

Di-Higgs production in HEFT and SMEFT



**Workshop on
EFT in Multi-Boson Production**

June 10, 2024

Gudrun Heinrich

*Institute for Theoretical Physics
Karlsruhe Institute of Technology*

based on work in collaboration with

Jens Braun, Ramona Gröber, Marius Höfer, Stephen Jones, Matthias Kerner,
Jannis Lang, Stefano Di Noi, Ludovic Scyboz, Marco Vitti

<https://arxiv.org/abs/2311.15004> GH, Jannis Lang

<https://arxiv.org/abs/2310.18221> Stefano Di Noi, Ramona Gröber, GH, Jannis Lang, Marco Vitti

<https://arxiv.org/abs/2204.13045> GH, Jannis Lang, Ludovic Scyboz

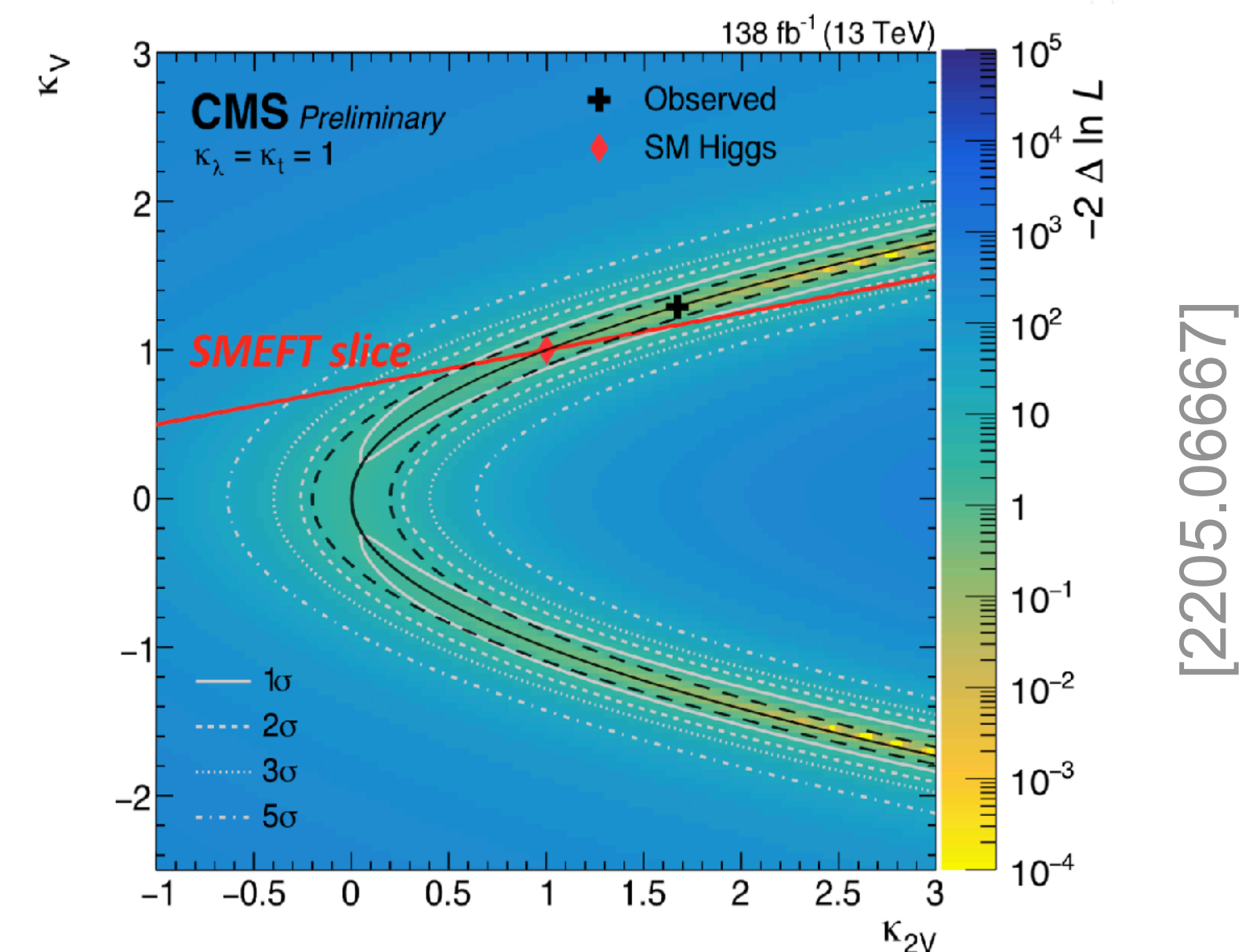
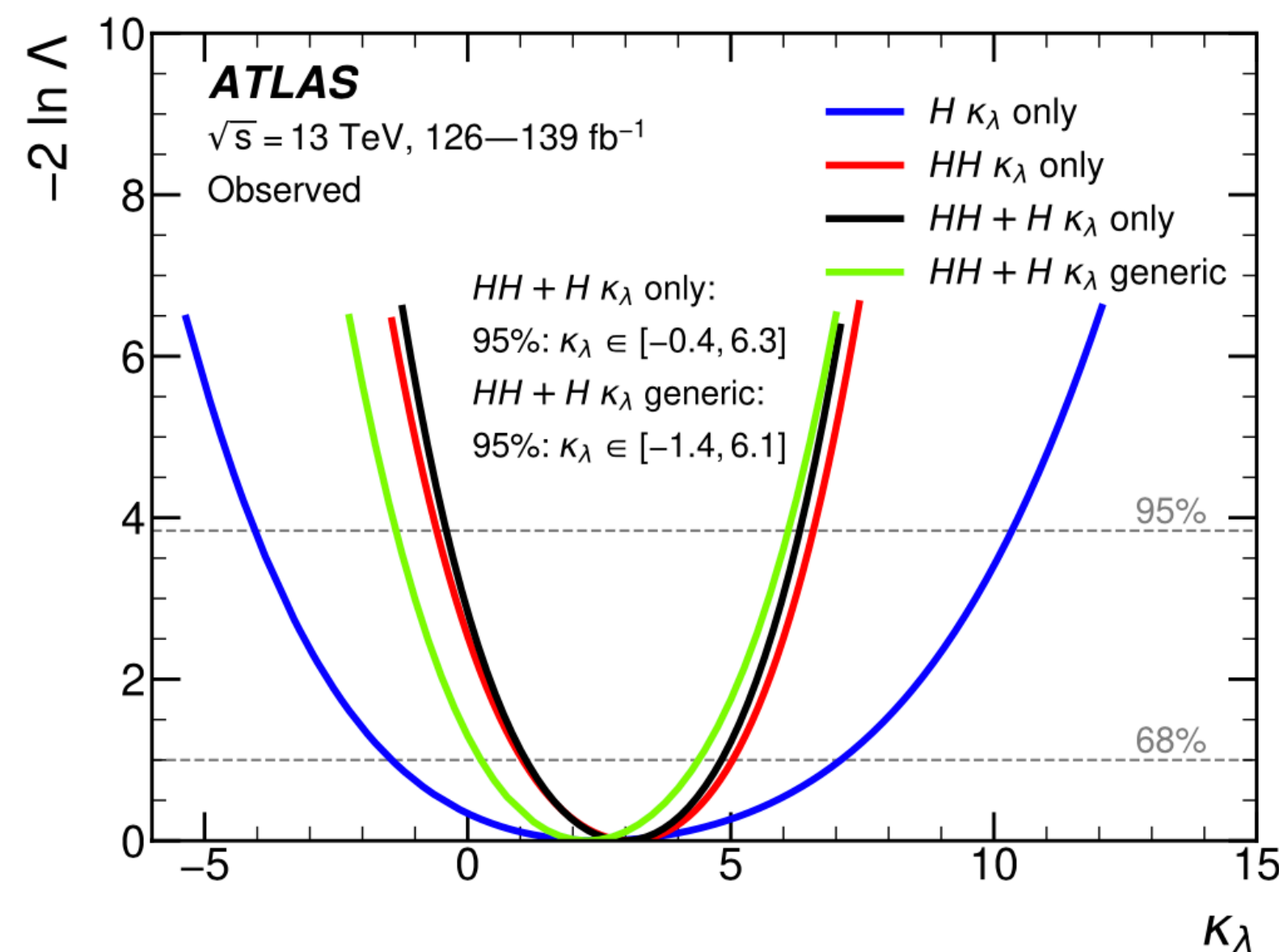
and work in progress Jens Braun, GH, Marius Höfer, Jannis Lang

Higgs boson pair production

prime process to explore the Higgs potential

at energies much larger than the electroweak scale: $V(\Phi) = ?$

after EW symmetry breaking: $V(h) \sim \frac{1}{2} \underbrace{(2v^2 \lambda)}_{m_h^2} h^2 + v\lambda h^3 + \frac{\lambda}{8} h^4 + \dots ?$



Effective Field Theory expansion schemes

SMEFT (Standard Model Effective Field Theory):

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{\text{dim}6} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

- assumes that Higgs field transforms linearly as a doublet under SU(2)_L
- canonical (mass) dimension counting
- weakly coupled UV completion

Effective Field Theory expansion schemes

HEFT (Higgs Effective Field Theory):

Feruglio '93; Grinstein, Trott '07; Contino et al. '10, Alonso et al. '13, Brivio et al. '13, Buchalla et al. '13

$$\mathcal{L}_{d_\chi} = \mathcal{L}_{(d_\chi=2)} + \sum_{L=1}^{\infty} \sum_i \left(\frac{1}{16\pi^2} \right)^L c_i^{(L)} O_i^{(L)}$$

counting of loop orders, expansion parameter $f^2/\Lambda^2 \approx 1/(16\pi^2)$

similar to chiral perturbation theory; $d_\chi = 2L + 2$ chiral dimension

$$\mathcal{L}_2 \supset \frac{v^2}{4} \langle D_\mu U^\dagger D^\mu U \rangle (1 + F_U(h)) + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h)$$

$$F_U(h) = \sum_{n=1}^{\infty} f_{U,n} \left(\frac{h}{v} \right)^n; \quad V(h) = v^4 \sum_{n=2}^{\infty} f_{V,n} \left(\frac{h}{v} \right)^n \quad \text{SM:} \quad f_{U,1} = 2, f_{U,2} = 1, f_{V,2} = f_{V,3} = 4f_{V,4} = \frac{m_h^2}{2v^2}$$

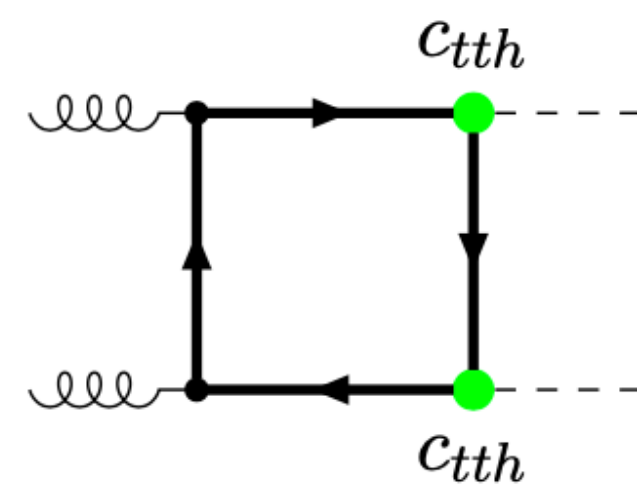
Higgs field h is an electroweak singlet \longrightarrow a priori no correlations between the $f_{U,n}; f_{V,n}$

different in SMEFT [Brivio et al. 1311.1832, 1604.06801, Gomez-Ambrosio et al. 2204.01763, 2207.09848, ...]

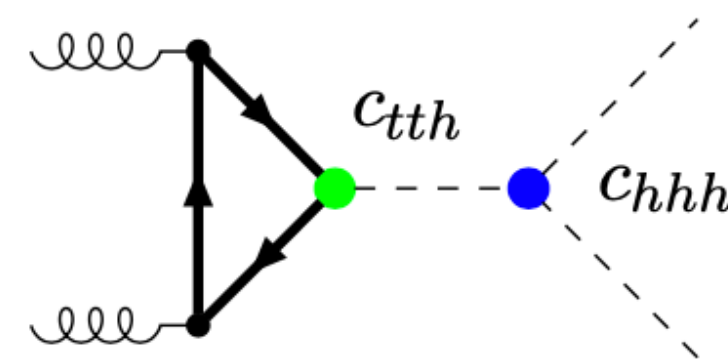
Lagrangians relevant for HH production

HEFT:

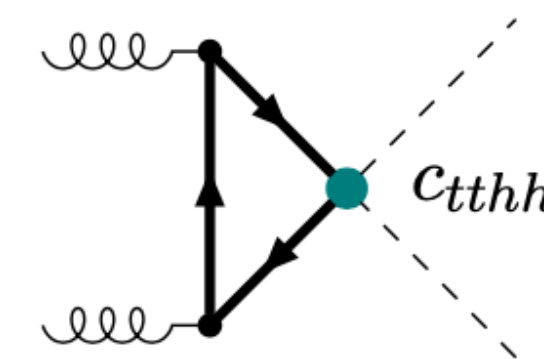
$$\mathcal{L}_{d_{\chi \leq 4}} \supset -m_t \left(c_t \frac{h}{v} + c_{tt} \frac{h^2}{v^2} \right) \bar{t} t - c_{hhh} \frac{m_h^2}{2v} h^3 + \frac{\alpha_s}{8\pi} \left(c_{ggh} \frac{h}{v} + c_{gggh} \frac{h^2}{v^2} \right) G_{\mu\nu}^a G^{a,\mu\nu}$$



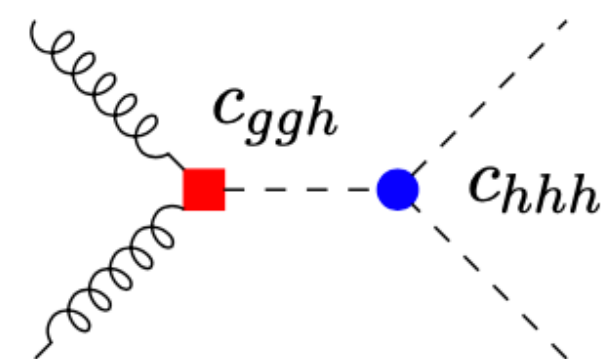
(a)



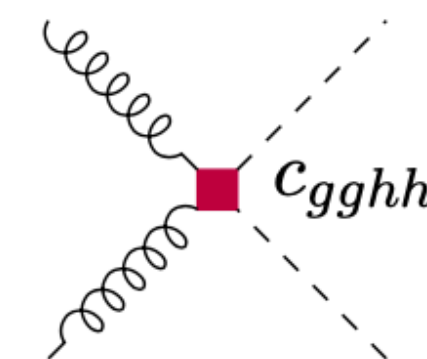
(b)



(c)



(d)



(e)

Leading SMEFT operators for HH production in gluon fusion

SMEFT: Warsaw basis Grzadkowski et al. 1008.4884

$$\begin{aligned}\Delta\mathcal{L}_{\text{Warsaw}} = & \frac{C_{H,\square}}{\Lambda^2} (\phi^\dagger\phi)\square(\phi^\dagger\phi) + \frac{C_{HD}}{\Lambda^2} (\phi^\dagger D_\mu\phi)^* (\phi^\dagger D^\mu\phi) + \frac{C_H}{\Lambda^2} (\phi^\dagger\phi)^3 \\ & + \left(\frac{C_{uH}}{\Lambda^2} \phi^\dagger\phi\bar{q}_L\phi^c t_R + h.c. \right) + \frac{C_{HG}}{\Lambda^2} \phi^\dagger\phi G_{\mu\nu}^a G^{\mu\nu,a}\end{aligned}$$

canonical normalisation

$$C_{H,\text{kin}} := C_{H,\square} - \frac{1}{4}C_{HD}$$

Leading SMEFT operators for HH production in gluon fusion

SMEFT: Warsaw basis Grzadkowski et al. 1008.4884

$$\Delta\mathcal{L}_{\text{Warsaw}} = \frac{C_{H,\square}}{\Lambda^2} (\phi^\dagger\phi)\square(\phi^\dagger\phi) + \frac{C_{HD}}{\Lambda^2} (\phi^\dagger D_\mu\phi)^* (\phi^\dagger D^\mu\phi) + \frac{C_H}{\Lambda^2} (\phi^\dagger\phi)^3$$

$$+ \left(\frac{C_{uH}}{\Lambda^2} \phi^\dagger\phi\bar{q}_L\phi^c t_R + h.c. \right) + \frac{C_{HG}}{\Lambda^2} \phi^\dagger\phi G_{\mu\nu}^a G^{\mu\nu,a}$$

canonical normalisation

$$C_{H,\text{kin}} := C_{H,\square} - \frac{1}{4}C_{HD}$$

$$? + \frac{C_{uG}}{\Lambda^2} (\bar{q}_L\sigma^{\mu\nu}T^a G_{\mu\nu}^a\phi^c t_R + h.c.)$$

(chromomagnetic operator)

Leading SMEFT operators for HH production in gluon fusion

SMEFT: Warsaw basis Grzadkowski et al. 1008.4884

$$\Delta\mathcal{L}_{\text{Warsaw}} = \frac{C_{H,\square}}{\Lambda^2} (\phi^\dagger\phi)\square(\phi^\dagger\phi) + \frac{C_{HD}}{\Lambda^2} (\phi^\dagger D_\mu\phi)^* (\phi^\dagger D^\mu\phi) + \frac{C_H}{\Lambda^2} (\phi^\dagger\phi)^3$$

$$+ \left(\frac{C_{uH}}{\Lambda^2} \phi^\dagger\phi\bar{q}_L\phi^c t_R + h.c. \right) + \frac{C_{HG}}{\Lambda^2} \phi^\dagger\phi G_{\mu\nu}^a G^{\mu\nu,a}$$

canonical normalisation

$$C_{H,\text{kin}} := C_{H,\square} - \frac{1}{4}C_{HD}$$

$$? + \frac{C_{uG}}{\Lambda^2} (\bar{q}_L\sigma^{\mu\nu}T^a G_{\mu\nu}^a\phi^c t_R + h.c.)$$

(chromomagnetic operator) + 4-fermion operators?

Leading SMEFT operators for HH production in gluon fusion

SMEFT: Warsaw basis Grzadkowski et al. 1008.4884

$$\Delta\mathcal{L}_{\text{Warsaw}} = \frac{C_{H,\square}}{\Lambda^2} (\phi^\dagger\phi)\square(\phi^\dagger\phi) + \frac{C_{HD}}{\Lambda^2} (\phi^\dagger D_\mu\phi)^* (\phi^\dagger D^\mu\phi) + \frac{C_H}{\Lambda^2} (\phi^\dagger\phi)^3$$

$$+ \left(\frac{C_{uH}}{\Lambda^2} \phi^\dagger\phi\bar{q}_L\phi^c t_R + h.c. \right) + \frac{C_{HG}}{\Lambda^2} \phi^\dagger\phi G_{\mu\nu}^a G^{\mu\nu,a}$$

canonical normalisation

$$C_{H,\text{kin}} := C_{H,\square} - \frac{1}{4}C_{HD}$$

? $+\frac{C_{uG}}{\Lambda^2} (\bar{q}_L\sigma^{\mu\nu}T^a G_{\mu\nu}^a\phi^c t_R + h.c.)$

(chromomagnetic operator) + 4-fermion operators?

sub-leading here if UV completion is a weakly coupled, renormalisable gauge theory

Wilson coefficients relevant for HH production

naive translation HEFT -> SMEFT at dim6 (comparing coefficients at Lagrangian level):

HEFT	Warsaw
C_{hhh}	$1 - 2 \frac{v^2}{\Lambda^2} \frac{v^2}{m_h^2} C_H + 3 \frac{v^2}{\Lambda^2} C_{H,\text{kin}}$
C_t	$1 + \frac{v^2}{\Lambda^2} C_{H,\text{kin}} - \frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2}m_t} C_{uH}$
C_{tt}	$-\frac{v^2}{\Lambda^2} \frac{3v}{2\sqrt{2}m_t} C_{uH} + \frac{v^2}{\Lambda^2} C_{H,\text{kin}}$
C_{ggh}	$\frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha_s} C_{HG}$
C_{gggh}	$\frac{v^2}{\Lambda^2} \frac{4\pi}{\alpha_s} C_{HG}$

problems:

- two field theories with different assumptions
- valid HEFT point can be invalid after translation to SMEFT
- translation depends on Λ
- treatment of strong coupling

Wilson coefficients relevant for HH production

naive translation HEFT -> SMEFT at dim6 (comparing coefficients at Lagrangian level):

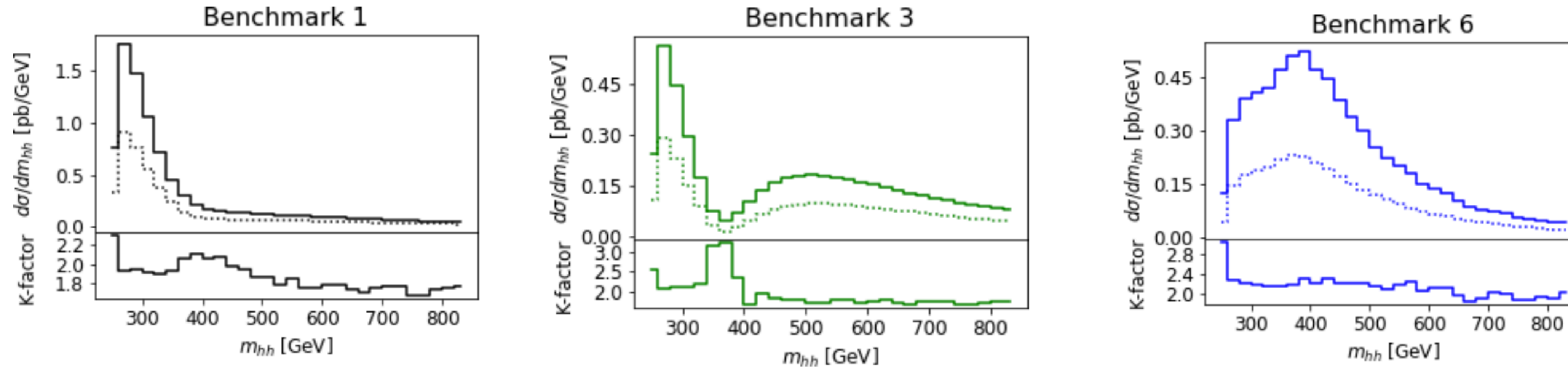
HEFT	Warsaw
C_{hhh}	$1 - 2 \frac{v^2}{\Lambda^2} \frac{v^2}{m_h^2} C_H + 3 \frac{v^2}{\Lambda^2} C_{H,\text{kin}}$
C_t	$1 + \frac{v^2}{\Lambda^2} C_{H,\text{kin}} - \frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2}m_t} C_{uH}$
C_{tt}	$-\frac{v^2}{\Lambda^2} \frac{3v}{2\sqrt{2}m_t} C_{uH} + \frac{v^2}{\Lambda^2} C_{H,\text{kin}}$
C_{ggh}	$\frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha_s} C_{HG}$
C_{gghh}	$\frac{v^2}{\Lambda^2} \frac{4\pi}{\alpha_s} C_{HG}$

problems:

- two field theories with different assumptions
- valid HEFT point can be invalid after translation to SMEFT
- translation depends on Λ
- treatment of strong coupling

HEFT benchmark points

consider benchmark points characteristic for a certain mHH **shape**



Capozi, GH,
1908.08923

benchmark (* = modified)	c_{hhh}	c_t	c_{tt}	c_{ggh}	c_{gggh}
SM	1	1	0	0	0
1*	5.105	1.1	0	0	0
3*	2.21	1.05	$-\frac{1}{3}$	0.5	0.25*
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25

- benchmark 1: enhanced low mHH
- benchmark 3: dip
- benchmark 6: SM-like except for shoulder left of peak

modified: to fulfil SMEFT relation $c_{ggh} = 2c_{gggh}$ and constraints after 2019

see also LHC Higgs WG4 note, 2304.01968

Naive translation HEFT to SMEFT

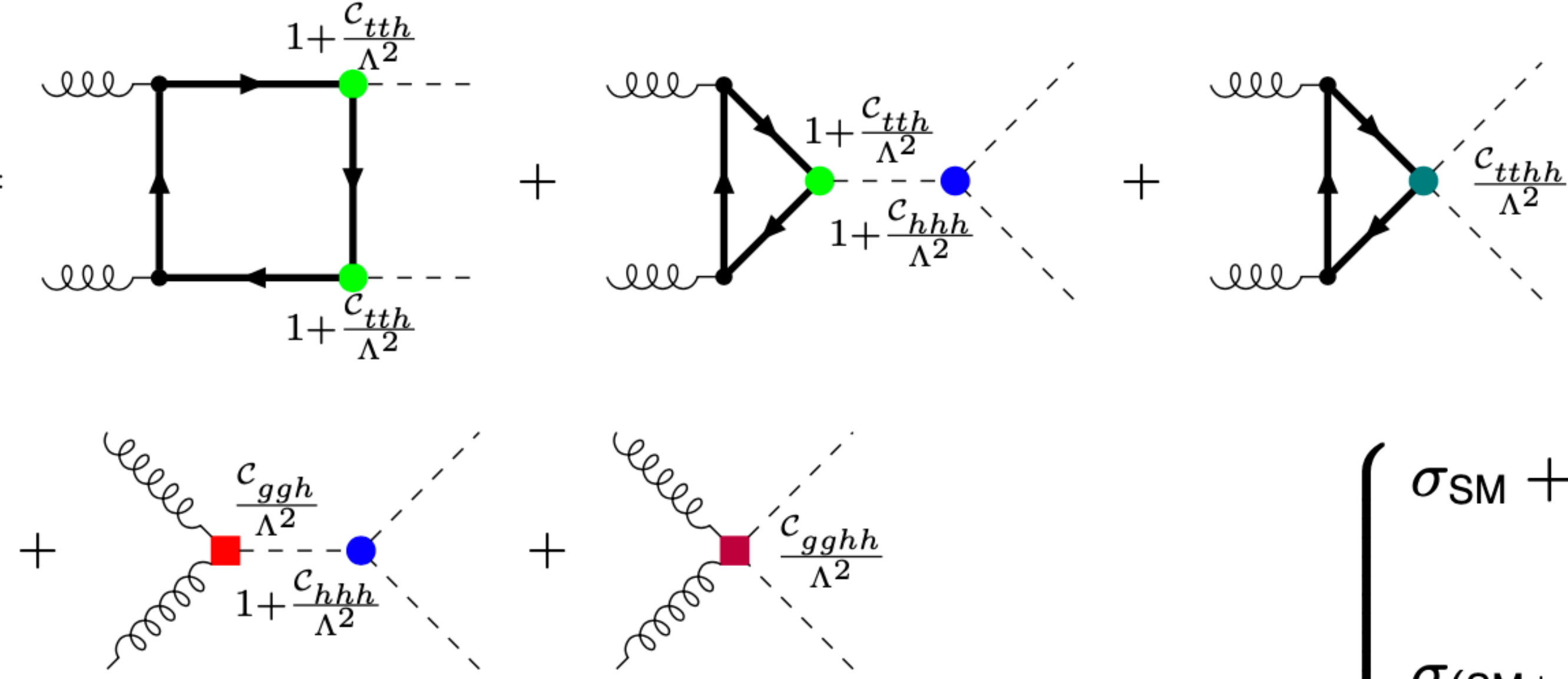
benchmark (* = modified)	c_{hhh}	c_t	c_{tt}	c_{ggh}	c_{gghh}	$C_{H,\text{kin}}$	C_H	C_{uH}	C_{HG}	Λ
SM	1	1	0	0	0	0	0	0	0	1 TeV
1*	5.105	1.1	0	0	0	4.95	-6.81	3.28	0	1 TeV
3*	2.21	1.05	$-\frac{1}{3}$	0.5	0.25*	13.5	2.64	12.6	0.0387	1 TeV
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV

benchmark	σ_{NLO} [fb] option (b)	K-factor option (b)	ratio to SM option (b)	σ_{NLO} [fb] option (a)	σ_{NLO} [fb] HEFT
SM	$27.94^{+13.7\%}_{-12.8\%}$	1.67	1	-	-
$\Lambda = 1 \text{ TeV}$					
1	$71.95^{+20.1\%}_{-15.7\%}$	2.06	2.58	-57.64	91.62
3	$68.69^{+9.4\%}_{-9.5\%}$	1.80	2.46	30.15	70.20
6	$70.18^{+18.8\%}_{-15.5\%}$	1.83	2.51	50.82	87.9

$$E^2 \frac{|C_i|}{\Lambda^2} \ll 1 \text{ not fulfilled for } \Lambda \simeq 1 \text{ TeV}$$

→ can lead to negative cross sections

SMEFT truncation

$$\mathcal{M}_{\text{SMEFT}}^{\text{LO}} =$$


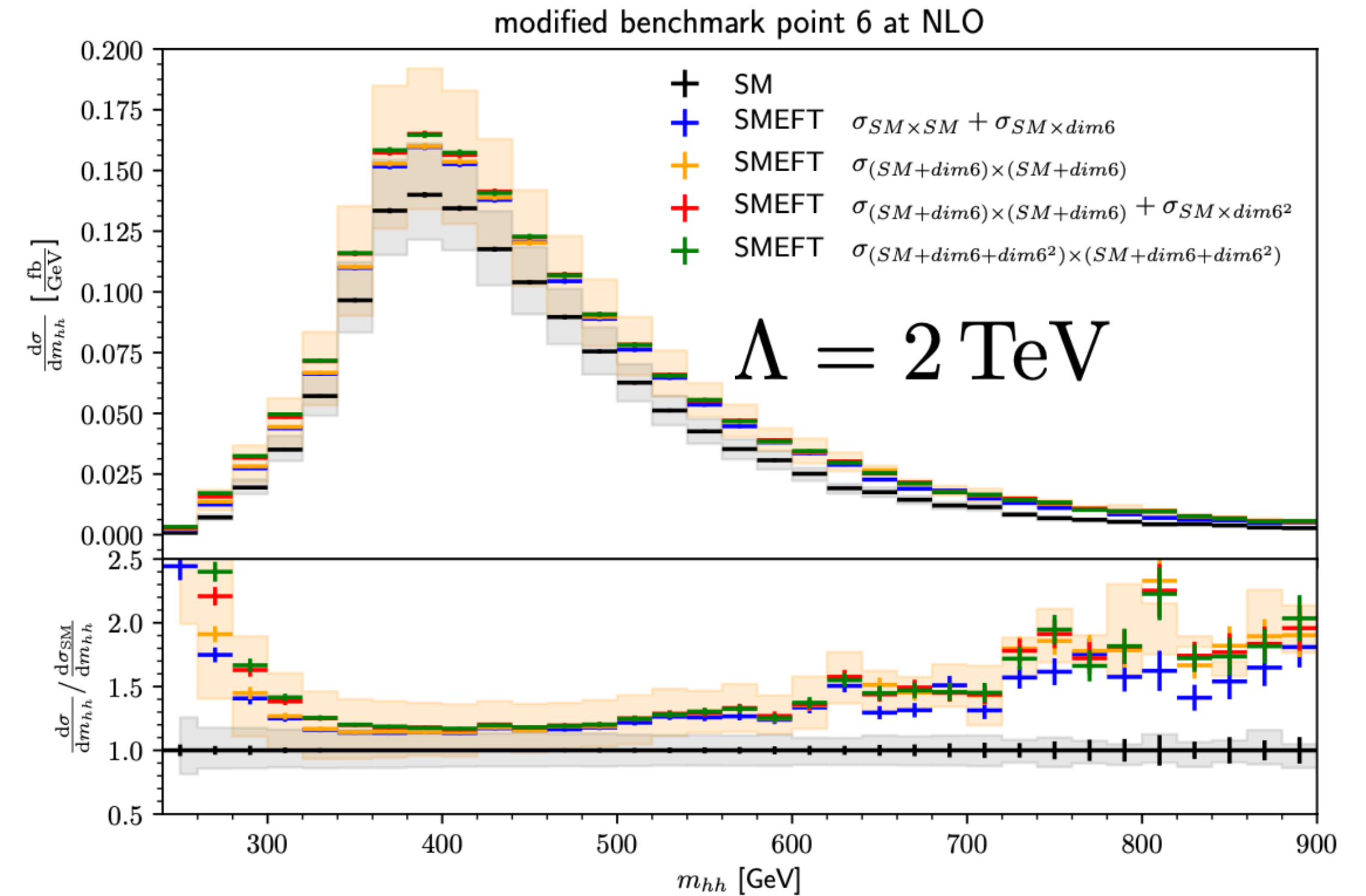
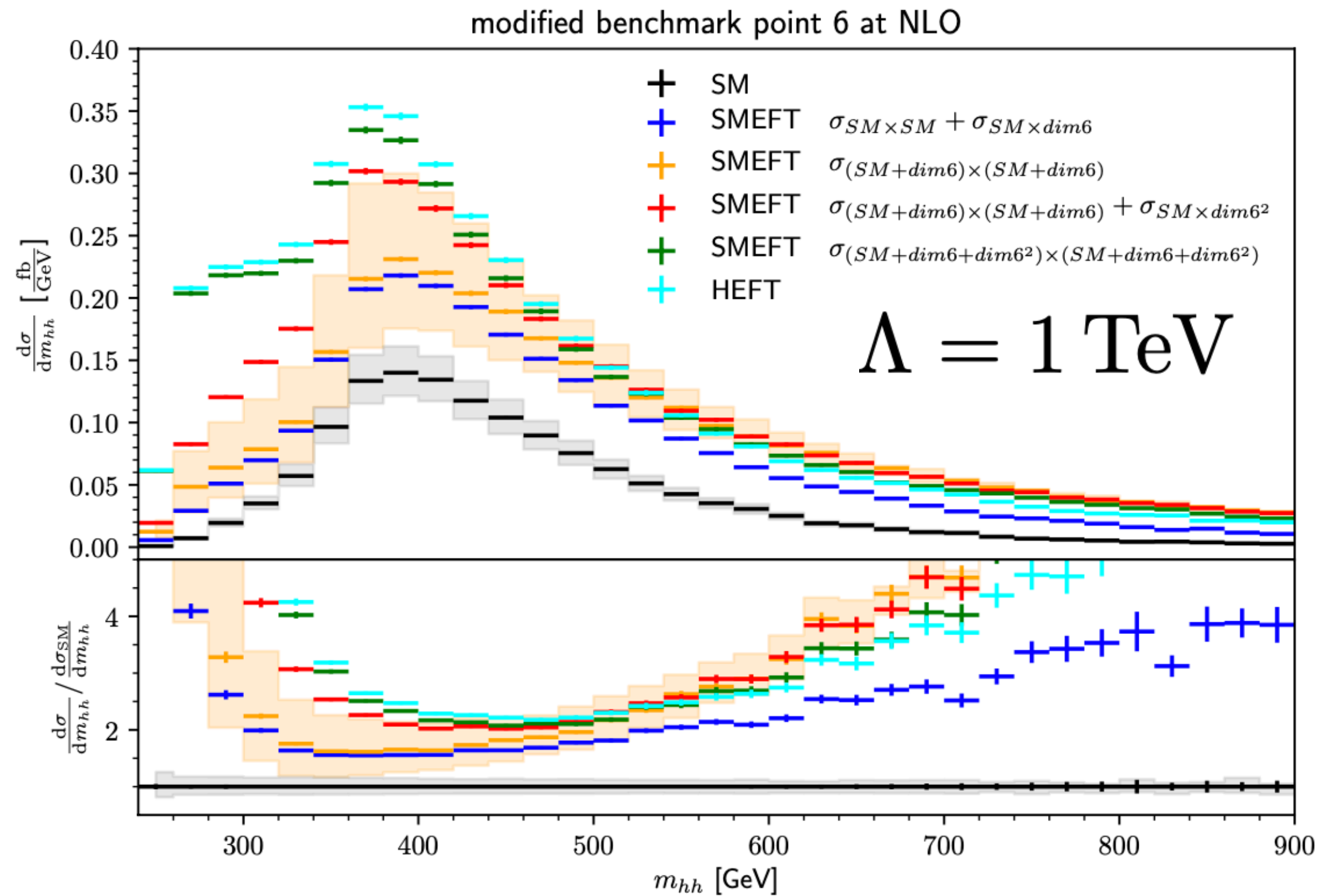
$$= \mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{dim6}} + \mathcal{M}_{(\text{dim6})^2}$$

$$\sigma \simeq \left\{ \begin{array}{ll} \sigma_{\text{SM}} + \sigma_{\text{SM} \times \text{dim6}} & \text{“linear”} \quad \text{(a)} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} & \text{“quadratic”} \quad \text{(b)} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} + \sigma_{\text{SM} \times \text{dim6}^2} & \text{(c)} \\ \sigma_{(\text{SM} + \text{dim6} + \text{dim6}^2) \times (\text{SM} + \text{dim6} + \text{dim6}^2)} & \text{(d)} \end{array} \right.$$

all options available in **ggHH_SMEFT** code

Truncation effects on Higgs boson pair invariant mass

benchmark point 6 $c_{hhh} = -0.684, c_t = 0.9, c_{tt} = -1/6, c_{ggh} = 0.5, c_{gghh} = 0.25$



figures: Jannis Lang

characteristic shape not present in SMEFT,
large difference between linear and quadratic truncation

differences between truncation options smaller, but
can hardly be distinguished from SM
within NLO scale uncertainties

EFT expansion + higher orders in QCD

(SM)EFT expansion parameters:

$$\Lambda^{-d_c} (g_s^2 L)^{l_{\text{QCD}}} \mathbf{L}^{l_{\text{not_QCD}}}$$

d_c : canonical dimension

This is an expansion in several parameters

g_s : strong coupling

$L = (16\pi)^{-1}$: loop factor (QCD)

$\mathbf{L} = (16\pi)^{-1}$: loop factor (new physics)

l_{QCD} : number of QCD loops

$l_{\text{not_QCD}}$: number of loops involving new particles or new interactions (or EW corrections)

EFT expansion + higher orders in QCD

(SM)EFT expansion parameters:

$$\Lambda^{-d_c} (g_s^2 L)^{l_{\text{QCD}}} \mathbf{L}^{l_{\text{not_QCD}}}$$

d_c : canonical dimension

This is an expansion in several parameters

g_s : strong coupling

$L = (16\pi)^{-1}$: loop factor (QCD)

$\mathbf{L} = (16\pi)^{-1}$: loop factor (new physics)

l_{QCD} : number of QCD loops

$l_{\text{not_QCD}}$: number of loops involving new particles or new interactions (or EW corrections)

In renormalisable, weakly coupled UV completions:

Operators containing field strength tensors are loop-generated \Rightarrow get a loop suppression factor

Arzt, Einhorn Wudka '94; Buchalla, GH, Müller-Salditt, Pandler 2204.11808

Loop-generated operators

Isidori, Wilsch, Wyler, Review Mod. Phys. 2303.16922

PTG: Potentially Tree Generated

LG: Loop Generated

5–7: Fermion Bilinears (ψ^2)

non-hermitian ($\bar{L}R$)			
5: $\psi^2 H^3 + \text{h.c.}$ [PTG]	6: $\psi^2 XH + \text{h.c.}$ [LG]		
$Q_{eH} (H^\dagger H)(\bar{\ell}_p e_r H)$	$Q_{eW} (\bar{\ell}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$	$Q_{uG} (\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{H} G_{\mu\nu}^A$	$Q_{dG} (\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G_{\mu\nu}^A$
$Q_{uH} (H^\dagger H)(\bar{q}_p u_r \tilde{H})$	$Q_{eB} (\bar{\ell}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$Q_{uW} (\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$	$Q_{dW} (\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H W_{\mu\nu}^I$
$Q_{dH} (H^\dagger H)(\bar{q}_p d_r H)$		$Q_{uB} (\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{H} B_{\mu\nu}$	$Q_{dB} (\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$

7: $\psi^2 H^2 D$ – hermitian + Q_{Hud} [PTG]		
($\bar{L}L$)	($\bar{R}R$)	($\bar{R}R'$) + h.c.
$Q_{H\ell}^{(1)} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\ell}_p \gamma^\mu \ell_r)$	$Q_{He} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$	$Q_{Hud} i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$
$Q_{H\ell}^{(3)} (H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{\ell}_p \tau^I \gamma^\mu \ell_r)$	$Q_{Hu} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$	
$Q_{Hq}^{(1)} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$	$Q_{Hd} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$	
$Q_{Hq}^{(3)} (H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$		

Loop-generated operators

Isidori, Wilsch, Wyler, Review Mod. Phys. 2303.16922

PTG: Potentially Tree Generated

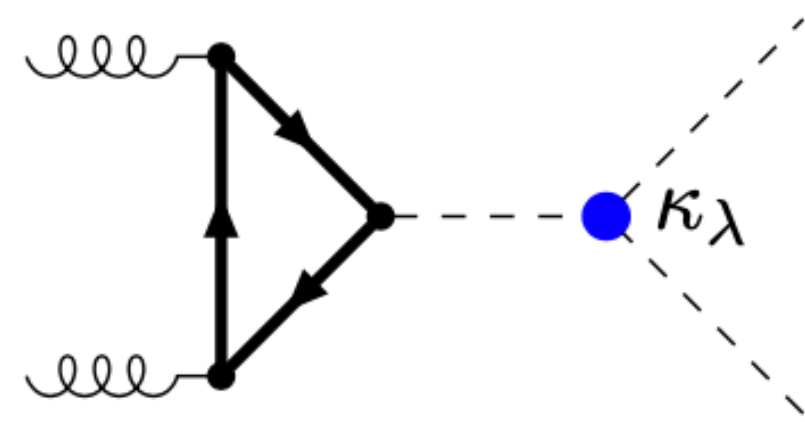
LG: Loop Generated

5–7: Fermion Bilinears (ψ^2)

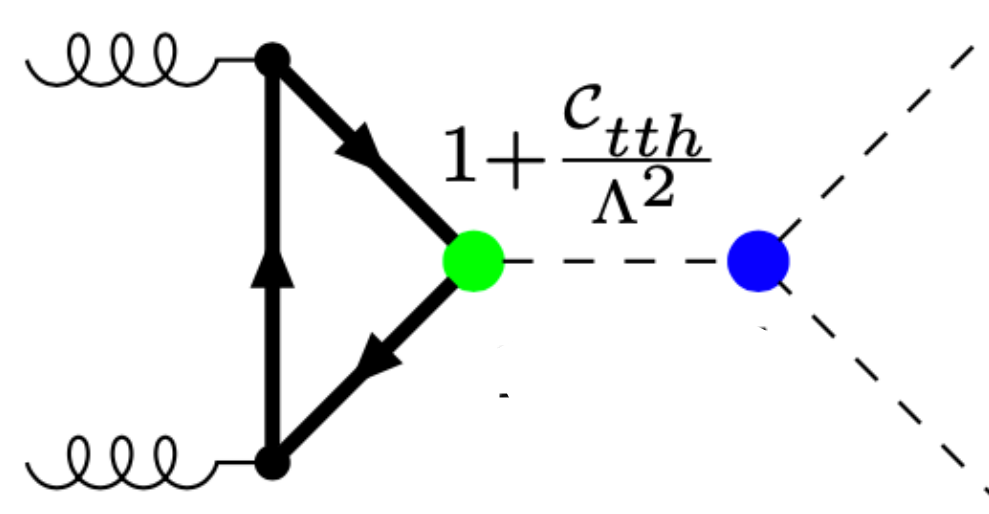
non-hermitian ($\bar{L}R$)			
5: $\psi^2 H^3 + \text{h.c.}$ [PTG]	6: $\psi^2 XH + \text{h.c.}$ [LG]		
$Q_{eH} (H^\dagger H)(\bar{\ell}_p e_r H)$	$Q_{eW} (\bar{\ell}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$	$Q_{uG} (\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{H} G_{\mu\nu}^A$	$Q_{dG} (\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G_{\mu\nu}^A$
$Q_{uH} (H^\dagger H)(\bar{q}_p u_r \tilde{H})$	$Q_{eB} (\bar{\ell}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$Q_{uW} (\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$	$Q_{dW} (\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H W_{\mu\nu}^I$
$Q_{dH} (H^\dagger H)(\bar{q}_p d_r H)$		$Q_{uB} (\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{H} B_{\mu\nu}$	$Q_{dB} (\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$

7: $\psi^2 H^2 D$ – hermitian + Q_{Hud} [PTG]		
($\bar{L}L$)	($\bar{R}R$)	($\bar{R}R'$) + h.c.
$Q_{H\ell}^{(1)} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\ell}_p \gamma^\mu \ell_r)$	$Q_{He} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$	$Q_{Hud} i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$
$Q_{H\ell}^{(3)} (H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{\ell}_p \tau^I \gamma^\mu \ell_r)$	$Q_{Hu} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$	
$Q_{Hq}^{(1)} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$	$Q_{Hd} (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$	
$Q_{Hq}^{(3)} (H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$		

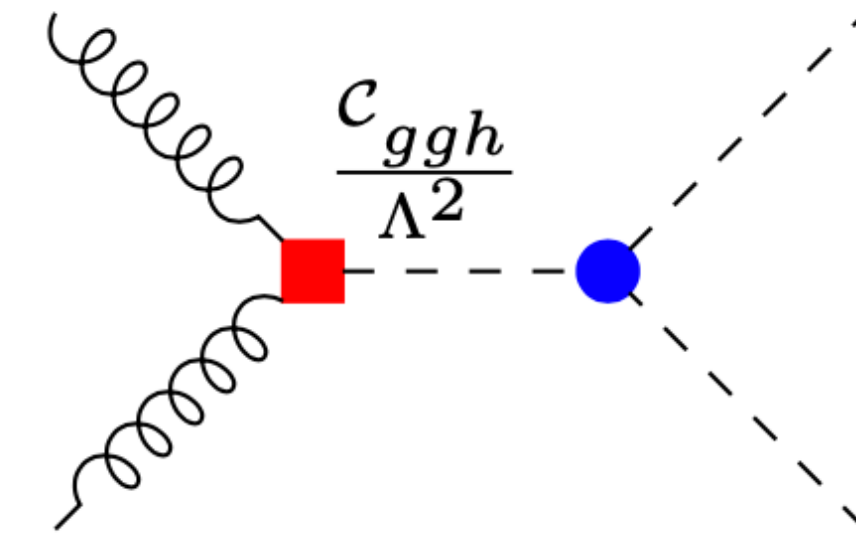
Loop counting in SMEFT



$$\frac{1}{16\pi^2} \quad l_{\text{QCD}} = 1$$

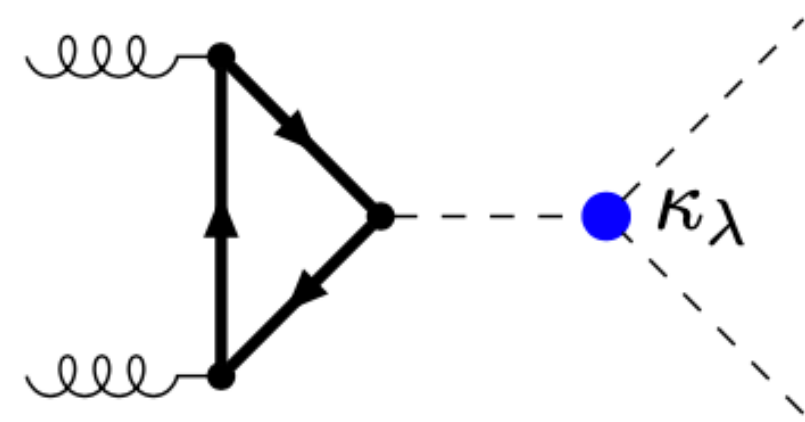


$$\frac{1}{\Lambda^2} \frac{1}{16\pi^2} \quad l_{\text{QCD}} = 1$$

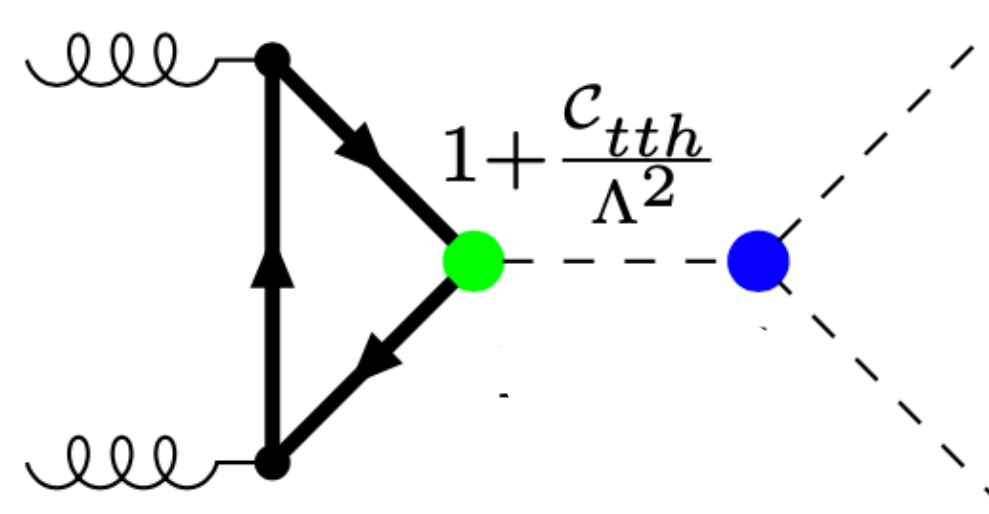


$$\frac{1}{\Lambda^2} \frac{1}{16\pi^2} \quad l_{\text{not-QCD}} = 1$$

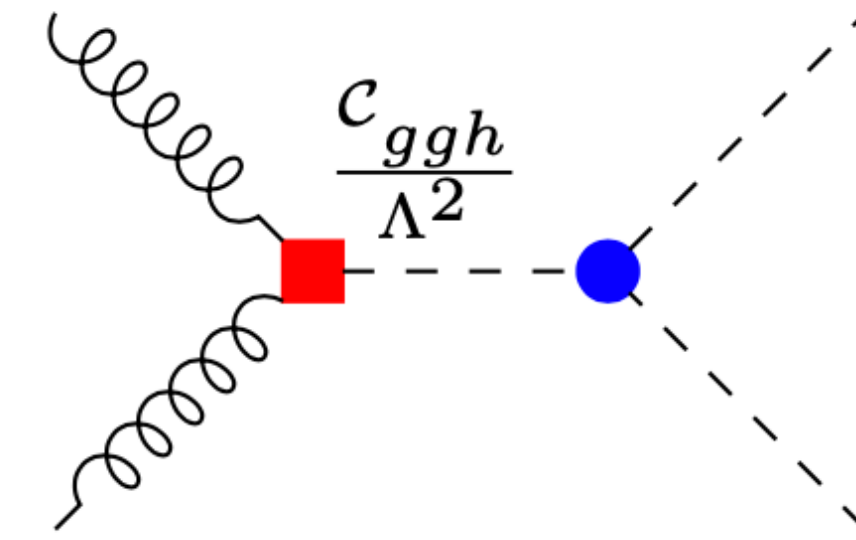
Loop counting in SMEFT



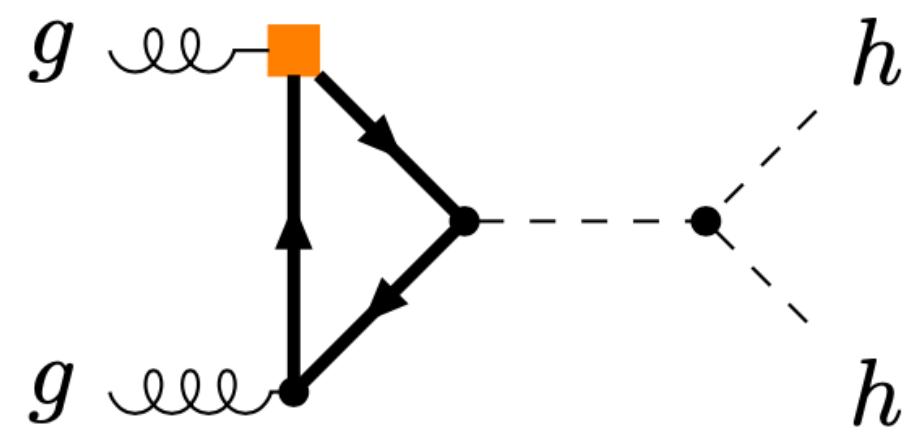
$$\frac{1}{16\pi^2} \quad l_{\text{QCD}} = 1$$



$$\frac{1}{\Lambda^2} \frac{1}{16\pi^2} \quad l_{\text{QCD}} = 1$$



$$\frac{1}{\Lambda^2} \frac{1}{16\pi^2} \quad l_{\text{not-QCD}} = 1$$

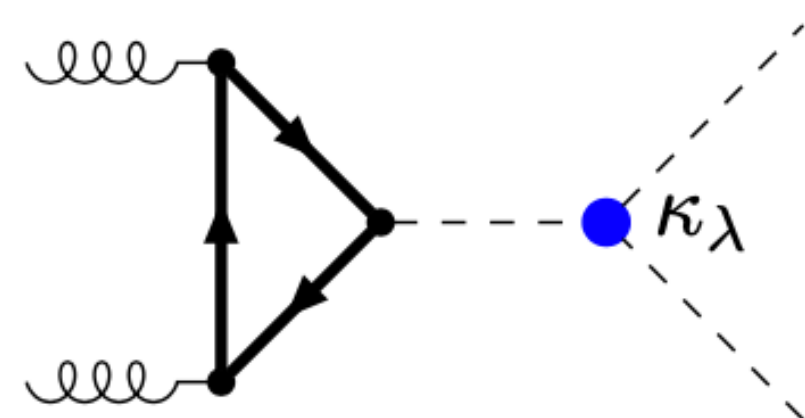


chromomagnetic operator

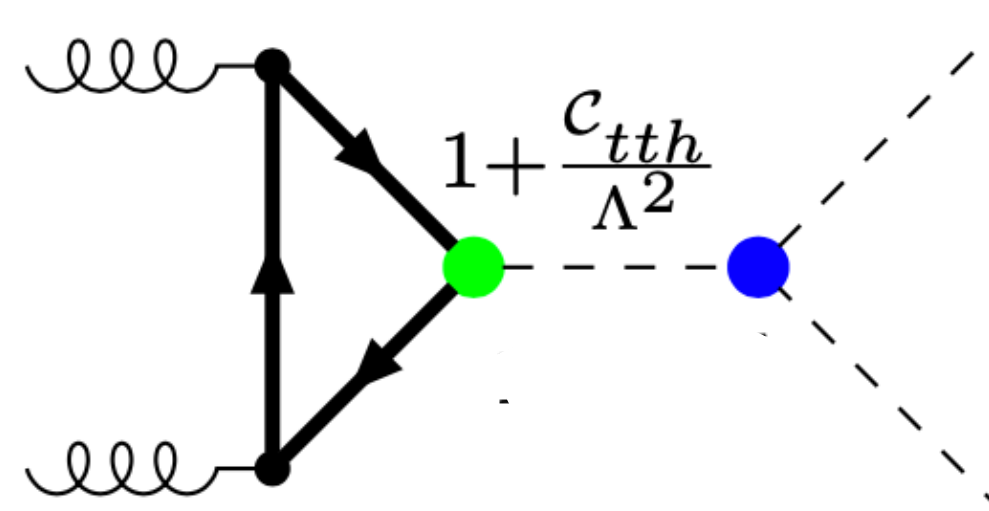
$$\frac{1}{\Lambda^2} \frac{1}{(16\pi^2)^2} \quad l_{\text{QCD}} = 1, \quad l_{\text{not-QCD}} = 1$$

explicit implicit

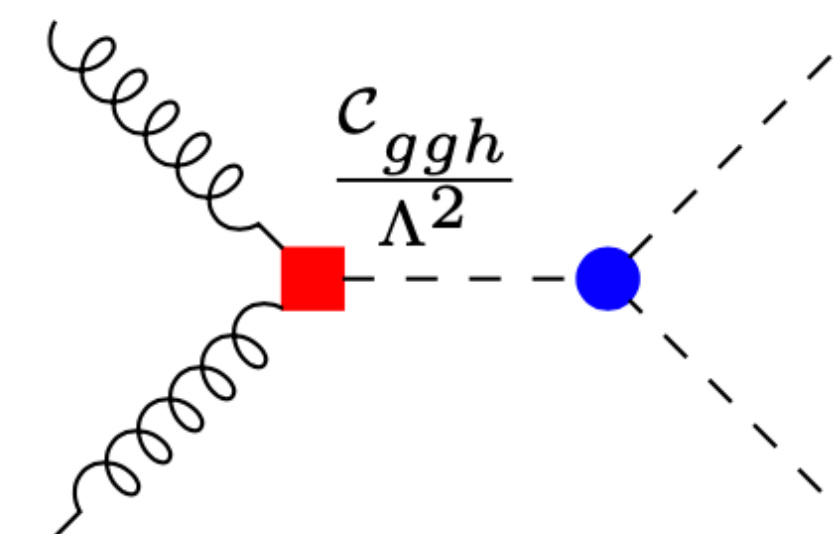
Loop counting in SMEFT



$$\frac{1}{16\pi^2} \quad l_{\text{QCD}} = 1$$



$$\frac{1}{\Lambda^2} \frac{1}{16\pi^2} \quad l_{\text{QCD}} = 1$$



$$\frac{1}{\Lambda^2} \frac{1}{16\pi^2} \quad l_{\text{not-QCD}} = 1$$

new boson

chromomagnetic operator

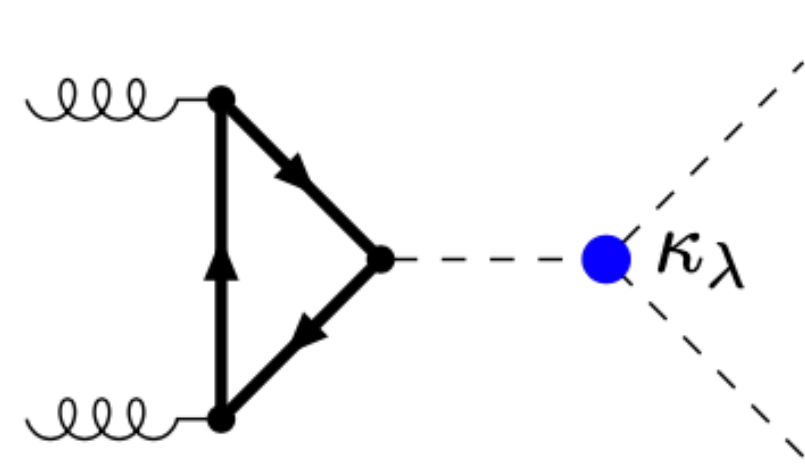
g

h

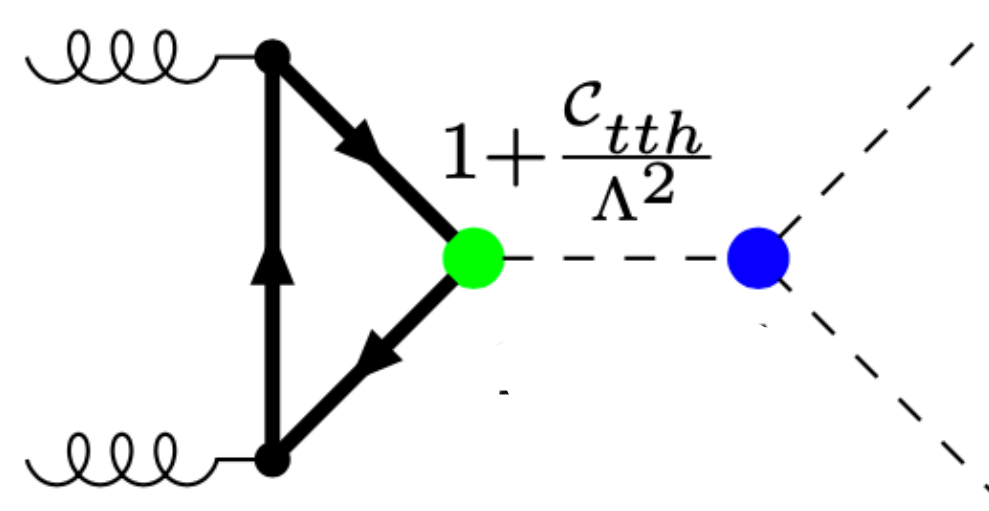
$$\frac{1}{\Lambda^2} \frac{1}{(16\pi^2)^2} \quad l_{\text{QCD}} = 1, \quad l_{\text{not-QCD}} = 1$$

explicit implicit

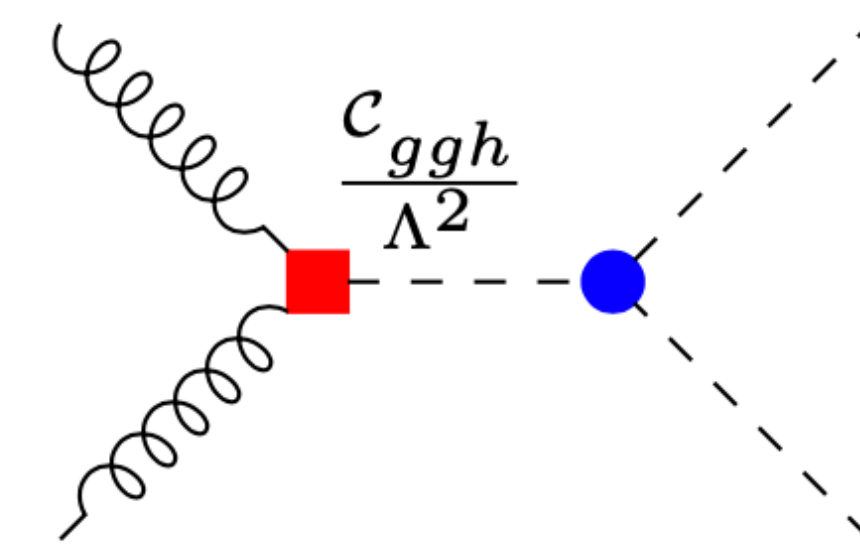
Loop counting in SMEFT



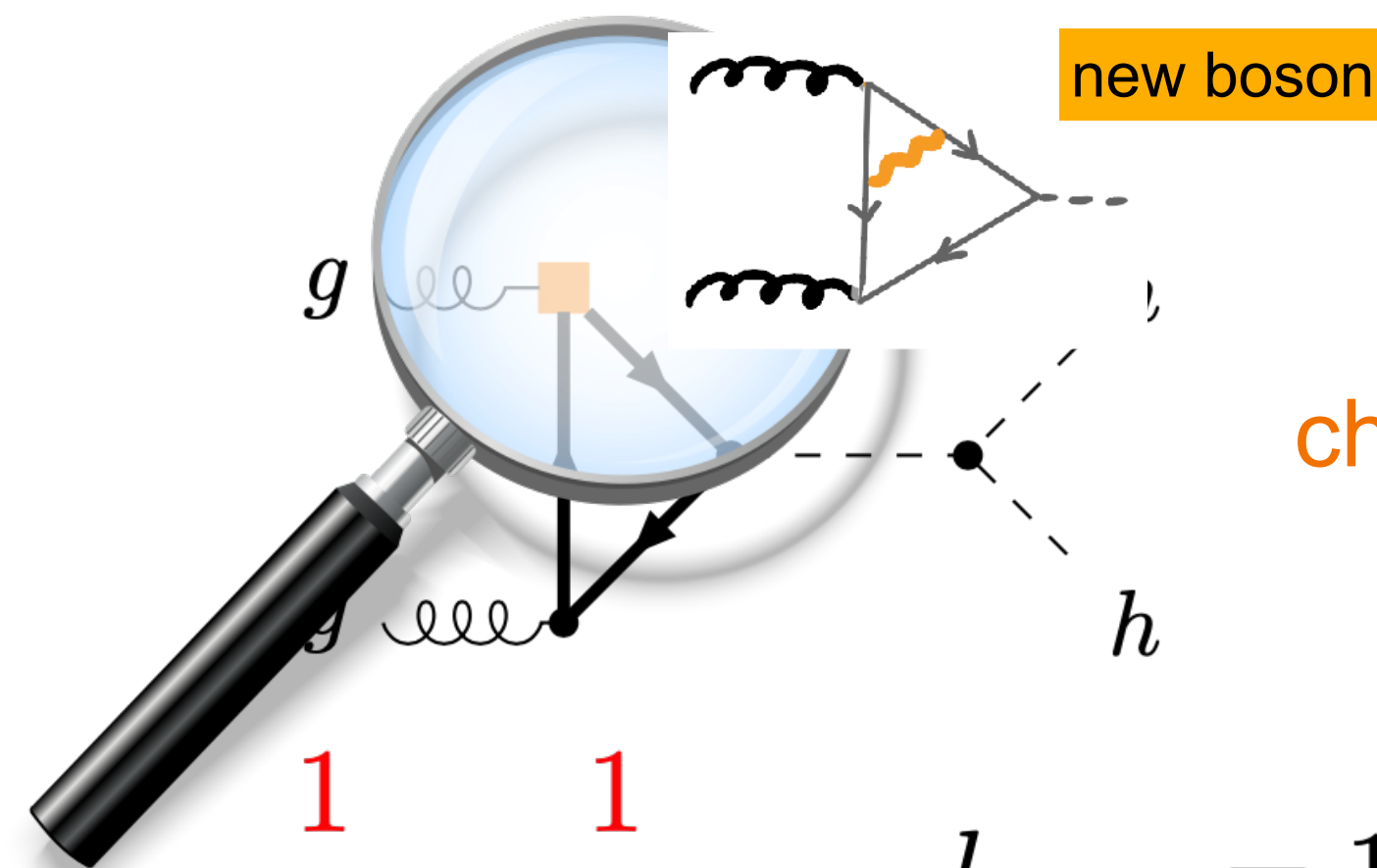
$$\frac{1}{16\pi^2} \quad l_{\text{QCD}} = 1$$



$$\frac{1}{\Lambda^2} \frac{1}{16\pi^2} \quad l_{\text{QCD}} = 1$$

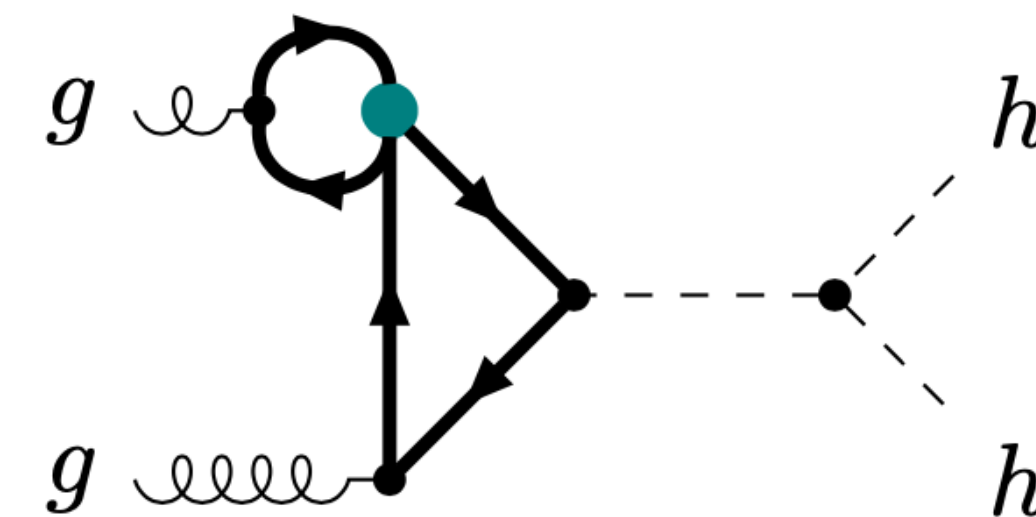


$$\frac{1}{\Lambda^2} \frac{1}{16\pi^2} \quad l_{\text{not-QCD}} = 1$$



$$\frac{1}{\Lambda^2} \frac{1}{(16\pi^2)^2} \quad l_{\text{QCD}} = 1, l_{\text{not-QCD}} = 1$$

explicit implicit



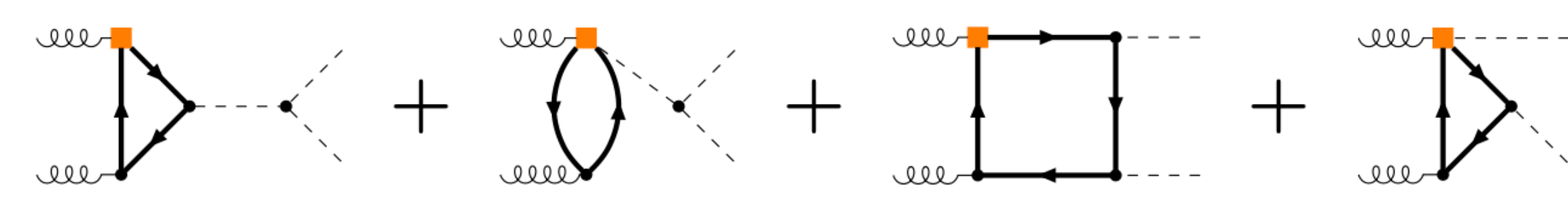
$$\frac{1}{\Lambda^2} \frac{1}{(16\pi^2)^2} \quad l_{\text{QCD}} = 1, l_{\text{not-QCD}} = 1$$

explicit explicit

4-top operators enter at the same order!

Subleading operators in SMEFT

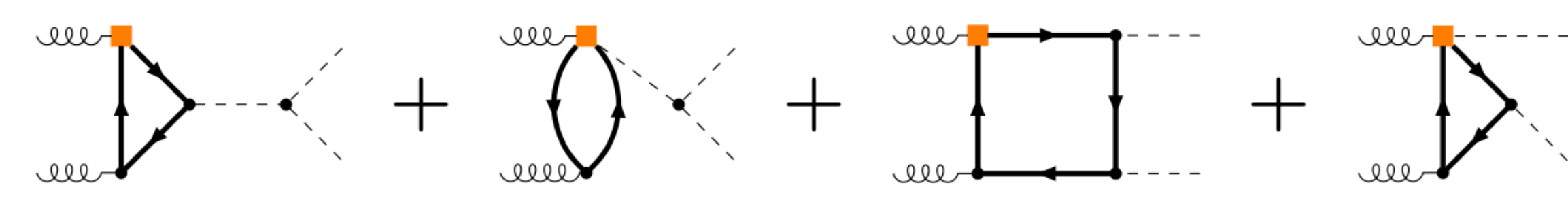
in a renormalisable, weakly coupling UV completion

$$\mathcal{L}_{tG} = \frac{C_{tG}}{\Lambda^2} \left(\bar{Q}_L \sigma^{\mu\nu} T^a G_{\mu\nu}^a \tilde{\phi} t_R + \text{h.c.} \right) \quad \mathcal{M}_{tG} =$$


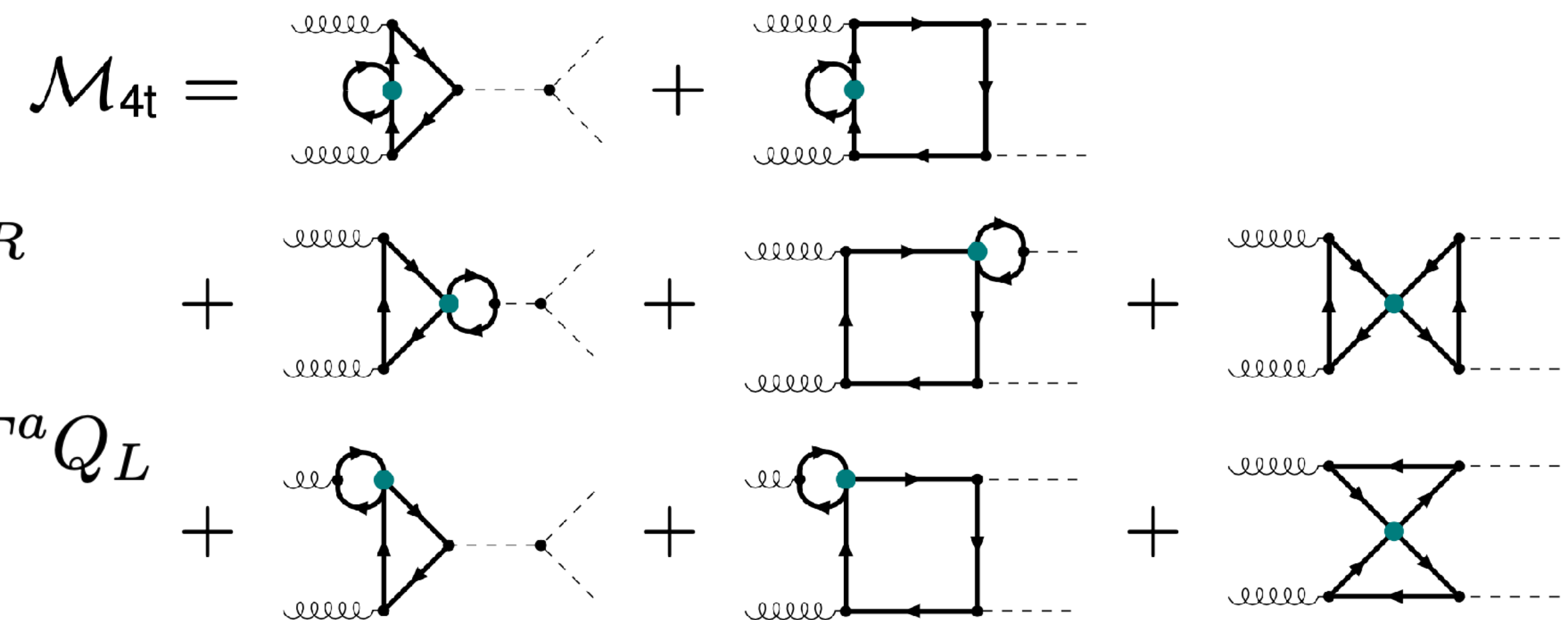
$$\begin{aligned} \mathcal{L}_{4t} = & \frac{C_{Qt}^{(1)}}{\Lambda^2} \bar{Q}_L \gamma^\mu Q_L \bar{t}_R \gamma_\mu t_R + \frac{C_{Qt}^{(8)}}{\Lambda^2} \bar{Q}_L \gamma^\mu T^a Q_L \bar{t}_R \gamma_\mu T^a t_R \\ & + \frac{C_{QQ}^{(1)}}{\Lambda^2} \bar{Q}_L \gamma^\mu Q_L \bar{Q}_L \gamma_\mu Q_L + \frac{C_{QQ}^{(8)}}{\Lambda^2} \bar{Q}_L \gamma^\mu T^a Q_L \bar{Q}_L \gamma_\mu T^a Q_L \\ & + \frac{C_{tt}}{\Lambda^2} \bar{t}_R \gamma^\mu t_R \bar{t}_R \gamma_\mu t_R \end{aligned}$$

Subleading operators in SMEFT

in a renormalisable, weakly coupling UV completion

$$\mathcal{L}_{tG} = \frac{C_{tG}}{\Lambda^2} \left(\bar{Q}_L \sigma^{\mu\nu} T^a G_{\mu\nu}^a \tilde{\phi} t_R + \text{h.c.} \right) \quad \mathcal{M}_{tG} =$$


$$\begin{aligned} \mathcal{L}_{4t} = & \frac{C_{Qt}^{(1)}}{\Lambda^2} \bar{Q}_L \gamma^\mu Q_L \bar{t}_R \gamma_\mu t_R + \frac{C_{Qt}^{(8)}}{\Lambda^2} \bar{Q}_L \gamma^\mu T^a Q_L \bar{t}_R \gamma_\mu T^a t_R \\ & + \frac{C_{QQ}^{(1)}}{\Lambda^2} \bar{Q}_L \gamma^\mu Q_L \bar{Q}_L \gamma_\mu Q_L + \frac{C_{QQ}^{(8)}}{\Lambda^2} \bar{Q}_L \gamma^\mu T^a Q_L \bar{Q}_L \gamma_\mu T^a Q_L \\ & + \frac{C_{tt}}{\Lambda^2} \bar{t}_R \gamma^\mu t_R \bar{t}_R \gamma_\mu t_R \end{aligned}$$

$$\mathcal{M}_{4t} =$$


Four-top operators

$$\mathcal{L}_{4t} = \frac{C_{Qt}^{(1)}}{\Lambda^2} \underbrace{\bar{t}_L \gamma^\mu t_L \bar{t}_R \gamma_\mu t_R}_{\bar{t} \mathbb{P}_R \gamma^\mu \mathbb{P}_L t \bar{t} \mathbb{P}_L \gamma_\mu \mathbb{P}_R t} + \frac{C_{Qt}^{(8)}}{\Lambda^2} \bar{t}_L \gamma^\mu T^a t_L \bar{t}_R \gamma_\mu T^a t_R + \dots$$

$\bar{t} \mathbb{P}_R \gamma^\mu \mathbb{P}_L t \bar{t} \mathbb{P}_L \gamma_\mu \mathbb{P}_R t \quad ; \quad \mathbb{P}_{L/R} = (\mathbb{I} \mp \gamma_5)/2$

- 4-top operators occur in 2-loop diagrams
- treatment of γ_5 matters!
- translation between schemes also affects other operators and parameters

Four-top operators

$$\mathcal{L}_{4t} = \frac{C_{Qt}^{(1)}}{\Lambda^2} \underbrace{\bar{t}_L \gamma^\mu t_L \bar{t}_R \gamma_\mu t_R}_{\bar{t} \mathbb{P}_R \gamma^\mu \mathbb{P}_L t \bar{t} \mathbb{P}_L \gamma_\mu \mathbb{P}_R t} + \frac{C_{Qt}^{(8)}}{\Lambda^2} \bar{t}_L \gamma^\mu T^a t_L \bar{t}_R \gamma_\mu T^a t_R + \dots$$

$\bar{t} \mathbb{P}_R \gamma^\mu \mathbb{P}_L t \bar{t} \mathbb{P}_L \gamma_\mu \mathbb{P}_R t$; $\mathbb{P}_{L/R} = (\mathbb{I} \mp \gamma_5)/2$

- 4-top operators occur in 2-loop diagrams

talk by Stefano Di Noi at HEFT

- treatment of γ_5 matters!

- translation between schemes also affects other operators and parameters

gamma5 in 4 dimensions

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \quad \text{definition in 4 space-time dimensions}$$

in 4 dimensions:

$$\{\gamma_5, \gamma^\mu\} = 0 \quad (1)$$

$$\text{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5] = -4i\epsilon^{\mu\nu\rho\sigma} \quad (2)$$

$$\text{Tr}[\Gamma_1\Gamma_2\gamma_5] = \text{Tr}[\gamma_5\Gamma_1\Gamma_2] \quad \text{cyclicity of Traces} \quad (3)$$

in $D = 4 - 2\epsilon$ dimensions: (1), (2) and (3) cannot be maintained simultaneously

gamma5 in D dimensions

different schemes to extend γ_5 to D dimensions:

“naive dimensional regularisation” (NDR):

Breitenlohner, Maison; ‘t Hooft, Veltman (BMHV):

keep $\{\gamma_5, \gamma^\mu\} = 0$

$$\gamma^\mu = \underbrace{\bar{\gamma}^\mu}_{4\text{-dim.}} + \underbrace{\hat{\gamma}^\mu}_{(D-4)\text{ dim.}} ; \quad \{\gamma_5, \bar{\gamma}^\mu\} = 0 ; \quad [\gamma_5, \hat{\gamma}^\mu] = 0$$

abandon cyclicity of trace (or fix inconsistencies by hand)

reading point for traces: “Kreimer scheme”

but: ambiguities observed at high loop orders

L. Chen, 2304.13814, J. Davies et al 2110.05496, ...

- spurious breaking of gauge invariance
- needs symmetry restoring counterterms
- the latter can be derived algorithmically

Scheme dependence induced by 4t operators

scheme dependent part

$$\begin{aligned}
 & \text{Diagram 1: } t \text{ line with a self-energy loop (blue dot)} = \frac{C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}}{\Lambda^2} (B_{m_t} + K_{m_t}) \times \text{Diagram 2: } t \text{ line with a cross (black dot)} ; \quad K_{m_t} = \begin{cases} -\frac{m_t^2}{8\pi^2} & \text{(NDR)} \\ 0 & \text{(BMHV)} \end{cases} \\
 & \text{Diagram 3: } h \text{ line with a self-energy loop (blue dot)} = \frac{C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}}{\Lambda^2} \left(B_{ht\bar{t}} + K_{m_t} - \frac{v^3}{\sqrt{2}m_t} K_{tH} \right) \times \text{Diagram 4: } h \text{ line with a vertex (black dot)} ; \quad K_{tH} = \begin{cases} \frac{\sqrt{2}m_t(4m_t^2 - m_h^2)}{16\pi^2 v^3} & \text{(NDR)} \\ 0 & \text{(BMHV)} \end{cases} \\
 & \text{Diagram 5: } g \text{ line with a self-energy loop (blue dot)} = \frac{C_{Qt}^{(1)} + (c_F - \frac{c_A}{2}) C_{Qt}^{(8)}}{C_{tG}} K_{tG} \times \text{Diagram 6: } g \text{ line with a vertex (orange square)} ; \quad K_{tG} = \begin{cases} -\frac{\sqrt{2}m_t g_s}{16\pi^2 v} & \text{(NDR)} \\ 0 & \text{(BMHV)} \end{cases}
 \end{aligned}$$

Scheme (in)dependence

The renormalised physical amplitude must be scheme-independent

$$\mathcal{M}^{\text{ren}} = \mathcal{M}^{\text{scheme indep.}}$$

⇒ scheme dependence of K-terms
must be cancelled by
scheme dependence of
Wilson coefficients and parameters

$$\begin{aligned}
 & + \left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right) \frac{1}{\Lambda^2} K_{m_t} \frac{\partial \mathcal{M}_{\text{SM}}}{\partial m_t} \times m_t \\
 & + \left[1 - \frac{v^3}{\sqrt{2}m_t} \left(\frac{C_{tH}}{\Lambda^2} + K_{tH} \frac{C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}}{\Lambda^2} \right) \right] \mathcal{M}_{\text{SM}} \\
 & + \underbrace{\left[C_{tG} + \left(C_{Qt}^{(1)} + \left(c_F - \frac{c_A}{2} \right) C_{Qt}^{(8)} \right) K_{tG} \right]}_{\tilde{C}_{tG}} \frac{1}{\Lambda^2} \mathcal{M}_{tG}|_{\text{FIN}}
 \end{aligned}$$

Scheme (in)dependence

The renormalised physical amplitude must be scheme-independent

$$\mathcal{M}^{\text{ren}} = \mathcal{M}^{\text{scheme indep.}}$$

$$+ \left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right) \frac{1}{\Lambda^2} K_{m_t} \frac{\partial \mathcal{M}_{\text{SM}}}{\partial m_t} \times m_t$$

$$+ \left[1 - \frac{v^3}{\sqrt{2}m_t} \left(\frac{C_{tH}}{\Lambda^2} + K_{tH} \frac{C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}}{\Lambda^2} \right) \right] \mathcal{M}_{\text{SM}}$$

$$+ \left[C_{tG} + \left(C_{Qt}^{(1)} + \left(c_F - \frac{c_A}{2} \right) C_{Qt}^{(8)} \right) K_{tG} \right] \frac{1}{\Lambda^2} \mathcal{M}_{tG}|_{\text{FIN}}$$

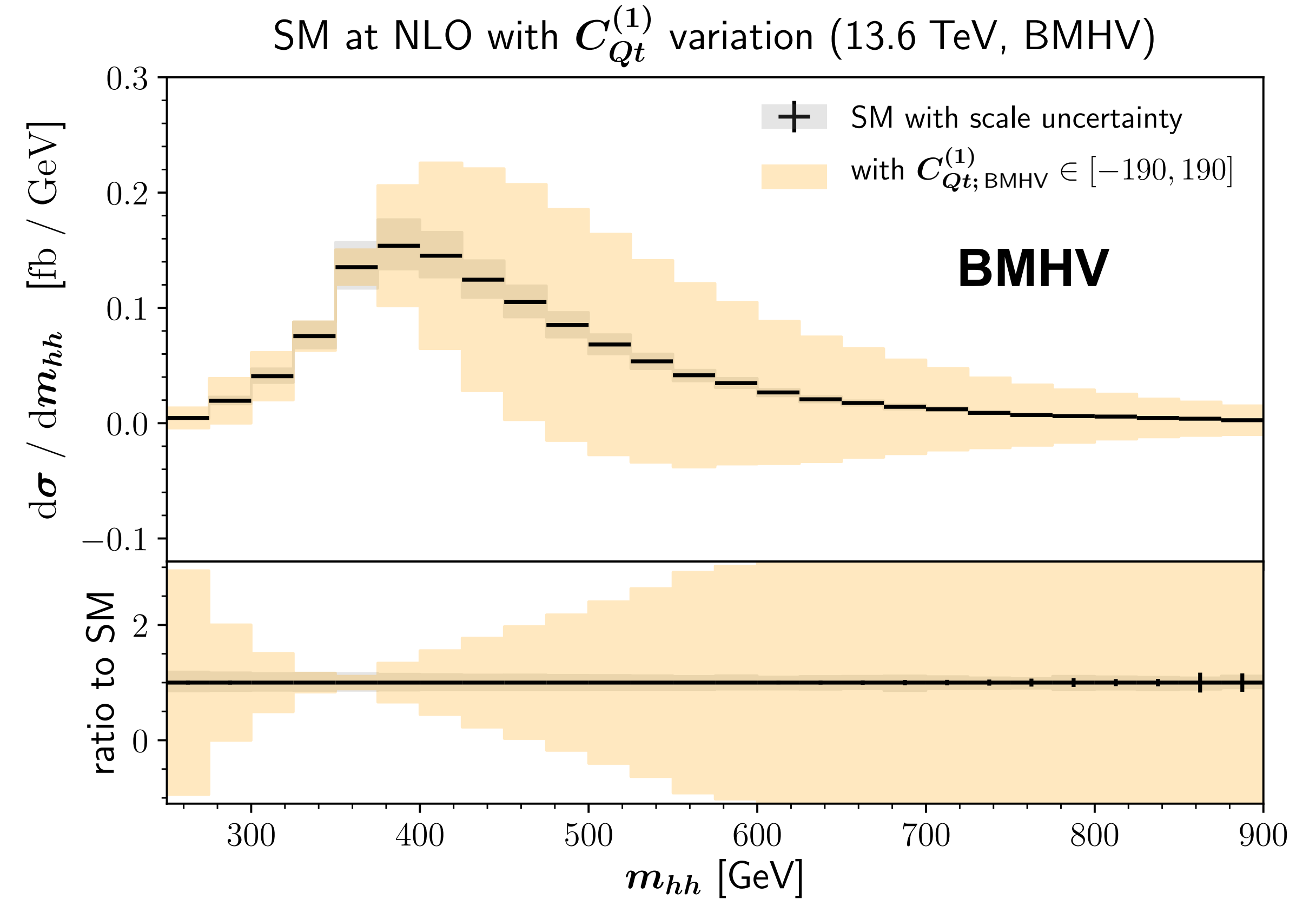
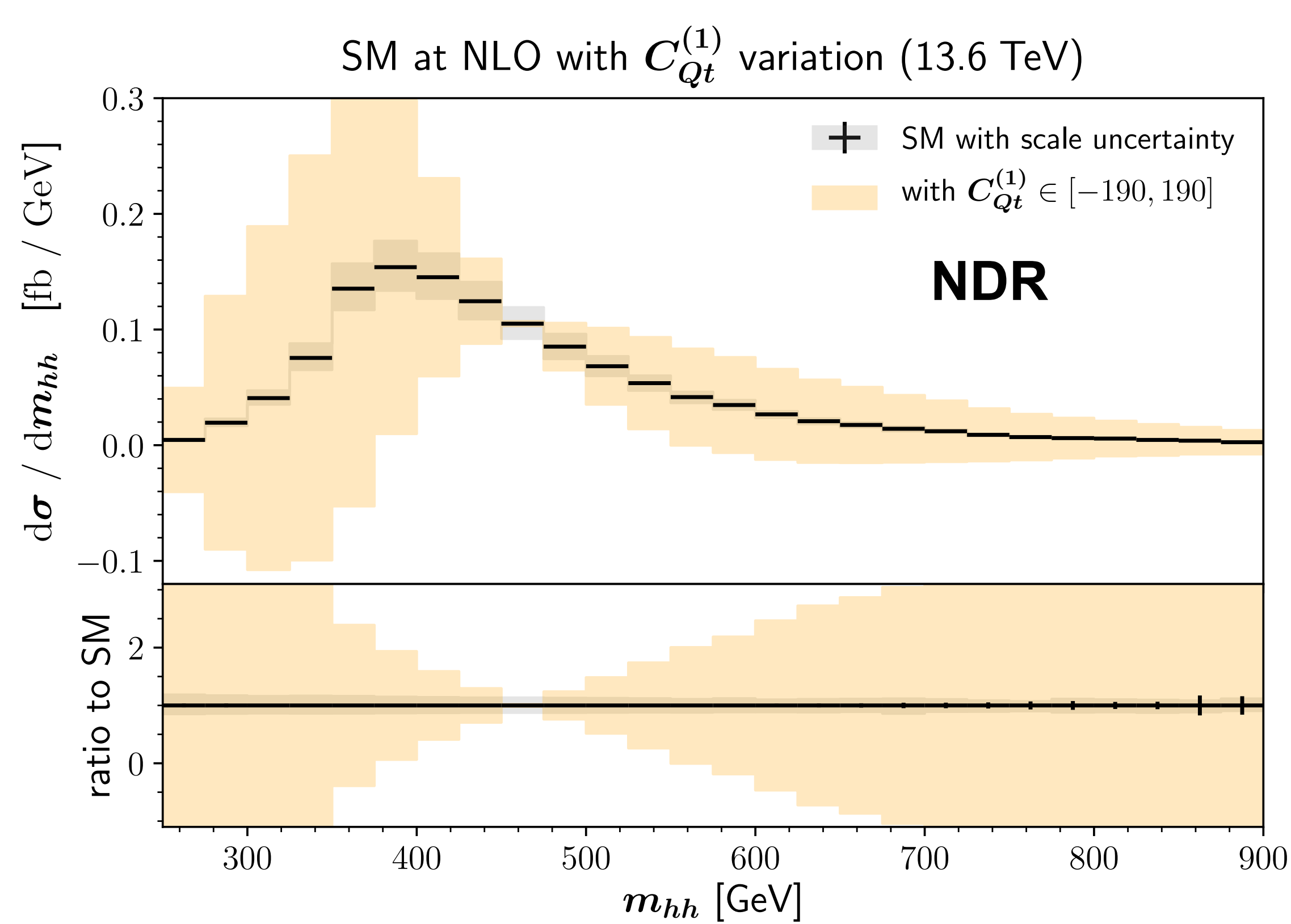
$$\underbrace{\hspace{15em}}_{\tilde{C}_{tG}}$$

⇒ scheme dependence of K-terms must be cancelled by scheme dependence of Wilson coefficients and parameters

⇒ define combinations absorbing the scheme dependence or translation table

Di Noi, Gröber, GH, Lang, Vitti
2310.18221

Example: $C_{Qt}^{(1)}$ in different gamma5 schemes



GH, J. Lang, 2311.15004

large effect and very different behaviour in the two schemes

Running Wilson coefficients

coming soon in the `ggHH_SMEFT` code (Powheg-Box-V2): [\[GH, Jannis Lang\]](#)

$$\mu \frac{\partial \mathcal{C}_i}{\partial \mu} = \frac{\gamma_{\mathcal{C}_i}^{\mathcal{C}_j}}{16\pi^2} \mathcal{C}_j, \quad \gamma_{\mathcal{C}_i}^{\mathcal{C}_j} : \text{anomalous dimension}$$

see also Gröber, Di Noi 2312.11327

Aoude et al. 2212.05067

Battaglia, Grazzini, Spira, Wiesemann 2109.02987

Deutschmann, Duhr, Maltoni, Vryonidou 1708.00460

Maltoni, Vryonidou, Zhang 1607.05330

$$\begin{aligned} \mu \frac{d\mathcal{C}_{tH}}{d\mu} &= -2\beta_0^y \frac{\alpha_s}{4\pi} \mathcal{C}_{tH} + \gamma_{\mathcal{C}_{tH}}^{\mathcal{C}_{Qt}} \frac{1}{16\pi^2} \left(\mathcal{C}_{Qt}^{(1)} + c_F \mathcal{C}_{Qt}^{(8)} \right) \\ \mu \frac{d\mathcal{C}_{HG}}{d\mu} &= -2\beta_0 \frac{\alpha_s}{4\pi} \mathcal{C}_{HG} + \gamma_{\mathcal{C}_{HG}}^{\mathcal{C}_{tG}} \frac{g_s}{16\pi^2} \mathcal{C}_{tG} + \gamma_{\mathcal{C}_{HG}}^{\mathcal{C}_{Qt}} \frac{\alpha_s}{4\pi} \left(\mathcal{C}_{Qt}^{(1)} + \left(c_F + \frac{c_A}{2} \right) \mathcal{C}_{Qt}^{(8)} \right) \\ \mu \frac{d\mathcal{C}_{tG}}{d\mu} &= -\beta_{tG} \frac{\alpha_s}{4\pi} \mathcal{C}_{tG} + \underbrace{\gamma_{\mathcal{C}_{tG}}^{\mathcal{C}_{HG}} \frac{g_s}{16\pi^2} \mathcal{C}_{HG}}_{\text{neglected}} \end{aligned}$$

$$\begin{pmatrix} \mu \frac{d\mathcal{C}_{Qt}^{(1)}}{d\mu} \\ \mu \frac{d\mathcal{C}_{Qt}^{(8)}}{d\mu} \end{pmatrix} = -\hat{\beta}_{Qt} \frac{\alpha_s}{4\pi} \begin{pmatrix} \mathcal{C}_{Qt}^{(1)} \\ \mathcal{C}_{Qt}^{(8)} \end{pmatrix} \quad \hat{\beta}_{Qt} = \begin{pmatrix} 0 & 3 \frac{N_c^2 - 1}{N_c^2} \\ 12 & \frac{6N_c^2 - 2N_c - 12}{N_c} \end{pmatrix}$$

new options for users:

WCscaledependence: Switches the WC scaledependence between three modes,
 0: $\mu_{EFT} = \mu_R$ but without any running effects (default, represents previous implementation)
 1: static EFT scale $\mu_{EFT} = \mu_{EFTinput} \times \text{EFTscfact}$ with running for $\text{EFTscfact} \neq 1$
 2: dynamic EFT scale $\mu_{EFT} = \frac{m_{hh}}{2} \times \text{EFTscfact}$ with running.

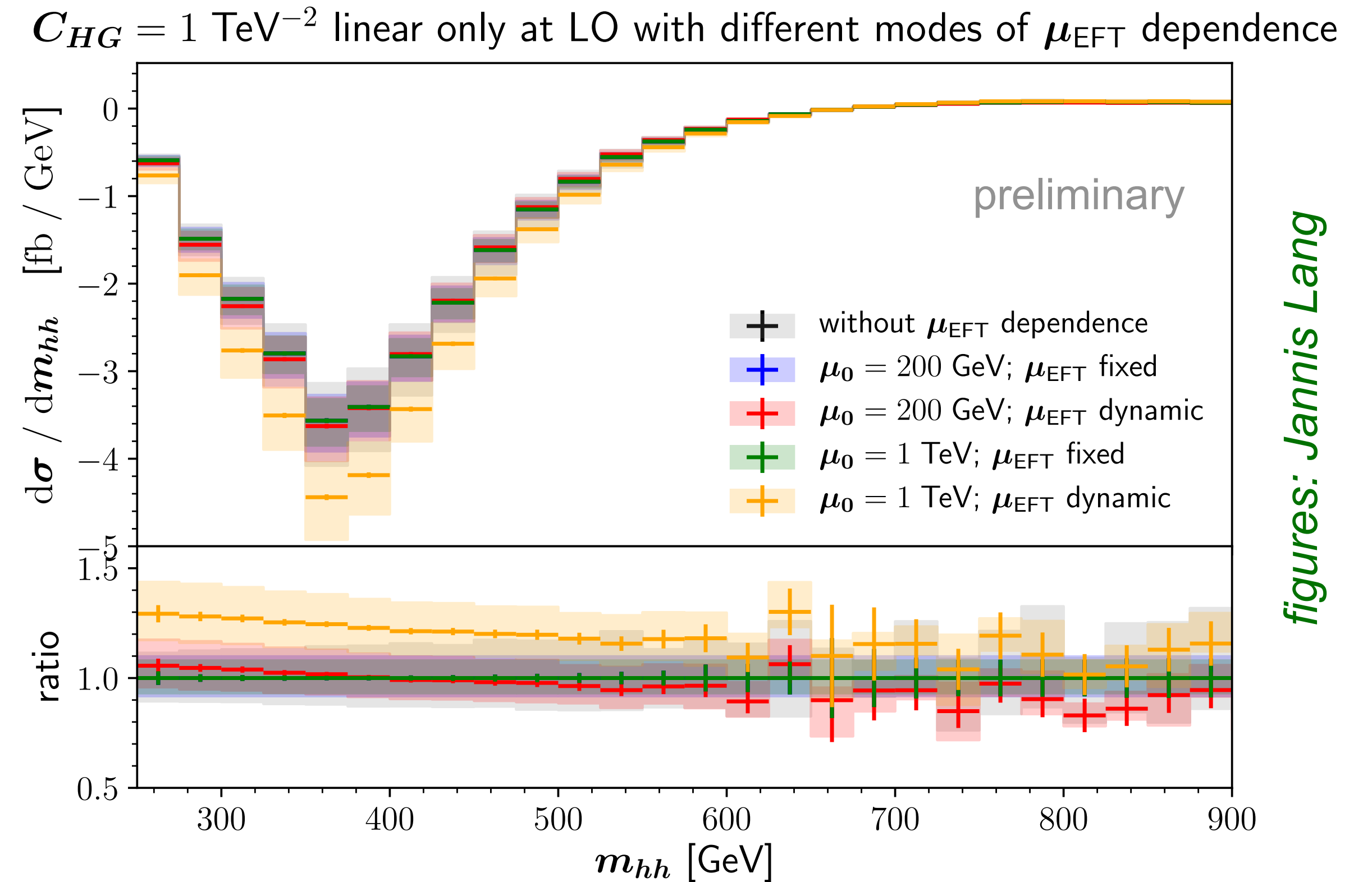
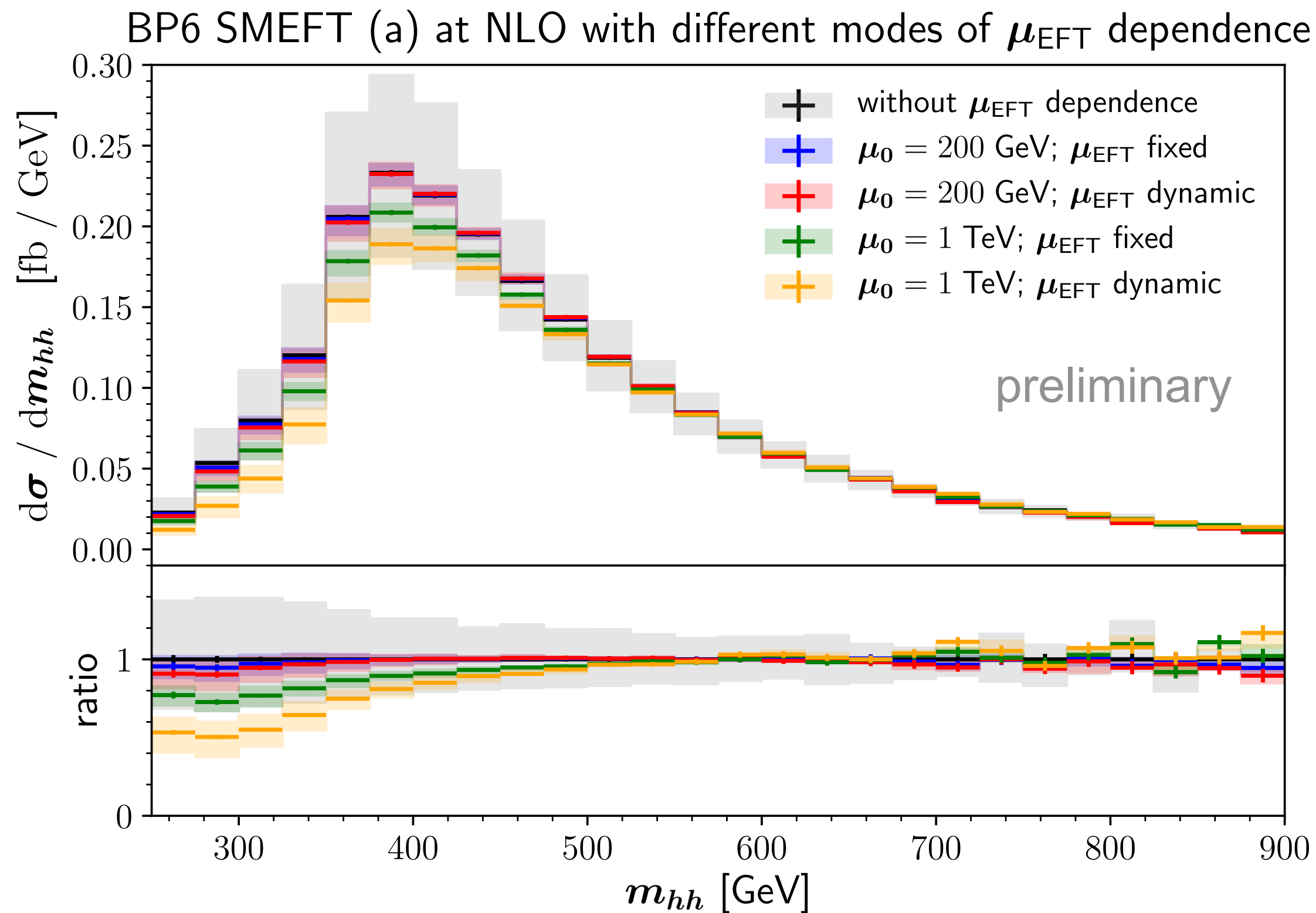
inputscaleEFT: defines the input scale/measurement scale $\mu_{EFTinput}$ of the Wilson coefficients, from which the running starts (only relevant for $\text{WCscaledependence} > 0$)

EFTscfact: varies the EFT scale μ_{EFT} around the central scale, to be used for uncertainty assessment.

Running Wilson coefficients

benchmark point 6:

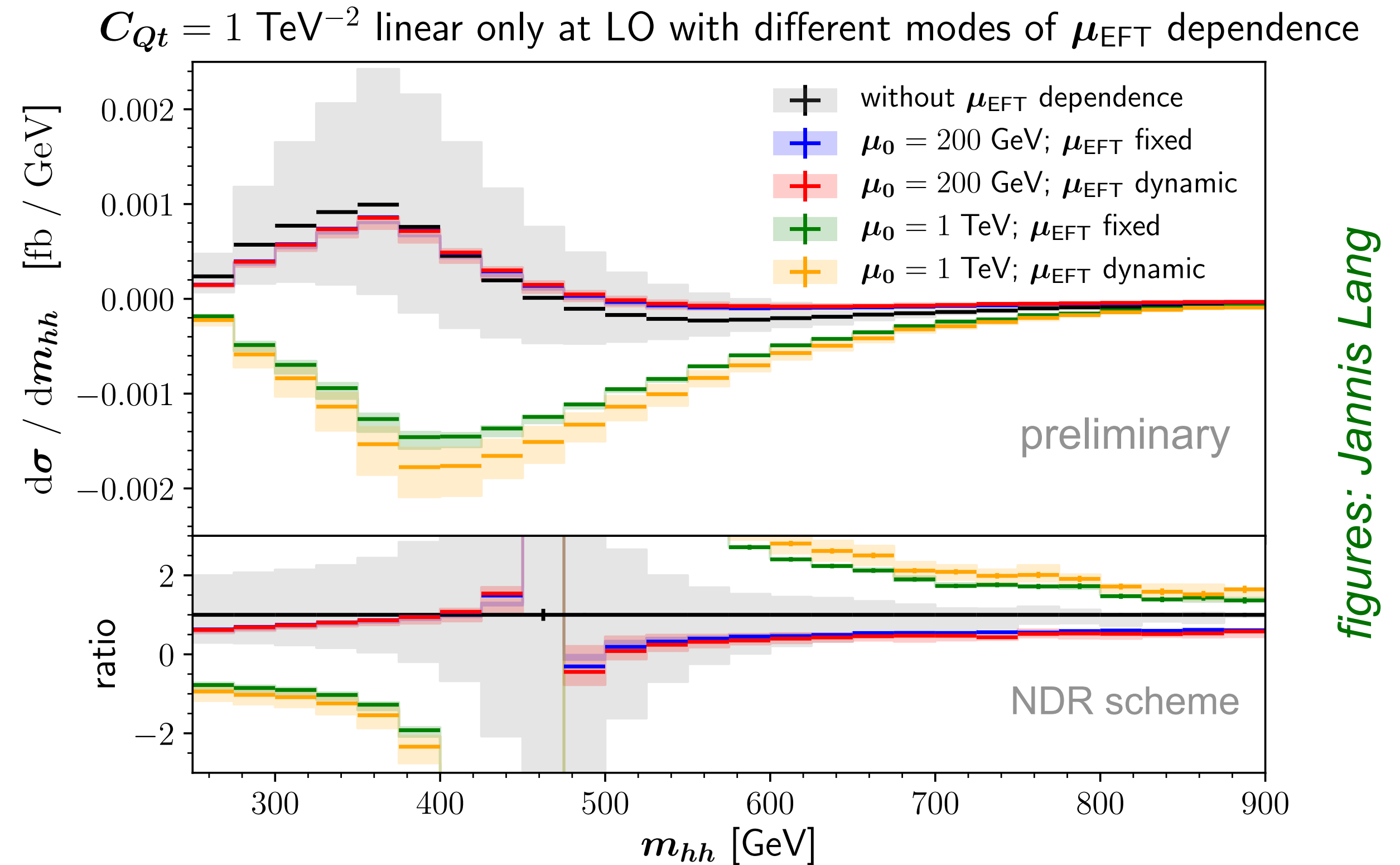
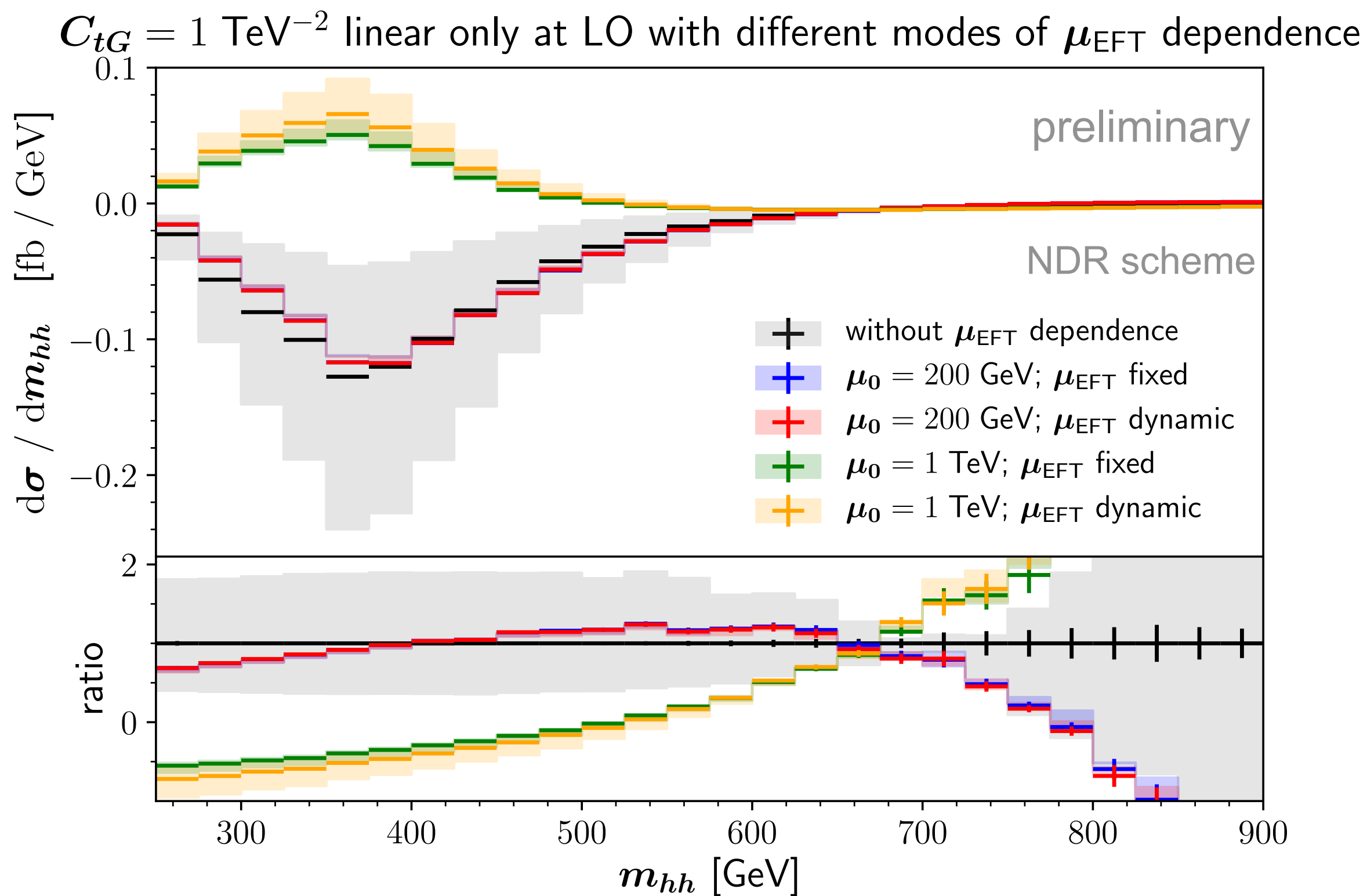
effect of running on CHG only:



figures: Jannis Lang

grey band: usual μ_r, μ_f variations; dynamic: $\mu_{\text{EFT}} = \frac{m_{hh}}{2}$; large effects only for “inconvenient” starting scale 1 TeV

Running Wilson coefficients



figures: Jannis Lang

sign change for large μ_0

VBF HH production in HEFT

uncorrelated in HEFT (LO): C_{ggH} and C_{ggHH} ; C_{VVH} and C_{VVHH}

gluon fusion VBF

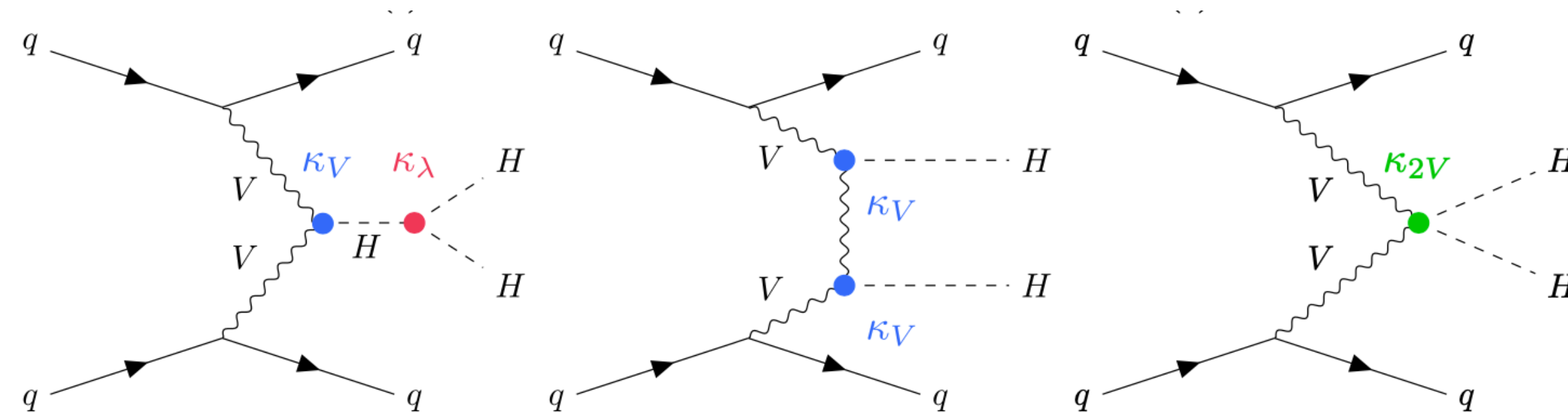
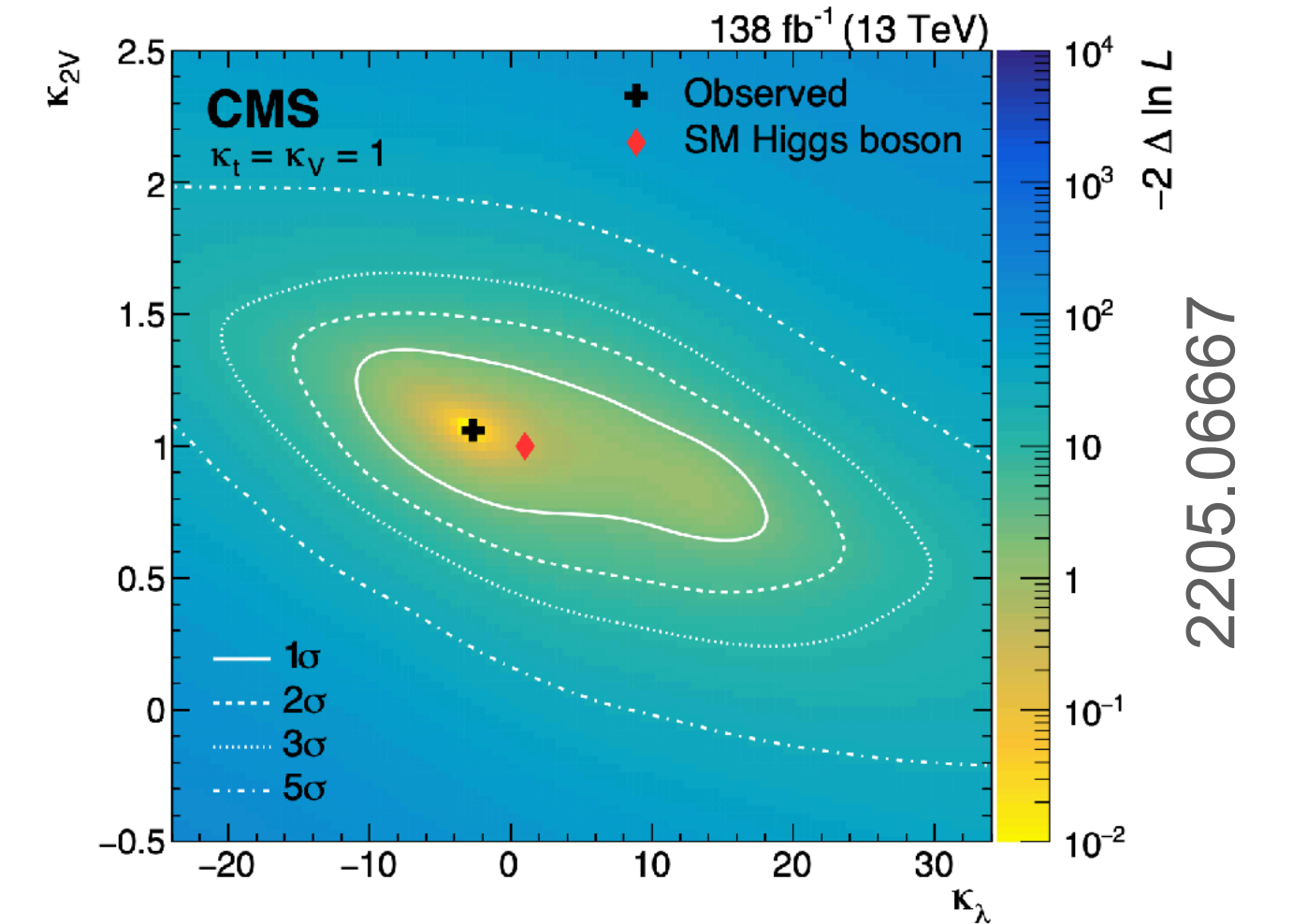


image: 2404.17193

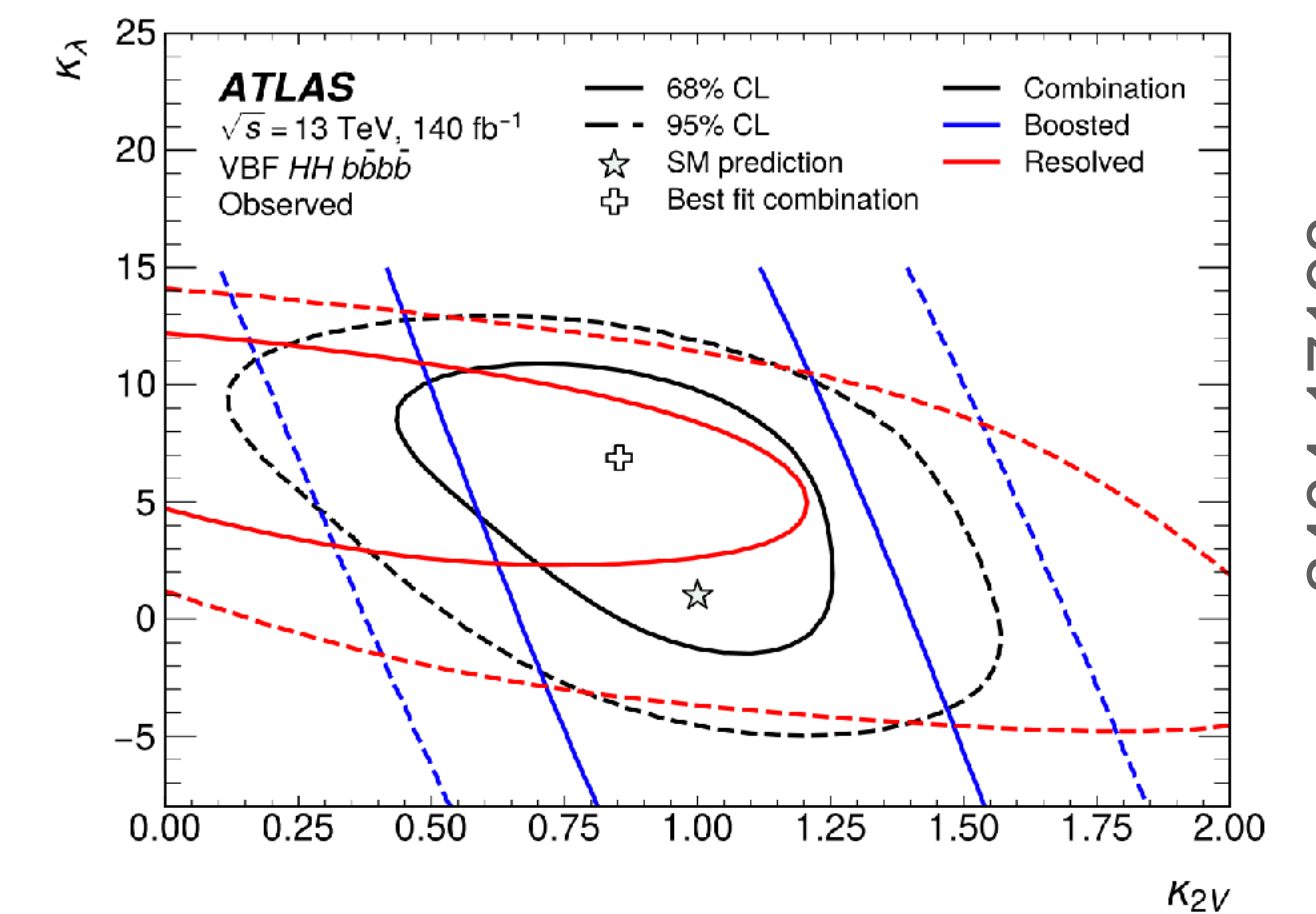
- The figure shows the kappa-framework
- A priori O(20) operators in HEFT



Buchalla et al. 1307.5017, Brivio et al. 1604.06891



2205.06667



2404.17193

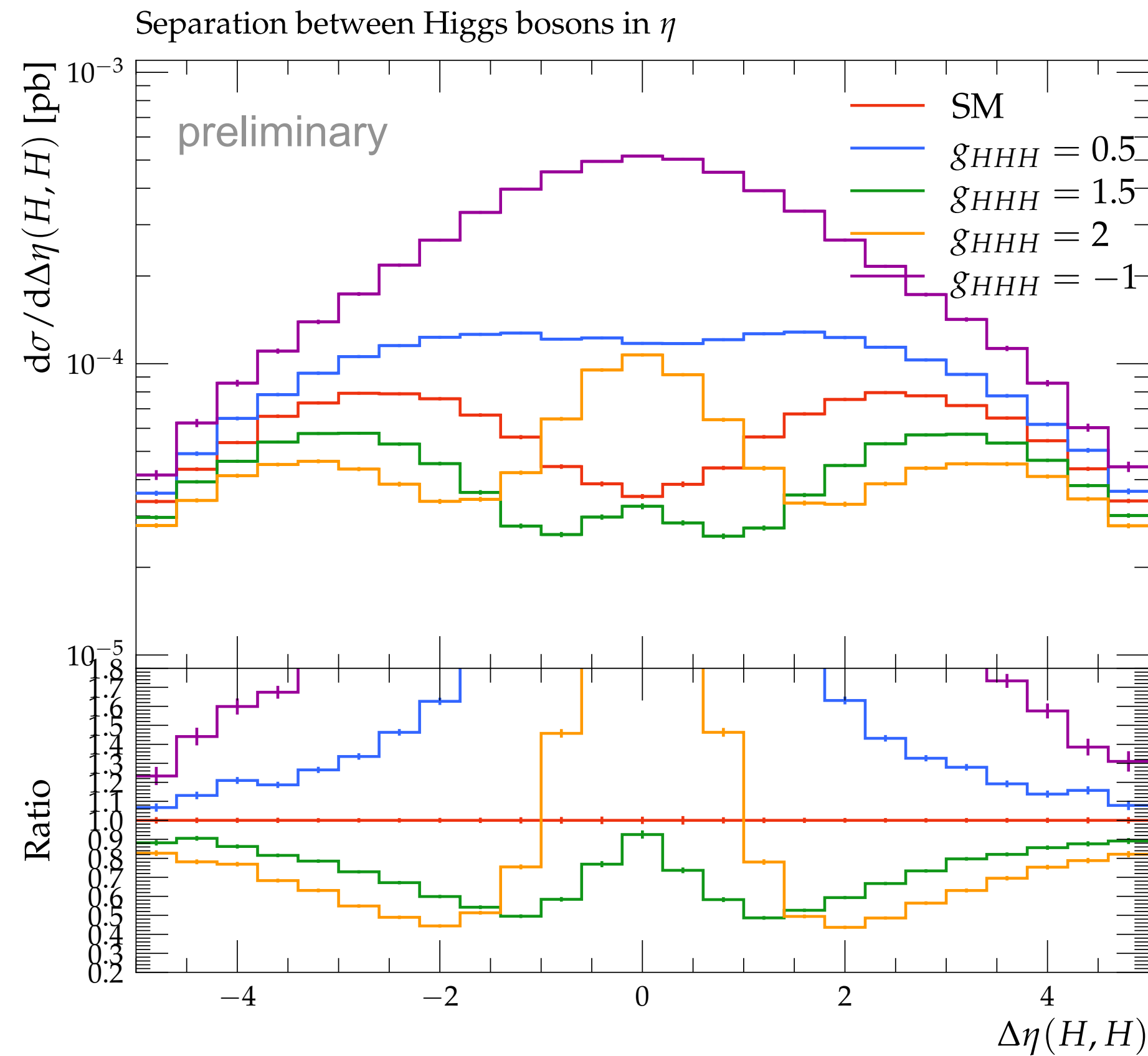
VBF HH production in HEFT

Jens Braun, GH, Marius Höfer (work on NLO in progress)

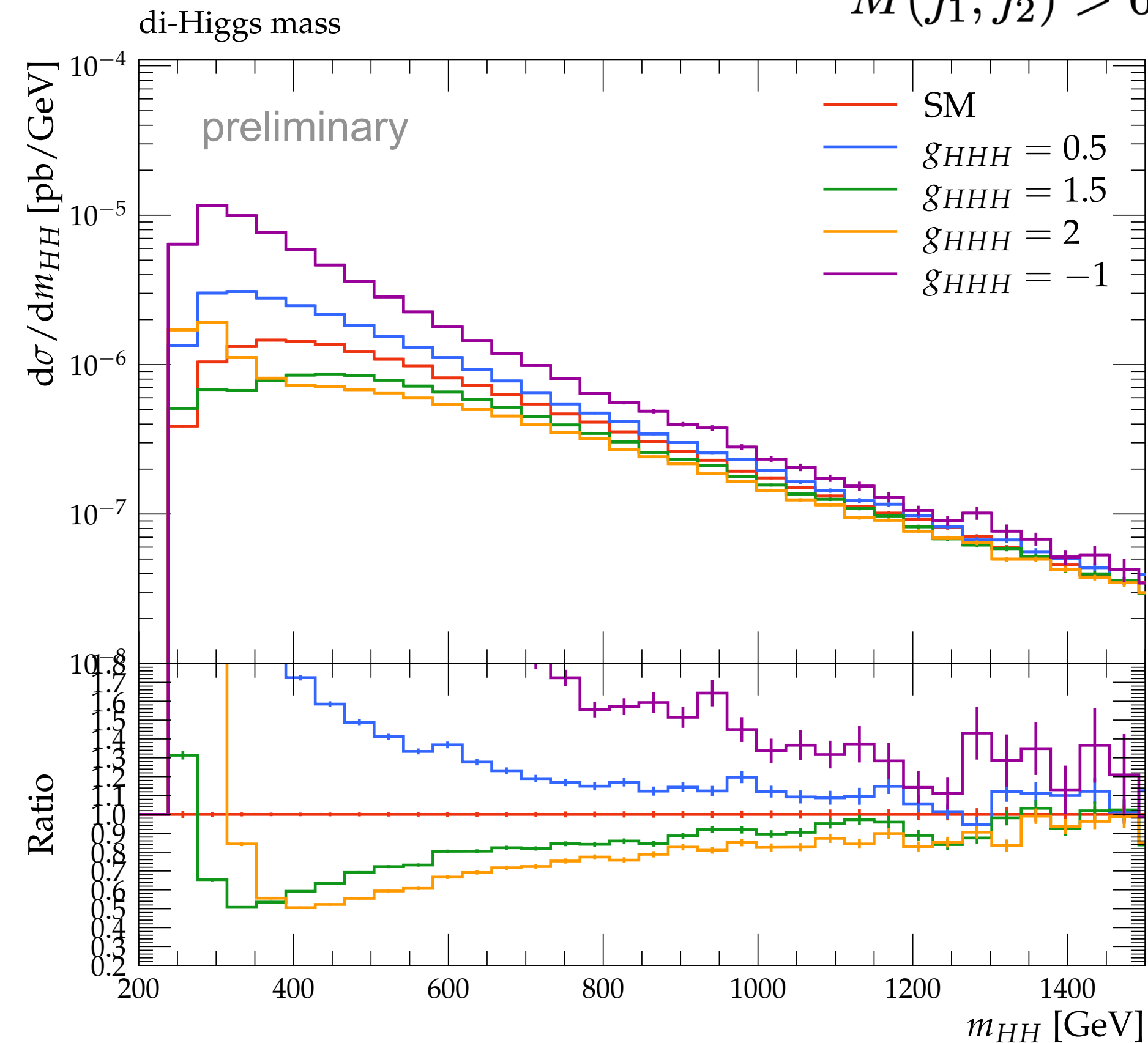
$\sqrt{s} = 13 \text{ TeV}, \mu = 2m_h$

VBF cuts $|\eta(j_1, j_2)| > 4$

$M(j_1, j_2) > 600 \text{ GeV}$



GoSam+Whizard (LO)



figures: Jens Braun

shape very sensitive to trilinear coupling (other couplings set to SM values)

Summary & outlook

- mHH shape benchmarks in HEFT: partially invalid in SMEFT and depend on truncation scheme and on value of Λ
- chromomagnetic operator and 4-top operators are linked through renormalisation $\Rightarrow \gamma_5$ scheme dependence talk by Stefano Di Noi at HEFT
- \Rightarrow consider O_{tG} and $O_{Qt}^{(1),(8)}$ together or document the scheme
- coming soon in ggHH_SMEFT: running of Wilson coefficients
- work in progress: HH in vector boson fusion at NLO QCD+HEFT

Thank you for your attention !



image: Laura Vogiatzis

ggHH and ggHH_SMEFT codes

- both codes: NLO QCD with full top quark mass dependence Borowka et al. 2016

HEFT: **ggHH** code available at

<http://powhegbox.mib.infn.it/User-Process-V2>

5 anomalous couplings: GH, Jones, Kerner, Scyboz, 2006.16877

SMEFT:

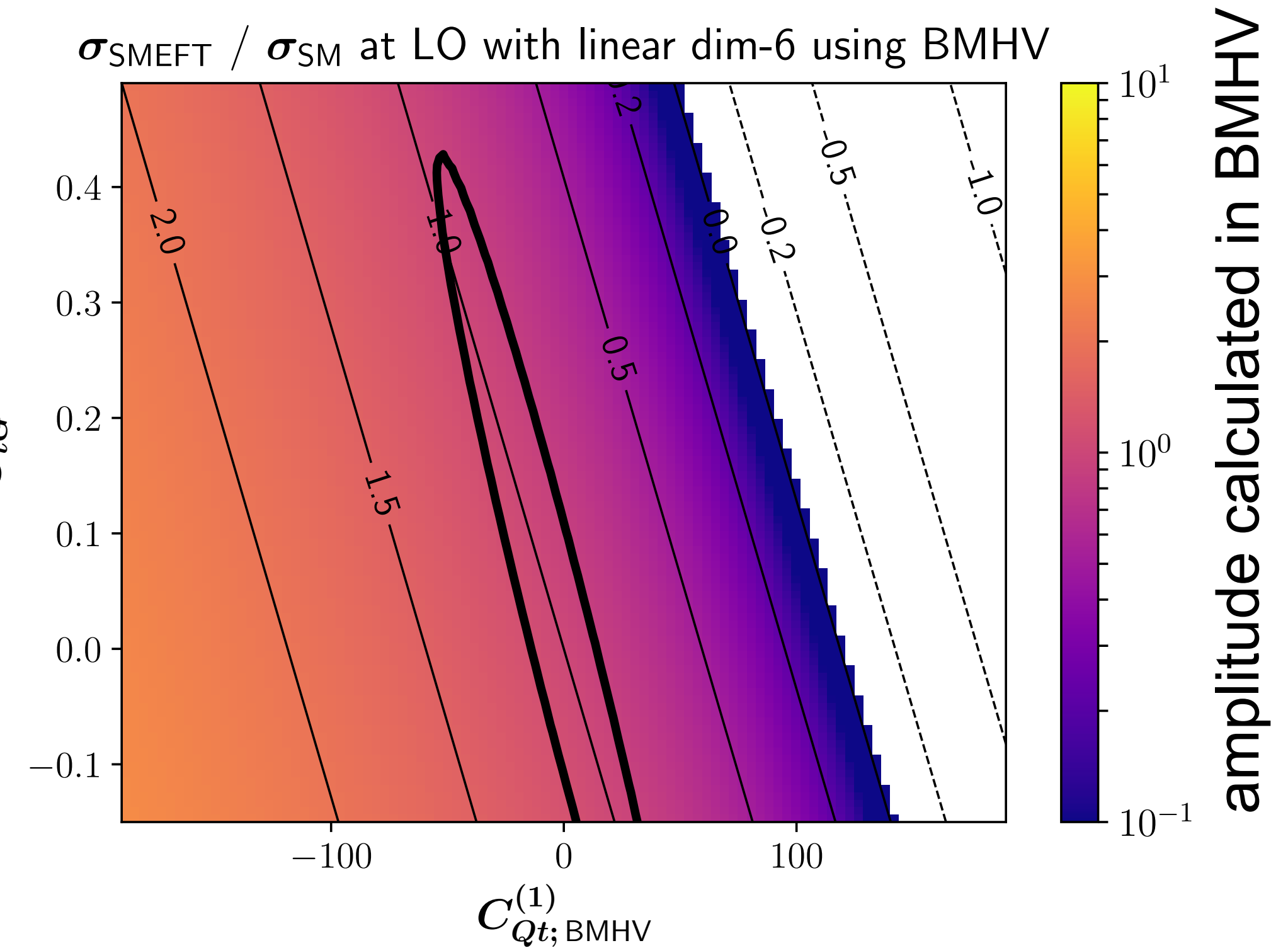
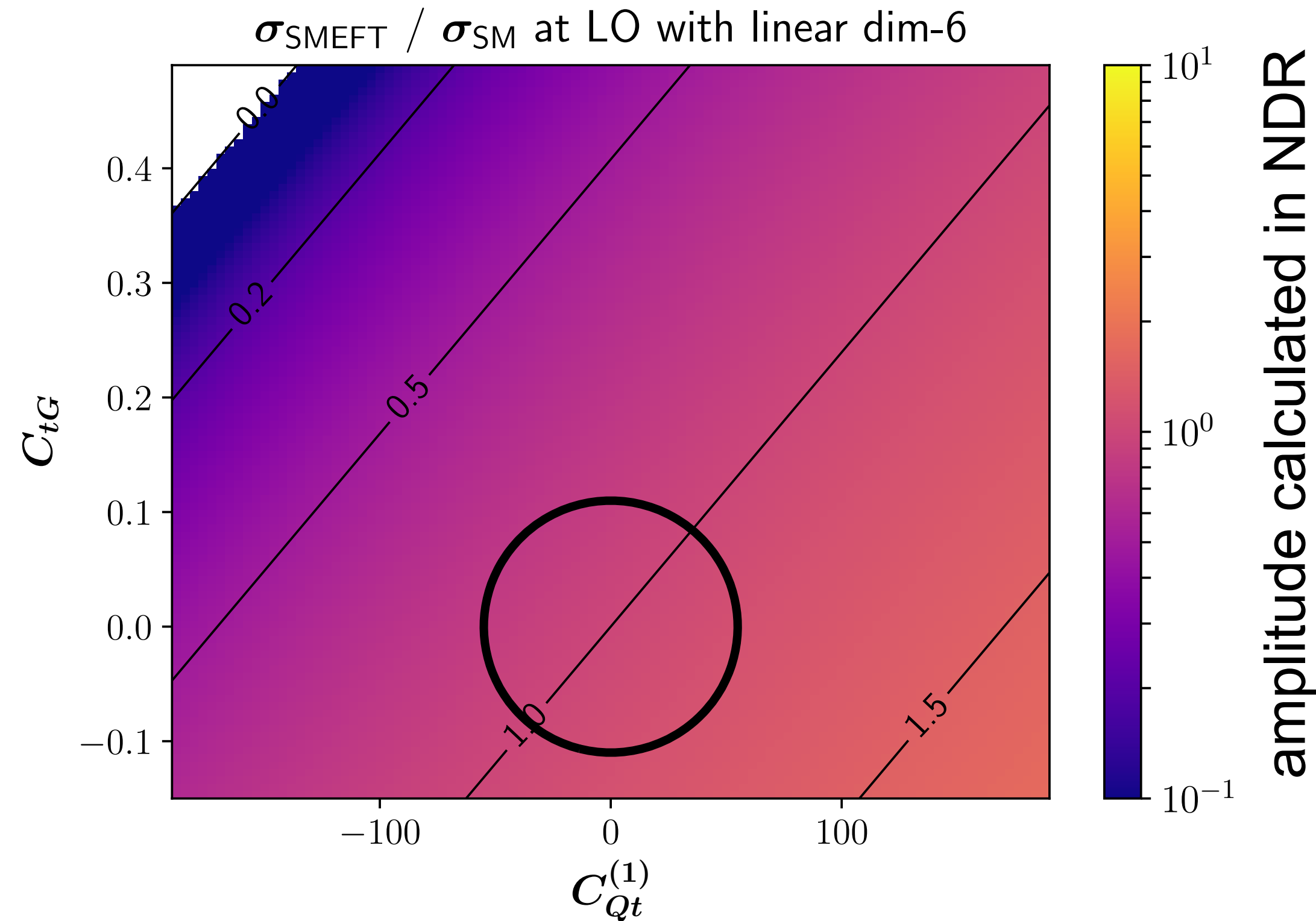
leading operators, different truncation options **ggHH_SMEFT** GH, J. Lang, L. Scyboz, 2204.13045

+ 6 subleading operators GH, J. Lang, 2311.15004

note: bug in 2-loop triangle contribution (in both codes) corrected September 2023

(thanks to Ramona Gröber, Emanuele Bagnaschi, Guiseppa Degrandi, 2309.10525)

Effect of different gamma5-schemes



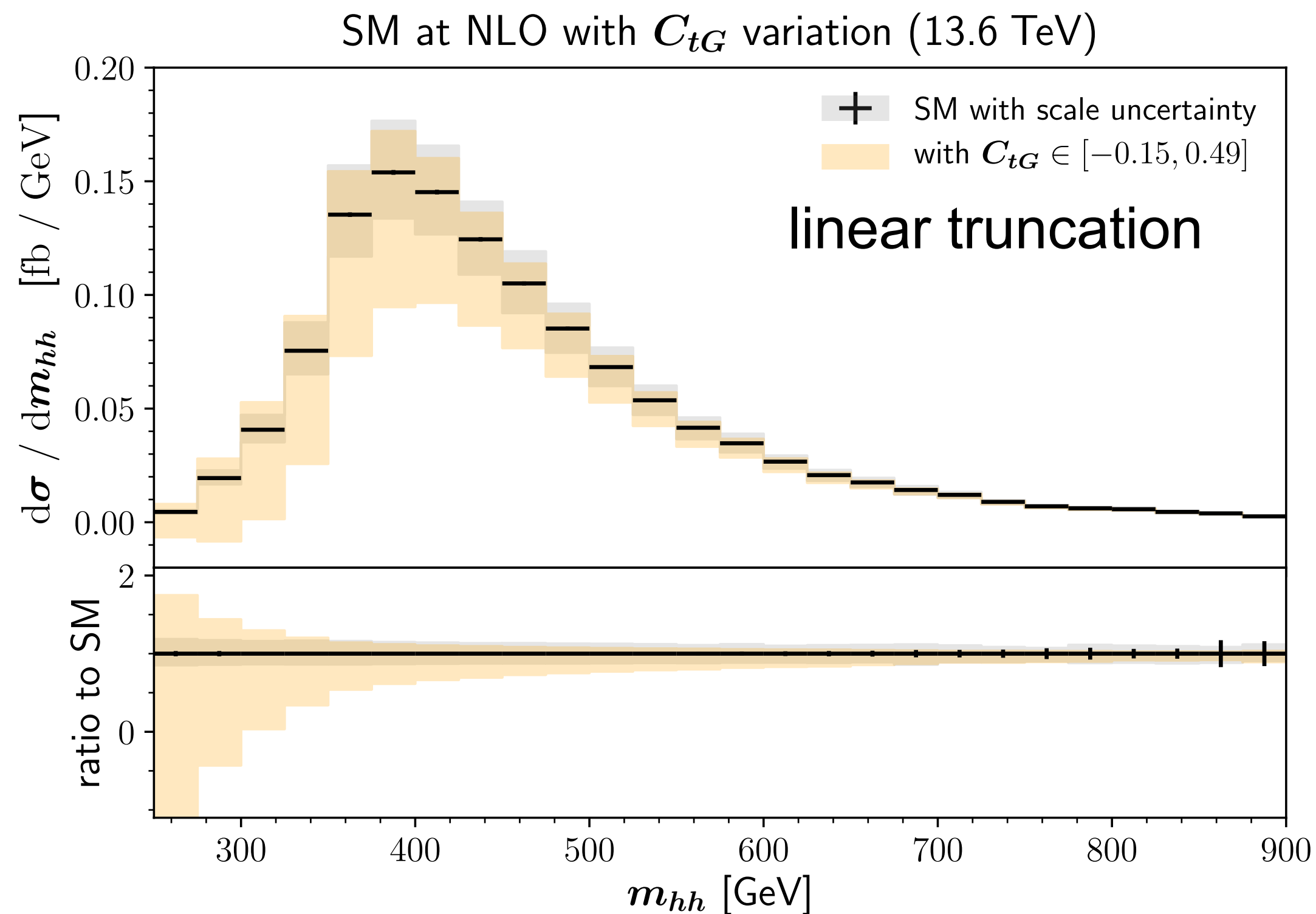
value pairs of $(C_{Qt}^{(1)}, C_{tG}^{\text{NDR}})$ within the circle are mapped to value pairs of $(C_{Qt}^{(1)}, C_{tG}^{\text{BMHV}})$ within the ellipse

$$C_{tG}^{\text{BMHV}} = C_{tG}^{\text{NDR}} - \frac{\sqrt{2}m_t g_s}{16\pi^2 v} \left(C_{Qt}^{(1)} + \left(c_F - \frac{c_A}{2} \right) C_{Qt}^{(8)} \right)$$

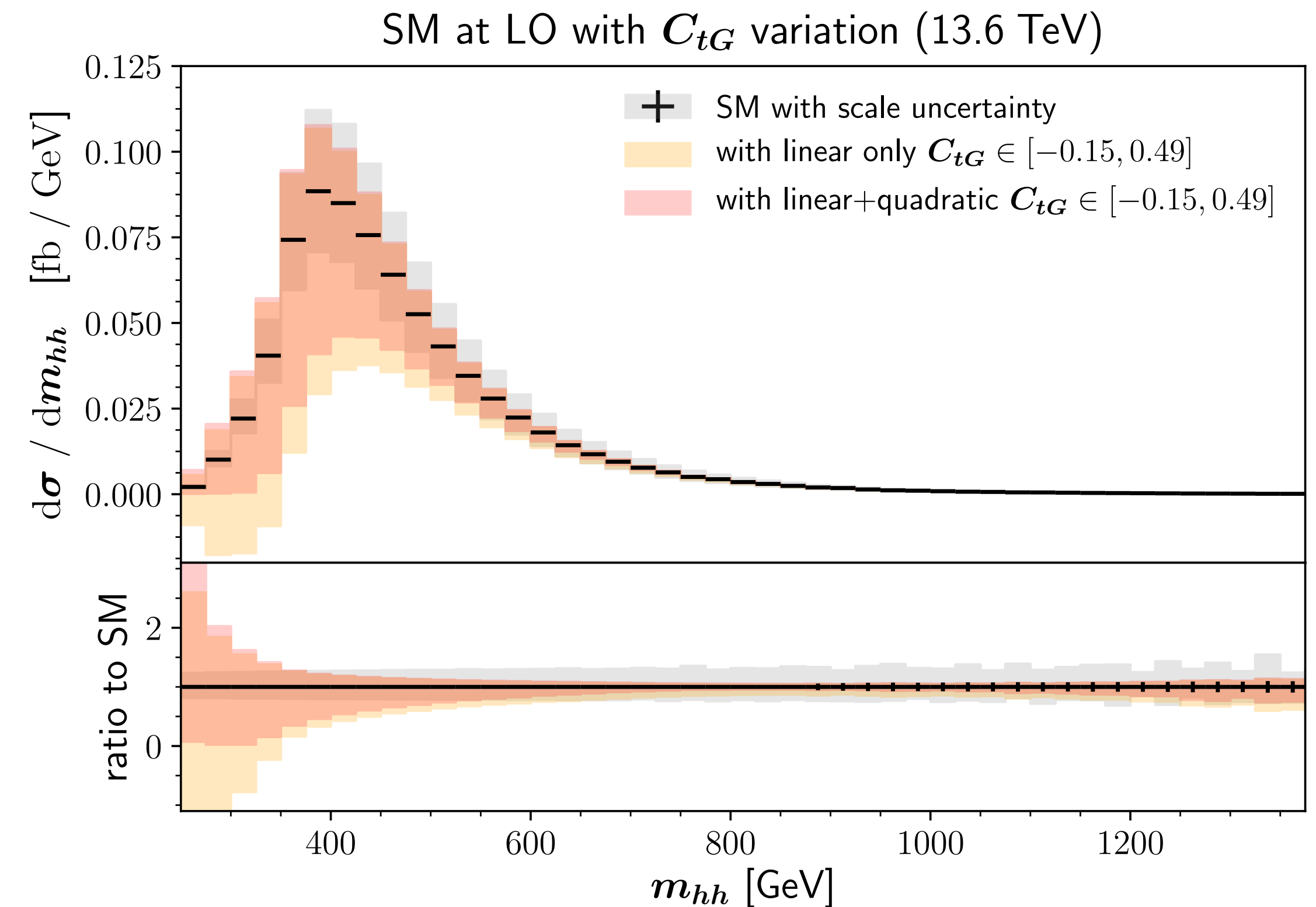
total cross section is the same in both schemes, a single Wilson coefficient is not an observable

Effects of chromomagnetic operator

variation ranges: from global fit (marginalised), Ethier et al, 2105.00006 [SMEFiT coll.]



Effect larger than SM scale uncertainties



Effect of linear+quadratic truncation smaller than linear only due to destructive interference

Operators contributing to VBF HH before application of custodial symmetry arguments

$$\begin{aligned}
 \Delta\mathcal{L} = & -\frac{m_h^2}{2v} g_{hhhh} h^3 \\
 & + g_{h\gamma\gamma} A_{\mu\nu} A^{\mu\nu} \frac{h}{v} \\
 & + g_{hZ\gamma}^{(1)} A_{\mu\nu} Z^\mu \frac{\partial^\nu h}{v} + g_{hZ\gamma}^{(2)} A_{\mu\nu} Z^{\mu\nu} \frac{h}{v} \\
 & + g_{hZZ}^{(1)} Z_{\mu\nu} Z^\mu \frac{\partial^\nu h}{v} + g_{hZZ}^{(2)} Z_{\mu\nu} Z^{\mu\nu} \frac{h}{v} + M_Z^2 g_{hZZ}^{(3)} Z_\mu Z^\mu \frac{h}{v} \\
 & + g_{hWW}^{(1)} (W_{\mu\nu}^+ W^{-,\mu} + \text{h.c.}) \frac{\partial^\nu h}{v} + g_{hWW}^{(2)} W_{\mu\nu}^+ W^{-,\mu\nu} \frac{h}{v} + 2M_W^2 g_{hWW}^{(3)} W_\mu^+ W^{-,\mu} \frac{h}{v} \\
 & + g_{hh\gamma\gamma} A_{\mu\nu} A^{\mu\nu} \frac{h^2}{v^2} \\
 & + g_{hhZ\gamma}^{(1)} A_{\mu\nu} Z^\mu \frac{h\partial^\nu h}{v^2} + g_{hhZ\gamma}^{(2)} A_{\mu\nu} Z^{\mu\nu} \frac{h^2}{v^2} \\
 & + g_{hhZZ}^{(1)} Z_{\mu\nu} Z^\mu \frac{h\partial^\nu h}{v^2} + g_{hhZZ}^{(2)} Z_{\mu\nu} Z^{\mu\nu} \frac{h^2}{v^2} + \frac{M_Z^2}{2} g_{hhZZ}^{(3)} Z_\mu Z^\mu \frac{h^2}{v^2} + g_{hhZZ}^{(4)} Z_\mu Z^\mu \frac{\partial_\nu h \partial^\nu h}{v^2} + g_{hhZZ}^{(5)} Z_\mu Z^\nu \frac{\partial_\nu h \partial^\mu h}{v^2} \\
 & + g_{hhWW}^{(1)} (W_{\mu\nu}^+ W^{-,\mu} + \text{h.c.}) \frac{h\partial^\nu h}{v^2} + g_{hhWW}^{(2)} W_{\mu\nu}^+ W^{-,\mu\nu} \frac{h^2}{v^2} + M_W^2 g_{hhWW}^{(3)} W_\mu^+ W^{-,\mu} \frac{h^2}{v^2} + g_{hhWW}^{(4)} W_\mu^+ W^{-,\mu} \frac{\partial_\nu h \partial^\nu h}{v^2} + g_{hhWW}^{(5)} W_\mu^+ W^{-,\nu} \frac{\partial_\nu h \partial^\mu h}{v^2}
 \end{aligned}$$

Scheme (in)dependence

possible solution: redefine parameters, absorbing scheme dependent parts

$$\tilde{C}_{tG} = C_{tG} + \left(C_{Qt}^{(1)} + \left(c_F - \frac{c_A}{2} \right) C_{Qt}^{(8)} \right) K_{tG}$$

$$\tilde{C}_{tH} = C_{tH} + \left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right) K_{tH}$$

known e.g. in flavour physics
 Ciuchini et al. '93
 Herrlich, Nierste '94

$$\tilde{m}_t = m_t \left(1 + \frac{C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}}{\Lambda^2} K_{m_t} \right)$$

Scheme (in)dependence

possible solution: redefine parameters, absorbing scheme dependent parts

$$\tilde{C}_{tG} = C_{tG} + \left(C_{Qt}^{(1)} + \left(c_F - \frac{c_A}{2} \right) C_{Qt}^{(8)} \right) K_{tG}$$

$$\tilde{C}_{tH} = C_{tH} + \left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right) K_{tH}$$

known e.g. in flavour physics
 Ciuchini et al. '93
 Herrlich, Nierste '94

$$\tilde{m}_t = m_t \left(1 + \frac{C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}}{\Lambda^2} K_{m_t} \right)$$

more flexible: derive a **translation dictionary** by requiring $\tilde{X}^{\text{NDR}} \stackrel{!}{=} \tilde{X}^{\text{BMHV}}$

Translation between BMHV and NDR

4-top operators are linked to other operators through a scheme translation

$$m_t^{\text{BMHV}} = m_t^{\text{NDR}} - \frac{m_t^3}{8\pi^2 \Lambda^2} \left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right)$$

$$C_{tH}^{\text{BMHV}} = C_{tH}^{\text{NDR}} + \frac{\sqrt{2}m_t(4m_t^2 - m_h^2)}{16\pi^2 v^3} \left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right)$$

$$C_{tG}^{\text{BMHV}} = C_{tG}^{\text{NDR}} + \frac{\sqrt{2}m_t g_s}{16\pi^2 v} \left(C_{Qt}^{(1)} + \left(c_F - \frac{c_A}{2} \right) C_{Qt}^{(8)} \right)$$

note: loop suppression factor for C_{tG} not included here (Warsaw basis conventions)

Translation between BMHV and NDR

4-top operators are linked to other operators through a scheme translation

$$m_t^{\text{BMHV}} = m_t^{\text{NDR}} - \frac{m_t^3}{8\pi^2 \Lambda^2} \left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right)$$

$$C_{tH}^{\text{BMHV}} = C_{tH}^{\text{NDR}} + \frac{\sqrt{2}m_t(4m_t^2 - m_h^2)}{16\pi^2 v^3} \left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right)$$

$$\frac{C_{tG}^{\text{BMHV}}}{16\pi^2} = \frac{C_{tG}^{\text{NDR}}}{16\pi^2} + \frac{\sqrt{2}m_t g_s}{16\pi^2 v} \left(C_{Qt}^{(1)} + \left(c_F - \frac{c_A}{2} \right) C_{Qt}^{(8)} \right)$$

note: loop suppression factor for C_{tG} not included here (Warsaw basis conventions)

shift can be of same order as Wilson coefficient itself