CAS SCHOOL, SEP 2024

INTRODUCTION TO ELECTROMAGNETISM

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MATERIALS

CAS Website

These slides and the video will be available the CAS school website

Books

- J. David Jackson, "Classical Electrodynamics"
- David J. Griffiths, "Introduction to Electrodynamics"
- **Chabay, Sherwood "Matter & Interactions"**

VARIABLES AND UNITS

$\mathbf{E}$		electric field [V/m]
$\mathbf{B}$		magnetic field [T]

q		electric charge [C]
ρ		electric charge density [C/m^3]
$\mathbf{J}$	$= \rho \mathbf{v}$	current density [A/m^2]

ϵ_0		permittivity of vacuum, $8.854 \cdot 10^{-12}$ [F/m]
μ_0	$= \frac{1}{\epsilon_0 c^2}$	permeability of vacuum, $4\pi \cdot 10^{-7}$ [H/m or N/A^2]
c		speed of light in vacuum, $2.99792458 \cdot 10^8$ [m/s]

WHY EM?

EM is our first example of a field theory

To work in the accelerator physics field you really should understand field theory and understand that well

EM teaches us about special relativity

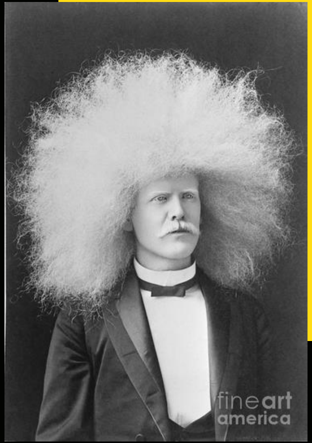
See Special Relativity lecture

Modern physics

Electromagnetism is the first example of using theories unification

EXAMPLES

Electric Force



Magnetic Force

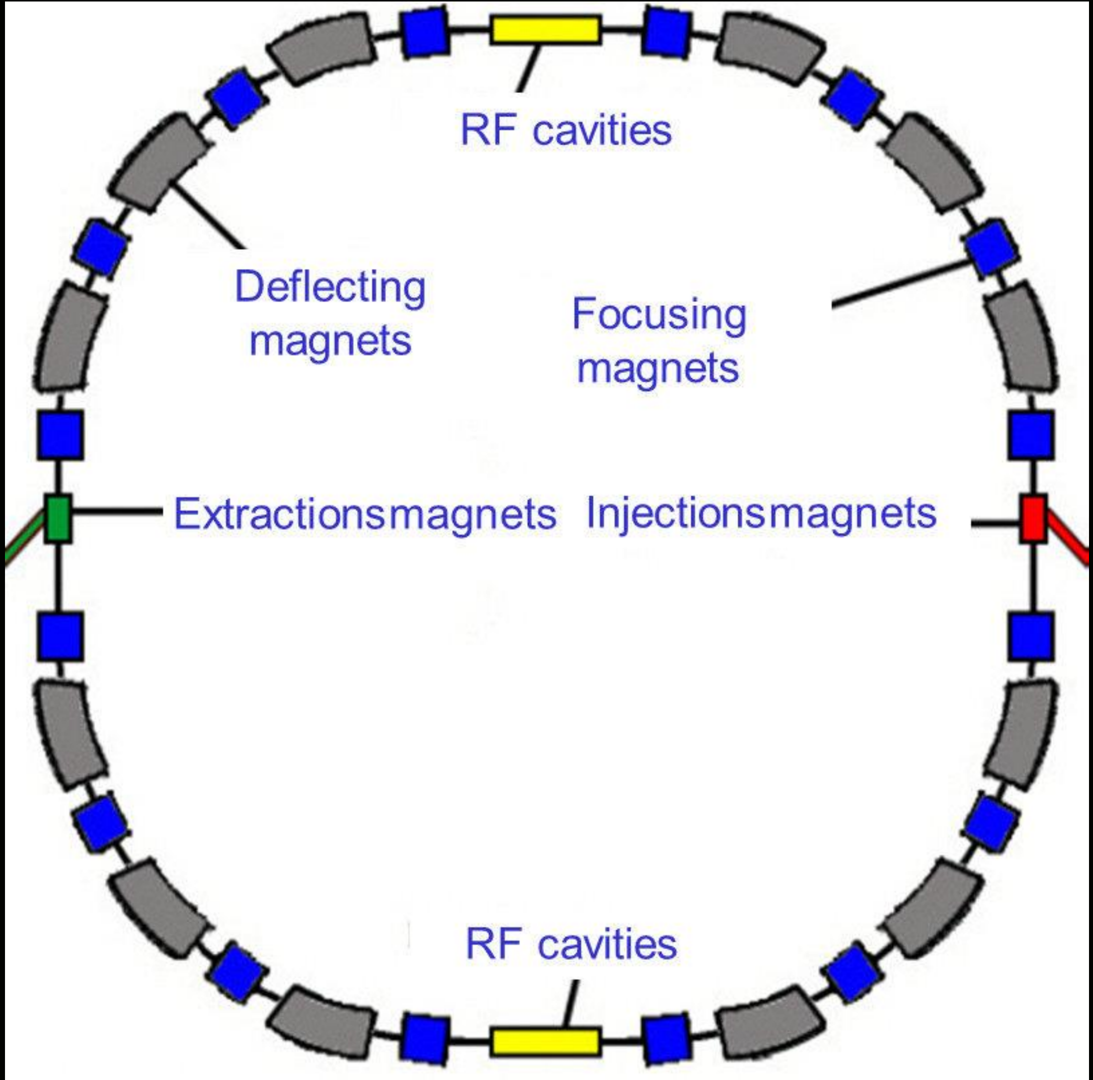
WHY EM FOR ACCELERATORS?

Acceleration of Particles: this is achieved by creating a potential difference using an **electric field** which imparts energy to the particles

Steering and Focusing of Particles: once the particles are accelerated, they need to be guided along the desired path. This is done using **magnetic fields**.

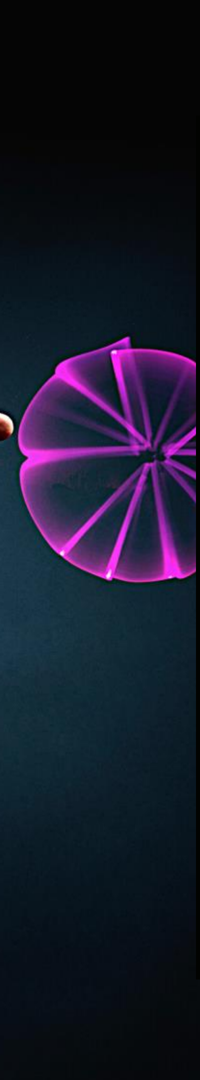
Synchrotron Radiation: In circular accelerators, charged particles emit electromagnetic radiation known as synchrotron radiation when they are deflected by a magnetic field. This is a key consideration **in the design and operation of accelerators**, as it leads to energy loss that must be compensated for.

Particle Detection: after a collision, the resulting particles are detected based on their **electromagnetic interactions** and the **electromagnetic responses** of the detector materials.


RF cavities
Deflecting magnets
Focusing magnets
Extractionsmagnets
Injectionsmagnets
RF cavities

CONTENT OF THE COURSE

- INTRODUCTION TO FIELDS
- MAXWELL EQUATIONS
- ELECTROSTATICS
- MAGNETOSTATICS
- ELECTROMAGNETISM

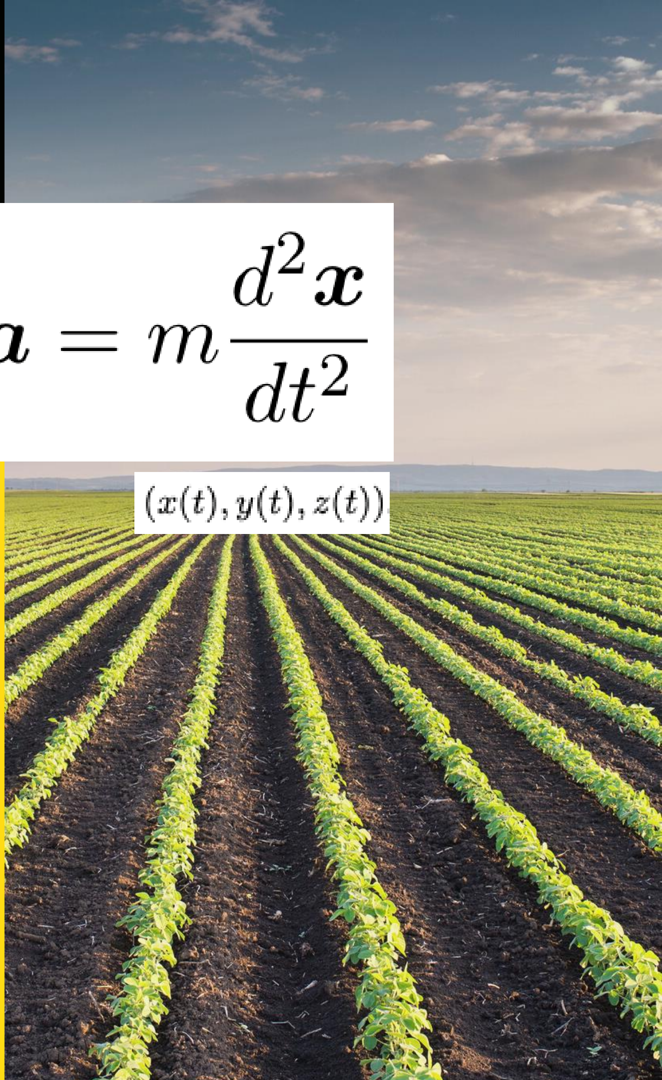


INTRODUCTION

- **WHAT IS FIELD?**
- CHARGE AND CURRENT
- LORENTZ FORCE

INTRODUCTION TO FIELDS

$$\boldsymbol{F} = m\boldsymbol{a} = m\frac{d^2\boldsymbol{x}}{dt^2}$$



$$(x(t), y(t), z(t))$$

GRAVITATIONAL FORCE

The force exerted by the earth on a particle.

GRAVITATIONAL FIELD

Instead of saying that the earth exerts a force on a falling object, it is more useful to say that the earth sets up **a gravitational force field**.

Any object near the earth is acted upon by the gravitational force field at that location.

INTRODUCTION TO FIELDS

F is the force acting on a particle of mass m and g – the acceleration due to gravity.

- *F and g are fields;*
- *the mass of the particle* m *is not a field*

$$F = mg$$

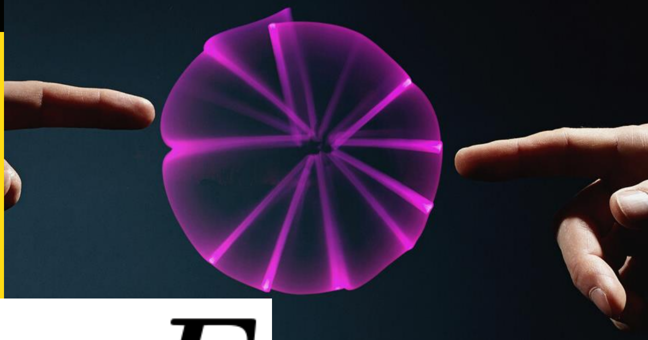
GRAVITATIONAL FORCE

- *We can split the system into a source which produces the field and an object which reacts to the field*
- *We treat both pieces separately*

INTRODUCTION TO FIELDS

ELECTRIC FORCE

The force between charged particles. Charged particles exert forces on each other



$$F = qE$$

$$F = mg$$

ELECTRIC FIELD

- The charge q of our particle replaces the mass m of our particle. q is a single number associated with the object that experiences the field.
- The electric field **E** replaces the gravitational field **g**

We are splitting things up into a source that produces a field and an object that experiences the field

INTRODUCTION TO FIELDS

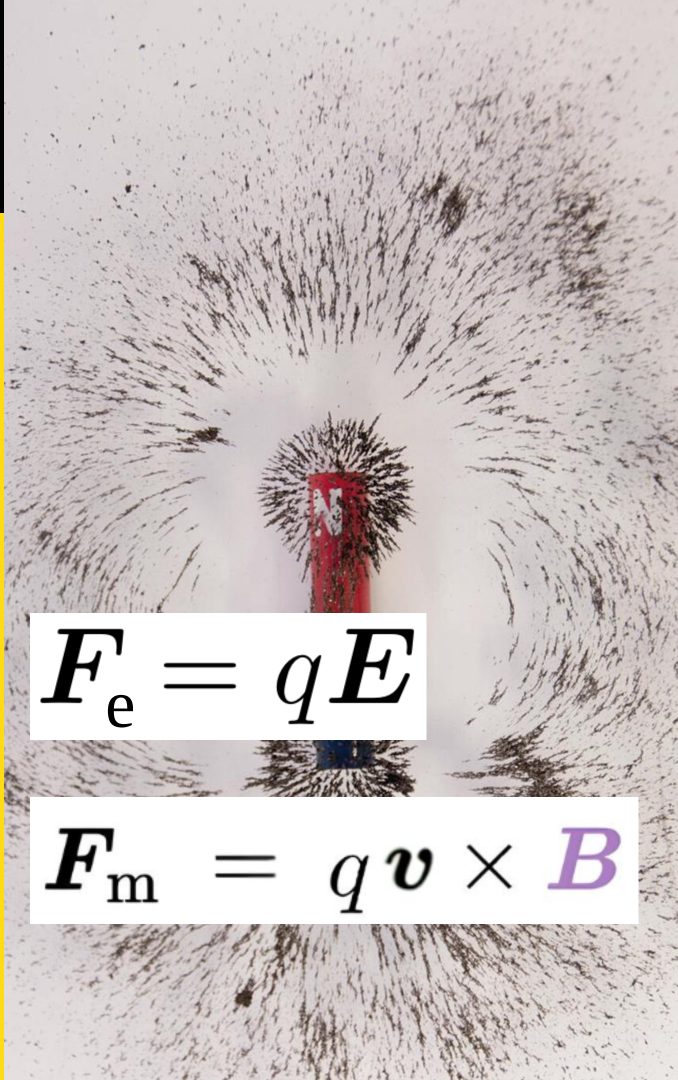
ELECTROMAGNETIC FORCE

To describe the **force of electromagnetism**, we need to introduce **two fields**:

ELECTRIC FIELD , E $\mathbf{E}(\mathbf{x}, t)$

AND

MAGNETIC FIELD , B $\mathbf{B}(\mathbf{x}, t)$



$$\boldsymbol{F}_{\mathrm{e}} = q\boldsymbol{E}$$

$$\boldsymbol{F}_{\mathrm{m}} = q\boldsymbol{v} \times \boldsymbol{B}$$

INTRODUCTION

- WHAT IS FIELD?
- **CHARGE AND CURRENT**
- LORENTZ FORCE

CHARGE AND CURRENT

$$e = 1.602176634 \times 10^{-19}\ C$$

$$q = ne \qquad n \in \mathbf{Z}$$

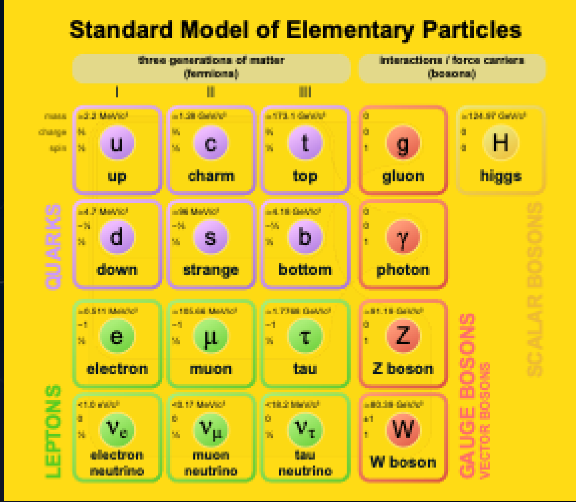
The SI unit of charge is the Coulomb, denoted by C

A much more natural unit . Then, proton/electron: n = ±1

$$q = -e/3$$

$$q = 2e/3$$

the charge of quarks



CHARGE AND CURRENT

$$\rho(\mathbf{x}, t)$$

$$Q = \int_V d^3x \; \rho(\mathbf{x}, t)$$

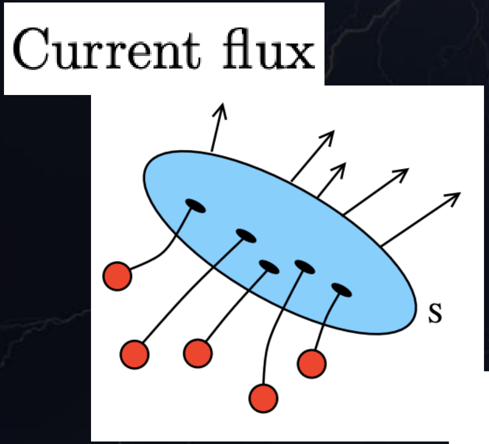
the charge density – charge per unit volume

the total charge Q in a given region V

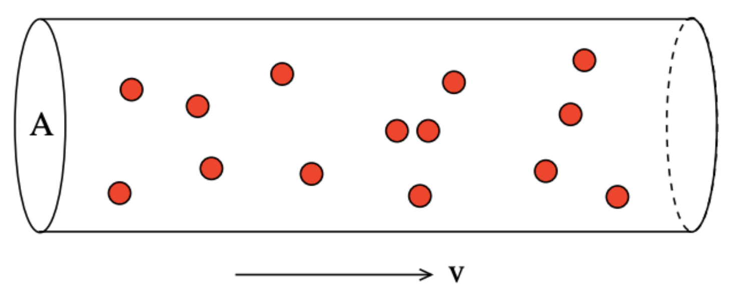
$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

the movement of charge from one place to another is captured by the current density J.
I is called current.
The current density is the current-per-unit-area

CHARGE AND CURRENT



$$\mathbf{J} = \rho\mathbf{v}$$



Move intuitive way:

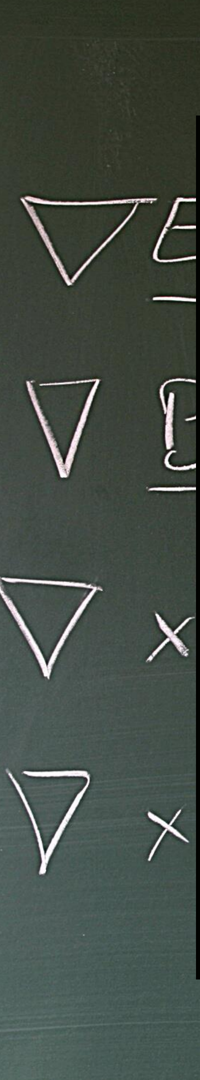
A continuous charge distribution in which the velocity of a small volume, at point **x**, is given by

v(x, t)

Electrons moving along a wire

$$\mathbf{J} = nq\mathbf{v}$$

$$I = |\mathbf{J}|A$$



INTRODUCTION

- WHAT IS FIELD?
- CHARGE AND CURRENT
- **LORENTZ FORCE**

LORENTZ FORCE

$$F = mg \rightarrow F = qE \rightarrow \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Lorentz Force

an electric field accelerates a particle in the direction **E**, while a magnetic field causes a particle to move in circles in the plane perpendicular to **B.**

$$m\frac{d^2\mathbf{r}}{dt^2} = q\,\mathbf{E} + q[\mathbf{v},\mathbf{B}]$$

Now we talk in terms of the force density **f**(**x**, t), which is the force acting on a small volume at point **x**

$$\mathbf{f} = \rho\mathbf{E} + \mathbf{J} \times \mathbf{B}$$

DIFFERENTIATION WITH VECTORS

We define operator "nabla" which we treat as a special vector

$$\nabla \stackrel{\text{def}}{=} \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

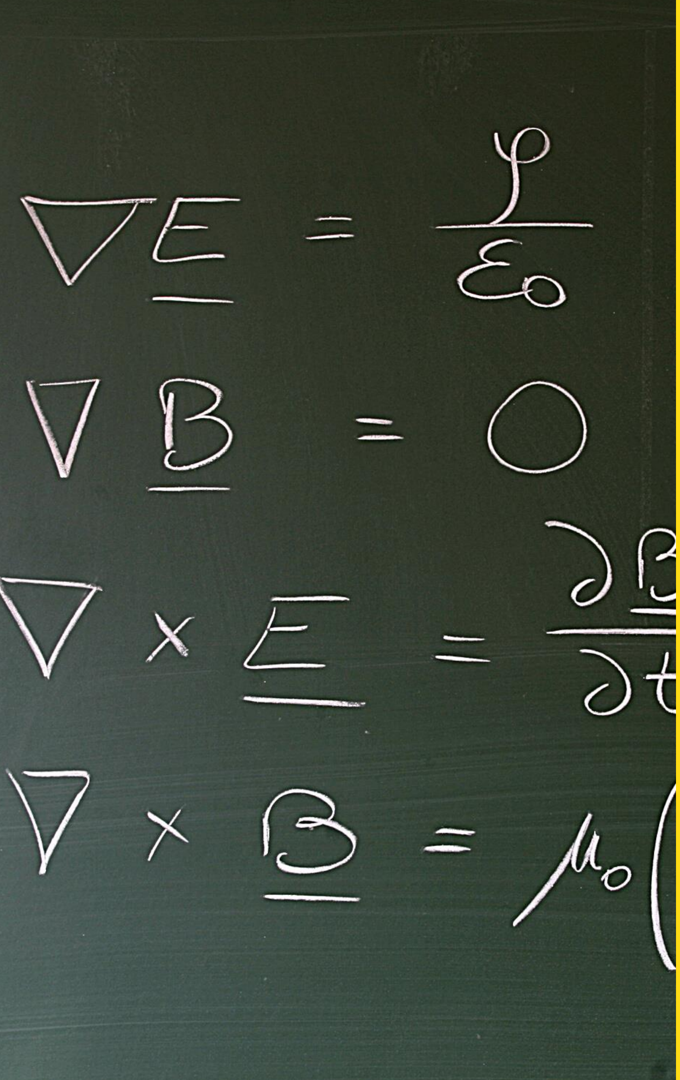
Divergence

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$$

Curl

$$\nabla \phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \right)$$

Gradient



RECAP OF THE INTRODUCTION

- Introduction to Fields

$$\boldsymbol{F}_{\mathrm{e}} = q\boldsymbol{E} \qquad \boldsymbol{F}_{\mathrm{m}} = q\,\boldsymbol{v} \times \boldsymbol{B}$$

$$\rho(\mathbf{x}, t) \qquad Q = \int_V d^3x\, \rho(\mathbf{x}, t)$$

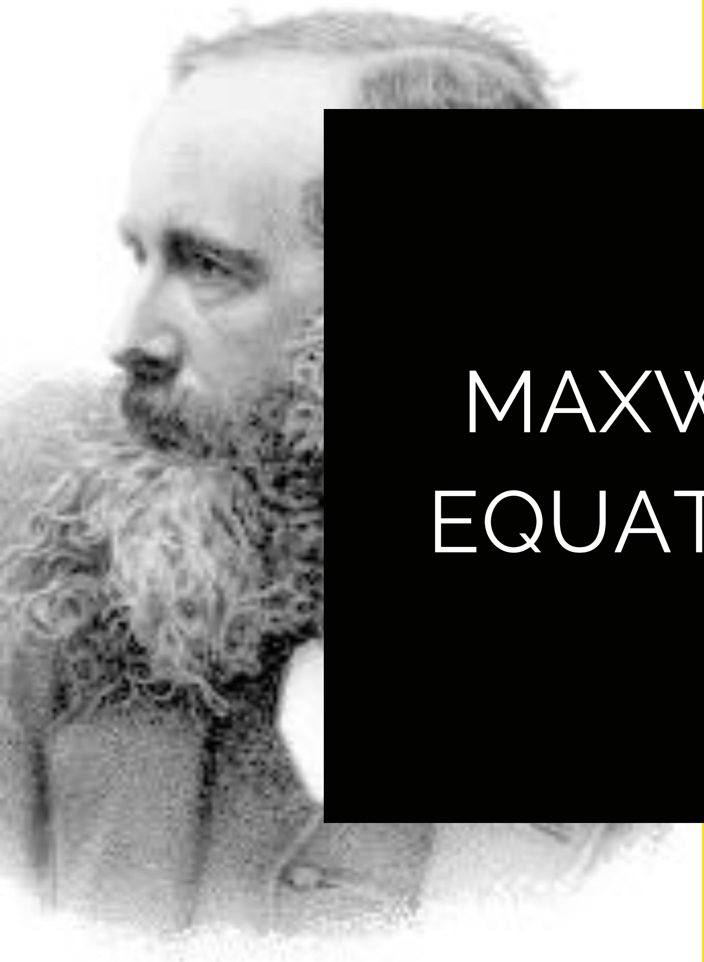
- Charge and Current

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

- Lorentz Force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{f} = \rho\mathbf{E} + \mathbf{J} \times \mathbf{B}$$



MAXWELL EQUATIONS

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

MAXWELL EQUATIONS

DIFFERENTIAL FORM

GAUSSIAN SYSTEM OF UNITS

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

GAUSS'S LAW FOR **E**
(Coulomb's law)

$$\nabla \cdot \mathbf{B} = 0$$

GAUSS'S LAW FOR **B**

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

FARADAY'S LAW
for time-varying magnetic fields

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

AMPERE(-MAXWELL) LAW
for time-varying electric fields

MAXWELL EQUATIONS

DIFFERENTIAL FORM

SI system of units

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

GAUSS'S LAW FOR **E** ***(Coulomb's law)***

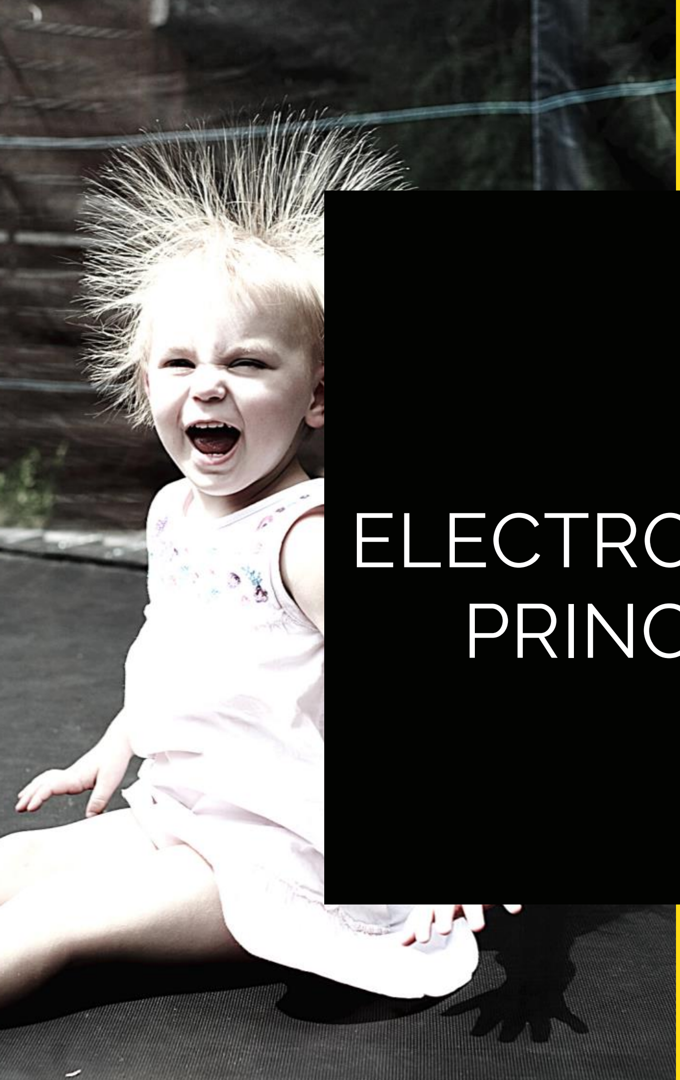
GAUSS'S LAW FOR **B**

FARADAY'S LAW for time-varying magnetic fields

AMPERE(-MAXWELL) LAW for time-varying electric fields

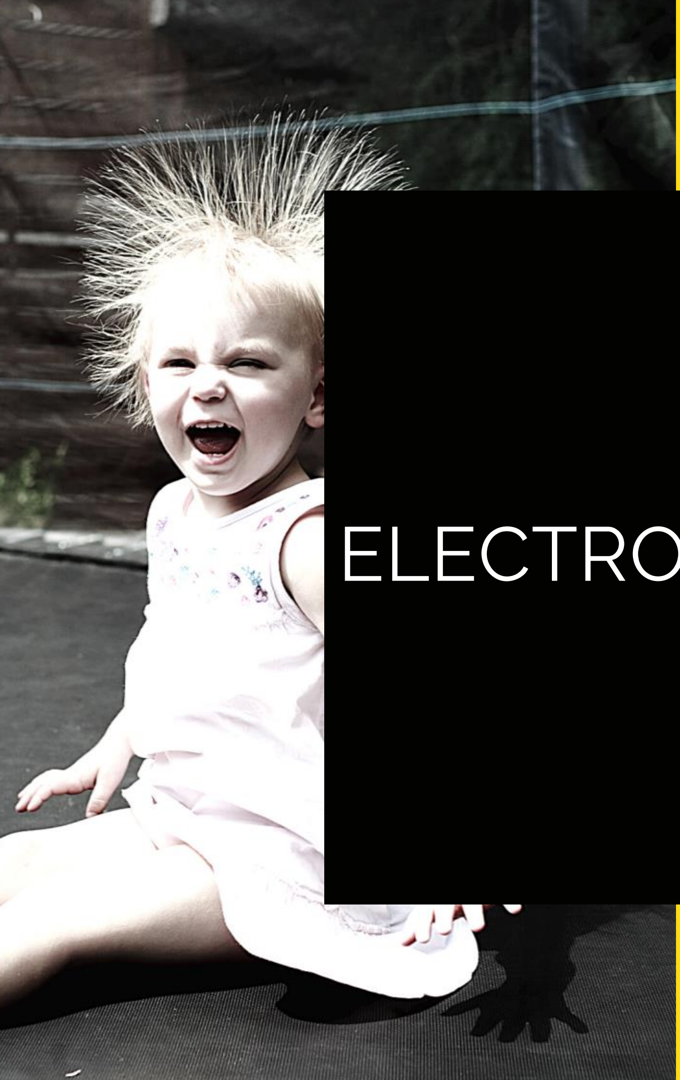
CONVERSION TABLE

Quantity	SI	Gaussian
Velocity of light	$(\mu_0\epsilon_0)^{-1/2}$	c
Electric field, potential	$\boldsymbol{E}, \phi$	$\frac{\boldsymbol{E}}{\sqrt{4\pi\epsilon_0}}, \frac{\phi}{\sqrt{4\pi\epsilon_0}}$
Charge density, current	$q, \rho, \boldsymbol{J}$	$q\sqrt{4\pi\epsilon_0}, \rho\sqrt{4\pi\epsilon_0}, \boldsymbol{J}\sqrt{4\pi\epsilon_0}$
Magnetic induction	$\boldsymbol{B}$	$\boldsymbol{B}\sqrt{\frac{\mu_0}{4\pi}}$



ELECTROSTATICS PRINCIPLES

- **MAXWELL EQUATIONS**
- COULOMB FORCE
- ELECTROSTATIC POTENTIAL
- PRINCIPLE OF SUPERPOSITION



ELECTROSTATICS

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

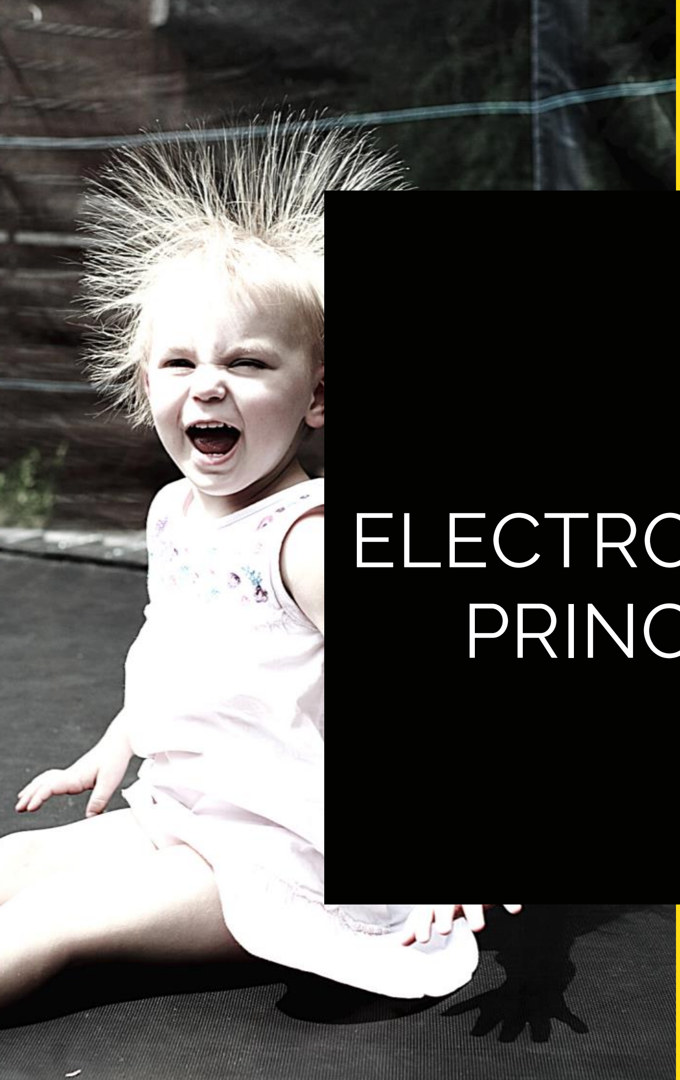
GAUSS'S LAW FOR E

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad 0$$

FARADAY'S LAW

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$



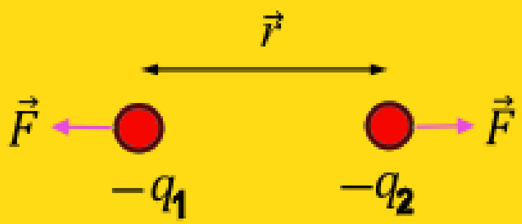
ELECTROSTATICS PRINCIPLES

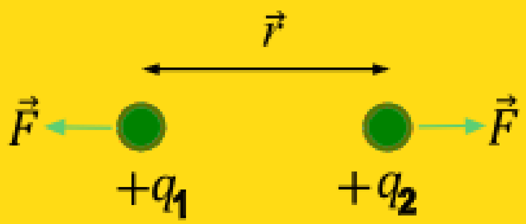
- MAXWELL EQUATIONS
- **COULOMB FORCE**
- ELECTROSTATIC POTENTIAL
- PRINCIPLE OF SUPERPOSITION

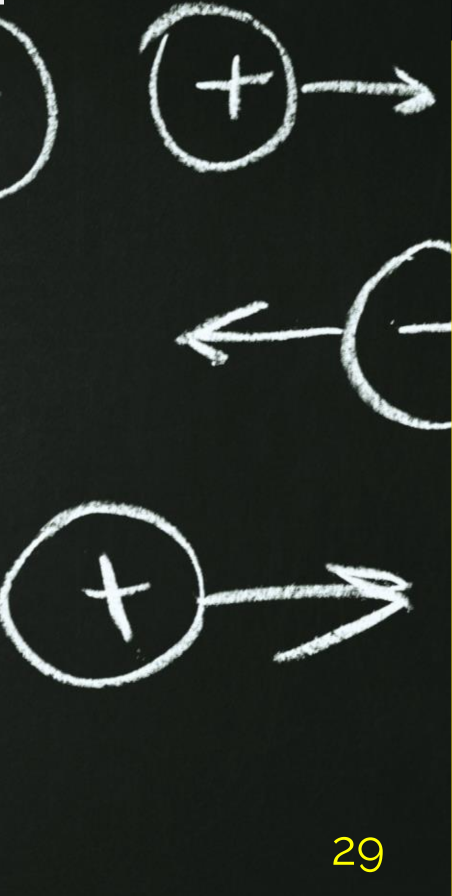
COULOMB FORCE

$$F_E = K \cdot \frac{q_1 \cdot q_2}{r^2}$$

- Like charges repel and unlike charges attract;
- The force acts along the line joining the two-point charges

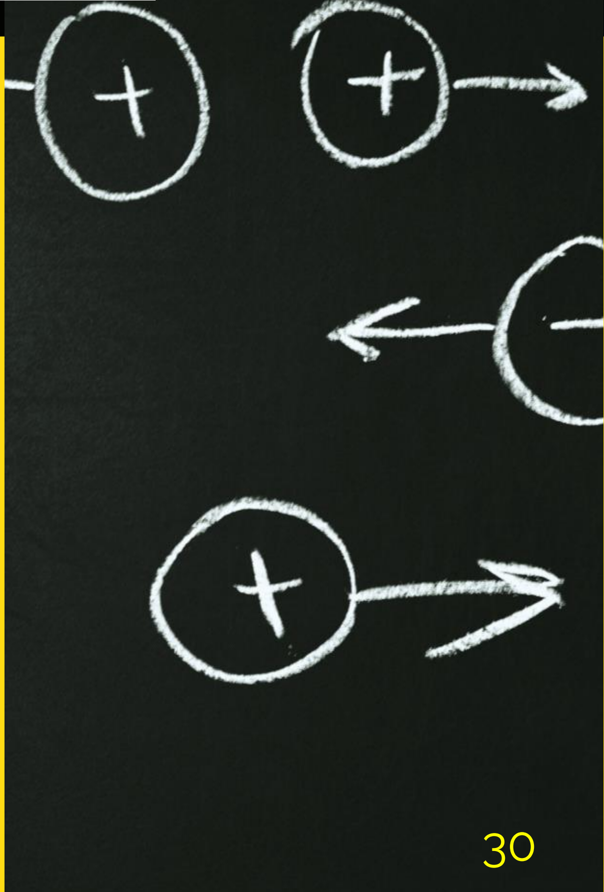






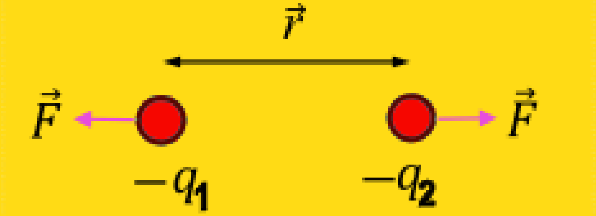
COULOMB FORCE

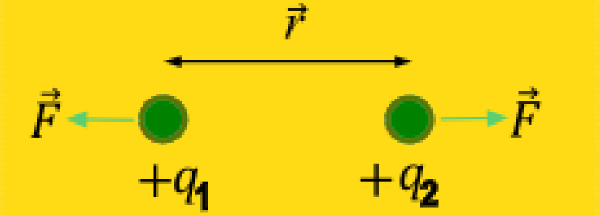
$$F_E = K \cdot \frac{q_1 \cdot q_2}{r^2}$$



Coulomb's Law, the basis of the electric force

The force between two charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.





COULOMB FORCE

$$F_E = K \cdot \frac{q_1 \cdot q_2}{r^2}$$

ELECTROSTATIC FORCE

- Proportional to electric charge of each of the two interacting objects
- Inversely proportional to square of the distance
- Proportional to Coulomb constant **K,** which depends on medium type (vacuum, air, water, etc)

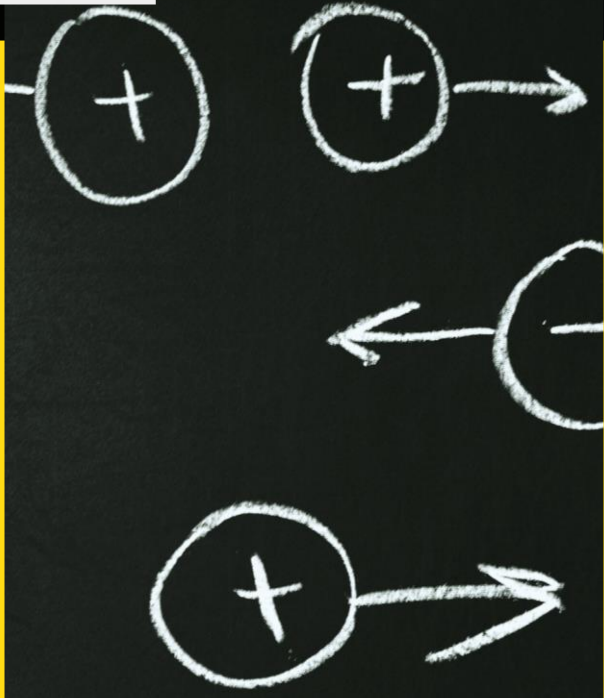
$$K = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \frac{C}{N \cdot m}$$

$$K = \frac{1}{4\pi\varepsilon}$$

$$\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi)\varepsilon_0$$

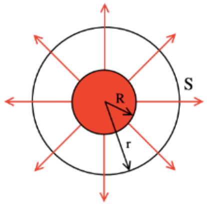
ε_r - relative permittivity

χ - susceptibility of the material



COULOMB FORCE VS GAUSS LAW

$$\mathbf{E}(\mathbf{x}) = \frac{Q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$$



electric field outside a spherically symmetric distribution of charge **Q**

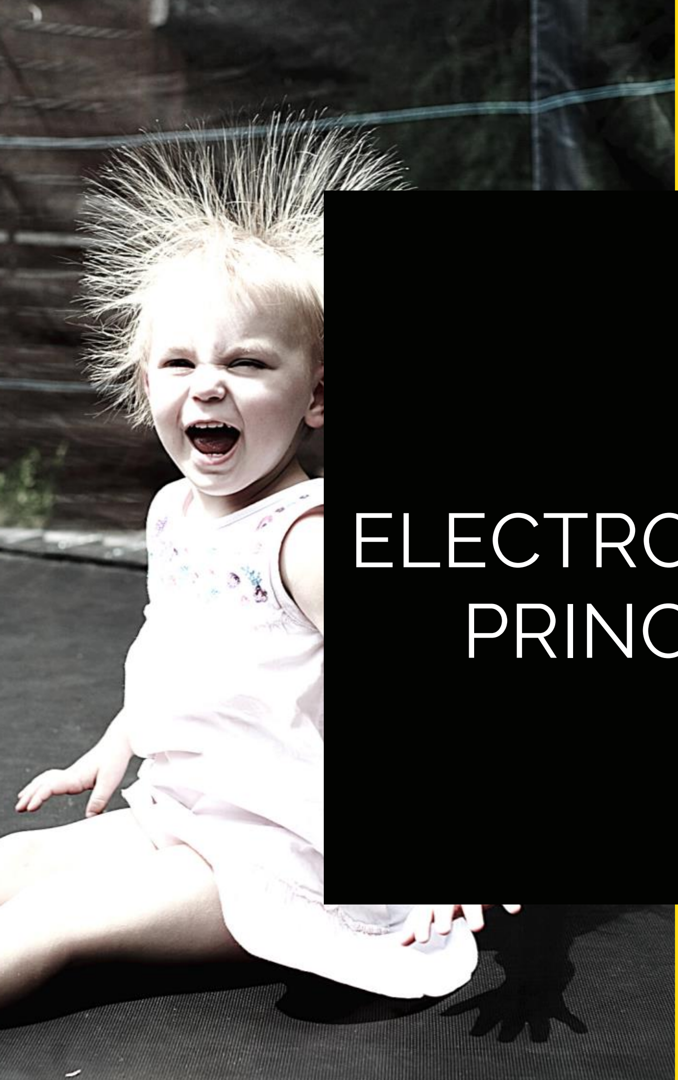
$$\mathbf{E}(\mathbf{x}) = E(r)\hat{\mathbf{r}}$$

$$\boldsymbol{F} = q\boldsymbol{E}$$

It can be used to calculate the force between two particles in a cyclotron or other particle accelerator.

$$\mathbf{F} = \frac{Qq}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$$

$$F_E = K \cdot \frac{q_1 \cdot q_2}{r^2}$$



ELECTROSTATICS PRINCIPLES

- MAXWELL EQUATIONS
- COULOMB FORCE
- **ELECTROSTATIC POTENTIAL**
- PRINCIPLE OF SUPERPOSITION
- CONTINUOUS DISTRIBUTION OF CHARGES

$$U = q\phi$$

volts

Electric Potential

- The **electrical potential energy per charge** is the electric potential.
- The scalar is called the **electrostatic potential** or **scalar potential** (or, sometimes, just the **potential)**.

The two ME for electrostatics can be combined into the **Poisson equation**

$$\nabla \times \mathbf{E} = 0$$

$$\mathbf{E} = -\nabla\phi$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0}$$

The Poisson's equation allows to compute the electric field generated by arbitrary charge distributions.

ELECTROSTATIC POTENTIAL

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0} \longrightarrow \nabla^2\phi = 0$$

Laplace equation

$$\nabla \cdot \nabla = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Laplacian

Both Poisson's and Laplace's equations are essential tools in electrostatics for **determining the electric potential (and, subsequently, the electric field) from a given charge distribution or boundary conditions.**

APPLICATION OF THE POISSON'S OR LAPLACE'S EQUATION

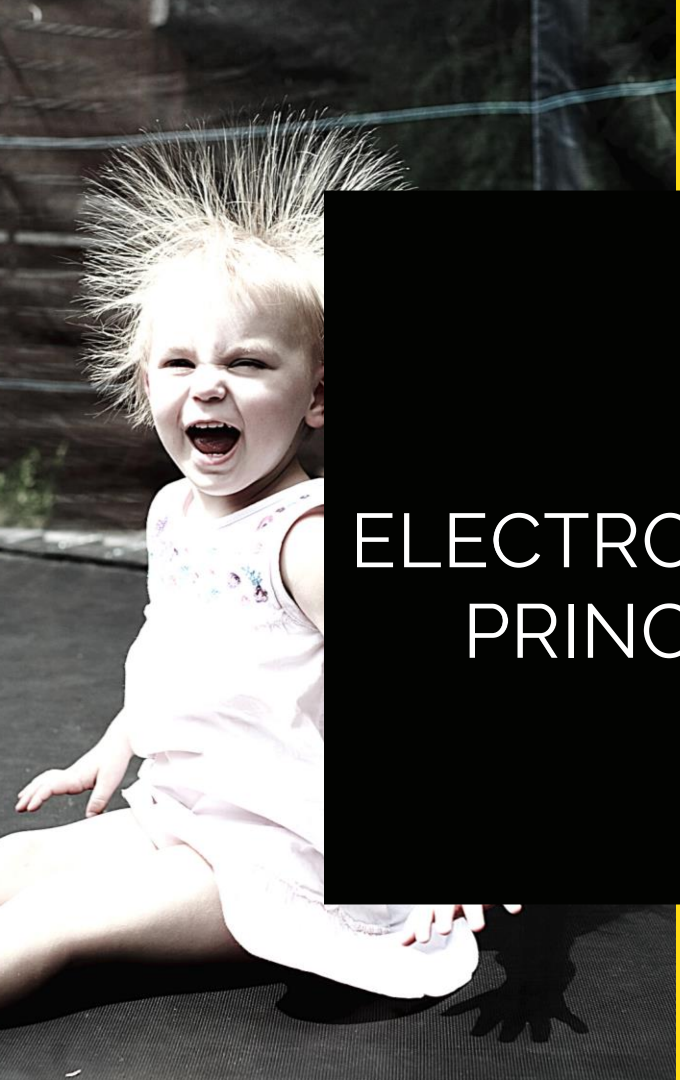
Boundary Value Problems: allow to solve boundary value problems to determine the electric potential everywhere in a region based on some boundary conditions.

Beam Dynamics: predict how space charge effects will impact the beam (poisson eq). For high-intensity beams, these effects can cause beam spreading, limiting the performance of the accelerator.

RF Cavity Design: The shape and structure of RF cavities are often optimized based on solving Laplace's and Poisson's equations to produce desired electric field configurations for efficient acceleration.

Beam Steering and Focusing

Diagnostics: understanding electric potential in BPM



ELECTROSTATICS PRINCIPLES

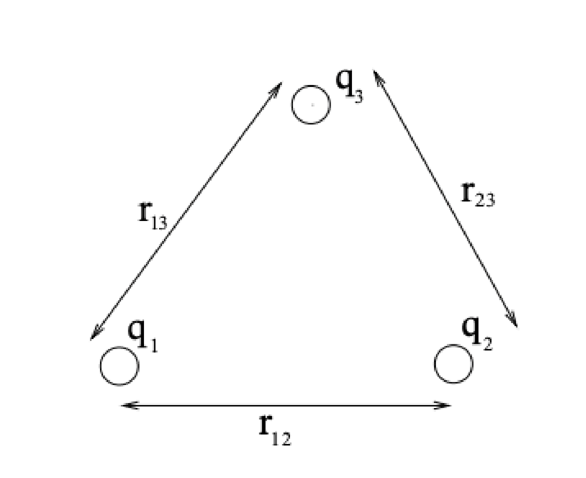
- MAXWELL EQUATIONS
- COULOMB FORCE
- ELECTROSTATIC POTENTIAL
- **PRINCIPLE OF SUPERPOSITION**

PRINCIPLE OF SUPERPOSITION

The net electric field at a location in space is equal to the vector sum of individual electric fields contributed by all charged particles located elsewhere.

Thus, the electric field contributed by a charged particle is unaffected by the presence of other charged particles.

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{r_{23}}$$



RECAP: ELECTROSTATICS

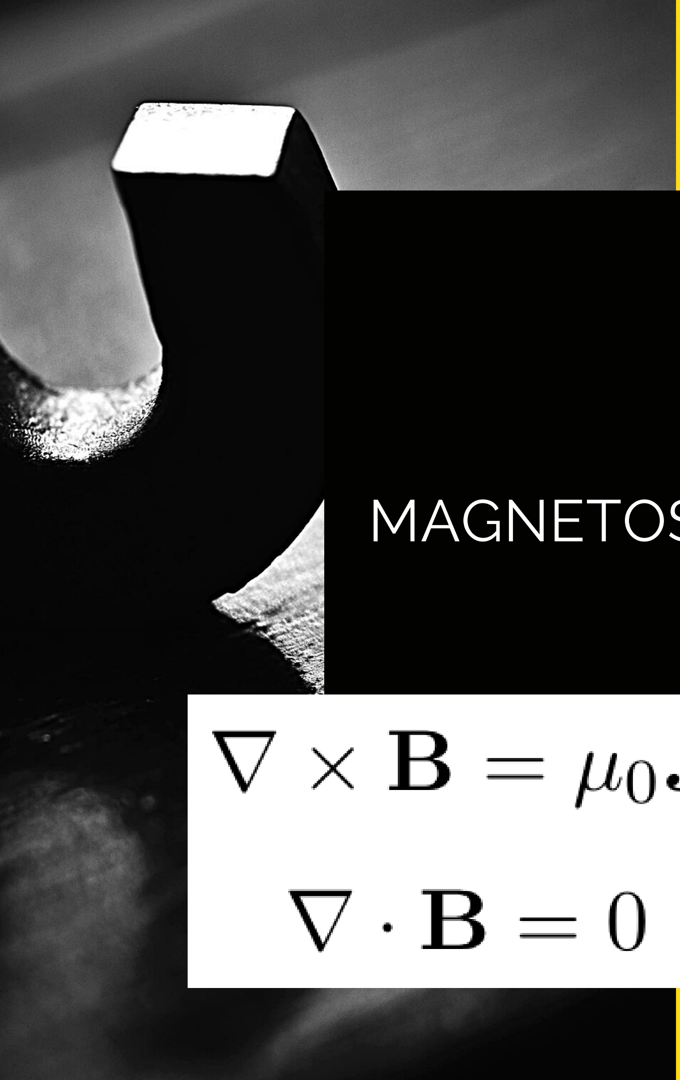
- Maxwell equations $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ $\nabla \times \mathbf{E} = 0$

- Coulomb force vs Gauss law $F_E = K \cdot \frac{q_1 \cdot q_2}{r^2}$ $\mathbf{F} = \frac{Qq}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$

- Electrostatic potential, Poisson eq $\nabla^2\phi = -\frac{\rho}{\epsilon_0}$

- Principle of superposition

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}} + \frac{1}{4\pi\epsilon_0}\frac{q_1q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0}\frac{q_2q_3}{r_{23}}$$



MAGNETOSTATICS

$$\nabla \times \mathbf{B} = \mu_0 \boldsymbol{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

- **MAXWELL EQUATIONS**
- STEADY CURRENT
- AMPÈRE'S LAW
- BIOT-SAVART LAW

MAGNETOSTATICS

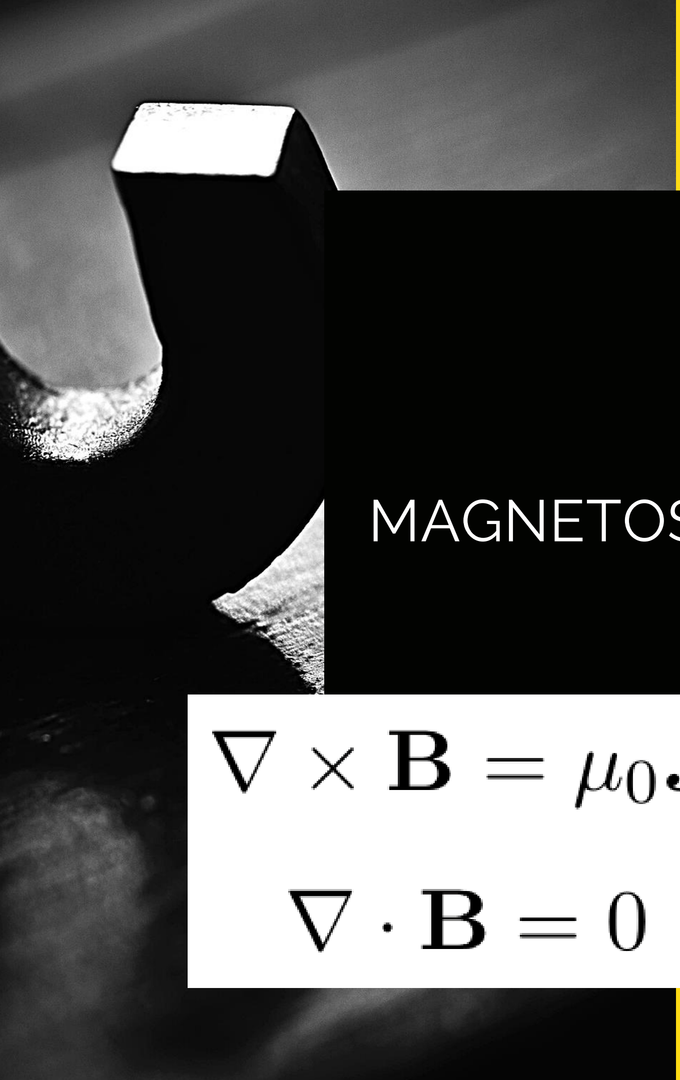
- Charges give rise to electric fields.
- Current give rise to magnetic fields.
- Moving charge particles make a magnetic field which is different from the electric field
- The magnetic field is induced by steady currents - continuous flow of charge

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$



MAGNETOSTATICS

$$\nabla \times \mathbf{B} = \mu_0 \boldsymbol{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

- MAXWELL EQUATIONS
- **STEADY CURRENT**
- AMPÈRE'S LAW
- BIOT-SAVART LAW

STEADY CURRENT

Bending Magnets: steady current-generated magnetic fields are used to bend the paths of charged particles.

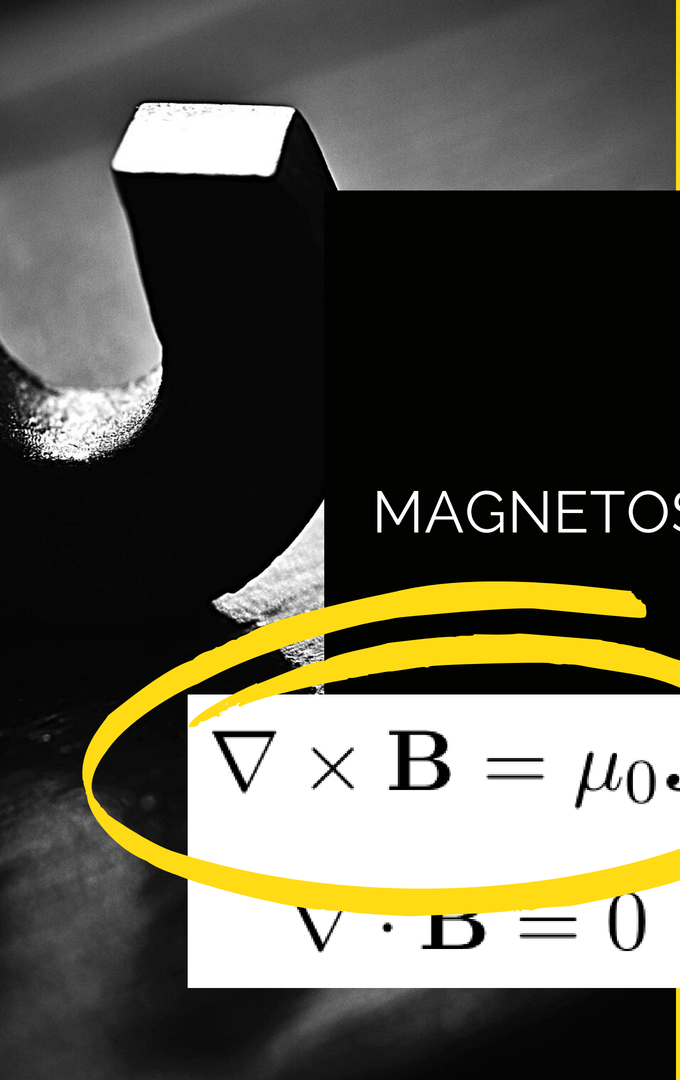
Quadrupole Magnets: steady currents are used in quadrupole magnets to focus the beam.

Sextupole and Higher Order Magnets: used to correct chromatic aberrations and other higher-order distortions in the beam's trajectory.

Beam Steering: dipole magnets with steady currents can be employed to make small adjustments to the trajectory of the beam

Stability: Time-independent magnetic fields, as opposed to oscillating ones, do not induce eddy currents in surrounding structures.

Magnetic Shielding and Corrections: magnetostatics principles are crucial when designing the shielding to prevent unwanted magnetic fields from affecting the accelerator's operation.



MAGNETOSTATICS

$$\nabla \times \mathbf{B} = \mu_0 \boldsymbol{J}$$

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- MAXWELL EQUATIONS
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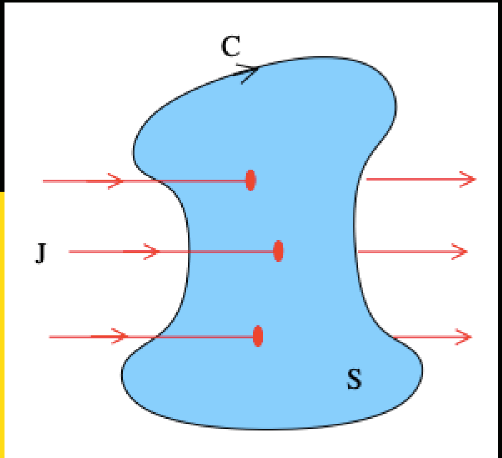
AMPÈRE LAW

$$\nabla \times \mathbf{B} = \mu_0 \boldsymbol{J}$$



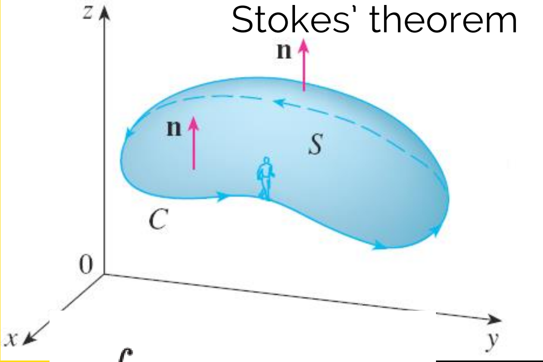
RELATIONSHIP BETWEEN A CURRENT AND THE MAGNETIC FIELD IT GENERATES

Ampere's law states that the integral of the magnetic field around the contour **C** equals

$$\int_S \nabla \times \mathbf{B} \cdot d\mathbf{S} = \oint_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{S}$$

= I

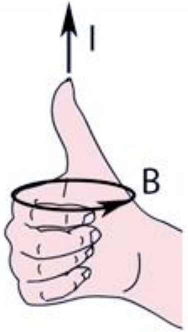
Stokes' theorem



$$\oint_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I$$

AMPÈRE LAW

Right hand thumb rule



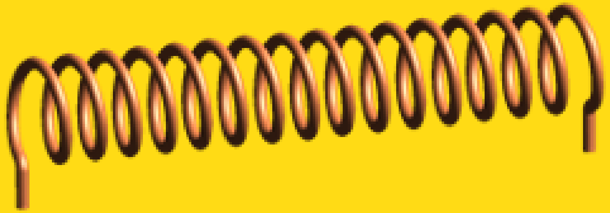
Thumb points in the direction of the electric current and fingers curl around the current indicating the direction of the magnetic field

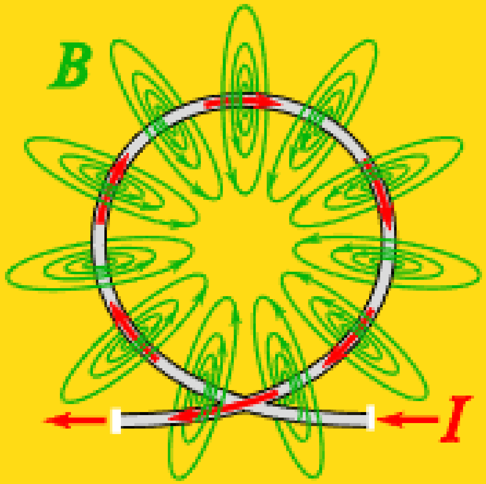
For positive current direction of magnetic field is determined with rule of right hand

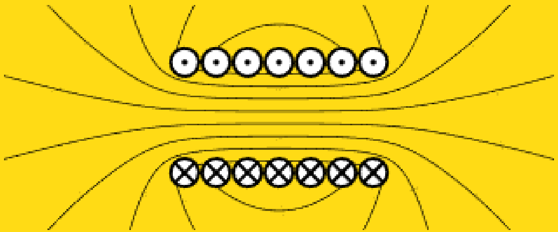
AMPÈRE LAW

THE PRIMARY USAGE OF THE AMPERE LAW IS
CALCULATING THE MAGNETIC FIELD GENERATED BY AN ELECTRIC CURRENT

Ex: a long straight conducting wire, coaxial cable, cylindrical conductor, solenoid, and toroid





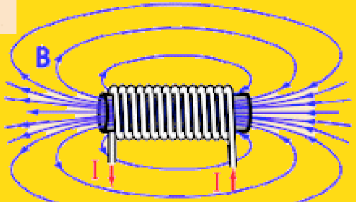


AMPÈRE LAW: KEY IMPLICATIONS AND APPLICATIONS

Solenoids and Toroids: Ampère's Law is commonly applied to find the magnetic field inside and outside long solenoids and.

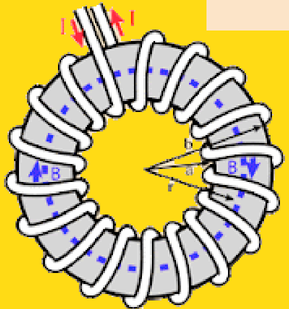
Wire Configurations: to determine the magnetic field around straight current-carrying wires, as well as more complex wire configurations.

$$BL = \mu N I$$
$$B = \mu \frac{N}{L} I$$
$$B = \mu n I$$

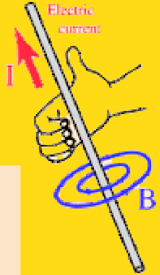


Magnetic field inside a long solenoic.

$$B = \frac{\mu N I}{2\pi r}$$

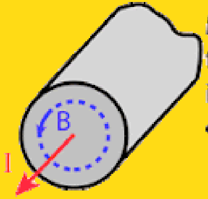


Magnetic field inside a toroidal coil.



Magnetic field from a long straight wire.

$$B = \frac{\mu_0 I}{2\pi r}$$



Magnetic field inside a conductor.

$$B = \frac{\mu J r}{2} = \frac{\mu r I}{2\pi R^2}$$

which at the surface approaches:

$$B_{surface} = \frac{\mu I}{2\pi R}$$

AMPÈRE LAW:
KEY IMPLICATIONS AND APPLICATIONS

Boundary Conditions: Ampère's Law (in conjunction with other principles) helps determine the behavior of magnetic fields at material interfaces.

Limitation: The primary limitation of Ampere's law is that it is applicable in magnetostatics and is valid for steady current. However, Maxwell modified Ampere's law by introducing displacement current.

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

MAGNETOSTATICS

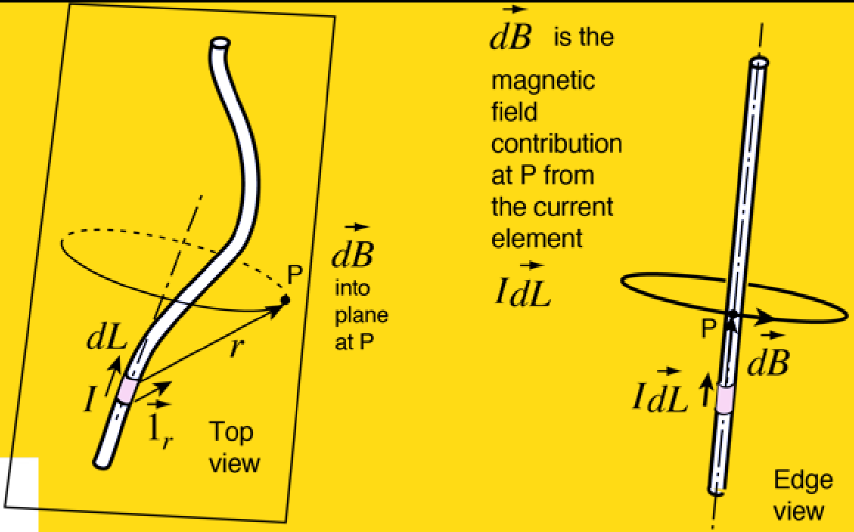
- MAXWELL EQUATIONS
- STEADY CURRENT
- AMPÈRE'S LAW
- **BIOT-SAVART LAW**

BIOT-SAVART LAW

THE ANALOGOUS OF COULOMB LAW

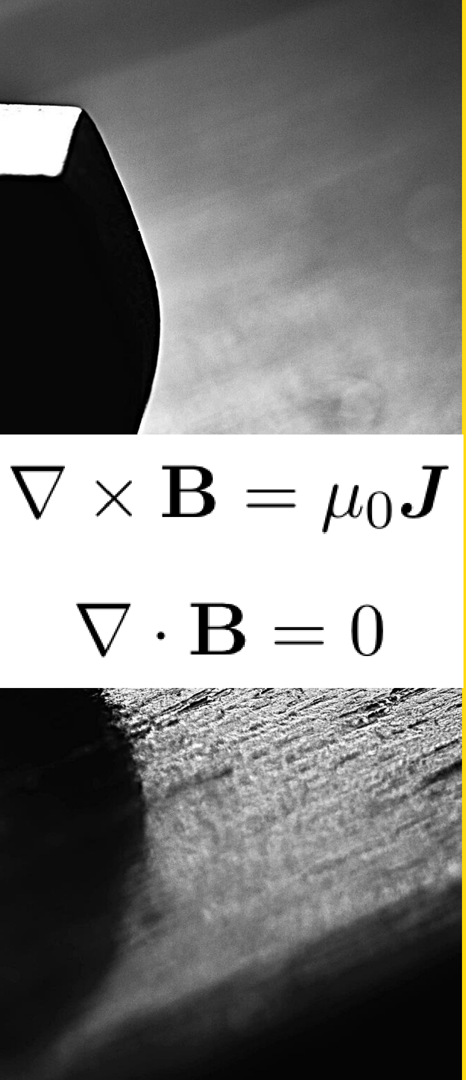
A segment of wire of length dl, carrying a current **I** sets up a magnetic field

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$



Biot-Savart law for currents

http://hyperphysics.phy-astr.gsu.edu



$$\nabla \times \mathbf{B} = \mu_0 \boldsymbol{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

RECAP: MAGNETOSTATICS

- **Steady Current**

  if a current flows into some region of space, an equal current must flow out to avoid the build up of charge.

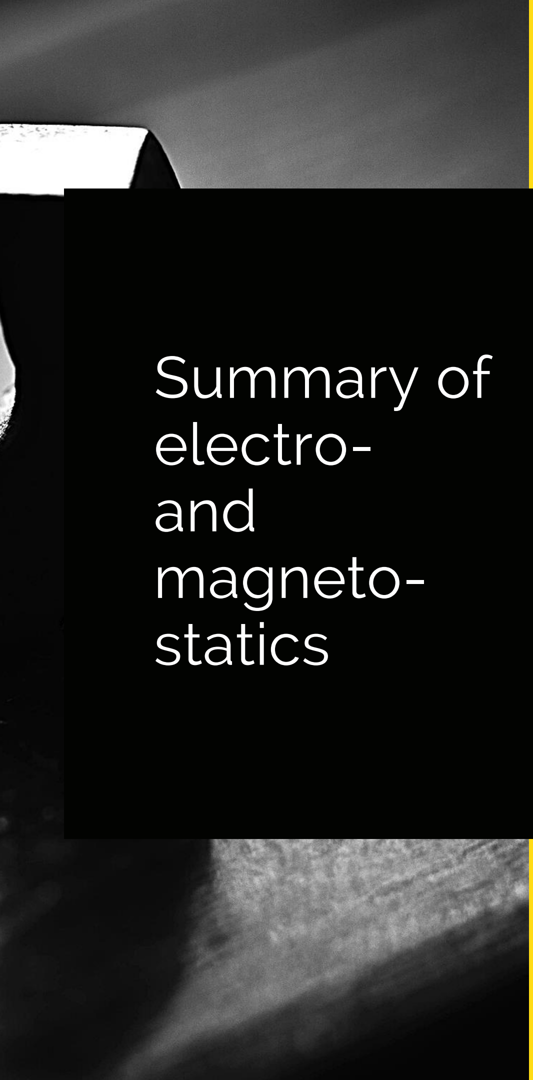
- **Ampère Law** $\nabla \times \mathbf{B} = \mu_0 \boldsymbol{J}$

  relationship between a current and the magnetic field it generates

- **Biot-Savart law**

  It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$



Summary of electro- and magneto-statics

One can compute the electric and the magnetic fields from the scalar and the vector potentials:

$$\vec{E} = -\nabla\phi$$

$$\vec{B} = \nabla \times \vec{A}$$

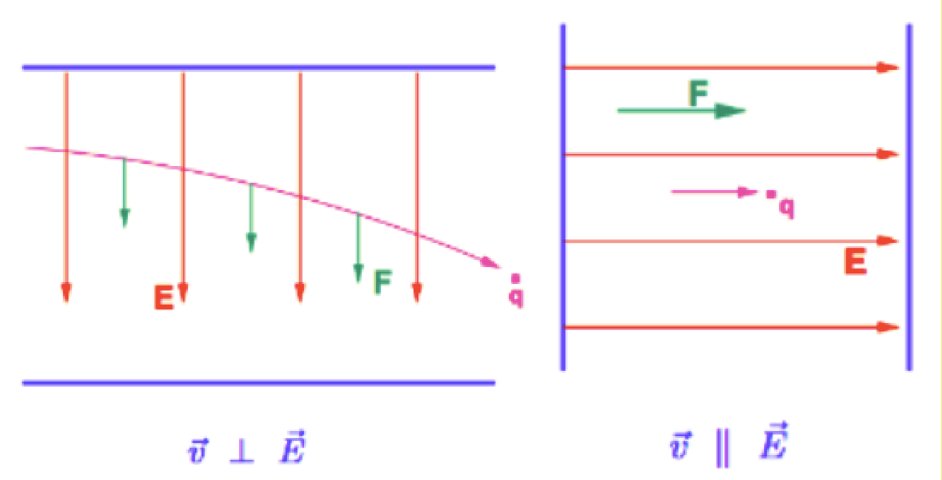
MOTION OF A CHARGED PARTICLE

Lorentz force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

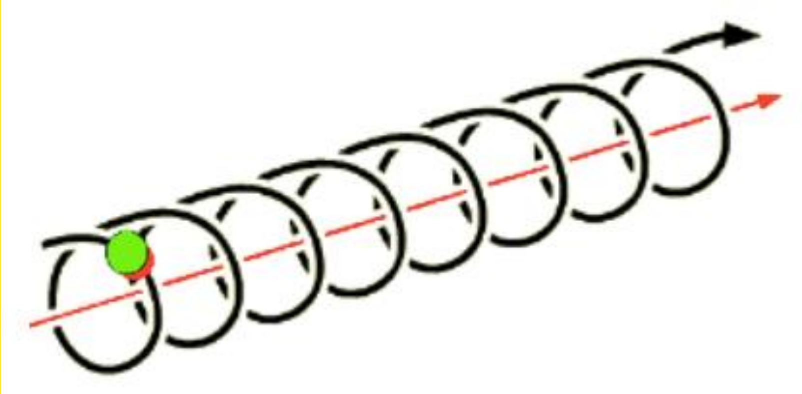
In case of an electric field, the force is always in the direction of the field, also for particles in rest.



MOTION OF A CHARGED PARTICLE

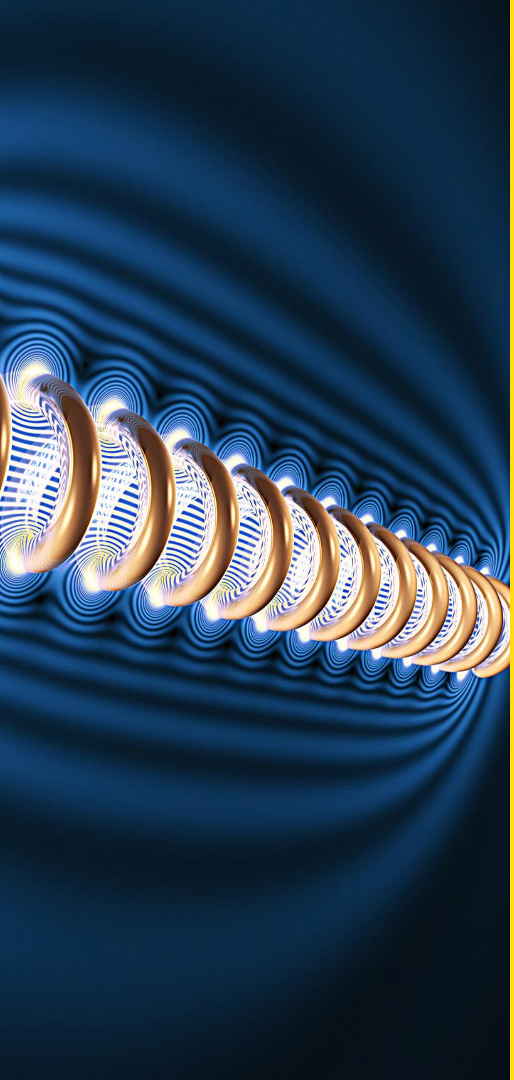
Lorentz force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$



$$\mathbf{F} = q(\cancel{\mathbf{E}} + \mathbf{v} \times \mathbf{B})$$

In this case the force is perpendicular to both, **v** and **B**



ELECTRIC FORCE VS MAGNETIC FORCE

Lorentz force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- To control a charged particle beam we use electromagnetic fields.
- In particle accelerators, transverse deflection is usually given by magnetic fields, whereas acceleration can only be given by electric fields.

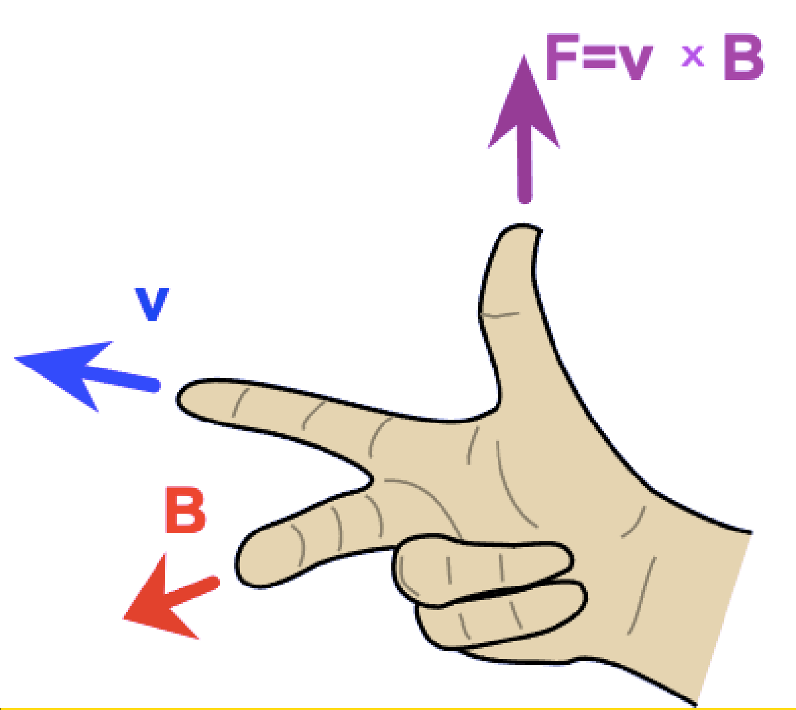
$$\left|\vec{E}\right| = 1 \quad \mathrm{MV/m}$$

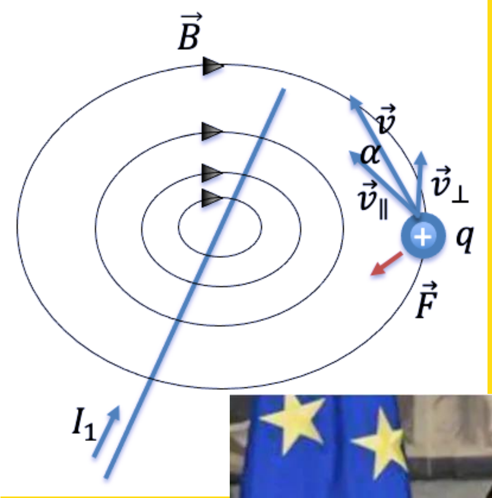
$$\left|\vec{B}\right| = 1 \quad \mathrm{T}$$

$$\frac{F_{\mathrm{magnetic}}}{F_{\mathrm{electric}}} = \frac{evB}{eE} = \frac{\beta cB}{E} \simeq \beta\frac{3 \cdot 10^8}{10^6} = 300\,\beta$$

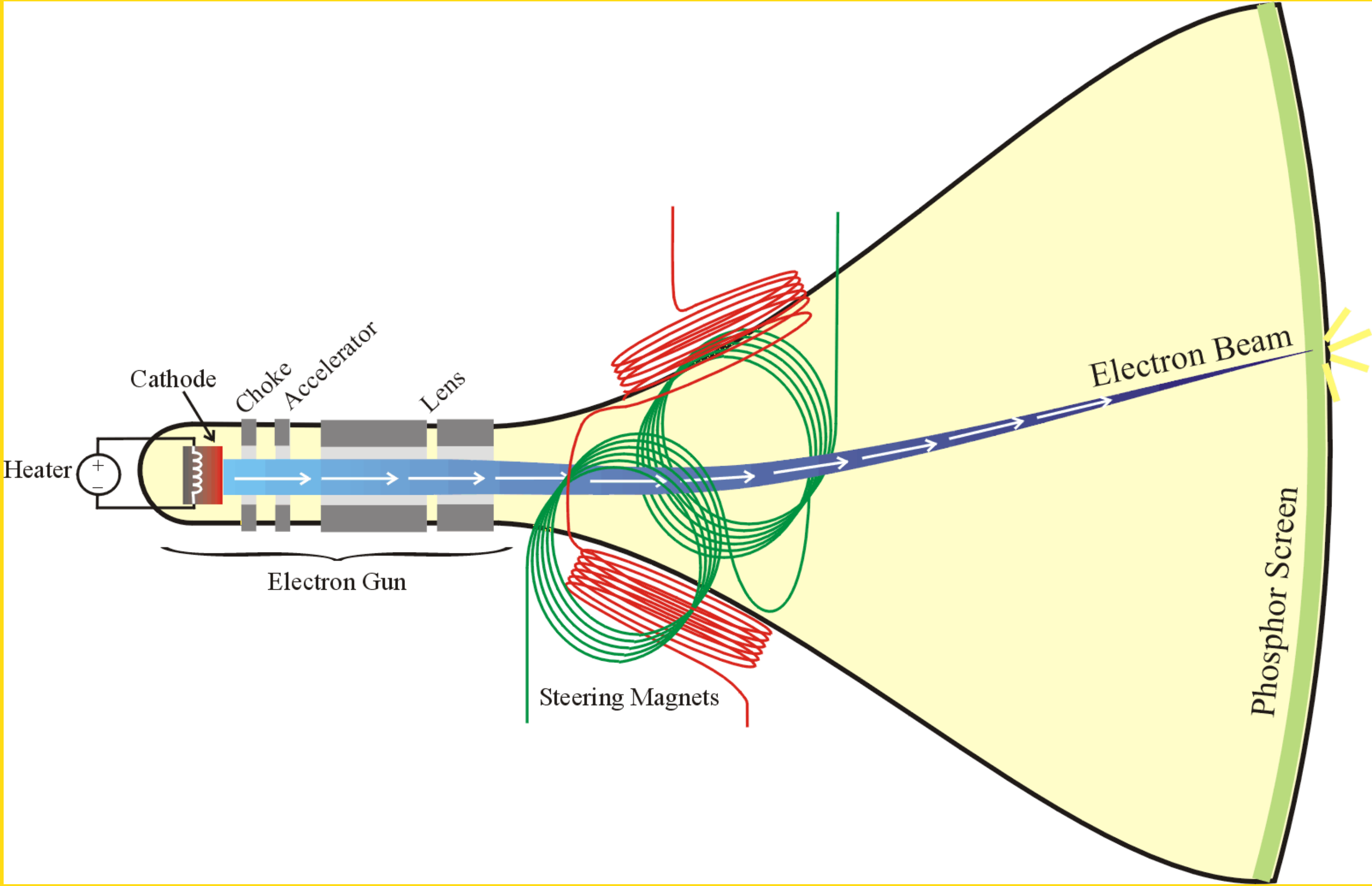
- The magnetic force is much stronger then the electric one: in an accelerator, use magnetic fields whenever possible.

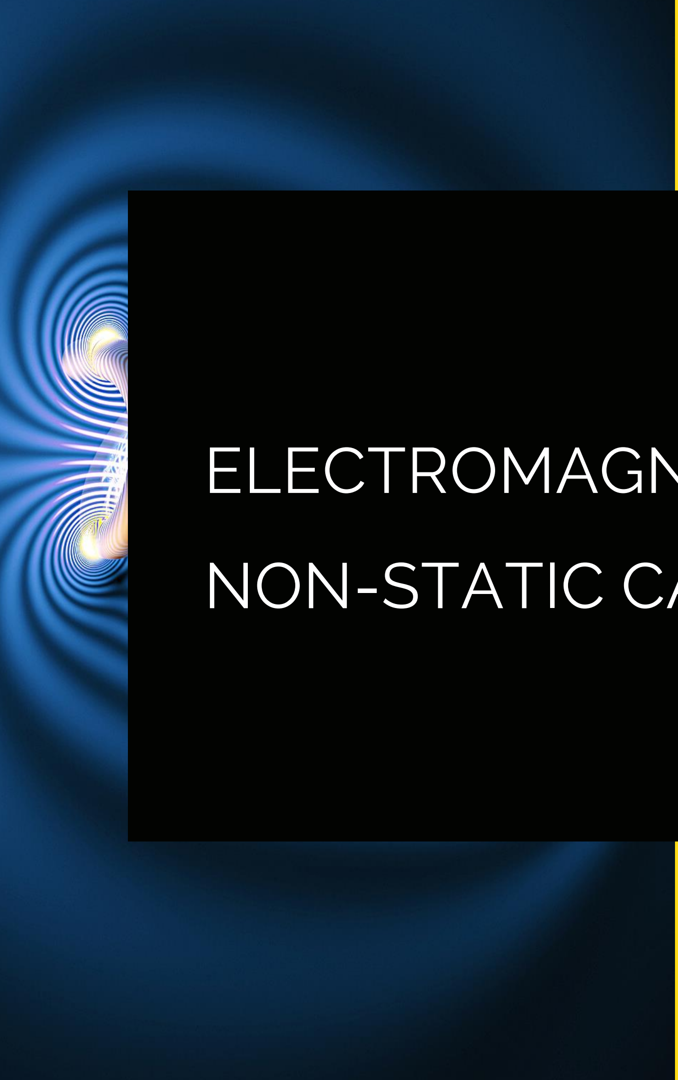
MOTION OF A CHARGED PARTICLE





MOTION OF A CHARGED PARTICLE

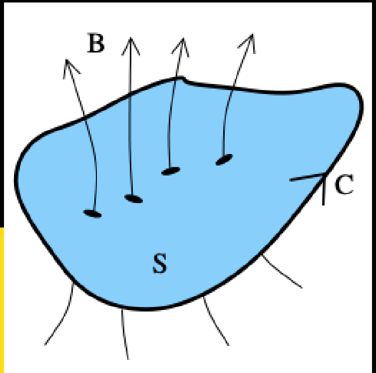




ELECTROMAGNETISM: NON-STATIC CASE

- **FARADAY'S LAW OF INDUCTION**
- WAVE FUNCTION
- PROPAGATION OF ELECTROMAGNETIC WAVES

FARADAY'S LAW OF INDUCTION



The process of creating a current through changing magnetic fields is called INDUCTION.

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\mathcal{E} = \int_C \mathbf{E} \cdot d\mathbf{r}$$

electromotive force

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

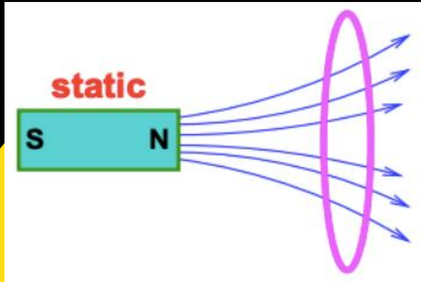
Faraday's Law

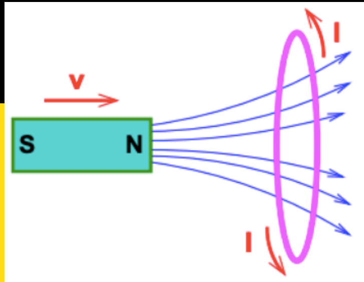
$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

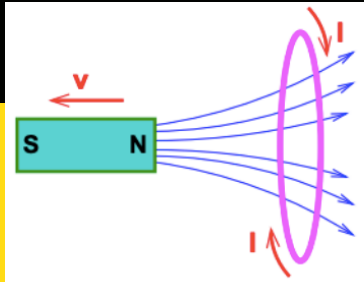
magnetic flux

FARADAY'S LAW OF INDUCTION

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

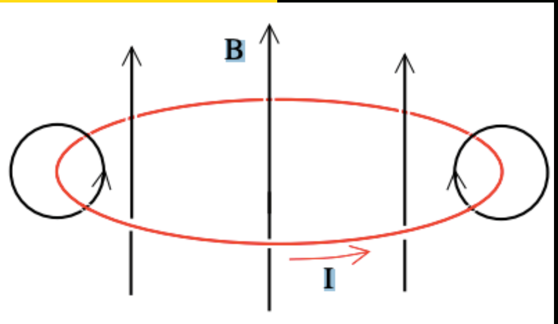






The electromotive force around a closed path is equal to the negative of the time rate of change of the magnetic flux enclosed by the path.

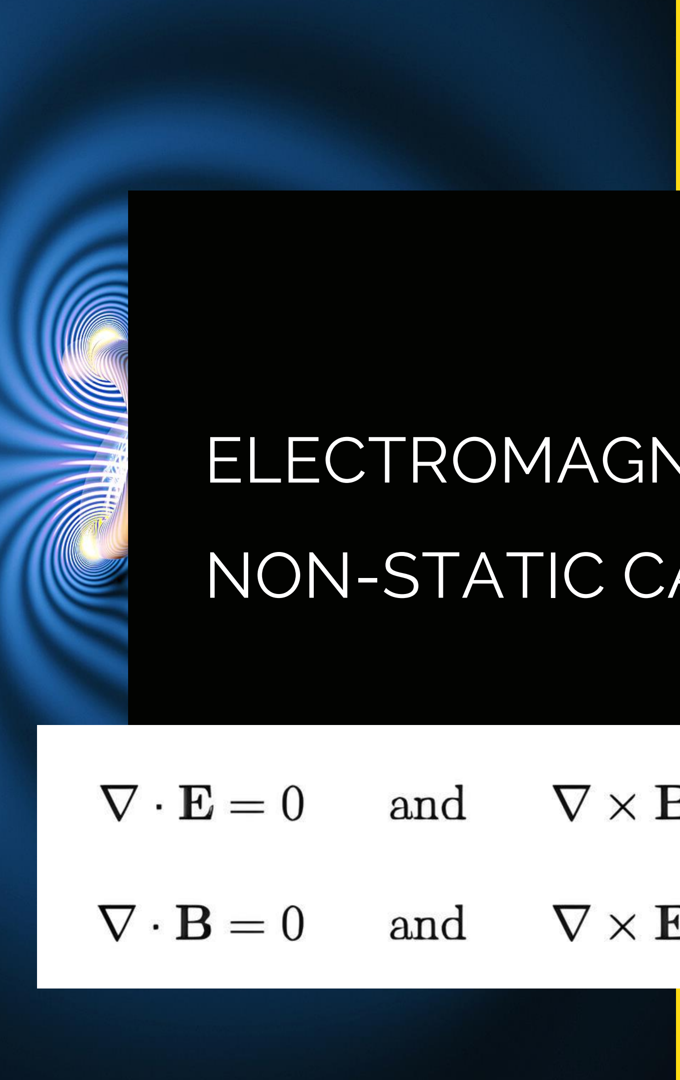
Secondary effect: When a current flows in C, it will create its own magnetic field. This induced magnetic field will always be in the direction that opposes the change. This is called **Lenz's law.**



FARADAY'S LAW OF INDUCTION

- **Induction Acceleration**
  - Betatrons
  - Induction Linacs
- **Beam Diagnostics**
  - Beam Current Monitors
  - Beam Position Monitors

- **Energy Loss and Compensation**
  - Synchrotron Radiation
- **Other Applications**
  - Eddy Current Effects
  - Magnetic Field Measurements



ELECTROMAGNETISM: NON-STATIC CASE

$$\nabla \cdot \mathbf{E} = 0 \qquad \text{and} \qquad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \text{and} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- FARADAY'S LAW OF INDUCTION
- **WAVE FUNCTION**
- PROPAGATION OF ELECTROMAGNETIC WAVES

WAVE FUNCTION

$$\nabla \cdot \mathbf{E} = 0 \quad \text{and} \quad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{and} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

ELECTRIC FIELD

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0$$

The wave equation

MAGNETIC FIELD

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

$$c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

SPEED OF LIGHT

AND THERE WAS LIGHT

The velocity of transverse undulations in our hypothetical medium, calculated from the electro-magnetic experiments of MM. Kohlrausch and Weber, agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau, that we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.

James Clerk Maxwell

WAVE FUNCTION

$\left\|\vec{k}\right\| = \frac{2\pi}{\lambda} = \frac{\omega}{c}$	wave-number vector
$\lambda = \frac{c}{f}$	wave length
f	frequency
$\omega = 2\pi f$	angular frequency

$$\mathbf{E} = \mathbf{E}_0\, e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \quad \text{and} \quad \mathbf{B} = \mathbf{B}_0\, e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$$

k – the wave-number vector with **|k|** = **k**, which gives the direction of propagation of the wave.

ω is more properly called the angular frequency **(f –** frequency)

$$\omega^2 = c^2 k^2$$

dispersion relation

$$c = \frac{\omega}{|k|} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

E0, B0 – constant vectors, the amplitude of the wave

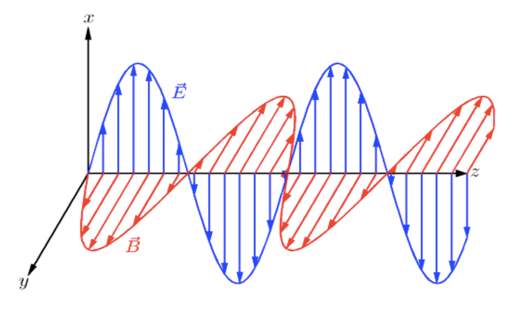
λ = **2π/k** - the wavelength of the wave

Short wavelength →high frequency → high energy

WAVE FUNCTION. KEY CONSTRAINS

Transverse Waves:

$$\boldsymbol{E}_0 \times \boldsymbol{k} = 0$$
$$\boldsymbol{B}_0 \times \boldsymbol{k} = 0$$



Orthogonal Fields:

$$\boldsymbol{E}_0 \times \boldsymbol{B}_0 = 0$$

Magnitude Relationship:

$$|\mathbf{B}_0| = |\mathbf{E}_0|/c$$

Direction Relationship:

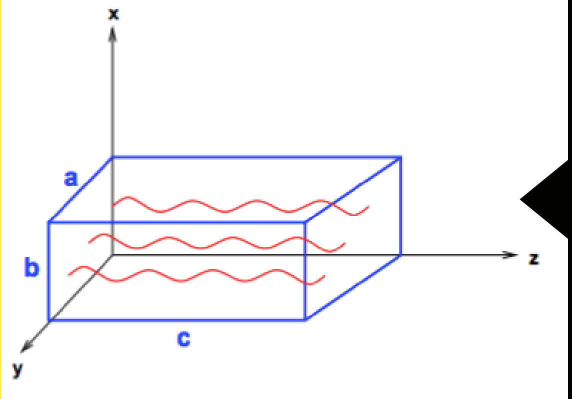
$$\boldsymbol{k} \times \boldsymbol{B}_0 = \omega \boldsymbol{B}_0$$

ELECTROMAGNETISM: NON-STATIC CASE

- FARADAY'S LAW OF INDUCTION
- WAVE FUNCTION
- **PROPAGATION OF ELECTROMAGNETIC WAVES**

PROPAGATION OF ELECTROMAGNETIC WAVES IN A CONDUCTOR

- **ATTENUATION:** The wave's amplitude decreases exponentially as it penetrates the conductor
- **SKIN DEPTH:** The wave's energy is concentrated near the surface of the conductor, within a characteristic distance called the skin depth.
- **PHASE SHIFT:** The electric and magnetic fields of the wave experience a phase shift as they propagate through the conductor.
- **DISPERSION:** The wave's velocity and wavelength can change within the conductor, depending on the frequency

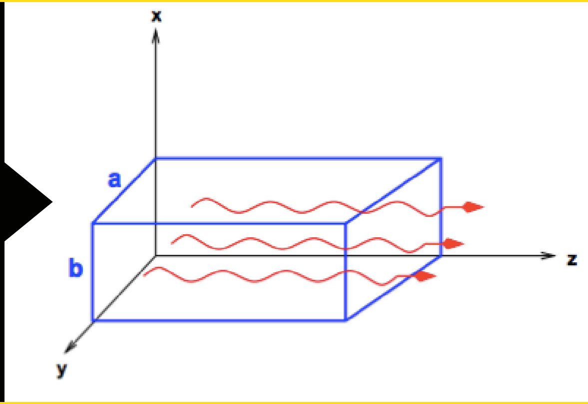


RF CAVITIES

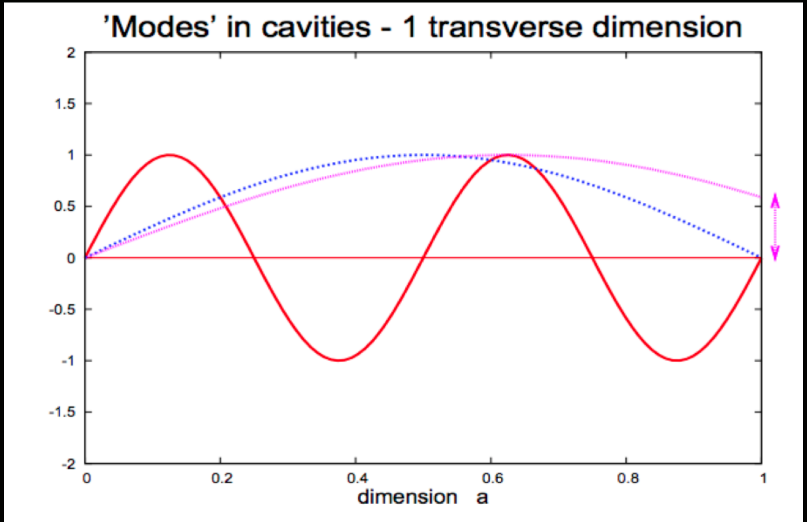
Field can persist and be stored

WAVEGUIDES

Plane waves can propagate along waveguides



EXAMPLE: FIELDS IN RF CAVITIES



No electric field at boundaries, wave must have "nodes" = zero fields at the boundaries

Only modes which 'fit' into the cavity are allowed

In the example: $\frac{\lambda}{2} = \frac{a}{4}$, $\frac{\lambda}{2} = \frac{a}{1}$, $\frac{\lambda}{2} = \frac{a}{0.8}$

(then either "sin" or "cos" is 0)

Consequences: RF cavities

Field must be zero at conductor boundary, only possible if:

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

and for k_x, k_y, k_z we can write, (then they all fit):

$$k_x = \frac{m_x \pi}{a}, \qquad k_y = \frac{m_y \pi}{b}, \qquad k_z = \frac{m_z \pi}{c},$$

The integer numbers m_x, m_y, m_z are called mode numbers, important for design of cavity !

→ half wave length $\lambda/2$ must always fit exactly the size of the cavity.

(For cylindrical cavities: use cylindrical coordinates)

Consequences: wave guides

Similar considerations as for cavities, no field at boundary.

We must satisfy again the condition:

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

This leads to modes like (no boundaries in direction of propagation z):

$$k_x = \frac{m_x \pi}{a}, \qquad k_y = \frac{m_y \pi}{b},$$

In z direction: No Boundary - No Boundary Condition ...

Consequences: wave guides

Re-writing the condition as:

$$k_z^2 = \frac{\omega^2}{c^2} - k_x^2 - k_y^2 \quad \rightarrow \quad k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}$$

Propagation without losses requires k_z to be real, i.e.:

$$\frac{\omega^2}{c^2} > k_x^2 + k_y^2 = (\frac{m_x \pi}{a})^2 + (\frac{m_y \pi}{b})^2$$

which defines a cut-off frequency ω_c. For lowest order mode:

$$\omega_c = \frac{\pi \cdot c}{a}$$

- Above cut-off frequency: propagation without loss
- At cut-off frequency: standing wave
- Below cut-off frequency: attenuated wave (it does not "fit in").

There is a very easy way to show that very high frequencies easily propagate

RECAP:

ELECTROMAGNETISM:

NON-STATIC CASE

- **Faraday's Law of Induction**

The electromotive force around a closed path is equal to the negative of the time rate of change of the magnetic flux enclosed by the path.

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

- **Wave Function**

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0 \qquad \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0 \qquad c = \frac{\omega}{|k|} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\mathbf{E} = \mathbf{E}_0\, e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} \quad \text{and} \quad \mathbf{B} = \mathbf{B}_0\, e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$$

- **Propagation of electromagnetic waves**

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

THE END

Laboratory of High Energy Physics Data Analysis
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Thank you for your attention!

I would like to thank my colleagues who gave the EM course previously (and I could profit from it while preparing the lecture (A. Latina, A. Wolski, P. Skowronski, W. Herr)

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