CAS SCHOOL, SEP 2024

INTRODUCTION TO SPECIAL RELATIVITY

Dr. Irina Shreyber, PhD

Laboratory of High Energy Physics Data Analysis
Tomsk State University

Every day example:

plane, airport, bad weather

MATERIALS

CAS Website

These slides and the video will be available the CAS school website

Books

- Jurgen Freund, "Special Relativity For Beginners"
- James H. Smith, "Introduction to Special Relativity"
- Mario Conte, William W. MacKay, "An Introduction to the Physics of Particle Accelerators"

IN THIS LECTURE, WE WILL LEARN…

THE TRANSITION

- in thinking that led from **Galilean Relativity** to the **Special Theory of Relativity** in 1905

THE POSTULATES

- of **Special Relativity**, which are the basis of the mathematics of the framework.

THE CONSEQUENCES

- of the postulates and **application to accelerators physics**

NEWTON'S PRINCIPLE OF RELATIVITY

GALILEO GALILEI IN 1632, AND LATER BY NEWTON

"The motions of bodies included in a given space are the same among themselves, whether that space is at rest or moves uniformly forward in a straight line."

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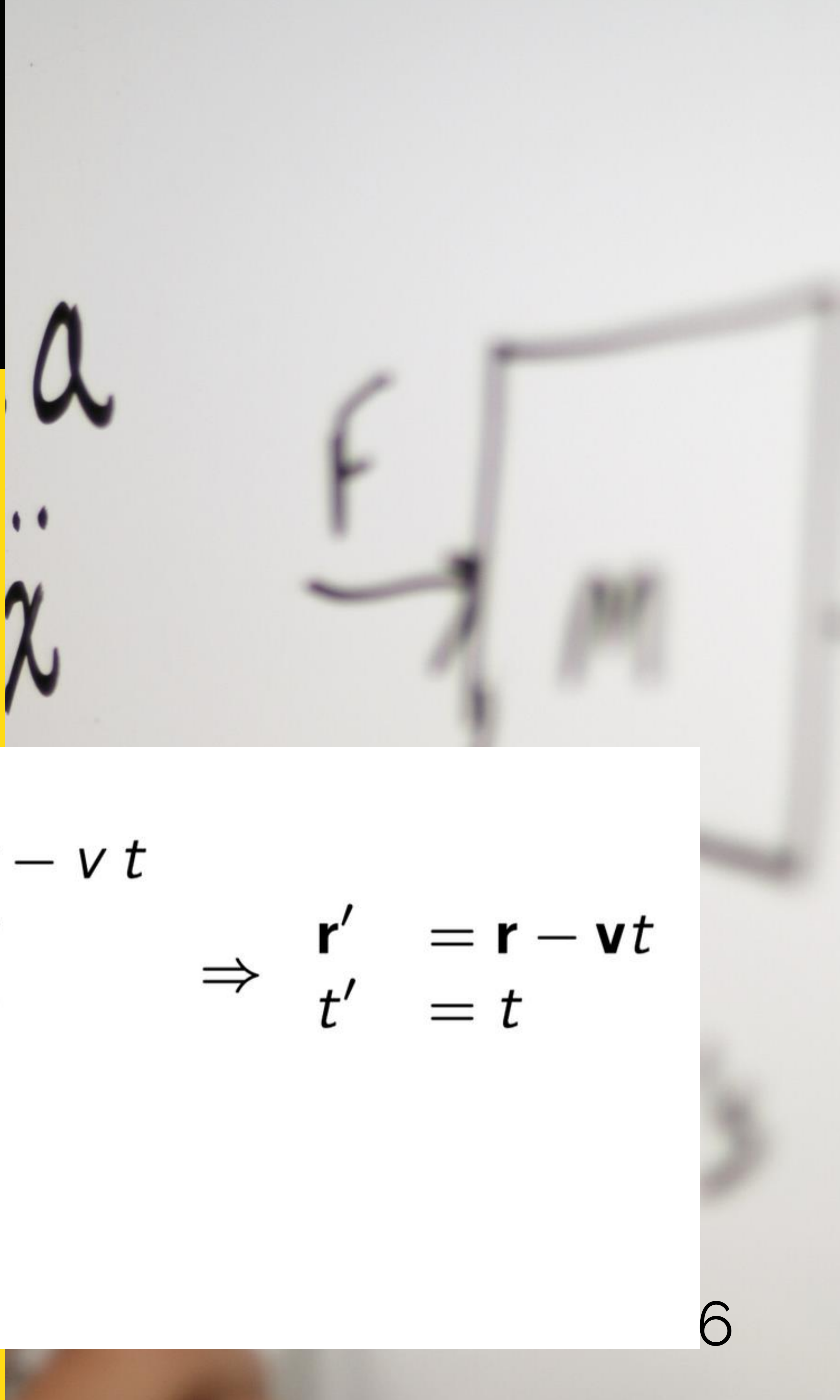
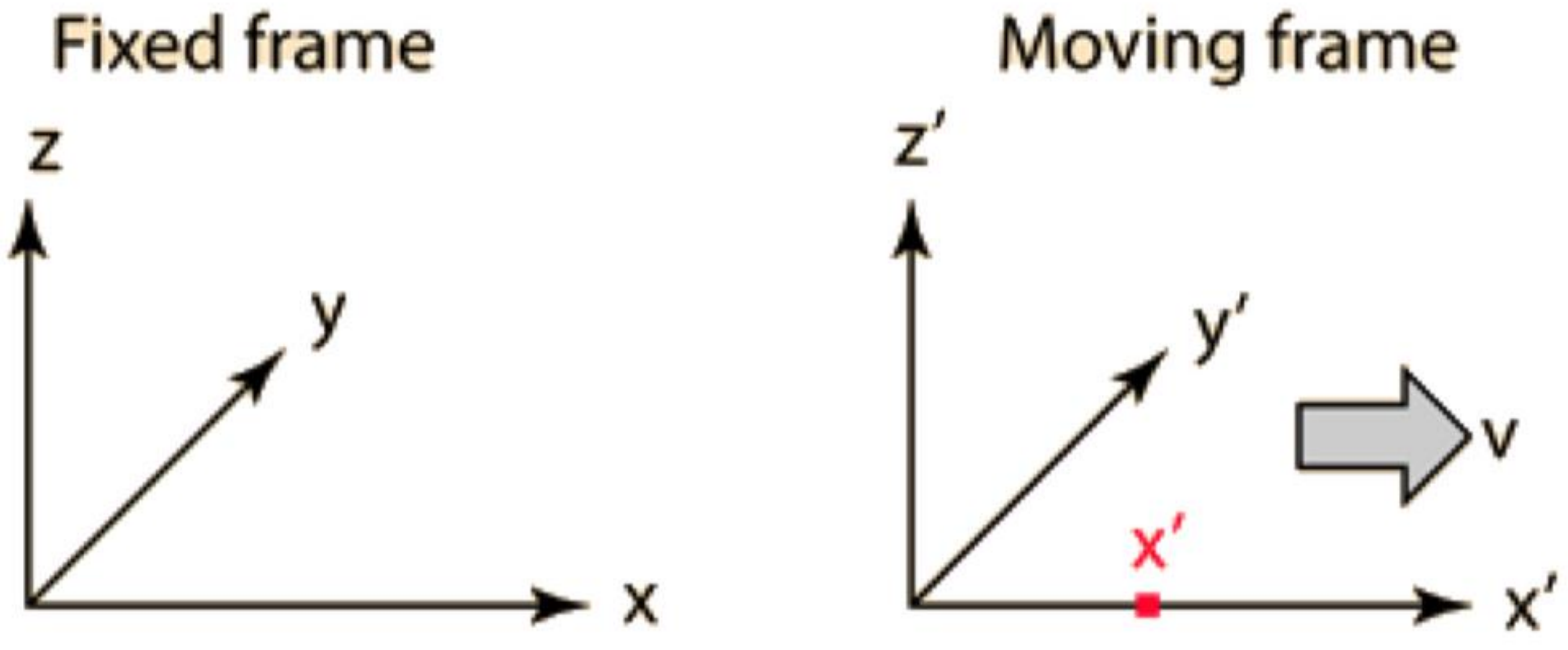
GALILEAN TRANSFORMATION



At the time of Newton the relation of the coordinates between two systems in motion with relative velocity v, was defined by the Galilean transformation of motion



$$
\begin{aligned}
x' &= x - v\,t \\
y' &= y \\
z' &= z \\
t' &= t
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
\mathbf{r}' &= \mathbf{r} - \mathbf{v}t \\
t' &= t
\end{aligned}
$$

with $\mathbf{r} = (x, \quad y, \quad z)$.

MAXWELL EQUATIONS

DIFFERENTIAL FORM

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

GAUSS'S LAW FOR E

$$\nabla \cdot \mathbf{B} = 0$$

GAUSS'S LAW FOR **B**

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

FARADAY'S LAW
for time-varying magnetic fields

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

AMPERE(-MAXWELL) LAW
for time-varying electric fields

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THE PROBLEM WITH GALILEAN TRANSFORMATION

- $$\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\right)\Psi = 0$$ Maxwell wave equation
- $$x = x' - vt,\ \ y' = y,\ \ z' = z,\ \ t' = t$$ Galilean transformation
- $$\left(\left[1-\frac{v^2}{c^2}\right]\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{2v}{c^2}\frac{\partial^2}{\partial x \partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi = 0$$ Maxwell wave equation

MAXWELL EQUATIONS: CONSEQUENCES

IF THERE IS A DISTURBANCE IN THE FIELD SUCH THAT LIGHT IS GENERATED, THESE ELECTROMAGNETIC WAVES GO OUT IN ALL DIRECTIONS EQUALLY AND AT THE SAME SPEED:

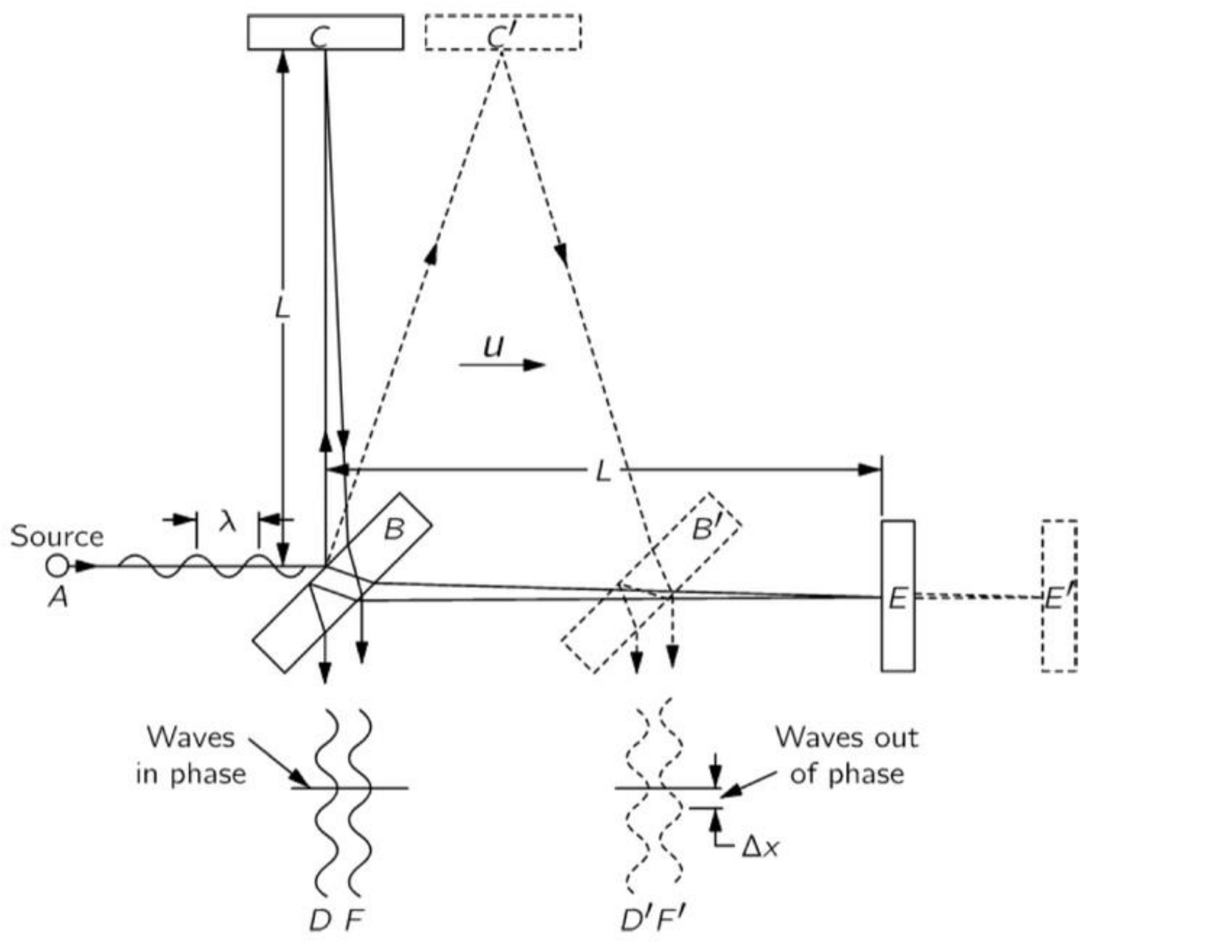
c = 299 792 458 m/s in vacuum ("celeritas" = speed)

IF THE SOURCE OF THE DISTURBANCE IS MOVING, THE LIGHT EMITTED GOES THROUGH SPACE AT THE **SAME SPEED C**.

This is analogous to the case of sound, the speed of sound waves being likewise independent of the motion of the source.

MICHELSON-MORLEY EXPERIMENT (1887)

The goal was to determine the absolute velocity of the earth through this hypothetical "ether":



A: light source; B: partially silvered glass plate; C and E: mirrors; D and F: superimposed light beams

$$B \to E: \quad ct_1 = L + ut_1 \Rightarrow t_1 = L/(c-u)$$
$$E \to B: \quad ct_2 = L - ut_2 \Rightarrow t_2 = L/(c+u)$$

$$t_1 + t_2 = \frac{2L}{(1 - u^2/c^2)}$$

$$B \to C: \quad (ct_3)^2 = L^2 + (ut_3)^2 \Rightarrow t_3 = L/\sqrt{c^2 - u^2}$$
$$C \to B: \quad t_4 = t_3$$

If there is an "ether drift" then

$$t_1 + t_2 \neq t_3 + t_4$$

The apparatus was amply sensitive to observe such an effect, but no time difference was found — the velocity of the earth through the aether could not be detected. The result of the experiment was null.

THE LESSONS OF THE MICHELSON-MORLEY EXPERIMENT

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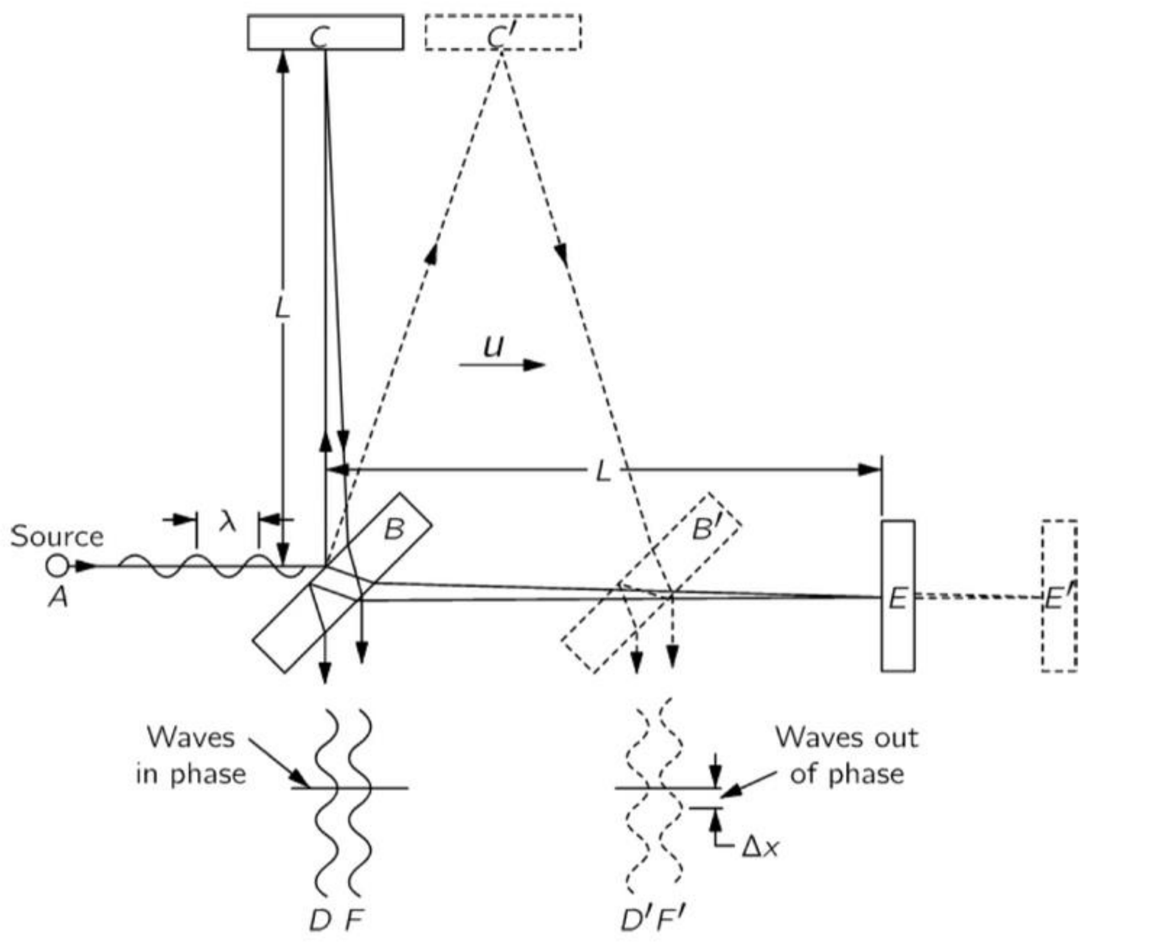

A: light source; B: partially silvered glass plate; C and E: mirrors; D and F: superimposed light beams

Light travels at a fixed and constant speed in any medium, regardless of the relative velocity of the light-source and the light-observer →

THIS IS UNLIKE ANY OTHER PHENOMENON DESCRIBED IN MECHANICS, AND IMPLIES THAT NEWTON'S MECHANICS IS INCOMPLETE.

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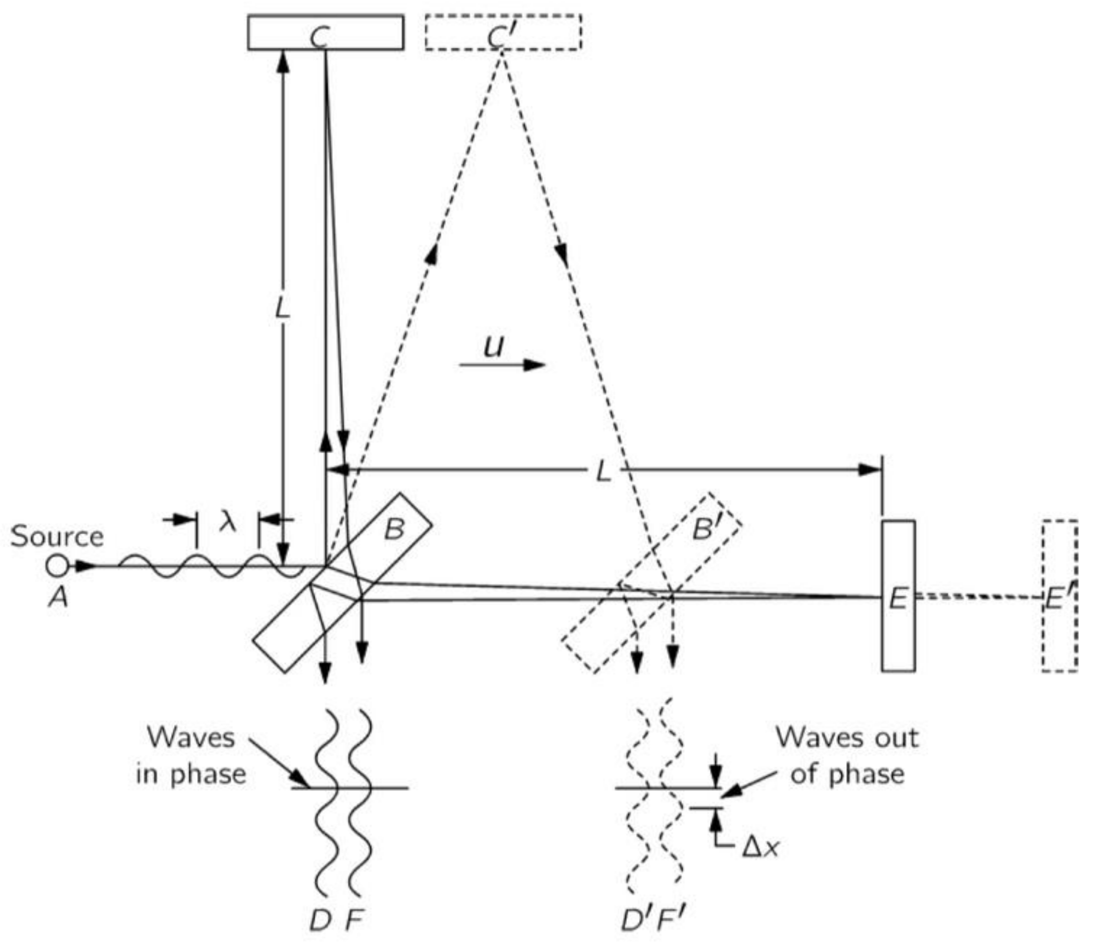
THE LESSONS OF THE MICHELSON-MORLEY EXPERIMENT

The goal was to determine the absolute velocity of the earth through this hypothetical "ether":



A: light source; B: partially silvered glass plate; C and E: mirrors; D and F: superimposed light beams

No medium is required for light to propagate; unlike a mechanical oscillatory phenomenon (wave), to exist light requires no medium to be distorted →

THIS IMPLIES MAXWELL'S EQUATIONS ARE COMPLETE

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THE LESSONS OF THE MICHELSON-MORLEY EXPERIMENT

These lessons would not be absorbed fully until 1905, when Albert Einstein published the definitive papers explaining how to reconcile mechanics, electricity and magnetism, and the Michelson-Morley experiment

"If the Michelson–Morley experiment had not brought us into serious embarrassment, no one would have regarded the relativity theory as a (halfway) redemption."

Einstein

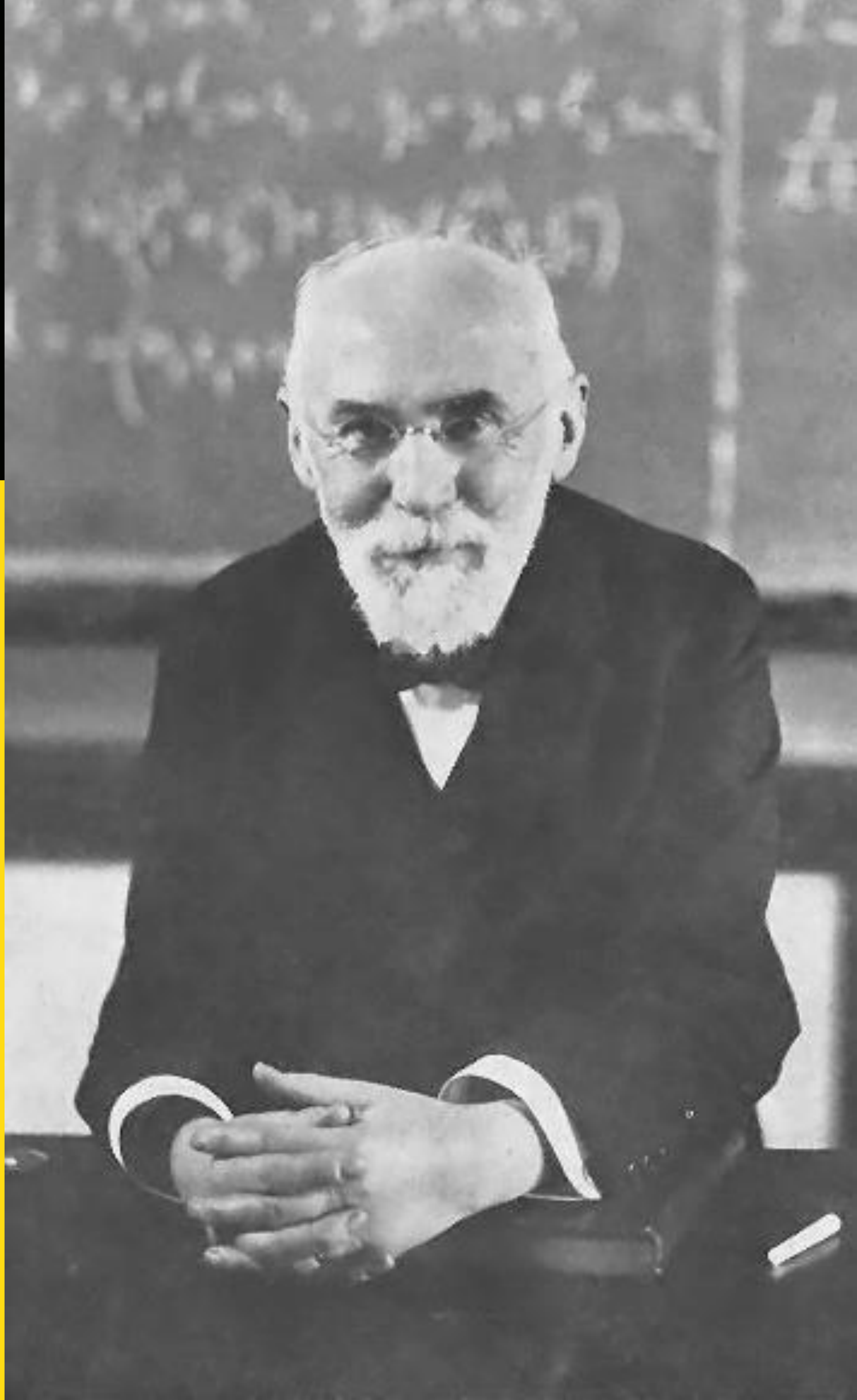
"COMPRESSION OF BODIES IN THE ETHER"

HENDRIK LORENTZ

replacement for the Galilean Relativity equations

THE EFFECTS OF THE AETHER ON BODIES IN MOTION

- ·Mechanical **bodies would compress** along the direction of motion in the ether, with a precise mathematical description for the process
- In transforming observations from the ether frame to other frames of reference, he would conceive of an **alteration of time** that also had a mathematical description



LORENTZ TRANSFORMATION

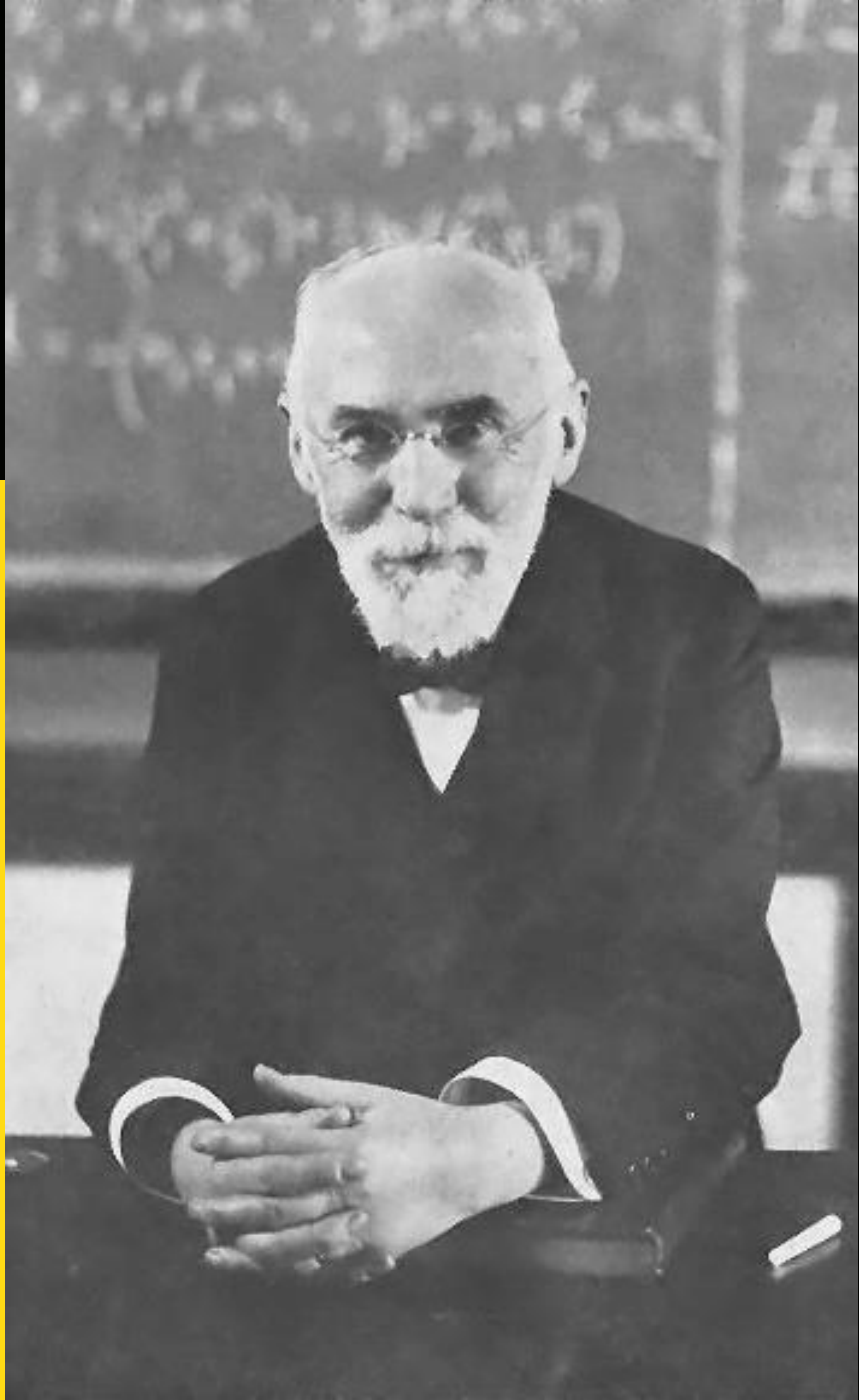
$$L_{\parallel} = L_0 \sqrt{1 - v^2/c^2}$$

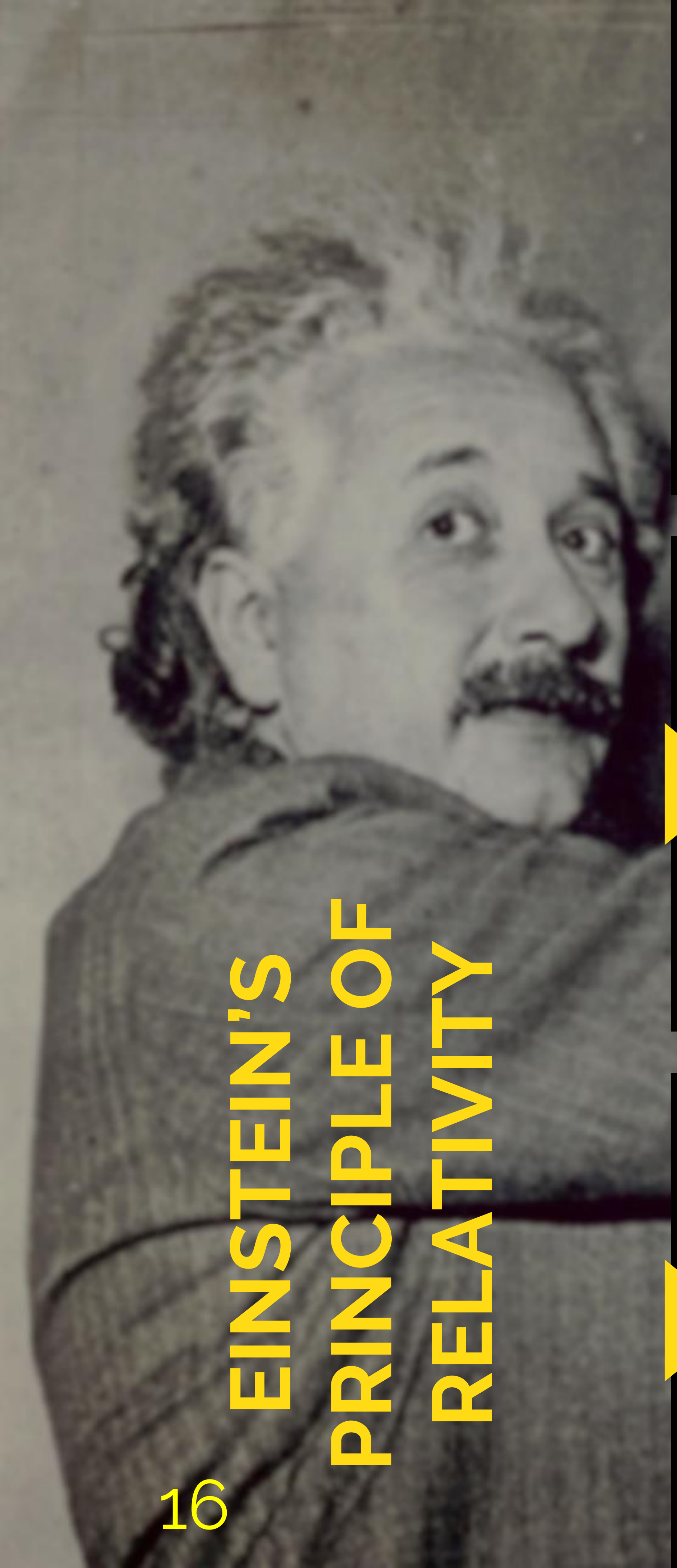
$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$





EINSTEIN'S PRINCIPLE OF RELATIVITY

IN ALBERT EINSTEIN'S ORIGINAL TREATMENT, IN 1905, THE PRINCIPLE OF RELATIVITY IS BASED ON **TWO POSTULATES**:

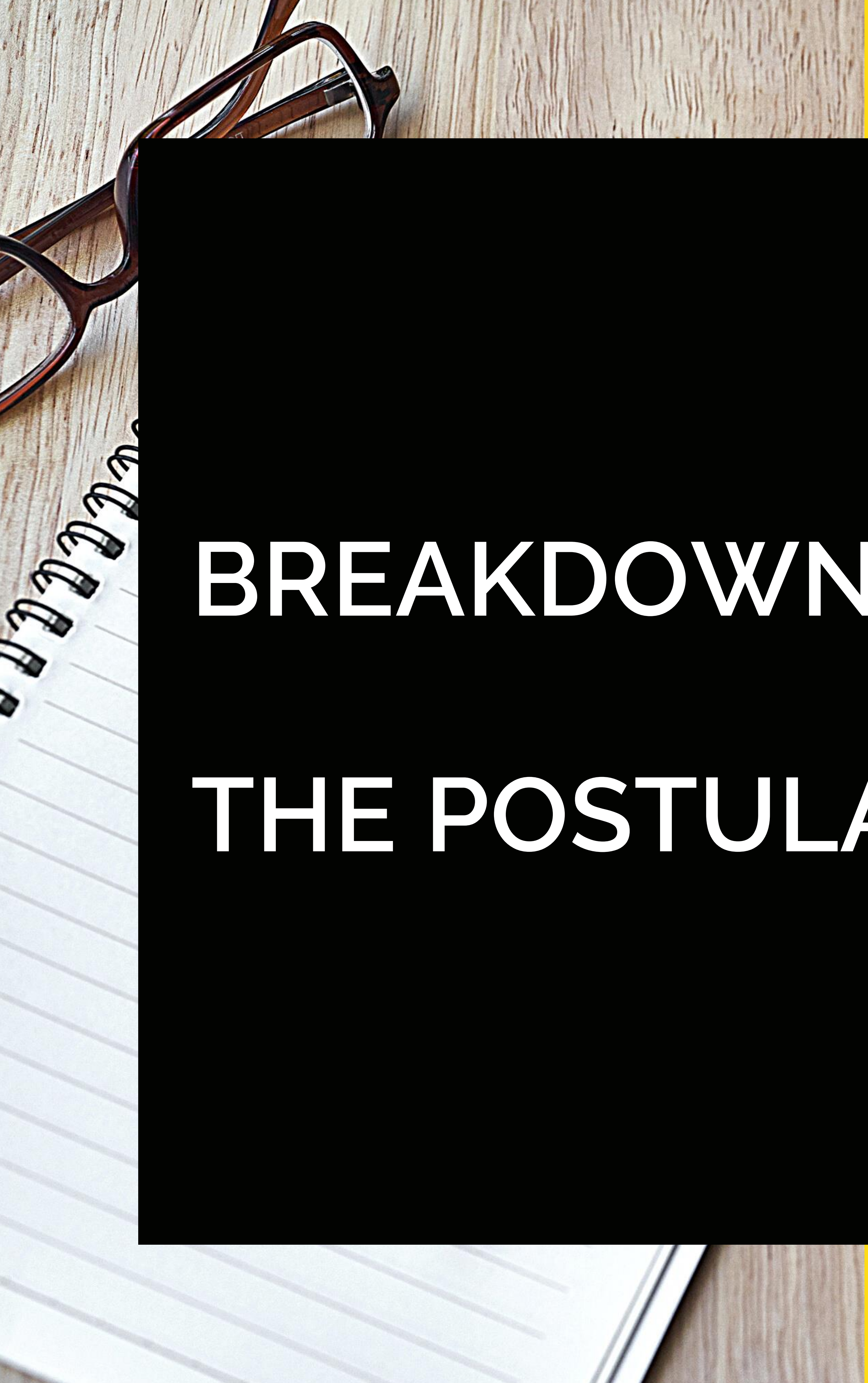
1. SPECIAL PRINCIPLE OF RELATIVITY:

The laws of physics are invariant (i.e. identical) in all inertial frames of reference (i.e. non-accelerating frames of reference).

2. INVARIANCE OF C:

The speed of light in a vacuum is the same for all observers, regardless of the motion of the light source or observer.

1905, Albert Einstein, "On the Electrodynamics of Moving Bodies".

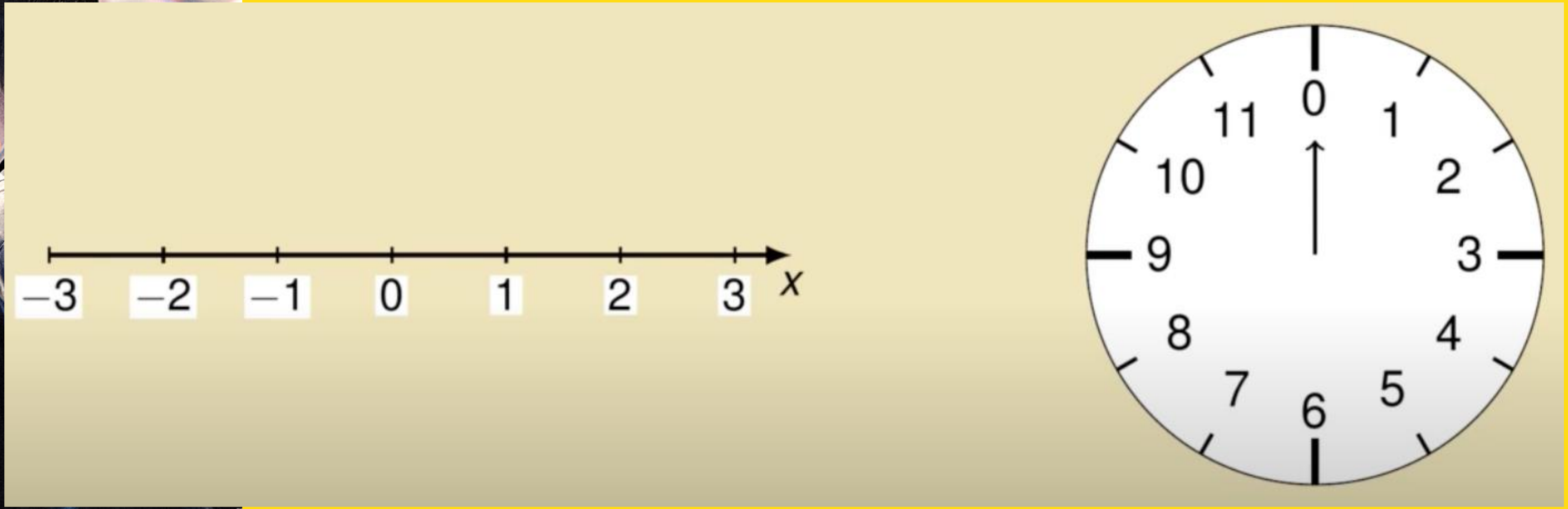


BREAKDOWN THE POSTULATES

- "EVENT"
- "FRAME OF REFERENCE"
- "SIMULTANEITY"
- "SPEED OF LIGHT"

"EVENT"

IS ANYTHING WITH A LOCATION IN SPACE AND TIME



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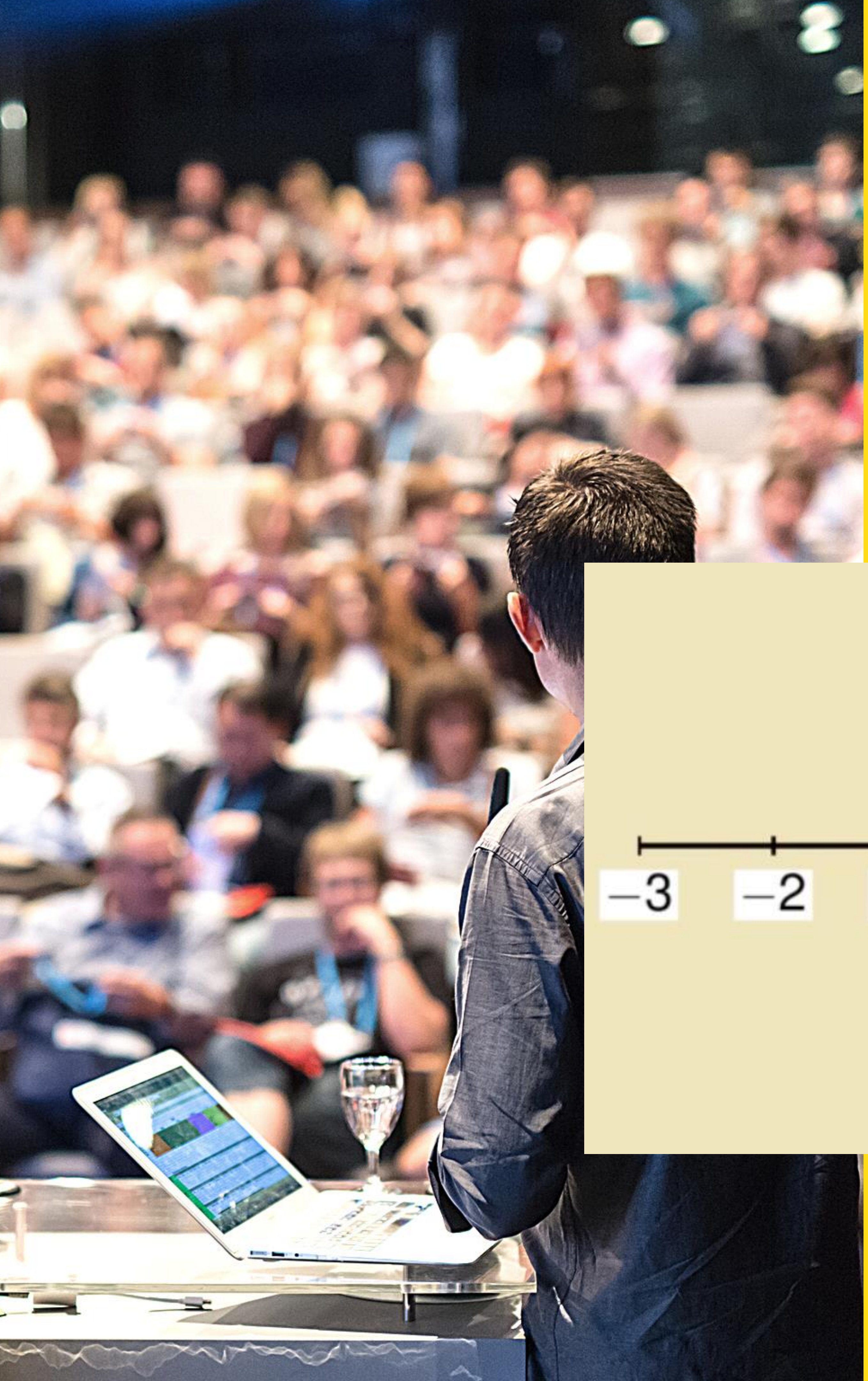

"EVENT"

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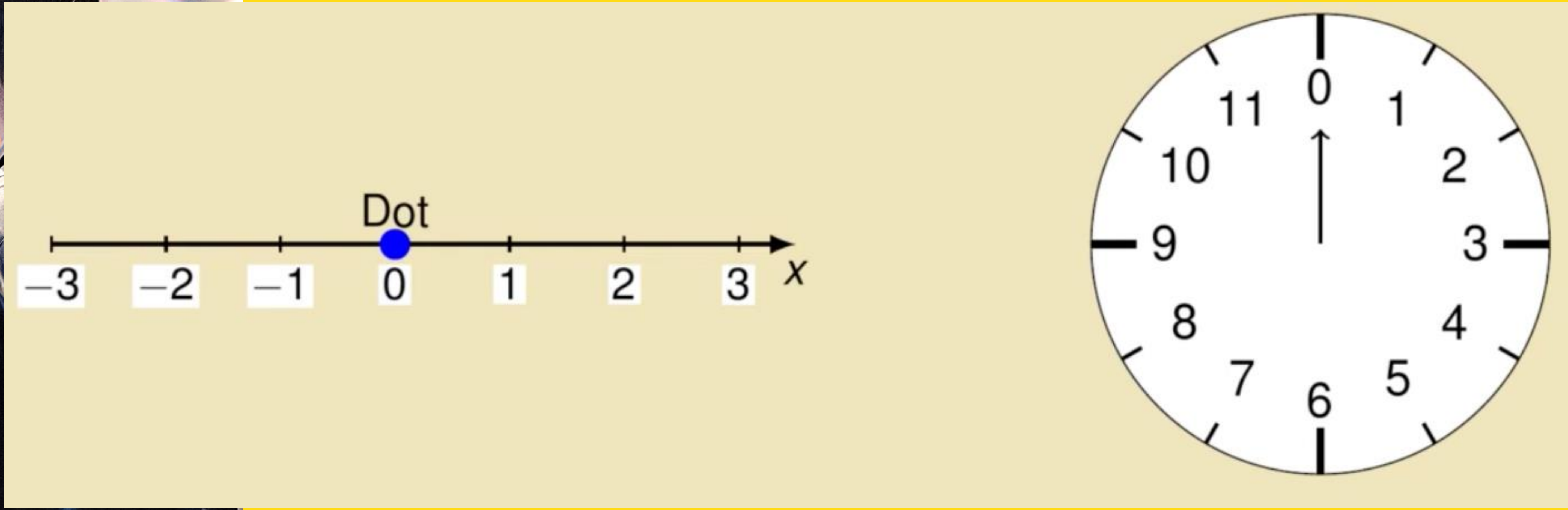

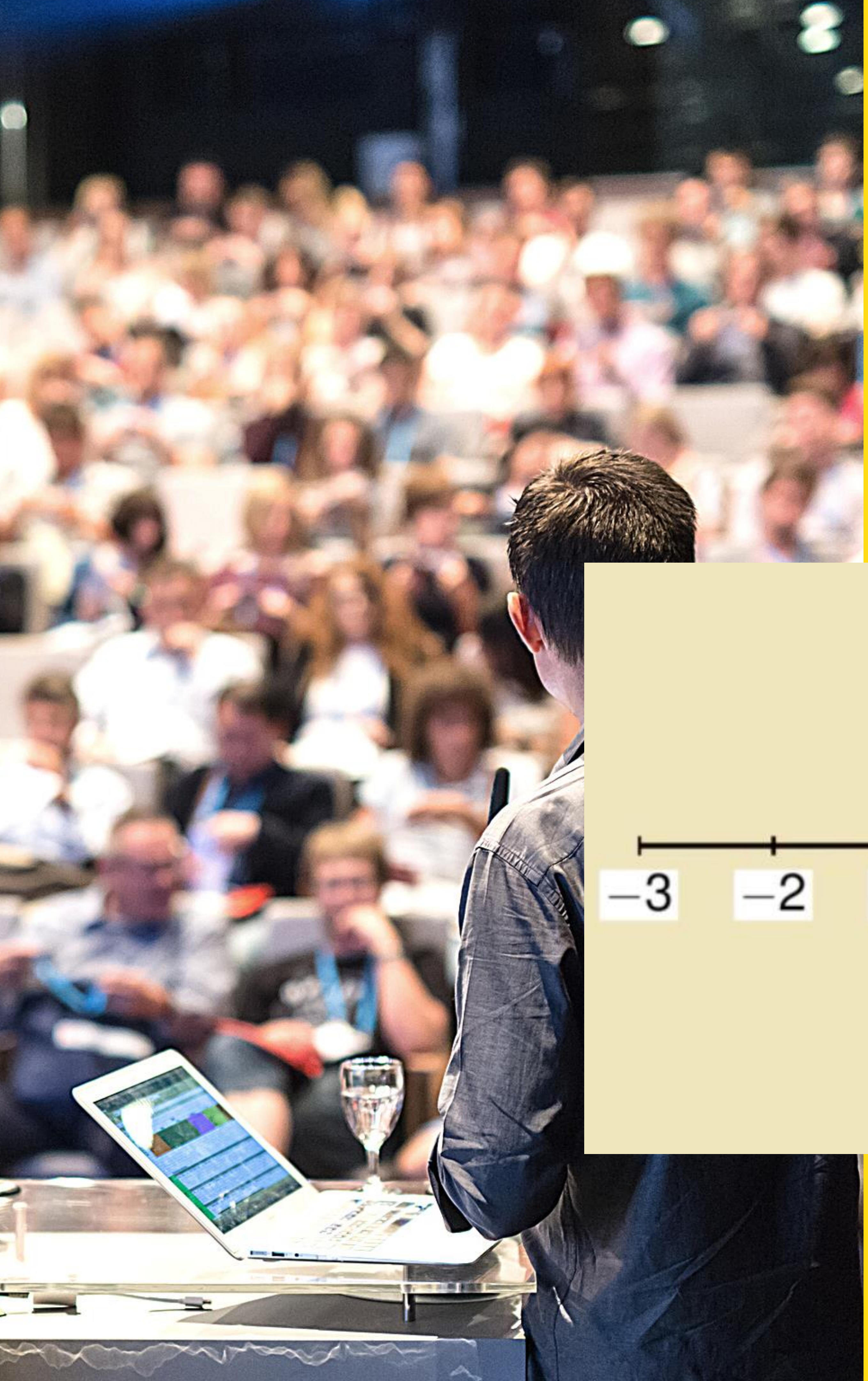

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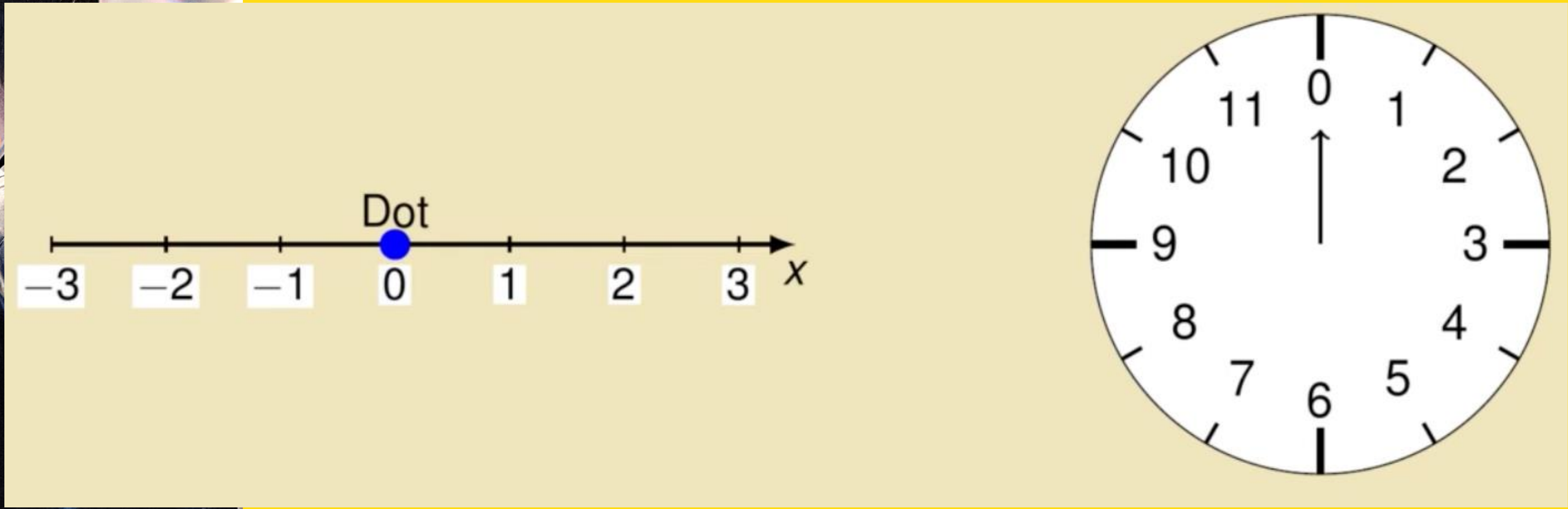
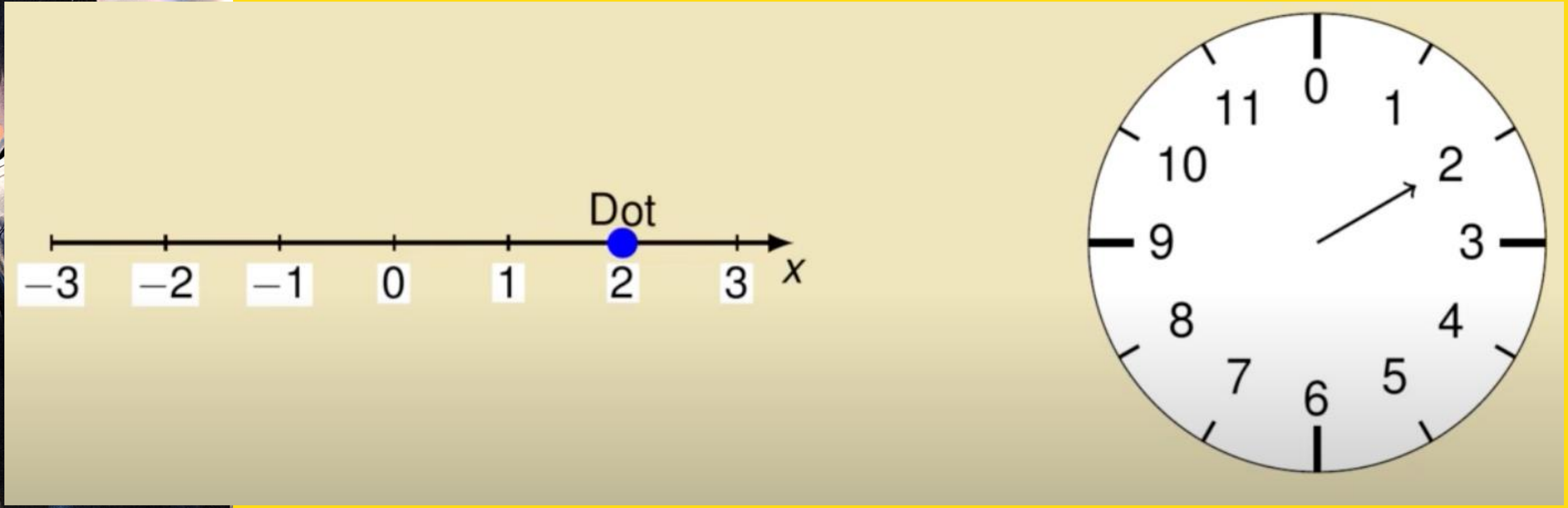


The dot is at position x=0 m ath time t=0 s

"EVENT"

IS ANYTHING WITH A LOCATION IN SPACE AND TIME



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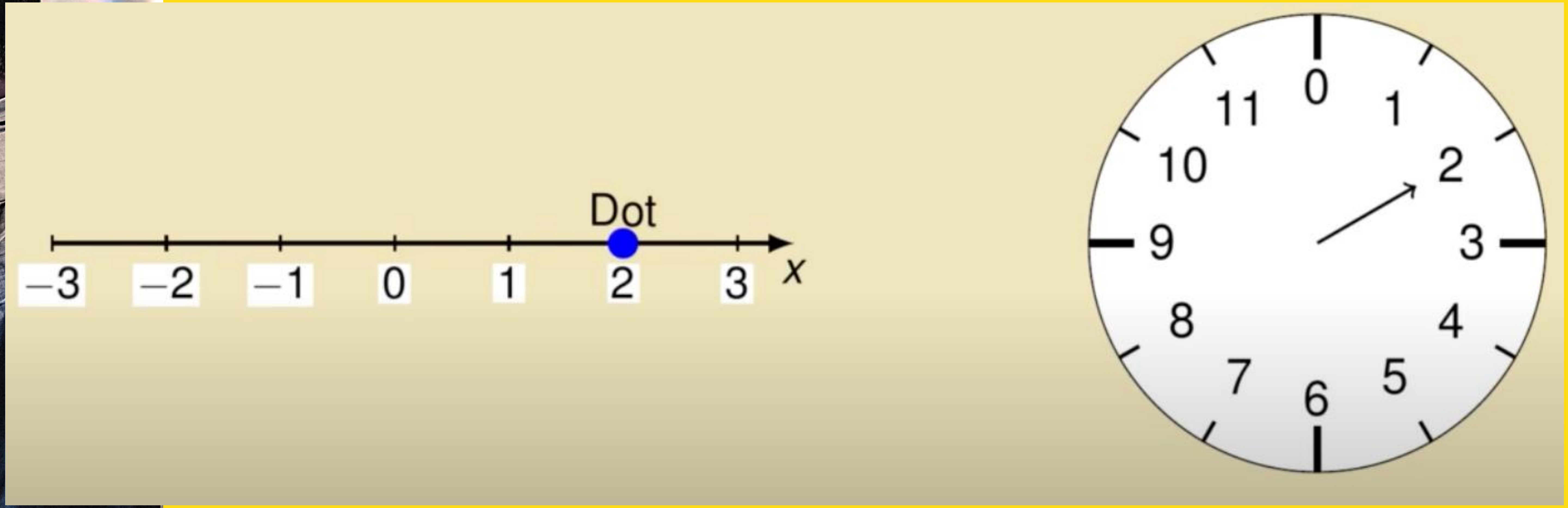
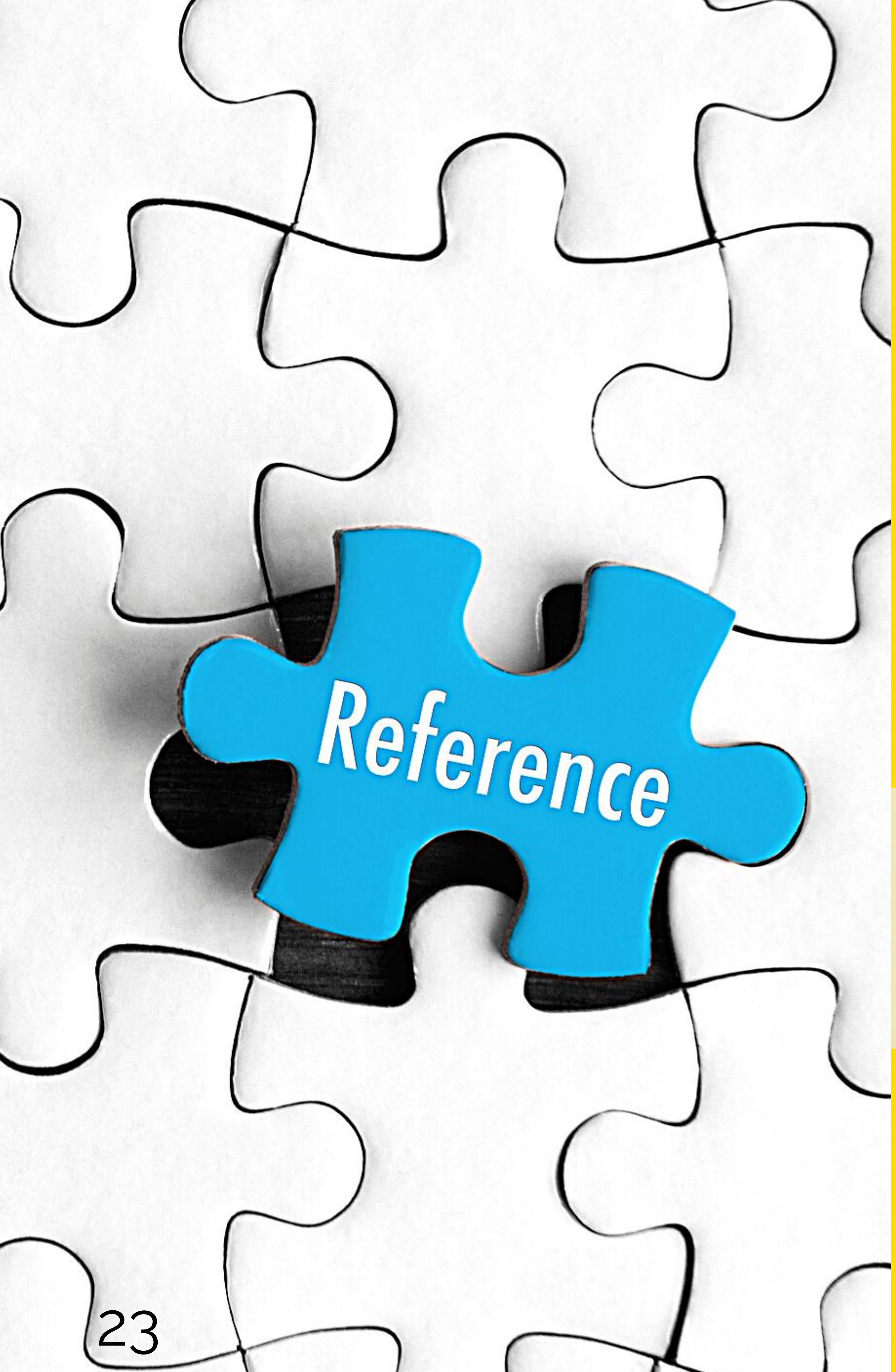
"EVENT"

IS ANYTHING WITH A LOCATION IN SPACE AND TIME

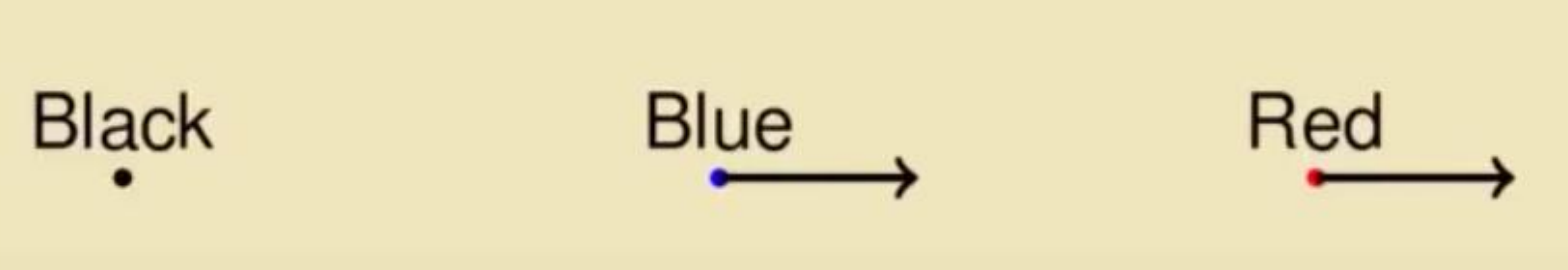


The dot is at position x=2m ath time t=2s



"FRAME OF REFERENCE"

IS ANY OBJECT OR SYSTEM ALL OF WHOSE PARTS MOVE AT THE SAME VELOCITY WITH RESPECT TO AN AGREED-UPON REFERENCE POINT IN SPACE.



Do the red dot and blue dot share the same or different frames of reference?

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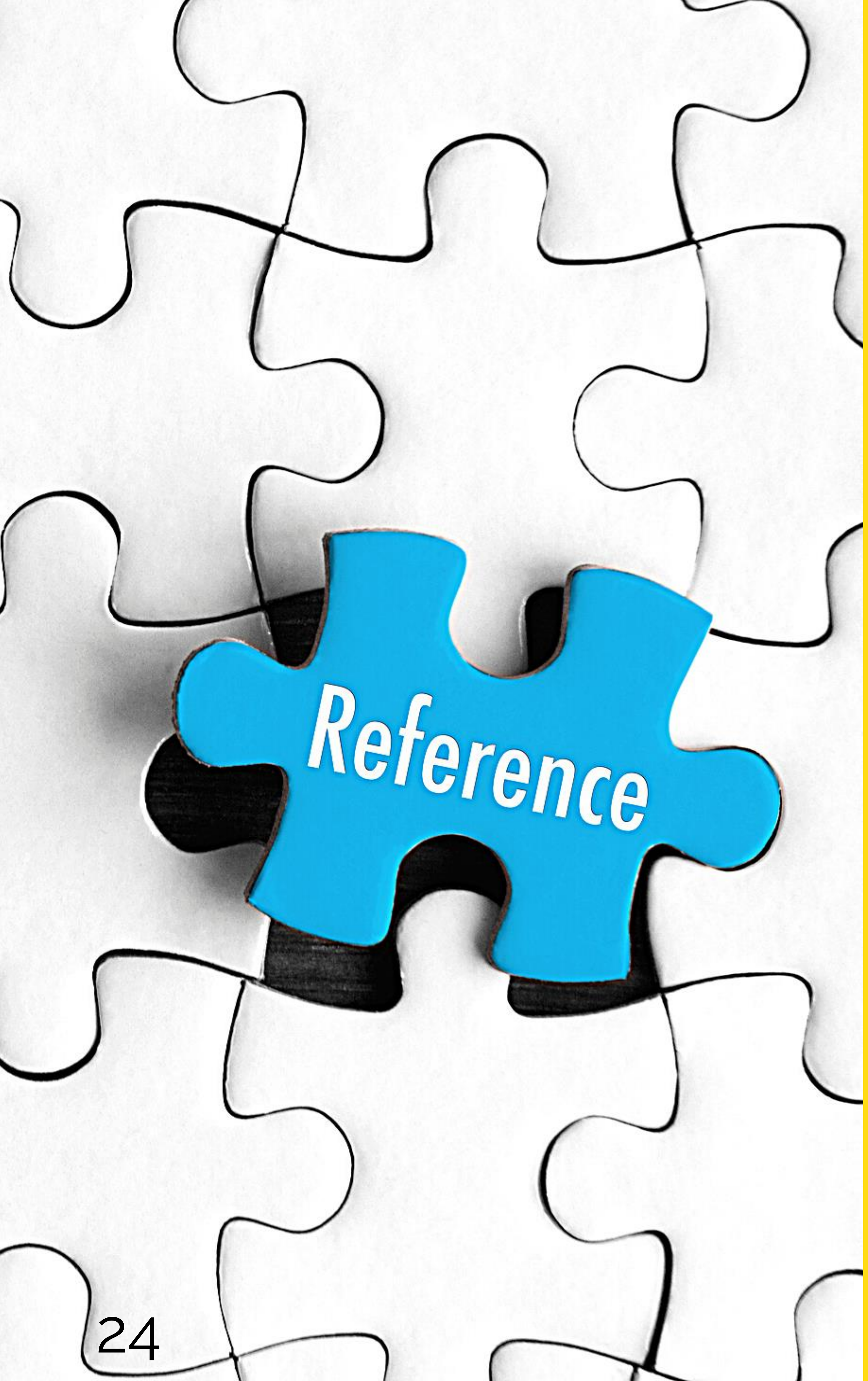

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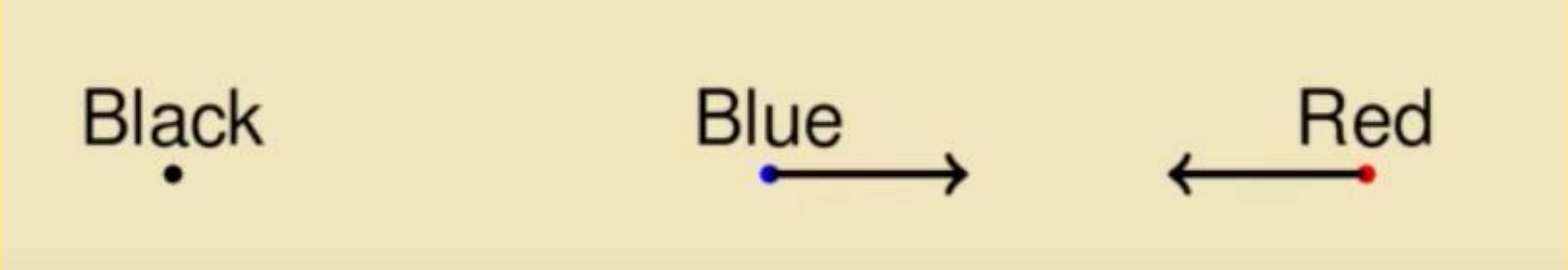

Do the red dot and blue dot share the same or different frames of reference NOW?

"SIMULTANEITY"

TWO EVENTS (OR MORE) ARE SAID TO BE SIMULTANEOUS (THAT IS, TO POSSESS OF SIMULTANEITY), IF THEY ARE OBSERVED TO OCCUR AT THE SAME MOMENT IN TIME

Think really hard about **whether** events are simultaneous, and **for whom (which observers in which frames of reference)** they are simultaneous.

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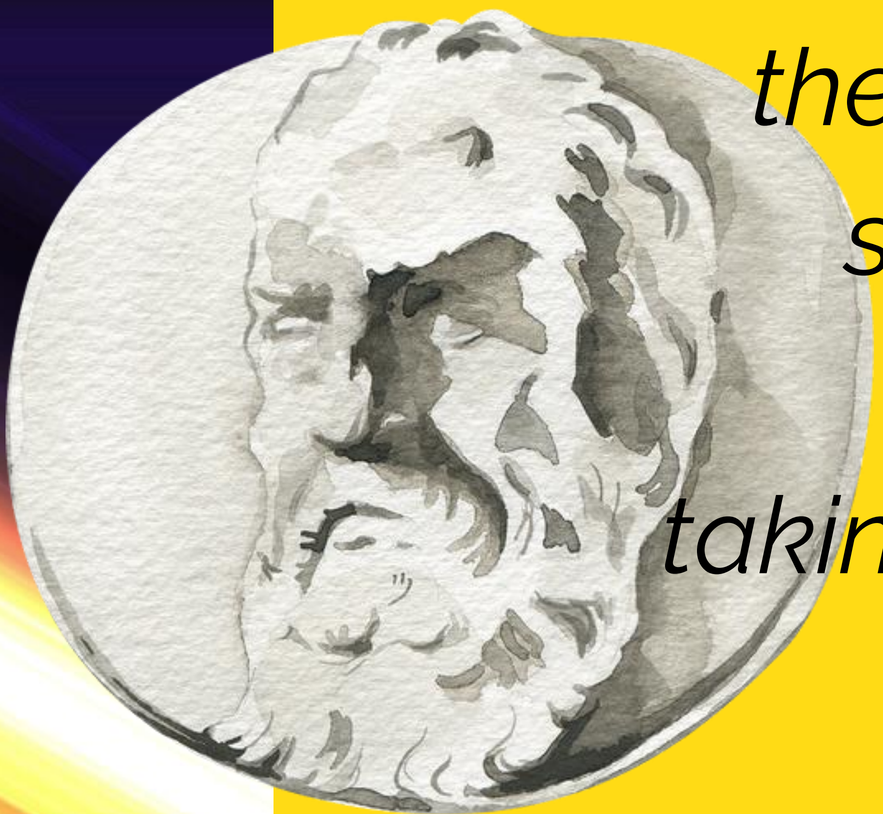
Modern Speed of Light

The speed of light, based on modern definitions of the meter and the second, is defined to be exactly 299, 792, 458m/s. Light travels roughly one foot in one billionth of a second (1ft/ns).

"THE SPEED OF LIGHT"

IT IS THE NUMBER OF METERS LIGHT CAN TRAVEL, ONCE EMITTED BY A SOURCE, IN A CERTAIN AMOUNT OF TIME.

Galileo Galilei: *attempted to measure this by uncovering a lantern, having an assistant on a distant hill who uncovers their lantern upon seeing his, and upon seeing the assistant's lantern light he recorded the time for the round trip, taking into account human reaction time*



LORENTZ TRANSFORMATION

VELOCITY:

$$\beta = \frac{v}{c} \qquad \in [0, 1]$$

LORENTZ FACTOR:

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \qquad \in [1, \infty)$$

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LORENTZ TRANSFORMATION

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

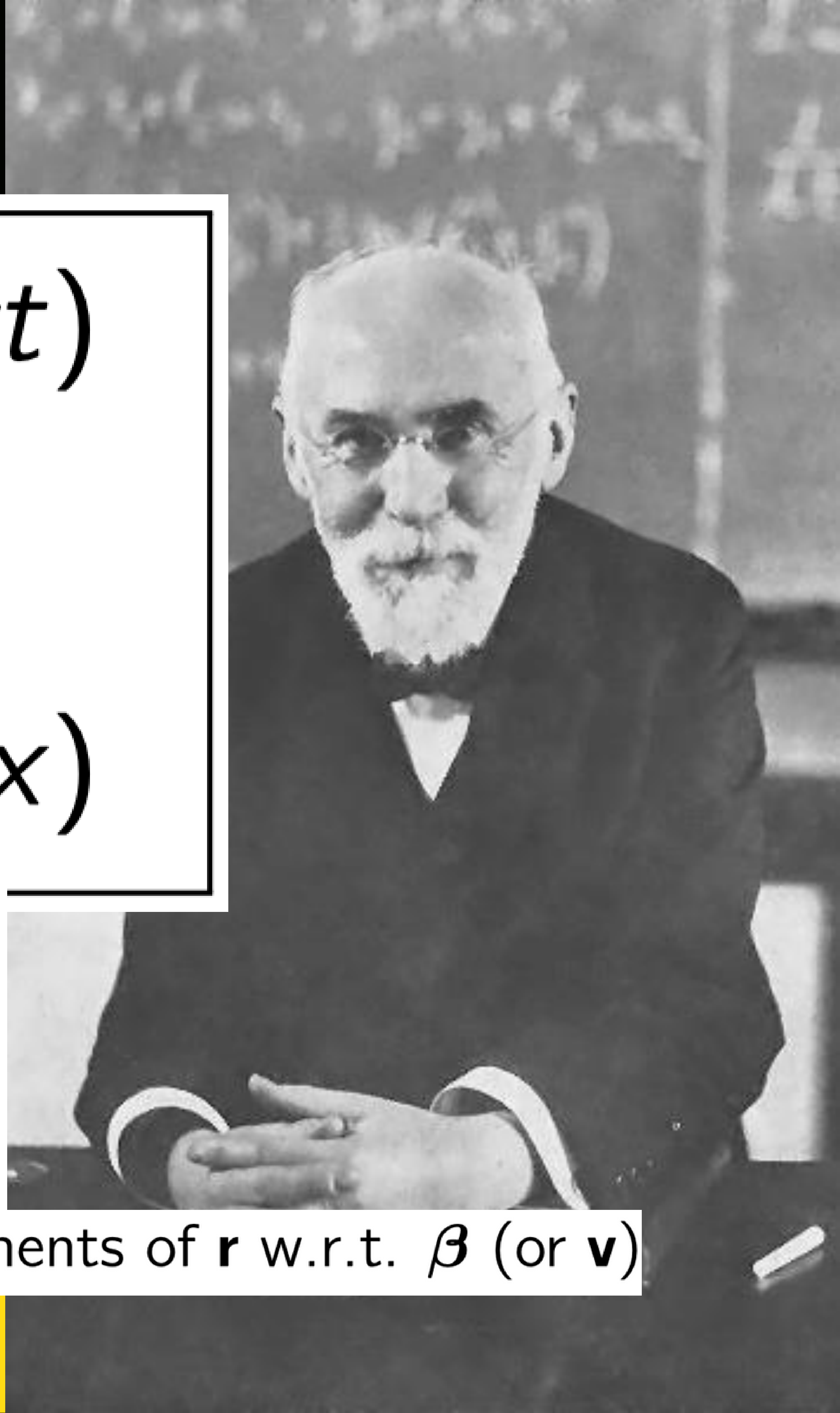
$$z' = z$$

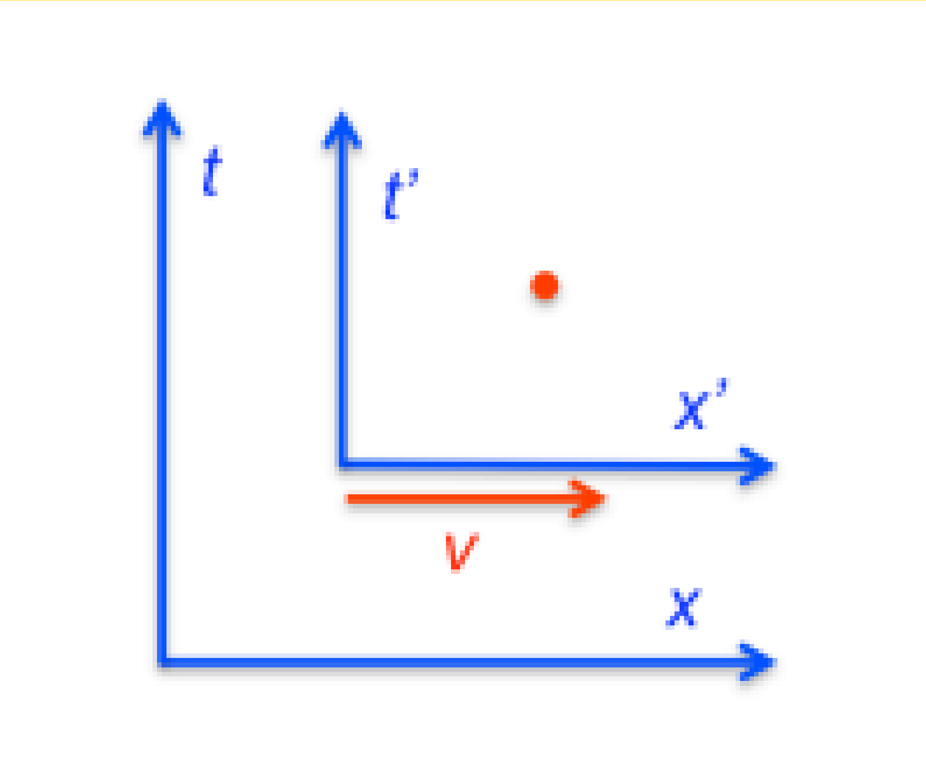
$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\begin{aligned} x' &= \gamma\,(x - \beta ct) \\ y' &= y \\ z' &= z \\ ct' &= \gamma\,(ct - \beta x) \end{aligned}$$

$$\begin{aligned} \mathbf{r}'_{\parallel} &= \gamma\left(\mathbf{r}_{\parallel} - \boldsymbol{\beta} ct\right) \\ \mathbf{r}'_{\perp} &= \mathbf{r}_{\perp} \\ ct' &= \gamma\left(ct - \mathbf{r}_{\parallel} \cdot \boldsymbol{\beta}\right) \end{aligned}$$

where $\mathbf{r}_{\parallel}$ and $\mathbf{r}_{\perp}$ are the components of $\mathbf{r}$ w.r.t. $\boldsymbol{\beta}$ (or $\mathbf{v}$)

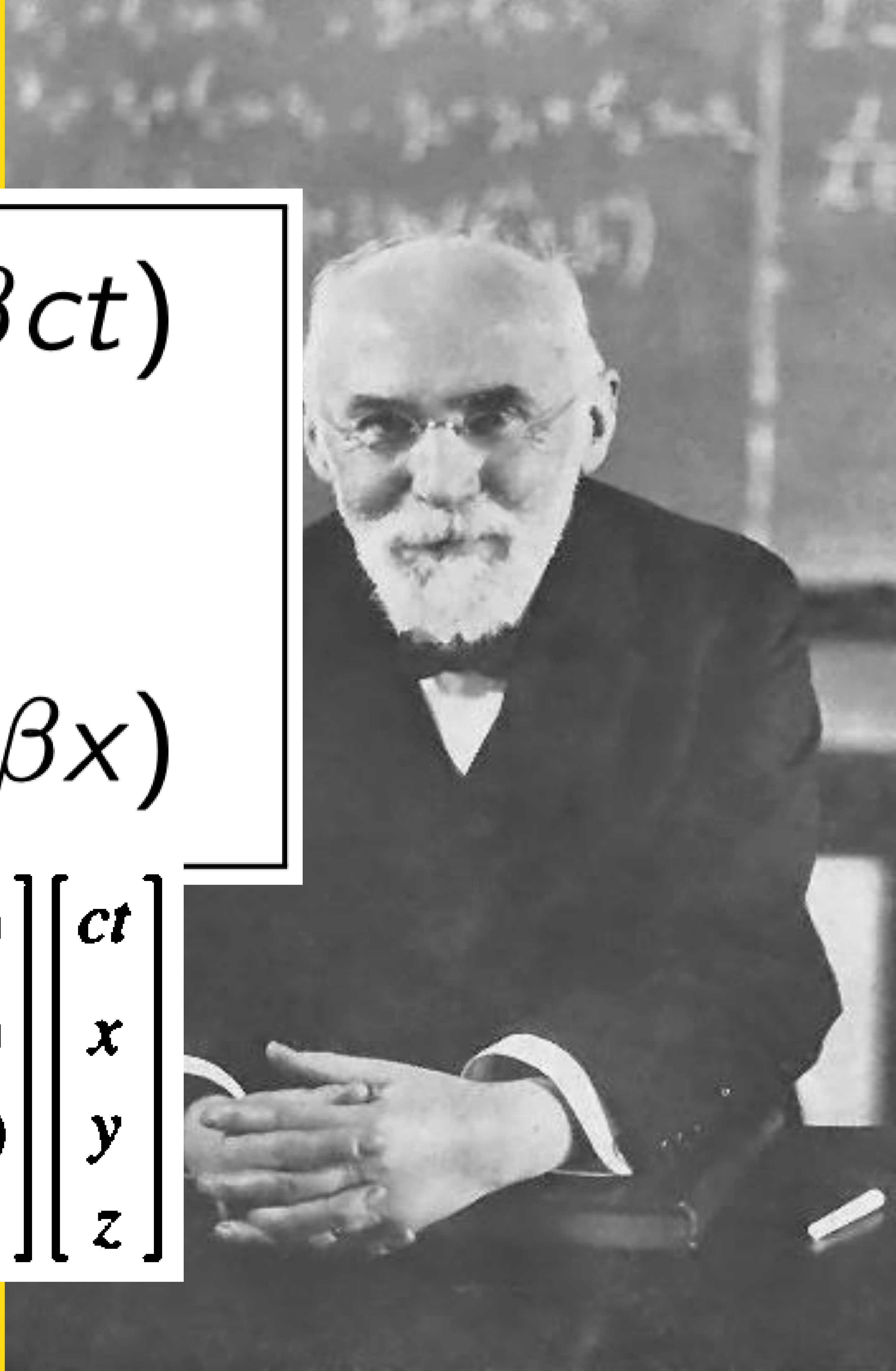




$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$
$$ct' = \gamma(ct - \beta x)$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$
$$y' = y$$
$$z' = z$$
$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

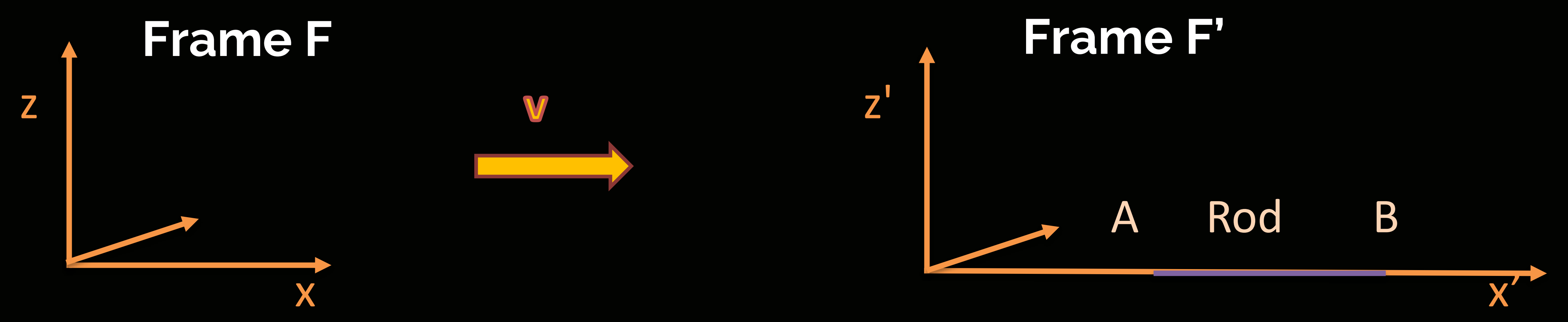


Lorentz contraction, is the solution that Lorentz proposed to solve the Michelson-Morley experiment:

is the phenomenon that a moving object's length is measured to be shorter than its proper length, which is the length as measured in the object's own rest frame

$$\Delta x' = \frac{\Delta x}{\gamma}$$

*Rod **AB** of length L' fixed in F' at x'_A, x'_B.*



What is its length measured in F?

Must measure positions of ends in F at the same time t!

From Lorentz:

$$x'_A = \gamma(x_A - vt) \qquad x'_B = \gamma(x_B - vt)$$

$$L' = x'_B - x'_A = \gamma(x_B - x_A) = \boldsymbol{\gamma L > L}$$

Moving objects appear contracted in the direction of the motion

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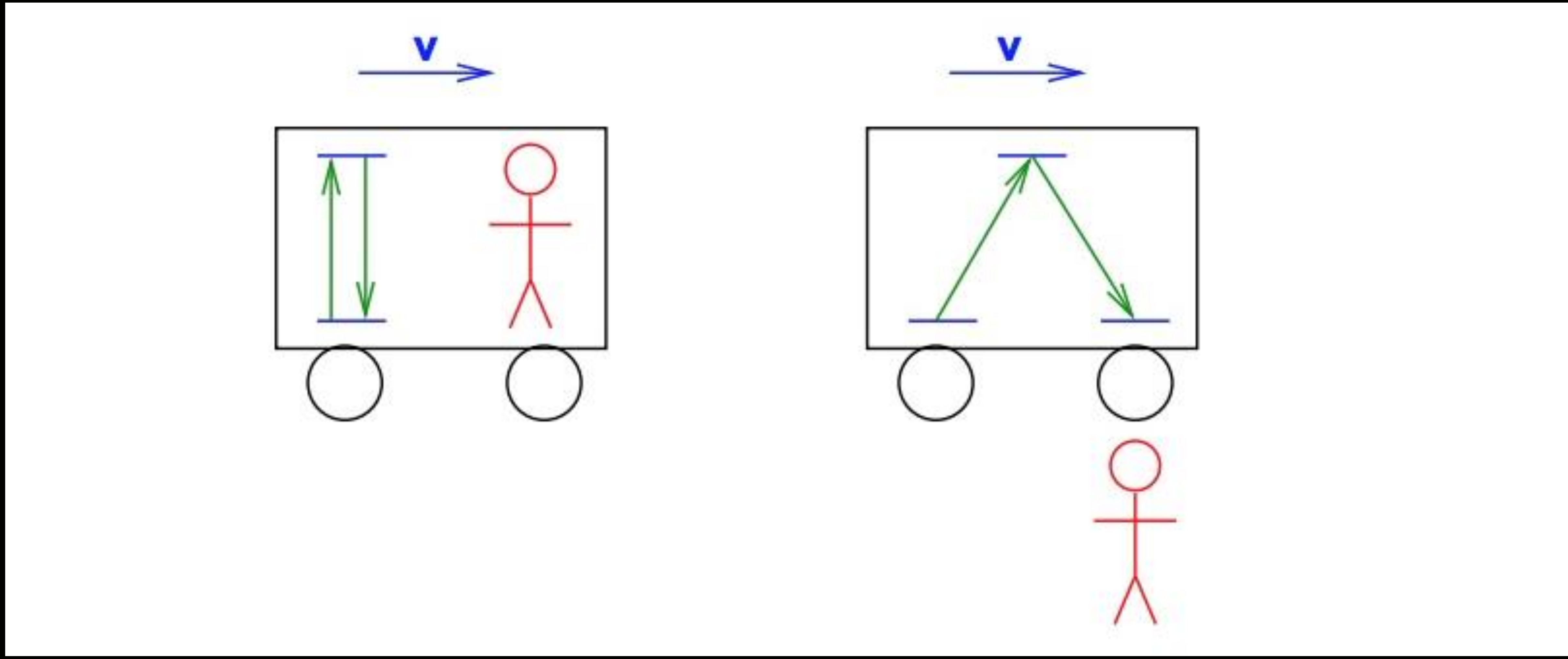
TIME DILATION

- is a difference in the elapsed time measured by two clocks, due to them having a velocity relative to each other

$$\Delta t' = \gamma \Delta t$$

Reflection of light between 2 mirrors seen inside moving frame and from outside



Frame moving with velocity *v*
Seen from outside the path is longer, but *c* must be the same…

Clock in frame F at point with coordinates *(x,y,z)* at different times t_A and t_B

In frame F' moving with speed v, Lorentz transformation gives

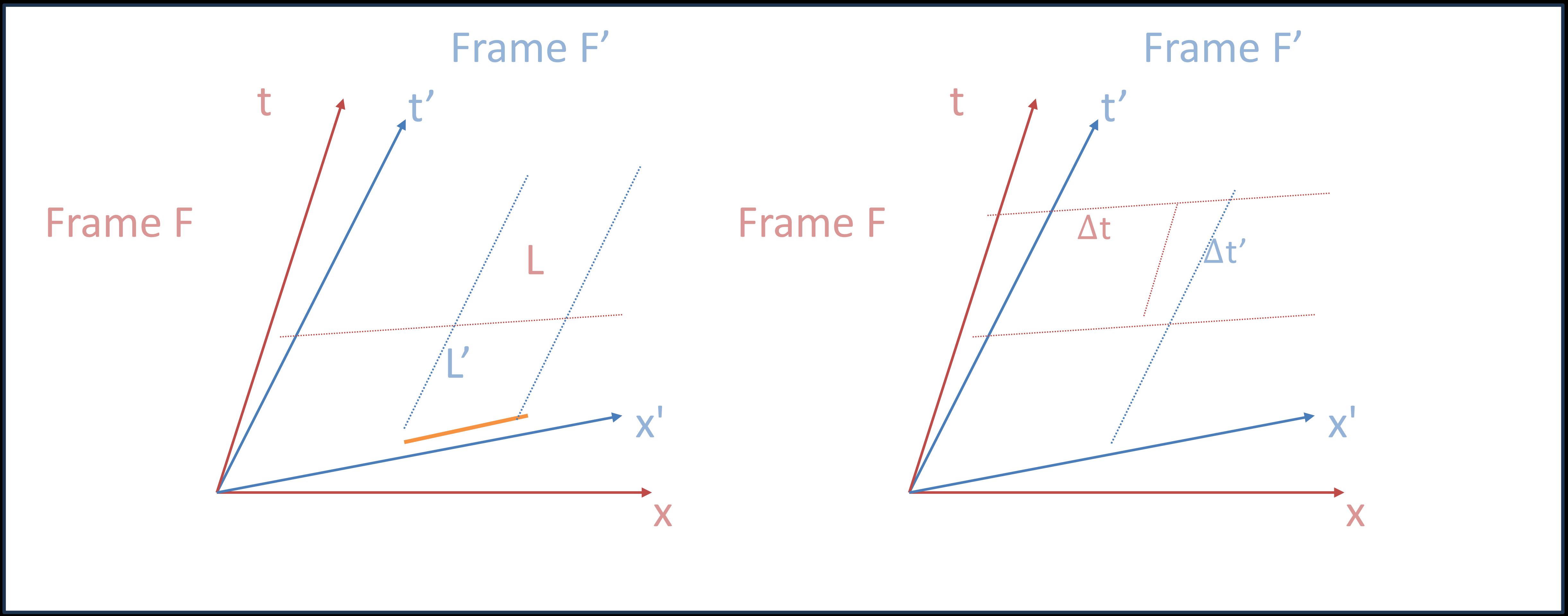
$$t'_A = \gamma\left(t_A - \frac{vx}{c^2}\right) \qquad t'_B = \gamma\left(t_B - \frac{vx}{c^2}\right)$$

$$\Delta t' = t'_B - t'_A = \gamma(t_B - t_A) = \gamma\Delta t > \Delta t$$

Moving clocks appear to run slow

CONSEQUENCES: TIME DILATION

SCHEMATIC REPRESENTATION OF THE LORENTZ TRANSFORMATION



Length contraction L<L'

Rod at rest in F'. Measurement in F at fixed time t, along a line parallel to x-axis

Time dilatation: Δt<Δt'

Clock at rest in F. Time difference in F'
from line parallel to x'-axis

EXAMPLE: ROCKET IN TUNNEL

$$v = \frac{\sqrt{3}}{2}c$$

Tunnel 100m long

Rocket length 100m

- All clocks synchronised.
- Observers X and Y at exit and entrance of tunnel say the rocket is moving, has contracted and has length

$$\frac{100}{\gamma} = 100 \times \left(1 - \frac{v^2}{c^2}\right)^{1/2} = 100 \times \left(1 - \frac{3}{4}\right)^{1/2} = 50m$$

- But the tunnel is moving relative to the ends A and B of the rocket and observers here say the rocket is 100 m in length, but the tunnel has contracted to 50 m

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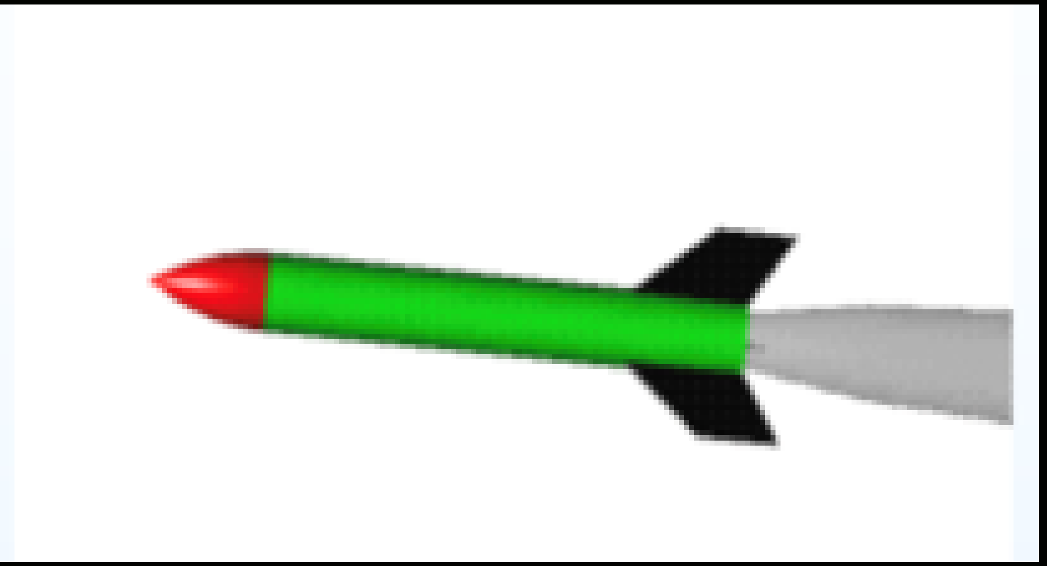


1. If X's clock reads zero as the A exits tunnel, what does Y's clock read when the B goes in?

Moving rocket length 50m, so B has still 50m to travel before his clock reads 0. Hence clock reading is

$$-\frac{50}{v} = -\frac{100}{\sqrt{3}c} \approx -200ns$$

2. What does the B's clock read as he goes in?

To the B, tunnel is only 50m long, so A is 50m past the exit as B goes in. Hence clock reading is

$$+\frac{50}{v} = +\frac{100}{\sqrt{3}c} \approx +200ns$$

3. Where is the B when his clock reads 0?

B's clock reads 0 when A's clock reads 0, which is as A exits the tunnel. To A and B, tunnel is 50m, so B is 50m from the entrance in the rocket's frame, or 100m in tunnel frame.

QUESTIONS

MUON DECAY IN THE ATMOSPHERE

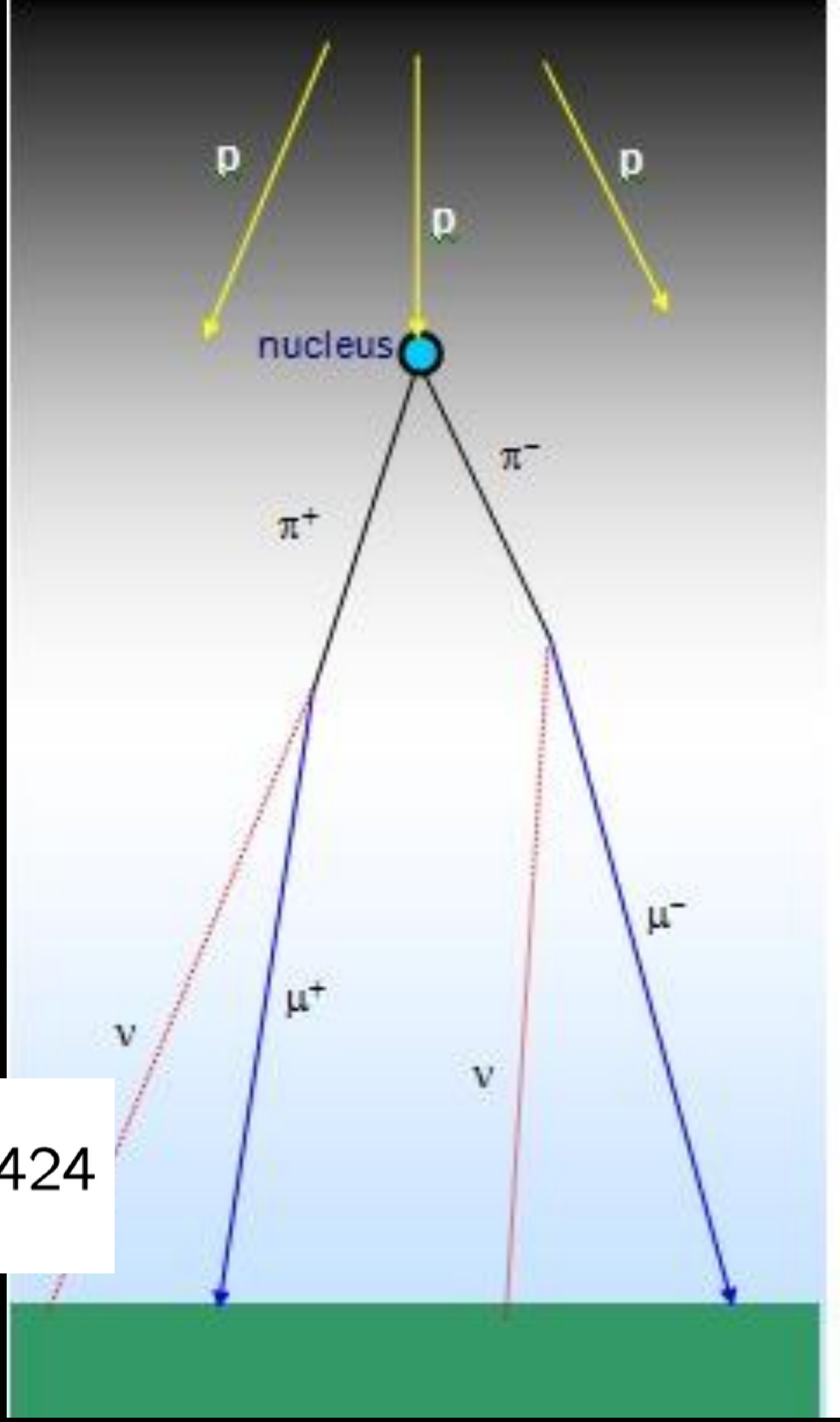
MUONS ARE FORMED IN COLLISIONS OF COSMIC RAYS WITH NUCLEI OF ATMOSPHERE'S ATOMS, AT HEIGHTS OF ABOUT 12000

- The half-life of a muon is 2.2 microseconds and so even moving at 0.994 c they would only expect to travel about 660 m before half of them decayed.

- As they are formed at 12000 m altitude it would take 40 μs, or about 20 half lives, to reach the ground.. So, they almost would not reach the ground
- But they do! This means that the muons are living longer???

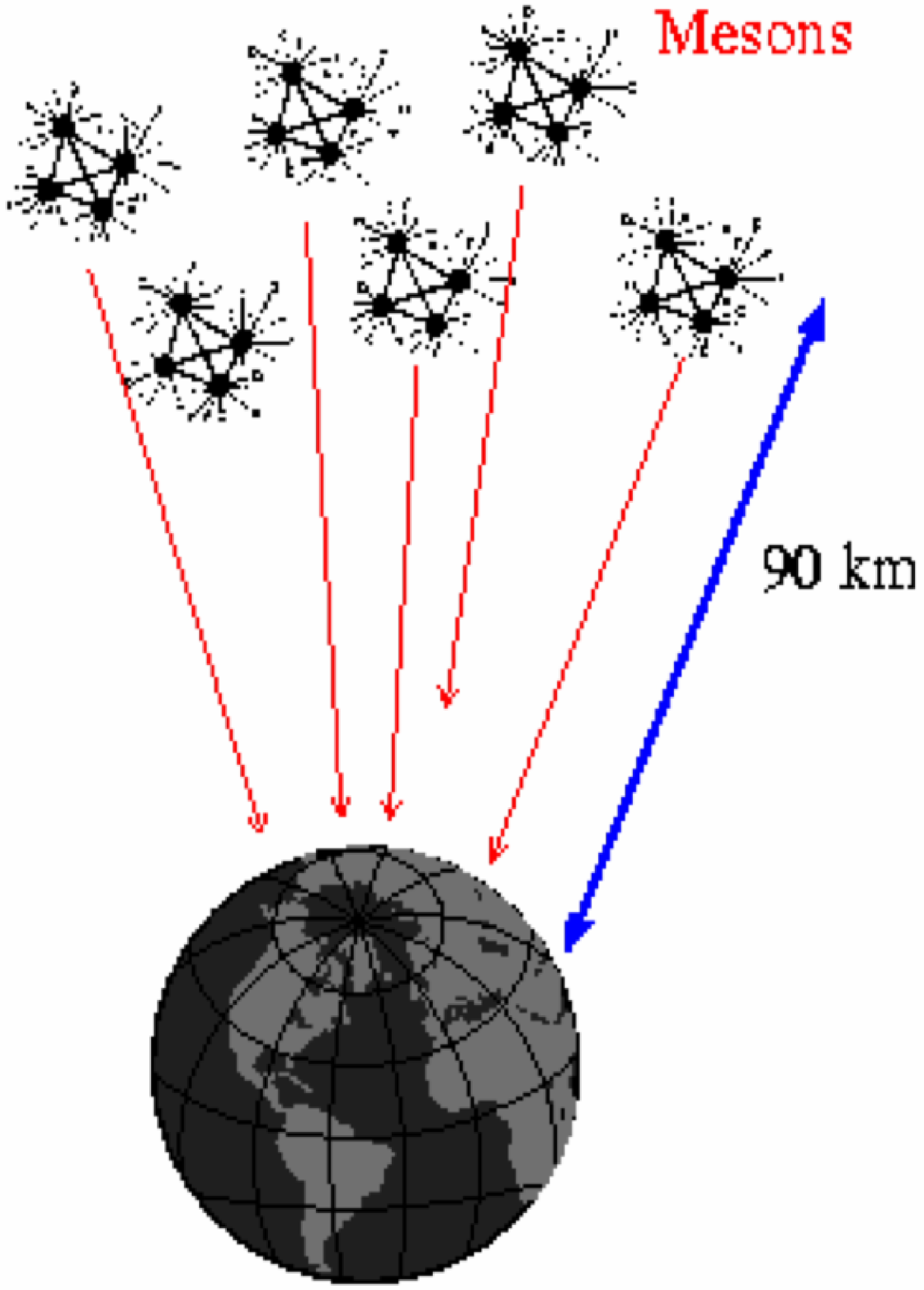
- Their relativistic factor is:

$$\gamma = \frac{1}{\sqrt{1 - 0.994^2}} = 9.1424$$



Their time slows down, and 2.2 μs become about γ times longer, or Lengths contract and the 12000m become 12000/γ m.

MESONS DECAY IN THE ATMOSPHERE



Half-life = 2×10^{-6} sec

so required velocity $= 90/2 \times 10^{-6} = 4.5 \times 10^{7}$ km/sec

$= 150$ c

SOME CLARIFICATION

- **Lorentz Contraction:**
  - It is not the matter that is compressed (what Lorentz thought)
  - It is the space that is modified (Einstein)
- **Time Dilation**
  - It is not the clock that is changed (what Lorentz and others thought
  - It is the time that is modified (Einstein)
- **EINSTEIN'S MAIN CONTRIBUTION: TO BELIEVE IT!**

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PROPER DEFINITIONS

PROPER MASS:

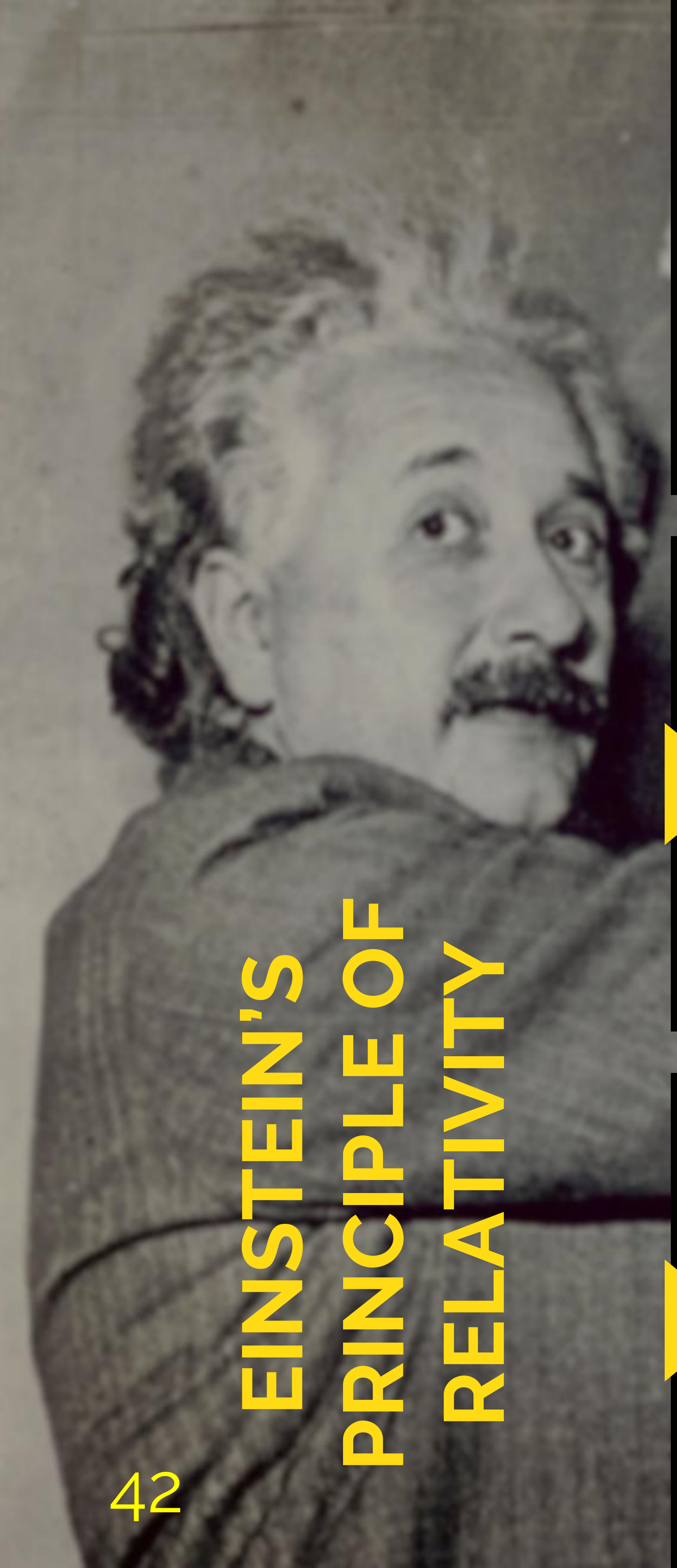
mass of a body at rest

PROPER TIME:

time as measured in its own frame

PROPER LENGHT:

length as measured in its own frame



EINSTEIN'S PRINCIPLE OF RELATIVITY

IN ALBERT EINSTEIN'S ORIGINAL TREATMENT, IN 1905, THE PRINCIPLE OF RELATIVITY IS BASED ON **TWO POSTULATES**:

1. SPECIAL PRINCIPLE OF RELATIVITY:

The laws of physics are invariant (i.e. identical) in all inertial frames of reference (i.e. non-accelerating frames of reference).

2. INVARIANCE OF C:

The speed of light in a vacuum is the same for all observers, regardless of the motion of the light source or observer.

1905, Albert Einstein , "On the Electrodynamics of Moving Bodies".

EINSTEIN POSTULATES CONSEQUENCES: SPECIAL PRINCIPLE OF RELATIVITY

- All physical laws (e.g. Newton's Laws or Maxwell's Equations) all have the same observed form in all inertial reference frames. This is "helpful" in that the basic laws of physics are not dependent on your state of motion.
- But as a consequence of this, it is impossible to tell from the laws of physics in your frame whether you are in motion or not.

There is no such thing as an absolute state of rest or motion - all motion is relative.

EINSTEIN POSTULATES CONSEQUENCES: INVARIANCE OF C

- All observers agree that light moves at a fixed speed - this is the singular invariant independent of states of relative motion;
- But as a consequence of this, the belief that time or space or both are experienced in the same way by observers in different states of motion must be abandoned.

There is no such thing as an absolute measure of time or space; measurements in one frame of reference need not agree with those in another, but all observers will agree that light signals travel at a fixed speed.

EINSTEIN POSTULATES CONSEQUENCES

- Space and time are NOT independent quantities
- Relativistic phenomena (with relevance for accelerators):
  - No speed of moving objects can exceed speed of light
  - (Non-) Simultaneity of events in independent frames
  - Lorentz Contraction and Time Dilation
  - Relativistic Doppler effect – change in frequency (and wavelength) of light, caused by the relative motion of the source and the observer
- There are no absolute time and space, no absolute motion

Inertial system: It is not possible to know whether one is moving or not

APPLICATION IN ACCELERATOR PHYSICS

TIME DILATION

Relativistic Effect: Particles moving at speeds near speed of light experience slower internal clocks.

Impact:

- **Extended Particle Lifetimes**: Unstable particles like muons, mesons live longer when accelerated, allowing for detailed study.
- **Design Consideration**: Extended particle lifetimes improve beam stability in accelerators like the **LHC**.

TIME DILATION: PROLONGED PARTICLE LIFETIMES

APPLICATION IN ACCELERATOR PHYSICS

LENGTH CONTRACTION

Relativistic Effect: Moving objects appear contracted along the direction of motion.

Impact:

- **Compressed Paths:** Particles experience shorter distances inside accelerators.
- **Example:** Particles moving near ccc see the circumference of the LHC ring contracted in their frame.
- **Beam Compression**: Bunches of particles contract, leading to better control over particle collisions.

LENGTH CONTRACTION: SHORTENED PATH LENGTHS

Two particles have equal rest mass m_0.

Laboratory Frame (LF): one particle at rest, total energy is E.

$p_1 = (E_1/c\,, p^1)$ $\quad$ $p_2 = (m_0 c, 0)$

Centre of Mass Frame (CMF): Velocities are equal and opposite, total energy is E_{cm}.

$p_1 = (E_{cm}/(2c)\,, p)$ $\quad$ $p_2 = (E_{cm}/(2c)\,, -p)$

- The quantity $(p_1 + p_2)^2$ is invariant.
- In the **CMF**, we have $(p_1 + p_2)^2 = E_{cm}^2 / c^2$.
- In general $(p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 = 2m_0^2 c^2 + 2p_1 \cdot p_2$.
- In the **LF**, we have and $p_1 \cdot p_2 = \boldsymbol{E_1 m_0}$ and $(p_1 + p_2)^2 = 2m_0 E$.
- And finally $E_{cm}^2 = 2m_0 c^2 E$

ENERGY AND MOMENTUM

Rest Energy, $E_o : E_o = m_0 c^2$

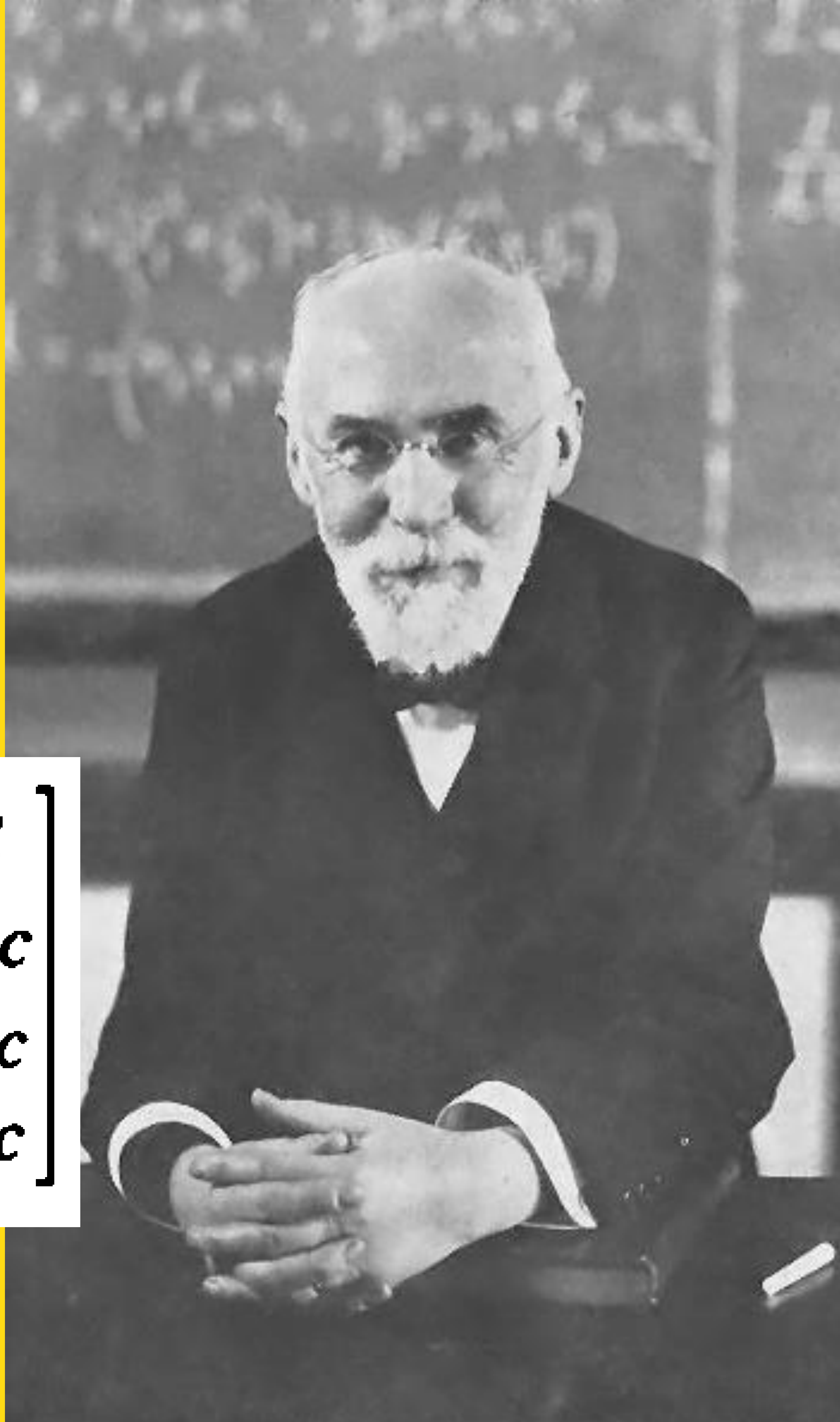
Total Energy, $E : E = \gamma m_0 c^2$

Momentum, $p : p = \gamma m_0 v$

$$E^2 = E_0^2 + p^2 c^2$$

$$\begin{bmatrix} E' \\ p_x' c \\ p_y' c \\ p_z' c \end{bmatrix} = \begin{bmatrix} \gamma E - \beta\gamma p_x c \\ -\beta\gamma E + \gamma p_x c \\ p_y c \\ p_z c \end{bmatrix}$$

$$\begin{bmatrix} E' \\ p_x' c \\ p_y' c \\ p_z' c \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{bmatrix}$$



EXAMPLE: LHC ENERGY

The Einstein relationship for energy includes both the kinetic energy and rest mass energy for a particle.

$$E = mc^2$$

The kinetic energy of a high-speed particle can be calculated from $E_K = mc^2 - m_0c^2$

The mass of proton is 938,3 MeV/c2

1) **What would be "relativistic mass", *m*?**
2) **Calculate Lorenz factor, γ?**
3) **Calculate velocity, v?**
4) **Calculate the energy of the rest of proton, E_0?**

Let´s take a look at γ (gamma) when the proton reaches LHC energy (7TeV per beam).

$E_k = \gamma \cdot m_0 \cdot c^2 - m_0 \cdot c^2$ Kinetic energy

$E_k = m_0 \cdot c^2(\gamma - 1)$

$M_p = 938.3$ MeV/c² $m_0 \cdot c^2 = 9{,}383 \cdot 10^{-4}$ TeV

7 TeV $= 9{,}383 \cdot 10^{-4} (\gamma - 1)$

$\gamma \sim 7461$

$\gamma \gg 1$, therefore we are nearing Special Relativity

EXAMPLE

We can now verify the of the proton with that energy comes close to that of the speed of light.

$$\gamma = 1/[1-(v/c)^2]^{1/2}$$

$$\gamma = 7461 \quad \Rightarrow \quad v = 0{,}999999991 \cdot c$$

$$v \sim c$$

EXAMPLE

DEFINITIONS AND PRACTICAL UNITS

$$\beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\gamma = \frac{1}{\sqrt{(1 - \frac{v^2}{c^2})}} = \frac{1}{\sqrt{(1 - \beta_r^2)}}$$

β_r **relativistic speed:** $\beta_r = [0,\ 1]$

LHC: $\beta_r \approx 0.999999991$

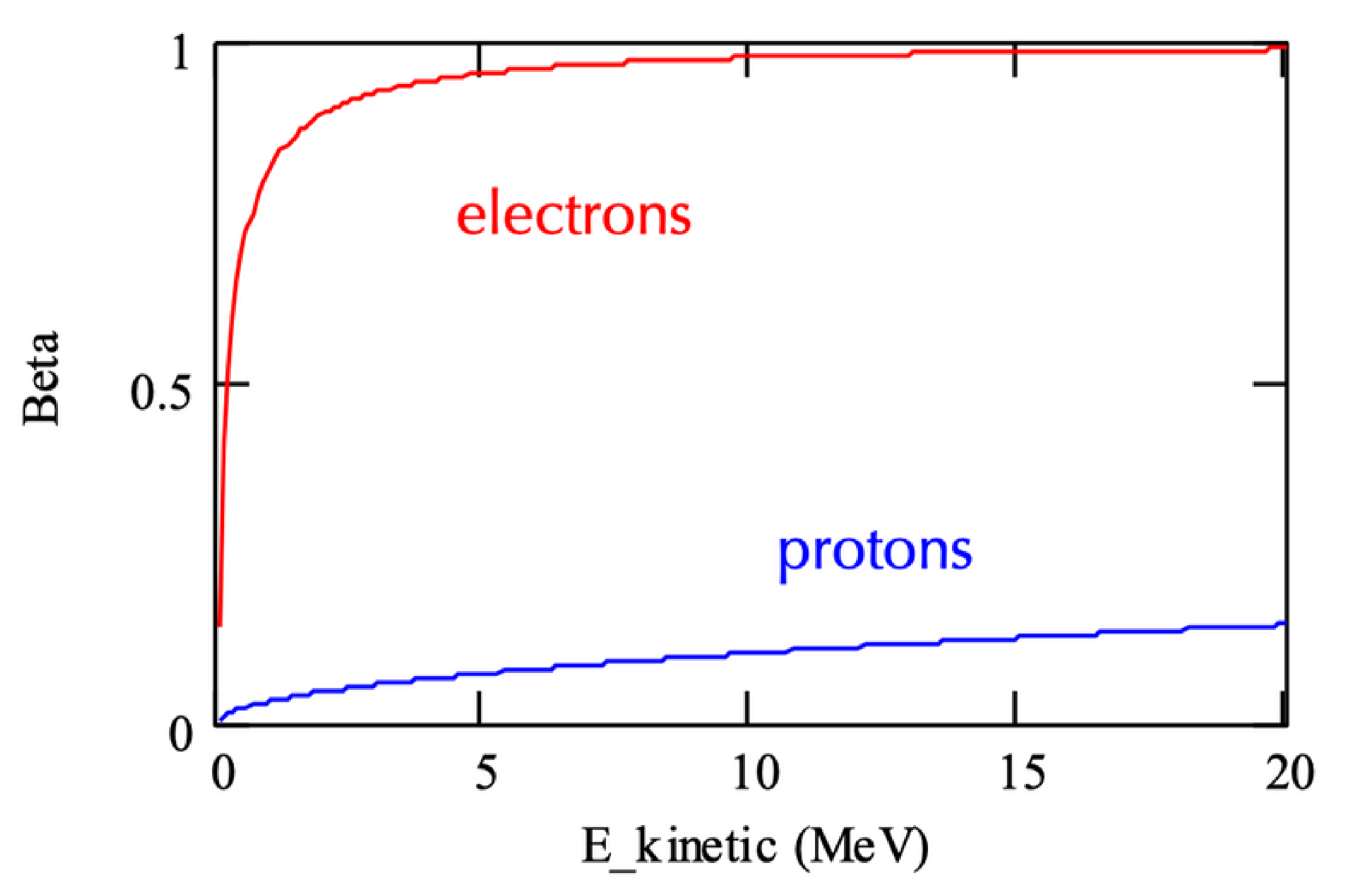
γ **Lorentz factor:** $\gamma = [1,\ \infty)$

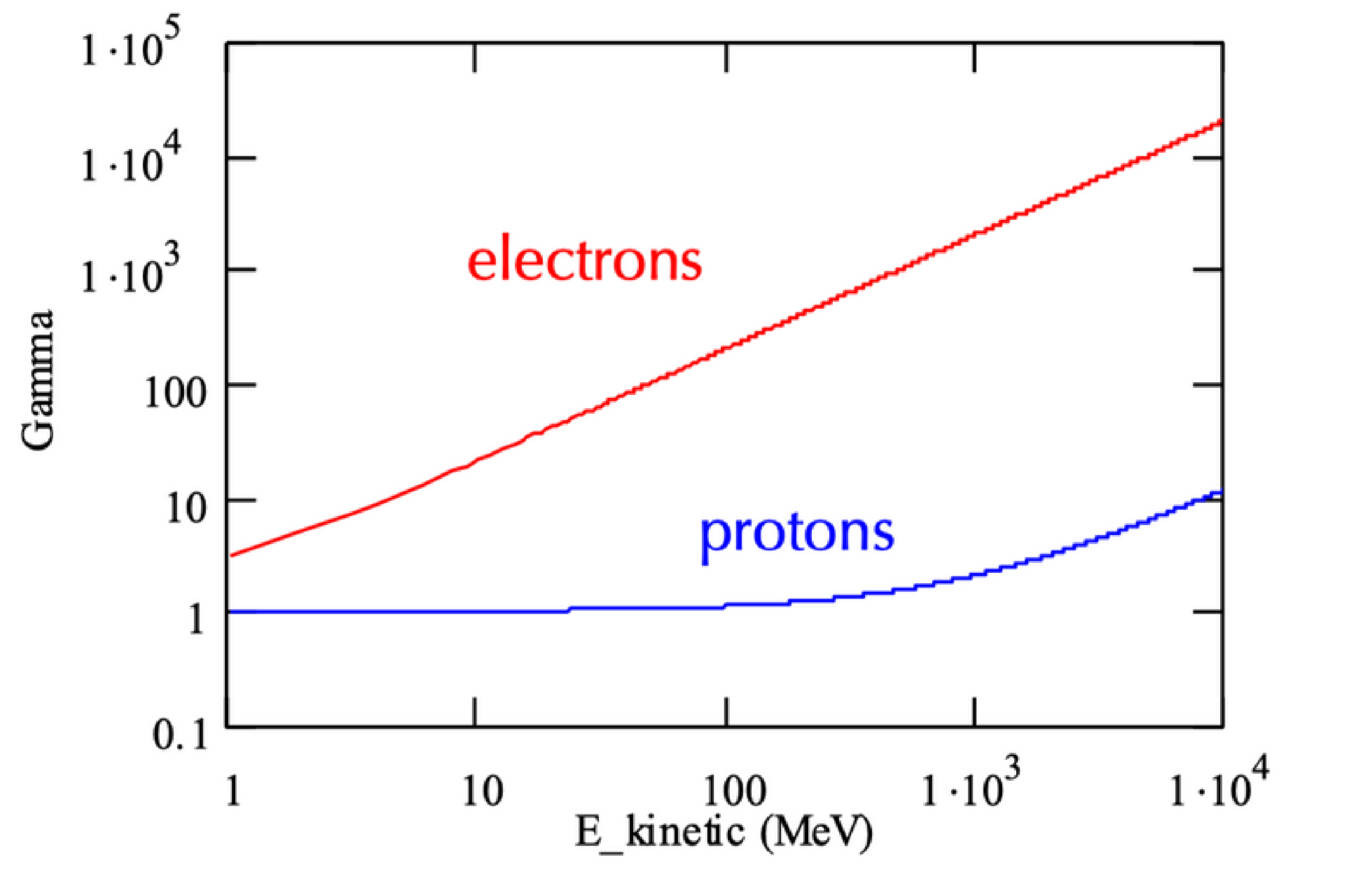
LHC: $\gamma \approx 7461$

EXAMPLES

$$\gamma = \frac{\text{total energy}}{\text{rest energy}}$$

$$\gamma = \frac{E}{E_0} = \frac{m}{m_0} = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}}$$





$$\beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

Normalized velocity

Dr. Irina Shreyber, PhD

TRANSFORMATIONS OF ELECTROMAGNETIC FIELDS

$$\begin{cases} E_x = \gamma\left(E'_x + vB'_y\right) \\ E_y = \gamma\left(E'_y - vB'_x\right) \\ E_z = E'_z \end{cases}$$

Unprimed quantities are in the lab frame, primed quantities in the a frame moving with velocity **v** along the **z** axis

$$\begin{cases} B_x = \gamma\left(B'_x - vE'_y/c\right) \\ B_y = \gamma\left(B'_y + vE'_x/c\right) \\ B_z = B'_z \end{cases}$$

$$\mathbf{E} = \gamma\left(\mathbf{E}' - \mathbf{v} \times \mathbf{B}'\right) - \frac{\gamma^2}{1+\gamma}\left(\mathbf{v} \cdot \mathbf{E}'\right)\mathbf{v}$$

$$\gamma = 1/\sqrt{1 - v^2/c^2}$$

In compact 3d vector form for a frame moving with arbitrary velocity **v**

"For a charge moving in an electromagnetic field, the force experienced by the charge is equal to the electric force, transformed into the rest frame of the charge"
A. Einstein

DEFINITIONS

m_0	rest mass	MeV/c^2
$E_0 = m_o c^2$	rest energy	MeV
$E = \gamma\, m_o c^2$	total energy	MeV
$K = E - m_o c^2$	kinetic energy	MeV
$\mathbf{v}$	velocity	m/s
$\boldsymbol{\beta} = \mathbf{v}/c$	relativistic velocity	–
$\gamma = 1/\sqrt{1 - \boldsymbol{\beta} \cdot \boldsymbol{\beta}}$	lorentz factor	–
$\mathbf{P} = \boldsymbol{\beta}\gamma\, m_o c^2$	momentum	MeV/c
$E^2 = (Pc)^2 + (m_o c^2)^2$	total energy	MeV

USEFUL RELATIONS AND QUANTITIES

$E^2 = P^2c^2 + m_o{}^2c^4$	total energy	MeV
$\mathbf{P}c = E\boldsymbol{\beta}$	total momentum times c	
$m_e = 0.510999$	rest mass of the electron	MeV/c^2
$m_p = 938.272$	rest mass of the proton	MeV/c^2
$m_\mu = 105.66$	rest mass of the muon	MeV/c^2

Frequent subdivisions

$\gamma \simeq 1$	non-relativistic
$\gamma > 1$	relativistic
$\gamma \gg 1$	ultra-relativistic

Dr. Irina Shreyber, PhD

EVERY DAY EXAMPLE: GPS SATELLITE

Orbital speed 14000 km/h ≈3.9 km/s

-> $\beta \approx$ **$1.3x10^{-5}$**, $\gamma \approx$ **1.000000000084**

Small, but accumulates **7 µs** during one day compared to reference time on earth!

After one day: your position wrong by ≈2 km !! (including general relativity error is 10 km per day)

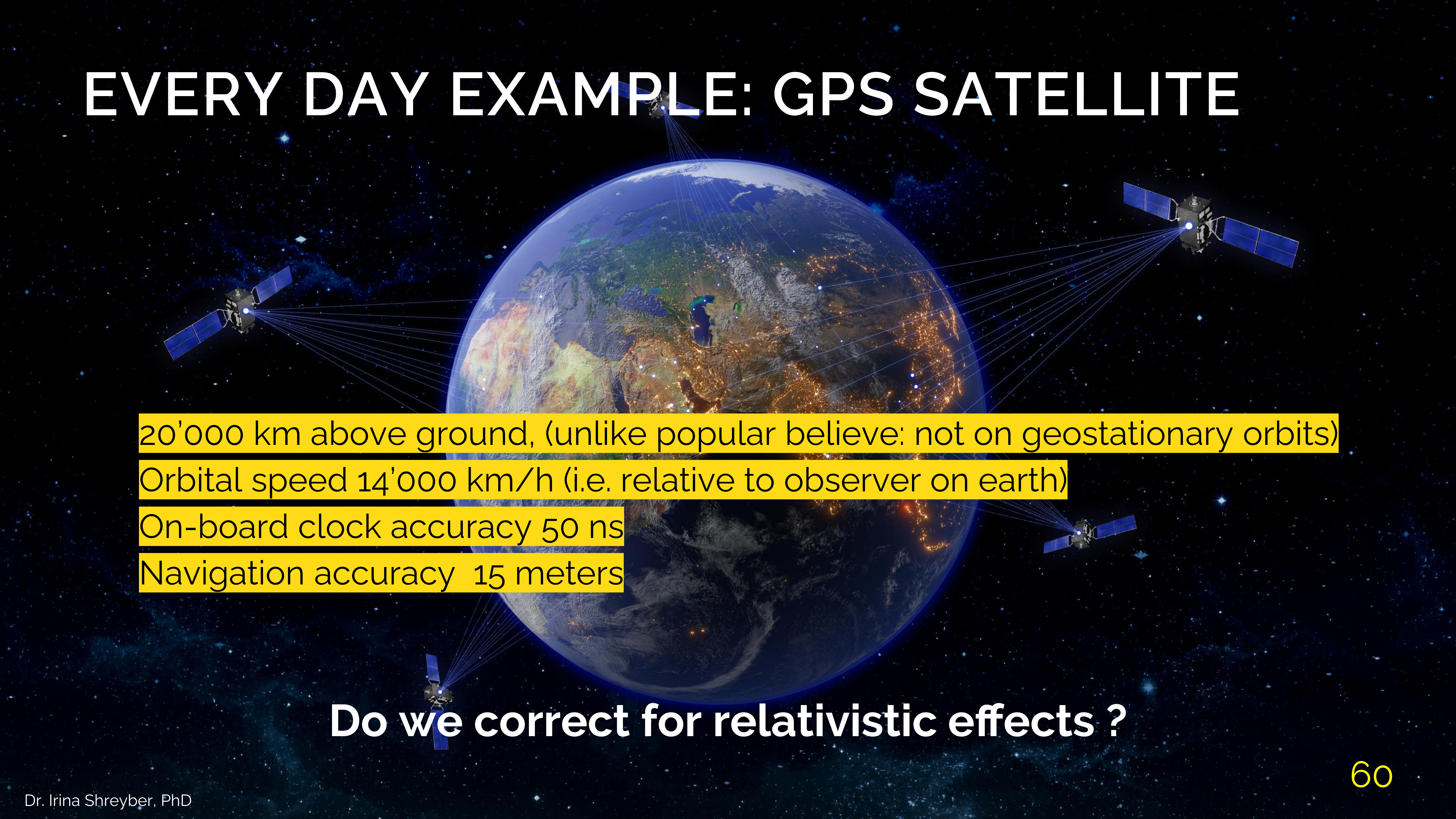
Special relativity: 7µs slower, general relativity: 45 µs faster

Countermeasures:

(1) Minimum **4 satellites** (avoid reference time on earth)

(2) Detune data transmission frequency from **1.023 MHz** to **1.022999999543 MHz** prior to launch

EVERY DAY EXAMPLE: GPS SATELLITE



- 20’000 km above ground, (unlike popular believe: not on geostationary orbits)
- Orbital speed 14’000 km/h (i.e. relative to observer on earth)
- On-board clock accuracy 50 ns
- Navigation accuracy 15 meters

Do we correct for relativistic effects ?

Laboratory of High Energy Physics Data Analysis
Tomsk State University

THE END

Thank you for your attention!

CONTACT INFORMATION

irina.shreyber@cern.ch

https://www.linkedin.com/in/ishreyber/