CERN

Injection and extraction

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with material from past and current members of the ABT group as well as F. Tecker

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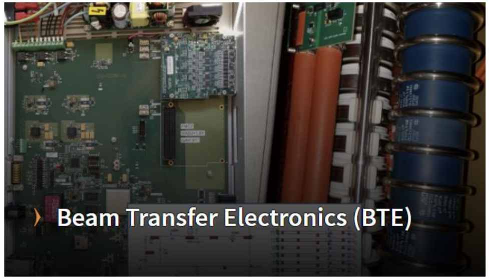
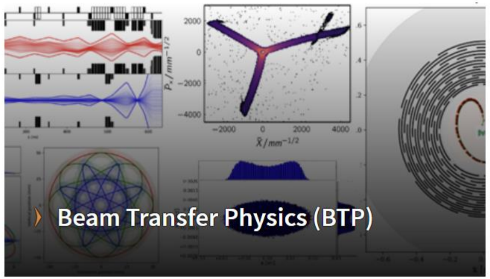
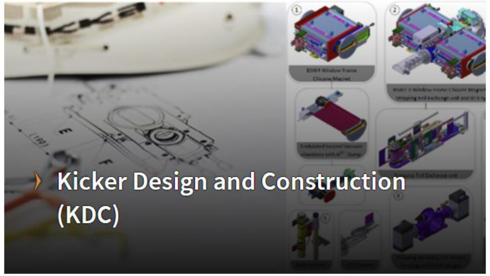
The CERN Accelerator School

Injection-Extraction @CERN

- CERN Accelerator Beam Transfer group: website

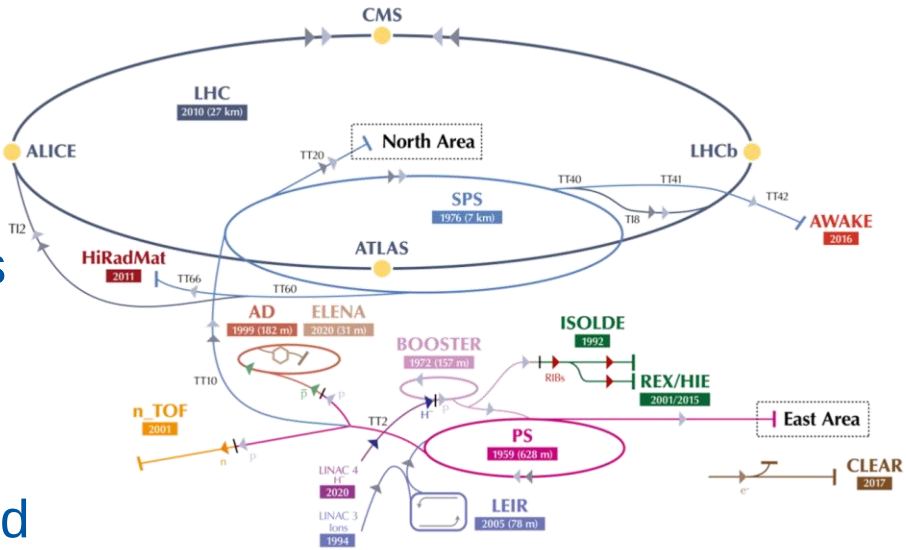






Introduction

- Accelerators have limited ranges of energies: particles get accelerated in a chain of accelerators.
- Injection and extraction between stages needs careful consideration
  - To deliver beams with required properties to users.
- The limits of this lecture: mostly focused on synchrotrons!



Layout

- Basic principle + hardware
- Injection techniques
  - Fast injection
    - Normalized phase space
    - Imperfect injection
  - Multi-turn injection
  - Charge exchange technique
  - Lepton injection
- Extraction techniques
  - Fast extraction
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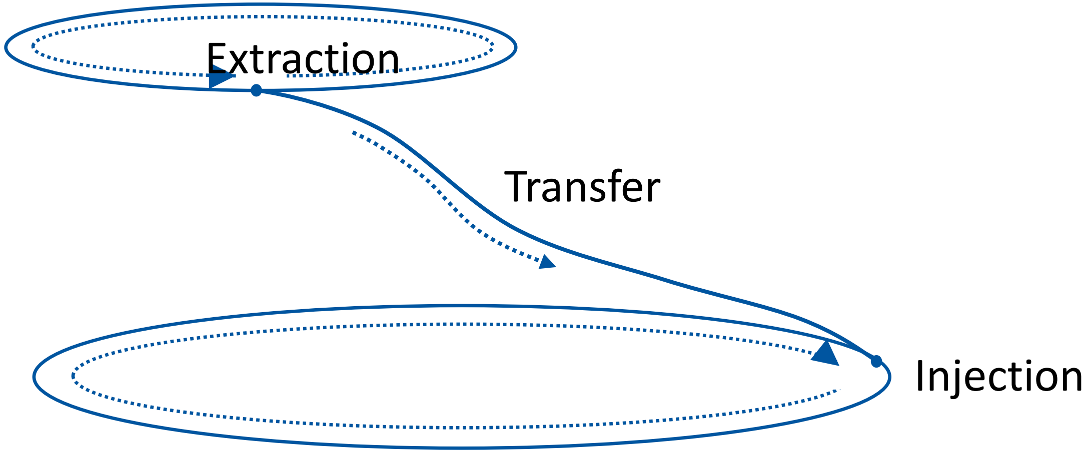
  - Resonant multi-turn extraction
  - Resonant slow extraction
- Transfer between machines
- Conclusion



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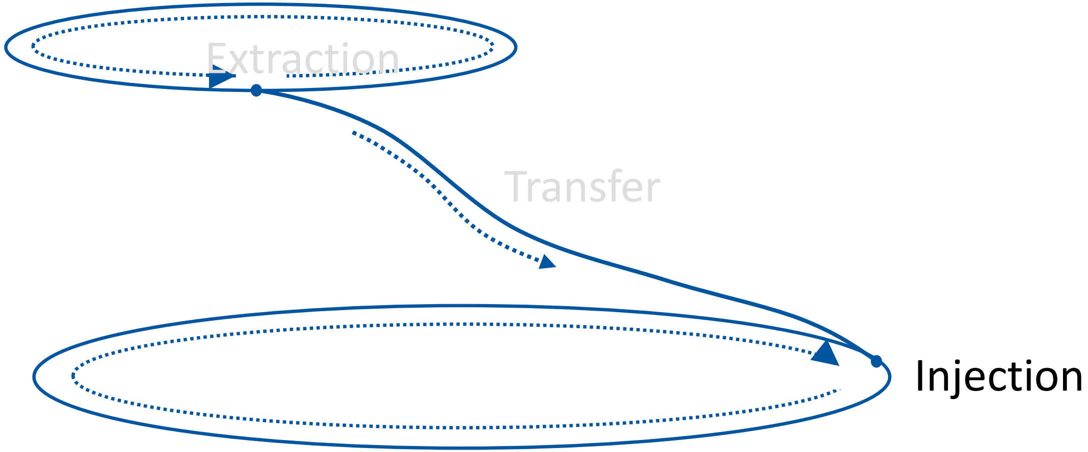
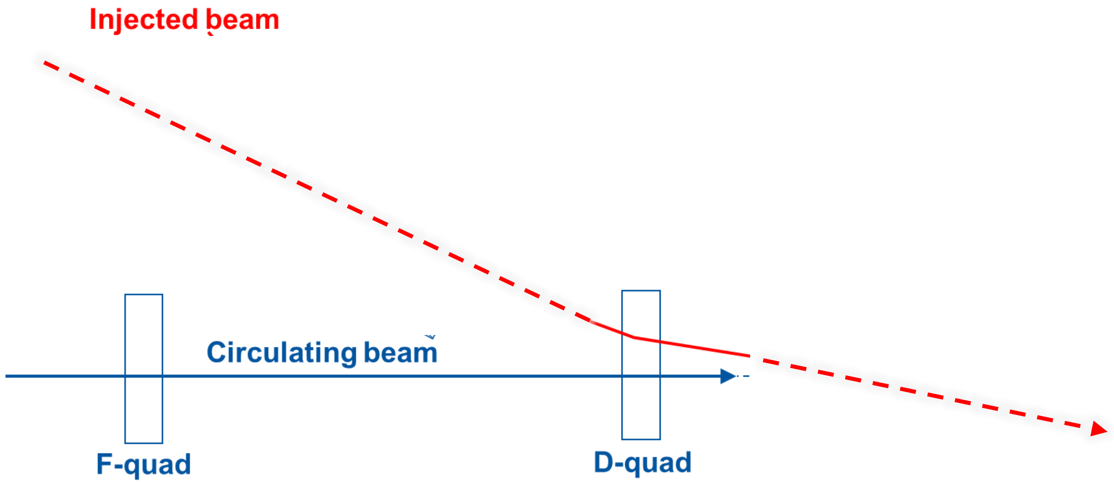
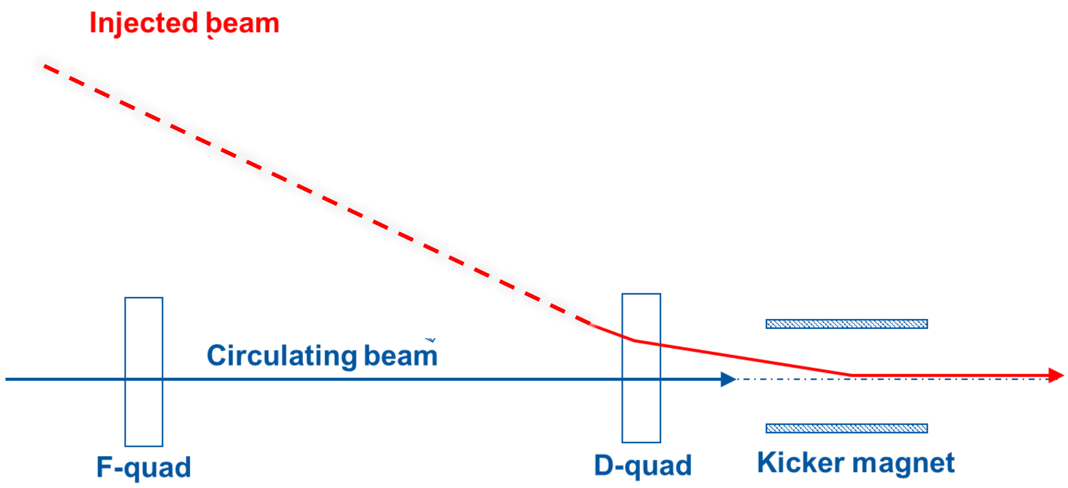
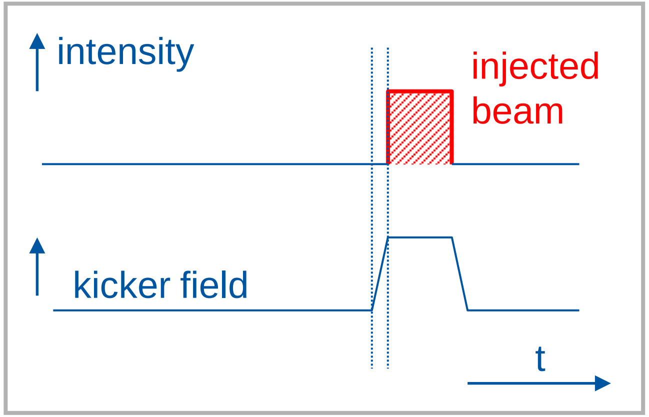


Basic principle



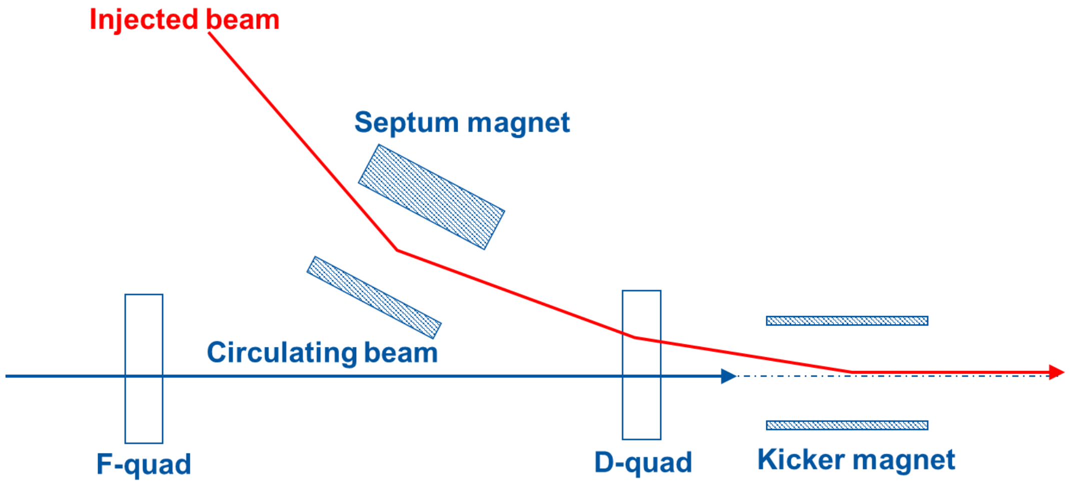
Basic principle

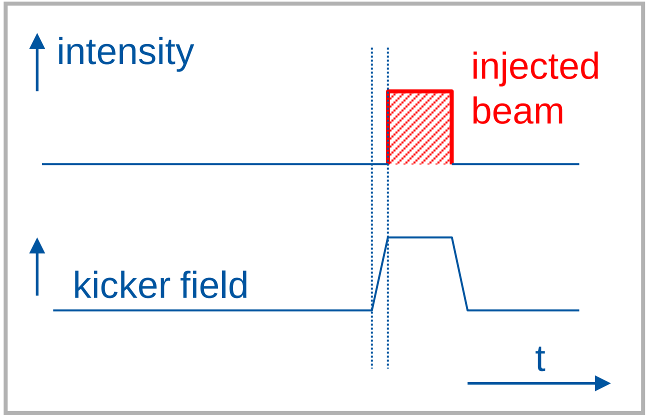




- **Kickers** produce fast pulses, rising their field within the particle-free gap in the circulating beam **(temporal separation)**

Basic principle

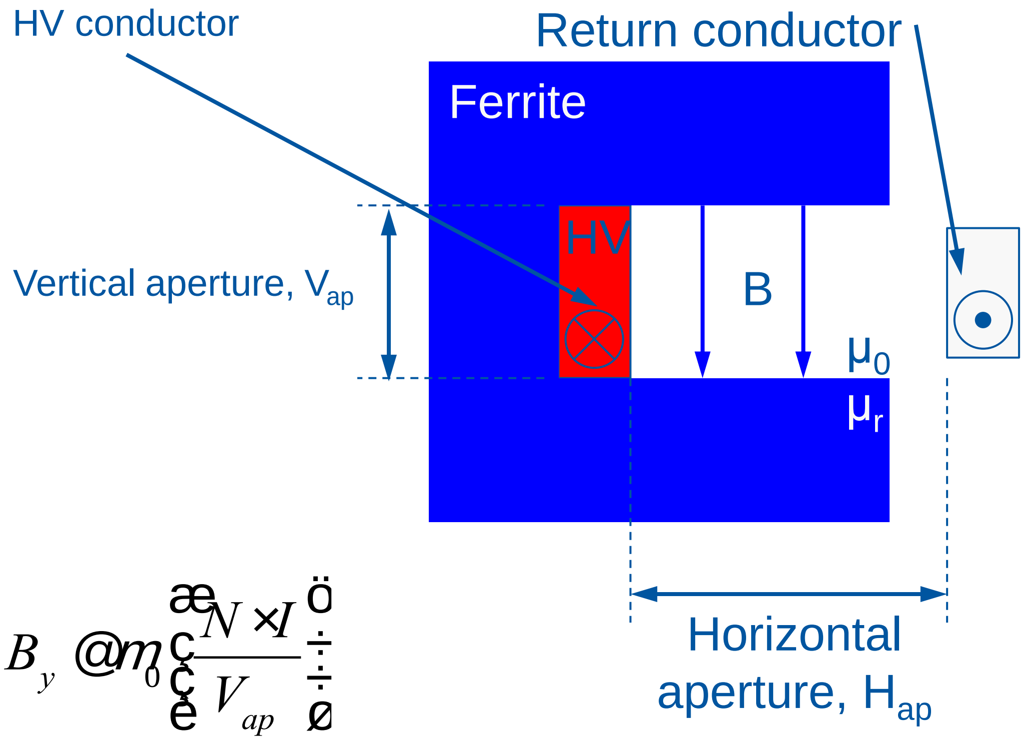




- **Kickers** produce fast pulses, rising their field within the particle-free gap in the circulating beam **(temporal separation)**
- **Septa** compensate for the relatively low kicker strength, and approach closely the circulating beam **(spatial separation)**

Basic principle: Kicker

- Aperture dimensions constrained by beam size at kicker location.
- Short rise time: < 100 ns –few ms: low inductance.
- Maximise horizontal kick: small vertical aperture.



$$L_{mag/m} \approx \mu_0 \left(\frac{N^2 \cdot H_{ap}}{V_{ap}} \right)$$

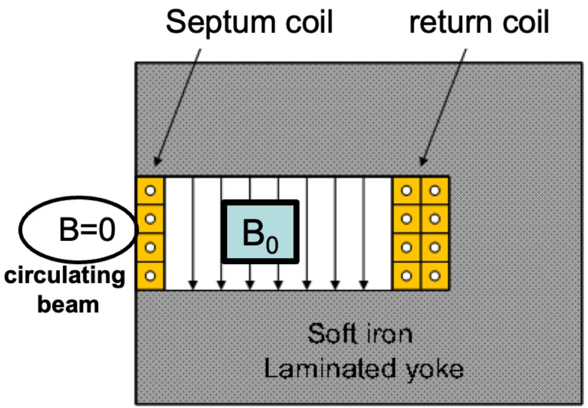
$$B_y \approx \mu_0 \left(\frac{N \cdot I}{V_{ap}} \right)$$

Basic principle: septum

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Magnetic

Septum coil: **2 – 20 mm**

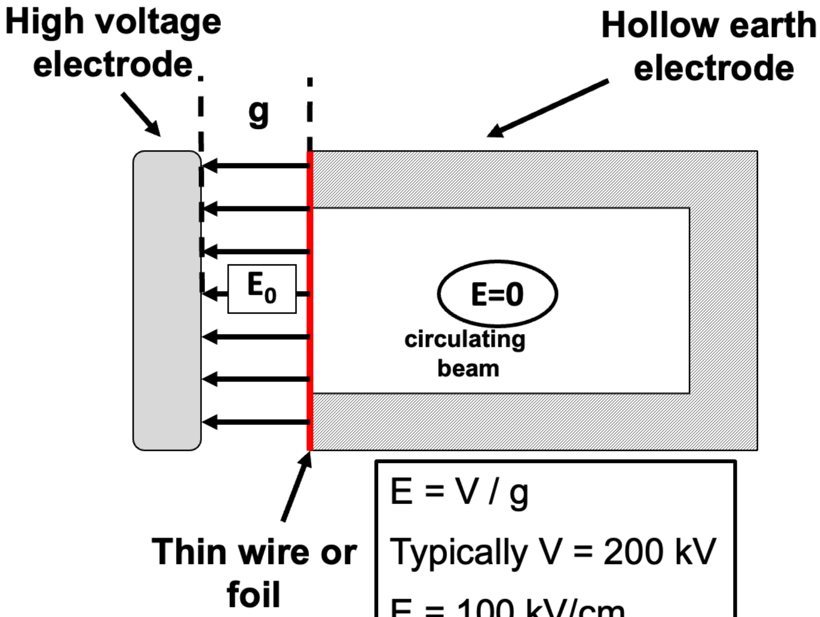


$B_o = \mu_0 I / g$

Typically I 5 - 25 kA

Electrostatic

Thin wire or coil: **~0.1 mm**



$E = V / g$

Typically V = 200 kV

E = 100 kV/cm

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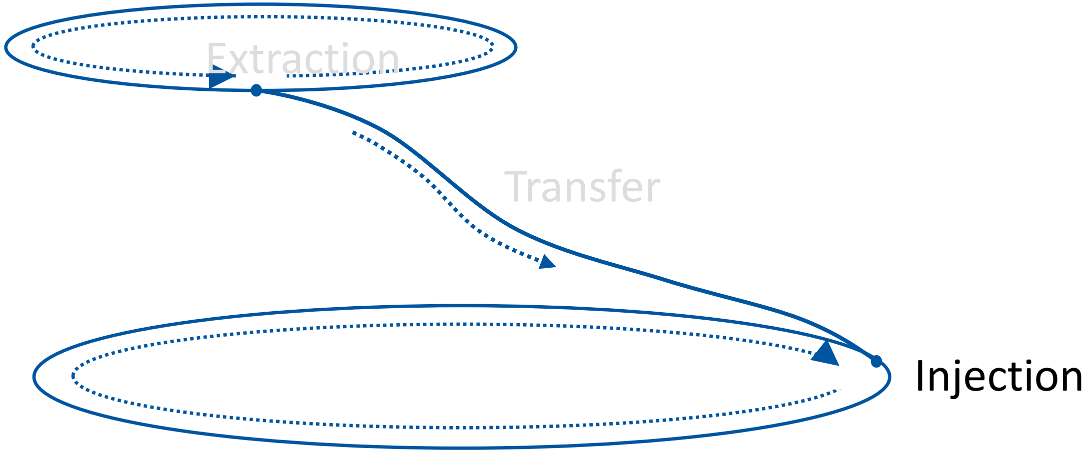
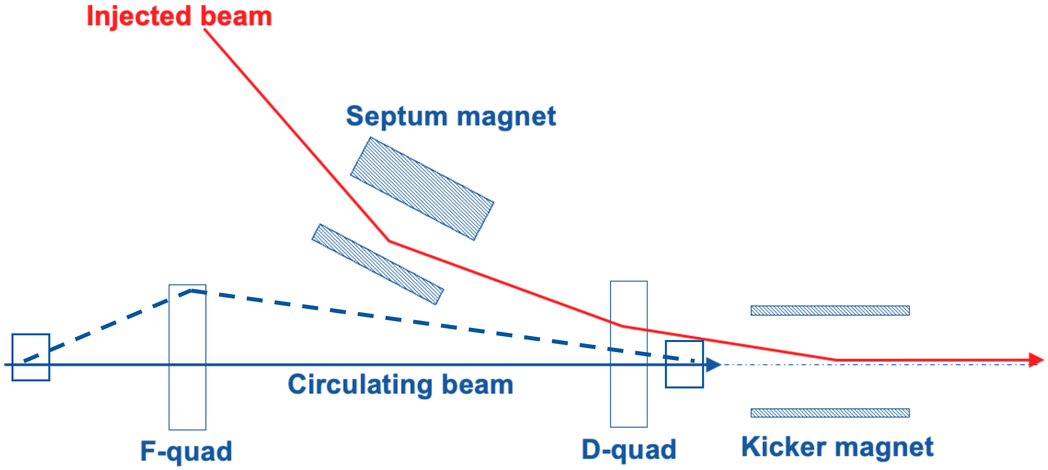
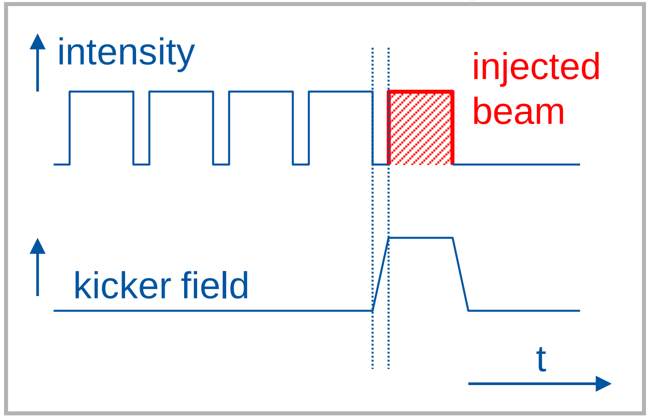


Fast injection: concept



'boxcar' stacking



- Septum + kicker work together to put beam onto reference trajectory.
- Several trains can be stacked (e.g. from smaller ring to larger).
- Slow bumpers may be used to minimize necessary kicks.

- Applications: PS, SPS, LHC, etc. ("Bread and butter" of injection.)

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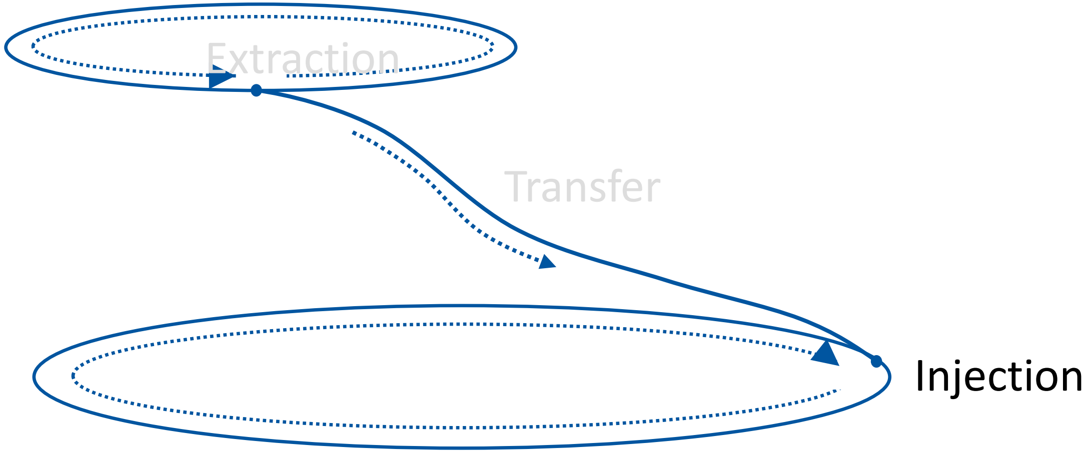
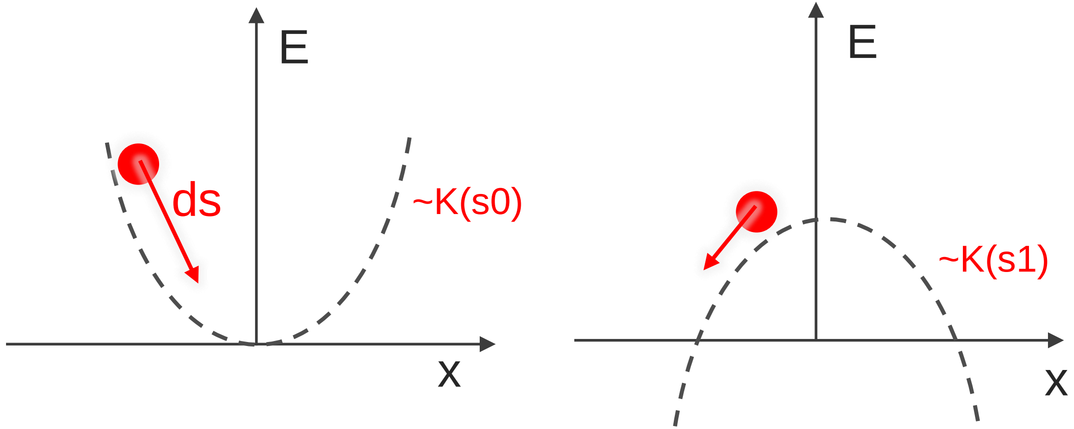


Normalized phase space: concept

Hill's equation describes oscillator whose amplitude and frequency vary with s coordinate.

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}x + K(s)x = 0$$

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos\left[\mu(s) + \mu_0\right]$$



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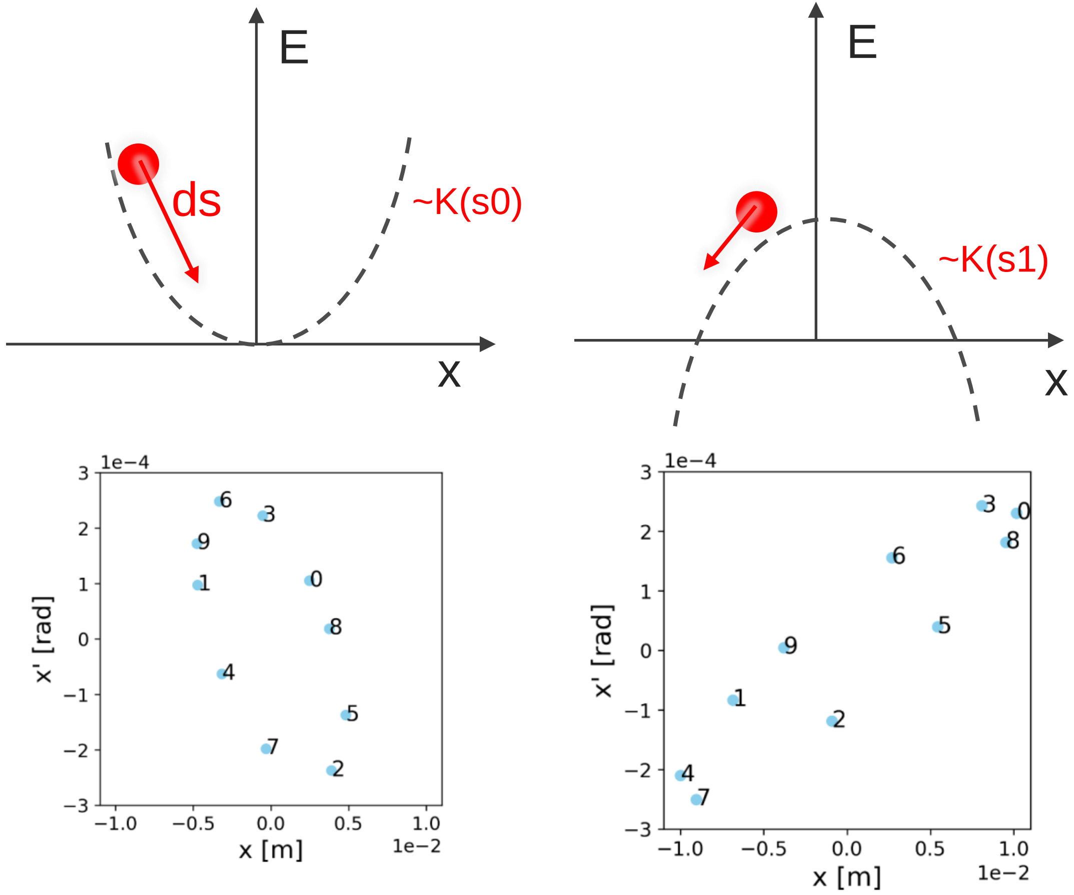

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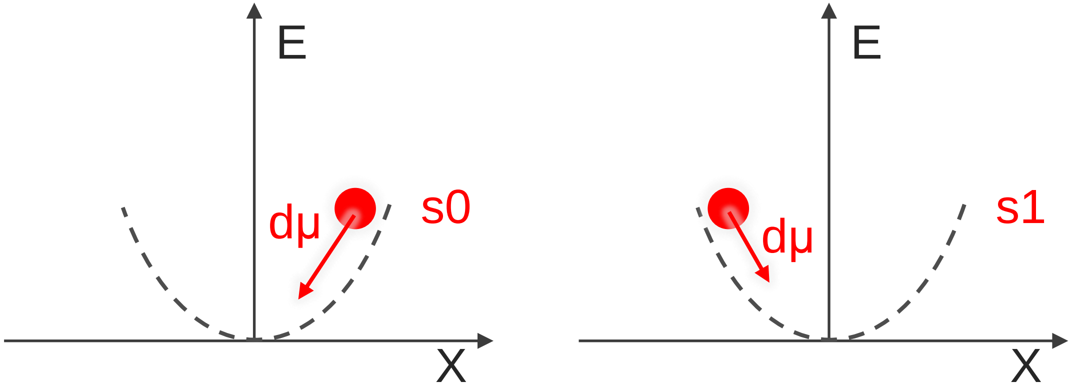
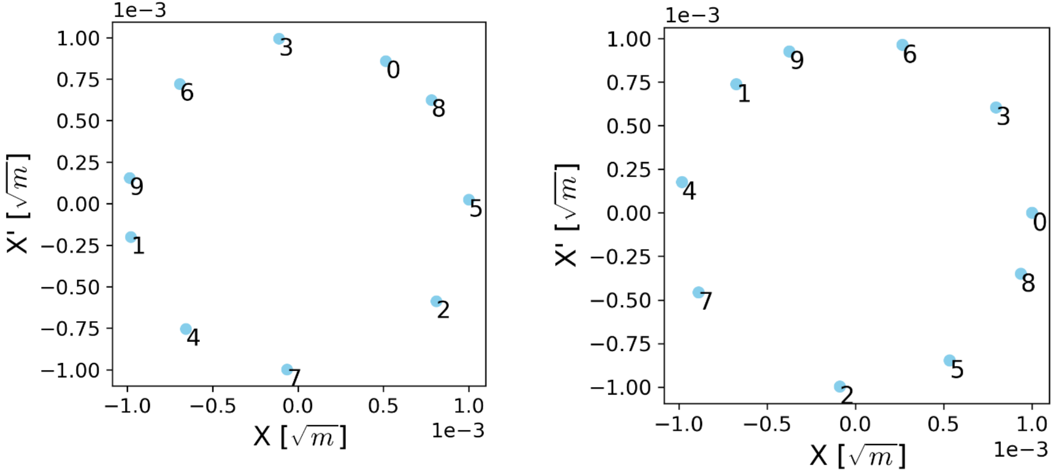
$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}x + K(s)x = 0$$

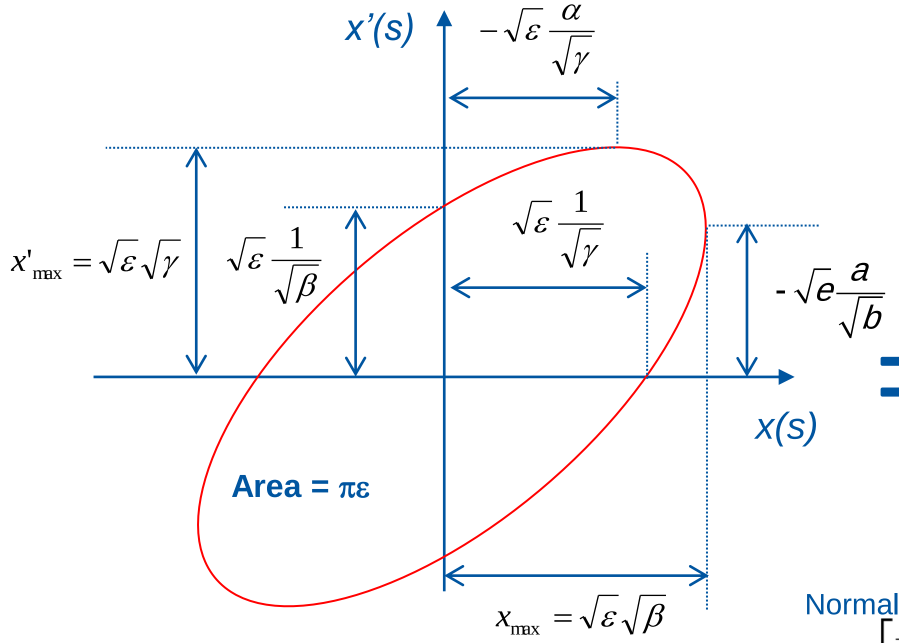
$$x(s) = \sqrt{e}\sqrt{b(s)}\cos[m(s) + m_0]$$

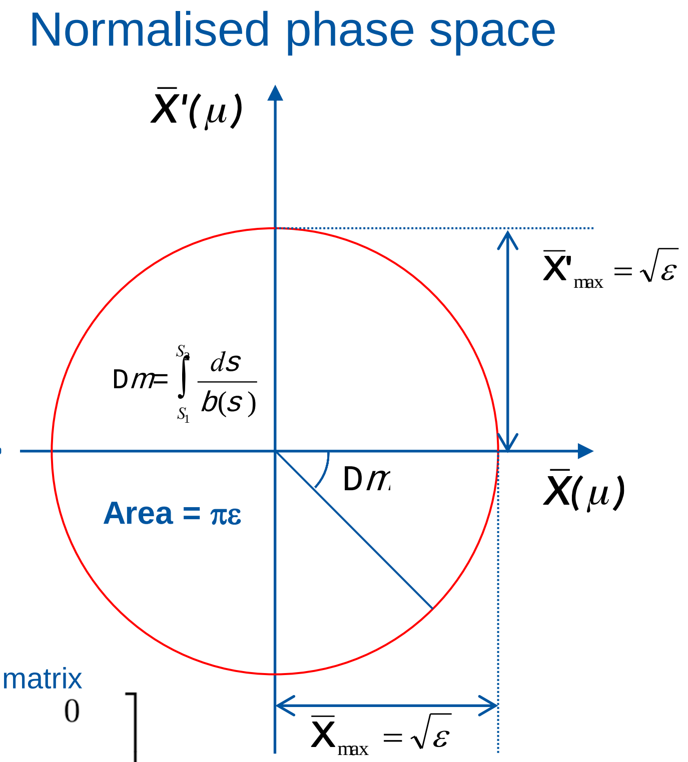
$$X(\mu_x) = \frac{1}{\sqrt{\beta_x}}x = A_x \cos(\mu_x + \mu_0)$$





Normalized phase space: math





$$\varepsilon = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

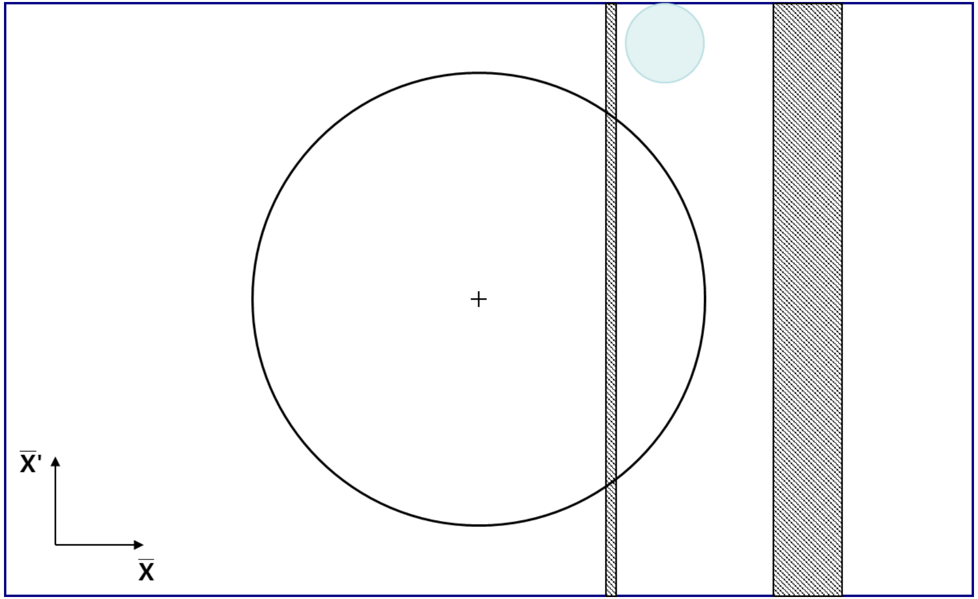
Normalization matrix

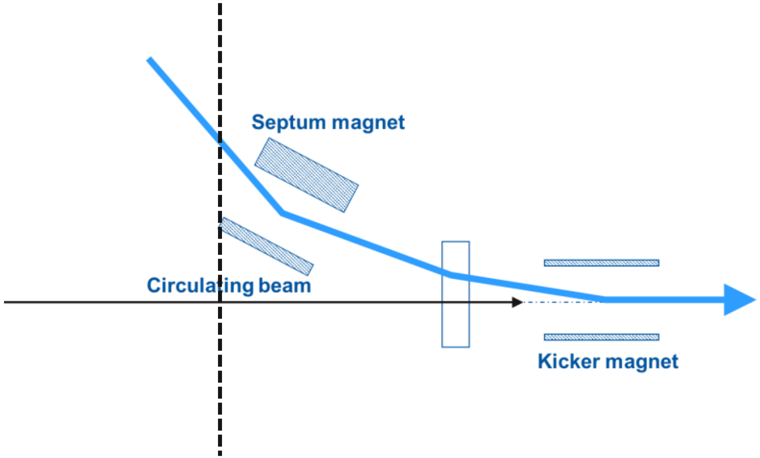
$$L^{-1}(s) = \begin{bmatrix} \frac{1}{\sqrt{\beta_x(s)}} & 0 \\ \frac{\alpha_x(s)}{\sqrt{\beta_x(s)}} & \sqrt{\beta_x(s)} \end{bmatrix}$$

$$\varepsilon = \bar{X}^2 + \bar{X}'^2$$

Fast injection: normalized phase space

- Seen from the ring
  - in the normalized transverse phase space in the plane of injection





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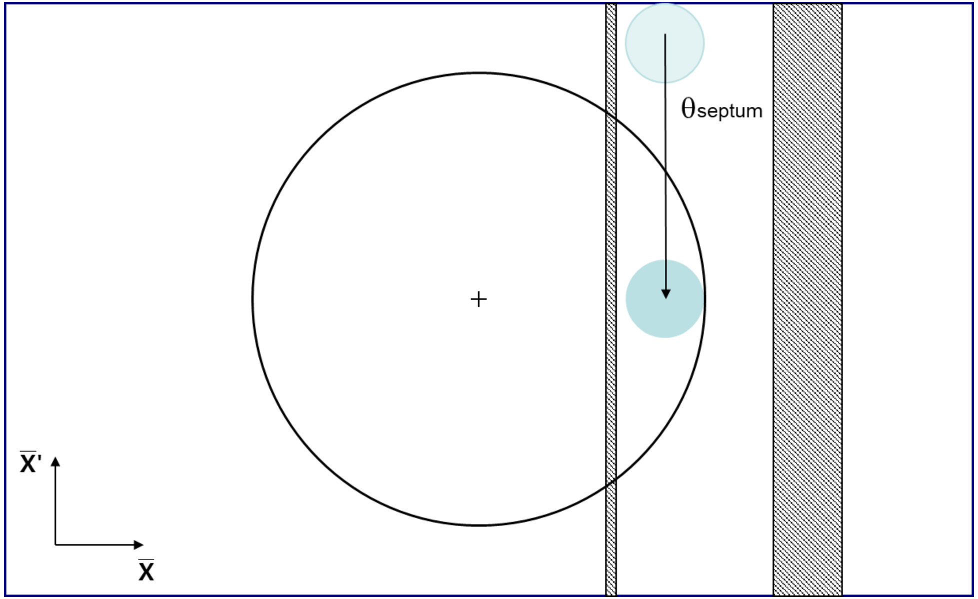

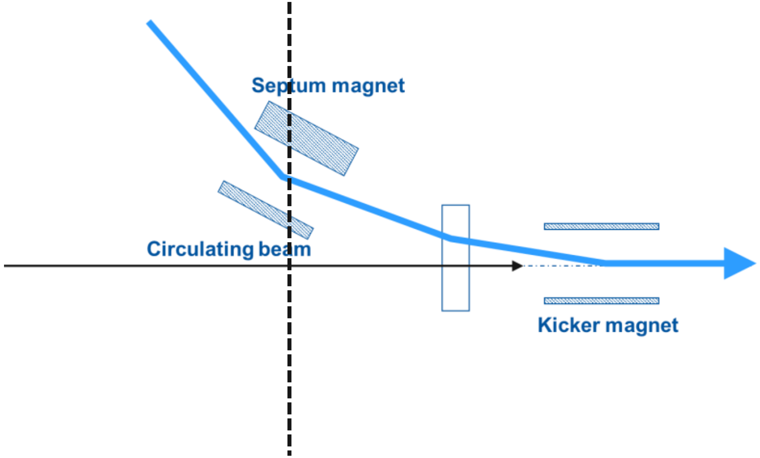

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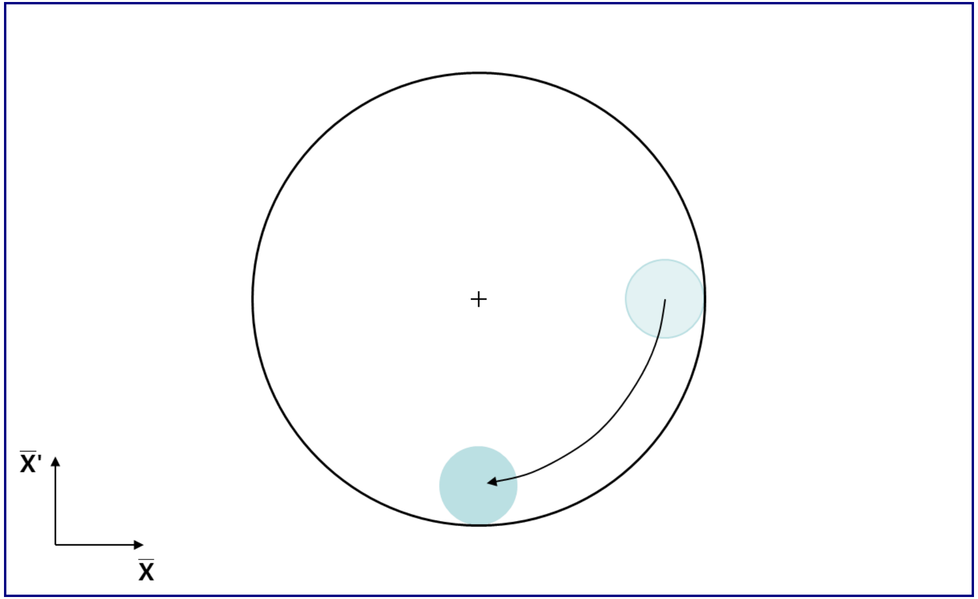

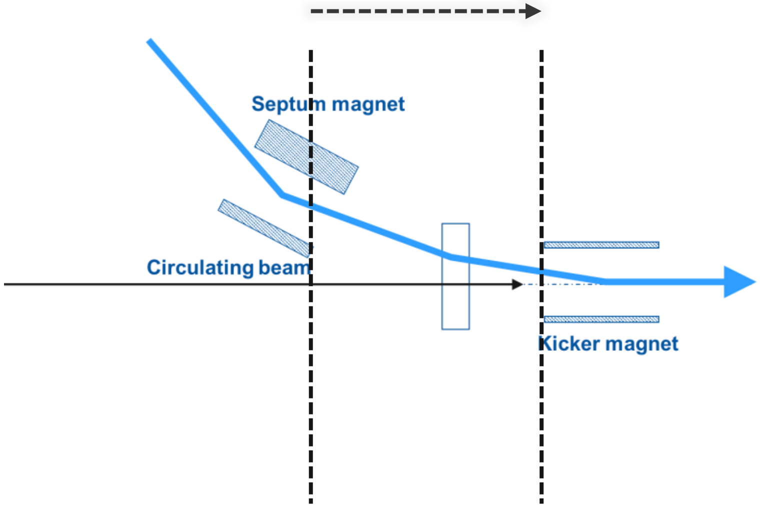

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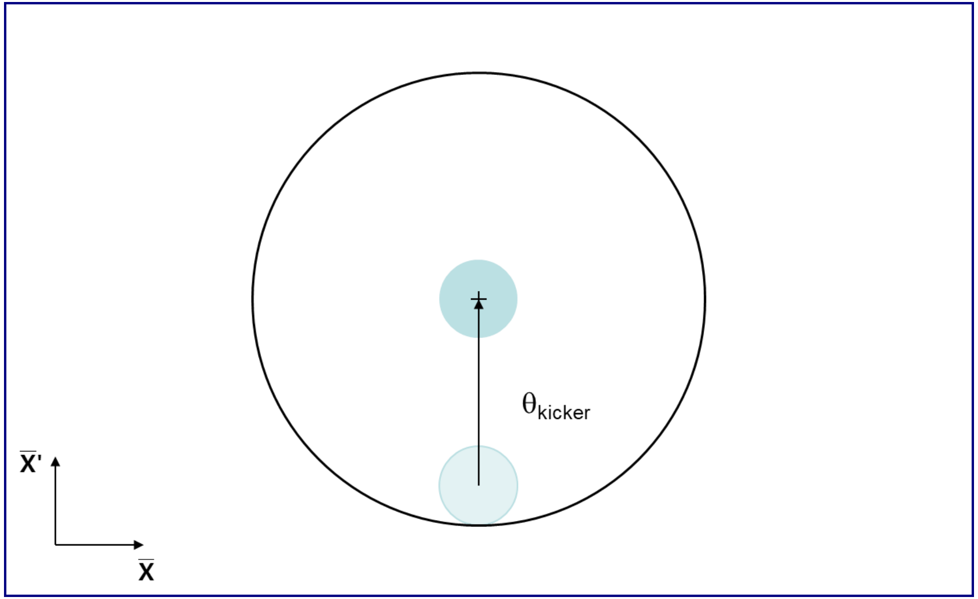

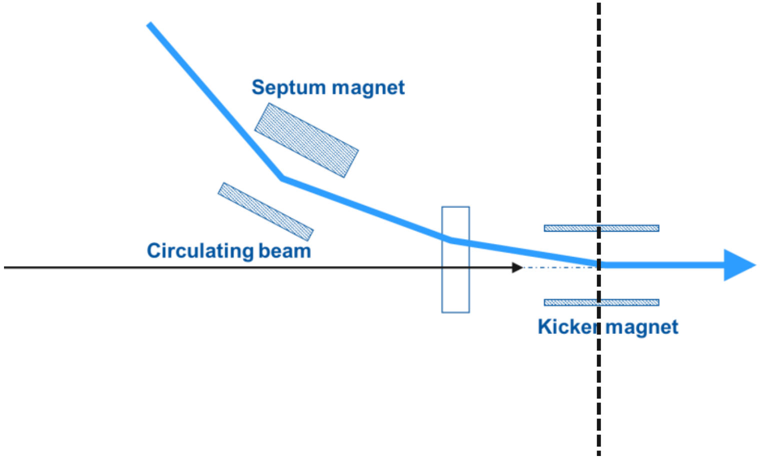

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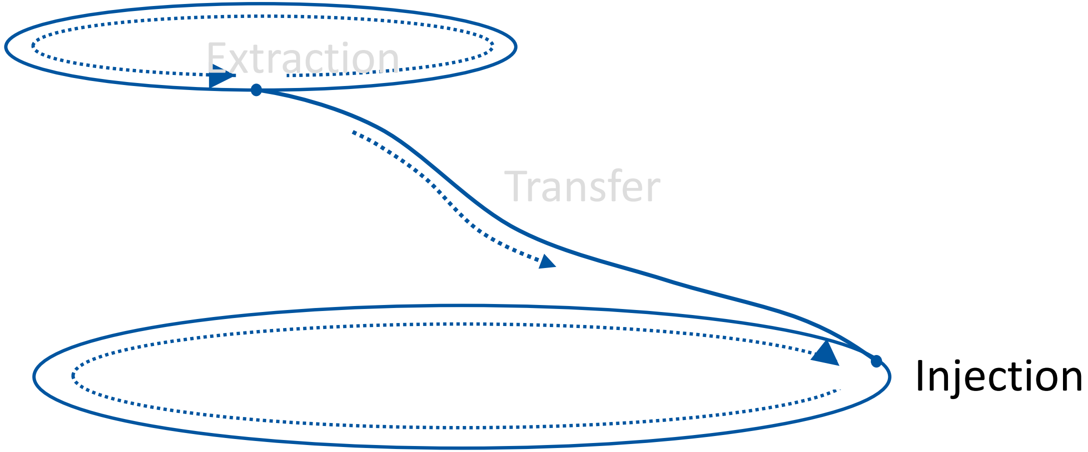

Imperfect injection: injection oscillations

Challenges

- Injected beam needs to be matched and aligned to the lattice and trajectory of the ring.
- Injection kicker(s) needs to be fast enough to deflect only the injected beam.

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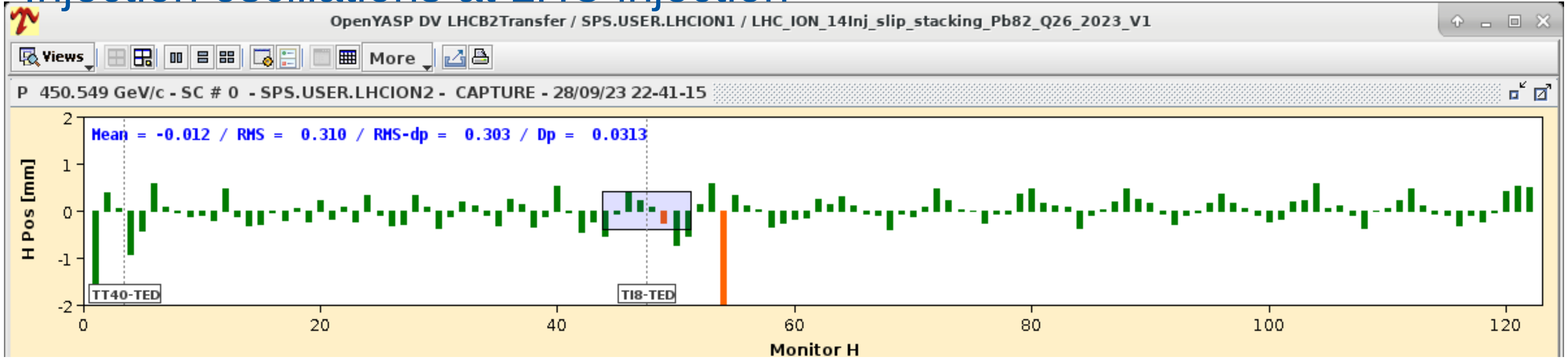
Injection oscillations at LHC injection



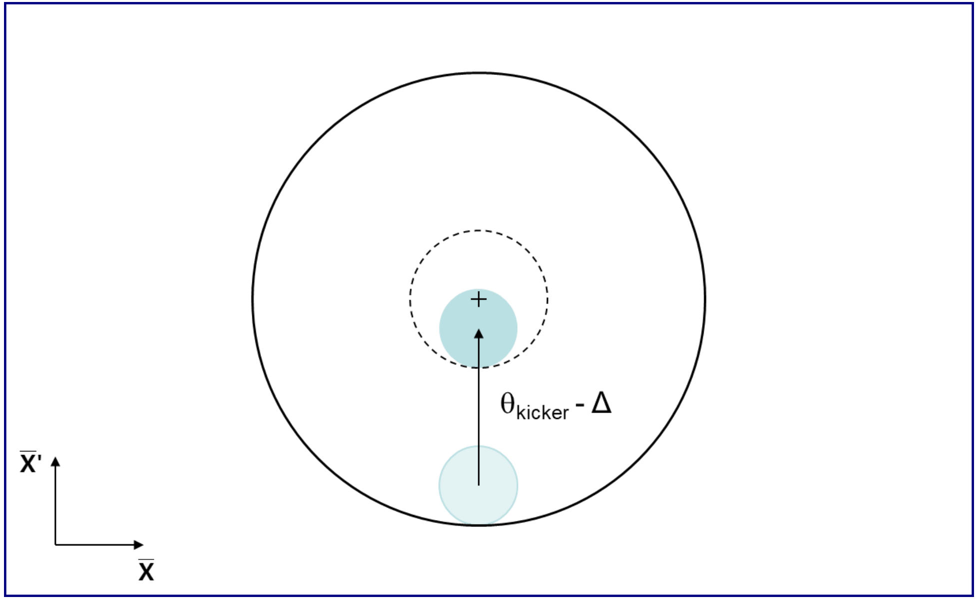
SPS

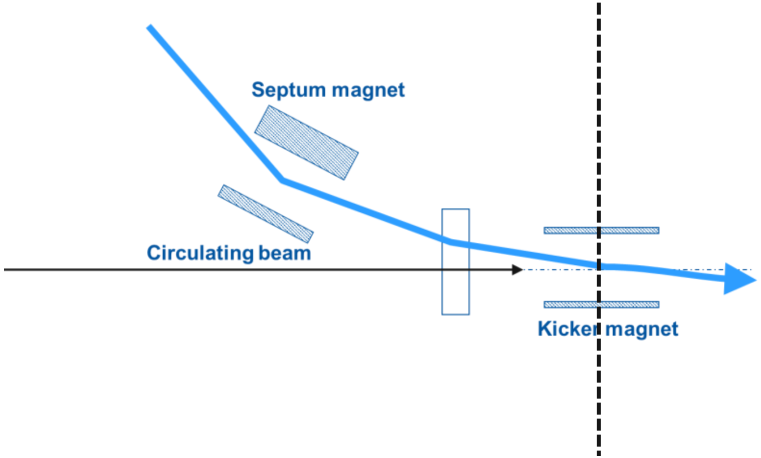
TI8 transfer line

First arc of LHC (P8 to P7)

Imperfect injection: injection oscillations

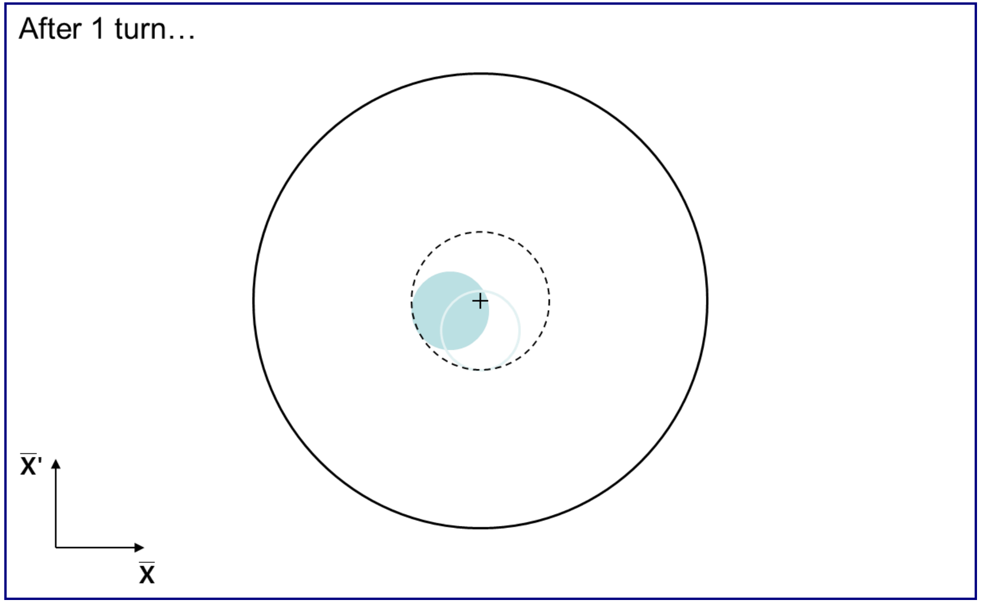
- Injection with a dipole error, here of Δ in angle at the kicker

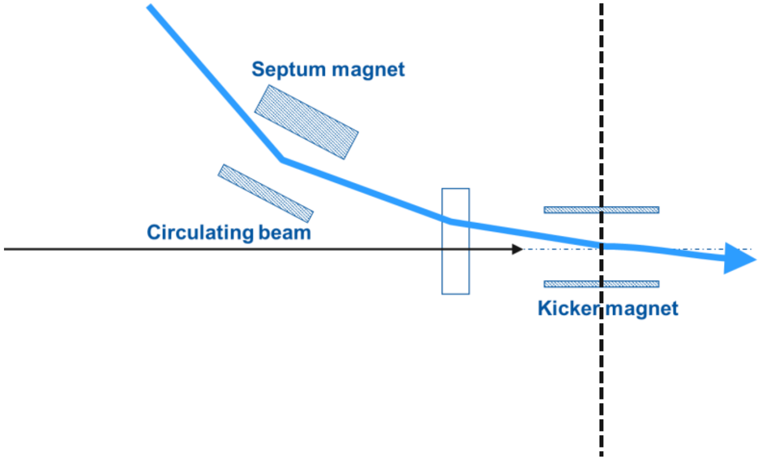




Imperfect injection: injection oscillations

- In the normalized phase space
  - The beam rotates around the closed orbit





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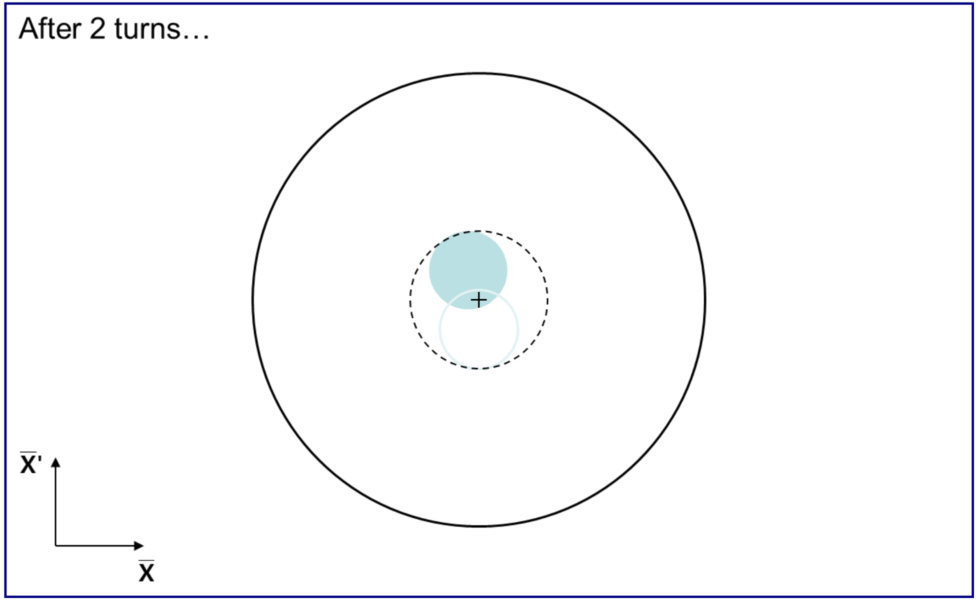

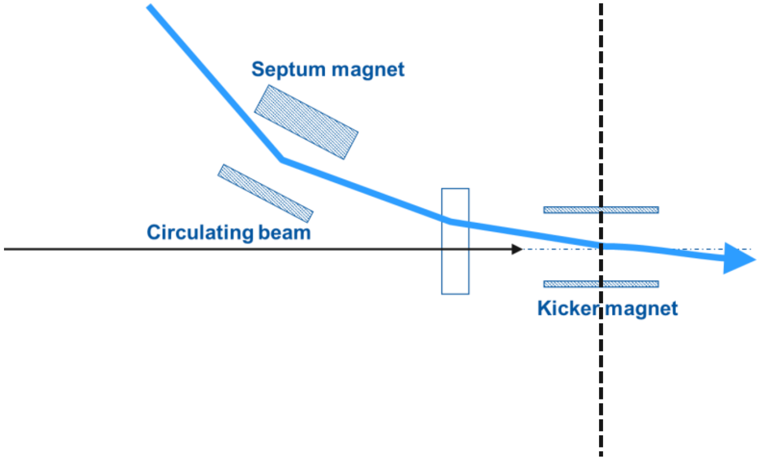

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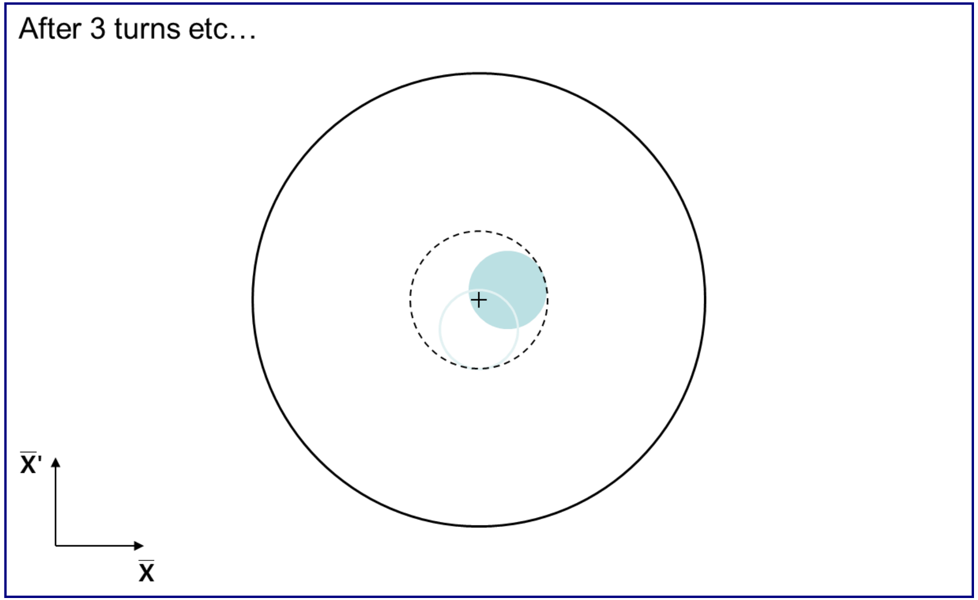
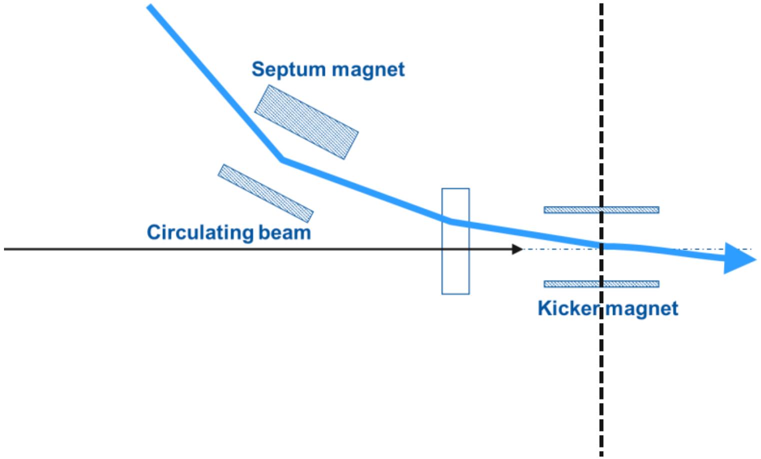
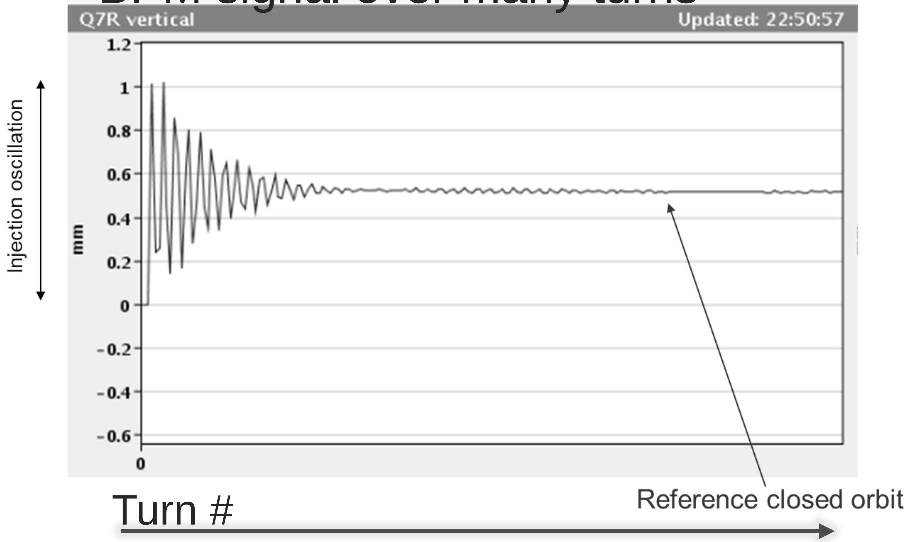




Imperfect injection: Filamentation

- If injection oscillation is left uncorrected, it might disappear after several turns.

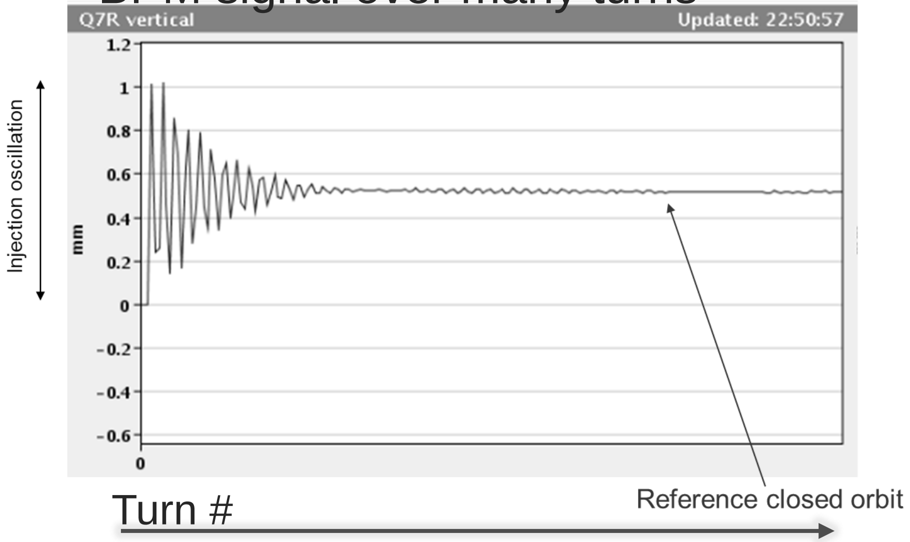
BPM signal over many turns

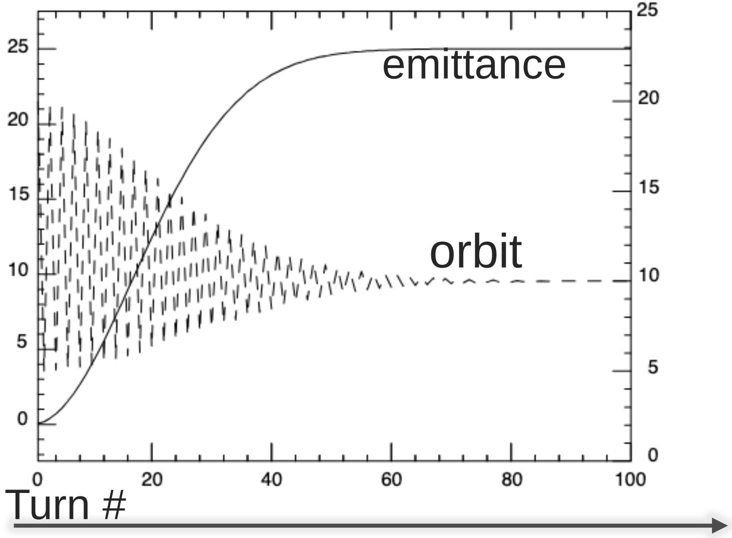


Imperfect injection: Filamentation

- If injection oscillation is left uncorrected, it might disappear after several turns.
- However, it has been “exchanged” for an r.m.s. emittance blow-up.

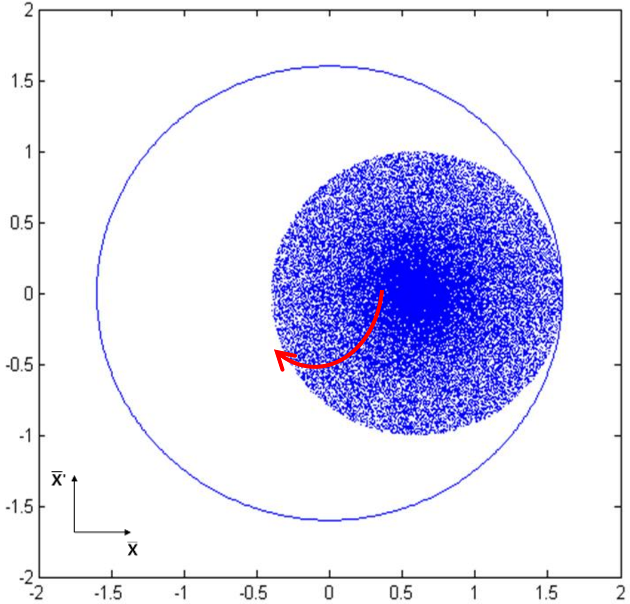
BPM signal over many turns





Imperfect injection: Filamentation

1. Mismatch between the injected beam and closed orbit -> injection oscillations



Imperfect injection: Filamentation

1. Mismatch between the injected beam and closed orbit -> injection oscillations
2. Non-linearities -> particles at different transverse amplitudes rotate at different speeds.

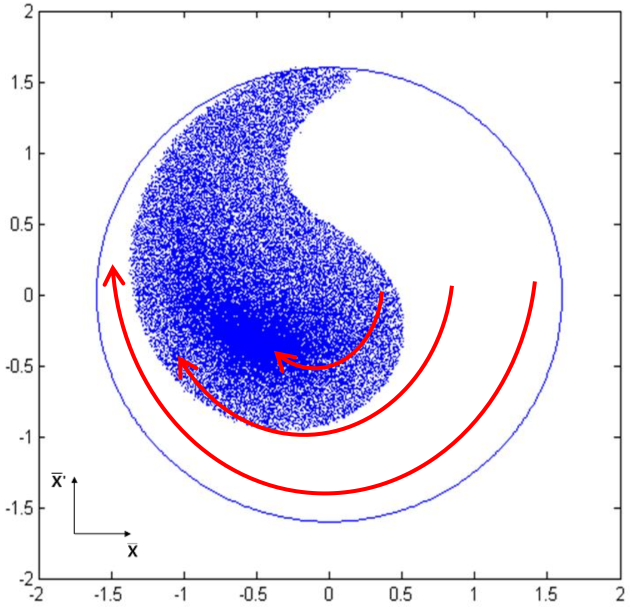
A first taste of Non- Linear Beam Dynamics I

3 Oct 2024, 09:35

1h

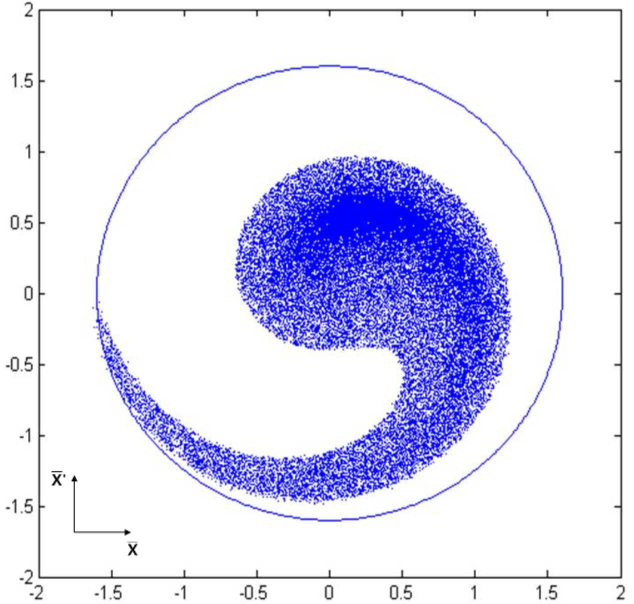
Speaker

Hannes Bartosik (CERN)



Imperfect injection: Filamentation

1. Mismatch between the injected beam and closed orbit -> injection oscillations
2. Non-linearities -> particles at different transverse amplitudes rotate at different speeds.
3. Beam distribution “wraps around” closed orbit.



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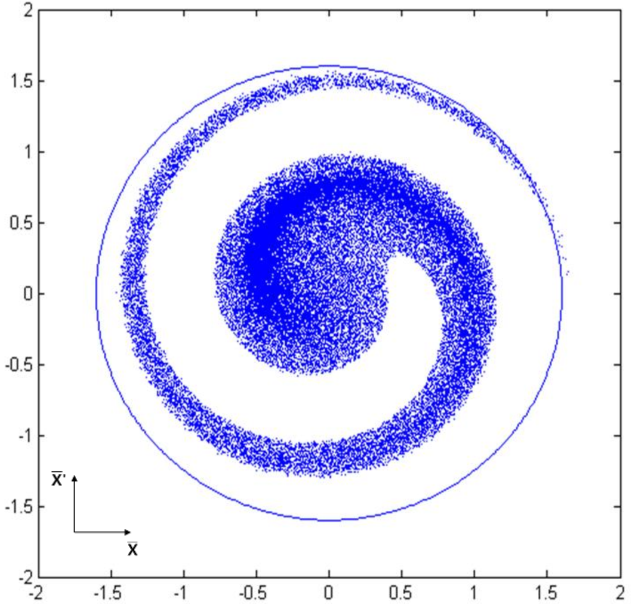

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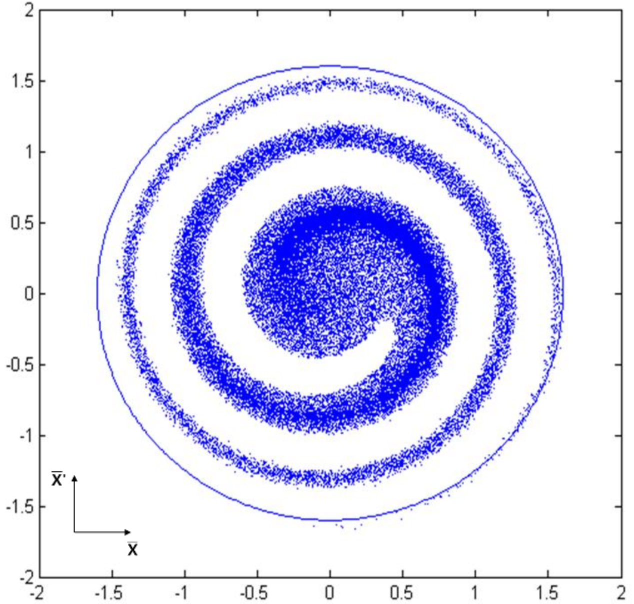

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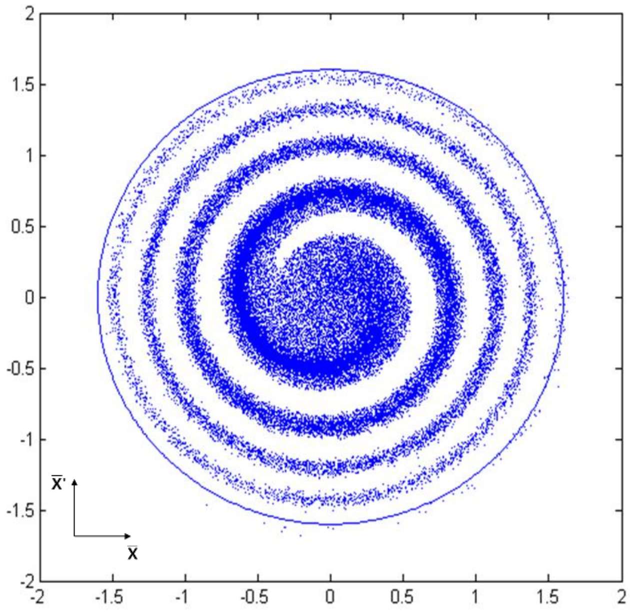

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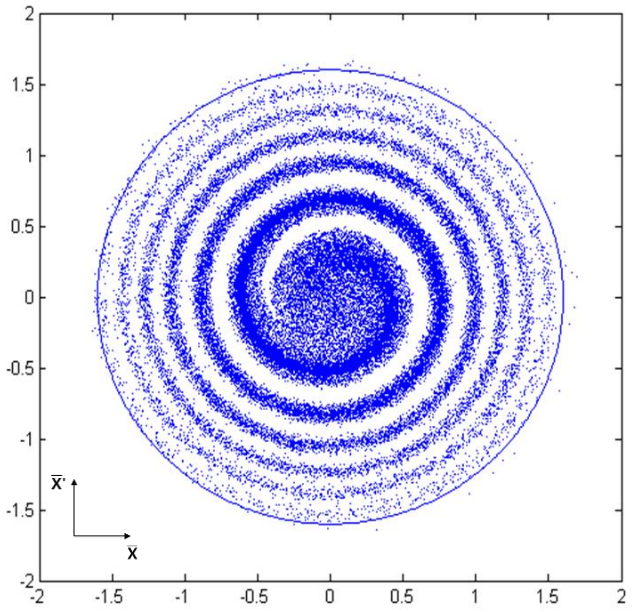

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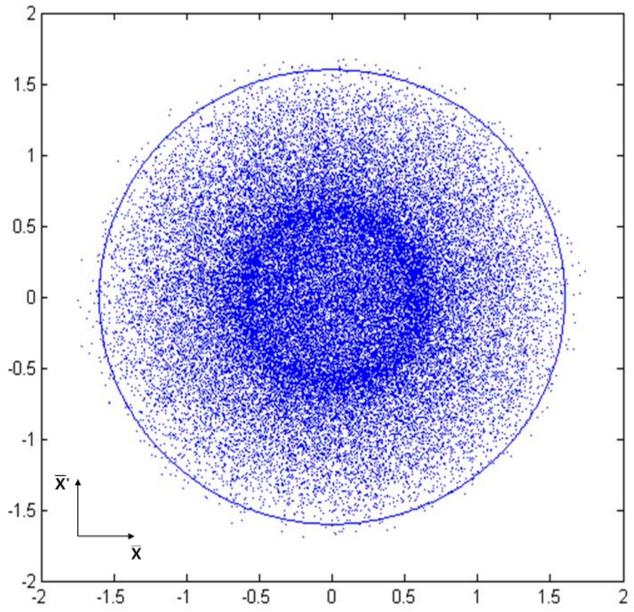

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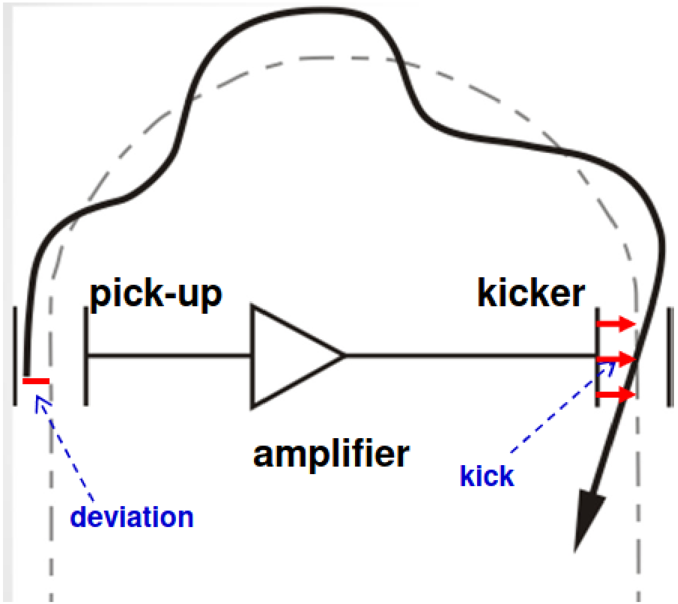
4. Rms emittance increases + injection oscillations are damped.

Effect can be partially mitigated by using transverse damper -> pick-up + kicker feedback system inside ring.



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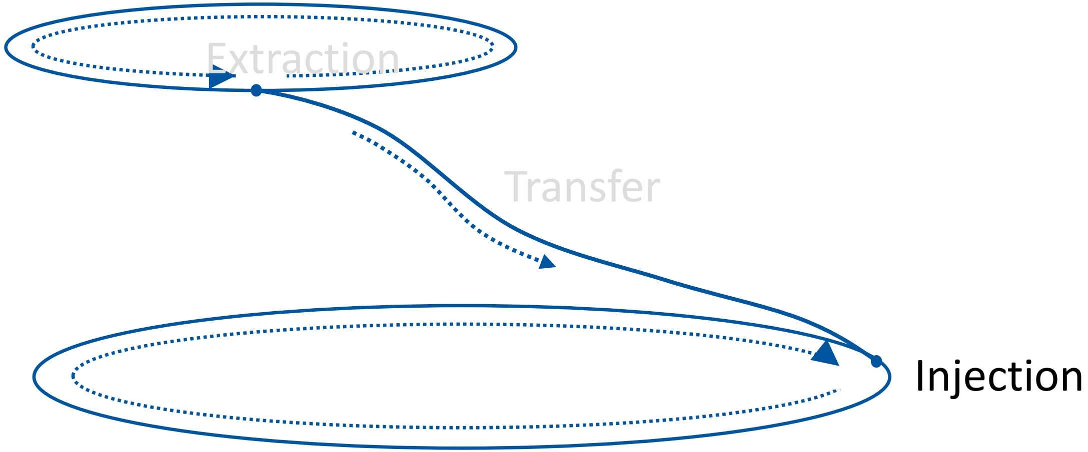
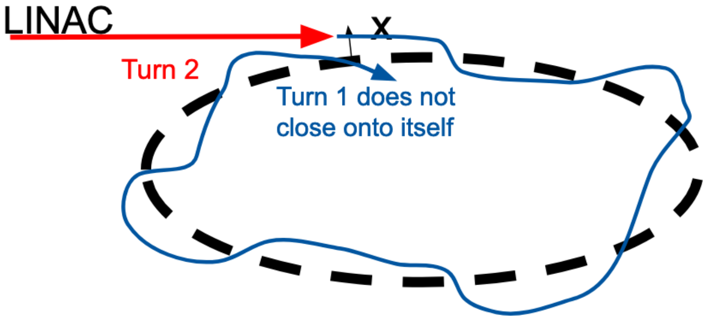
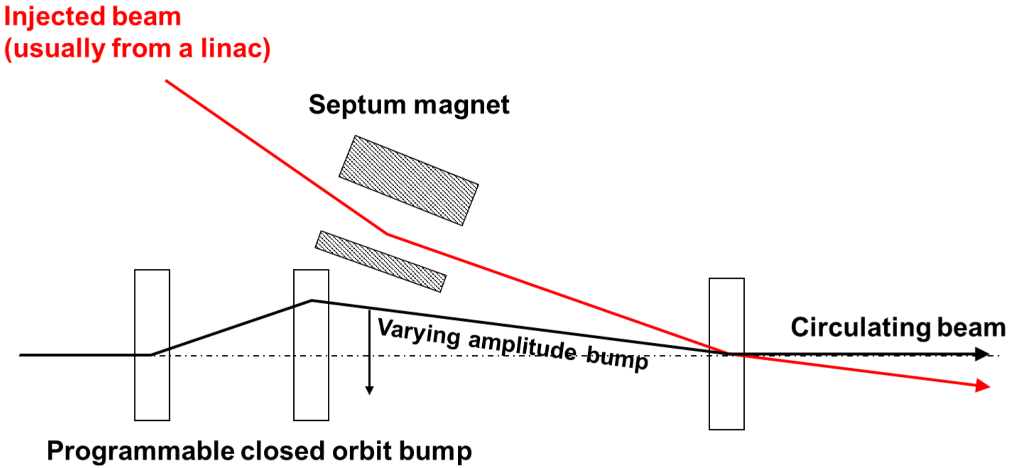
  - Resonant slow extraction
- Transfer between machines
- Conclusion



Multi-turn injection: concept

- Principle
  - Injected beam is injected in a small part of the available phase space.
  - The injected trajectory is moved relative to the closed orbit during the injection process.
  - Allows to accumulate intensity and `paint` the phase space with a beam smaller than the available ring acceptance, typically from a linac to a ring.



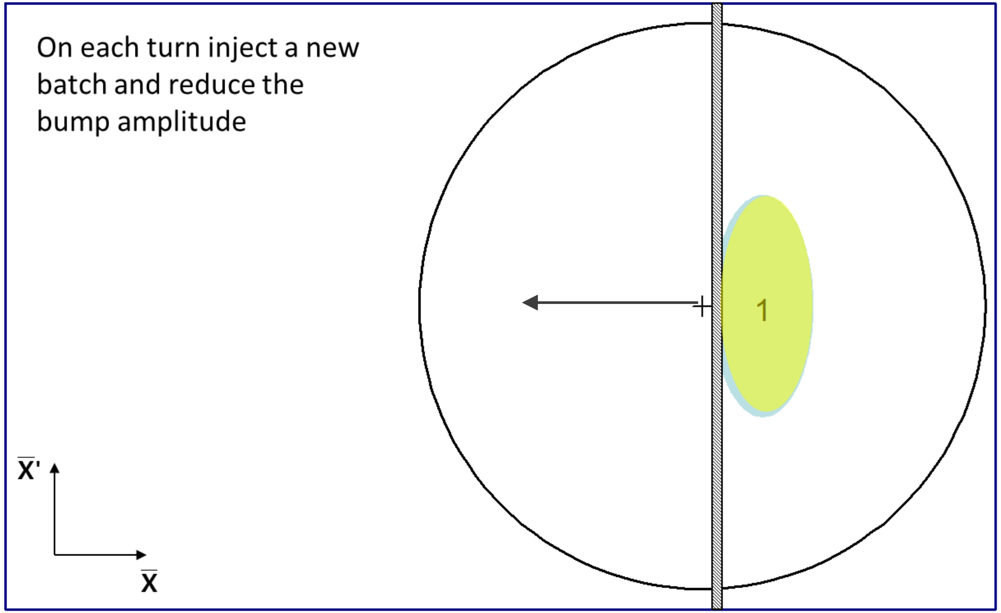


- Applications
  - At CERN PSB until LS2 and at LEIR presently.
  - At GSI SIS18 heavy ions injection.

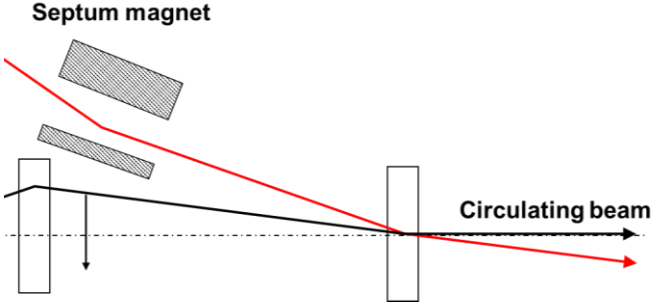
Multi-turn injection: normalized phase space

PSB (pre-LS2): horizontal injection with tune ~ 0.25

Turn 1



Septum



Multi-turn injection: normalized phase space

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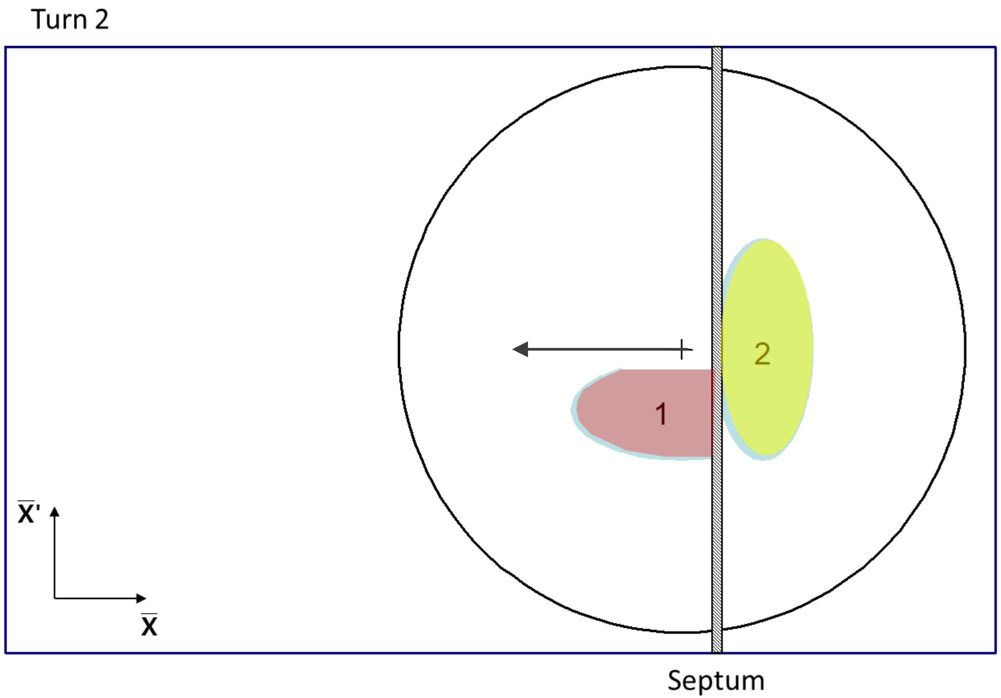
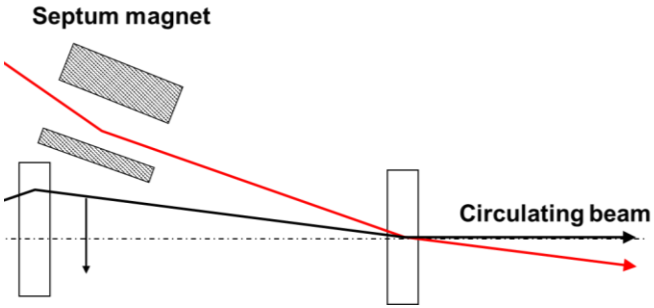
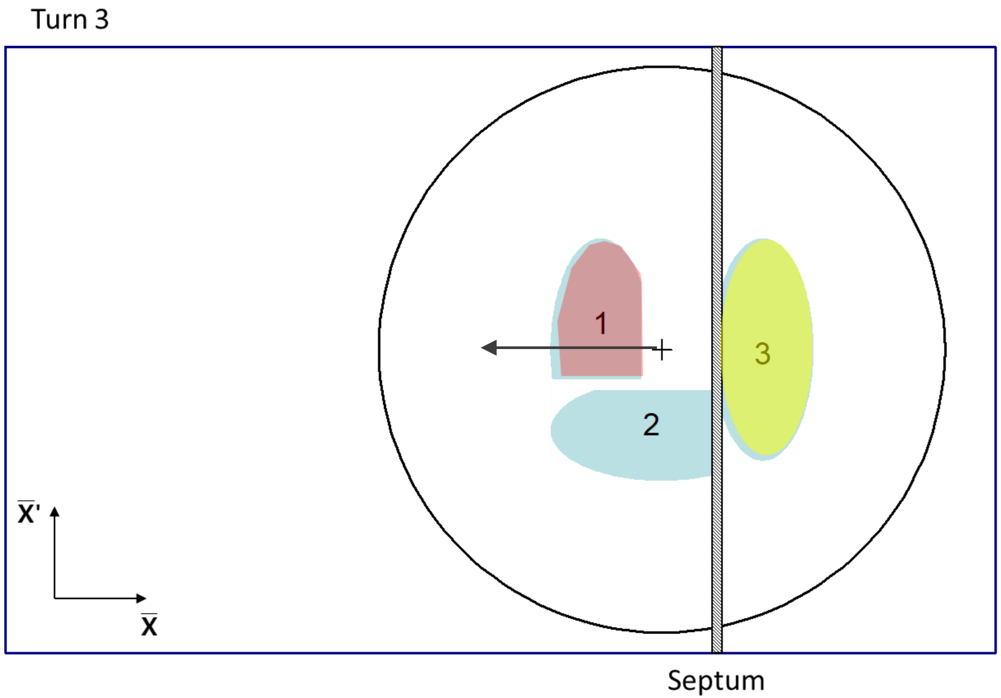
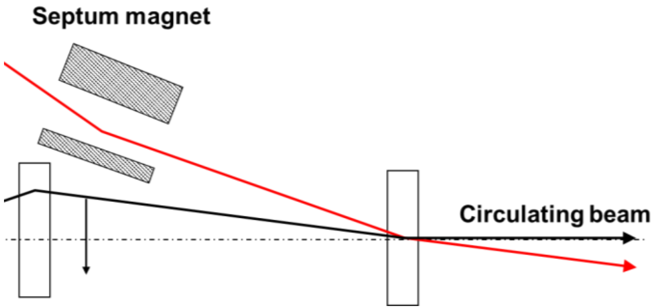




Multi-turn injection: normalized phase space

PSB (pre-LS2): horizontal injection with tune ~ 0.25

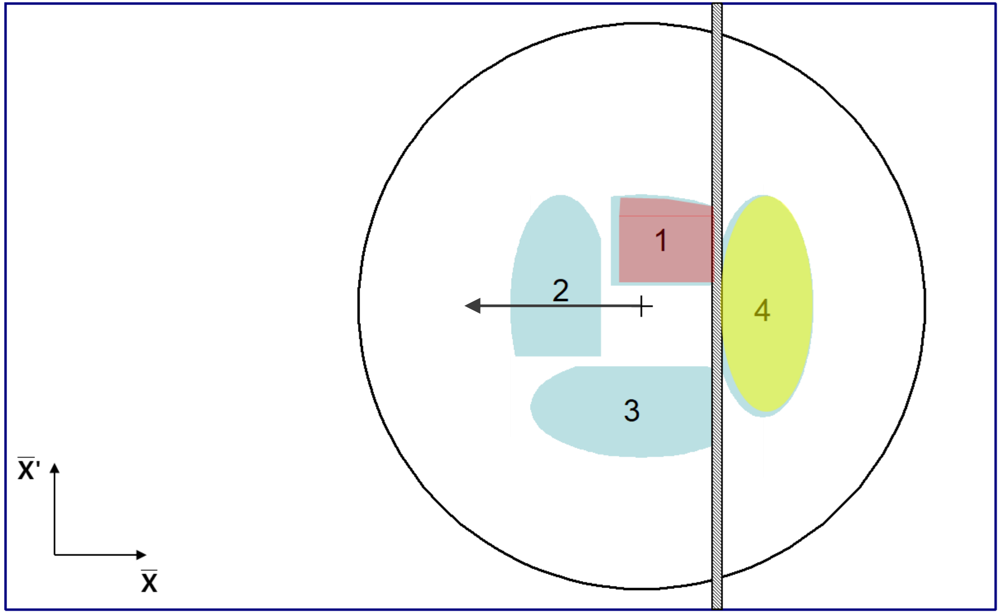


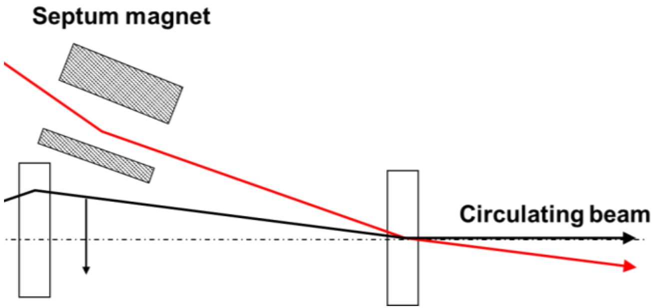


Multi-turn injection: normalized phase space

PSB (pre-LS2): horizontal injection with tune ~ 0.25

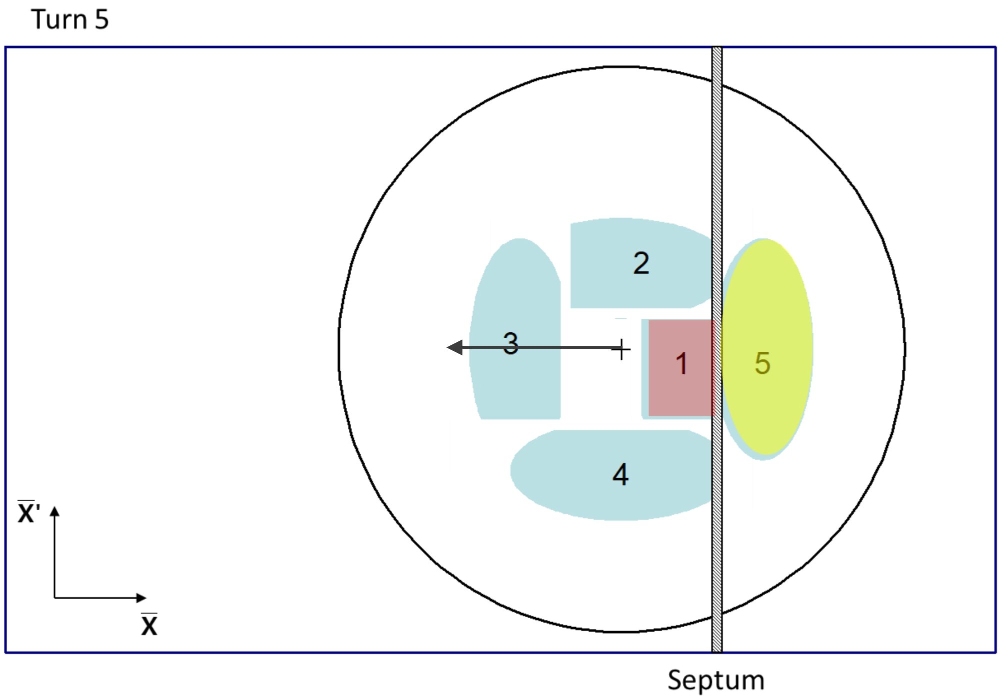
Turn 4

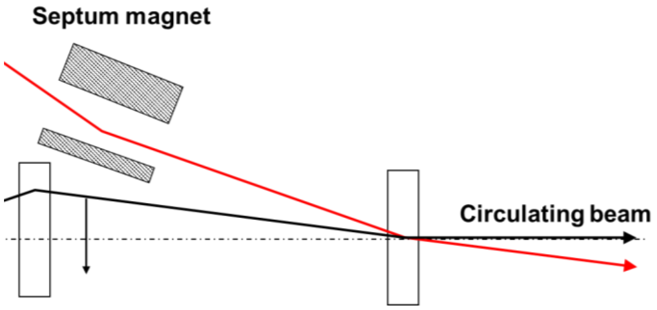




Multi-turn injection: normalized phase space

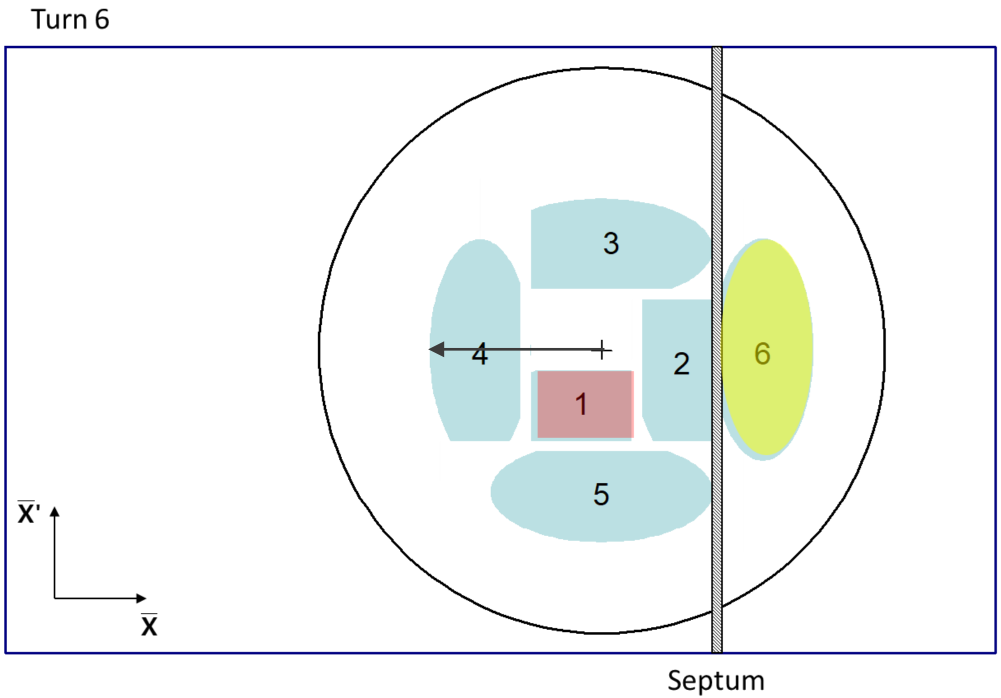
PSB (pre-LS2): horizontal injection with tune ~ 0.25

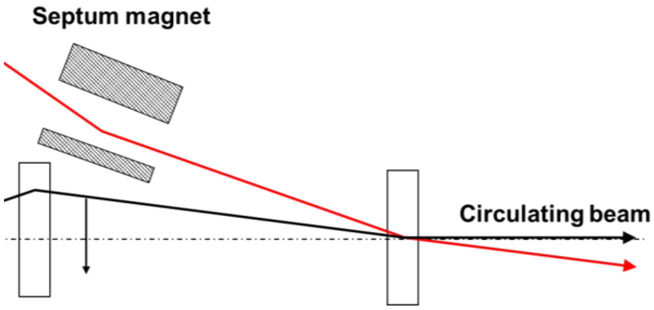




Multi-turn injection: normalized phase space

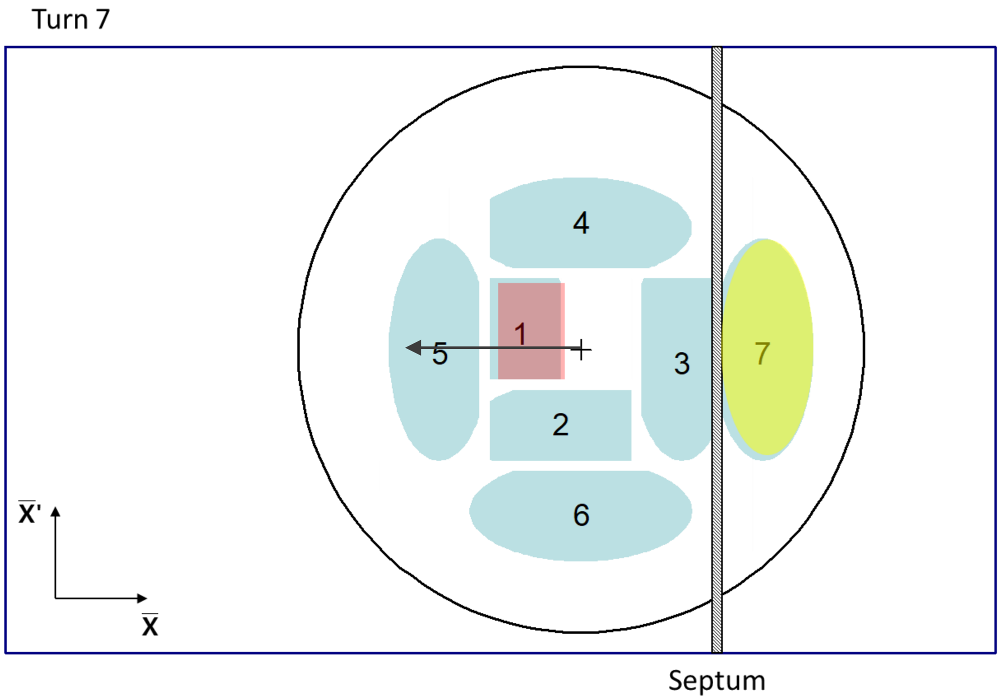
PSB (pre-LS2): horizontal injection with tune ~ 0.25

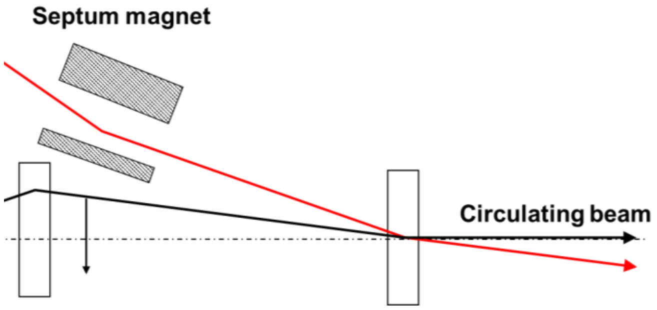




Multi-turn injection: normalized phase space

PSB (pre-LS2): horizontal injection with tune ~ 0.25





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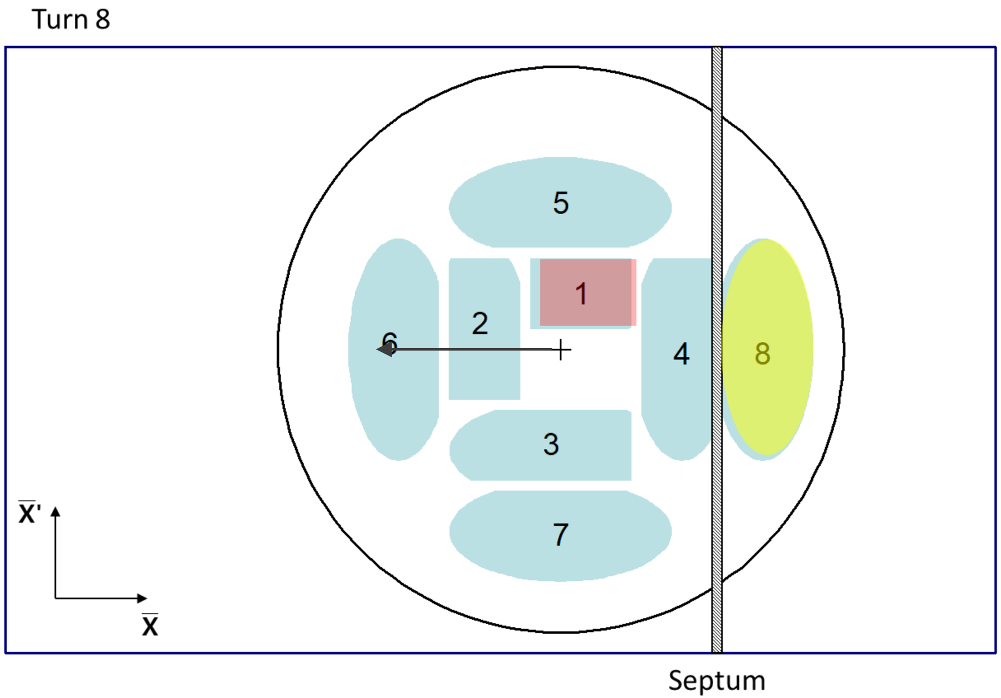
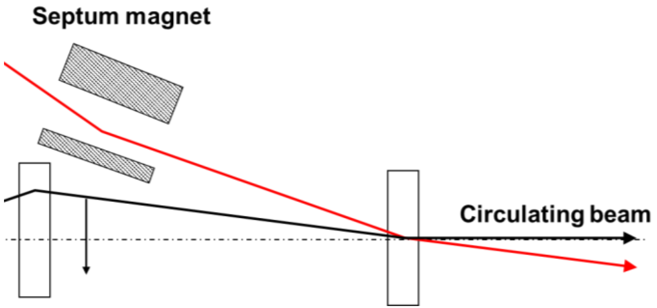
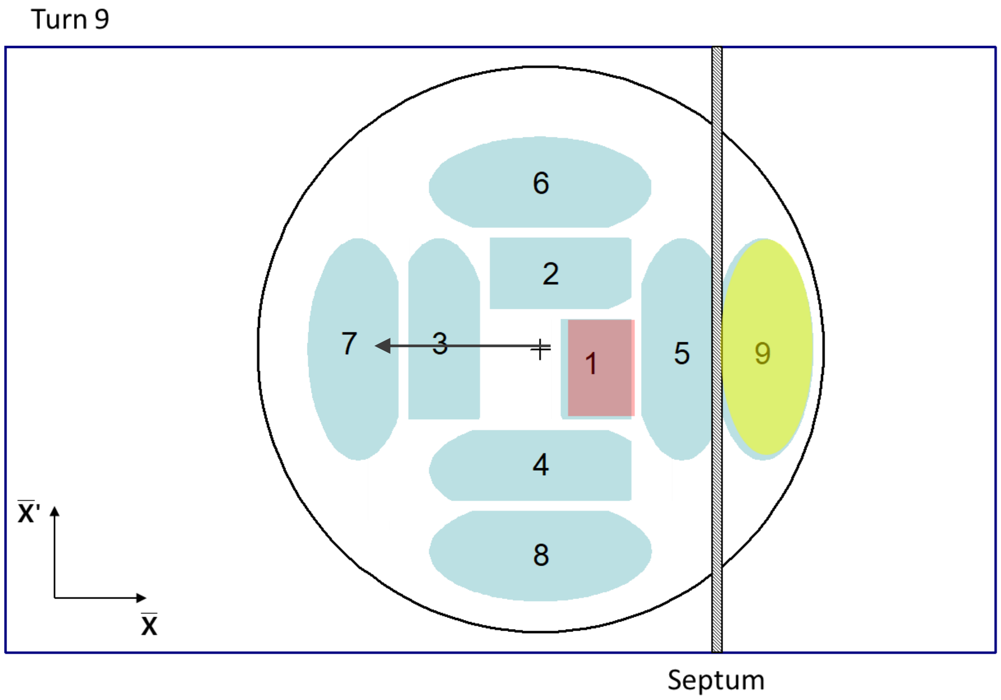
PSB (pre-LS2): horizontal injection with tune ~ 0.25

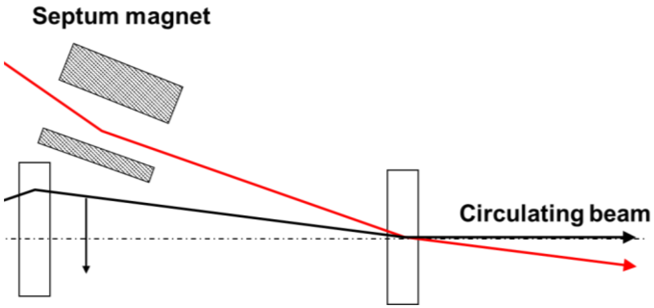




Multi-turn injection: normalized phase space

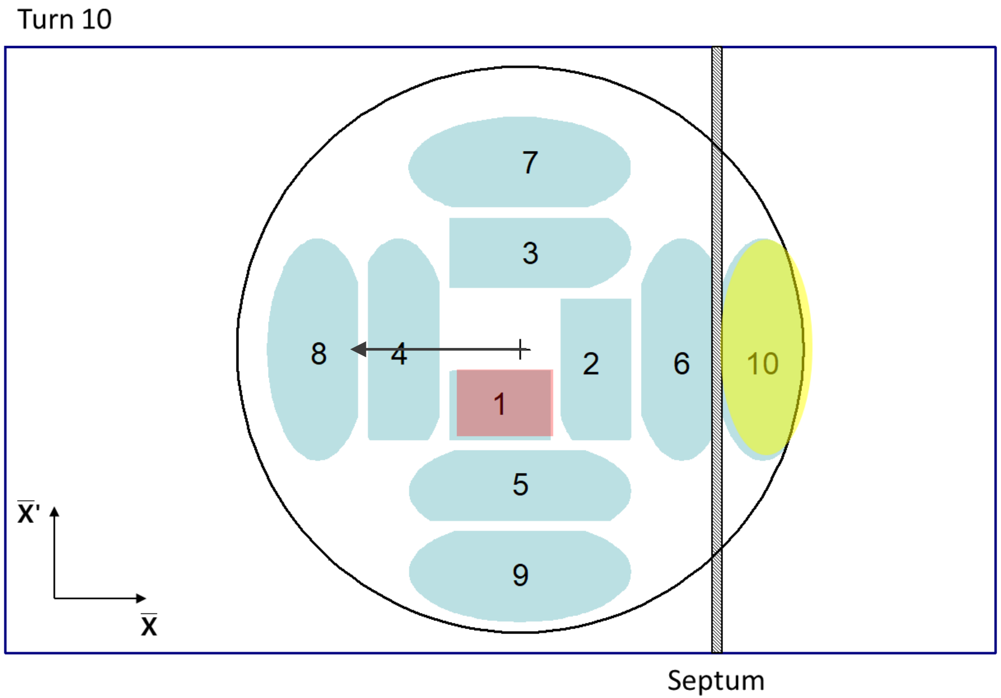
PSB (pre-LS2): horizontal injection with tune ~ 0.25

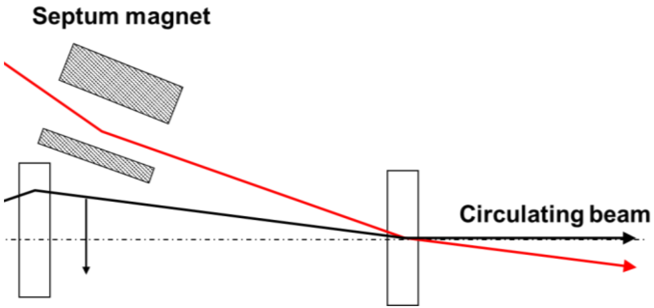




Multi-turn injection: normalized phase space

PSB (pre-LS2): horizontal injection with tune ~ 0.25

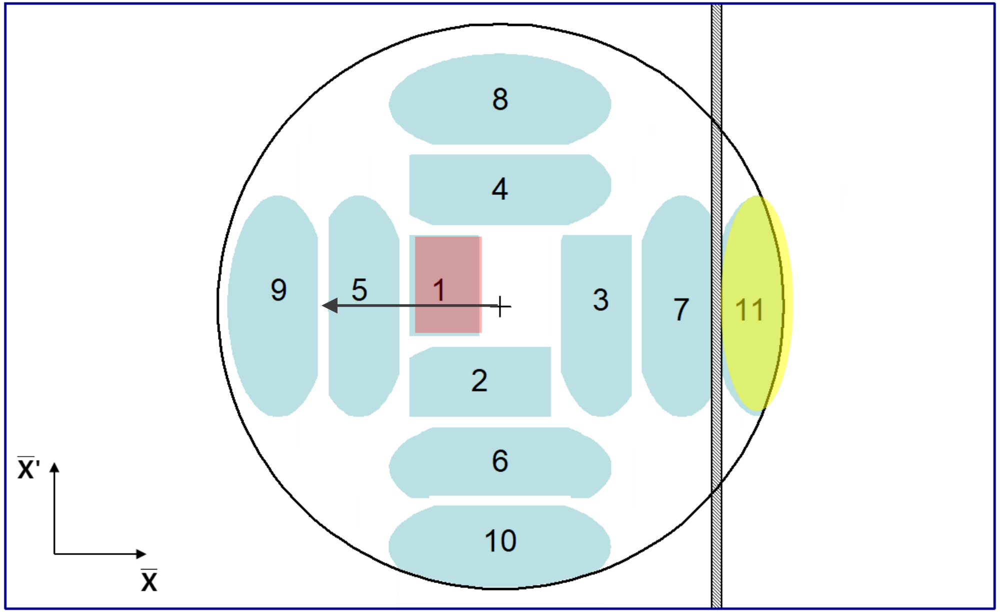




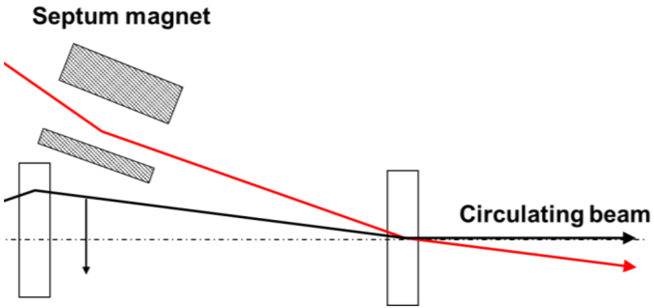
Multi-turn injection: normalized phase space

PSB (pre-LS2): horizontal injection with tune ~ 0.25

Turn 11

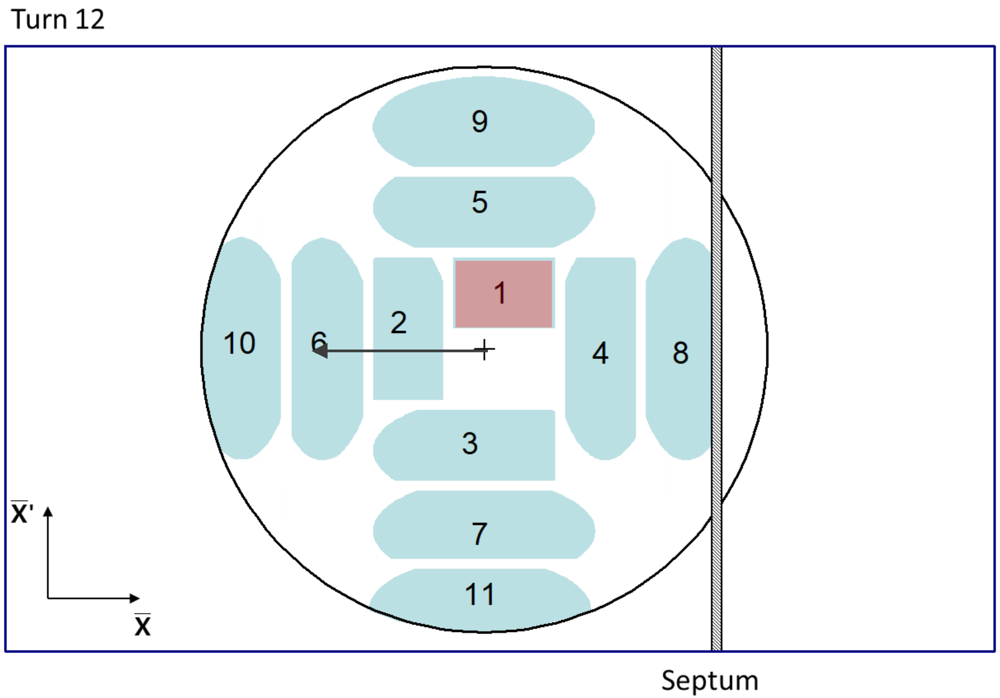


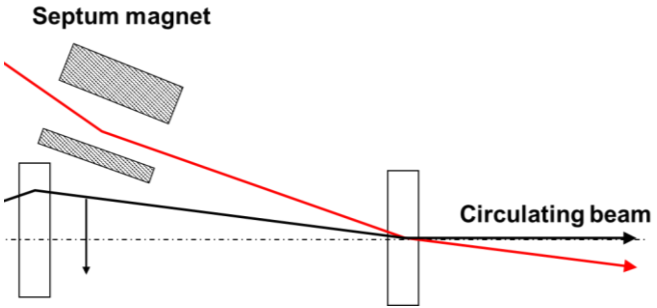
Septum



Multi-turn injection: normalized phase space

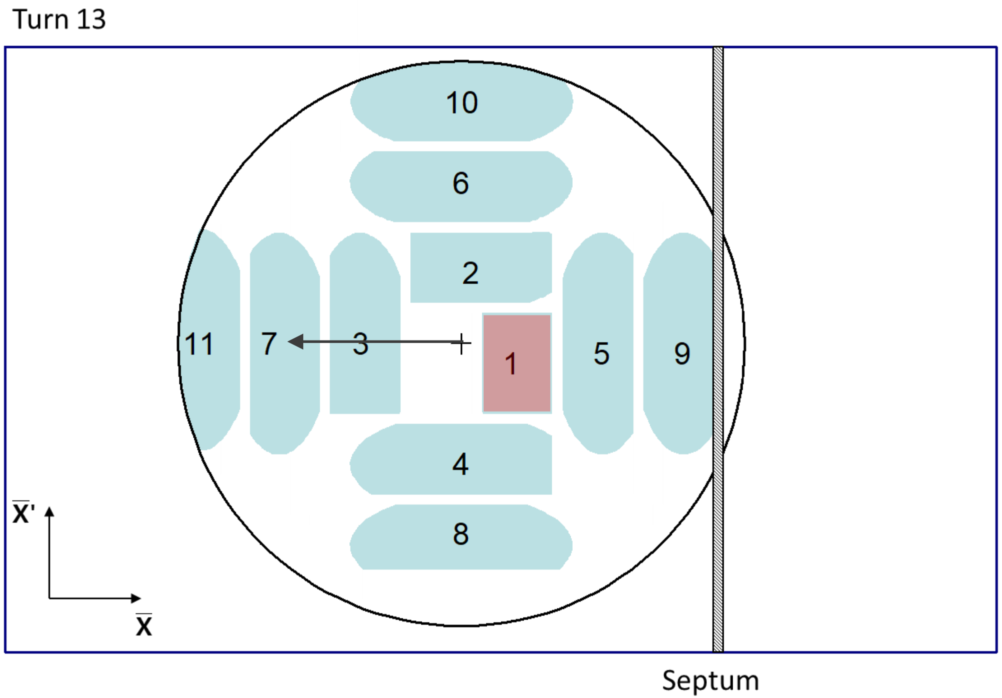
PSB (pre-LS2): horizontal injection with tune ~ 0.25

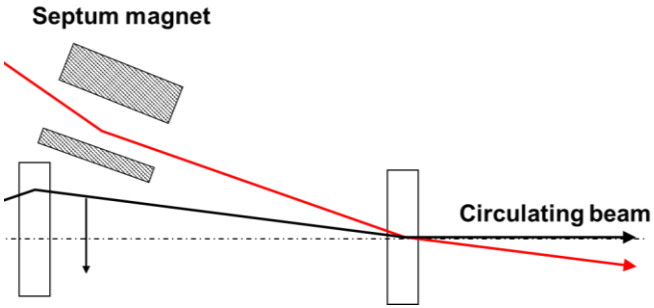




Multi-turn injection: normalized phase space

PSB (pre-LS2): horizontal injection with tune ~ 0.25





Multi-turn injection: normalized phase space

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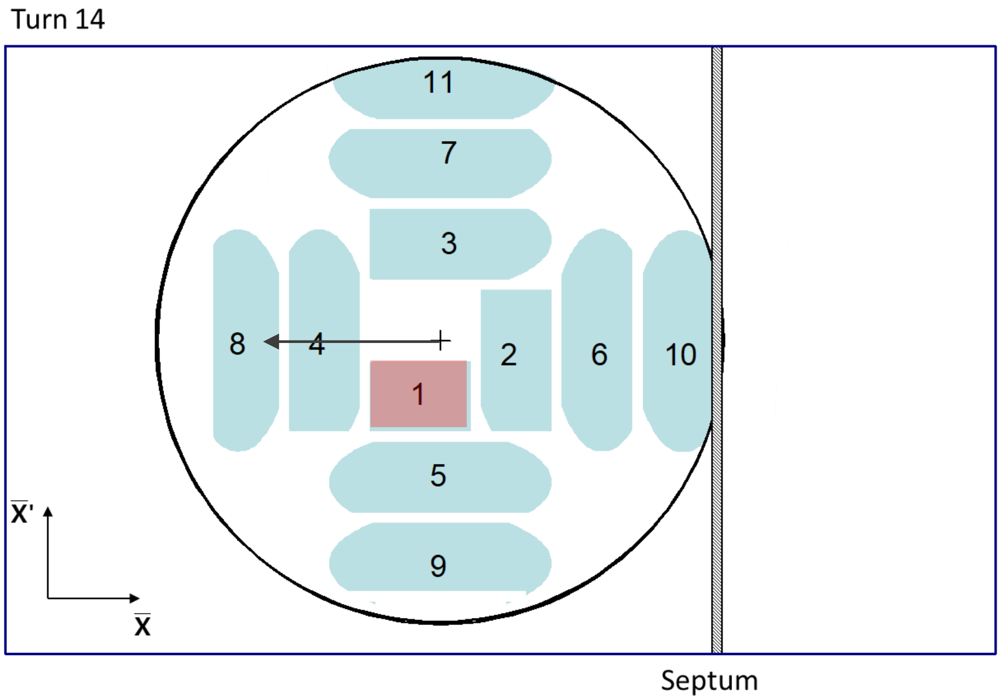
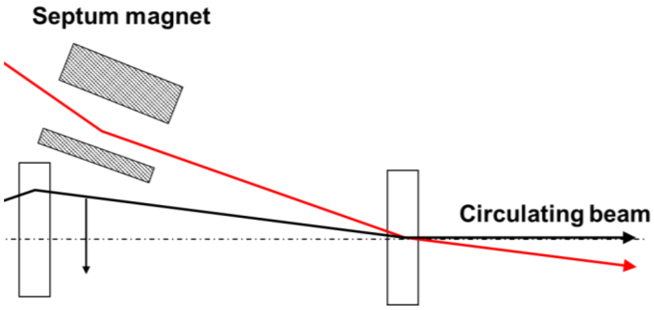
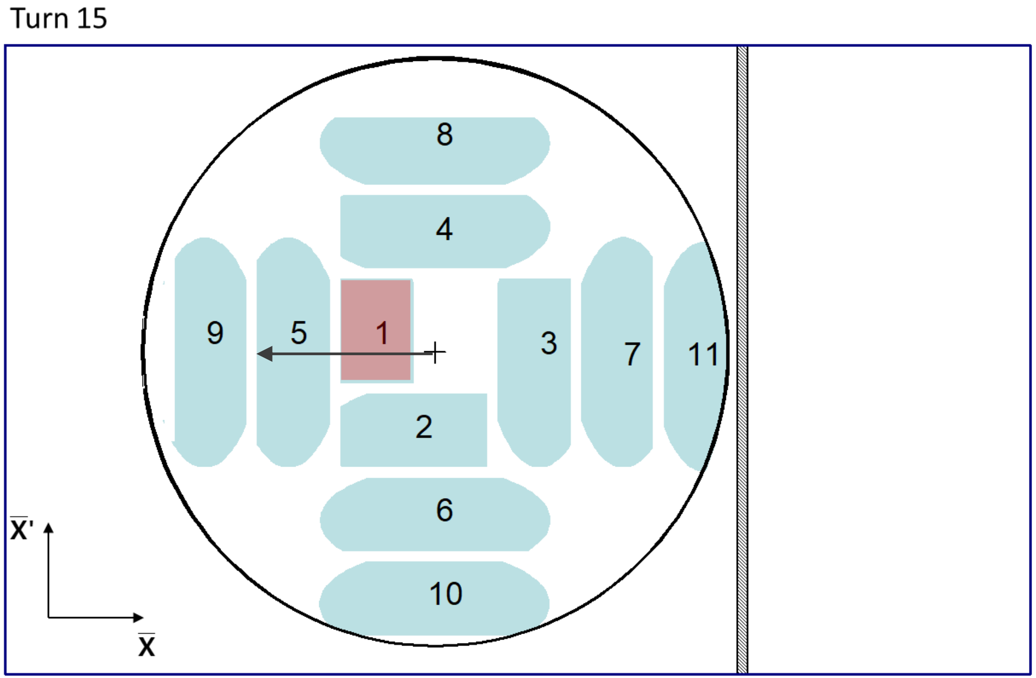
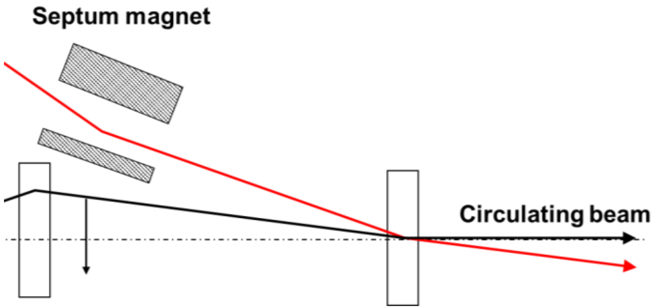




Multi-turn injection: normalized phase space

PSB (pre-LS2): horizontal injection with tune ~ 0.25

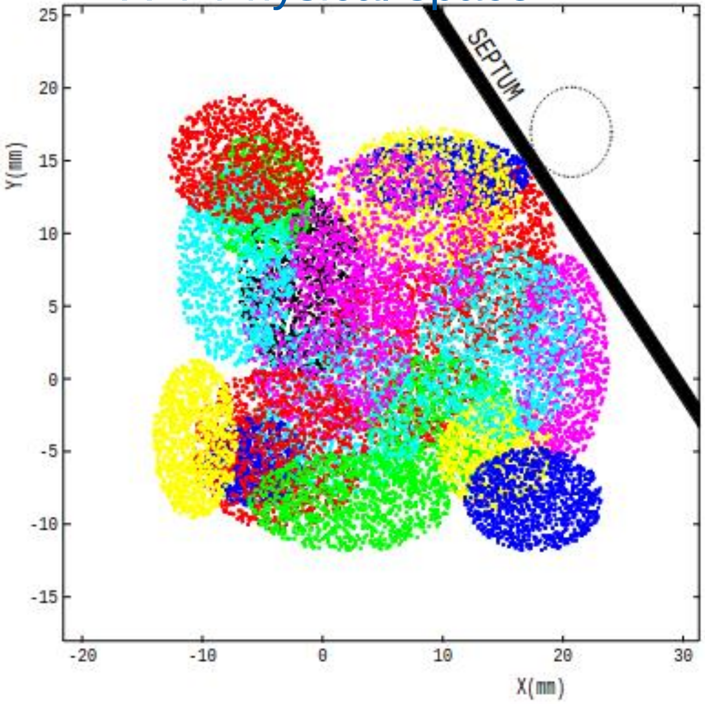




Example: Multi-turn injection in LEIR

- By tilting the septum, both horizontal and vertical planes can be filled.
- Challenges:
  - Very complicated optimization processes.
  - Inevitably lossy at injection septum.
  - LEIR efficiency ~70%.
- Limitations:
  - Phase space density must be conserved, so resulting beam will inevitably be much larger than injected beam.

X-Y Physical space



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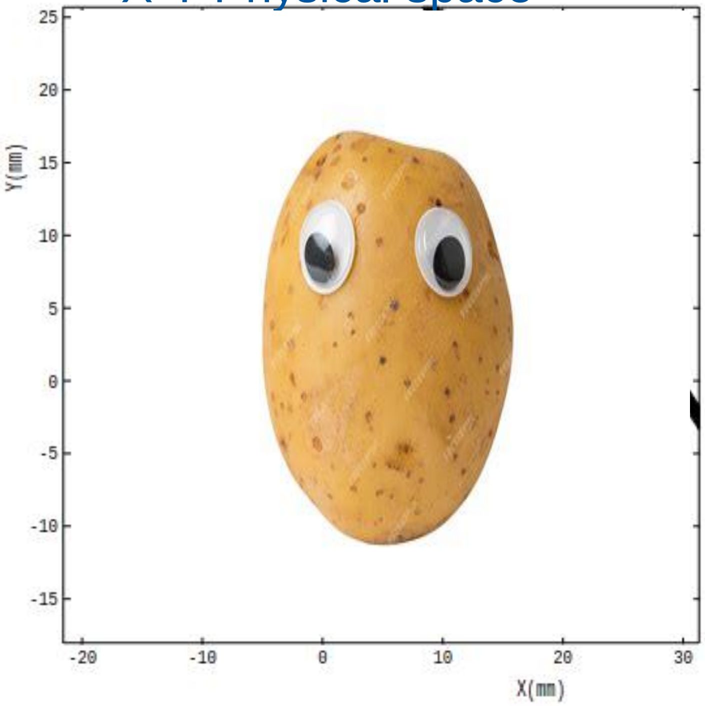

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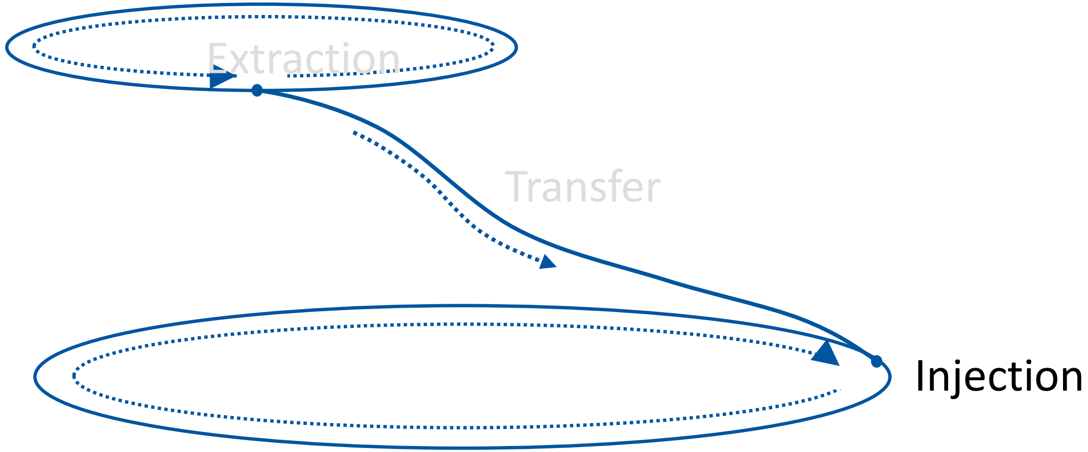
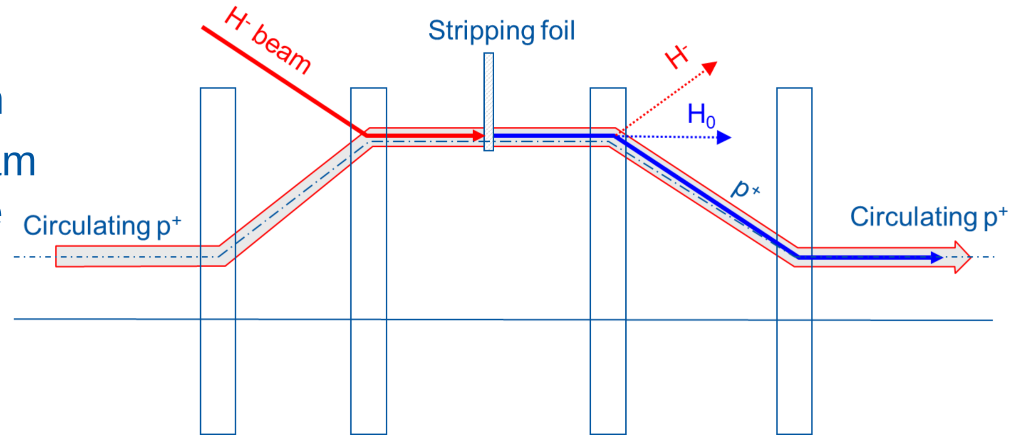


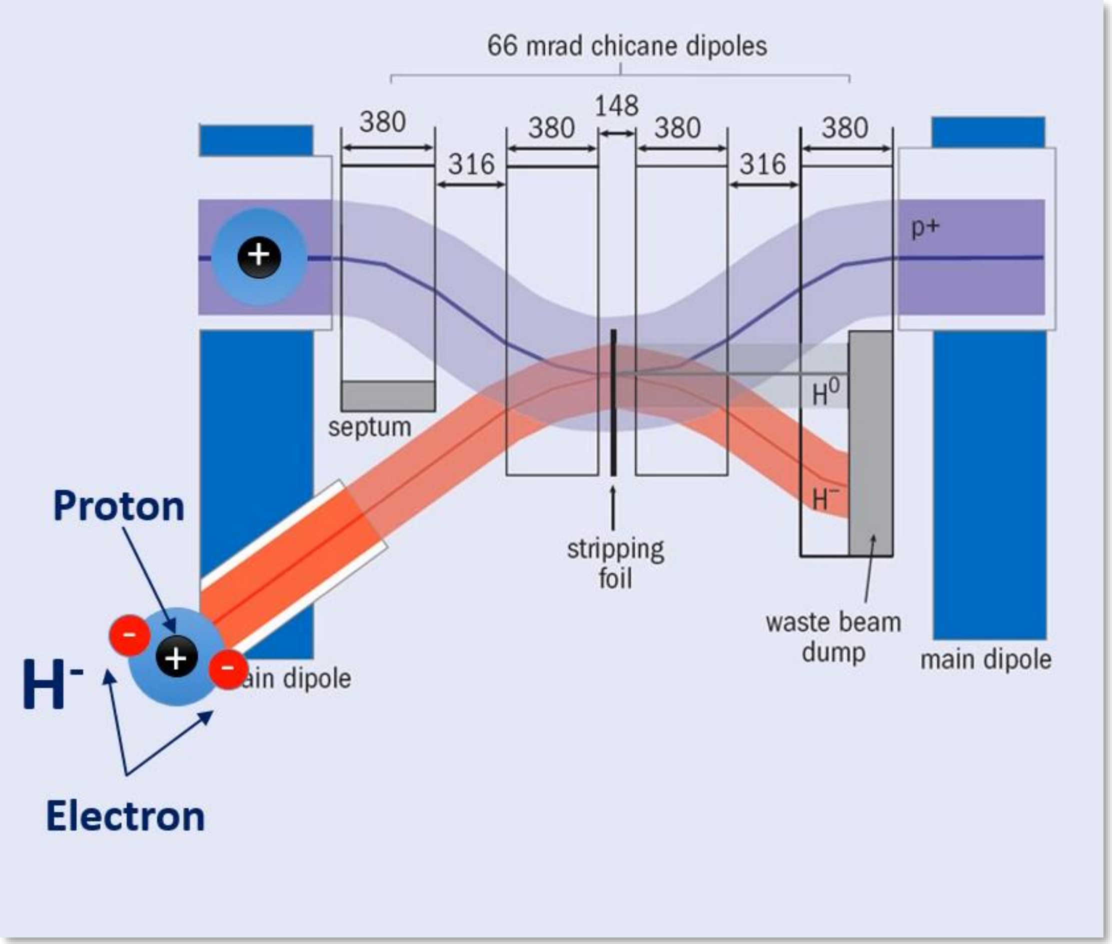
Charge exchange: concept

Phase space density must be conserved **under conservative forces**

- Principle
  - Injected beam changes charge states through stripping in the injection region
  - Since both injected and circulating beam can cross the same space at the same time, longer injection times and higher brightness are reachable



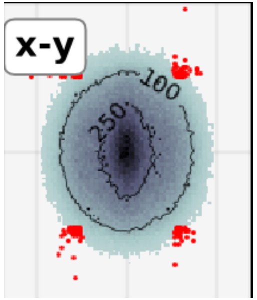
- Applications
  - At CERN PSB since 2020
  - At BNL AGS-Booster since the 1992

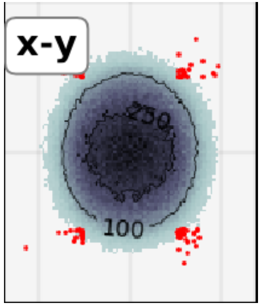


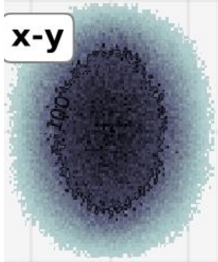
E. Renner, CERN proton booster (PSB), SY-ABT-BTP (2018)

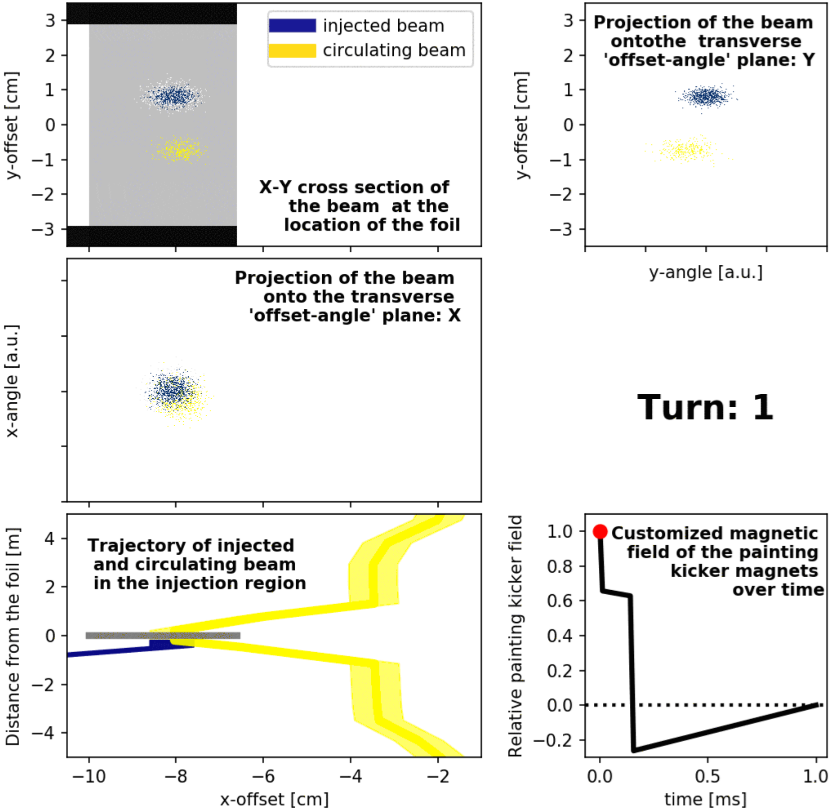
Charge exchange: painting

- By carefully controlling the bumper settings, different phase space paitings can be produced.









E. Renner

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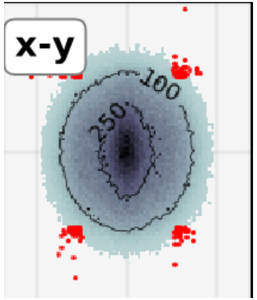

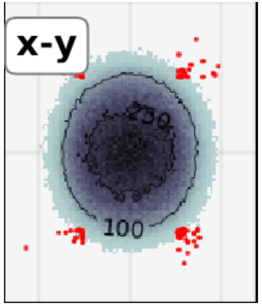

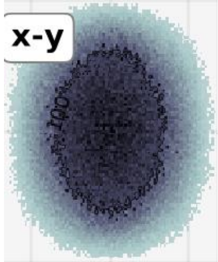

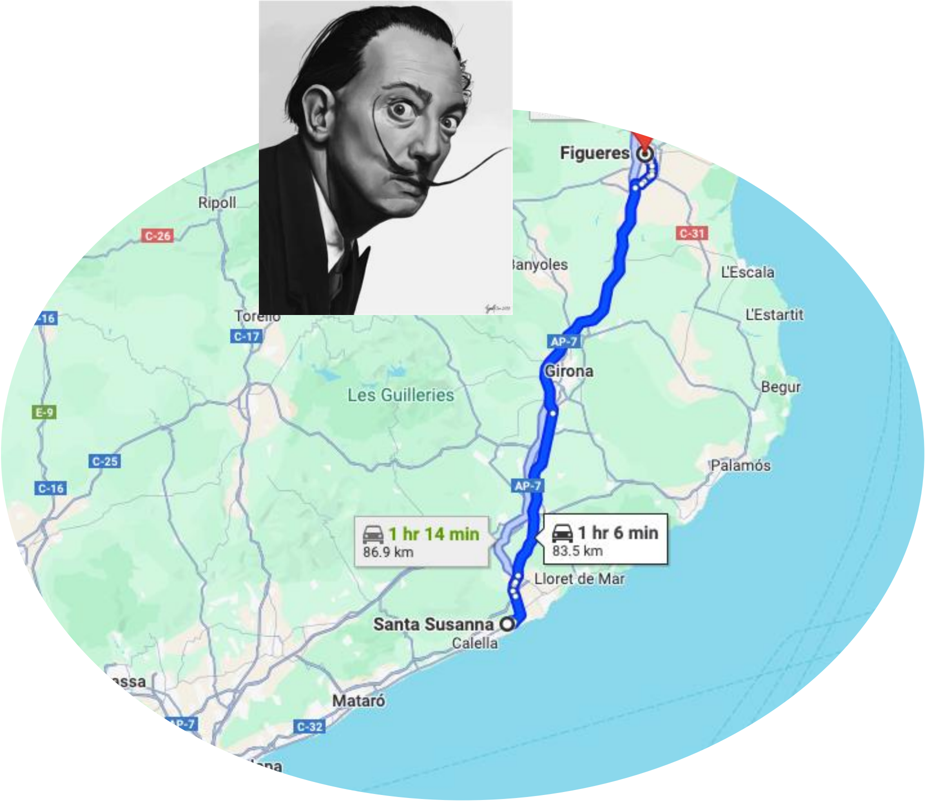

E. Renner

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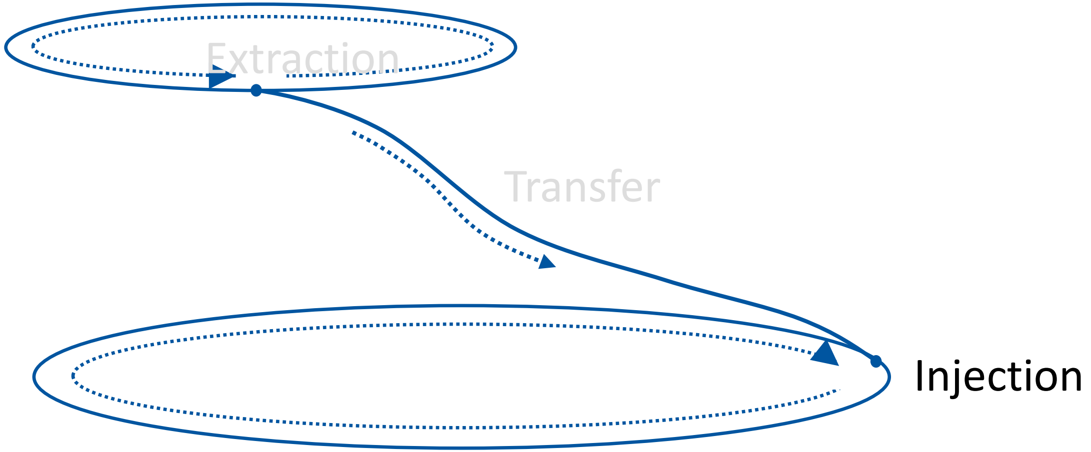
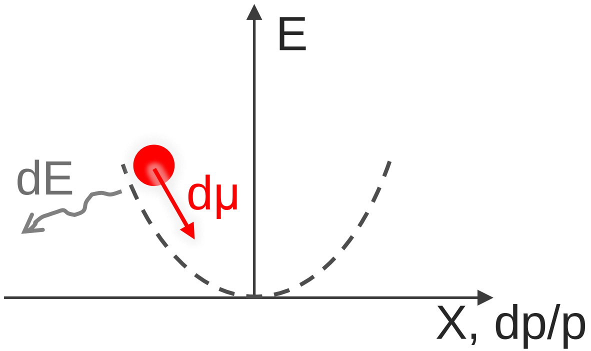


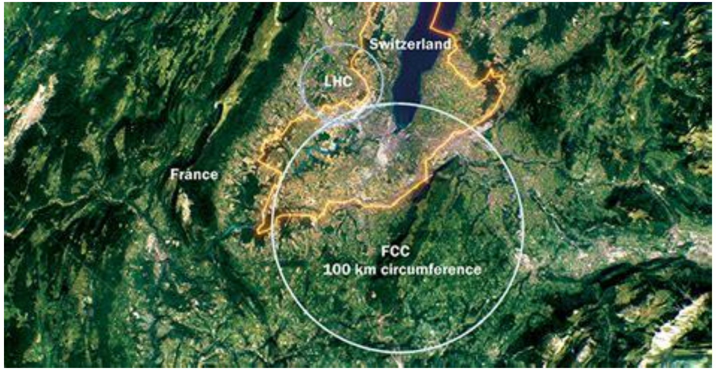
Lepton injection: concept

Phase space density must be conserved **under conservative forces**

- Principle
  - In a synchrontron-radiation dominated lepton ring one can take advantage of the fast damping of oscillations (transverse and/or longitudinal).
  - Possible to perform “top-up” injection for continuous operation.

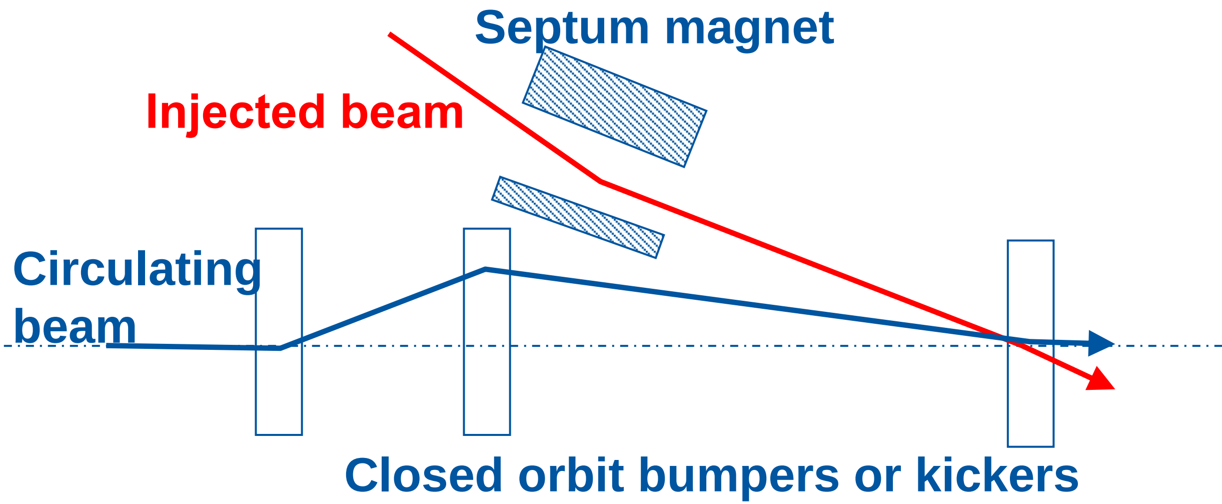


- Applications
  - At CERN LEP injection
  - In most synchrotron light sources
  - In FCCee

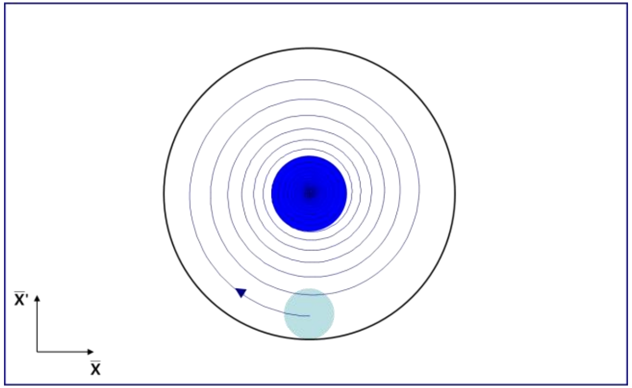


Lepton injection: off-axis

- Beam is injected with an angle with respect to the closed orbit.



- Injected beam performs damped betatron oscillations about the closed orbit.



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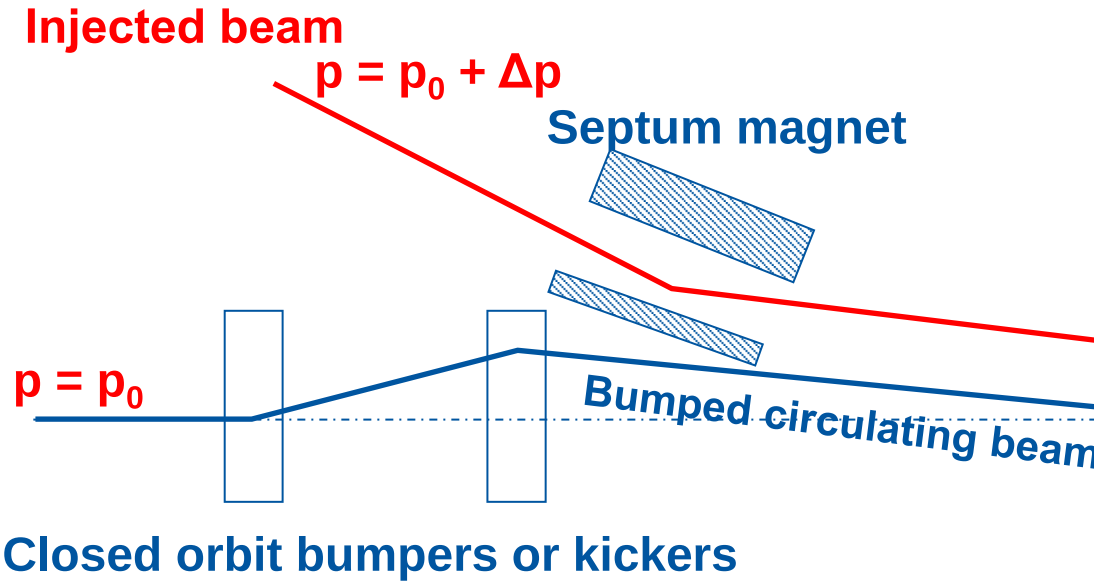

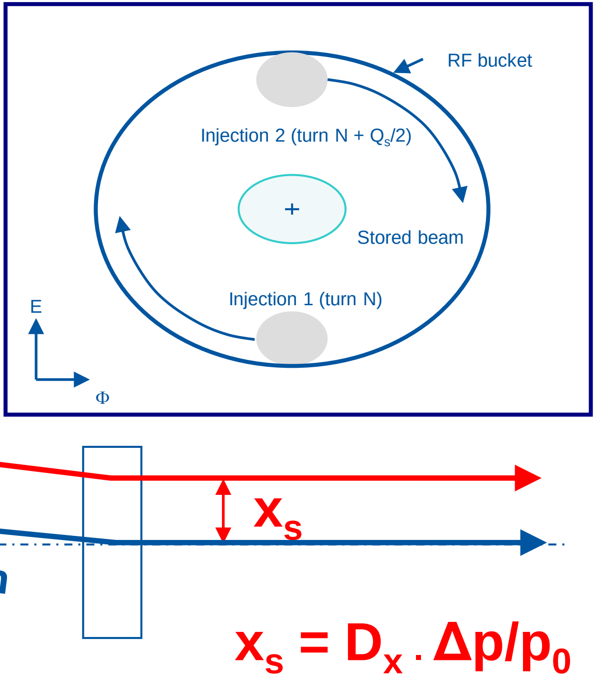

- Beam injected parallel to circulating beam, onto dispersive orbit of offset Δp/p.
- Injected beam makes damped synchrotron oscillations (2x faster damping than transverse).
- No betatron oscillations -> No offset at locations with Dx=0, e.g. interaction points.

Injection techniques : Summary

- Several different techniques using kickers, septa and bumpers:
  - Single-turn injection for hadrons
    - Boxcar stacking: transfer between machines in accelerator chain
    - Angle / position errors $\Rightarrow$ injection oscillations
    - Uncorrected errors $\Rightarrow$ filamentation $\Rightarrow$ emittance increase
  - Multi-turn injection for hadrons
    - Phase space painting to increase intensity
    - H- injection allows injection into same phase space area
  - Lepton injection
    - May take advantage of SR damping
    - Injection errors translates into lower efficiency

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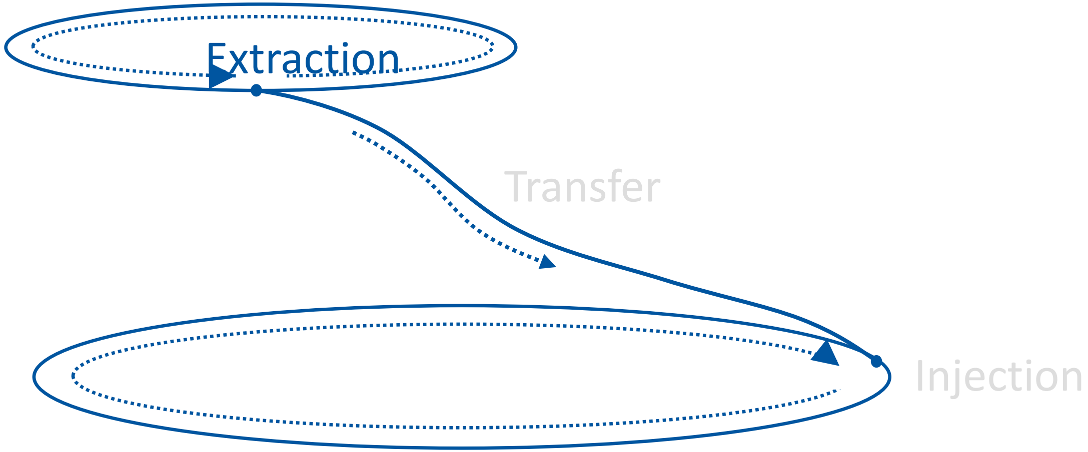
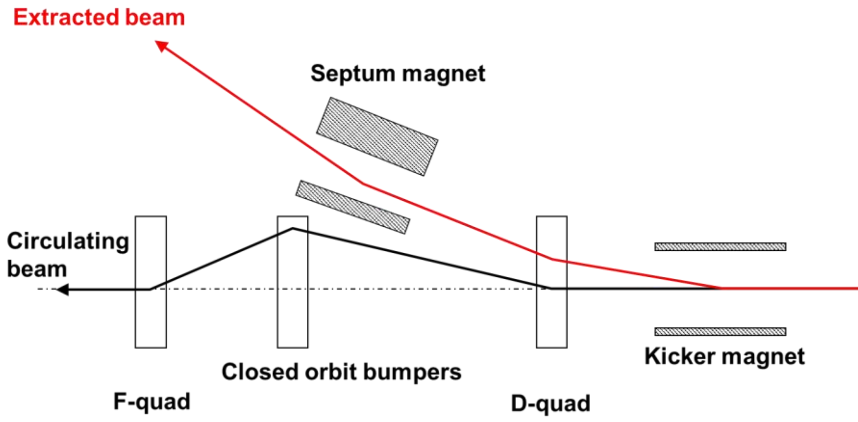
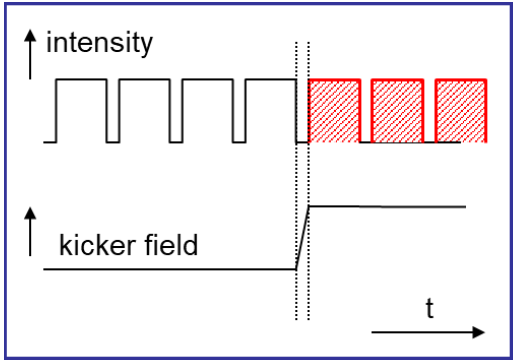


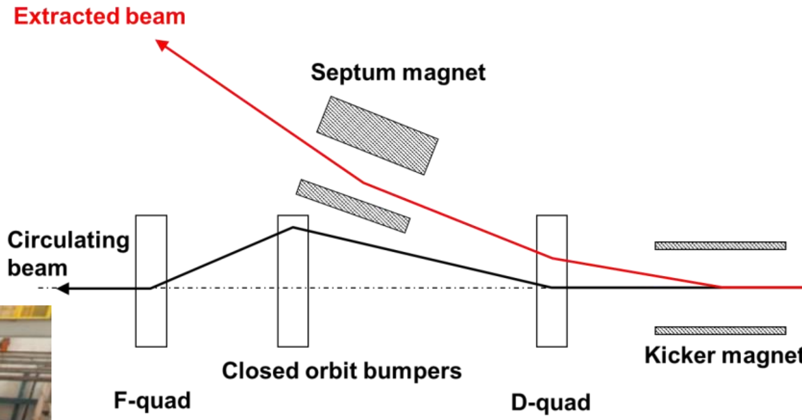
Fast extraction: concept

- Principle
  - Mirror of the fast injection technique.
  - Circulating beam is moved close to the septum magnet.
  - A fast kicker magnet imparts a final deflection to channel the beam towards the septum aperture and the extraction line.
- Challenges
  - Higher energies require stronger elements than for injection.
- Applications
  - At CERN PSB, SPS, AD and more.
  - At many other synchrotrons around the world.

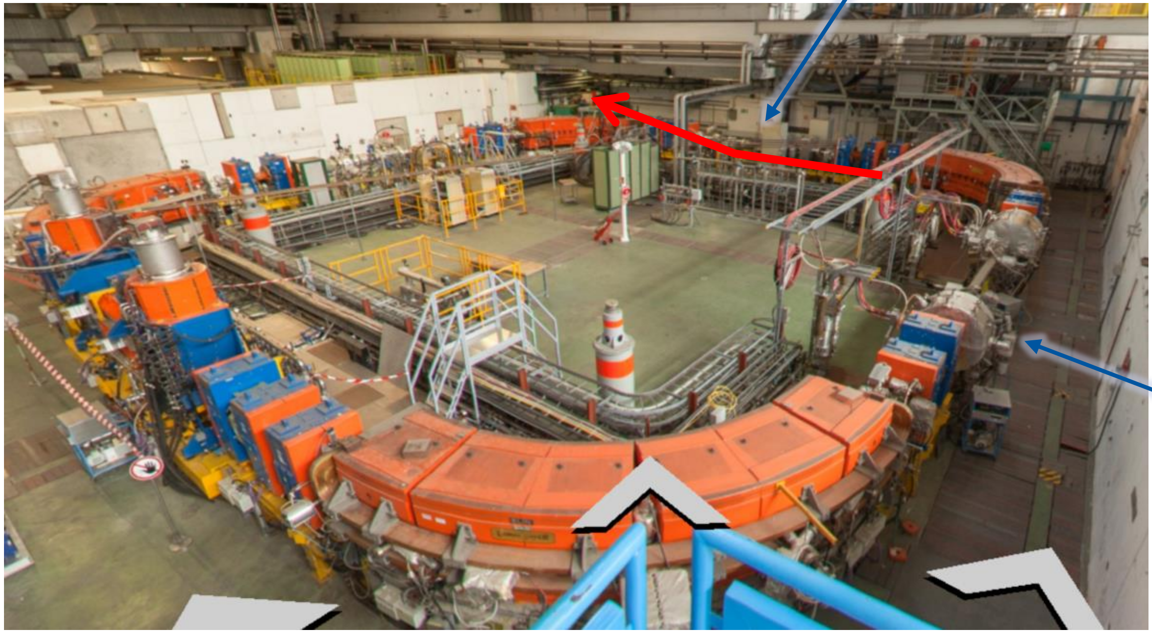




Example: LEIR extraction



Septum



Kicker

Gis link and panoramic

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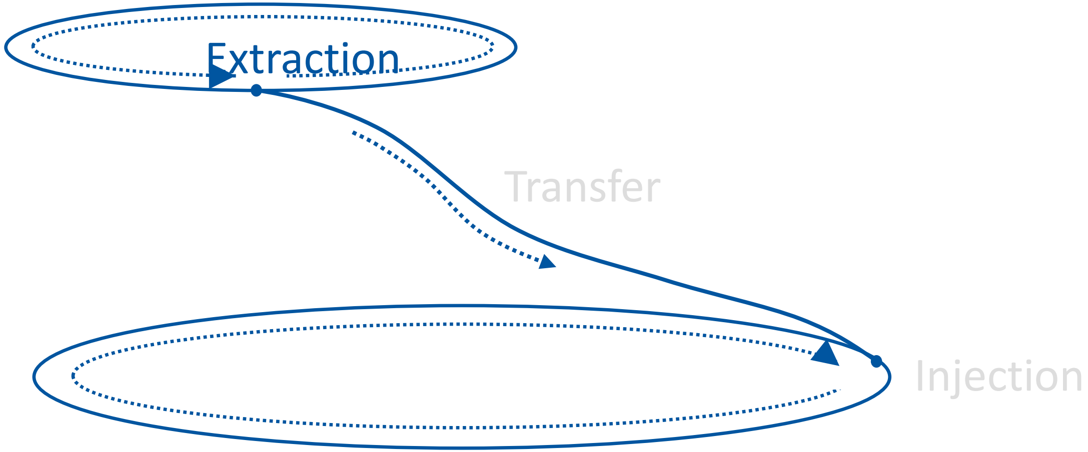
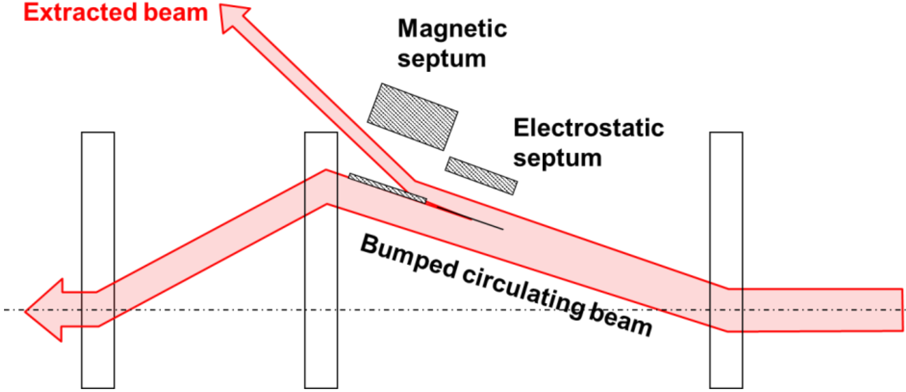


Multi-turn extraction: concept

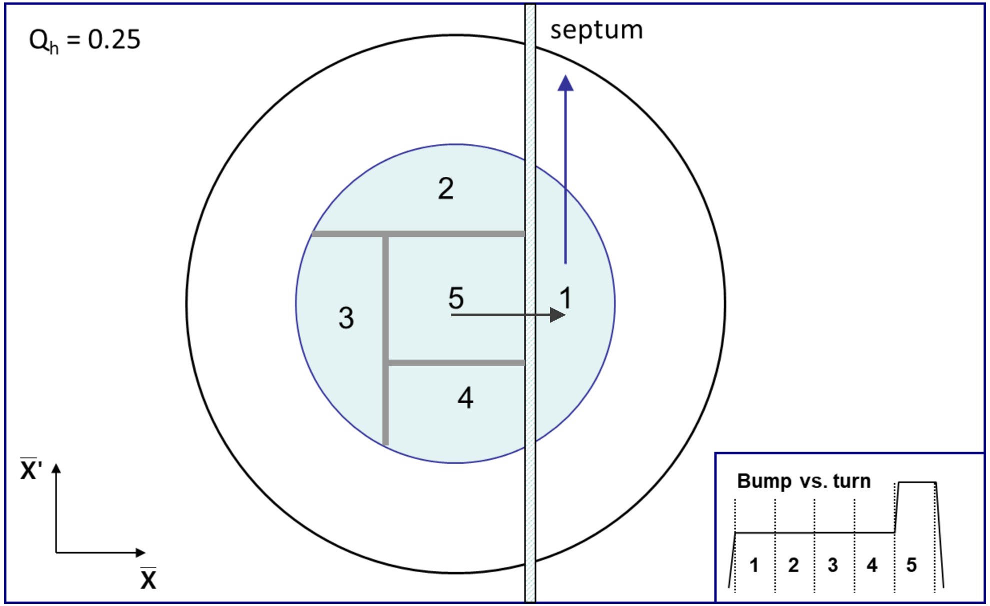
- Principle
  - Somewhat mirrors the principle of multi-turn injection.
  - Circulating beam is brought close to an electrostatic septum and partially pushed through using fast kicker.
  - The beam is shaved over a few turns .

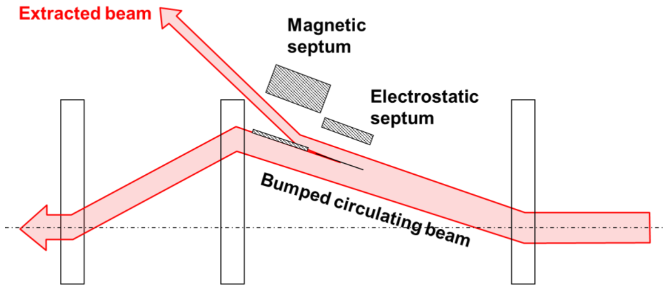


- Applications
  - At CERN PS until 2015

Multi-turn extraction: normalized phase space

- CERN PS to SPS: 5-turn continuous transfer





Multi-turn extraction: normalized phase space

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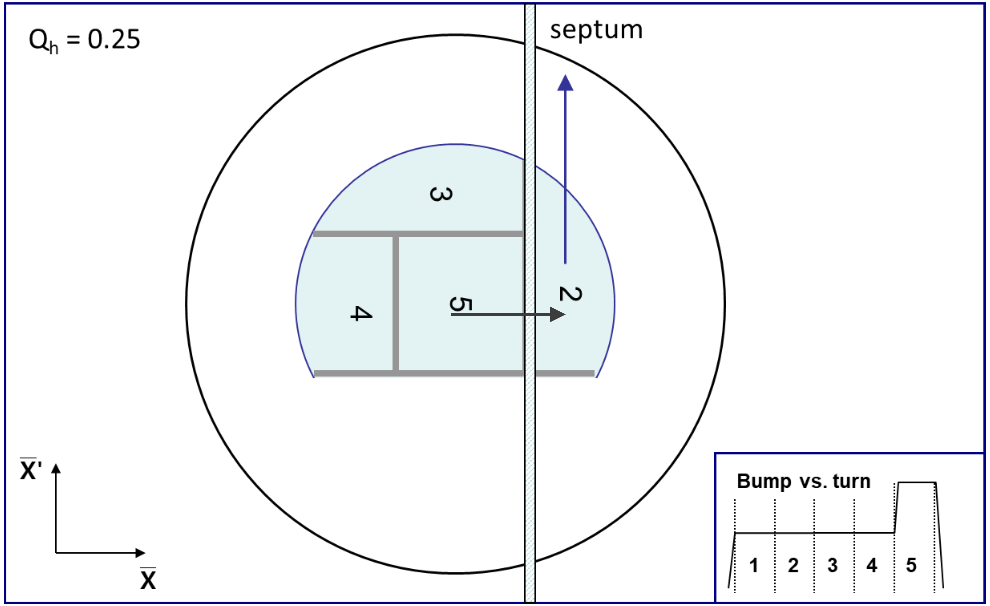

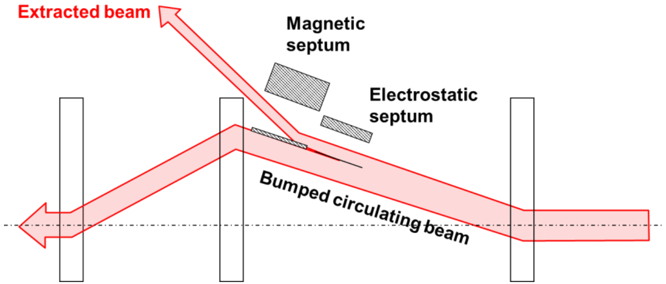

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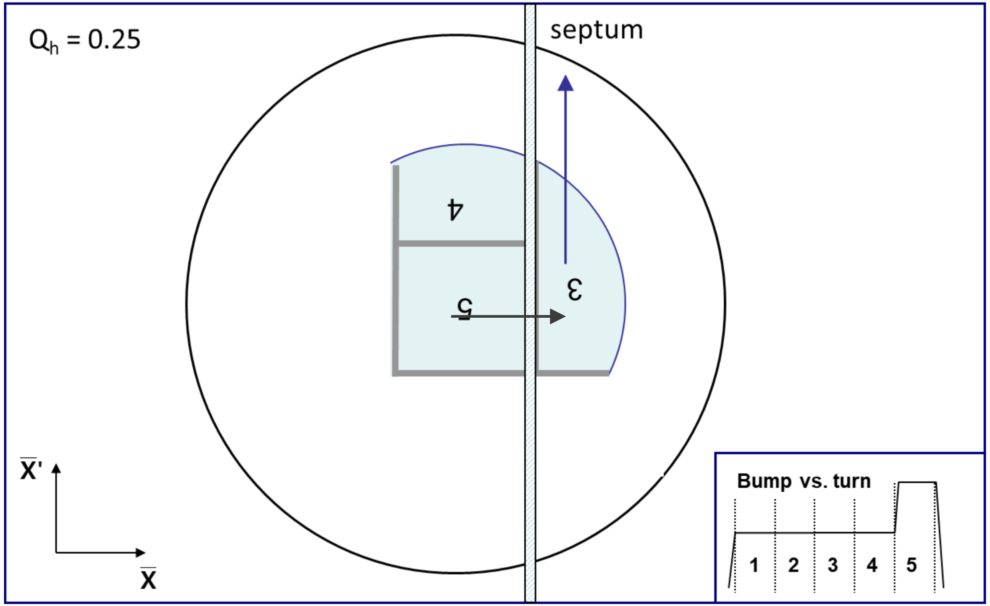

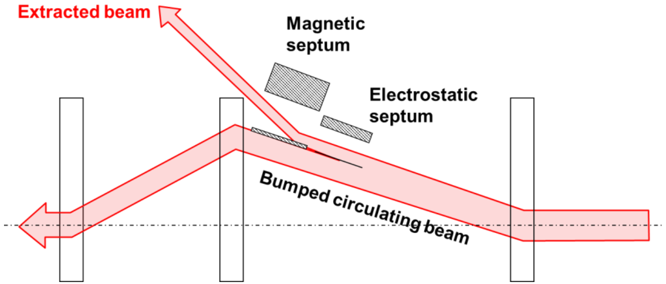

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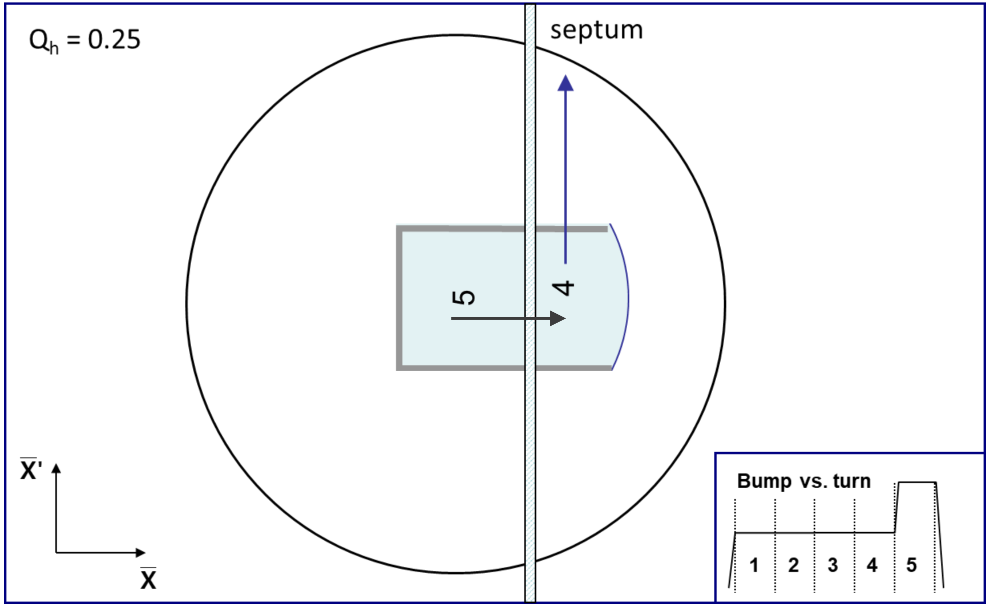

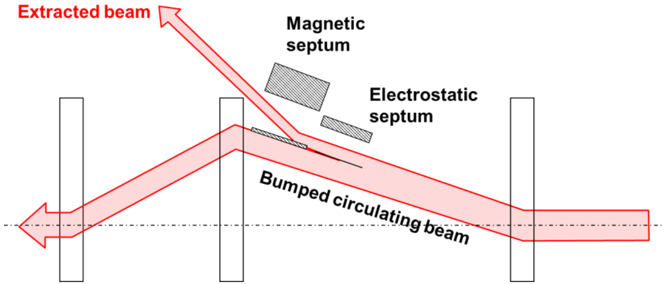

Multi-turn extraction: normalized phase space

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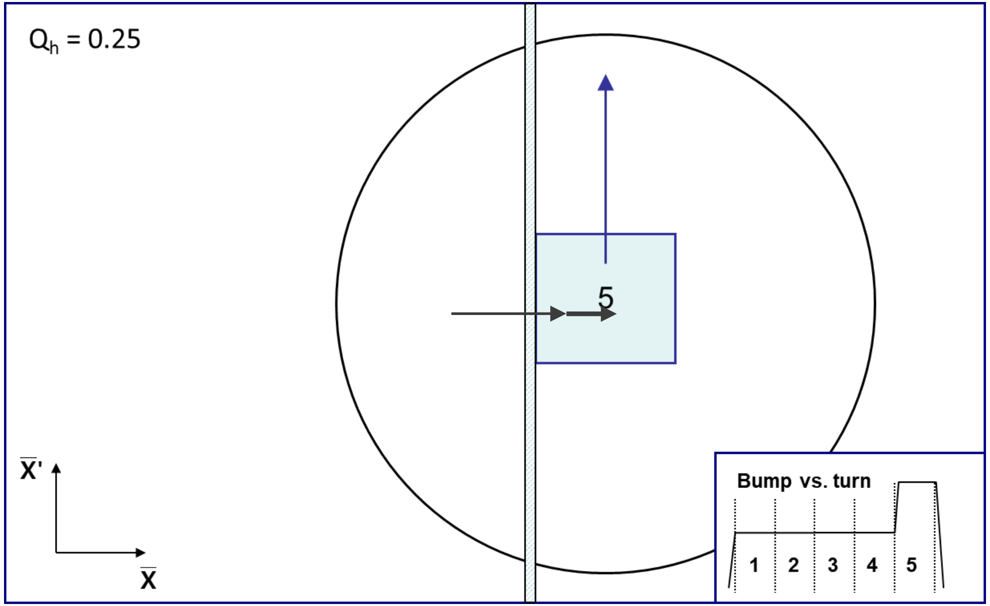

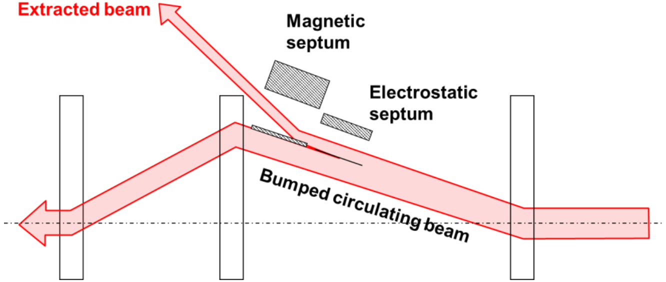

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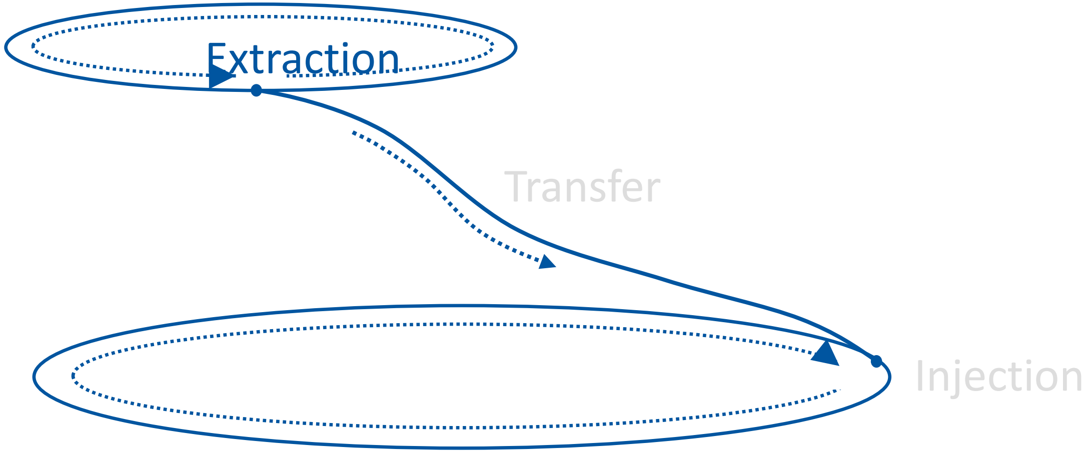
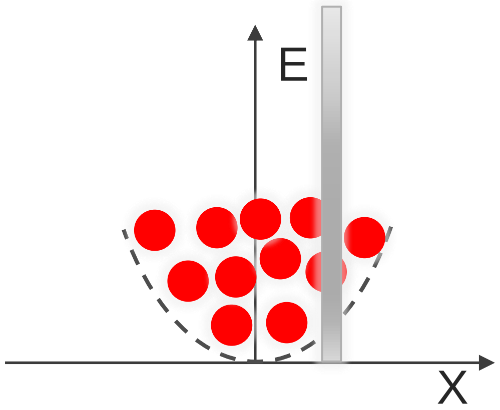
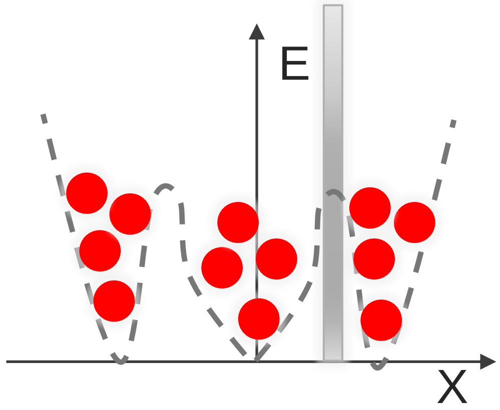


Resonant multi-turn extraction: concept

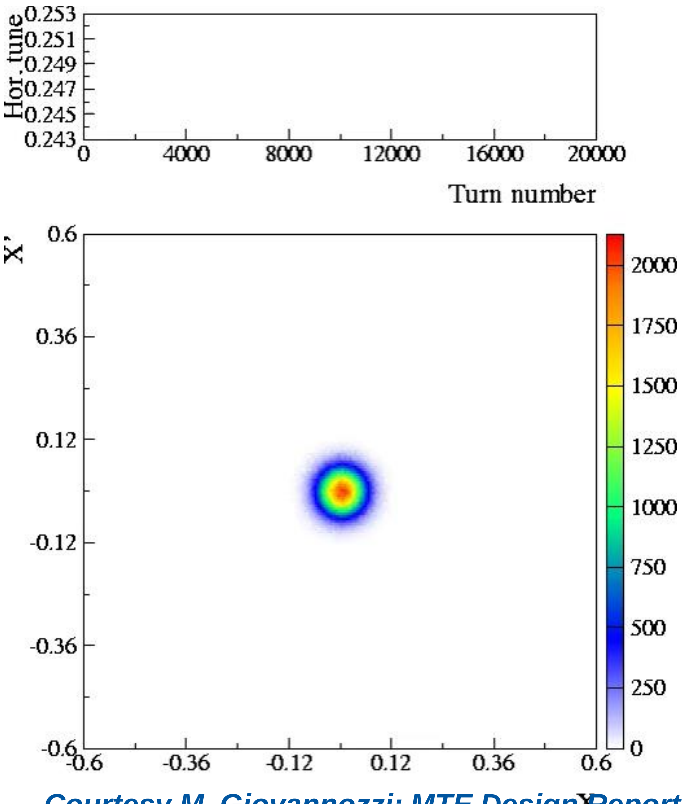
- Challenges from non-resonant multi-turn extraction
  - The electrostatic septum is used for its thin blade (down to a few tens of um), but limited effect on high energy beams.
  - This scheme is intrinsically lossy and will create activation around the extraction elements, and in particular the electrostatic septum.
  - Large inefficiency, ~15% losses.



- Principle of stable "islands"
  - Use non-linear fields (sextupoles and octupoles) to create islands of stability in phase space.
  - A slow crossing of a transverse resonance drives particles into the islands (capture).



Resonant MTE: normalized phase space



Courtesy M. Giovannozzi: MTE Design Report, CERN-2006-011, 2006

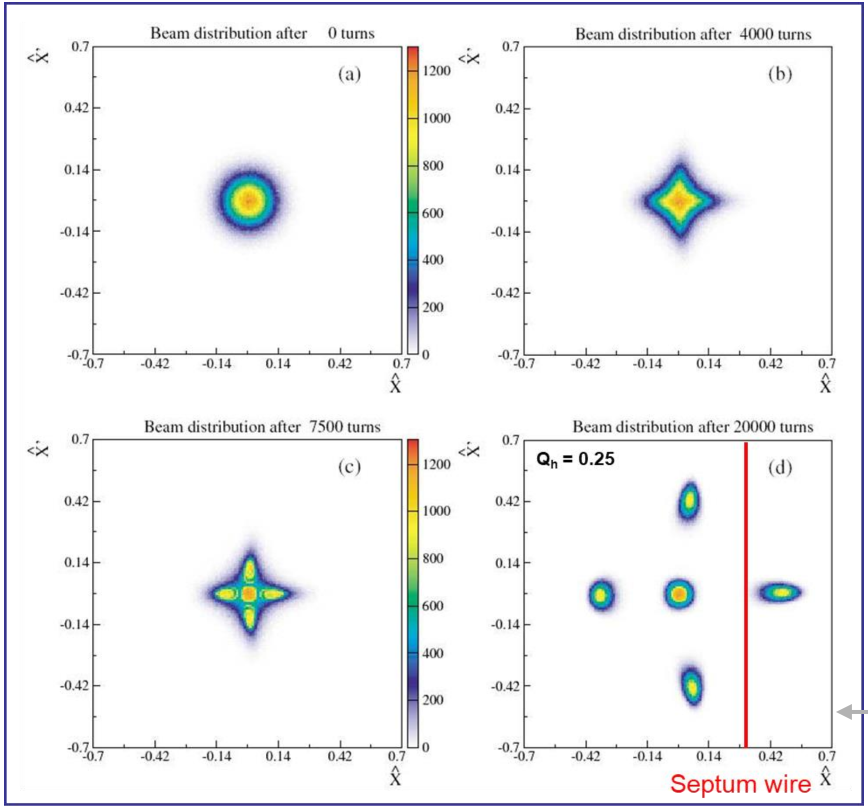
a. Unperturbed beam

b. Increasing non-linear fields

a. Beam captured in stable islands

b. Islands separated and beam bumped across septum – extracted in 5 turns

Resonant MTE: normalized phase space



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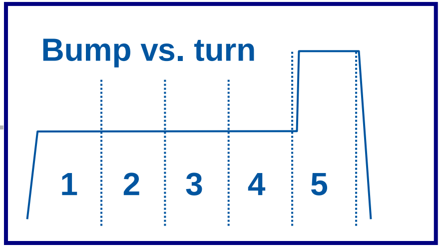

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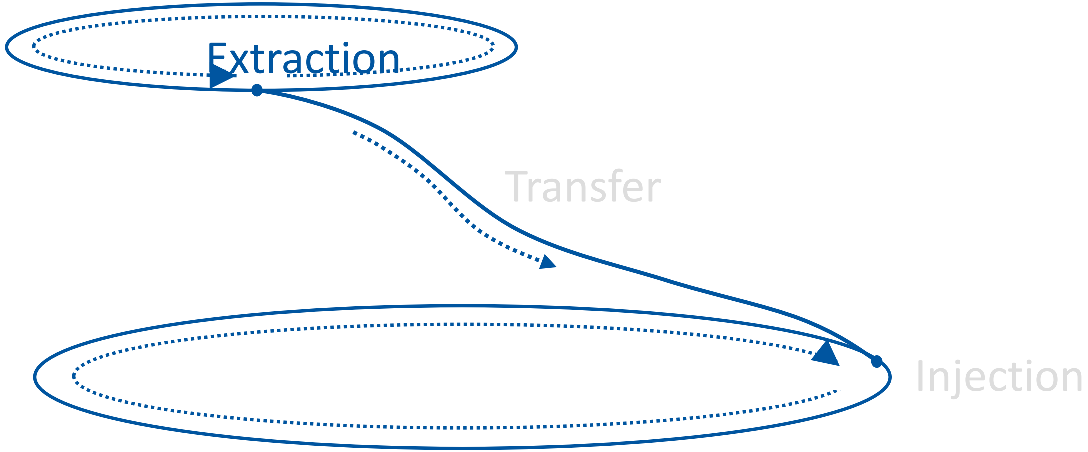
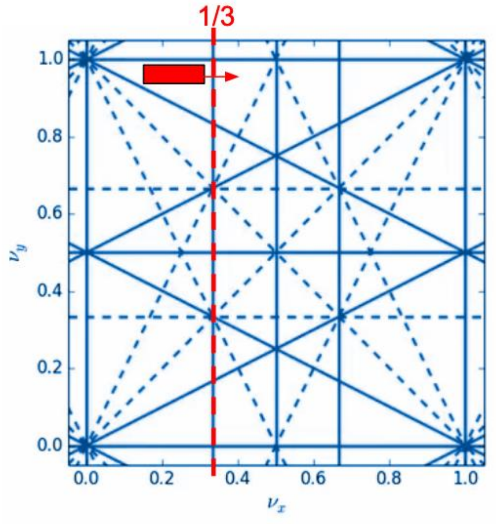
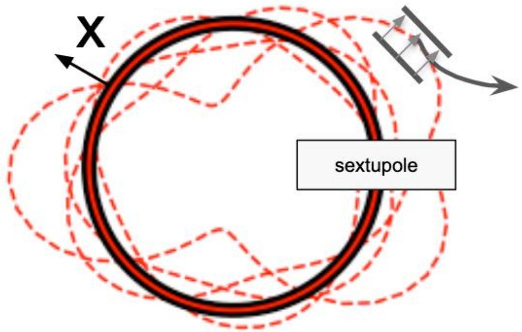


Resonant slow extraction: concept

- Principle
  - Tune adjusted close to nth order betatron resonance (typically q=1/3).
  - Multipole magnets excite resonance strength and beam is slowly pushed into instability.
  - Particles are “peeled” from circulating beam and jump over septum to provide a continuous spill.

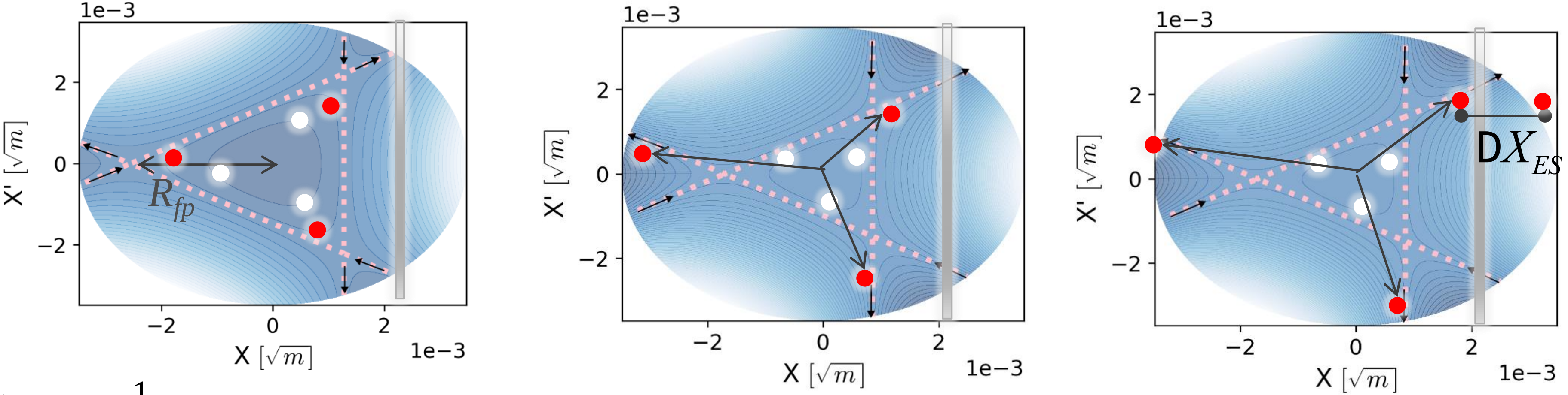
- Applications
  - At CERN PS and SPS
  - At BNL AGS until 2000 and Booster
  - At medical synchrotrons





Slow extraction: normalized phase space

- 3rd order resonances: Sextupole fields distort the circular normalised phase space.
- Stable area defined, delimited by unstable Fixed Points.
- Stable area can be reduced by scanning the machine tune Qh (or sextupole str.)
- Unstable particles grow in amplitude and jump over the septum (spiral step).

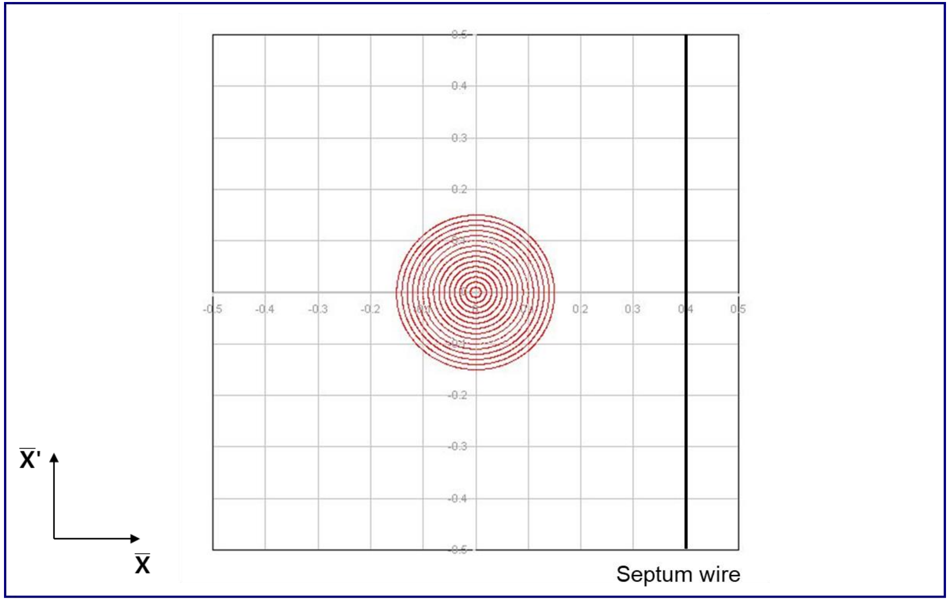


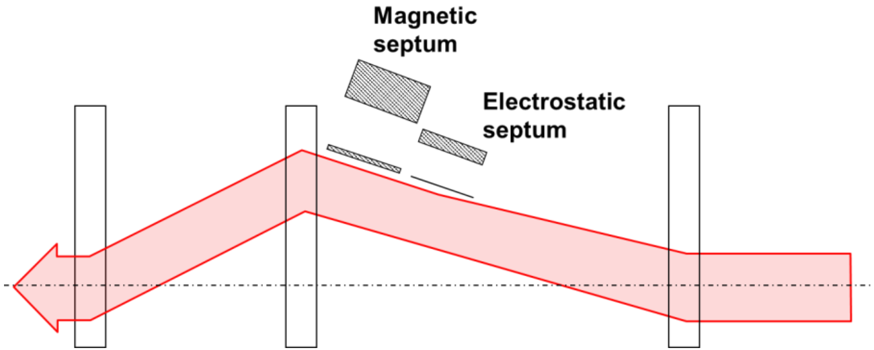
$$R_{fp}^{\ 1/2} \propto \Delta Q \cdot \frac{1}{k_2}$$

$$DX_{ES} \propto |k_2| \frac{X_{ES}^2}{\cos q}$$

Slow extraction: normalized phase space

- Evolution of the phase space for slow extraction





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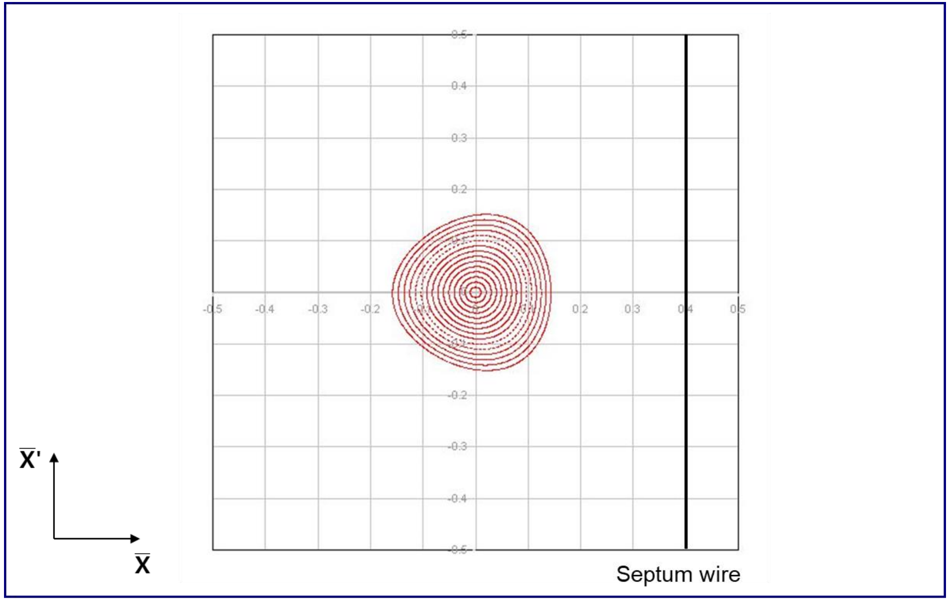

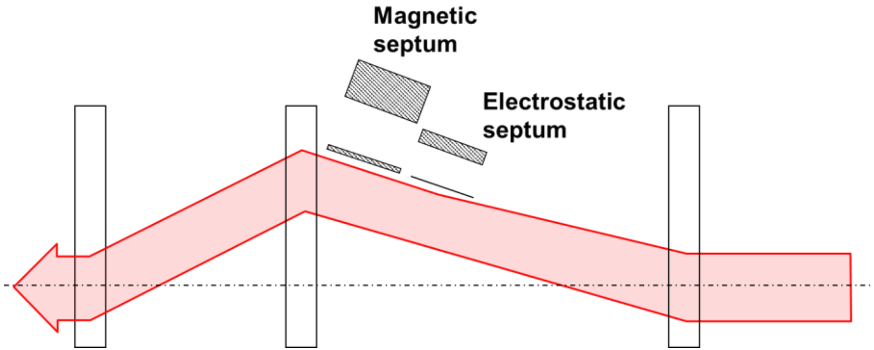

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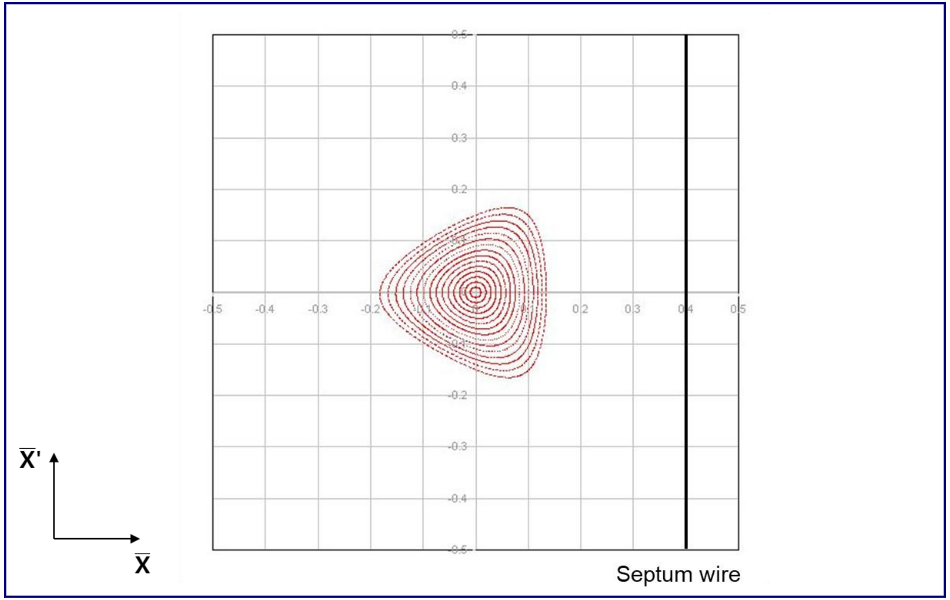

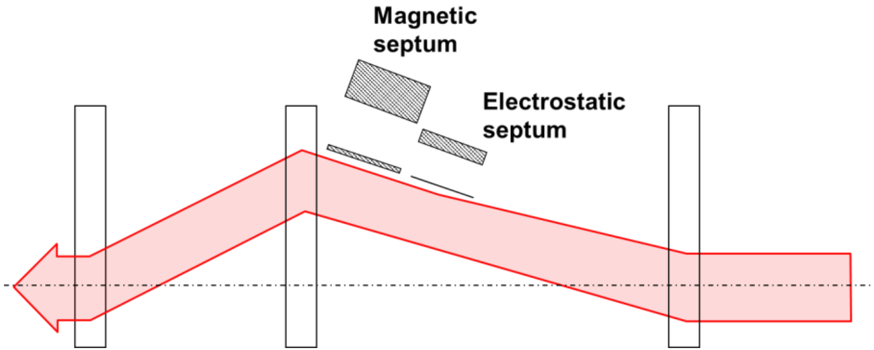

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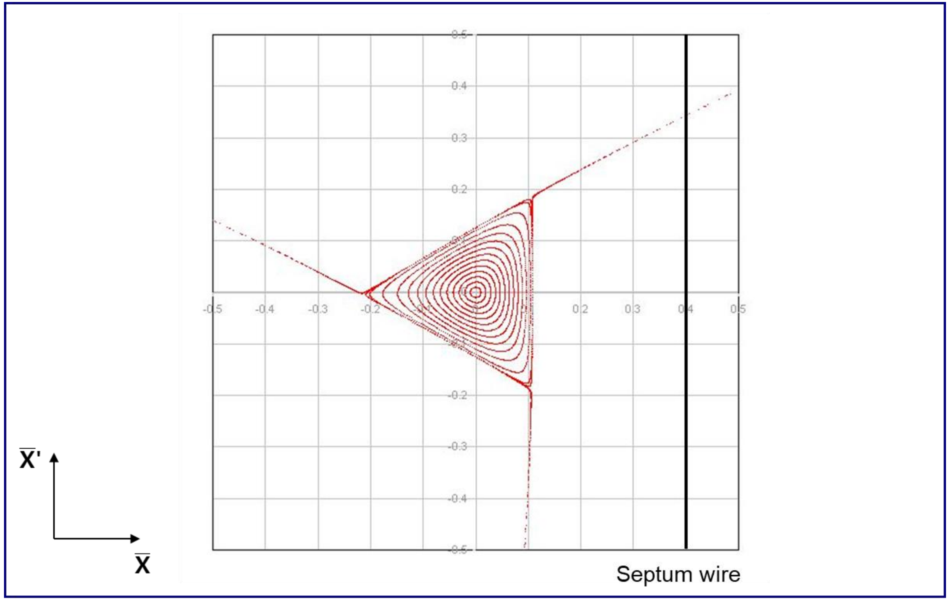

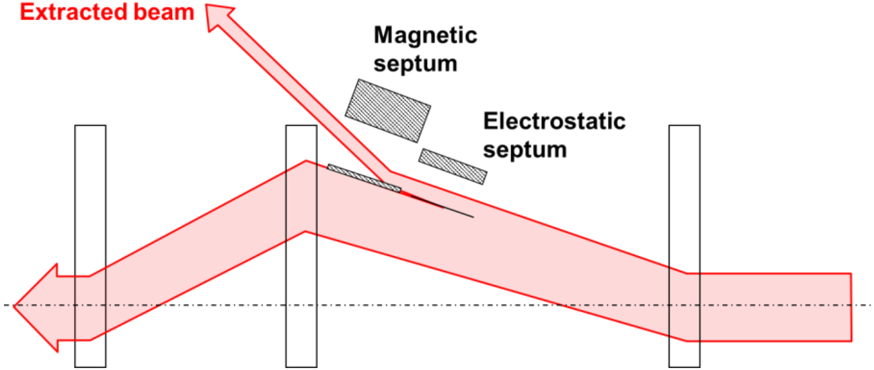

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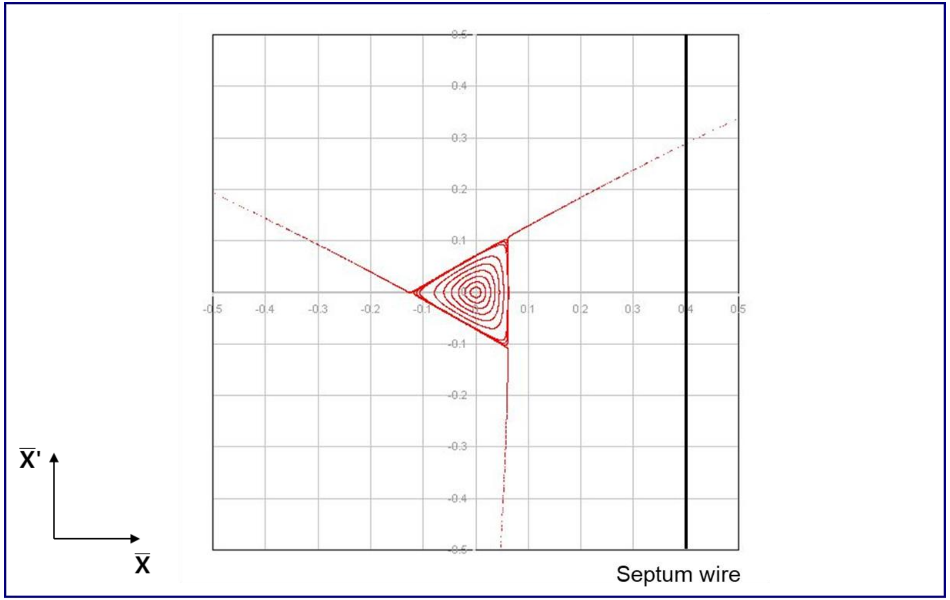

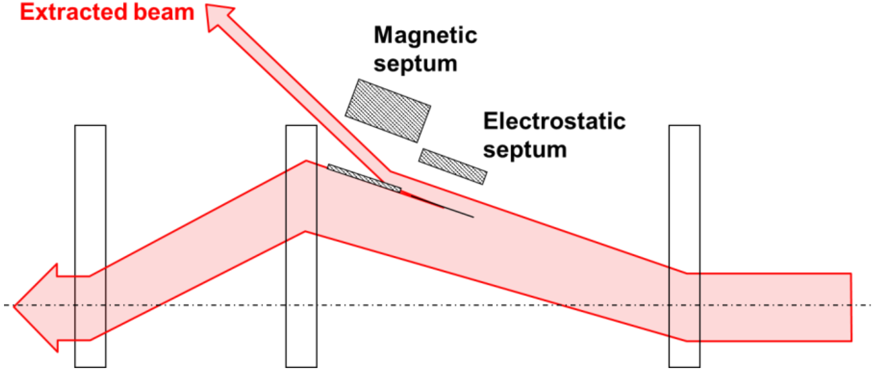

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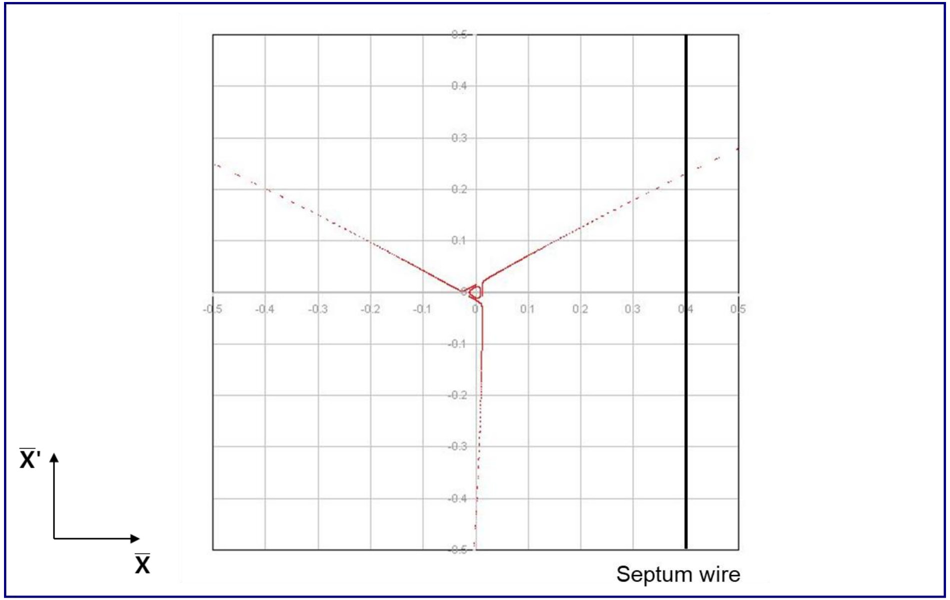

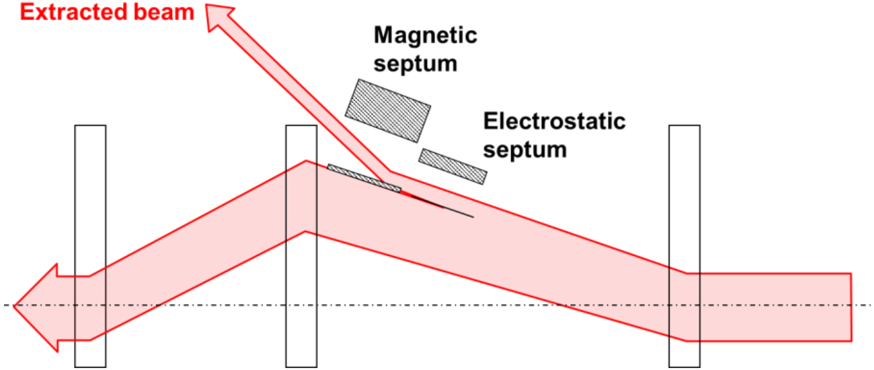

Extraction techniques : Summary

- Single-turn fast extraction:
  - for transfer between machines in accelerator chain, beam abort, etc.
- Non-resonant (fast) multi-turn extraction
  - slice beam into equal parts for transfer between machine over a few turns.
- Resonant low-loss (fast) multi-turn extraction
  - create stable islands in phase space: slice off over a few turns.
- Resonant (slow) multi-turn extraction
  - create stable area in phase space and slowly drive particles into resonance →long spill over many thousand turns.

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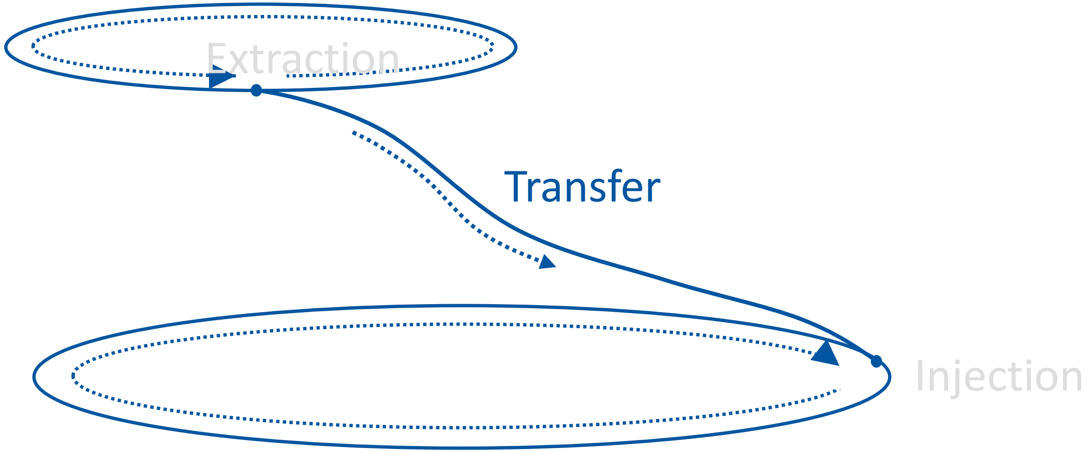
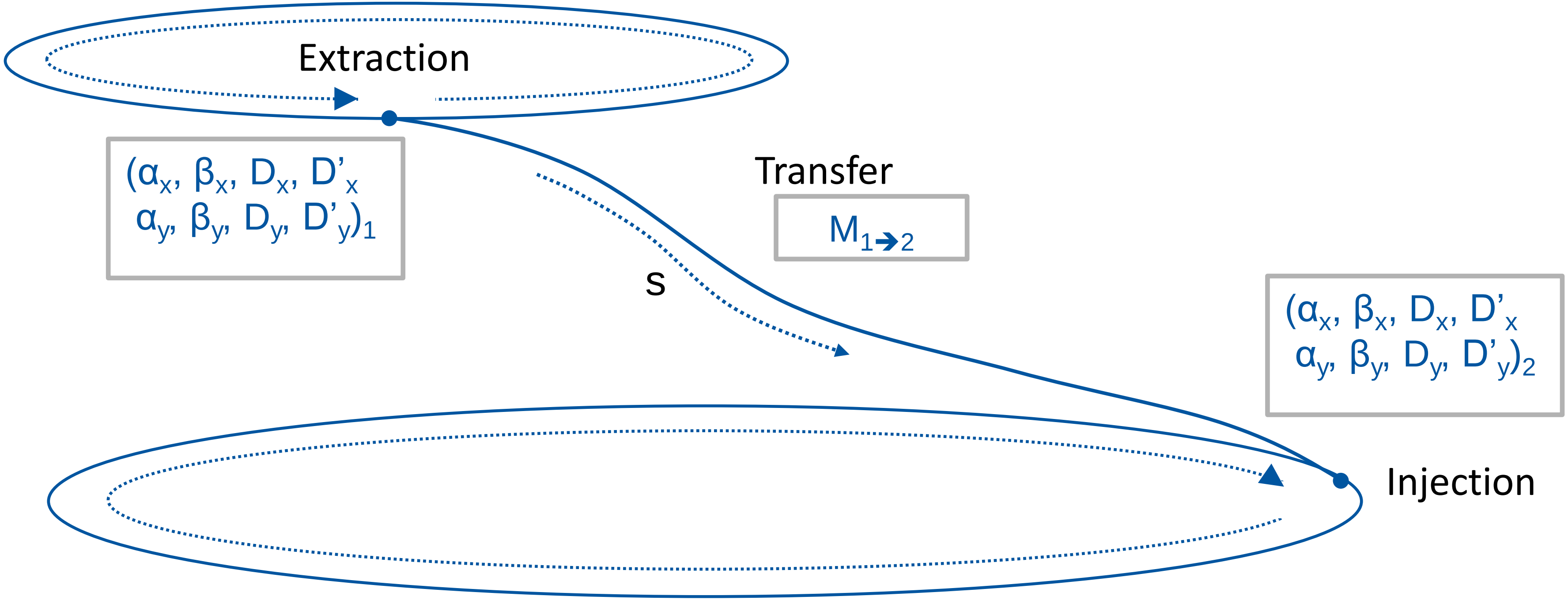


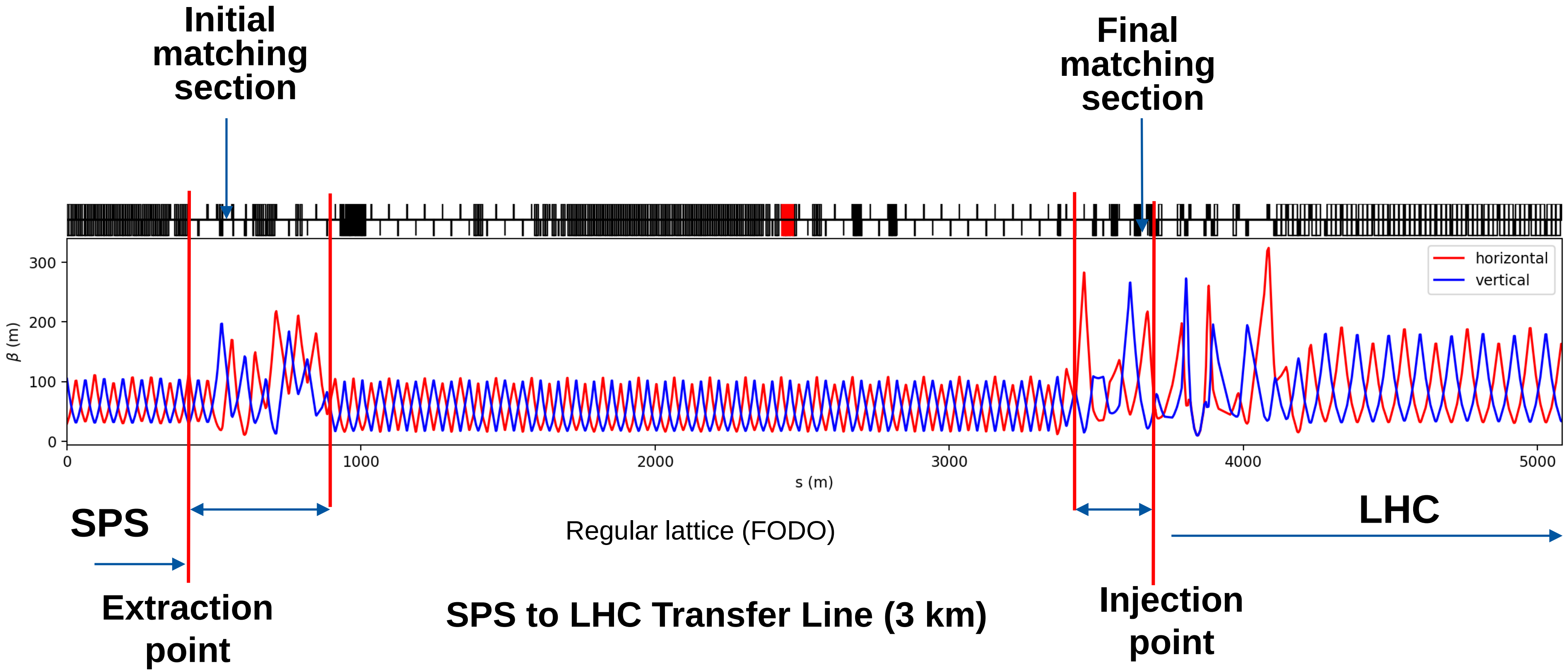
Transfer between machines

1. **Extract** a beam out of one machine ➔ initial beam parameters
2. **Transport** this beam towards the following machine (or experiment)
3. **Inject** this beam into a following machine with a predefined optics

➔ Transfer line optics has to produce required beam parameters for matching

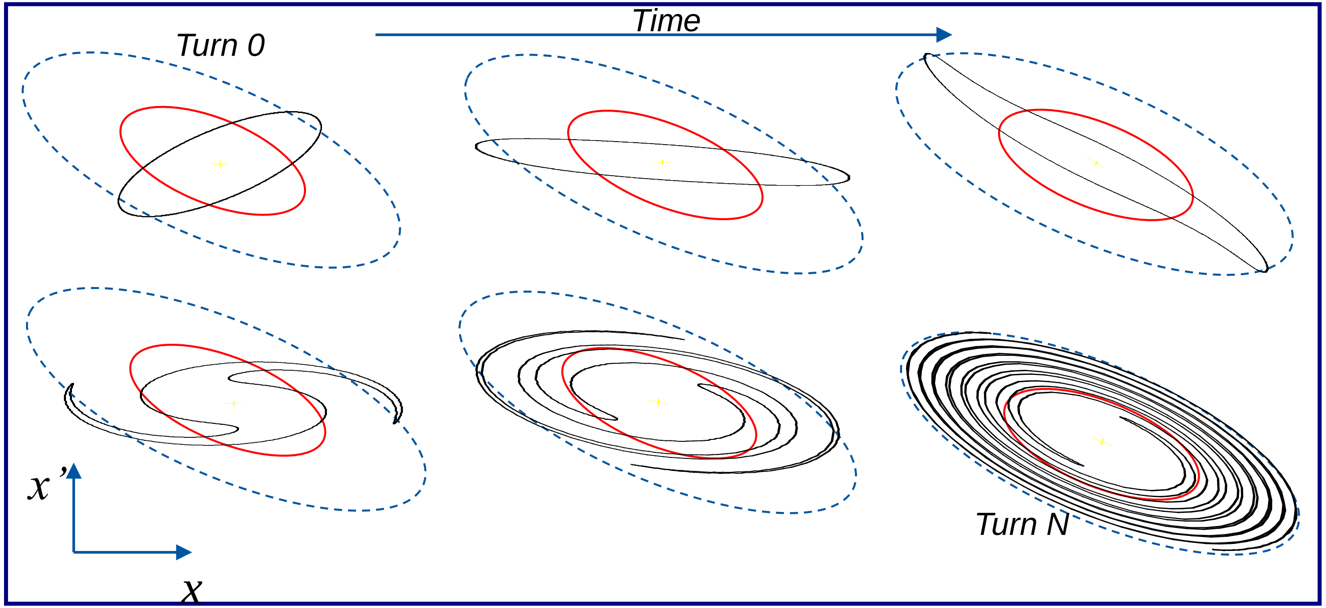


Example: SPS to LHC



Imperfect matching

- Mismatch in optical parameters will lead to emittance blow-up via filamentation:



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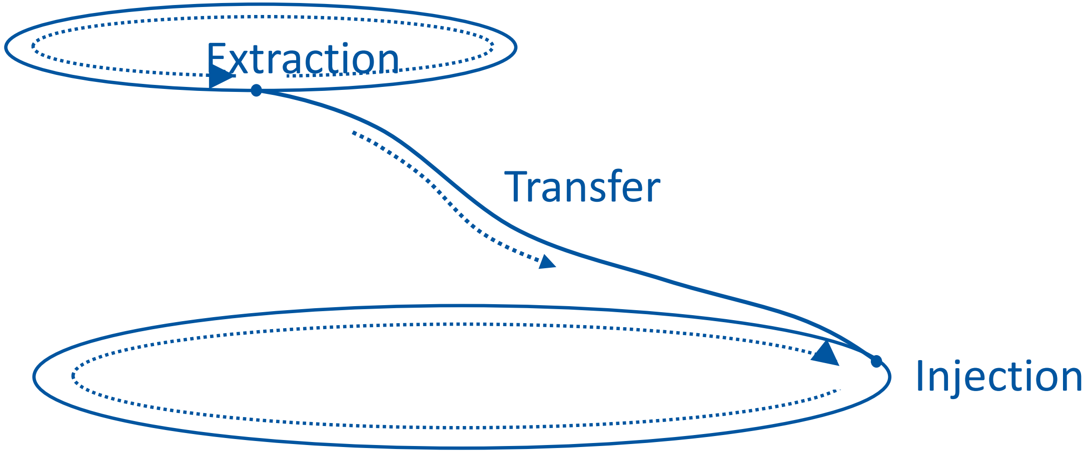
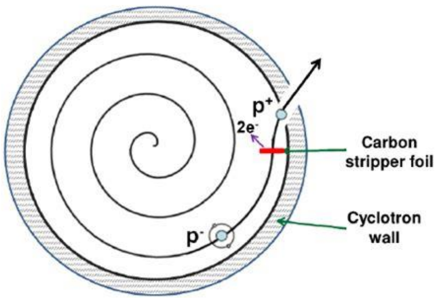
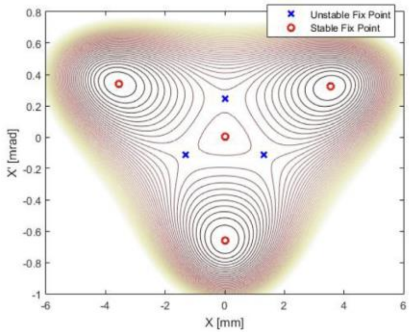


Conclusion

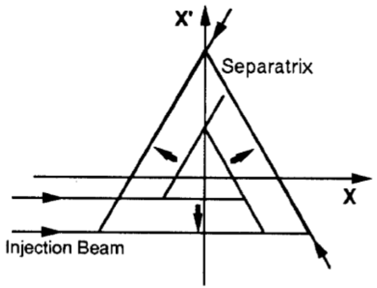
- Reviewed injection and extraction techniques, from tried-and-true fast injection/extraction to more sophisticated techniques.
- There is often a trade-off between robustness and flexibility.
- Key figures of merit include emittance preservation and beam loss minimization.
- Many aspects we haven't discussed, e.g. can we mirror all injection/extraction techniques?



Charge exchange extraction



Island injection



Slow injection?

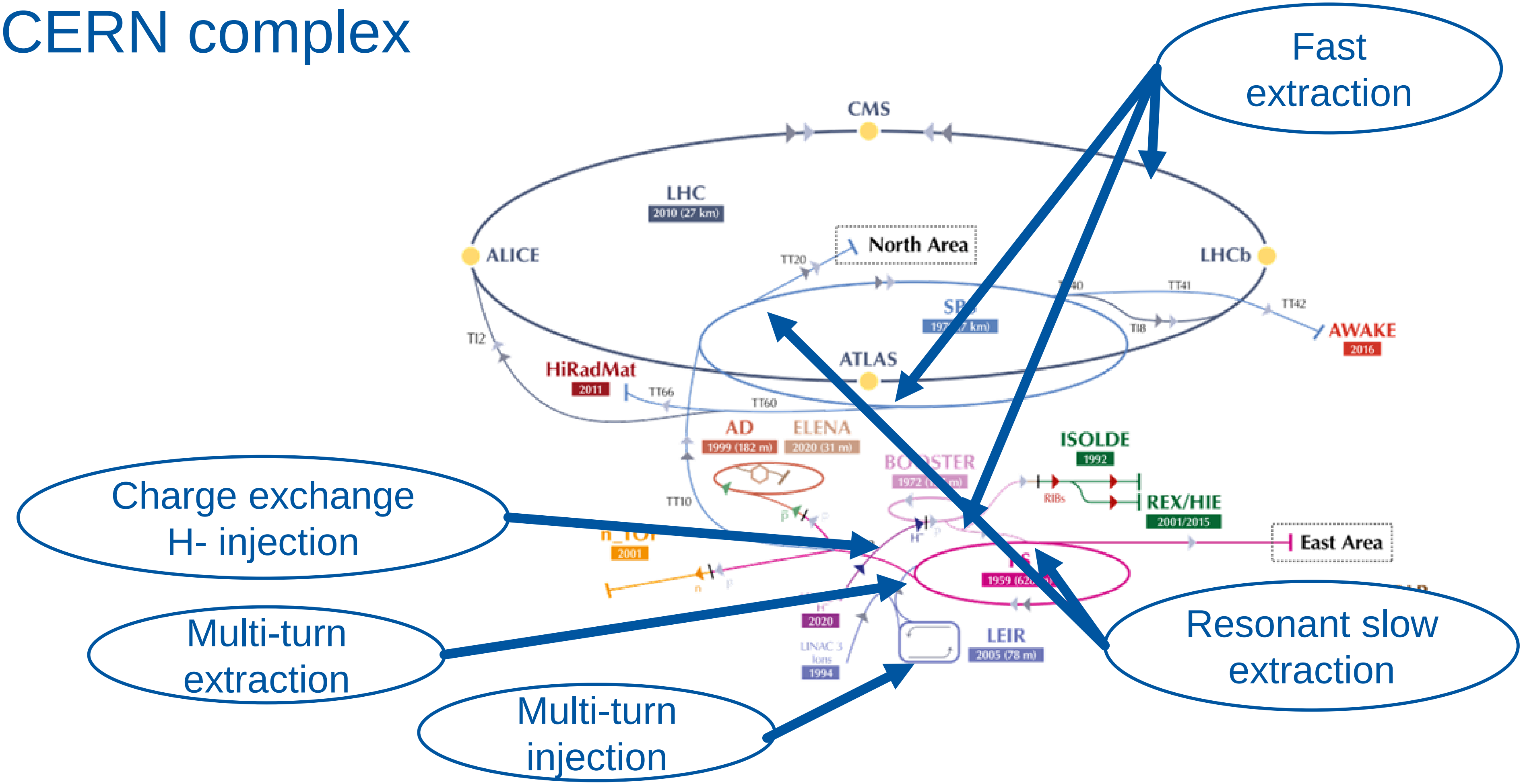
Further reading (with the formulae !), and resources

- ABT group is on the 865 building, my office 865/1D26
- CAS 2017 on Beam injection, extraction and transfer at https://indico.cern.ch/event/451905/
  - Overlooked here, timing and synchronization by RF expert H. Damerau next week
  - Detailed talk on resonant slow extraction by P. Bryant https://indico.cern.ch/event/451905/contributions/2159064/
  - Exotic extraction methods that discuss all the possibilities overlooked here by B. Goddard https://indico.cern.ch/event/451905/contributions/2159103
- This CAS lecture on Kickers and Septa by M. Barnes https://indico.cern.ch/event/1018359/contributions/4312229
- CERN GIS machine portal https://gis.cern.ch/gisportal/Machine.htm

Thank you

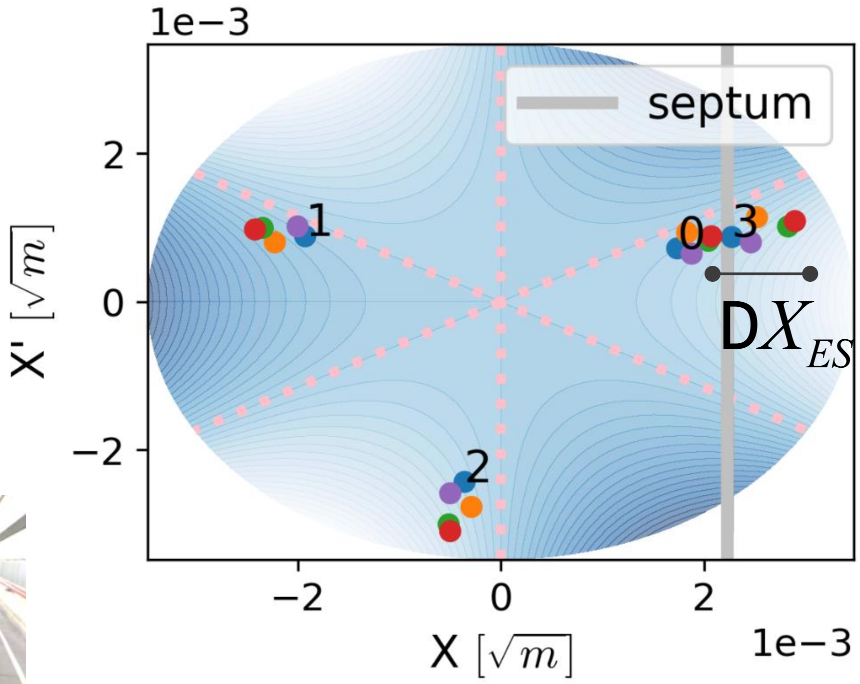
EXTRA SLIDES

CERN complex



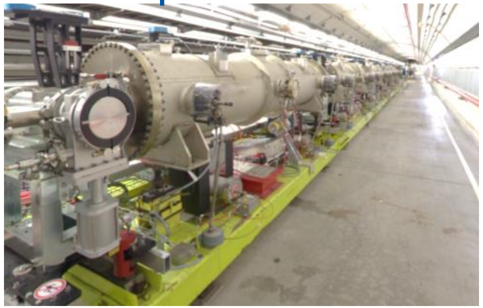
Slow extraction: spiral step and beam loss

- Eventually, the resonant kicks add up and particles are pushed over the septum.
- Distance travelled over last 3 turns is called "spiral step".
- Still, a few particles interact with the septum blade and are lost.



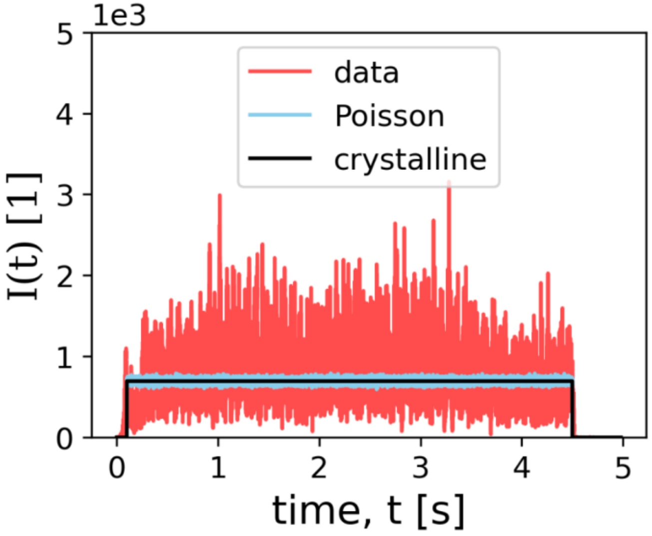
$$\Delta X_{ES} \propto |k_2| \frac{X_{ES}^2}{\cos\theta}$$

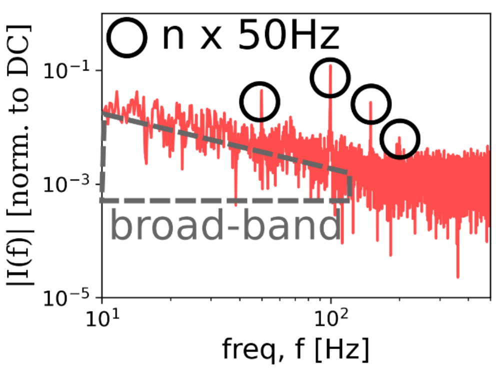
Septa tanks



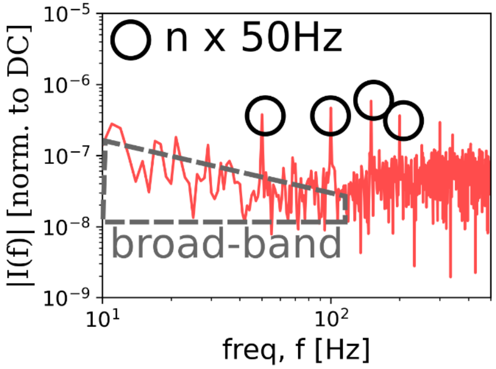
Slow extraction: spill quality

- Slow extraction is a resonant process -> amplifies any perturbation.
- Ripple of a few parts per million on quadrupole current can significantly modulate extracted intensity.





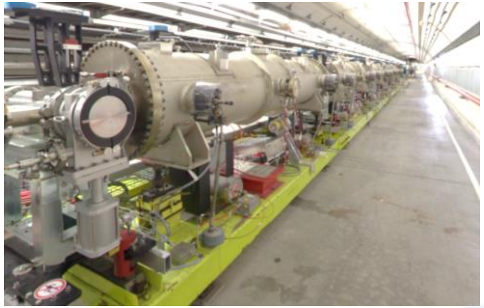
(a) Spill.

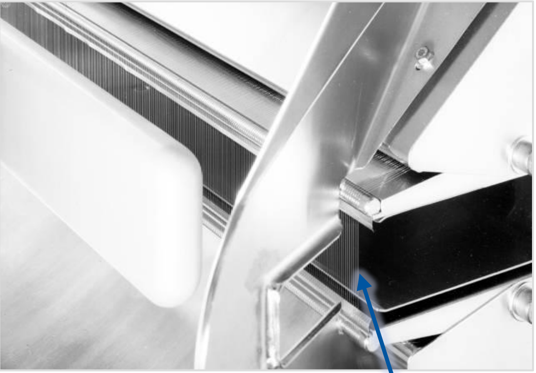


(b) F-quad current.

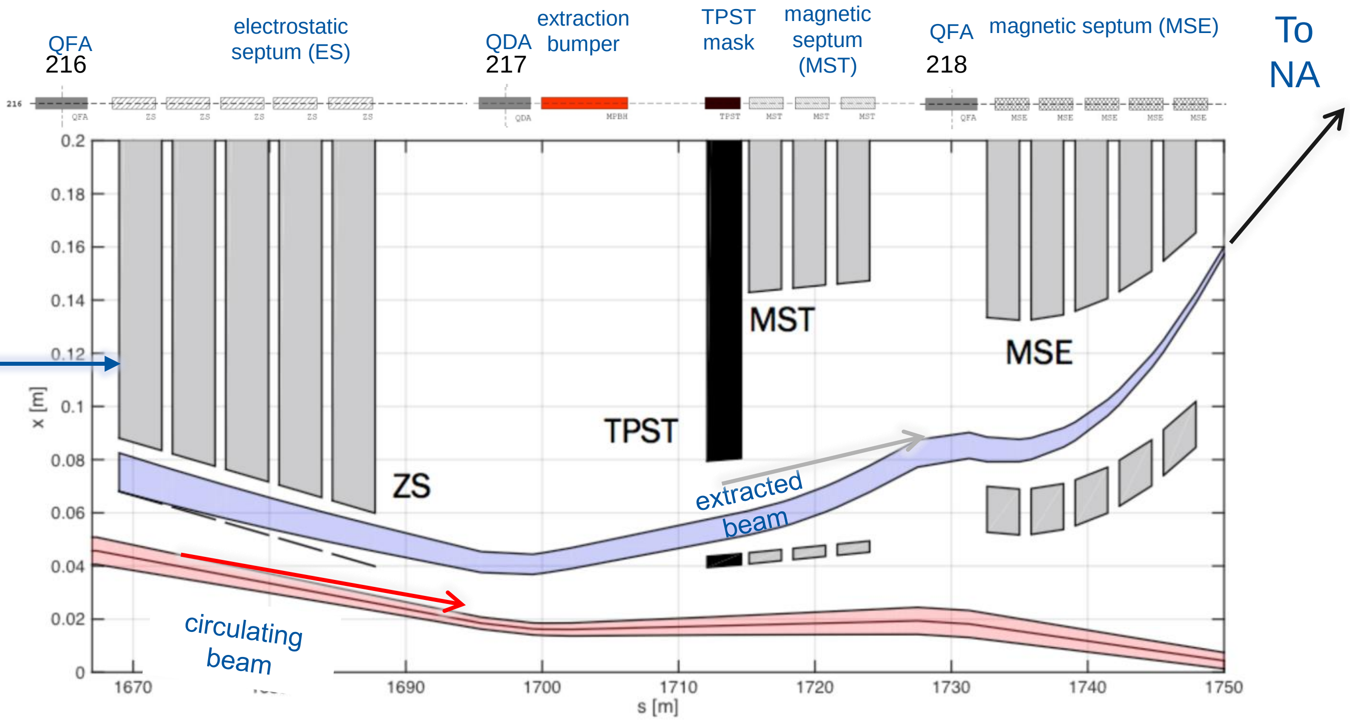
Extraction techniques : Resonant extraction, slow extraction from the SPS

Septa tanks





Septum wires



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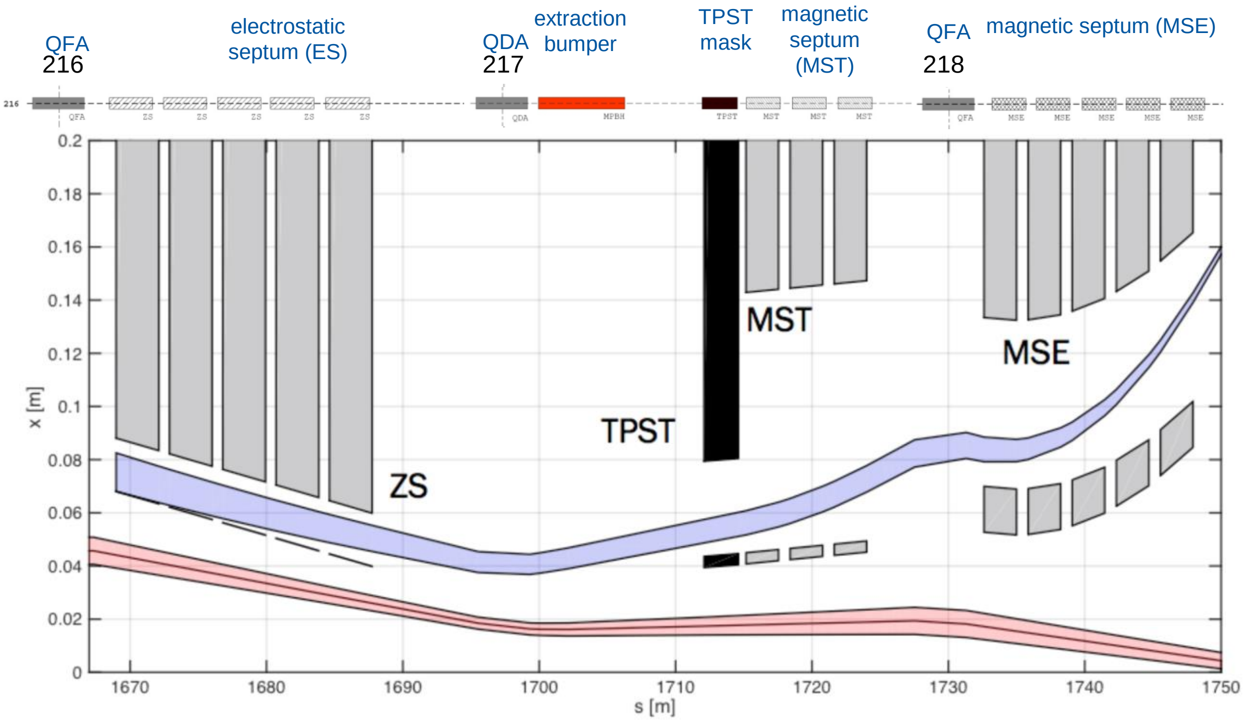
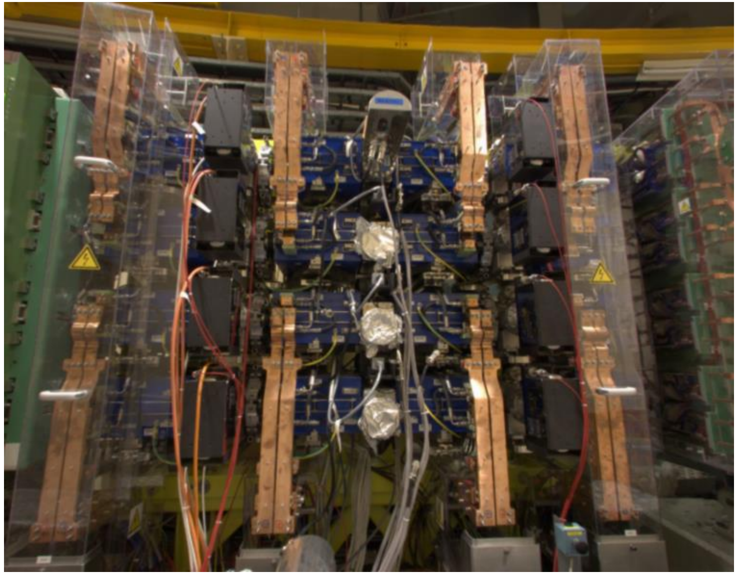


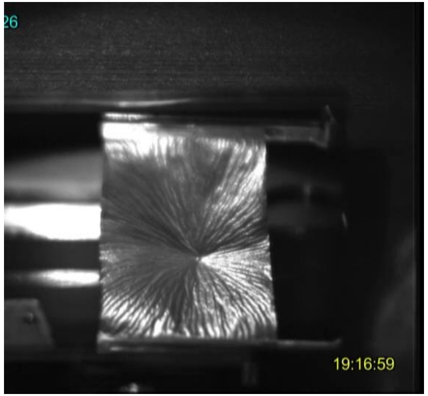
Injection techniques : Charge exchange

- Example of the PSB
  - Paint uniform transverse phase space density by modifying closed orbit bump and steering injected beam
  - Foil thickness calculated to double-strip most ions (≈99%)
    - 200 µg.cm^{-2} (≈ 1 µm of C!)
    - Carbon foils generally used – very fragile
  - Injection chicane reduced or switched off after injection, to avoid excessive foil heating and beam blow-up

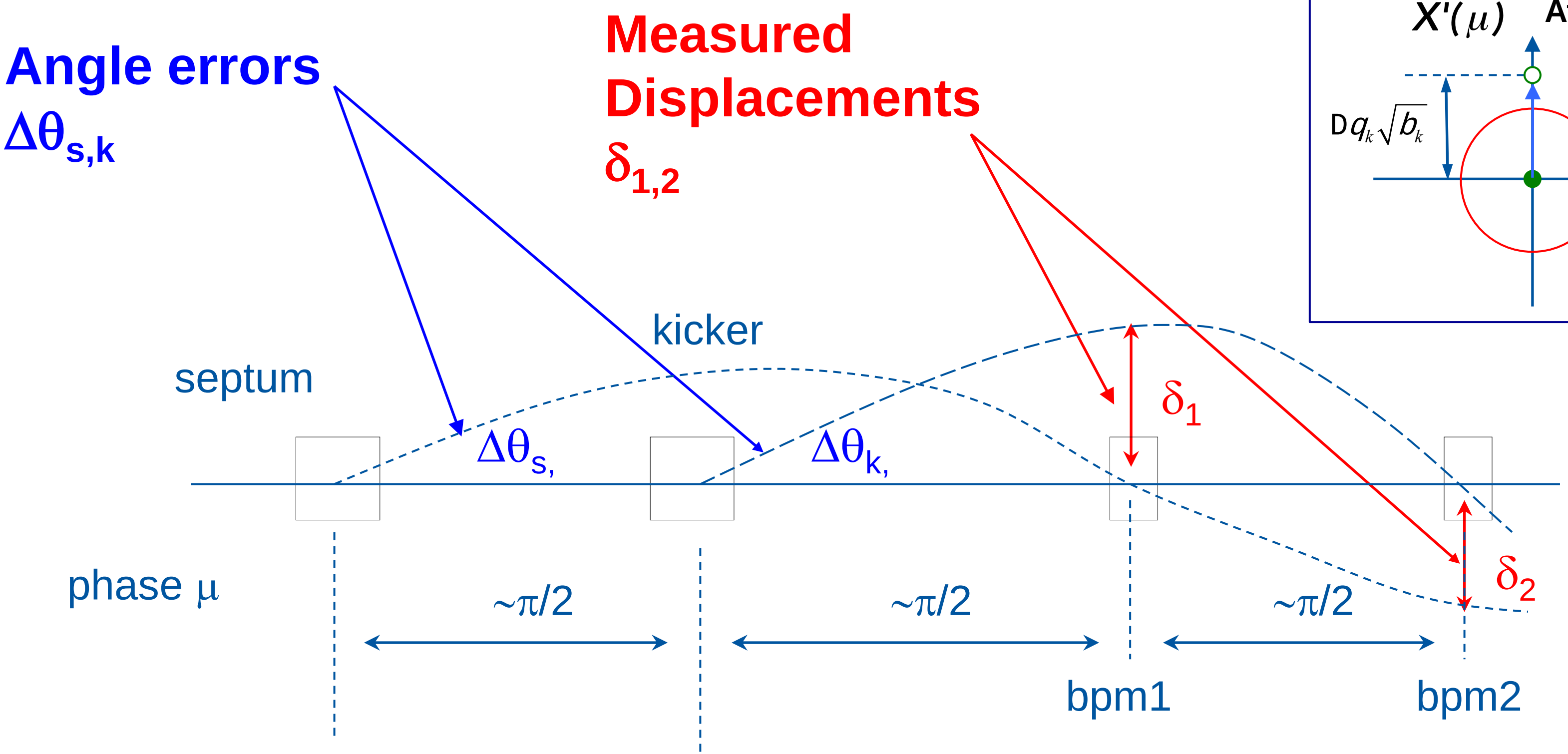
PSB injection 4 rings stacked

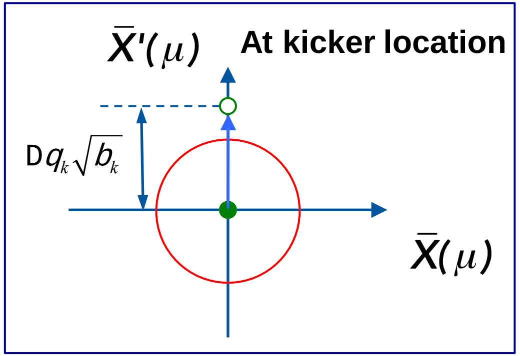


Foils cassette developed by SY-ABT

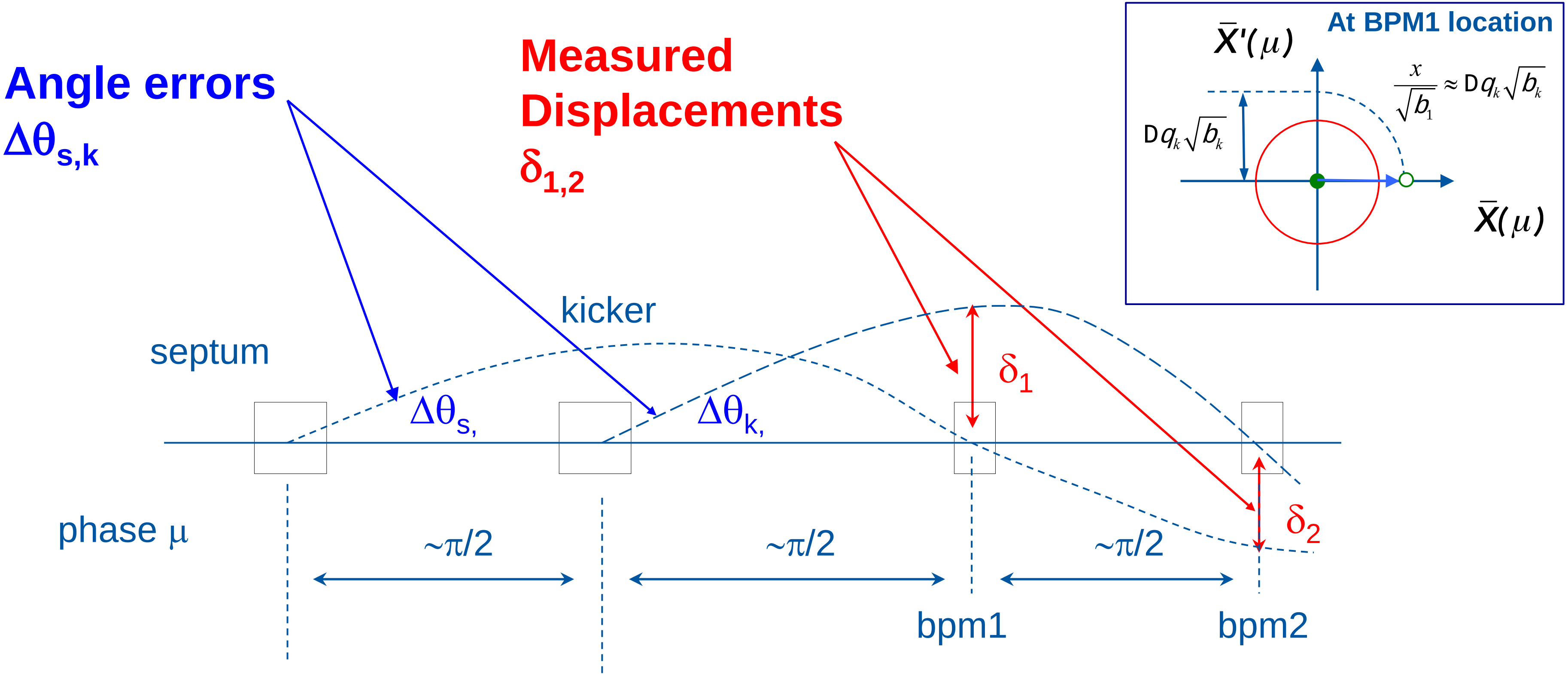


Injection techniques : Fast injection, dipole error and correction





Injection techniques : Fast injection, dipole error and correction



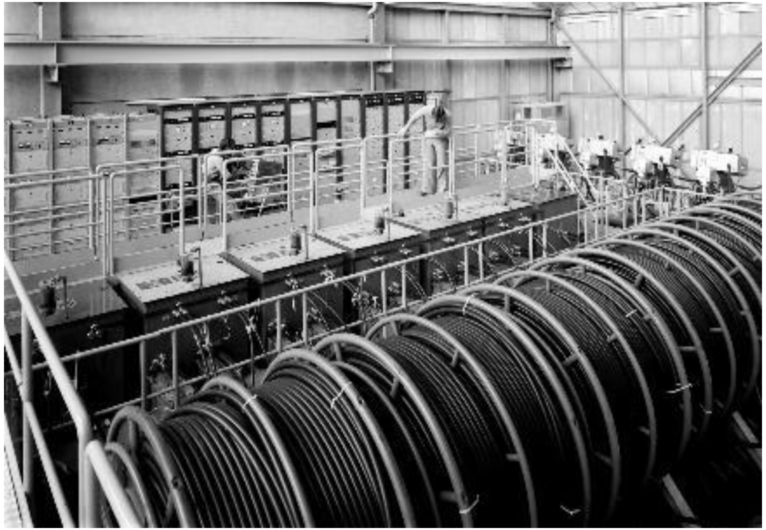
$$\delta_1 = \Delta\theta_s \sqrt{(\beta_s\beta_1)} \sin(\mu_1 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k\beta_1)} \sin(\mu_1 - \mu_k) \quad \approx \Delta\theta_k \sqrt{(\beta_k\beta_1)}$$

$$\delta_2 = \Delta\theta_s \sqrt{(\beta_s\beta_2)} \sin(\mu_2 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k\beta_2)} \sin(\mu_2 - \mu_k) \quad \approx -\Delta\theta_s \sqrt{(\beta_s\beta_2)}$$

Hardware system : PFL & PFN

Pulse Forming Line (PFL)

- Low-loss coaxial cable
- Fast and ripple-free pulses
- Attenuation & droop becomes problematic for pulses > 3 µs
- Above 40 kV SF6 pressurized PE tape cables are used at CERN
- Bulky: 3 µs pulse ~ 300 m of cable



Pulse Forming Network (PFN)

- Artificial coaxial cable made of lumped elements
- For low droop and long pulses > 3 µs
- Each cell individually adjustable: adjustment of pulse flat-top difficult and time consuming.