

Longitudinal Hands-on Calculations Longitudinal Tracking



H. Damerou, L. Intelisano,
F. Tecker, M. Zampetakis

CERN



Introduction to Accelerator Physics

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Agenda of the afternoon

14h50 – 15h20

Introduction to exercises

15h20 – 15h50

Longitudinal tracking

Coffee break

16h20 – 16h35

Intermediate wrap-up

16h35 – 17h40

Longitudinal tracking

17h40 – 18h25

Discussion on solutions of exercises

Outline

- **Introduction**
 - Interaction between beam and RF system
- **Longitudinal particle tracking**
 - Basic tracking equations
 - Single and multiple particle tracking
- **Summary**

Introduction

Study interaction between beam and RF

Complementary approaches for the same problem

(Semi-)Analytical

- Describe particle motion by **differential equations**
 - **Continuous trajectories** of particle motion
 - Deduce useful parameters for **stable acceleration**:
 - RF bucket
 - Synchrotron frequency
 - Stable phase
 - ...



Study interaction between beam and RF

Complementary approaches for the same problem

(Semi-)Analytical	Numerical: tracking
<ul style="list-style-type: none">• Describe particle motion by differential equations→ Continuous trajectories of particle motion→ Deduce useful parameters for stable acceleration:<ul style="list-style-type: none">→ RF bucket→ Synchrotron frequency→ Stable phase→ ...	<ul style="list-style-type: none">• Track particle parameters from turn to turn→ Profit from discretization of motion: turn-by-turn, RF station-by-RF station→ No notion of RF bucket, synchrotron frequency, stable phase, etc.→ Follow ensemble of particles to study evolution of bunch

Study interaction between beam and RF

Complementary approaches for the same problem

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<p style="text-align: center;"></p> <ul style="list-style-type: none"> → Classical introduction of longitudinal beam dynamics 	<p style="text-align: center;"></p> <ul style="list-style-type: none"> → Flexible Today approach

Objectives of longitudinal hands-on

1. Design RF system (upgrade)

`LongitudinalHandsOnRFSystemCalculations_empty.ipynb`

- Study boundary constraints
- Derive requirements for RF system
- Choose main components
- Compare with existing facilities

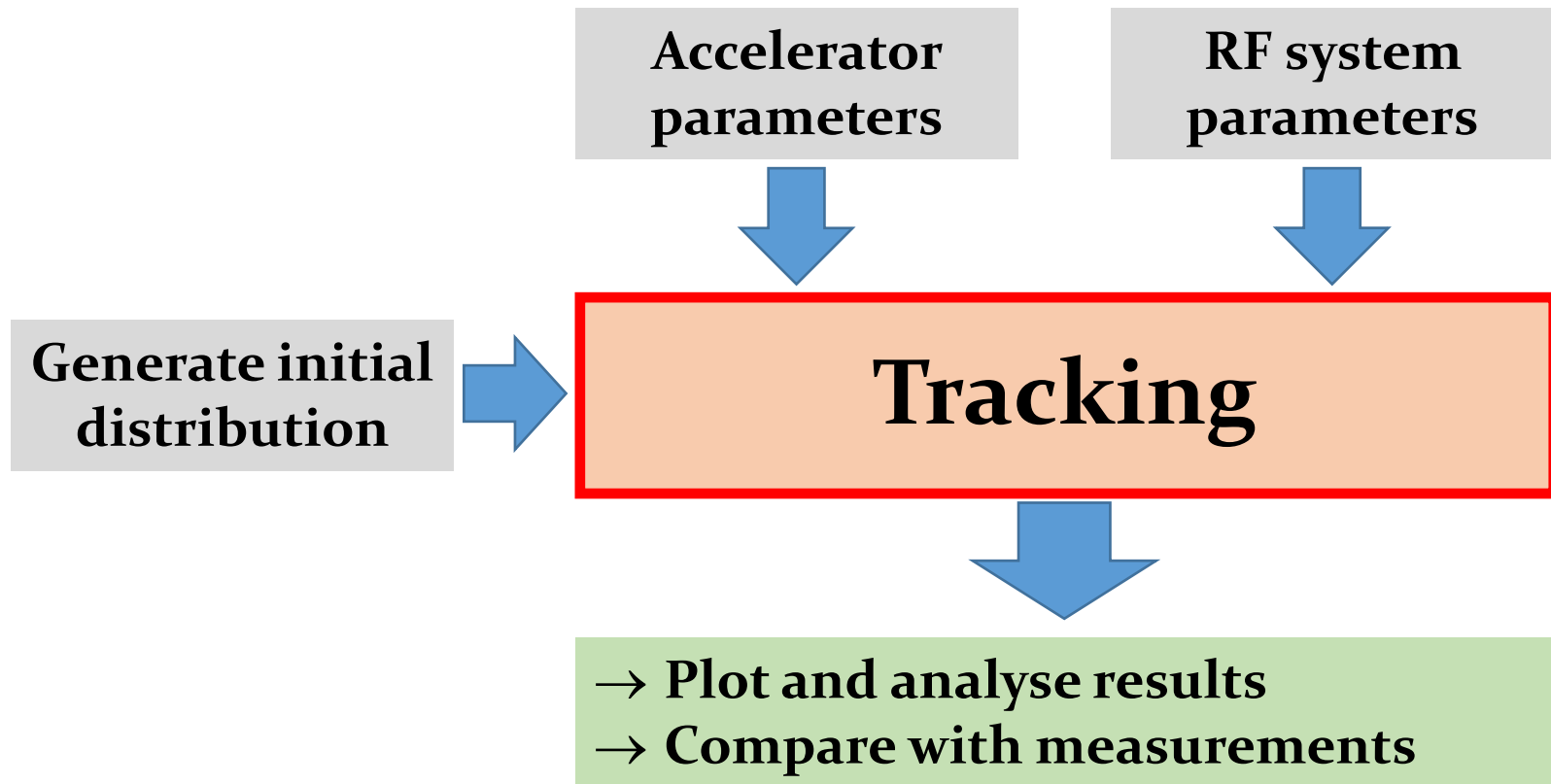
2. Play with longitudinal beam dynamics

`LongitudinalHandsOnTracking_empty.ipynb`

- **Build your own particle tracker**
- **Understand motion of particles in longitudinal phase space**
- **Transition from single particle motion to evolution of an entire bunch**

Longitudinal tracking

Tracking simulation flow



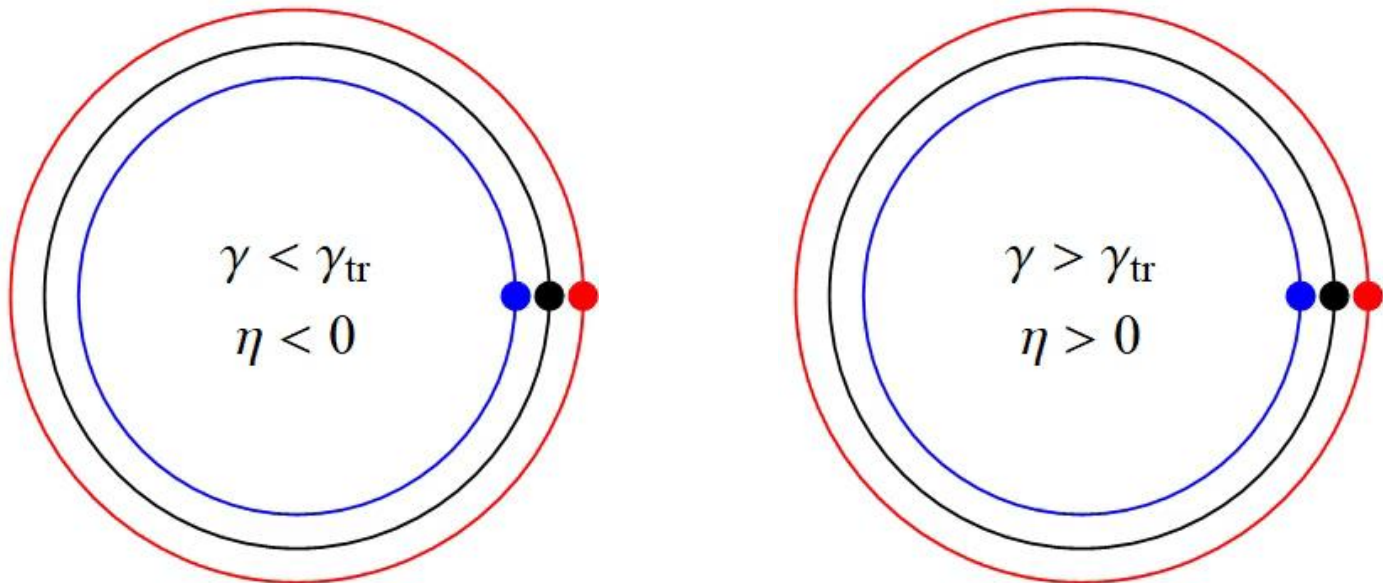
→ Follow the coordinates of one or more particles
determine its behaviour

Circular ~~accelerator~~ without RF system

↳ ring

- Particles with **higher** or **lower momentum** have a different orbit compared to a reference particle

→ **Arrival time/phase depends on energy** $\frac{\Delta L}{L} = \frac{1}{\gamma_{\text{tr}}^2} \frac{\Delta p}{p}$



$$\frac{\Delta f}{f} = -\eta \frac{\Delta p}{p}, \text{ phase slip factor: } \eta = \frac{1}{\gamma_{\text{tr}}^2} - \frac{1}{\gamma^2}$$

Arrival phase of a particle at next turn

$$\frac{\Delta f}{f} = -\eta \frac{\Delta p}{p} \quad \frac{\Delta f}{f} = -\frac{\eta}{\beta^2} \frac{\Delta E}{E}$$

$$\frac{\Delta p}{p} = \frac{1}{\beta^2} \frac{\Delta E}{E}$$

$$\frac{\Delta \theta}{\theta} = \frac{\eta}{\beta^2} \frac{\Delta E}{E}$$

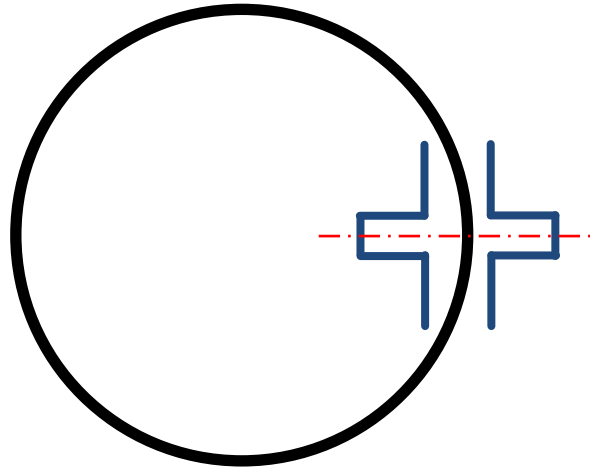
→ Turn-by-turn drift equation

$$\theta_{n+1} = \theta_n + 2\pi \frac{\eta}{\beta^2} \frac{\Delta E}{E}$$

$$\phi_{n+1} = \phi_n + 2\pi h \frac{\eta}{\beta^2} \frac{\Delta E}{E}$$

→ Azimuth, θ or **phase, ϕ** a particles arrives next turn

Circular accelerator with RF station



- Particle energy changes at passage through cavity

→ For sinusoidal RF voltage:

$$\Delta E_{n+1} = \Delta E_n + qV \sin \phi_{n+1}$$

→ With acceleration:

$$\Delta E_{n+1} = \Delta E_n + qV [\sin \phi_{n+1} - \sin \phi_S]$$

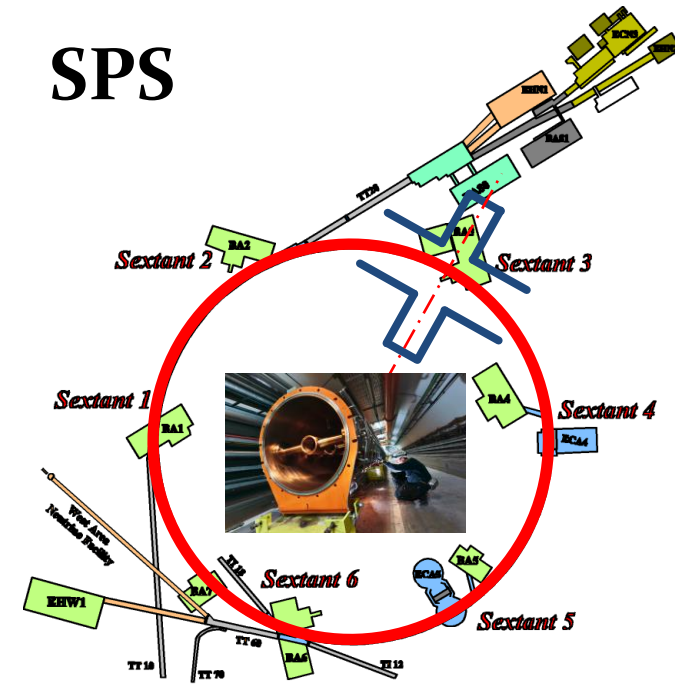
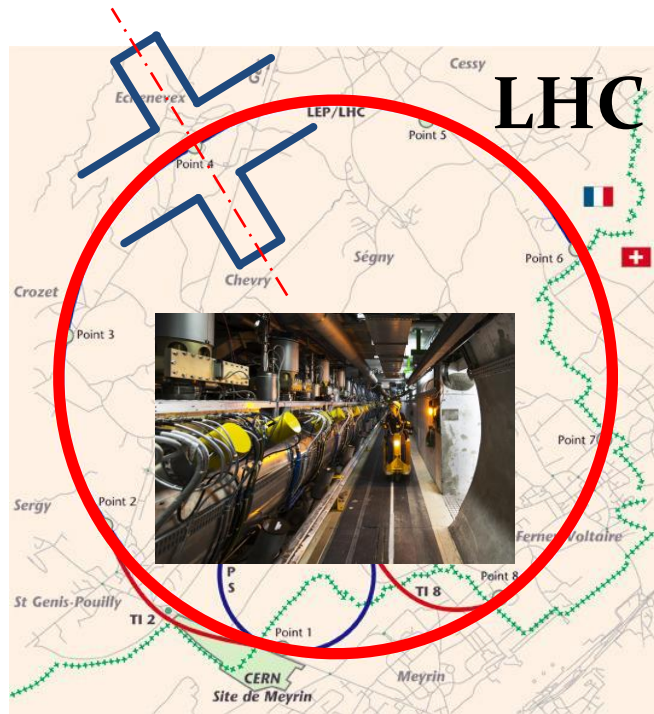
Reference particle: $\phi = \phi_S$ ←

→ General energy change:

$$\Delta E_{n+1} = \Delta E_n + qV [g(\phi_{n+1}) - g(\phi_S)] + \Delta E_{\text{ext}} + \Delta E_{\text{self}}$$

Multiple RF stations

- RF systems modelled point-like mostly valid approximation
→ **Valid in most cases**



Exceptions:

- Large synchrotron tune f_S/f_{rev}
- Strong intensity effects: interaction within one turn
- Beam energy changing during turn

Multiple RF stations



2.8 – 10 MHz

Acceleration

to SPS

Booster

TT2

Y

Linac III

Linac II

PS

Earth hall

LEIR

FA58

TT70

RF Manipulations



40 MHz



80 MHz



200 MHz

Longitudinal blow-up



0.4 – 5 MHz

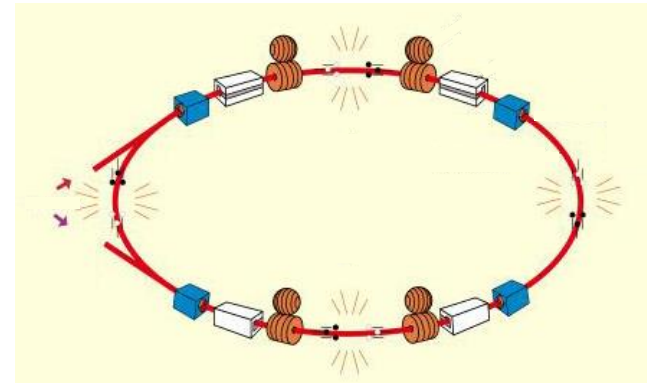
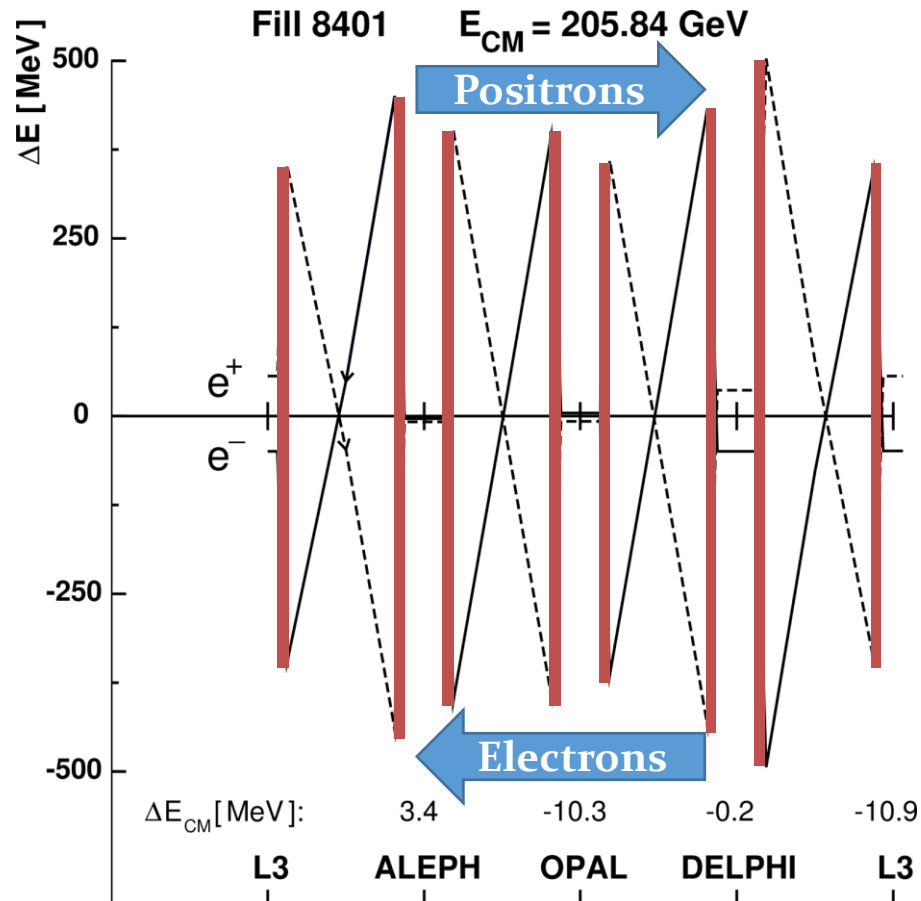


20 MHz

→ Small $Q_S = f_S / f_{\text{rev}}$: Single kick per turn fully sufficient

Example: Electrons and positrons in LEP

- Beam energy changed in LEP along turn due to strong synchrotron radiation



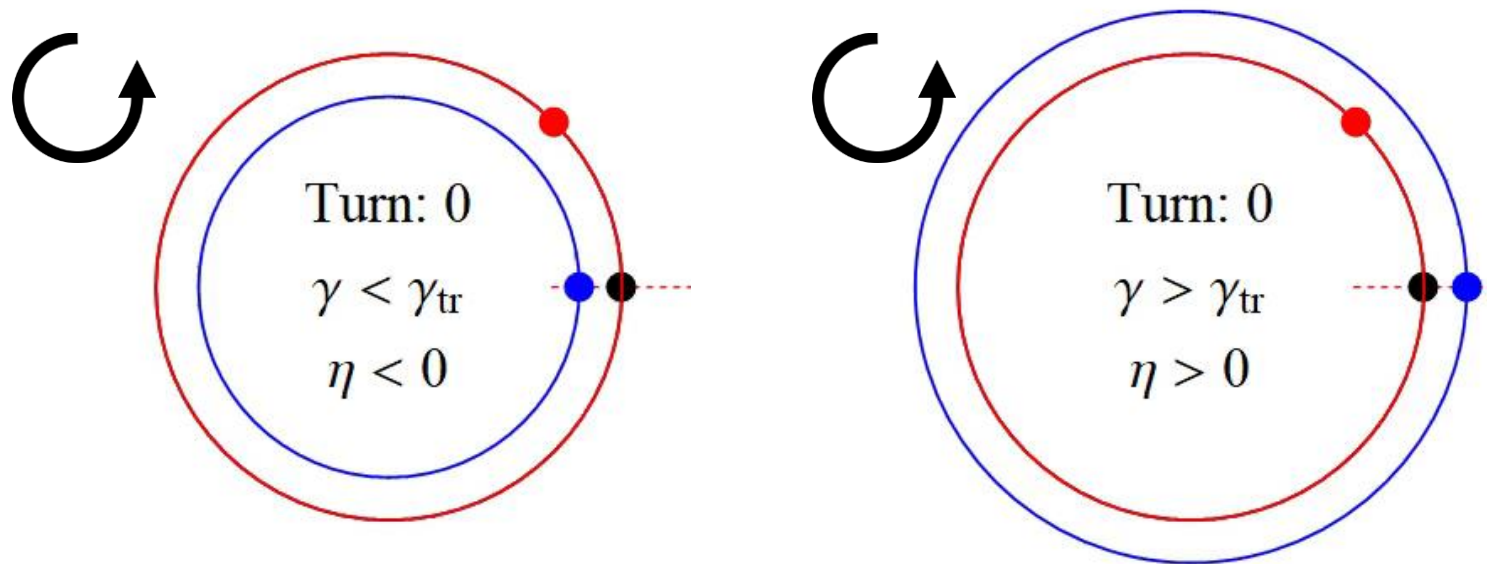
- 4×2 RF sections
- Energy loss in bending magnets
- Track from RF section to RF section

Combining both tracking equations

- Observe phase and energy error at each turn with respect to reference particle

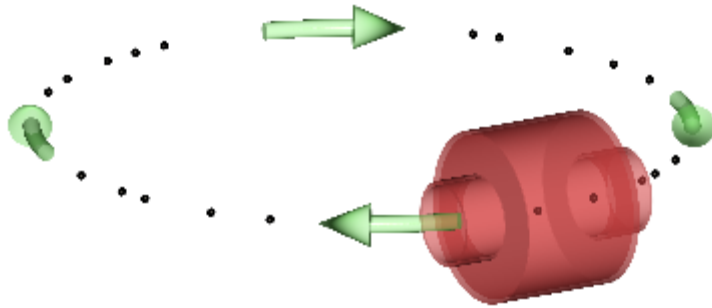
- Test particles:

$\Delta\phi = \phi - \phi_S = 0$	$\Delta E = 0$
$\Delta\phi \neq 0$	$\Delta E = 0$
$\Delta\phi = 0$	$\Delta E \neq 0$



Longitudinal phase space

Simple accelerator model:

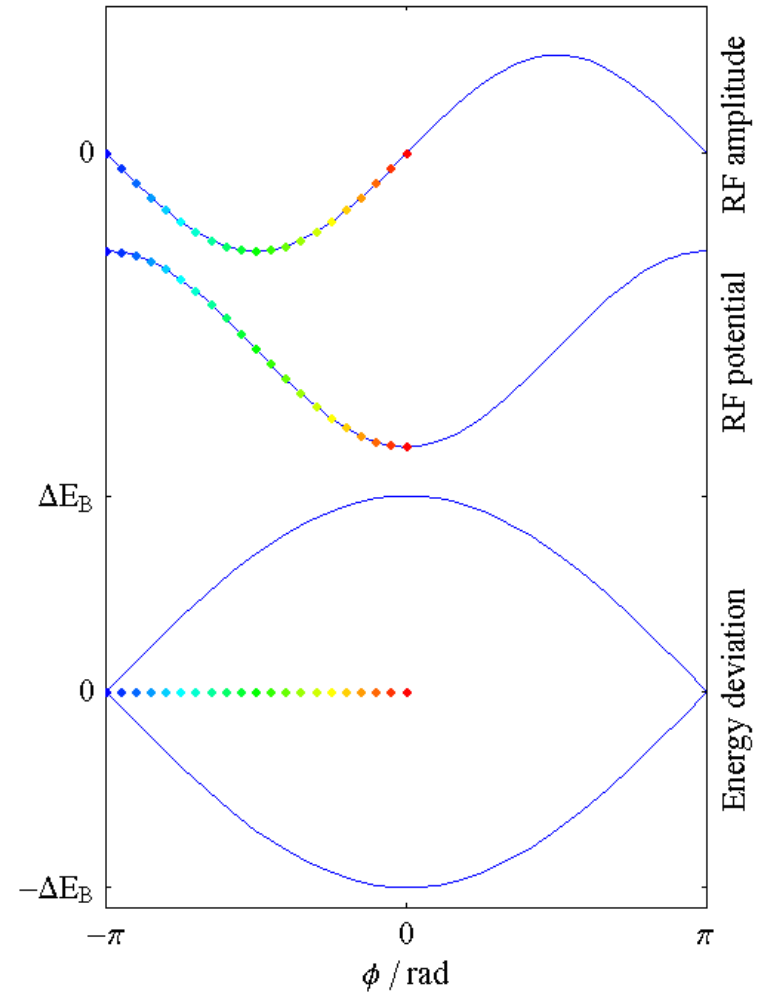


Energy dependent phase advance, ϕ :

$$\phi_{n+1} = \phi_n + 2\pi h\eta \frac{\Delta E_n}{\beta^2 E}, \quad \eta = \frac{1}{\gamma_{\text{tr}}^2} - \frac{1}{\gamma^2}$$

Phase dependent energy gain, ΔE :

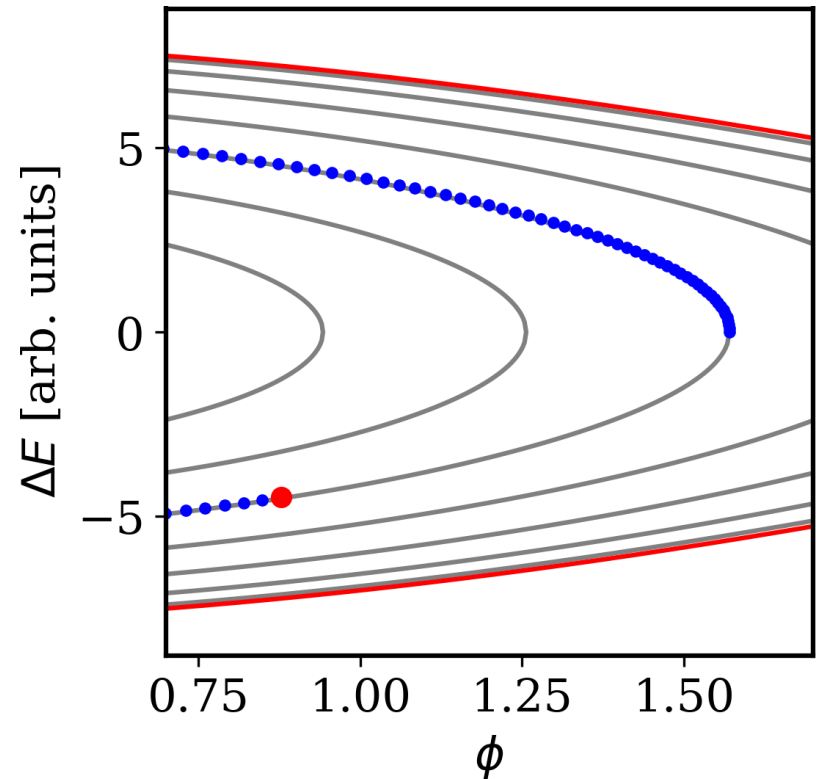
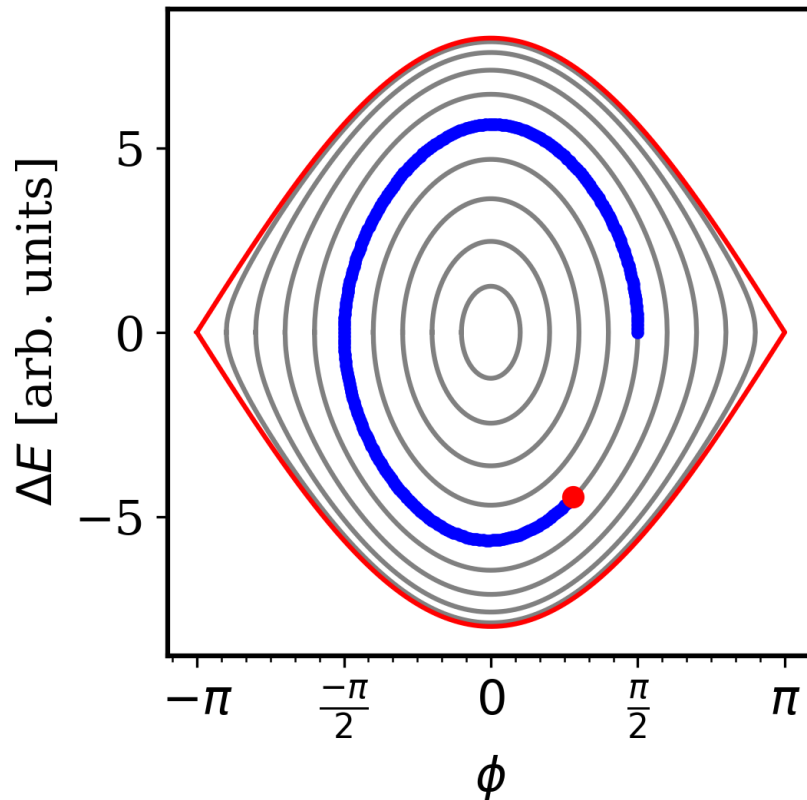
$$\Delta E_{n+1} = \Delta E_n + qVg(\phi_{n+1})$$



Works for arbitrary shape of acceleration amplitude $g(\phi)$

Continuous versus discrete

- Analytical solution describes static condition
- No notion of turn-by-turn evolution



→ Same result with both approaches for $Q_S = f_S/f_{\text{rev}} \ll 1$

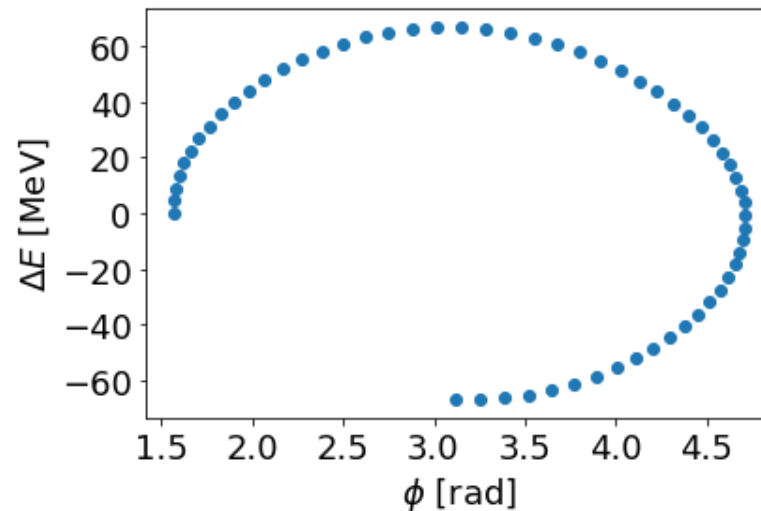
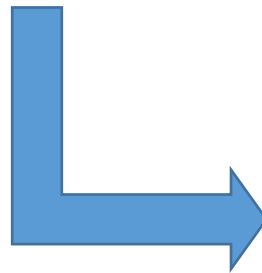
Example: simple tracking in Python

- Follow the trajectory of a single particle

```
# Tracking functions
def drift(phaseInitial, energyInitial, harmonic, eta, beta, energy):
    newPhase = phaseInitial + 2*np.pi*harmonic*eta*energyInitial/(beta**2*energy)
    return newPhase
def kick(energyInitial, phaseInitial, charge, voltage):
    newEnergy = energyInitial + charge*voltage*np.sin(phaseInitial)
    return newEnergy
```

Turn $n \rightarrow n+1$

```
n_turns = 60
particlePhase = 0.5*np.pi # Initial particle phase
particleEnergy = 0 # Initial particle energy offset
particlePhaseArray = np.zeros(n_turns) # Initialize phase array
particleEnergyArray = np.zeros(n_turns) # Initialize energy offset array
for i in range(n_turns):
    particleEnergyArray[i] = np.array(particleEnergy)
    particlePhaseArray[i] = np.array(particlePhase)
    particlePhase = drift(particlePhase, particleEnergy, harmonic, eta, beta, energy) # Track phase
    particleEnergy = kick(particleEnergy, particlePhase, charge, voltage) # Track energy
```

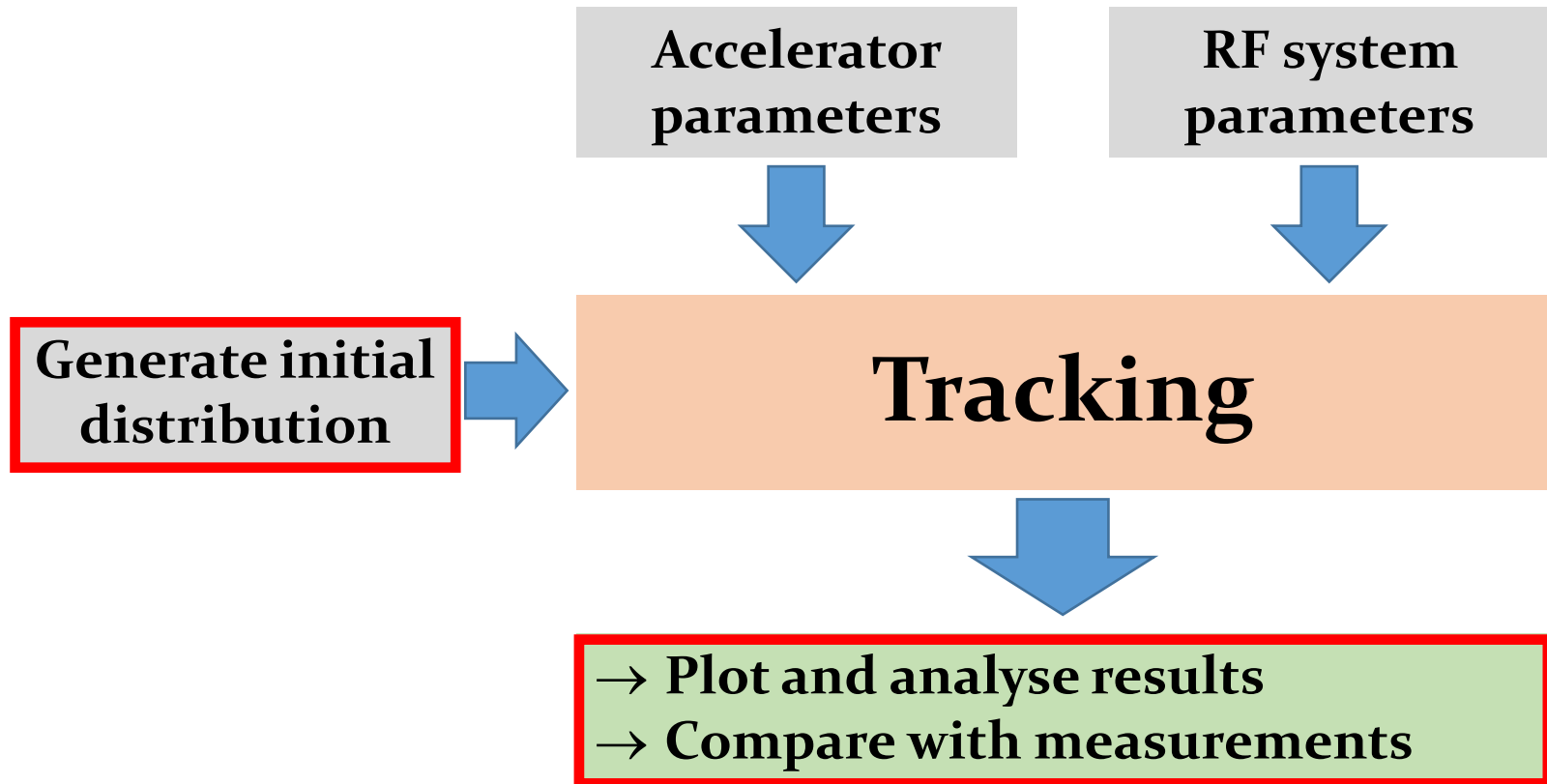


Choice of particle coordinates

- Time or phase? Momentum or energy?
- Absolute or relative coordinates

		Advantages	Disadvantages
t	E	<ul style="list-style-type: none"> • Most universal • Suitable for any tracking • Canonically conjugated 	<ul style="list-style-type: none"> • Numerical precision: large absolute value • Relative bunch motion more difficult to follow
Δt	ΔE	<ul style="list-style-type: none"> • Relevant deviations only • Canonically conjugated • Most suited for multiple h 	<ul style="list-style-type: none"> • Required synchronous particle as reference • Duration of turn may change
Φ	$E, \Delta E$	<ul style="list-style-type: none"> • Turn length always 2π • Relevant deviations only 	<ul style="list-style-type: none"> • Requires synchronous particle as reference • Not canonically conjugated
ϕ	$E, \Delta E$	<ul style="list-style-type: none"> • RF bucket length always 2π • Relevant deviations only • Most suited for single h 	<ul style="list-style-type: none"> • Requires synchronous particle as reference • Not canonically conjugated

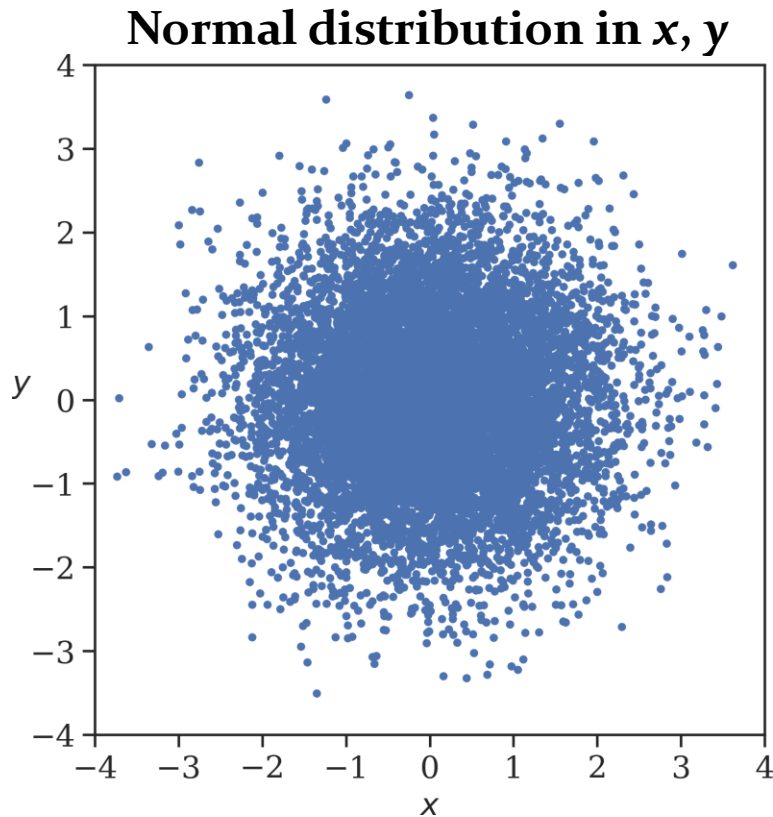
Tracking simulation flow



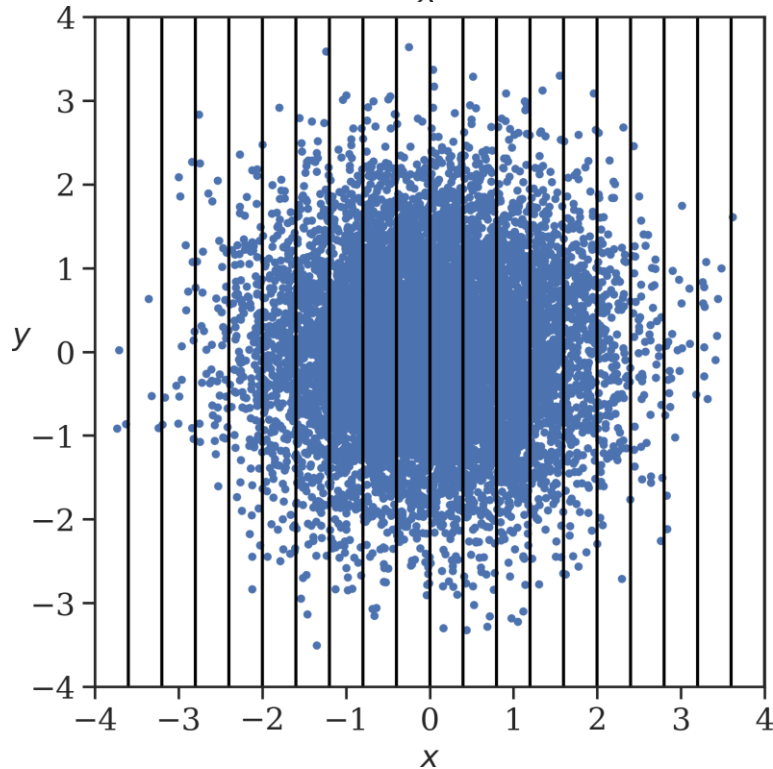
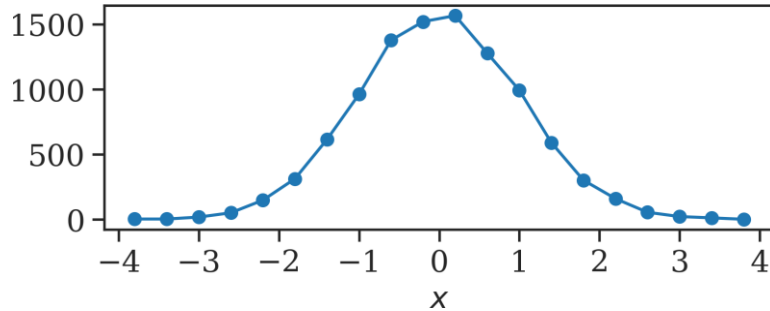
Distributions and projections

From single particle tracking to distribution

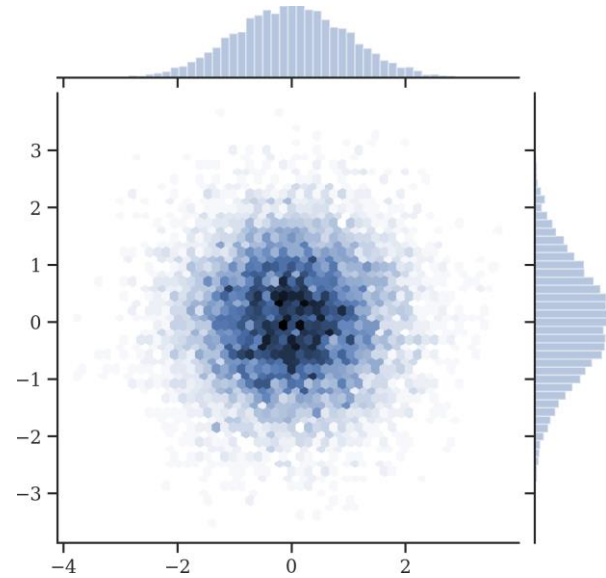
- $10^{10} \dots 10^{12}$ particles per bunch → too much computing power
- Macro-particles to reduce → up to few 10^6 per bunch



Projections of distributions



- **Very common task:**
 - e.g. Python seaborn
 - `plot_phase_space_distribution`

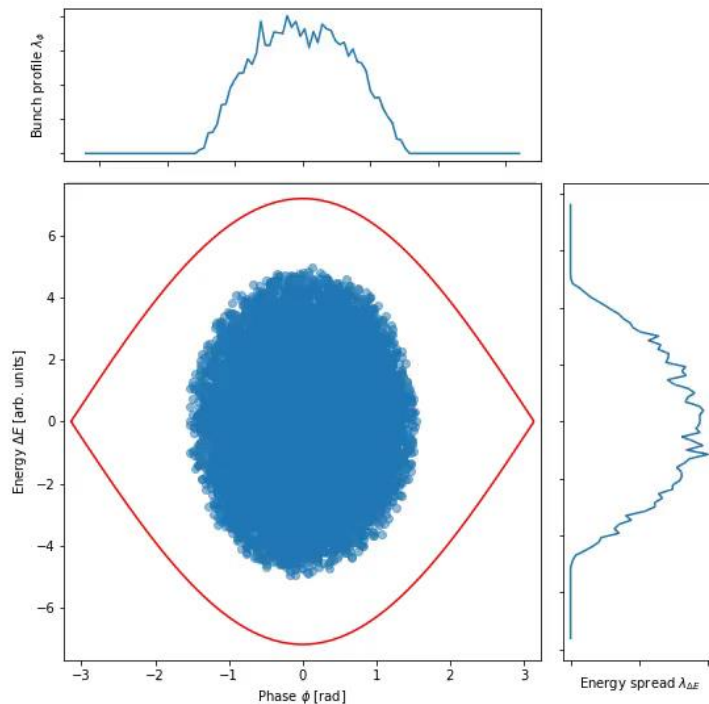


- **Time projection directly observable: bunch profile**

Example: Tracking of a single bunch

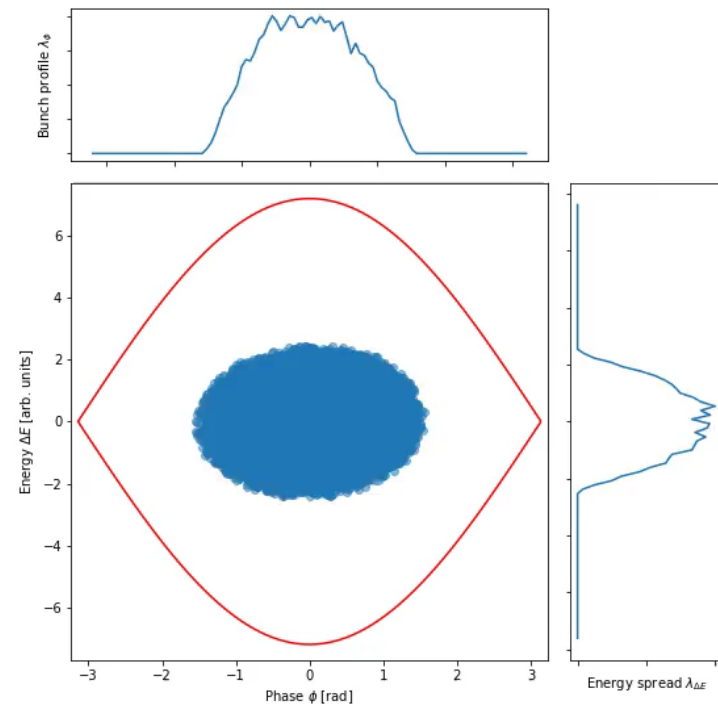
- Set-up bunch with parabolic distribution: `generate_bunch`
- Most simple case: single harmonic RF without acceleration

Correct voltage at correct phase



→ Matched bunch

Wrong voltage at correct phase

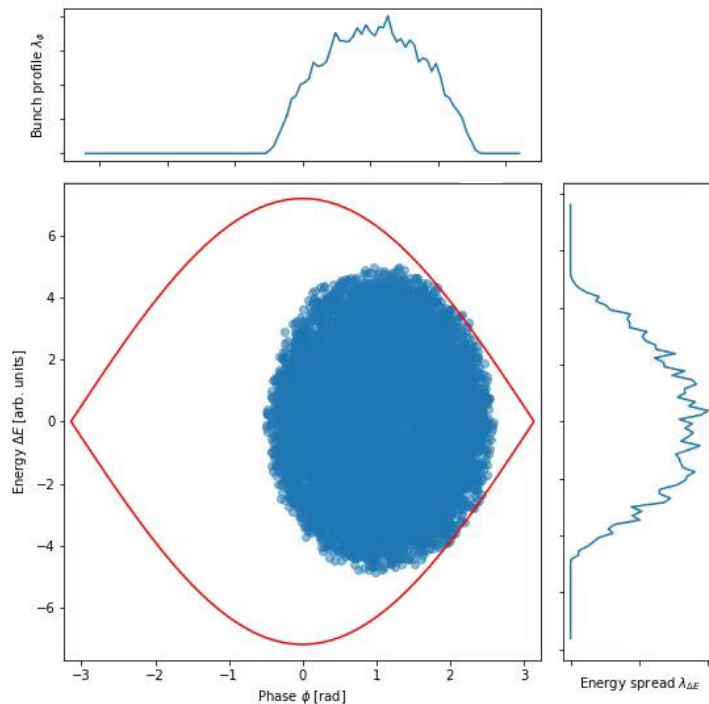


→ Breathing bunch
(quadrupole)

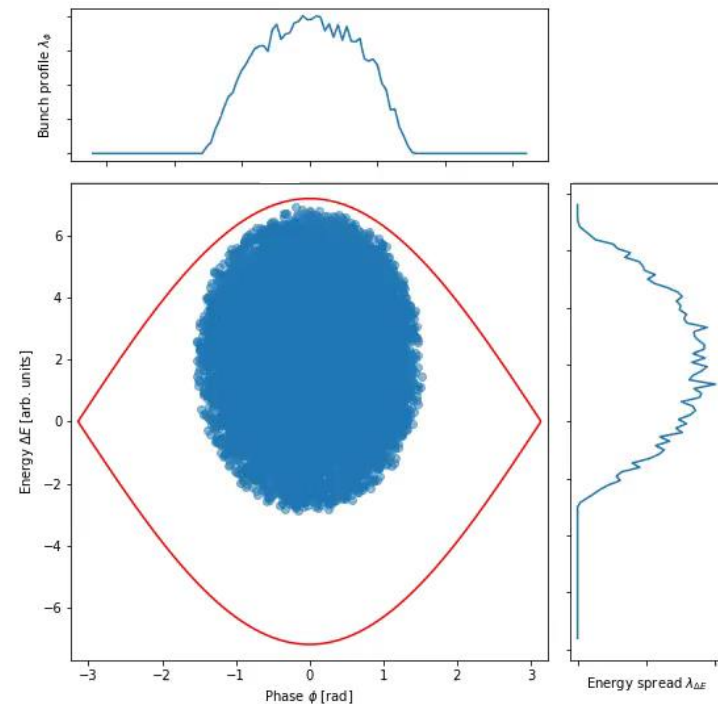
Example: Tracking of a single bunch

- Set-up bunch with parabolic distribution: `generate_bunch`
- Most simple case: single harmonic RF without acceleration

Correct voltage at **wrong phase**



All correct, but **wrong energy**

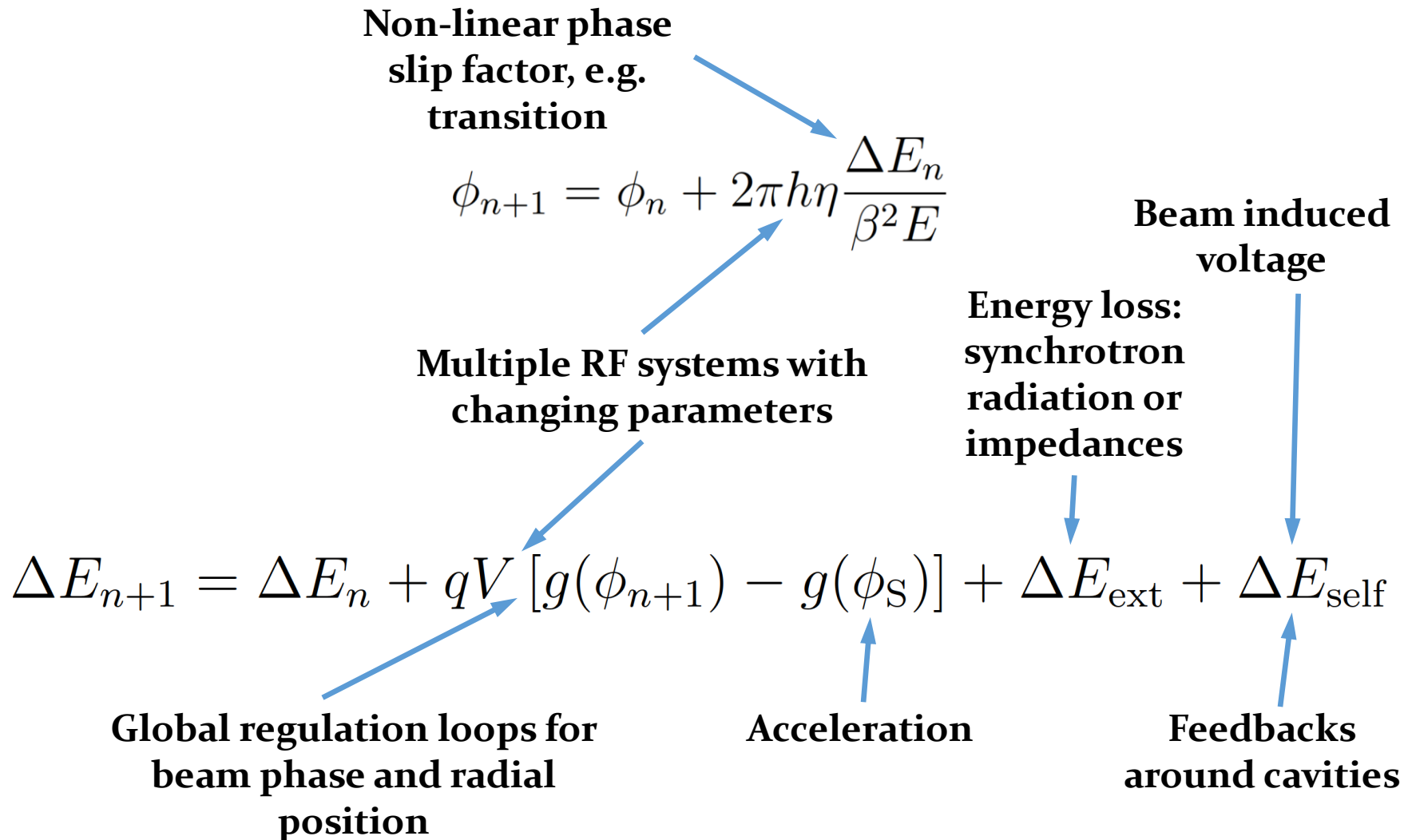


→ Dipole oscillations

→ Phase and energy offset for example at injection


Getting closer to reality

→ State-of-the-art tracking may include much more



Longitudinal tracking codes

- **Dedicated to longitudinal dynamics: fast and focussed on RF aspects**
- **Combined transverse and longitudinal tracking**

Name	Remarks	
BLonD	<ul style="list-style-type: none"> • Widely used at CERN • Complex RF manipulations and feedbacks • Longitudinal intensity effects 	 http://blond.web.cern.ch/
ESME	<ul style="list-style-type: none"> • Longitudinal work horse code for many years • RF manipulations with multiple RF systems • Intensity effects 	esme.fnal.gov
PyHeadTail	<ul style="list-style-type: none"> • Longitudinal and transverse combined simulation 	Longitudinal, 1D
PyOrbit	<ul style="list-style-type: none"> • Longitudinal and transverse combined simulation 	
elegant	<ul style="list-style-type: none"> • Longitudinal and transverse combined simulation • Mainly used for electron accelerators 	
...	...	Combined, 3D

Summary

- **Longitudinal simulations using particle tracking**
 - **Complementary approach** to longitudinal beam dynamics
 - **Flexibility** to change parameters during tracking
 - **Powerful technique** to study
 - **Multi-harmonic RF systems**
 - **Complicated intensity effects**
 - **Longitudinal dynamics with feedbacks and RF loops**



Google
...is your friend!

**You will build a (small)
longitudinal tracking code**

A big Thank You

to all colleagues providing support, material and feedback

**Simon Albright, Maria-Elena Angoletta,
Philippe Baudrenghien, Thomas Bohl, Wolfgang Höfle, Erk
Jensen, Alexander Lasheen, Elena Shaposhnikova,
Frank Tecker, Daniel Valuch, Manfred Wendt, Jörg Wenninger
and many more...**

**Thank you very much
for your attention!**

References

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