Accelerator & Technology Sector
Beams Department
Accelerator Beam Physics Group

Particle Accelerators and Beam Dynamics

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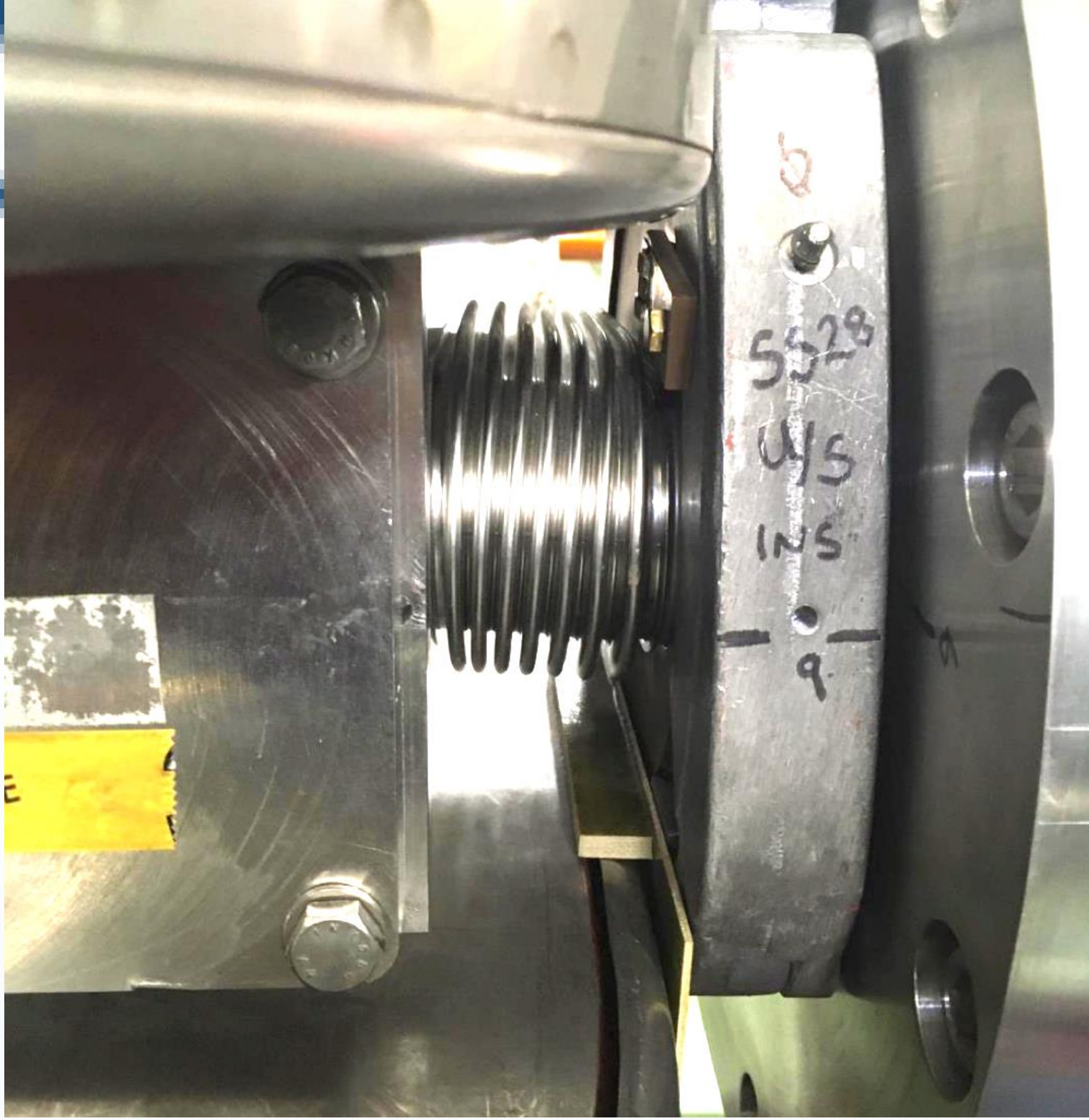
based on: H. Bartosik, D. Gamba, Y. Papaphilipou

Eurolabs 2024

Outline

- Introduction
- Closed orbit distortion (steering error)
  - Beam orbit stability
  - Imperfections leading to closed orbit distortion
- Optics function distortion (gradient error)
  - Imperfections leading to optics distortion
  - Tune-shift and beta distortion due to gradient errors
- Coupling error
  - Coupling errors and their effect
  - Coupling correction
- Summary



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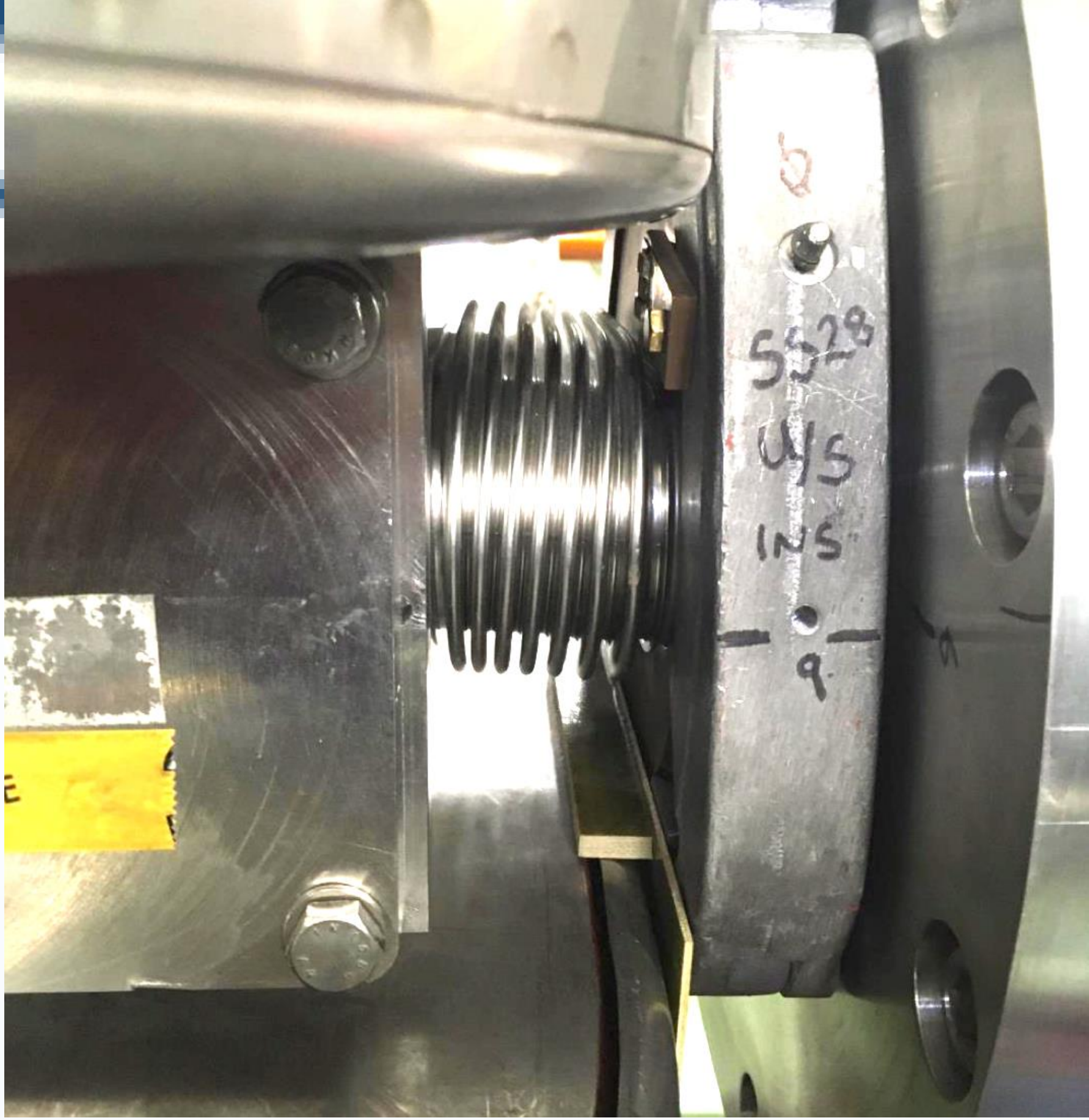
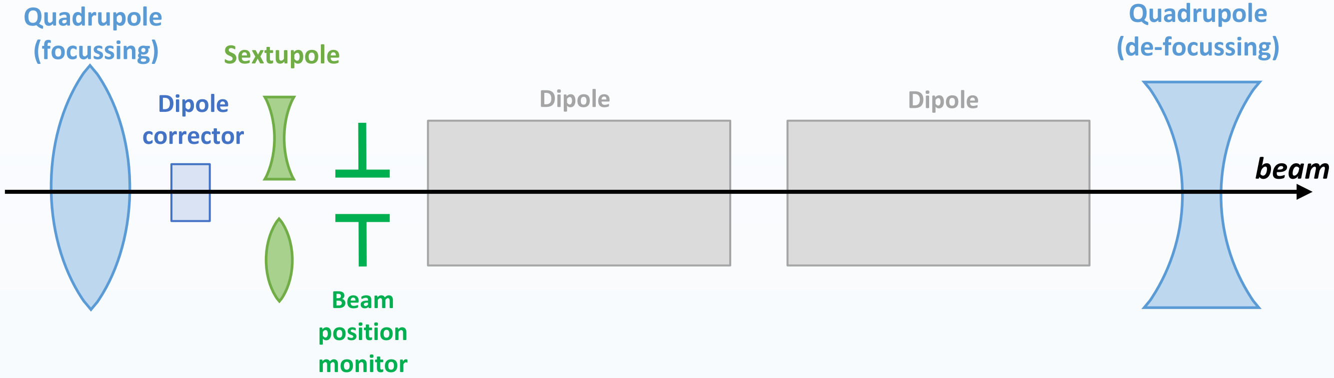


Basics: accelerator lattice

- An accelerator is usually build using a repetitio
- A simple <u>FODO cell</u> usually contains:
    - **Dipole** magnets to bend the beams
    - **Quadrupole** magnets to focus the beams
    - Beam position monitors (**BPM**) to measure the beam position
    - Small dipole **corrector** magnets for beam steering
    - (**Sextupole** magnets to control off-energy focusing)



Quadrupole (focussing)

Dipole corrector

Sextupole

Beam position monitor

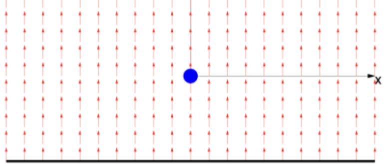
Dipole

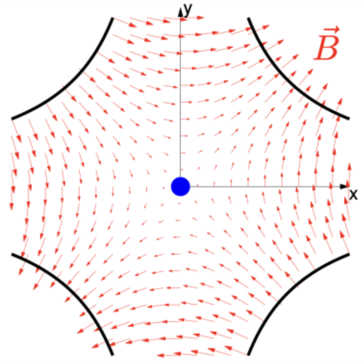
Dipole

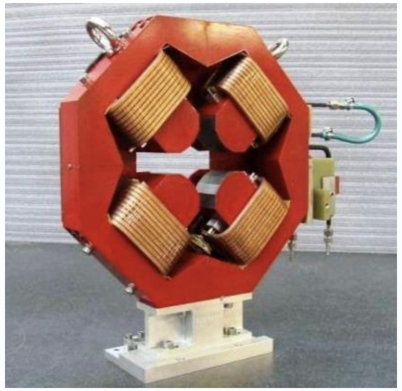
Quadrupole (de-focussing)

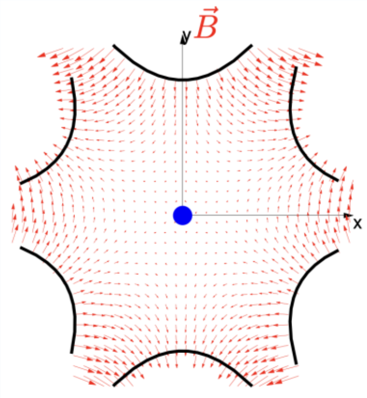
beam

Schematic of a ½ cell (not to scale)



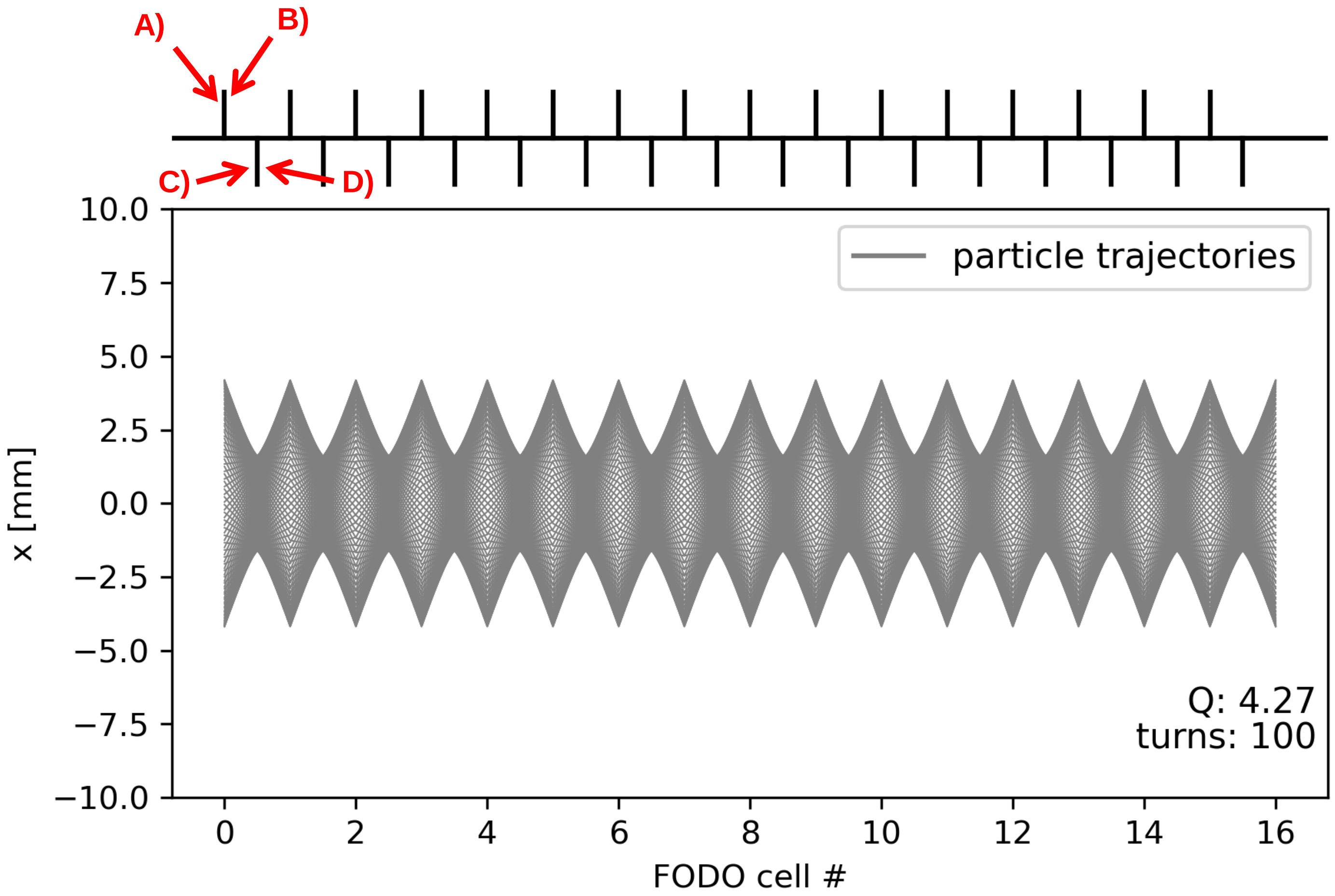


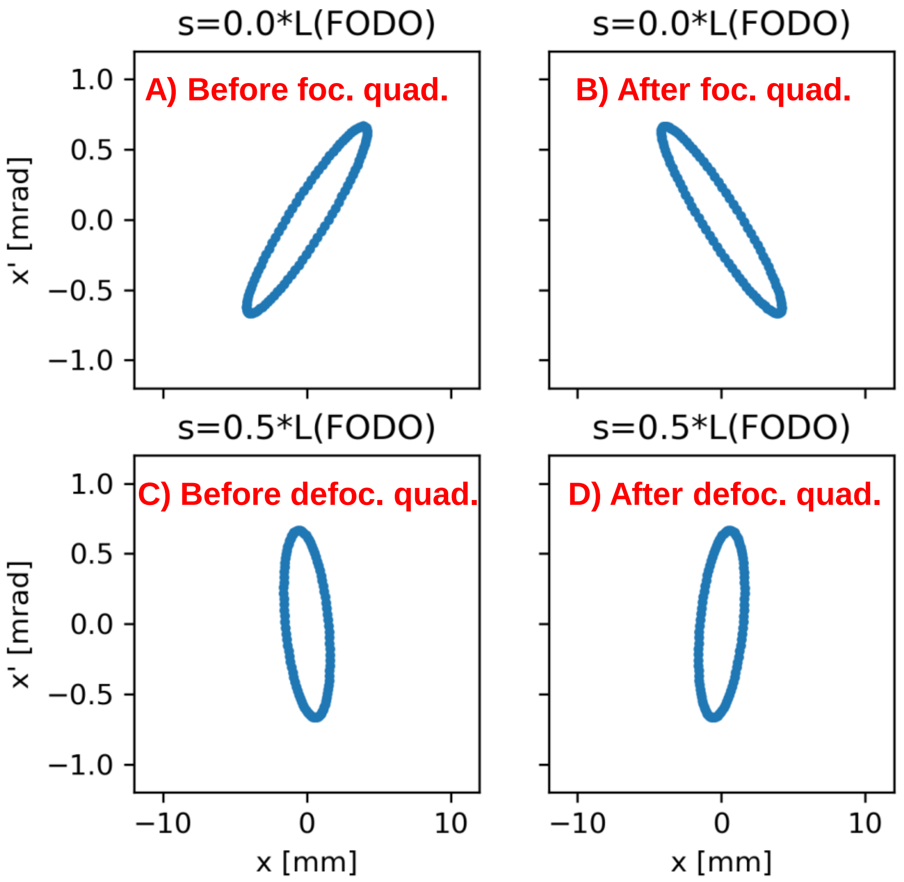




Toy FODO lattice – no errors

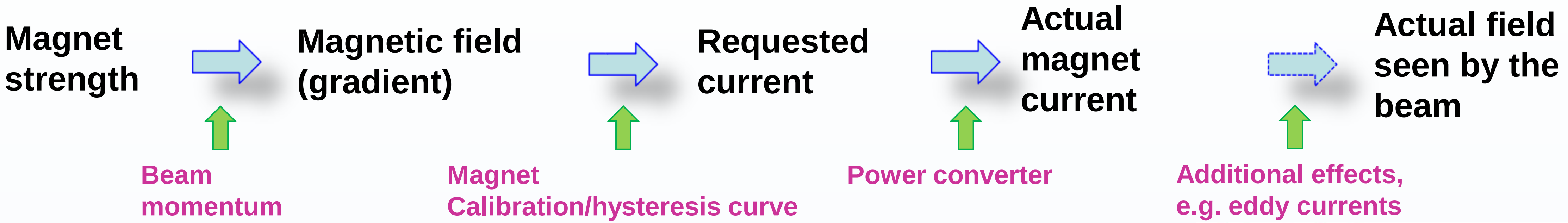
- Tracking a single particle, for many turns





From model to reality - fields

- The physical units of the machine model defined by the accelerator physicist must be converted into magnetic fields and eventually into currents for the power converters that feed the magnet circuits.
- **Imperfections** (= errors) in the real accelerator optics can be introduced by uncertainties or errors on:
  - Actual **beam momentum**, **magnet calibration** and **hysteresis, power converter regulation, …**



Magnet strength

Magnetic field (gradient)

Requested current

Actual magnet current

Actual field seen by the beam

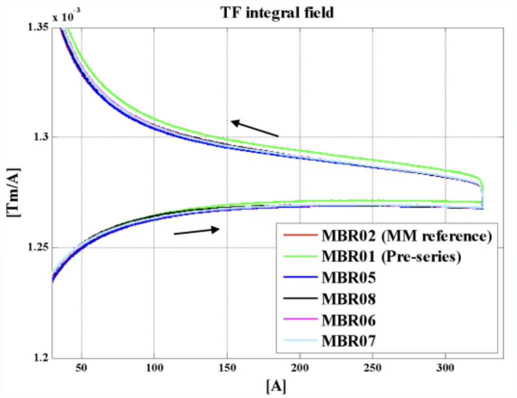
Beam momentum

Magnet Calibration/hysteresis curve

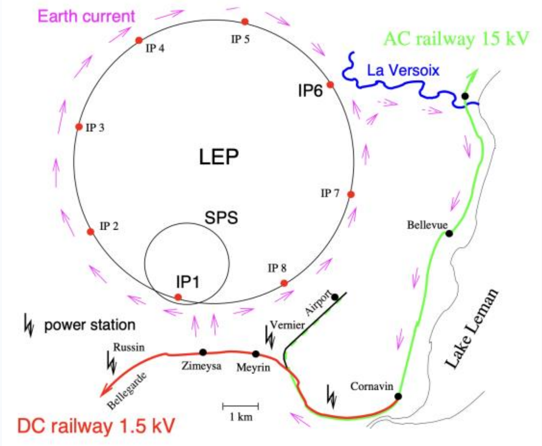
(transfer function)

Power converter

Additional effects, e.g. eddy currents



Example of the ELENA main dipoles hysteresis curve



Earth currents flowing over the LEP vacuum chamber that were generated by the DC railway line near Geneva

From model to reality - alignment

- To ensure that the accelerator elements are in the correct position the alignment must be precise – to the level of nanometer for the CLIC final focusing !
  - For CERN hadron machines we aim for accuracies of about 0.1 mm.
- The **alignment process** implies:
  - Precise measurements of the **magnetic axis** in the laboratory with reference to the element **alignment markers** used by the survey group.
  - Precise **in-situ alignment** (position and angle) of the element in the tunnel.
- **Alignment errors are a common source of imperfections**

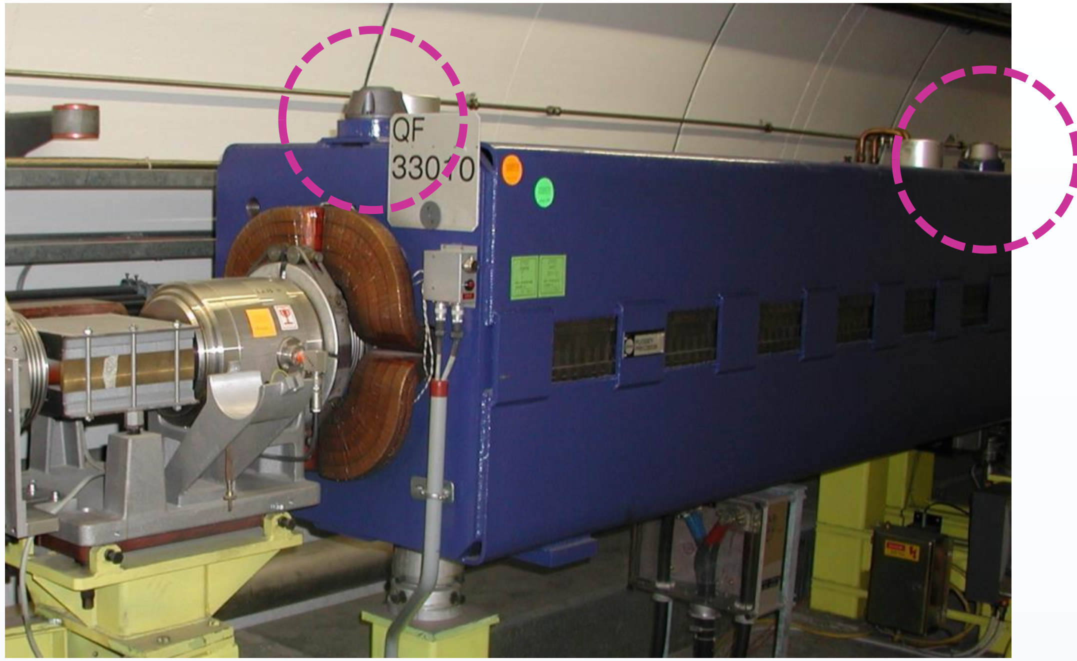


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Illustration of closed orbit distortion

1. Ideal machine toy model (no errors)
    a) Particle injected on the design (or reference) orbit … remains on the design orbit turn after turn

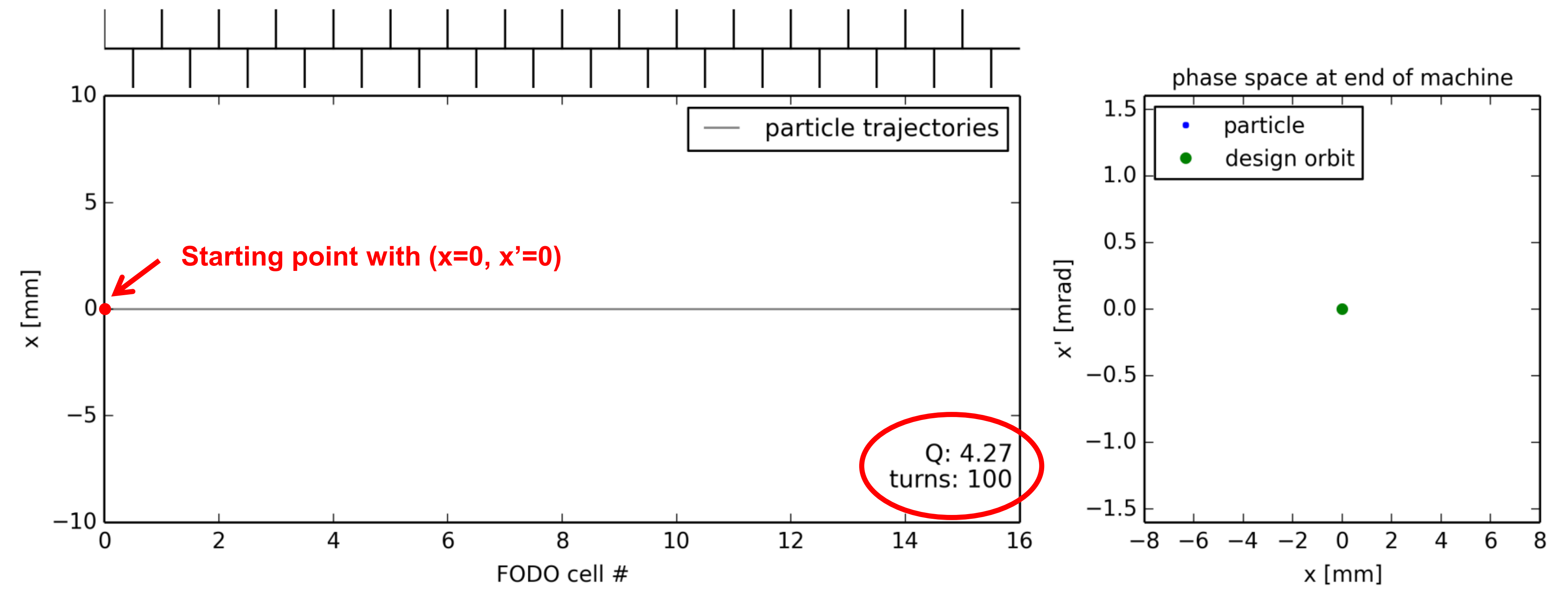


Illustration of closed orbit distortion

1. Ideal machine toy model (no errors)
   1. a) Particle injected on the design (or reference) orbit … remains on the design orbit turn after turn
   2. b) Particle injected with offset …

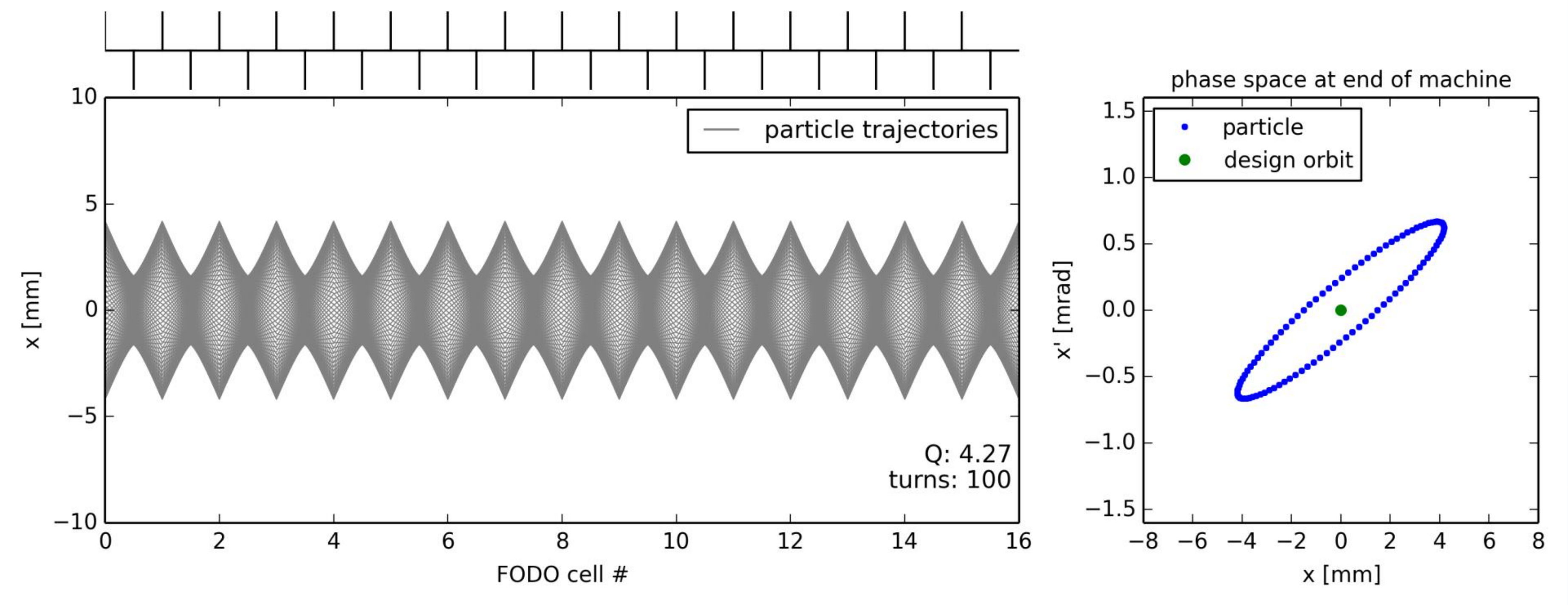


Illustration of closed orbit distortion

1. Ideal machine toy model (no errors)
    a) Particle injected on the design (or reference) orbit … remains on the design orbit turn after turn
    b) Particle injected with offset … performs betatron oscillations around the **closed orbit** which is the same as the design orbit as long as there are no imperfections

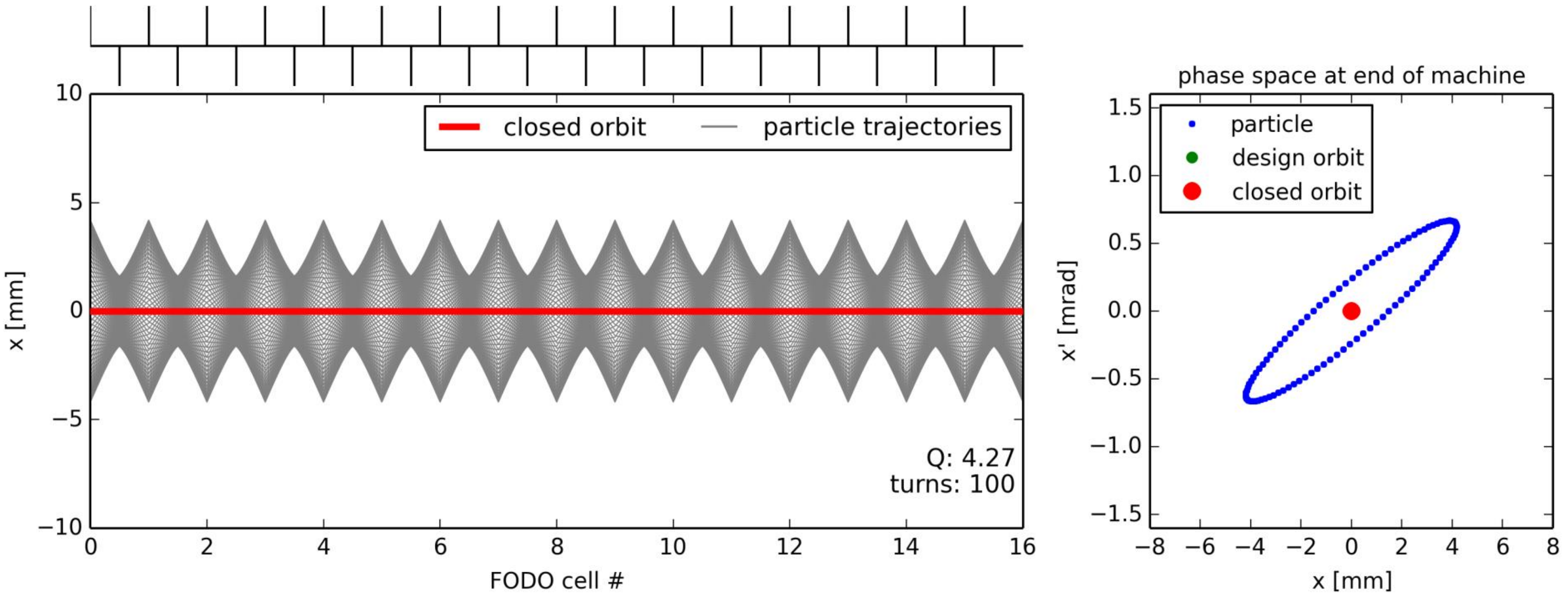
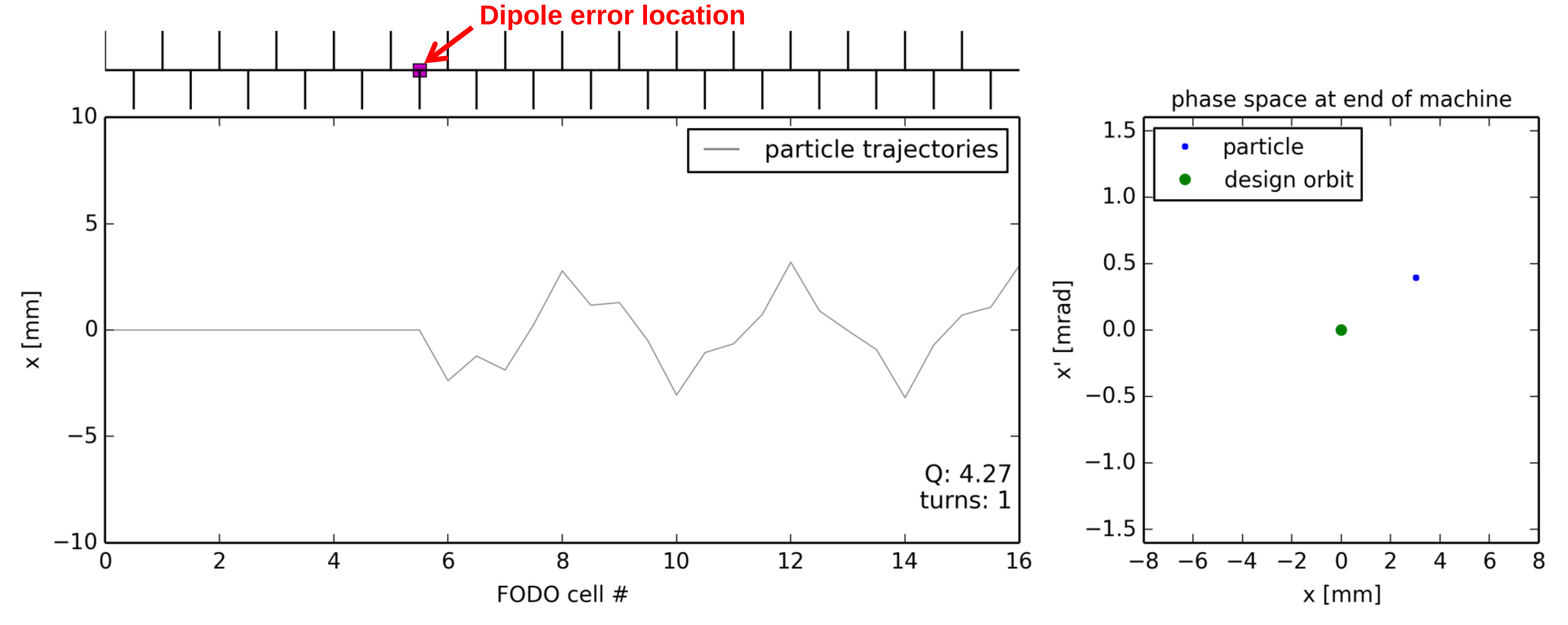


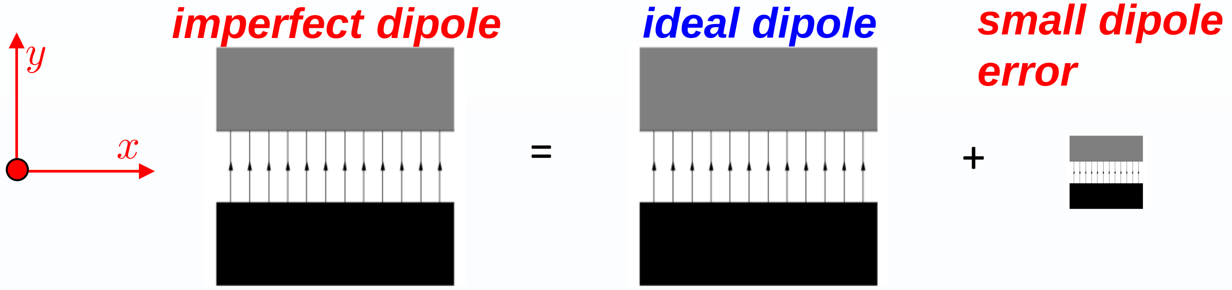
Illustration of closed orbit distortion

2. Ideal machine toy model with dipole error (unintended deflection) somewhere
    a) Particle injected on the design orbit … receives dipole kick every turn



Sources of unintended deflections

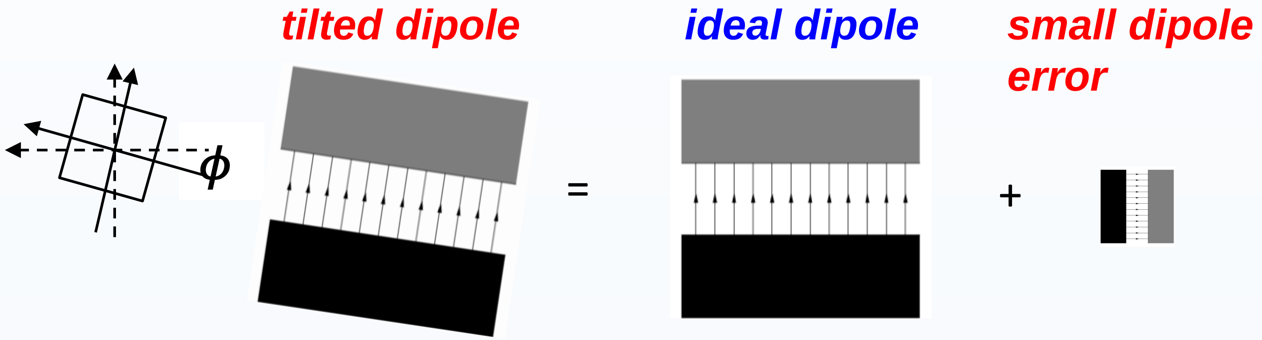
- **Field error** (deflection error) of a dipole magnet
  - This can be due to an **error** in the **magnet current** or in the **calibration table** (measurement accuracy etc.)
  - The imperfect dipole can be expressed as the ideal one + a small error



→ horizontal kick

$$\theta = \frac{\delta(Bl)}{B\rho}$$

- A small **rotation (misalignment)** of a dipole magnet has the same effect, but (mostly) in the “other” plane

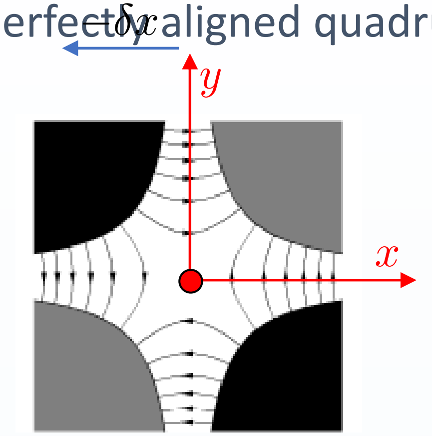


→ vertical kick

$$\theta = \frac{Bl\sin\phi}{B\rho}$$

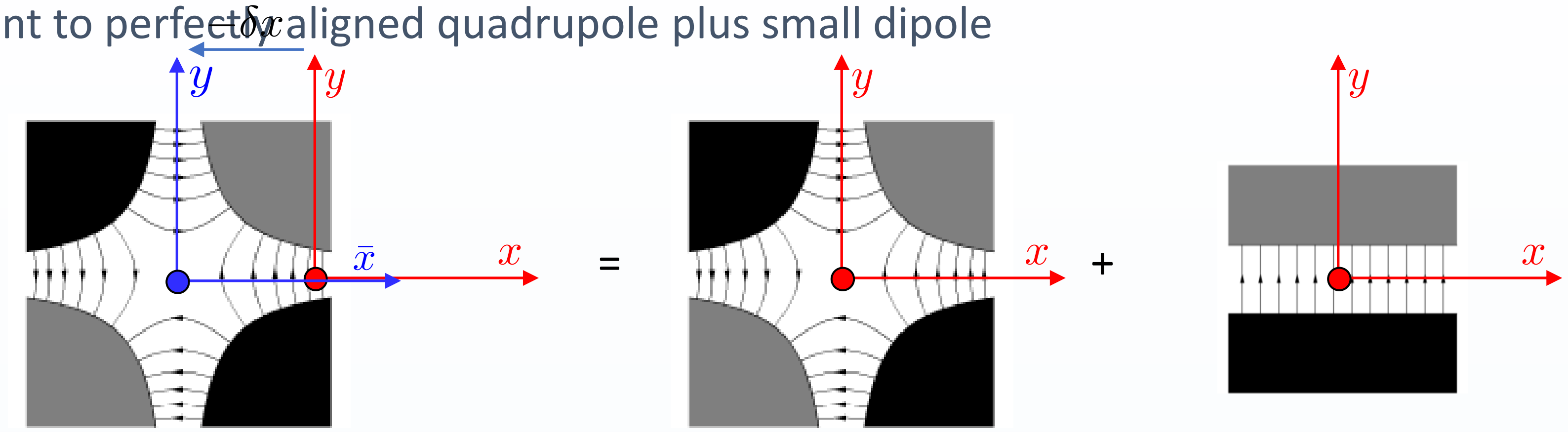
Misalignments causing feed-down

- Horizontal misalignment of a quadrupole magnet
- Equivalent to perfectly aligned quadrupole plus small dipole



Misalignments causing feed-down

- Horizontal misalignment of a <u>quadrupole</u> magnet
  - Equivalent to perfectly aligned quadrupole plus small dipole



$$B_x(\bar{x}, y) = B_x(x + \delta x, y) = G(y) \qquad = \underbrace{Gy}$$

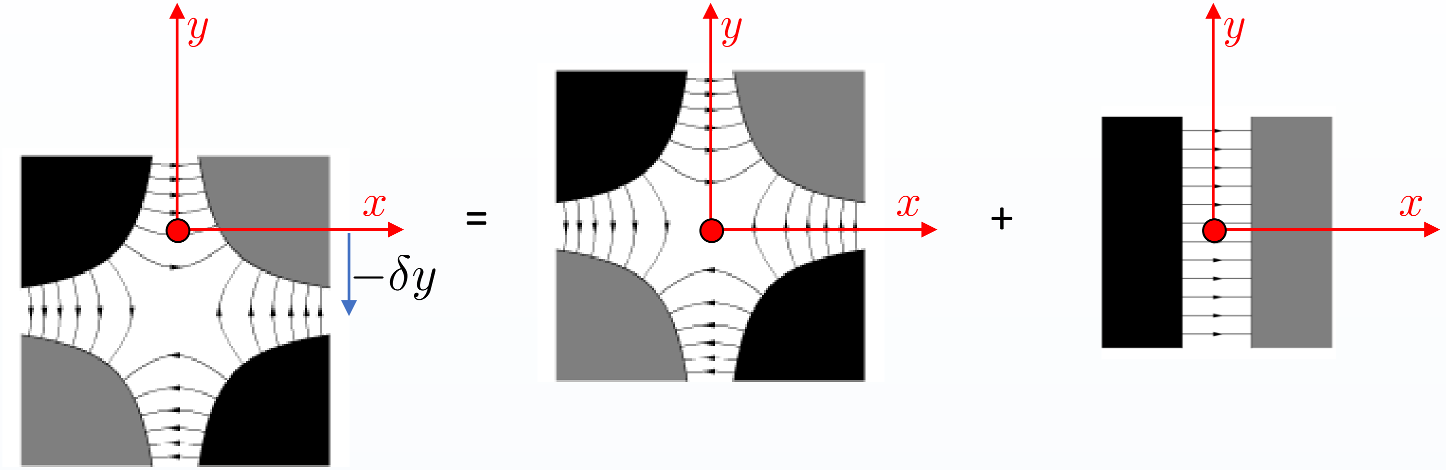
quadrupole dipole

$$B_y(\bar{x}, y) = B_y(x + \delta x, y) = G(x + \delta x) = \overbrace{Gx} + \overbrace{G\delta x}$$

horizontal offset creates horizontal (normal) dipole

Misalignments causing feed-down

- Vertical misalignment of a quadrupole magnet
  - Equivalent to perfectly aligned quadrupole plus small dipole



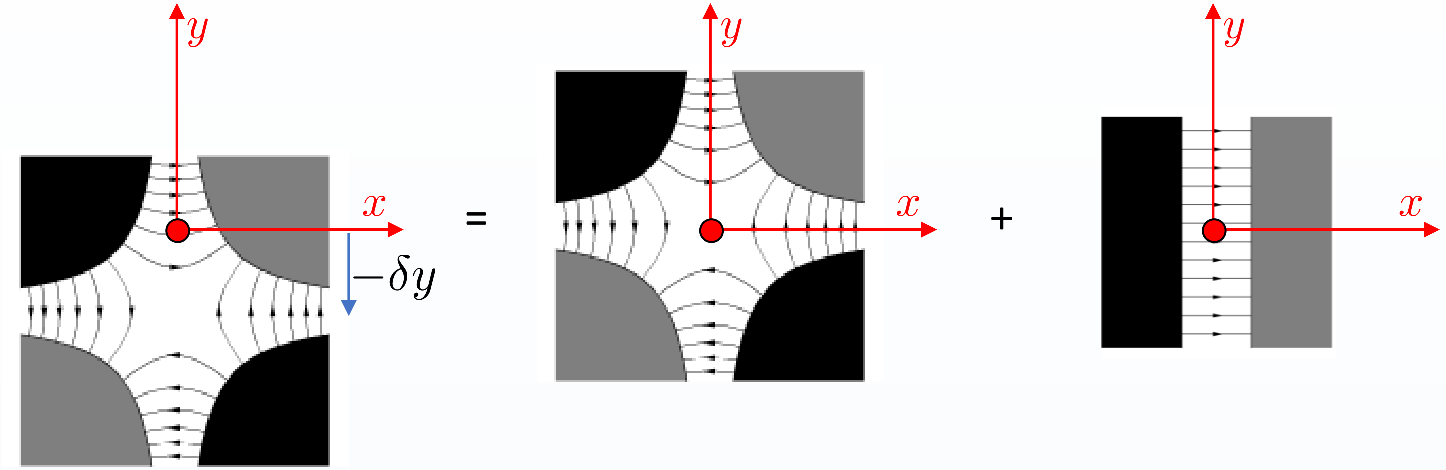
$$B_x(x, y+\delta y) = G(y+\delta y) = \underbrace{Gy}_{\text{quadrupole}} + \underbrace{G\delta y}_{\text{dipole}}$$

$$B_y(x, y+\delta y) = G(x) \qquad = \overbrace{Gx}$$

vertical offset creates vertical (skew) dipole

Misalignments causing feed-down

- Vertical misalignment of a quadrupole magnet
  - Equivalent to perfectly aligned quadrupole plus small dipole



$$B_x(x, y+\delta y) = G(y+\delta y) = \underbrace{Gy}_{\text{quadrupole}} + \underbrace{G\delta y}_{\text{dipole}}$$

$$B_y(x, y+\delta y) = G(x) \qquad = \overbrace{Gx}$$

vertical offset creates vertical (skew) dipole

Illustration of closed orbit distortion

2. Ideal machine toy model with dipole error (unintended deflection) somewhere
   a) Particle injected on the design orbit … receives dipole kick every turn

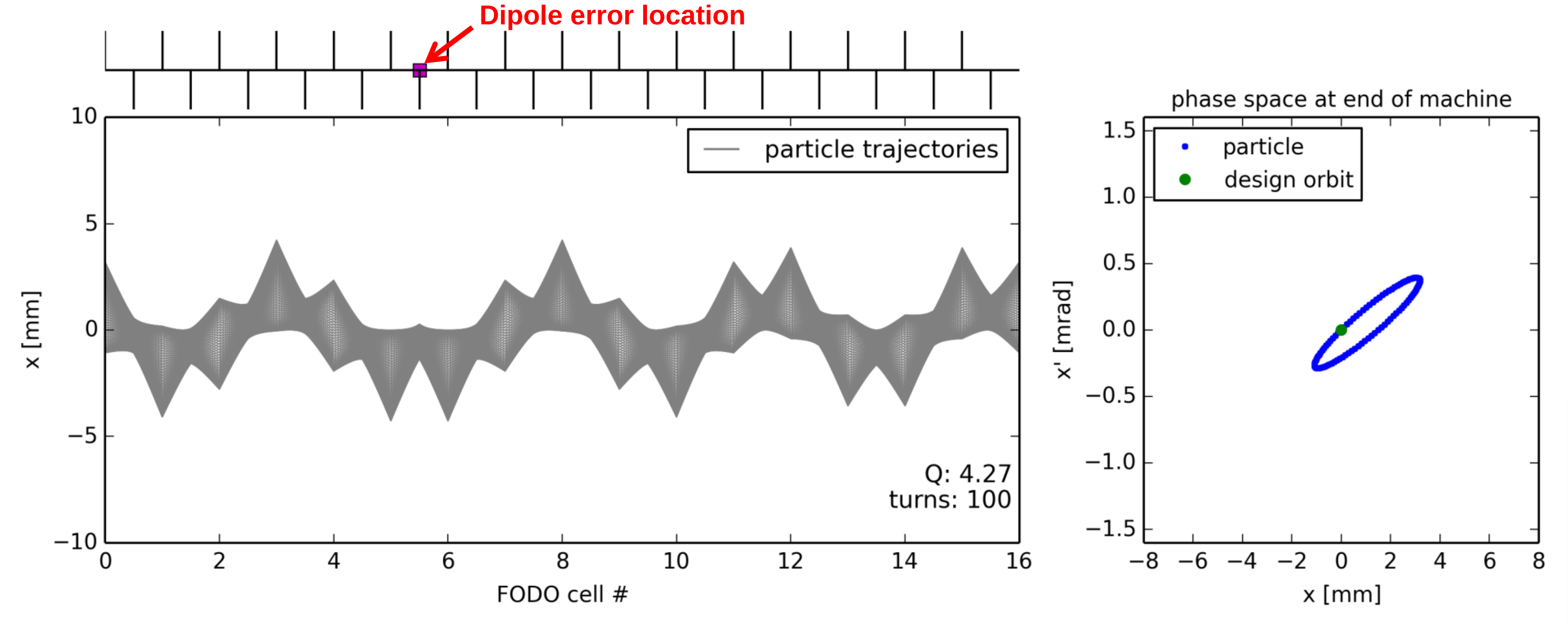


Illustration of closed orbit distortion

2. Ideal machine toy model with dipole error (unintended deflection) somewhere
   a) Particle injected on the design orbit … receives dipole kick every turn … and consequently performs betatron oscillation around a **distorted closed orbit**

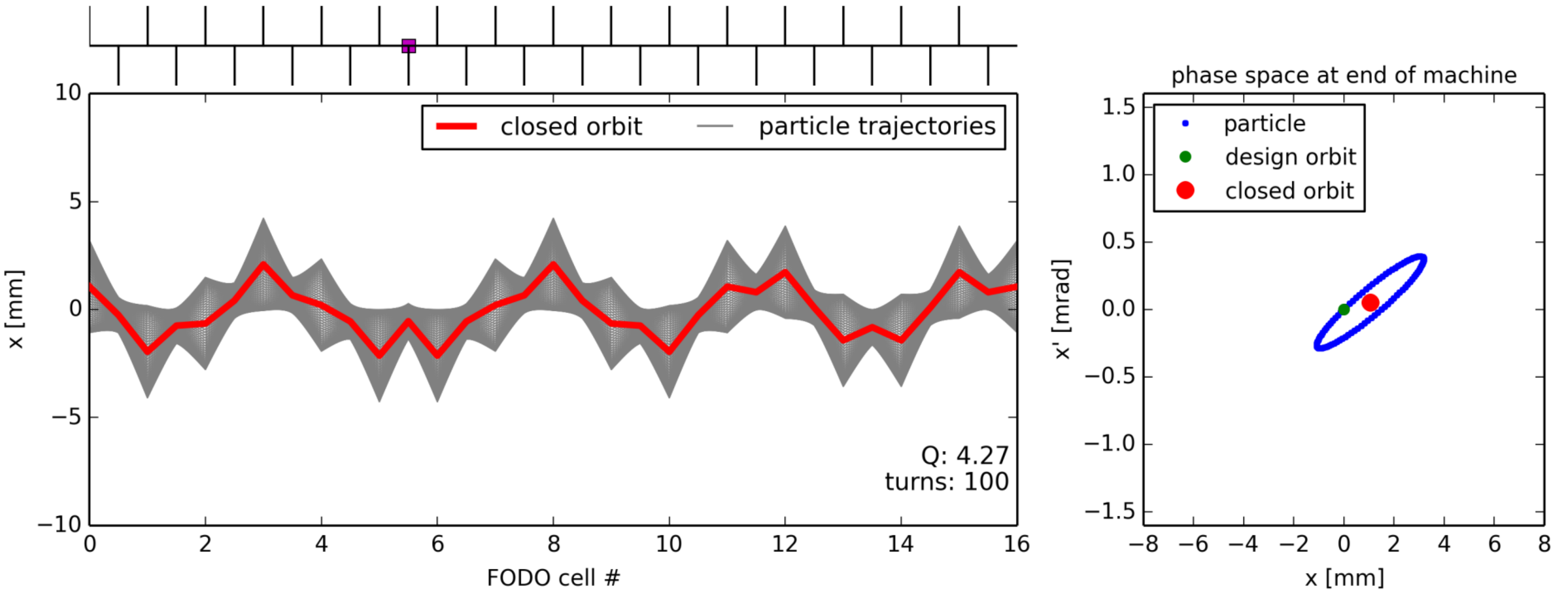


Illustration of closed orbit distortion

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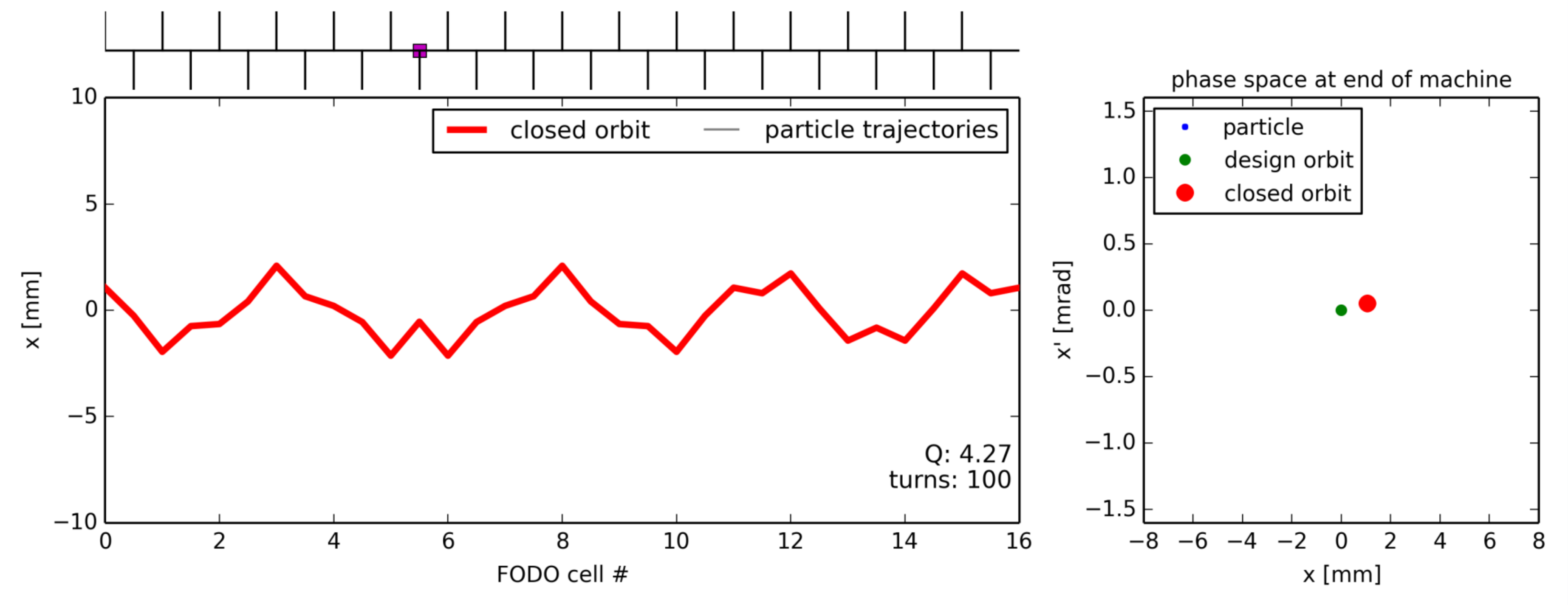
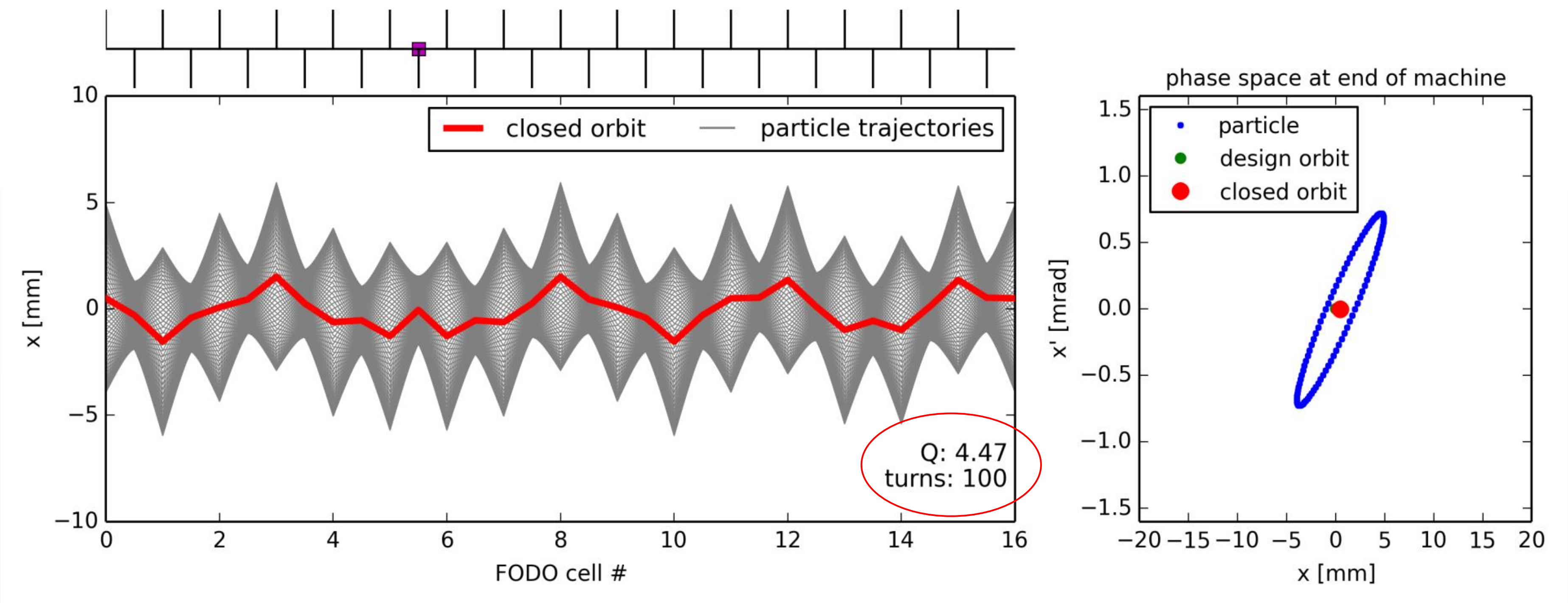
   b)

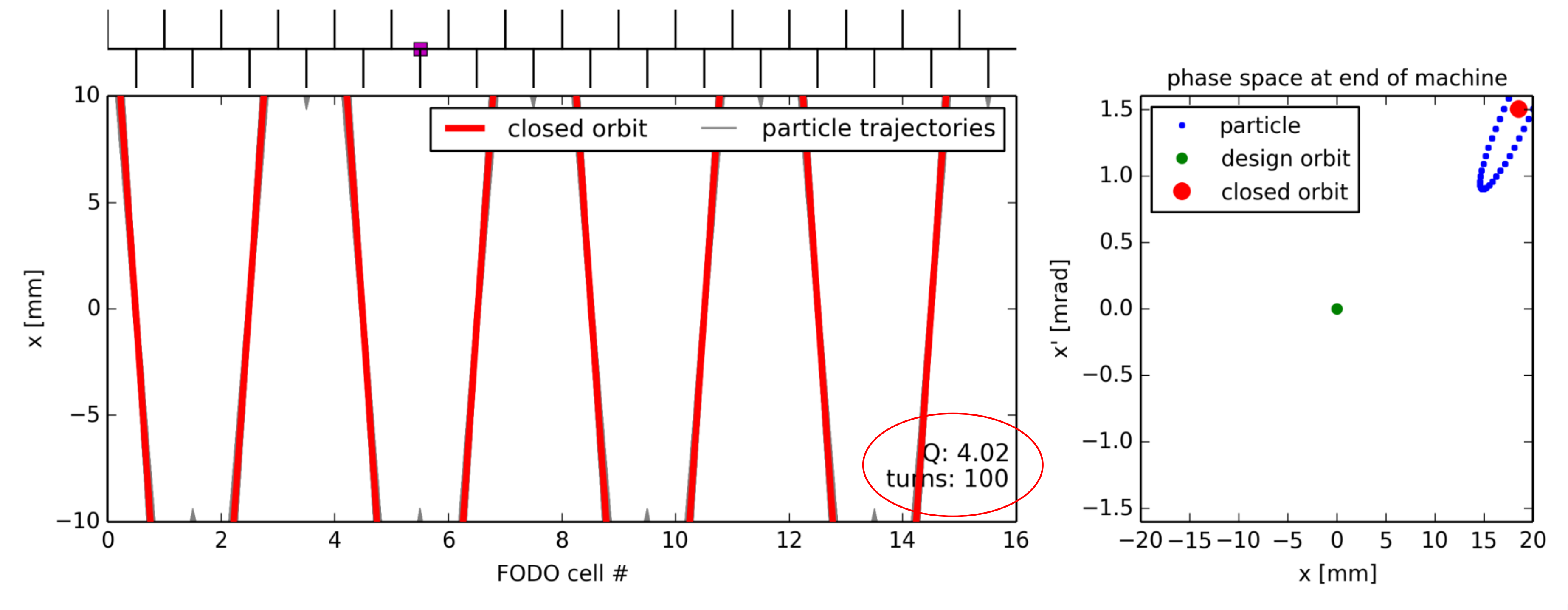


Single dipole kick vs. tune



Single dipole kick vs. tune

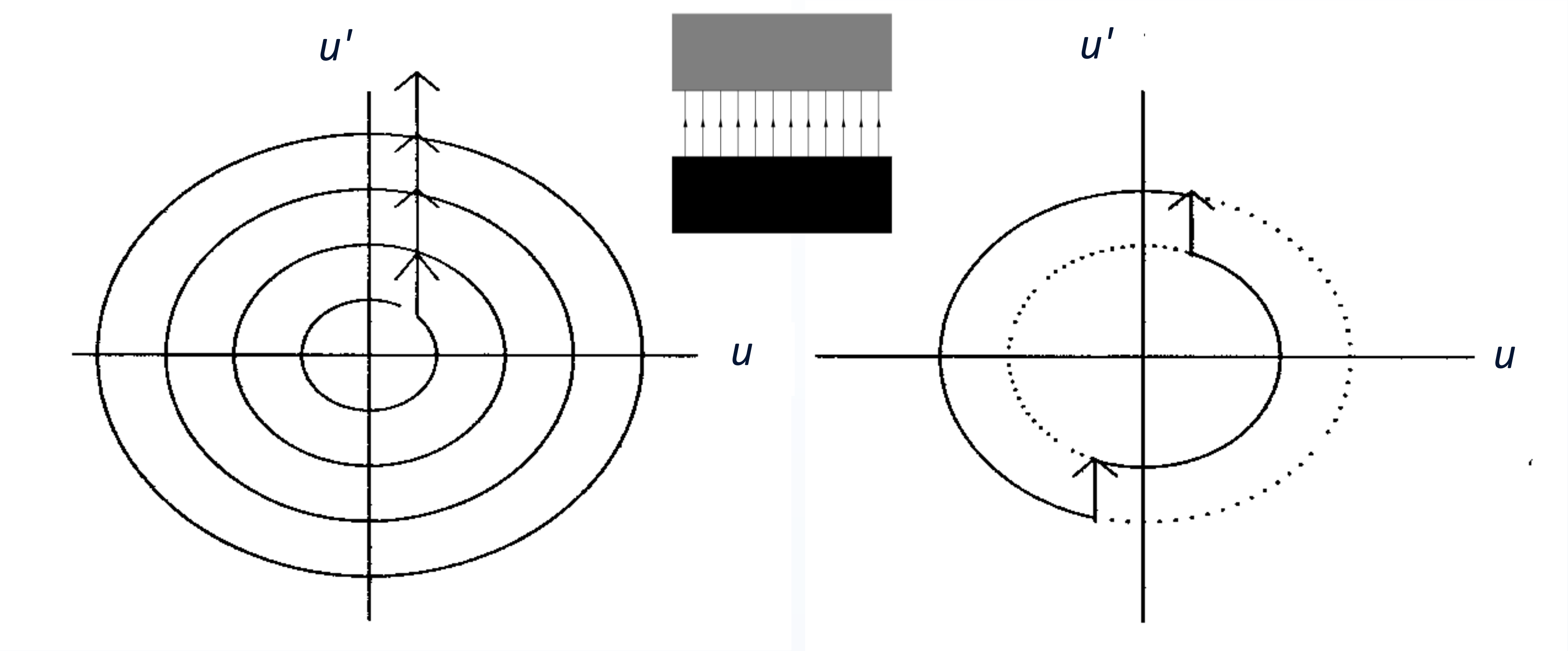
- ***Note:*** the closed orbit distortion propagates with the betatron phase advance (e.g. **single kick induces 4 oscillations for a tune of Q=4.x**)
- Closed orbit distortion is most critical for **tunes close to integer → closed orbit becomes unstable (but beam size not affected)**



Integer and half integer resonance

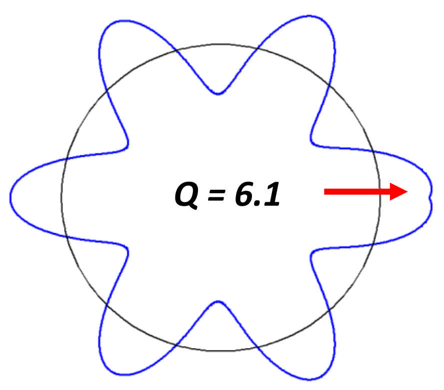
- Dipole kicks add-up in consecutive turns for **Q = n**
- **Integer tune** excites orbit oscillations (**resonance**)
    - **orbit becomes unstable!**

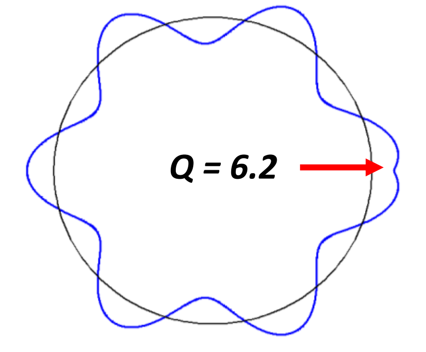
- Dipole kicks get cancelled in consecutive turns for **Q = n+1/2**
- **Half-integer tune cancels orbit oscillations**

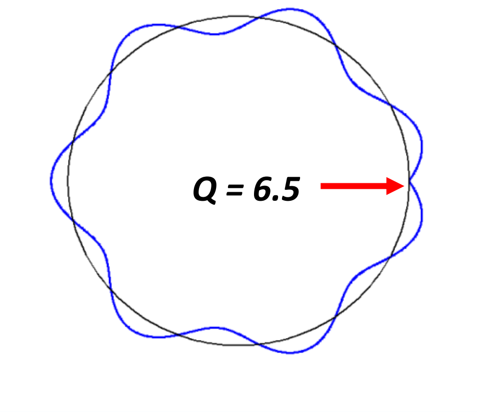


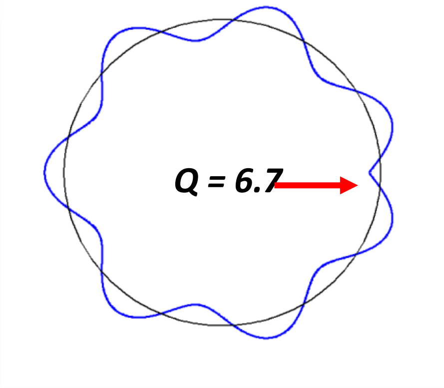
Closed orbit examples

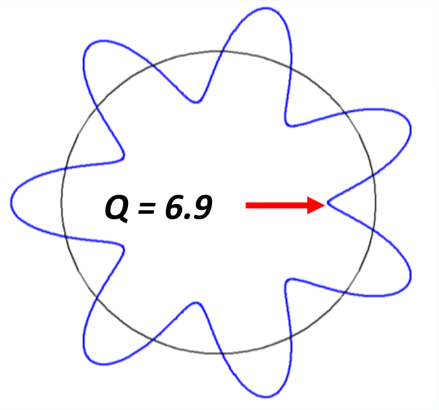
- Example of horizontal closed orbit for a machine with tune Q = 6.x
- The kink at the location of the deflection (→) can be used to localize the deflection (if it is not known) → can be used for orbit correction.





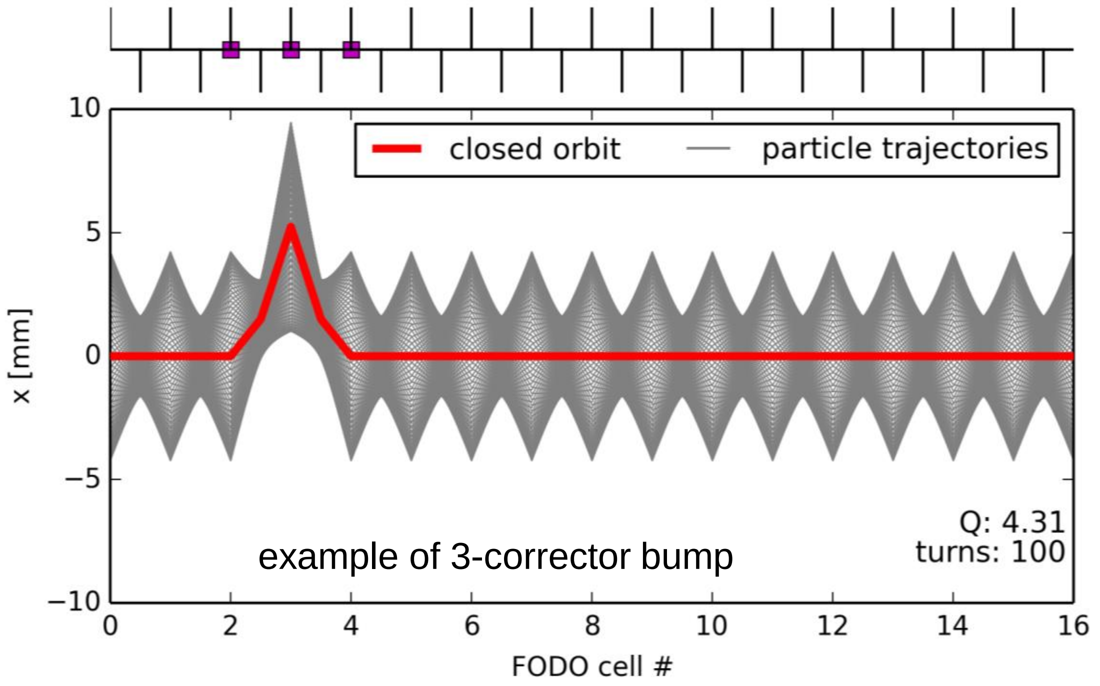




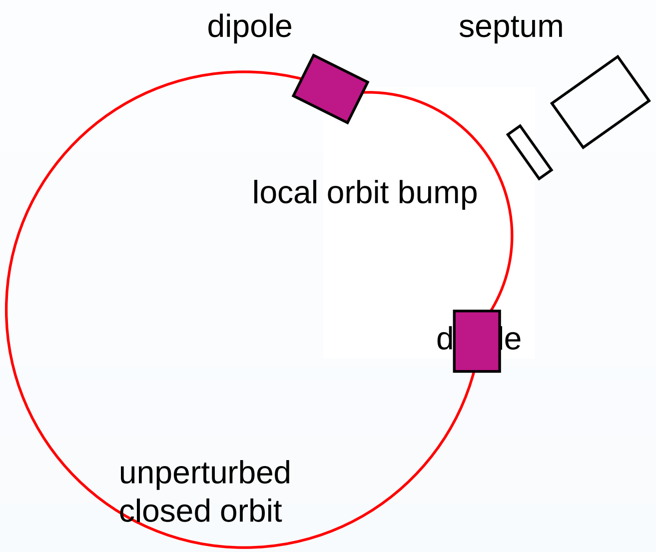


Closed orbit bumps

- Often it is needed to steer the closed-orbit away from the nominal trajectory in a localized part of a synchrotron
  - Injection / extraction
  - Local orbit correction (or steering around local aperture restrictions)
- Standard bump configurations exist
  - π-bump (with 2 correctors)
  - 3 and 4-corrector bumps



example of 2-corrector bump



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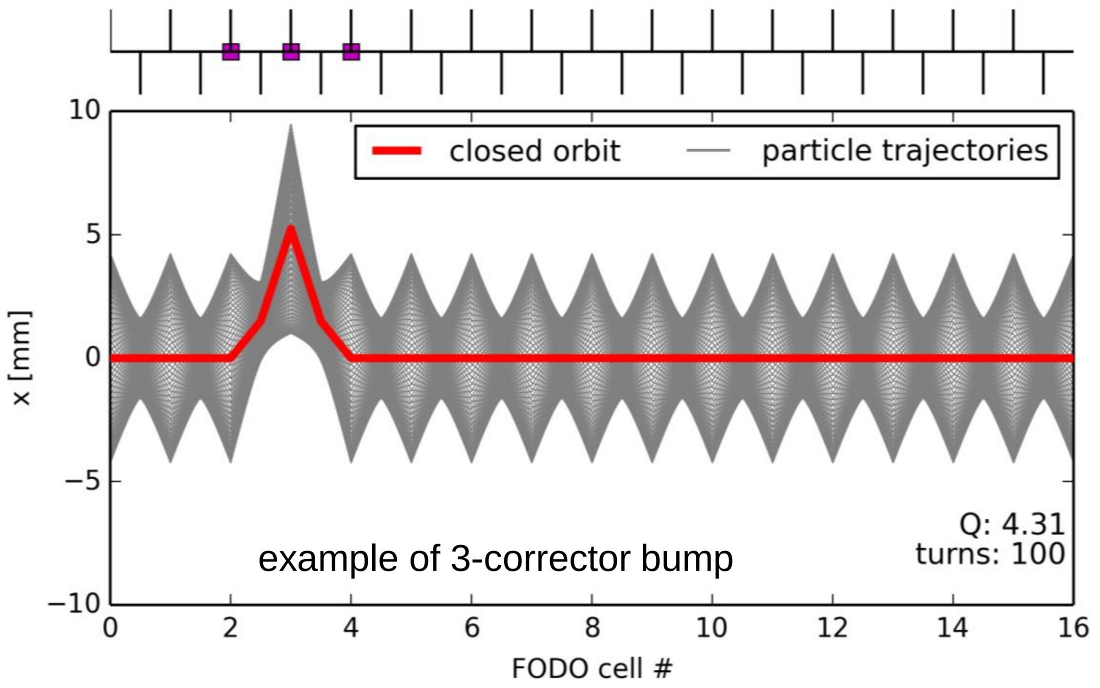
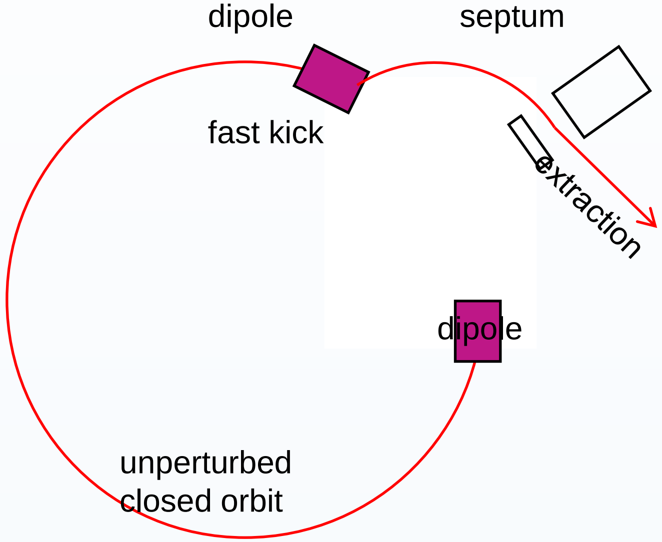
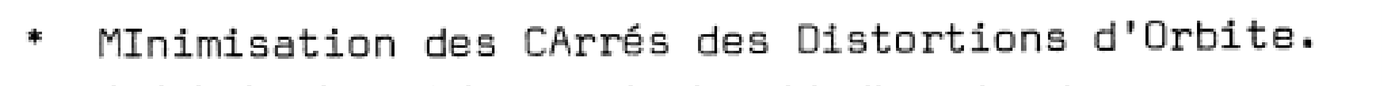


example of 2-corrector bump



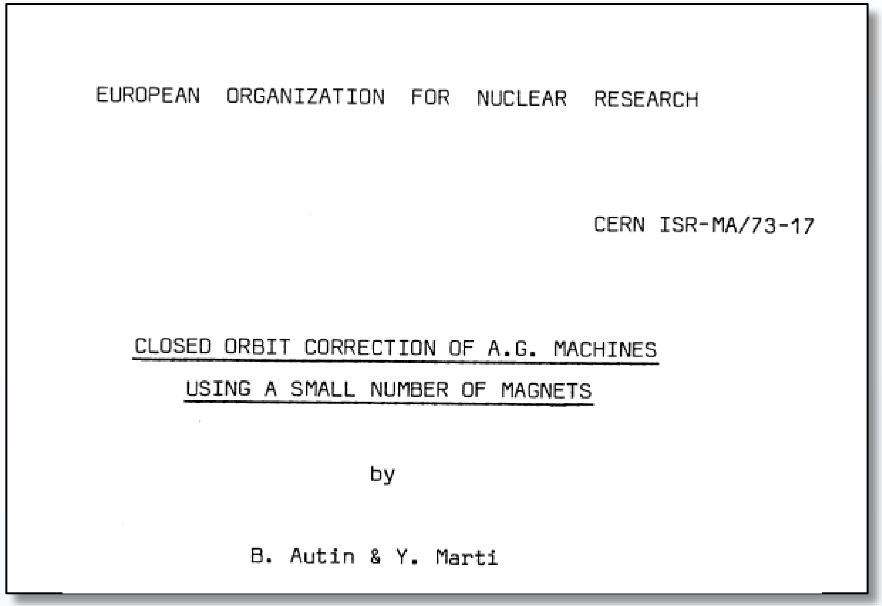
Closed orbit correction: MICADO

- The problem of correcting the orbit deterministically came up a long time ago in the first CERN machines.
- B. Autin and Y. Marti published a note in 1973 describing an algorithm that is still in use today (but in JAVA/C/C++ instead of FORTRAN) at ALL CERN machines: **MICADO***



* MInimisation des CArrés des Distortions d'Orbite.

(Minimization of the quadratic orbit distortions)



EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN ISR-MA/73-17

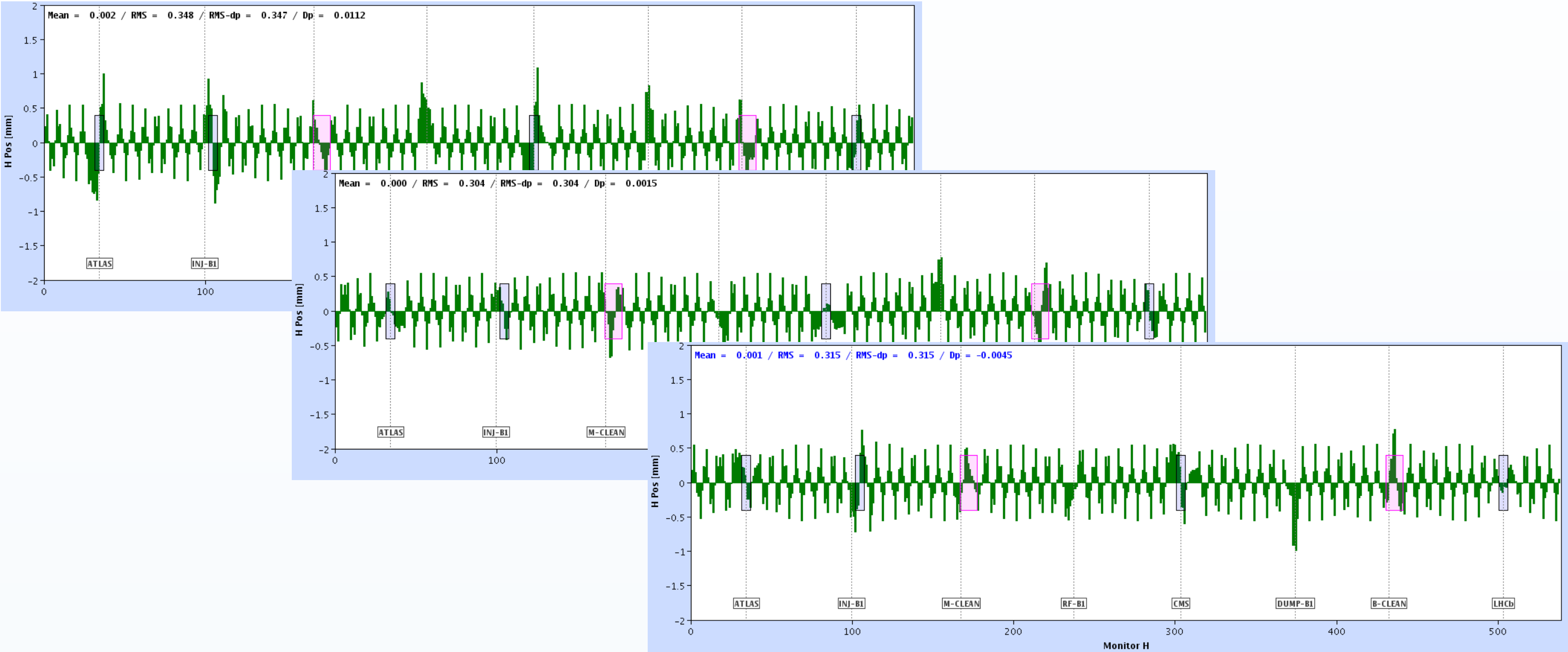
CLOSED ORBIT CORRECTION OF A.G. MACHINES
USING A SMALL NUMBER OF MAGNETS

by

B. Autin & Y. Marti

MICADO – how does it work?

- The intuitive principle of MICADO is rather simple:
  1. It requires a **model of the machine**
  2. It computes for **each orbit corrector** what the effect (**response**) is expected to be on the orbit



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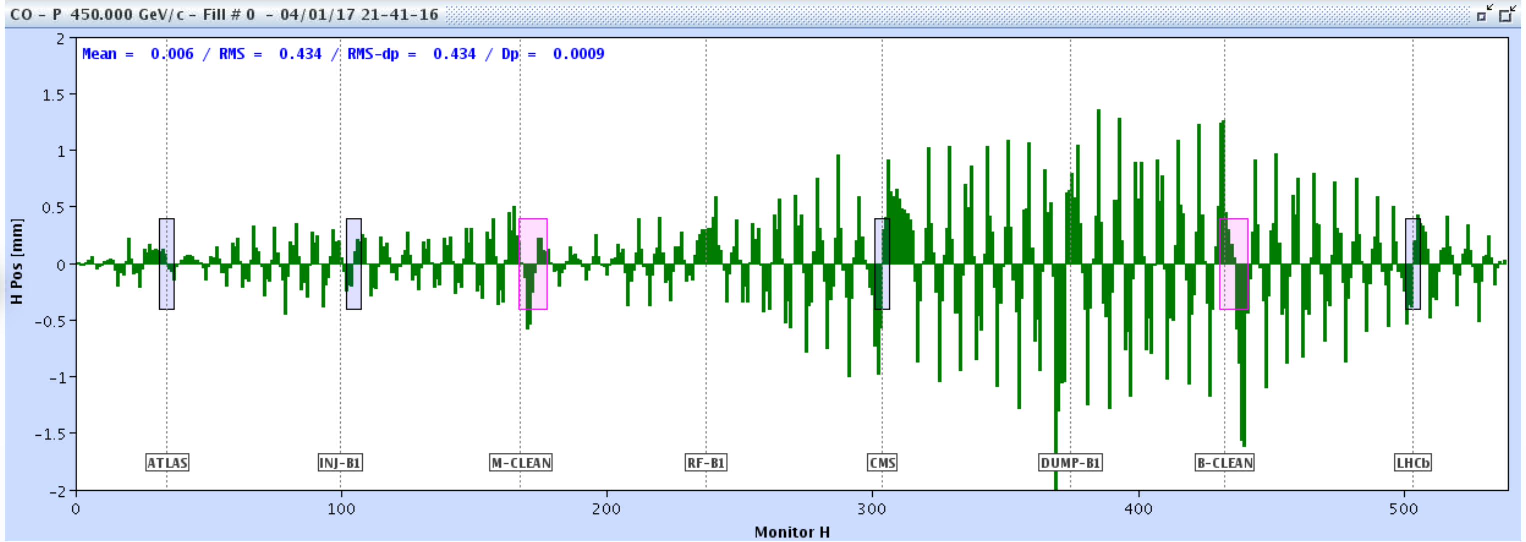
MICADO – how does it work?

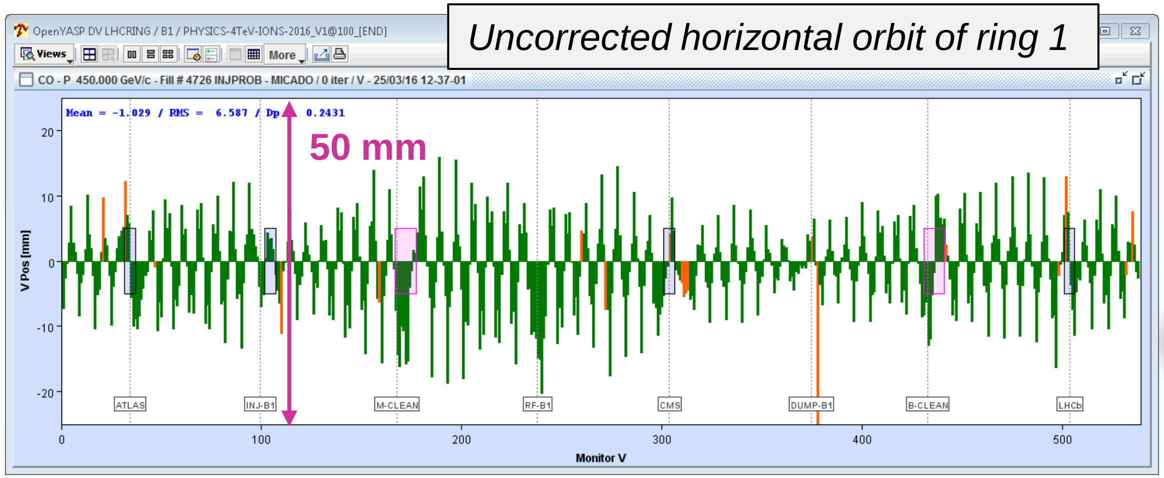
3. MICADO compares the response of every corrector with the raw orbit in the machine



4. MICADO **picks the corrector that has the best match with the raw orbit**, i.e. that will give the largest improvement to the rms orbit deviation
5. The procedure can be **iterated** to the second-best corrector and so on, until the orbit is good enough (or as good as it can be)

MICADO – LHC Orbit example

- The raw orbit at the LHC can have huge errors, but the correction (based partly on MICADO) brings the deviations down by a factor 20

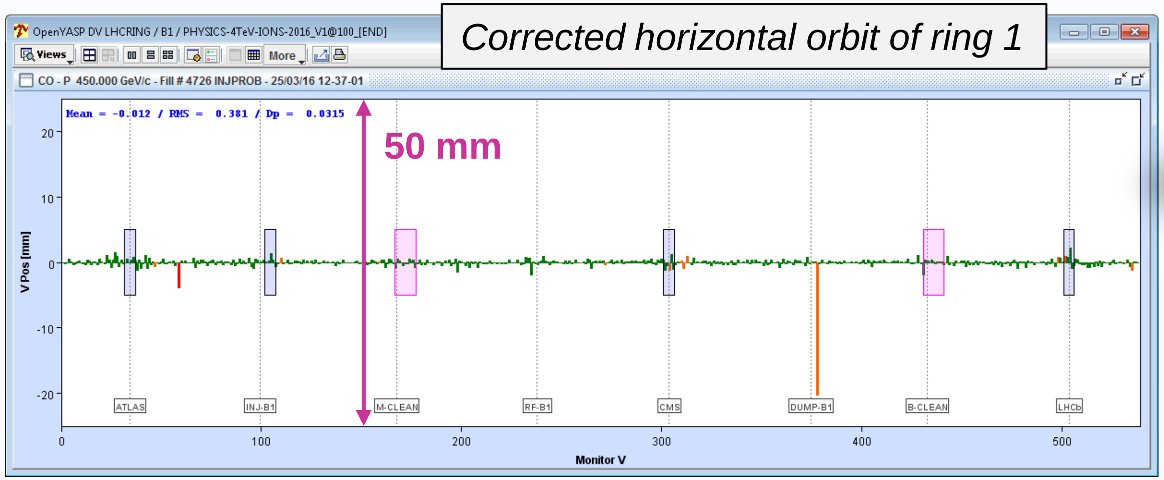


Uncorrected horizontal orbit of ring 1

OpenYASP DV LHCRING / B1 / PHYSICS-4TeV-IONS-2016_V1@100_[END]

CO - P 450.000 GeV/c - Fill # 4726 INJPROB - MICADO / 0 iter / V - 25/03/16 12-37-01

Mean = -1.029 / RMS = 6.587 / Dp 0.2431



Corrected horizontal orbit of ring 1

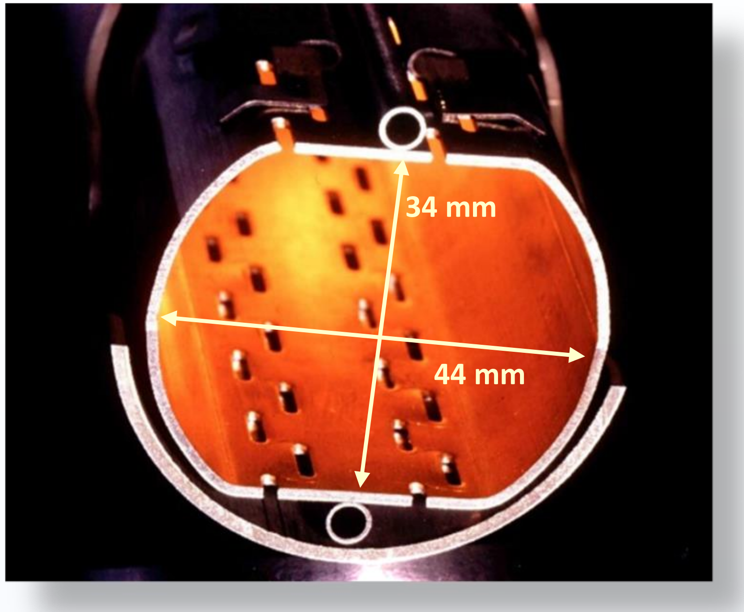
OpenYASP DV LHCRING / B1 / PHYSICS-4TeV-IONS-2016_V1@100_[END]

CO - P 450.000 GeV/c - Fill # 4726 INJPROB - 25/03/16 12-37-01

Mean = -0.012 / RMS = 0.381 / Dp = 0.0315

MICADO & Co

LHC vacuum chamber



At the LHC a good orbit correction is vital !

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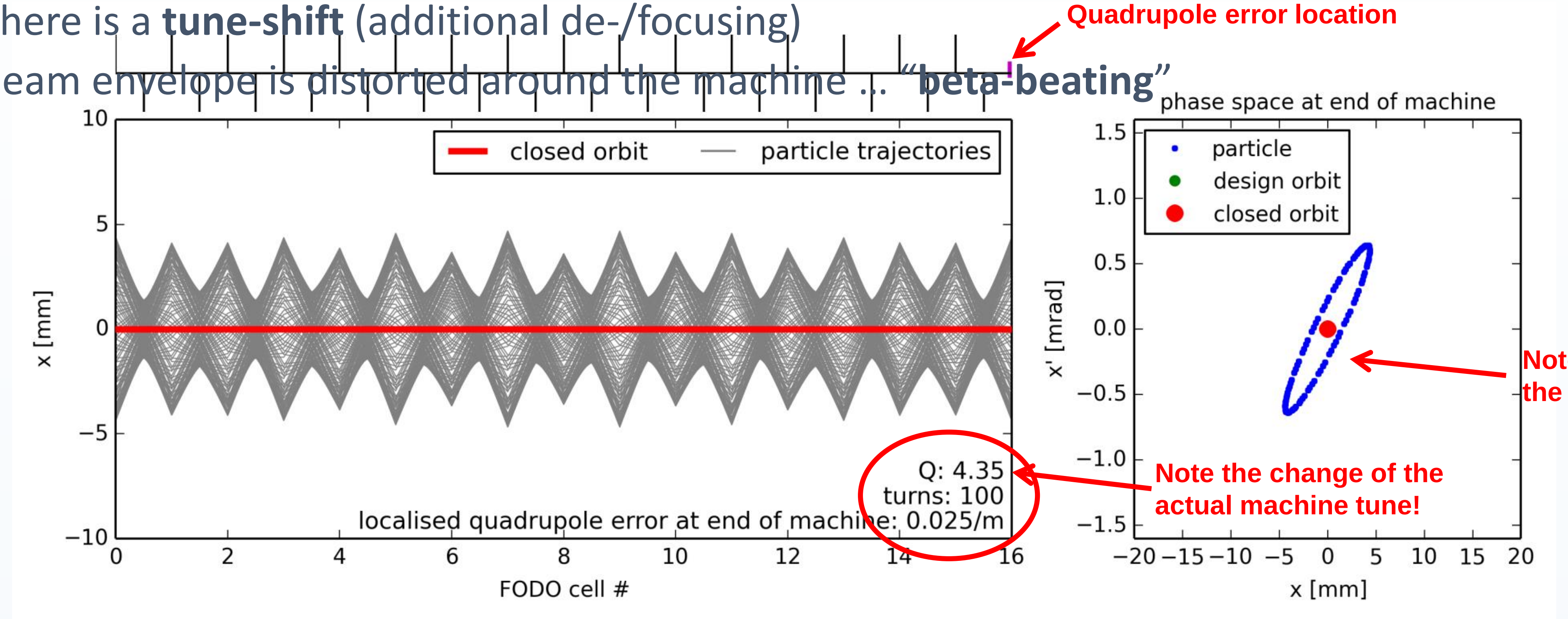
Illustration of optics distortion

- Ideal machine toy model with regular FODO lattice and quadrupole error at the end of circumference
- Particle injected with offset performs betatron oscillations but gets additional focusing from quadrupole error
  - There is a **tune-shift** (additional de-/focusing)
  - Beam envelope is distorted around the machine … “**beta-beating**”



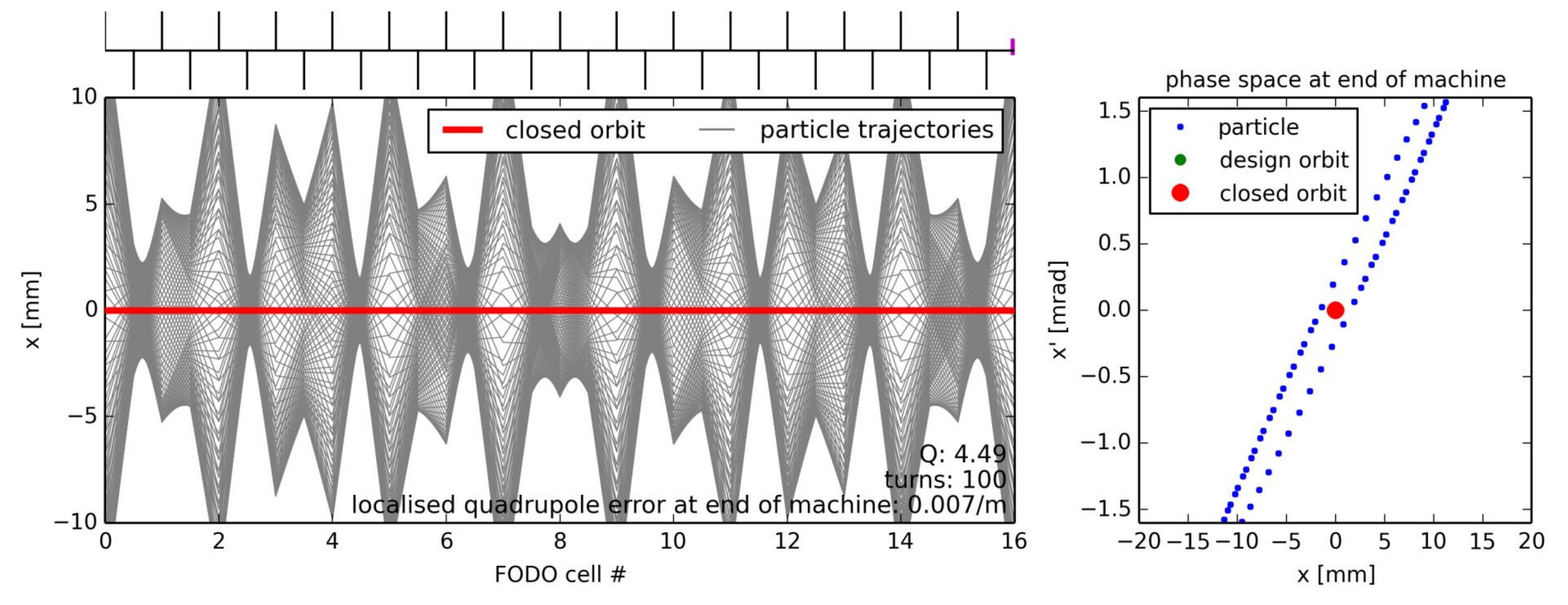
Quadrupole error location

Note the change of the ellipse shape!

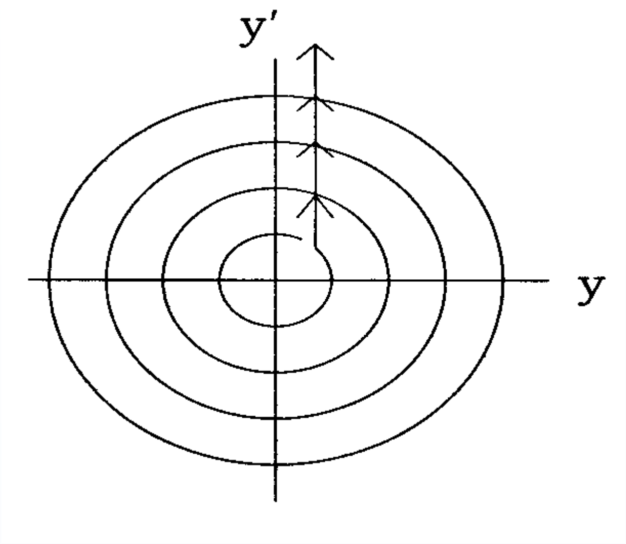
Note the change of the actual machine tune!

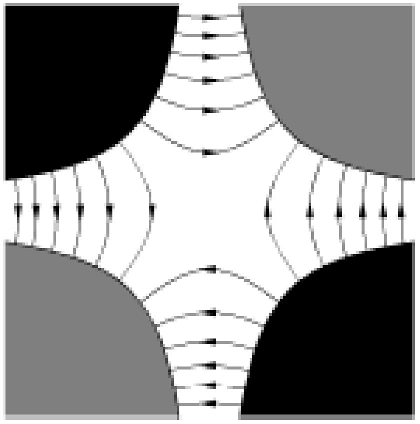
Optics distortion vs. tune

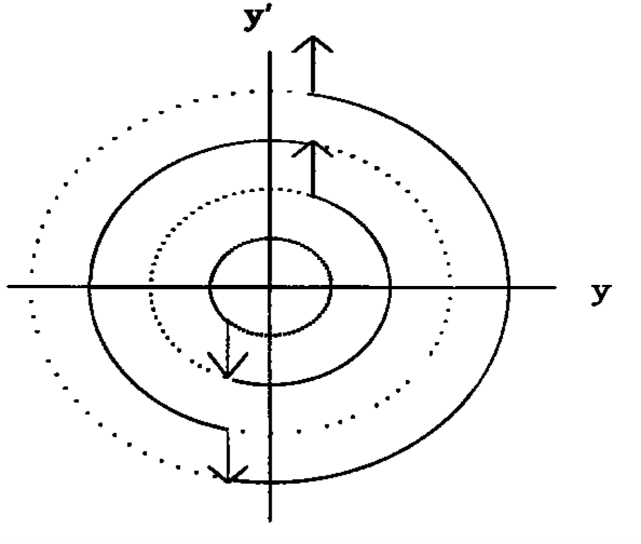
- For which configuration quadrupole errors have biggest impact on optics?
  - **Close to integer and half integer tunes → envelope (or beam size) becomes unstable**



Quadrupole error in phase space







Q = n (integer)
→ kicks from quadrupoles add up
(same as for kicks from dipoles)

Q = n/2 (half integer)
→ kicks from quadrupoles also add up
(while kicks from dipoles cancel)

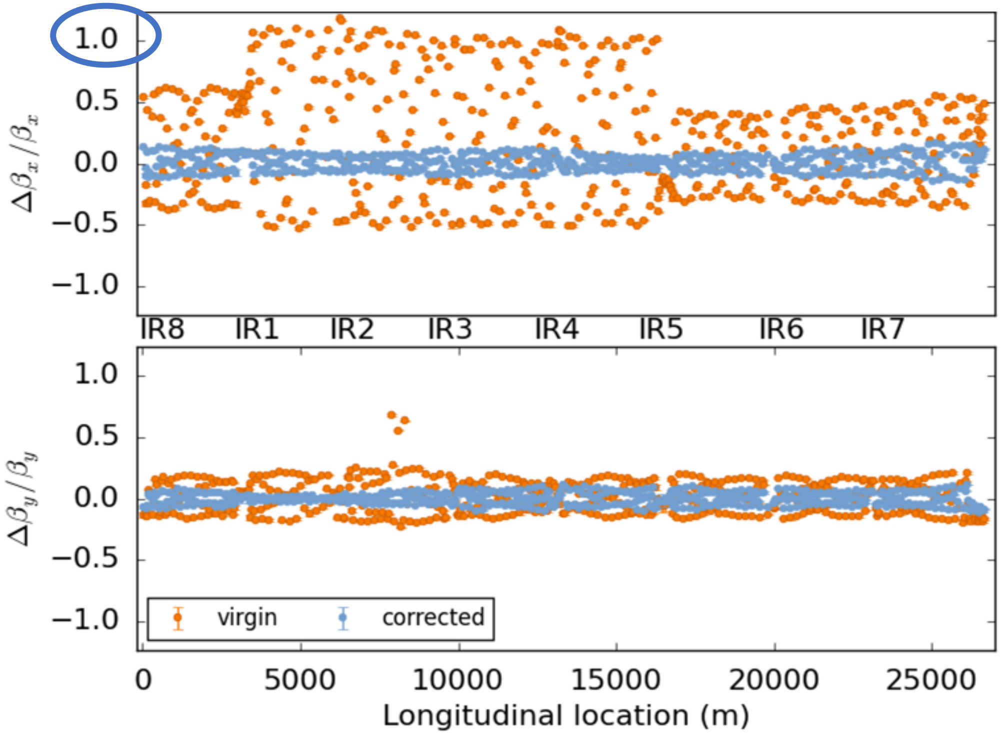
- Therefore, **integer tunes and half integer tunes need to be avoided for machine operation** to avoid beam envelope becoming unstable due to quadrupole errors
- *Recall:* for integer tunes dipole errors drive the closed orbit unstable, but for half integer tunes they have minimum effect

Example: LHC optics corrections

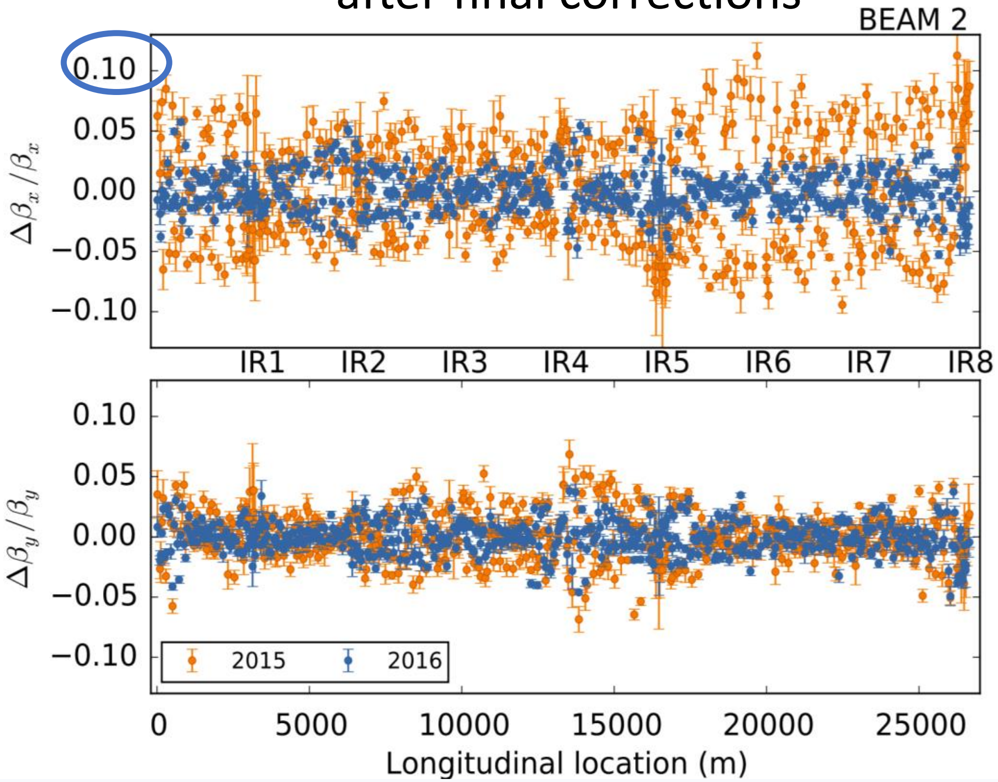
- At β*=40cm, the bare machine has a beta-beat of more than 100%
- After global and local corrections, β-beating was reduced to few %

R. Tomas et al.

before and after local correction



after final corrections

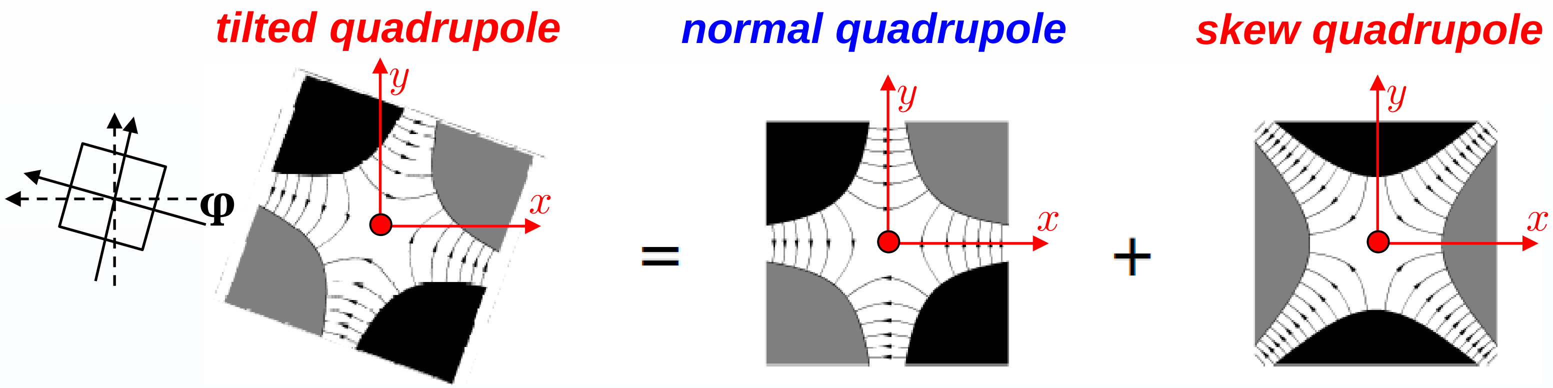


Outline

- Introduction
- Closed orbit distortion (steering error)
  - ❑Beam orbit stability
  - ❑Imperfections leading to closed orbit distortion
- Optics function distortion (gradient error)
  - ❑Imperfections leading to optics distortion
  - ❑Tune-shift and beta distortion due to gradient errors
- Coupling error
  - ❑Coupling errors and their effect
  - ❑Coupling correction
- Summary

Coupling errors

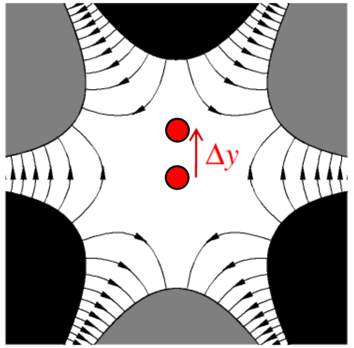
- Coupling may result from **rotation of a quadrupole**, so that the field contains a skew quadrupole component



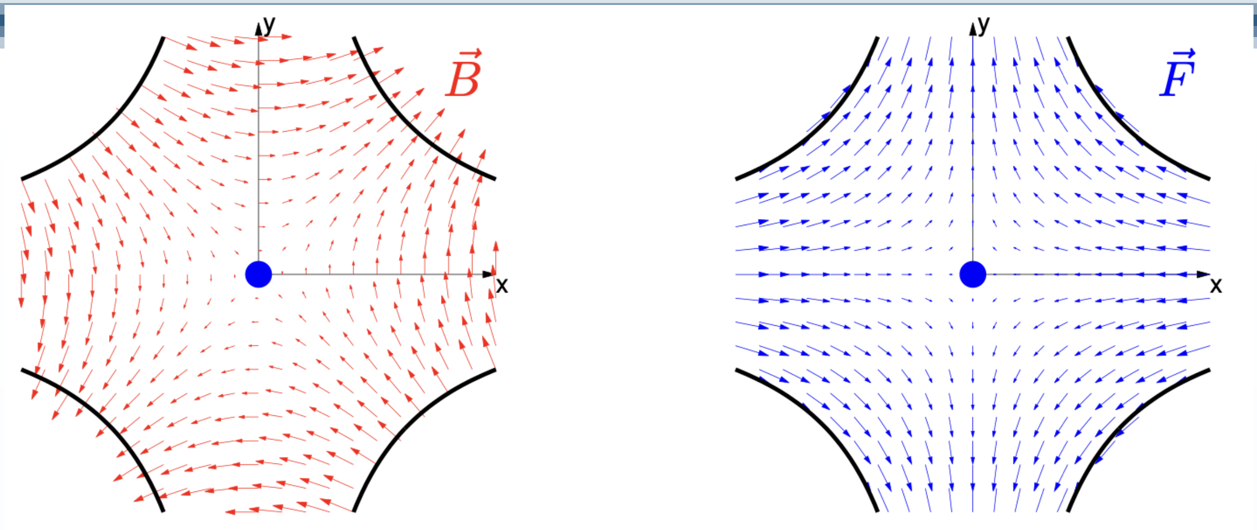
- A **systematic vertical offset in a sextupole** has the same effect as a **skew quadrupole**.
  - For a displacement of δy the field becomes

$$B_x = 2B_2x\bar{y} = 2B_2xy + \underbrace{2B_2x\delta y}_{\text{skew quadrupole}}$$

$$B_y = B_2(x^2 - \bar{y}^2) = \overbrace{-2B_2y\delta y} + B_2(x^2 - y^2) - B_2(\delta y)^2$$



Normal vs. skew quadrupole

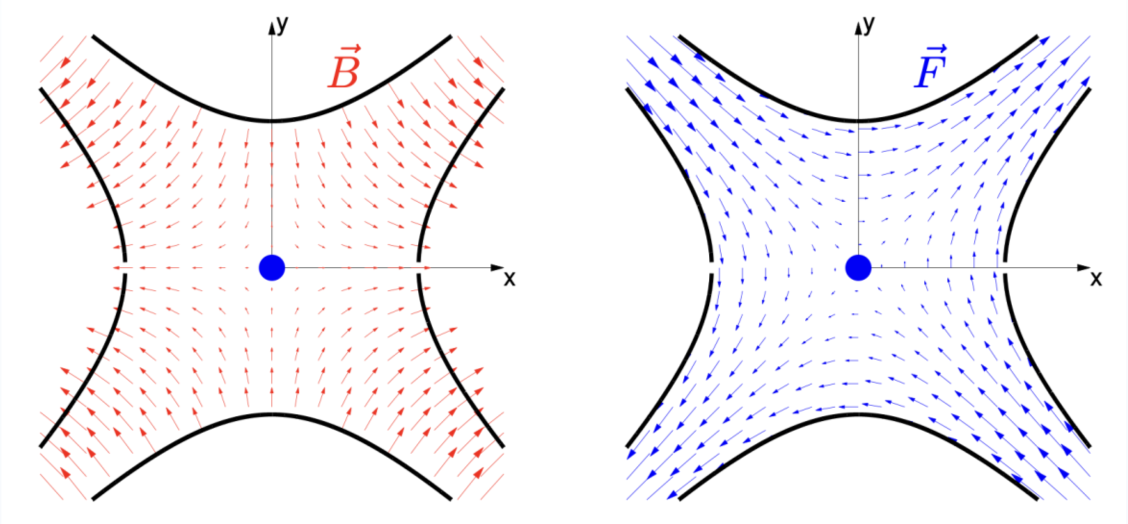


normal quadrupole

$$F_x = -kx$$

$$F_y = +ky$$

horizontal force depends on horizontal position (and likewise for vertical)



skew quadrupole

$$F_x = k_s y$$

$$F_y = k_s x$$

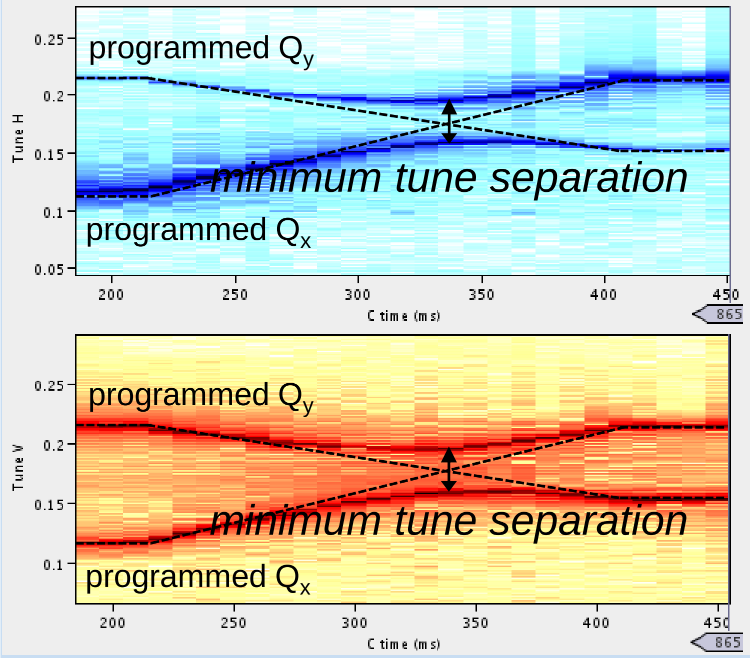
horizontal force depends on vertical position (and vice versa)

→ resulting forces couples the motion in the two planes

Closest tune approach

Tune measurement in the CERN PS

quadrupole setting
changed dynamically
during storage time



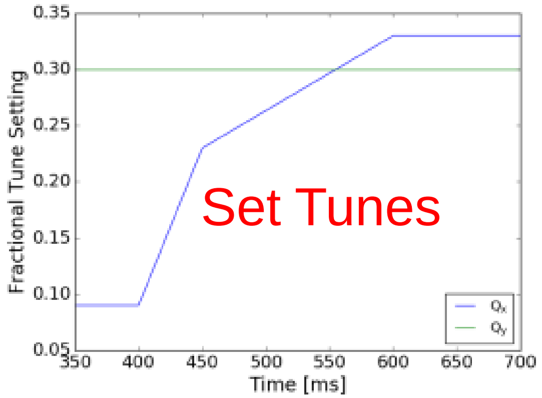
tune peaks from both planes
visible in Fourier spectra of
horizontal and vertical motion

$$\Delta Q_{min} = |C^-|$$

- Coupling makes it impossible to approach the tunes below where C^- is the coupling coefficient
- The coupling coefficient C^- can be measured by trying to approach the tunes and measure the minimum distance

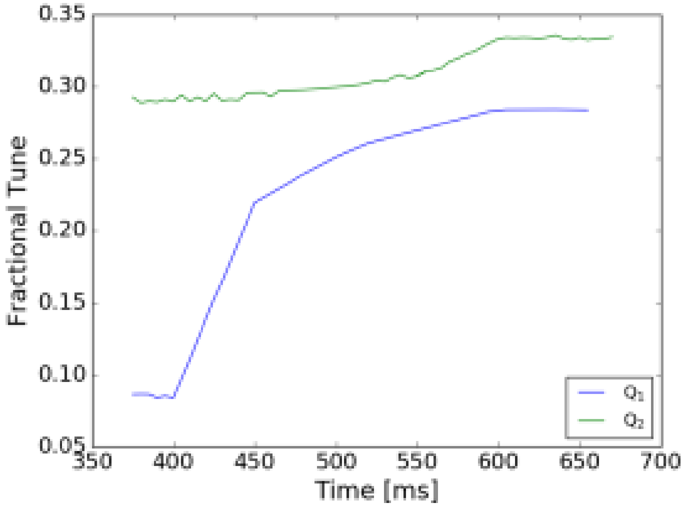
Closest tune approach

Tune measurement & coupling correction in the CERN PS



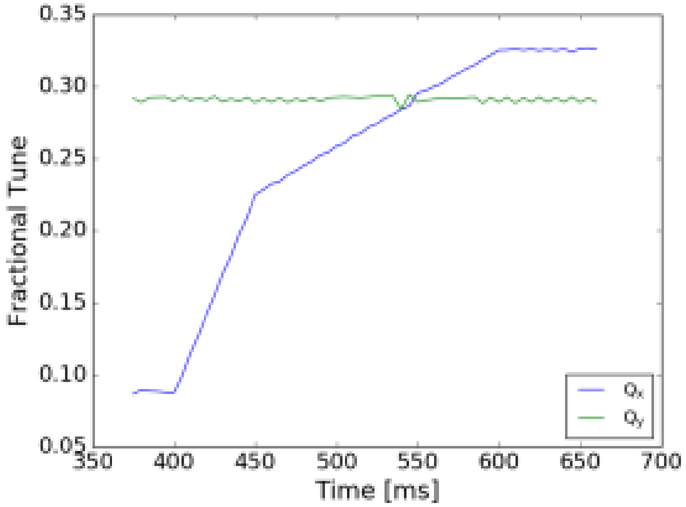
Set Tunes

Measured Tunes



Using
Skew Quads

Measured Tunes
+ coupling correction



Summary of linear imperfections

- **Linear imperfections,** such as magnet misalignments and field errors, **are unavoidable** in a real accelerator, but they can be corrected to some extent as summarized in this table:

Error	Effect	Cure
fabrication imperfections	unwanted multipolar components	better fabrication / multipolar correctors coils
transverse misalignments	feed-down effect	better alignment / correctors
dipole kicks	orbit distortion / residual dispersion	corrector dipoles
quadrupole field errors	tune shift, beta-beating	trim special quadrupoles
quadrupole tilts	coupling x – y	better alignment / skew quads
power supplies	closed orbit distortion / tune shift / modulation	improve power supplies and their calibration

Interlude: multipole expansion

- Multipole expansion of transverse magnetic field
- Start from the general expression for the transverse magnetic field in terms of multipole coefficients

$$\mathbf{B}(x,y) = B_y(x,y) + iB_x(x,y) = \sum_{n=0}^{\infty}(B_n + iA_n)(x+iy)^n$$

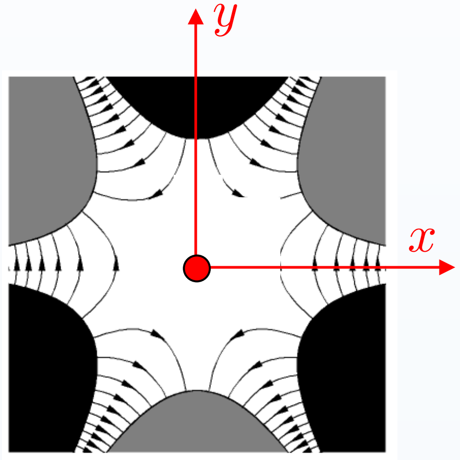
Normal components ("upright" magnets)

Skew components (magnets rotated by $\frac{\pi}{2(n+1)}$)

Note that: $B_n = \frac{1}{n!}\frac{\partial^n B_y}{\partial x^n}\Big|_{(0,0)}$

and: $A_n = \frac{1}{n!}\frac{\partial^n B_x}{\partial y^n}\Big|_{(0,0)}$

e.g. The magnetic field normal sextupole is $\mathbf{B}(x,y) = B_2(x^2 - y^2) + i2B_2xy$



$$\frac{1}{1!}\frac{\partial B_y(x,y)}{\partial x}\Big|_{(0,0)} = 2B_2x|_{(0,0)} = 0$$

$$\frac{1}{2!}\frac{\partial^2 B_y(x,y)}{\partial x^2}\Big|_{(0,0)} = \frac{1}{2!}2B_2\Big|_{(0,0)} = B_2$$

$$\frac{1}{3!}\frac{\partial^3 B_y(x,y)}{\partial x^3}\Big|_{(0,0)} = 0$$

…

$$\frac{1}{1!}\frac{\partial B_x(x,y)}{\partial y}\Big|_{(0,0)} = 2B_2x|_{(0,0)} = 0$$

$$\frac{1}{2!}\frac{\partial^2 B_x(x,y)}{\partial y^2}\Big|_{(0,0)} = 0$$

…

Feed-down from multipoles

- Let’s **explicitly** write the vertical/horizontal **field** as the **sum of all multipole** components

normal skew

$$B_y = \underbrace{B_0}_{\text{dipole}} + \underbrace{B_1 x - A_1 y}_{\text{quadrupole}} + \underbrace{B_2(x^2 - y^2) - 2A_2xy}_{\text{sextupole}} + \underbrace{B_3(x^3 - 3xy^2) - A_3(-y^3 + 3x^2y)}_{\text{octupole}} + \dots$$

$$B_x = \overbrace{A_0}^{} + \overbrace{A_1 x + B_1 y}^{} + \overbrace{A_2(x^2 - y^2) + 2B_2xy}^{} + \overbrace{A_3(x^3 - 3xy^2) + B_3(-y^3 + 3x^2y)}^{} + \dots$$

- A **horizontal offset** (-*δx*) in a normal(skew) magnet of **order *n*** creates normal(skew) **feed-down components** at y=0 **of all lower orders!**:

(normal)

$$\begin{cases} B_x(y=0) = 0 \\ B_y(y=0) = \underbrace{B_n\bar{x}^n}_{2(n+1)\text{-pole}} = B_n(x+\delta x)^n = B_n(\underbrace{x^n}_{2(n+1)\text{-pole}} + \underbrace{n\delta x x^{n-1}}_{2n\text{-pole}} + \underbrace{\frac{n(n-1)}{2}\delta x^2 x^{n-2}}_{2(n-1)\text{-pole}} + \dots + \underbrace{(\delta x)^n}_{\text{dipole}}) \end{cases}$$

(skew)

$$\begin{cases} B_x(y=0) = \overbrace{A_n\bar{x}^n}^{} = A_n(x+\delta x)^n = A_n(\overbrace{x^n}^{} + \overbrace{n\delta x x^{n-1}}^{} + \overbrace{\frac{n(n-1)}{2}\delta x^2 x^{n-2}}^{} + \dots + \overbrace{(\delta x)^n}^{}) \\ B_y(y=0) = 0 \end{cases}$$

Feed-down from multipoles

- Let’s **explicitly** write the vertical/horizontal **field** as the **sum of all multipole** components

$$B_y = \underbrace{B_0}_{\text{dipole}} + \underbrace{\overset{\text{normal}}{B_1 x} - \overset{\text{skew}}{A_1 y}}_{\text{quadrupole}} + \underbrace{B_2(x^2 - y^2) - 2A_2 xy}_{\text{sextupole}} + \underbrace{B_3(x^3 - 3xy^2) - A_3(-y^3 + 3x^2 y)}_{\text{octupole}} + \dots$$

$$B_x = \overbrace{A_0} + \overbrace{A_1 x + B_1 y} + \overbrace{A_2(x^2 - y^2) + 2B_2 xy} + \overbrace{A_3(x^3 - 3xy^2) + B_3(-y^3 + 3x^2 y)} + \dots$$

- A **vertical offset** (-*δy*) in normal(skew) magnets of **order *n*** results in **alternating** skew(normal) and normal(skew) **feed-down components** for x=0 **of all lower orders!**, as can be worked out looking at n-order terms are defined (for x=0):

$$\text{for n = even} \begin{cases} B_y(x=0) = i^n B_n \bar{y}^n \\ B_x(x=0) = i^n A_n \bar{y}^n \end{cases} \qquad \text{for n = odd} \begin{cases} B_y(x=0) = i^{n+1} A_n \bar{y}^n \\ B_x(x=0) = i^{n-1} B_n \bar{y}^n \end{cases}$$

- E.g. **n even**, normal magnet:

Odd exponent,
i.e. it must be a skew component

$$B_y(x=0) = i^n B_n (y + \delta y)^n = i^n B_n (\overset{\text{normal}}{y^n} + n\delta y y^{n-1} + \frac{n(n-1)}{2}\delta y^2 y^{n-2} + \dots + (\delta y)^n)$$

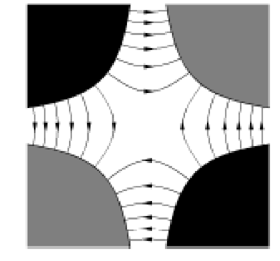
***Even exponent*, i.e. a normal component again …**

Multipole expansion

- Multipole expansion of transverse magnetic field
  - Start from the general expression for the transverse magnetic flux in terms of multipole coefficients

$$\mathbf{B} = B_y + iB_x = \sum_{n=0}^{\infty} (B_n + iA_n) \cdot (x + iy)^n$$

e.g. normal quad



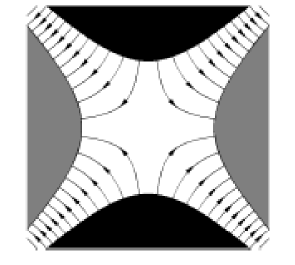
Normal components
(“upright” magnets)

$$B_n = \frac{1}{n!} \frac{\partial^n B_y}{\partial x^n}\bigg|_{(0,0)}$$

Skew components
(magnets rotated by $\frac{\pi}{2(n+1)}$)

$$A_n = \frac{1}{n!} \frac{\partial^n B_x}{\partial y^n}\bigg|_{(0,0)}$$

e.g. skew quad



- In some cases it is more convenient to use “normalized” components:

Normalized normal components

$$k_n = \frac{1}{B_0\rho_0} \frac{\partial^n B_y}{\partial x^n}\bigg|_{(0,0)} = \frac{n!}{B_0\rho_0} B_n\bigg|_{(0,0)}$$

Normalized skew components

$$j_n = \frac{1}{B_0\rho_0} \frac{\partial^n B_x}{\partial y^n}\bigg|_{(0,0)} = \frac{n!}{B_0\rho_0} A_n\bigg|_{(0,0)}$$

so that:

$$B_y + iB_x = B_0\rho_0 \sum_{n=0}^{M} (k_n + ij_n) \frac{(x + iy)^n}{n!}$$

Feed-down from multipoles

- Explicitly: the vertical field is the sum of all multipole components

$$B_y = \underbrace{B_0}_{\text{dipole}} + \underbrace{B_1 x - A_1 y}_{\text{quadrupole}} + \underbrace{B_2(x^2 - y^2) - 2A_2 xy}_{\text{sextupole}} + \underbrace{B_3(x^3 - 3xy^2) + A_3(y^3 - 3x^2 y)}_{\text{octupole}} + \dots$$

- Feed-down: lower order field components from misalignments
  - Systematic horizontal offset in normal (skew) magnets creates normal (skew) feed-down components as seen with $\bar{x} = x + \delta x$ at y=0:

$$B_x(y\!=\!0) = A_n \bar{x}^n = A_n (x + \delta x)^n = A_n\left(x^n + n\delta x x^{n-1} + \tfrac{n(n-1)}{2}\delta x^2 x^{n-2} + \dots + (\delta x)^n\right)$$

$$B_y(y\!=\!0) = B_n \bar{x}^n = B_n (x + \delta x)^n = B_n(\underbrace{x^n}_{\text{2(n+1)-pole}} + \underbrace{n\delta x x^{n-1}}_{\text{2n-pole}} + \underbrace{\tfrac{n(n-1)}{2}\delta x^2 x^{n-2}}_{\text{2(n-1)-pole}} + \dots + \underbrace{(\delta x)^n}_{\text{dipole}})$$

  - Systematic vertical offset in normal magnets results in alternating skew and normal feed-down components (and vice-versa for skew magnets), as can be worked out from

$$\text{for n = even}\begin{cases} B_y(x\!=\!0) = i^n B_n \bar{y}^n \\ B_x(x\!=\!0) = i^n A_n \bar{y}^n \end{cases} \qquad \text{for n = odd}\begin{cases} B_y(x\!=\!0) = i^{n+1} A_n \bar{y}^n \\ B_x(x\!=\!0) = i^{n-1} B_n \bar{y}^n \end{cases}$$