Lectures 9-10
Instabilities

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Aims of the Lectures

- Provide an overview of the mechanisms that drive instabilities.
- Study the beam coupling with the accelerator environment through the concepts of impedance and wakefields.
- Identify various types of instabilities in the transverse and longitudinal planes.
- Identify single-bunch and multi-bunch instabilities.

- **Approach**
  - The deep treatment of instabilities requires a detailed mathematical analysis. In these lectures, a qualitative approach is given and details can be found in the references.

References

- P. Bryant, K. Johnsen, *Principles of Circular Accelerators and Storage Rings*, Cambridge University Press
- D. Edwards, M. Syphers, *Introduction to the Physics of High Energy Accelerators*, Wiley-VCH
- M. Migliorati, *Instabilities and Wakefields*, JUAS Lectures
- G. Romulo, *Beam Instabilities*, CAS Lectures
- H. Wiedemann, *Particle Accelerator Physics*, Springer
- E. Wilson, *Introduction to Particle Accelerators*, Oxford University Press

Special thanks to the JAI Lecturers

- H. Garcia-Morales
- S. Sheehy

Introduction

So you've carefully tuned the machine to produce the best possible performance at the highest intensity and you've invited the lab director to the control room to observe, for the first time, the machine working at design intensity. The intensity comes up, the pulse goes up and just when you're about to get there…there's a sudden loss of beam. You try again, and it repeats. This is characteristic of an instability.

E. J. N. Wilson, CERN

Ted Wilson in the SPS control room in 1977

Instabilities in Accelerators

- When pushed in terms of performance, accelerators tend to reach an **intensity limit**.
- With analysis, understanding and mitigation, a new limit emerges.
- The same pattern can be seen with **many high-intensity and high-energy accelerators**.

- Why does this happen?
  - **Electromagnetic interactions** of the beam with the accelerator environment can affect both **individual particles** and the **collective motion** of the whole bunch.

There are both **transverse** and **longitudinal** instabilities.

There are both **single-bunch** and **multi-bunch** instabilities.

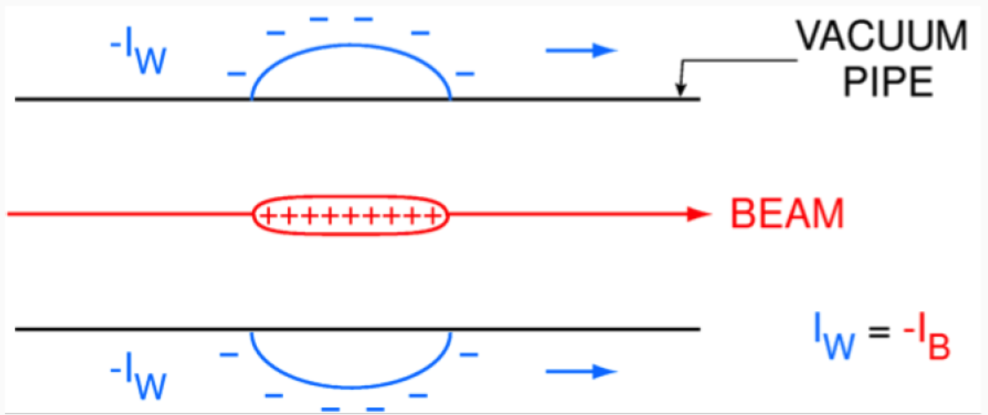
Impedance Introduction

- We have studied the single particle dynamics (transverse dynamics lectures).
- We have seen what happens when the bunches are treated as distribution of self-interacting charges (beam-beam and space charge lectures).
- A circulating beam resembles an electric circuit.
- Impedance plays an important role in determining the induced voltage on circulating current.
- **Impedance**
  - Fourier transform of the electromagnetic waves induced by the passing charged particle beam (wakefield).

Impedance of a Resistive Wall

- There exists a wall current I_W due to the circulating beam.
- Vacuum pipe is not smooth, so I_W sees an impedance.

$$Z = Z_r + iZ_i$$

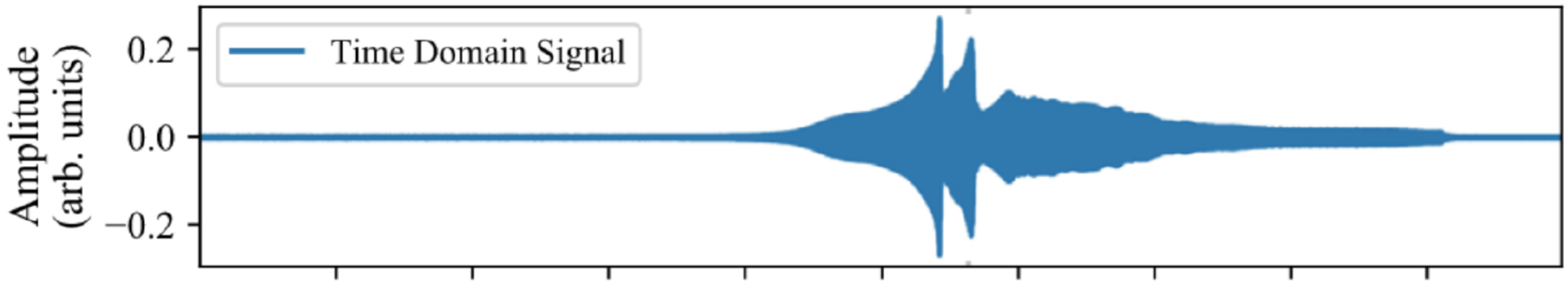


- The induced voltage is $V = I_w Z = -I_B Z$, which acts back on the beam.

Instabilities are therefore intensity dependent.

Description of an Instability

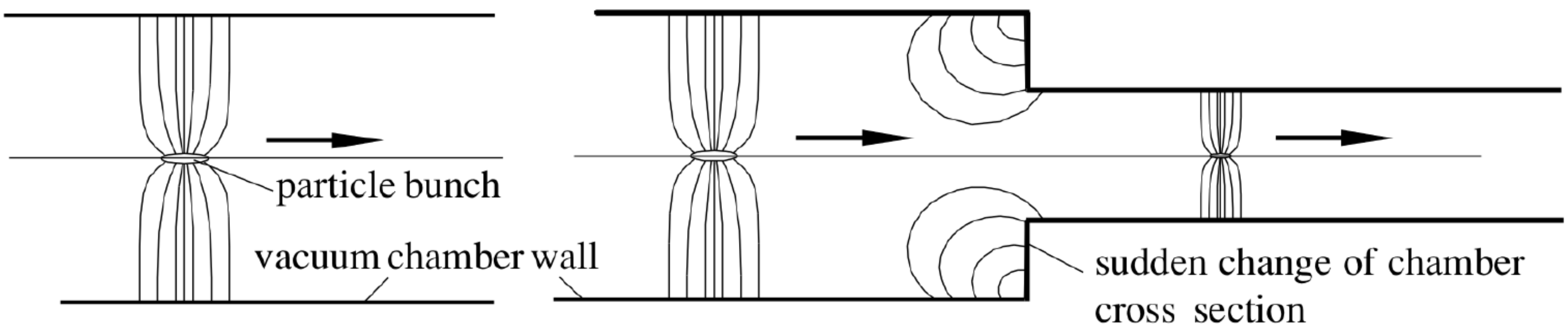
- From initial small perturbation (i.e. impedance), observe if the perturbation is:
  - Increased, thus giving rise to an **Instability.**
  - Decreased, thus **Stability**.



- **Example:** Perturbation in local line density of charge around a synchrotron.
  - If the forces set up by a pattern of perturbation reinforce the shape, it is sure to grow exponentially.
- **Observables**
  - Increase of oscillation amplitude.
  - Beam losses.
  - Intensity losses.

Impedance – Origin/Source

- Accelerator components such as resistive wall vacuum chamber, space charge, image charge on vacuum chamber, broad-band impedance due to bellows, vacuum ports, and BPMs, and narrow-band impedance due to high-Q resonance modes in RF cavities, septum and kicker tanks.



Impedance

- The particle beam covers a wide frequency spectrum from the kHz regime to the order of the revolution frequency up to many GHz, limited only by bunch length.
- The vacuum chamber environment constitutes an impedance which can become significant in the same frequency regime and efficient coupling can occur leading to collective effects.
- Strong coupling between RF-cavity and beam.
- The ultimate beam intensity is limited by instabilities caused by electromagnetic interaction of the beam current with the environment of the vacuum chamber.

Impedance and Wakefields

- Wakefields are created after the interaction of the beam charge with the accelerator environment.
- They have the ability to pull or push the charges of the distribution.
- Energy losses and gains of a single or collection of particles can cause modifications in the beam dynamics, eventually driving a beam instability.
- Transverse wakefields and Longitudinal wakefields

Impedance & Wakefields

- In time domain, the interaction is described by wakefields that act on charges.
- In frequency domain, vacuum chamber components can be represented as a frequency dependent complex impedance ***Z(ω)***.

$$V(\omega) = -Z(\omega)I(\omega)$$

- Impedance os the counterpoint of wakefields in the frequency domain.
- The negative sign indicates that the induced voltage leads to an energy loss.
- All elements can be seen as accidental cavities. Z depends on their shape, material and frequency under consideration.

Impedance and Instabilities

- In general, impedances, Z, are complex functions and are related to the frequency, ω

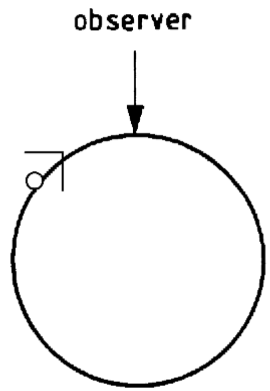
$$Z(\omega) = Z(\omega)_{real} + iZ(\omega)_{imag}$$

- Strong coupling between beam and vacuum chamber if the impedance and particle beam have a significant component at the same frequency.
- Impedance depends on each piece of vacuum chamber including cavities, or changes in beam pipe diameter, material, shape etc...

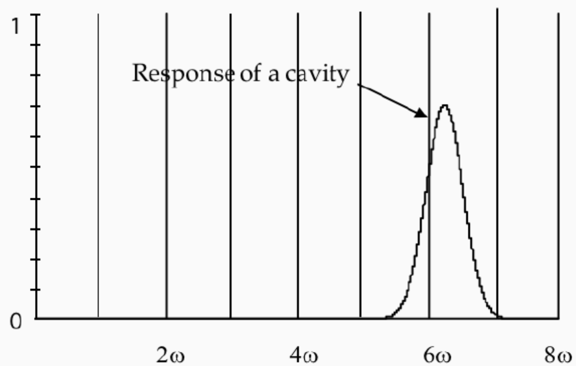
Impedances for a particular component can be narrow band quality factor $Q >> 1$ as in an accelerating cavity

OR they can be broadband with $Q \approx 1$ due to change in vacuum chamber cross section.

Impedance Driving Terms



Delta function of bunch passing an observer



Spectrum from a bunch showing response of an r.f. cavity

Fourier analysis of a circulating delta function bunch of charge passing an observer.

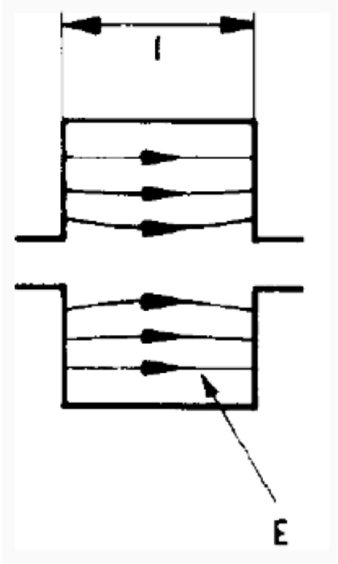
$$I = \Sigma I_n e^{in\omega(0)t}$$

Produces a fundamental at the revolution frequency plus all higher harmonics in equal strength.

Impedance at a Cavity

- The voltage experienced in local enlargement in the beam pipe (which acts like a cavity) has the form:

$$I = \hat{I}e^{-i\omega t}, V = \hat{V}e^{-i\omega t}$$



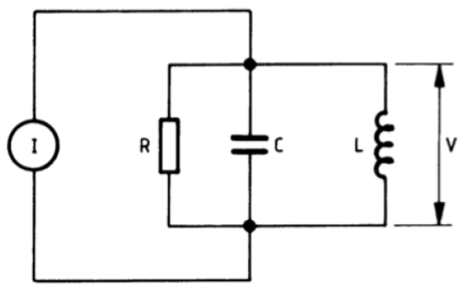
- We can relate force on particles to the Fourier component of the beam current which excites the force.
- The impedance is a complex quantity. ***V(ω) = - Z(ω) I (ω)***
  - REAL if voltage and current are in phase
  - IMAGINARY if 90 degrees or *i* between voltage and current.

    (Inductive = +, Capacitive = -)
- Differs from RF wave by 90 degrees

The resistive part of the impedance can lead to a shift in the betatron oscillation frequency of the particles while the reactive or imaginary part may cause damping or anti-damping.

Interlude: RLC Circuit Impedance

- A cavity can be modelled as an AC resonant circuit:



Resonant frequency of cavity

$$\omega_r = \frac{1}{\sqrt{LC}}$$

where the quality factor

$$Q = R\sqrt{C/L} = R/L\omega_r = RC\omega_r$$

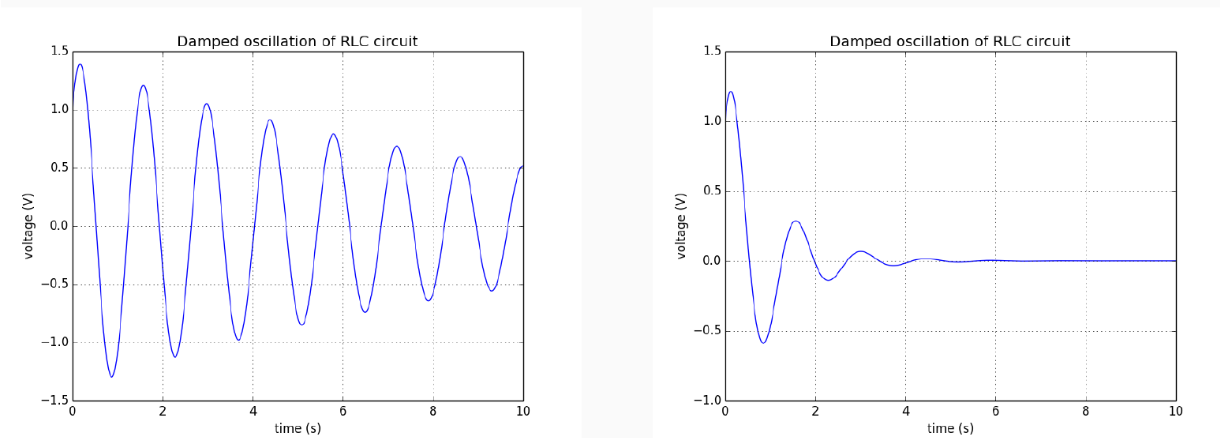
and a differential equation can be written down for voltage and current:

$$\ddot{V} + \frac{\omega_r}{Q}\dot{V} + \omega_r^2 V = \omega_r \frac{R}{Q}\dot{i}$$

Interlude: RLC Circuit Impedance

- The solution is a damped resonant circuit with a damping rate $\boldsymbol{\alpha = \omega_r / 2Q}$

$$V = V_0 e^{-\alpha t} \sin\left[\omega_r \sqrt{1 - \frac{1}{4Q^2}}\, t\right] + \phi$$



http://www.amanogawa.com/archive/CircuitsA.html

RLC Circuit Impedance

- If the current in the circuit is ***I = I' e$^{i\omega t}$*** then the impedance seen is

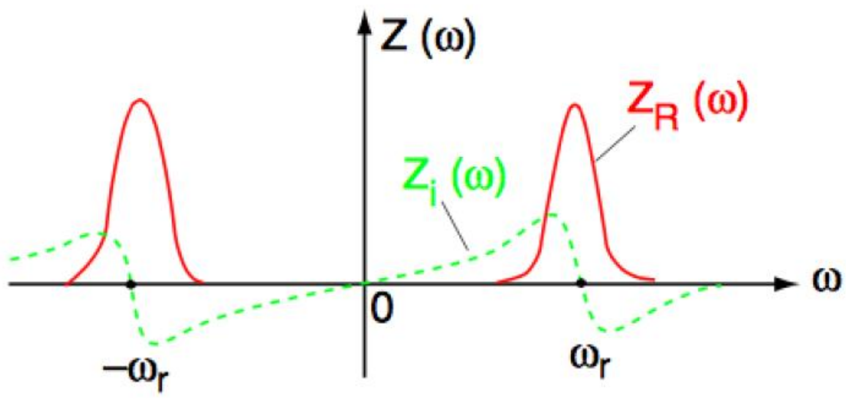
$$Z(\omega) = Z_r(\omega) + iZ_i(\omega) = R\left[\frac{1 - iQ\left(\frac{\omega^2-\omega_r^2}{\omega\omega_r}\right)}{1 + Q^2\left(\frac{\omega^2-\omega_r^2}{\omega\omega_r}\right)^2}\right]$$

  - When the driving frequency ω is below the resonant frequency, the reactive component is inductive or positive.
  - When the driving frequency is above the resonant frequency, it becomes capacitive and negative.

RLC Circuit Impedance

- For a high-Q cavity (narrow-band resonator), this can be simplified near the resonance frequency with $\boldsymbol{\Delta\omega = \omega - \omega_r}$, to:

$$Z(\omega) \approx R_s \frac{1 - i2Q\frac{\Delta\omega}{\omega_r}}{1 + \left(2Q\frac{\Delta\omega}{\omega_r}\right)^2}$$



Impedance Effects

- For a narrow-band impedance (RF Cavities), the Q factor is high and the damping rate α is low.
  - Signal will oscillate for many turns and produce *multi-bunch effects*.
- For a broad-band cavity, Q is low, α is large.
  - The fields collapse and they do not affect subsequent bunches.
  - Discontinuities in cross section of vacuum chamber (RF Cavities, flanges, kicker magnets, BPMs,...)
  - May produce *single-bunch effects*.

Impedance – Longitudinal vs Transverse

- Longitudinal
- Transverse

$$Z_{||}(\omega) = -\frac{\int \vec{E}_{||}(\omega) d\vec{z}}{I(\omega)}$$

(

$$Z_{\perp}(\omega) = i\frac{\int (\vec{E}(\omega) + [\vec{v} \times \vec{B}(\omega)])_{\perp} d\vec{z}}{I(\omega)\Delta x}$$

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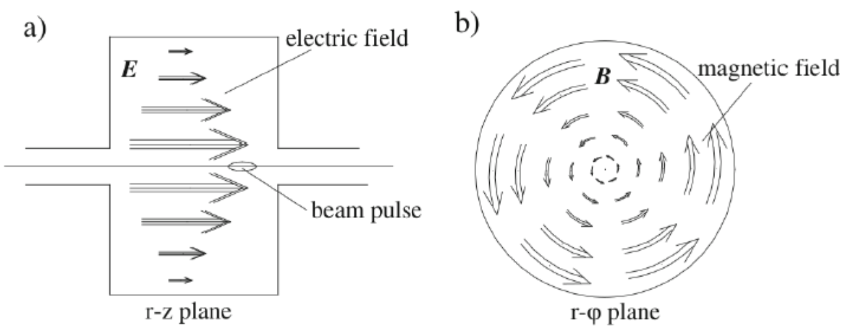
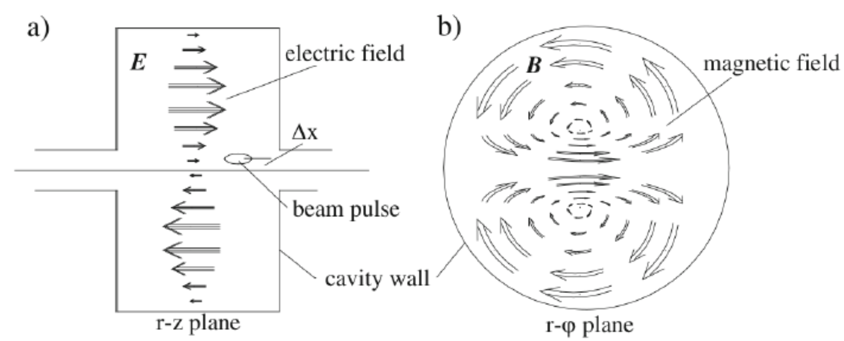




A bunch passing through a structure on axis excites
a longitudinal electrical field and a transverse magnetic field.

Impedance

- In general:

$$Z(\omega) = Z_{\text{Re}}(\omega) + iZ_{\text{Im}}(\omega) = R\left[\frac{1 - iQ\frac{\omega^2-\omega_r^2}{\omega_r\omega}}{1 + iQ\frac{(\omega^2-\omega_r^2)^2}{\omega_r^2\omega^2}}\right]$$

- Resistive part (Z_{Re}): Tune-shift
- Reactive part (Z_{Im}): Damping or anti-damping
  - $\omega < \omega_r$: Inductive: $Z_{Im} > 0$.
  - $\omega > \omega_r$: Capacitive: $Z_{Im} < 0$.

Impedance in the Accelerator Environment

- The vacuum chamber of an accelerator is too complicated in geometry to allow an analytical expression for its impedance.
- Each section must be treated independently.
- Examples of impedances:
  - Resistive wall impedance.
  - Space-charge impedance.
  - Cavity-like impedance.
- In earlier machines, the impedance was as much as 20 – 50 Ω, whereas it is < 1 Ω in modern synchrotrons.

Resistive Wall Impedance

- The particle beam induces an image current in the vacuum chamber wall in a thin layer.
- Since conductivity is not perfect, resistive losses apply a pull or decelerate field on the particle beam.
- The pull is proportional to the beam current:

$$\frac{Z_{||}(\omega_n)}{n} = \frac{1-i}{n}\frac{\bar{R}}{cr_w}\sqrt{\frac{\mu_r \omega_n}{2\epsilon_0 \sigma}} = \frac{1-i}{n}\frac{\bar{R}}{r_w \sigma \delta_{\mathrm{skin}}}$$

- Important role only for low frequencies.
- The transverse resistive wall impedance for a round beam pipe is:

$$Z_{\perp}(\omega_n) = \frac{2\bar{R}}{r_w^2}\frac{Z_{||}(\omega_n)}{n}$$

Space-charge Impedance

- There is an induced voltage leading to an energy gain or loss due to a collection of charged particles.

$$Z_{||,sc}(\omega) = -\frac{i}{\epsilon_0 c}\frac{n}{2\beta\gamma^2}\left(1 + 2\ln\frac{r_w}{r_0}\right)$$

- Correct for long wavelengths. Purely reactive.
- Especially strong for low energy particle beams.

$$Z_{\perp,sc}(\omega) = -\frac{i}{\epsilon_0 c}\frac{\bar{R}}{\beta^2\gamma^2}\left(\frac{1}{r_0^2} - \frac{1}{r_w^2}\right)$$

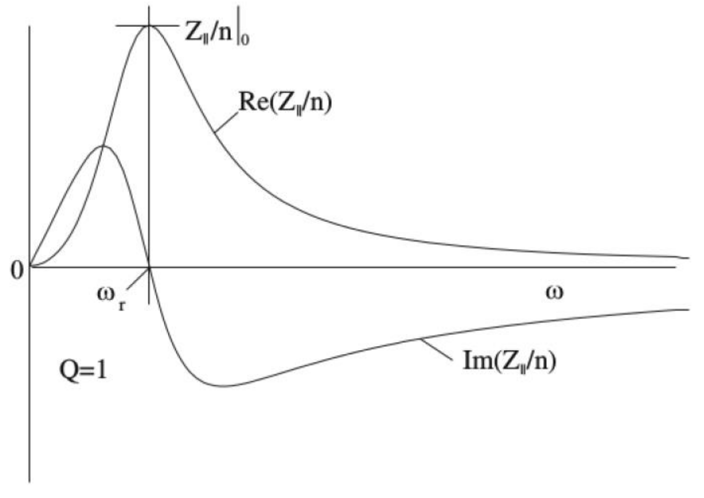
Cavity-like Structure Impedance

- Vacuum chamber impedances occur due to sudden changes of cross section.
- Collectively described by a cavity-like impedance with quality factor Q = 1.
- Broad-band impedance.

$$\frac{Z_{||}}{n}_{\rm bb}(\omega) = \left|\frac{Z_{||}}{n}\right|_0 \frac{1 - i\frac{\omega^2-\omega_r^2}{\omega_r\omega}}{1 + i\frac{(\omega^2-\omega_r^2)^2}{\omega_r^2\omega^2}}$$

Cavity-like Structure Impedance

- At low frequencies the impedance is almost purely reactive and inductive.
- At high frequencies it becomes capacitive.
- At resonance, it is purely resistive.



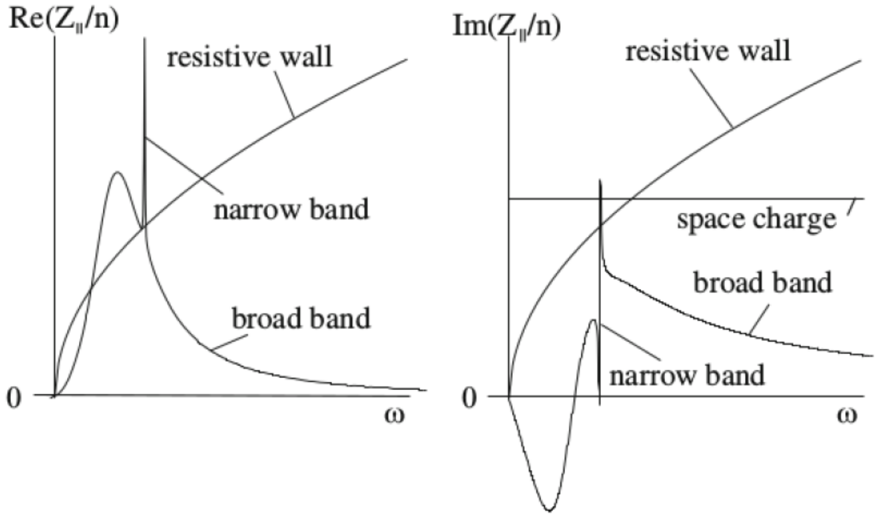
Cavity-like Structure Impedance

- Transverse broad-band impedance:

$$Z_{\perp}(\omega_n) \approx \frac{2\bar{R}}{r_w^2} Z_{||}(\omega_n)$$

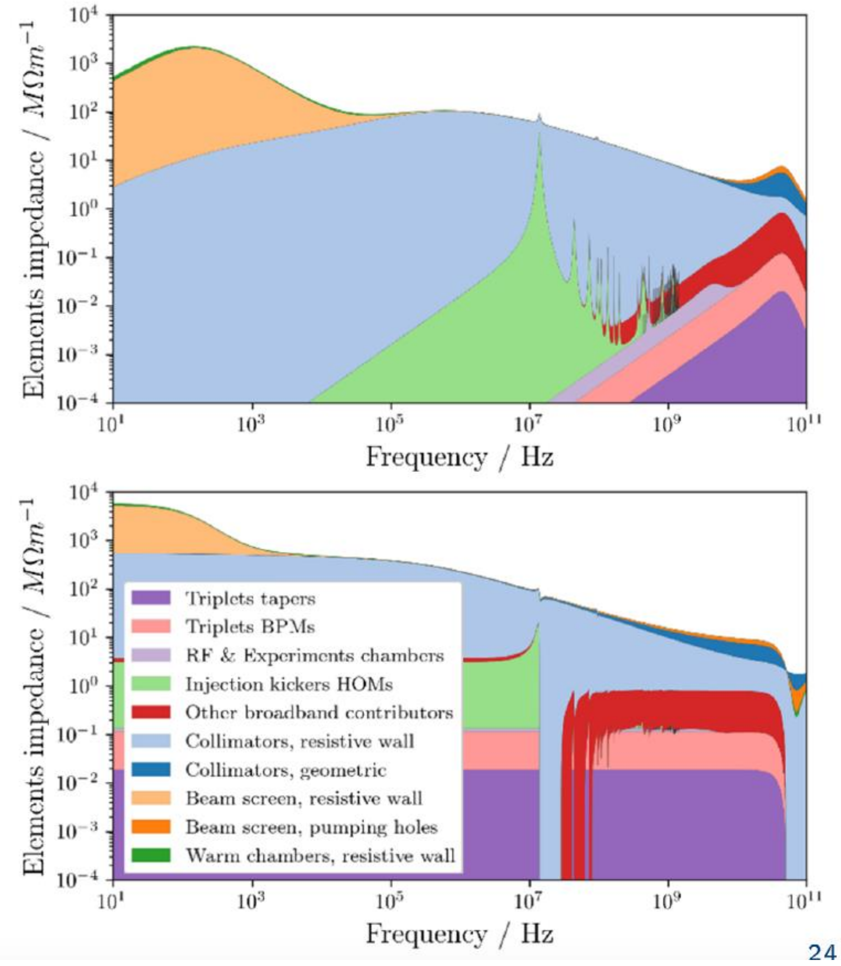
Overall Accelerator Impedance

- At low frequencies the reactive and the resistive component of the resistive wall impedance dominates.
- Space-charge impedance is independent of frequency.
- The narrow-band cavity spectrum includes the high impedances at the fundamental and higher-mode frequencies.



LHC Impedance Model

- The impedance model summarises the main impedance contributions from the different elements of the machine.
- In the LHC, the main contribution (for high frequencies) is coming from the collimators.
- New low-impedance collimators under implementation.



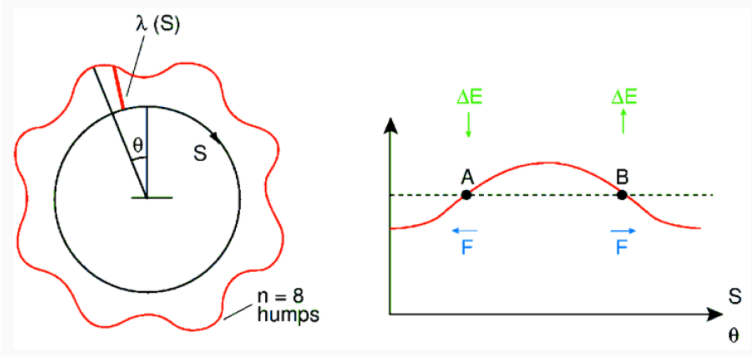
Types of Instabilities

- A non-exhaustive list of instabilities

	Transverse	Longitudinal
Single bunch	Rigid bunch instability	Negative mass instability Head tail instability Robinson instability Longitudinal microwave instability
Multi-bunch	Coupled bunch modes Resistive wall instability	Coupled bunch modes

Negative Mass Instability

- Imagine a ring with modulation in the line density λ(s) around ring.



- What is the result?

$$E = -\left[\frac{q}{4\pi\epsilon_0\gamma^2}\right]\frac{\partial\lambda}{\partial s}$$

Field due to space-charge force

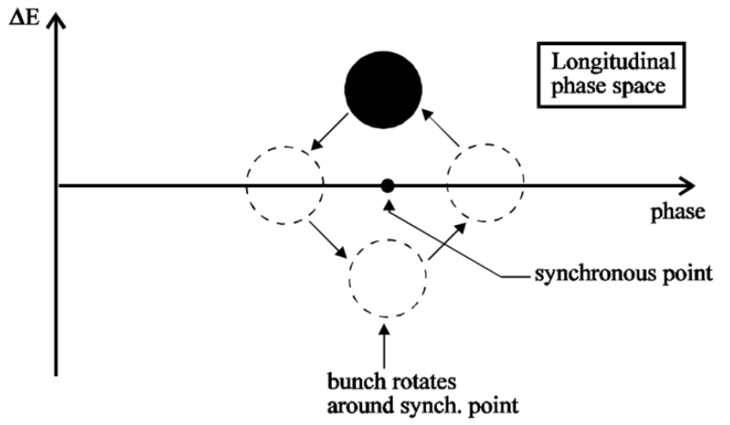
  - Particle B finds itself with a larger charge density behind it than in front of it, pushing it forward and accelerating.
  - Particle A will be decelerated by large charge in front of it.

Negative Mass Instability

- Observed in coasting beams.
- So is this stable or unstable? Depends on $\boldsymbol{\gamma_t}$.
  - **If $\gamma<\gamma_t$**: Revolution frequency increases. Repulsive EM field from a 'lump' in the distribution causes particles ahead of the 'lump' to be accelerated and particles behind to be decelerated. Stabilizing situation and the 'lump' smoothes out – **STABLE**.
  - **If $\gamma>\gamma_t$**: Revolution frequency decreases. Particles A and B move toward the 'lump' of charge. Instabilty due to **'negative mass'** – **UNSTABLE**.

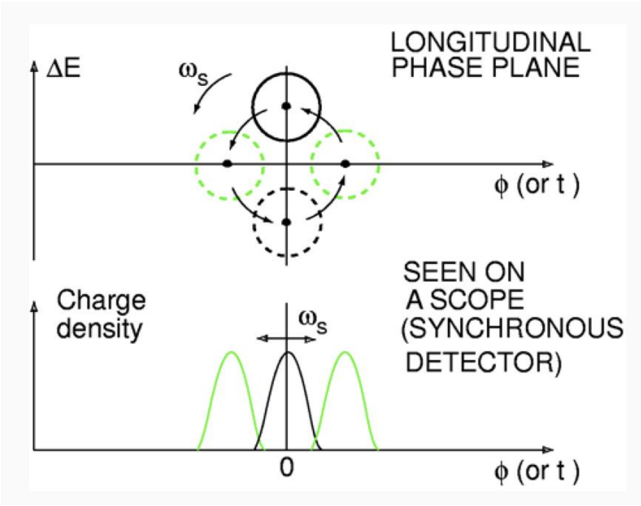
Robinson Instability

- Most basic longitudinal instability mechanism that occurs in circular accelerators.
- Induced by longitudinal impedance due to RF cavities (ω_R).
- Dipole mode oscillation. The whole bunch is moving back and forth around the synchronous position.
- Above transition, the beam will be unstable if ω_R is slightly above $h\omega_0$ and stable if slightly below. Below transition, it is the other way around.



Robinson Instability

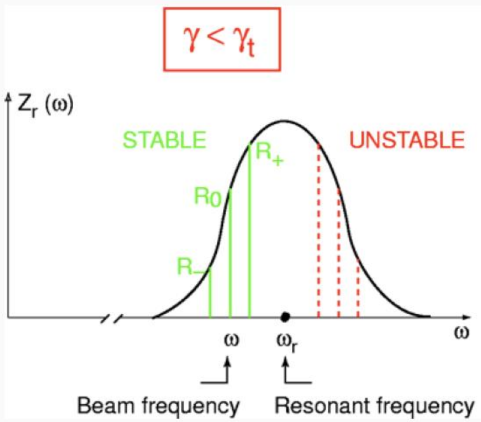
- Single bunch and resonator over multiple turns.



- The single bunch in 'dipole' or 'rigid bunch' mode rotates in longitudinal phase plane with $\boldsymbol{\omega_s}$.
  - Phase $\boldsymbol{\varphi}$ and energy $\boldsymbol{\Delta E}$ also vary with $\boldsymbol{\omega_s}$.

Robinson Instability

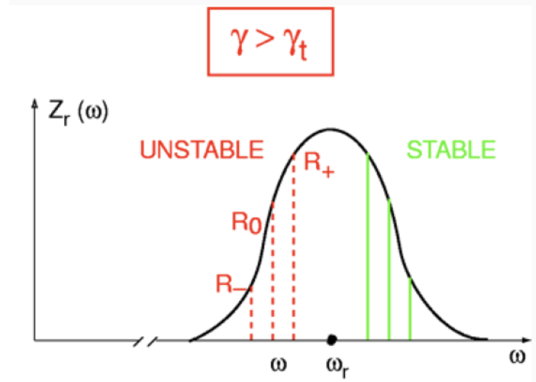
The bunch sees a resonator impedance at $\boldsymbol{\omega_r = \omega_0}$



Whenever $\Delta E > 0$, for $\omega < \omega_r$:

- ω increases
- sees a larger real impedance $R+$
- more energy taken from the beam
- STABLE

The opposite is true for $\omega > \omega_r$.



Whenever $\Delta E > 0$, for $\omega < \omega_r$:

- ω decreases (above transition)
- sees a smaller $R+$
- less energy taken from the beam
- UNSTABLE

Robinson Instability

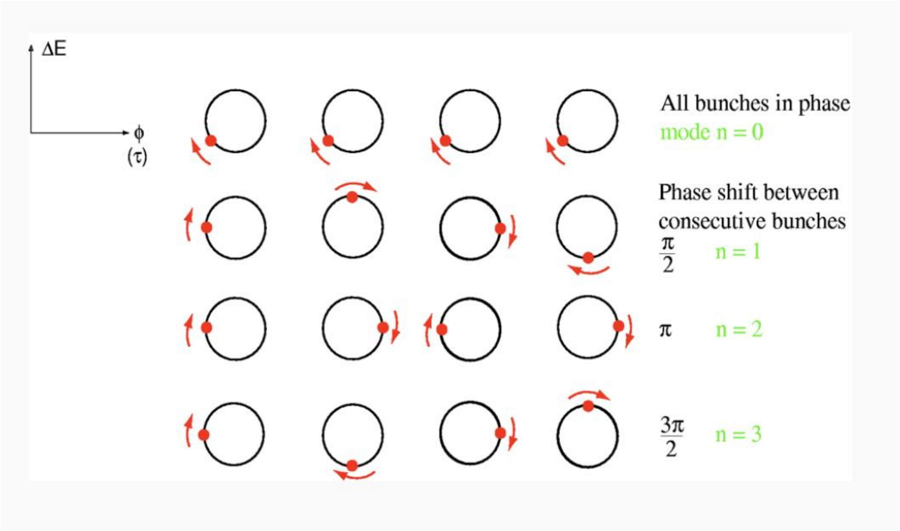
- Robinson instabilities were removed just by fine tuning the cavity's resonant frequency ω_r slightly away from the beam frequency ω_0

$$\boldsymbol{\omega_r = n\omega_0}$$

- Nowadays a feedback system on the cavity tune is an efficient way of removing it for increased performance.

Multi-bunch Coupling Instability

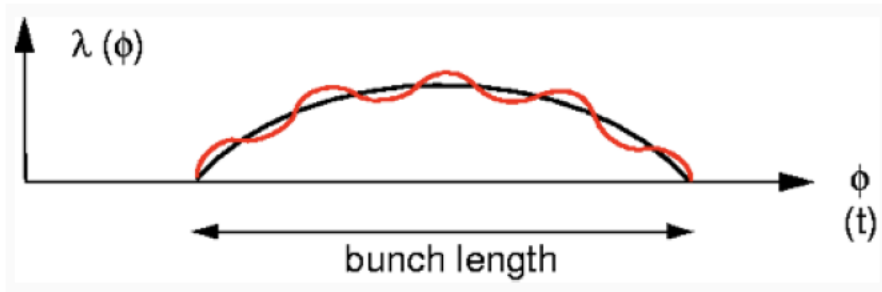
- Fields that are induced in the resonator remain long enough to influence subsequent bunches.
- Example: For *M* = *4* bunches, they can couple together in 4 ways.



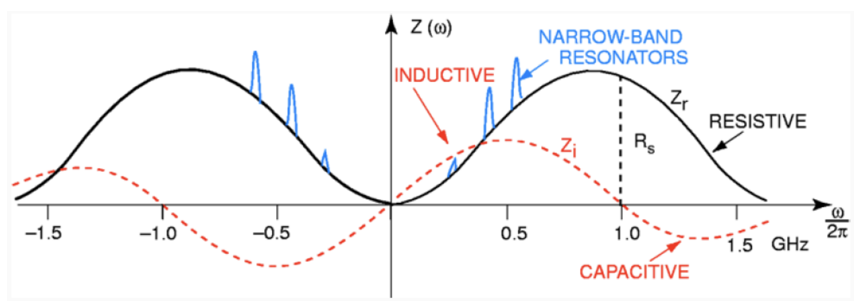
- With 4 possible phase shifts between the four bunches, above transition, n = 1 is UNSTABLE.

Longitudinal Microwave Instability

- This is a single-bunch effect, driven by the broad-band impedance, which is caused by discontinuities in the beam pipe.



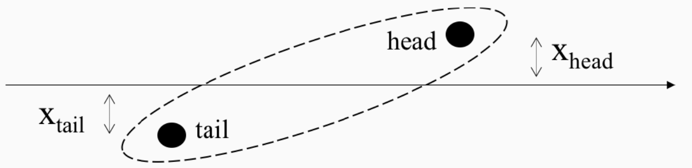
- Typically results in a high-frequency density modulation superimposed on the bunch shape. Has fast growth rates and also affects leption machines.



Head – Tail Instability

- Single-bunch effect of transverse wakefields generated by head of the bunch on its own tail.
- Occurs for broad-based impedances, which act very quickly and decay quickly, so only affects a single bunch.
- In linear accelerators, can lead to beam break-up, as they have many cavities.

Represent the head and tail as a two macro-particle model:



This produces a single bunch current $I_b = qNI_bf_{rev}$ limit of:

$$I_b \leq \frac{4\pi q\gamma\omega_0\nu_\beta\nu_s}{r_c\beta c\,(Z_\perp/n)_{imag}}$$

Severe limitation on single-bunch currents in storage rings – special care much be taken to minimise transverse impedance of the vacuum chamber.

Head – Tail Instability

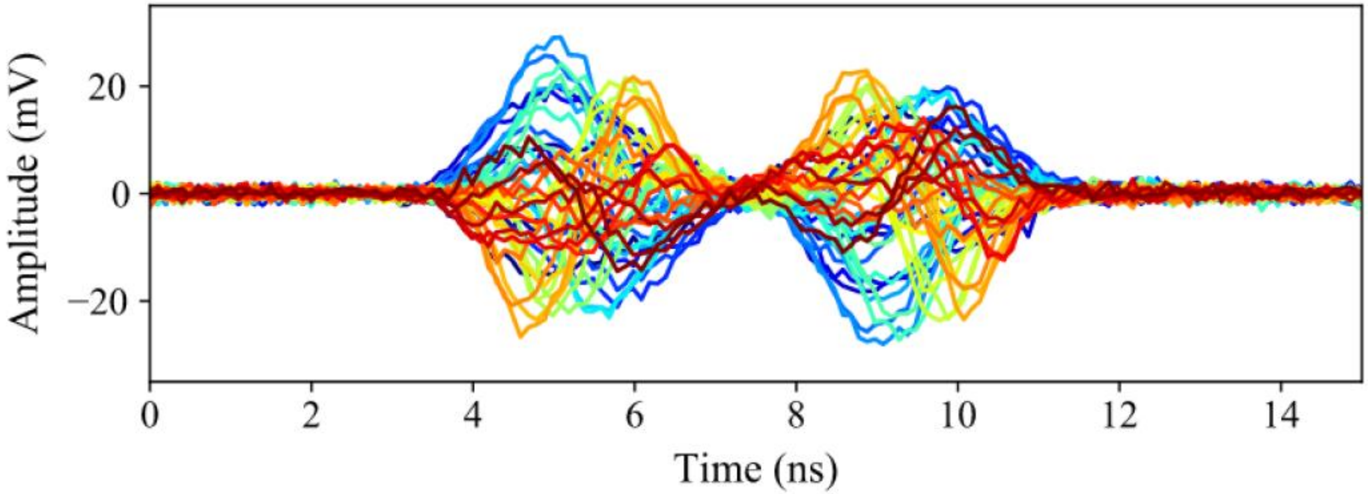
- In a circular proton accelerator:
  - No radiation damping.
  - Weak transverse wakefields can lead to transverse bunch blow up and beam loss.
  - Particles in the head of a bunch oscillate due to synchrotron oscillations.
  - There is an intensity $\boldsymbol{I_b}$ from which the perturbation from wakefields is strong enough to produce instabilities.

$$I_b \leq \frac{4\pi q \gamma \omega_0 \nu_\beta \nu_s}{r_c \beta c \, \mathrm{Im}(Z_\perp / n)}$$

- Exceeding this limit leads to an immediate loss of the excess current.
- Most severe instability.

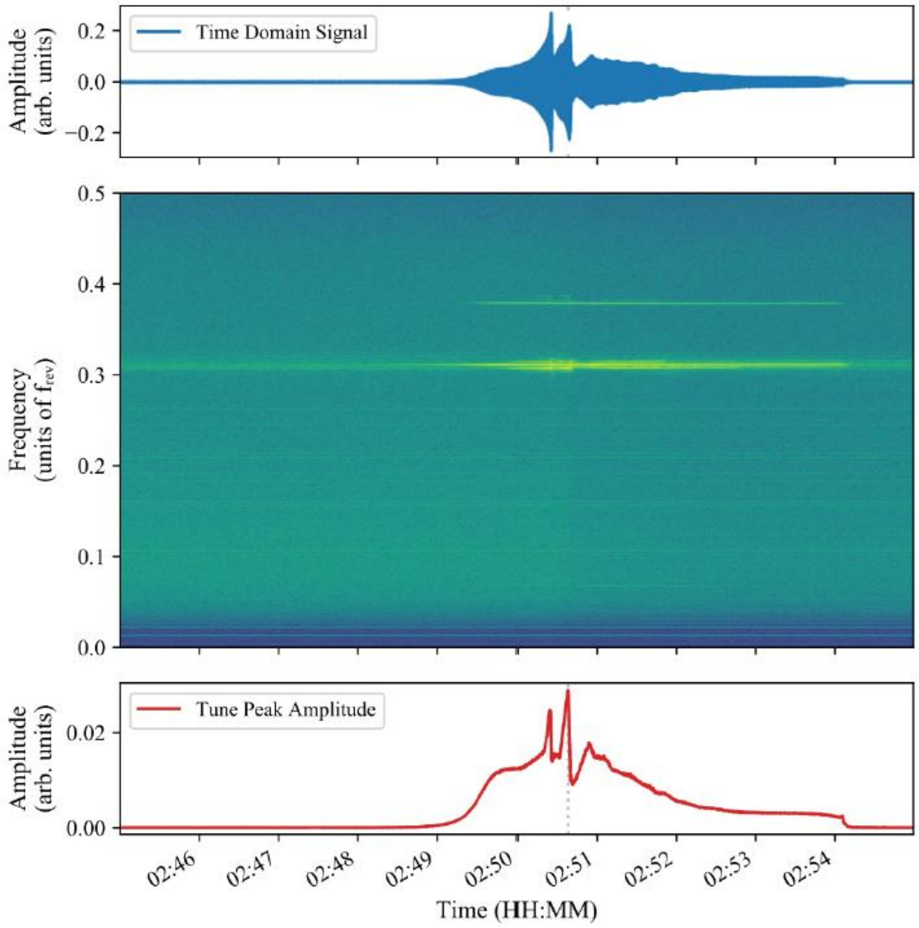
Head – Tail Instability

- Head and tail start to oscillate while the centre of the bunch remains unperturbed.



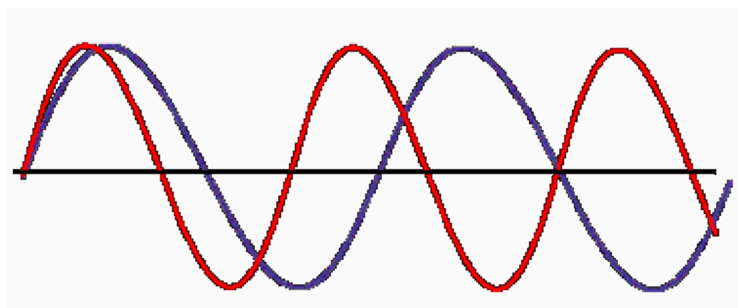
(a) 8-bit oscillscope

Typical Instability in the LHC



Landau Damping

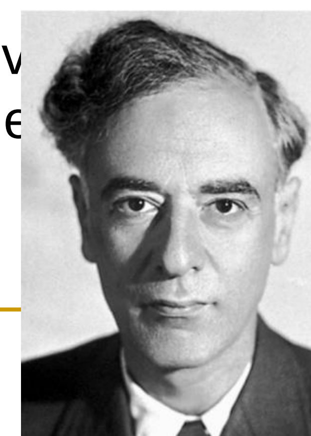
- In real machines, not all particles in the beam have same momentum (frequency).
  - Hence have a revolution frequency spread that may be sufficient to stabilise beam.
- The coherent motion from an instability therefore de-coheres over time, potentially dampening the instability.



Two particles
of different frequency
move out of step.

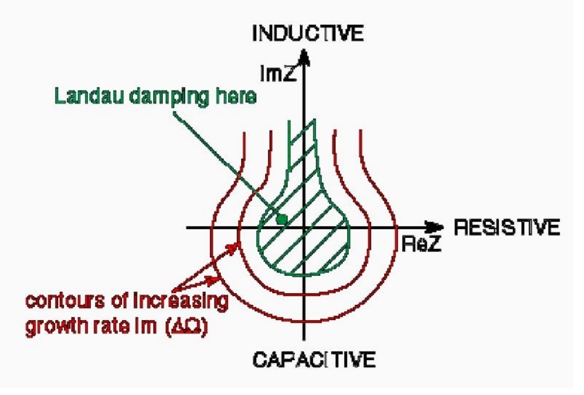
- Two oscillators excited together become incoherent and giv
  centre of charge motion after a number of turns comparable
  reciprocal of their frequency difference.
- Landau damping is the absence of oscillations.

Lev. D. Landau



Landau Damping

- Landau damping applies not just to **longitudinal** but also **transverse**, **single-bunch** and **multi-bunch** instabilities.
- Along with active feedback systems, it is a powerful way to overcome coherent beam instabilities.



Instability diagramme with damping

- The line defining zero growth rate leads us to a handy approximation for the stability limit of unbunched beams – the '**Keil-Schnell Stability Criterion**':

$$\left|\frac{Z}{n}\right| \leq \frac{F m_0 c^2 \beta^2 \gamma \eta}{I_0} \left(\frac{\Delta p}{p}\right)^2_{FWHH}$$

General Remarks

- Transverse and longitudinal planes treated in basically same way and results are very similar.
- Some differences:
  - Longitudinal impedance depends predominantly on induced electric fields.
    - $Z_{long.}$ Is order of a few Ω.
  - Transverse impedance depends on both electric and magnetic fields, with the latter being more important.
    - $Z_{trans.}$ Is millions of Ω/m

General Remarks

- Impedance of machine varies with frequency.
  - Low frequencies – Impedance dominated by skin effect of vacuum chanmber.
  - High frequencies – Impedance behaves like broadband resonator (Q = 1), due to numerous small resonators (bellows, cavities, tanks, etc.)
- Mitigation factors against coherent instabilities are limited.
  - In longitudinal case, design vacuum chamber with low coupling impedance.
  - In transverse case, enhance Landau damping by use of sextupoles and octupoles.
  - Active feedback system as last resort.

Summary

- Impedance is one of the main sources of beam instability.
  - Generated by the interaction/coupling between the beam and the accelerator environment.
  - Produce wakefields that perturb the beam motion.
- Eventually, this perturbation might lead to instabilities.
  - Broad-band impedances mainly responsible for single-bunch beam instabilities.
  - Narrow-band impedances can cause multi-bunch instabilities but usually do not affect single-bunch intensity limits.
  - Both can cause longitudinal and transverse instabilities.
- Various ways exist to mitigate instabiities (radiation damping, octupoles etc.)