Cyclotrons

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CERN Accelerator School – Introductory Course

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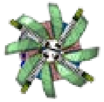
Santa Susanna

Cyclotrons - Outline

- the classical cyclotron
  - **history of the cyclotron, basic concepts and scalings, focusing, stepwidth, classification of cyclotron-like accelerators**
- synchro-cyclotrons
  - **concept, synchronous phase, example**
- isochronous cyclotrons **(→ sector cyclotrons)**
  - **isochronous condition, focusing in Thomas-cyclotrons, spiral angle, classical extraction: pattern/stepwidth, space charge**
- applications and examples of existing cyclotrons
  - **TRIUMF, RIKEN SRC, PSI Ring, PSI medical cyclotron**
- discussion
  - **classification of circular accelerators, Pro’s and Con’s of cyclotrons / FFA for different applications**



The Classical Cyclotron

two capacitive electrodes „Dees“, two gaps per turn

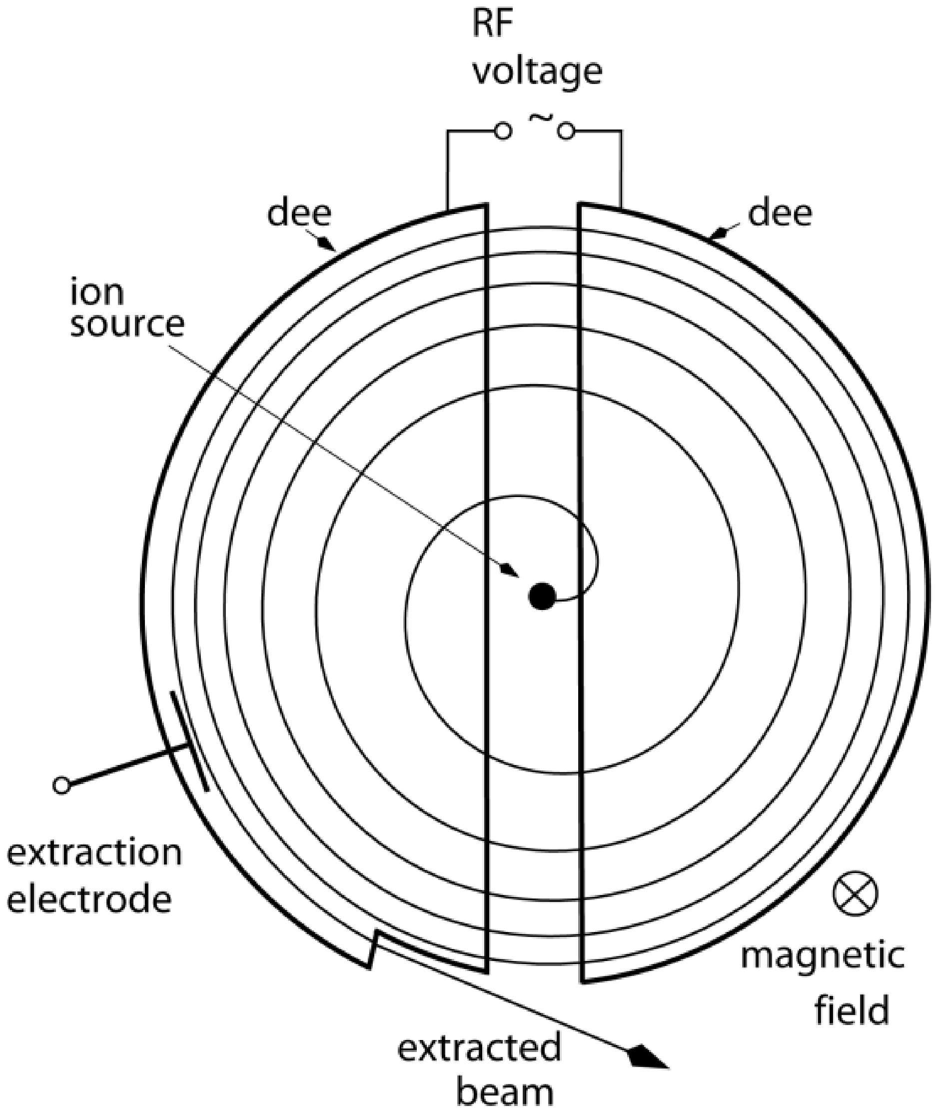
internal ion source

homogenous B field

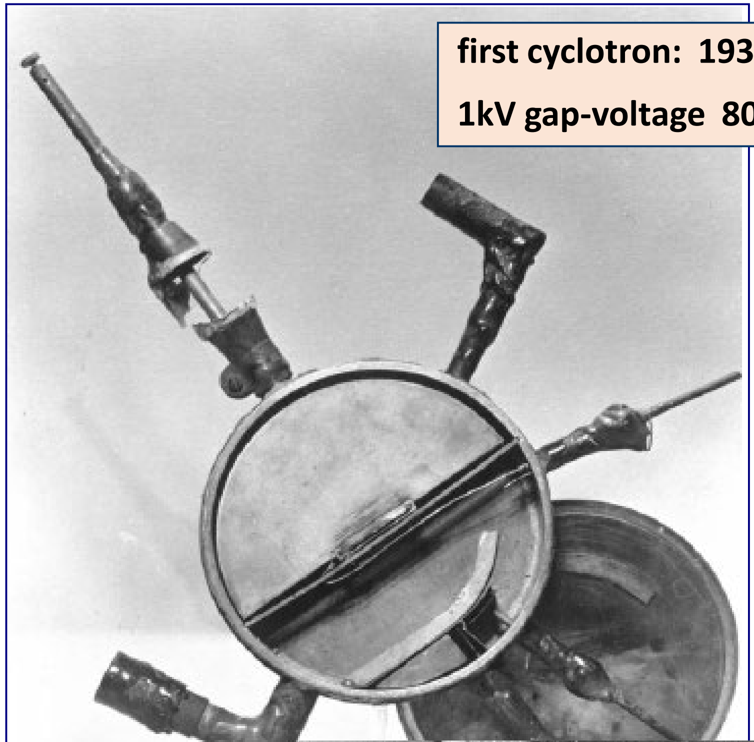
works for low energy, <≈20MeV (p)

a powerful concept:

- ➔ **simplicity, compactness**
- ➔ **continuous injection/extraction**
- ➔ **multiple usage of accelerating voltage**



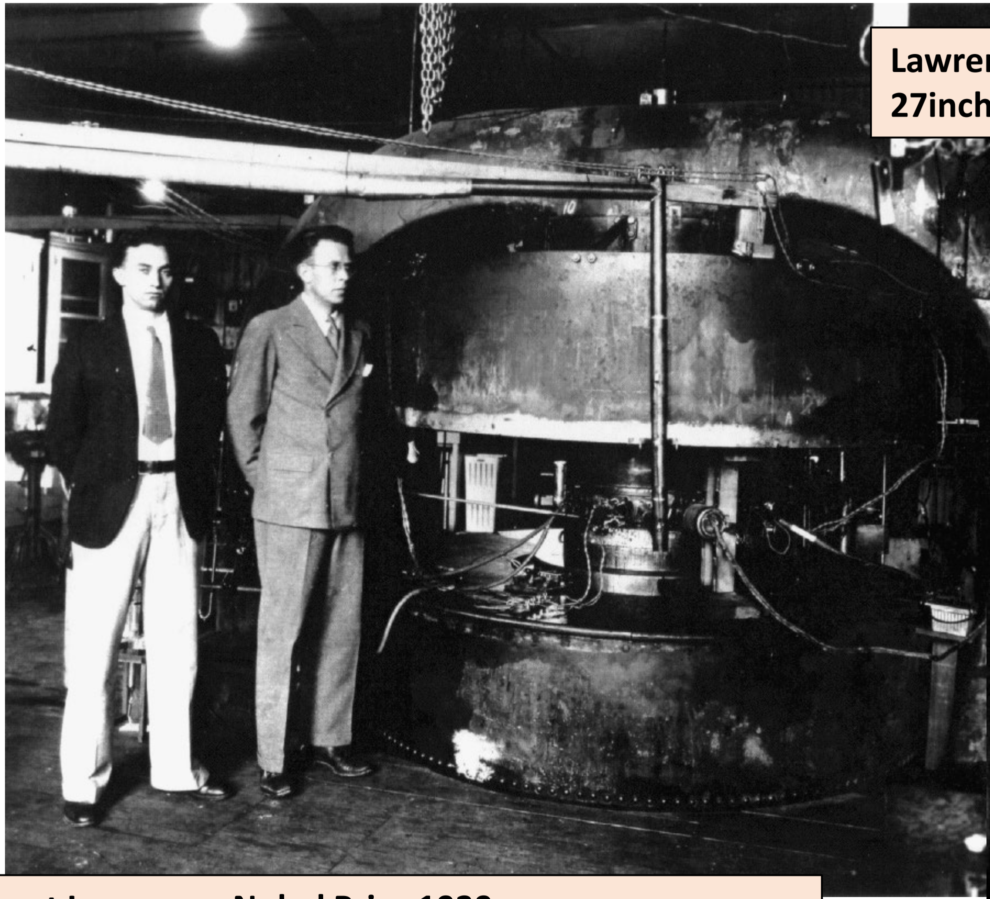
some History …



first cyclotron: 1931, Berkeley

1kV gap-voltage 80keV Protons

John Lawrence (center), 1940'ies
first medical applications: treating patients with neutrons generated in the 60inch cyclotron



Lawrence & Livingston, 27inch Zyklotron

Ernest Lawrence, Nobel Prize 1939
"for the invention and development of the cyclotron and for results obtained with it, especially with regard to artificial radioactive elements"

[images: Lawrence Berkeley National Laboratory]

The Key to the Cyclotron?

$$\vec{F}_{\mathrm{Lorentz}} = \vec{F}_{\mathrm{centrifugal}}$$

$$q\omega \underset{\uparrow}{R} B = m \underset{\uparrow}{R} \omega^2, \quad \omega = v/R \qquad \Longrightarrow \qquad \omega_c = \frac{eB_z}{m}$$

R cancels *R* !

circulation time is constant,
independent of energy or radius

Lawrence's graduate student J. J. Brady later recalled his young supervisor's excitement following his eureka moment in early 1929:

He came bursting into the lab. . . , his eyes glowing with enthusiasm, and pulled me over to the blackboard. He drew the equations of motion in a magnetic field.

'Notice that *R* appears on both sides,' he said. 'Cancels out. *R* cancels *R*. Do you see what that means? The resonance condition is not dependent on the radius. . . *Any* acceleration!'. . . '*R* cancels *R*' he said again. 'Do you see?' . . . He left in a rush, I suppose to tell other people that *R* canceled *R*.

cited from Craddock, Symon, Reviews of Accelerator Science and Technology, 2008, p. 65

PSI Ring Cyclotron & Crew

cyclotron frequency and *K* value

- **cyclotron frequency** (homogeneous) B-field:

$$\omega_c = \frac{eB}{\gamma m_0}$$

- **cyclotron *K*-value**: → *K* is the **energy reach** for protons (1/12 C) **from bending strength** in non-relativistic approximation:

$$K = \frac{e^2}{2m_0}(B\rho)^2$$

→ *K* can be used to rescale the energy reach of protons to other charge-to-mass ratios:

$$\frac{E_k}{A} = K\left(\frac{Q}{A}\right)^2$$

→ *K* in [MeV] is often used for naming cyclotrons

examples: **K-130 cyclotron / Jyväskylä**

cyclone C230 / IBA

cyclotron - isochronicity and scalings

continuous acceleration → revolution time should stay constant, though E_k, R vary

magnetic rigidity: $BR = \frac{p}{e} = \beta\gamma\frac{m_0 c}{e}$

orbit radius from isochronicity: $R = \frac{c}{\omega_c}\beta = R_\infty\beta$

deduced scaling of *B*: $\longrightarrow B(R) \propto \gamma(R)$

to be isochronous, *B* must be raised ∝ γ(*R*)
→ this contradicts the focusing requirements!

main difficulty to be overcome by cyclotron & FFA variants.

field index

the field index describes the (normalized) radial slope of the bending field:

$$\begin{aligned} k &= \frac{R}{B}\frac{dB}{dR} \\ &= \frac{\beta}{\gamma}\frac{d\gamma}{d\beta} \\ &= \gamma^2 - 1 \end{aligned}$$

from isochronous condition:
$B \propto \gamma; \; R \propto \beta$

→ thus k > 0 (positive slope of field) to keep beam isochronous!

focusing in a classical cyclotron

centrifugal force mv²/r

Lorentz force qv×B

$$m\ddot{r} = mr\dot{\theta}^2 - qr\dot{\theta}B_z$$

focusing: consider small deviations *x* from beam orbit *R* (*r = R+x*):

$$\begin{aligned}
\ddot{x} + \frac{q}{m} v B_z(R+x) - \frac{v^2}{R+x} &= 0, \\
\ddot{x} + \frac{q}{m} v \left(B_z(R) + \frac{\mathrm{d}B_z}{\mathrm{d}R} x \right) - \frac{v^2}{R} \left(1 - \frac{x}{R} \right) &= 0, \\
\ddot{x} + \omega_\mathrm{c}^2 (1+k) x &= 0.
\end{aligned}$$

$$\text{using: } \omega_\mathrm{c} = qB_z/m = v/R,\ r\dot{\theta} \approx v, k = \tfrac{R}{B}\tfrac{dB}{dR}$$

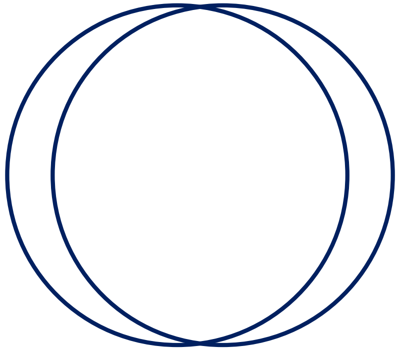
betatron tunes in cyclotrons

thus in radial plane:

$$\begin{aligned}\omega_r &= \omega_c\sqrt{1+k} = \omega_c\nu_r \\ \nu_r &= \sqrt{1+k} \\ &\approx \gamma\end{aligned}$$

using isochronicity condition

note: simple case for *k = 0*: $\nu_r = 1$ (one circular orbit oscillates w.r.t the other)



using Maxwell to relate B_z and B_R: $\mathrm{rot}\,\vec{B} = \frac{dB_R}{dz} - \frac{dB_z}{dR} = 0$

in vertical plane: $\nu_z = \sqrt{-k}$ ⇐ *k*<0 to obtain vertical focus.

thus: in classical cyclotron *k < 0* required for vert. focus; however this violates isochronous condition *k = γ²-1 > 0*

naming conventions of cyclotrons …

1.) resonant acceleration

classical cyclotron
limit energy / ignore problem

synchro- cyclotron
frequency is varied

isochronous cyclotron
avg. field slope positive

2.) transverse focusing

classical cyclotron
negative field slope

AVF-/Thomas-/sector cyclotron
focusing by flutter, spiral angle

classification of cyclotron like accelerators

Classical Cyclotron
[B(θ) = const]

Thomas cyclotron
[Azimuthally Varying Field,
e.g. B(θ) ∝ b+cos(3θ), one pol]

complexity

Separated Sector Cyclotron
[separated magnets, resonators]

high intensity

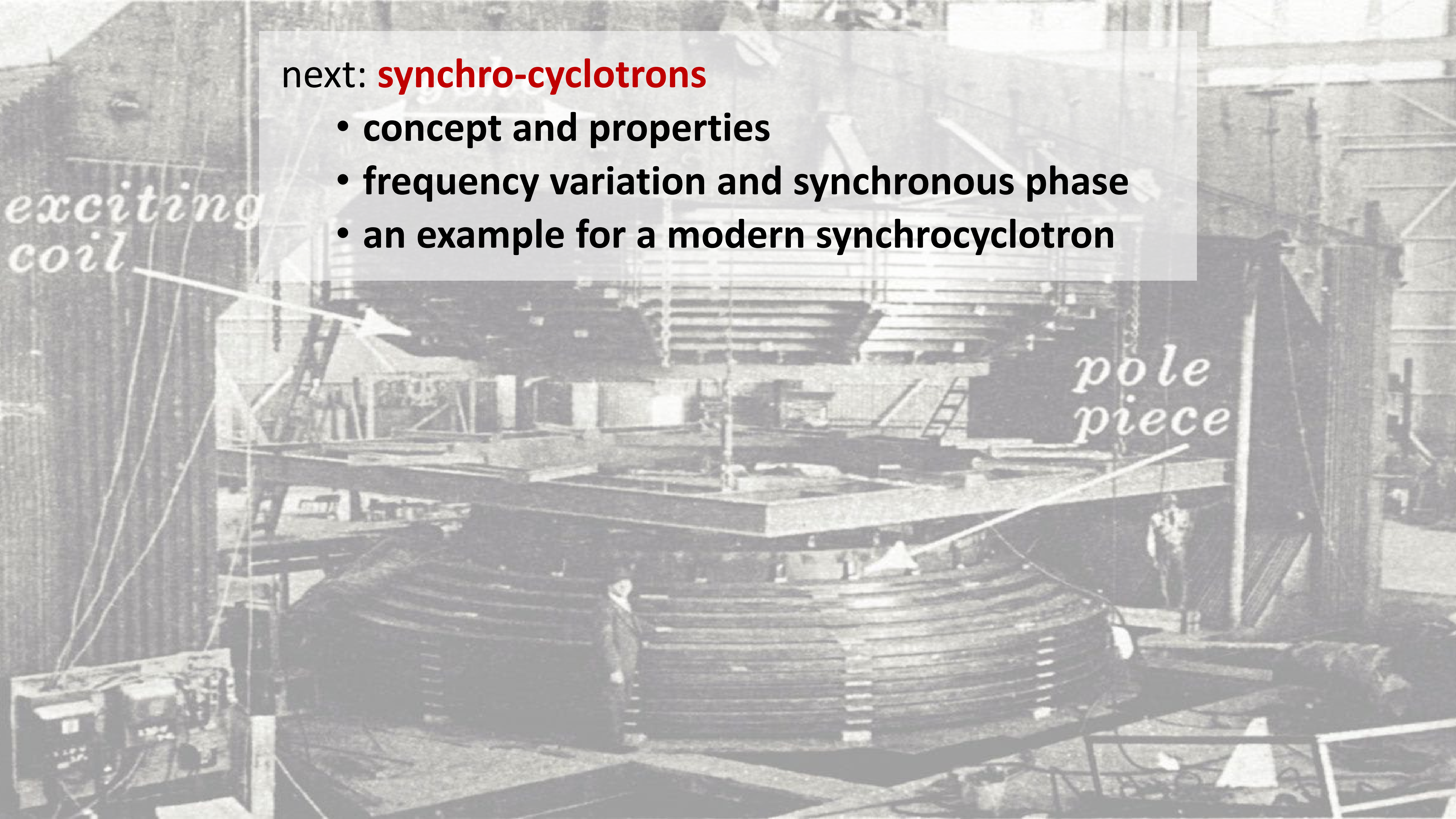
Synchro-Cyclotron
[varying RF frequency]

high energy

Fixed Field Alternating Gradient Accelerator FFAG
[varying RF, strong focusing]

compact machine

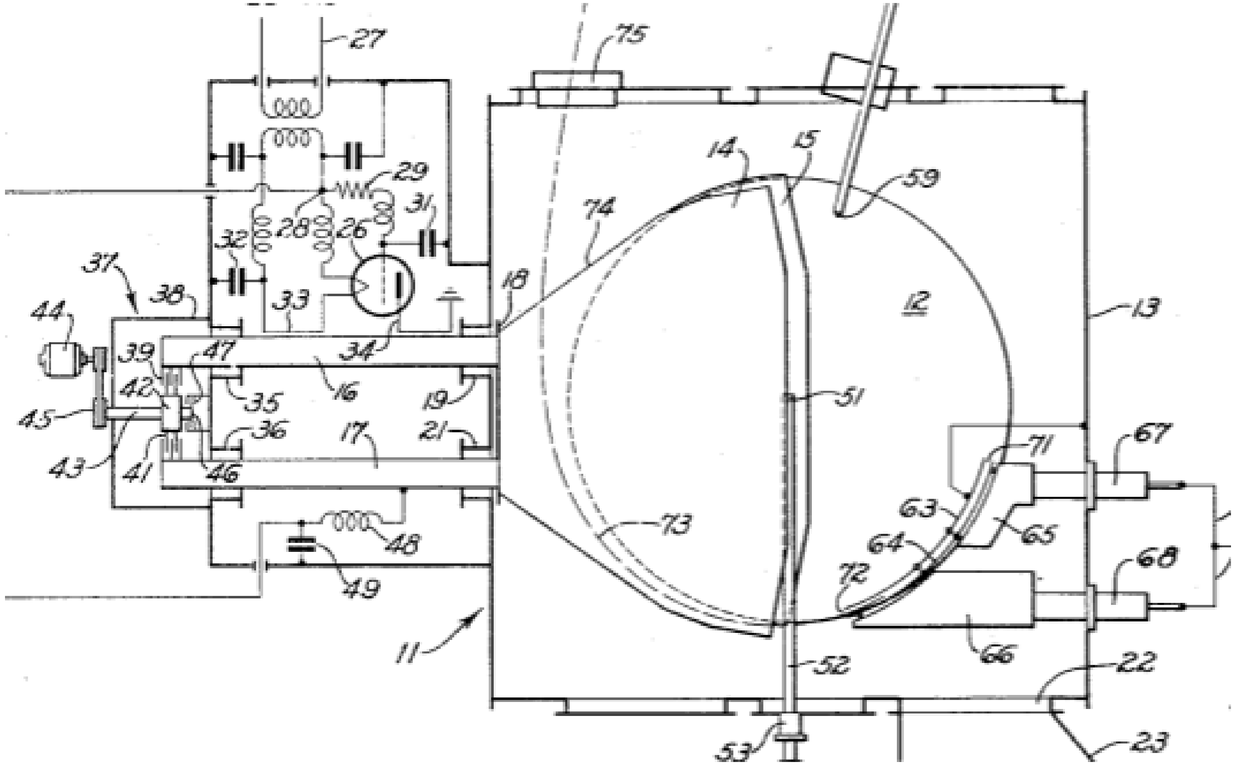
AVF concept – harmonic pole shaping,
electron model, Richardson et al (1950),
courtesy of Lawrence Berkeley National Laboratory



next: **synchro-cyclotrons**

- **concept and properties**
- **frequency variation and synchronous phase**
- **an example for a modern synchrocyclotron**

Synchrocyclotron -concept



first proposal by Mc.Millan, Berkeley

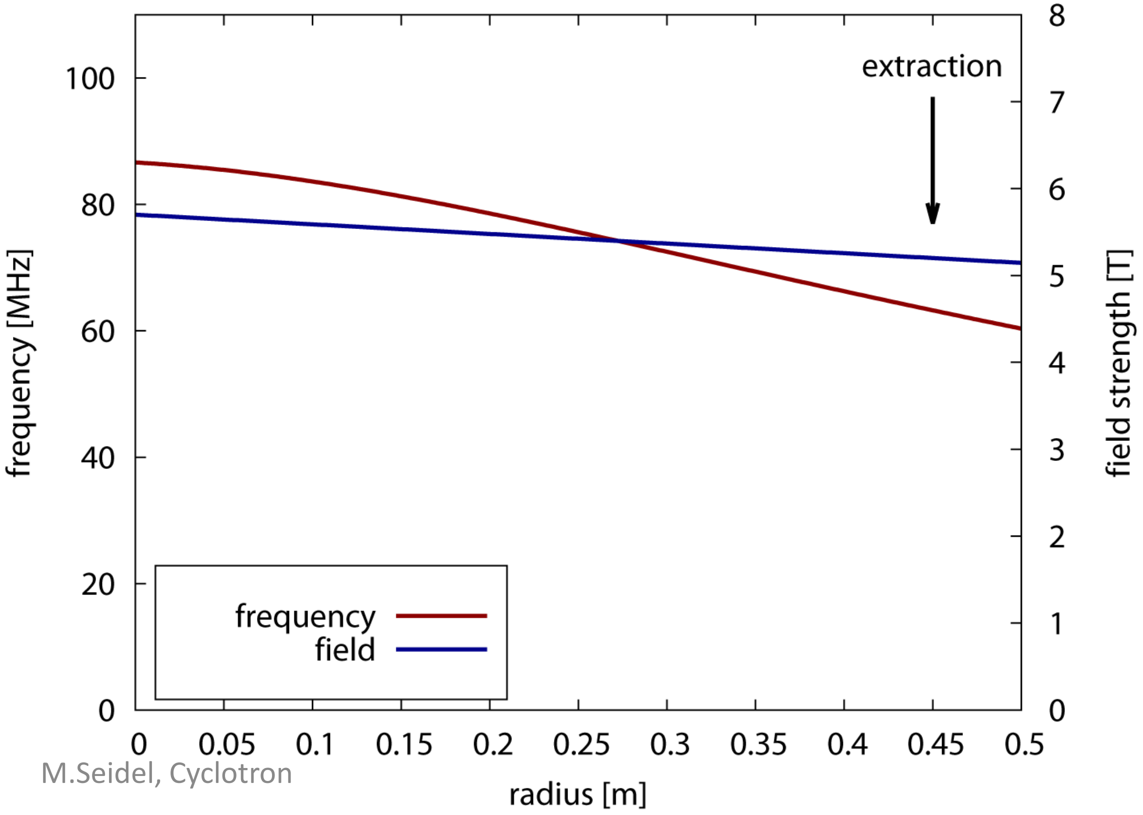
- accelerating frequency is variable, is reduced during acceleration
- negative field index (= negative slope) ensures sufficient focusing
- operation is pulsed, thus avg. intensity is low
- bending field constant in time, thus rep. rate high, e.g. 1kHz

Synchrocyclotron continued

advantages	disadvantages
- high energies possible (≥1Gev) - focusing by field gradient, no flutter required → thus compact magnet - only RF is cycled, fast repetition as compared to synchrotron	- low intensity, at least factor 100 less than CW cyclotron - complicated RF control required - weak focusing, large beam

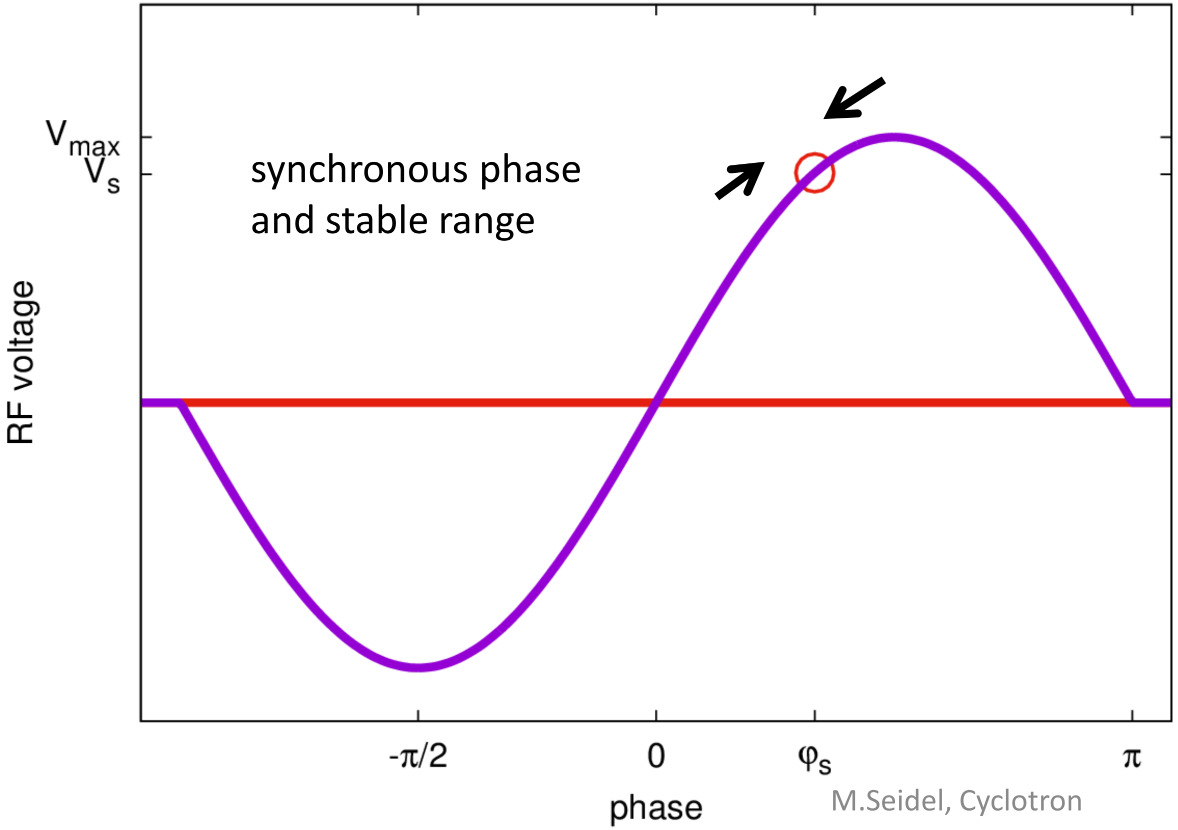
numerical example
field and frequency vs. radius:

- 230MeV p, strong field
- RF curve must be programmed in some way



Synchrocyclotron and synchronous phase

- internal source generates continuous beam; only a fraction is captured by RF wave in a phase range around a synchronous particle
- compared to a synchrotron the “storage time” is short → in practice no synchrotron oscillations



relation of

energy gain per turn and

rate of frequency change

$$\frac{qU_0N\cos\varphi_s}{E_k+E_0} = -\frac{2\pi}{\omega^2}\frac{d\omega}{dt}$$

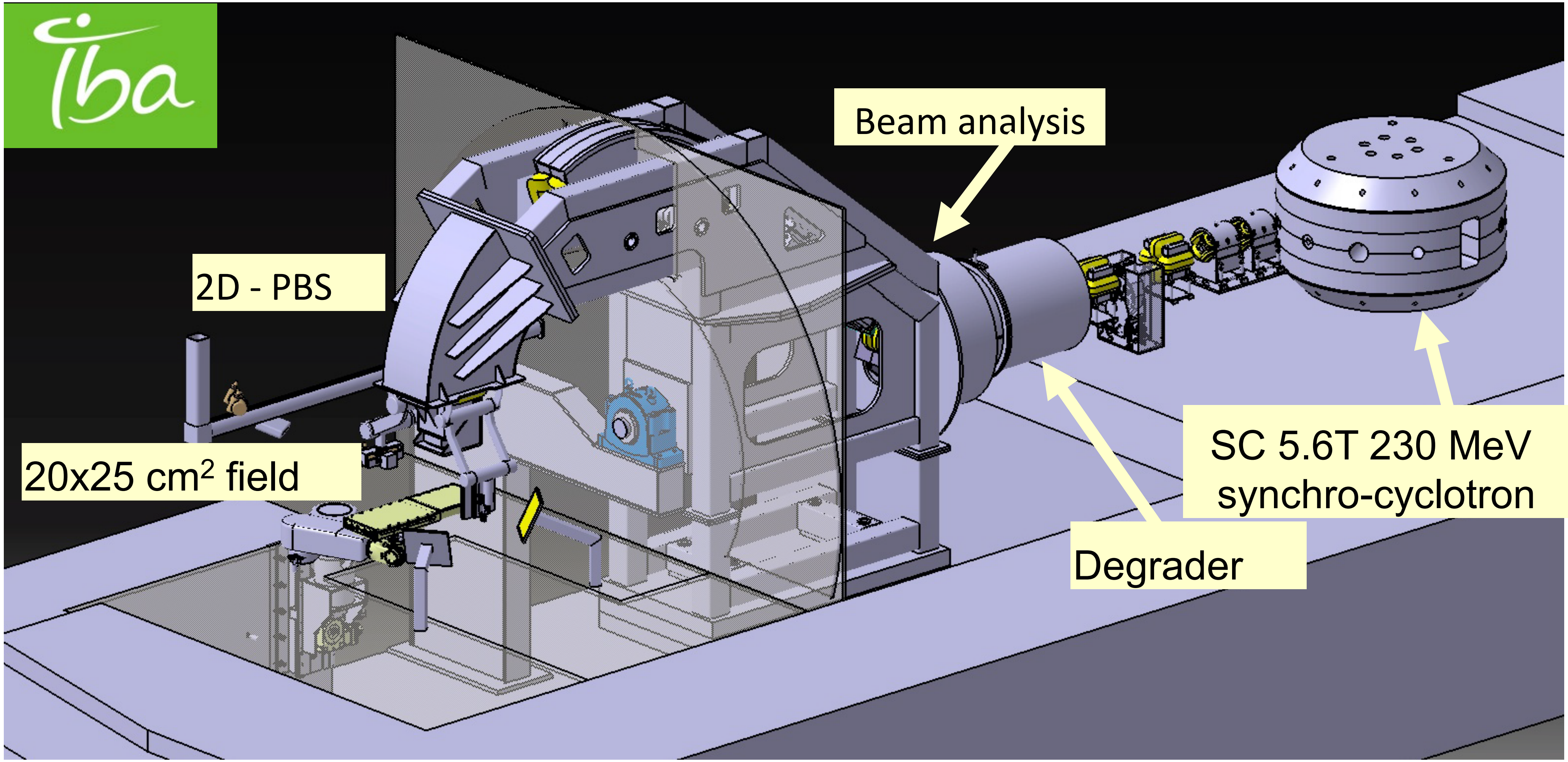
A modern synchrocyclotron for medical application – IBA S2C2

→ at the same energy synchrocyclotrons can be build more compact and with lower cost than sector cyclotrons; however, the achievable current is significantly lower

energy	230 MeV
current	130 nA
dimensions	∅2.5 m x 2 m
weight	< 50 t
extraction radius	0.45 m
s.c. coil strength	5.6 Tesla
RF frequency	90…60 MHz
repetition rate	1 kHz

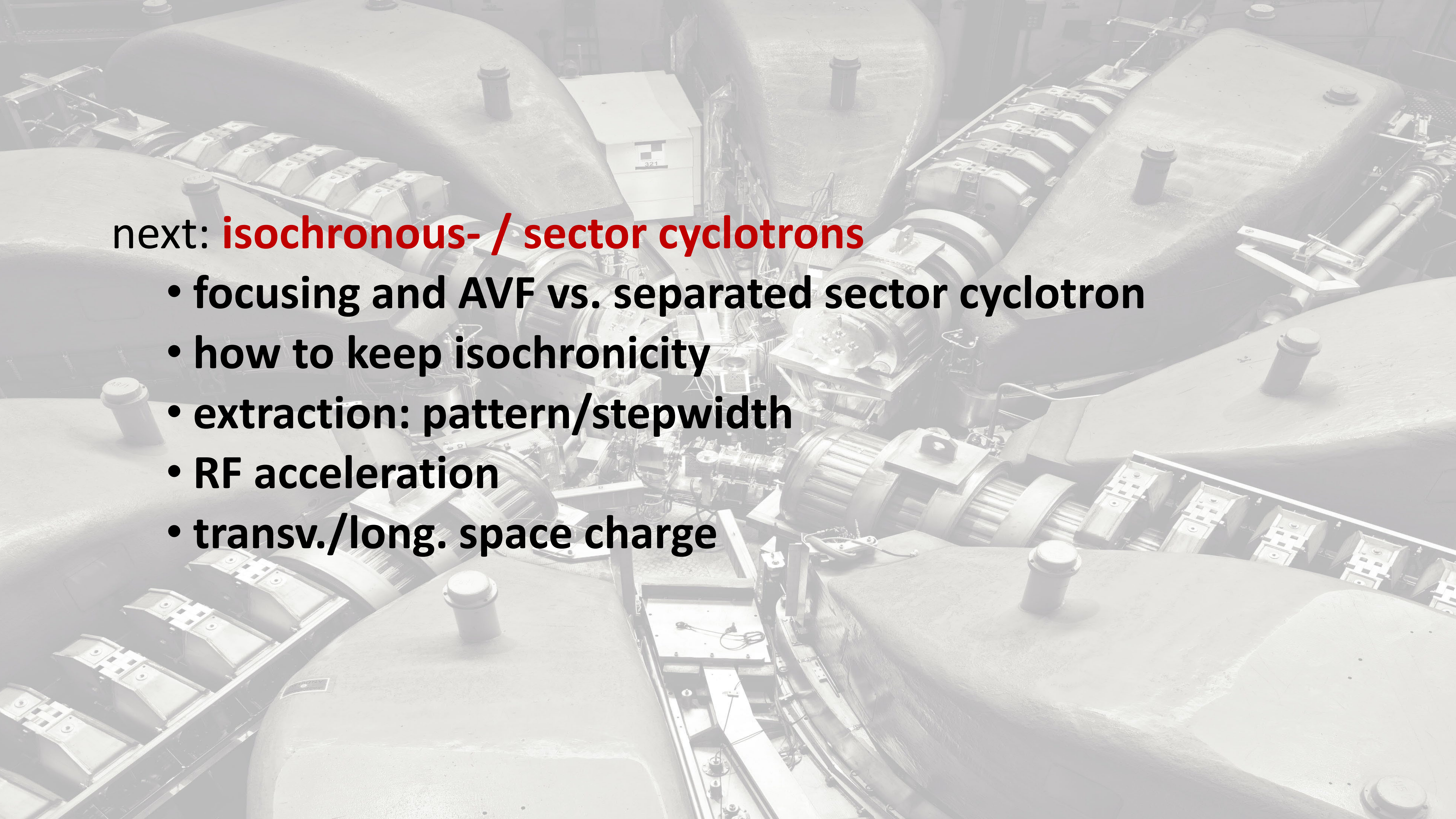
courtesy:
P.Verbruggen,IBA

compact treatment facility using the high field synchro-cyclotron



[image courtesy: IBA]

- required area: $24\text{x}13.5\text{m}^2$ (small)
- 2-dim pencil beam scanning



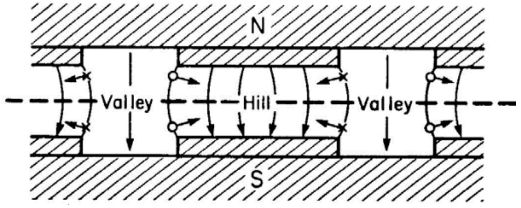
next: **isochronous- / sector cyclotrons**

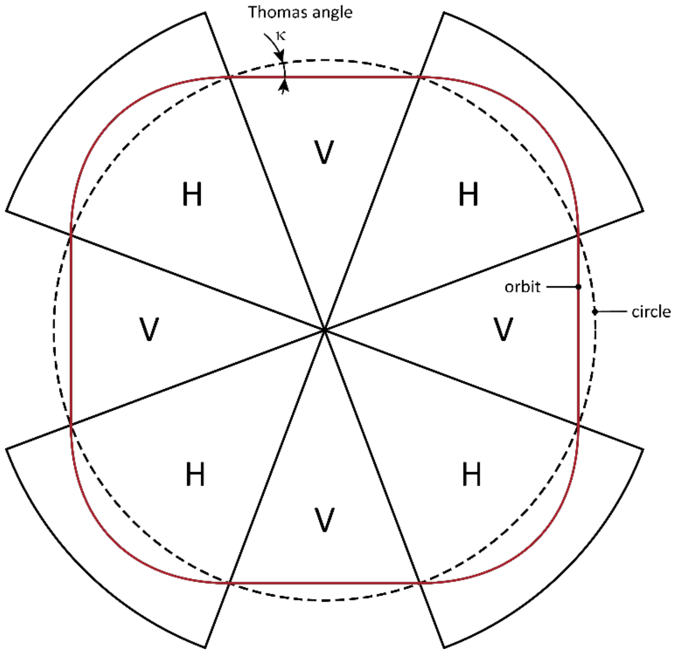
- **focusing and AVF vs. separated sector cyclotron**
- **how to keep isochronicity**
- **extraction: pattern/stepwidth**
- **RF acceleration**
- **transv./long. space charge**

focusing in sector cyclotrons

hill / valley variation of magnetic field (Thomas focusing) makes it possible to design cyclotrons for higher energies

Illustration of focusing at edges





vertical lens at boundary:

$$\frac{1}{f_z} = \frac{q}{\beta\gamma m_0 c}(B_H - B_V)\tan\kappa$$

resulting vertical tune:

$$\nu_z^2 = -\frac{R}{B_z}\frac{dB_z}{dR} + F$$

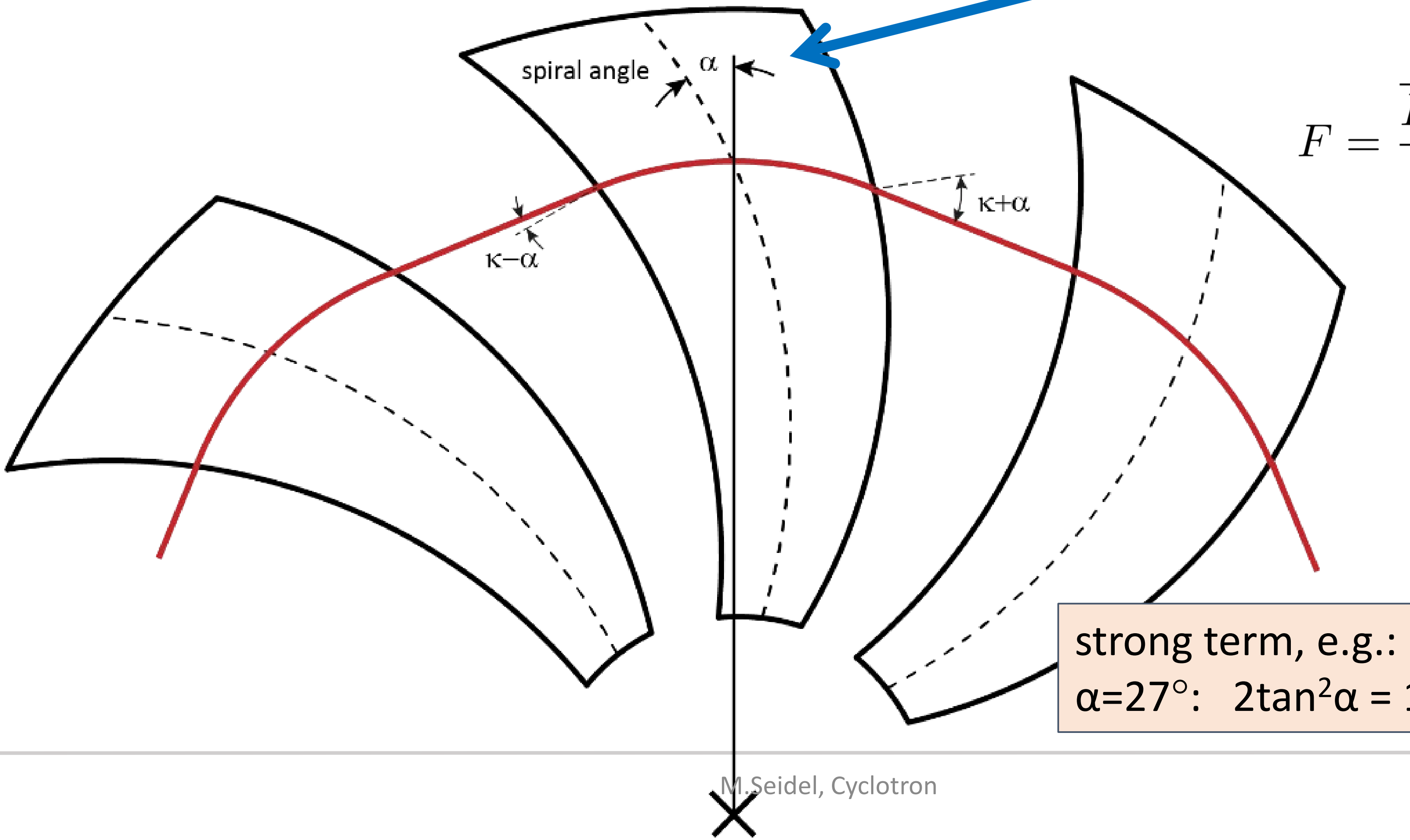
flutter factor describes modulation depth:

$$F = \frac{\overline{B_z^2} - \overline{B_z}^2}{\overline{B_z}^2}$$

adding a spiral angle

the spiral angle introduces additional focusing **with alternating contribution** at entry and exit of the sector fields:

$$\nu_z^2 = -\frac{R}{B_z}\frac{dB_z}{dR} + F(1 + 2\tan^2\alpha)$$

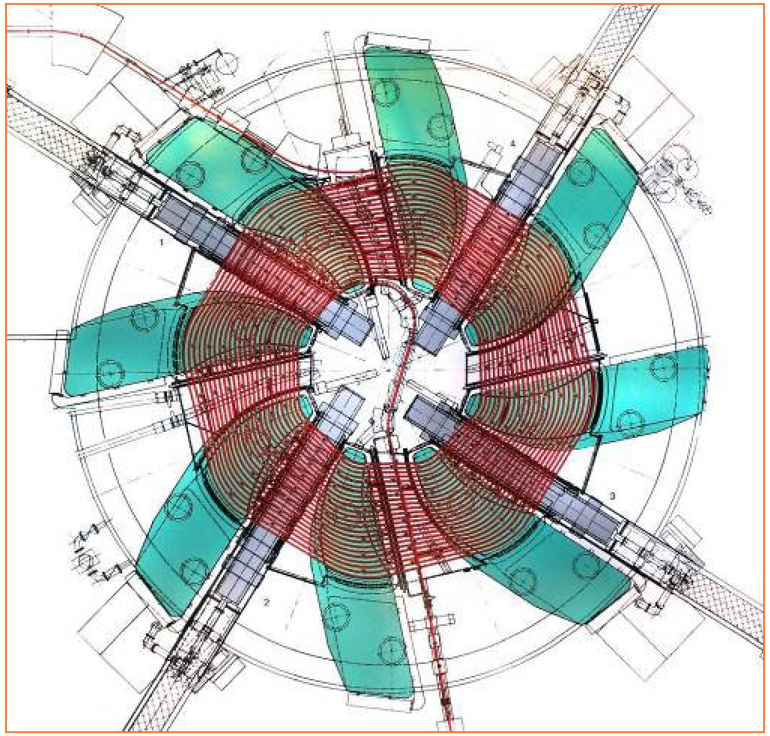


$$F = \frac{\overline{B_z^2} - \overline{B_z}^2}{\overline{B_z}^2}$$

strong term, e.g.:
α=27°: $2\tan^2\alpha = 1.0$

Azimuthally Varying Field vs. Separated Sector Cyclotrons

PSI/Varian comet: 250MeV sc. medical cyclotron



PSI Ring cyclotron

- **AVF = single pole with shaping,** often **spiral poles**
- **internal source** possible
- **D-type RF electrodes**, rel. low energy gain
- **compact**, cost effective
- depicted Varian cyclotron: 80% extraction efficiency; **not suited for high power**

- **modular layout,** larger cyclotrons possible, sector magnets, box resonators, stronger focusing, injection/extraction in straight sections
- **external injection** required, i.e. pre-accelerator
- **box-resonators** (high voltage gain)
- high **extraction efficiency** possible: e.g. PSI: 99.98%

three methods to raise the average magnetic field with γ

remember:

$$\begin{aligned} \text{rev.time}: \quad R &\propto \beta \\ \text{momentum}: \quad BR &\propto \beta\gamma \\ \text{thus}: \quad B &\propto \gamma \end{aligned}$$

1.) broader hills (poles) with radius

2.) **decrease pole gap with radius**

3.) s.c. coil arrangement to enhance field at large radius (in addition to iron dominated field)

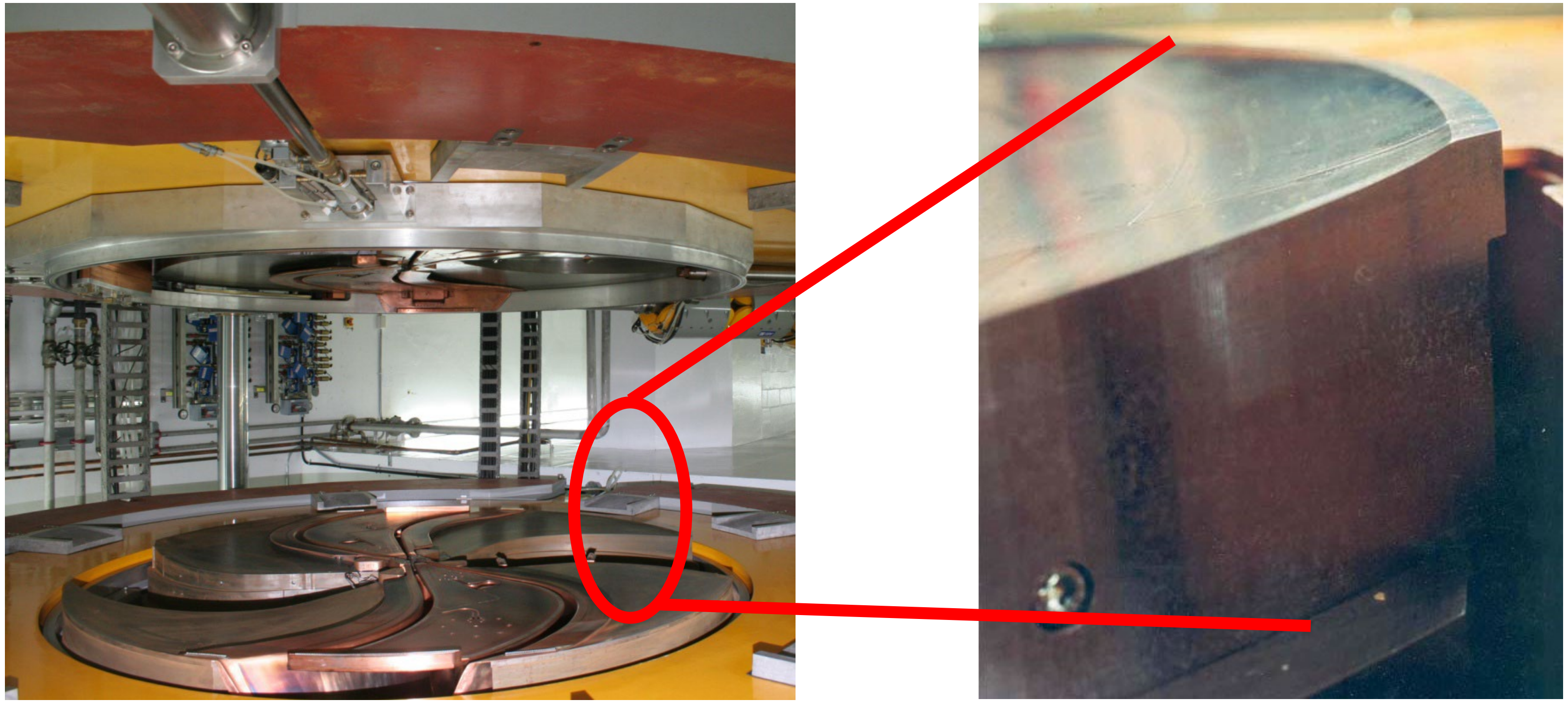


photo:

S. Zaremba, IBA

derivation of (relativistic) turn separation in a cyclotron

starting point: **bending strength**
→ compute total log.differential
→ use field index *k* = *R/B·dB/dR*

$$BR = \sqrt{\gamma^2 - 1}\,\frac{m_0 c}{e}$$

$$\frac{dB}{B} + \frac{dR}{R} = \frac{\gamma\, d\gamma}{\gamma^2 - 1}$$

$$\frac{dR}{d\gamma} = \frac{\gamma R}{\gamma^2 - 1}\,\frac{1}{1+k}$$

radius change
per turn

$$\begin{aligned} \frac{dR}{dn_t} &= \frac{dR}{d\gamma}\,\frac{d\gamma}{dn_t} \\ &= \frac{U_t}{m_0 c^2}\,\frac{\gamma R}{(\gamma^2 - 1)(1+k)} \quad \Big\} \\ &= \frac{U_t}{m_0 c^2}\,\frac{R}{(\gamma^2 - 1)\gamma} \quad \Big\} \end{aligned}$$

isochronicity not conserved (last turns)

isochronicity conserved (general scaling)

[U_t = energy gain per turn]

turn separation - discussion

for clean extraction a large stepwidth (turn separation) is of utmost importance;
in the PSI Ring most efforts were directed towards maximizing the turn separation

general scaling at extraction:

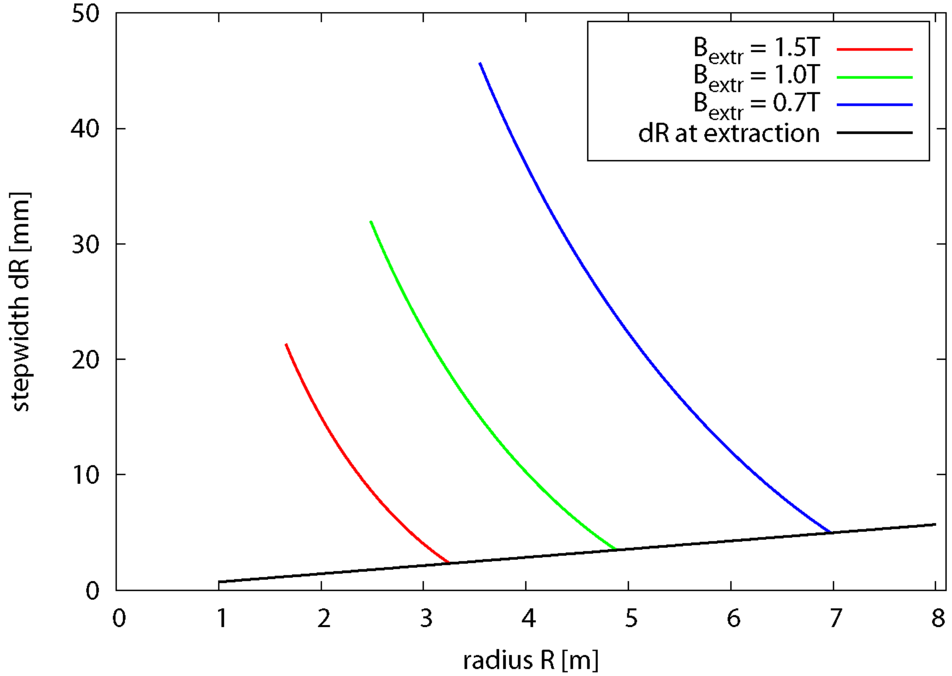
$$\Delta R(R_{\mathrm{extr}}) = \frac{U_t}{m_0 c^2} \frac{R_{\mathrm{extr}}}{(\gamma^2 - 1)\gamma}$$

scaling during acceleration:

$$\frac{dR}{dn_t} \approx \frac{U_t}{m_0 c^2} \frac{R}{\beta^2} \rightarrow \Delta R(R) \propto \frac{1}{R}$$

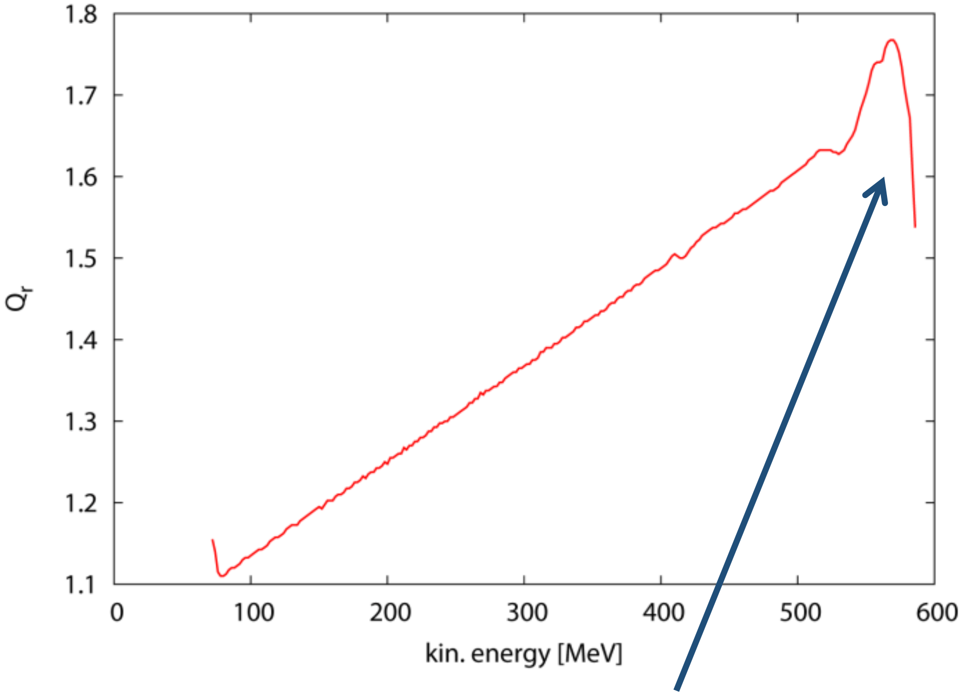
desirable:
- limited energy (< 1GeV)
- large radius R_{extr}
- high energy gain U_t

illustration:
stepwidth vs. radius in cyclotrons of different sizes but same energy;
100MeV inj → 800MeV extr



extraction with off-center orbits

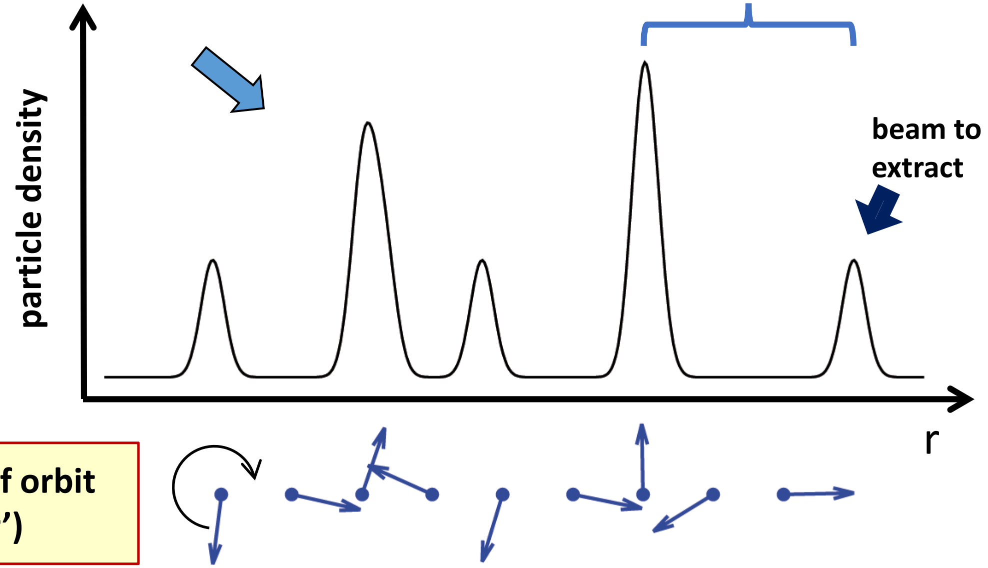
betatron oscillations around the "closed orbit" can be used to increase the radial stepwidth by a factor 3 !



radial tune vs. energy (PSI Ring)
typically νr ≈ γ during acceleration;
but decrease in outer fringe field

without orbit oscillations: stepwidth from E_k-gain

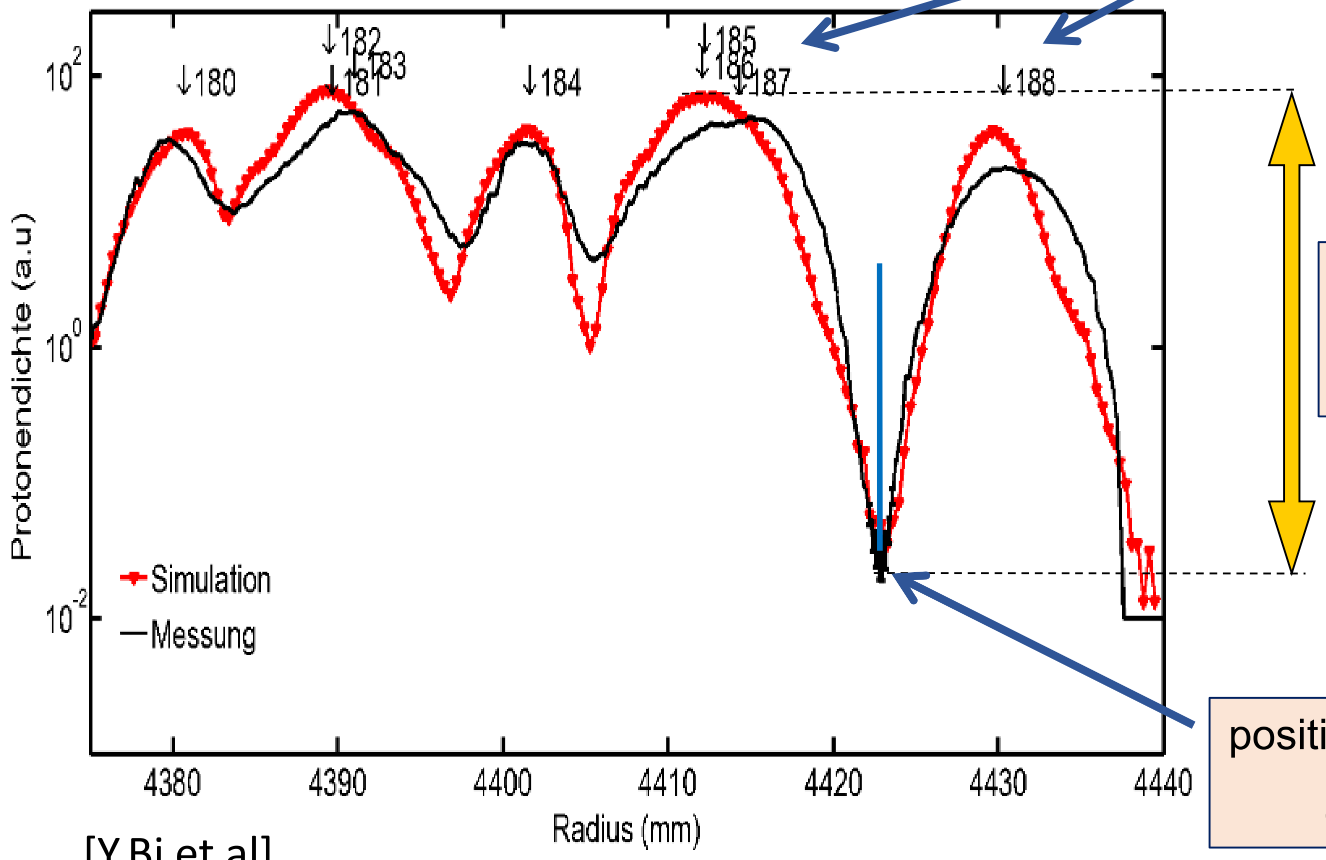
with orbit oscillations: extraction gap; up to 3 x stepwidth possible for ν_r=1.5π (phase advance)



phase vector of orbit oscillations (r,r')

extraction profile measured at PSI Ring Cyclotron

red: tracking simulation [OPAL]
black: measurement



turn numbers

from simulation

dynamic range:
factor 2.000 in
particle density

position of extraction septum

d=50μm

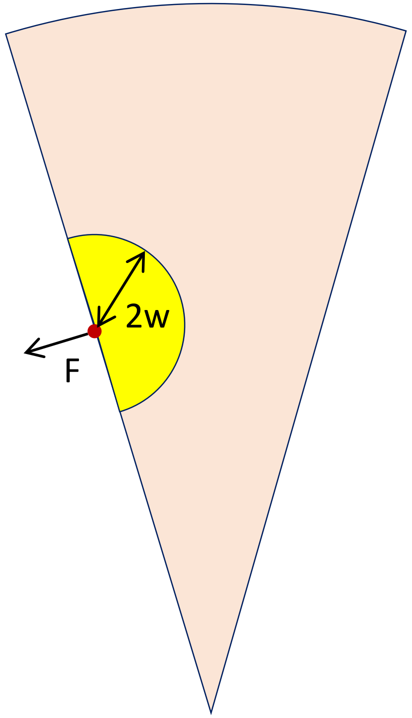
[Y.Bi et al]

longitudinal space charge

sector model (W.Joho, 1981):

→ accumulated energy spread transforms into transverse tails

- consider rotating uniform sectors of charge (overlapping turns)
- test particle "sees" only fraction of sector due to shielding of vacuum chamber with gap height 2*w*



two factors are proportional to the number of turns:

1) the charge density in the sector
2) the time span the force acts

$$\Delta U_{sc} \quad = \quad \frac{8}{3} e I_p Z_0 \ln\left(4\frac{w}{a}\right) \cdot \frac{n_{\rm max}^2}{\beta_{\rm max}} \approx 2.800\Omega \cdot e I_p \cdot \frac{n_{\rm max}^2}{\beta_{\rm max}}$$

derivation see: **High Intensity Aspects of Cyclotrons, ECPM-2012, PSI**

in addition:

3) the inverse of turn separation at extraction: $\dfrac{1}{\Delta R_{\rm extr}} \quad \propto \quad n_{\rm max}$

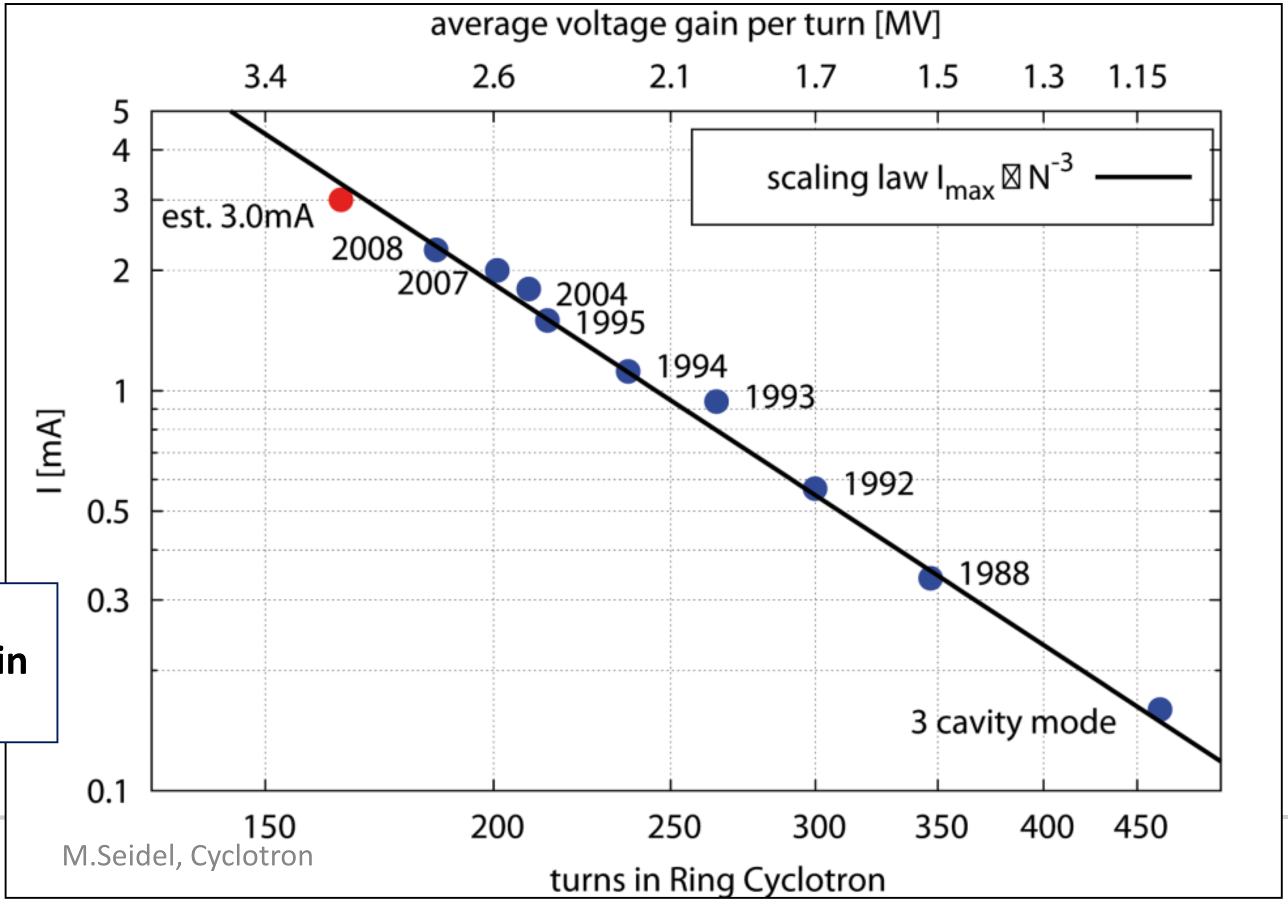
→ thus the attainable current at constant losses scales as $n_{\rm max}^{-3}$

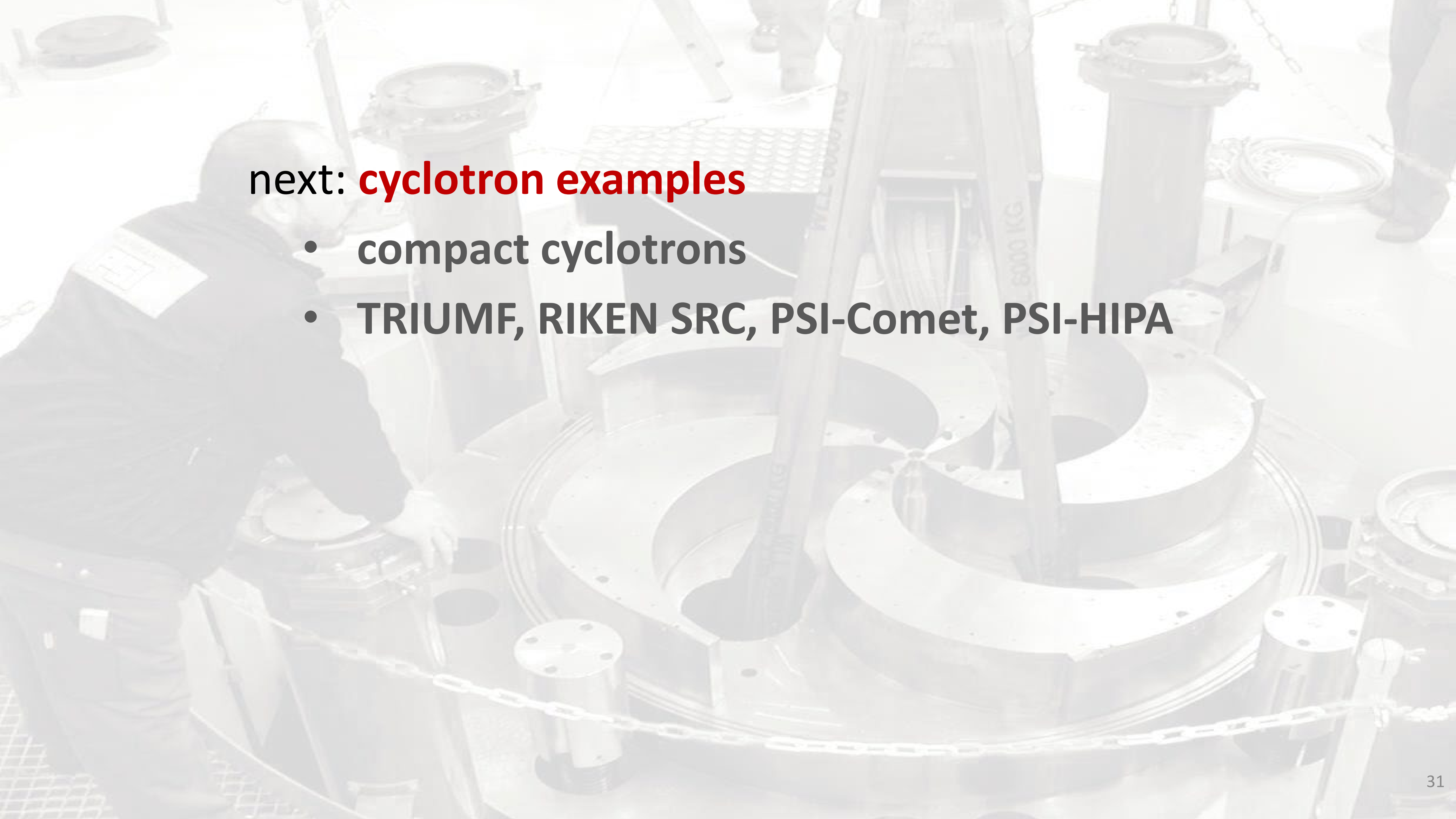
longitudinal space charge; evidence for third power law

- at PSI the maximum attainable current indeed scales with the third power of the turn number
- maximum energy gain per turn is of utmost importance in this type of high intensity cyclotron

→ with constant losses at the extraction electrode the maximum attainable current indeed scales as: $I_{\max} \propto n_t^{-3}$

historical development of current and turn numbers in PSI Ring Cyclotron



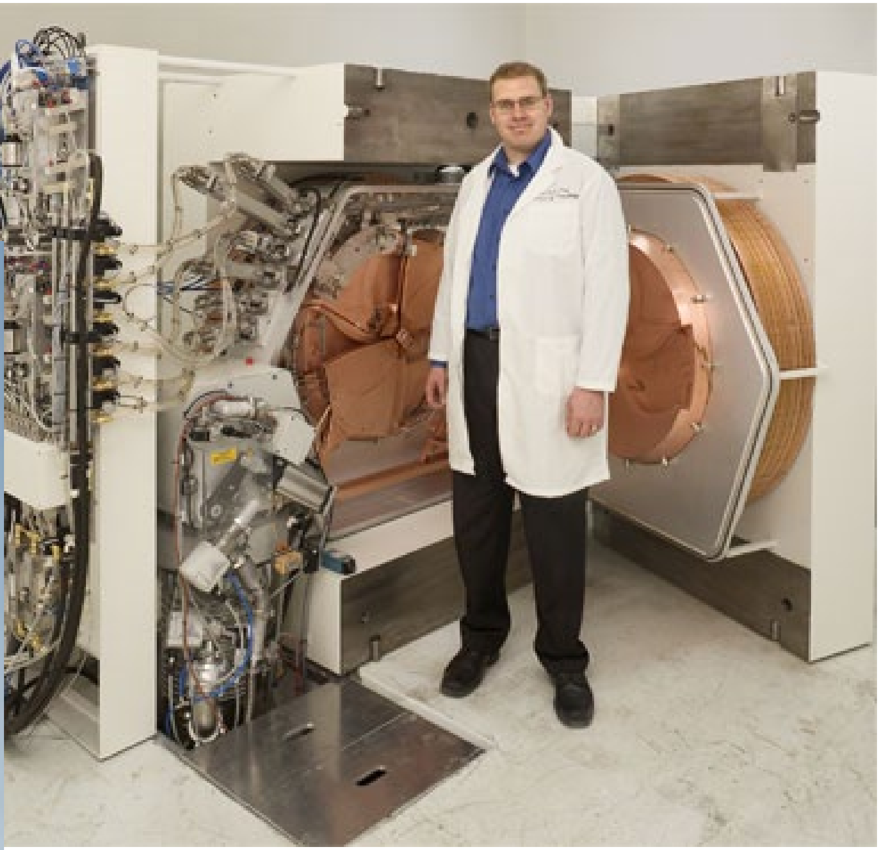


next: **cyclotron examples**

- **compact cyclotrons**
- **TRIUMF, RIKEN SRC, PSI-Comet, PSI-HIPA**

compact cyclotrons for Isotope production

CYCLONE 30 (IBA) : H^- 15 à 30 MeV



vertical setup

some cyclotrons

	TRIUMF	**RIKEN SRC (supercond.)**	**PSI Ring**	**PSI medical (supercond.)**
particles	H- → p	ions	p	p
K [MeV]	520	2600	592	250
magnets (poles)	(6)	6	8	(4)
peak field strength [T]	0.6	3.8	2.1	3.8
R_{inj}/R_{extr} [m]	0.25/3.8…7.9	3.6/5.4	2.4/4.5	-/0.8
P_{max} [kW]	110	1 (86Kr)	1300	0.25
extraction efficiency (tot. transmission)	0.9995 (0.70)	(0.63)	0.9998	0.80
extraction method	stripping foil	electrostatic deflector	electrostatic deflector	electrostatic deflector
comment	variable energy	ions, flexible	high intensity	compact

cyclotron examples: TRIUMF / Vancouver

- p, 520MeV, up to 110kW beam power
- diameter: 18m (largest n.c. cyclotron worldwide)
- extraction by stripping H^- → variable energy; multiple extraction points possible

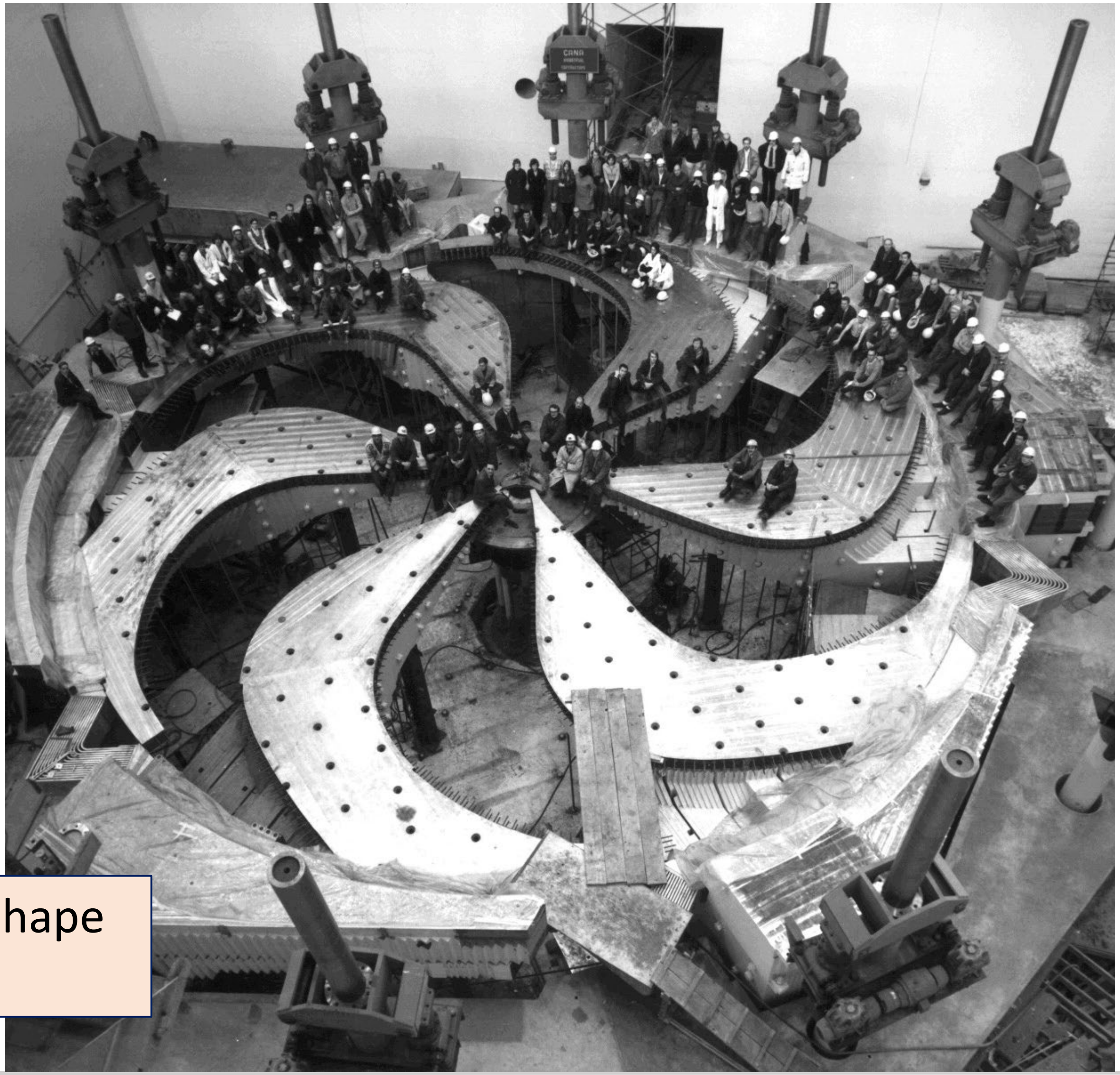


photo: iron poles with spiral shape

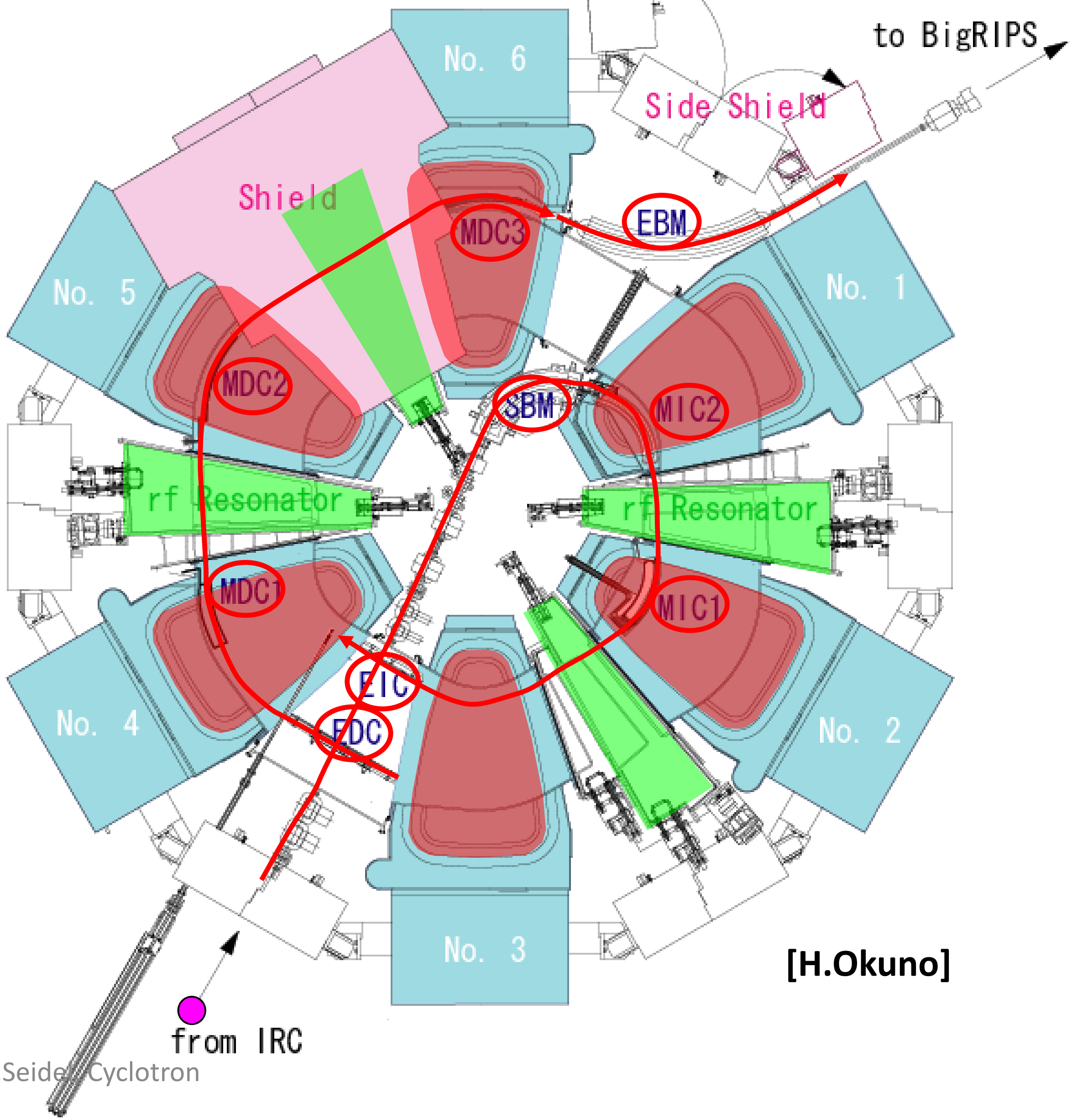
δ_{max}=70deg

example: RIKEN (Jp) superconducting cyclotron

K = 2,600 MeV
Max. Field: 3.8T (235 MJ)
RF frequency: 18-38 MHz
Weight: 8,300 tons
Diameter: 19m
Height: 8m

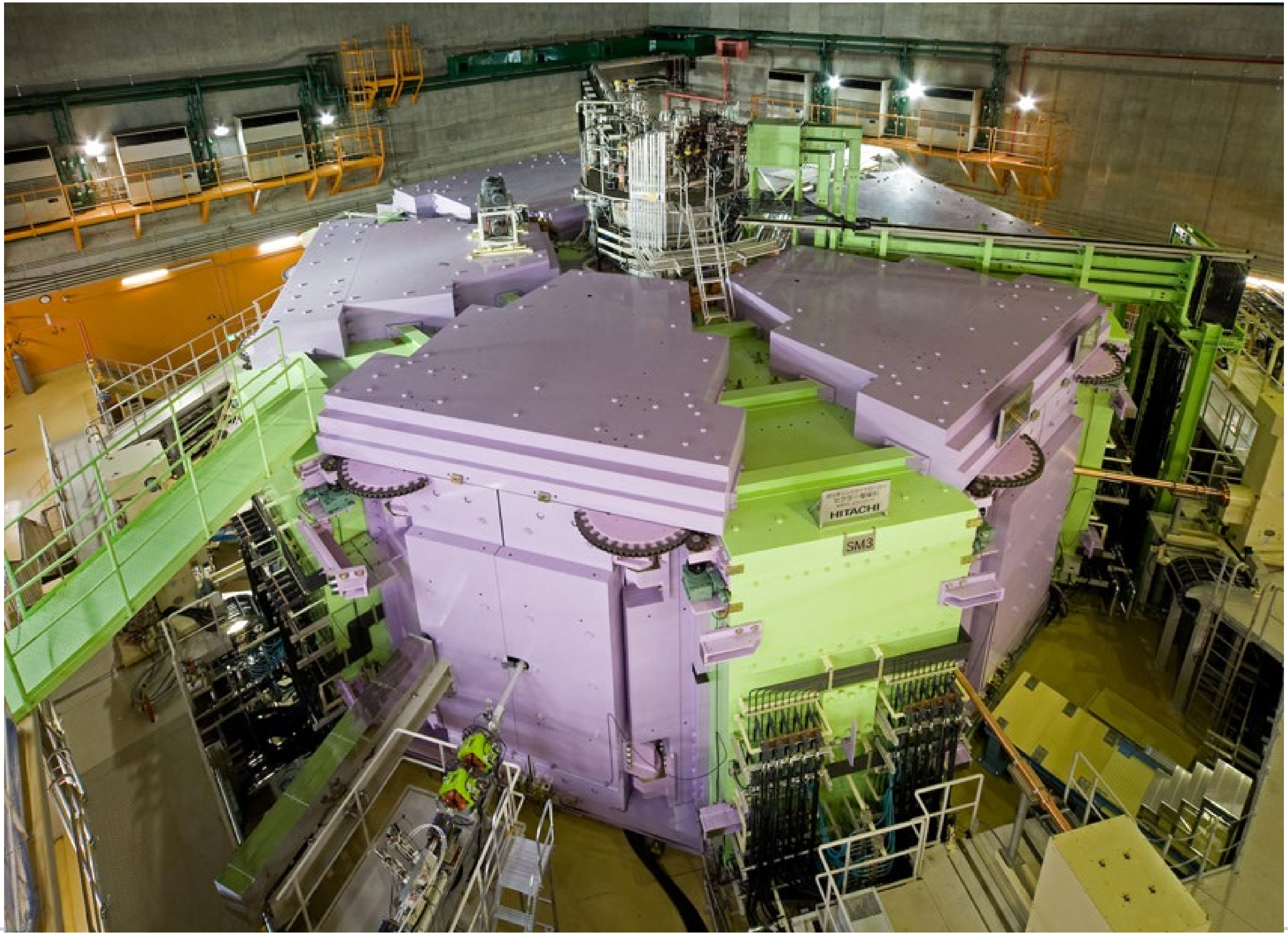
superconducting Sector Magnets :6
RF Resonator :4
Injection elements.
Extraction elements.

utilization:
broad spectrum of ions up to Uranium

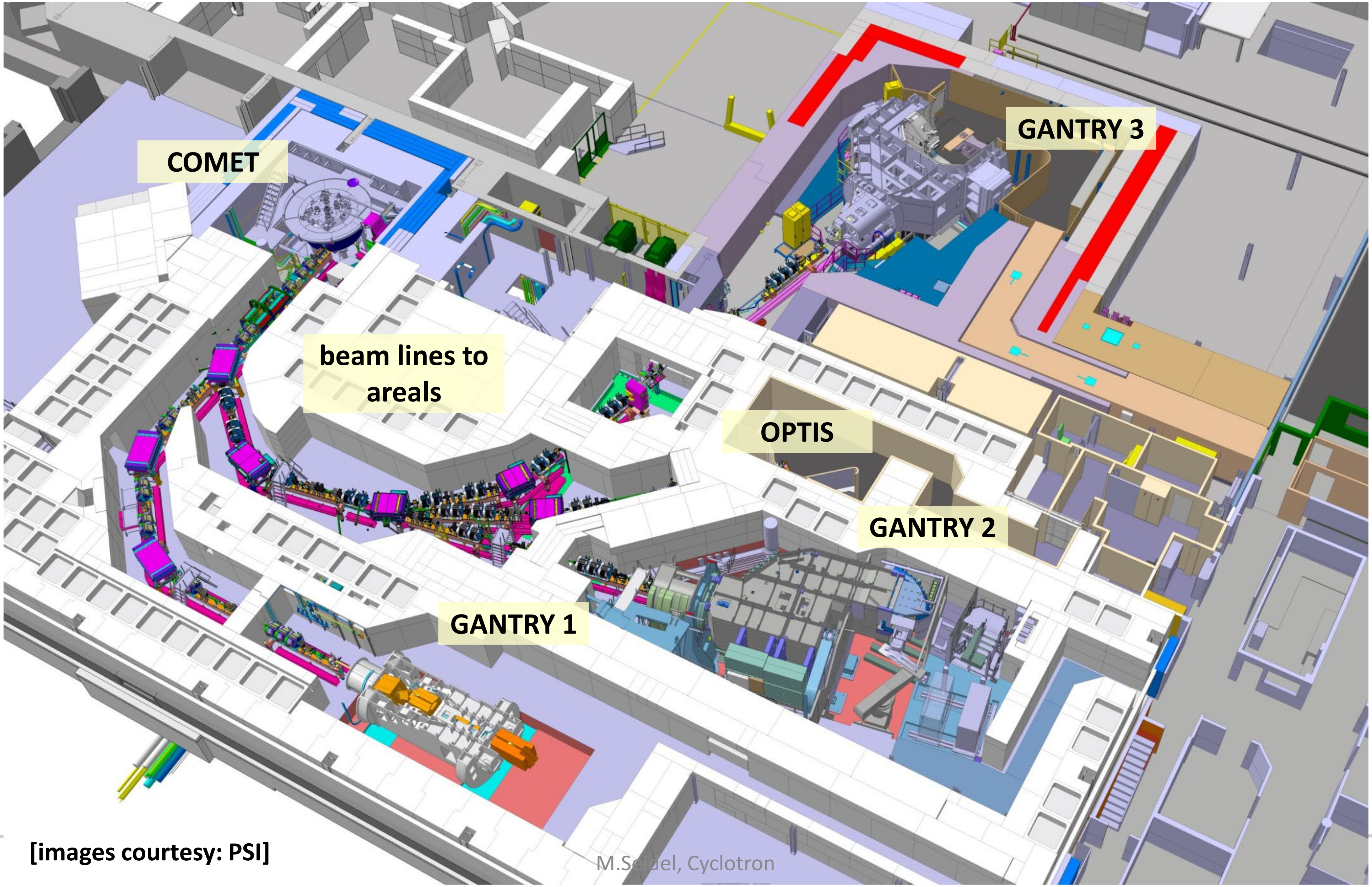


[H.Okuno]

RIKEN SRC in the vault

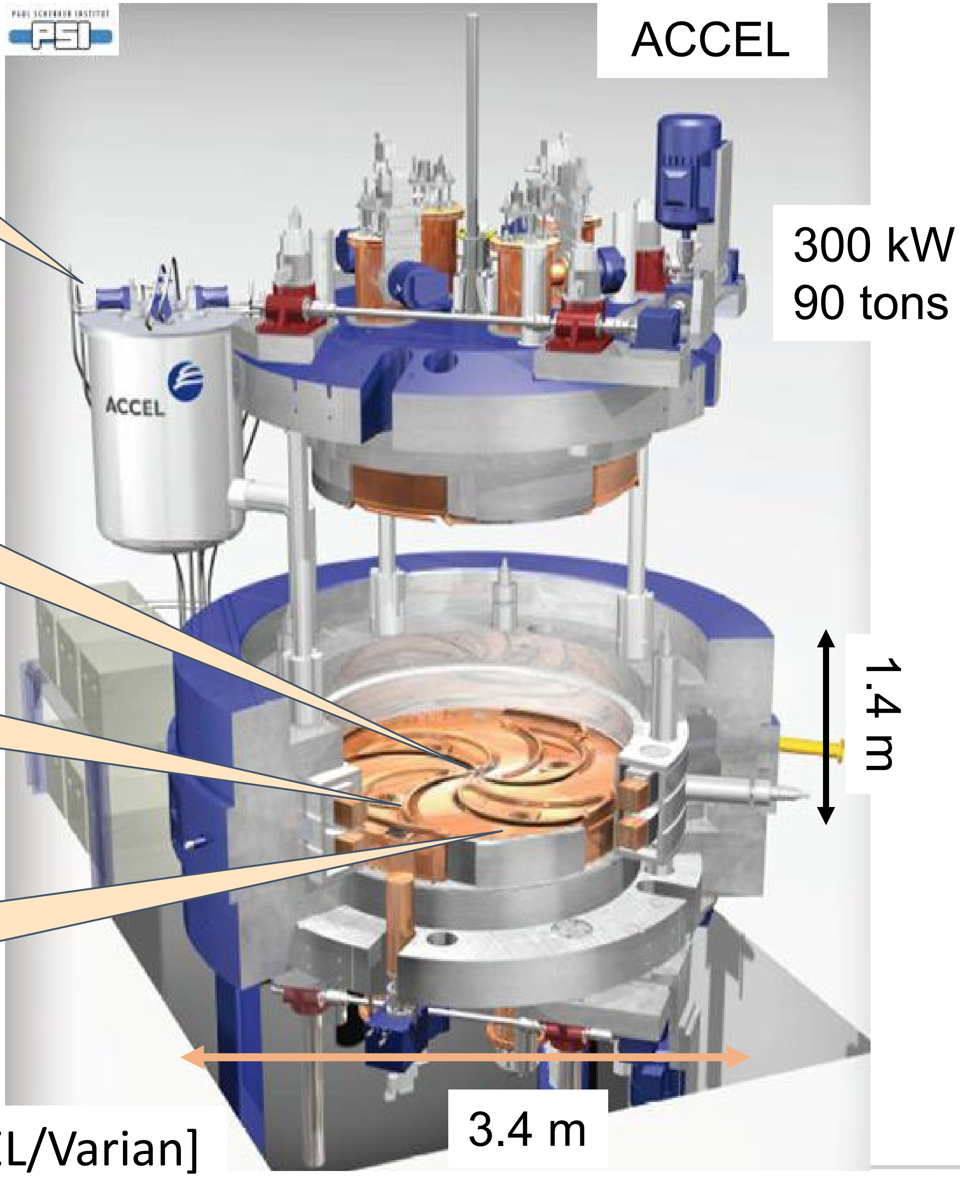


PSI Proton Therapy Facility



[images courtesy: PSI]

250 MeV isochronous proton cyclotron



Closed He system
4 x 1.5 W @4K

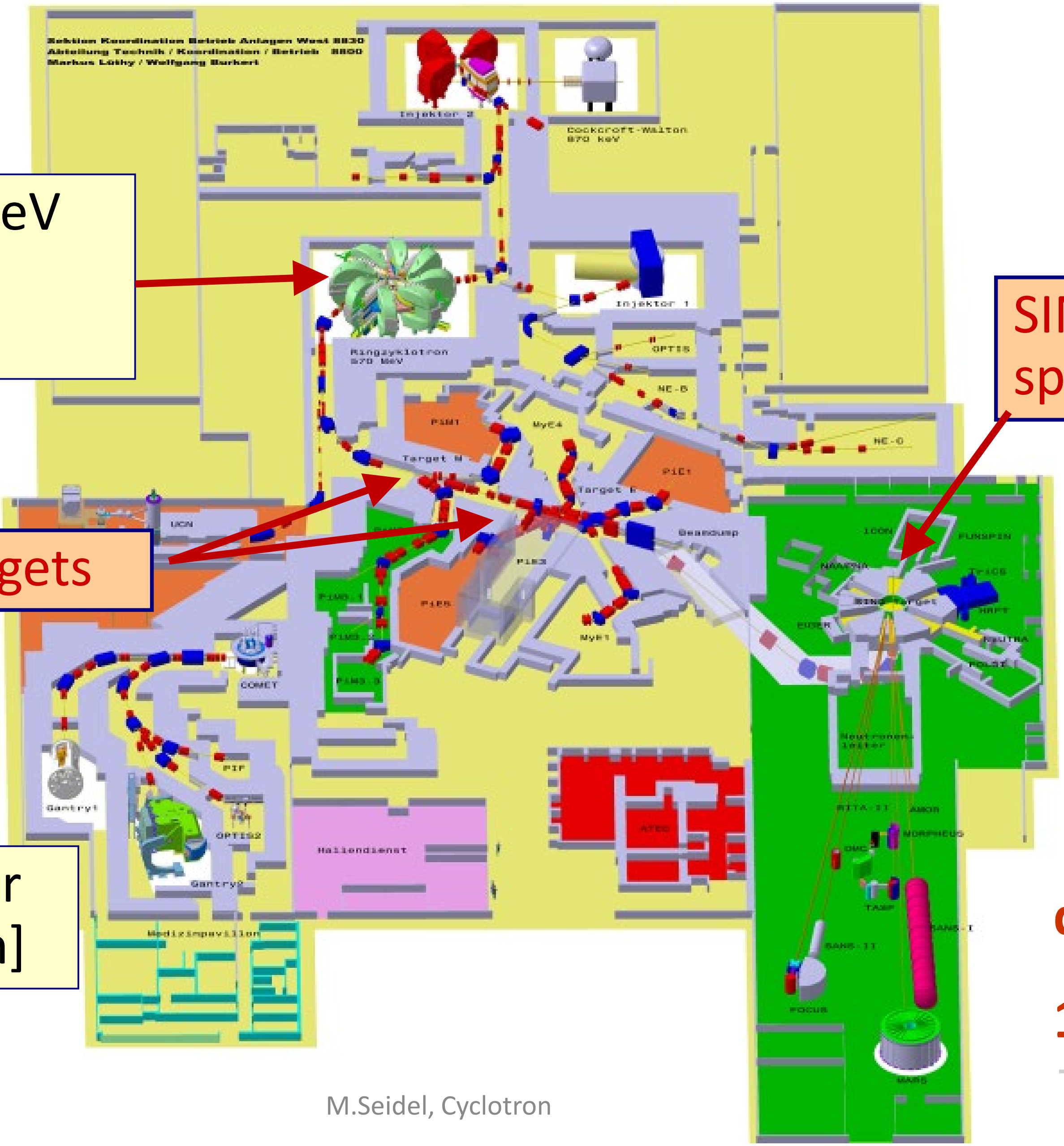
Proton source

superconducting coils
=> 2.4 - 3.8 T

4 RF-cavities
≈100 kV on 4 Dees

[image courtesy: ACCEL/Varian]

examples: PSI High Intensity Proton Accelerator



Ring Cyclotron 590 MeV
2.4mA / 1.4MW
diameter: 15m

SINQ
spallation source

meson production targets

proton therapie center
[250MeV sc. cyclotron]

dimensions:

120 x 220m^2

pro and contra cyclotron / FFA

limitations of cyclotrons	typical utilization of cyclotrons
• energy ≤1GeV (relat. effects) • weak focusing: space charge, 10mA? • tuning difficult; limited diagnostics • wide vacuum vessel (radius variation)	• medical applications; plenty intensity • acceleration of heavy ions (e.g. RIKEN) • very high intensity proton beams (PSI:1.4MW, TRIUMF: 100kW)

Fixed Focus Alternating Gradient Accelerator (FFA)

- strong focusing, compact magnets & chamber
- large acceptance, e.g. 10.000 mm mrad

but:

- high intensity difficult (no CW and extraction!), no demonstrator for high intensity after many years of discussion …

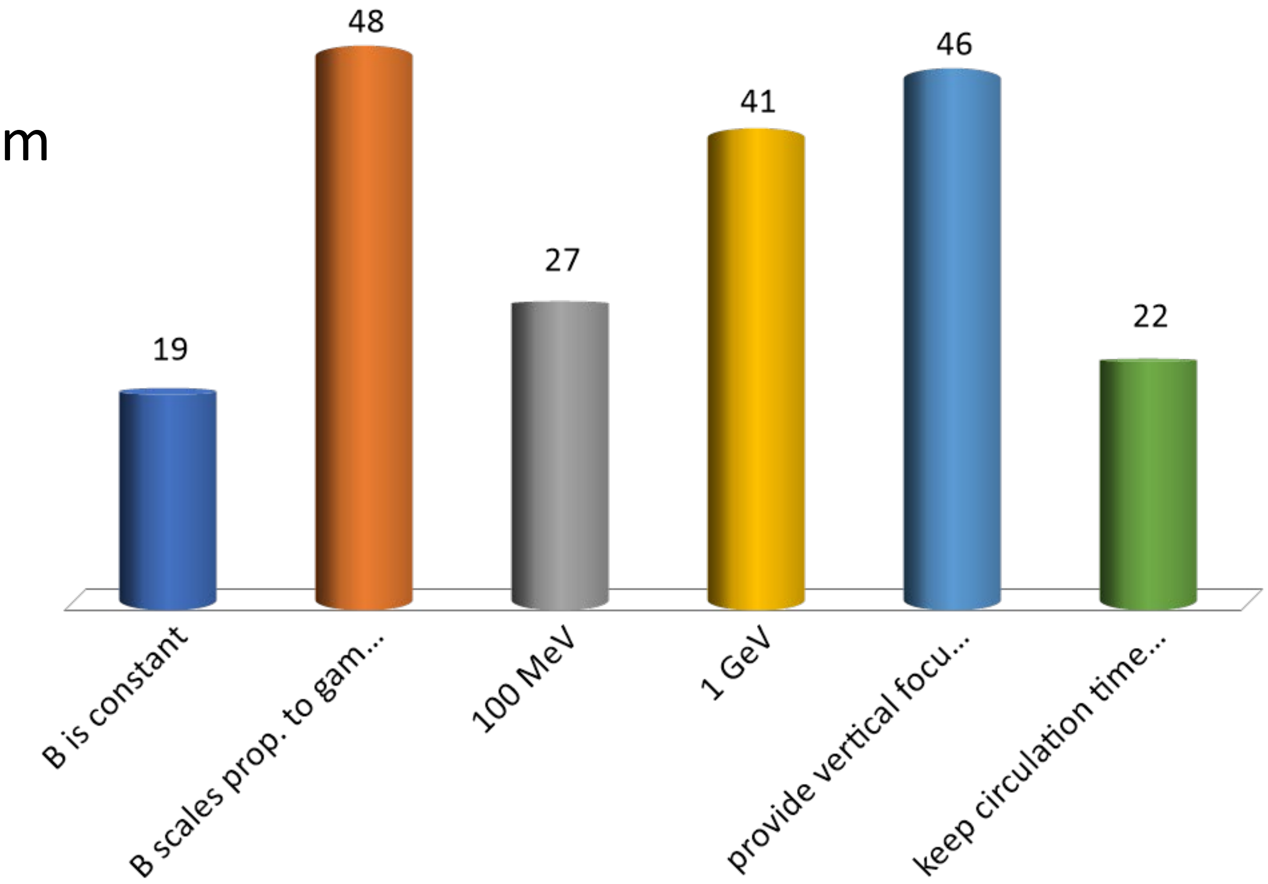
open „ttpoll.eu“, session: „CAS24“

In an isochronous cyclotron, how is the B field scaled towards larger radii: A or B ?
Approximately what proton energy is reachable in an isochronous cyclotron: C or D?
Why do cyclotrons have spiral magnets: E or F?

A. B is constant
✓ B. B scales prop. to gamma of beam

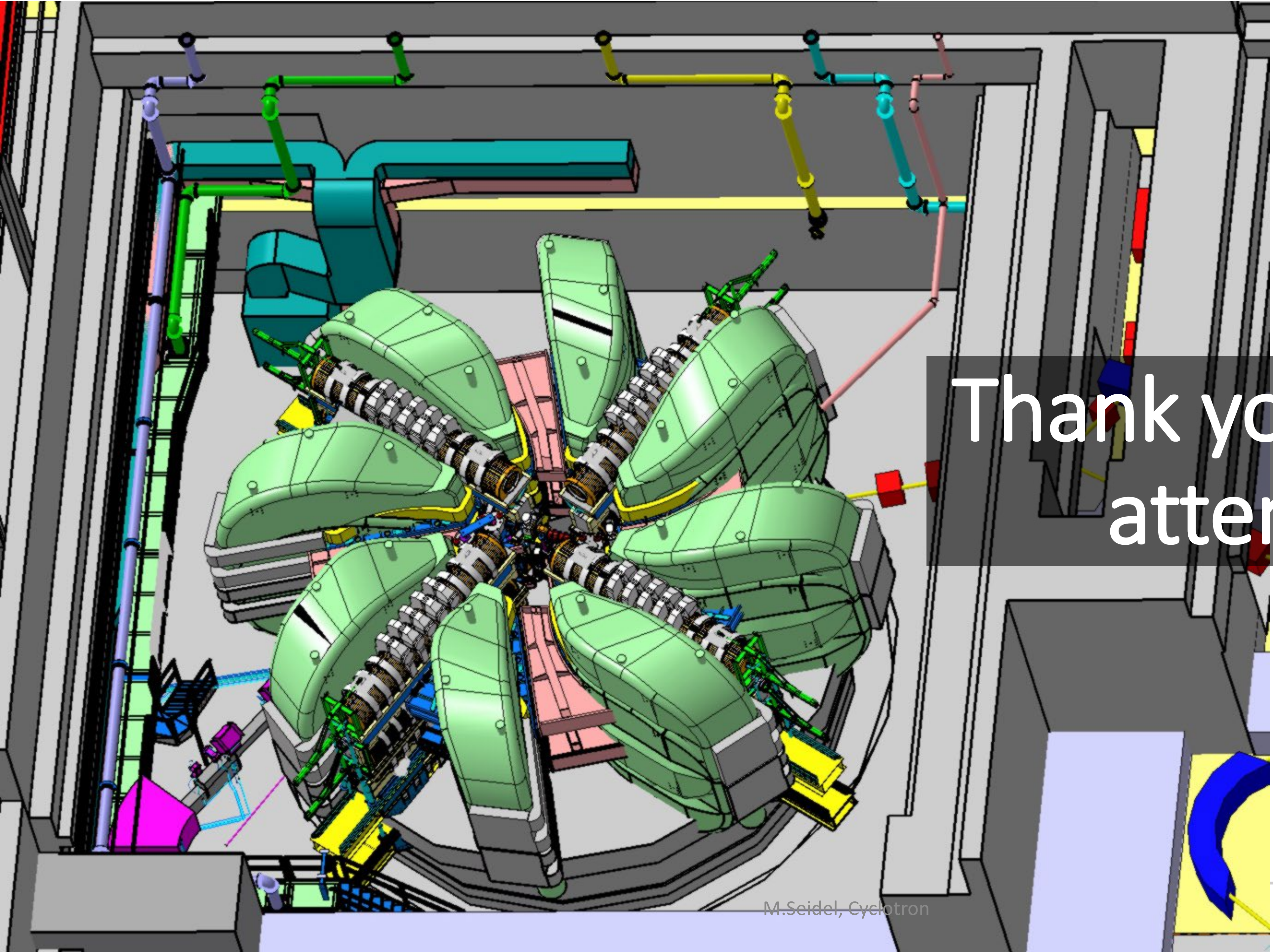
C. 100 MeV
✓ D. 1 GeV

✓ E. provide vertical focusing
F. keep circulation time const



some literature w.r.t. cyclotrons & FFA

comprehensive overview on cyclotrons	L.M.Onishchenko, Cyclotrons: A Survey, Physics of Particles and Nuclei 39, 950 (2008) **http://www.springerlink.com/content/k61mg262vng17411/fulltext.pdf**
50 Years of Cyclotron Development	L. Calabretta, M. Seidel IEEE Transactions on Nuclear Science, Vol. 63, No. 2, 965 – 991(2016) http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7410111
space charge effects and scalings	W.Joho, High Intensity Problems in Cyclotrons, Proc. 5th intl. Conf. on Cyclotrons and their Applications, Caen, 337-347 (1981) http://accelconf.web.cern.ch/AccelConf/c81/papers/ei-03.pdf
ICFA BDN Nr 43	series of high level FFA articles (2007) https://www-bd.fnal.gov/icfabd/Newsletter43.pdf
FFA Optics	M. Craddock, FFA Optics (2011) https://www.cockcroft.ac.uk/events/ffag11/FFAG_talks/11/5.Craddock.pdf
comparison of cyclotron and FFA	M. Craddock, Was the Thomas cyclotron of 1938 a proto-FFAG? https://www.cockcroft.ac.uk/events/FFAG08/presentations/Craddock/Thomas-FFAG.pdf



Thank you for your attention !