# An introduction to Magnets for Accelerators

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John Adams Institute
Accelerator Course

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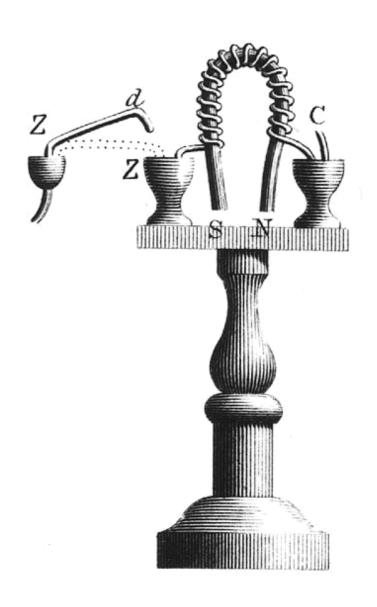
### This is an introduction to magnets as building blocks of synchrotrons / transfer lines

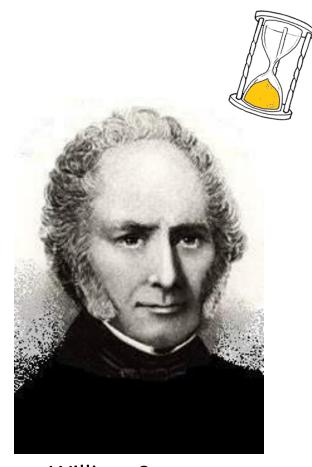
```
//
// MADX Example 2: FODO cell with dipoles
// Author: V. Ziemann, Uppsala University
// Date: 060911
TITLE, 'Example 2: FODO2.MADX';
BEAM, PARTICLE=ELECTRON, PC=3.0;
DEGREE:=PI/180.0;
OF: OUADRUPOLE, L=0.5, K1=0.2;
OD: QUADRUPOLE, L=1.0, K1=-0.2;
B: SBEND, L=1.0, ANGLE=15.0*DEGREE;
FODO: SEQUENCE, REFER=ENTRY, L=12.0;
 QF1: QF, AT=0.0;
 B1: B, AT=2.5;
 QD1: QD, AT=5.5;
 B2: B, AT=8.5;
 QF2: QF, AT=11.5;
ENDSEQUENCE;
```

#### If you want to know more...

- 1. Special CAS edition on magnets, St. Pölten, Nov.-Dec. 2023
- 2. Special CAS edition on magnets, Bruges, Jun. 2009
- 3. N. Marks, Magnets for Accelerators, JAI (John Adams Institute) course, Jan. 2015
- 4. D. Tommasini, Practical Definitions & Formulae for Normal Conducting Magnets
- 5. Lectures about magnets in CERN Accelerator Schools
- 6. Superconducting magnets for particle accelerators in USPAS (U.S. Particle Accelerator Schools)
- 7. J. Tanabe, Iron Dominated Electromagnets
- 8. P. Campbell, Permanent Magnet Materials and their Application
- 9. K.-H. Mess, P. Schmüser, S. Wolff, Superconducting Accelerator Magnets
- 10. M. N. Wilson, Superconducting Magnets
- 11. A. Devred, Practical Low-Temperature Superconductors for Electromagnets
- 12. L. Rossi and E. Todesco, Electromagnetic design of superconducting dipoles based on sector coils

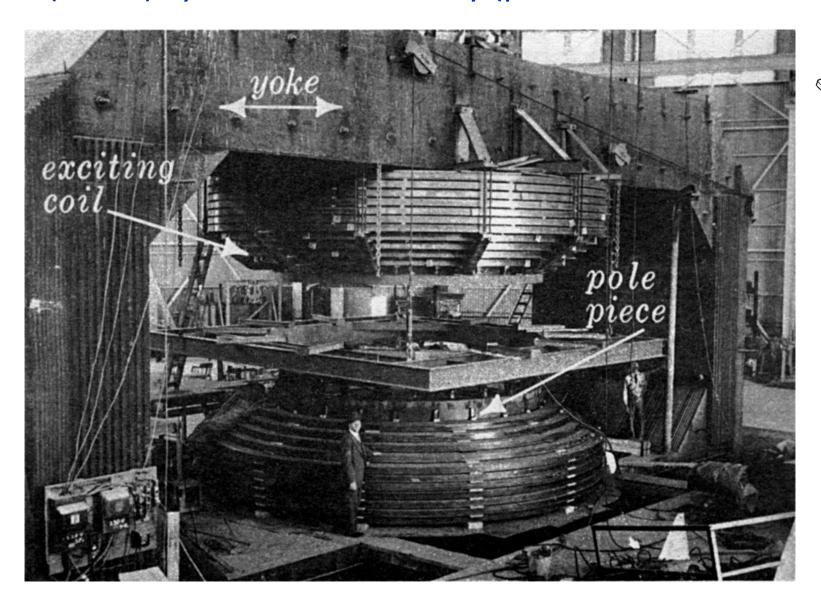
### According to history, the first electromagnet (not for an accelerator) was built in England in 1824 by William Sturgeon





William Sturgeon

The working principle is the same as this large magnet, of the 184" (4.7 m) cyclotron at Berkeley (picture taken in 1942)



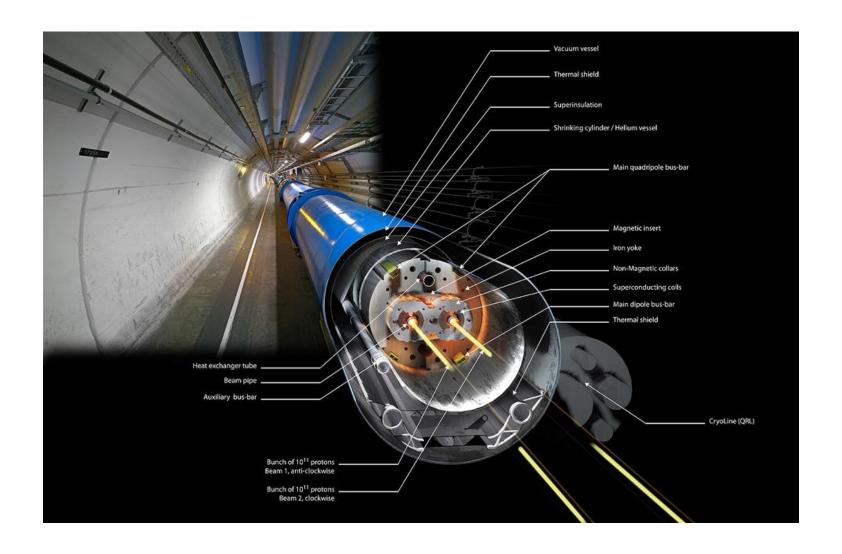
#### This short course is organized in several blocks

- 1. Introduction, jargon, general concepts and formulae
- 2. Resistive magnets
- 3. Superconducting magnets
- 4. Tutorial with FEMM

### Magnets can be classified based on their geometry / what they do to the beam

dipole solenoid combined function quadrupole bending sextupole corrector octupole skew magnet undulator / kicker wiggler

#### This is a main dipole of the LHC at CERN: 8.3 T $\times$ 14.3 m



#### These are main dipoles of the SPS at CERN: 2.0 T $\times$ 6.3 m



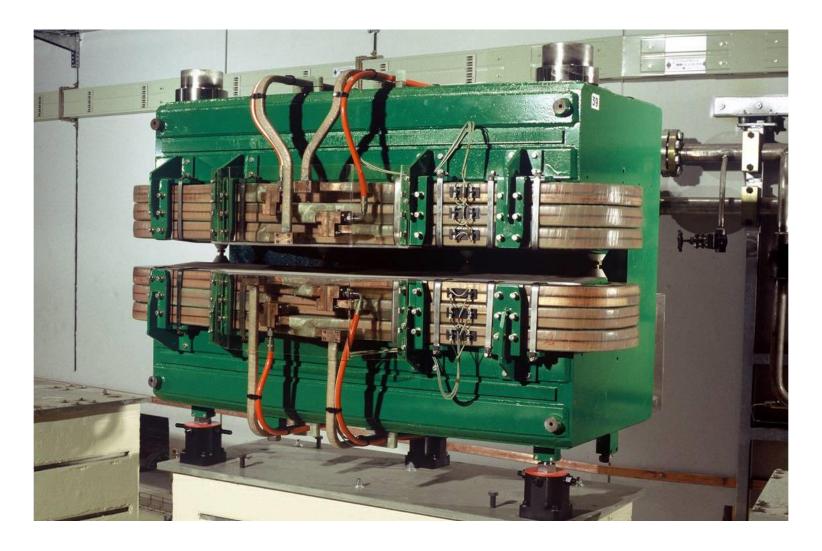
### This is a cross section of a main quadrupole of the LHC at CERN: $223 \text{ T/m} \times 3.2 \text{ m}$



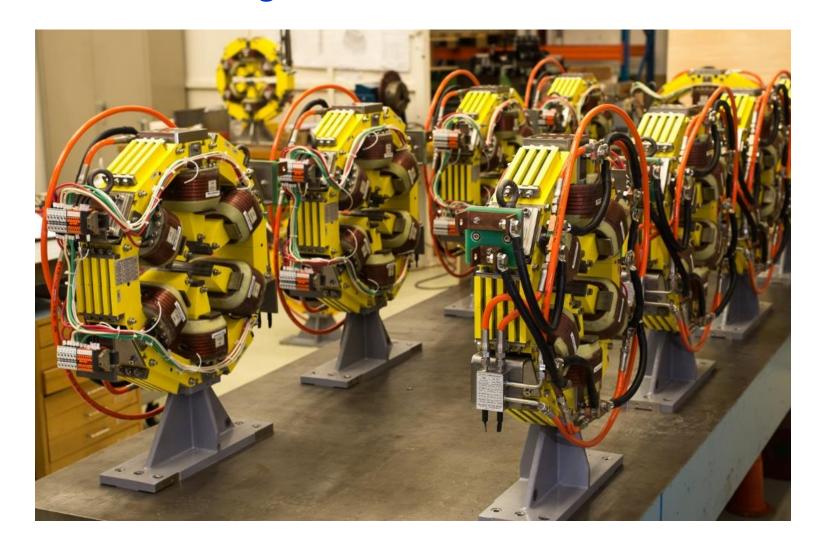
# These are main quadrupoles of the SPS at CERN: $22 \text{ T/m} \times 3.2 \text{ m}$



### This is a combined function bending magnet of the ELETTRA light source



### These are sextupoles (with embedded correctors) of the main ring of the SESAME light source



### Magnets can be classified also differently, looking for example at their technology

electromagnet

permanent magnet

iron dominated

coil dominated

normal conducting (resistive)

superconducting

static

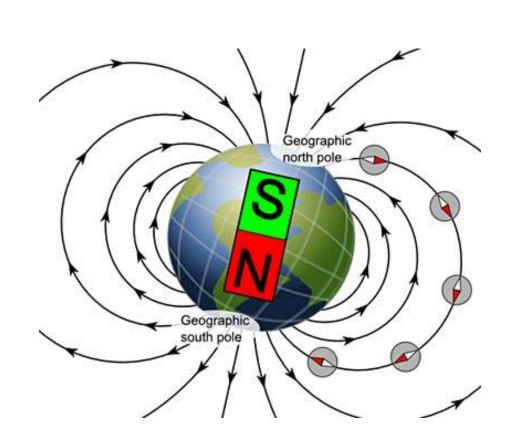
cycled / ramped slow pulsed

fast pulsed

#### Nomenclature

В	magnetic field B field magnetic flux density magnetic induction	T (Tesla)
Н	H field magnetic field strength magnetic field	A/m (Ampere/m)
$\mu_0$	vacuum permeability	1.25663706212(19)·10 <sup>-6</sup> H/m $4\pi\cdot10^{-7}$ H/m (Henry/m)
$\mu_{r}$	relative permeability	dimensionless
μ	permeability, $\mu = \mu_0 \mu_r$	H/m

### The polarity comes from the direction of the flux lines, that go from a North to a South pole





in Oxford, on 25/01/2017 |B| = 48728 nT = 0.048728 mT = 0.000048728 T

### Magnetostatic fields are described by Maxwell's equations, coupled with a law describing the material

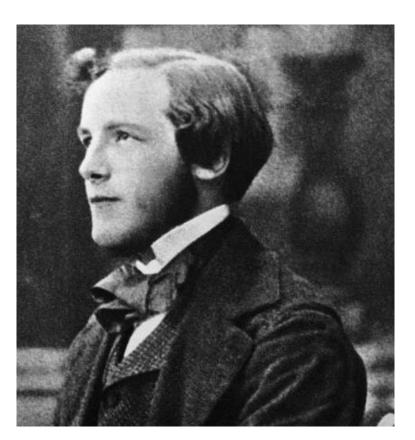
$$\operatorname{div} \vec{B} = 0$$

$$\oint_{S} \vec{B} \cdot \vec{dS} = 0$$

$$\operatorname{rot} \vec{H} = \vec{J}$$

$$\oint_C \vec{H} \cdot \vec{dl} = \int_S \vec{J} \cdot \vec{dS} = NI$$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

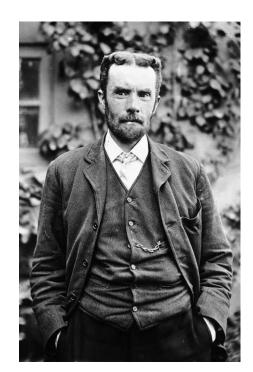


James Clerk Maxwell

### The Lorentz force is the link between electromagnetism and mechanics

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$
  
for charged beams

$$\vec{F} = I \vec{\ell} \times \vec{B}$$
 for conductors



Oliver Heaviside



Hendrik Lorentz

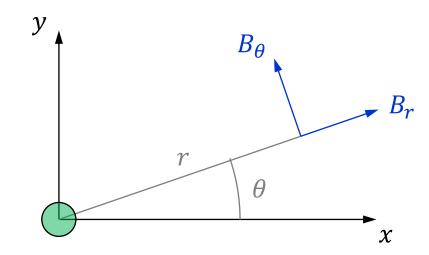


Pierre-Simon, marquis de Laplace

### In synchrotrons / transfer lines magnets, the B field seen from the beam is often expressed as a series of multipoles

$$B_r = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} \left[B_n \sin(n\theta) + A_n \cos(n\theta)\right]$$

$$B_{\theta} = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} \left[B_n \cos(n\theta) - A_n \sin(n\theta)\right]$$



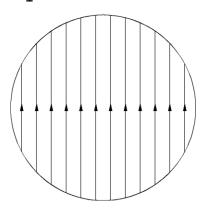
direction of the beam (orthogonal to plane)

$$B_{y}(z) + iB_{x}(z) = \sum_{n=0}^{\infty} (B_{n} + iA_{n}) \left(\frac{z}{R}\right)^{n-1}$$

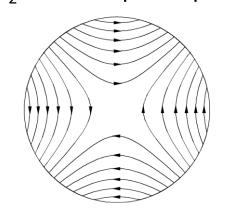
$$z = x + iy = re^{i\theta}$$

### Each multipole term corresponds to a field distribution; they can be added up (exploiting linear superposition)

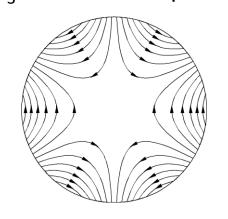
B<sub>1</sub>: normal dipole



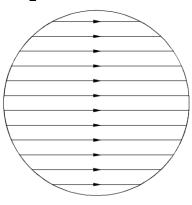
B<sub>2</sub>: normal quadrupole



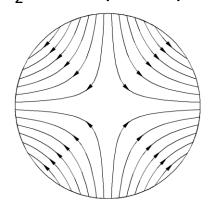
B<sub>3</sub>: normal sextupole



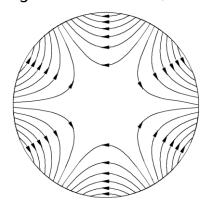
A<sub>1</sub>: skew dipole



A<sub>2</sub>: skew quadrupole

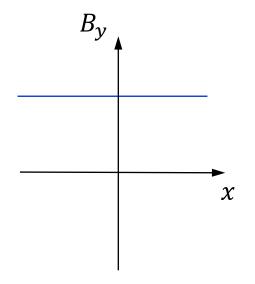


A<sub>3</sub>: skew sextupole

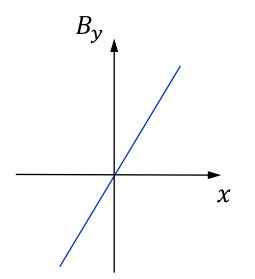


### The field profile in the horizontal plane follows a polynomial expansion

$$B_y(x) = \sum_{n=1}^{\infty} B_n \left(\frac{x}{R}\right)^{n-1} = B_1 + B_2 \frac{x}{R} + B_3 \frac{x^2}{R^2} + \cdots$$

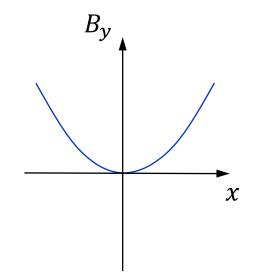


B<sub>1</sub>: dipole



B<sub>2</sub>: quadrupole

$$G = \frac{B_2}{R} = \frac{\partial B_y}{\partial x}$$



B<sub>3</sub>: sextupole

$$B^{\prime\prime} = \frac{2B_3}{R^2}$$

### For the optics, usually the field or multipole component is given, together with the (magnetic) length: ex. from MAD-X

#### <u>Dipole</u>

bend angle a [rad] & length L [m]

$$k_0$$
 [1/m] & length L [m] obsolete  
 $k_0 = B / (B\rho)$   $B = B_1$ 



#### <u>Quadrupole</u>

quadrupole coefficient 
$$k_1 [1/m^2] \times length L [m]$$
  
 $k_1 = (dB_y/dx) / (B\rho)$   
 $G = dB_y/dx = B_2/R$ 

#### <u>Sextupole</u>

sextupole coefficient 
$$k_2$$
 [1/m³] × length L [m]  
 $k_2$  = (d²B<sub>y</sub>/dx²) / (B $\rho$ )  
(d²B<sub>y</sub>/dx²)/2! = B<sub>3</sub>/R²

### Here is how to compute magnetic quantities from MAD-X entries, and vice versa

```
BEAM, PARTICLE=ELECTRON, PC=3.0;

DEGREE:=PI/180.0;

QF: QUADRUPOLE, L=0.5, K1=0.2;

QD: QUADRUPOLE, L=1.0, K1=-0.2;

B: SBEND, L=1.0, ANGLE=15.0*DEGREE;
```

$$(B\rho) = 10^9/c*PC = 10^9/299792485*3.0 = 10.01 Tm$$

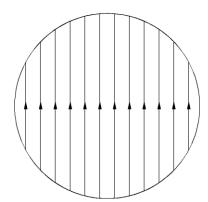
dipole (SBEND)  
B = 
$$|ANGLE|/L^*(B\rho) = (15*pi/180)/1.0*10.01 = 2.62 T$$

#### <u>quadrupole</u>

$$G = |K1|*(B\rho) = 0.2*10.01 = 2.00 T/m$$

The harmonic decomposition is used also to describe the field quality (or field homogeneity), that is, the deviations of the actual B with respect to the ideal one

(normal) dipole



$$\vec{B}_{id}(x,y) = B_1 \vec{J}$$

$$B_{y}(z) + iB_{x}(z) =$$

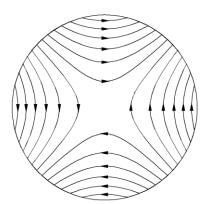
$$= B_{1} + \frac{B_{1}}{10000} \left[ ia_{1} + (b_{2} + ia_{2}) \left( \frac{z}{R} \right) + (b_{3} + ia_{3}) \left( \frac{z}{R} \right)^{2} + (b_{4} + ia_{4}) \left( \frac{z}{R} \right)^{3} + \cdots \right]$$

$$b_2 = 10000 \frac{B_2}{B_1}$$
  $b_3 = 10000 \frac{B_3}{B_1}$   $a_1 = 10000 \frac{A_1}{B_1}$   $a_2 = 10000 \frac{A_2}{B_1}$  ...

#### The same expression can be written for a quadrupole



#### (normal) quadrupole



$$\vec{B}_{id}(x,y) = B_2[x\vec{j} + y\vec{i}]\frac{1}{R}$$

$$B_{y}(z) + iB_{x}(z) =$$

$$= B_{2} \frac{z}{R} + \frac{B_{2}}{10000} \left[ ia_{2} \left( \frac{z}{R} \right) + (b_{3} + ia_{3}) \left( \frac{z}{R} \right)^{2} + (b_{4} + ia_{4}) \left( \frac{z}{R} \right)^{3} + \cdots \right]$$

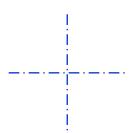
$$b_3 = 10000 \frac{B_3}{B_2}$$
  $b_4 = 10000 \frac{B_4}{B_2}$   $a_2 = 10000 \frac{A_2}{B_2}$  ...

### The *allowed / not-allowed* harmonics refer to some terms that shall / shall not cancel out thanks to design symmetries

#### fully symmetric dipoles

allowed:  $B_1$ ,  $b_3$ ,  $b_5$ ,  $b_7$ ,  $b_9$ , etc.

not-allowed: all the others





\_.\_...

half symmetric dipoles

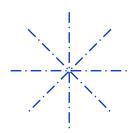
allowed:  $B_1$ ,  $b_2$ ,  $b_3$ ,  $b_4$ ,  $b_5$ , etc.

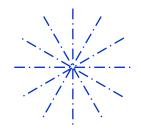
not-allowed: all the others

#### fully symmetric quadrupoles

allowed:  $B_2$ ,  $b_6$ ,  $b_{10}$ ,  $b_{14}$ ,  $b_{18}$ , etc.

not-allowed: all the others





fully symmetric sextupoles

allowed:  $B_3$ ,  $b_9$ ,  $b_{15}$ ,  $b_{21}$ , etc.

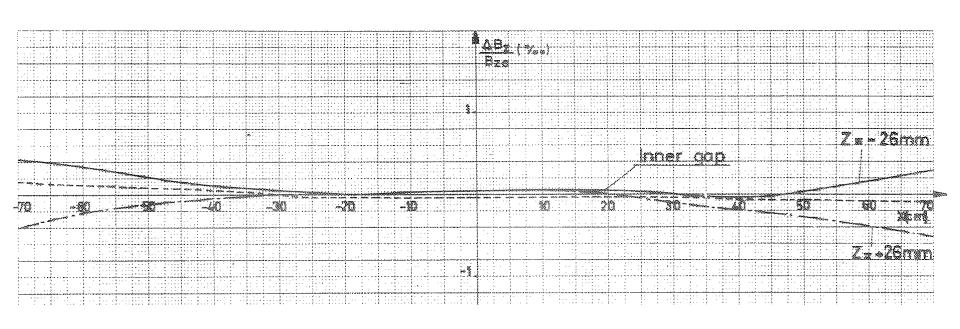
not-allowed: all the others

#### The field quality is often also shown with a $\Delta B/B$ plot



$$\frac{\Delta B}{B} = \frac{B(x, y) - B_{id}(x, y)}{B_{id}(x, y)}$$

done on one component, usually B<sub>y</sub> for a dipole



# $\Delta B/B$ can (at least locally) be expressed from the harmonics: this is the expansion for a dipole



$$B_{y,id}(x) = B_1$$

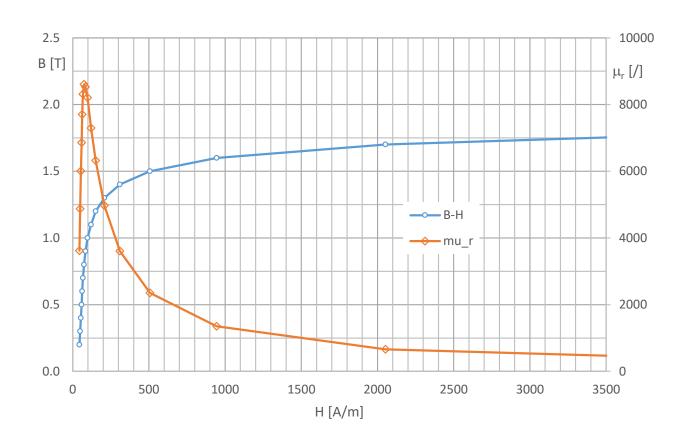
$$B_y(x) = B_1 + \frac{B_1}{10000} \left[ b_2 \left( \frac{x}{R} \right) + b_3 \left( \frac{x}{R} \right)^2 + b_4 \left( \frac{x}{R} \right)^3 + \cdots \right]$$

$$\frac{\Delta B}{B}(x) = \frac{1}{10000} \left[ b_2 \left( \frac{x}{R} \right) + b_3 \left( \frac{x}{R} \right)^2 + b_4 \left( \frac{x}{R} \right)^3 + \cdots \right]$$

1. Introduction, jargon, general concepts and formulae

- 2. Resistive magnets
- 3. Superconducting magnets
- 4. Tutorial with FEMM

### Resistive magnets are in most cases "iron-dominated": the BH response of the yoke material is important



curves for typical M1200-100 A electrical steel

### These are typical fields for resistive dipoles and quadrupoles, taken from machines at CERN

#### PS @ 26 GeV

combined function bending B = 1.5 T

#### SPS @ 450 GeV

bending B = 2.0 T

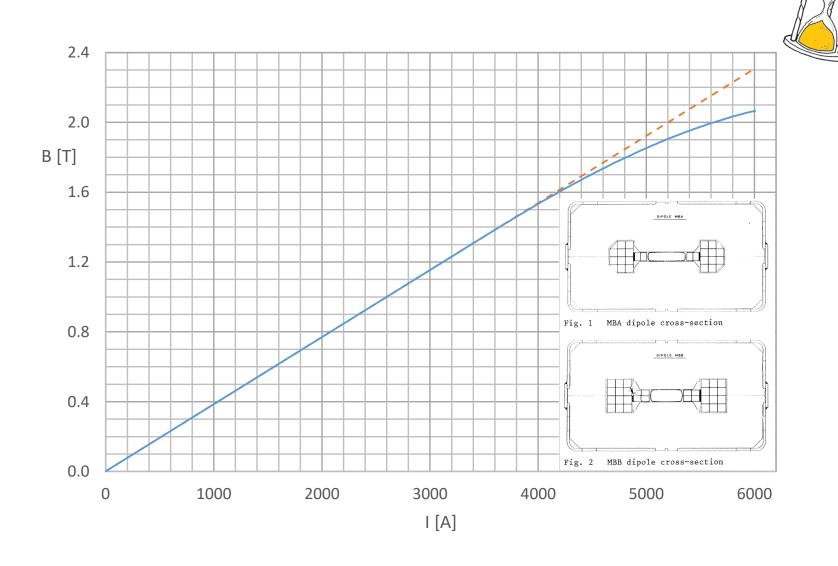
quadrupole  $B_{pole} = 21.7*0.044 = 0.95 T$ 

#### TI2 / TI8 (transfer lines SPS to LHC, @ 450 GeV)

bending B = 1.8 T

quadrupole  $B_{pole} = 53.5*0.016 = 0.86 T$ 

This is the (average) transfer function field B vs. current I for the SPS main dipoles



If the magnet is not dc, then an rms power / current is taken, considering the duty cycle



$$P_{rms} = RI_{rms}^2 = \frac{1}{T} \int_0^T R[I(t)]^2 dt$$

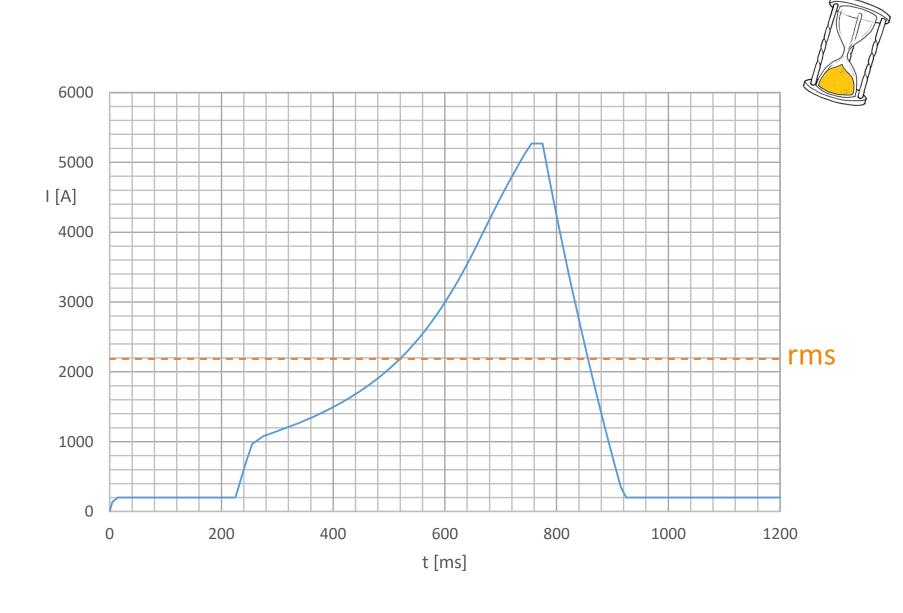
for a pure sine wave

$$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$$

for a linear ramp from 0

$$I_{rms} = \frac{I_{peak}}{\sqrt{3}}$$

#### This is a cycle to 2.0 GeV of the PSB at CERN



### For resistive coils, the material is most often copper,

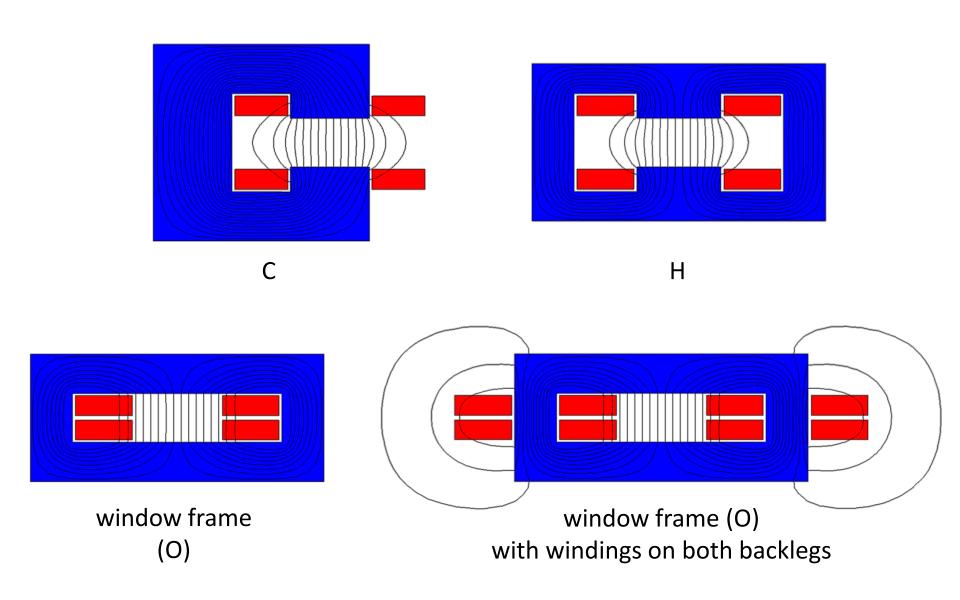
#### sometimes aluminum

CuAlraw metal price $\approx 8400 \ \$/\text{ton}$  $\approx 2300 \ \$/\text{ton}$ electrical resistivity $1.72 \cdot 10^{-8} \Omega/\text{m}$  $2.65 \cdot 10^{-8} \Omega/\text{m}$ density $8.9 \ \text{kg/dm}^3$  $2.7 \ \text{kg/dm}^3$ 

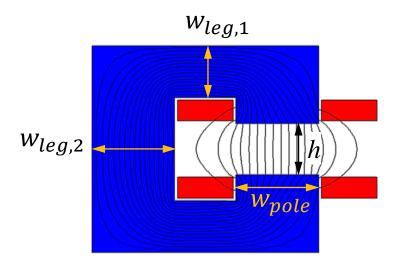


LHCb detector dipole
Al coils
coil mass 2 × 25 t
power 2 × 2.1 MW

#### These are the most common types of resistive dipoles



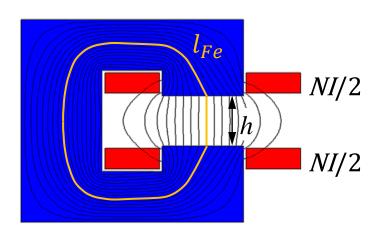
The magnetic circuit is dimensioned so that the pole is wide enough for field quality, and there is enough room for the flux in the return legs



$$w_{pole} \cong w_{GFR} + 2.5h$$

$$B_{leg} \cong B_{gap} \frac{w_{pole} + 1.2h}{w_{leg}}$$

# The Ampere-turns are a linear function of the gap and of the B field (at least up to saturation)



$$NI = \oint \vec{H} \cdot \vec{dl} = \frac{B_{Fe}}{\mu_0 \mu_r} \cdot l_{Fe} + \frac{B_{gap}}{\mu_0} \cdot h \cong \frac{B_{gap}h}{\mu_0}$$

$$NI = \frac{Bh}{\eta \mu_0} \qquad \eta = \frac{1}{1 + \frac{1}{\mu_r} \frac{l_{Fe}}{h}}$$

### The same can be solved using magnetic reluctances and Hopkinson's law, which is a parallel of Ohm's law



$$\mathcal{R} = \frac{\text{NI}}{\Phi}$$

$$R = \frac{V}{I}$$

$$\mathcal{R} = \frac{l}{\mu_0 \mu_r A} \qquad \qquad \mathbf{R} = \frac{l}{\sigma S}$$

$$R = \frac{l}{\sigma S}$$

$$\eta = \frac{1}{1 + \frac{\mathcal{R}_{Fe}}{\mathcal{R}_{gap}}}$$

### Example of computation of Ampere-turns and current

$$\eta \cong 0.97$$

$$NI = (1.5*0.080)/(0.97*4*pi*10^-7) = 98446 A total$$

### low inductance option

64 turns,  $I \cong 98500/64 = 1540 A$ 

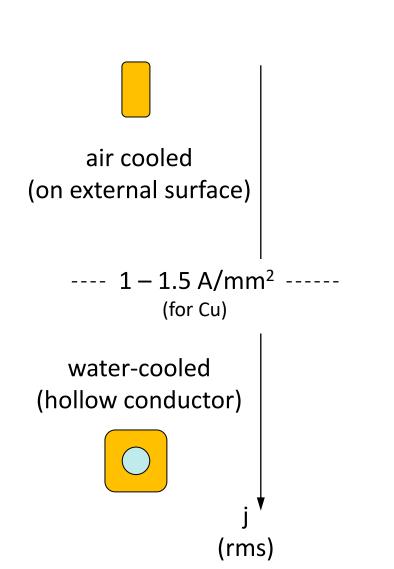
 $L = 62.9 \text{ mH}, R = 15.0 \text{ m}\Omega$ 

### low current option

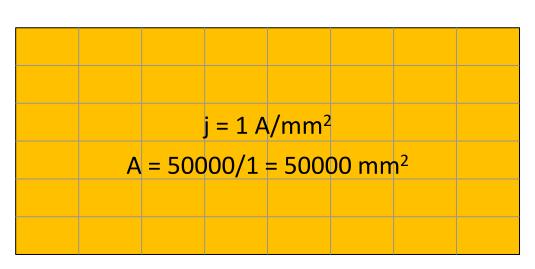
204 turns,  $I \cong 98500/204 = 483 A$ 

 $L = 639 \text{ mH}, R = 160 \text{ m}\Omega$ 

Besides the number of turns, the overall size of the coil depends on the current density, which drives the resistive power consumption (linearly)



ex. NI = 50000 A (rms)



j = 5 A/mm<sup>2</sup> A = 50000/5 = = 10000 mm<sup>2</sup>

### These are common formulae for the main electric parameters of a resistive dipole (1/2)

Ampere-turns (total)  $NI = \frac{Bh}{\eta \mu_0}$ current

$$NI = \frac{Bh}{\eta \mu_0}$$

$$I = \frac{(NI)}{N}$$

$$R = \frac{\rho N L_{turn}}{A_{cond}}$$

$$L \cong \eta \mu_0 N^2 A/h$$

$$A \cong (w_{pole} + 1.2h)(l_{Fe} + h)$$

### These are common formulae for the main electric parameters of a resistive dipole (2/2)

voltage

$$V = RI + L\frac{dI}{dt}$$

resistive power (rms)  $P_{rms} = RI_{rms}^2$ 

$$\begin{aligned} P_{rms} &= RI_{rms}^2 \\ &= \rho j_{rms}^2 V_{cond} \\ &= \frac{\rho L_{turn} B_{rms} h}{\eta \mu_0} j_{rms} \end{aligned}$$

magnetic stored energy  $E_m = \frac{1}{2}LI^2$ 

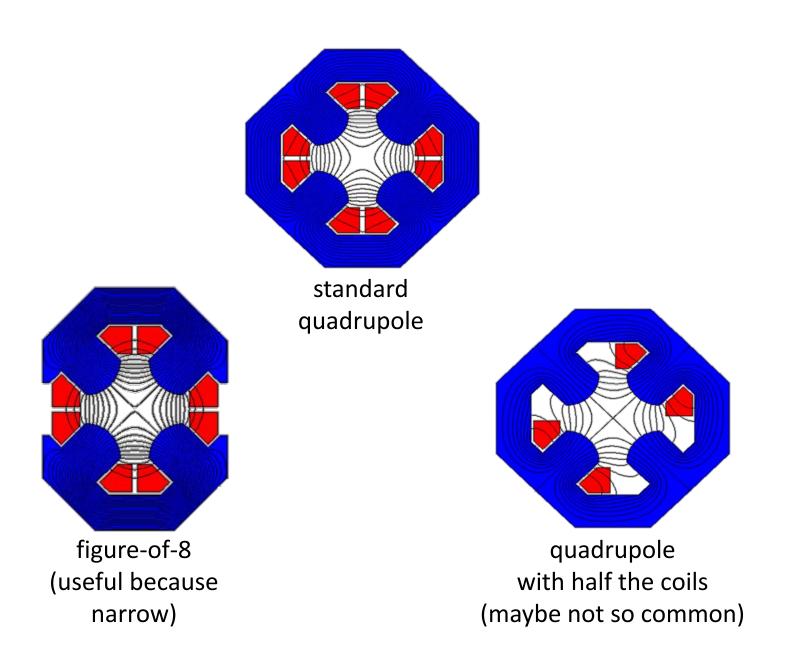
$$E_m = \frac{1}{2}LI^2$$

# The table describes the field quality – in terms of allowed multipoles – for the different layouts of these examples

	C-shaped	H-shaped	O-shaped	
b <sub>2</sub>	1.4	0	0	
b <sub>3</sub>	-88.2	-87.0	0.2	
b <sub>4</sub>	0.7	0	0	
<b>b</b> <sub>5</sub>	-31.6	-31.4	-0.1	
$b_6$	0.1	0	0	
b <sub>7</sub>	-3.8	-3.8	-0.1	
b <sub>8</sub>	0.0	0	0	
b <sub>9</sub>	0.0	0.0	0.0	

 $b_n$  multipoles in units of  $10^{-4}$  at R = 17 mm NI = 20 kA, h = 50 mm,  $w_{pole}$  = 80 mm

### These are the most common types of resistive quadrupoles



### These are useful formulae for standard resistive quadrupoles



pole tip field

$$B_{pole} = Gr$$

Ampere-turns (per pole)  $NI = \frac{Gr^2}{2\eta\mu_0}$ 

$$NI = \frac{Gr^2}{2\eta\mu_0}$$

current

$$I = \frac{(NI)}{N}$$

resistance (total)

$$R = 4 \frac{\rho N L_{turn}}{A_{cond}}$$

### These are useful formulae for the main cooling parameters of a water-cooled resistive magnet



$$Q_{tot} \cong 14.3 \frac{P}{\Delta T}$$
  $Q_{tot} \cong N_{hydr}Q$ 

$$Q_{tot} \cong N_{hydr}Q$$

$$v = \frac{1000}{15\pi d^2} Q$$

$$Re \cong 1400 dv$$

$$\Delta p = 60 L_{hydr} \frac{Q^{1.75}}{d^{4.75}}$$

# The *ideal* poles for dipole, quadrupole, sextupole, etc. are lines of constant scalar potential

#### dipole

$$\rho \sin(\theta) = \pm h/2$$

$$y = \pm h/2$$

straight line

quadrupole

$$\rho^2 \sin(2\theta) = \pm r^2$$

$$2xy = \pm r^2$$

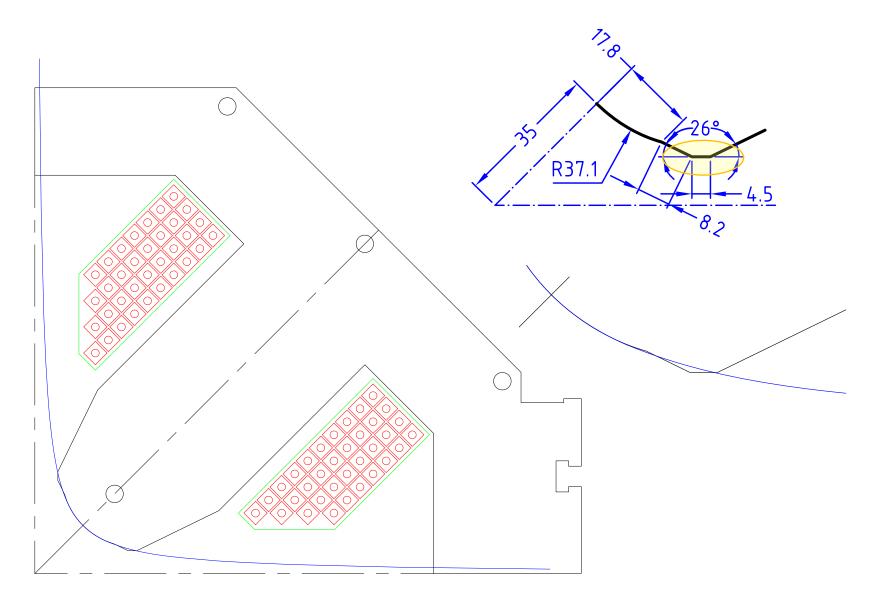
hyperbola

sextupole

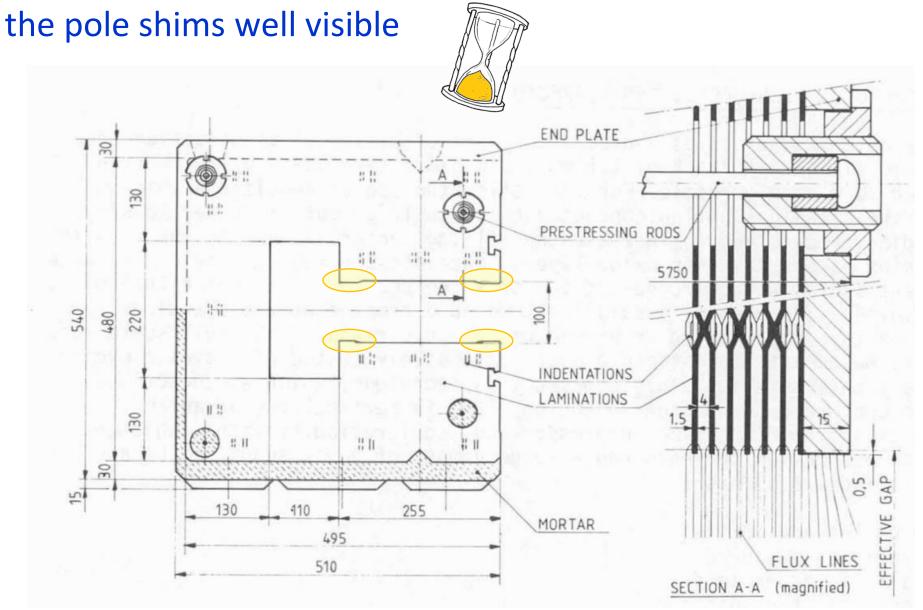
$$\rho^3 \sin(3\theta) = \pm r^3$$

$$3x^2y - y^3 = \pm r^3$$

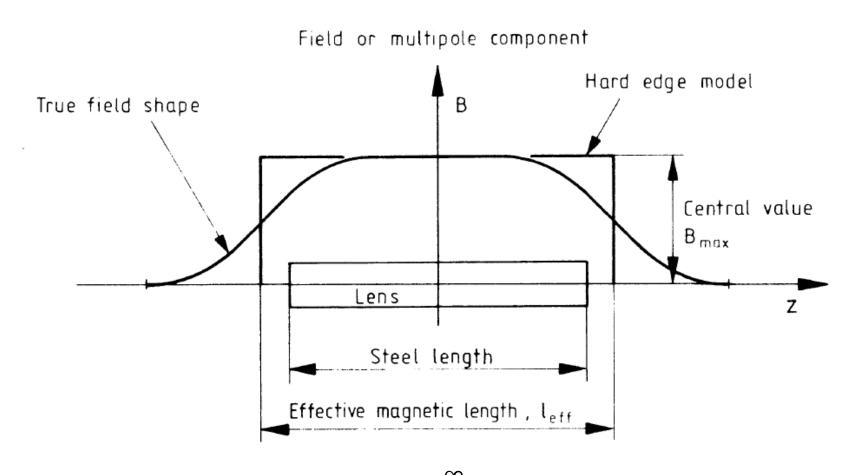
# As an example, this is the pole tip used in the SESAME quadrupoles vs. the theoretical hyperbola



This is the lamination of the LEP main bending magnets, with

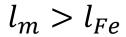


# In 3D, the longitudinal dimension of the magnet is described by a magnetic length



$$l_m B_0 = \int_{-\infty}^{\infty} B(z) dz$$

# The magnetic length can be estimated at first order with simple formulae





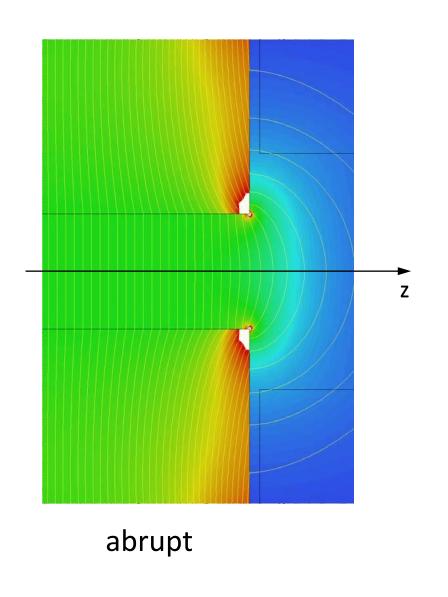
dipole

$$l_m \cong l_{Fe} + h$$

quadrupole

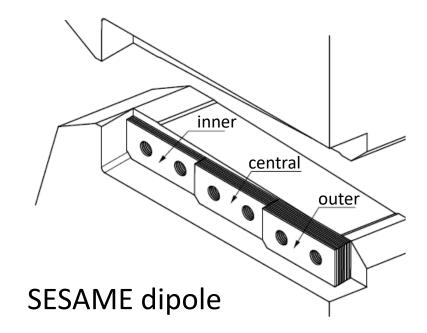
$$l_m \cong l_{Fe} + 0.80r$$

There are many different options to terminate the pole ends, depending on the type of magnet, its field level, etc.

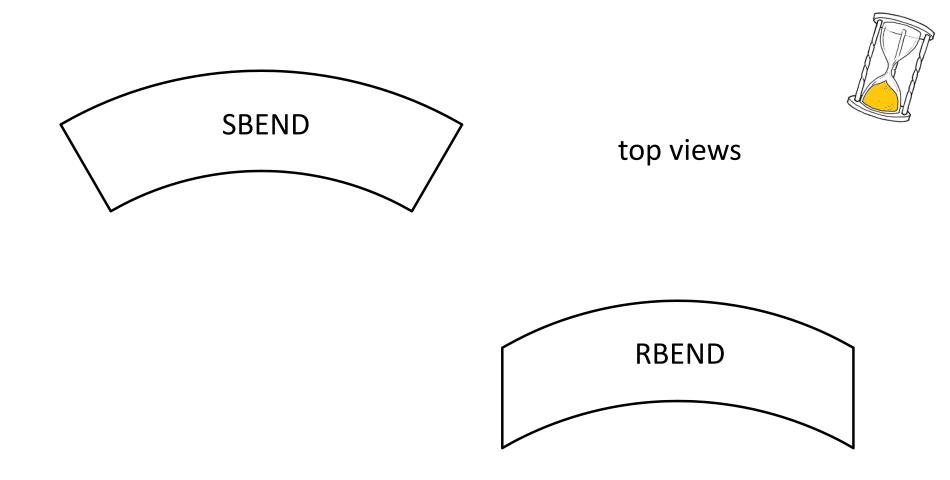




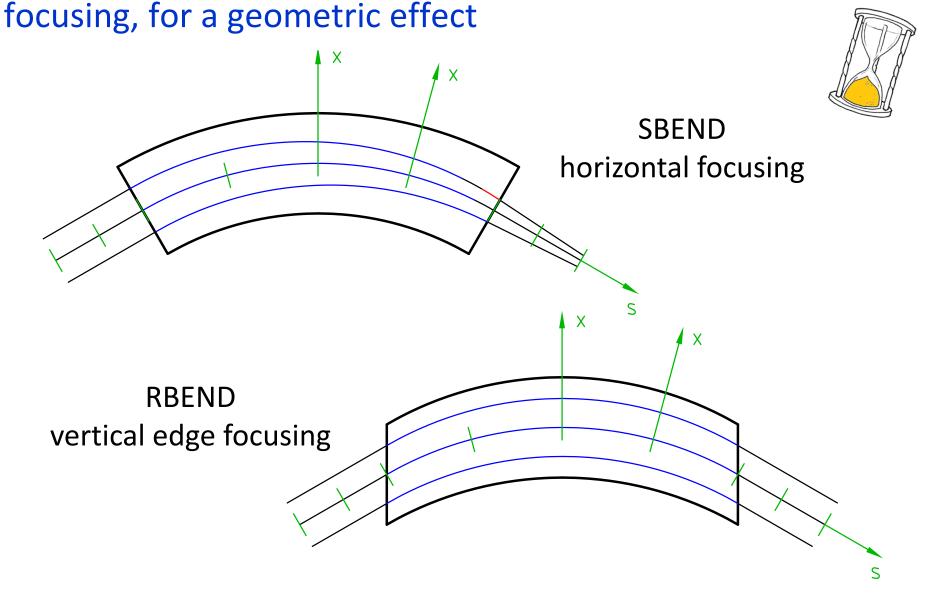
DIAMOND dipole



Usually two dipole elements are found in lattice codes: the sector dipole (SBEND) and the parallel faces dipole (RBEND)



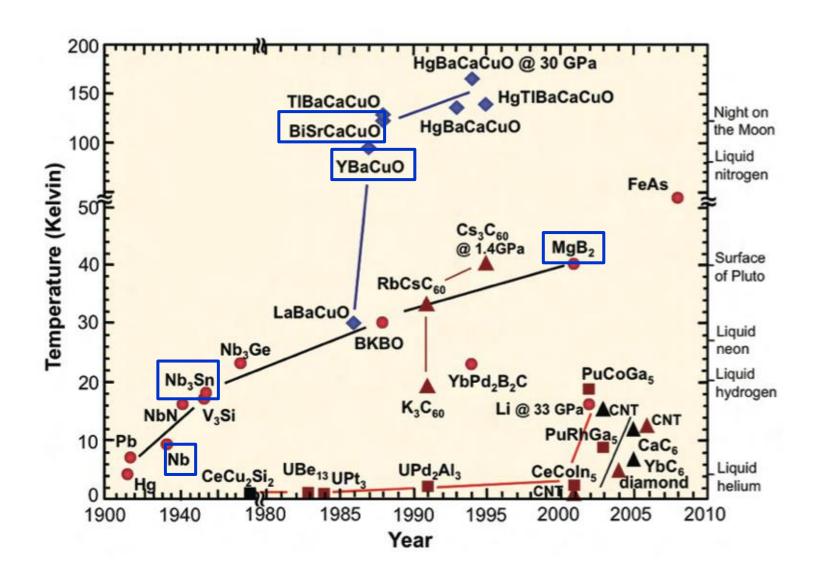
The two types of dipoles are slightly different in terms of



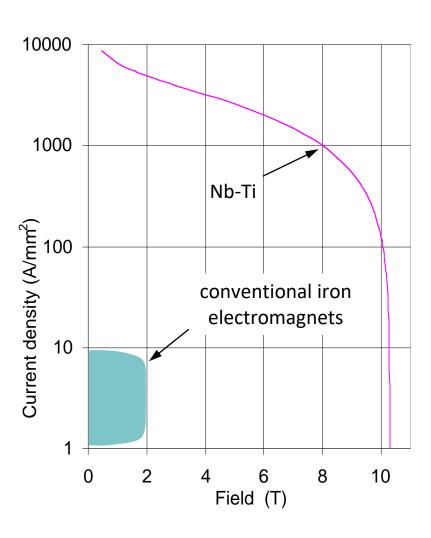
- and anything in between (playing with the edge angles) -

- 1. Introduction, jargon, general concepts and formulae
- 2. Resistive magnets
- 3. Superconducting magnets (thanks to Luca Bottura for the material of many slides)
- 4. Tutorial with FEMM

# This is a history chart of superconductors, starting with Hg all the way to HTS (High Temperature Superconductors)



## Superconductivity makes possible large accelerators with fields well above 2 T



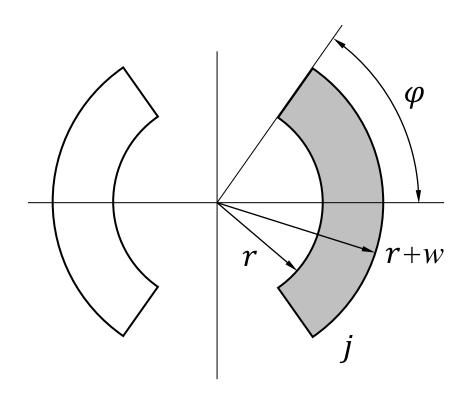
### This is a summary of (somehow) practical superconductors

	LTS			HTS		
material	Nb-Ti	Nb₃Sn	MgB <sub>2</sub>	REBCO	BSCCO	Fe based
year of discovery	1961	1954	2001	1987	1988	2008
T <sub>c</sub> [K]	9.2	18.2	39	≈93	95 / 108	up to 58
B <sub>c2</sub> [T]	≈14.5	≈30	>30	120250	≈200	>100

# The field in the aperture of a superconducting dipole can be derived using Biot-Savart law (in 2D)

$$B_{\theta} = \frac{\mu_0 I}{2\pi\rho}$$

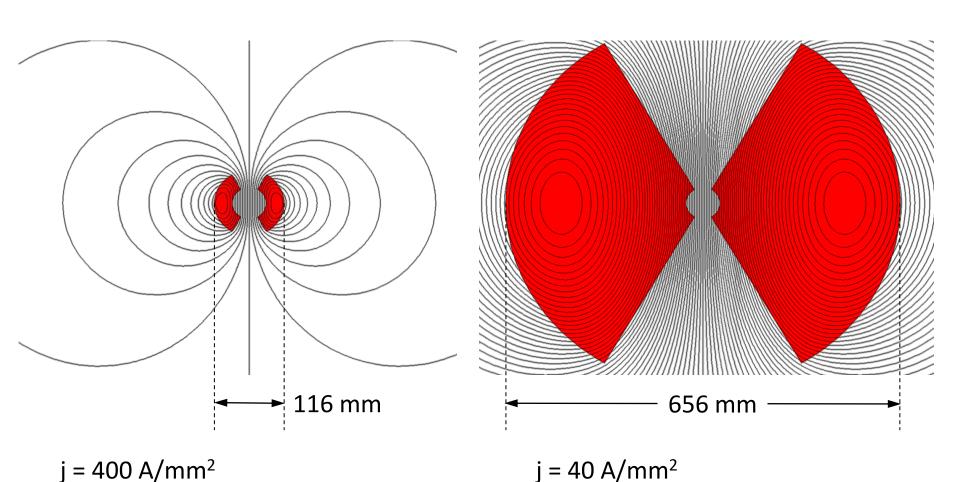
Biot-Savart law for an infinite wire



$$B = \frac{2\mu_0 \sin \varphi}{\pi} jw$$
  
for a sector coil

$$B = \frac{\sqrt{3}\mu_0}{\pi} jw$$
 for a 60 deg sector coil

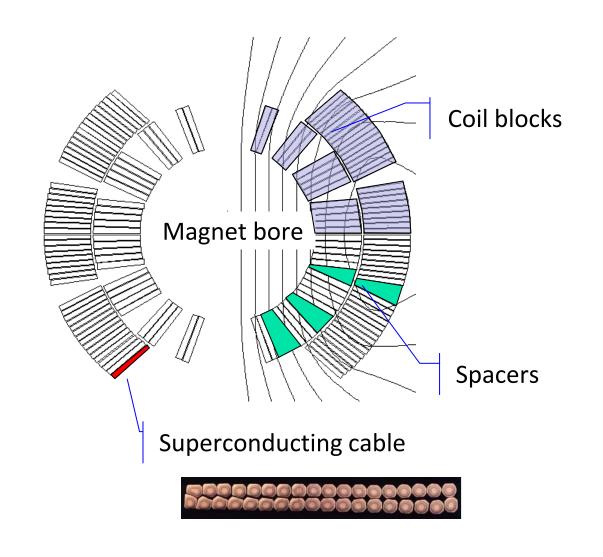
## This is how it would look like one aperture of the LHC dipoles at 8.3 T, with two different current densities (without iron)



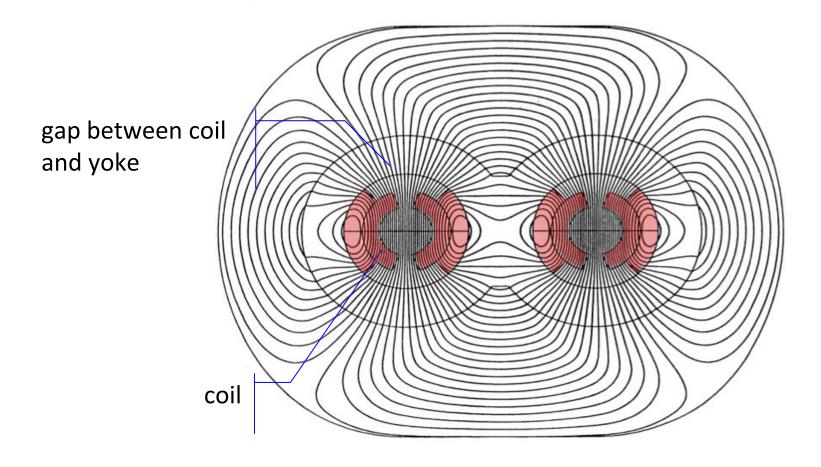
j = 400 A/mm<sup>2</sup> w = 30 mm NI = 1.2 MA P = 14.9 MW/m (if Cu at room temp.)

w = 300 mm NI = 4.5 MA P = 6.2 MW/m (if Cu at room temp.)

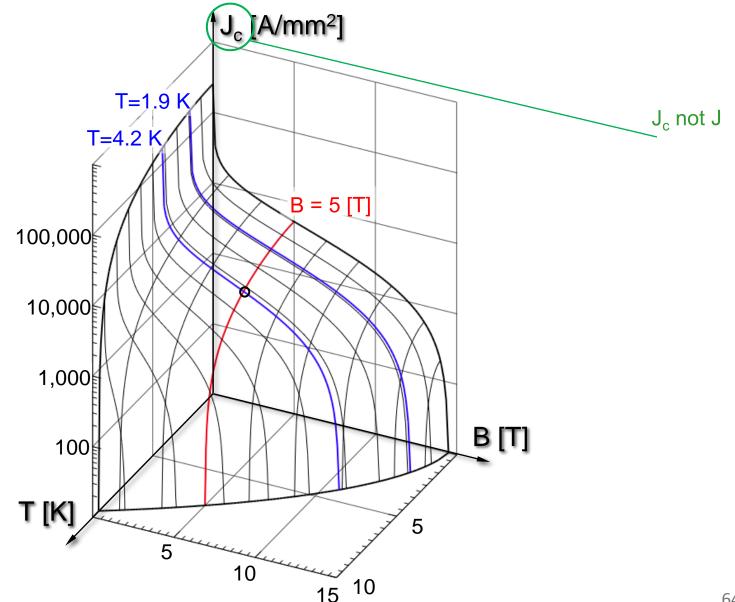
## This is the actual coil of the LHC main dipoles (one aperture), showing the position of the superconducting cables



### Around the coils, iron is used to close the magnetic circuit

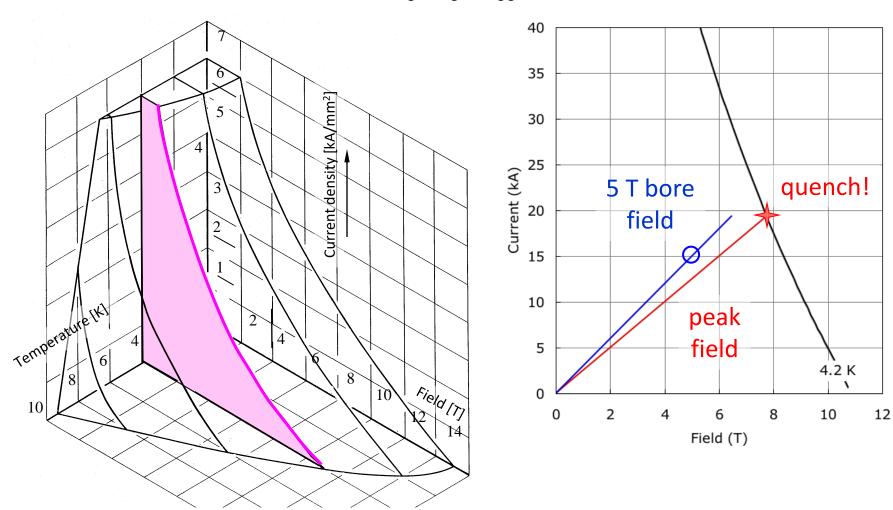


# The allowable current density is high – though finite – and it depends on the temperature and the field

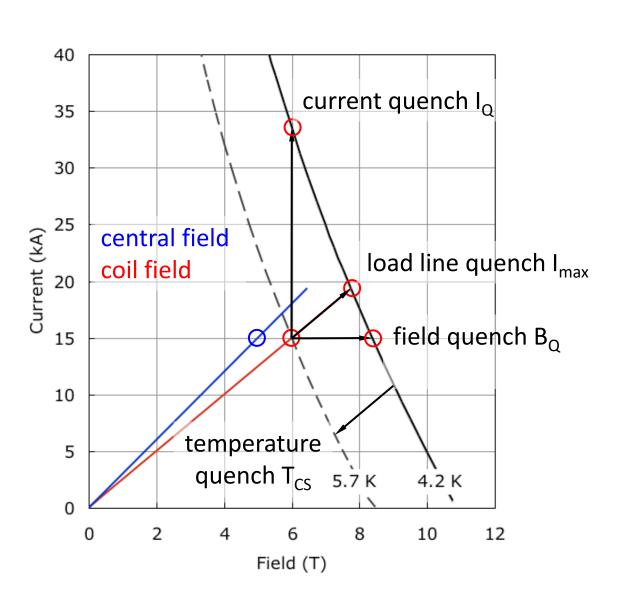


### The maximum achievable field (on paper) depends on the amount of conductor and on the superconductor's critical line

Nb-Ti critical surface ---  $I_C = J_C \times A_{SC} ---$  Nb-Ti critical current  $I_C(B)$ 



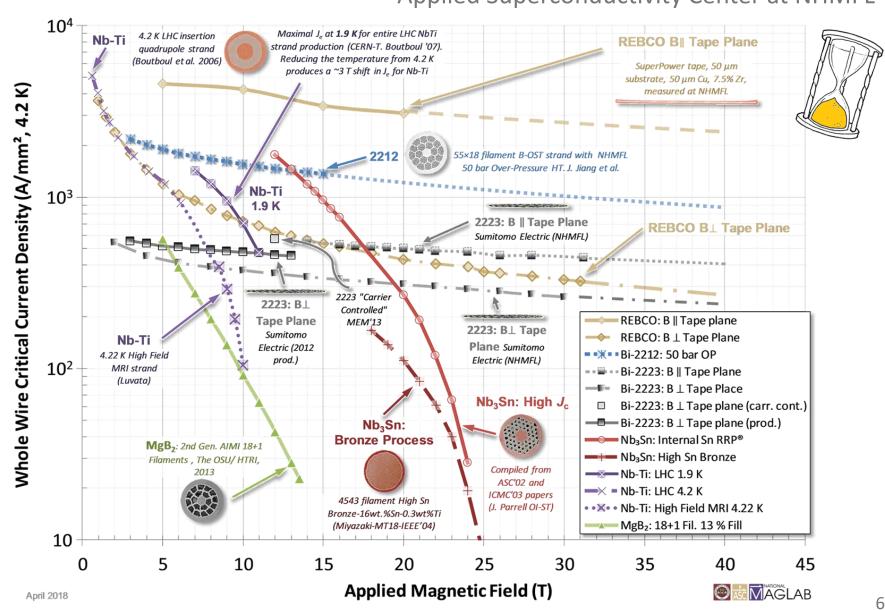
### In practical operation, margins are needed with respect to this short sample limit





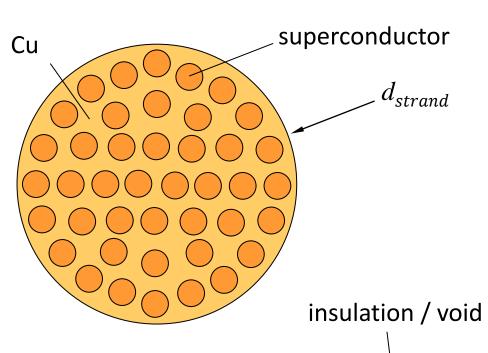
### This is the best (Apr. 2018) critical current for several superconductors

Applied Superconductivity Center at NHMFL



# The overall current density is lower than the current density on the superconductor



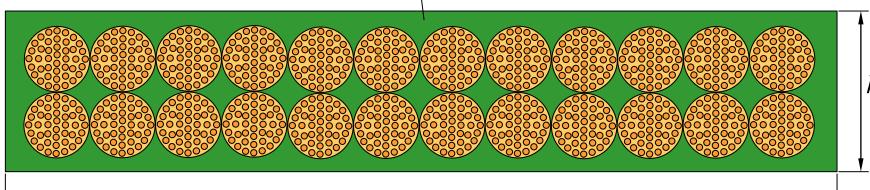


$$j_{overall} = \frac{I}{w_{cable}t_{cable}}$$

$$j_{cond} = \frac{I}{N_{strand} \frac{\pi d_{strand}^2}{4}}$$

$$j_{sc} = (1 + v_{Cu-sc})j_{cond}$$

$$v_{Cu-sc} = \frac{A_{Cu}}{A_{sc}}$$



 $h_{cable}$ 

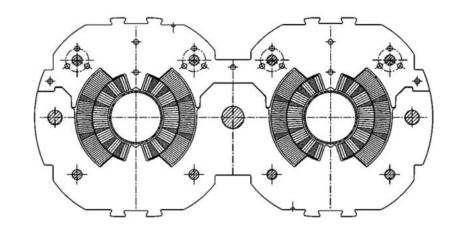
# The forces can be very large, so the mechanical design is important



Nb-Ti LHC MB @ 8.3 T

 $F_x \approx 350 \text{ t per meter}$ 

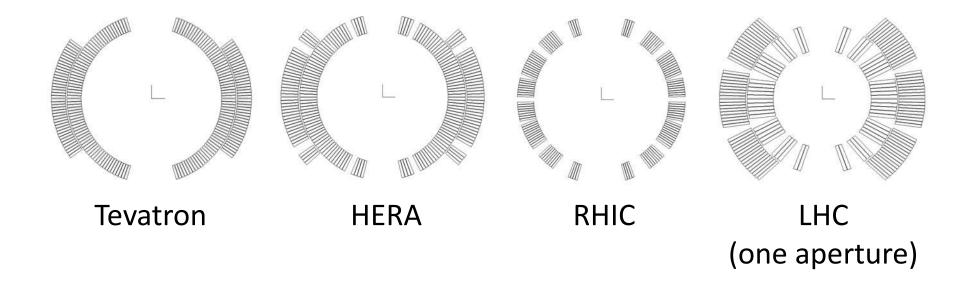
precision of coil positioning: 20-50 µm



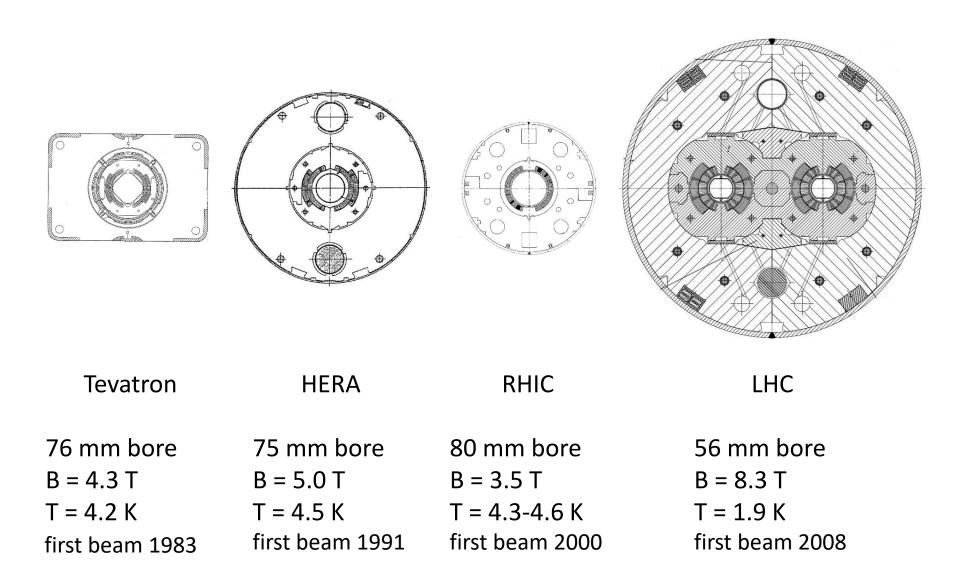
$$F_z \approx 40 \text{ t}$$



# The coil cross sections of several superconducting dipoles show a certain evolution; all were (are) based on Nb-Ti



## Also the iron, the mechanical structure and the operating temperature can be quite diverse



### This is how they look in their machines









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See Jérémie Bauche, this afternoon

### There are different programs used for magnetic simulations



- 1. OPERA-2D and OPERA-3D, by Dassault Systèmes
- 2. ROXIE, by CERN
- 3. POISSON, by Los Alamos
- 4. FEMM
- 5. RADIA, by ESRF
- 6. ANSYS
- 7. Mermaid, by BINP
- 8. COMSOL