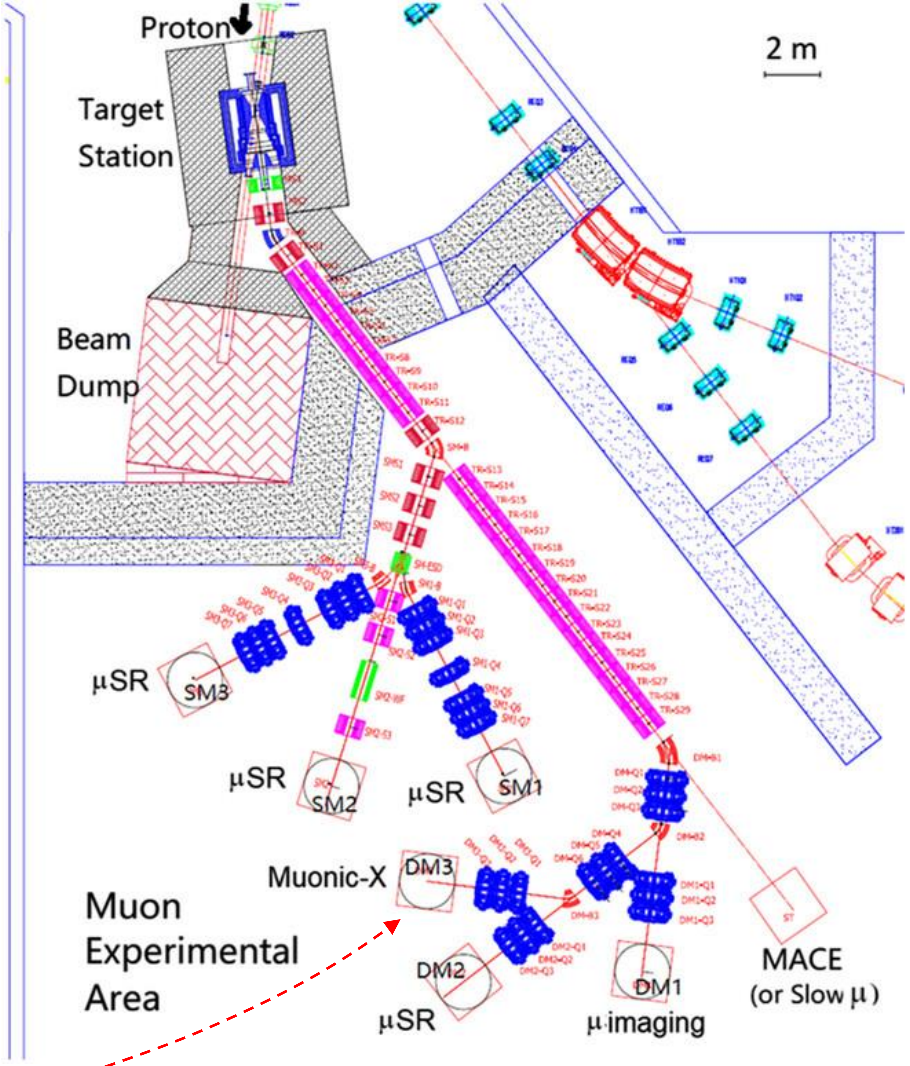
Reconstructed image of muon beam CT

Fit to the Line Spread Function of the sample edge yields a resolution of ~1.6 mm (σ)

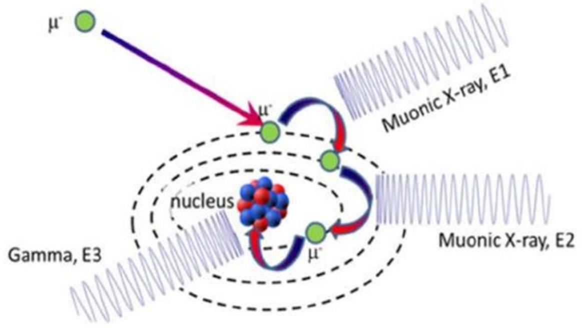
Summary and Outlook

- Geant4 simulation and image reconstruction study in the EMuS baseline scheme demonstrate the potential of muon beam CT.
- Simulation results suggest that bulk sample imaging with muon CT could be a promising application for muon facilities with high-momentum muon beams.
- Current EMuS baseline design would require approximately one week to image a single cross-section of the sample studied in this work.
- Improvements:
  - Simultaneous irradiation with more beam spots
  - Improve the track reconstruction algorithm
- Continuous-wave muon beam
  - Capability to measure individual muon

Contents



EMuS baseline scheme layout



- DM1 endstation: high-momentum muon beam imaging
- DM3 endstation: muonic X-ray analysis

Study of Muonic X-ray with Geant4

- **Background**: start-to-end simulation of muonic X-ray analysis on EMuS DM3 is performed with G4beamline (a Geant4-based program)
- **Problem**: muonic X-ray emission lines are incorrectly calculated by Geant4, especially for high-energy transition (e.g., K-shell)

G4MuonMinusCapture
G4EmCaptureCascade

Preliminary **solution**:

1. Use **MuDirac*** to produce muonic X-ray database (energies and probabilities) for all elements
2. **Modify** *G4EmCaptureCascade* to **extract** the transition energy and probability from database
3. Create a new **messenger class** to manage whether to replace the original G4 process (for G4beamline or other Geant4-based program)

Progress:

1. Developed Python scripts to run MuDirac in multi-process mode (< 1 hour for all nuclides using a 64-core server)
2.
   - ✓ Finished: transition energy extraction
   - ☐ Ongoing: implementation of transition probability
3. Finished

Git repository

Transition	Kα (2p-1s) [keV]		
Element	G4bl before	G4bl now	Exp.
^{29}Cu	1491	1502	1507

*__MuDirac__ is a software developed by the ISIS muon group, which calculates the transition energies and probabilities of muon cascade with a precision of a few keV.
https://doi.org/10.1002/xrs.3212
https://doi.org/10.3390/condmat8040101

Future Improvement

- Intensities are also important for quantitative elemental analysis.
- The intensities of the simulated spectra are not correct (either by MuDirac or Geant4).
  - Need to consider the initial muon population on initial energy level, i.e. the angular momentum distribution at capture, **L-distribution**.
  - Try to collaborate with MuDirac team and other groups.

Thank You!

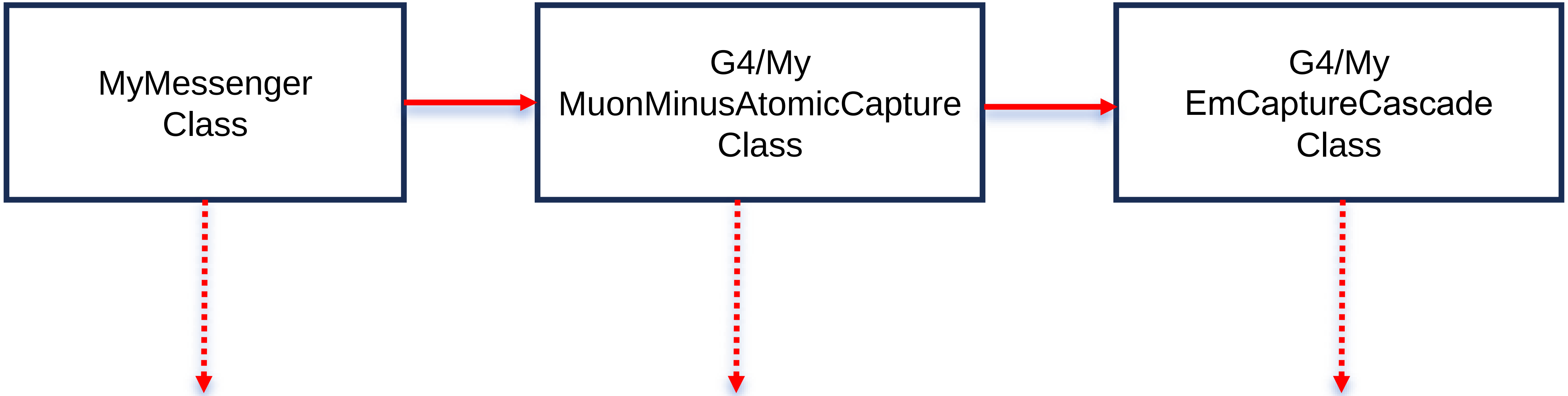
Backup

Summary of variables used in Bethe-Bloch formula.

Symbol	Definition	Units or Value
K	$4\pi N_A r_e^2 m_e c^2/A$	0.307075 MeV·g^{-1}·cm^2
N_A	Avogadro's number	6.022 1415(10) × 10^{23} mol^{-1}
$m_e c^2$	Electron mass × c^2	0.510 998 918(44) MeV
Z	Atomic number of absorber	
A	Atomic mass of absorber	
β	v/c	
γ	$\gamma = 1/\sqrt{1-\beta^2}$	
I	Mean excitation energy	
δ	Density effffect correction	
M	The mass of incident particles	

T_{max} is the maximum energy transfer possible in a single collision.

$$T_{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}.$$



Interact with users and switch between the default process and the customized process.

A class that handles a series of processes following the capture of a muon by an atom, including the formation of muonic atoms, muon cascade processes, and so on.

A class that handles muon cascade processes. This class also handles meson cascade processes, but we only modify the mu- process in the MyMuonMinusAtomicCapture class.