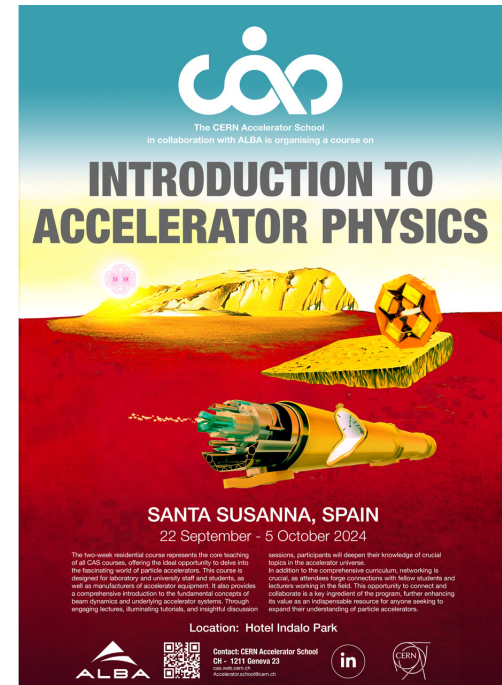


# Warm Magnets

**Gijs de Rijk**  
CERN

**CAS**  
Santa Susanna, Spain  
24<sup>th</sup> September 2024



The CERN Accelerator School  
in collaboration with ALBA is organising a course on





## INTRODUCTION TO ACCELERATOR PHYSICS

**SANTA SUSANNA, SPAIN**  
22 September - 5 October 2024

The two-week residential course represents the core teaching of all CAS courses, offering the ideal opportunity to delve into the fascinating world of particle accelerators. This course is designed for laboratory and university staff and students, as well as manufacturers of accelerator equipment. It also provides a comprehensive introduction to the fundamental concepts of beam dynamics and underlying accelerator systems. Through engaging lectures, illuminating tutorials, and insightful discussion sessions, participants will deepen their knowledge of crucial topics in the accelerator universe. In addition to the comprehensive curriculum, networking is crucial, as attendees forge connections with fellow students and collaborators to a key ingredient of the program, further enhancing its value as an indispensable resource for anyone seeking to expand their understanding of particle accelerators.

Location: Hotel Indalo Park

Contact: CERN Accelerator School  
CH - 1211 Geneva 23  
acceleratorschool@cern.ch



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- Introduction: magnetic field and warm magnet principles
- Field description and magnet types
- Practical magnet design & manufacturing
- Permanent magnets
- Examples of accelerator magnets from the early times until the present
- Literature on warm Magnets

We can also classify magnets based on their technology

electromagnet

permanent magnet

iron dominated

coil dominated

normal conducting  
(resistive)

superconducting

static

cycled / ramped  
slow pulsed

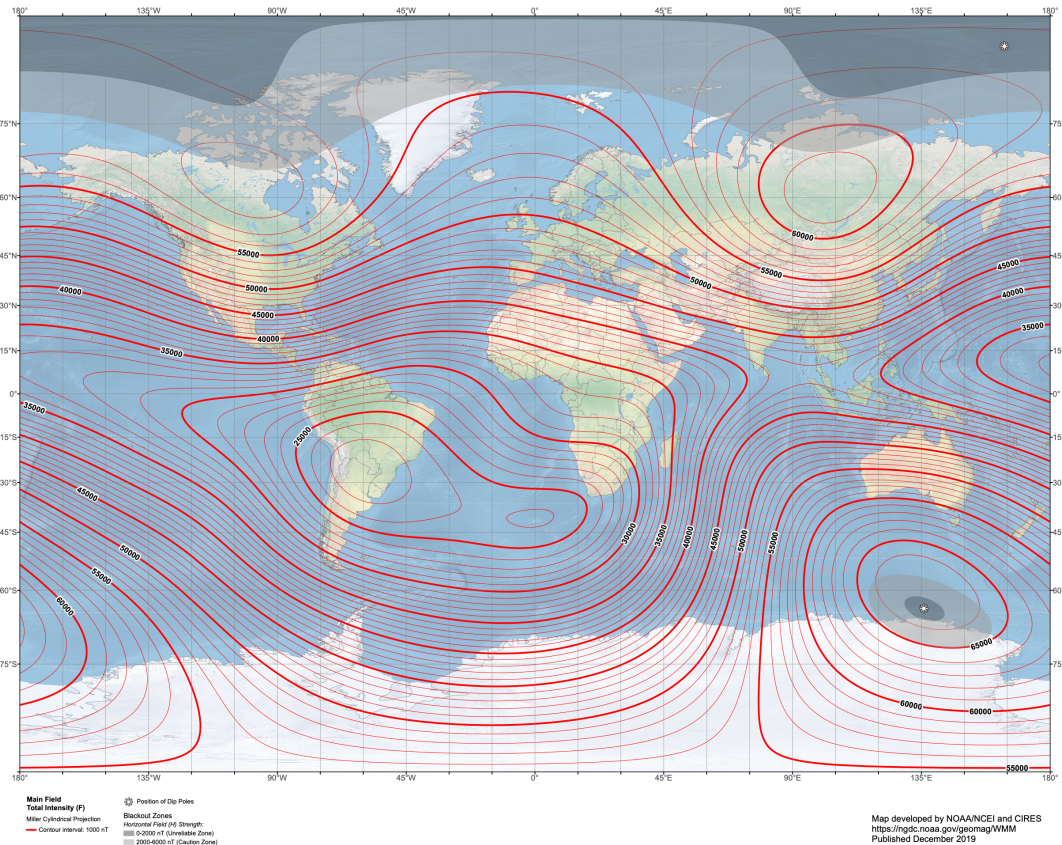
fast pulsed

In Geneva, on 2/08/2023, the (estimated) flux density is

$$|B| = 47814 \text{ nT} = 0.047814 \text{ mT} = 4.7814 \cdot 10^{-5} \text{ T} \approx 0.5 \text{ Gauss.}$$

$$B_{\text{horizontal}} = 22311 \text{ nT}$$

US/UK World Magnetic Model - Epoch 2020.0  
Main Field Total Intensity (F)



For comparison:  $1.6 \cdot 10^9 \text{ T}$  in the ultraluminous pulsar in the Milky Way J0243.6+6124.  
see: <https://www.universetoday.com/> 17th July 2022

Macroscopic formulation aka ‘Maxwell’s equations in matter’

Integral form

Differential form

$$\oint \vec{H} d\vec{s} = \int_A \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) d\vec{A}$$

Ampere’s law

$$\text{rot} \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\oint \vec{E} d\vec{s} = - \frac{\partial}{\partial t} \int_A \vec{B} d\vec{A}$$

Faraday’s equation

$$\text{rot} \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\int_A \vec{B} d\vec{A} = 0$$

Gauss’s law for magnetism

$$\text{div} \vec{B} = 0$$

$$\int_A \vec{D} d\vec{A} = \int_V \rho dV$$

Gauss’s law

$$\text{div} \vec{D} = \rho$$

With:  $\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H} = \mu_0 (\vec{H} + \vec{M})$

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 (\vec{E} + \vec{P})$$

$$\vec{J} = \kappa \vec{E} + \vec{J}_{imp.}$$

H: magnetic field  
 B: flux density  
 E: electric field  
 D: displacement field  
 P: electric polarisation  
 J: current density

Let's have a closer look at the 3 equations that describe magnetostatics

Gauss law of magnetism

$$(1) \quad \operatorname{div} \vec{B} = 0 \quad \text{always holds}$$

Ampere's law with no time dependencies

$$(2) \quad \operatorname{rot} \vec{H} = \vec{J} \quad \text{holds for magnetostatics}$$

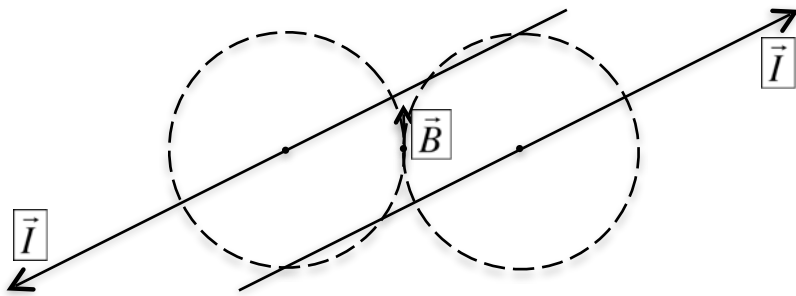
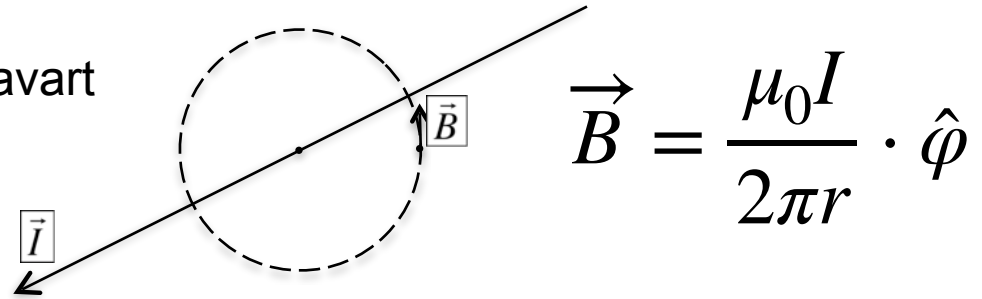
Relation between  $\vec{H}$  field and the flux density  $\vec{B}$

$$(3) \quad \vec{B} = \mu_0 \mu_r \vec{H} \quad \text{holds for linear materials}$$

From Ampere's law with no time dependencies

(Integral form) 
$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

We can derive the law of Biot and Savart



If you wanted to make a  $B = 1.5$  T magnet with just two infinitely thin wires placed at 100 mm distance in air one needs :  
 $I = 187500$  A

- To get reasonable fields ( $B > 1$  T) one needs large currents
- Moreover, the field homogeneity will be poor



With the help of an iron yoke we can get fields with less current

$$\oint_c \vec{H} \cdot d\vec{l} = NI$$

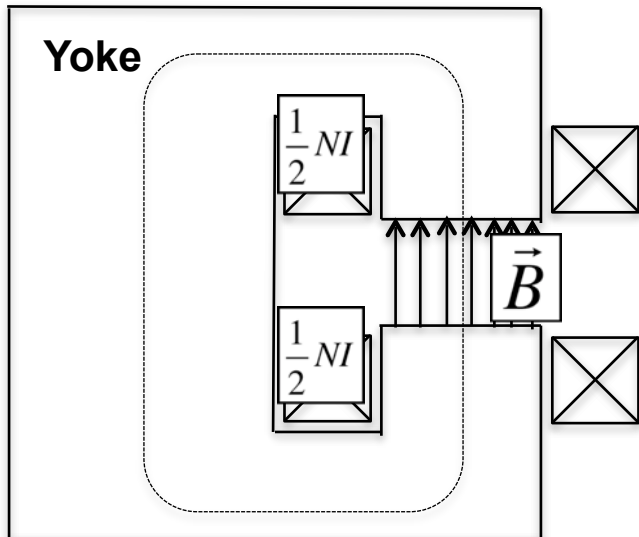
$$N \cdot I = H_{\text{iron}} \cdot l_{\text{iron}} + H_{\text{air gap}} \cdot l_{\text{air gap}}$$

$$N \cdot I = \frac{B}{\mu_0 \mu_r} \cdot l_{\text{iron}} + \frac{B}{\mu_0} \cdot l_{\text{air gap}}$$

$$N \cdot I = \frac{B}{\mu_0} \cdot l_{\text{air gap}}$$

This is valid as  $\mu_r \gg 1$  in the iron : limited to  $B < 2$  T

Example: C shaped dipole for accelerators



coil

$B = 1.5$  T

Gap = 50 mm

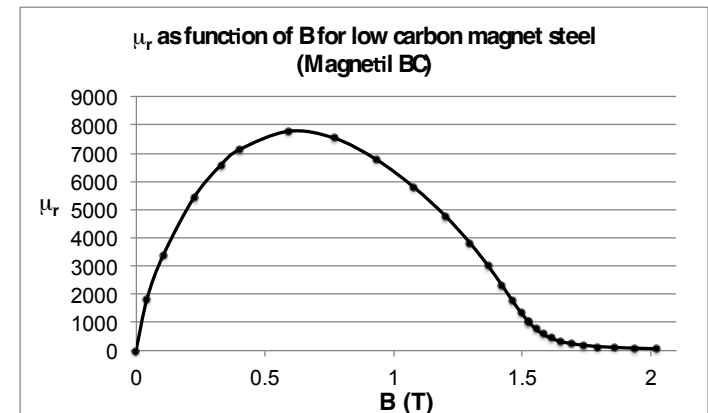
$N \cdot I = 59683$  A

2 x 30 turn coil

$I = 994$  A

@5 A/mm<sup>2</sup>, 200 mm<sup>2</sup>

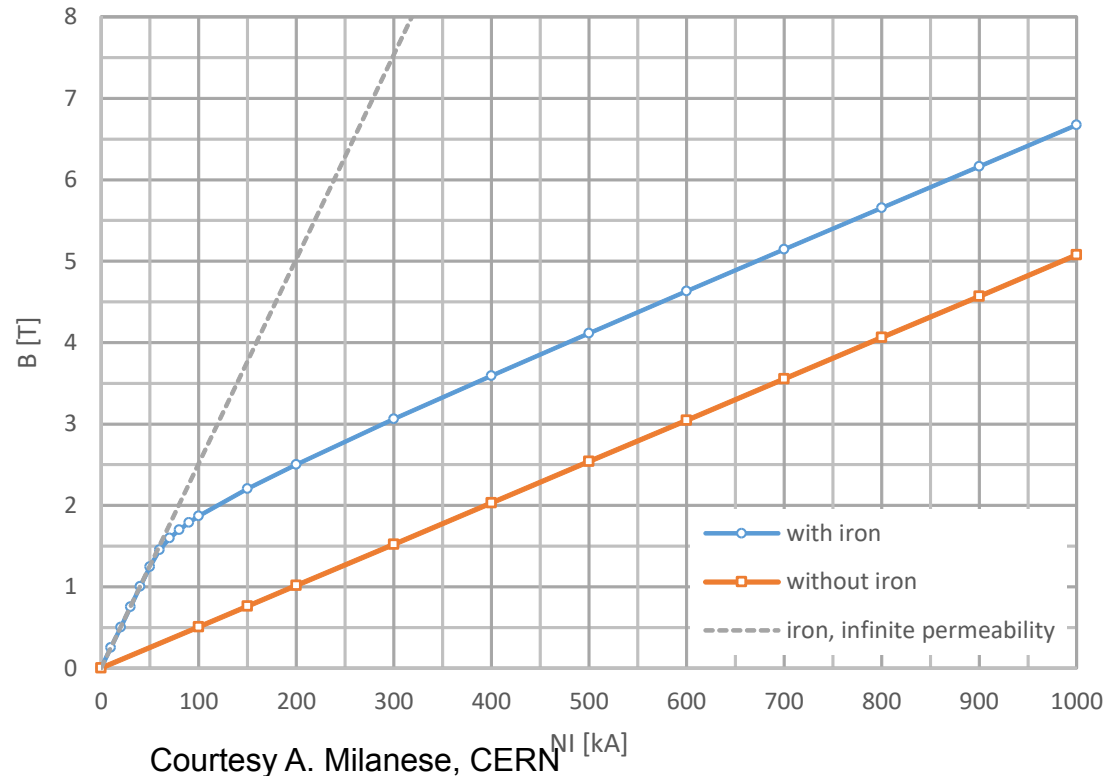
14 x 14 mm Cu



Imagine a magnet with a 50 mm vertical gap ( horizontal width ~100 mm)  
 Iron magnet wrt to an air coil:

- Up to 1.5 T we get ~6 times the field
- Between 1.5 T and 2 T the gain flattens of : the iron saturates
- Above 2 T the slope is like for an air-coil: currents become too large to use resistive coils

These two curves are the transfer functions – B field vs. current – for the two cases



$$B_y(z) + iB_x(z) = 10^{-4} B_1 \sum_{n=1}^{\infty} (b_n + ia_n) \left( \frac{x + iy}{R_{ref}} \right)^{n-1}$$

with:

$$z = x + iy,$$

$B_x$  and  $B_y$  the flux density components in the  $x$  and  $y$  direction,

$R_{ref}$  the radius of the reference circle,

$B_1$  the dipole field component at the reference circle,

$b_n$  the normal  $n$ th multipole component,

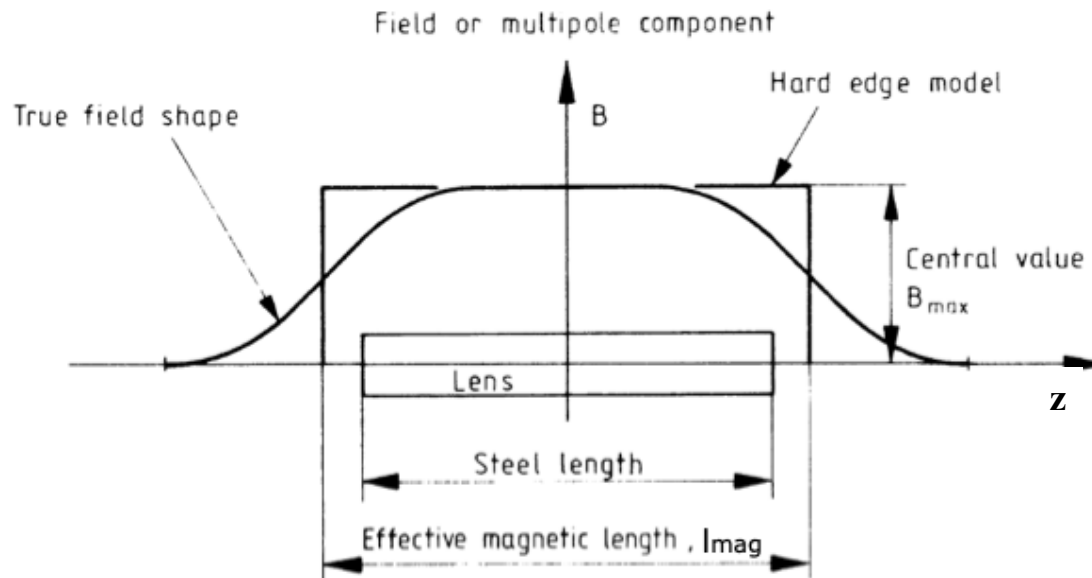
$a_n$  the skew  $n$ th multipole component .

The “wanted”  $b_n$  or  $a_n$  is equal to 1

In a ring-shaped accelerator, where the beam does multiple passes, one typically demands :

$$a_n, b_n \leq 1 \text{ unit } 10^{-4}$$

In 3D, the longitudinal dimension of the magnet is described by a magnetic length

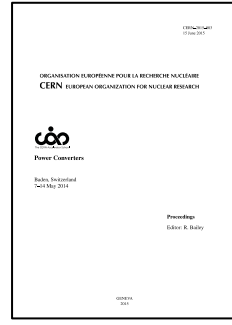
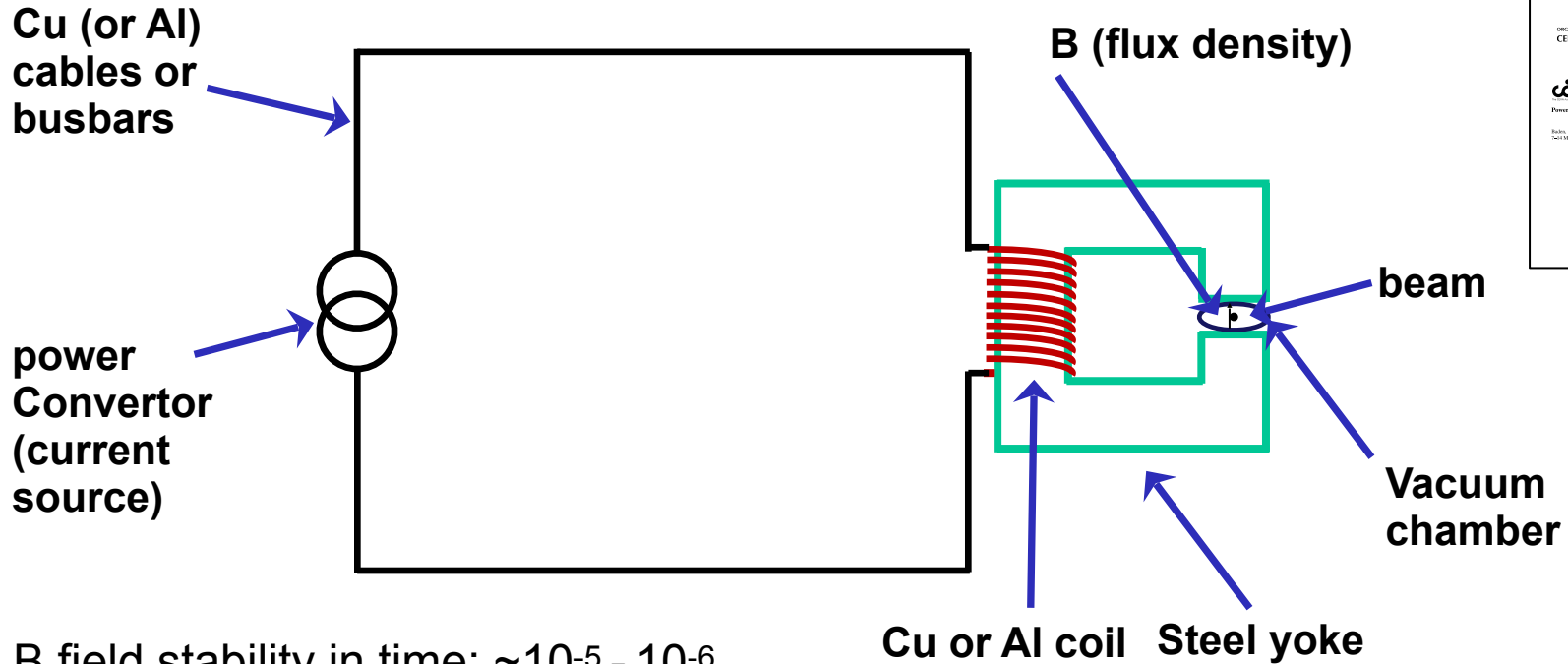


$$l_{mag} B_0 = \int_{-\infty}^{\infty} B(z) dz$$

Courtesy A. Milanese, CERN

magnetic length  $L_{mag}$  as a first approximation:

- For dipoles  $L_{mag} = L_{yoke} + d$   $d = \text{pole distance}$
- For quadrupoles:  $L_{mag} = L_{yoke} + r$   $r = \text{radius of the inscribed circle between the 4 poles}$



- B field stability in time:  $\sim 10^{-5} - 10^{-6}$
- Typical R of a magnet  $\sim 20\text{m}\Omega - 60\text{m}\Omega$
- Typical L of a magnet  $\sim 20\text{mH} - 200\text{mH}$
- Powering cable (for 500A): Cu  $250\text{ mm}^2$  (Cu:  $17\text{ n}\Omega\cdot\text{m}$ )  $R = 70\text{ }\mu\Omega/\text{m}$ , for 200m:  $R = 13\text{m}\Omega$
- Take a typical rise time 1s

Then the Power Convertor has to Supply : 0-500 A with a stability of a few ppm.

Voltage up to 40 V (resistive)

And 100 V (inductive)



When building, manipulating, testing and using magnets in accelerators a number of safety considerations have to be observed.

Magnets have safety issues with:

1. magnetic field
2. currents
3. voltage
4. induction: can cause high voltages
5. temperature
6. magnetic forces
7. magnet weight
8. liquid pressures

International and local laboratory safety standards have to be observed.

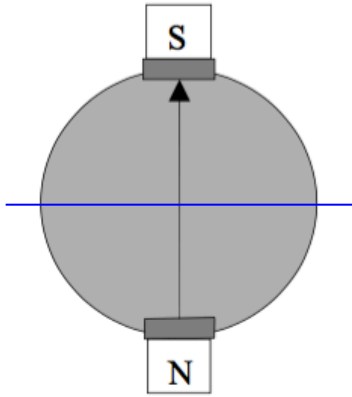
The documentation is to be read before starting anything !

for 1) at CERN see:

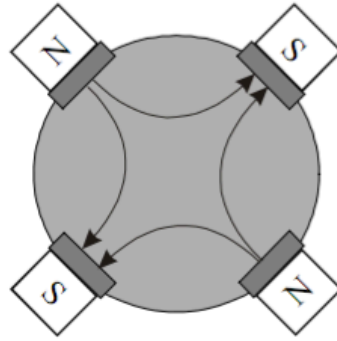
[https://edms.cern.ch/ui/file/2974732/LAST\\_RELEASED/GSI-NIR-1\\_EN.pdf](https://edms.cern.ch/ui/file/2974732/LAST_RELEASED/GSI-NIR-1_EN.pdf)

Common sense to be always used (cowboys stay away)

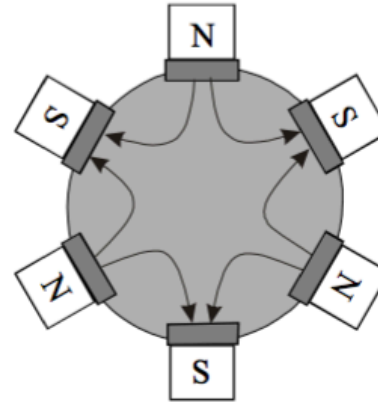
## NORMAL : vertical field on mid-plane



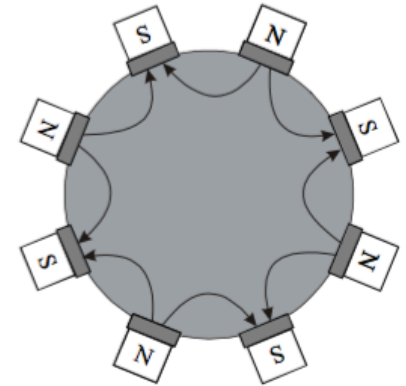
Dipole  
 $|B|=const$



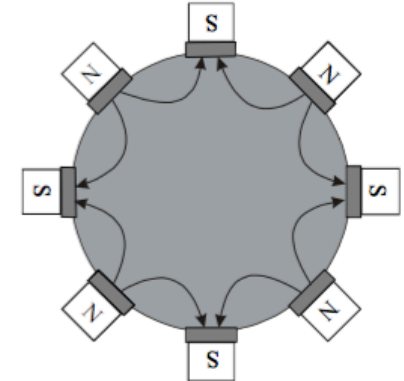
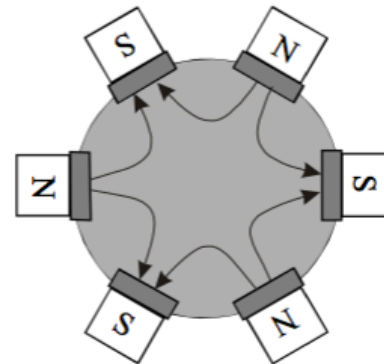
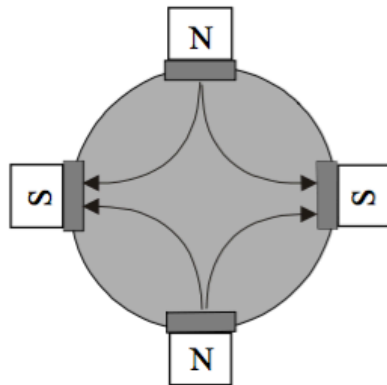
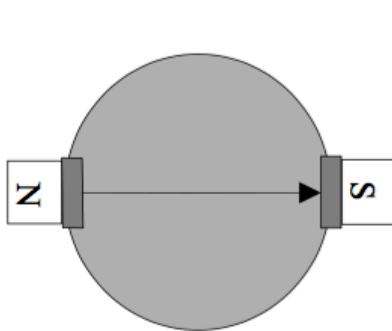
Quadrupole  
 $|B|=G \cdot r$



Sextupole  
 $|B|=1/2 \cdot B'' \cdot r^2$

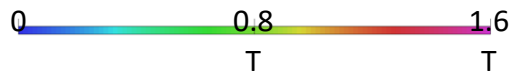
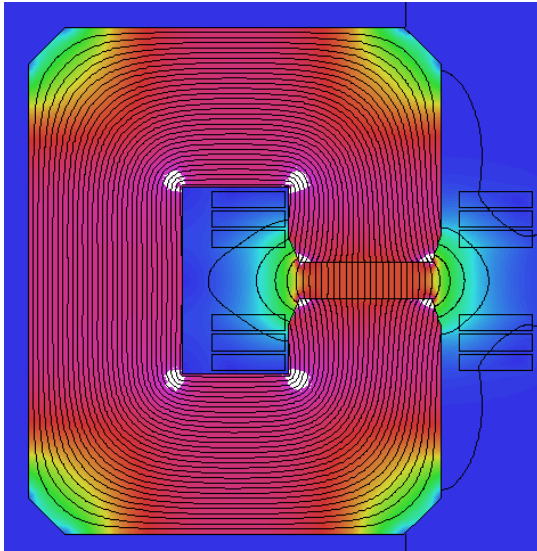


Octupole  
 $|B|=1/6 \cdot B''' \cdot r^3$

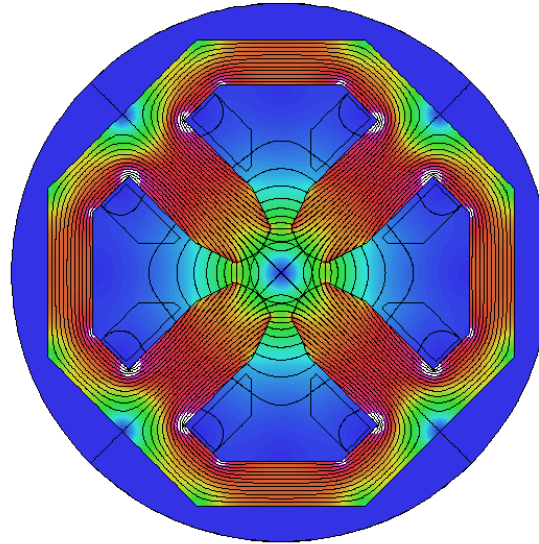


## Courtesy D. Tommasini, CERN **SKREW : horizontal field on mid-plane**

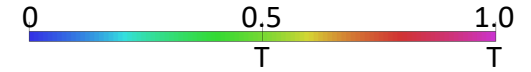
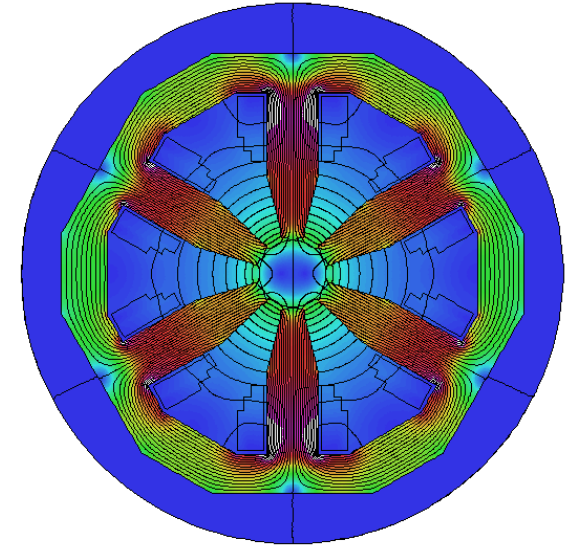
Dipole



Quadrupole



sextupole





In a fully symmetric magnet certain field harmonics are natural.

<b>Magnet type</b>	<b>Allowed harmonics <math>b_n</math></b>
n=1 Dipole	n=3,5,7,...
n=2 Quadrupole	n=6,10,14
n=3 Sextupole	n=9,15,21
n=4 Octupole	n=12,20,28

Non-symmetric designs and fabrication errors give rise to non allowed harmonics:  $b_n$  with n other than listed above and  $a_n$  with any n

Magnet	Pole shape	Transfer function	Inductance (H)
<p> <math>w</math> : pole width  <math>g</math> : vertical gap                 </p>	parallel	$B = \mu_0 NI / g$	$L = \mu_0 N^2 A / g$ $A \approx (w + 1.2 \cdot g) \cdot (l + g)$
<p> <math>w</math> : pole width  <math>g</math> : vertical gap                 </p>	parallel	$B = \mu_0 NI / g$	$L = \mu_0 N^2 A / g$ $A \approx (w + 1.2 \cdot g) \cdot (l + g)$
<p> <math>w</math> : pole width  <math>g</math> : pole gap  <math>t</math> : coil width                 </p>	parallel	$B = \mu_0 NI / g$	$L = 2\mu_0 N^2 A / g$ $A \approx (d + 2/3t) \cdot (l + g)$
<p> <math>w</math> : pole width  <math>g</math> : pole gap  <math>t</math> : coil width                 </p>	parallel	$B = \mu_0 NI / g$	$L = \mu_0 N^2 A / g$ $A \approx (d + 2/3t) \cdot (l + g)$
<p> <math>R</math> : aperture radius  <math>d</math> : coil distance  <math>t</math> : coil width                 </p>	$2xy = R^2$	$B(r) = G \cdot r$ $G = 2\mu_0 NI / R^2$	$L = 8\mu_0 N^2 A / R$ $A \approx (d + 1/3t) \cdot (l + 2/3R)$
<p> <math>R</math> : aperture radius  <math>d</math> : coil distance  <math>t</math> : coil width                 </p>	$3x^2y - y^3 = R^3$	$B(r) = S \cdot r^2 = 1/2 B'' \cdot r^2$ $S = 3\mu_0 NI / R^3$	$L = 20\mu_0 N^2 A / R$ $A \approx (d + 1/3t) \cdot (l + 1/2R)$



Steps in the process:

1. Specification
2. Conceptual design
3. Detailed design
  1. Yoke: yoke size, pole shape, FE model optimization
  2. Coils: cross-section, geometry, cooling
  3. Raw material choice
  4. Yoke ends, coil ends design
4. Yoke manufacturing, tolerances, alignment, structure
5. Coil manufacturing, insulation, impregnation type
6. Magnetic field measurements

Before you start designing you need to get from the accelerator designers:

- B(T) or G (T/m) (higher orders:  $G_3(T/m^2)$ , etc)
- Magnet type: C-type, H-type, DC (slow ramp) or AC (fast ramp)
- Aperture:
  - Dipole : “good field region“ → air-gap height and width
  - Quads and higher order: “good field region“ → aperture inscribed circle
- Magnetic length and estimated real length
- Current range of the power convertor (and the voltage range: watch out for the cables )

- Field quality:

$$dipole: \frac{\Delta B}{B} (ref\ volume), \quad quadrupole: \frac{\Delta G}{G} (reference\ circle)$$

$$or \quad b_n, a_n \quad for \quad n = 1, 2, 3, 4, 5, \dots$$

- Cooling type: air, water ( $P_{max}$ ,  $\Delta p_{max}$  and  $Q_{max}$  (l/min))
- Jacks and Alignment features
- Vacuum chamber to be used → fixations, bake-out specifics

These need careful negotiations and often iterations after the conceptual (and detailed) design, and the result will probably be a compromise.

- From  $B$  and  $l$  you get  $NI$  (A)

$$NI = \frac{l_{airgap} B}{\mu_0}$$

- From  $NI$  (A) and the power converter  $I_{max}$  you get  $N$
- Then you decide on a coil X-section using:

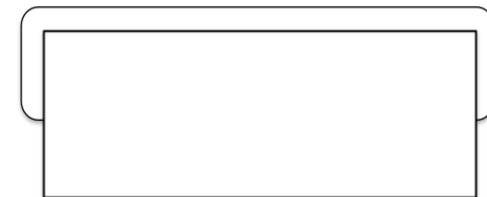
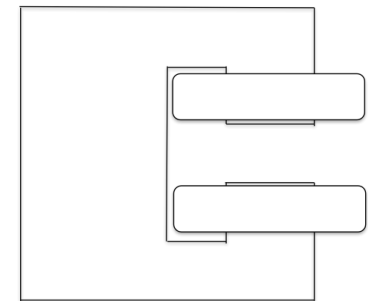
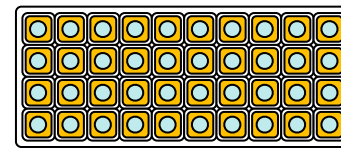
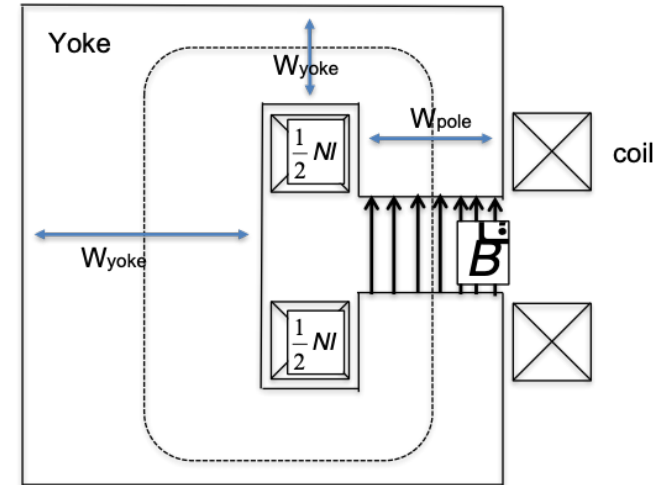
$$j_{coil} = 5 \frac{A}{mm^2} \text{ for water cooled}$$

$$\text{or } j_{coil} = 1 \frac{A}{mm^2} \text{ for air cooled}$$

- This defines the coil cavity in the yoke (you add 0.5 mm insulation around each conductor and 1 mm ground insulation around the coil) and select the best fitting rectangular
- You can the draw the draft X-section using:

$$W_{yoke} = W_{pole} \frac{B}{B_{sat}} \quad \text{with } 1.5 T < B_{sat} < 2 T$$

- Decide on the coil ends: racetrack, bedstead
- You now have the rough magnet cross section and envelope



## Power generated by coil

- DC: from the length of the conductor  $N \cdot L_{turn}$ , the cross-section  $\sigma$  and the specific resistivity  $\rho$  of the material one gets the spent Power in the coil

$$P/l [W/m] = \frac{\rho}{S} I^2 \quad \text{with:} \quad \begin{aligned} \rho_{Cu} &= 1.72(1 + 0.0039(T - 20))10^{-8} \Omega m \\ \rho_{Al} &= 2.65(1 + 0.0039(T - 20))10^{-8} \Omega m \end{aligned}$$

For AC: take the average  $I^2$  for the duty cycle

Power losses due to hysteresis in the yoke: (Steinmetz law up to 1.5 T)

$$P [W/kg] = \eta f B^{1.6} \quad \text{with } \eta = 0.01 \text{ to } 0.1, \quad \eta_{Si \text{ steel}} \approx 0.02$$

Power losses due to eddy currents in the yoke

$$P [W/kg] = 0.05 \left( d_{lam.} \frac{f}{10} B_{av} \right)^2$$

with  $d_{lam.}$  the lamination thickness in mm,  $B_{av}$  the average flux density

Aim: to design  $d_{cooling}$ ,  $P_{water}$  [bar],  $\Delta P$  [bar],  $Q$  [l/min]

- Choose a desired  $\Delta T$  (20°C or 30°C depending on the  $T_{cooling\ water}$  )
- with the heat capacity of water (4.186 kJ/kg°C) we now know the required water flow rate:  $Q$  (l/min)
- The cooling water needs to be in moderately turbulent regime (with laminar flow the flow speed is zero on the wall !):  $Reynolds > 2000$

$$Re_e = \frac{dv}{\nu} \sim 140 d[mm] v[m/s] \quad \text{for } T_{water} \sim 40^\circ C$$

- A good approximation for the pressure drop in smooth pipes can be derived from the Blasius law, giving:

$$\Delta P[bar] = 60 L[m] \frac{Q[l/min]^{1.75}}{d[mm]^{4.75}}$$

The ideal poles for dipole, quadrupole, sextupole, etc. are lines of constant scalar potential

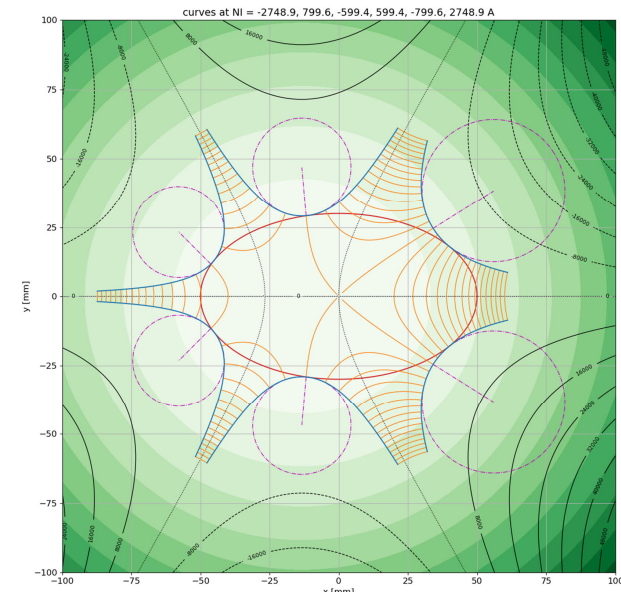
Dipole	$y = \pm h/2$	straight line
quadrupole	$2xy = \pm r^2$	hyperbola
sextupole	$3x^2y - y^3 = \pm r^3$	

To calculate pole shapes of pure and combined function magnet poles a convenient new code exist, the note describing the method and the Python code can be found on:

A. Milanese, CERN, "Tracking magnetic equipotential curves for general combinations of multipolar fields" : <https://edms.cern.ch/document/2792136/1>

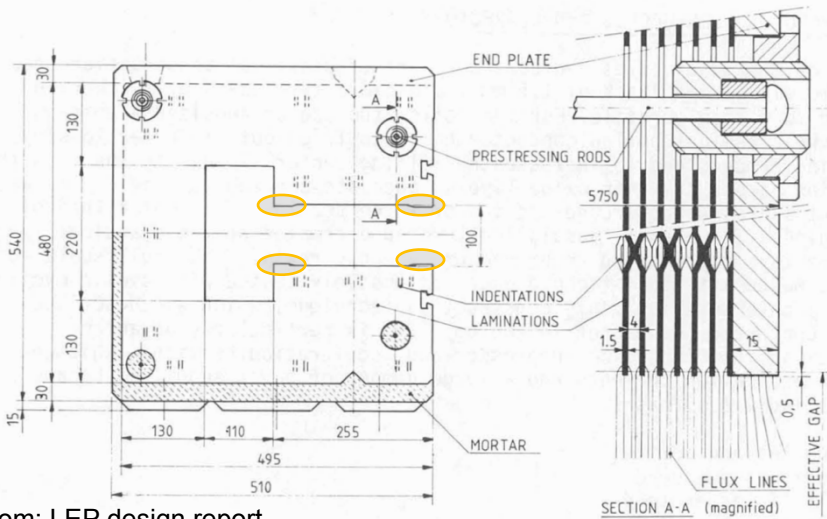
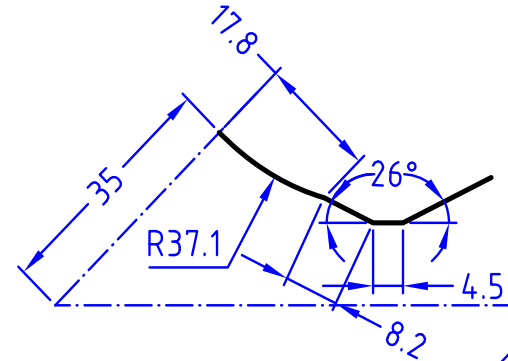
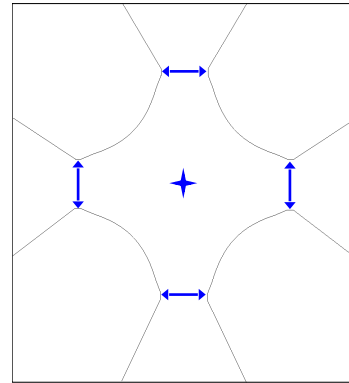
example of combined quadrupole-sextupole

4.7 Quadrupole  $B' = 2 \text{ T/m}$  + Sextupole  $B'' = 150 \text{ T/m}^2$ , tangent poles





- Dipole example: below a lamination of the LEP main bending magnets, with the pole shims well visible

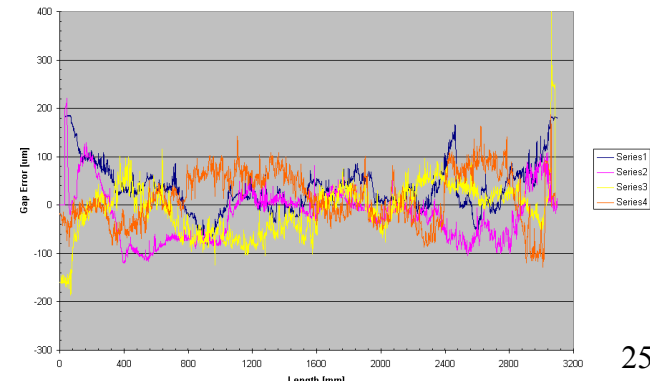


from: LEP design report



- Quadrupoles: at the edge of the pole one can put a combination a shim and alignment feature (examples: LHC-MQW, SESAME quads, etc)
- This then also allows to measure the pole distances : special instrumentation can be made for this

left\_final\_40\_40p\_adjus



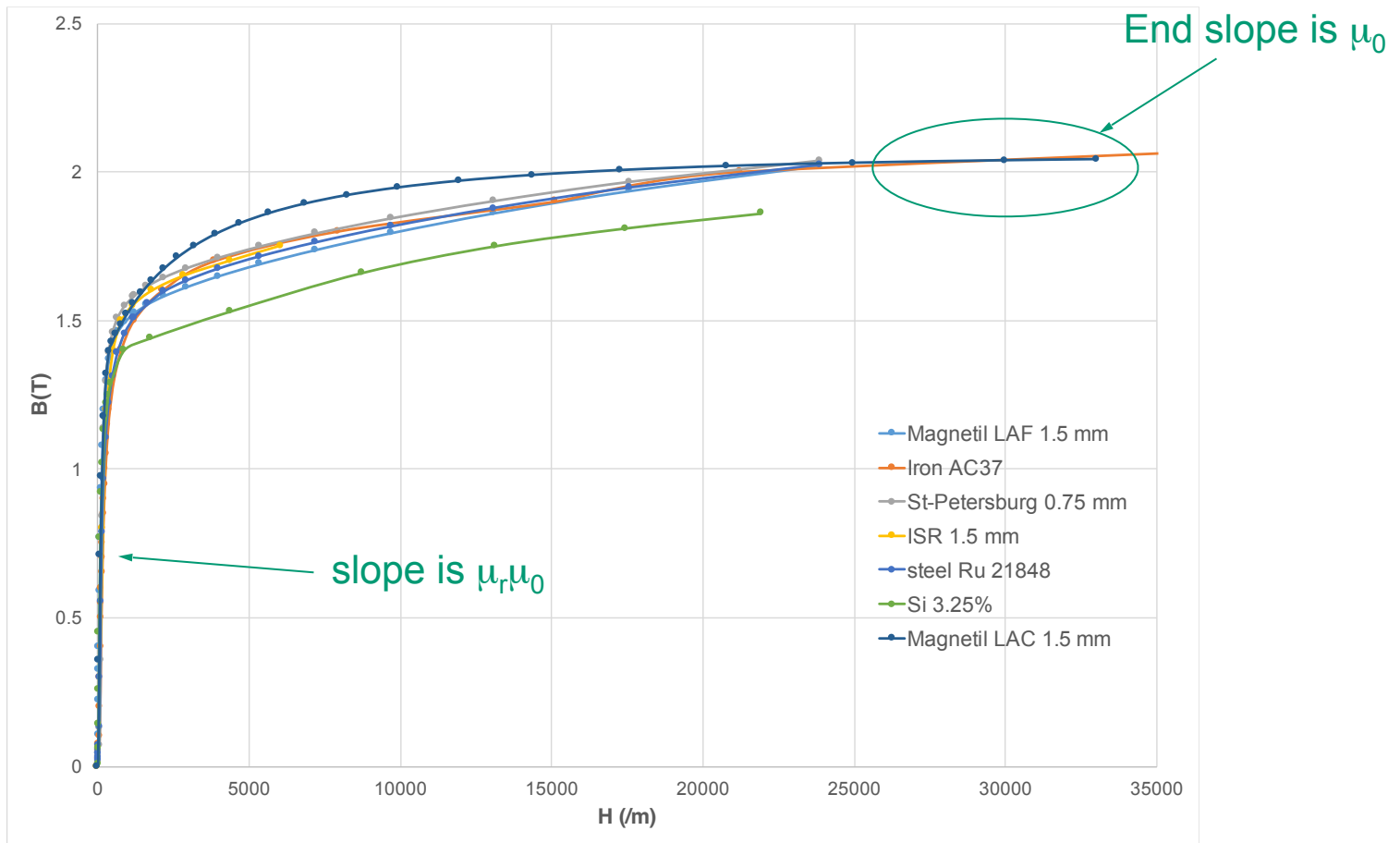
- Aim of the electromagnetic FE models:
  - The exact shape of the yoke needs to be designed
    - Optimise field quality: adjust pole shape, minimise high saturation zones
    - Minimise the total steel amount (magnet weight, raw material cost)
  - Calculate the field: needed for the optics and dynamic aperture modelling
    - transfer function  $B_{xsection}(l)$  ,  $\int Bdl$  , magnetic length
    - multipoles (in the centre of the magnet and integrated)  $b_n$  and  $a_n$
  
- Some Electromagnetic FE software packages that are often used:
  - **Opera** from Cobham: 2D and 3D commercial software see: <http://operafea.com/>
  - “Good old” **Poisson**, 2D: now distributed by LANL-LAACG see: [http://laacg.lanl.gov/laacg/services/download\\_sf.phtml](http://laacg.lanl.gov/laacg/services/download_sf.phtml) contact: [jbillen@lanl.gov](mailto:jbillen@lanl.gov)
  - **ROXIE** (CERN) 2D and 3D, specialised for accelerator magnets; single fee license for labs & universities see: <https://espace.cern.ch/roxie/default.aspx>
  - **ANSYS Maxwell**: 2D and 3D commercial software see: <http://www.ansys.com/Products/Electronics/ANSYS-Maxwell>
  - **RAT** (Little Beast Engineering) : 3D magnetic field solver for any coil geometry, see: <https://www.littlebeastengineering.com/>

You can use a close 'generic' B(H) curve for a first cut design

You **HAVE** to use a measured, and smoothed, curve to properly calculate

$$B_{xsection}(I), \quad \int Bdl, \quad b_n \quad \text{and} \quad a_n$$

As illustration the curves for several types of steel:

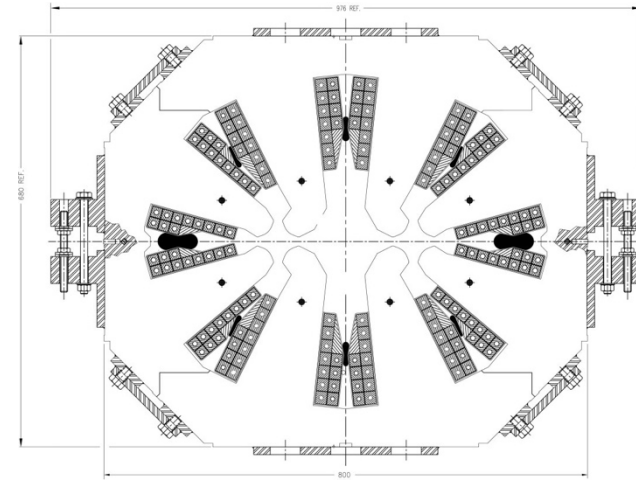


Use symmetry and the thus appropriate boundary conditions to model only  $\frac{1}{4}$ <sup>th</sup> (dipoles, quadrupoles) or even  $\frac{1}{6}$ <sup>th</sup> sextupoles.

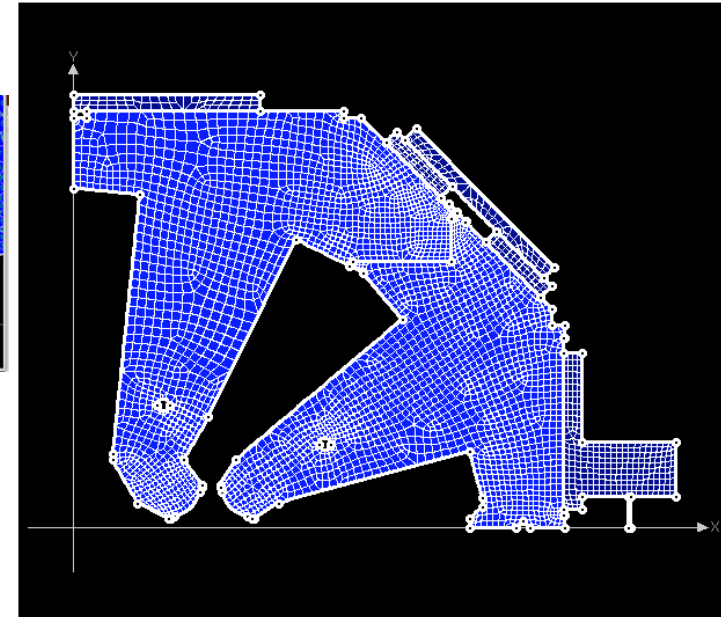
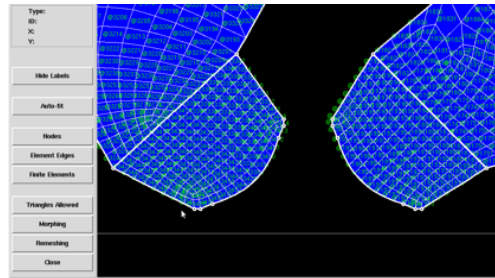
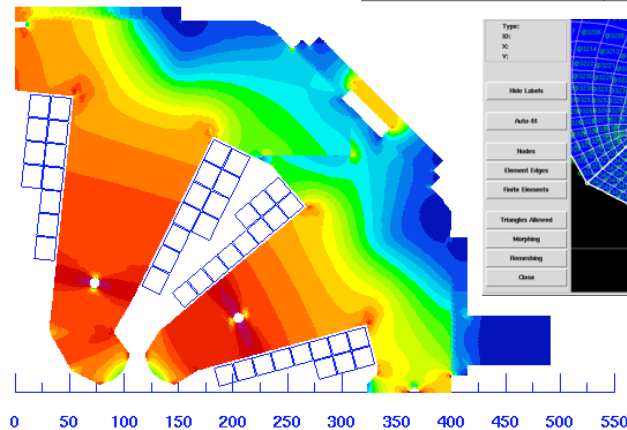
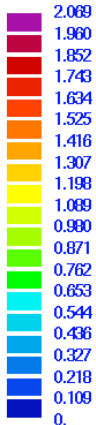
Meshing needs attention in the detailed areas like poles, slits, etc

Table 8.6: Main parameters of the MQW normal conducting quadrupole magnet

Magnet type	MQWA	MQWB
Magnetic length		3.1 m
Beam separation		224 mm
Aperture diameter		46 mm
Operating temperature		< 65° C
Nominal gradient	35 T/m	30 T/m
Nominal current	710 A	600 A
Inductance		28 mH
Resistance		37 mΩ
Conductor X-section	20.5 x 18.0 mm <sup>2</sup> inner poles 17.0 x 17.0 mm <sup>2</sup> outer poles	
Cooling hole diameter	7 mm inner poles, 8 mm outer poles	
Number of turns per magnet	8 x 11	
Minimum water flow	28 l/min	
Dissipated power at $I_{nom}$	19 kW	14 kW
Mass	11700 kg	



|Btot| (T)



- Yokes are nearly always laminated to reduce eddy currents during ramping
- Laminations can be coated with an inorganic (oxidation, phosphating, Carlite) or organic (epoxy) layer to increase the resistance
- Magnetic properties: depend on chemical composition + temperature and mechanical history
- Important parameters: coercive field  $H_c$  and the saturation induction.
  - $H_c$  has an impact on the remnant field at low current
    - $H_c < 80$  A/m typical
    - $H_c < 20$  A/m for magnets ranging down also to low field  $B < 0.05$  T
  - low carbon steel (C content  $< 0.006\%$ ) is best for higher fields  $B > 1$  T
- Yoke laminations can be glued together (small to medium size magnets), bolted or welded into stacks

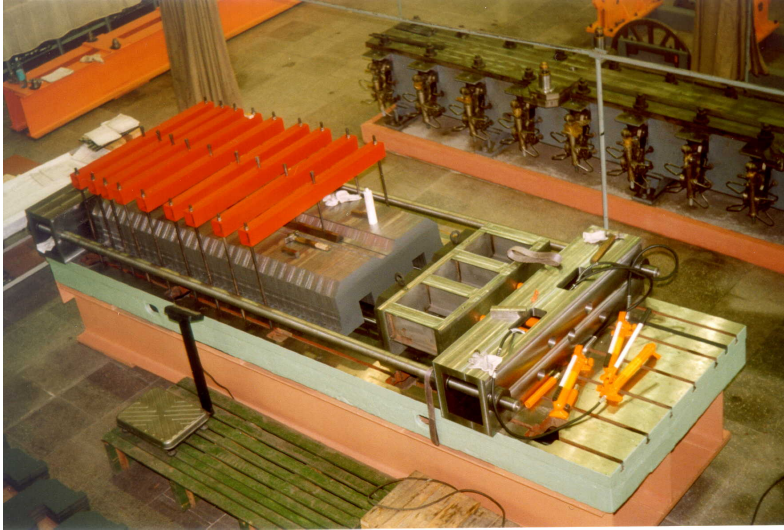
Field Strength [A/m]	Minimum Induction [T]
40	0.20
60	0.50
120	0.95
500	1.4
1 200	1.5
2 500	1.62
5 000	1.71
10 000	1.81
24 000	2.00

Example specification for 1.5 mm thick oxide coated steel for the LHC warm separation magnets,  $B_{max} = 1.53$  T

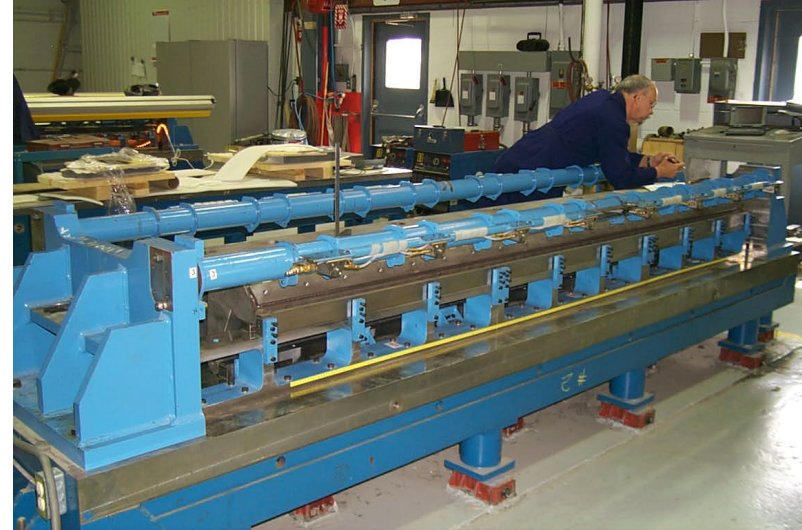
Field Strength H [A/m]	Minimum Induction B [T]
100	0.07
300	1.05
500	1.35
1000	1.50
2500	1.62
5000	1.72
10000	1.82

Example specification for 0.5 mm thick epoxy coated steel for LHC transfer line corrector magnet  $B_{max} = 0.3$  T

## Stacking an MBW dipole yoke stack



## Stacking an MQW quadrupole yoke stack

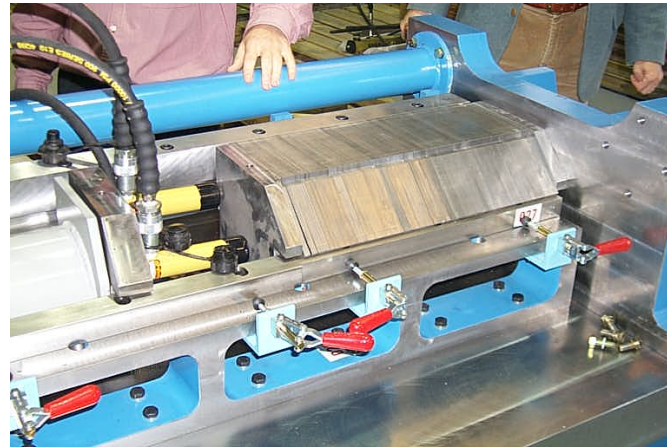


## MQW yoke assembly

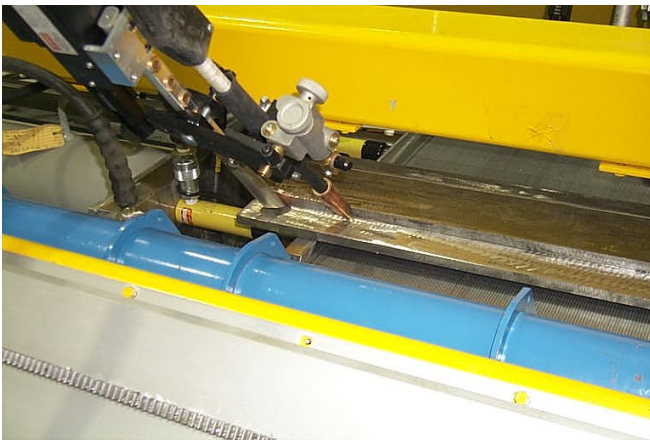


## Double aperture LHC quadrupole MQW

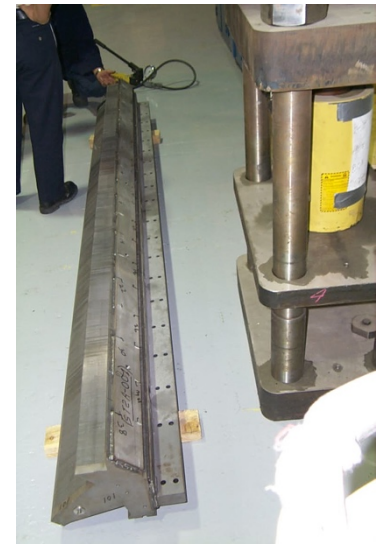
Stacking on a precision table



Welding the structural plates



Finished stack



- Yokes are either
  - Glued , using epoxy coated laminations
  - Welded, full length plates are welded on the outside
  - Compressed by tie rods in holes
 or a combination of all these
- To be able to keep the yoke (or yoke stack) stable you probably need end plates (can range from  $\pm 1$  cm to 5 cm depending on the size)
- The end plates have pole chamfers and often carry end shims

Glued yoke (MCIA LHC TL)



Welded stack



Tie rod

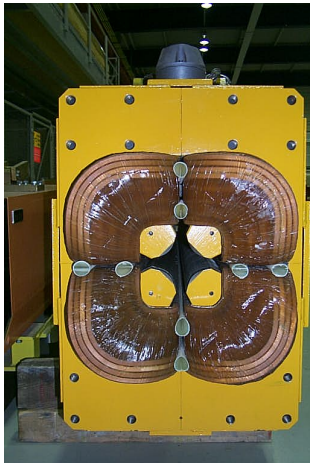
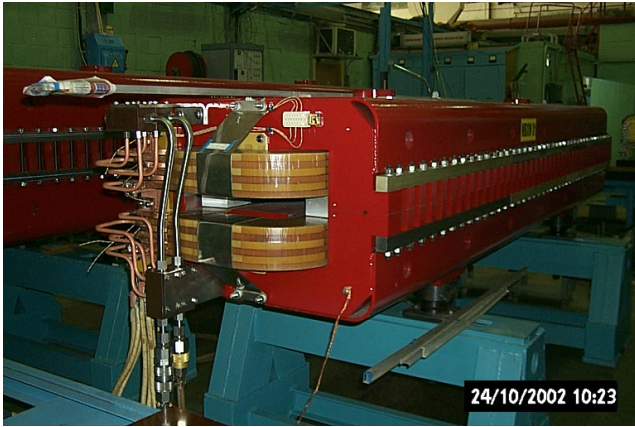




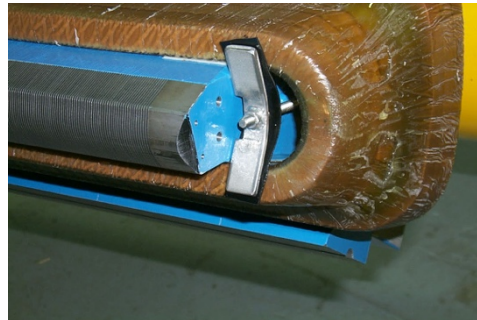
# Coil manufacturing, insulation, impregnation type

- Winding Cu conductors is an well established technique
- When the Cu conductor is thick it is best to use “dead soft” Cu (T treatment)
- Insulation of the coil
  - Glass fibre – epoxy impregnated
    - Individual conductor 0.5 mm glass fibre, 0.25 mm tape wound half lapped
    - Impregnated with radiation resistant epoxy, total glass volume ratio >50%
  - For thin conductors: Cu enamel coated, possibly epoxy impregnated afterwards

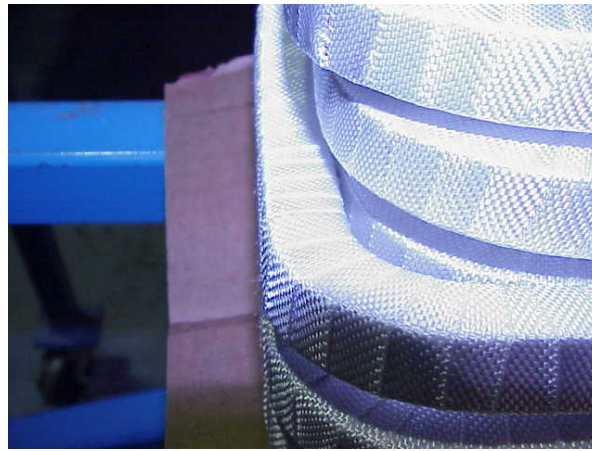
For dipoles some main types are racetrack of bedstead



Quadrupoles



## MQW Glass fibre tape wrapping.



## Winding the hollow Cu conductor



Mounted coil



coil electrical test (under water !)



Dipoles racetrack coil



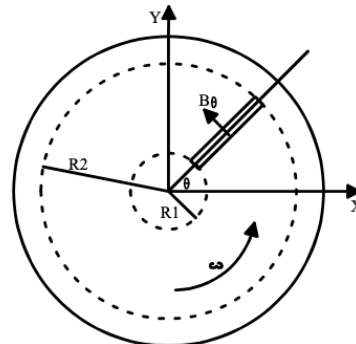
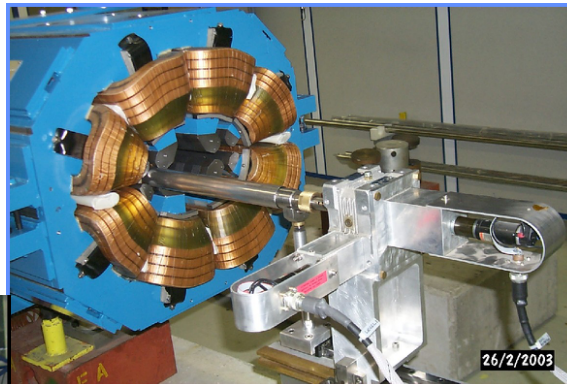
MBXW Coil winding

Finished MBXW coil

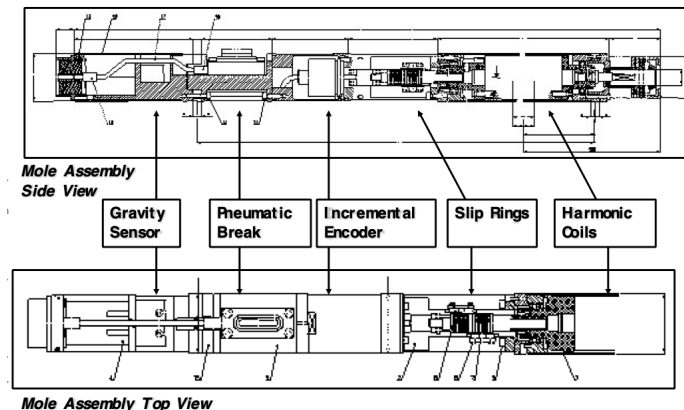


Several Magnetic Measurements techniques can be applied, e.g.:

- Rotating coils: multipoles and integrated field or gradient in all magnets
- Stretched wire: magnetic centre and integrated gradient for  $n > 1$  magnets
- Hall probes: field map
- Pickup coils: field on a current ramp
- Example below : MQW : double aperture quadrupole for the LHC.

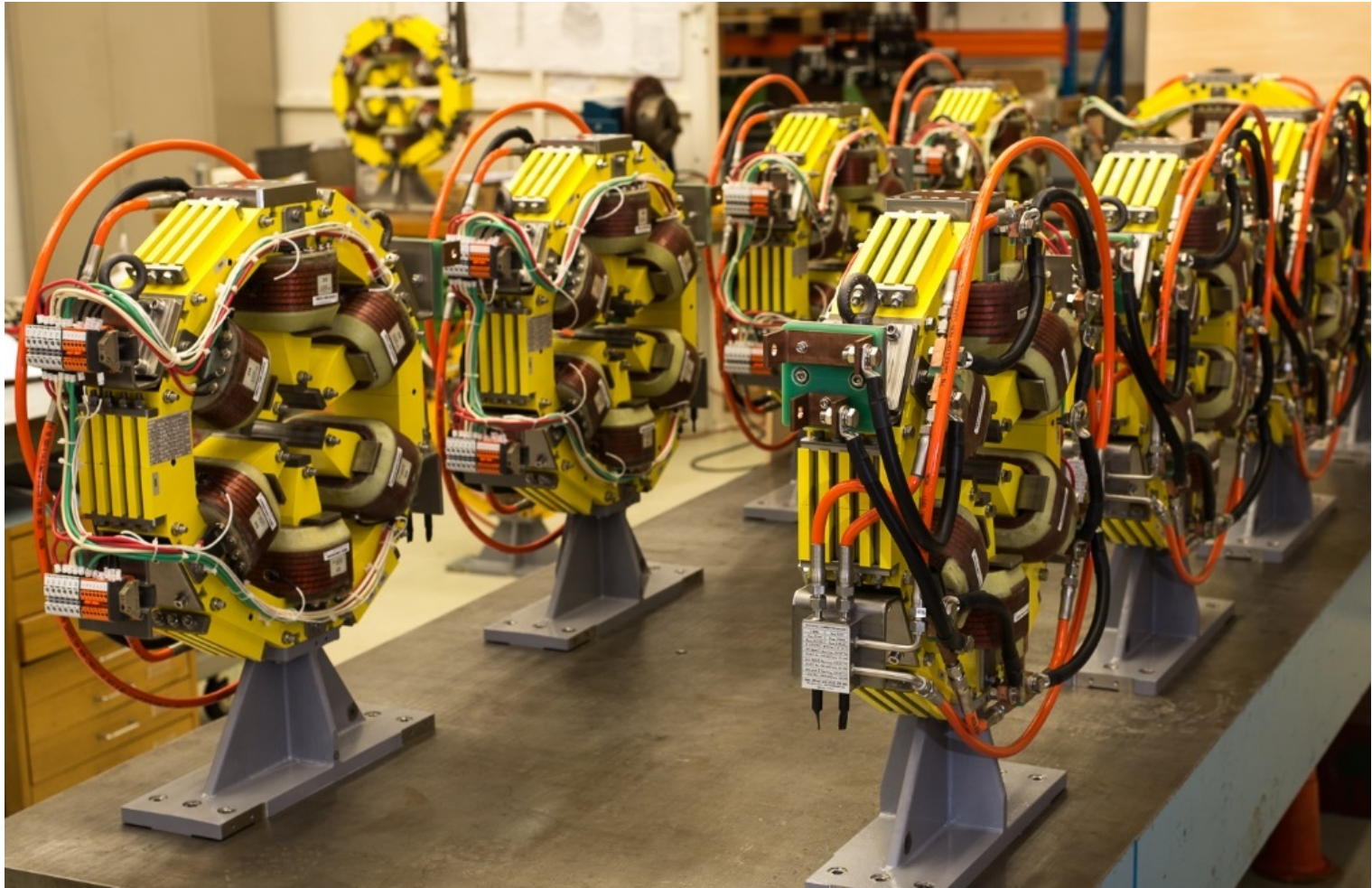


Rotating radial coil



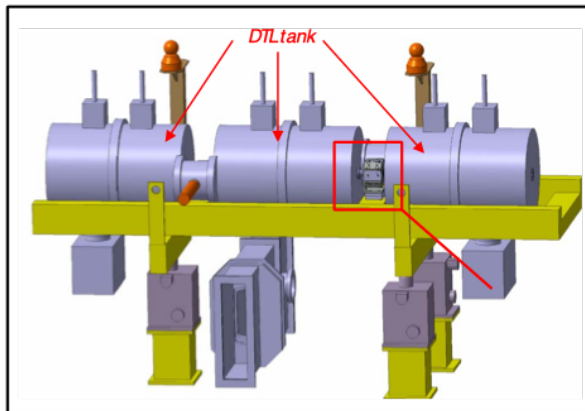
Mole Assembly Top View

- These are sextupoles (with embedded correctors) of the main ring of the SESAME light source



Remark: The field calculation of a PM follows a similar logic as for a resistive magnet (slide 9) using the path integral of Ampere's law supplemented with the magnetic flux conservation in the yoke. This can be found in Ref[5], section 4.5 page 138

Linac4 @ CERN  
permanent magnets ,  
quadrupoles



*Pictured: Cell-Coupled Drift Tube Linac module.*

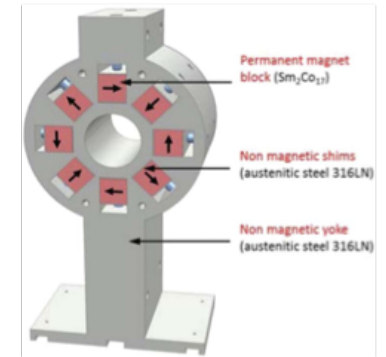
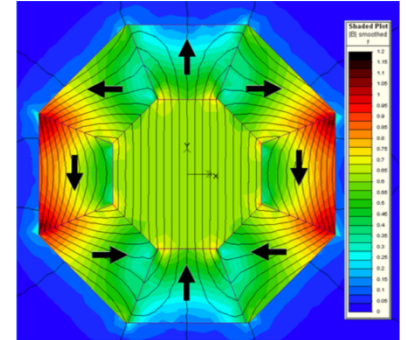
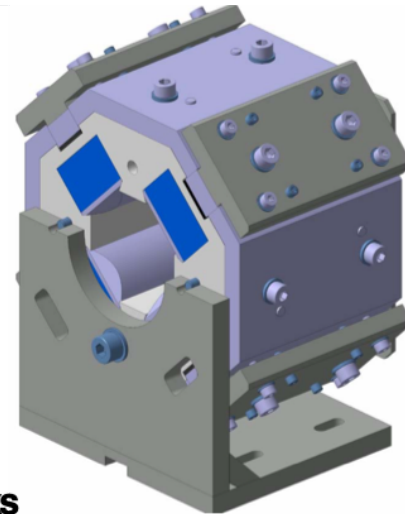
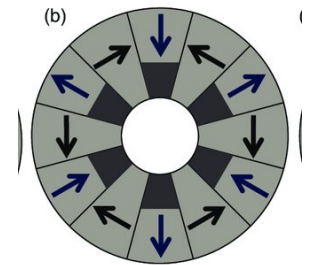
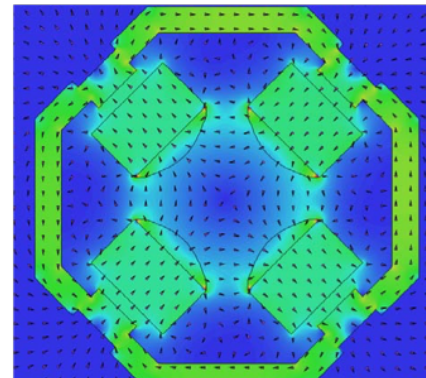


Fig. 1. Schematic layout of the Linac4 permanent-magnet quadrupole.

- Permanent magnet because of space between DTL tanks
- $\text{Sm}_2\text{Co}_{17}$  permanent magnets
- Integrated gradient of 1.3 to 1.6 Tesla
- 15 magnets
- Magnet length 0.100 m
- Field quality/ amplitude tuning blocks



Sextupole Hallback Array 39

Courtesy D. Tommasini, CERN

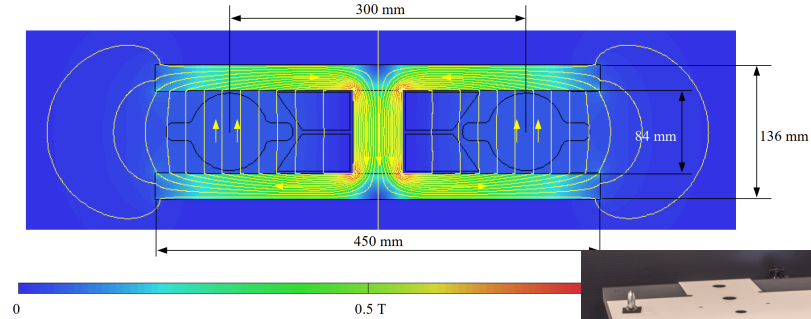




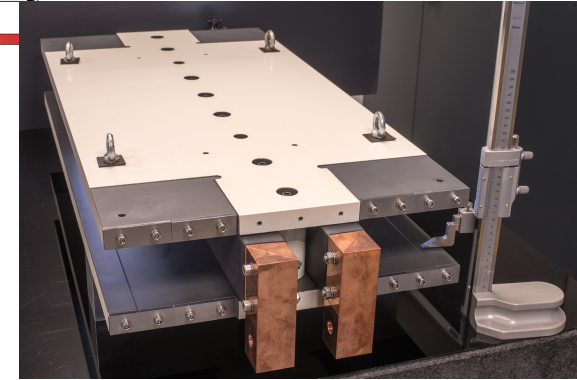
## Twin aperture dipole

Table 3.1: Parameters of the main bending magnets.

Strength, 45.6 GeV to 182.5 GeV	mT	14.1 to 56.6
Magnetic length	m	21.94 / 23.94
Number of units per ring		2900
Aperture (horizontal×vertical)	mm	130× 84
Good field region (GFR) in horizontal plane	mm	±10
Field quality in GFR (not counting quadrupole term)	$10^{-4}$	≈1
Central field	mT	57
Expected $b_2$ at 10 mm	$10^{-4}$	≈3
Expected higher order harmonics at 10 mm	$10^{-4}$	<1
Maximum operating current	kA	1.9
Maximum current density	A/mm <sup>2</sup>	0.79
Number of busbars per side		2
Resistance per unit length (twin magnet)	μΩ/m	22.7
Maximum power per unit length (twin magnet)	W/m	164
Maximum total power, 81.0 km (interconnections included)	MW	13.3
Inter-beam distance	mm	300
Iron mass per unit length	kg/m	219
Aluminium mass per unit length	kg/m	19.9



1 m model magnet

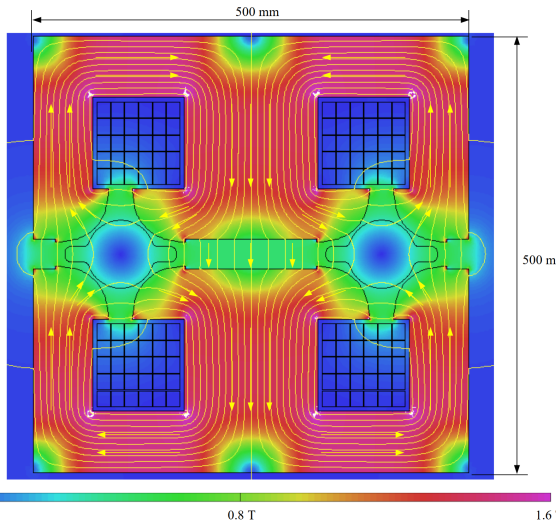


from: FCC-ee CDR: Eur. Phys. J. Special Topics 228, 261–623 (2019), A. Milanese et al

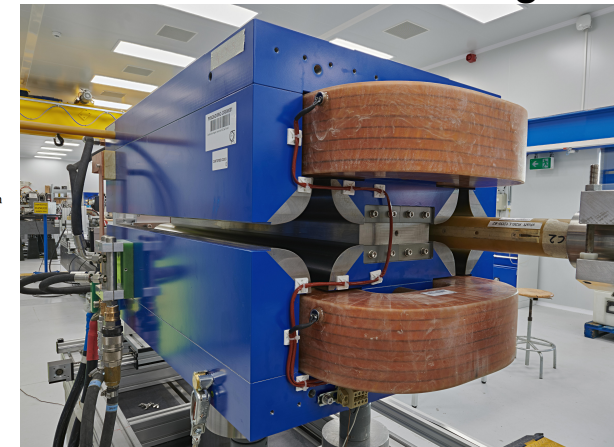
## Twin aperture quadrupole

Table 3.2: Parameters of the main quadrupole magnets.

Maximum gradient	T/m	10.0
Magnetic length	m	3.1
Number of twin units per ring		2900
Aperture diameter	mm	84
Radius for good field region	mm	10
Field quality in GFR (not counting dip. term)	$10^{-4}$	≈1
Maximum operating current	A	474
Maximum current density	A/mm <sup>2</sup>	2.1
Number of turns		2×30
Resistance per twin magnet	mΩ	33.3
Inductance per twin magnet	mH	81
Maximum power per twin magnet	kW	7.4
Maximum power, 2900 units (with 5% cable losses)	MW	22.6
Iron mass per magnet	kg	4400
Copper mass per magnet (two coils)	kg	820

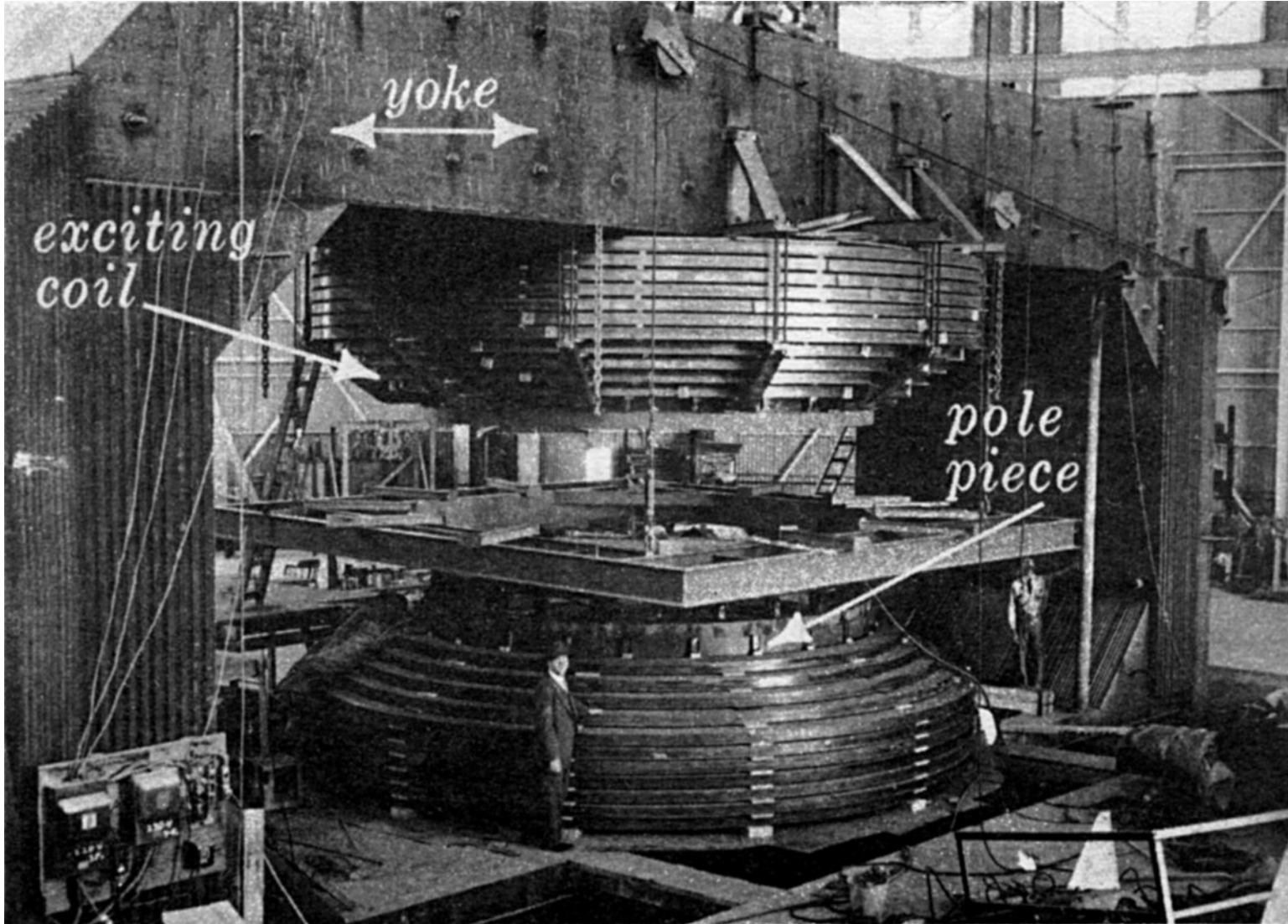


1 m model magnet



## Examples;

Some history, some modern regular magnets and some special cases



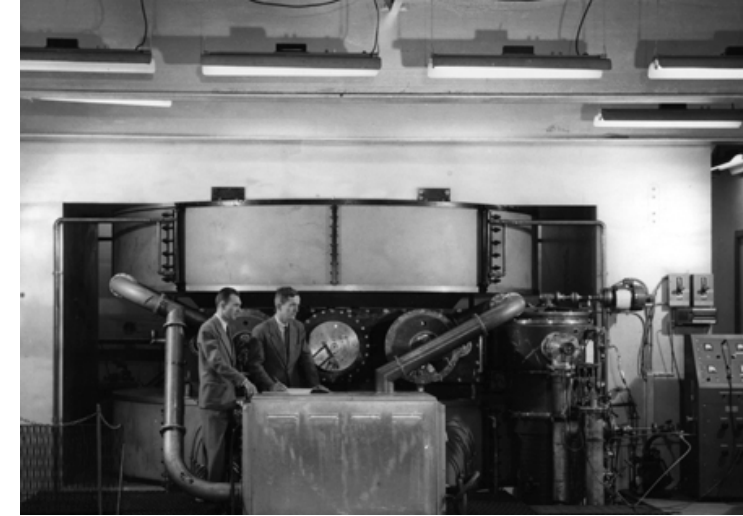
$B=1.6\text{ T}$

Courtesy A. Milanese, CERN



PSI= 590 MeV proton 1974

$B_{\text{peak}}=2 \text{ T}$

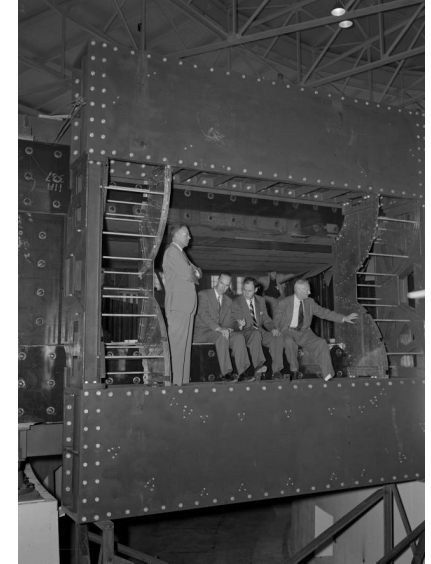


Harvard 1948

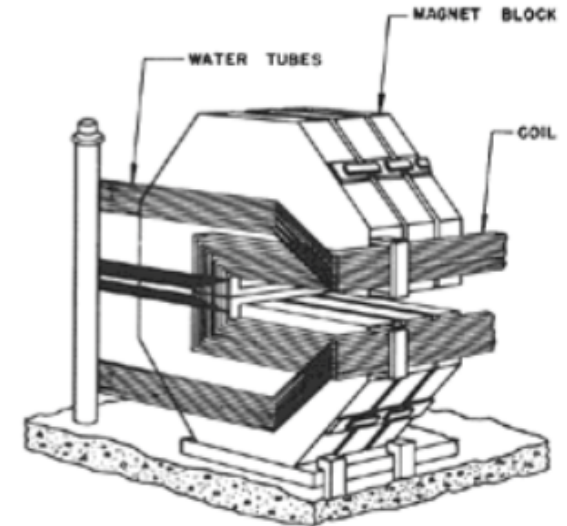
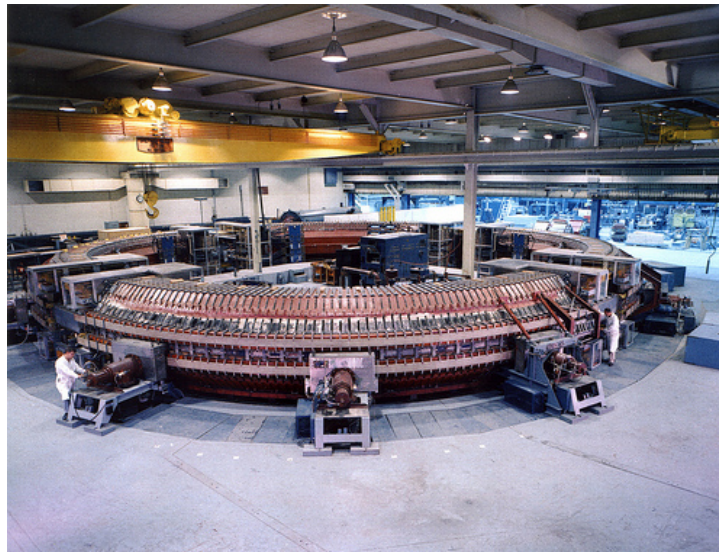
picture: CERN courier



Bevatron  
(Berkeley)  
1954, 6.2 GeV  
 $B=1.55\text{T}$



Cosmotron  
(Brookhaven)  
1953, 3.3 GeV  
 $B=1.4\text{T}$   
Aperture:  
20 cm x 60 cm



## Magnetic field:

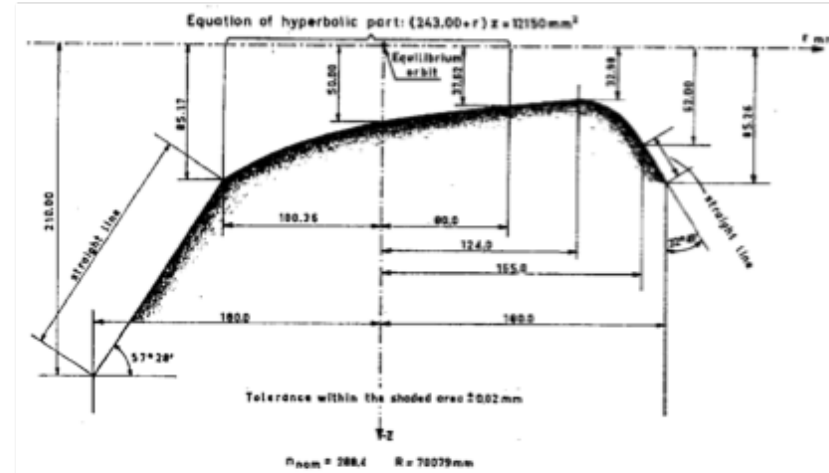
at injection  
for 24.3 GeV  
maximum  
Weight of one magnet unit

147 G  
1.2 T  
1.4 T  
38 t

Gradient @1.2 T : 5 T/m

Equipped with pole-face windings for higher order corrections

Water cooled Al race-track coils



FINAL POLE PROFILE.

Fig. 9: Final pole profile.

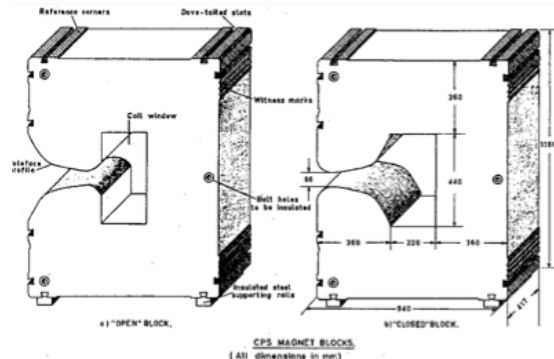
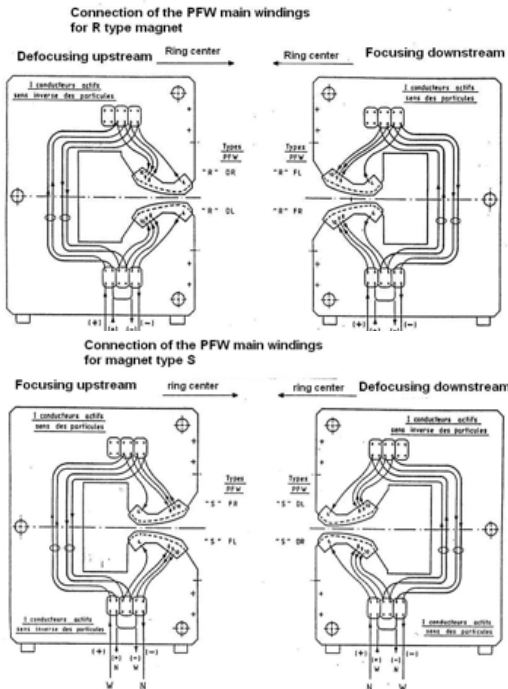


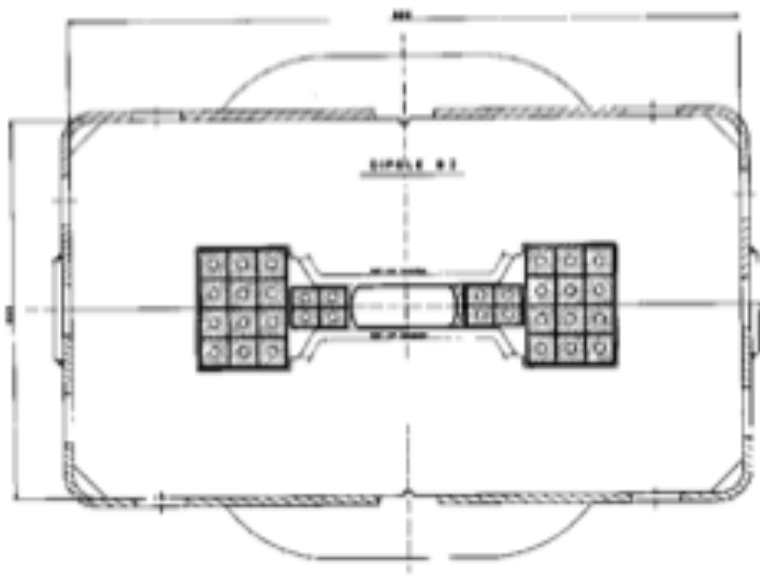
Fig. 12: Final form of the magnet blocks.

4 accelerator rings in a common yoke. (increase total beam intensity by 4 in presence of space charge limitation at low energy):  $B=1.48\text{ T @ }2\text{ GeV}$   
 Was originally designed for 0.8 GeV !



Courtesy D. Tommasini, CERN



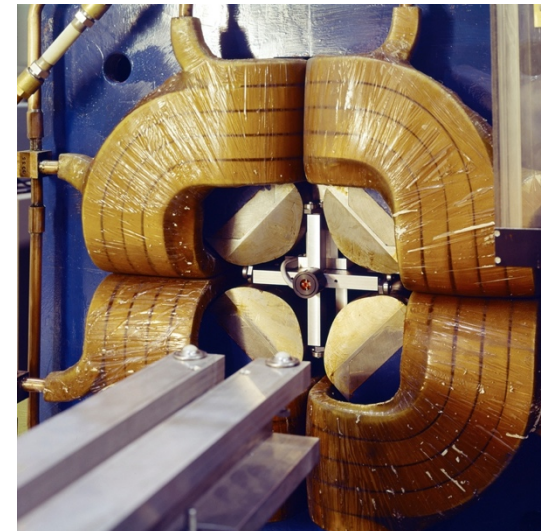
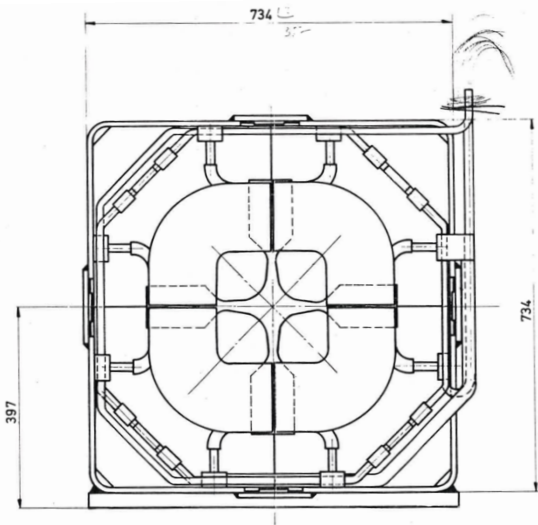


H magnet type MBB  
 $B = 2.05 \text{ T}$   
Coil : 16 turns  
 $I_{max} = 4900 \text{ A}$   
Aperture =  $52 \times 92 \text{ mm}^2$   
 $L = 6.26 \text{ m}$   
Weight = 17 t



CAS Santa Susanna, 24-Sept-2024, warm magnets, GdR





type MQ

$G = 20.7 \text{ T/m}$

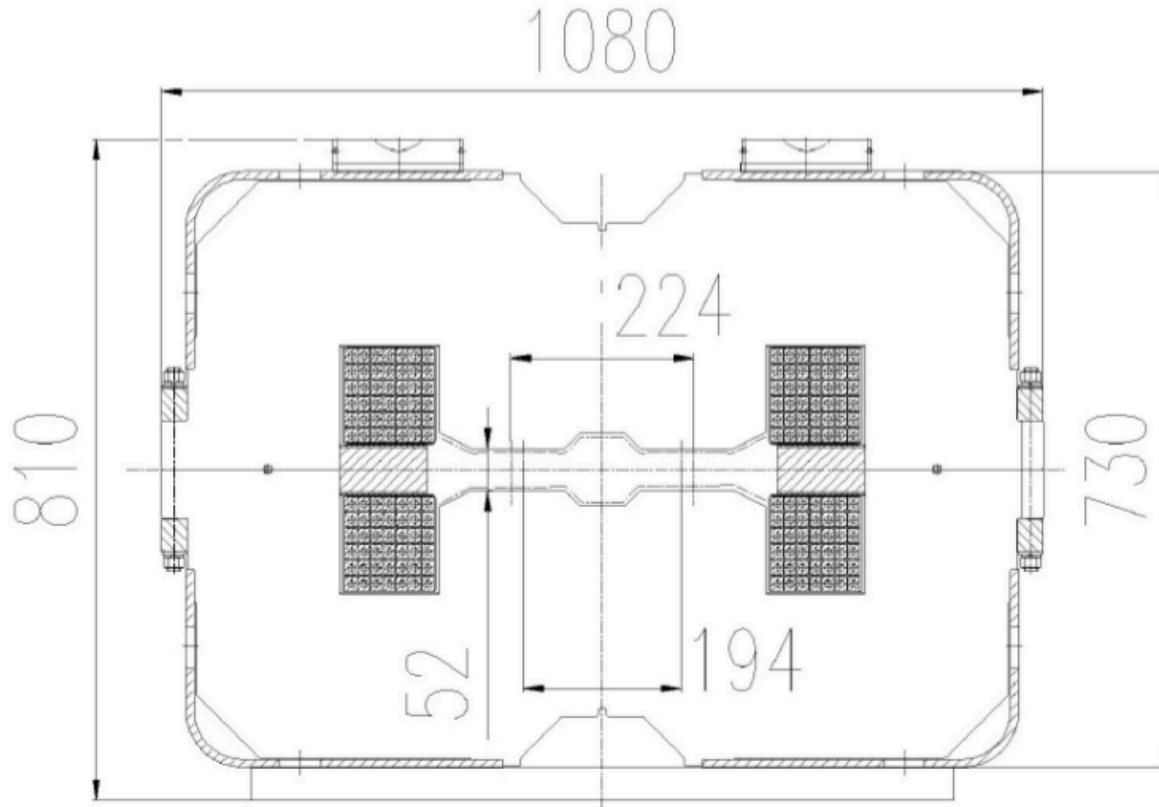
Coil : 16 turns

$I_{max} = 1938 \text{ A}$

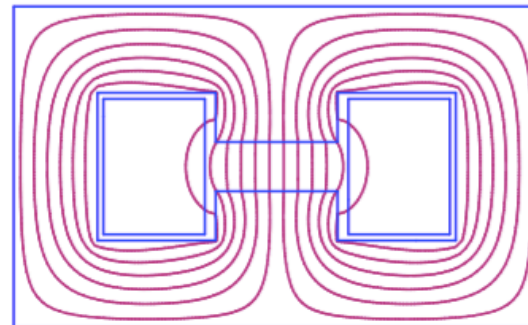
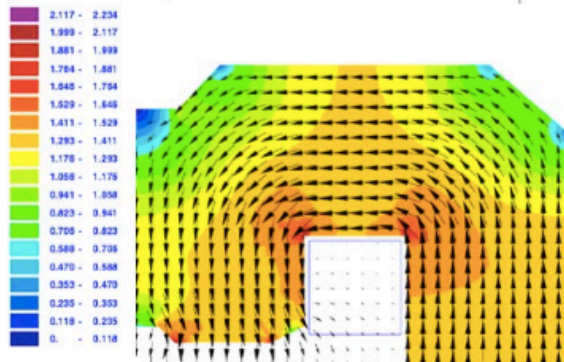
Aperture inscribed radius = 44 mm

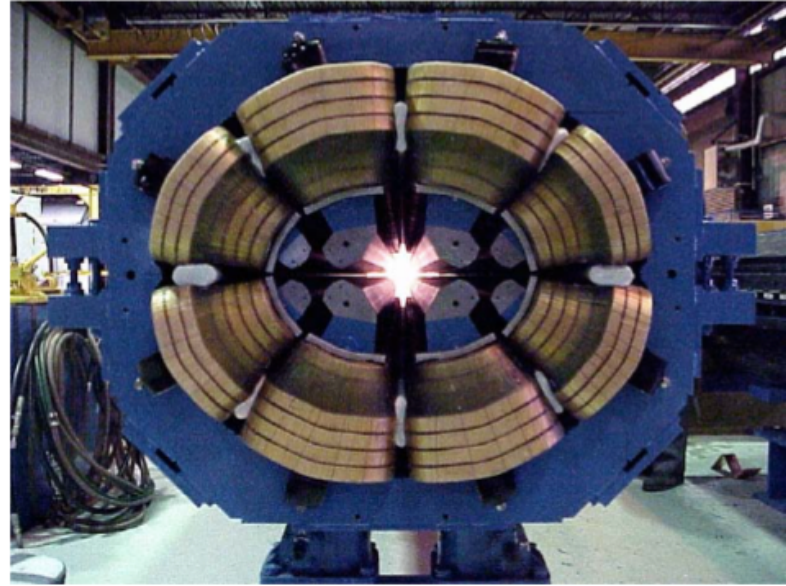
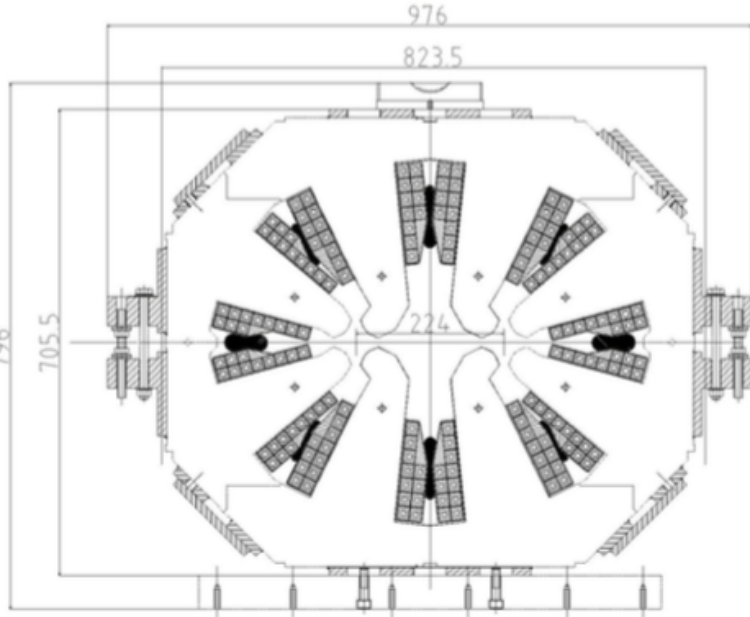
$L_{coil} = 3.2 \text{ m}$

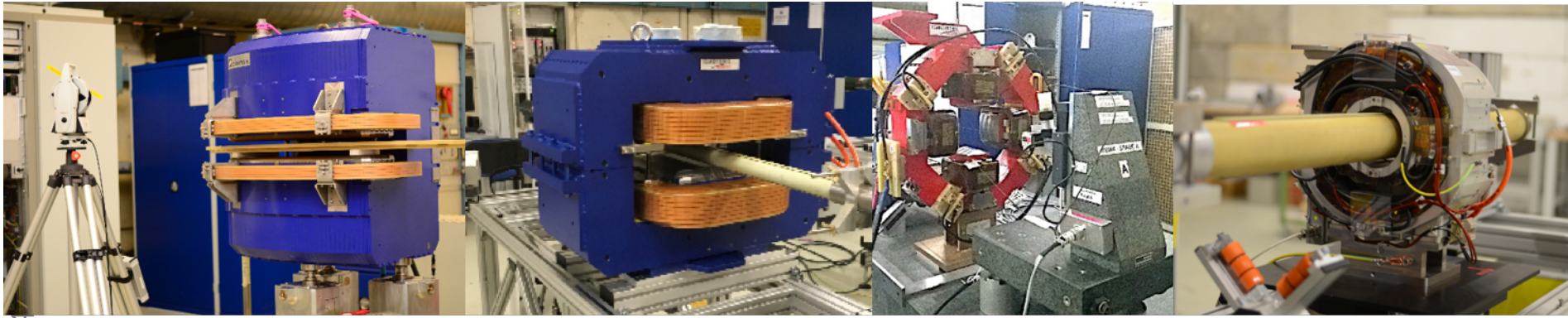
Weight = 8.4 t



Parameter	Value
Aperture	52 mm
Nominal field	1.42 T
Magnetic length	3.4 m
Weight	18 t
Water flow	19 l/min
Power	29 kW







Ring dipoles 8/8

TL dipoles 3/3

Skew quads 3/3

HV correctors 3/14



Courtesy A. Dael, CEA

- Books

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2. J. Tanabe, "Iron Dominated Electromagnets", World Scientific, ISBN 978-981-256-381-1, May 2005
3. P. Campbell, Permanent Magnet Materials and their Application, ISBN-13: 978-0521566889
4. S. Russenschuck, Field computation for accelerator magnets : analytical and numerical methods for electromagnetic design and optimization / Weinheim : Wiley, 2010. - 757 p.
5. R. Appleby, et al., The Science and Technology of Particle Accelerators. 2022, Open access: <https://library.oapen.org/handle/20.500.12657/53311>
6. A. Jain, "Basic theory of magnets", CERN 98-05 (1998) 1-26

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7. CAS Bruges, 2009, specialized course on magnets, 2009, CERN-2010-004
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9. CAS Varna 2010, Magnets (Warm) by D. Tommasini
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11. D. Tommasini, "Practical definitions and formulae for magnets," CERN, Tech. Rep. EDMS 1162401, 2011
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# Acknowledgements



For this lecture I used material from lectures, seminars, reports, etc. from the many colleagues. Special thanks goes to:

Davide Tommasini, Attilio Milanese, Antoine Dael, Stephan Russenschuck, Thomas Zickler

And to the people who taught me, years ago, all the fine details about magnets !



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