RF Systems

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CAS - Introduction to Accelerator Physics

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Outline – Part 1

- **Introduction – what is RF for accelerators?**
- **Simplified RF system and first RF-accelerator principles**

- **Choice of parameters for an RF system**
  - Frequency and voltage

- **RF cavity types and accelerating fields in an RF cavity**
  - $\lambda/2$ **or** $\lambda/4$ resonator
  - Principle of tuneable cavities

Introduction

What is Radio Frequency (for accelerators)?

Hz “Hertz” is the unit of frequency: sec^{-1}

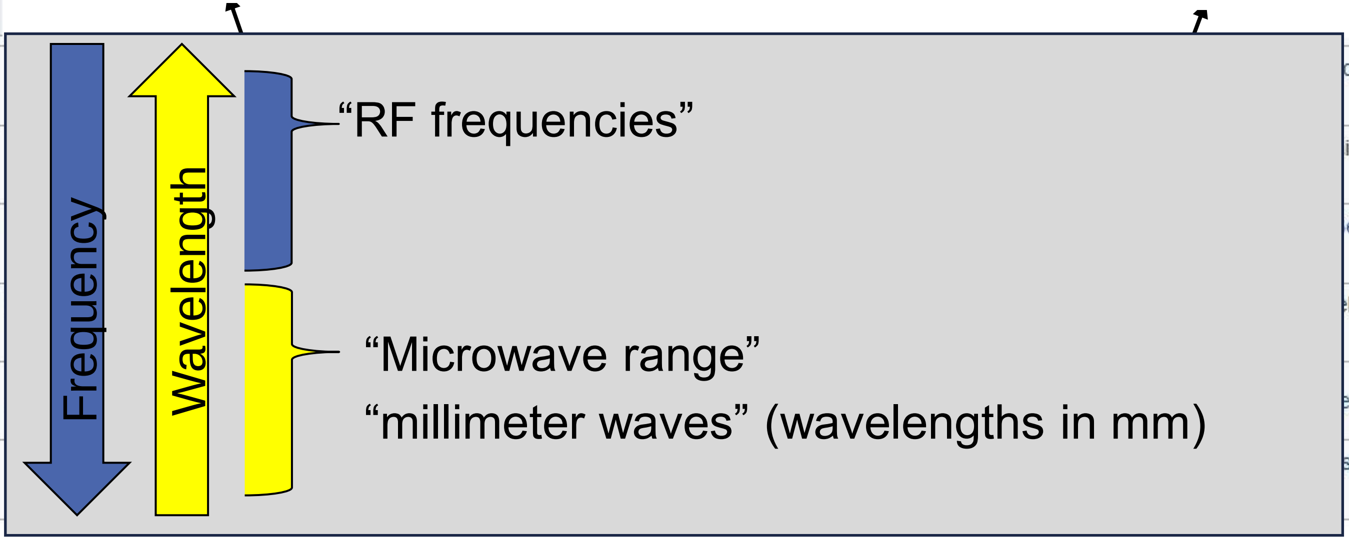
Frequency f and wavelength λ are inversely proportional:

Large frequency means small wavelength.

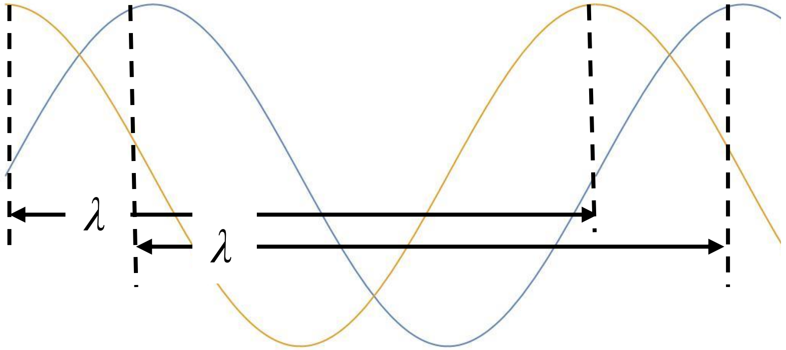
1 MHz = 10^6 Hz (Megahertz)

1 GHz = 10^9 Hz (Gigahertz)

Band name	Abbreviation	ITU band number	Frequency and Wavelength	
High frequency	HF	7	3–30 MHz 100–10 m	omatic link establishment (ALE) /
Very high frequency	VHF	8	30–300 MHz 10–1 m	ications, amateur radio, weather
Ultra high frequency	UHF	9	300–3,000 MHz 1–0.1 m	ee, GPS and two-way radios such
Super high frequency	SHF	10	3–30 GHz 100–10 mm	llite television broadcasting, DBS,
Extremely high frequency	EHF	11	30–300 GHz 10–1 mm	scanner, Wireless Lan 802.11ad.
Terahertz or Tremendously high frequency	THz or THF	12	300–3,000 GHz 1–0.1 mm	spectroscopy, terahertz



Source: en.wikipedia.org/wiki/Radio_spectrum



Speed of light in vacuum…

$$\lambda f = c_0 = v_p$$

Phase velocity “everywhere else”.

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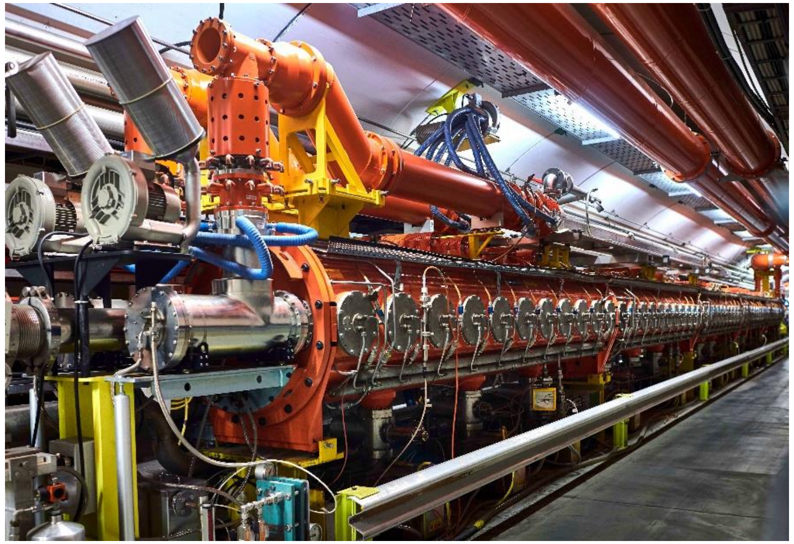
What is Radio Frequency (for accelerators)?

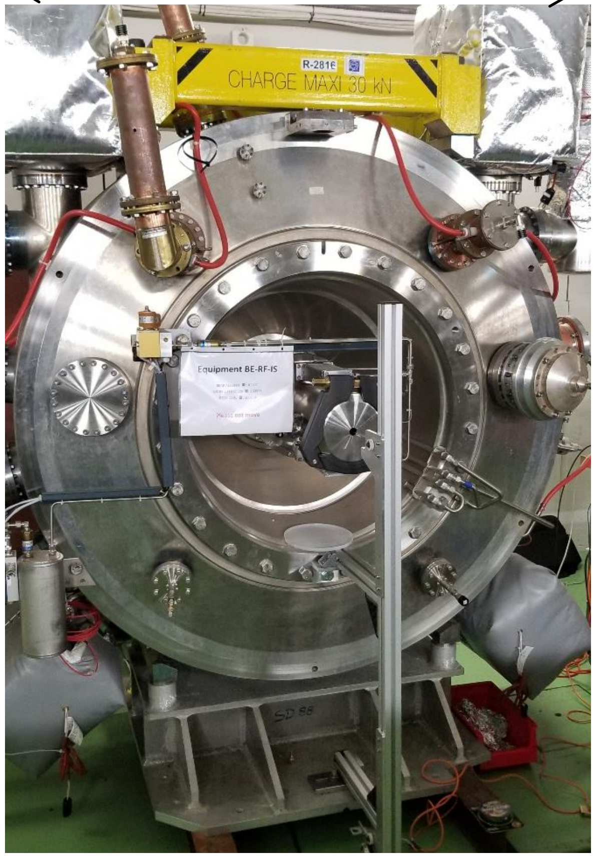
Source: en.wikipedia.org/wiki/Radio_spectrum

Band name	Abbreviation	ITU band number	Frequency and Wavelength
High frequency	HF	7	3–30 MHz 100–10 m
Very high frequency	VHF	8	30–300 MHz 10–1 m
Ultra high frequency	UHF	9	300–3,000 MHz 1–0.1 m
Super high frequency	SHF	10	3–30 GHz 100–10 mm
Extremely high frequency	EHF	11	30–300 GHz 10–1 mm

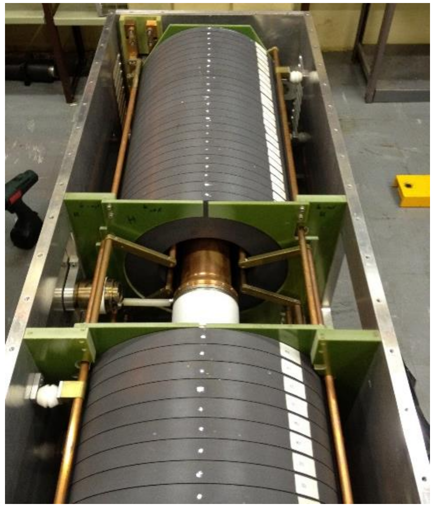


Travelling wave cavity, freq = 200 MHz
Total length: 12 & 16 m. (CERN SPS)

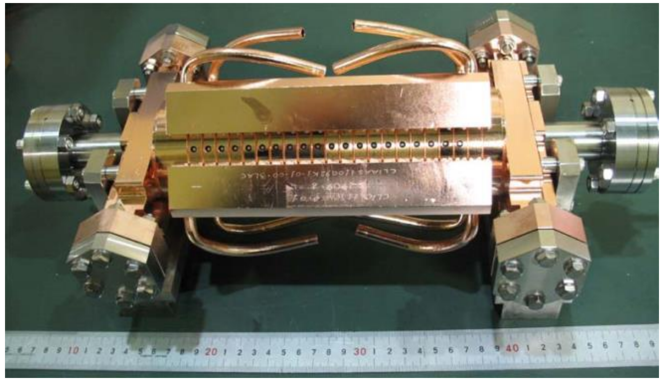
approx. 2 m



Accelerating Cavity, freq = 80 MHz
(CERN PS)



Ferrite Loaded Cavity,
freq = 3 – 8 MHz
(CERN PS Booster)



CLIC structure, freq = 12 GHz

What is Radio Frequency (for accelerators)?

- **RF domain is huge!**
- **The radiofrequency (RF) system does the actual acceleration transforms a string of magnets into an accelerator.**
- **Cavity is the most visible part of an RF system**
  - → **On top of the RF system food chain**
  - → **Interacts directly with beam**
  - → **Provides acceleration and also RF manipulations, but also some unwanted effects (e.g. beam coupling impedance contribution)**
  - → **Cavity is no “stand-alone” element…**
  - → **… requires RF signals generated (LLRF system) to be operated successfully and power input to transfer energy to the beam.**

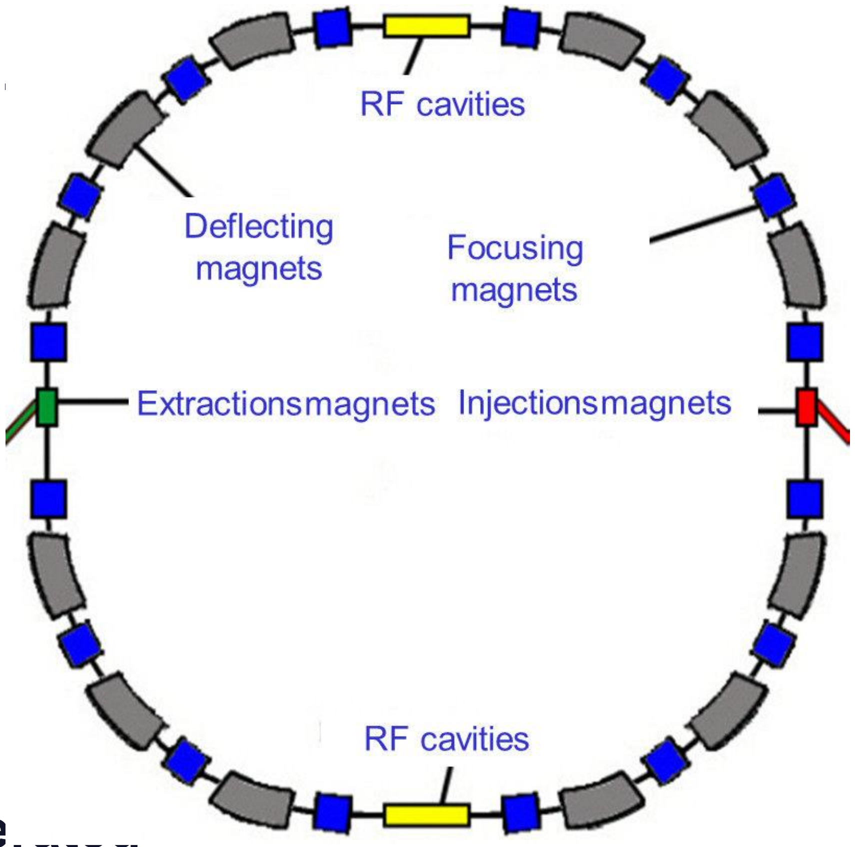
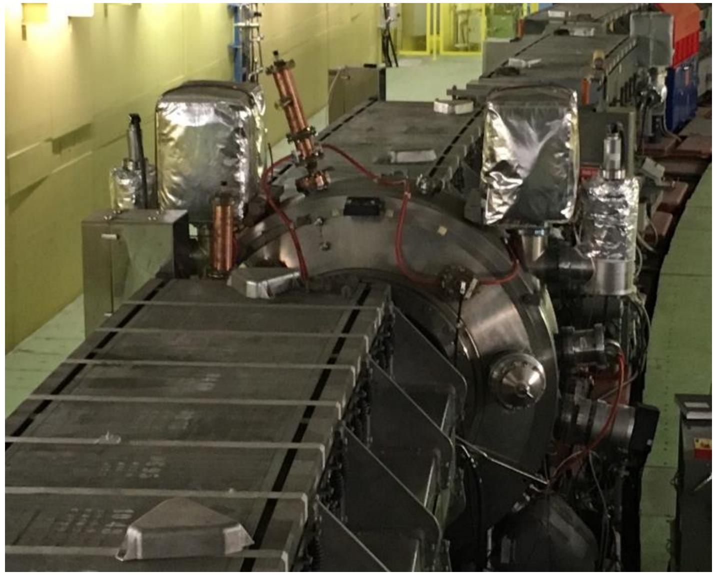


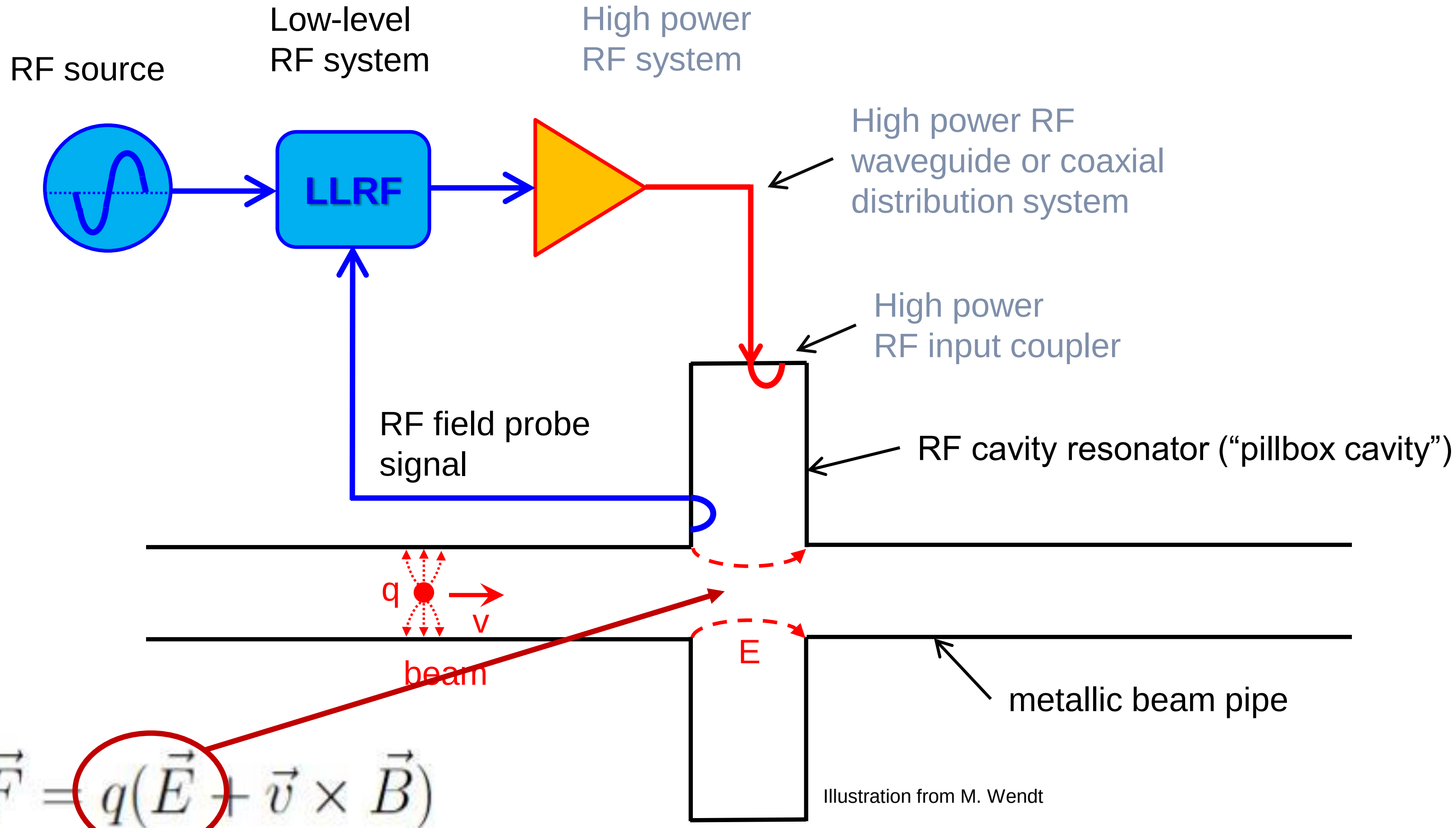
Image from I. Shreyber

→ **The LLRF system picks up a signal or several signals from the cavity pick-up and a signal from the phase reference line system to calculate the required amplitude and phase in the cavity.**

A simplified RF System



PS single cell cavity (“pillbox”)

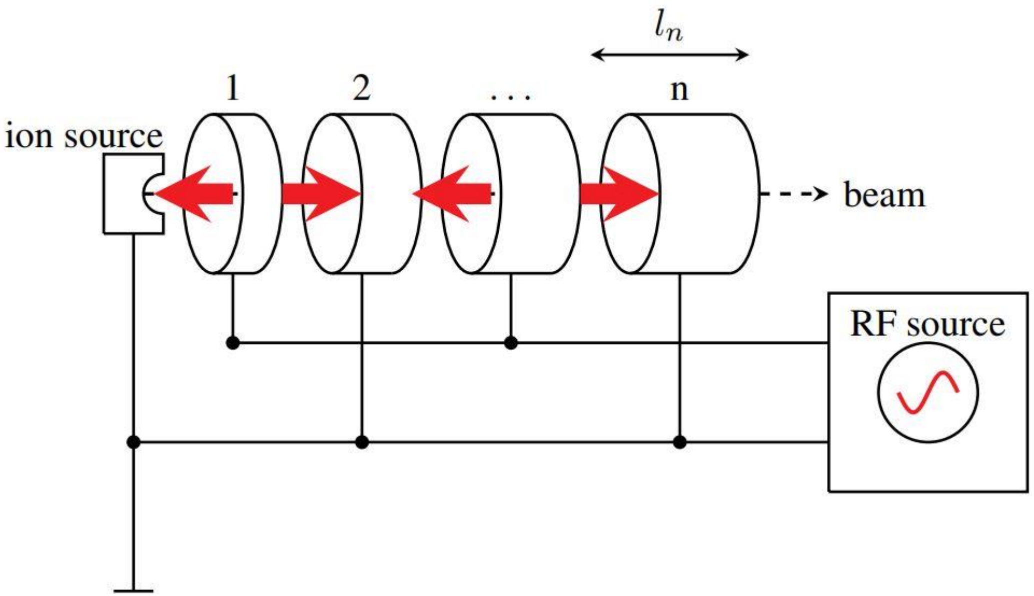


$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

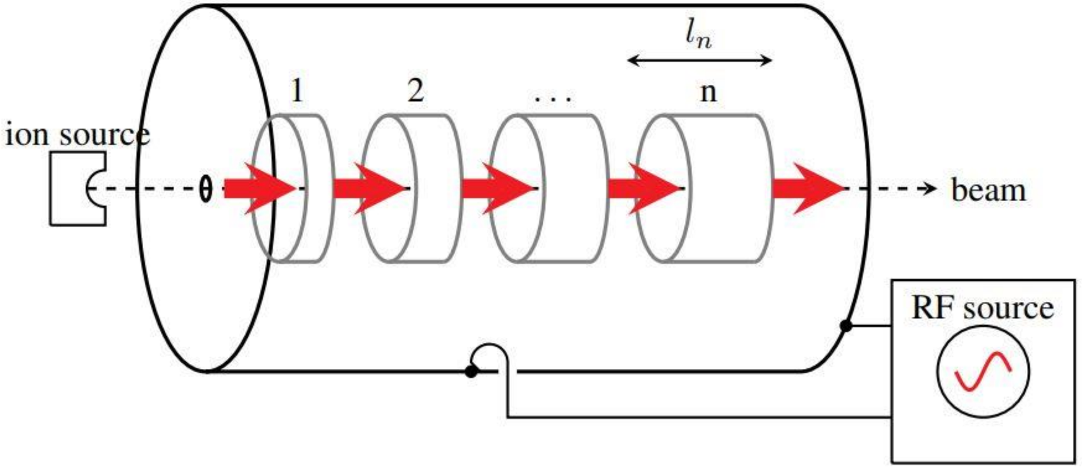
Recall: Lorenz force will only accelerate if the E-field is synchronized with the beam (synchronicity condition).

First accelerators – drift tube linacs type

Images: F. Gerigk, CAS Ebeltoft, *Cavity Types*, 2010



Wideroe-type linac (= drift-tube-linac)

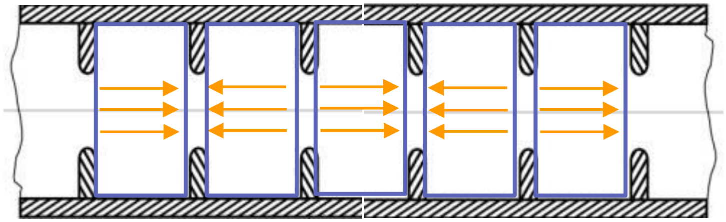


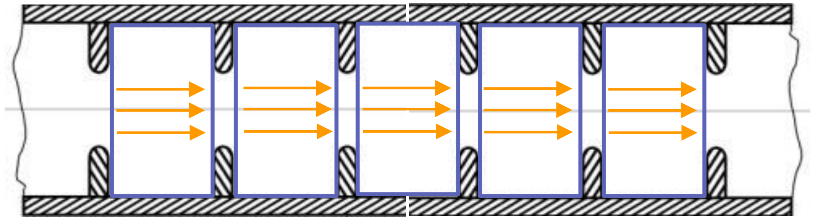
Alvarez-type linac (= drift-tube-linac)

Pi-mode structure

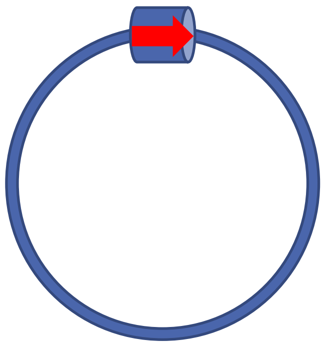
0-mode structure

Naming: The name of the resonant mode is given by the phase advance between two consecutive cells.





RF system for high-energy accelerators



Accelerator type

- **Linear (single pass)**
  - **Electrons** → Maximum RF voltage, constant velocity
  - **Hadrons** → Maximum RF voltage, variable velocity
- **Circular (multi-pass)**
  - **Electrons** → Compensate synchrotron radiation losses
  - **Hadrons** → Increase particle energy & Sweep frequency with beam*

$$\boxed{L_n = v_n \cdot T_{RF}}$$

$$\boxed{f_{RF} = h \cdot f_{rev}}^{*}$$

Choice of RF parameters includes:

- frequency (range), or: shall we apply high or low RF frequency?
- Minimum voltage requirement, within limitations…

*Exceptions (rare) exist,
e.g. "fixed frequency" acceleration in SPS

L_n = Distance between two adjacent gaps

Why choose a low RF frequency?

Advantages	Disadvantages
• Large beam aperture • Long RF buckets, large acceptance • Wide-band or wide range tunable cavities possible • Power amplification and transmission straightforward	• Bulky cavities, size scales $\propto 1/f$, volume $\propto 1/f^3$ • To downsize cavities, often lossy material used • Moderate or low acceleration gradient • Short particle bunches difficult to generate

RF frequencies below ~200 MHz for

→ Mainly electrons, some hadron linear accelerators
→ Cyclotrons
→ Low- and medium energy hadron synchrotrons

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Why choose a high RF frequency?

Advantages	Disadvantages
• Reasonable cavity size scales $\propto 1/f$, volume $\propto 1/f^3$ • Break down voltage increases • High gradient per length • Particle bunches are short	• Maximum beam available aperture scales $\propto 1/f$ • No technology for wide-band or tunable cavities • Power amplifiers more difficult • Power transmission losses

RF frequencies above ~200 MHz used for

→ Linear accelerators
→ Electron storage rings
→ High energy hadron storage rings

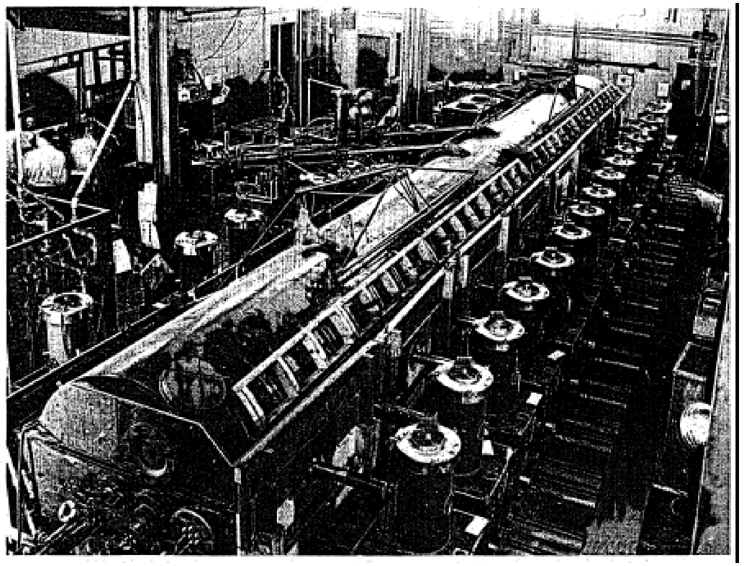
Small side remark:

Luis Alvarez and the Drift Tube Linac

The war effort forced to develop the competences and gave the components to go to higher frequencies (in the MHz – GHz range) .

Alvarez tried acceleration of a proton beam to the MeV range using the Wideröe principle.

He worked at MIT on radar during the war. In 1945, he had the tools and the competences to build his own accelerator.



The 1st Drift Tube Linac by L. Alvarez and his team at Berkeley, reaches 32 MeV in 1947.

Following M. Vretenar, RF CAS 2023

Choice of Frequency :
Alvarez received from the US Army a stock of 2'000 (!) surplus **202.56 MHz** transmitters, produced for a radar surveillance system.
26 were installed to power the DTL with a total of 2.2 MW.

They were soon replaced because unreliable, but this frequency remained as the standard linac frequency.

Some standard frequencies = your choice

If exact RF frequency not critical, choose standard value

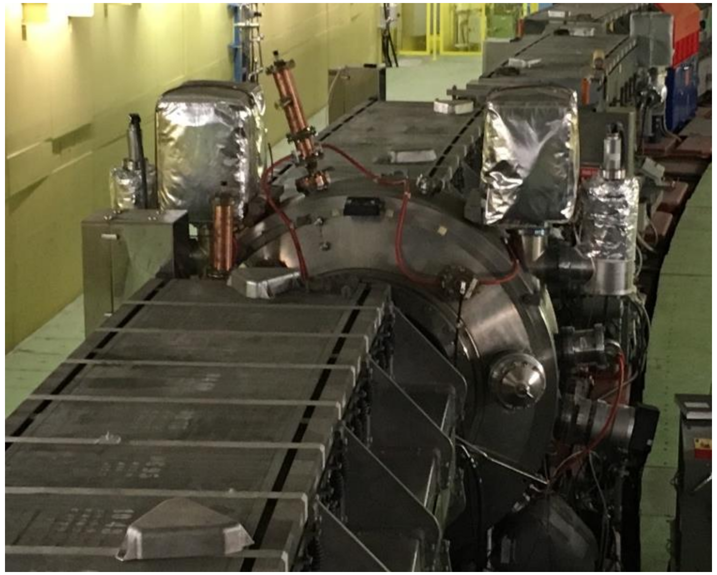
Accelerator	Frequency
Hadron synchrotrons (PSB, PS, JPARC RCS, MR)	<10 MHz
Hadron accelerators and storage rings (RHIC, SPS)	~200 MHz
Electron storage rings (LEP, ESRF, Soleil)	352 MHz
Electron storage rings (DORIS, BESSY, SLS,…)	499.6…499.8 MHz
Superconducting electron linacs and FELs (X-FEL, ILC)	1300 MHz
Normal conducting electron linacs (SLAC)	2856 MHz
High-gradient electron linac (CLIC)	11.99 GHz

→ **Off-the-shelf RF components easily available in frequency ranges used by industry.**

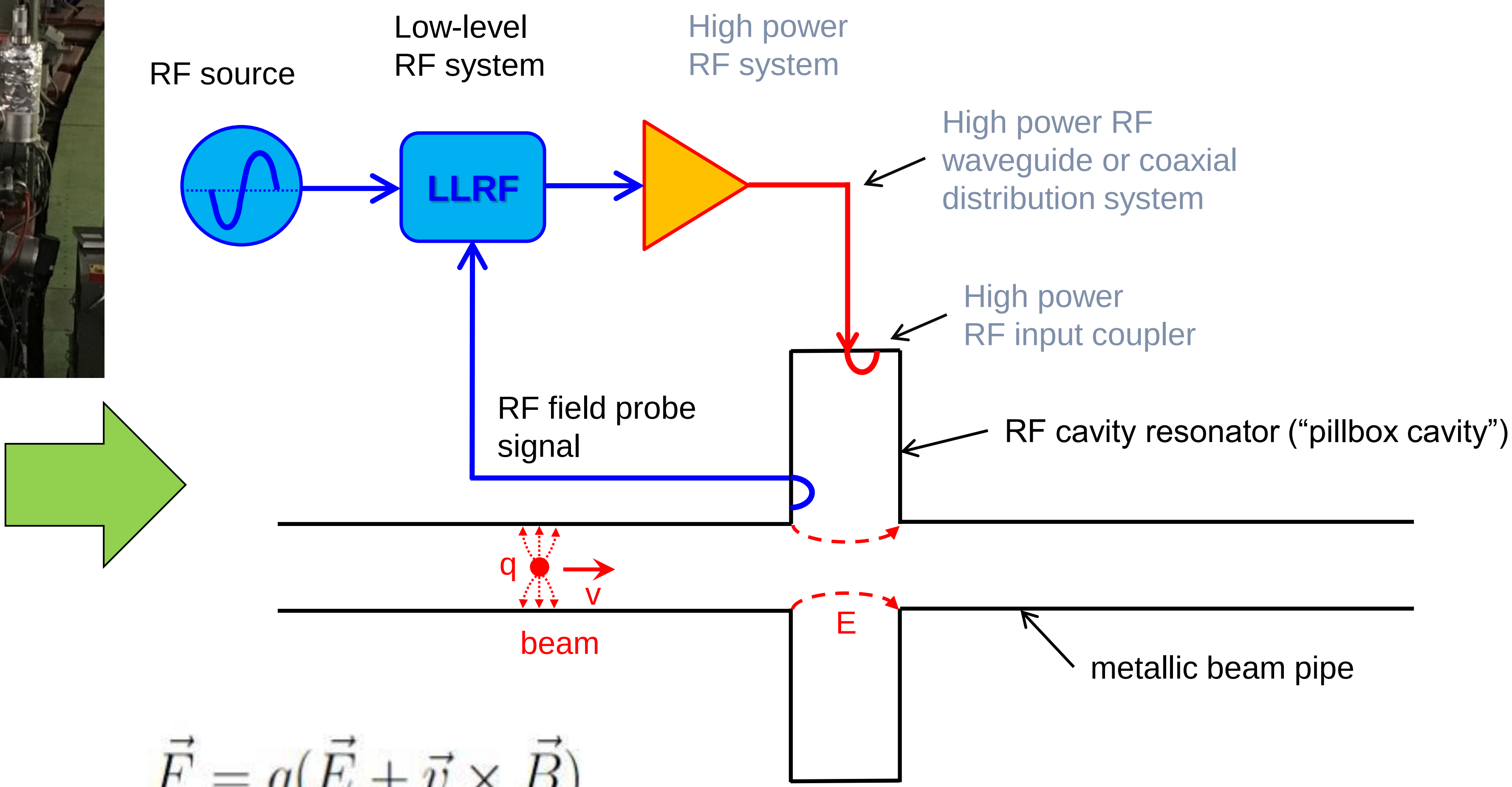
→ **Exchange of developments and equipment amongst research laboratories.**

→ **Exchange expertise with colleagues from other labs!**

How to get to the electric field of this RF cavity?



PS single cell cavity
("pillbox")



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Illustration from M. Wendt

How to get to the electric field of this RF cavity?

"Just apply Maxwell's Equations*…
... Simply superpose a forward and backward travelling wave to get a standing wave…
… terminate a waveguide with two conducting walls…
… use the EM-fields of a waveguide…
… cut the inner conductor of a coaxial line…"

*for source-free, time-harmonic, linear, isotropic media.

or maybe not?

$$\nabla \cdot \vec{D} = \rho \quad \text{or} \quad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

Wave equation (=Helmholtz' eq.)!

$$\nabla^2 \vec{E} + \omega^2 \mu\epsilon \vec{E} = 0$$

$$\nabla^2 \vec{H} + \omega^2 \mu\epsilon \vec{H} = 0$$

define

$$\boxed{k = \omega\sqrt{\mu\epsilon}}$$

Propagation constant k
(or: phase constant, or wave number, or separation constant), with unit 1/m

Basic Wave Equation – Plane wave

$$\nabla^2 \vec{E} + \omega^2 \mu \epsilon \vec{E} = 0$$

This is the equation to solve…

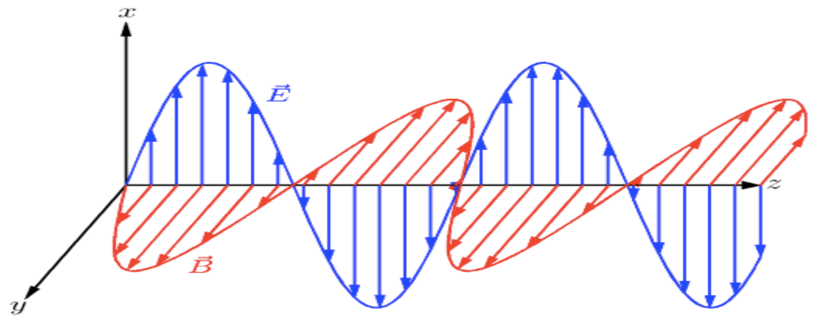


Image from I. Shreyber

Simplest solution is: Plane wave (no variation in x and y).
We assume propagation in z-direction:

E^+ and E^- denote the wave amplitudes for travelling in positive and negative z-direction.

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

Exponential function in frequency domain

$$E_x(z,\omega) = E^+ e^{-jkz} + E^- e^{jkz}$$

Real part of e-function in time domain

$$E_x(z,t) = Re\{E_x(x,\omega) e^{j\omega t}\} = E^+ \underbrace{\cos(\omega t - kz)}_{\text{wave travelling in positive z-direction}} + E^- \underbrace{\cos(\omega t + kz)}_{\text{wave travelling in negative z-direction}}$$

If you don't see this so quickly:

$$(\omega t - kz)$$

Keep the cosine argument constant to maintain a fixed point of the wave: for increasing time, z has to increase as well → this describes a propagation in positive z-direction.

Basic Wave Equation – Plane wave

- Plane wave = E-field and H-field are in phase.
- In our equipment, boundary conditions lead to a phase difference in the EM-fields.

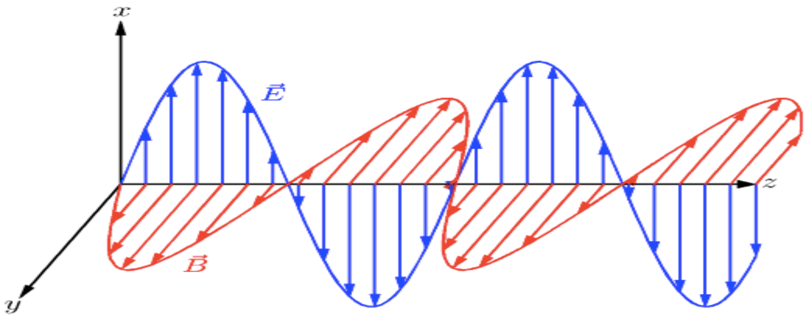


Image from I. Shreyber

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

Exponential function in frequency domain

$$E_x(z,\omega) = E^+ e^{-jkz} + E^- e^{jkz}$$

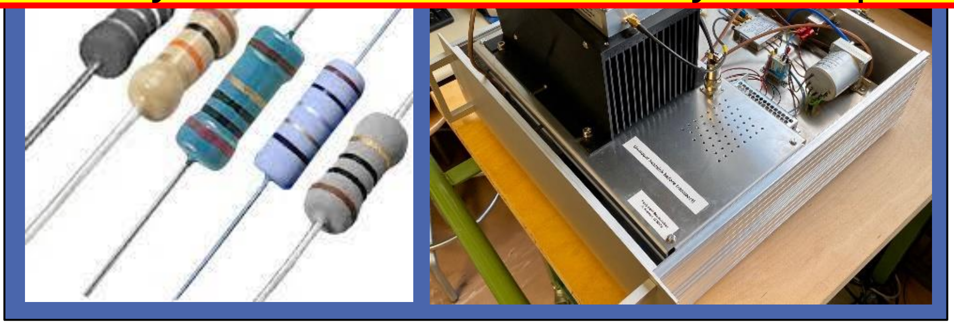
Real part of e-function in time domain

$$E_x(z,t) = Re\{E_x(x,\omega)e^{j\omega t}\} = E^+\cos(\omega t - kz) + E^-\cos(\omega t + kz)$$

In RF, almost all equipment sees EM-field effects, and needs to be treated accordingly.

This leads to Transmission Line Theory or Classical Field Theory concepts.

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial D}{\partial t}}$$



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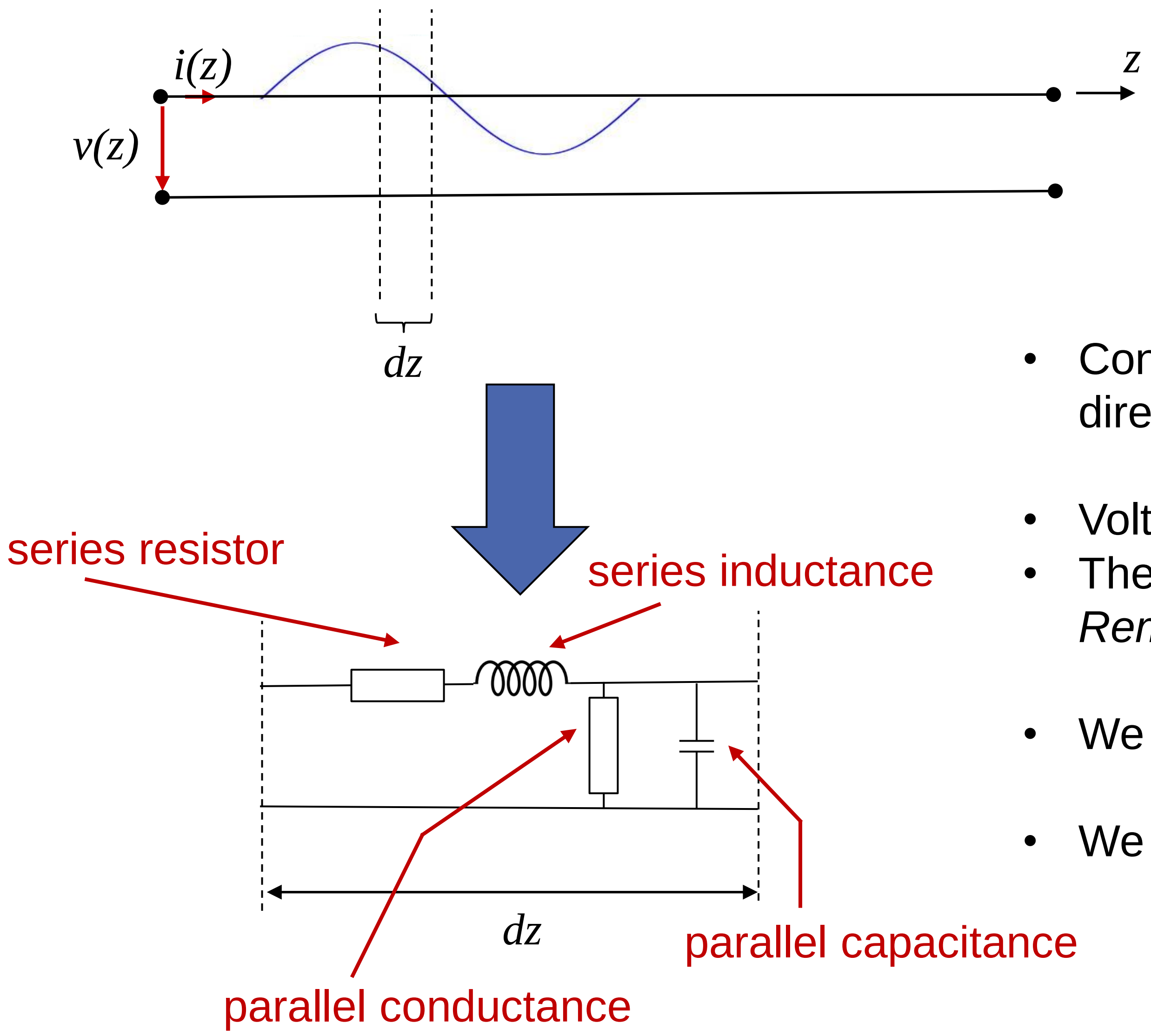
Basic Transmission Line Theory



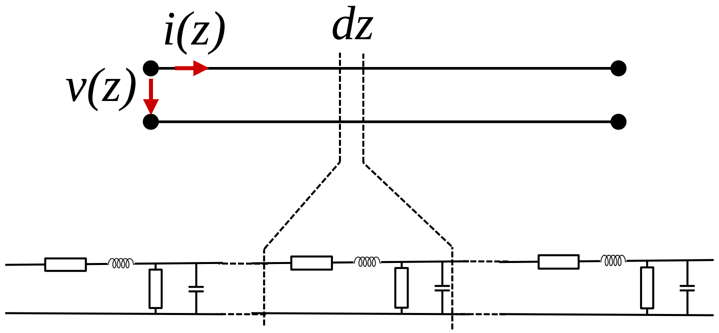
- Consider a transmission line of a certain length aligned in z-direction.

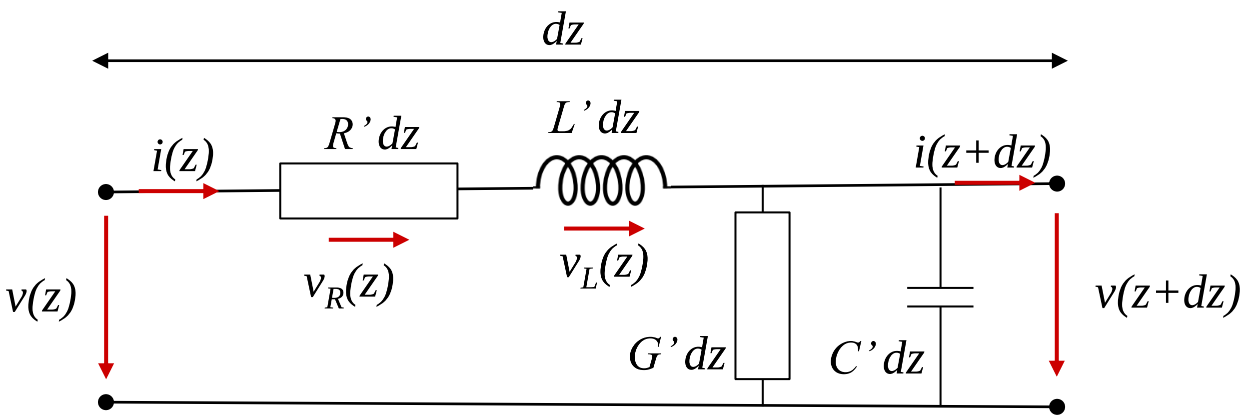
- Voltage and current along the line depend on z.
- Their variation along the line depends on their wavelength. *Remember? High frequency comes with short wavelength.*

- We split the line in infinitesimal increments of length dz.

- We model the length dz with lumped elements.

Basic Transmission Line Theory





- Consider a transmission line of length dz.
- Voltage and current along the line depend on z.
- We model the line length dz with lumped elements.
- We express all lumped elements per unit length dz :

$$R' = \frac{R}{dz}$$

resistance in $[\Omega/m]$

$$L' = \frac{L}{dz}$$

inductance in $[H/m]$

$$G' = \frac{G}{dz}$$

conductance in $[S/m]$

$$C' = \frac{C}{dz}$$

capacitance in $[F/m]$

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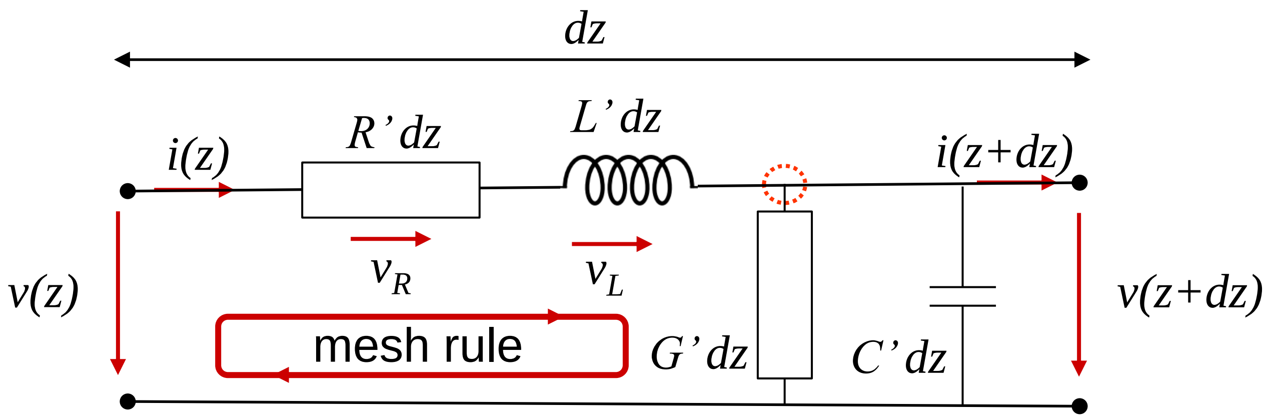
Basic Transmission Line Theory



$$i(z) = i_{\mathrm{C}} + i_{\mathrm{G}} + i(z+dz)$$

$$i(z) = j\omega C'\, dz\, v(z+dz) + G'\, dz\, v(z+dz) + i(z+dz)$$

$$v(z) = v_{\mathrm{R}} + v_{\mathrm{L}} + v(z+dz)$$

$$v(z) = R'\, dz\, i(z) + j\omega L'\, dz\, i(z) + v(z+dz)$$

$$\underbrace{v(z+dz) - v(z)}_{\frac{dv}{dz} dz} = -R'\, dz\, i(z) - j\omega L'\, dz\, i(z)$$

$$\frac{dv}{dz} dz = -(R'\, dz + j\omega L'\, dz) i(z)$$

$$\boxed{\frac{di}{dz} = -(G' + j\omega C')\, v(z)}$$

$$\boxed{\frac{dv}{dz} = -(R' + j\omega L')\, i(z)}$$

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Basic Transmission Line Theory

Transmission line equations

$$\boxed{\frac{di}{dz} = -(G' + j\omega C')\, v(z)}$$

$$\boxed{\frac{dv}{dz} = -(R' + j\omega L')\, i(z)}$$

2nd derivative

& combine…

$$\frac{d^2 i}{dz^2} = (R' + j\omega L')(G' + j\omega C')\, i(z)$$

$$\frac{d^2 v}{dz^2} = \underbrace{(R' + j\omega L')(G' + j\omega C')}_{\gamma^2}\, v(z)$$

This equation, we know already!
Mathematically, it is the same type as the one-dimensional, scalar Helmholtz' equation for EM-fields.

$$I(z,t) = I(z)e^{j\omega t} = (I^+ e^{-\gamma z} + I^- e^{+\gamma z})e^{j\omega t}$$

$$V(z,t) = V(z)e^{j\omega t} = (V^+ e^{-\gamma z} + V^- e^{+\gamma z})e^{j\omega t}$$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\gamma = \alpha + j\beta$$

phase constant

attenuation constant

Complex propagation constant

Two categories of electromagnetic waves

Waves in unbounded media.

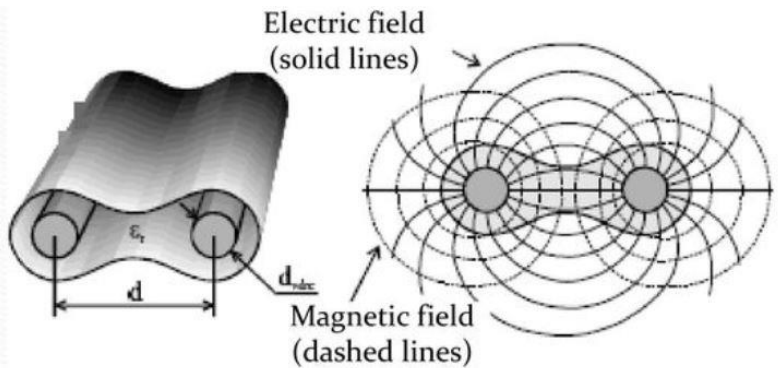
Waves confined by material boundaries, so-called guided waves.

Transmission Lines and Waveguides

Transmission Lines and Waveguides

- Wave patterns in guided wave systems depend on the number of conductors used.

"Open" two-wire system (TEM)

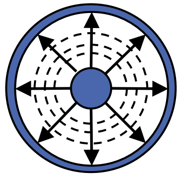


Source: Ma, *Electromagnetic Waves and Applications Part III, univ. lecture*

TEM-propagation = 2 conductor system

Coaxial line (TEM)

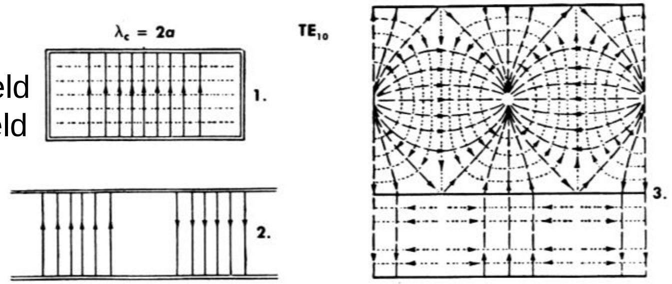
Picture: Coaxial loads for the SPS 200 MHz cavity © CERN



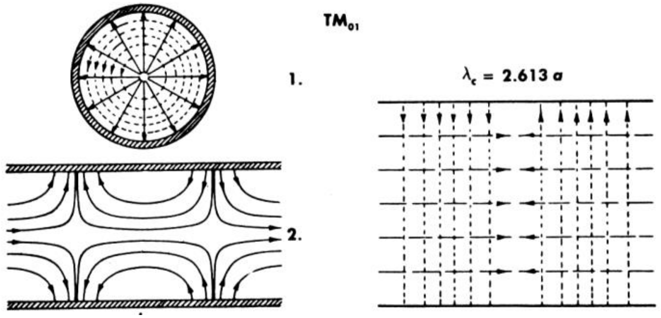
—— E-field ---H-field

Uniform waveguides rectangular, round, random cross-section (all non-TEM)

— E-field
---H-field



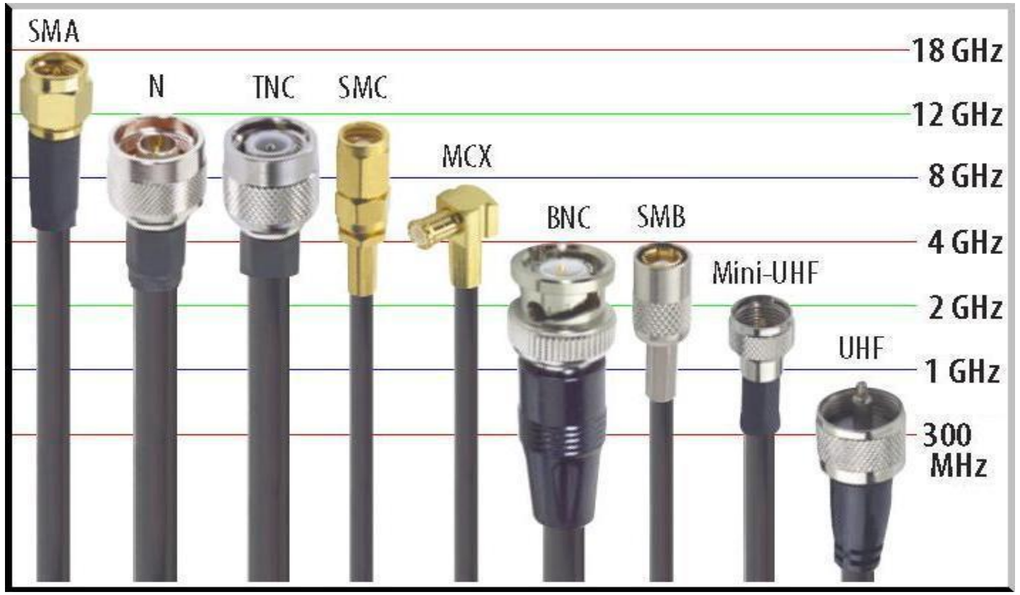
Source: Saad, *Microwave Engineers Handbook, vol. 1,* Artech House



Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, 2nd ed., Springer

Coaxial Lines

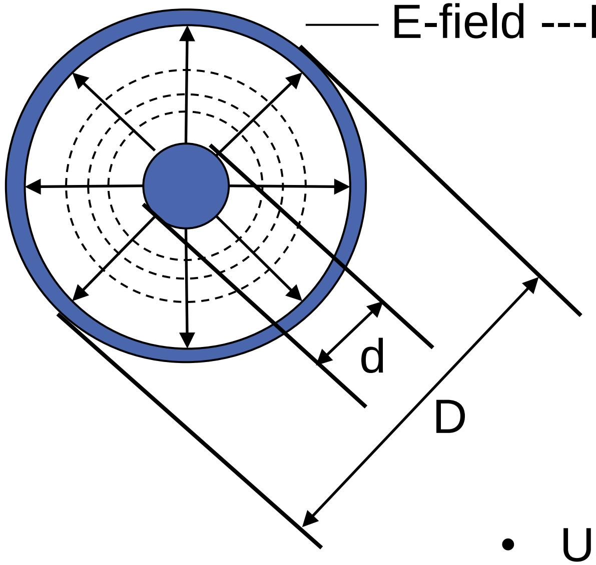
- Usually operated in TEM mode, but also carries waveguide modes if the frequency is sufficiently high.
- Different coaxial cables are used for measurements, or for power transport to the cavity. They need to be optimized for their purpose.



Source: www.winpoint.com.tw

SPS 200 MHz cavity feeder line.

Coaxial Line in TEM-mode



- Characteristic impedance for a lossless line: $Z_0 = \sqrt{\frac{L'}{C'}}$

- From textbook, we look up *(calculated from electrostatic equations…)*:

$$L' = \frac{\mu}{2\pi} \ln\left(\frac{D}{d}\right) \qquad C' = \frac{2\pi\epsilon}{\ln\left(\frac{D}{d}\right)}$$

$$\Rightarrow \boxed{Z_0 = \sqrt{\frac{\mu}{\epsilon}} \frac{\ln\left(\frac{D}{d}\right)}{2\pi}}$$

- Usually, there is low-loss dielectric between inner and outer conductor.

$$\epsilon = \epsilon_0 \epsilon_r$$
$$\mu = \mu_0 \mu_r \qquad \mu_r \approx 1$$

$$\boxed{Z_0 = \frac{60\Omega}{\sqrt{\epsilon_r}} \ln\left(\frac{D}{d}\right)}$$

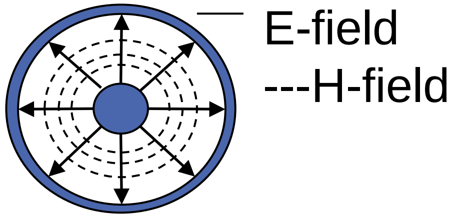
Coax characteristic impedance
with low-loss dielectric

$$v_p = \frac{c_0}{\sqrt{\mu_r \epsilon_r}} = \frac{c_0}{\sqrt{\epsilon_r}}$$

- Full shielding properties is making the coaxial line very popular for RF measurement equipment.

How about: we use a coaxial line for acceleration?

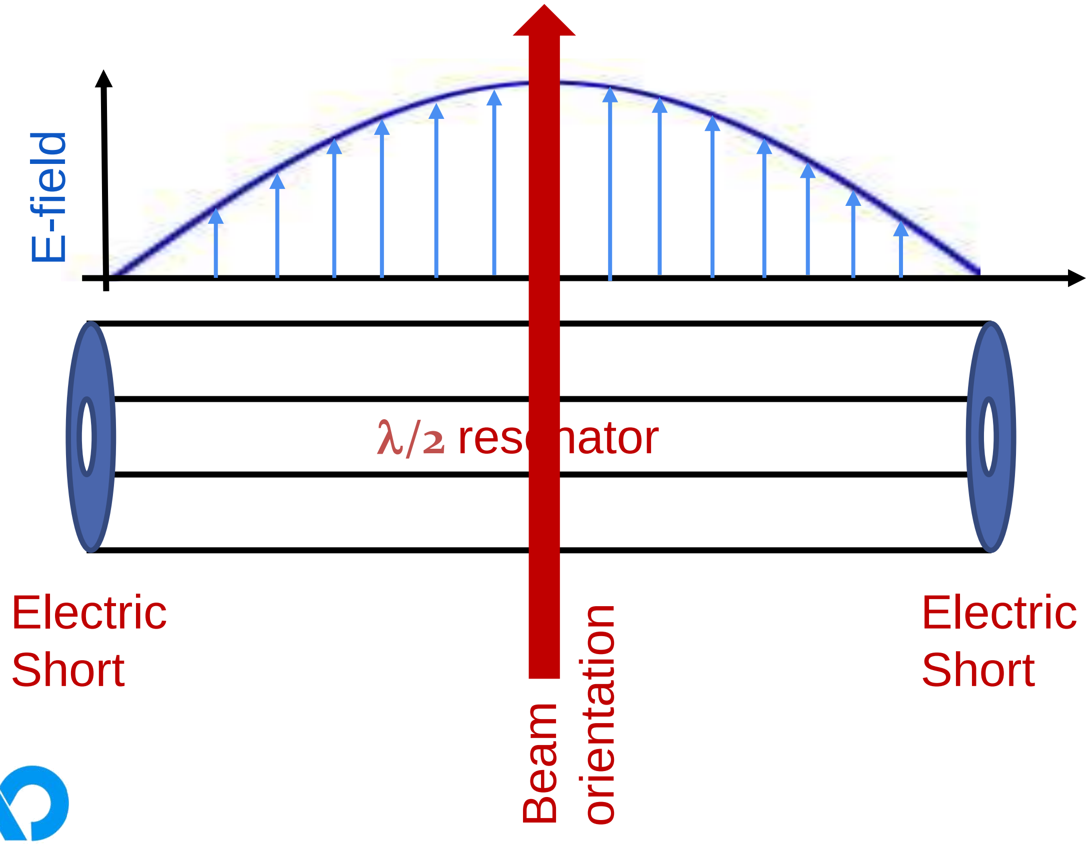
From Coaxial Line to RF Cavities in low frequency range

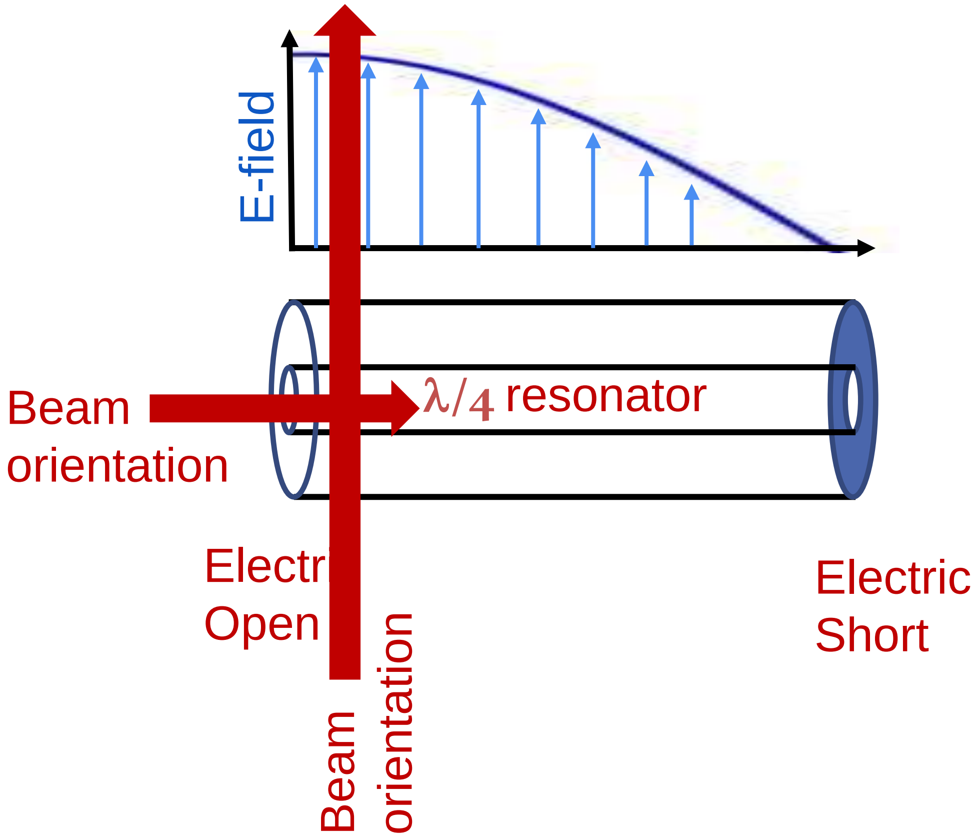


Frequency below ~10 MHz gives an RF wavelength largely above >30 m

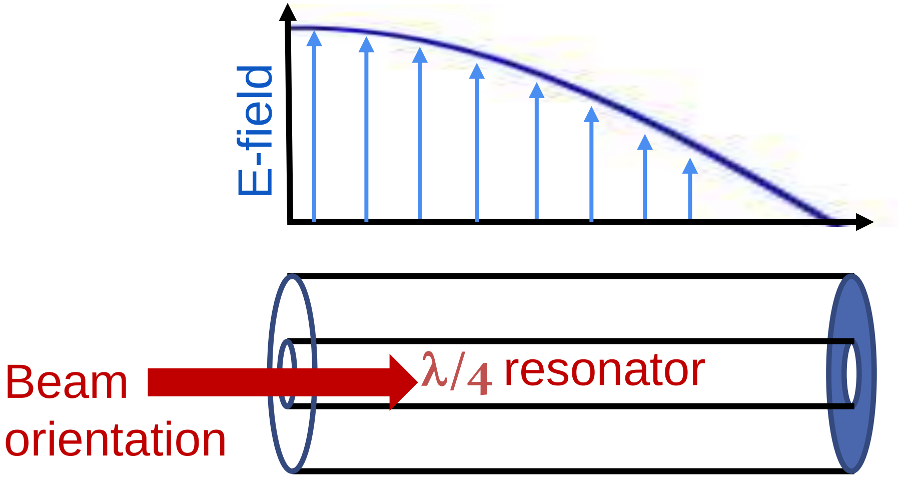
→ With classical cavity (pillbox design), we would need huge cavities → not suited for accelerators

→ Line resonators: $\lambda/2$ or $\lambda/4$ resonator



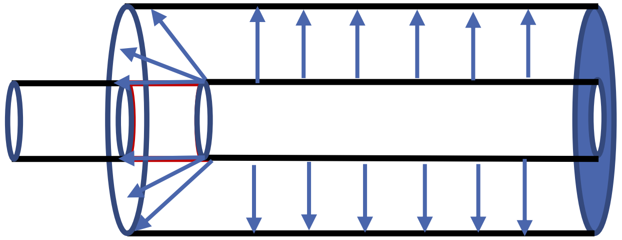


QW-Cavities in low frequency range



Electric Open

Electric Short



Ceramic gap = Electric Open

This side acts like a capacitor, we speak of capacitive loading.

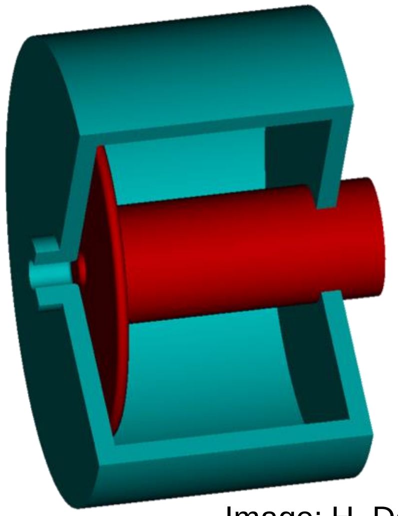


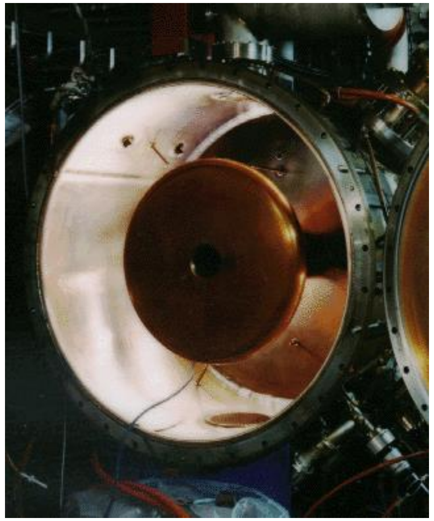
Image: H. Damerau

→ **Still rather long geometry, 7.5 m at 10 MHz**

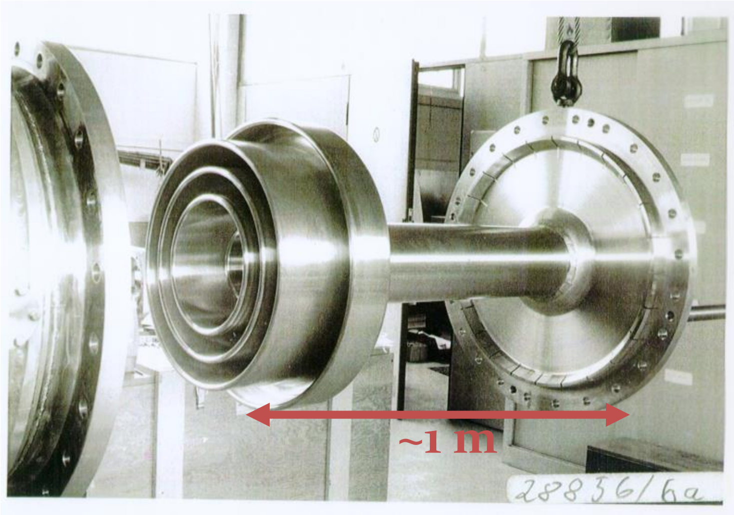
Capacitive loading in real world

→ **Add capacitor at gap of cavity to shorten the resonator**

NSLS, 52.88 MHz



DESY PIA, 10.4 MHz, inner cond.



Outer cond.

M. Nagl

→ **Significantly reduces cavity size (recall: at 10 MHz, QW is 7.5m)**

→ **Fixed frequency only**

→ **Adds small losses due to capacitor**

→ **Advantage: entire cavity in vacuum**

RF cavities in low frequency range

- Generally: with same geometric length, the frequency goes down, if inductance or capacitance goes up.
- Leads to an increase in electrical length.

$\rightarrow$ Add more capacitive or inductive loading

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

resonance frequency

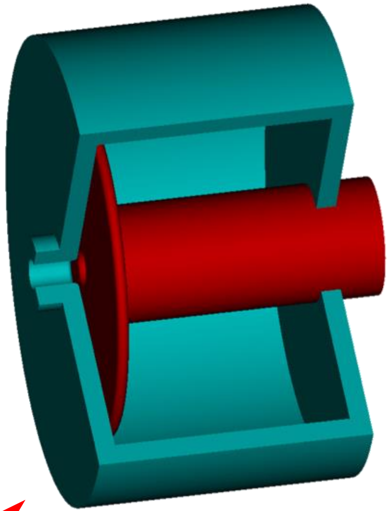
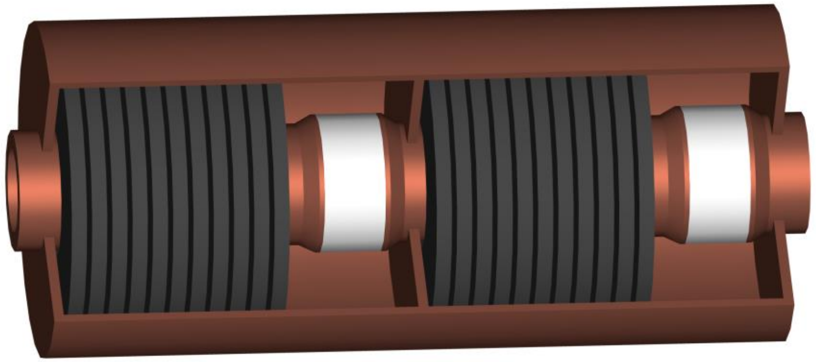


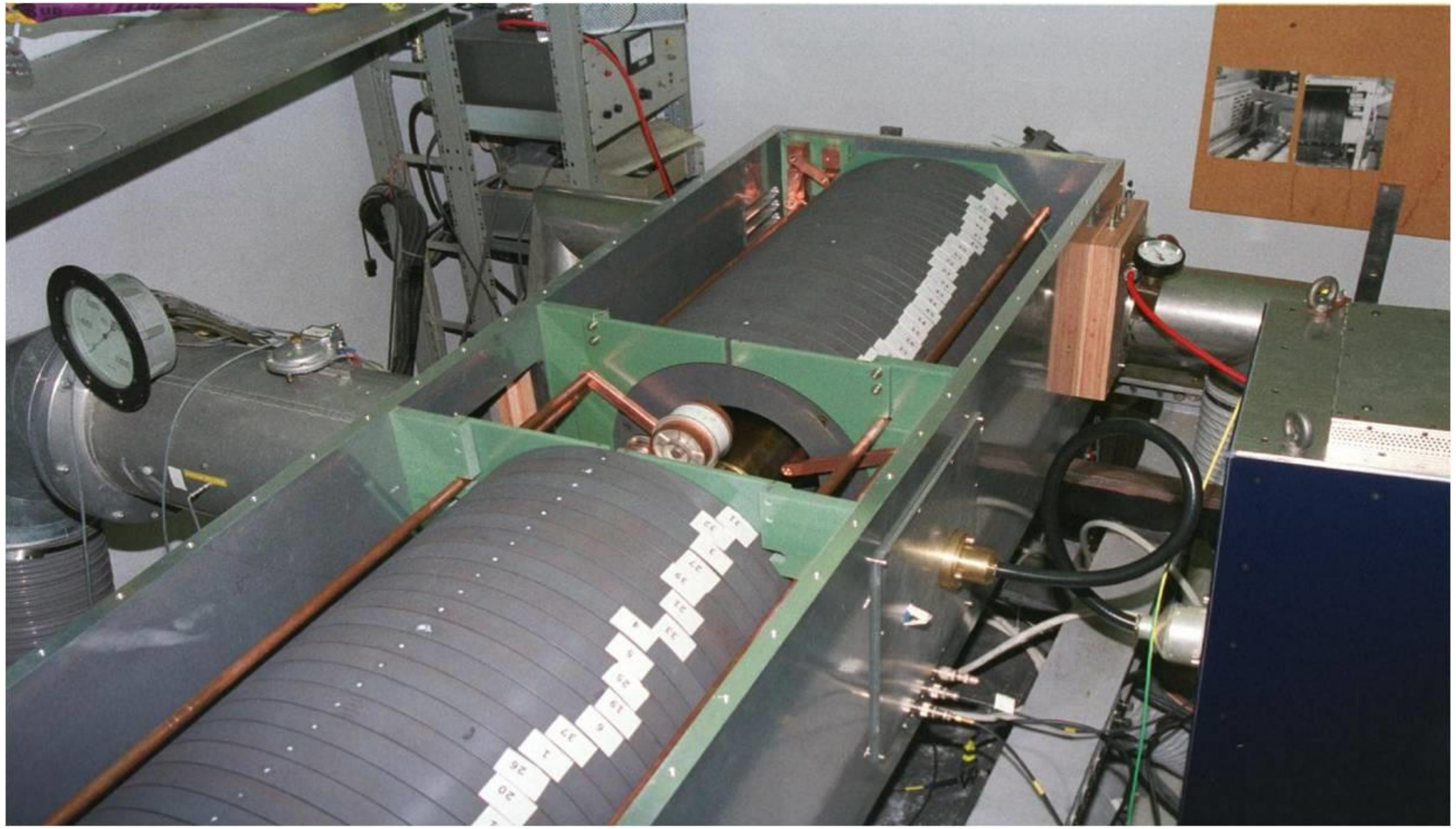
Plate capacitor



Ferrite inductivity (also allows frequency tuning)

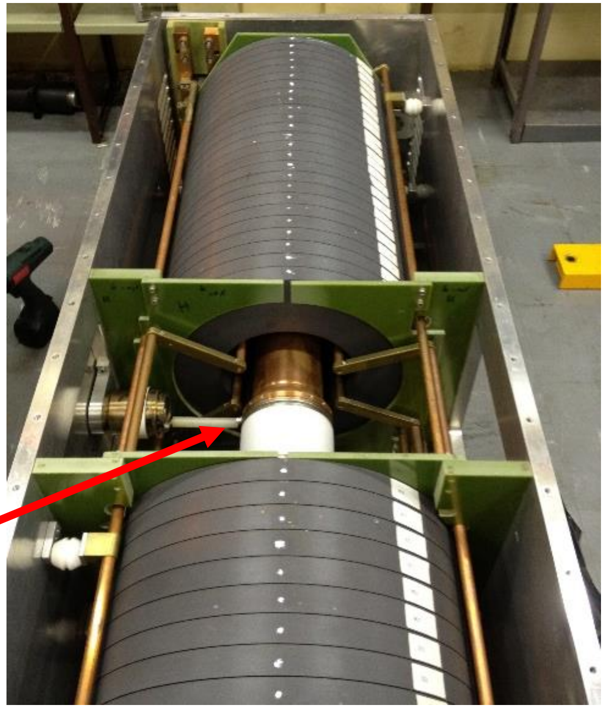
Images: H. Damerau

Tunable cavities – example (former) PSB



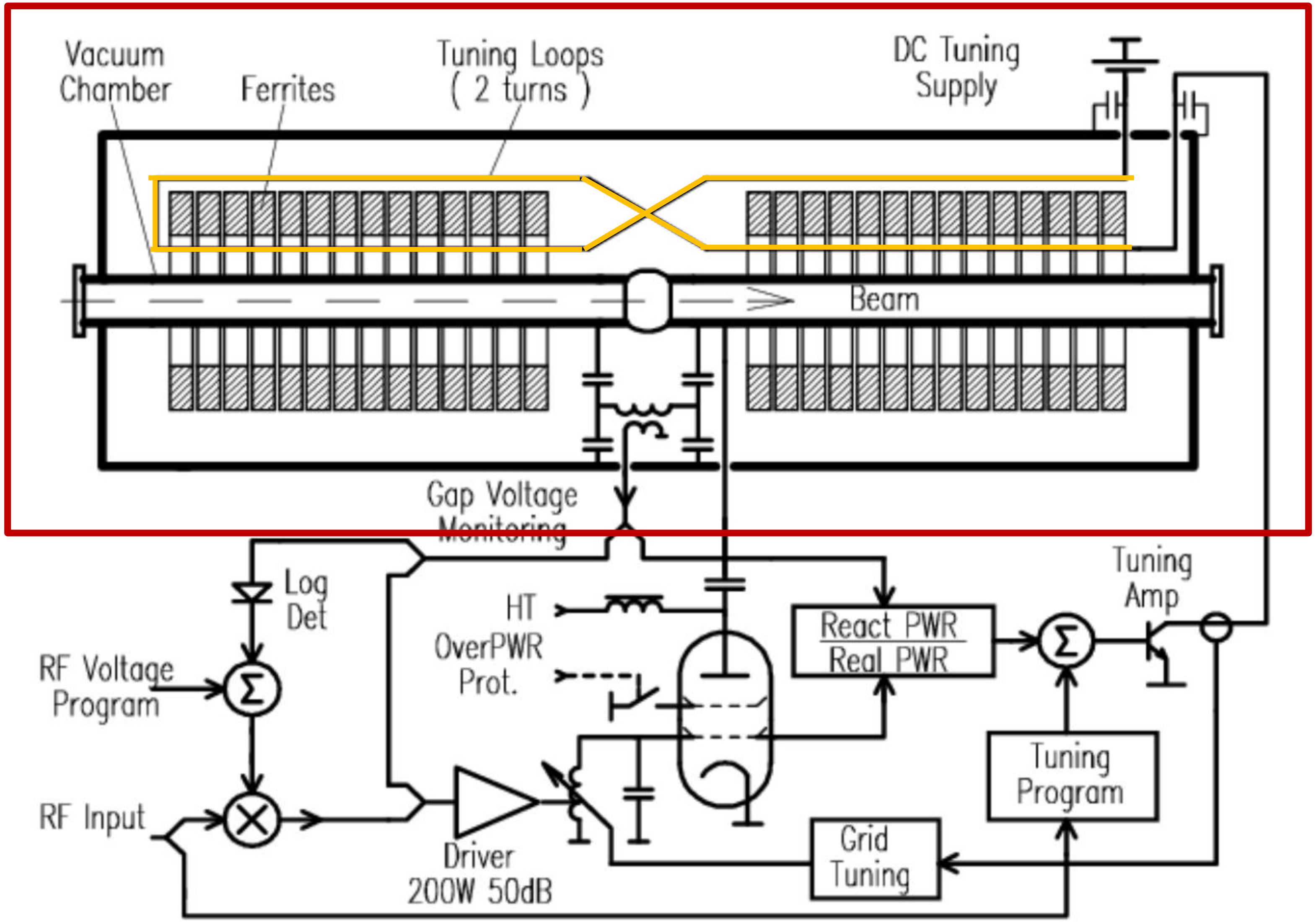
PSBooster Cavity, below 20 MHz

- Makes use of spinnel ferrites.
- Tuning via external magnetic field.
- So-called “figure-of-eight” biasing (= parallel biasing)
- Cavity, operating 8-12 MHz
- Ferrite material is lossy and brings the Q of the cavity down.
- $Q_0 = 50$



Beam vacuum pipe with ceramic gap.

Tunable cavities – figure-of-eight biasing



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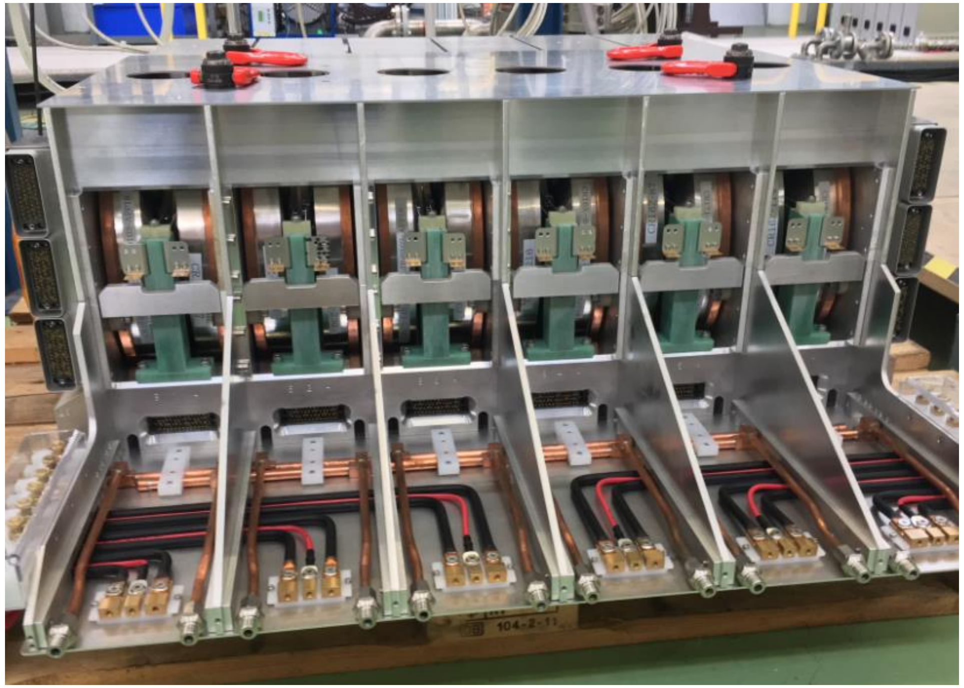
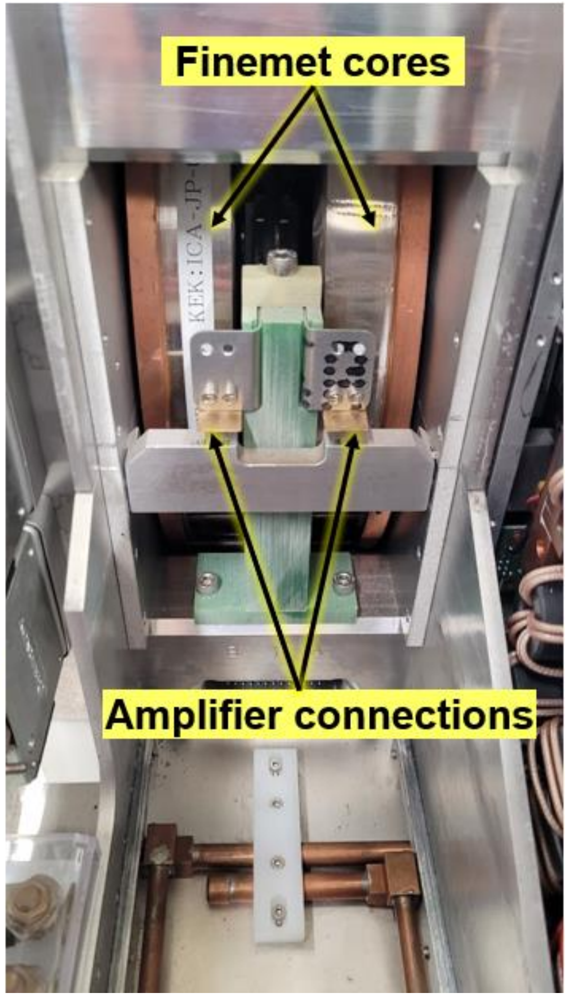
Tunable cavities – the PSB finemet cavity

Image: M. Paoluzzi



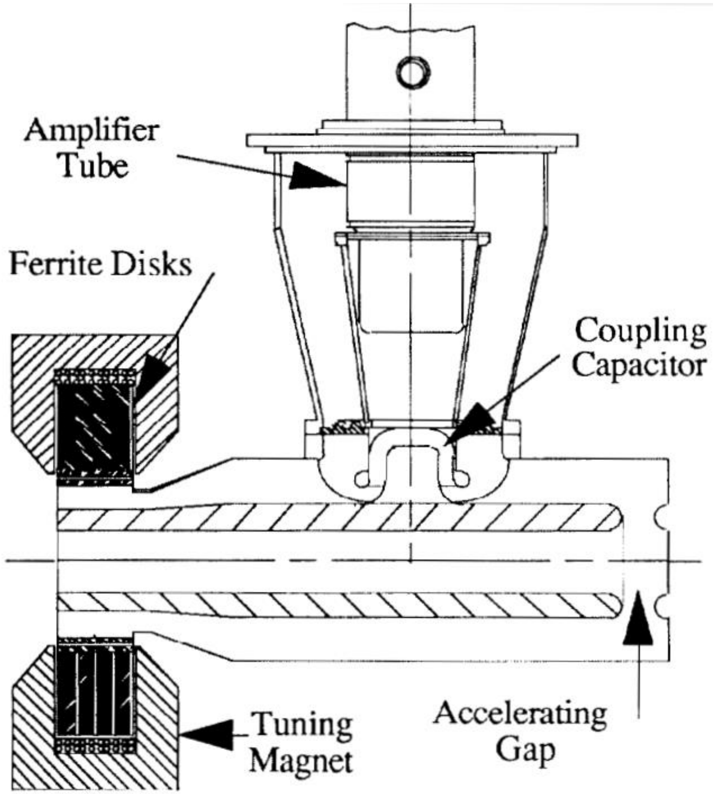
CERN PSB Finemet cav., 0.6-18 MHz



Tunable cavities – higher frequencies

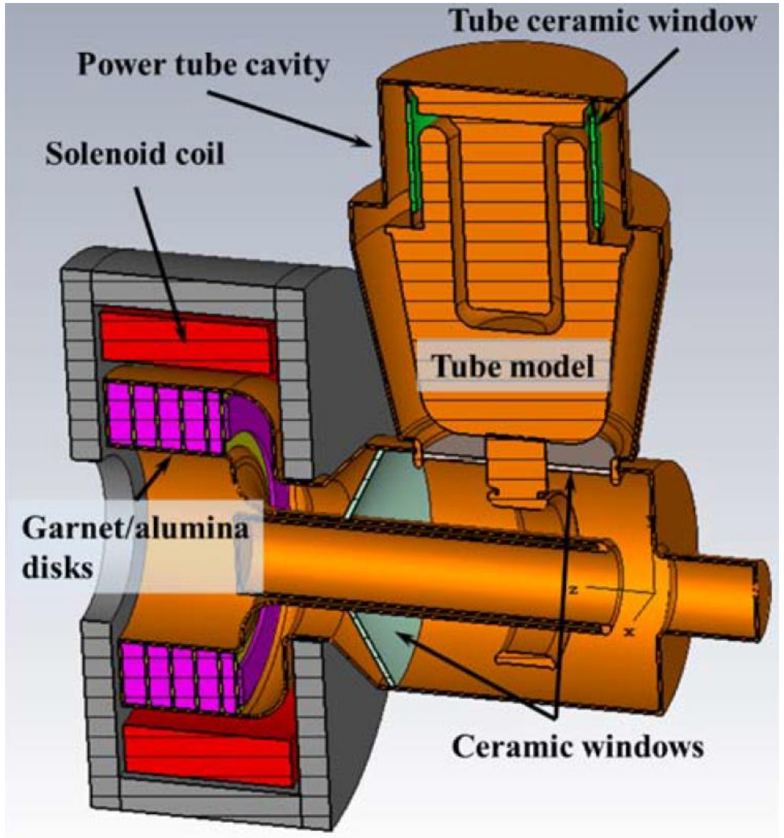
→ **These cavities work with perpendicular biasing, and new type of low loss ferrites (garnet).**

SSC Low Energy Booster,
~47 MHz to 60 MHz



C. C. Friedrichs et al., PAC91, p. 1020

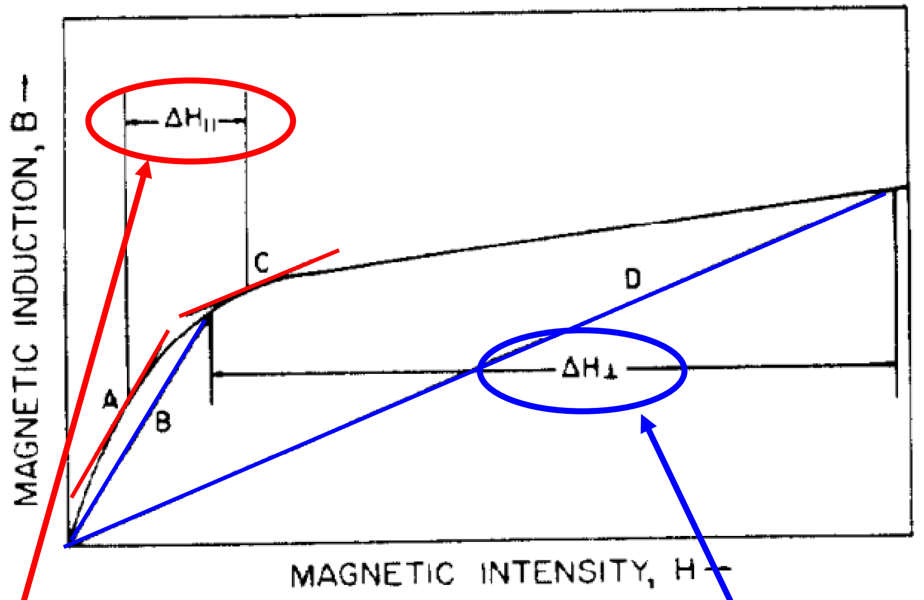
FNAL Booster 2nd harmonic,
76 MHz – 106 MHz, 100 kV



R. L. Madrak, IPAC16, p. 130

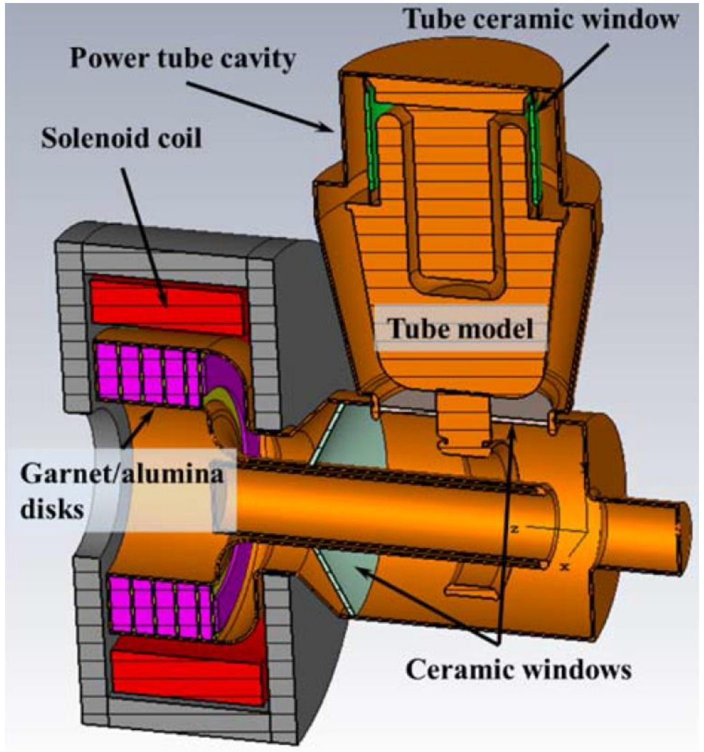
Tunable cavities at higher frequencies

From: Smythe, IEEE, TNS [3]



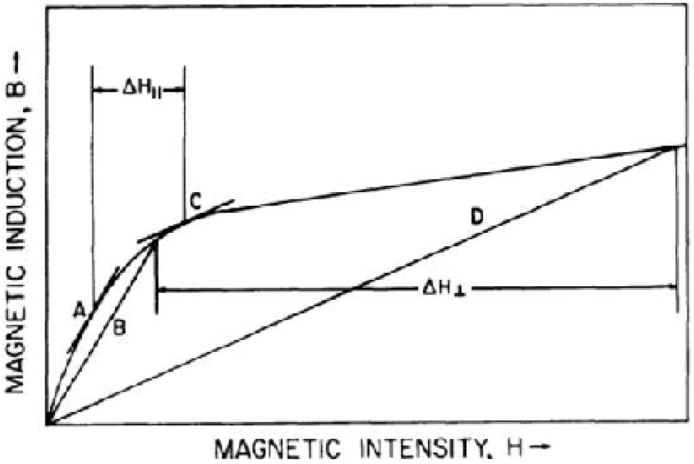
Parallel bias
(= $H_{mag,bias}$ is parallel H_{RF})

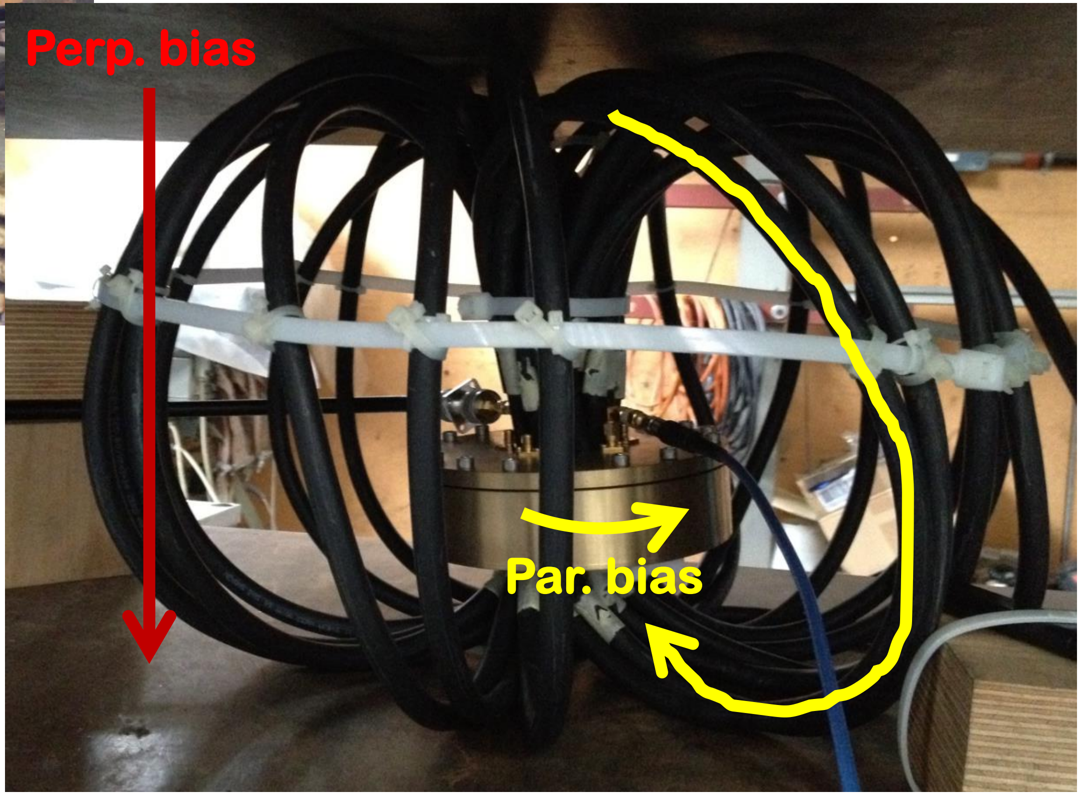
Perpendicular bias
(= $H_{mag,bias}$ is perpendicular H_{RF})



- Garnets are ferrites with a magnetic field-dependent relative permeability.
- Exposed to a (slowly varying) magnetic field, their µ-value covers a certain range until they saturate.
- Problems are the garnet outgassing (cannot be in vacuum), the limited Q-value of the the cavity due to material losses and the rather large H-fields needed.

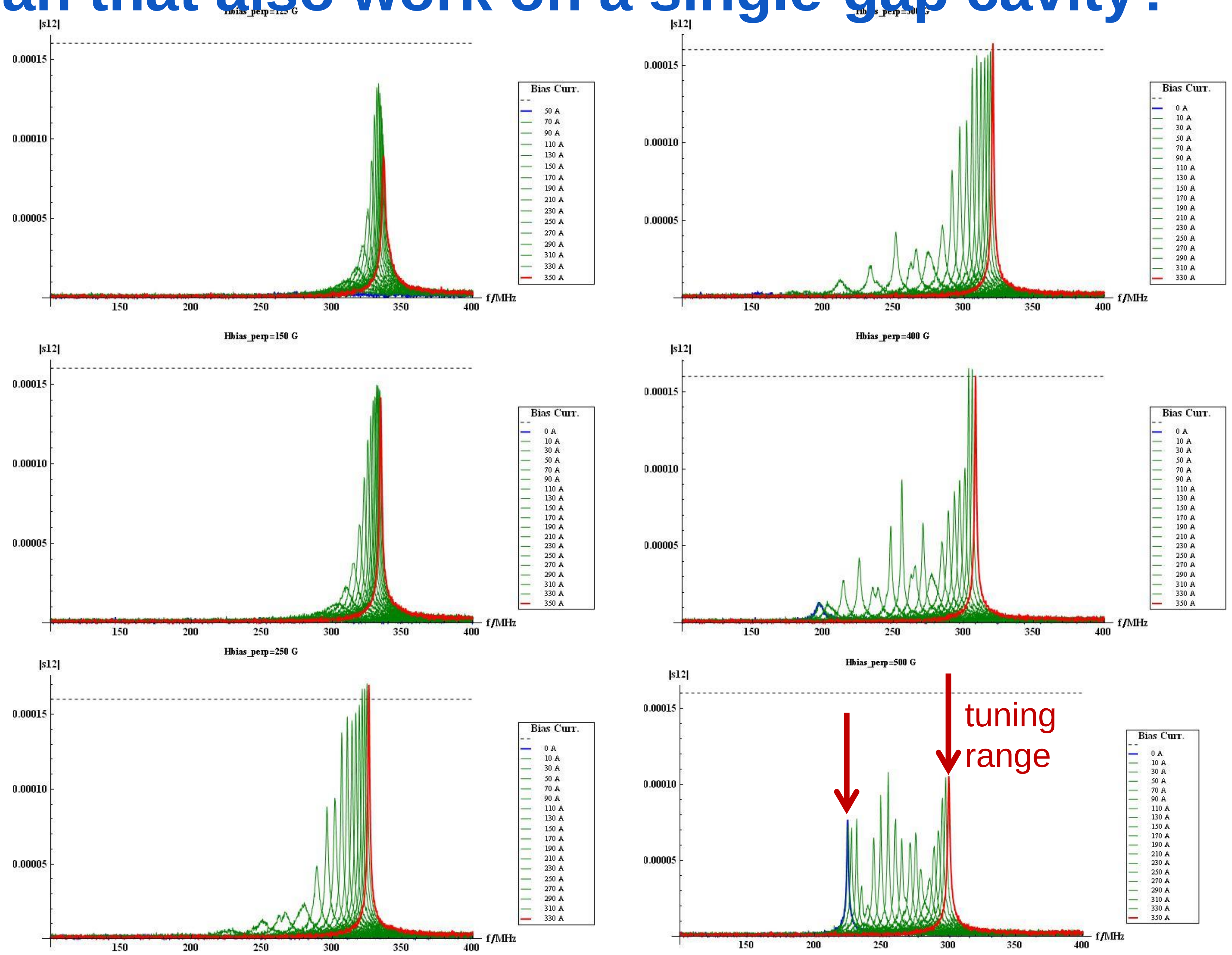
Can that also work on a single-gap cavity?





Can that also work on a single-gap cavity?

Resonance peaks due to different magnetic bias fields



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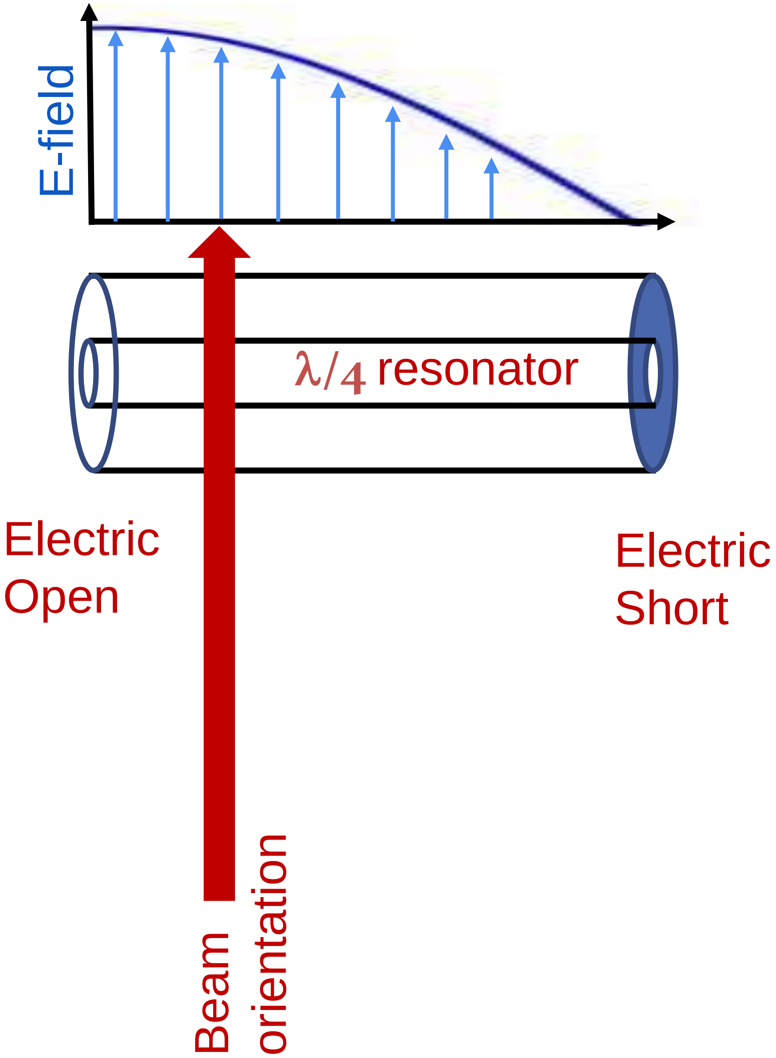
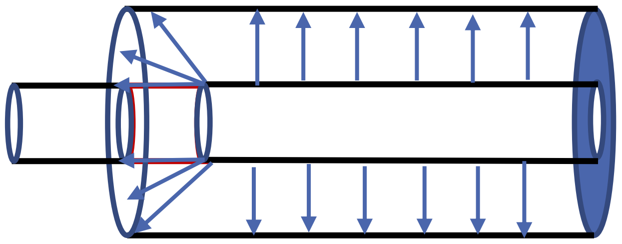
Applying the tuning principle to a single-gap cavity

QW-Cavities in low frequency range

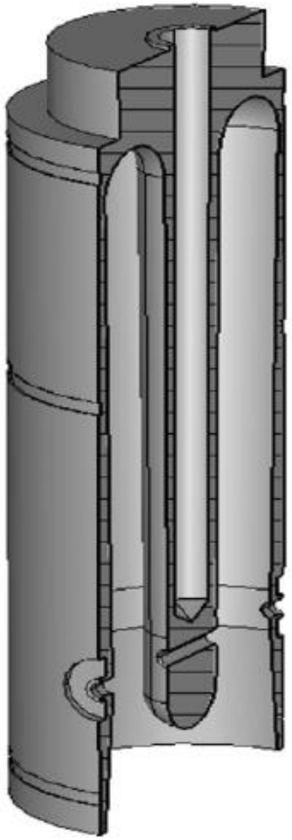


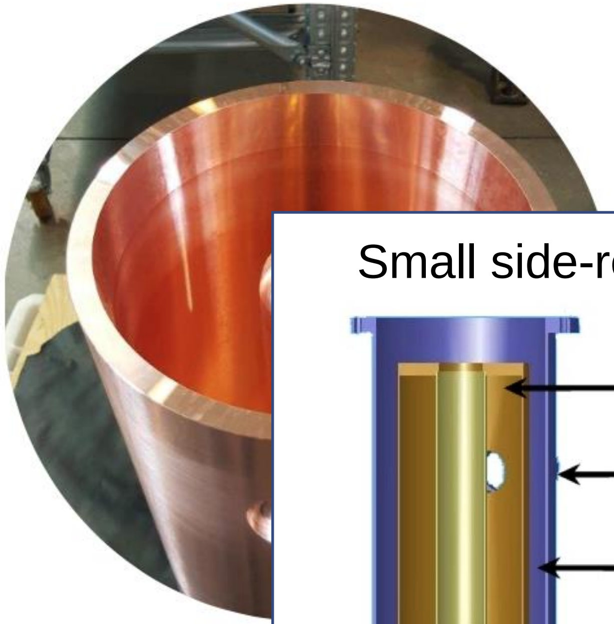


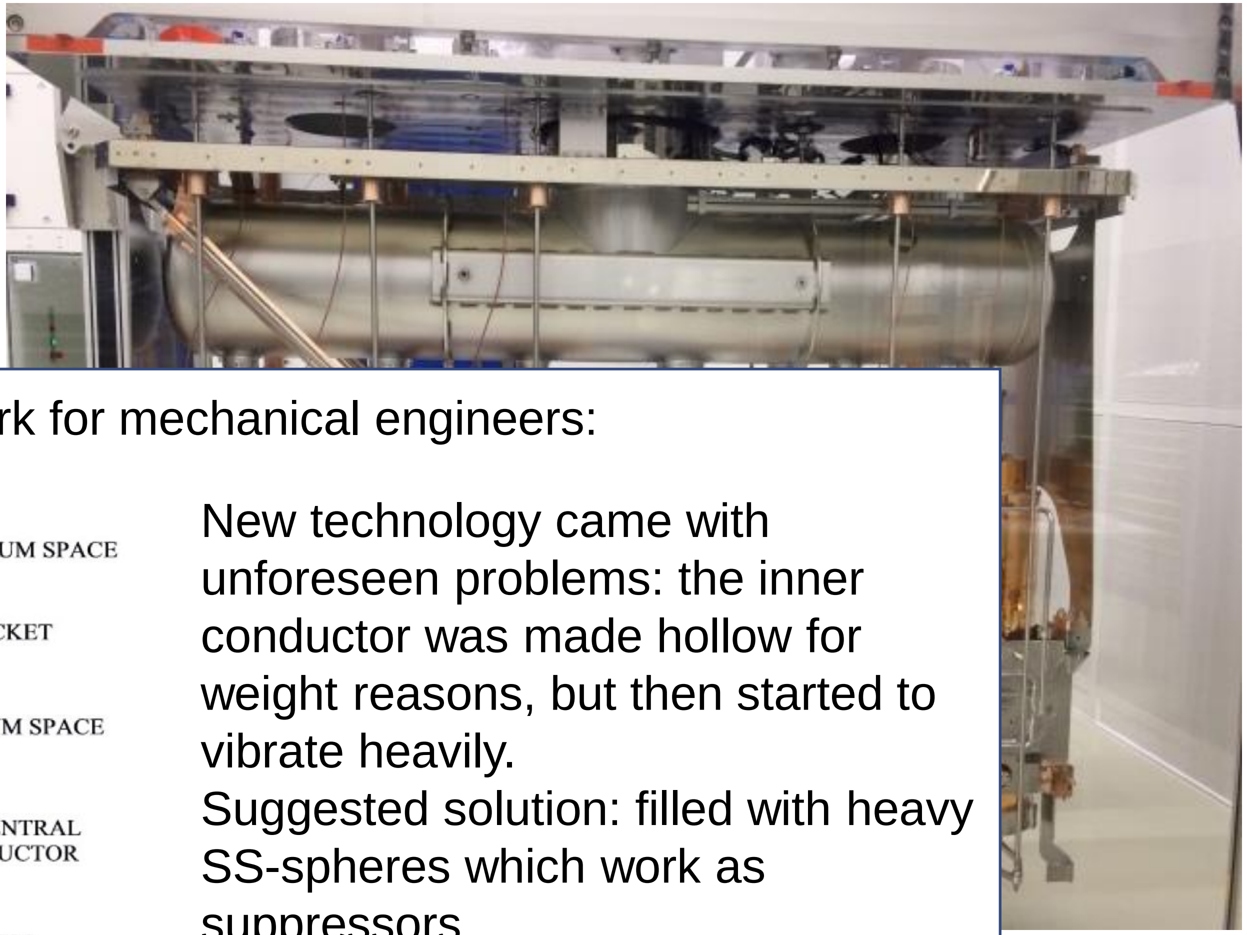
Ceramic gap =
electric Open

This side acts like a capacitor, we speak of capacitive loading.

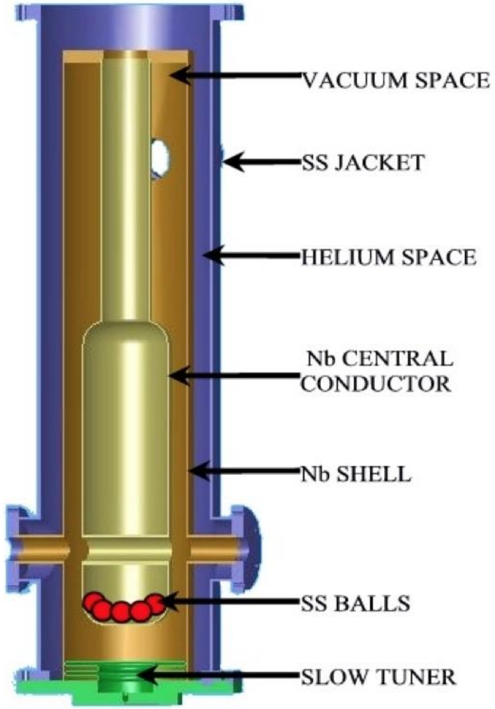
QW-Cavity at HIE-Isolde (CERN)







Small side-remark for mechanical engineers:



New technology came with unforeseen problems: the inner conductor was made hollow for weight reasons, but then started to vibrate heavily.
Suggested solution: filled with heavy SS-spheres which work as suppressors.

Image: A. Roy, APAC 2007, THYMA04

- Accelerator for radioactive ion beams of F
- 100 MHz superconducting Nb on Cu struc
- 6 MV with Q > 5 x 10^8

Images: F. Gerigk, CERN

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Exotic TEM-Cavity (Spoke Cavity)

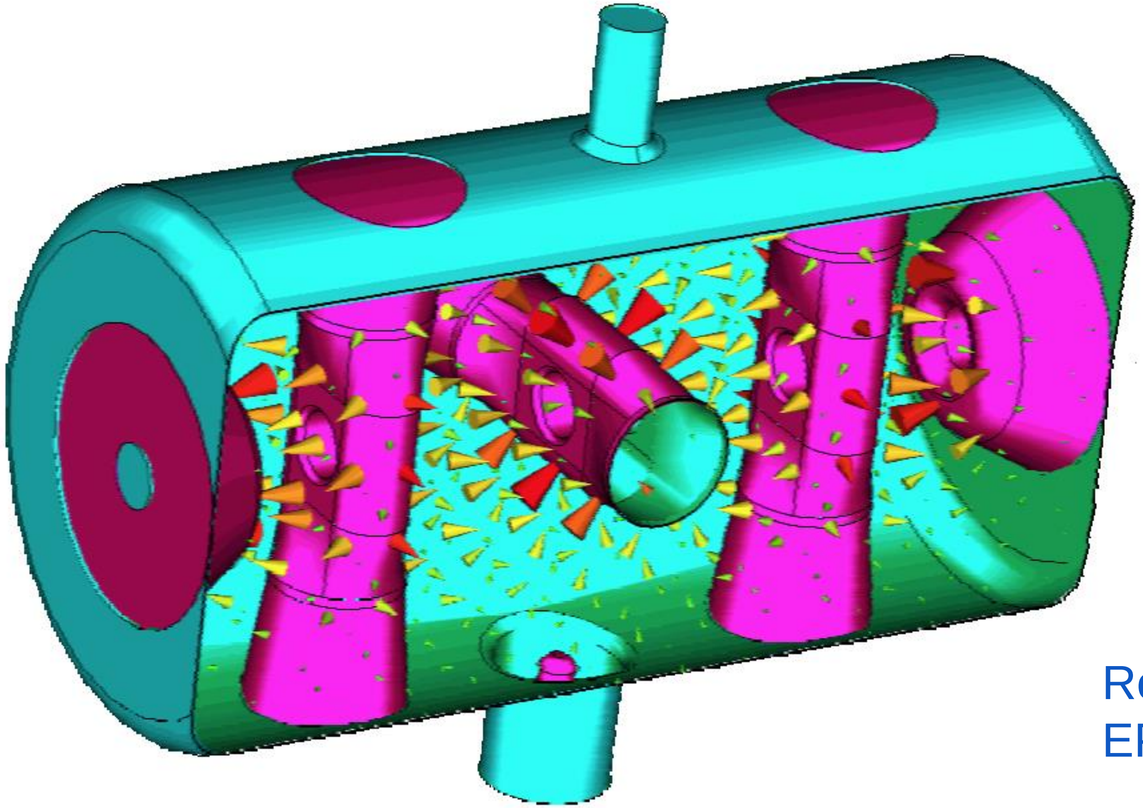


Image: F. Gerigk, RF CAS Berlin 2024.

- Spoke cavities are made of 1…n combined $\lambda/2$ –wave TEM cavities.
- Typically, they have 1…3 spokes and are superconducting.
- Used for lower to medium beta.

Reference: E. Zaplatin et al: “Triple Spoke Cavities at FZJ” EPAC 2004

End of Part 1!

From Waveguide to Cavity…

- Just apply Maxwell’s Equations*…
- ... superpose a forward and backward travelling wave to get a standing wave…
- … terminate a waveguide with two conducting walls…
- … use a waveguide for acceleration
- … cut the inner conductor of a coaxial line…

RF components:
Waveguide and Cavity

Waveguides

Rectangular waveguide

Circular waveguide

Arbitrarily shaped cross-section waveguide

Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, 2nd ed., Springer

- Waveguides are hollow metallic tubes with uniform cross-sections of different shapes.
- The metallic waveguide is a completely enclosed system without any radiation loss.
- Waveguides present a one-conductor system, so *no TEM-mode propagation* is possible.
  Propagation happens in TE-mode *(**T**ransverse-**E**lectric)* and TM-mode *(**T**ransverse-**M**agnetic).*

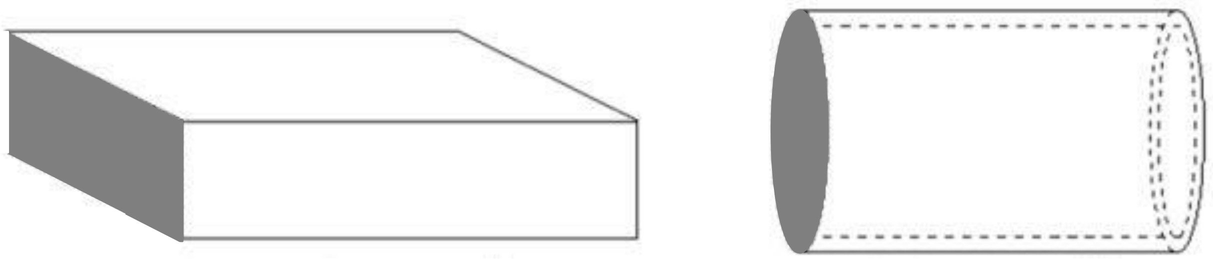
Images: www.pasternak.com

- Waves are following the waveguide shape, even if it is bend.
- Propagating waves need to fulfill boundary conditions on the waveguide walls.

Cavity resonators

Microwave resonators can be built from waveguides by closing the open ends.

We can build an RF-resonator or a *cavity*.



Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, 2nd ed., Springer

- A cavity stores electric and magnetic energy inside the hollow body.
- The frequency of the electromagnetic field resonance depends on the cavity dimensions.

- Same as with the rectangular and the circular waveguide, the electromagnetic field needs to fulfil the boundary conditions on the resonator walls. The field builds up in resonant modes.

- Cavities that are used for the acceleration of particles in our accelerators are mostly of a cylindrical and flat shape. This is why we call them pillbox cavities.

- We will later see how resonances can be excited in cavities by feeding RF-signal in, and how such a cavity can be used in accelerators for particle acceleration.

Cavity Resonators and Q-value (1/2)

- Resonators are classified by their quality factor *Q*.
- The quality factor (or *Q-value*) can be used as a measure of "how well the cavity is resonating".

define

$$Q_0 = 2\pi \frac{\text{energy stored in the resonator}}{\text{energy dissipated in the resonator}}$$

Q_0 = unloaded *Q*

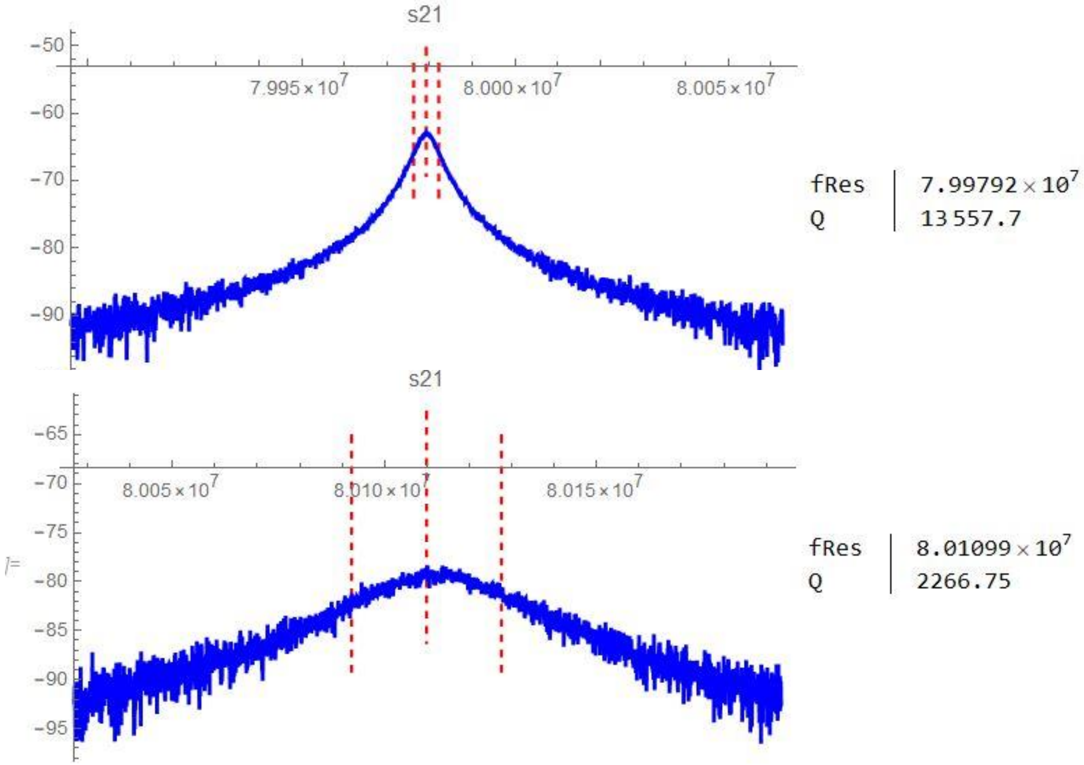
- High *Q*-value is desired in accelerating cavities; *Q*-value is one of the accelerator efficiency figures-of-merit.

- *Q*-value reduces e.g. due to the power dissipated in the metallic walls or other loss mechanisms.

- The connection of the cavity resonator to the outer world will reduce its *Q*-value as well *(we say "it loads" the cavity with an additional loss mechanism)*.

Cavity Resonators and Q-value (2/2)

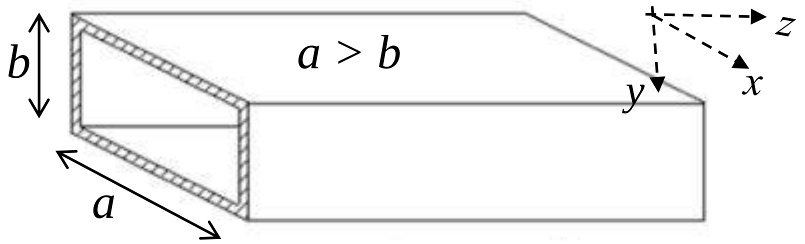
Example: measurements taken on the PS 80MHz pillbox cavity



- We tested this cavity for *Q*-deterioration and shifting of its fundamental mode (~80 MHz).
- *Q*-values obtained from 3-dB-measurement (see dashed red lines).
  → Plot on the top has a higher *Q*-value than the plot below.

Rectangular Waveguides (1/4)

TE-modes



Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, 2nd ed., Springer

- TE-modes are characterized by a zero *electric* field in propagation direction. The electric field is in the transverse plane, only.

- Each mode has a cut-off frequency:

$$f_{c,mn} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

- TE_{10}-mode is the mode with the lowest cut-off frequency:

$$\omega_{c,10} = \frac{1}{2a\sqrt{\mu\epsilon}}$$

The mode with the *lowest cut-off frequency* is called *the dominant mode*.

Note that if $n=m=0$, all field components become zero, there is no TE_{00} mode.

A waveguide is called *overmoded*, if more than one mode is propagating in the waveguide at the same time.

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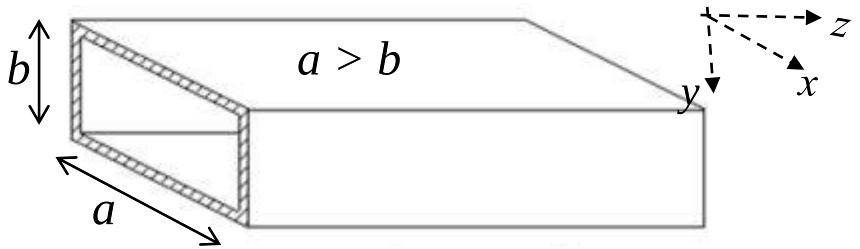
Rectangular Waveguides (2/4)

TM-modes

- TM-modes are characterized by a zero *magnetic* field in propagation direction. The magnetic field is in the transverse plane, only.



Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, 2nd ed., Springer

- Also in this case, each mode has a cut-off frequency (identical to TE):

$$f_{c,mn} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

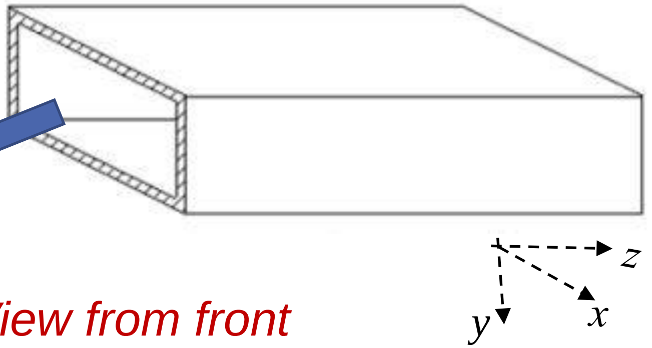
- TM_{11} has as cut-off frequency:

$$\omega_{c,11} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2}$$

There is no TM_{00}-mode, neither do TM_{01} or TM_{10} exist. For $n=m=0$, all field components become zero.

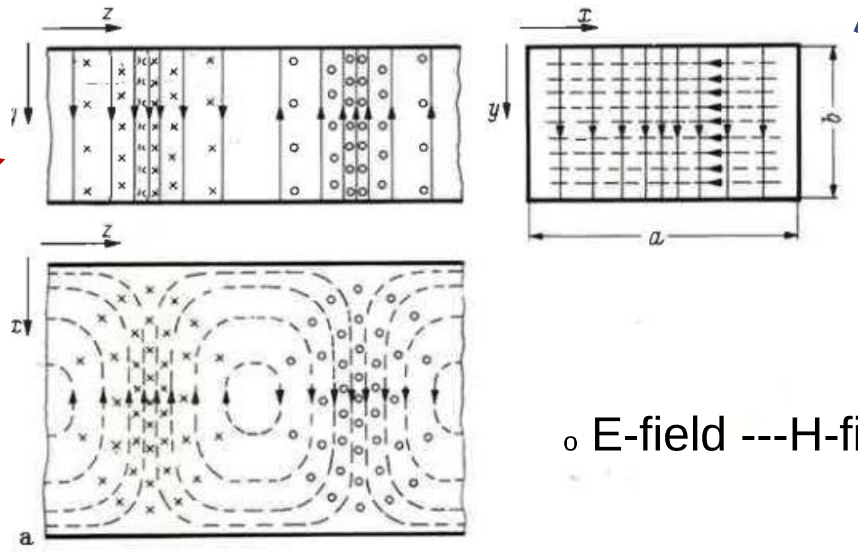
Note that TE_{10}-mode remains the dominant mode, it is the mode with the lowest cut-off frequency.

Rectangular Waveguides (3/4)



Field pattern for TE_{01}-mode

What do we actually see?

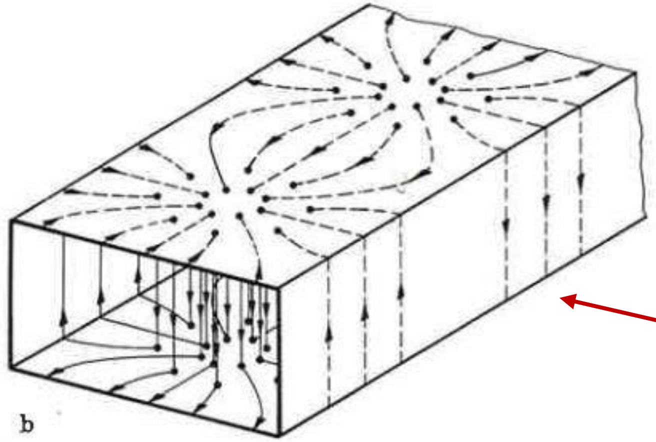


Side view narrow
(y/z-plane)

Side view broad
(x/z-plane)

View from front
(x/y-plane)

o E-field ---H-field



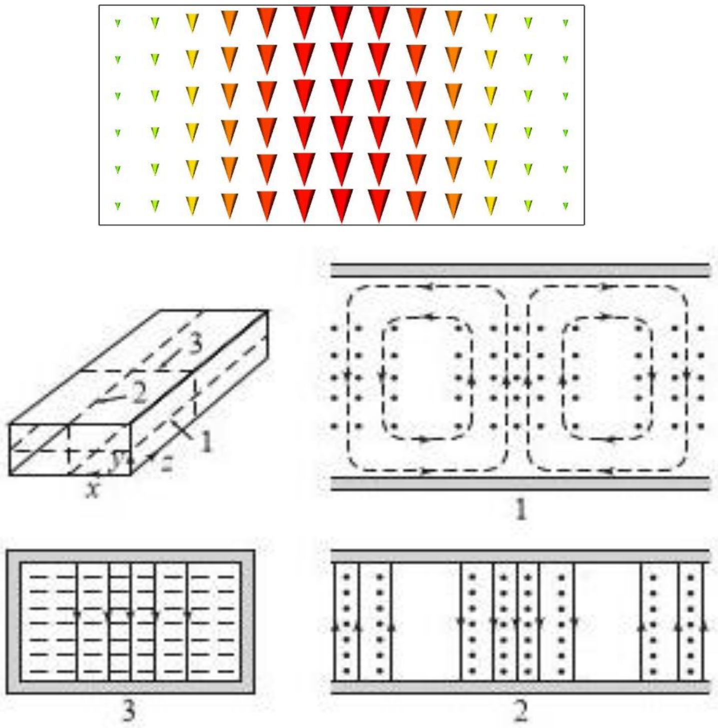
Wall currents and displacement currents inside the waveguide.

source: Zinke/Brunswig, *Hochfrequenztechnik,* 5th ed., Springer

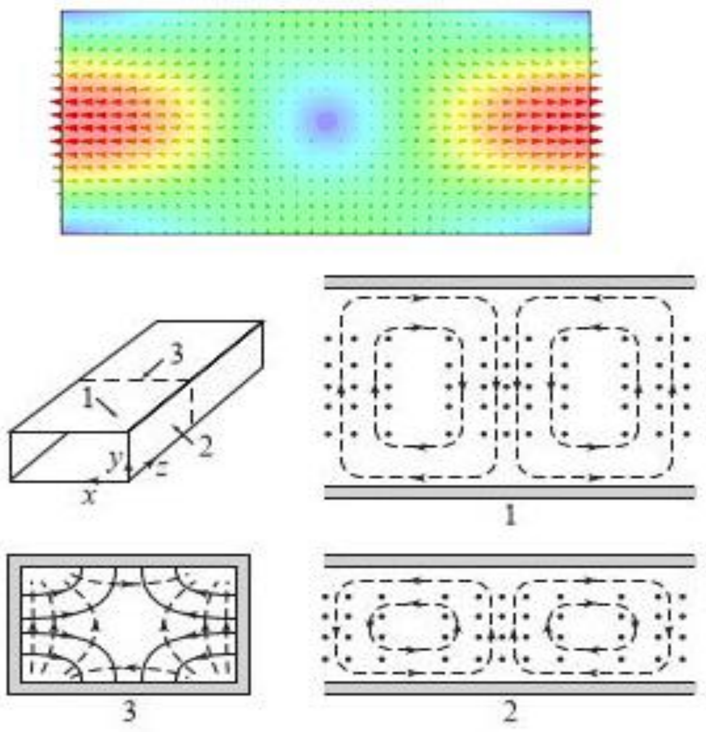
Rectangular Waveguides (4/4)

Fundamental and higher waveguide modes and field patterns for TE-modes

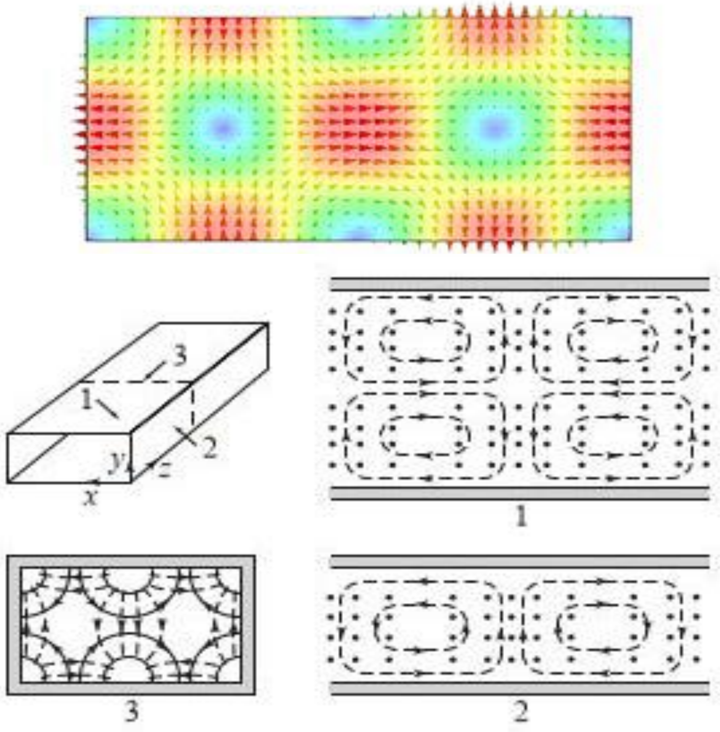
TE_{10}-mode



TE_{11}-mode



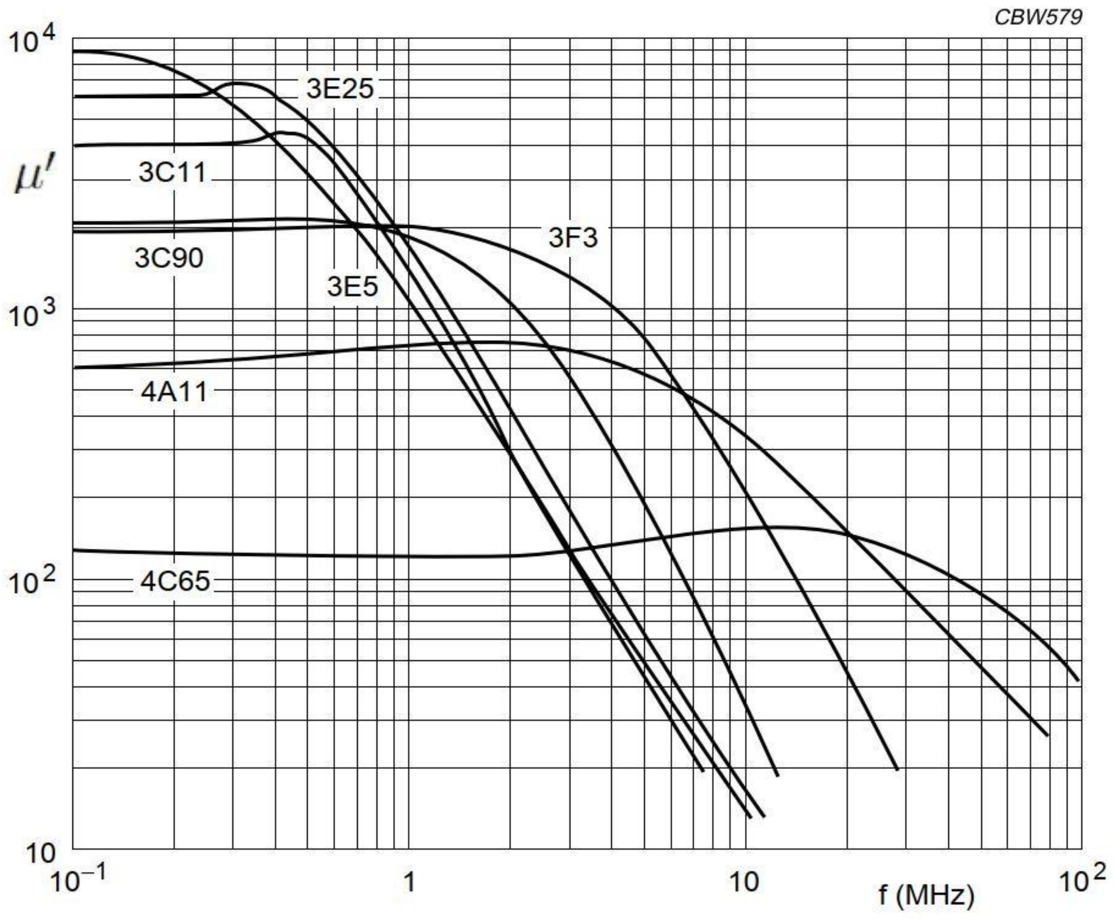
TE_{21}-mode



Simulaton pictures: courtesy E. Jensen, field pattern source: Pozar, *Microwave engineering*, 4^{th} ed., Wiley

Dispersion (1/3)

- *Dispersion* generally denotes frequency dependent behaviour.
  Best known example is dispersive media, changing its characteristics as a function of frequency.



Source: *Ferroxcube Data Handbook*

$$\boxed{\tan \delta_m = \frac{\mu''}{\mu'}}$$

magnetic loss tangent, frequency dependent as well

> Transmission lines and waveguides also show dispersive behaviour.
>
> Except: coaxial line!

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Dispersion (2/3)

If phase velocity and attenuation of a transmission line or wave guide are constants that do not change with frequency, *THEN* the phase of a signal that contains more than one frequency will not be distorted (= *no dispersion*).

If the phase velocity is different for different frequencies, *THEN* the line is dispersive.
This means that individual components of a wave will not maintain their original phase relationship when they propagate.
We will experience a signal distortion if the signal contains more than one frequency.

Dispersion = no single phase velocity can be attributed to the signal as a whole.

Different v_p means that some signal components travel faster than others.

If dispersion is small, a group velocity can be defined: v_g.
Group velocity = speed at which a signal propagates and at which power is transported.

Dispersion (3/3)

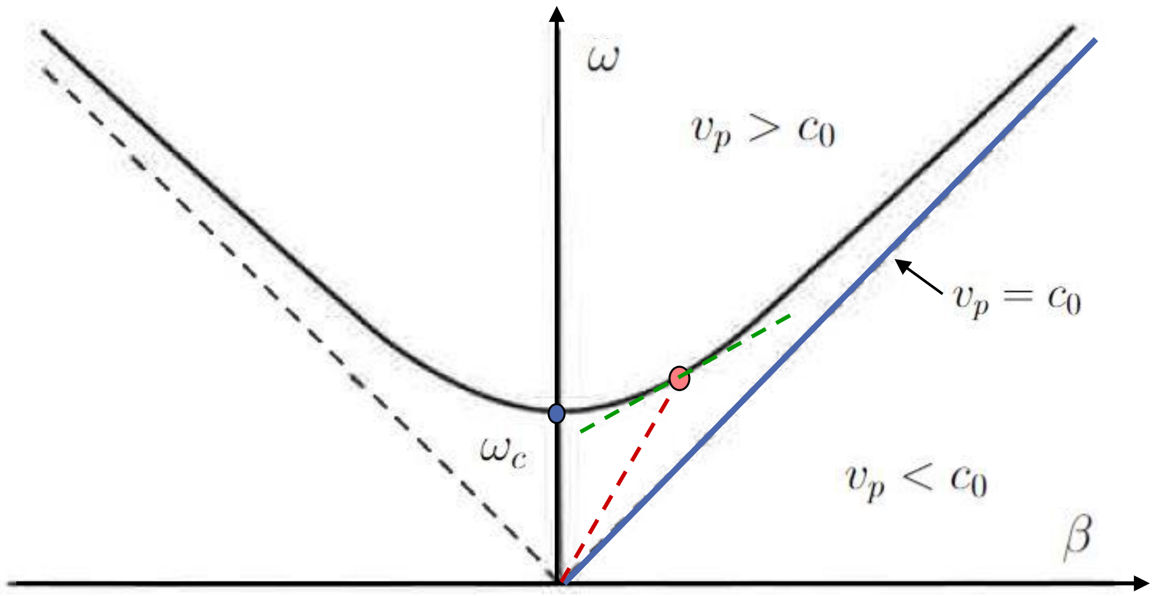
Comparison of Transmission lines and waveguides

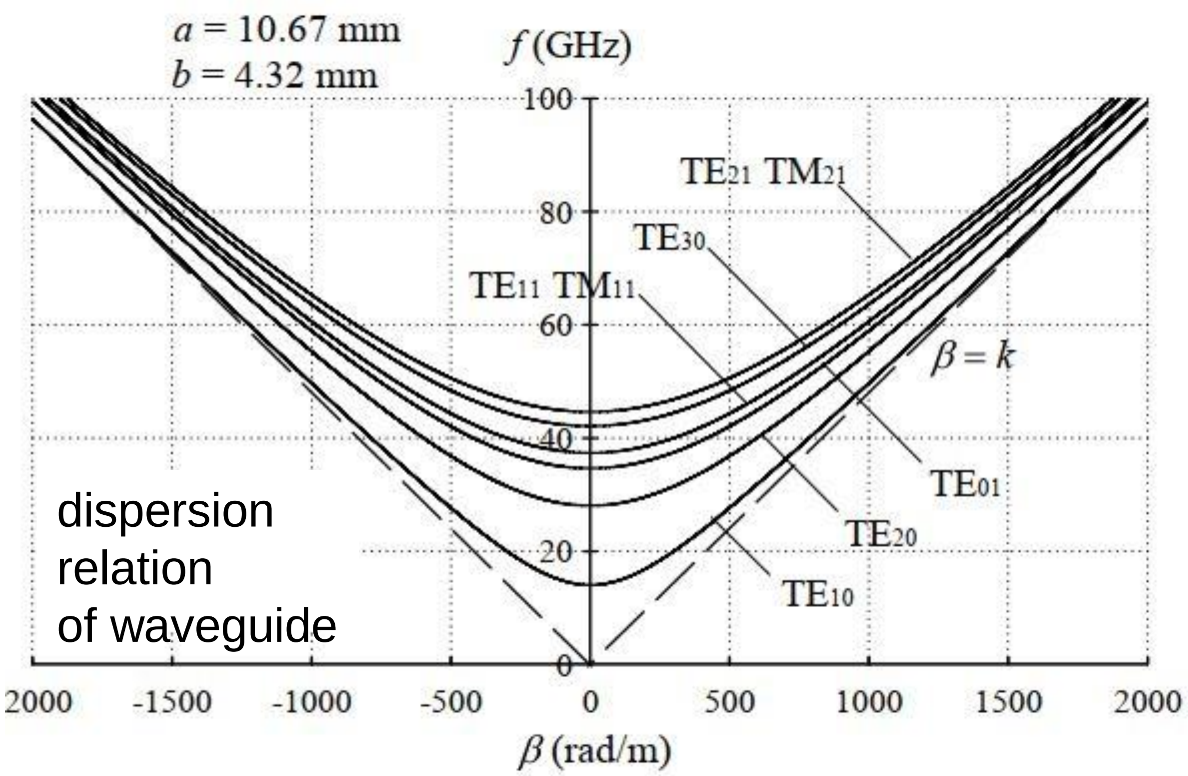
Characteristic	Coax	Waveguide	Stripline	Microstrip
Modes: Preferred	TEM	TE_{10}	TEM	Quasi-TEM
Other	TM,TE	TM,TE	TM,TE	Hybrid TM,TE
Dispersion	None	Medium	None	Low
Bandwidth	High	Low	High	High
Loss	Medium	Low	High	High
Power capacity	Medium	High	Low	Low
Physical size	Large	Large	Medium	Small
Ease of fabrication	Medium	Medium	Easy	Easy
Integration with	Hard	Hard	Fair	Easy

Source: Pozar, *Microwave engineering*, 4th ed., Wiley

Dispersion Diagram for Waveguides

general dispersion relation





dispersion relation of waveguide

Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, Springer

Tangent at point on the dispersion curve

$$\boxed{v_p = \frac{\omega}{\beta}}$$

From origin to point on the dispersion curve

$$\boxed{v_g = \frac{\partial\omega}{\partial\beta}}$$

- v_g is zero at cut-off frequency
- v_g is smaller than c_0
- v_p can be larger than c_0

β is propagation constant (!)

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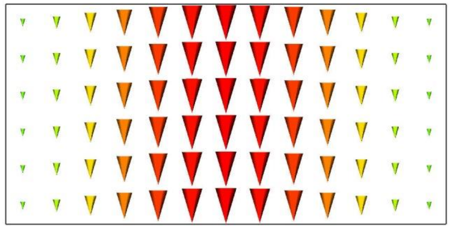
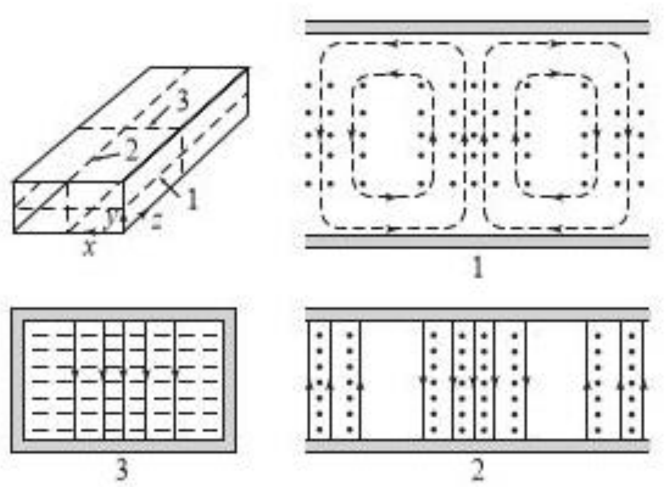
Rectangular Cavity Resonators (1/3)

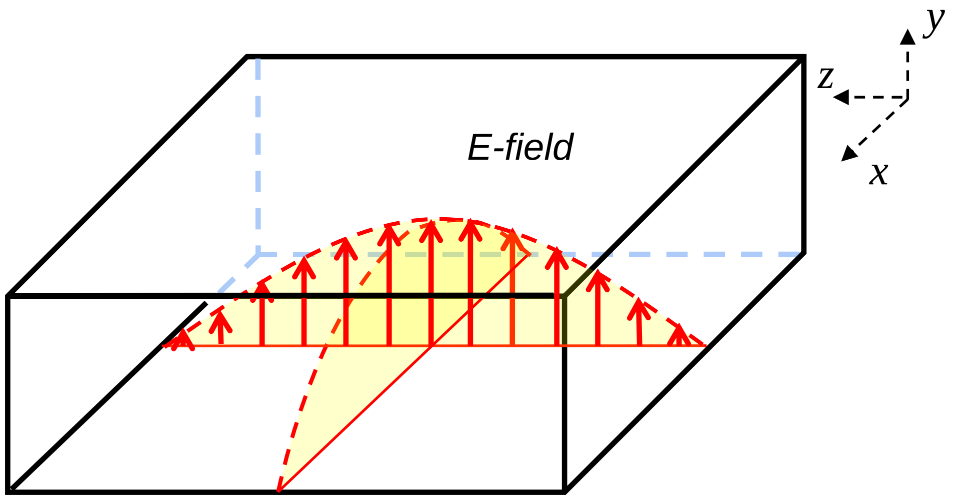
From the field pattern of the TE10-mode in the rectangular waveguide → imagine that the resonant field in a rectangular resonator will build up in a similar way.

Waveguide TE_{10}-mode





Source: Pozar, *Microwave engineering*, 4th ed., Wiley



- The field is building up in a standing wave pattern, using sine and cosine functions.
- From the boundary condition on the resonator wall, the electric field has to be zero, thus we can see multiples of electric field maxima.
- The *mode indices* are counting the maxima of the field along one axis (we see either a TE_{xyz} or a TM_{xyz}- mode).
- The mode shown is a TE_{101}-mode.

Rectangular Cavity Resonators (2/3)

Formulae for a rectangular resonator:

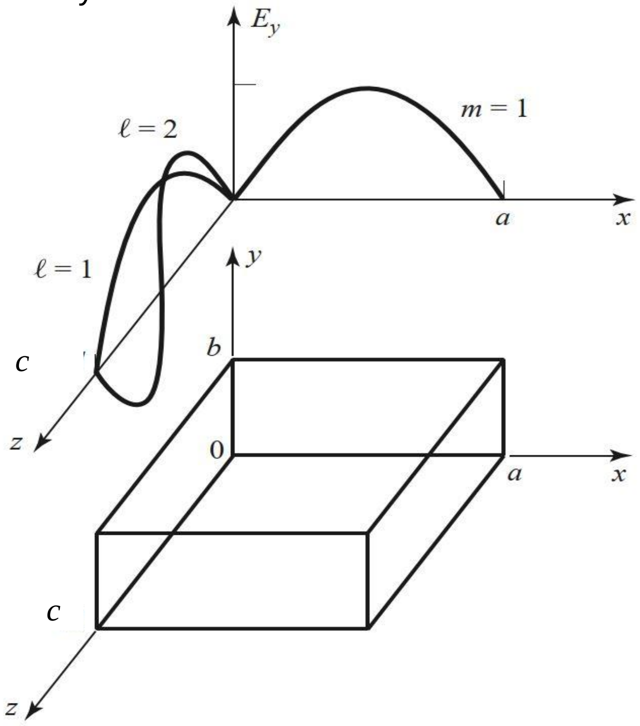
- resonant wavelength: $\lambda_0 = \dfrac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{c}\right)^2}}$

- Resonant frequencies for TE_{mnl}-mode and TM_{nml}-modes:

$$f_{mnl} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2}$$

- Q-value for TE_{101}: $Q_{TE101} = \dfrac{\lambda_0}{\delta}\dfrac{b}{2}\dfrac{(a^2+b^2)^{3/2}}{c^3(a+2b)+a^3(c+2b)}$

Source: Pozar, *Microwave Engineering*, 4th ed., Wiley



Electric field variation
for TE_{101} and TE_{102} modes
(counting the number of “zeros” along the length)

TE_{101}-mode is the lowest resonant mode, hence the **dominant mode** in this resonator.

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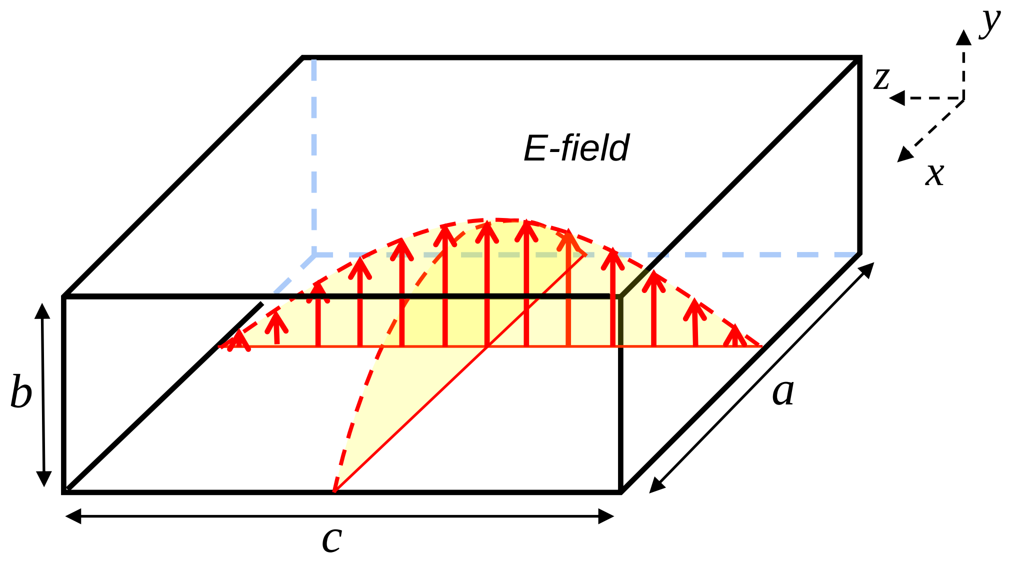
Rectangular Cavity Resonators (3/3)

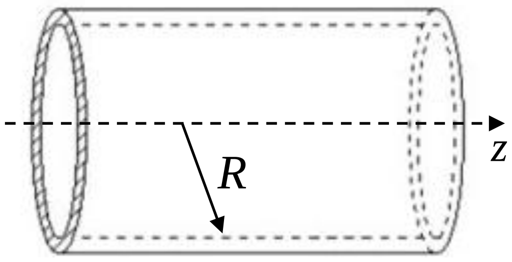
For the case $a=c$, the formulae simplifies strongly.

- resonant wavelength: $\lambda_0 = \sqrt{2}a$
- Q-value for TE_{101}: $Q_0 = \frac{1}{\delta}\frac{ab}{a+2b}$



If you had to use this resonator, where would you connect the vacuum pipe?

Circular (round) Waveguides (1/4)



- Like all transmission lines, also the circular waveguide is characterized by a propagation constant, an attenuation constant and a characteristic impedance.
- These quantities are derived by field theory analysis. Propagation is usually assumed in z-direction.

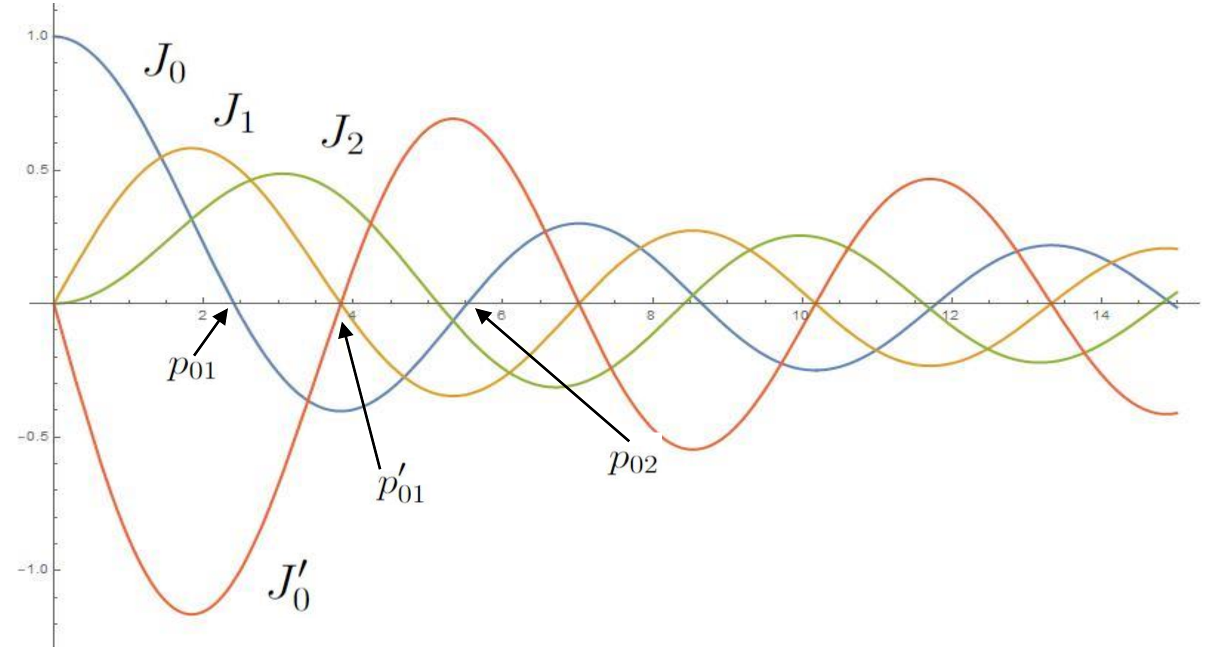


TABLE 3.3 Values of p'_{nm} for TE Modes of a Circular Waveguide

n	p'_{n1}	p'_{n2}	p'_{n3}
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970

TABLE 3.4 Values of p_{nm} for TM Modes of a Circular Waveguide

n	p_{n1}	p_{n2}	p_{n3}
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

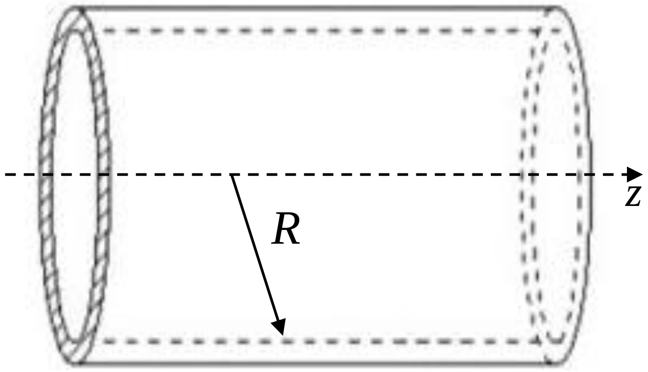
Source: Pozar, *Microwave Engineering*, 4th ed., Wiley

p_{01} = 1st root of Bessel function of first type J_0

p_{02} = 2nd root of Bessel function of first type J_0

p_{01}' = 1st root of derivative of Bessel function of first type J_0'

Circular (round) Waveguides (2/4)



- For a perfect conducting tube (no resistive attenuation) with radius R, we obtain propagation constants for the TE-mode and the TM-mode:

$$\beta_{nm} = \sqrt{\omega^2 \epsilon \mu - \left(\frac{p'_{nm} \text{ or } p_{nm}}{R} \right)^2}$$

p_{nm} → Roots of the Bessel-function for TM-mode $J_n(p_{nm}) = 0$

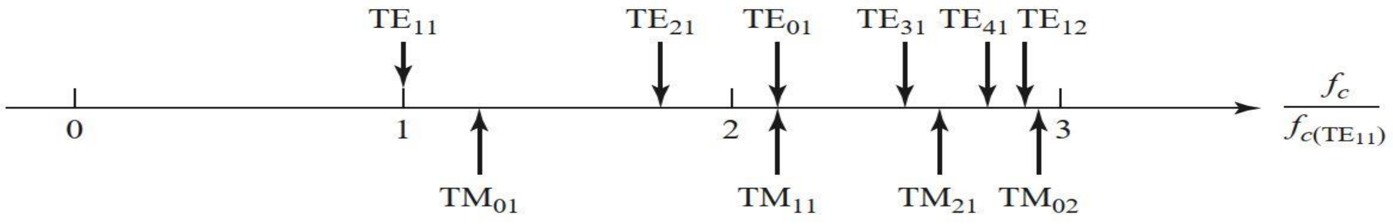
p'_{nm} → Roots of the derivative of the B.F. for TE-mode $J'_n(p'_{nm}) = 0$

- This leads to the cut-off frequencies for the different modes:

$$f_{c,nm} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \left(\frac{p'_{nm} \text{ or } p_{nm}}{R} \right)$$

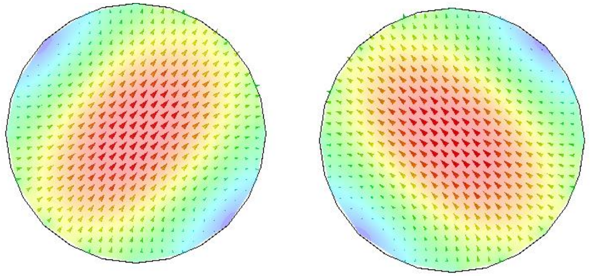
Circular (round) Waveguides (3/4)

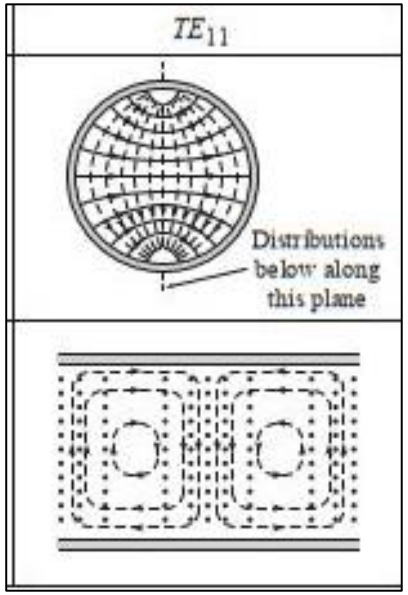
Waveguide modes and field patterns

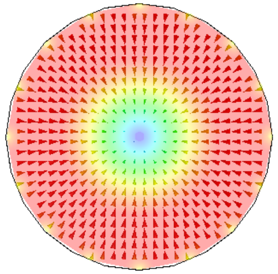


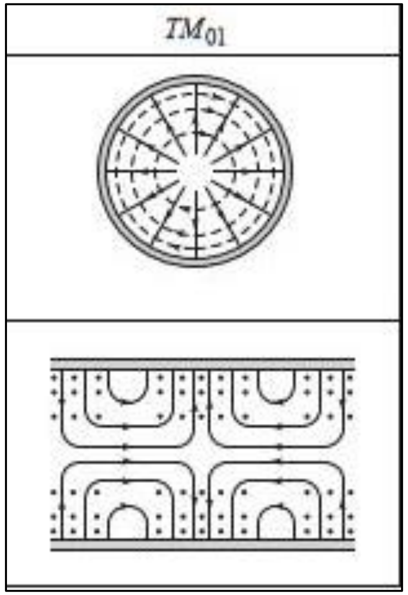
1st mode is TE_{11}. Electric field is transverse.
Mode has 2 polarisations (orientations of the electrical field).

- 2nd mode is TM_{01}. Magnetic field is transverse.





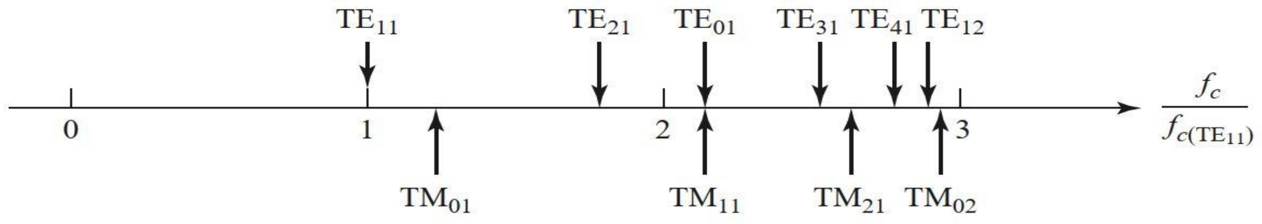




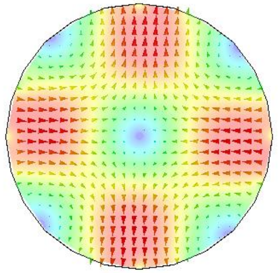
Simulaton pictures: courtesy E. Jensen, source pictures and chart: Pozar, *Microwave engineering*, 4th ed., Wiley

Circular (round) Waveguides (4/4)

Waveguide modes and field patterns



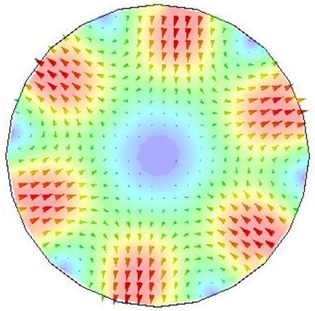
- 3^{rd} mode is TE_{21}.
  Electric field is transverse

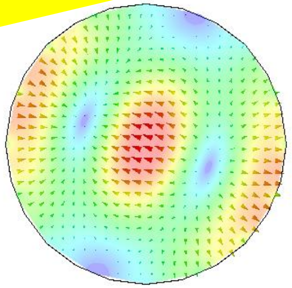


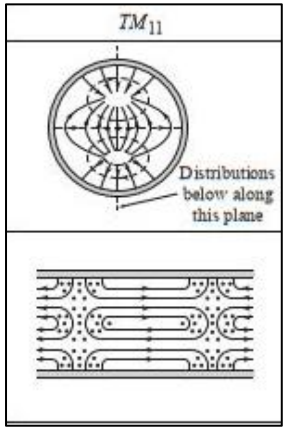
- 4^{th} … M_{11}. Magnetic field is transverse.

Next higher modes? Not important right now…

- … eld is transverse.







Further field plots can *e.g.* be found here: Lee, Lee and Chuang, *Plot of Modal Field Distribution in Rectangular and Circular Waveguides*. IEEE, Trans. Microwave Theory and Techniques, Vol. MTT-33, no. 3, March 1985

Simulaton pictures: courtesy E. Jensen, source pictures and chart: Pozar, *Microwave engineering*, 4^{th} ed., Wiley

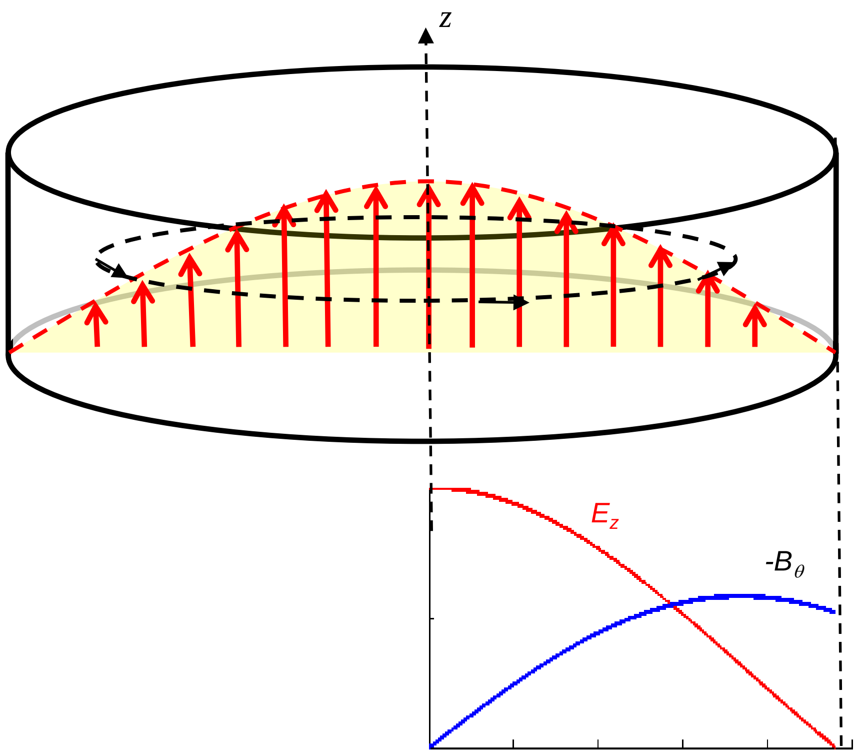
Pillbox Cavity Resonator (1/4)

We obtain a cylindrical cavity resonator by shortening the circular waveguide at both ends.

→ TE_{111}-mode is the dominant <u>TE-mode</u> for a cylindrical cavity.

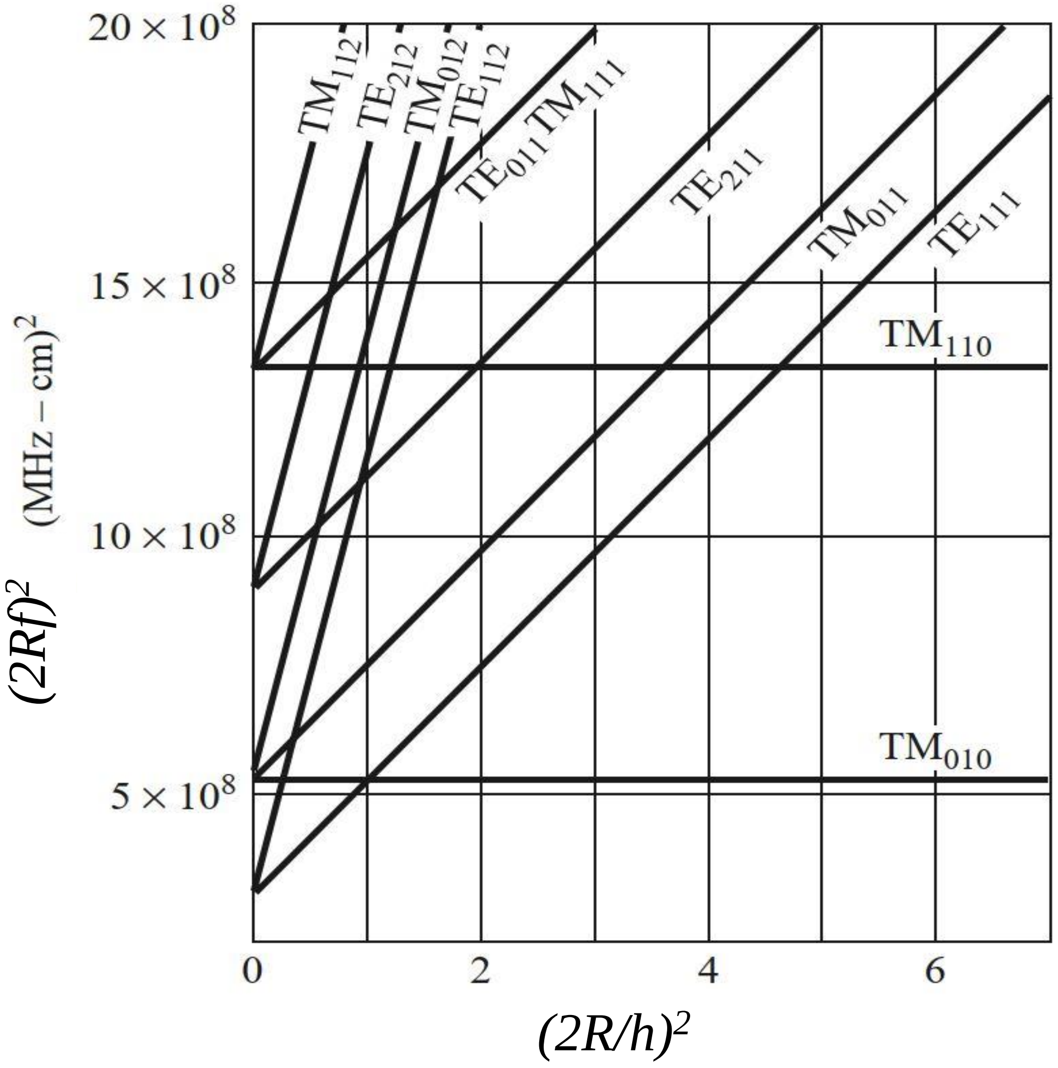
→ TM_{010}-mode is the dominant <u>TM-mode</u> for a cylindrical cavity.

→ *TM_{010}-mode mode is used for acceleration as it has a large electric field along the z-axis.*



FINE. What’s the problem?

Pillbox Cavity Resonator (2/4)



Source: Pozar, *Microwave engineering*, 4th ed., Wiley

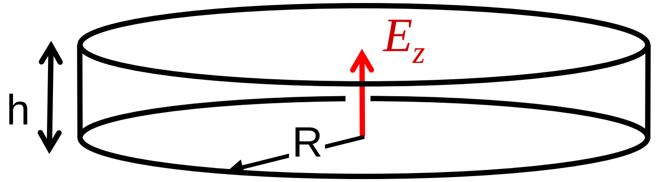
The problem is the mode chart.

Mode chart is a resonant chart for a general cylindrical cavity that shows the excited modes as a function of cavity dimensions.

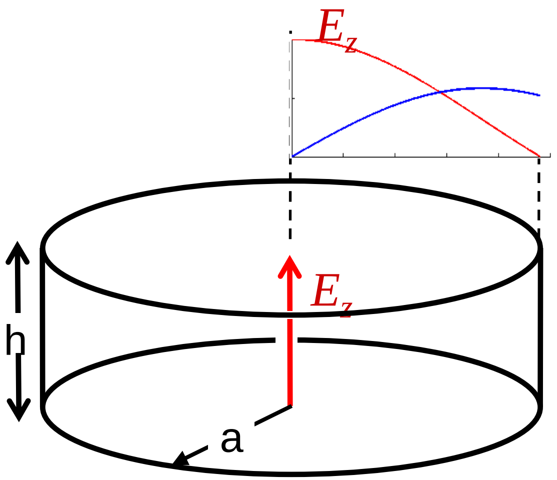
→ 1st TE-mode is TE_{111} (useless for acceleration)

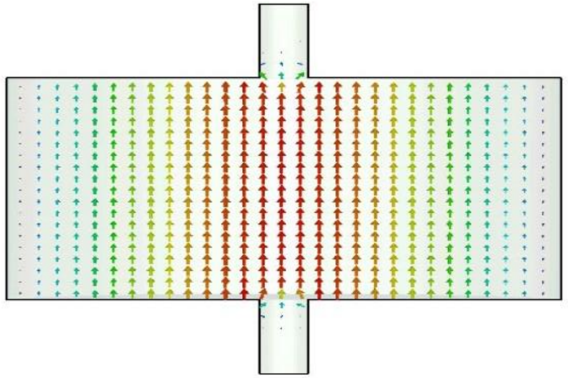
→ 1st TM-mode is TM_{010},
and shows up as first mode for ratios $(2R/h)^2 > 1$

The cavity needs to be flat to provide the accelerating mode.



Pillbox Cavity Resonator (3/4)





Pillbox cavity is flat compared to a “long” cylindrical cavity.
If the cavity is of a pillbox shape, all is easy!

- Mode used for acceleration is TM010.
- Resonant frequencies for TE_{nml}-mode:

$$f_{\mathrm{TE},nml} = \frac{c_0}{2\pi}\sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{l\pi}{h}\right)^2}$$

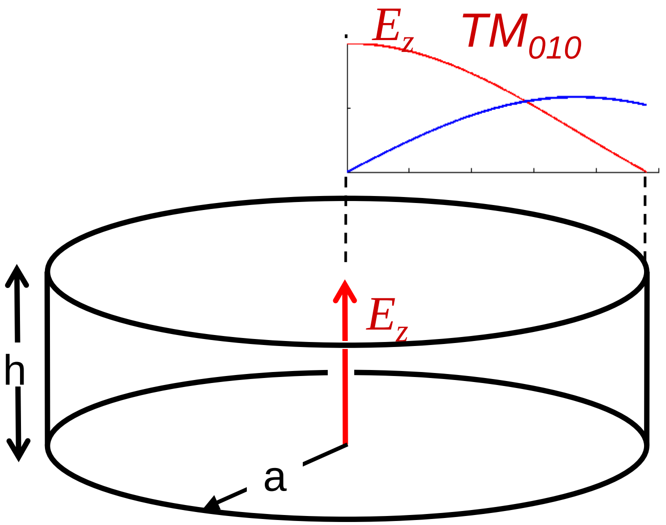
- Resonant frequency for TM_{nml}-modes:

$$f_{\mathrm{TM},nml} = \frac{c_0}{2\pi}\sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{l\pi}{h}\right)^2}$$

Roots of B.F.

Note that the TM_{010}- mode is *independent of the cavity height h* (until the TE_{111} mode shows up, roughly at *2a/h=1*).

Pillbox Cavity Resonators (4/4)



For the case of the TM_{010}-mode, when we have no dependence on cavity height, we can use simpler formulae for the wavelength:

$$\boxed{0.383\,\lambda_{\mathrm{TM},010} = a}$$

And we can use the Q-calculation (equally simplified!):

$$\boxed{Q = \frac{0.383\,\lambda_{\mathrm{TM},010}}{\delta}\left[1 + \frac{a}{h}\right]^{-1} = \frac{a}{\delta}\left[1 + \frac{a}{h}\right]^{-1}}$$

Skindepth: $\delta = \sqrt{\dfrac{2}{\omega \sigma \mu}}$

$\eta_0 = 120\,\pi\,\Omega$

Accelerator-figure-of-Merit:

$$\boxed{R/Q = \frac{4\eta_0}{\pi\, p_{01}^3\, J_1^2(p_{01})}\,\frac{\sin^2\left(\frac{p_{01}}{2}\frac{h}{a}\right)}{h/a}}$$

$$\boxed{R/Q = 128\,\frac{\sin^2\left(\frac{p_{01}}{2}\frac{h}{a}\right)}{h/a} \approx 185\,h/a}$$

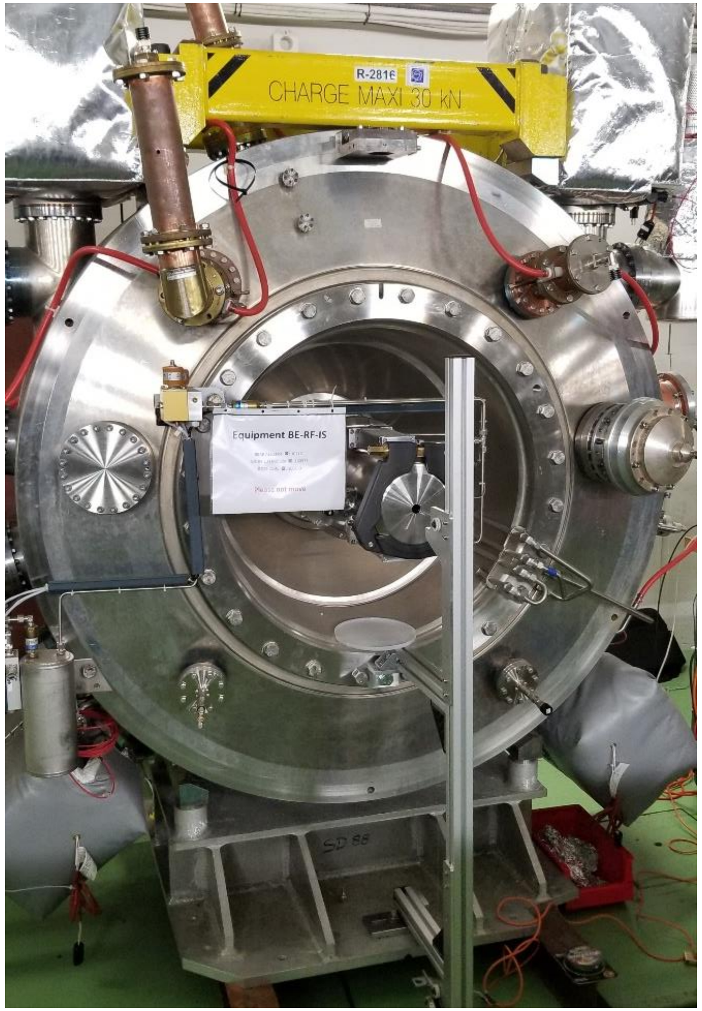
$p_{01} = 2.405$ $\quad J_1(p_{01}) = 0.51911$

For small arguments of Sinus-function

Source: Pozar, *Microwave Engineering*, 4th ed., Wiley

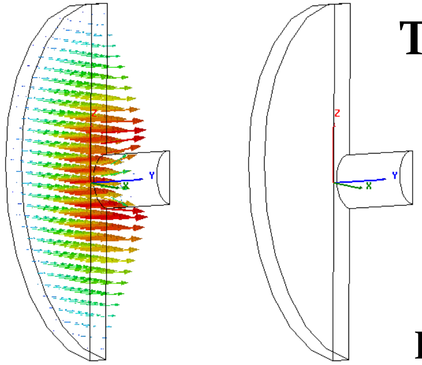
Example “true” Pillbox Cavity

Cavity from DORIS Storage ring
(1970-ish, very early
electron/positron collider)



Electric field,
TM_{010}-mode

Magnetic field,
TM_{010}-mode



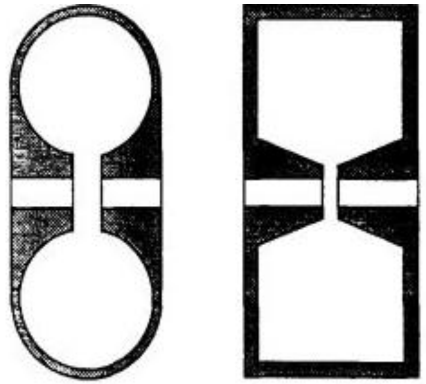
E. Jensen

Accelerating Cavity, freq = 80 MHz
(CERN PS)

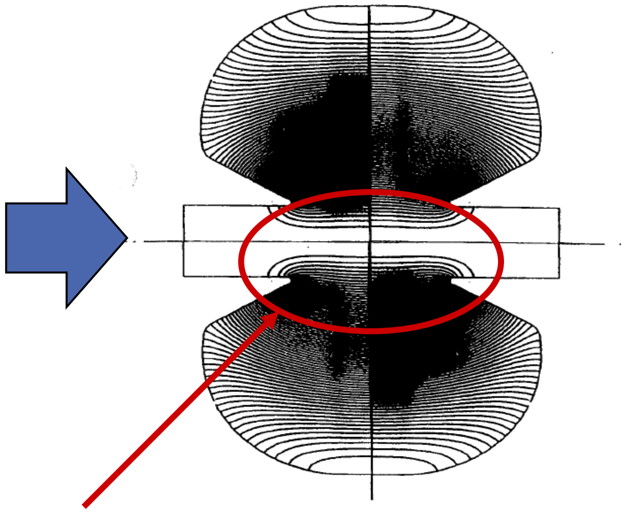
Pillbox Cavity Design Feature

In practice, a “pure” pillbox cavity is not very efficient for acceleration.
A simple shape modification can be done by using so-called “nose cones”.

Nose cone is a protrusion on the cavity wall that causes a concentration of the electrical field in the gap.

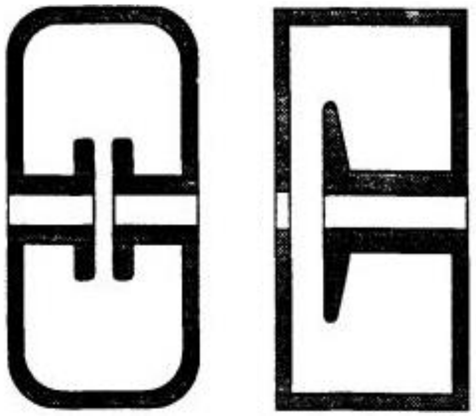


Source: Puglisi, *RF-CAS 92*, CERN

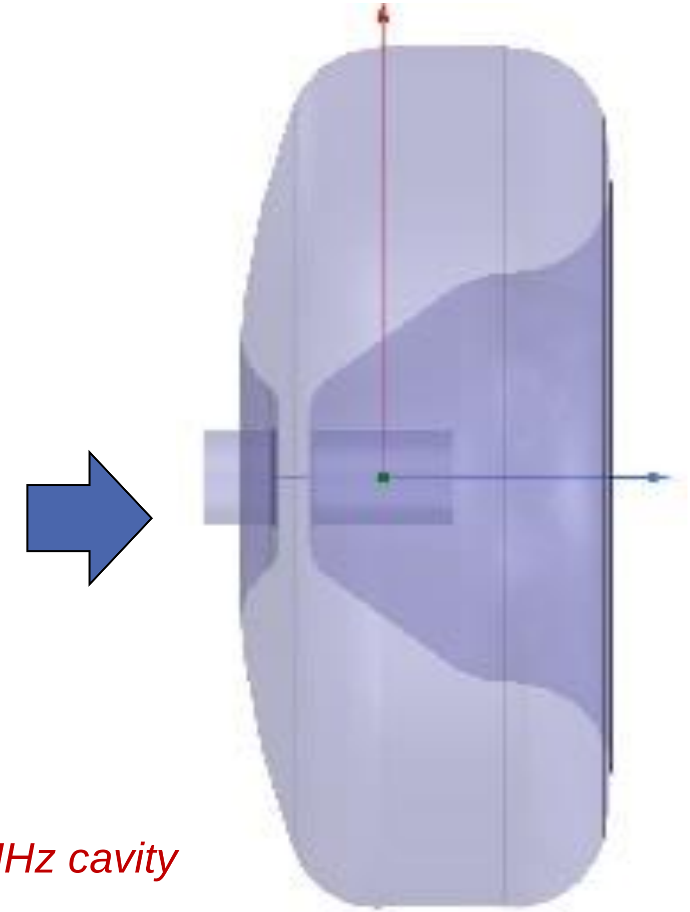


Enhancement of E-field

Nose cones also help to improve the Transit-time factor…

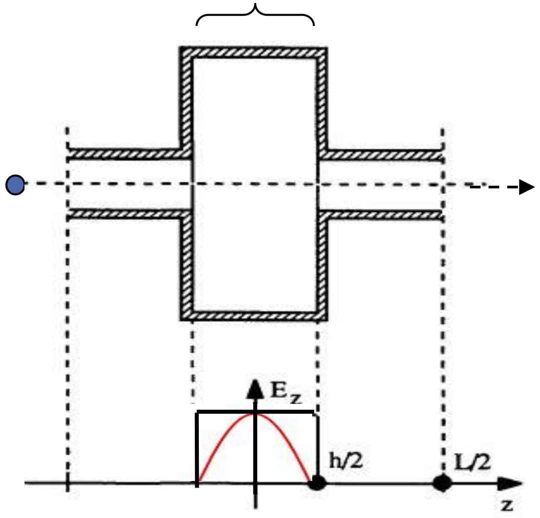


Source: Puglisi, *RF-CAS 92*, CERN



Inside PS 80 MHz cavity

Transit Time Factor (1/2)



transit-time factor:

$$T = \frac{\textit{energy gained in time-varying RF-field}}{\textit{energy gained in a DC field of voltage } V_0}$$

$$\boxed{T = \frac{\int_{-L/2}^{L/2} E(0,z)\cos(2\pi z/\beta\lambda)\,dz}{\int_{-L/2}^{L/2} E(0,z)\,dz}}$$

Distance the particle travelled in one RF-period; β=v/c

- Particle in the harmonic time-varying field will see less energy gain compared to a constant DC field.
- This is called transit-time effect and is described by a factor T (transit-time factor).

Transit Time Factor (2/2)

$$\boxed{T = \frac{\int_{-L/2}^{L/2} E(0,z)\cos(2\pi z/\beta\lambda)\,dz}{\int_{-L/2}^{L/2} E(0,z)\,dz}}$$

Transit time factor

$E(0,z)\,dz$ = constant
(as it is for an ideal pillbox cavity!)

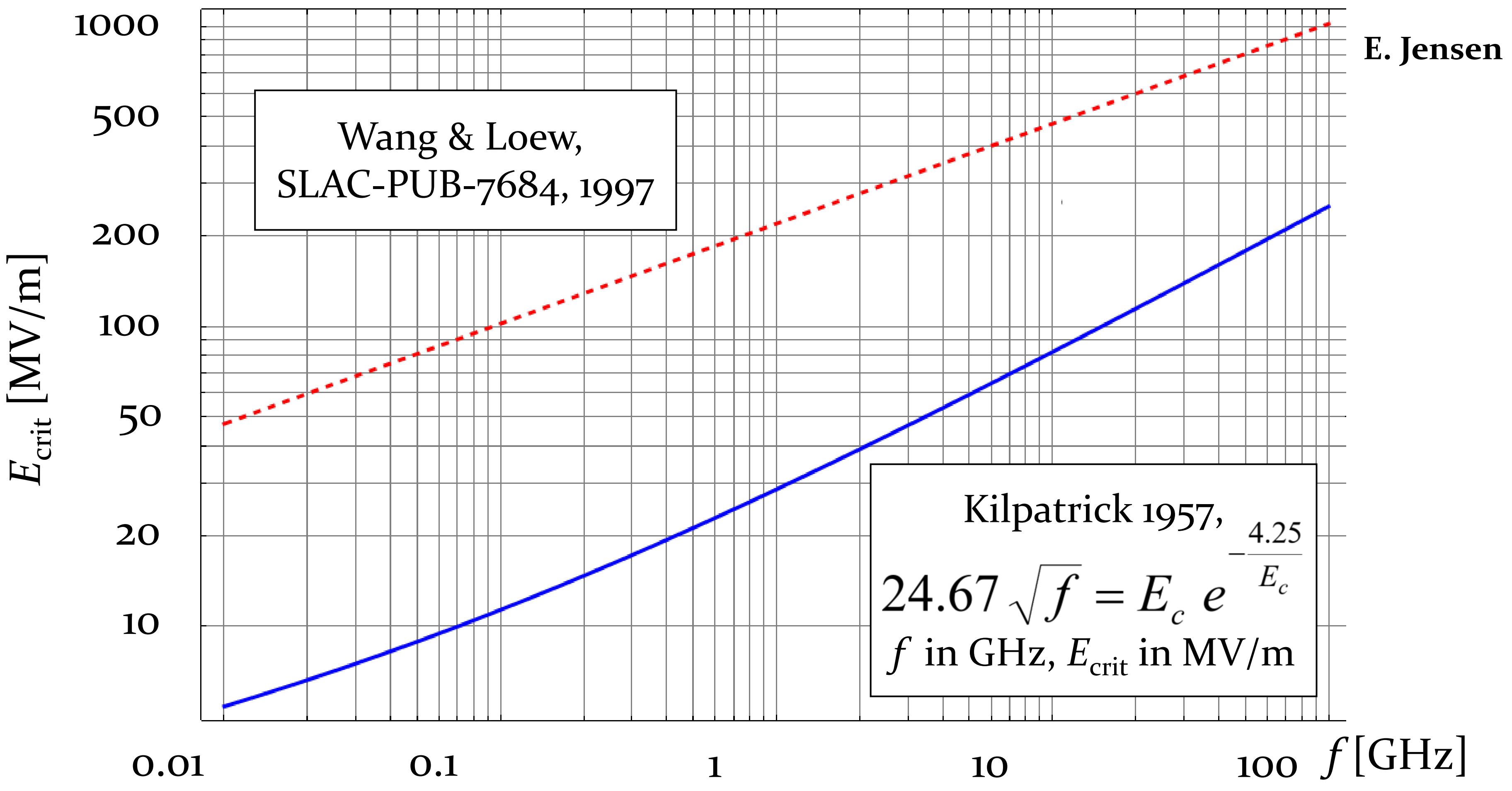
gap length

$$\boxed{T = \frac{\sin \pi g/\beta\lambda}{\pi g/\beta\lambda}}$$

- To achieve max. energy gain from this formulae, we want that $\boldsymbol{T = 1 \rightarrow g = 0}$
- Leads to design request: ***gap as small as possible*****.**
- But other considerations as: risk of RF electric breakdown also impact on optimum gap geometry.
- Note that it is assumed that the particle does not change velocity along the gap length.

Limits to maximum gradient

- **Surface electric field in vacuum**



→ **High frequencies preferred for large gradient.**

→ **Today also other criteria are used, e.g. considering local field quantity, see: Grudiev et al., PRAB #102001 , 2009**

Accelerator efficiency figures-of-merit (1/2)

Several figures-of-merit are commonly used to characterize accelerating cavities:

- *Q-value*
  unloaded Q_0 - measure of the resonance quality

$$Q_0 = \frac{\omega U}{P}$$

energy stored in the resonator

energy dissipated in the resonator

- *Shunt impedance [MΩ]*
  measure of effectiveness to produce an axial voltage V_0

$$r_s = \frac{V_0^2}{P}$$

Design goal is a high shunt impedance

- *Effective Shunt impedance [MΩ/m]*
  Measure of effectiveness per unit power loss to deliver energy to a particle

$$r_{s,\mathrm{eff}} = \frac{(V_0 \dot{T})^2}{P} = r_s T^2$$

- *"R-over-Q" [Ω]*
  Measure of cavity acceleration efficiency at a given frequency – geometry dependent only!

$$r/Q = \frac{(V_0 \dot{T})^2}{\omega U}$$

Accelerator efficiency figures-of-merit (2/2)

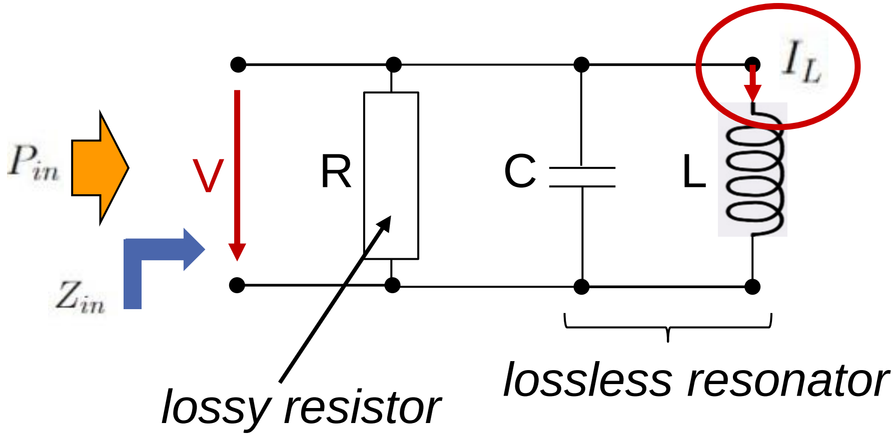
Typical values for different cavities:

Cavity type	***R/Q***	Q_0	***R***
Ferrite loaded cavity (low frequency, rapid cycling)	4 kΩ	50	200 kΩ
Room temperature copper cavity (type 1 with nose cone)	192 Ω	$30 * 10^3$	5.75 MΩ
Superconducting cavity (type 2 with large iris)	50 Ω	$1 * 10^{10}$	500 GΩ

Cavity equivalent circuit (1/4)

At frequency near resonance, the cavity resonator can be modeled by a lumped-element circuit.

For a cavity with the desired high shunt impedance, only a parallel resonant circuit is suited
→ require to model a large voltage.



$$P_{in} = \frac{1}{2}VI^* = \frac{1}{2}|V|^2\frac{1}{Z_{in}^*}$$

power to the resonator circuit

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1}$$

input impedance

$$P_{in} = |V|^2\left(\frac{1}{R} + j\frac{1}{\omega L} - j\omega C\right)$$

Power dissipated in the resistor: $P_{\text{loss}} = \frac{1}{2}\frac{|V|^2}{R}$

Energy stored in capacitor: $W_e = \frac{1}{4}|V|^2 C$

Energy stored in inductor: $W_m = \frac{1}{4}|I_L|^2 L = \frac{1}{4}|V|^2\frac{1}{\omega^2 L}$

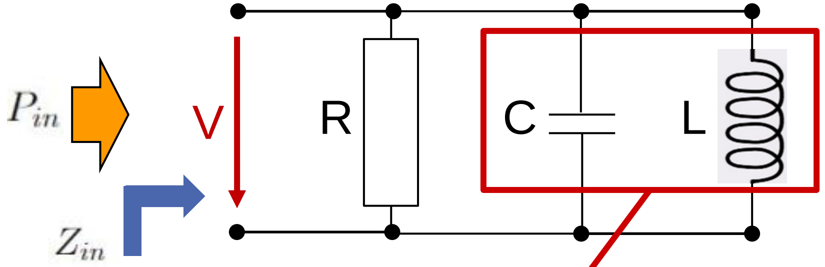
The CERN Accelerator School

Cavity equivalent circuit (2/4)

At frequency near resonance, the cavity resonator can be modeled by a lumped-element circuit.

For a cavity with the desired high shunt impedance, only a parallel resonant circuit is suited
→ require to model a large voltage.

resonance case is: $W_e = W_m$



$$P_{in} = |V|^2\left(\frac{1}{R} + j\frac{1}{\omega L} - j\omega C\right) = P_{\text{loss}} + 2j\omega\cancel{(W_e - W_m)}$$

power to the resonator circuit

Power dissipated in the resistor: $P_{\text{loss}} = \frac{1}{2}\frac{|V|^2}{R}$

$$Z_{in} = \frac{P_{\text{loss}}}{\frac{1}{2}|I|^2} = R$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

$$\boxed{Q_0 = \omega_0\frac{2W_m}{P_{\text{loss}}} = \frac{R}{\omega_0 L} = \omega_0 RC}$$

☺ trivial!

Cavity equivalent circuit (3/4)

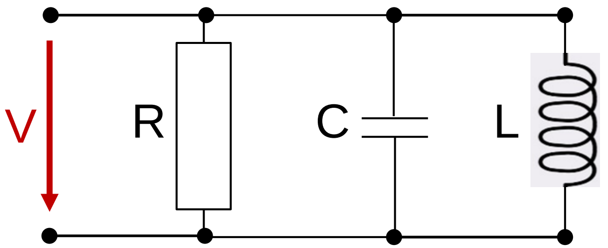
At frequency near resonance, the cavity resonator can be modeled by a lumped-element circuit.

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

resonance frequency

$$\boxed{Q_0 = \omega_0 \frac{2W_m}{P_{\text{loss}}} = \frac{R}{\omega_0 L} = \omega_0 RC}$$

unloaded Q



R in equivalent circuit == shunt impedance in cavity

C and L in equivalent circuit == resonance mechanism

$$\boxed{C_{\text{par}} = \frac{Q_0}{\omega_0 r_{\text{shunt}}}} \qquad \boxed{L_{\text{par}} = \frac{r_{\text{shunt}}}{\omega_0 Q_0}}$$

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Cavity equivalent circuit (4/4) – Cavity loading

Connecting the cavity to the outer world…

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

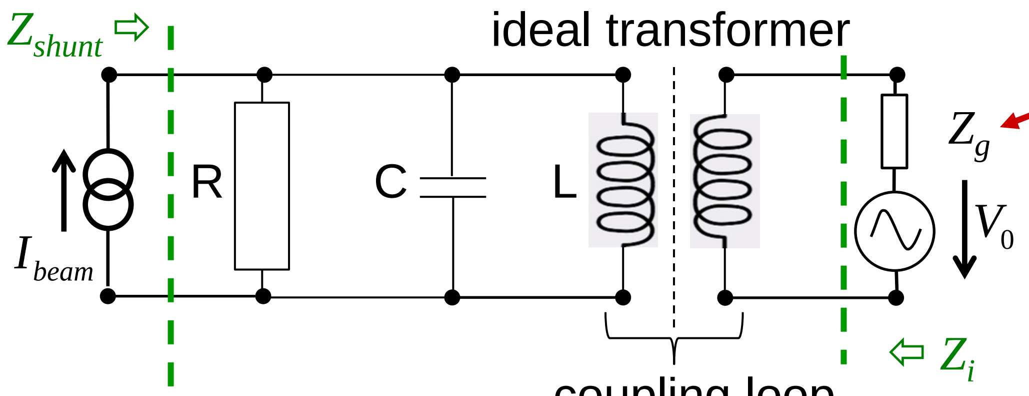
resonance frequency

$$\boxed{Q_0 = \omega_0 \frac{2W_m}{P_{\text{loss}}} = \frac{R}{\omega_0 L} = \omega_0 RC}$$

unloaded Q

$$\boxed{C_{\text{par}} = \frac{Q_0}{\omega_0 r_{\text{shunt}}}} \quad \boxed{L_{\text{par}} = \frac{r_{\text{shunt}}}{\omega_0 Q_0}}$$

parallel capacitance and inductance



Generator impedance
(generally complex due to matching requirement)

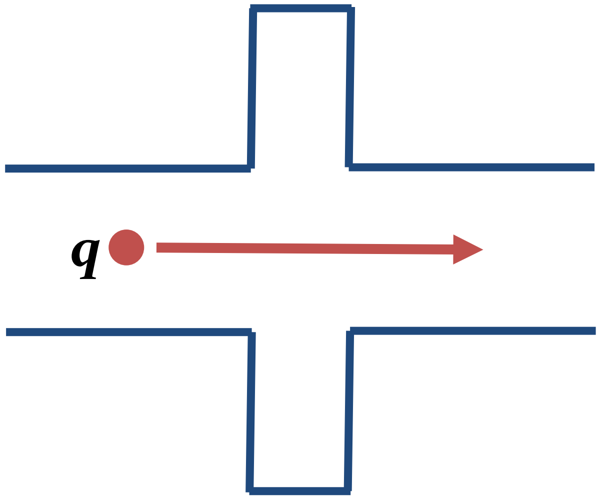
So-called: Cavity loading
(inductive in this case)

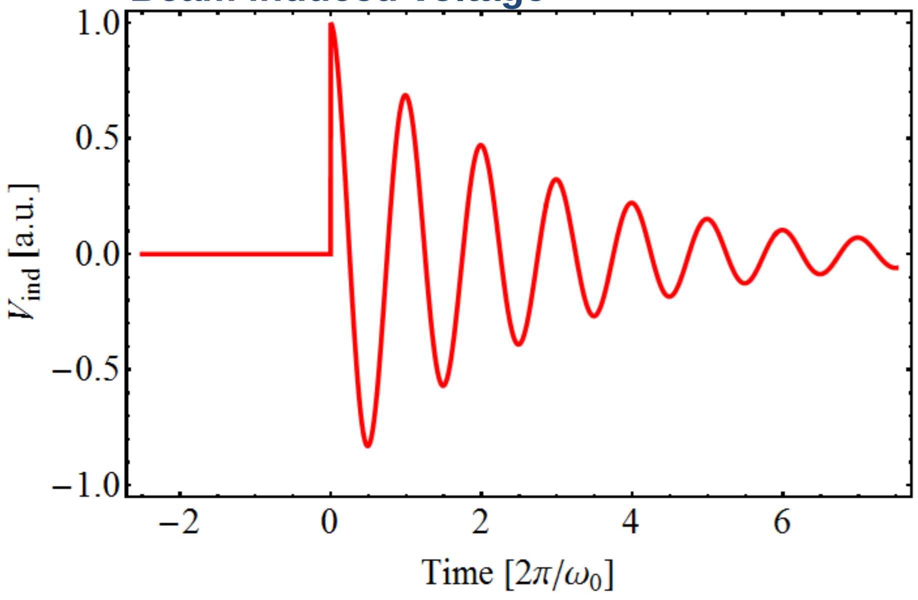
$$\frac{1}{Q} = \frac{1}{Q_{\text{cav}}} + \frac{1}{Q_{\text{ext}}}$$

- Beam is usually modelled as a current source and sees a (generally complex) Z_{shunt}.
- Via the transformer, the coupling to the cavity can be adjusted to *"matching"*.
  In practice, we would rotate our coupling loop to modify the coupling strength.

What about this R/Q-criteria?

→ **Charged particle experiences cavity gap as capacitor**





$$q = V_{\mathrm{ind}} C$$

$$Q = \omega_0 R C \quad \rightarrow \quad \frac{1}{C} = \left(\frac{R}{Q}\right)\omega_0$$

$$\boxed{V_{\mathrm{ind}}} = \frac{q}{C} \propto \boxed{\frac{R}{Q}}$$

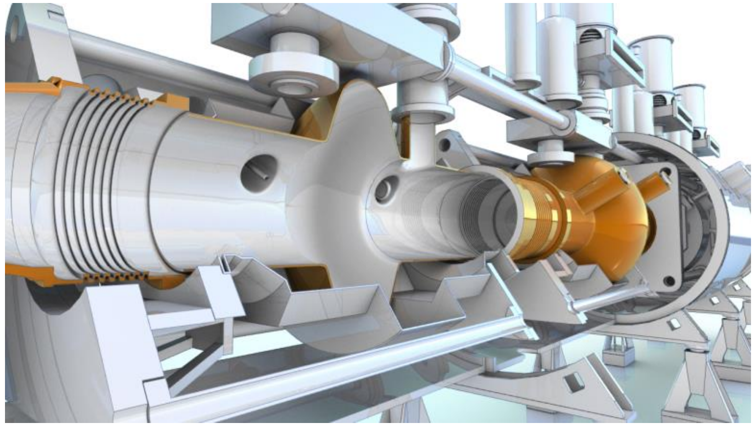
→ **Design goal: to reduce beam loading, aim for cavity geometry with small *R/Q***

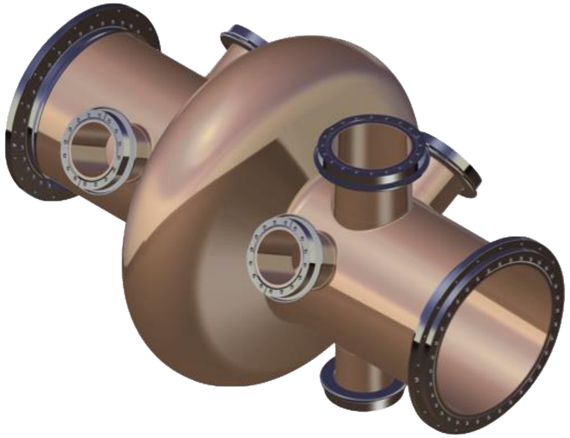
Example: 400 MHz cavities in LHC

→ Very high beam currents, need to reduce beam loading in RF cavities

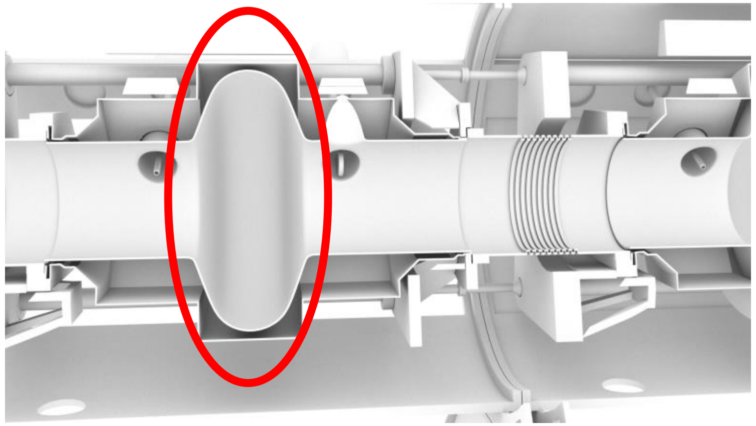
→ Shunt impedance *R, is high but* small R/Q is needed

→ Could be achieved with superconducting cavities in LHC





Bell shape: R/Q ~ 44 Ω, 400 MHz



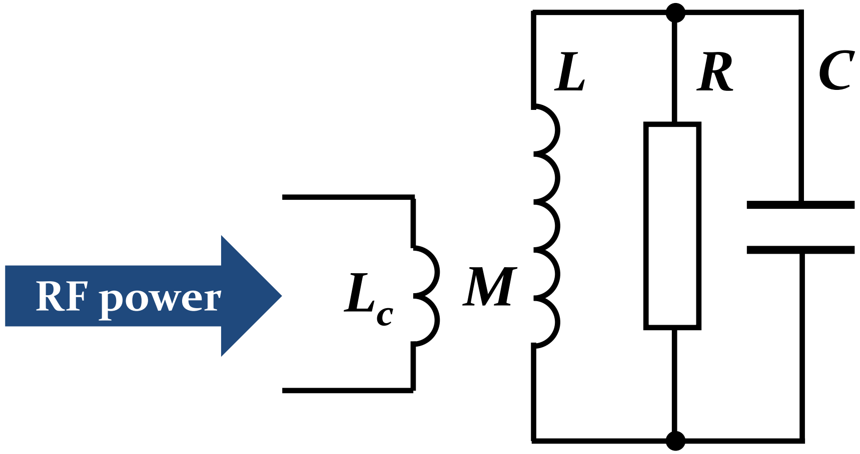
→ 2×8 cavities, 5.3 MV/m

$$\frac{1}{Q} = \cancel{\frac{1}{Q_{\mathrm{cav}}}}^{\sim 0} + \frac{1}{Q_{\mathrm{ext}}}$$

Principles of coupling power into a cavity

Coupling power into a cavity

- Attach inductivity or capacitance of resonator, or combined



$$\frac{1}{Q} = \frac{1}{Q_{\text{cav}}} + \frac{1}{Q_{\text{ext}}}$$

→ Coupling loop forms transformer with resonator inductivity

L. Stigelin

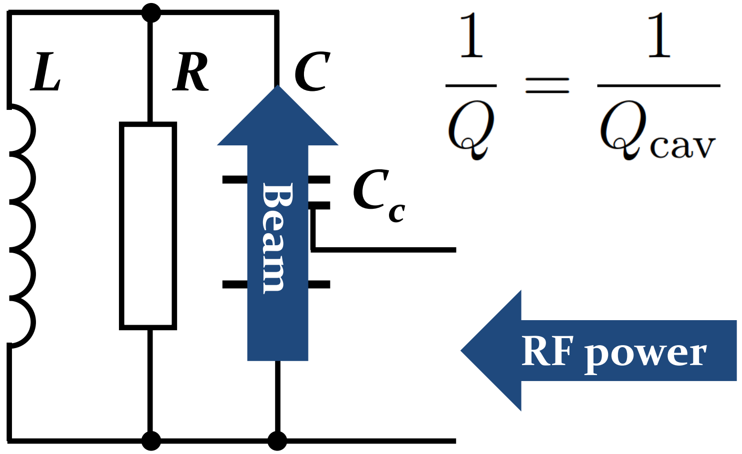
- Main coupler PSI cyclotron

→ ~1 MW at 50 MHz

- By far the most common method, allows also to adopt the matching.

Coupling power into a cavity

- Attach inductivity or capacitance of resonator, or combined



$$\frac{1}{Q} = \frac{1}{Q_{\text{cav}}} + \frac{1}{Q_{\text{ext}}}$$

→ Capacitive divider to gap to transform generator impedance to cavity shunt impedance

→ Advantage: allows to DC isolate the coupler (if required by amplifier).

→ Disadvantage: coupling is fixed.

→ Beam also couples capacitively via the gap

Coupler of CERN PS 40 MHz

→ Coupler forms one half of capacitor with the gap

Capacitive (electric) coupling

- Coupling through an electric antenna

Electrical coupler to space

https://en.wikipedia.org/wiki/Transmitter_Solt

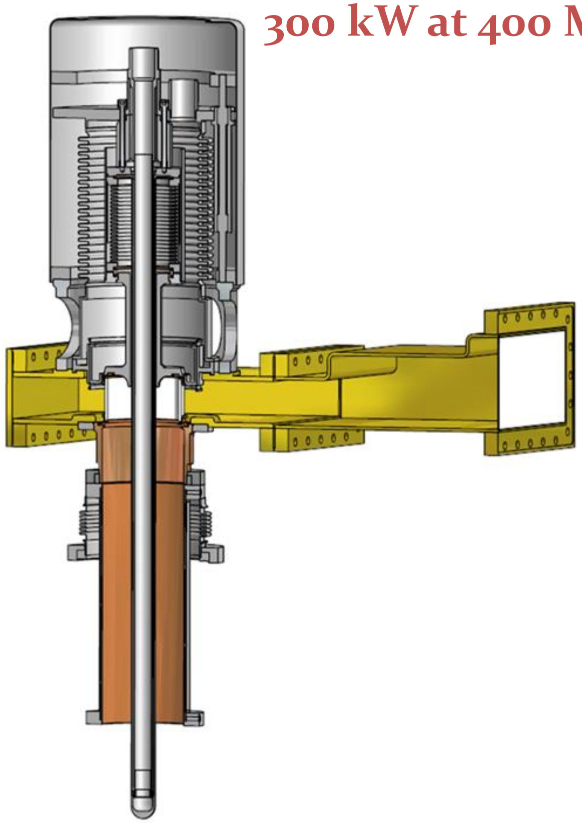
→ 2 MW at 540 kHz

→ Used to transmit radio broadcast in Hungarian language around the world.

→ *claims to have reached Michigan…

Power coupler of LHC cavities

300 kW at 400 MHz



→ Coupler antenna transmits directly into the cavity

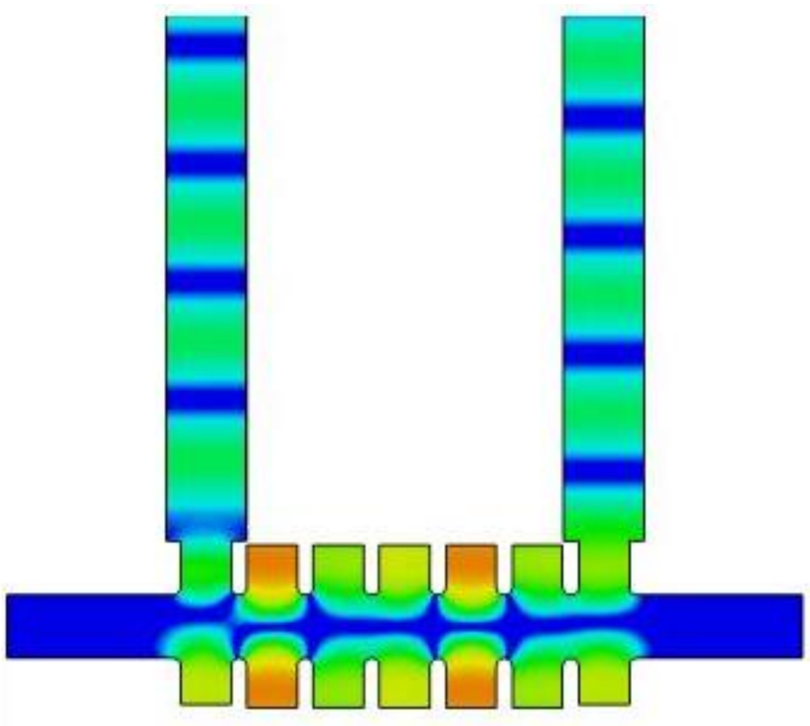
End of part 2

Travelling wave cavities

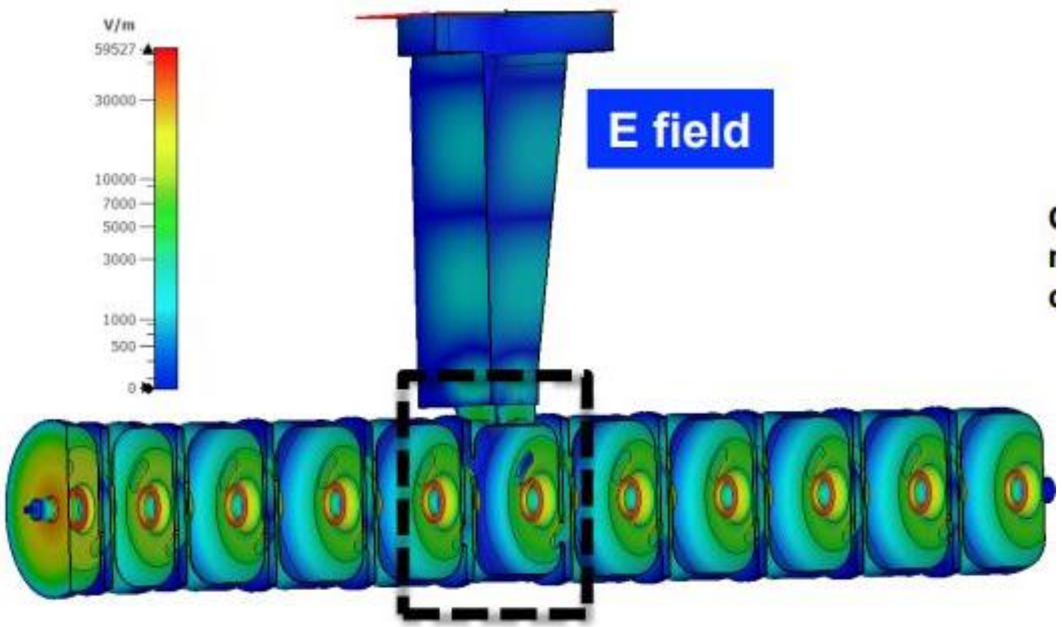
(Extra!)

Travelling vs. Standing wave

Just because we have a multi-cell structure, this does not mean that we are having a travelling wave cavity!



travelling wave



standing wave

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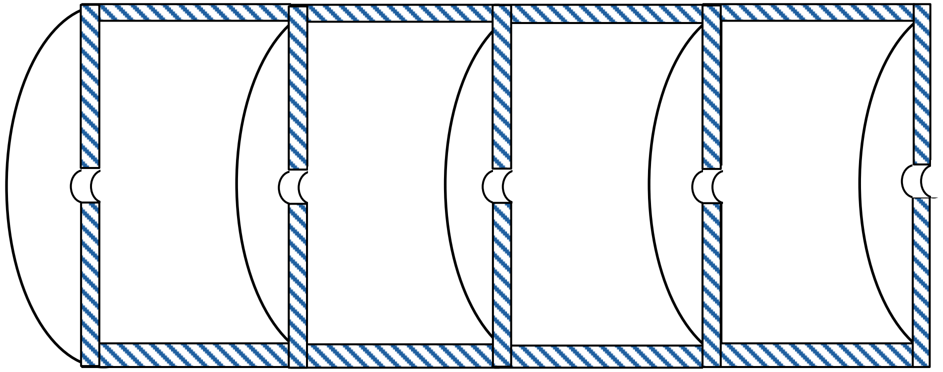
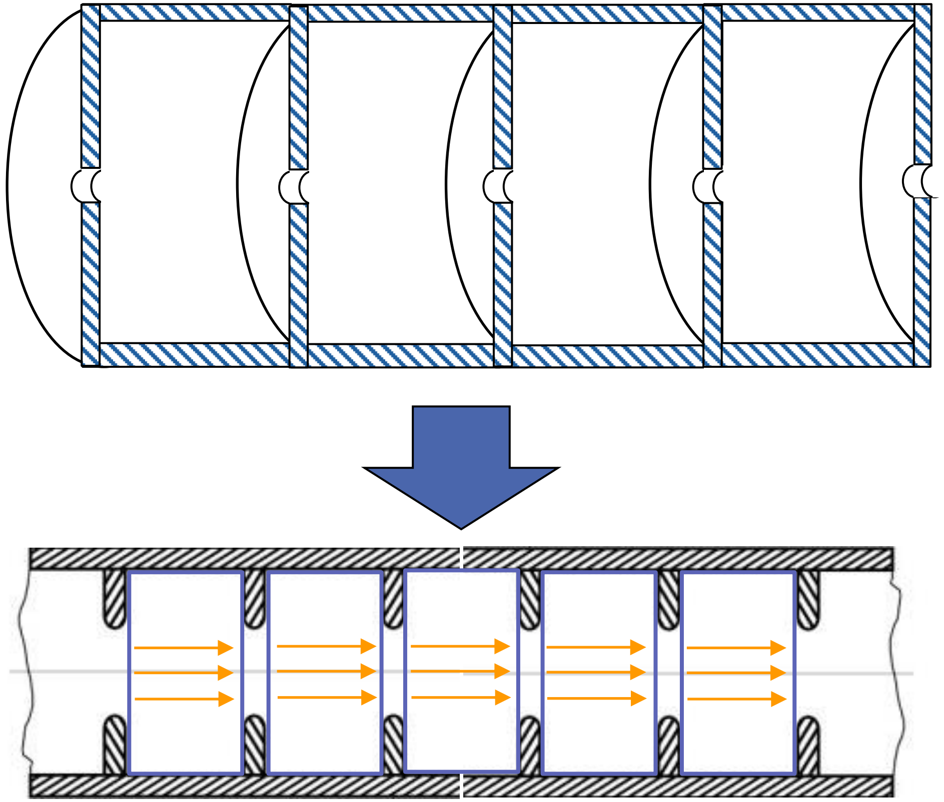
Disc-loaded circular waveguide (1/2)



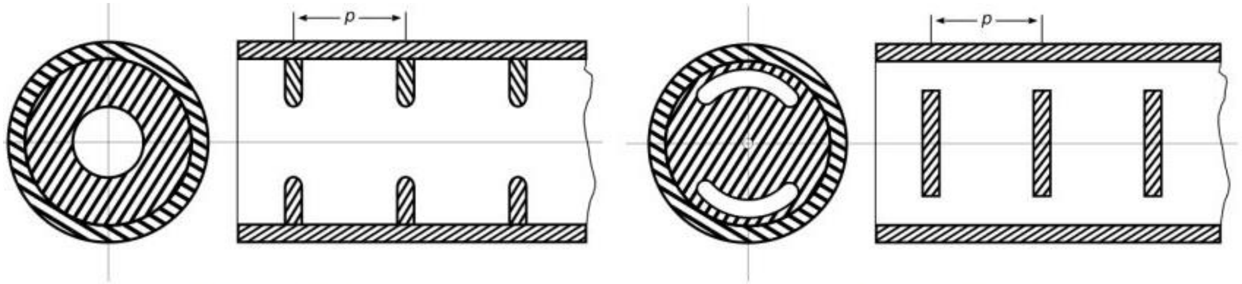
- Disc-loaded waveguide is a circular metallic waveguide with periodically added metallic discs and holes for particle passage and for coupling of the cells.
- We speak of a coupled-cavity chain structure.
- EM-fields need to fulfill boundary conditions on the metallic discs.
- Disc-loaded waveguide *can operate in travelling-wave as slow wave structure or in standing-wave mode*.
- In standing wave mode, the cells present a “concatenation” of TM_{010}-mode “pillbox”-type cavities.

Disc-loaded circular waveguide (2/2)



Example: 0-mode structure in standing wave (SW) … can be derived from pillbox TM_{010}-mode.

Source: Kramer, *Studies of HOM-couplers for the Upgrades Travelling Wave Acceleration System in the CERN SPS*, PhD, CERN-THESIS-2019-371



(a) Iris coupled structure. (b) Slot coupled structure.

- Different coupling mechanisms between the individual cells exits, either on the side or in the center.
- We speak of iris-coupled-structure and slot-coupled-structure.
- For slot-coupled-structures, the center opening for the beam can be very small and is often ignored in the EM-calculations.

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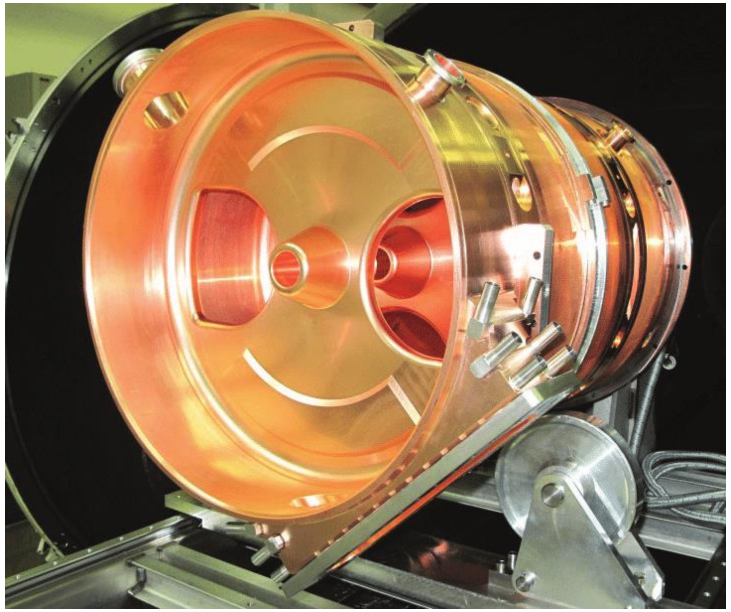
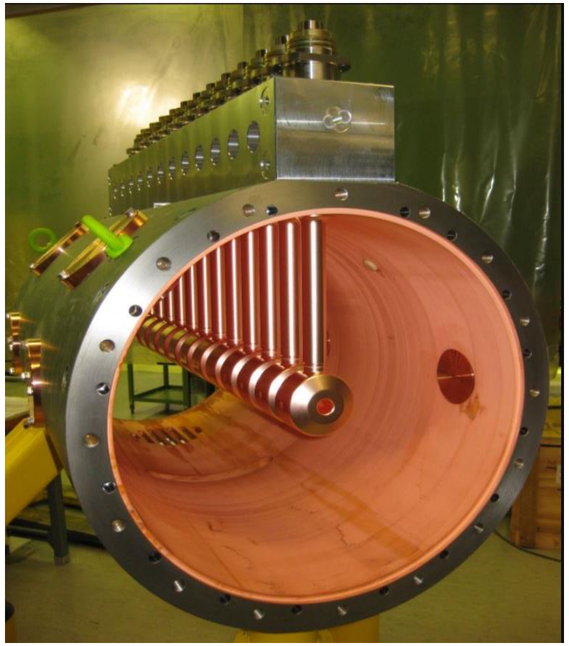
Example of Pi-mode Cavities (SW, 2/3)

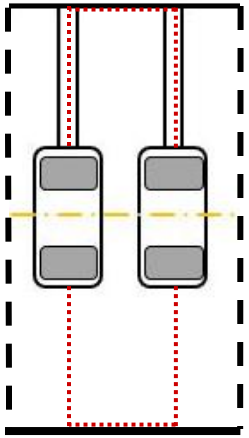
How does this look in reality?

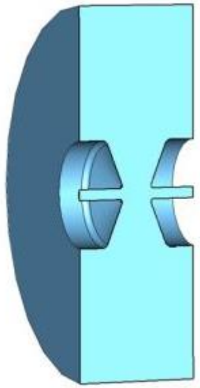


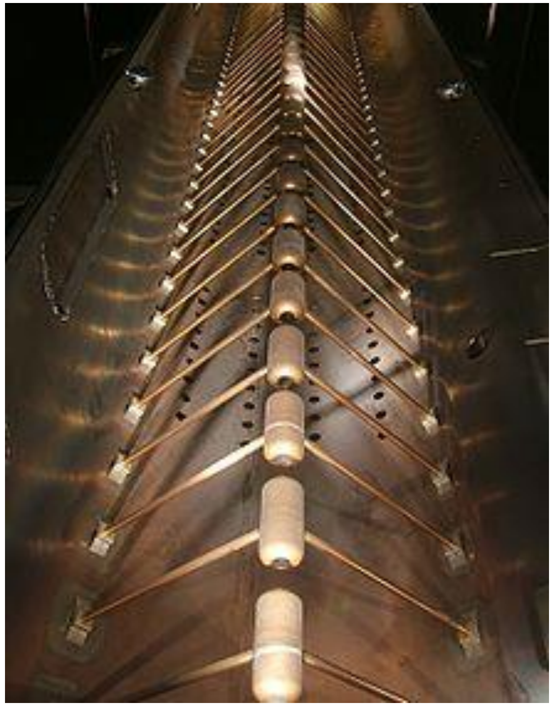
Slot-coupled-structure (PIMS) at CERN



Drift tube structure, CERN







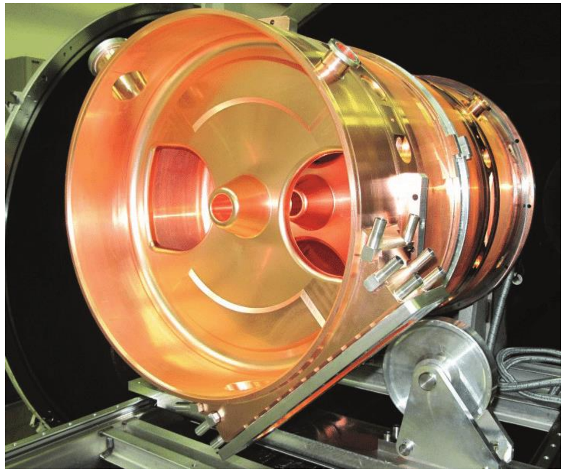
Picture of Linac1, CERN

Note that a ‘cell’ is not necessarily a pillbox-type shape. In drift tube structures, a cell is the area between two drifttubes and looks in principle like a pillbox with nose cones.

sources: P. Bourquin et al., *Development Status of the Pi-mode Accelerating Structure (PIMS) for LINAC4, Proc. Of LINAC08, Canada.*
C. Plostinar (ed.), Comparative Assessment of HIPPI Normal Conducting Structures, CARE-report-2002-0771

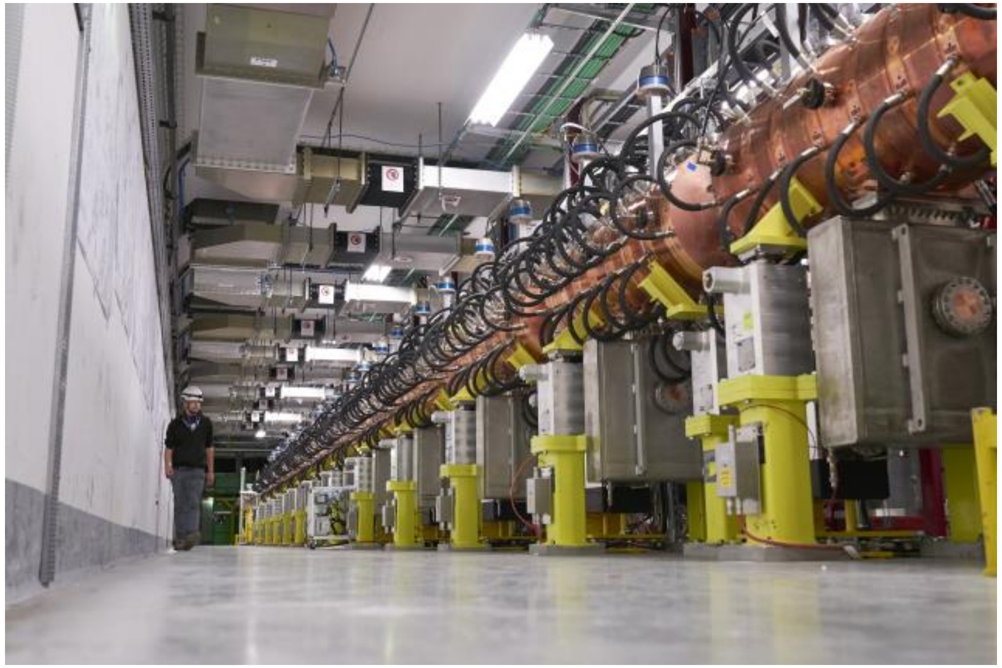
Example of Pi-mode Cavities (SW, 3/3)

Slot-coupled-structure (PIMS) at CERN



PIMS cell

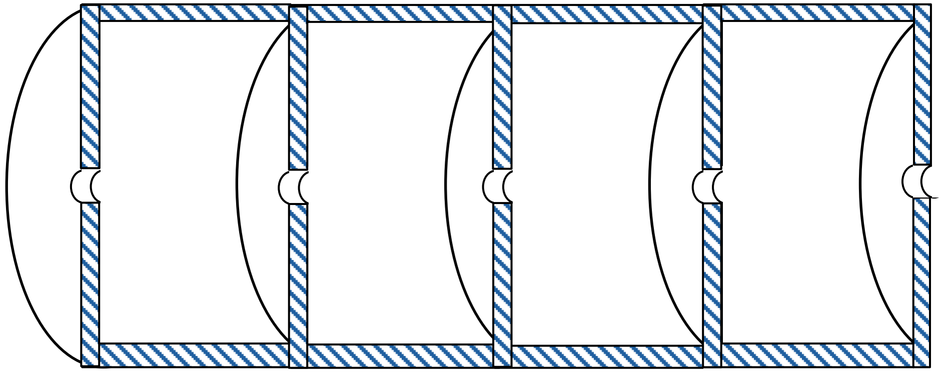
PIMS test set-up



Completed LINAC4, CERN

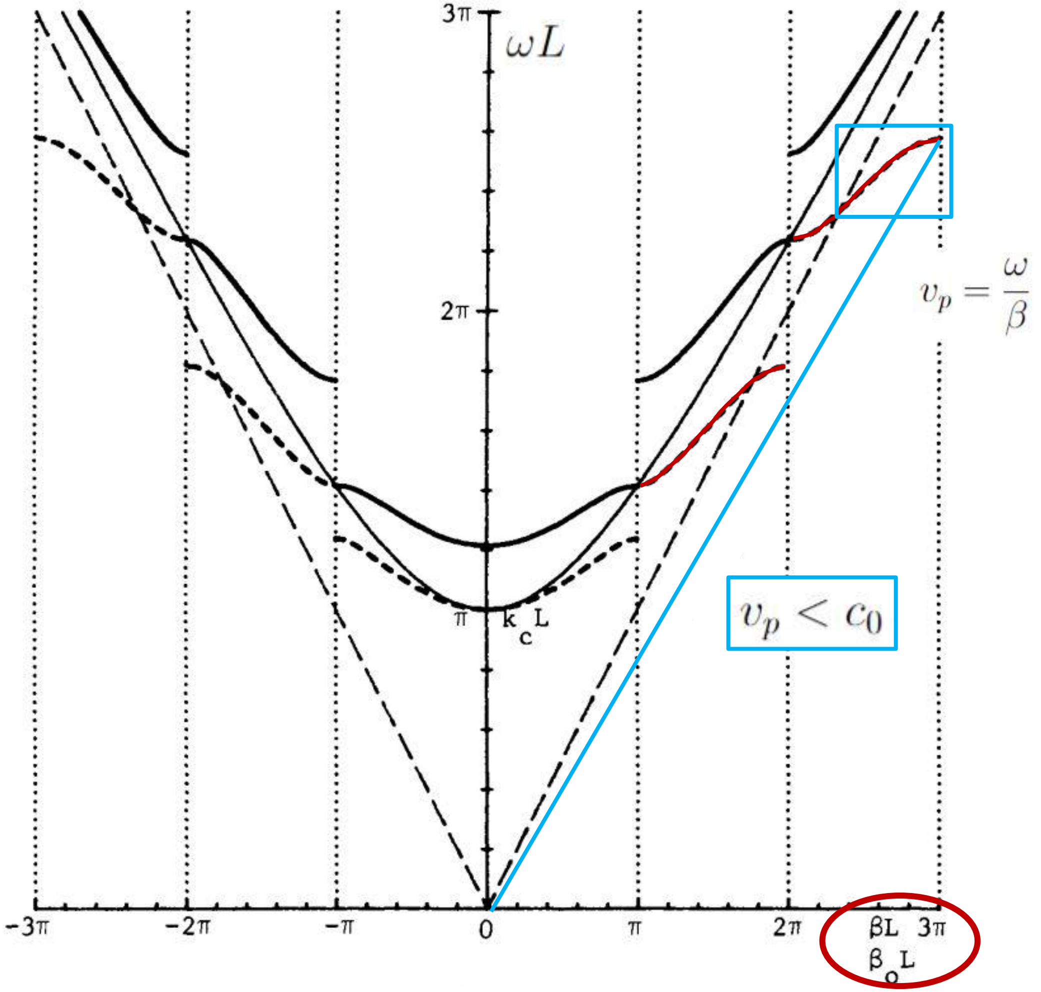
Picture sources: CERN cds
R. Wegner et al., *Linac4 PIMS Construction and First Operation*, IPAC2017, Copenhagen.
P. Bourquin et al., *Development Status of the Pi-mode Accelerating Structure (PIMS) for LINAC4, Proc. Of LINAC08, Canada*

Slow wave structure (1/7)



- Slow wave structures work in the range of $v_p < c_0$.
- The addition of discs inside the cylindrical waveguide induces multiple reflections between the discs and results in a change of the dispersion curve.
- The disc-loaded structure can then be used in travelling wave mode.
The dispersion curve changes from continuous to splitting up into different modes which are slowed down. We speak of pass-bands. These modes are separated by stopbands.

source: G. Dome, *RF Systems: Waveguides and Cavities, aip-conf-proc.153-1296*



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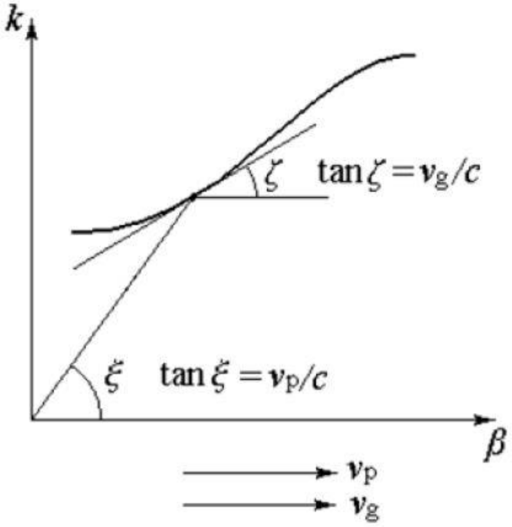
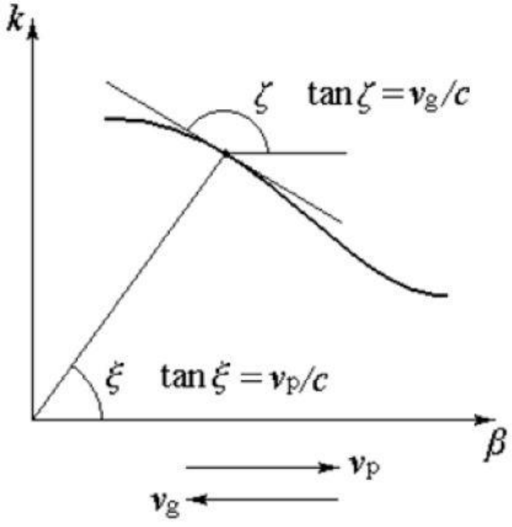
Slow wave structure (2/7)

Dispersion Curves, phase- and group velocity for FW and BW wave in periodic structure.

Forward wave (FW)
v_g and v_p point in the same direction.





Backward wave (BW)
v_g and v_p point in the opposite direction.

For the advanced RF-fans:

1. All guided modes in common transmission lines, metallic and dielectric waveguides are forward waves.
2. For transmission lines of “high-pass filter” type, the phase constant β decreases with increasing frequency.
3. “High-pass filter” type lines are modelled with a distributed series capacitance and a shunt inductance (i.e., opposite of what we did in the transmission line modelling!).
4. Forward and backward type of waves – this makes no difference for the direction of the beam.

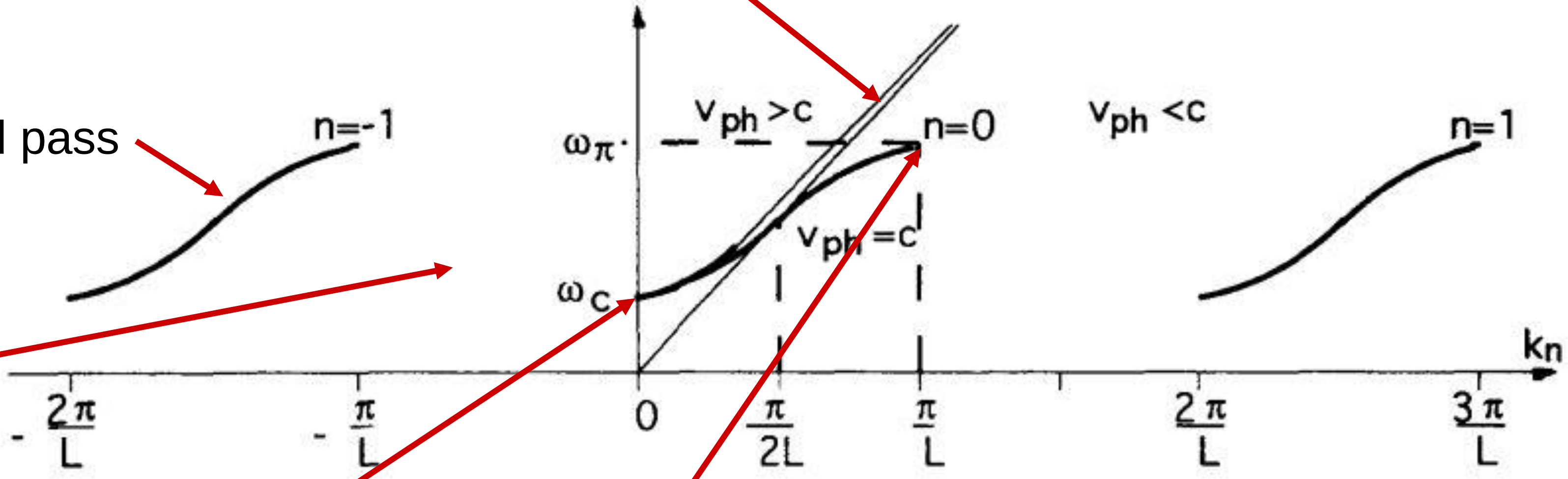
Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics,* Springer

Slow wave structure (3/7)

Dispersion diagram of periodic structure (uniform waveguide is shown for orientation)

source: M. Weiss, *Introduction to RF Linear Accelerators.*

- Dispersion diagram shows the periodicity of the structure.
- For a given mode, there is a limited pass band of possible frequencies.
- The passbands are separated by stopbands where the specific mode cannot propagate.
- At both ends of the passband, the group velocity is zero and wave propagation stops.

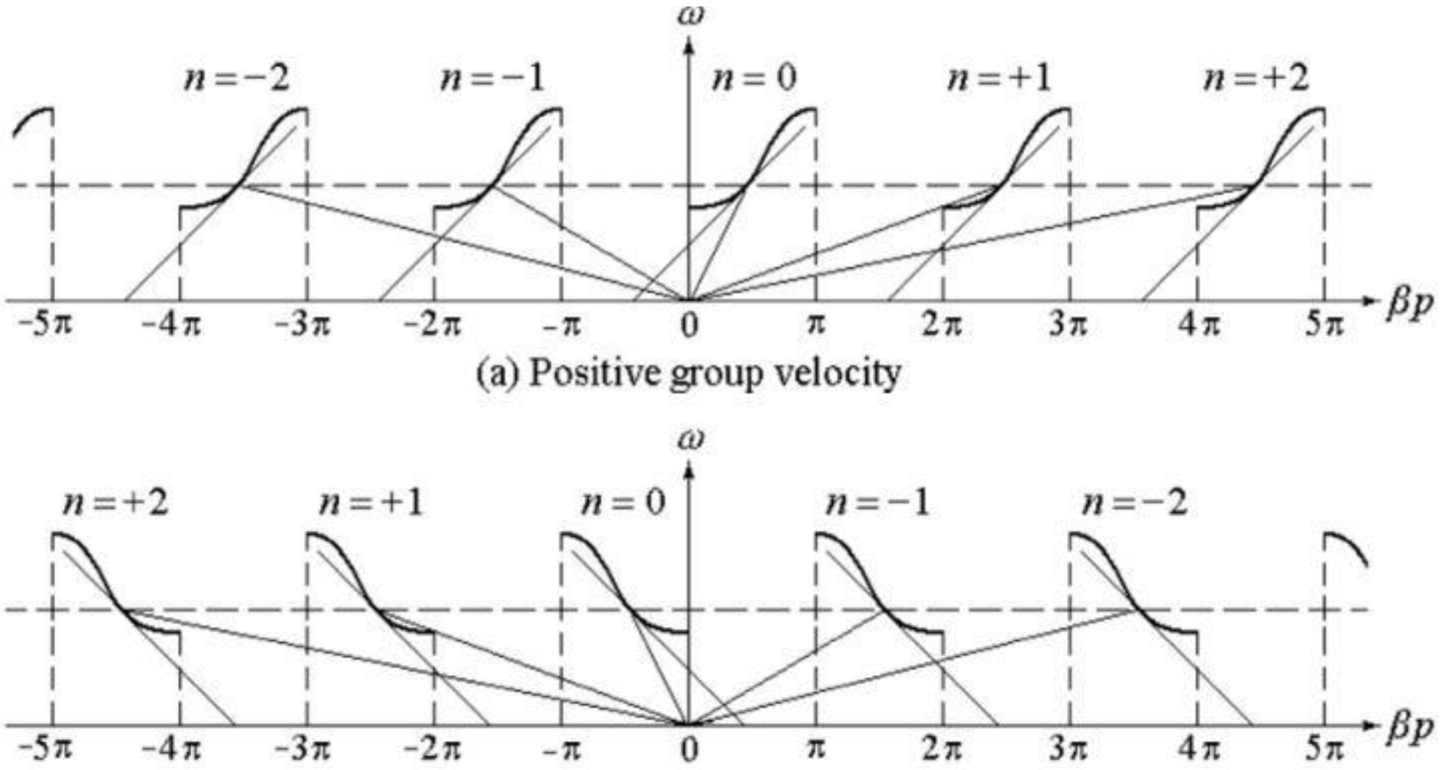


- When group velocity and phase velocity are in the same direction, we speak about forward waves, if they are in different direction, we speak about backward waves. This has nothing to do with the direction of the particle travelling.

Slow wave structure (4/7)

- The request to fulfill the periodic boundary conditions leads to the concept of *space harmonics* (instead of wave modes).

- Space harmonics are closely connected the so-called *Floquet Theorem* (we will not cover this here).



(a) Positive group velocity

(b) Negative group velocity

- The *Floquet Theorem* describes the relation between the phase constant of the n-th space harmonic and is a fundamental theorem of periodic structures.

Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics,* Springer

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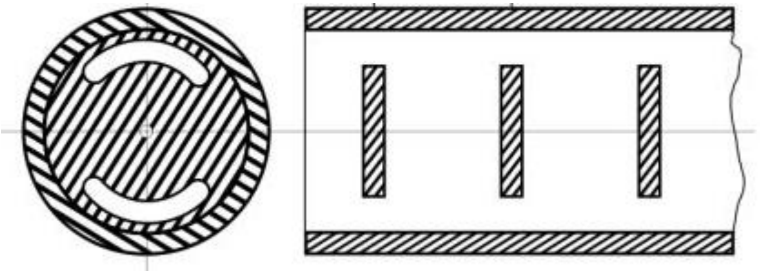
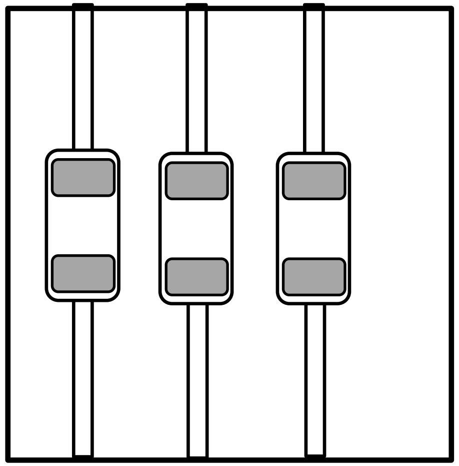
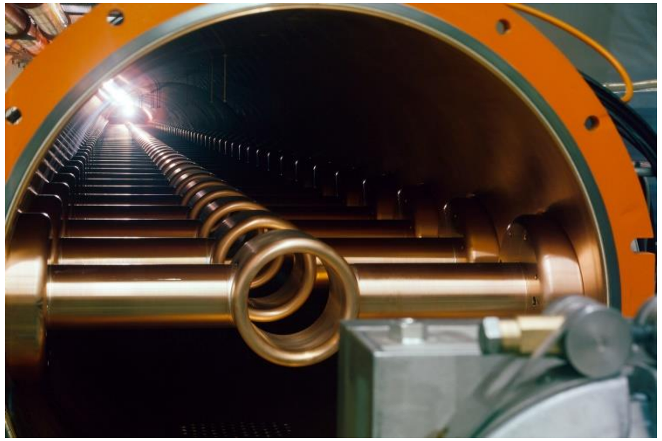
Slow wave structure (5/7)

- As was shown before, drift tubes can be used instead of separating discs to slow down the EM-field. The theory for drift tubes is very similar to calculating a slot-coupled structure. Just imagine that the slot is very large.







Inside view of the SPS travelling wave structure © CERN.

- In travelling wave (TW) mode, the field propagates through each cell.

- The phase advance per cell (distance between the discs, or periodic spacing of drift tubes) determines the length of the cell:

$$\Delta\varphi = \frac{2\pi}{\beta\lambda} l$$

Remember? $\beta\lambda$ was already introduced for the single gap cavity as distance that a particle travels during one RF-period.

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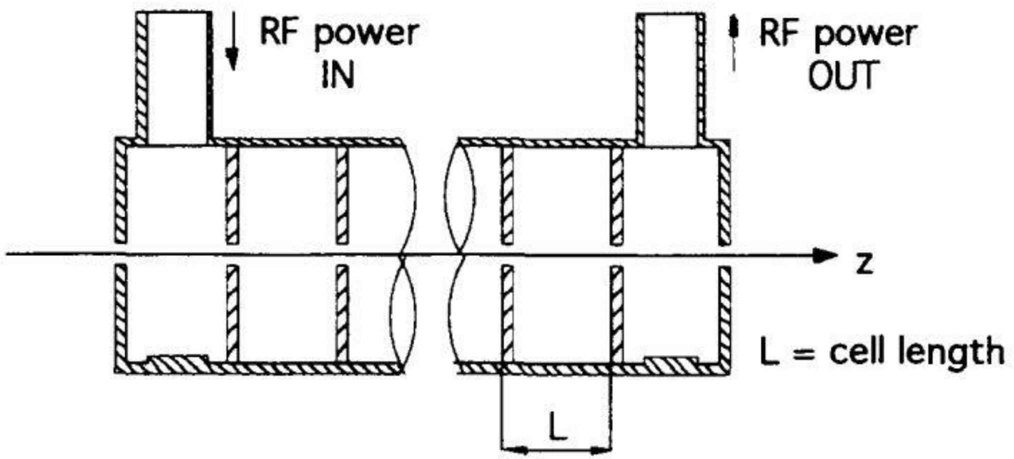
Slow wave structure (6/7)

- The only missing component for our travelling wave system are the input and output couplers.
- These have to be matching the structure to avoid that standing waves are building up inside the periodic structure.



- Often, higher order modes (HOMs) are building up in the structure due to the finite length (end covers are put there).

  Note that a matching network is used to obtain the travelling state. This network is only matching the fundamental mode and not the HOMs!
  I.e. the fundamental mode is travelling, others might be travelling, partially travelling or standing modes.

Slow wave structure (7/7)

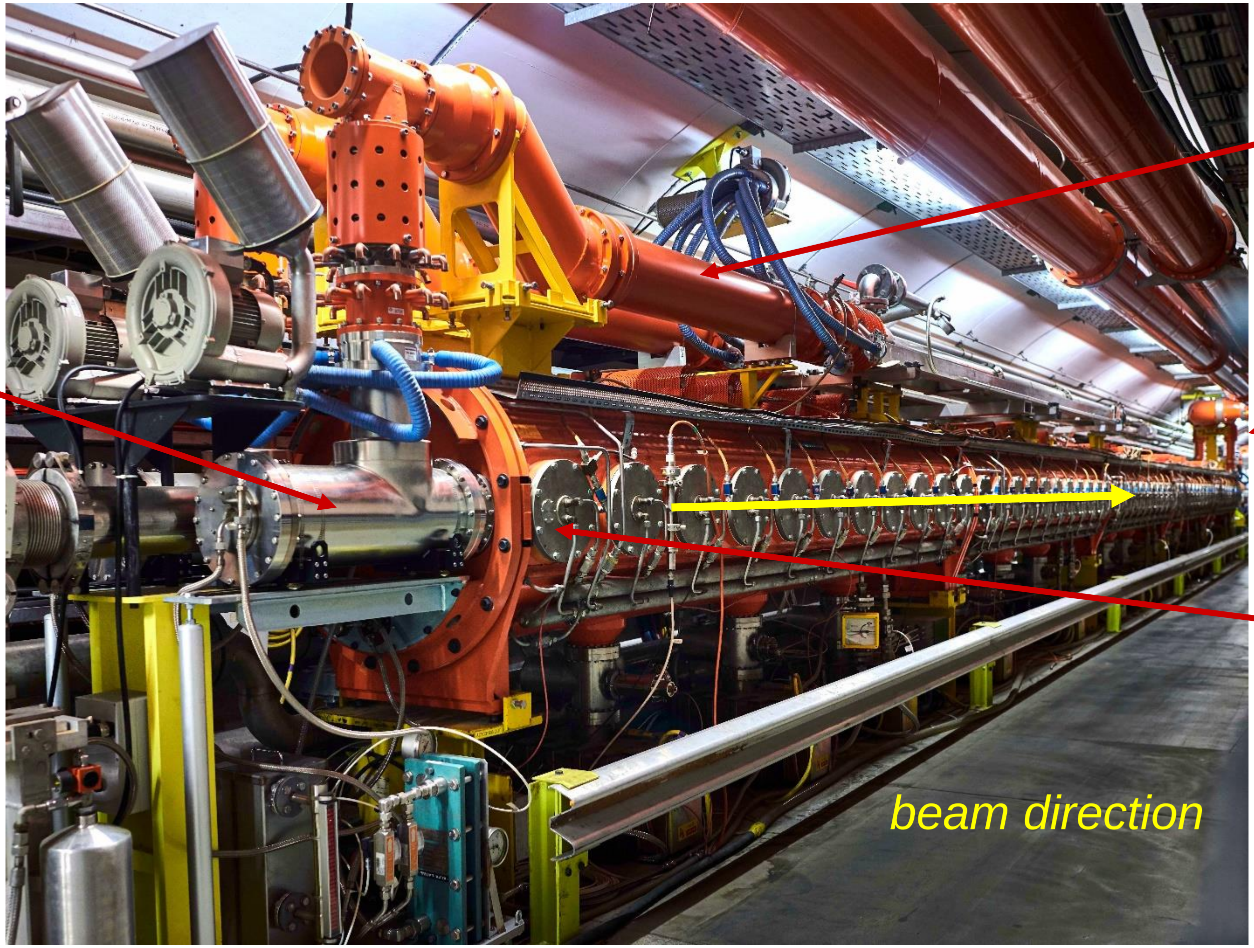
SPS 200 MHz travelling wave cavity (16m long).

Backward wave structure

Water cooled coaxial load

RF OUT side

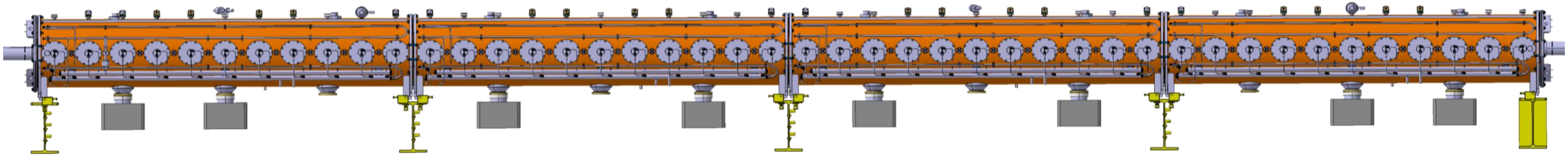
RF IN side



Drift tube with water cooling

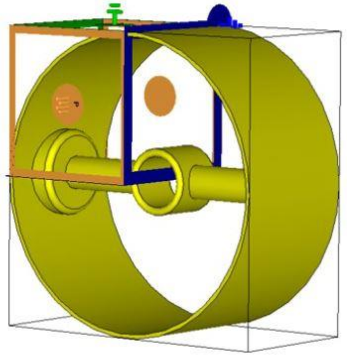
Example: 200 MHz TWC of SPS (1/3)

courtesy: E. Montesinos, CERN

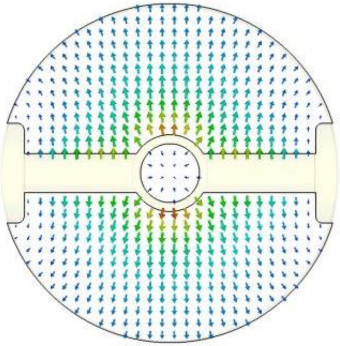


- Cavity is part of the LHC injector chain and was upgraded recently and equipped with new power amplifiers to reach a higher accelerating voltage.
- The cavity was entirely modelled in CST, so that we could well see the fields of the fundamental and the HOMs.

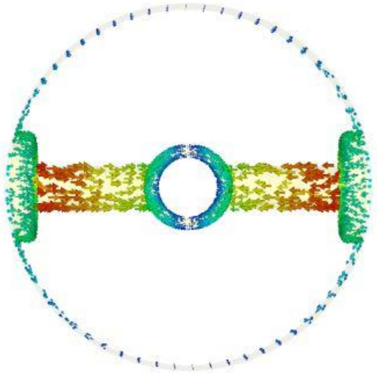
Modelling in CST (P. Kramer, CERN)



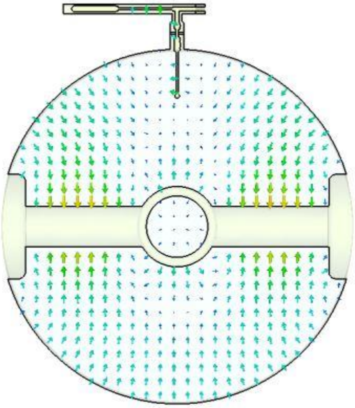
Single cell CST model with symmetries (no HOM coupler)



Electric field of fundamental mode in the Cross-section



Surface currents on the Drift tube



E-field in a cell with HOM-coupler (22π/33 mode)

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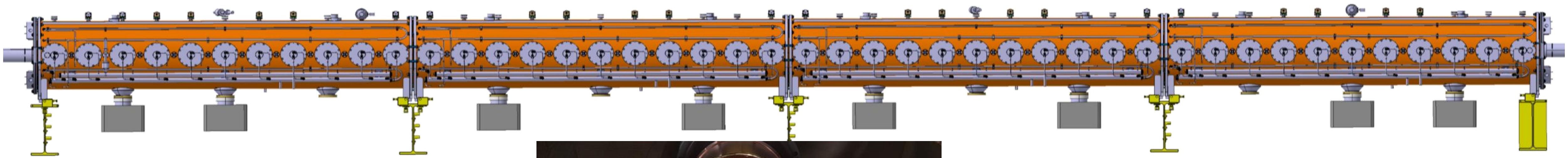
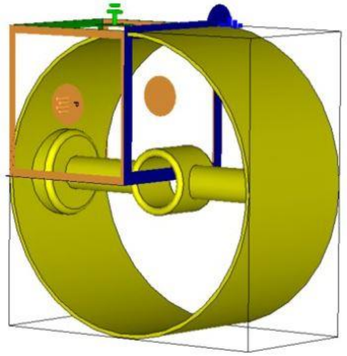
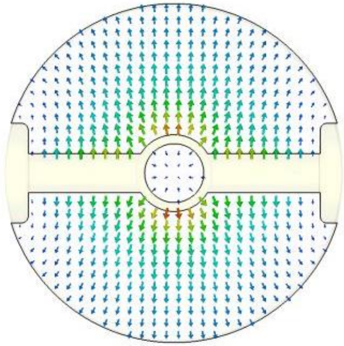
200 MHz TWC of SPS (2/3)

courtesy: E. Montesinos, CERN





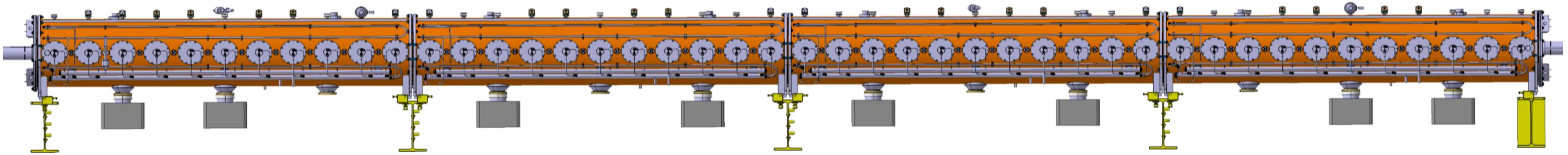
Single cell CST model with symmetries (no HOM coupler)



Electric field of fundamental mode in the Cross-section

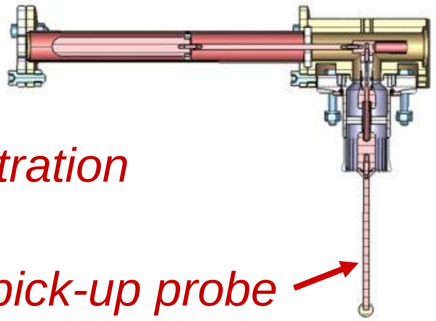
200 MHz TWC of SPS (3/3)

courtesy: E. Montesinos, CERN



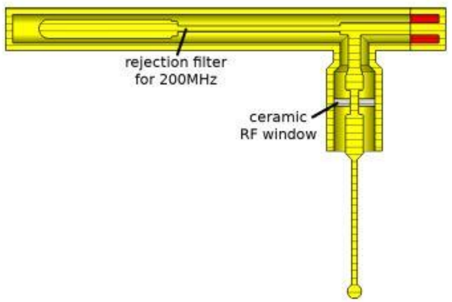
- Cavity is part of the LHC injector chain and was upgraded recently and equipped with new power amplifiers to reach a higher accelerating voltage.

- A number of HOMs are developing, mostly as standing wave and these were taken out by HOM-couplers.
  Most harmful was the mode at 630 MHz, each section of the cavity has a number of these Hom couplers installed:



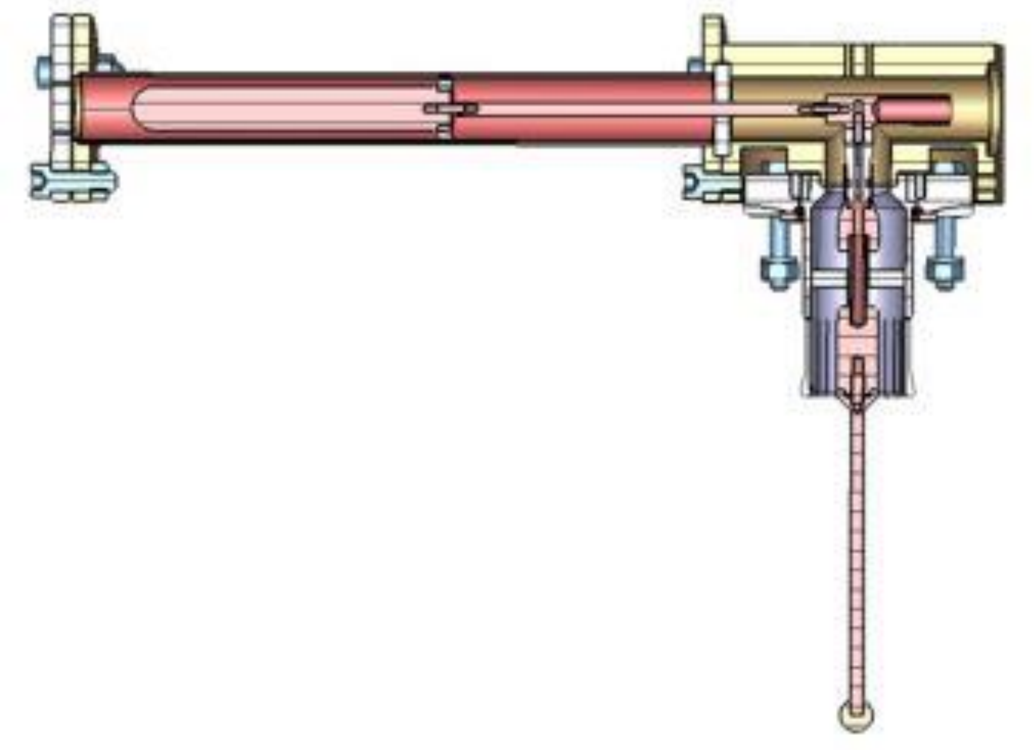
Schematic illustration

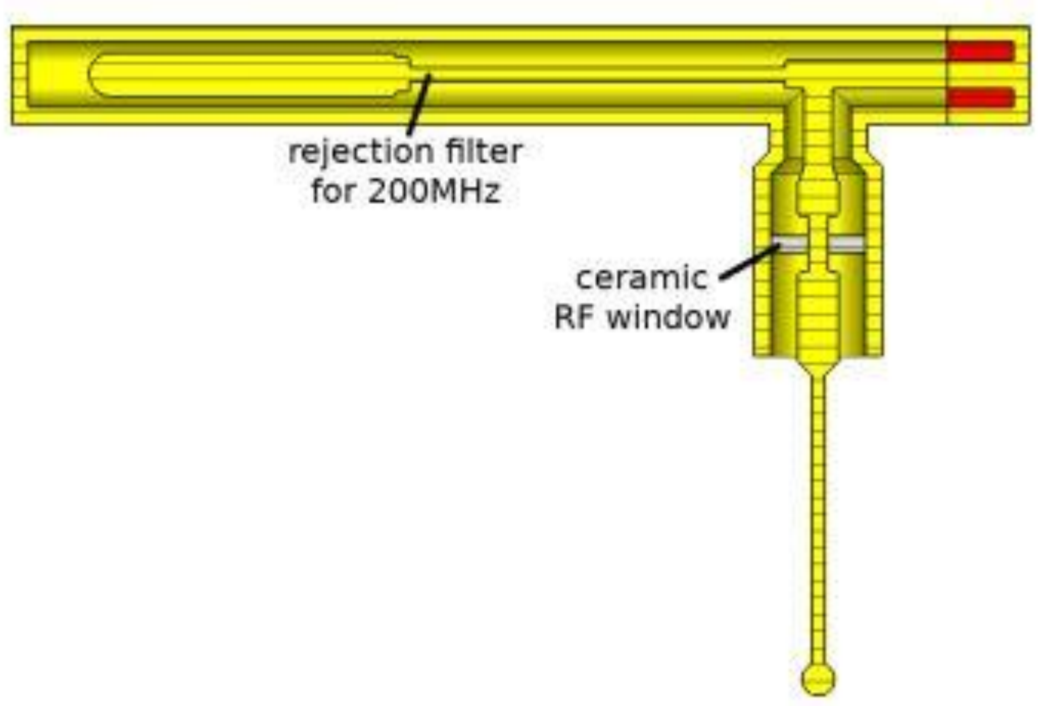
E-field pick-up probe



Modelling in CST
(P. Kramer, CERN)

630 MHz - HOM Couplers





courtesy: E. Montesinos, CERN

End of the extra part!

Many thanks to

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who all kindly provided material to this presentation!