HTE - UKO

INTRODUCTION: PARTICLE PHYSICS

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Physics

Particle Physics

Nuclear Physics

Solid state Physics

Chemistry-Biology

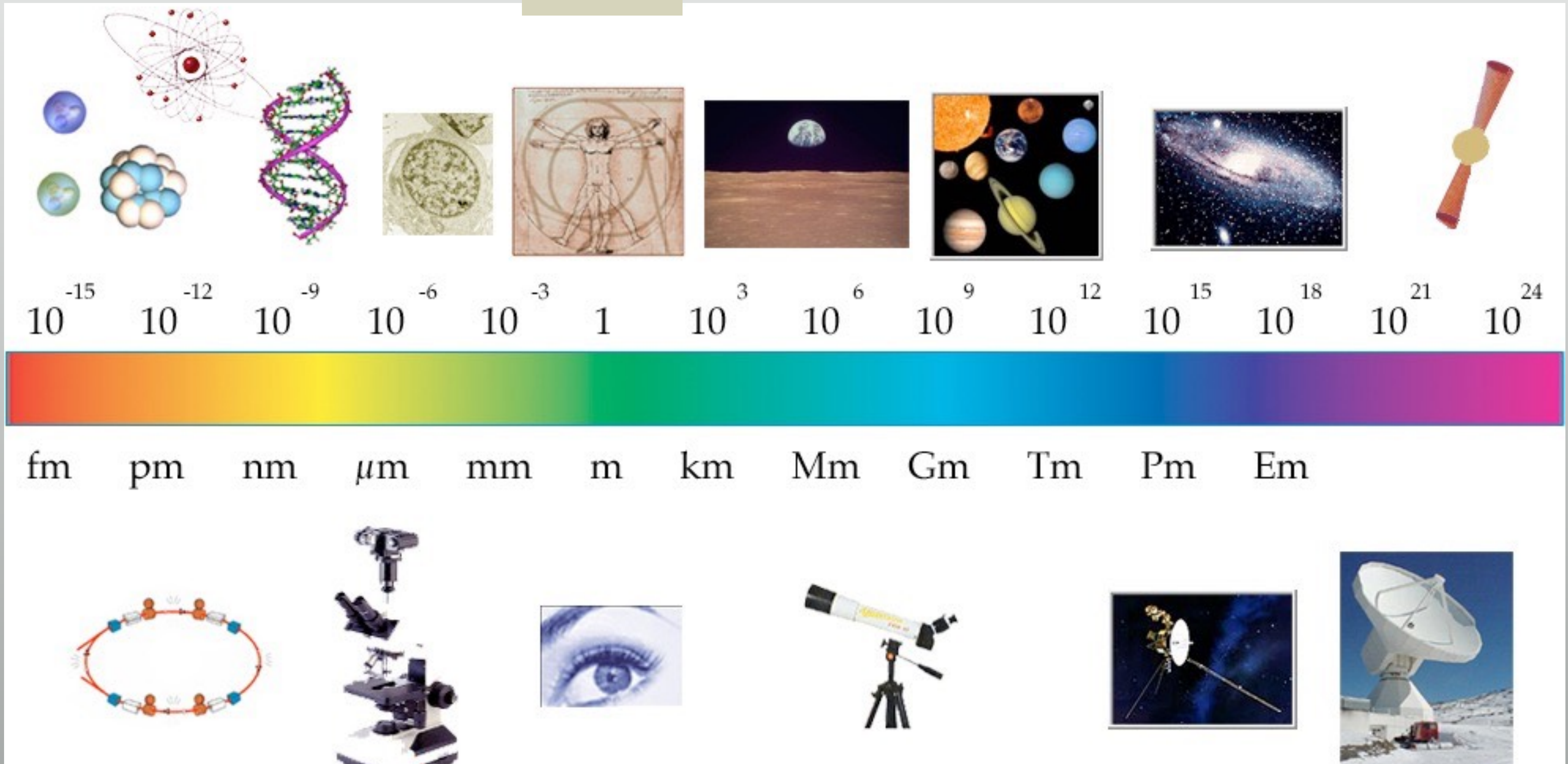
Mechanics

Geophysics

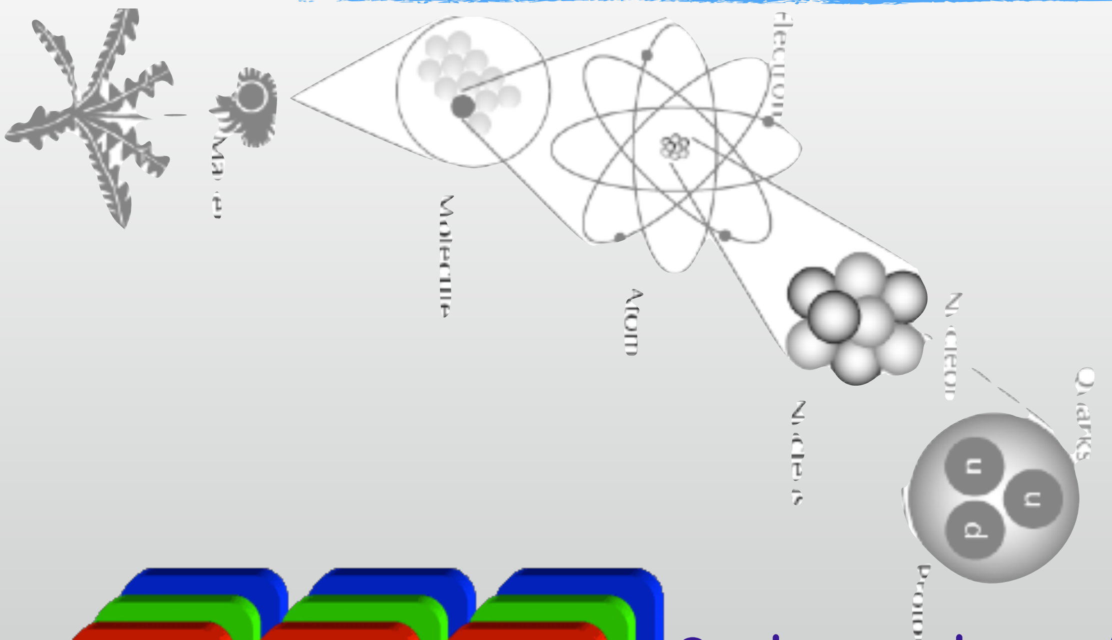
Astronomy

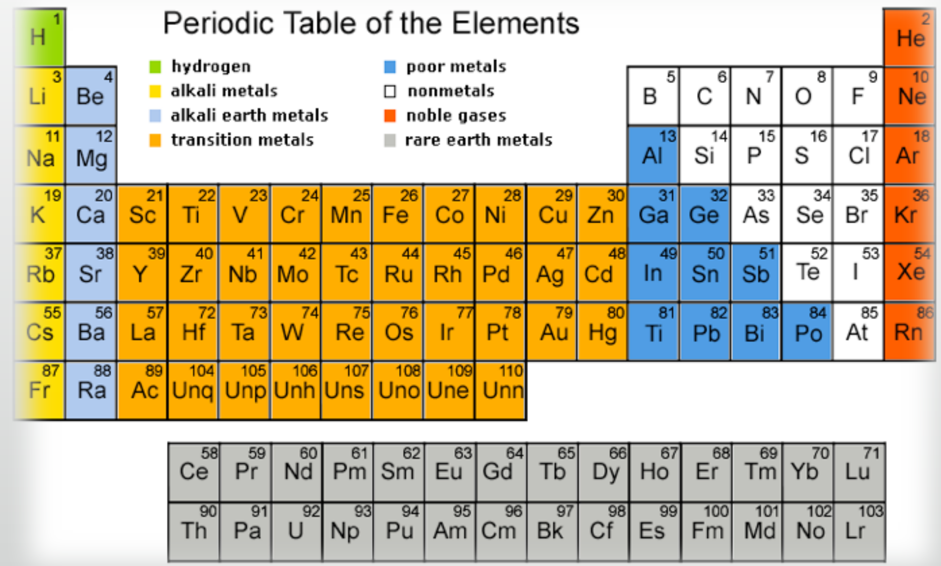
Astrophysics

Cosmology



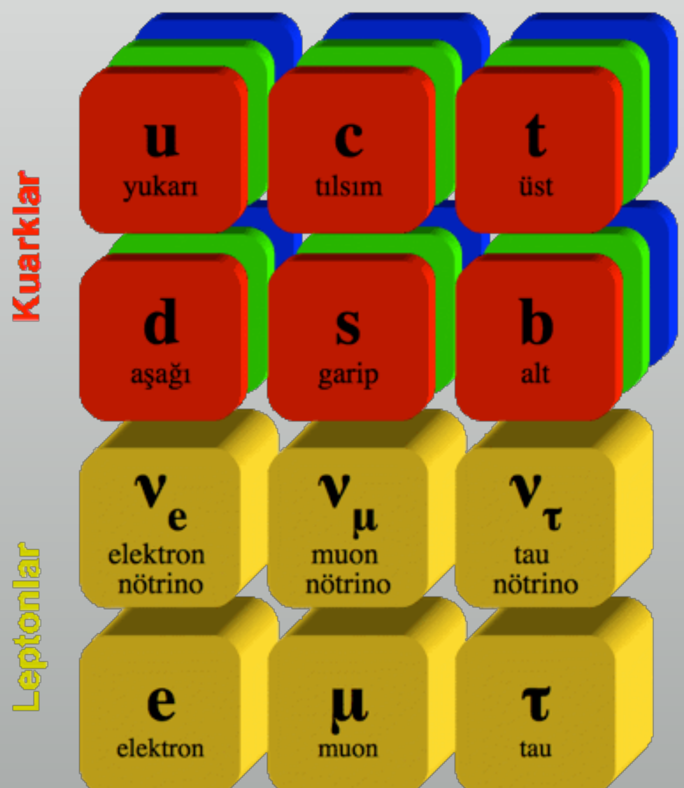
Particle Physics



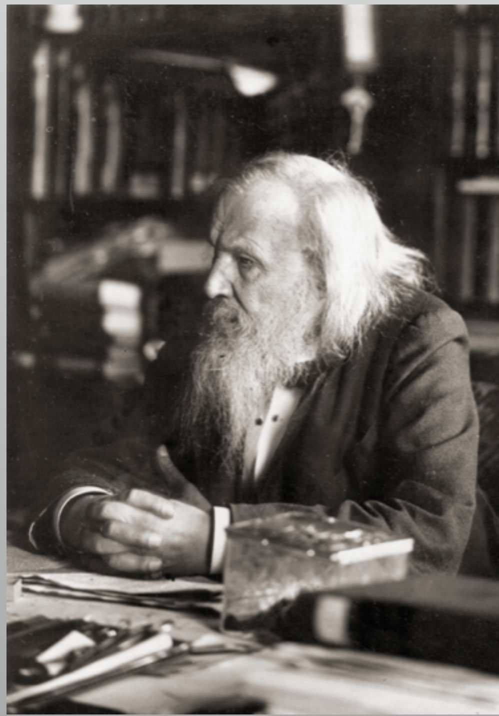

Periodic Table of the Elements

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Ti	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Unn								

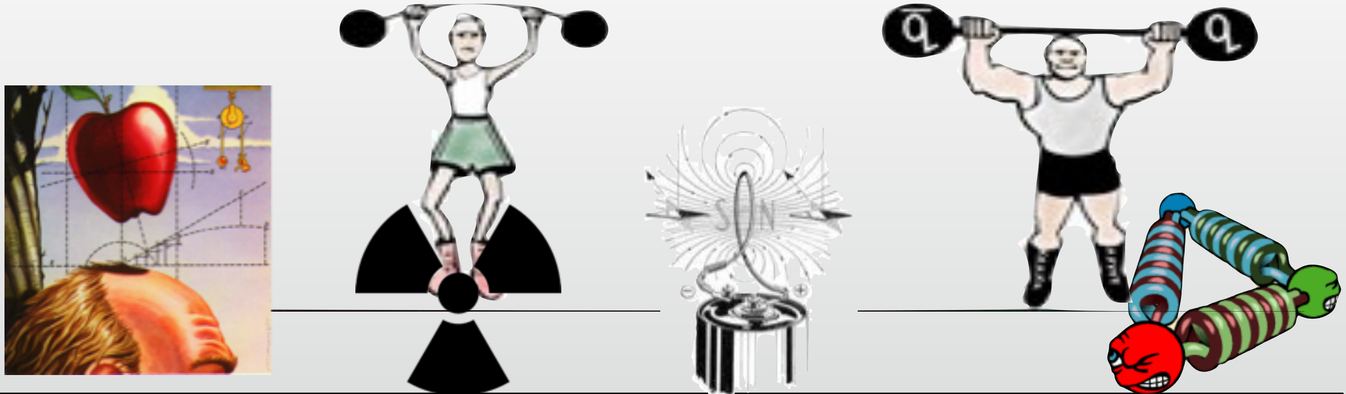
58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr



- elementary **fermions**: quarks and leptons.
  - ➡ can't have same quantum numbers
- These form all the known matter
  - ➡ p≈uud, n≈udd
  - ➡ Hydrogen=p+e

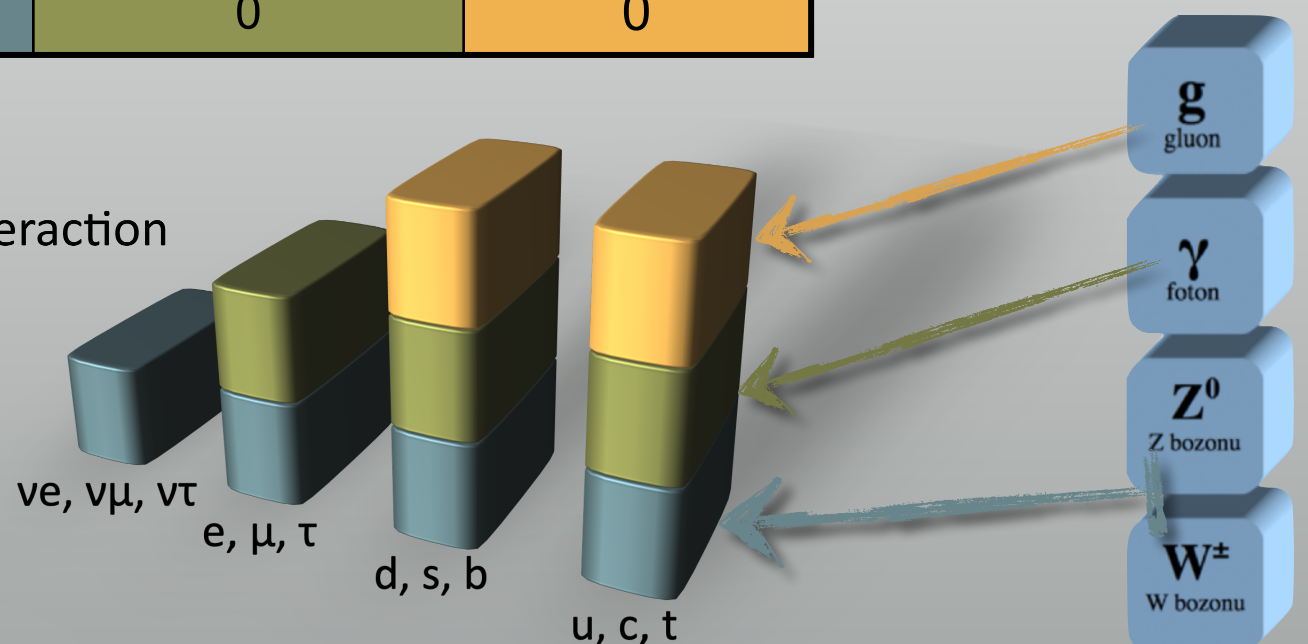


interactions via bosons



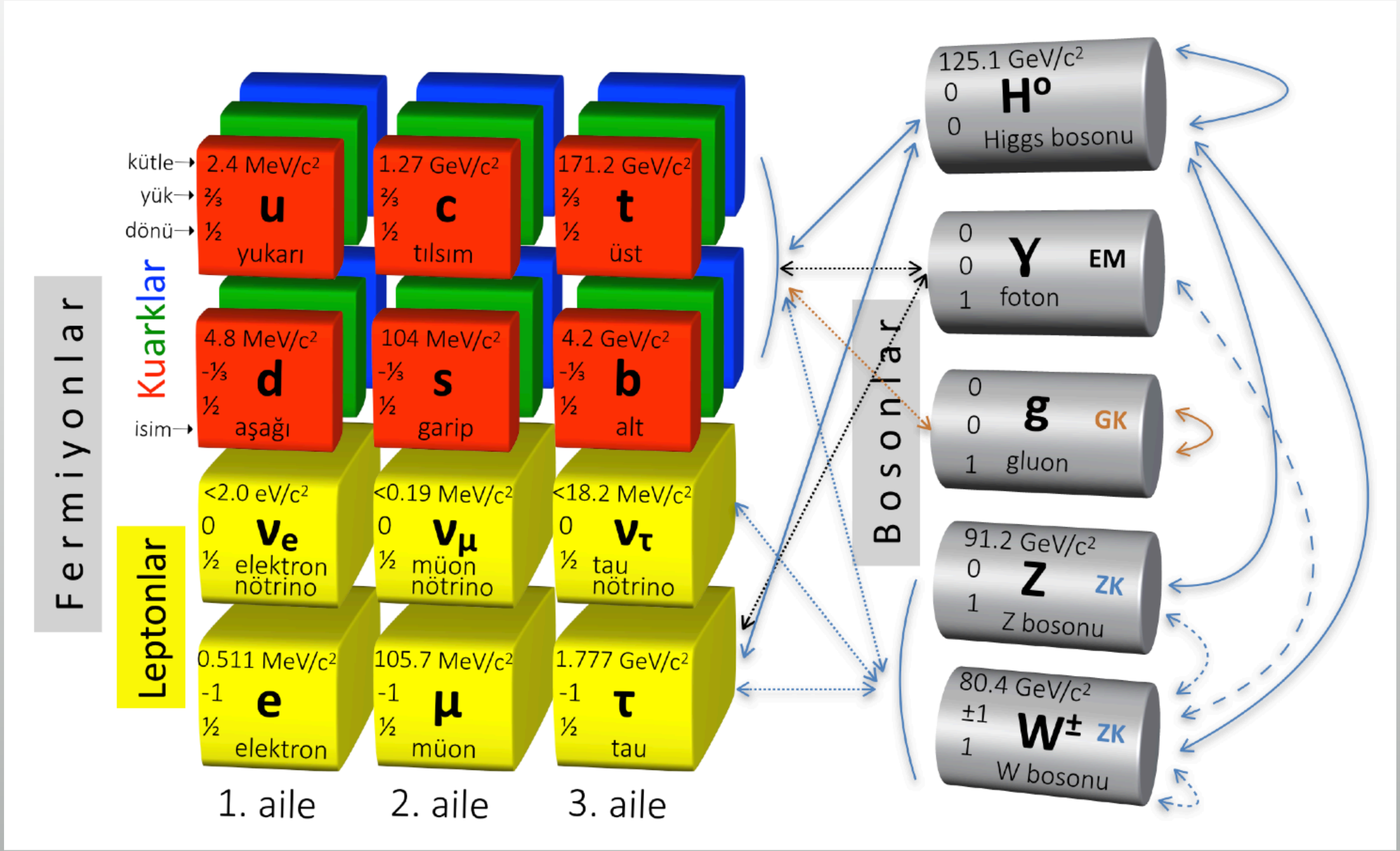
Force	Gravity	Weak	Electromagnetic	Strong
Carrier	Graviton (not yet observed)	**W+ W- Zo**	**Photon**	**Gluon**
Interacts with	all	quarks and leptons	quarks, charged leptons, W+ W-	quarks and gluons
mass (GeV)	0?	80 - 91	0	0

Strong Interaction
Electromagnetic Interaction
Weak Interaction



Bosons: <u>can have</u> same quantum numbers

Particles & Interactions



Bosons: can have same quantum numbers

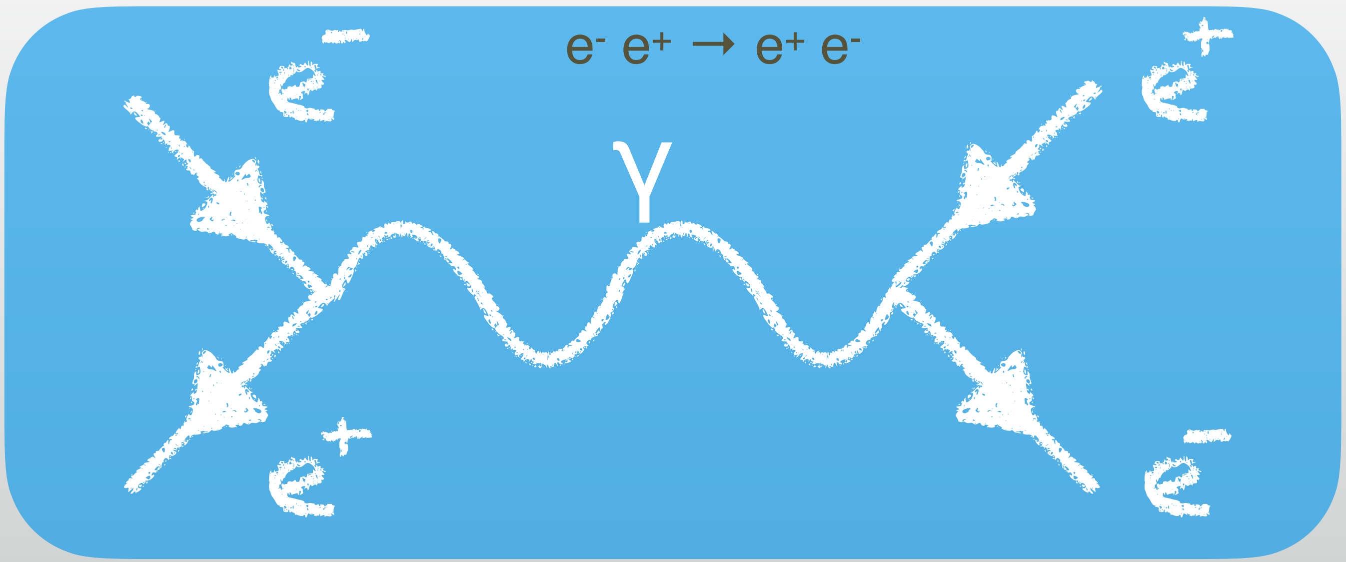
- Interactions = particle exchange

Feynman Diagrams

photon

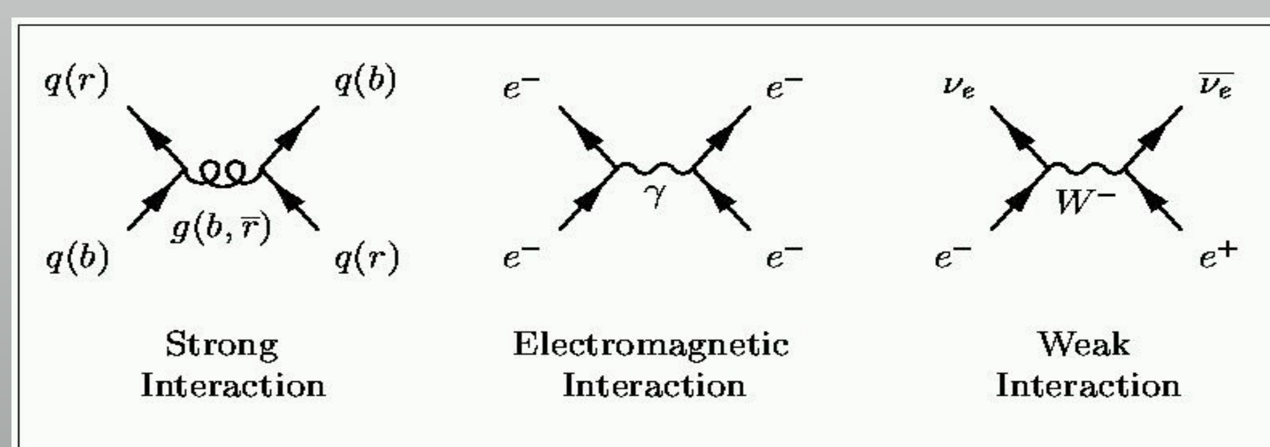
a particle

an anti-particle



- her parçacığın kendisine zıt kuantum mekaniksel özellikler taşıyan bir kardeşi var.
  - buna karşı-parçacık denir
  - parçacık + karşı-parçacık = enerji = yeni parçacık
- Örnekler:
  - elektron(-) & pozitron(+) en basiti: yük ters olmuş (e- e+)
  - u (2/3) & $\overline{u}$ (-2/3) benzer şekilde
  - ν_e & $\overline{\nu_e}$: lepton numarası ters olmuş
  - r & $\overline{r}$: renk yükü ters olmuş

- **Interactions = particle exchange**



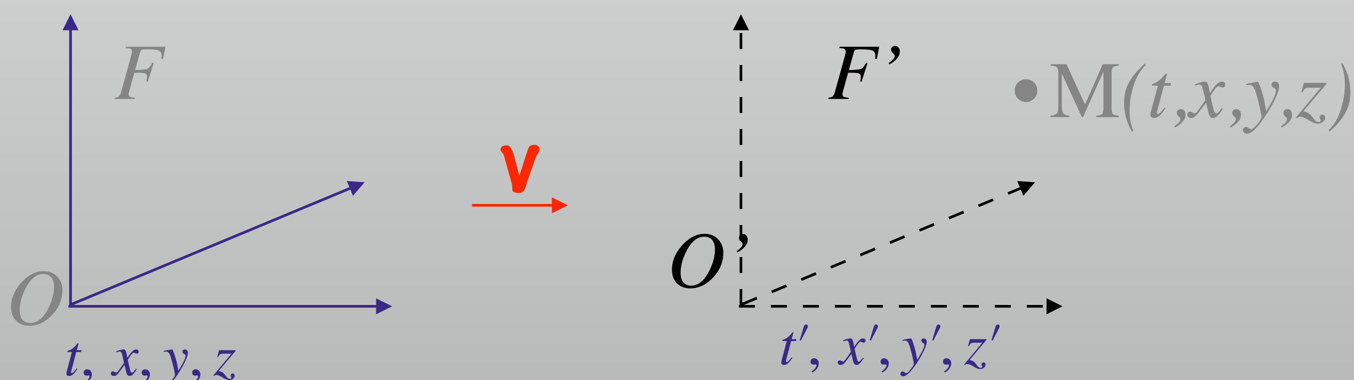
Relativity

- Einstein: “Speed of light is always fixed, **c**.”
  - what is the corrolary?

$\Delta t' = \gamma \Delta t$ Time dilation

$L = \frac{L_0}{\gamma}$ Length Contraction

- Generalization by Lorentz
  - called a Lorentz Transformation (v=const)



$$ct' = \gamma(ct - x\beta)$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

What quantities are invariant under Lorentz Transformations?
What are the details? --> **See Appendix**

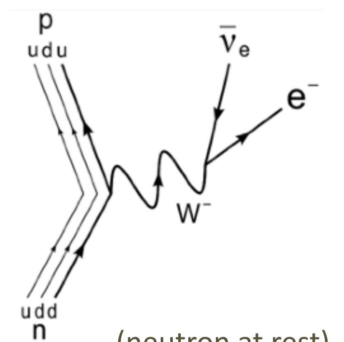
$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

LT in matrix form

Thanks to relativity: Muon Lifetime measurement, Muon Accelerator, LLP experiments, Channeling radiation, FEL....

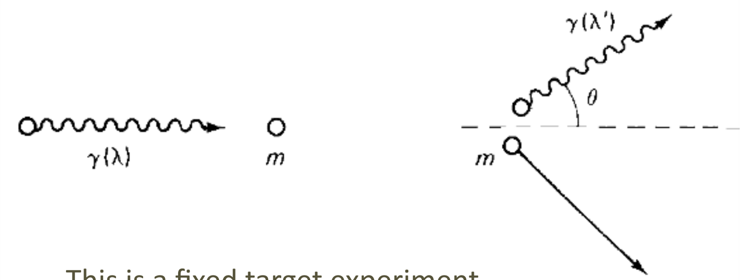
Decays & Scatterings

Ex: Neutron Decay



(neutron at rest)

Ex: Compton scattering



This is a fixed target experiment

Define and use Energy-momentum 4-vectors (LV)

$$\mathbf{P} = (E, \vec{p}) \qquad \eta = + - - -$$

E-m 4 vector is conserved during the process

$\mathbf{P}_{\text{initial}} = \mathbf{P}_{\text{final}}$

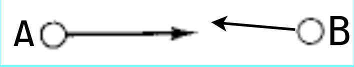
Use most convenient reference frame

magnitude of LV is invarant

Frequently used Lorentz Vectors

Coordinates :	$X^\mu = (ct, x, y, z) = (ct, \vec{x})$
Velocities :	$U^\mu = \frac{dX^\mu}{d\tau} = \gamma(c, \dot{\vec{x}}) = \gamma(c, \vec{u})$
Momenta :	$P^\mu = mU^\mu = m\gamma(c, \vec{u}) = \gamma(mc, \vec{p})$
Force :	$F^\mu = \frac{dP^\mu}{d\tau} = \gamma\frac{d}{d\tau}(mc, \vec{p})$
Wave propagation vector :	$K^\mu = (\frac{\omega}{c}, \vec{k})$
Also the Gradient :	$\partial^\mu = (\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}) = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right)$
	$\partial_\mu = (\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}) = \left(\frac{1}{c}\frac{\partial}{\partial t}, +\frac{\partial}{\partial x}, +\frac{\partial}{\partial y}, +\frac{\partial}{\partial z}\right)$

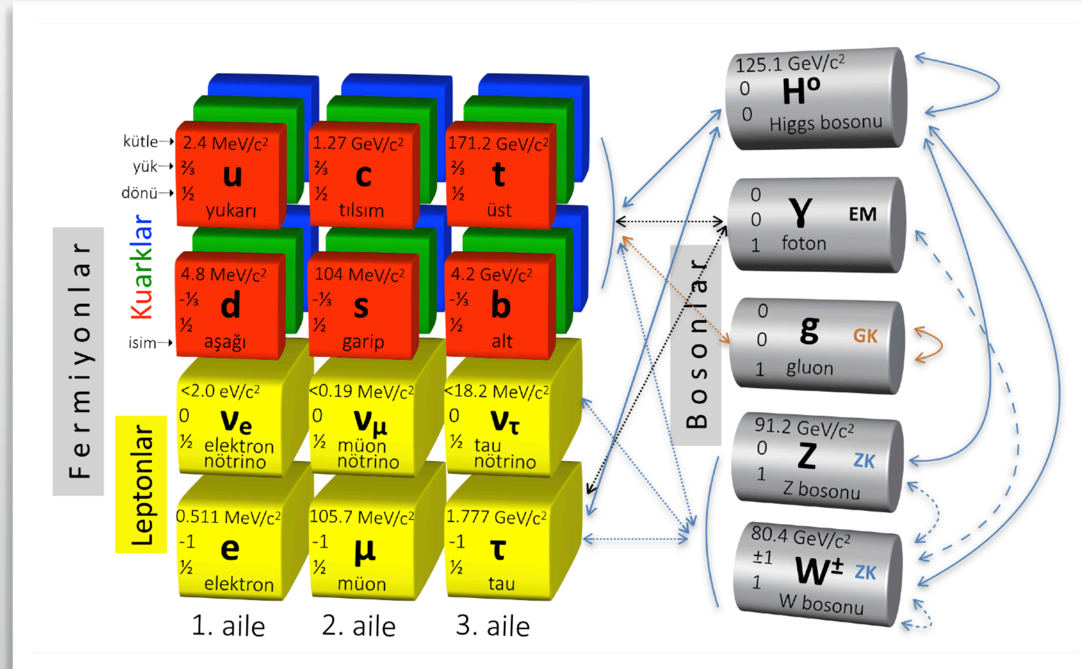
Example: a collider experiment



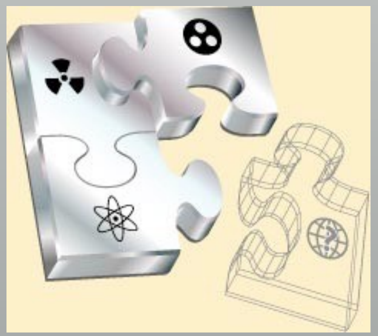
- A particle of energy E_A, of mass m_A & of momentum p_A, hits the particle B of E_B, mass m_B and momentum p_B arriving in opposite direction. (c=1)
- Lets call the total energy of the system $\sqrt{s}$
- $s=(\mathbf{P}_A+\mathbf{P}_B)^2 = \mathbf{P}_A^2+\mathbf{P}_B^2 +2\mathbf{P}_A.\mathbf{P}_B=m_A^2 +m_B^2 +2(E_AE_B-p_A.p_B)$
- $s=m_A^2 +m_B^2 +2(E_AE_B-|p_A||p_B|\cos\theta)$
  - if $m_A \sim m_B \sim 0$ compared to particle kinetic energy, and
  - if the particle are head-to-head colliding
  - $E_{CM} = \sqrt{4E_AE_B}$

Standard Model

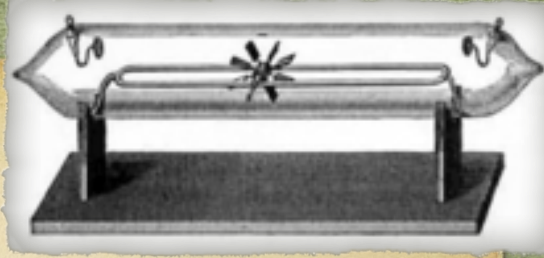
- **Electroweak theory**
  - Photon & two cousins W, Z particles (discovered at CERN).
- **QCD theory**
  - Protons & neutrons are held together by gluon exchange in the nuclei
- **Higgs theory**
  - To explain how fermions and EW bosons acquire mass



- **Results**:
  - A single theory can explain all the "electroweak and strong" interactions.
  - All atomic and sub atomic experiments & observations can be explained.
  - SM & General Relativity can almost explain the universe we observe.

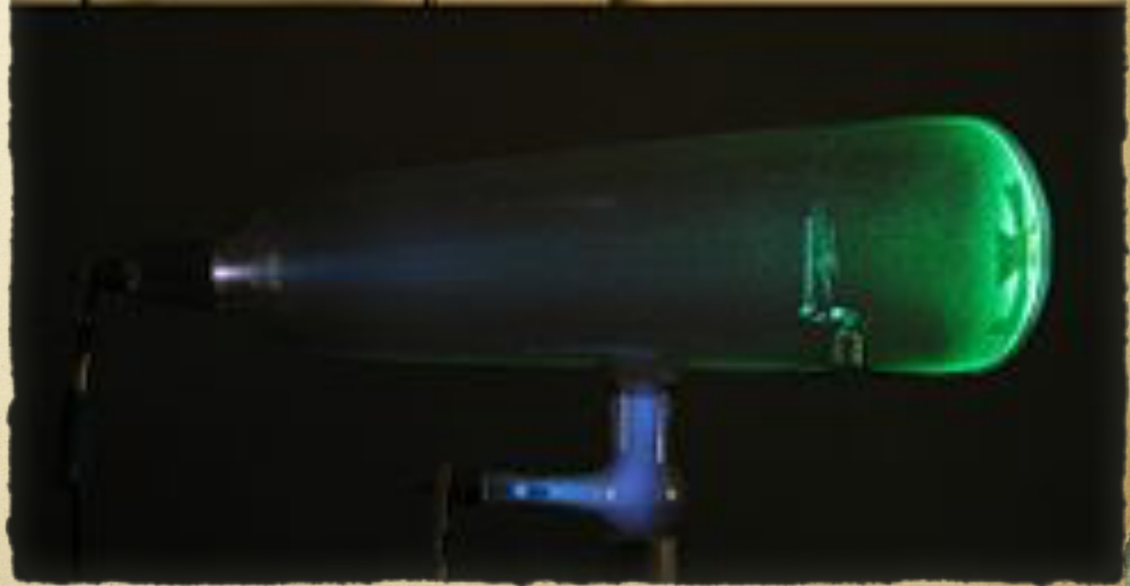


Crookes's tube



- 1869: cathode rays discovered
- 1895: X rays discovered

'01 Nobel



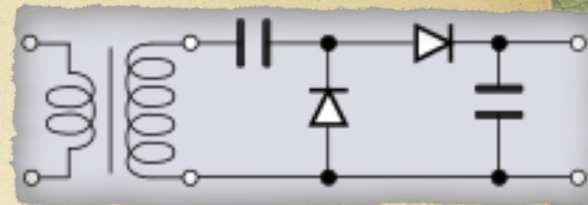
- 1897: electron discovered, & m/q ratio measured by JJ Thompson

'06 Nobel

early 1900s

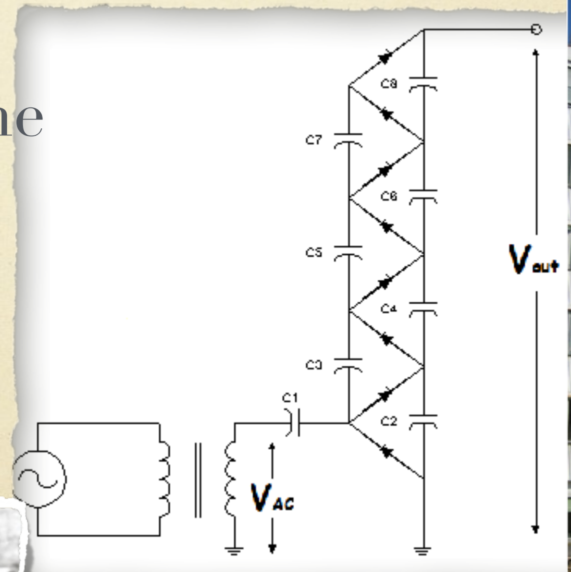
'08 Nobel

- 1911 Rutherford discovers the nucleus
- 1913 Greinacher invents multiplier circuit



'52 Nobel

- 1932 Cockroft Walton machine



- 1932 Atom (nucleus) is divided @400kV

$$p + Li \rightarrow He + He$$

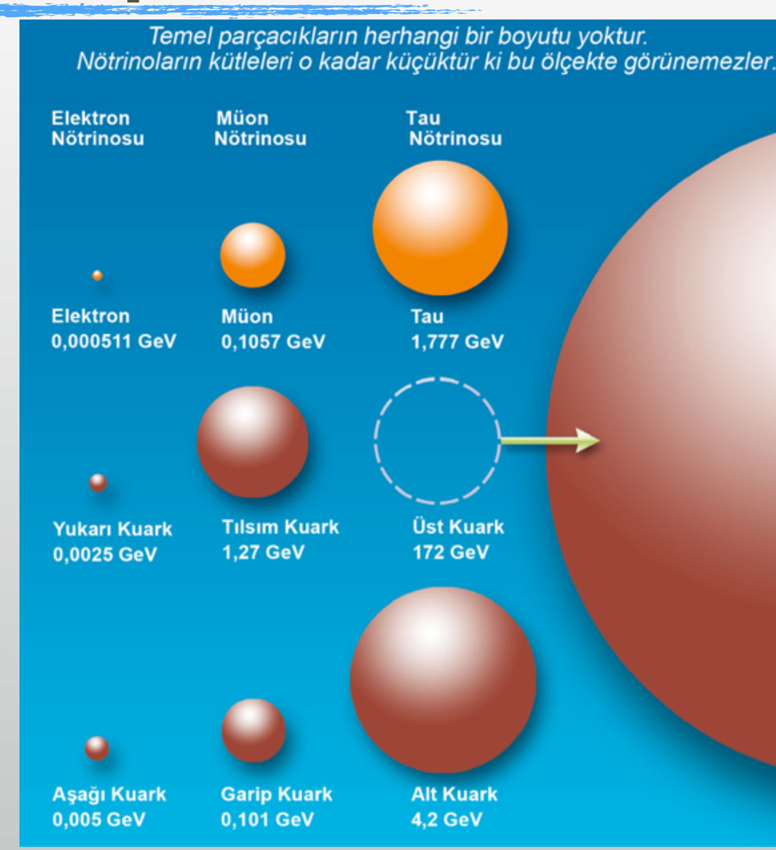
...rnest Rutherford (centre) encouraged Ernest Walton (left) and John Cockcroft (right) to build a high-voltage accelerator to split

’50s - ’60s

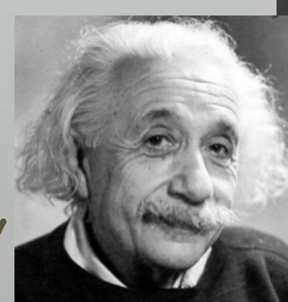
- 1954 CERN is founded
- 1955 Bevatron p @6.2 GeV 🏅 Nobel: antiproton is discovered
- 1959 CERN PS: first strong focusing accelerator 28GeV
- 1960 BNL AGS p@33 GeV
  3 🏅🏅🏅 Nobel: $\nu_\mu \quad J/\psi \; c \quad \not{CP}$
- 1961 AdA first collider
- 1966 electron cooling

a fly in the soup

- If simple is beautiful, SM is simpler with massless particles



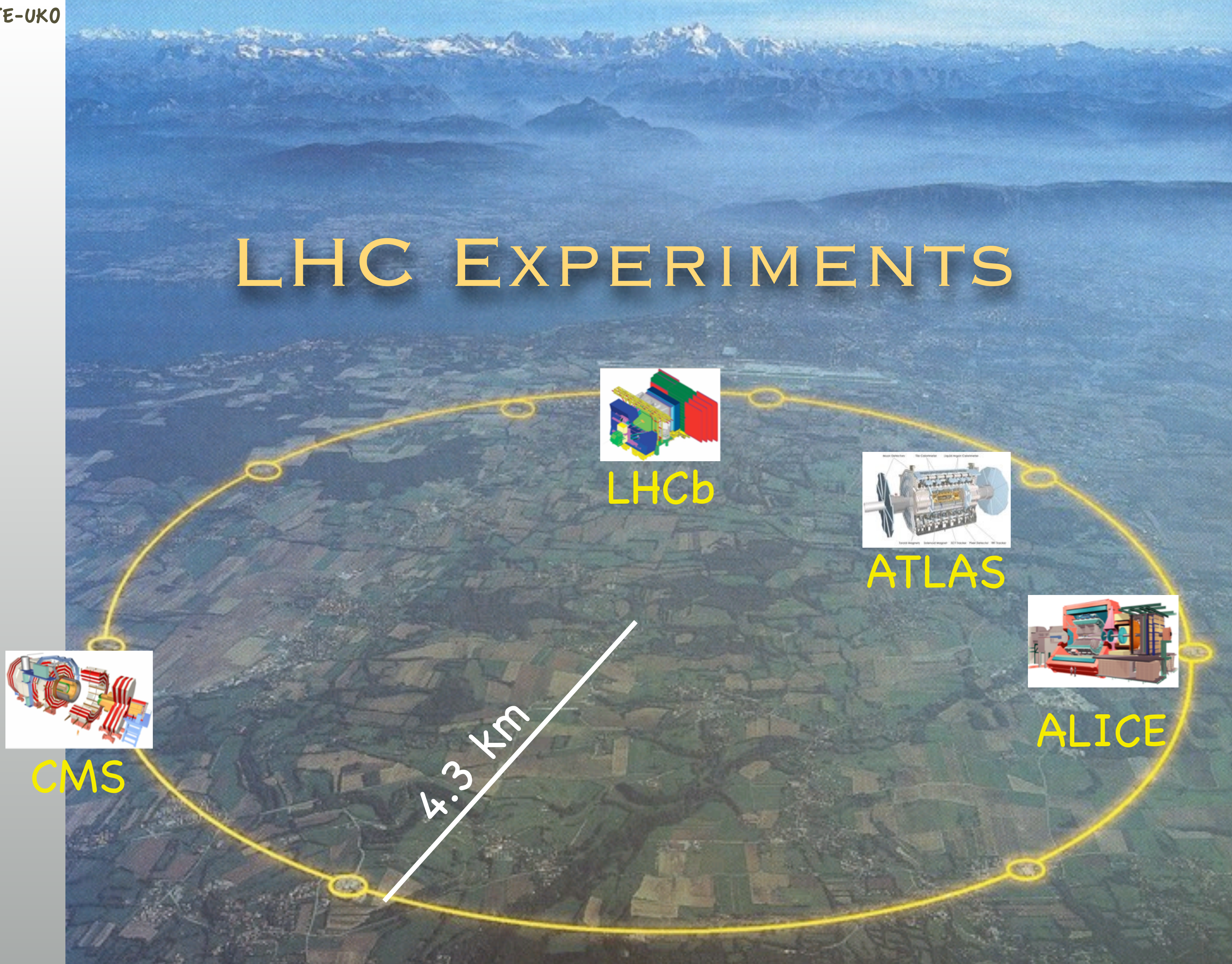
- But we observe mass:
  - ➡ Why different fermion masses?
    - $m_t >>> m_e$ why??????
  - ➡ Why different boson masses?
    - Z ∿ γ but mZ=90 mγ=0, why??????
- What is mass anyway?
  - ➡ Newton: $m=F/a$ and $m=Fr^2/MG$
  - ➡ Einstein: $m=E/c^2$



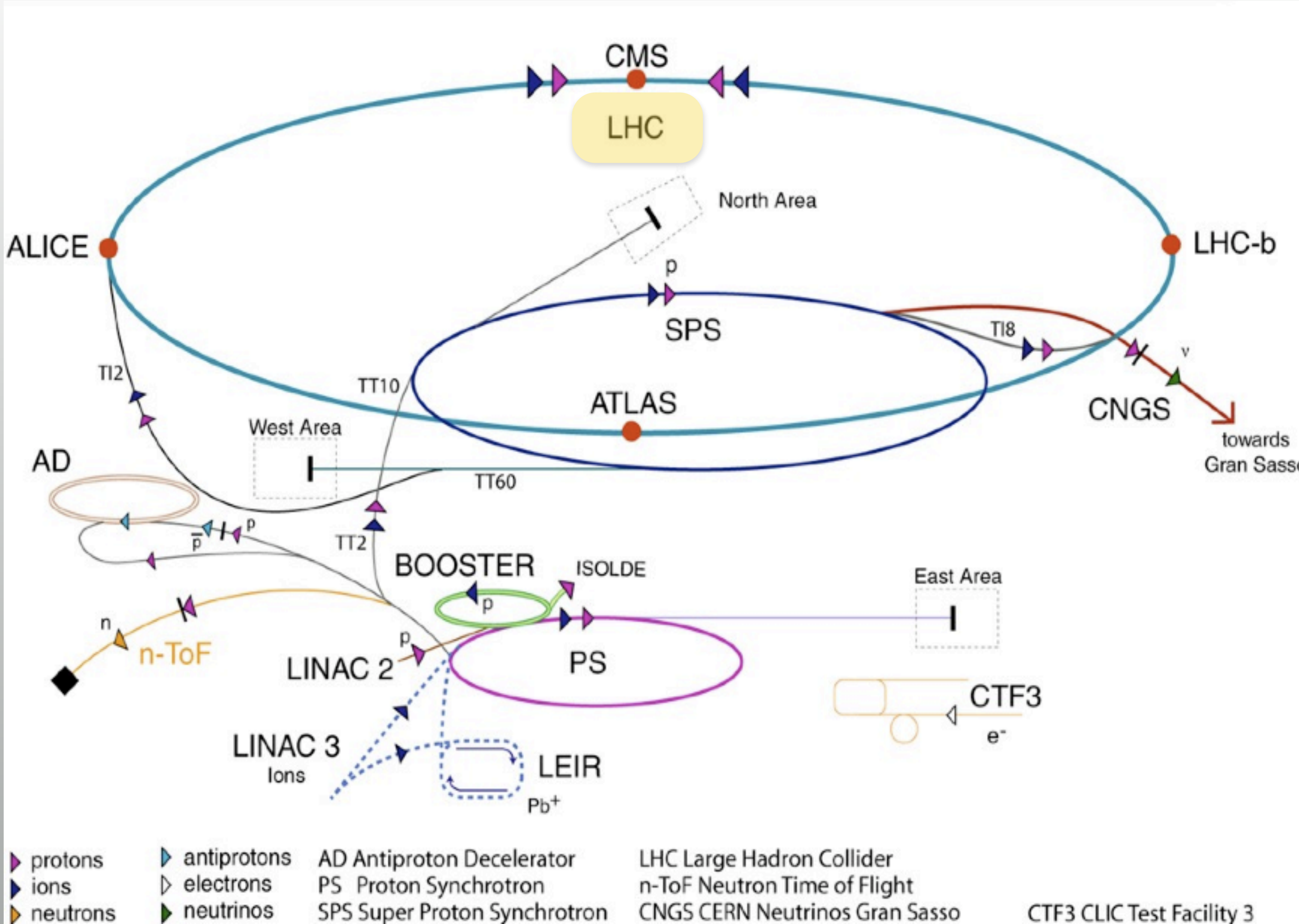
  - ➡ **Caution**! mp≈940MeV but m(u+u+d)≈9.4MeV
  - ➡ 99% proton mass= binding energy=QCD

*We are trying to understand the masses of **fundamental** particles!*

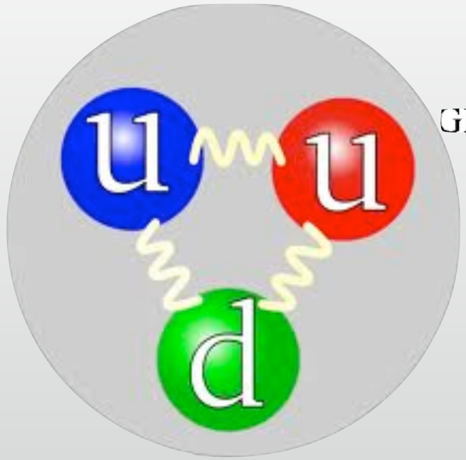
LHC Experiments

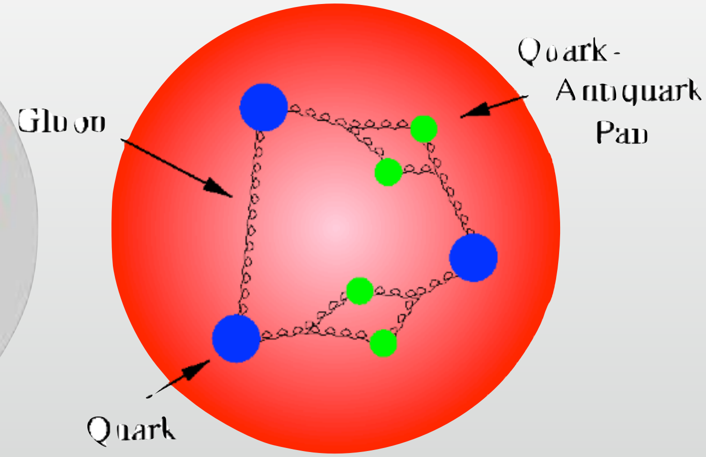


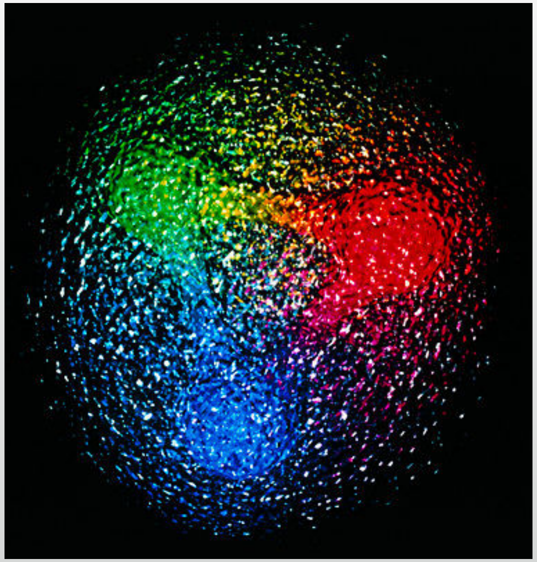
Colliders @CERN



why collide protons

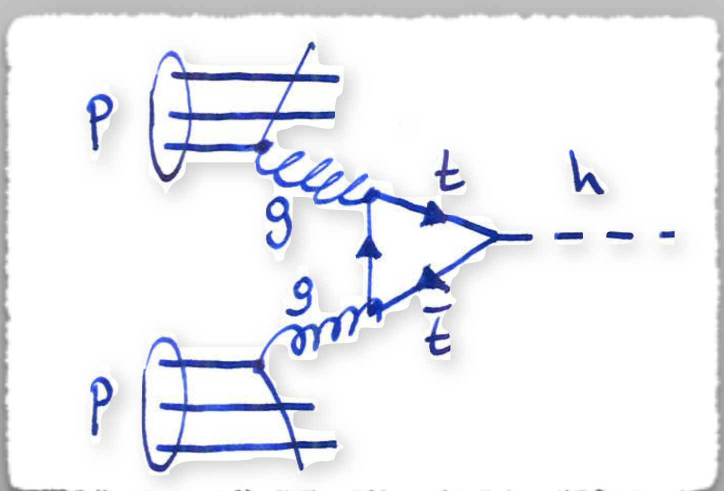


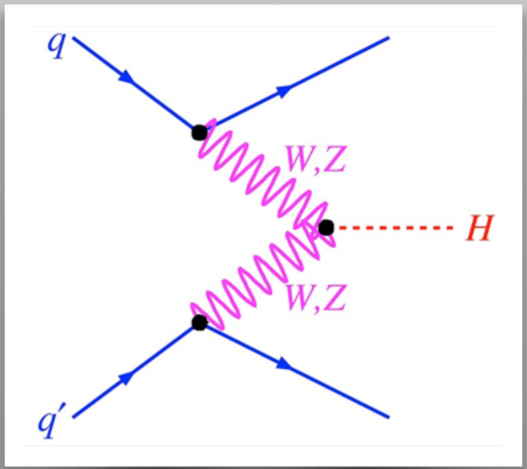




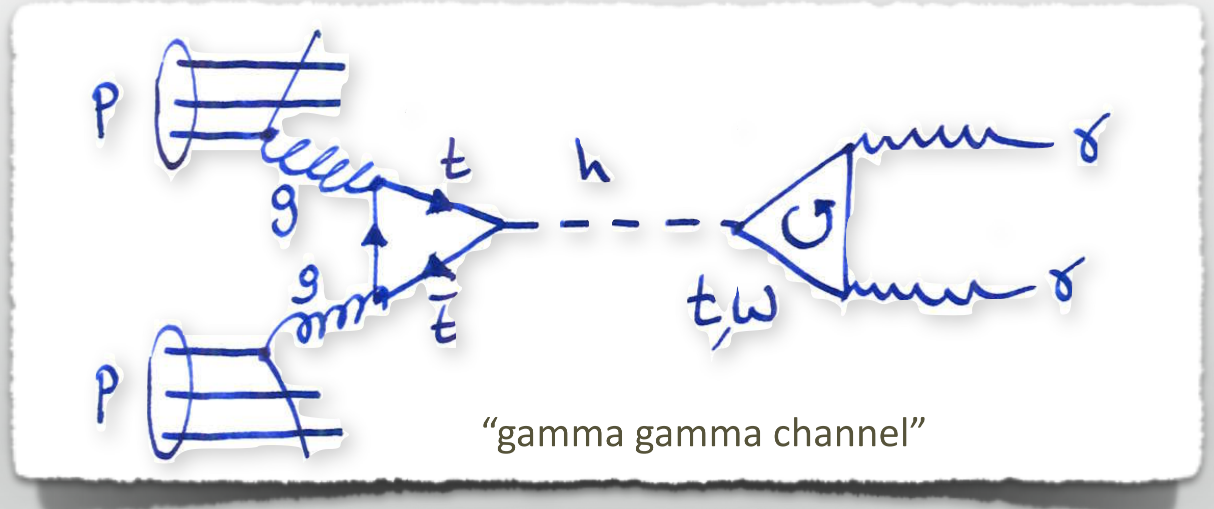
- Higher energy reveals more and more details inside the proton

- proton machines are discovery tools

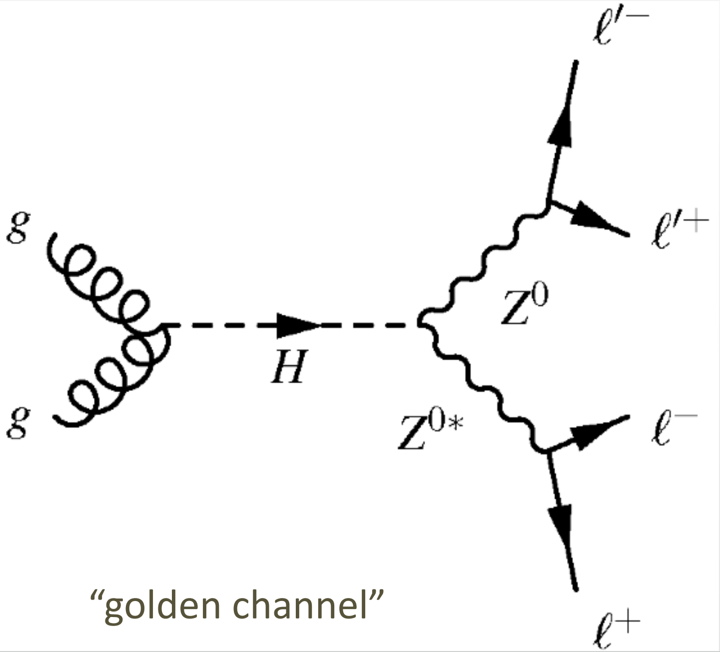




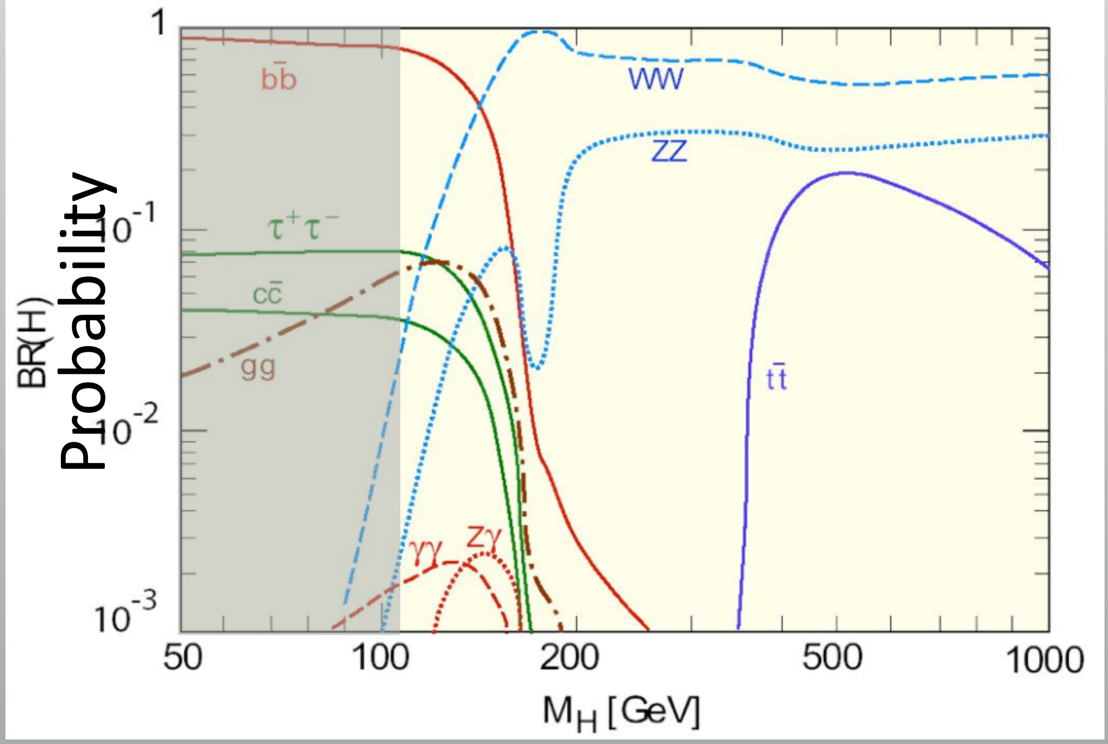
Higgs decays in 10^{-22} s to...



"gamma gamma channel"



"golden channel"



Higgs decays products are calculable in terms of the Higgs mass.

We need to find & measure the decay products!

Around 120 GeV H —>γγ 0.1%
& H —>ZZ 3%

the art of detection

- how do we "see" the final state particles?

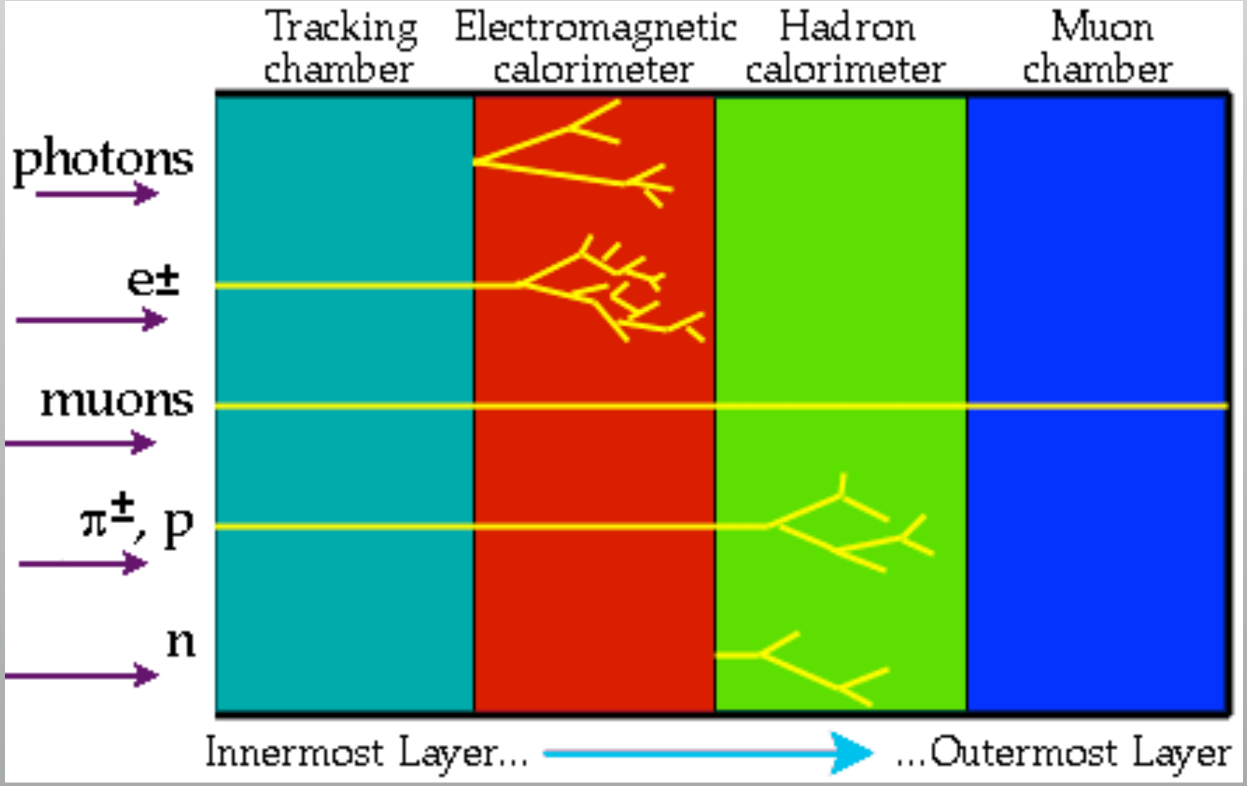
a truck passed by

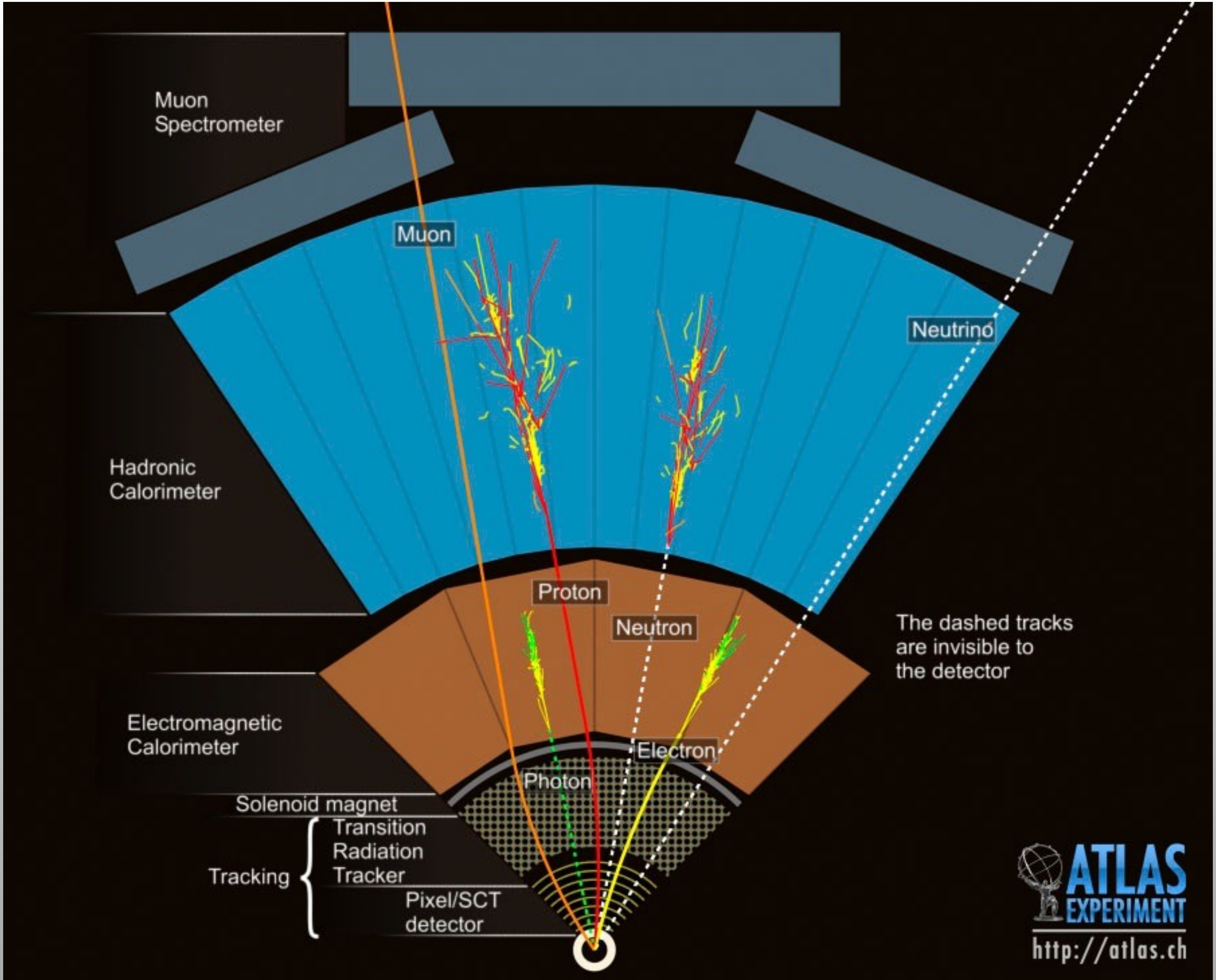
a bear walked forward

a bear walked backward

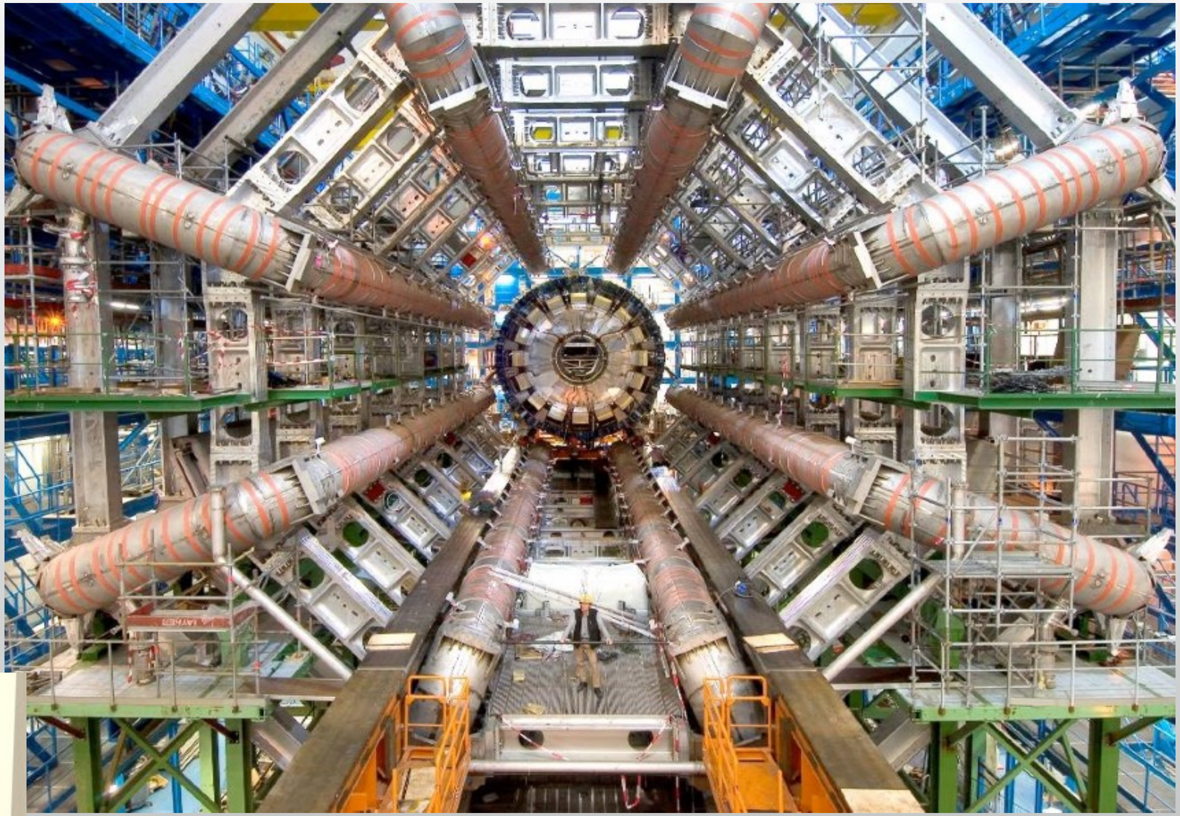
Identification

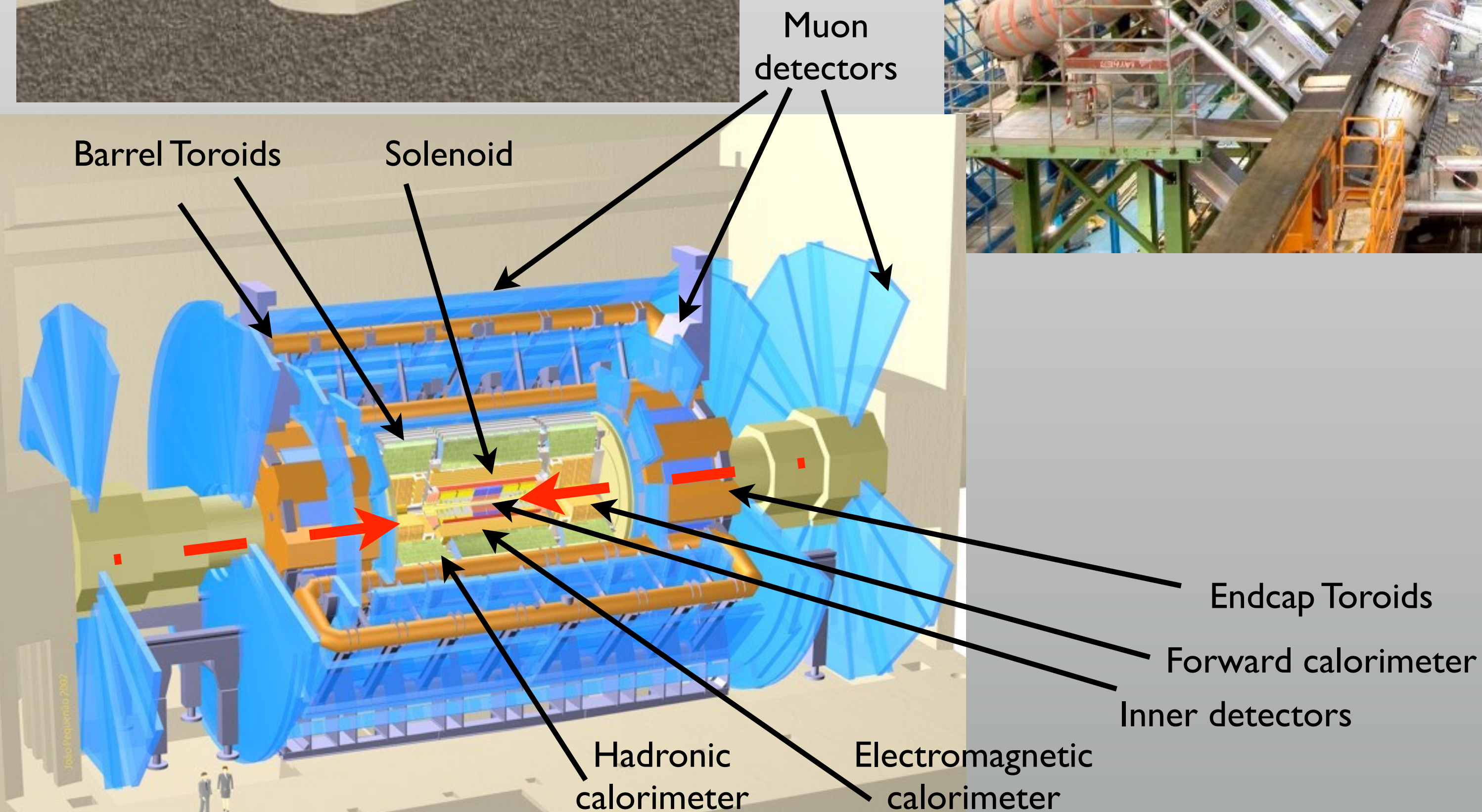
- ionisation: tracking
  - charged particle
  - under B field,
    - charge
    - momentum
- EM Calorimetry
  - e+, e−, γ
- HAD Calorimetry
  - mesons + hadrons
- Muons
  - Trigger
- under B field,
  - charge
  - momentum
- Δp=0
  - neutrinos




Muon
Spectrometer
Muon
Hadronic
Calorimeter
Neutrino
Proton
Neutron
The dashed tracks
are invisible to
the detector
Electromagnetic
Calorimeter
Electron
Photon
Solenoid magnet
Transition
Radiation
Tracker
Tracking
Pixel/SCT
detector
ATLAS
EXPERIMENT
http://atlas.ch

ATLAS

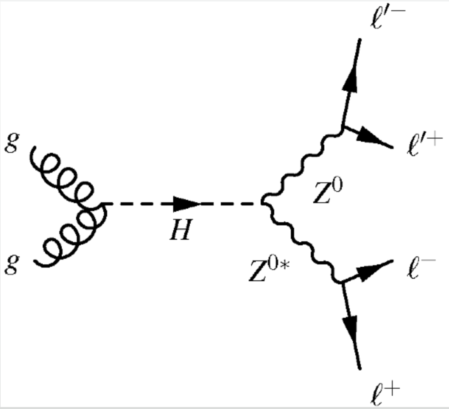




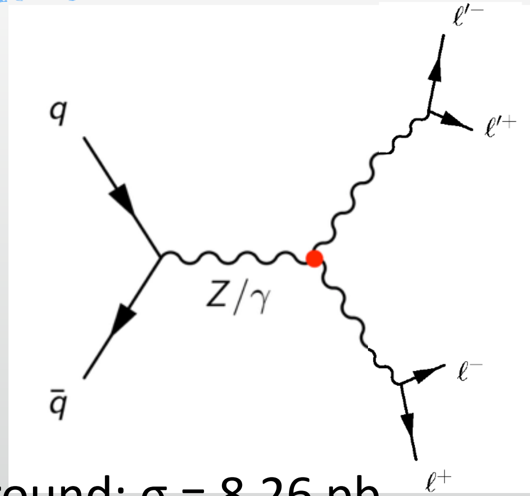
Length = 46 m
Diameter = 25 m
Weight = 7 kT

Find the hays from Higgs field in the haystack

finding Higgs took 48 years
= 1 514 764 800 s!



signal: σ = 0.001 pb



background: σ = 8.26 pb

- need to find an efficient way of selecting "Higgs" events
  - ➡Quick & efficient
  - ➡without biasing the data
- **Blind analysis**
  - ➡do not look at final distributions during the analysis
- 5σ significance needed for discovery
  - ➡P(statistical fluctuation)= 5.7×10^{-7}

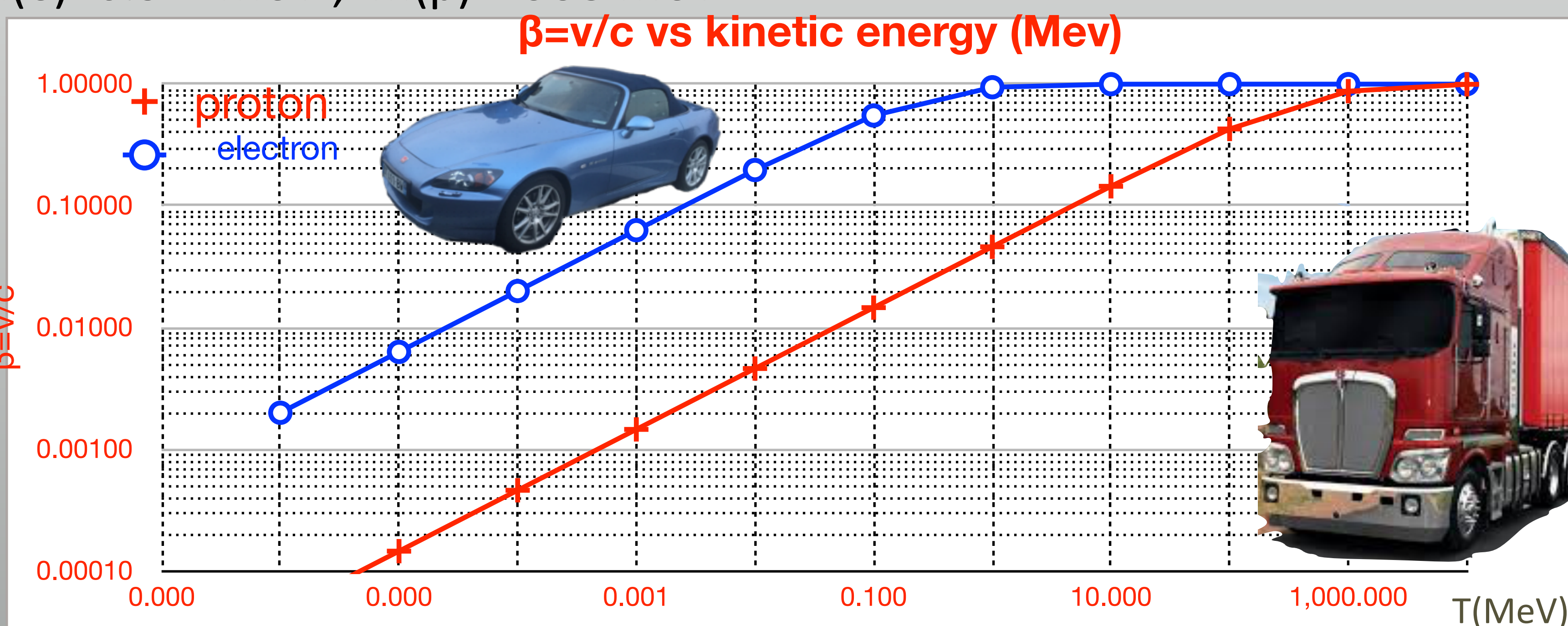
Units in PP

- Energy = Mass except a coefficient
  - Energy is measured in eV, the energy picked up by a particle of charge e, while going through a potential V. *Naturally we have keV, MeV, GeV…*
- For any particle: $E^2 = p^2c^2 + m^2c^4$
  - $[p]$ = MeV/c $[m]$ = GeV/c^2 this can be simplified with 'c=1' approach
  - $[E] = [p] = [m]$ => k/M/G/T eV
- $E = m_0 + T$ (T: Kinetic energy) $= \gamma$ m
  - where γ is Lorentz Factor
- Remember: m(e)=0.511MeV , m(p) = 938 MeV

1 eV = 1.6 x 10^{-19} J

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$



"natural" units

- Energy: GeV
- Time: (GeV/ħ)$^{-1}$
- Momentum: GeV/c
- Length: (GeV/ħc)$^{-1}$
- Mass: GeV/c^2
- Area: (GeV/ħc)$^{-2}$

- ħ=c=1 would simplify alot!!
- All physical quantities would be expresses as powers of GeV.
- ħ and c coefficients are used s appropriatelt to convert back to S.I.

plank units

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616\,199(97) \times 10^{-35}\ \mathrm{m}$$

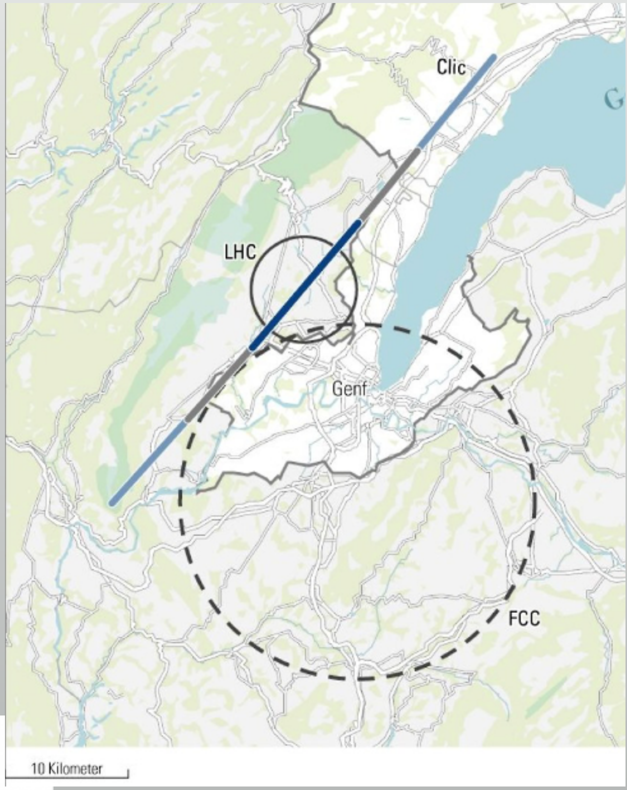
$$t_P \equiv \sqrt{\frac{\hbar G}{c^5}} \approx 5.391\,16(13) \times 10^{-44}\ \mathrm{s}$$

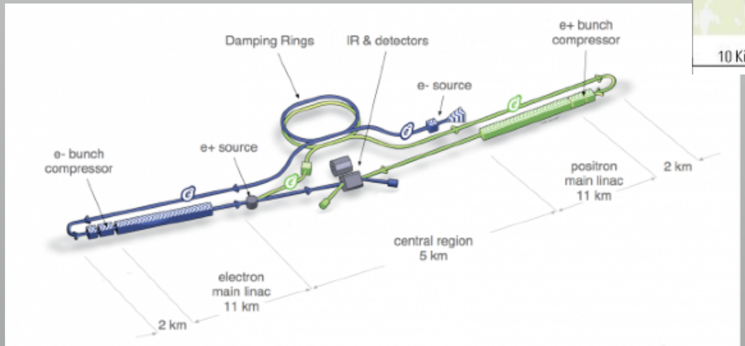
$$m_P = \sqrt{\frac{\hbar c}{G}} \approx 1.2209 \times 10^{19}\ \mathrm{GeV/c^2} = 2.176\,51(13) \times 10^{-8}\ \mathrm{kg}$$

$$E_P = \sqrt{\frac{\hbar c^5}{G}} \approx 1.956 \times 10^{9}\ \mathrm{J} \approx 1.22 \times 10^{28}\ \mathrm{eV}$$

outlook

- Need more data to answer questions in PP:
  - ➡ LHC data taking re-starts in March 2024
  - ➡ We are limited by the current LHC tunnel & superconductor (cooling) technology and PDFs
- New machines in the horizon
  - ➡ **Future Circular Collider** (if no new tech.)
    - **BIG** pp machine $\sqrt{}$=100TeV, 100km circumference
  - ➡ Dedicated Linear Higgs machine
    - ILC?
    - Compton back-scattering, γ-γ collider
    - We need new accelerator paradigm: **100GeV/m**
      - **wake field acceleration??**





Appendix

most economical solution

- 60's & minis: mini-cooper, mini-skirts, mini-computers
  - Brout & Englert; Guralnik, Hagen & Kibble; Higgs's papers offered a minimalistic & elegant solution.
  - There are other theories solving the mass problem, B-E-H is the simplest model.

- Higgs theory kills two birds with a single stone:
  - fermion masses can be explained $\mathcal{L}_{\text{Yukawa}}(\phi,\psi) = -g\bar{\psi}\phi\psi$
  - W & Z boson masses & their relationship explained (SSB)

- BUT
  - Higgs theory **doesn't predict** Higgs boson mass, says mH < 1TeV
  - It has to be found experimentally, by observing its decay products.
  - Higgs decay products depend on its mass. So:
    - One needs to search for the Higgs boson in all the mass range.

for the connaisseurs

1) **Prequel - field theory**
   a. **particles represented by fields ψ: fermion ϕ: scalar**
   b. **Lagrangian (density) formalism**
2) **simplest fermion Lagrangian**

$$\mathscr{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - \sout{m\bar{\psi}\psi}$$

$\psi \to U\psi \quad \bar{\psi}' \to \bar{\psi}U^{-1}$ This is not a good mass term, it is not gauge invariant. $-m\bar{\psi}\phi\psi$

3) **Consider a scalar field ϕ with this particular potential**

$$\mathscr{L}_s = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda^2\phi^4$$

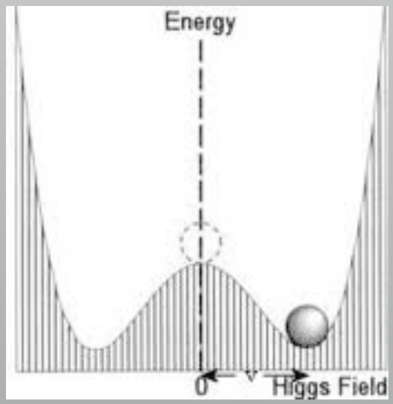
not a mass term

$$V = \frac{1}{4}\lambda^2\phi^4 - \frac{1}{2}\mu^2\phi^2$$
$$\frac{\partial V}{\partial \phi} = 0$$

$\lambda^2\phi^3 - \mu^2\phi = 0 \quad \phi(\lambda^2\phi^2 - \mu^2) = 0$

$\phi_{min} = \pm\sqrt{\frac{\mu^2}{\lambda^2}}$

Spontaneous Symmetry Breaking



$\phi \to \phi \pm \frac{\mu}{\lambda}$

$$\mathscr{L}_s^{new} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{2}\mu^2(\phi \pm \tfrac{\mu}{\lambda})^2 - \tfrac{1}{4}\lambda^2(\phi \pm \tfrac{\mu}{\lambda})^4$$

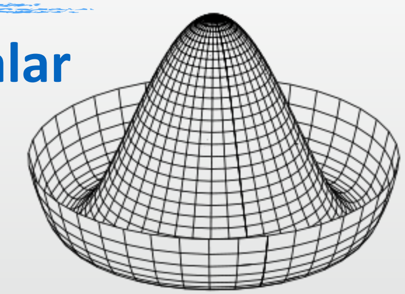
$$\mathscr{L}_s^{new} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{2}\mu^2(\phi^2 + \tfrac{\mu^2}{\lambda^2} \pm 2\phi\tfrac{\mu}{\lambda}) - \tfrac{1}{4}\lambda^2(\phi^4 \pm 4\phi^3\tfrac{\mu}{\lambda} + 6\phi^2\tfrac{\mu^2}{\lambda^2} \pm \phi\tfrac{\mu^3}{\lambda^3} + \tfrac{\mu^4}{\lambda^4})$$

$$\mathscr{L}_s^{new} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{2}\tfrac{\mu^4}{\lambda^2} \pm \phi\tfrac{\mu^3}{\lambda} - \tfrac{1}{4}\lambda^2\phi^4 \mp \phi^3\mu\lambda - \tfrac{6}{4}\phi^2\mu^2 \mp \phi\tfrac{\mu^3}{\lambda} - \tfrac{\mu^4}{4\lambda^2}$$

$$\mathscr{L}_s^{new} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{2-6}{4}\mu^2\phi^2 - \tfrac{1}{4}\lambda^2\phi^4 \mp \phi^3\mu\lambda + \tfrac{\mu^4}{\lambda^2}$$

ϕ mass term

for the connaisseurs



4) Consider a simple universe: a gauge field A (like EM), a complex scalar field ϕ and a fermion field ψ. (~ Z boson, Higgs, Electron)

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \qquad \phi_{min} = \pm\frac{\mu}{\lambda} \equiv \frac{v}{\sqrt{2}} \qquad \phi_1^2 + \phi_2^2 = v^2$$

complex scalar field has a degeneracy in the (φ_1, φ_2) plane. We arbitrarily choose (v, 0) as the minima.

$$\mathscr{L}_{g,s,f} = \tfrac{1}{2}(\partial_\mu - ieA_\mu)\phi^*(\partial^\mu + ieA^\mu)\phi + \tfrac{1}{2}\mu^2\phi^*\phi - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2 - g\bar{\psi}\phi\psi$$

massless fermion & photon

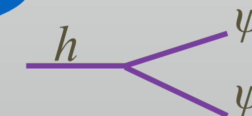
We benefit from SSB as before to “gauge out” φ_2 :

$$\phi_1 = h + \frac{\mu}{\lambda} \qquad \phi_2 = 0$$

massive Higgs

massive photon “Z”

massive fermion



$$\begin{aligned}\mathscr{L}_{g,s,f} \quad &= \quad \frac{1}{2}\partial_\mu h\partial^\mu h - \mu^2 h^2 - \frac{1}{2}\left(\frac{e\mu}{\lambda}\right)^2 A_\mu A^\mu - \frac{g\mu}{\lambda}\bar{\psi}\psi - g\bar{\psi}h\psi \\ &+ \quad \frac{e^2\mu}{\lambda}hA_\mu A^\mu + \frac{e^2}{2}h^2 A_\mu A^\mu - \lambda\mu h^3 - \frac{1}{4}\lambda^2 h^4 + \text{const}\end{aligned}$$

for the connaisseurs

What just happened?

A boson (ϕ_2) vanished to give mass to a vector boson (*A*): total number of degrees of freedom **is** preserved.

5) **Consider a realistically complex universe: a U(1) gauge field X, a SU(2) gauge field W, a complex scalar field doublet ϕ and a chiral fermion field doublet ψ_L, and a singlet ψ_R .**
    1) ϕ now has 4 fields, massless X has 1 field, and massless W has 3 fields.
    2) SSB as before, makes 3 scalar fields disappear.
    3) BEH as before, makes 3 vector bosons massive.
        - W^+, W^-, W^0
    4) SU(2) neutral boson and X neutral boson mix, by Weinberg angle θ_W.
        - we get one massive boson (Z) and one massless boson (γ).

This is exactly what happens in the electroweak sector of the Standard Model

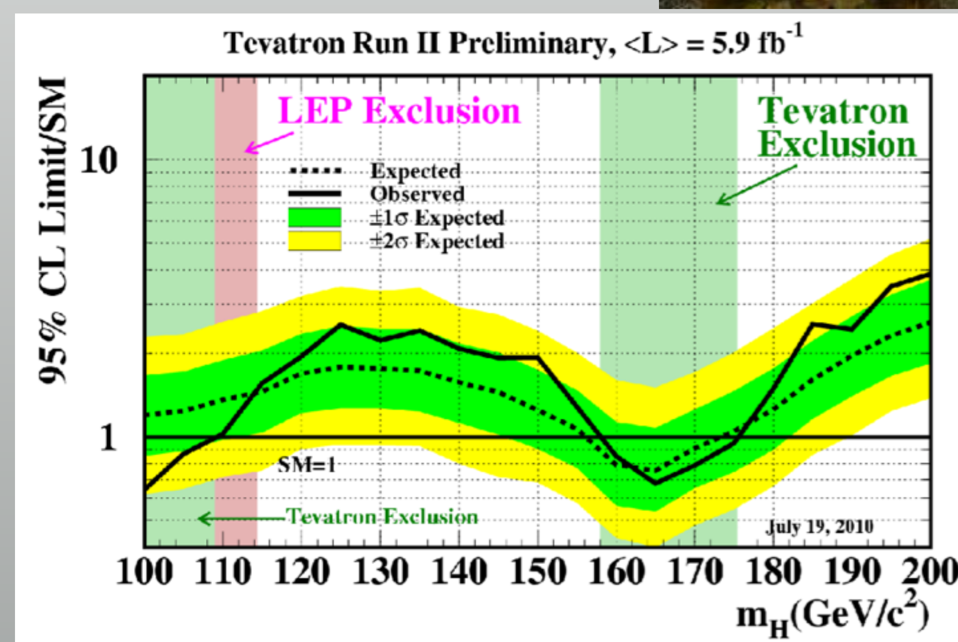
5) electron-neutrino scattering allows measurement of θ_W. Its value, combined with fermi constant, allows prediction of W and Z boson masses.
    - SPPS finds Z and W exactly at predicted masses @ 1983.

Our Precious Higgs

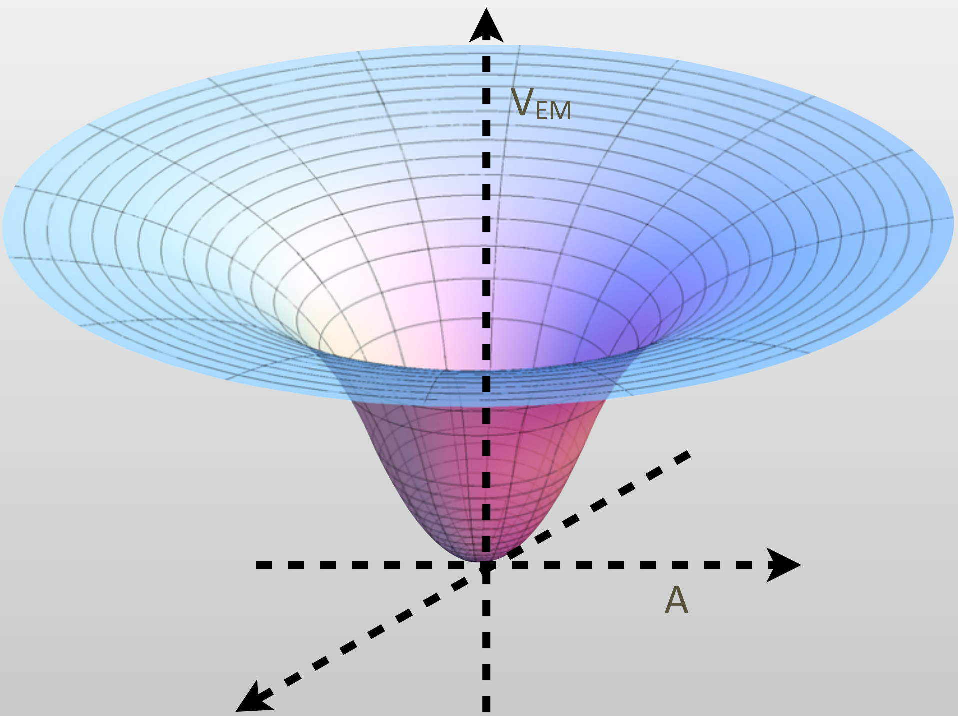
- One has to produce it to search for
  - ➡ heavier Higgs —> more Energy needed
  - ➡ build an accelerator and a collider "**ring**"

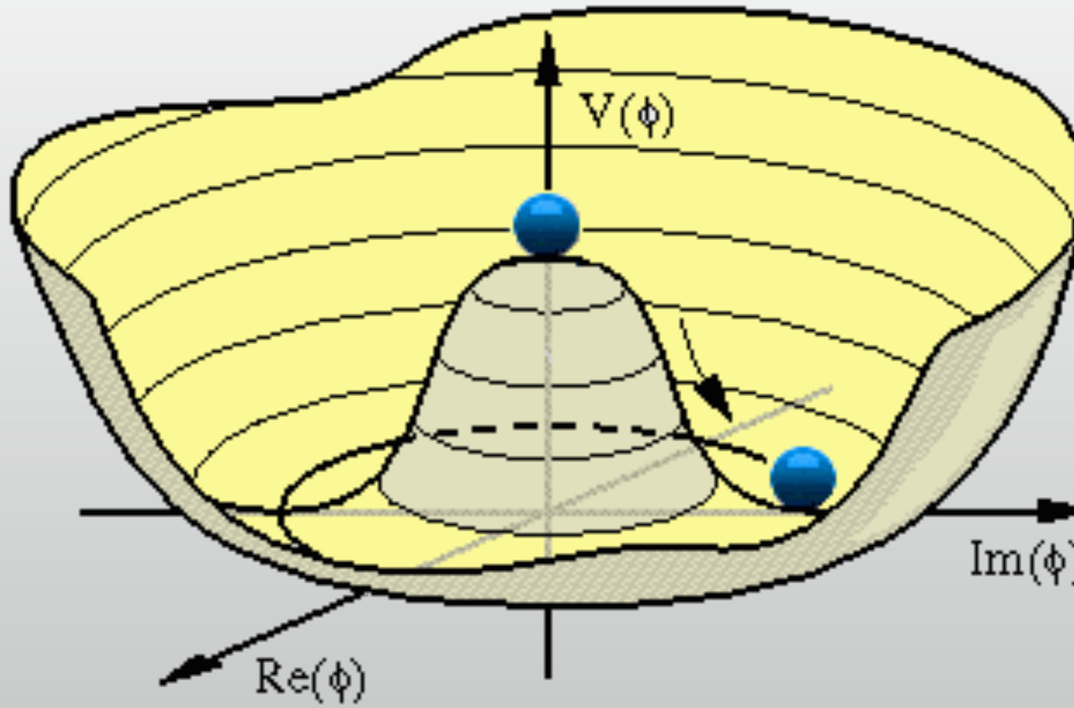
- Many have tried - and failed to find it.
  - ➡ SppS: 1981 - 1984, $\sqrt{s}$= 630GeV
  - ➡ LEP I,II: 1989 - 2000, $\sqrt{s}$= 90, 209GeV
  - ➡ Tevatron: 1987 - 2011, $\sqrt{s}$= 980GeV

- It was LHC's turn...
  - ➡ pp again
  - ➡ $\sqrt{s}$= 7000 & 8000 GeV



EM field vs Higgs field





if EM field amount is zero, the minimum potential is also zero.

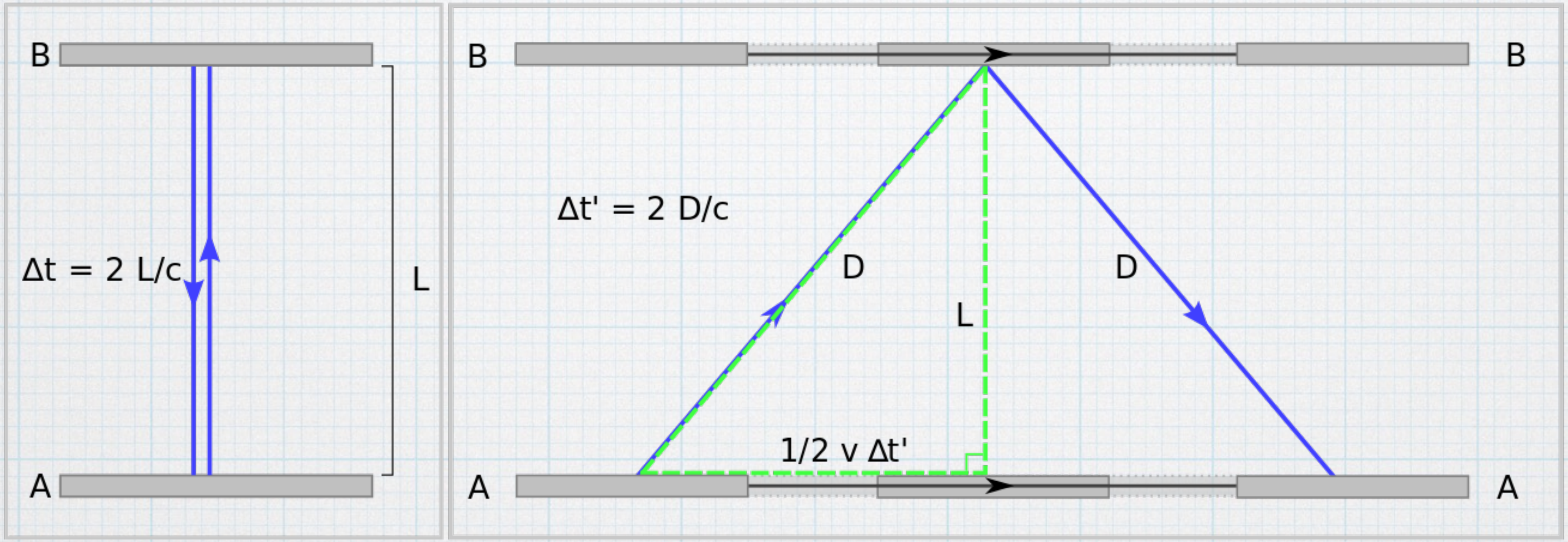
if Higgs field amo... zero, the minimum potential is non... minimize the pote... ...uum) ...d.

abo... ...ning

in a constant non-zero Higgs field since BB.

c ≈ 300000 km/s

- Einstein: "Speed of light is always fixed, **c**."
  - what is the corrolary?



$$c = \frac{2L}{\Delta t}$$

$$c = \frac{2D}{\Delta t'}$$

$$D^2 = L^2 + (v\Delta t'/2)^2$$

$$\frac{L}{\Delta t} = \frac{\sqrt{L^2 + v^2 \Delta t'^2/4}}{\Delta t'}$$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - v^2/c^2}}$$

Time dilation

TIME DILATION

$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

β=0..1

γ=1..∞

The moving object's own time τ

$$\Delta t' = \gamma \Delta t$$

The same object's observed time in a stationary lab

a coefficient that depends the speed of the the moving object

- Muon's real lifetime is 2.2 μs but we see in cosmic rays and in the experiments a much longer lifetime
  - this is the idea behind the muon collider!

Measurements

- The length of an object
  - for the man at rest (lab frame)

L_0 here time is t

time=0 time=t I measure the real length

  - for man going with speed v (spaceship frame)

v—>

L here time is τ

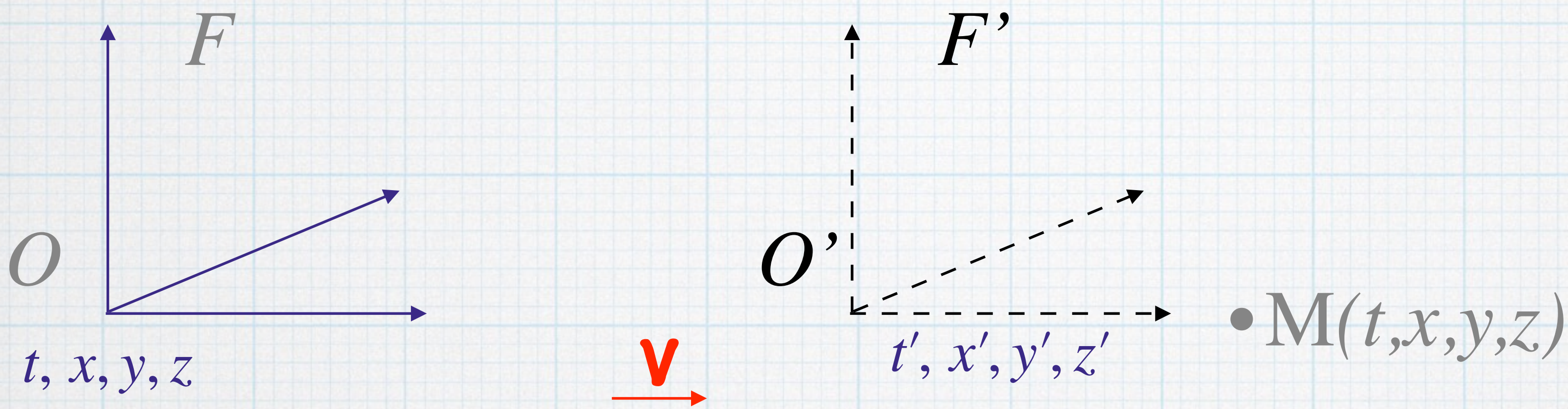
$$v = L/\tau = L_0/t \qquad t = \gamma\tau$$

$$L = \frac{L_0}{\gamma}$$

β=0..1
γ=1..∞

Length Contraction

Generalization

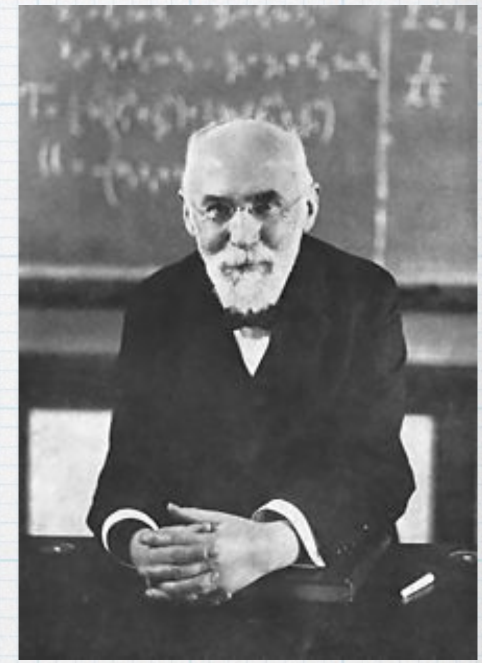


Ref. Frame F' moves with speed v.

An event M observed in O.

How to express it in O'?

1) go from O to O'
2) write M in O'



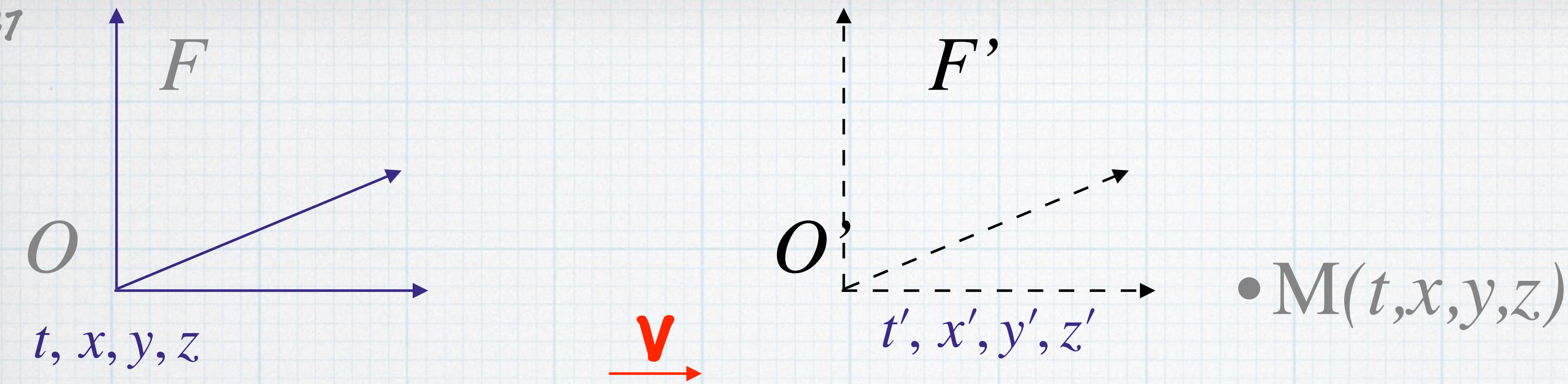
Lorentz 1902

$$\left.\begin{aligned} OM &= x \\ OO' &= vt \\ O'M &= x'/\gamma \end{aligned}\right\}$$

$$OM = OO' + O'M.$$

$$x = vt + x'/\gamma$$

$$x' = \gamma(x - vt)$$



Same event written in frame F' : **$OM = OO' + O'M$.**

$$\left.\begin{aligned} OM &= x/\gamma \\ OO' &= vt' \\ O'M &= x' \end{aligned}\right\}$$

$$x/\gamma = vt' + x'$$

$$x' = \gamma(x - vt)$$

$$x/\gamma = vt' + \gamma x - \gamma vt$$

$$x/\gamma - \gamma x + \gamma vt = vt'$$

$$\gamma[\ x/\gamma^2 - x + vt\ ] = vt'$$

$$\gamma[\ (x/\gamma^2 - x)/v + t\ ] = t'$$

$$t' = \gamma[\ t - x(1 - 1/\gamma^2)/v\ ]$$

$$t' = \gamma[\ t - x(\gamma^2 - 1)/v\gamma^2\ ]$$

$$t' = \gamma[\ t - xv/c^2\ ]$$

$$t' = \gamma[\ t - xv/c^2\ ]$$

$$ct' = \gamma[\ ct - x\beta\ ]$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

Lorentz transformation

$ct' = \gamma(ct - x\beta)$
$x' = \gamma(x - vt)$
$y' = y$
$z' = z$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Is there a quantity invariant under Lorentz transformations?

yes: distance in 4D: $s^2 = ct^2 - x^2 - y^2 - z^2$

beware: when we calculate 4D quantities the time-like and space-like components contribute with opposite signs: + - - - OR - + + + .
This is called a metric and it is related to the strucuture of spacetime.
Simplest form = Minkowski space = flat space

$M^2 = E^2 - P^2$ = Lorentz invariant — 4-Momenta

4-density (charge density, current density)
4-EMPot (scalar potential, vector potential)