Exploring HH and HHH at colliders with the Bosonic-HEFT

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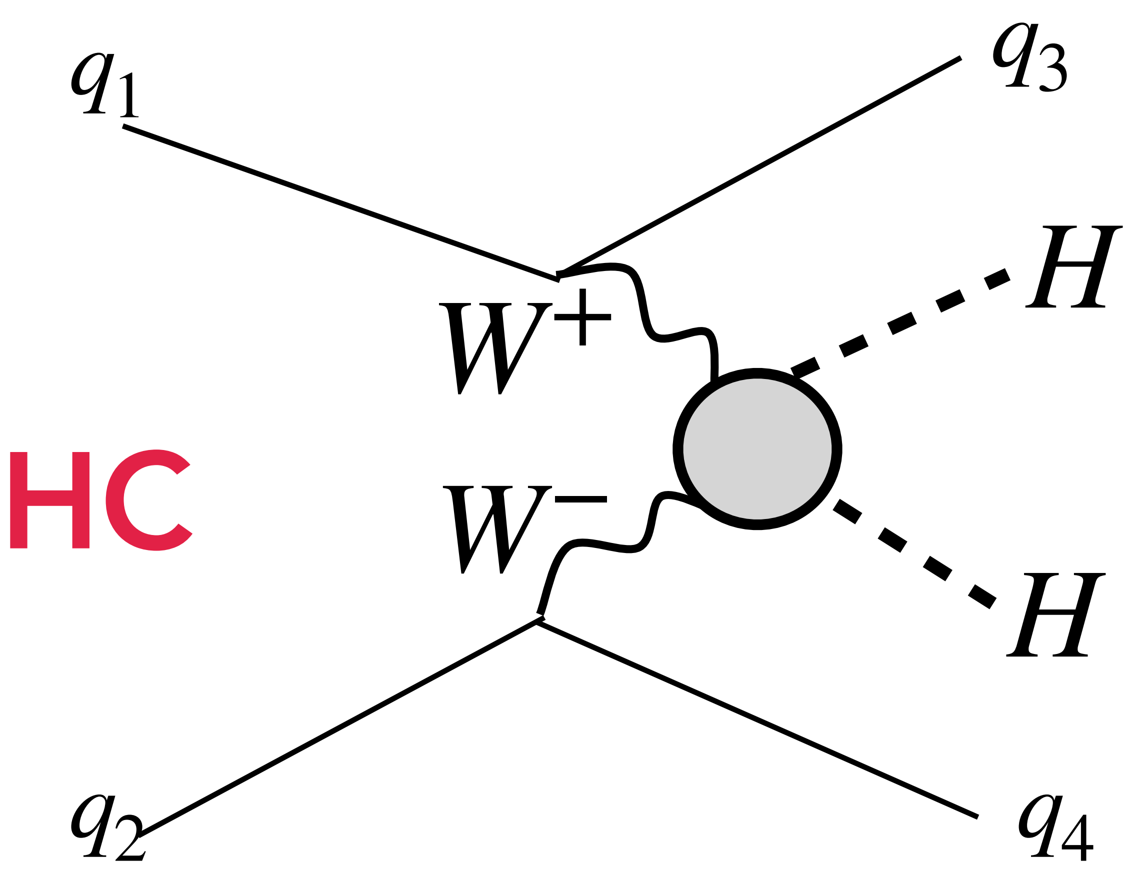
Content of this talk based on results in:

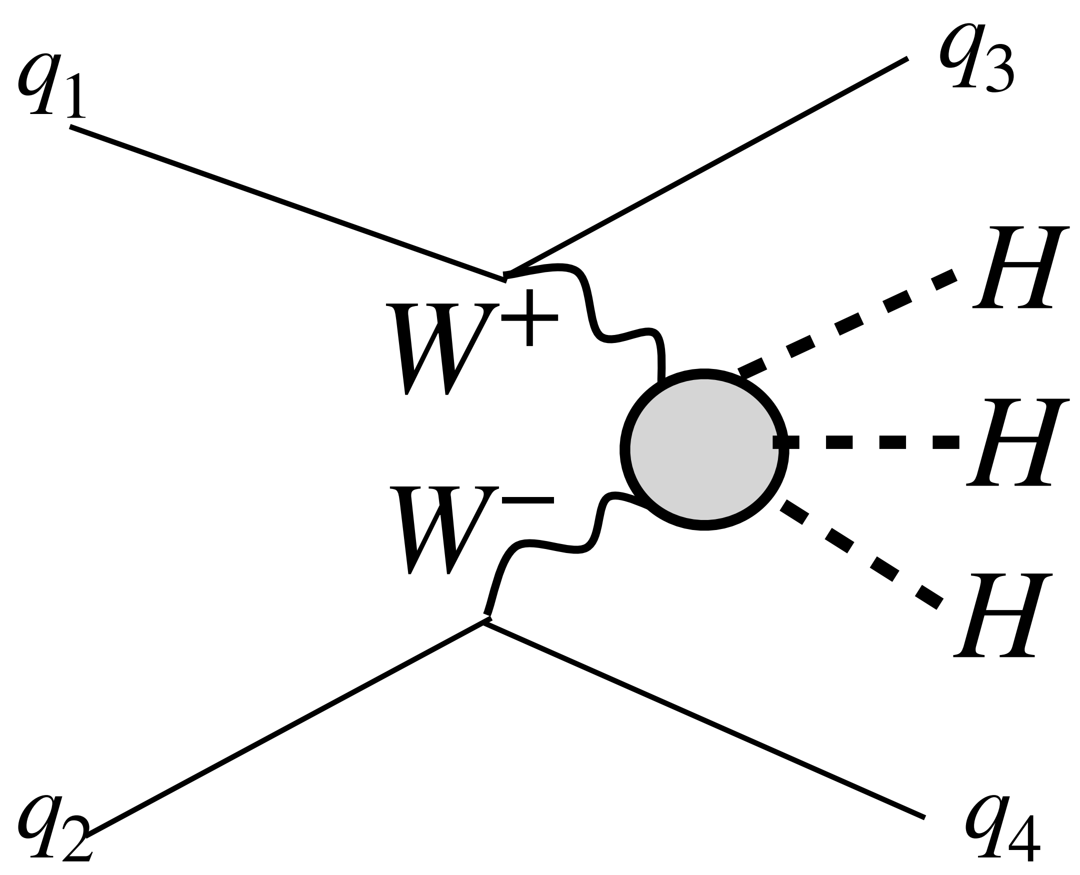
LO	2011.13195, EPJC 81 (2021)3, 260, González-López, Herrero, Martínez-Suárez	**HH and HHH within HEFT at e^+e^-**
(κ_V, κ_{2V})	2312.03877, EPJC 84 (2024)5, 503, Davila, Domenech, Herrero, Morales	
NLO(η, δ)	2208.05452, Phys. Rev. D 106 (2022)11, 115027, Domenech, Herrero, Morales, Ramos	
NLO	2405.05385 Anisha, Domenech, Englert, Herrero, Morales (gg to HH and HHH)	**HH and HHH within HEFT at pp**
(κ_V, κ_{2V})	Preliminar. Cepeda, Domenech, Garcia-Mir, Herrero. Work in progress (2024)	
(η, δ)	Preliminar. Domenech, Herrero, Morales. Work in progress (2024)	
Renorm. in R_ξ	2208.05900, Phy.Rev.D 106 (2022)7, 073008, Herrero, Morales →	**Rad. corrections via 1PIs**
Matching Amplitudes	2307.15693, Phys.Rev.D 108 (2023)9, 095013, Arco, Domenech, Herrero, Morales →	**Matching HEFT-2HDM**

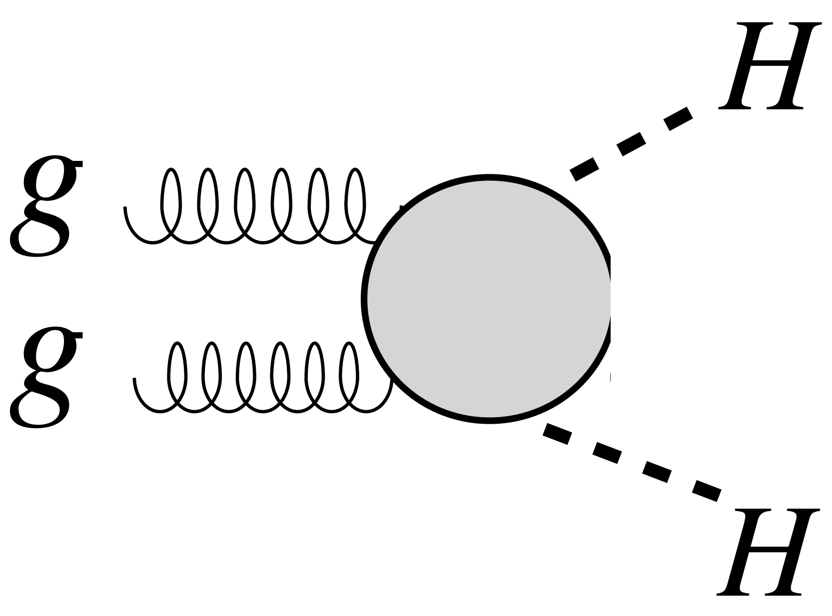
Tools used: FeynArts, FeynRules, FormCalc, Looptools, MG5, VBFNLO, HEFT model file included

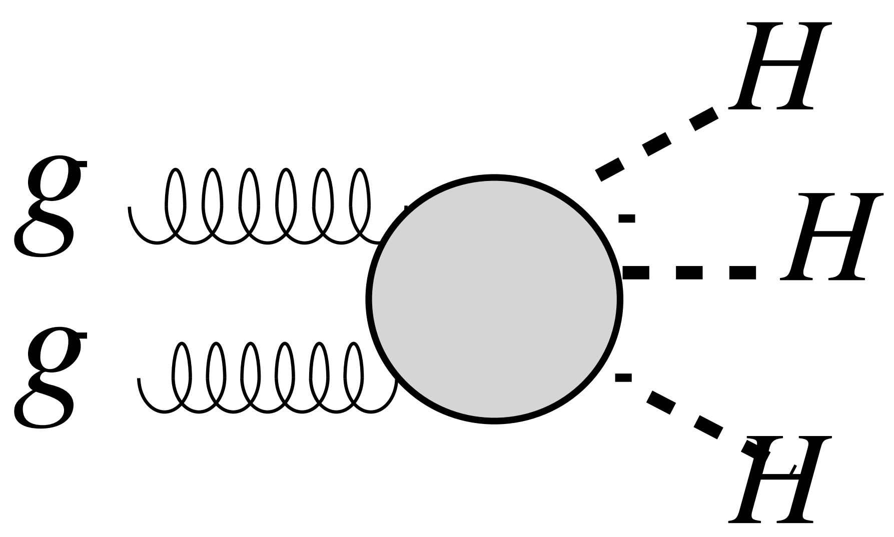
Main motivation: HH and HHH production at colliders

LHC

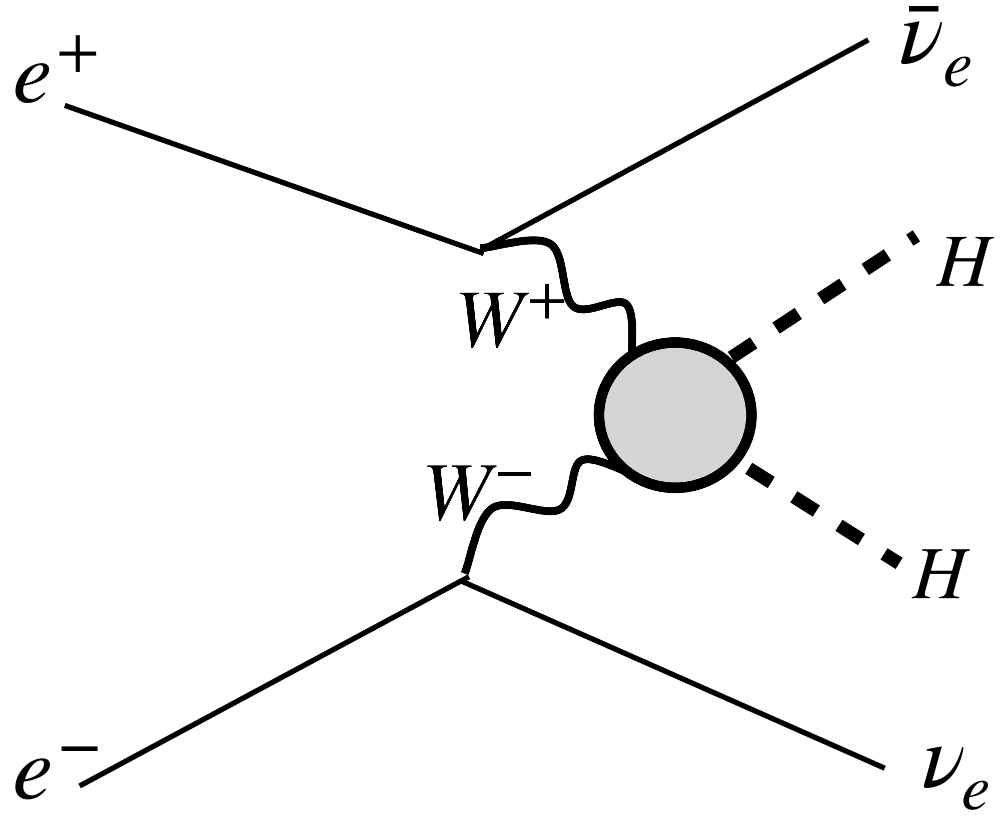


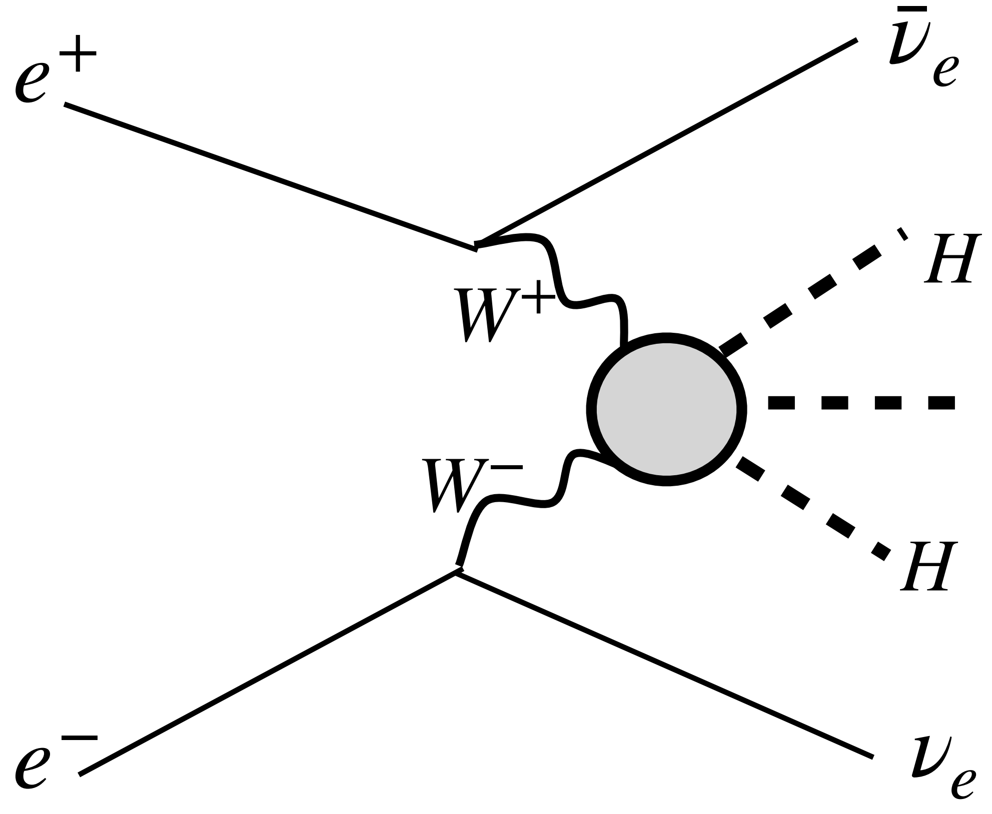






e^+e^-





Correlations within SM :

$V_{HWW} = vV_{HHWW}$

$V_{HHH} = vV_{HHHH}$

What about BSM?

Do we learn anything comparing HH and HHH production?

Bosonic HEFT (=EChL): proper tool for BSM MultiHiggs at pp and ee.
Easy connection of HEFT with kappa formalism. Fermionic sector assumed here to be as in the SM.
H being a singlet in HEFT gives uncorrelated interactions. In contrast to others (SM, SMEFT, 2HDM,…)
Our main focus: 1) sensitivity to LO $(\kappa_V, \kappa_{2V}), (\kappa_3, \kappa_4)$ 2) correlations 3) NLO ($a_i's$ and rad.corrections).
4) Tests of BSM in specific observables and with specific operators in contrast to global fits

HH and HHH (EW) production with LO-HEFT: $a, b, \kappa_3\ c, \kappa_4 \ldots$

$$\mathscr{L}^{\rm LO}_{\rm HEFT} = \frac{v^2}{4}\left[1 + 2a\left(\frac{H}{v}\right) + b\left(\frac{H}{v}\right)^2 + c\left(\frac{H}{v}\right)^3\right] {\rm Tr}\left[D_\mu U^\dagger D^\mu U\right] - \kappa_3 \lambda v H^3 - \frac{1}{4}\kappa_4 \lambda H^4 - \frac{1}{2} m_H^2 H^2 + \ldots$$

SM: $\mathbf{a} = \mathbf{b} = \kappa_3 = \kappa_4 = 1$

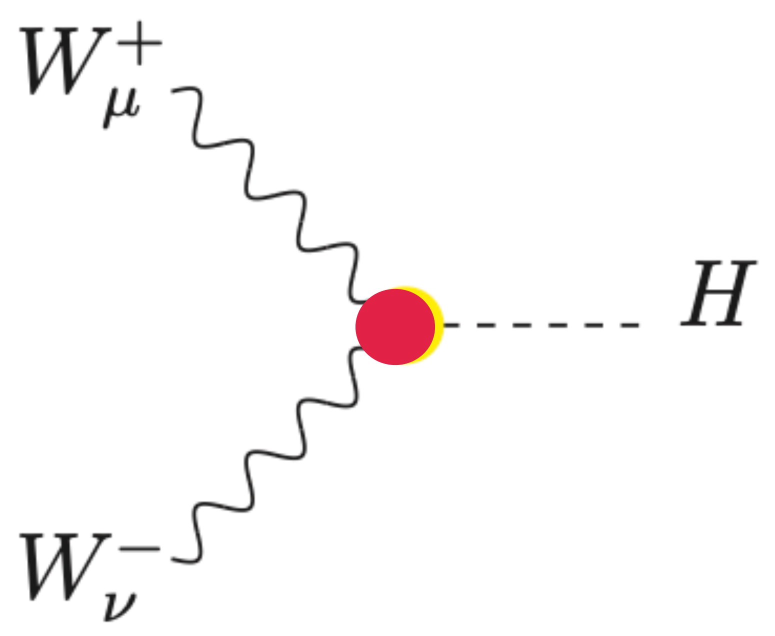
LECs=Anomalous couplings: parametrize possible BSM effects in LO-HEFT

$$U(\omega^a) = e^{\omega^a \tau^a / v}$$

$\omega^a = \pi^a$ — Non-Linear GBs

H — Higgs in Polynomials — Higgs is singlet

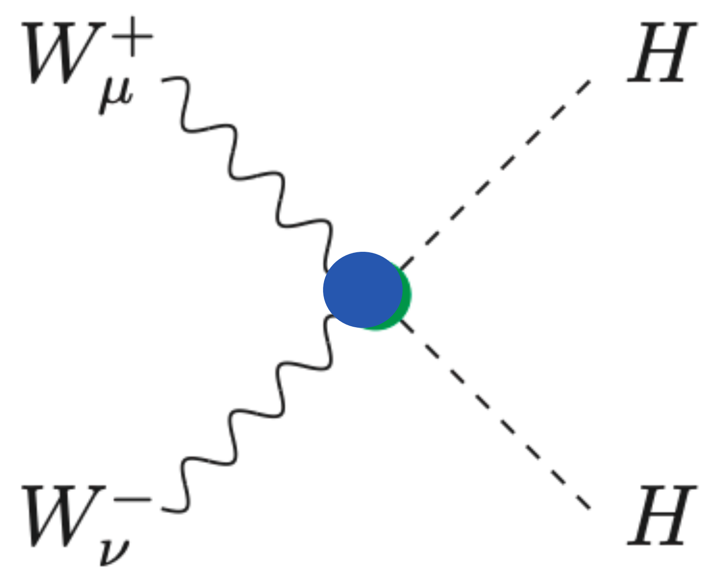
$m_H^2 = 2\lambda v^2$; $m_W = gv/2$; $m_Z = m_W / c_W$



$a = \kappa_V$

$$i\Gamma_{WWH} = iagm_W g_{\mu\nu}$$

$a^{\rm exp} \in [0.97, 1.13]$[1]



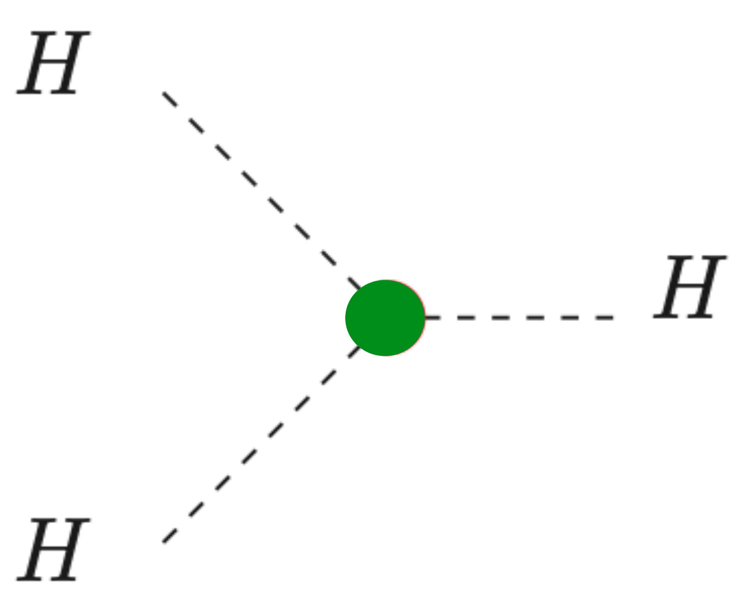
$b = \kappa_{2V}$

$$i\Gamma_{WWHH} = \frac{ibg^2}{2} g_{\mu\nu}$$

$b^{\rm exp} \in [0.0, 2.1]$[2a]

$b^{\rm exp} \in [-0.1, 2.2]$(2b)

$b^{\rm exp} \in [-0.03, 2.11]$(2c)

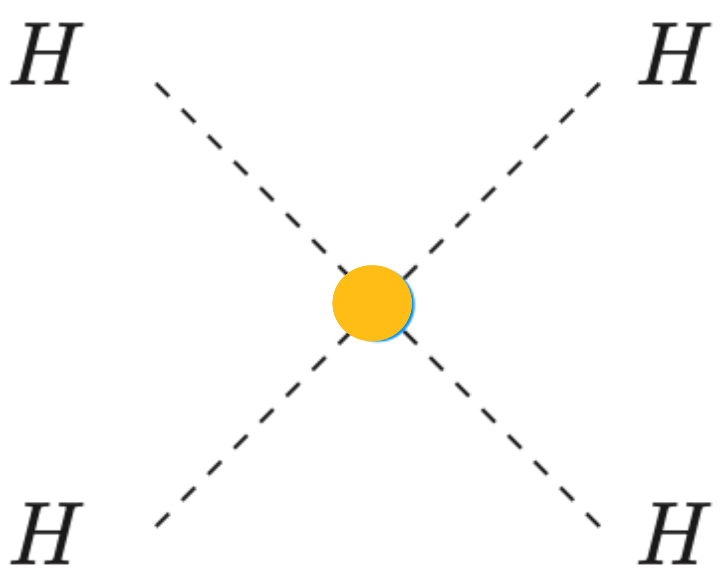


$\kappa_3 = \kappa_\lambda$

$$i\Gamma_{HHH} = -i\kappa_3 6\lambda v$$

$\kappa_3^{\rm exp} \in [-0.4, 6.3]$[3a]

$\kappa_3^{\rm exp} \in [-1.24, 6.49]$[3b]



κ_4

$$i\Gamma_{HHHH} = -i\kappa_4 6\lambda$$

$\kappa_4^{\rm exp}$ unconstrained

LO uncorrelated coeffs.

a versus b

κ_3 versus κ_4

In contrast to
SM, SMEFT, 2HDM,..
(where H is in a doublet)

[1]ATLAS, Phys. Rev. D **101** (2020) [1909.02845]

[2a]CMS, PLB 842,137531 (2023) [2206.09401]

(2b) CMS, PRL129, 081802(2022) [2202.09617]

(2c) ATLAS, PRD108, 052003(2023) [2301.03212]

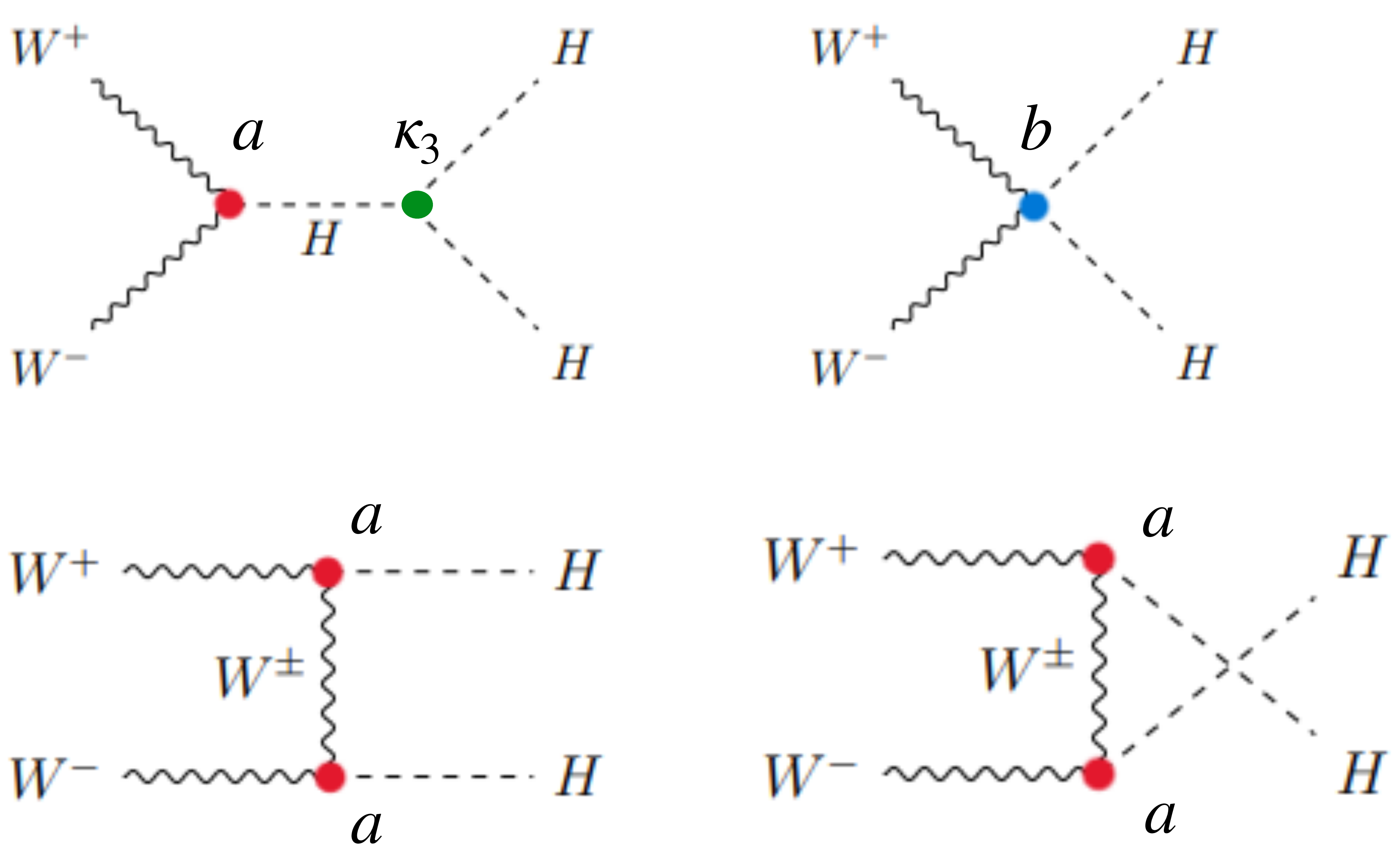
[3a]ATLAS (PLB 843 (2023)137745

[3b]CMS (Nature 607, 7917, 2022)

WW—>HH gives access to a, b, κ_3 (LO-HEFT)

Idem ZZ—>HH

Unitary Gauge



$$\mathcal{A} = \mathcal{A}_c + \mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u$$

$$\mathcal{A}_c = \frac{g^2 b}{2} \epsilon_+ \cdot \epsilon_- ,$$

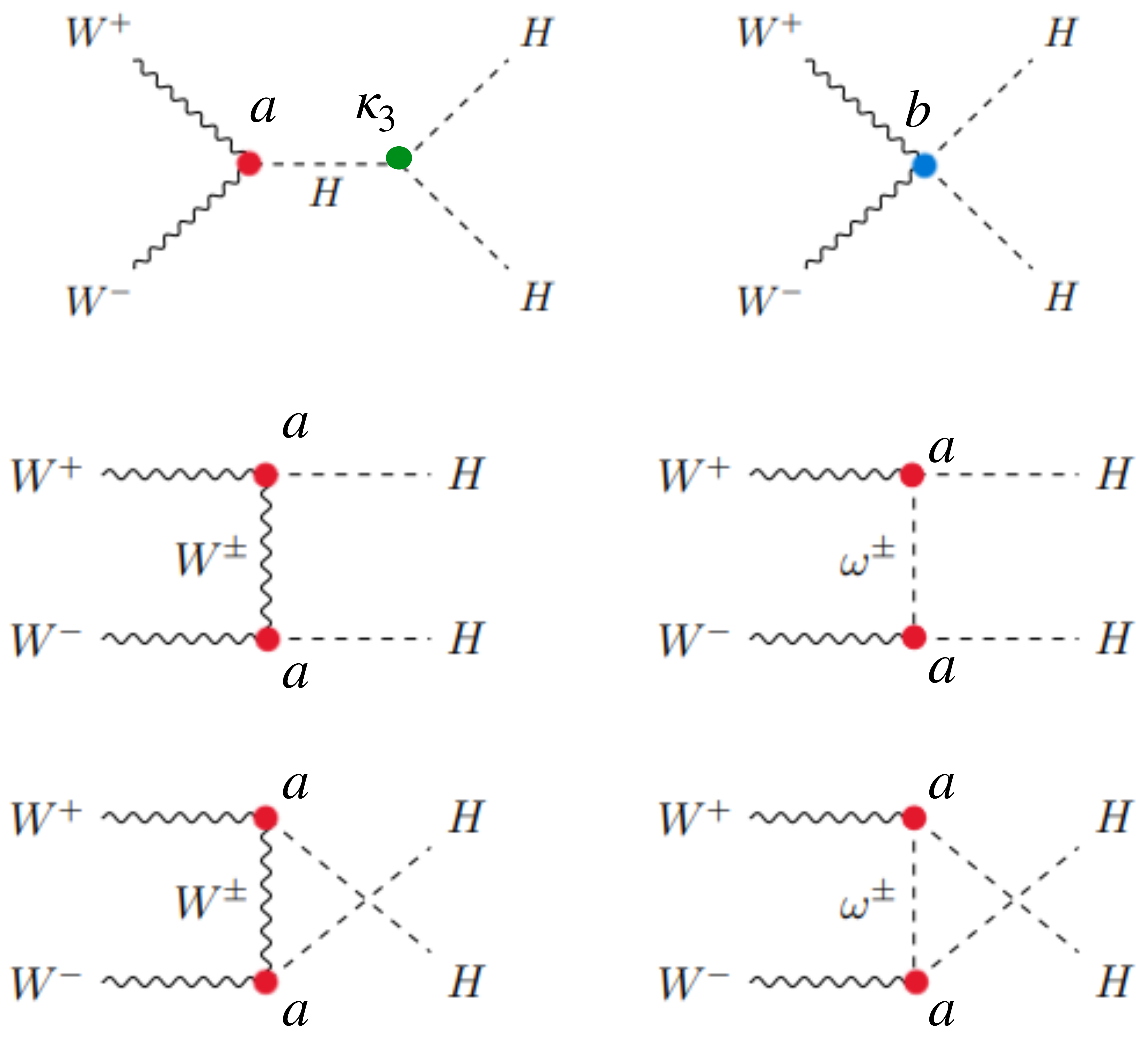
$$\mathcal{A}_s = \frac{3g^2 a}{2} \frac{\kappa_3 m_H^2}{s - m_H^2} \epsilon_+ \cdot \epsilon_- ,$$

$$\mathcal{A}_t = g^2 a^2 \frac{m_W^2 (\epsilon_+ \cdot \epsilon_-) + (\epsilon_+ \cdot k_1)(\epsilon_- \cdot k_2)}{t - m_W^2} ,$$

$$\mathcal{A}_u = g^2 a^2 \frac{m_W^2 (\epsilon_+ \cdot \epsilon_-) + (\epsilon_+ \cdot k_2)(\epsilon_- \cdot k_1)}{u - m_W^2} .$$

Covariant R_ξ Gauge

$$\mathcal{L}_{\rm GF} = -F_+ F_- - \frac{1}{2} F_Z^2 - \frac{1}{2} F_A^2, \quad F_\pm = \frac{1}{\sqrt{\xi}} (\partial^\mu W_\mu^\pm - \xi m_{\rm W} \pi^\pm), F_Z = \frac{1}{\sqrt{\xi}} (\partial^\mu Z_\mu - \xi m_Z \pi^3), F_A = \frac{1}{\sqrt{\xi}} (\partial^\mu A_\mu).$$



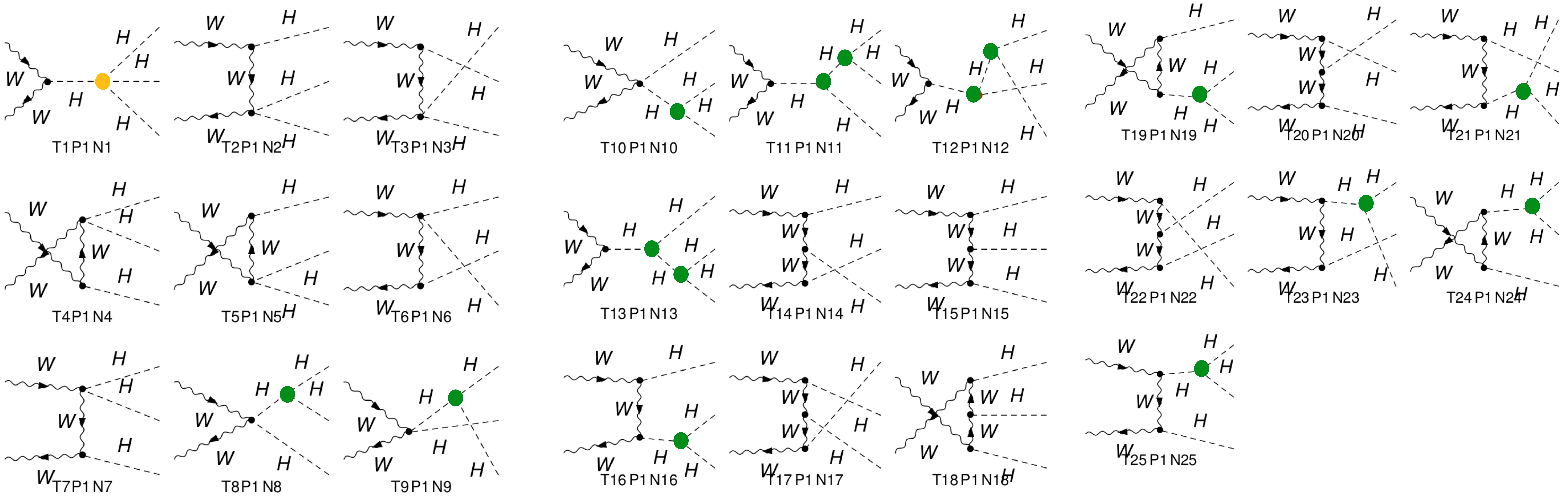
⟷ Identical results (GAUGE INVARIANT)

ξ dependence cancels in t and u channels separately

kappa-parametrization gives gauge invariant scattering amplitudes within HEFT: $\kappa_V = a$, $\kappa_{2V} = b$, $\kappa_3 = \kappa_\lambda$

WW—>HH gives relevant access to a , b and κ_3

WW—>HHH gives access to κ_3 and κ_4 (LO-HEFT)



Less available phase space ——-> smaller cross sections than for WW —>HH: But yet possible access to large BSM $\kappa's$

Very small SM ($\kappa_3 = \kappa_4 = 1$) rates: $\sigma^{\rm SM}(pp \to HHHjj)\,(14\,{\rm TeV}) = 10^{-7}\,{\rm pb}$ $\quad\sigma^{\rm SM}(e^+e^- \to HHH\nu\bar{\nu})\,(3\,{\rm TeV}) = 3\times 10^{-7}\,{\rm pb}$

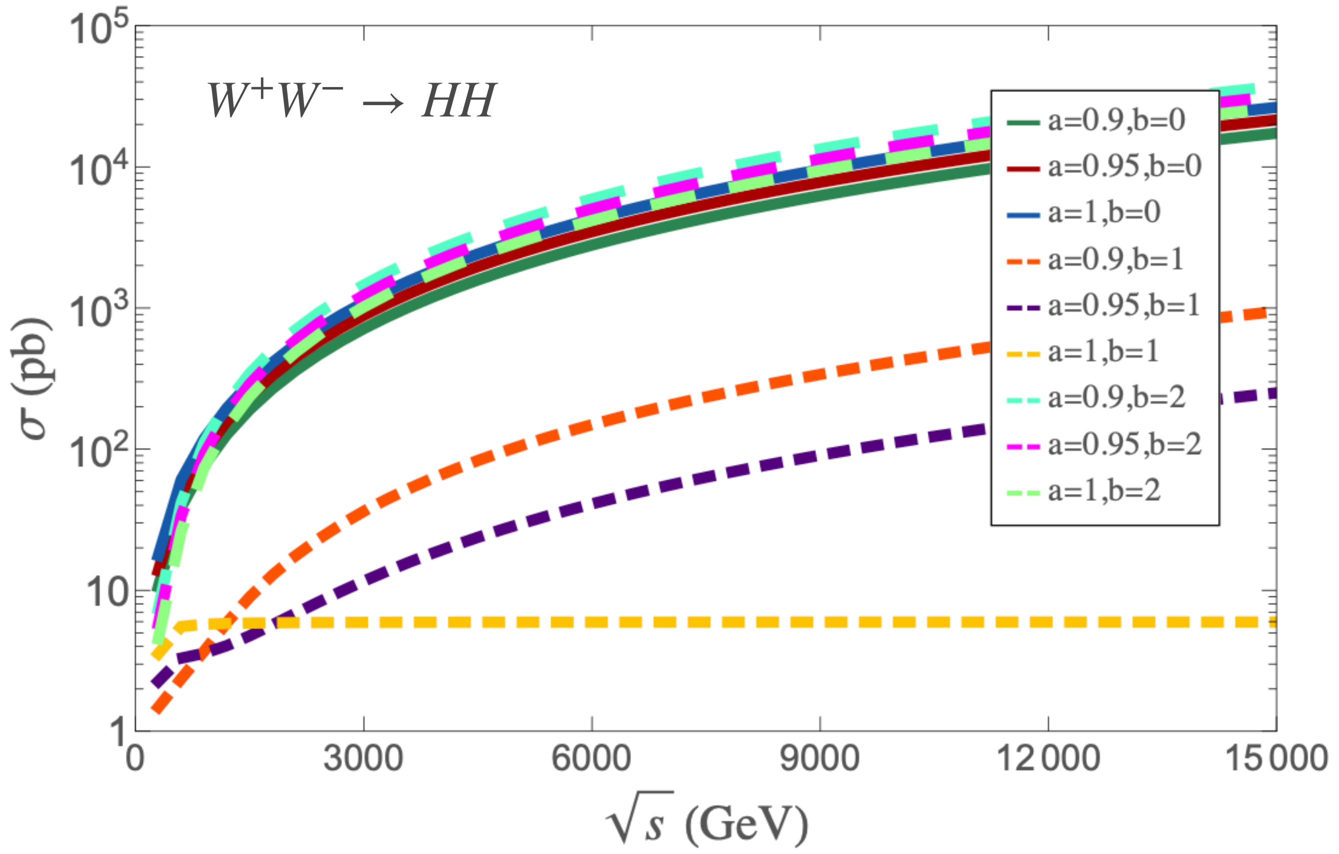
Behavior with energy: subprocess (LO-HEFT)

2011.13195, EPJC 81 (2021)3, 260, González-López, Herrero, Martínez-Suárez

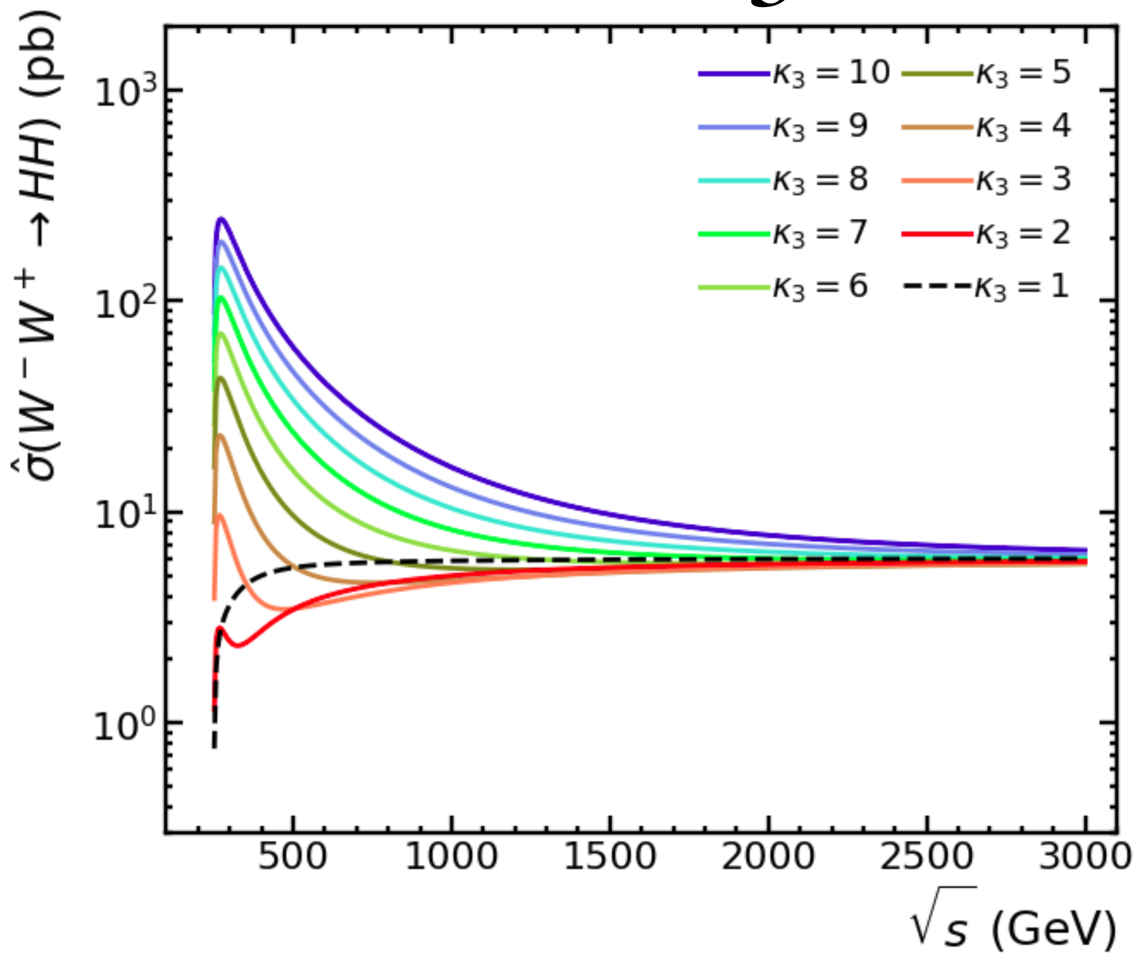
Subprocess level

$WW \to HH$

$b \neq a^2$

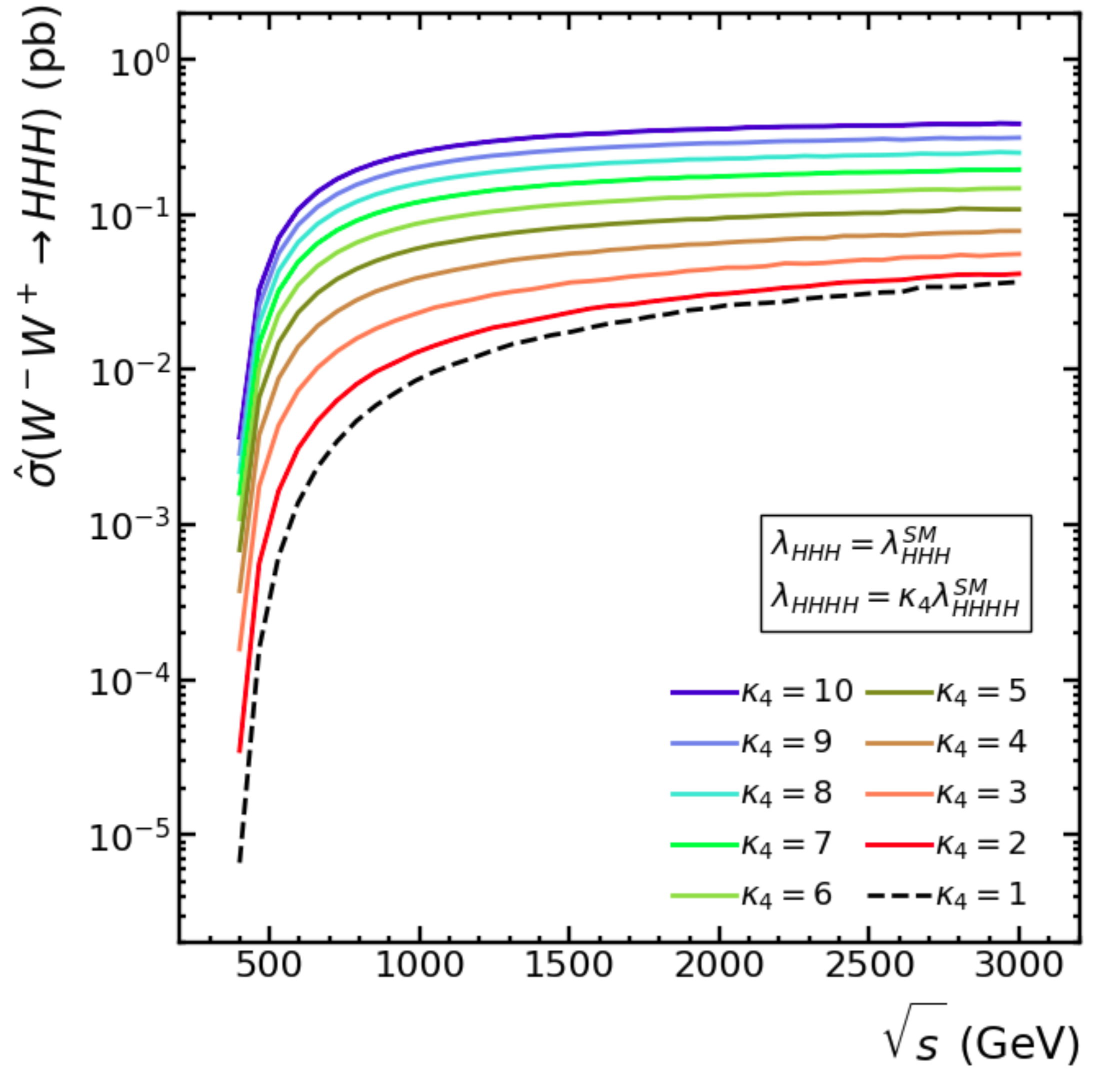


κ_3

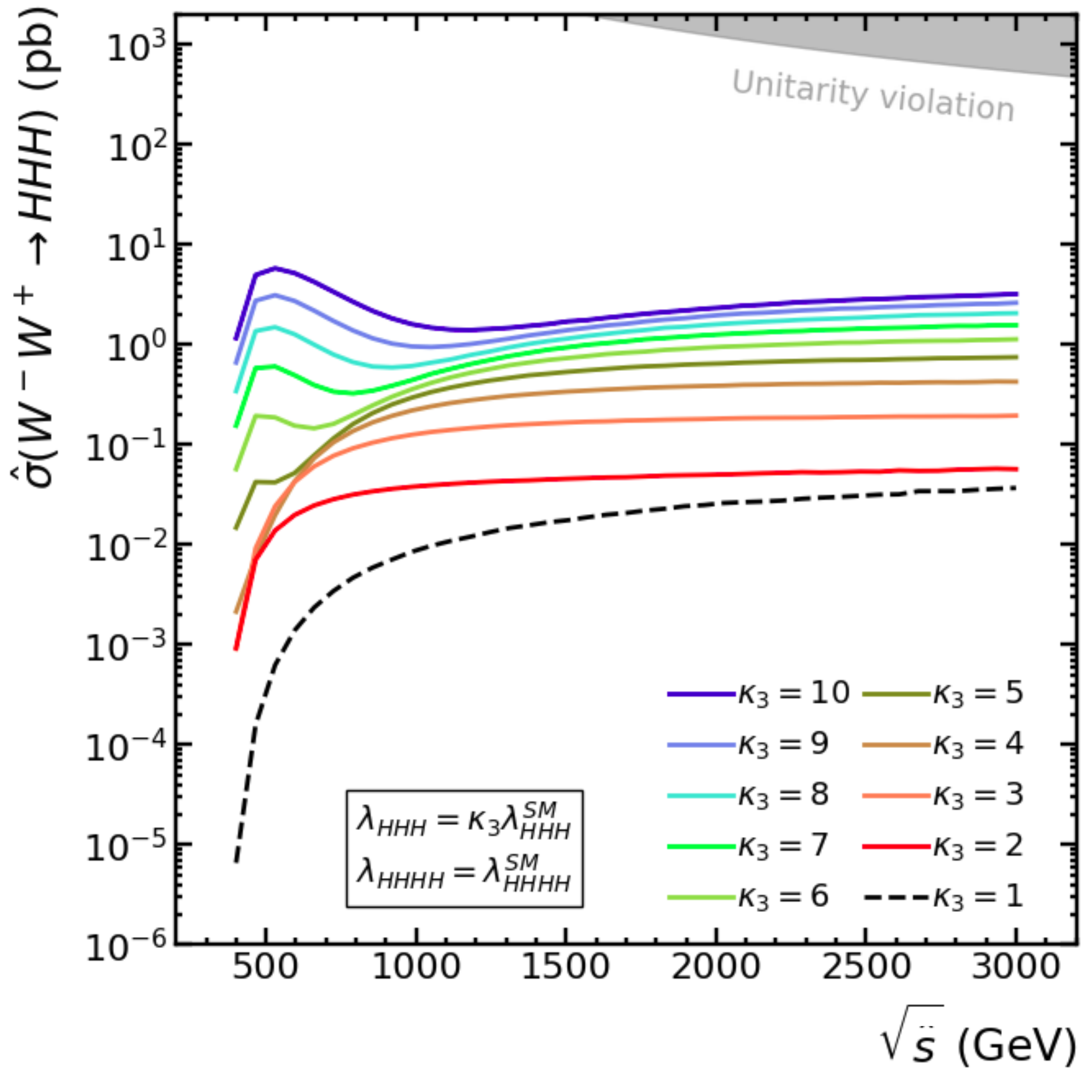


Largest deviations respect to SM in LL modes

$WW \to HHH$



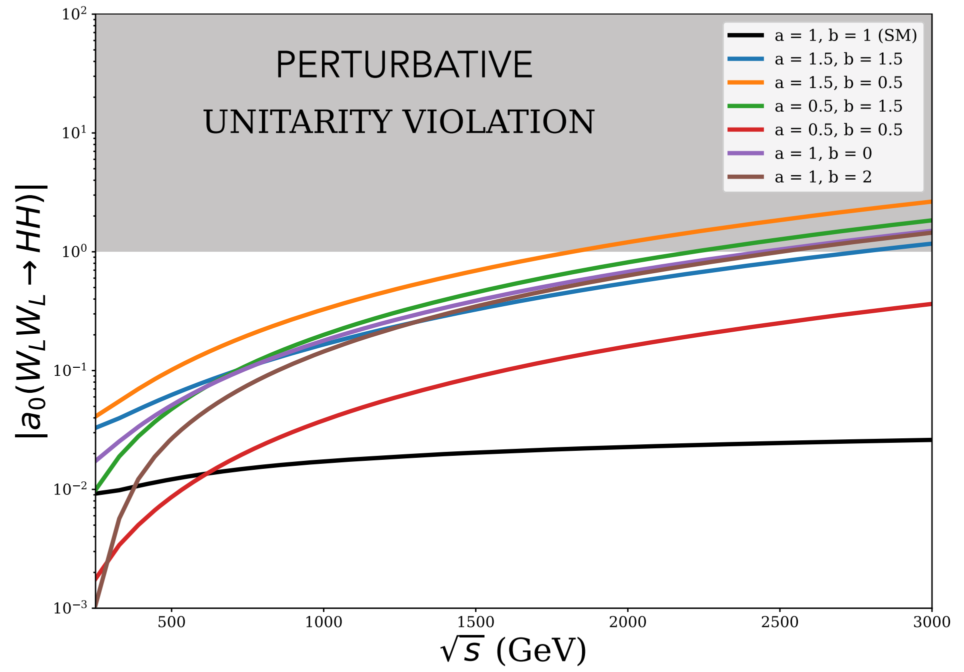
κ_4



κ_3

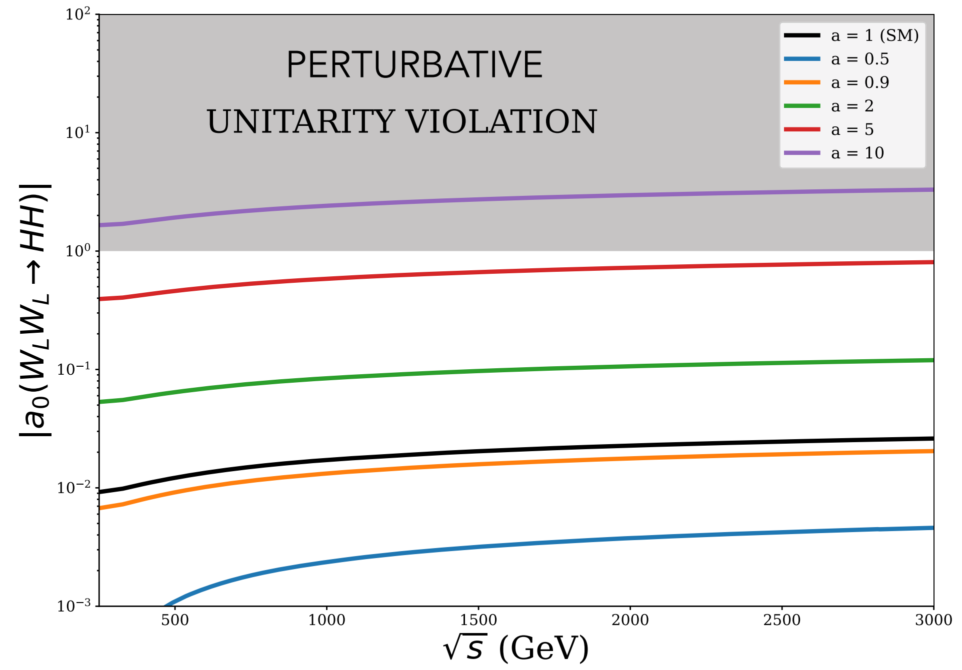
Idem ZZ—>HH(H)

$b \neq a^2$



HH

$b = a^2$



HH : Strong enhancement at large $\sqrt{s}$ for $b \neq a^2$
Pert. unitarity viol above few TeV
$\kappa_{2V} = 0$ viol unit. above 2.4 TeV !
Max sensitivity to κ_3 close to $2m_H$

HHH : Similar behavior at large $\sqrt{s}$ as in the SM (shifted upwards)
No unitarity constraints on κ_3, κ_4
Max sensitivity to κ_3 close to $3m_H$

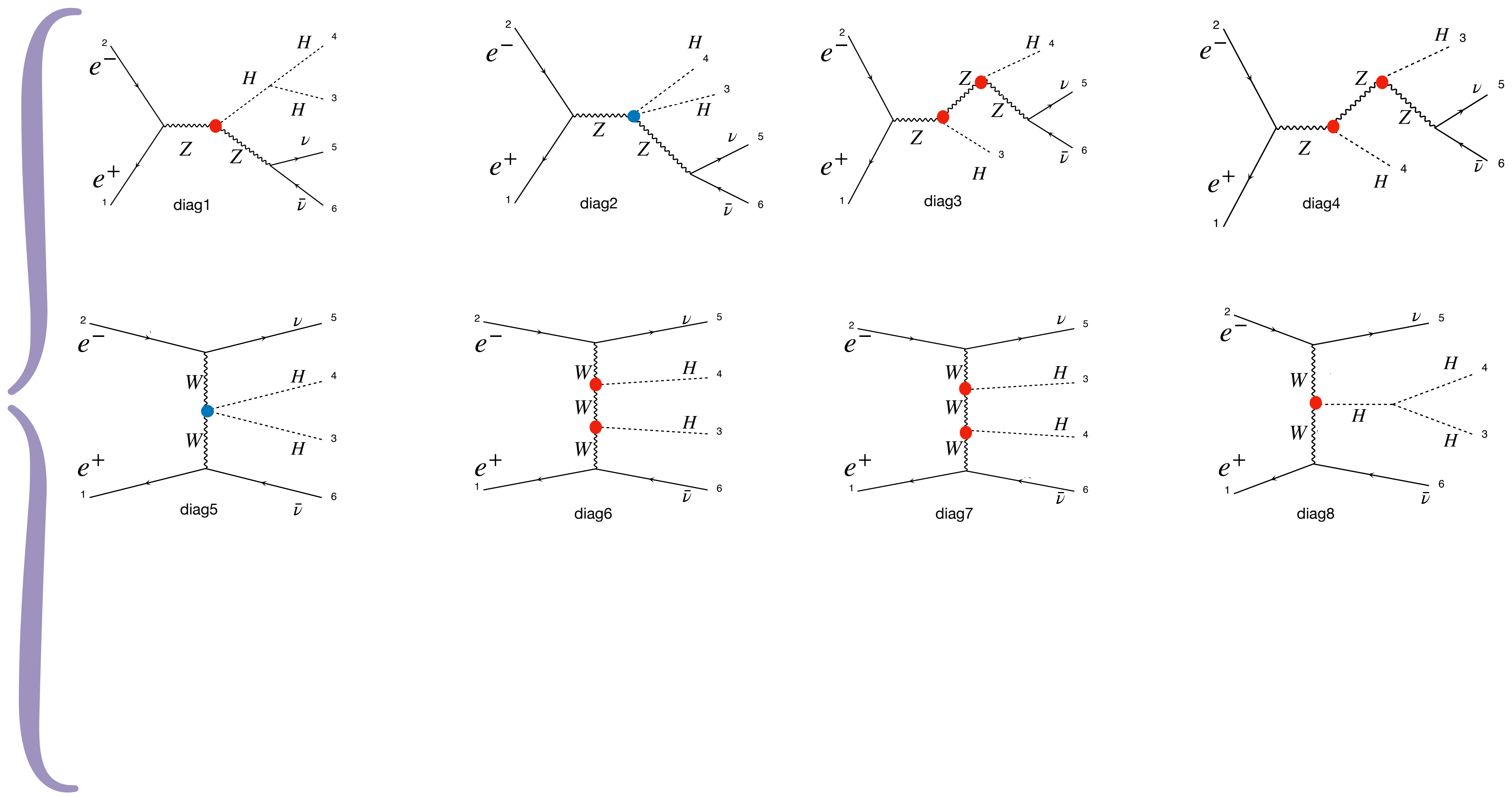
The largest sensitivity to k3 and k4 occurs in ggF, gg —>HH(HHH), see later

HH production: testing a=κ_V, b=κ_{2V} together at colliders (LO-HEFT)

Our Bosonic-HEFT model file is implemented in MG5

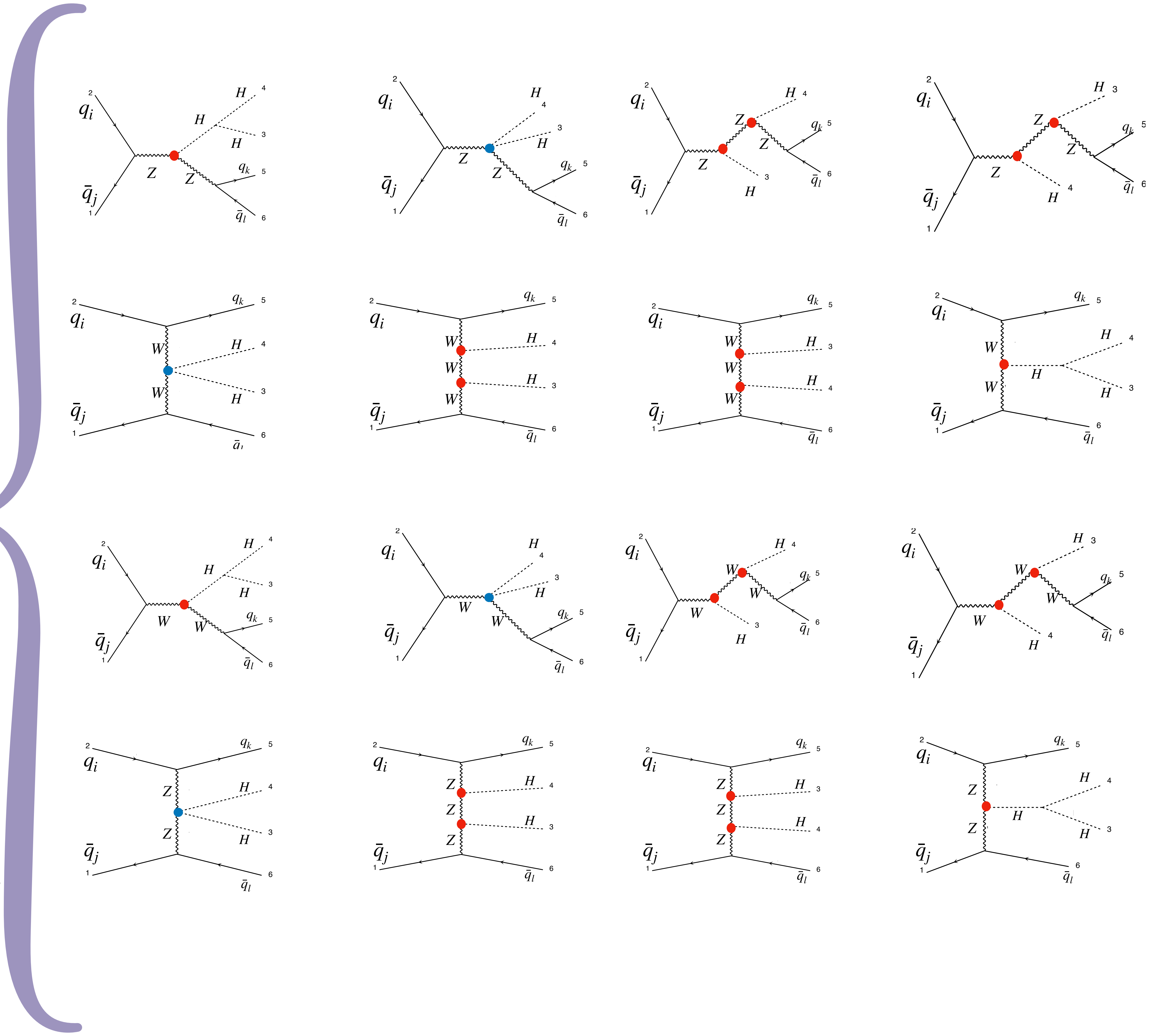
e^+e^-

$e^+e^- \to HH\nu\bar{\nu}$



LHC

$q_1\bar{q}_2 \to HHq_3\bar{q}_4$ (+ diags for $\bar{q}\bar{q}$ and for qq)



both $(a, b) = (\kappa_V, \kappa_{2V})$ involved

Explore in $HH\not{E}_T$ events. We require $\not{E}_T > 20\,\mathrm{GeV}$

Explore in $HHjj$ events. We require WBF cuts, $\Delta\eta_{jj}$> 4, $M_{jj} > 500$ GeV

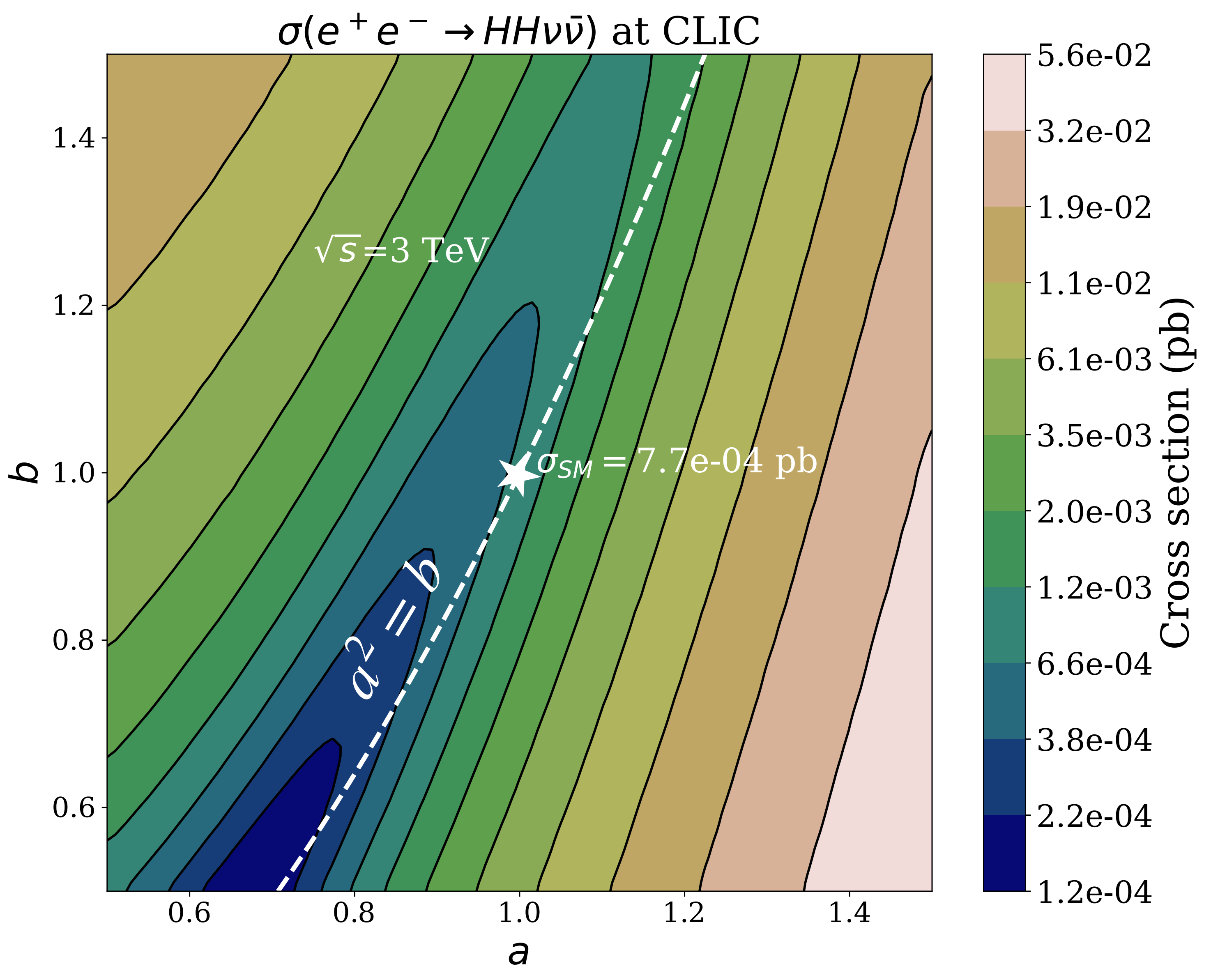
BSM signals means deviations in σ and in $d\sigma's$ respect the SM rates. We also explore correlations.

(a, b) in (EW) HH / HHH production : e^+e^- and pp

(all simulations with MG5)

e^+e^- (CLIC)

$HH \not{E}_T$



2312.03877 Davila, Domenech, Herrero, Morales, EPJC 84 (2024)5, 503

Largest sensitivity expected if

$$a^2 \neq b$$

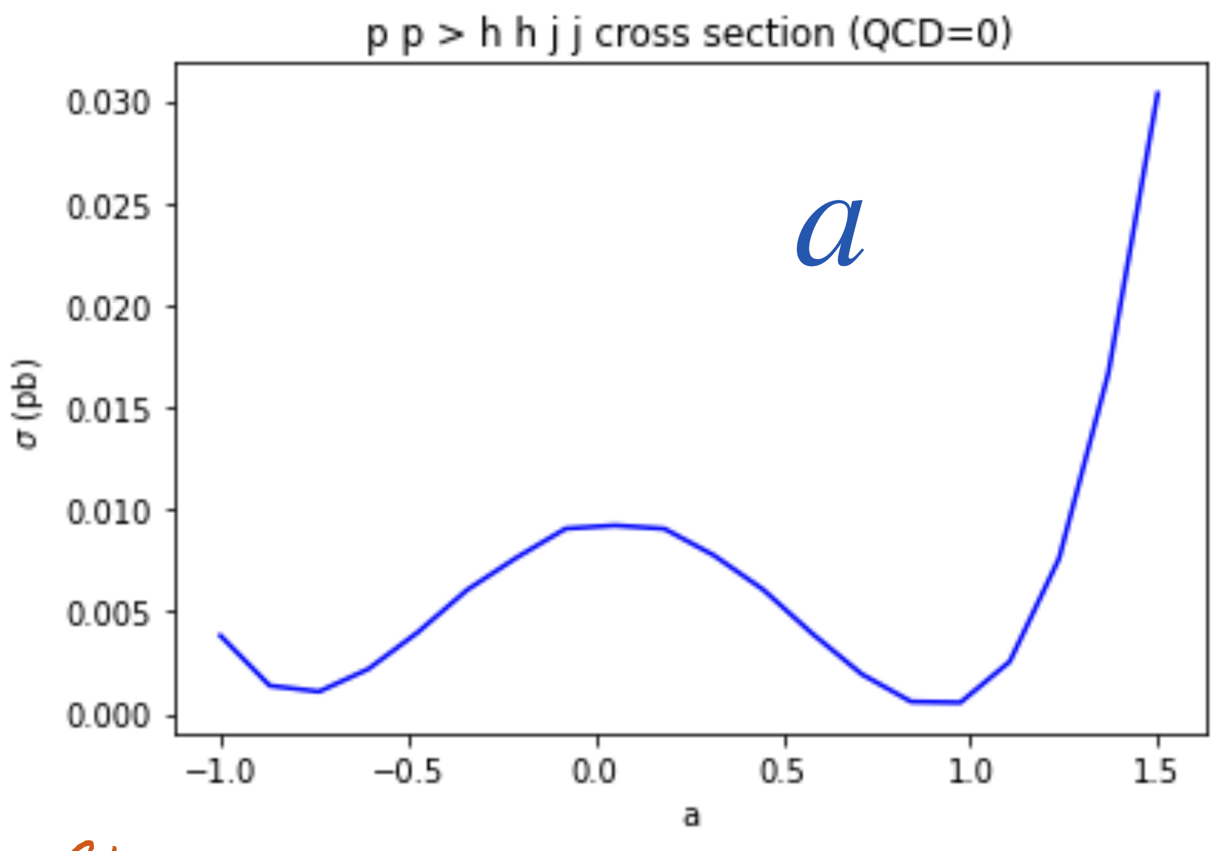
Expected sensitivity at CLIC. : $\Delta b \sim \mathcal{O}(10^{-1})$

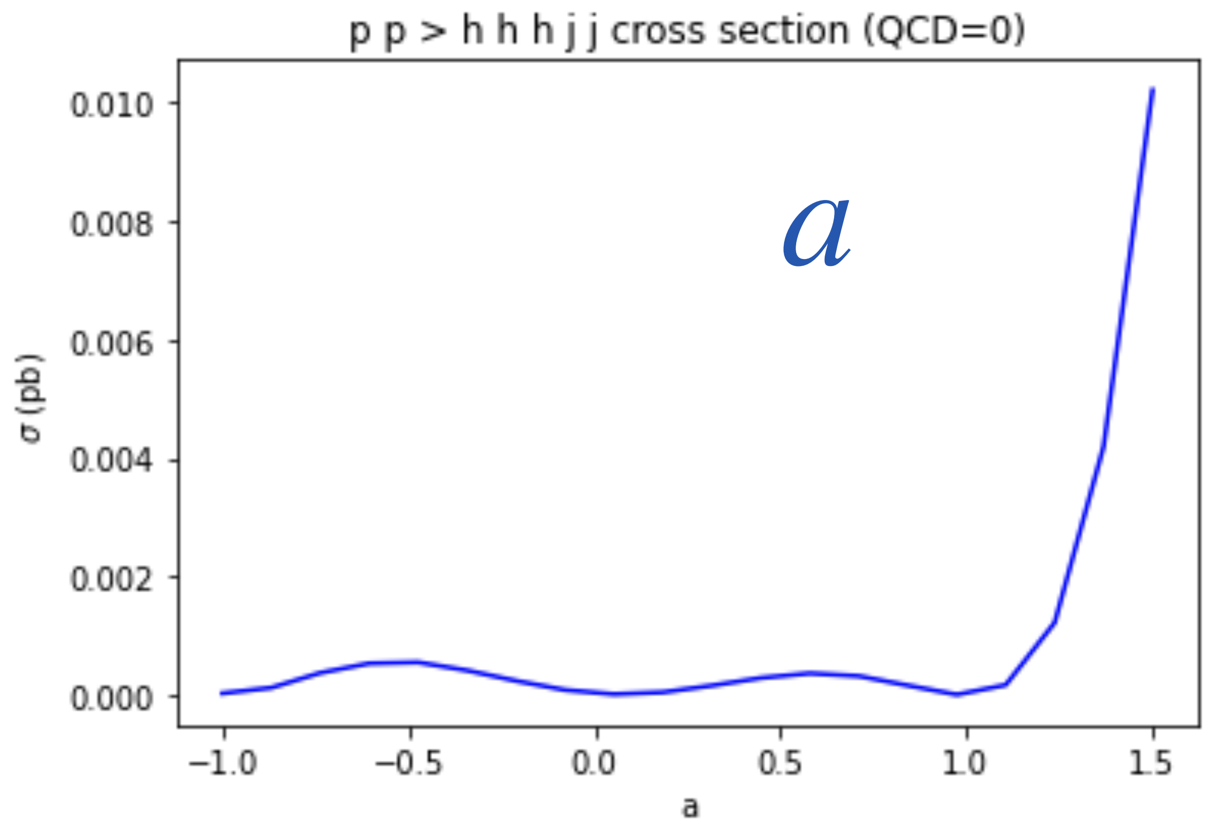
Cuts: $\not{E}_T > 20\,\mathrm{GeV}$ (see also ILC 0.5 and 1 TeV in paper)

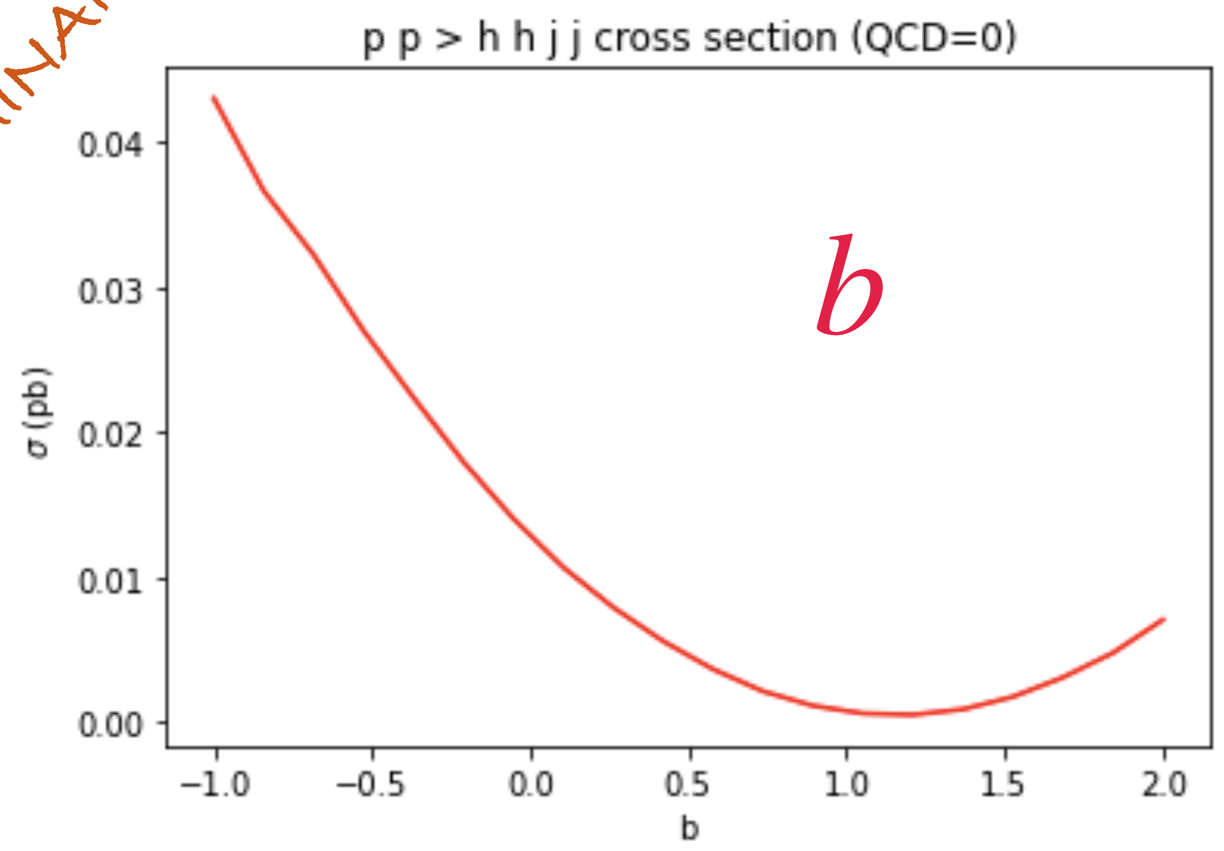
pp (LHC)

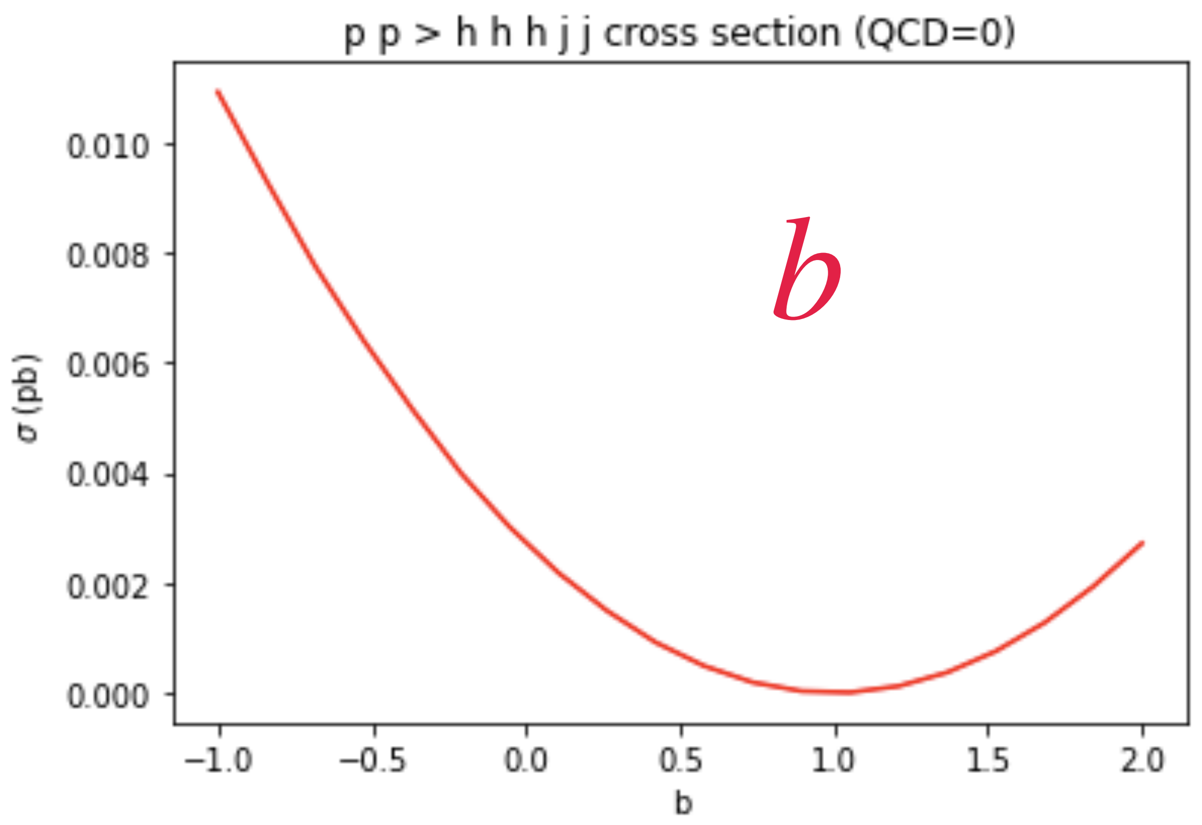
$HH\,jj$ | $HHH\,jj$

PRELIMINAR









WBF cuts:

$M_{jj} > 500\,\mathrm{GeV}$

$2 < \eta_j < 5$, $\eta_{j1} \times \eta_{j2} < 0$

$P_{Tj} > 20\,\mathrm{GeV}$, $\Delta R_{jj} > 0.4$

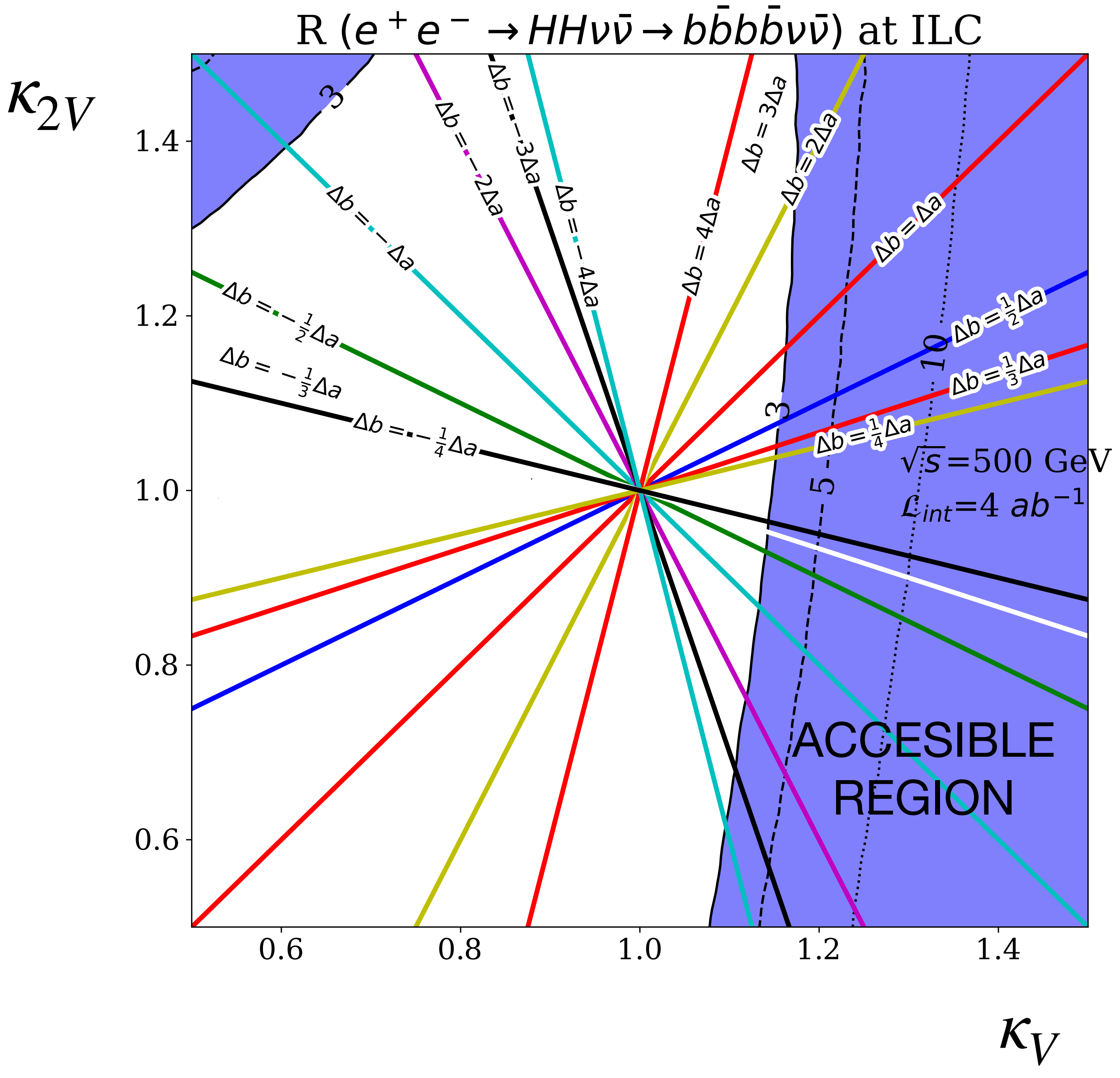
Interesting correlation in b !!

Except for factor suppression HHH/HH due to phase space

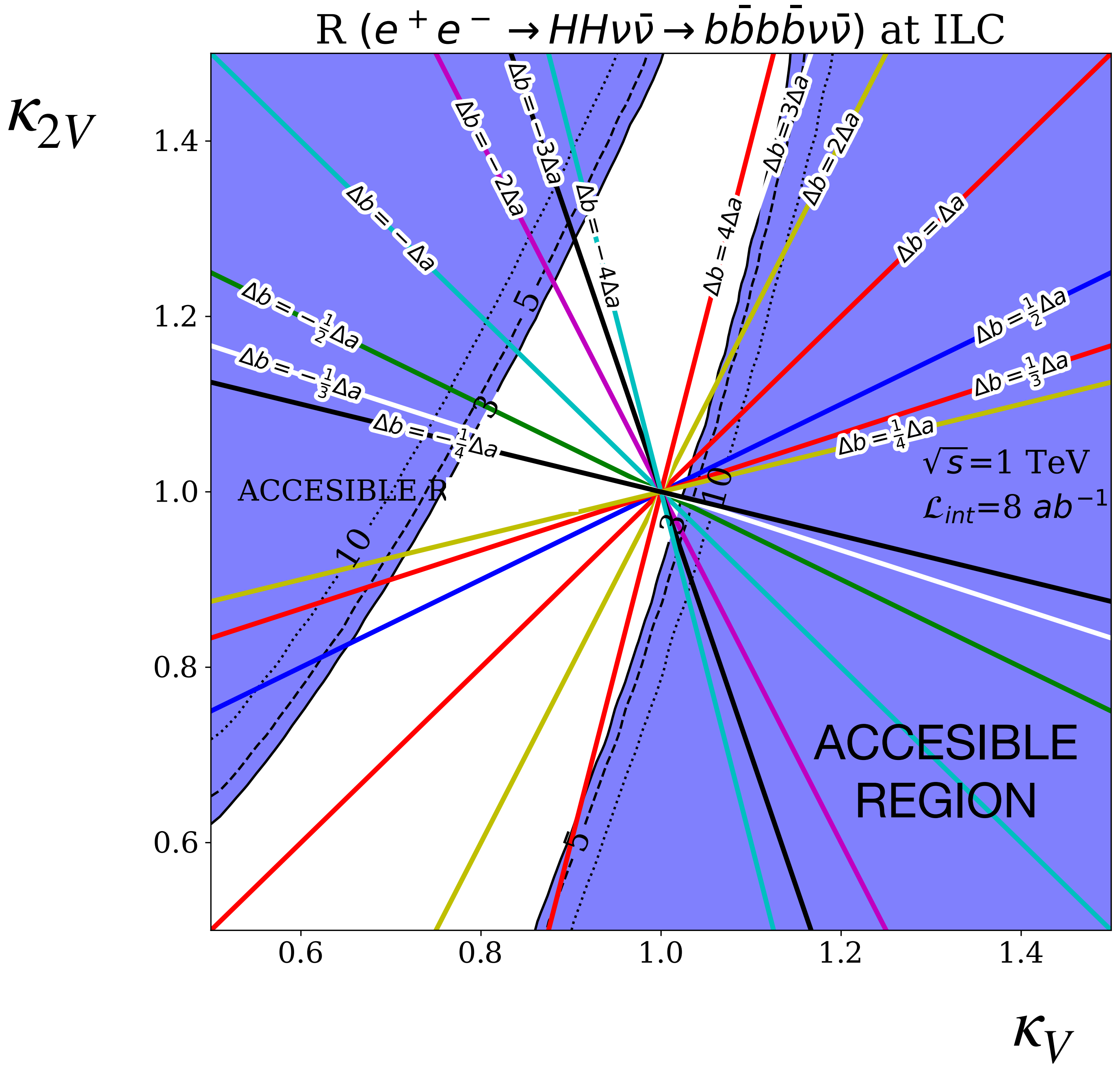
Work in progress: Herrero with Morales, Domenech, ...Englert, Anisha...

Access to (κ_V, κ_{2V}) and correlations in $e^+e^- \to HH\nu\bar{\nu} \to 4bjets + E_T^{miss}$

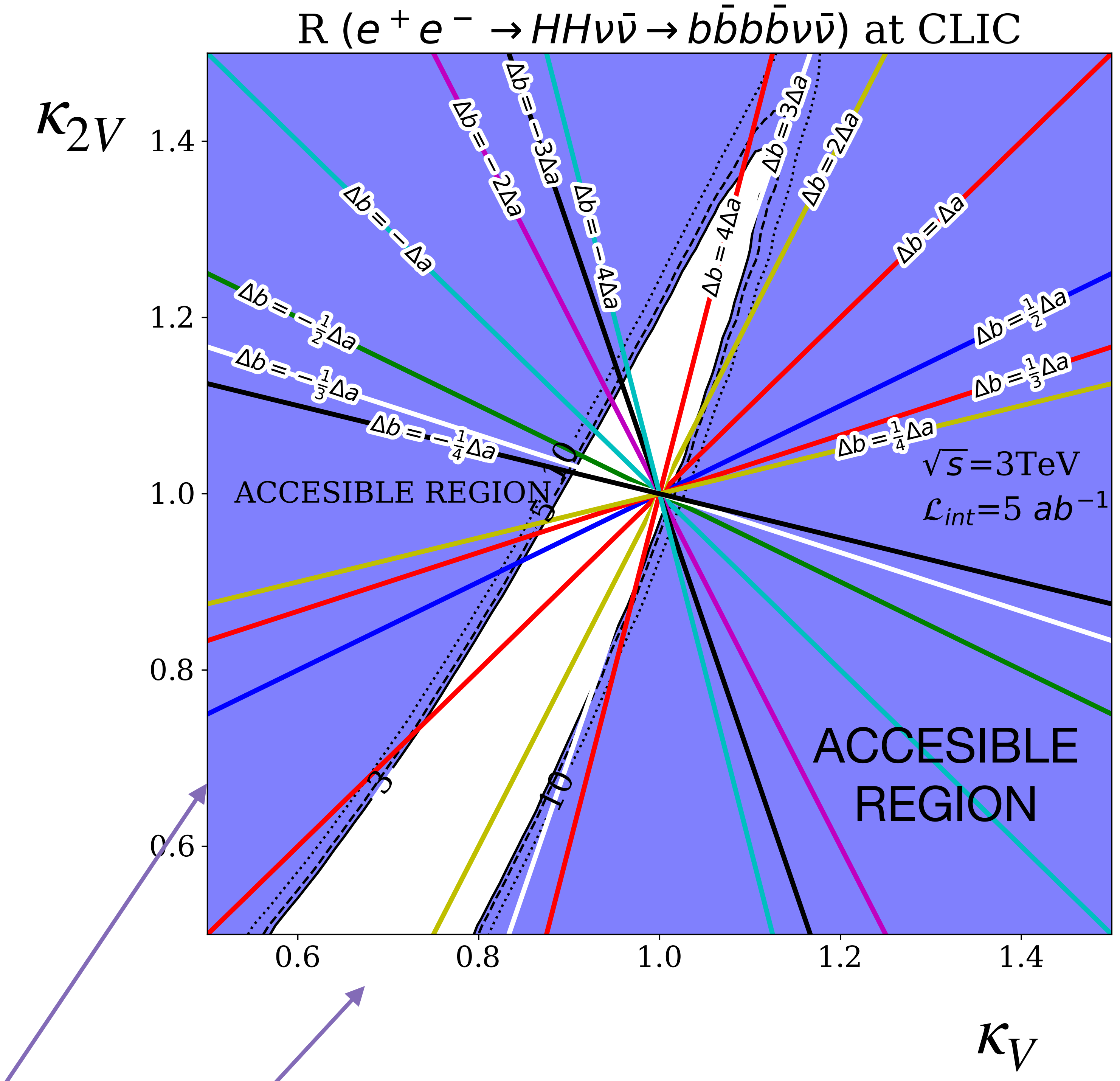
ILC 500 GeV



ILC 1 TeV



CLIC 3 TeV



Accessible Regions (in purple) defined as $R = \dfrac{N_{\rm BSM} - N_{\rm SM}}{\sqrt{N_{\rm SM}}} > 3$

Largest regions in $(a, b) = (\kappa_V, \kappa_{2V})$ in CLIC, up to $\Delta\kappa \sim \mathcal{O}(10^{-1})$

Correlations $b \neq a^2$ defined by lines $\Delta b = C\Delta a$; $b = 1 - \Delta b$; $a = 1 - \Delta a$

Some correlations better tested, for instance $C = 1/4, 1/3, 1/2, 1$ if $\kappa_{V,2V} < 1$

In contrast to moving in line $b = a^2$ (equiv to $\Delta b = 2\,\Delta a$, yellow lines)

N=EVENTS with 4b+ETmiss

4-btagged jets $\epsilon_b = 0.8$

- ✓ $p_T^j > 20$ GeV ✓ $\Delta R_{jj} > 0.4$
- ✓ $|\eta^j| < 2$ ✓ $E_T^{\rm mis} > 20$ GeV

Interesting cases: $\Delta b|_{\rm 2HDM} = -2\Delta a|_{\rm 2HDM}$ (magenta lines)
$\Delta b|_{\rm SMEFT} = 4\Delta a|_{\rm SMEFT}$ (red lines)

More difficult if $\kappa_{2V} > 1$

2312.03877 Davila, Domenech, Herrero, Morales, EPJC 84 (2024)5, 503 ; 2011.13195 2307.15693 2208.05452

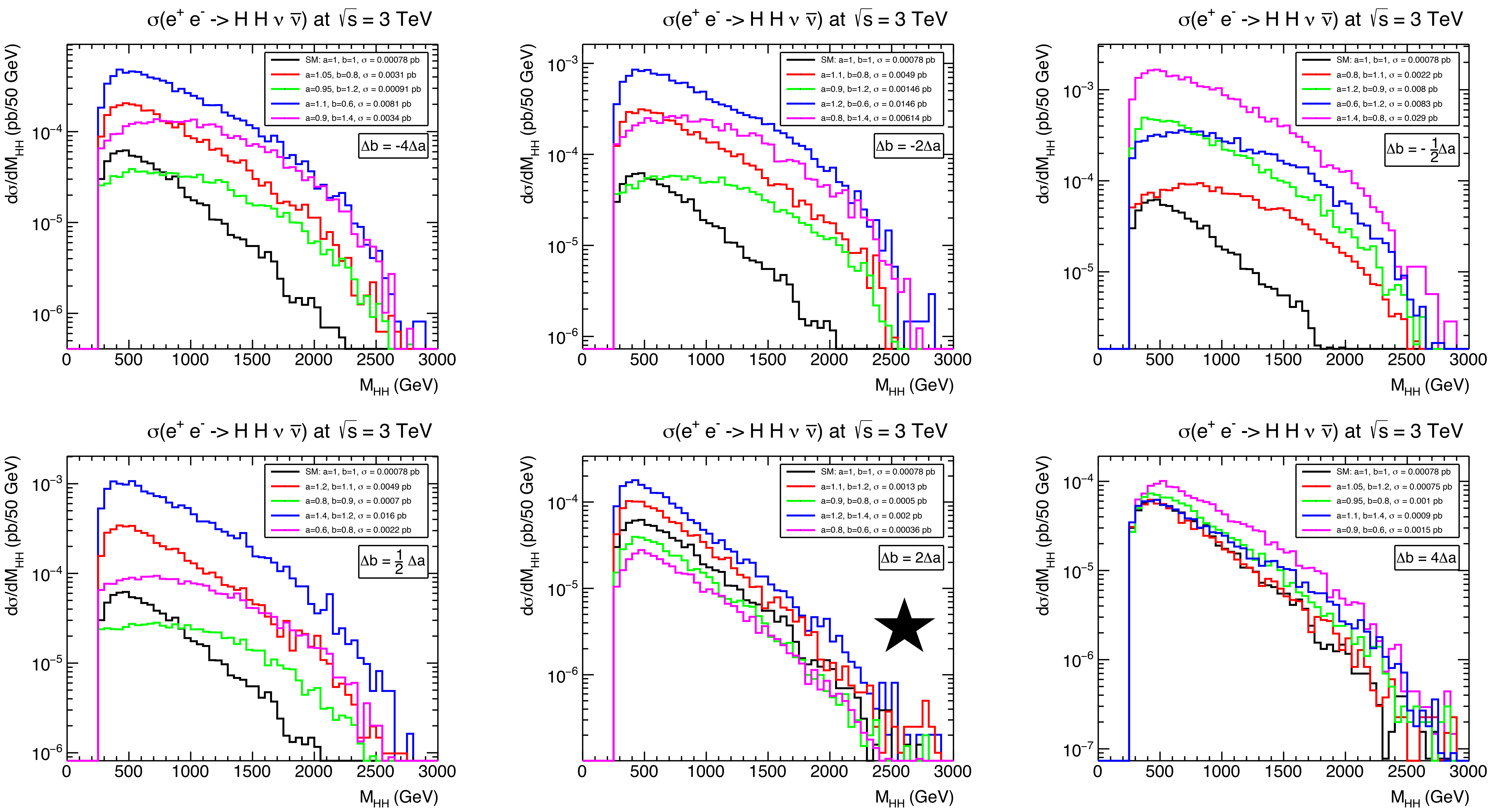
Exploring correlations (κ_V, κ_{2V}) at $e^+e^- \to HH\nu\bar{\nu}$ in $d\sigma/dM_{HH}$

$e^+e^-(3\,\text{TeV})$

Dávila, Domenech, Herrero, Morales [2312.03877] EPJC 84 (2024)5, 503

In general going BSM with $\kappa_{2V} \neq 1$; $\kappa_V \neq 1$ distorts the dist. in M_{HH} producing bumps,

Except close to $\kappa_{2V} = \kappa_V^2$ ★



Maximum sensitivity if

$\kappa_{2V} \neq \kappa_V^2$

Minimum sensitivity if

$\kappa_{2V} = \kappa_V^2$

$\Delta\kappa_{2V} = 2\Delta\kappa_V$

$(\Delta b = 2\Delta a)$

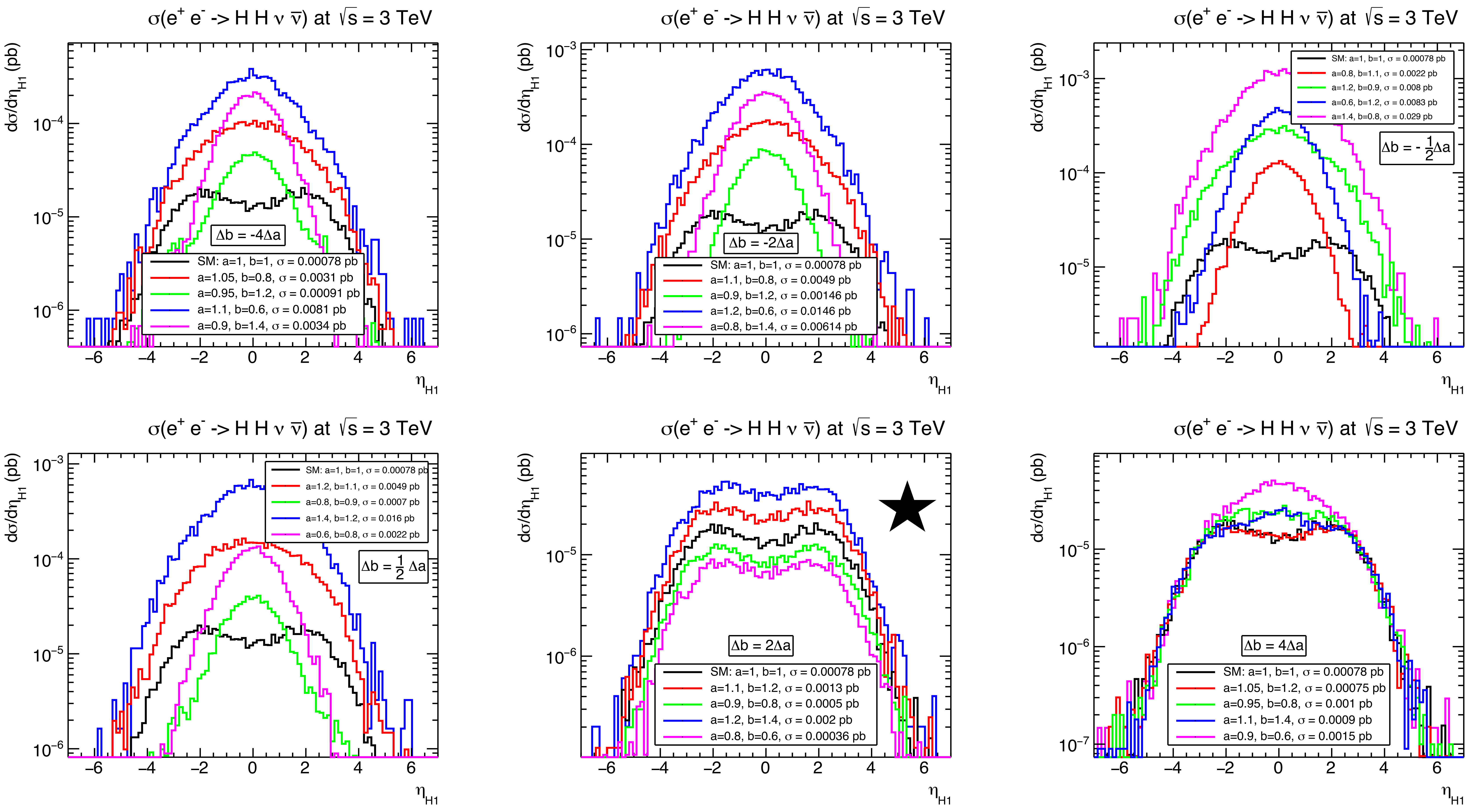
larger $(\kappa_V^2 - \kappa_{2V}) \neq 0 \to$ bigger bumps

Exploring correlations (κ_V, κ_{2V}) at $e^+e^- \to HH\nu\bar{\nu}$ in $d\sigma/d\eta_H$

In general going BSM with $\kappa_{2V} \neq 1$; $\kappa_V \neq 1$ distorts the dist. in η_H producing peaks at $\eta_H = 0$

Except close to $\kappa_{2V} = \kappa_V^2$ ★

$e^+e^-(3\,\text{TeV})$



Maximum sensitivity if

$$\kappa_{2V} \neq \kappa_V^2$$

Minimum sensitivity if

$$\kappa_{2V} = \kappa_V^2$$

Very characteristic BSM events with $(\kappa_V^2 - \kappa_{2V}) \neq 0$

larger $(\kappa_V^2 - \kappa_{2V}) \neq 0 \to$ higher peaks $\to$ more transverse Higgs !!!

Dávila, Domenech, Herrero, Morales [2312.03877] EPJC (2024)

Exploring correlations (κ_V, κ_{2V}) at $pp \to HHj_1j_2$ in $d\sigma/d\eta_H$

LHC (14TeV)

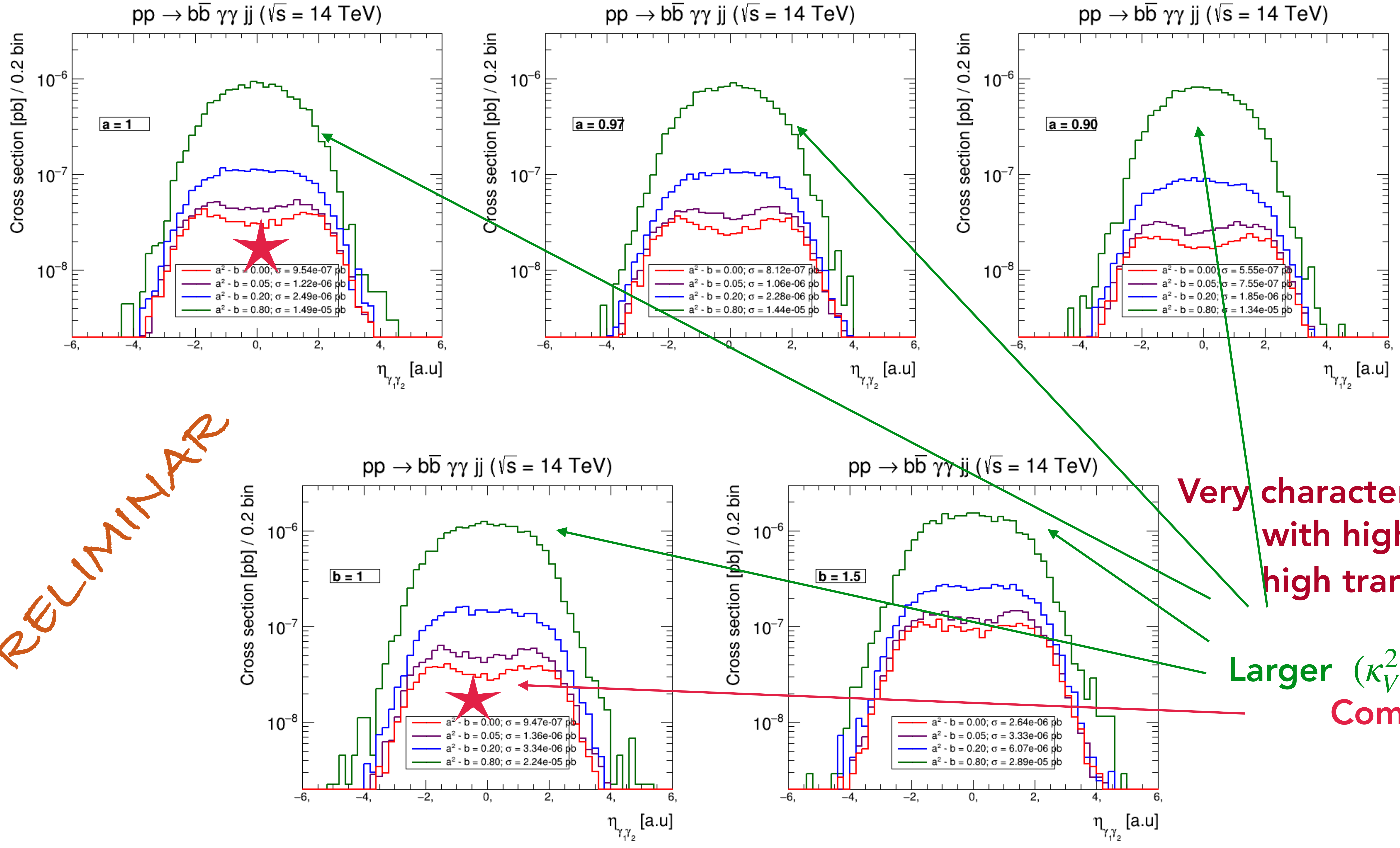
$P_{Tj_i}^{min} = 20$ GeV, $\eta_{j_1} \times \eta_{j_2} < 0$ $\Delta R_{jj}^{min} = 0.4$.

$2 \leq |\eta_{j_i}| \leq 5$, $M_{jj}^{min} = 500$ GeV

WBF jets !!

decays considered: $HH \to \gamma\gamma b\bar{b}$

PRELIMINAR



Very characteristic BSM events with $(\kappa_V^2 - \kappa_{2V}) \neq 0$ with high transversality in final H, hence high transversality in $(\gamma\gamma)$ and $(b\bar{b})$ pairs

Larger $(\kappa_V^2 - \kappa_{2V})$ give more transverse $(\gamma\gamma)$

Compared to SM typical shape ★

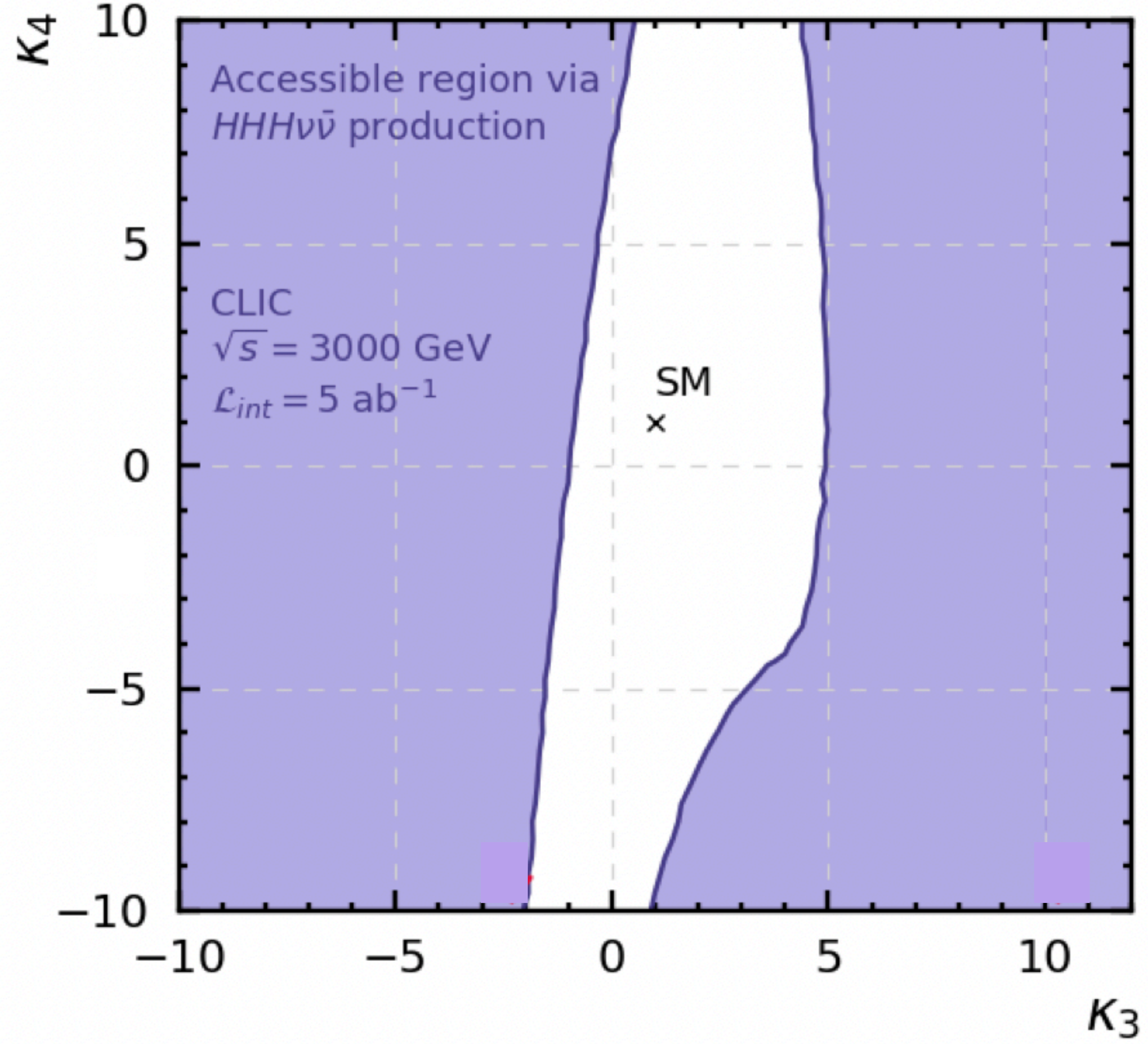
Cepeda, Domenech, Garcia-Mir, Herrero (Work in progress)

It looks promising : now we are including pythia and Delphes for a more realistic simulation

Access to (κ_3, κ_4) in $e^+e^- \to HHH\nu\bar{\nu} \to 6b\text{jets} + E_T^{miss}$

2011.13195, EPJC 81 (2021)3, 260, González-López, Herrero, Martínez-Suárez

(cases with at least 10 events after applying all cuts)



$$e^+e^- \to 6b + E_T^{mis}$$

10 events required for accessibility

At least 5 btagged jets $\epsilon_b = 0.8$

✓ $p_T^j > 20$ GeV ✓ $N_j \geq 6$

✓ $|\eta^j| < 2.72$ ✓ $E_T^{mis} > 20$ GeV

The best expectations are for CLIC (3 TeV) where

BSM/SM $\gtrsim 10$ for $\kappa_3 \gtrsim 2$ $(\kappa_4 = 1)$

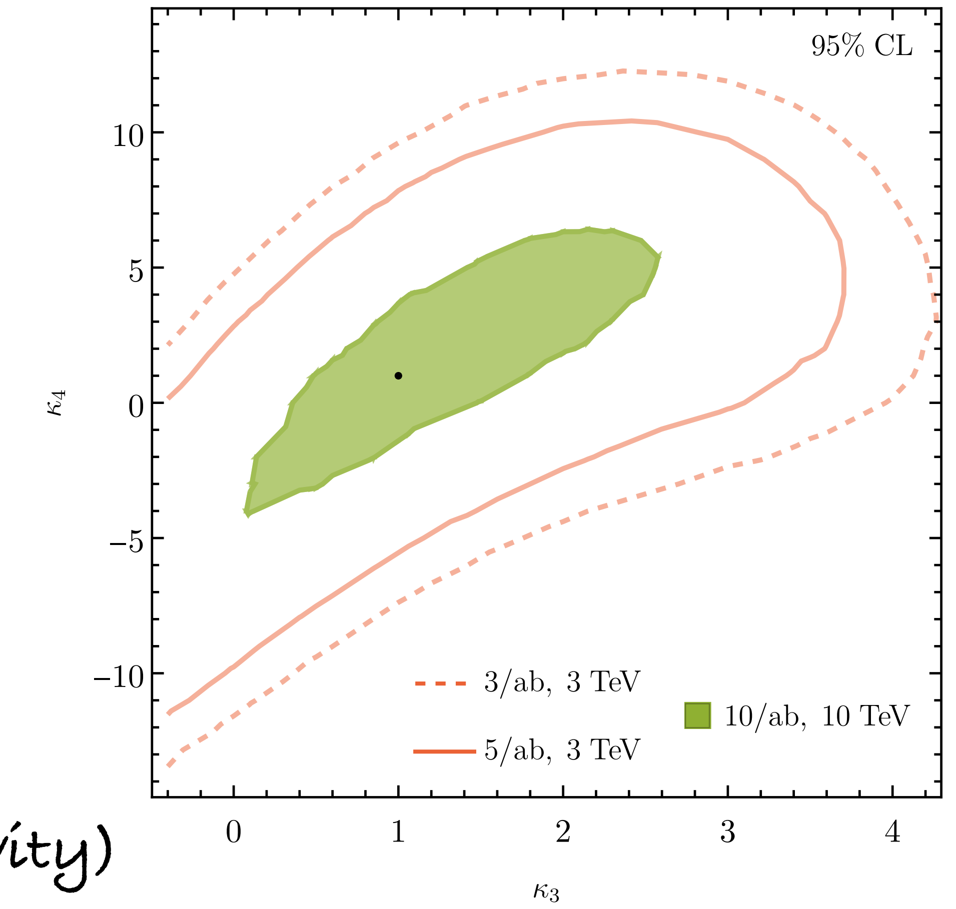
BSM/SM $\gtrsim 10$ for $\kappa_4 \gtrsim 4$ $(\kappa_3 = 1)$

$\sigma^{SM}(e^+e^- \to HHH\nu\bar{\nu})\,(3\,\text{TeV}) = 3 \times 10^{-7}\,\text{pb}$

Sensitivity at CLIC to both κ_3 and κ_4

Other studies of 5b's at CLIC

2312.04646 (Stylianou, Weiglein)



A recent study (more sophisticated and precise than ours) Is in agreement with our previous sensitivities found, solid red contours: reach at CLIC, $\kappa_3 \sim 3.5$, $\kappa_4 \sim 10$

Also compared with HL-LHC $3\,\text{ab}^{-1}$ (giving poorer sensitivity)

$\kappa_4 \sim 60$ already in the non-perturbative regime

Conclusions:
- Future expected sensitivity to κ_4 yet poor
- much higher sensitivity to κ_3 expected

NLO-HEFT Higgs operators involved in (EW) HH production

$$\mathscr{L}^{\rm NLO}_{\rm HEFT} = \ldots -a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\,\partial^\nu H}{v^2}\mathrm{Tr}\Big[\mathcal{V}_\mu\mathcal{V}_\nu\Big] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\,\partial_\mu H}{v^2}\mathrm{Tr}\Big[\mathcal{V}^\nu\mathcal{V}_\nu\Big] + a_{11}\mathrm{Tr}\Big[\mathcal{D}_\mu\mathcal{V}^\mu\mathcal{D}_\nu\mathcal{V}^\nu\Big]$$
$$-\frac{m_{\rm H}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v} + a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right)\mathrm{Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big]$$
$$-\left(a_{HWW}\frac{H}{v} + a_{HHWW}\frac{H^2}{v^2}\right)\mathrm{Tr}\Big[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Big] + i\left(a_{d2} + a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}\mathrm{Tr}\Big[\hat{W}_{\mu\nu}\mathcal{V}^\mu\Big]$$
$$+\left(a_{\Box\mathcal{V}\mathcal{V}} + a_{H\Box\mathcal{V}\mathcal{V}}\frac{H}{v}\right)\frac{\Box H}{v}\mathrm{Tr}\Big[\mathcal{V}_\mu\mathcal{V}^\mu\Big] + a_{d3}\frac{\partial^\nu H}{v}\mathrm{Tr}\Big[\mathcal{V}_\nu\mathcal{D}_\mu\mathcal{V}^\mu\Big]$$
$$+\left(a_{\Box\Box} + a_{H\Box\Box}\frac{H}{v}\right)\frac{\Box H\,\Box H}{v^2} + a_{dd\Box}\frac{\partial^\mu H\,\partial_\mu H\,\Box H}{v^3} + a_{Hdd}\frac{m_{\rm H}^2}{v^2}\frac{H}{v}\partial^\mu H\,\partial_\mu H$$

$$\mathcal{V}_\mu = (D_\mu U)U^\dagger$$

e.o.m

$$\Box H = -m_h^2 H - \tfrac{3}{2}\kappa_3 m_h^2\frac{H^2}{v} - \tfrac{a}{2}v\,\mathrm{Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big] - \tfrac{b}{2}H\,\mathrm{Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big]$$
$$\mathrm{Tr}\Big[\tau^j\mathcal{D}_\mu\mathcal{V}^\mu\Big] = -\mathrm{Tr}\Big[\tau^j\mathcal{V}^\mu\Big]\tfrac{2a}{v}\partial_\mu H$$

Full operators list given in the literature (see, for instance, Brivio et al 1311.1823)

summarized by: $a_{dd\mathcal{V}\mathcal{V}1} \leftrightarrow c_8$, $a_{dd\mathcal{V}\mathcal{V}2} \leftrightarrow c_{20}$, $a_{11} \leftrightarrow c_9$, $a_{HWW} \leftrightarrow a_W$, $a_{HHWW} \leftrightarrow b_W$, $a_{d2} \leftrightarrow c_5$, $a_{Hd2} \leftrightarrow a_5$, $a_{\Box\mathcal{V}\mathcal{V}} \leftrightarrow c_7$, $a_{H\Box\mathcal{V}\mathcal{V}} \leftrightarrow a_7$, $a_{d3} \leftrightarrow c_{10}$, $a_{Hd3} \leftrightarrow a_{10}$, $a_{\Box\Box} \leftrightarrow c_{\Box H}$, $a_{H\Box\Box} \leftrightarrow a_{\Box H}$, $a_{dd\Box} \leftrightarrow c_{\Delta H}$, $a_{H\mathcal{V}\mathcal{V}} \leftrightarrow a_C$ and $a_{HH\mathcal{V}\mathcal{V}} \leftrightarrow b_C$.

$$\mathscr{L}^{\rm NLO+e.o.m}_{\rm HEFT} = \ldots -a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\,\partial^\nu H}{v^2}\mathrm{Tr}\Big[\mathcal{V}_\mu\mathcal{V}_\nu\Big] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\,\partial_\mu H}{v^2}\mathrm{Tr}\Big[\mathcal{V}^\nu\mathcal{V}_\nu\Big]$$
$$-\frac{m_{\rm H}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v} + a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right)\mathrm{Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big] + a_{Hdd}\frac{m_{\rm H}^2}{v^2}\frac{H}{v}\partial^\mu H\,\partial_\mu H$$
$$-\left(a_{HWW}\frac{H}{v} + a_{HHWW}\frac{H^2}{v^2}\right)\mathrm{Tr}\Big[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Big] + i\left(a_{d2} + a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}\mathrm{Tr}\Big[\hat{W}_{\mu\nu}\mathcal{V}^\mu\Big]$$

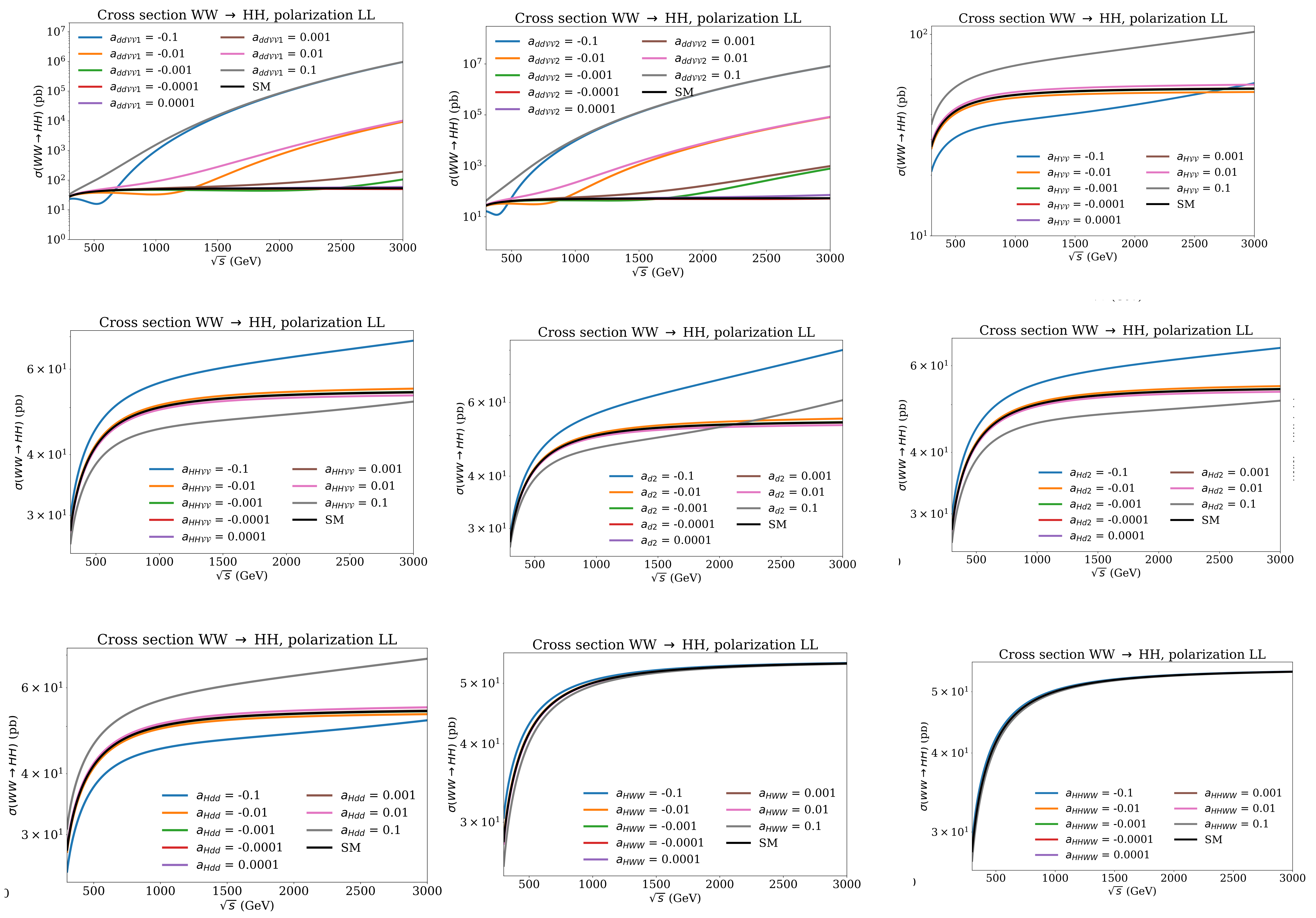
Reduction to 9 $a_i's$ NLO coefficients entering into $WW \to HH$

The most relevant are $a_{ddVV1} \equiv \eta$, $a_{ddVV2} \equiv \delta$

Comparing the relevance of the various NLO $a_i's$ at $WW \to HH$

$a_{ddVV1} \equiv \eta$ $\quad$ $a_{ddVV2} \equiv \delta$

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos



For similar size of the $a_i's$
we find the largest xsections for

$a_{ddVV1} \equiv \eta$, $a_{ddVV2} \equiv \delta$

by several orders of magnitud !!

At energies in the relevant interval

$$\sqrt{s} \in (500{,}3000)\ (\mathrm{GeV})$$

Due to dominance of LL polarizations

NLO: faster growth $\quad A \sim \mathcal{O}(s^2)$

than LO: $\quad A \sim \mathcal{O}(s)$

$$a_{ddVV1}(1/v^2)\, \partial^\mu H \partial^\nu H \mathrm{Tr}\,[(D_\mu U^+)\,(D_\nu U)] + a_{ddVV2}(1/v^2)\, \partial^\mu H \partial_\mu H \mathrm{Tr}\,[(D^\nu U^+)\,(D_\nu U)]$$

Largest deviations respect to SM in LL modes
Relevance of (η, δ) easily understood in ET

Accessibility to NLO-HEFT (η, δ) at e^+e^- (4b+ETmiss)

Minimal detection cuts

$p_T^b > 20$ GeV $|\eta^b| < 2$

$\Delta R_{bb} > 0.4$ $\not{E}_T > 20$ GeV

b-tagging efficiency of 80%

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos

Signal with greater statistics: $e^+e^- \to HH\nu\bar{\nu} \to b\bar{b}b\bar{b}\nu\bar{\nu}$

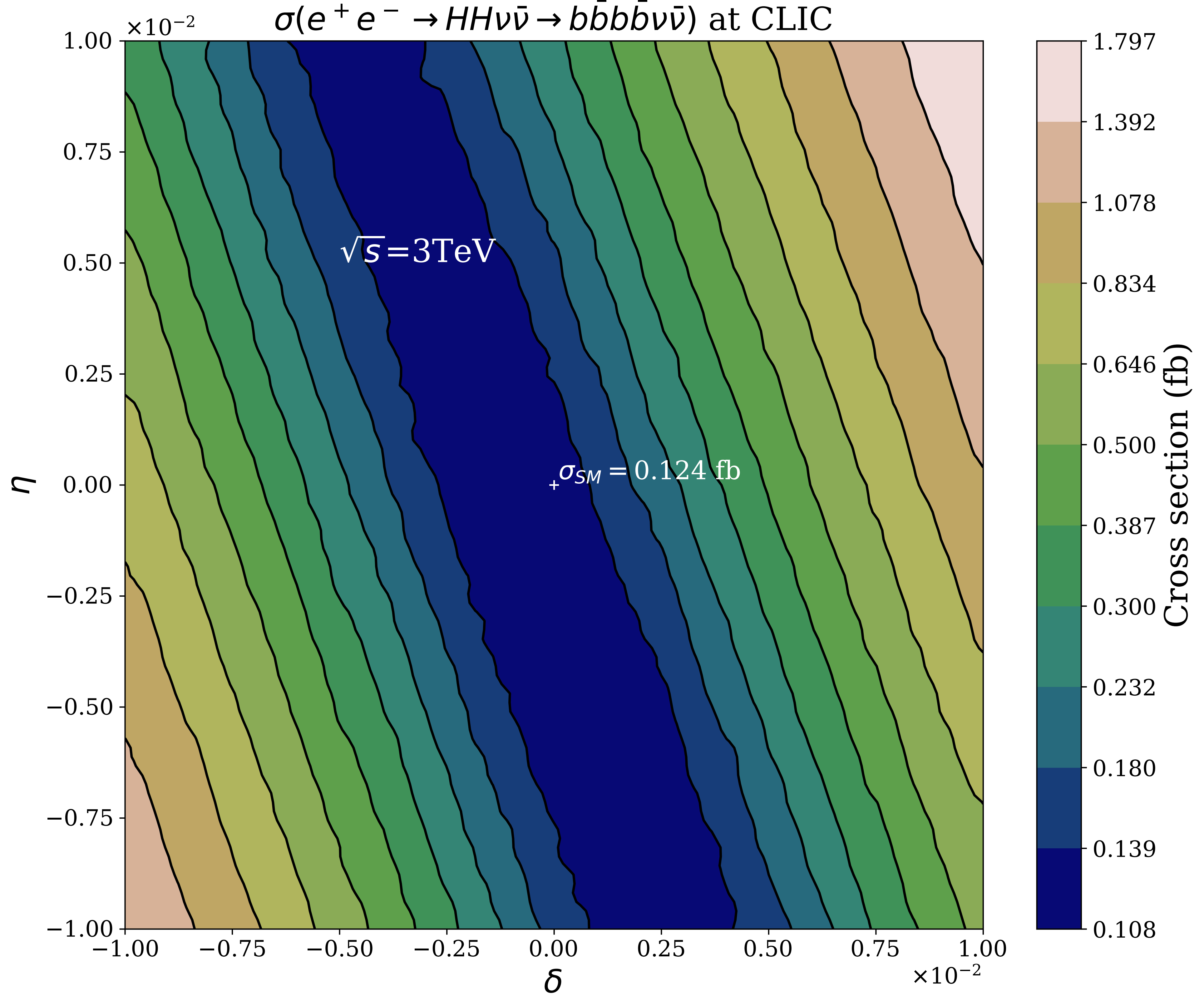
Accessibility parameter

$$R = \frac{N_{BSM} - N_{SM}}{\sqrt{N_{SM}}}$$

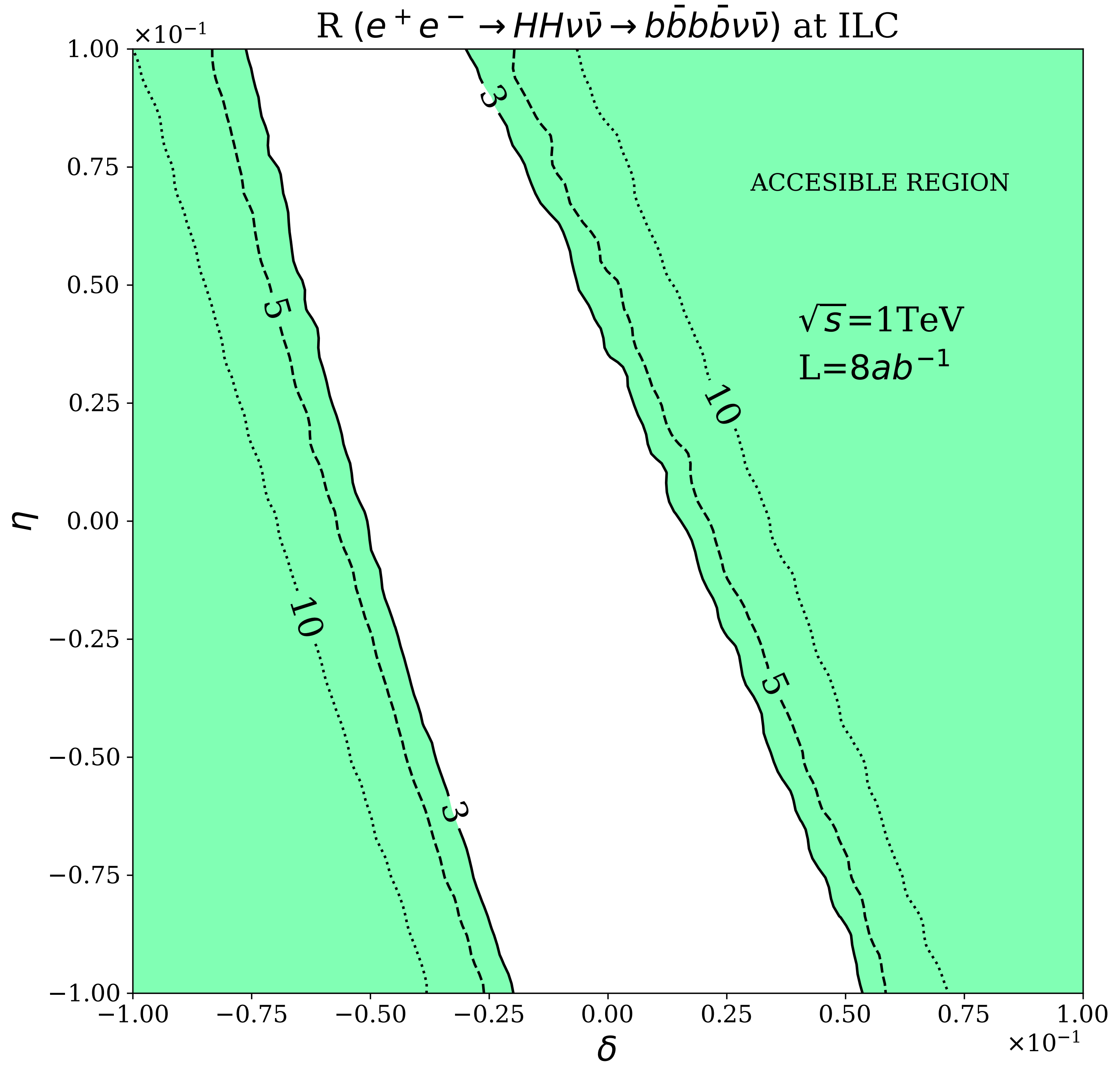
Accesible region: R > 3

Greater accessibility in CLIC (3TeV)

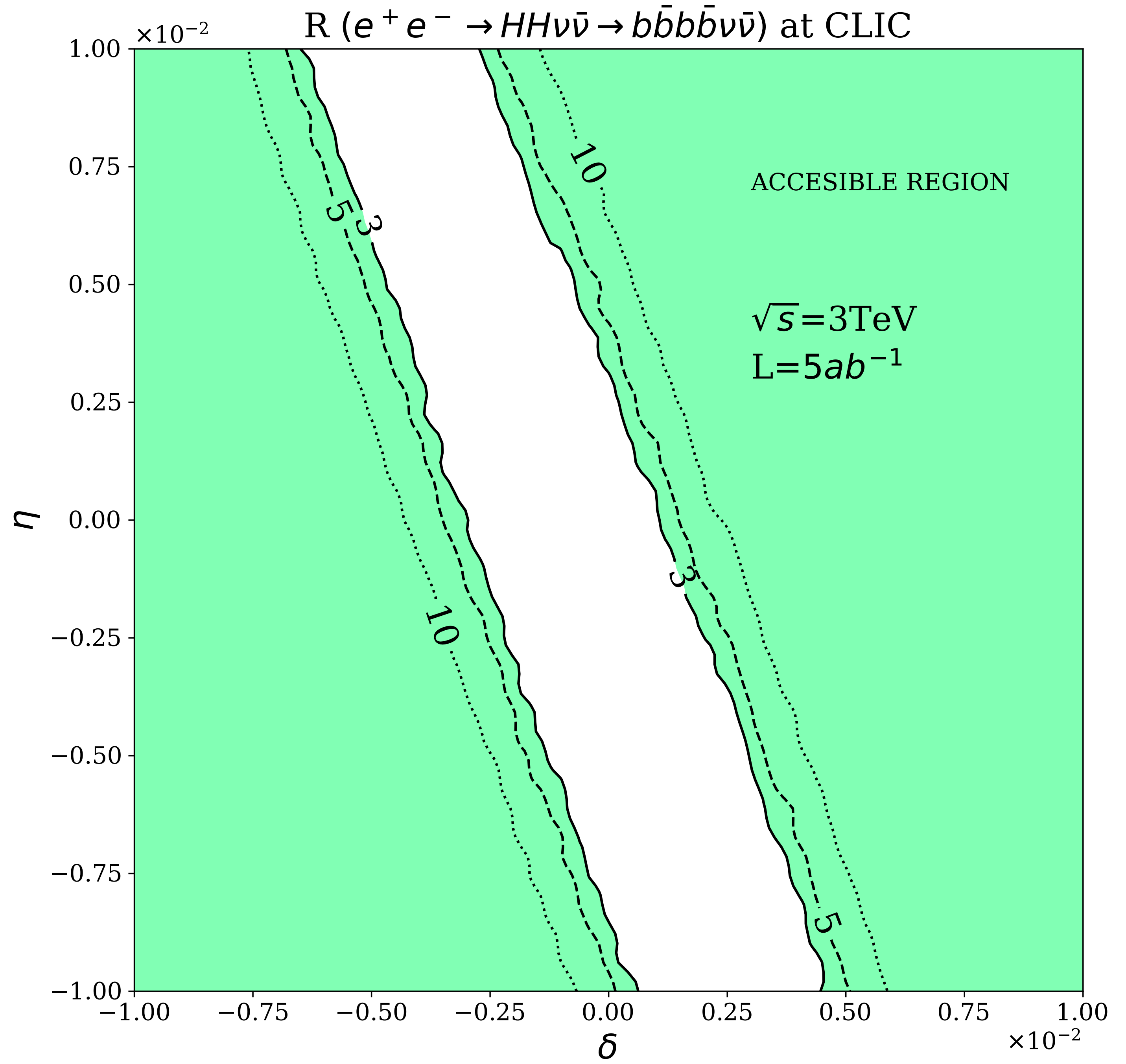
Expected reach $\eta, \delta \sim \mathcal{O}(10^{-3})$



ILC



CLIC



Similar work in progress for HL-LHC via $pp \to HHjj \to \gamma\gamma b\bar{b}jj$
Preliminar: BSM expected reach in this channel $\eta, \delta \lesssim \mathcal{O}(10^{-2})$

Including radiative corrections within bosonic-HEFT

★ Developed a practical program to include one-loop HEFT radiative corrections via insertions of 1PI's

★ Easy to implement in physical scattering procceses

★ Based on computation of one-loop FDs (graphical/intuitive) easy to implement with usual tools FeynRules, FormCalc, Looptools etc..

★ Renormalization of the involved 1PI Green functions in generic R_ξ gauges, with generic off-shell legs (renormalization of the Lagrangian is not enough, running Wilson coffs. is not enough)

★ Master equation for renormalized 1PI function within NLO HEFT

$$\hat{\Gamma}^{\mathrm{NLO}} = \Gamma^{\mathrm{LO}} + \Gamma^{a_i} + \Gamma^{\mathrm{Loop}} + \Gamma^{\mathrm{CT}}$$

$\hat{\Gamma}^{\mathrm{NLO}}$: Finite for all external (off-shell) momenta

Better not to use e.o.m, all operators needed

We use renorm. conditions: OS for W, Z,H…, MSbar for HEFT coefficients

Γ^{LO}: From $\mathscr{L}^{\mathrm{LO}}$ FRs $a, b, \kappa_3, \kappa_4, \ldots$

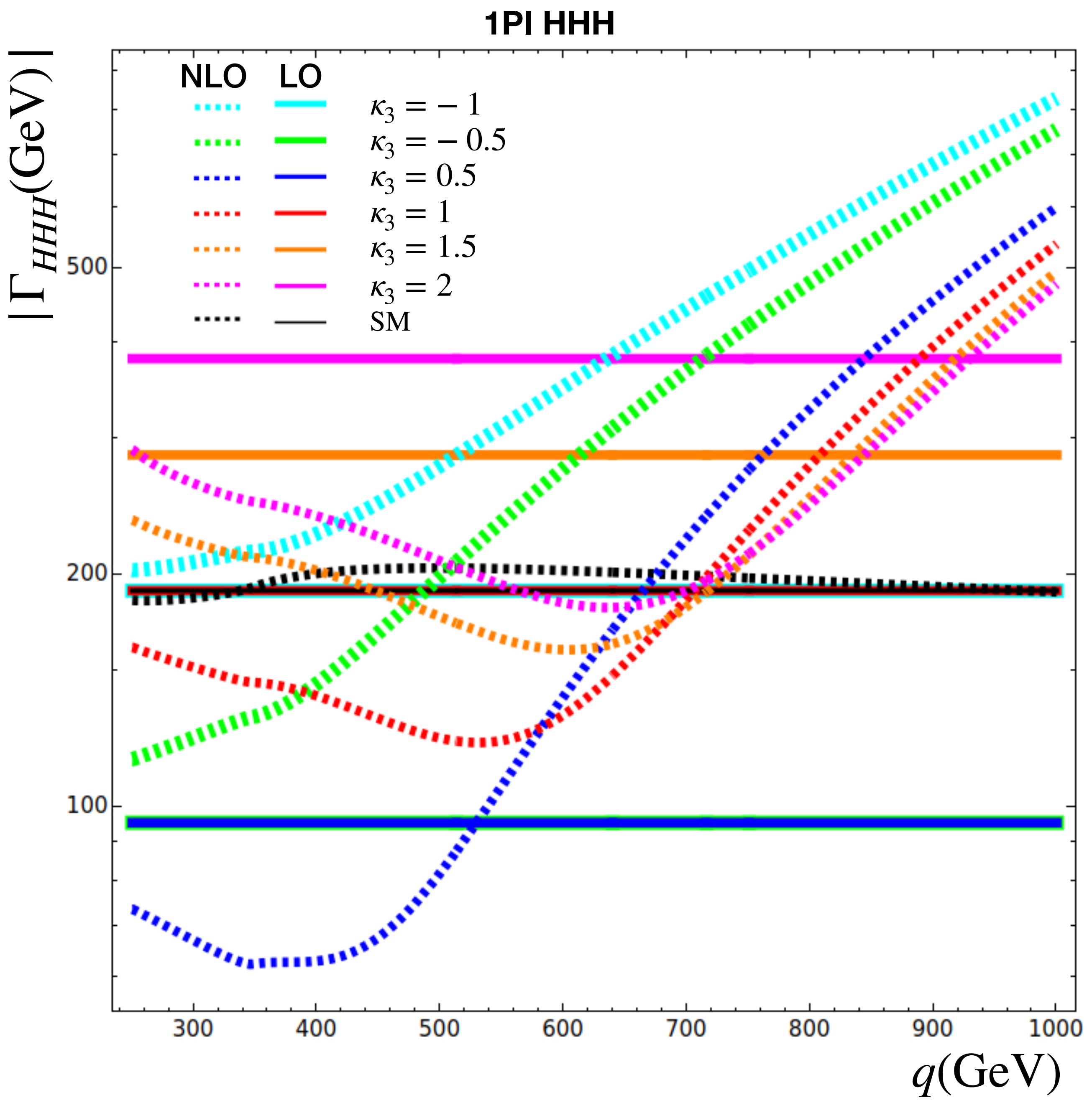
Γ^{a_i}: From $\mathscr{L}^{\mathrm{NLO}}$ FRs $a_i \to a_i + \delta a_i$ needed as new CTs to cancel new divergences from loops

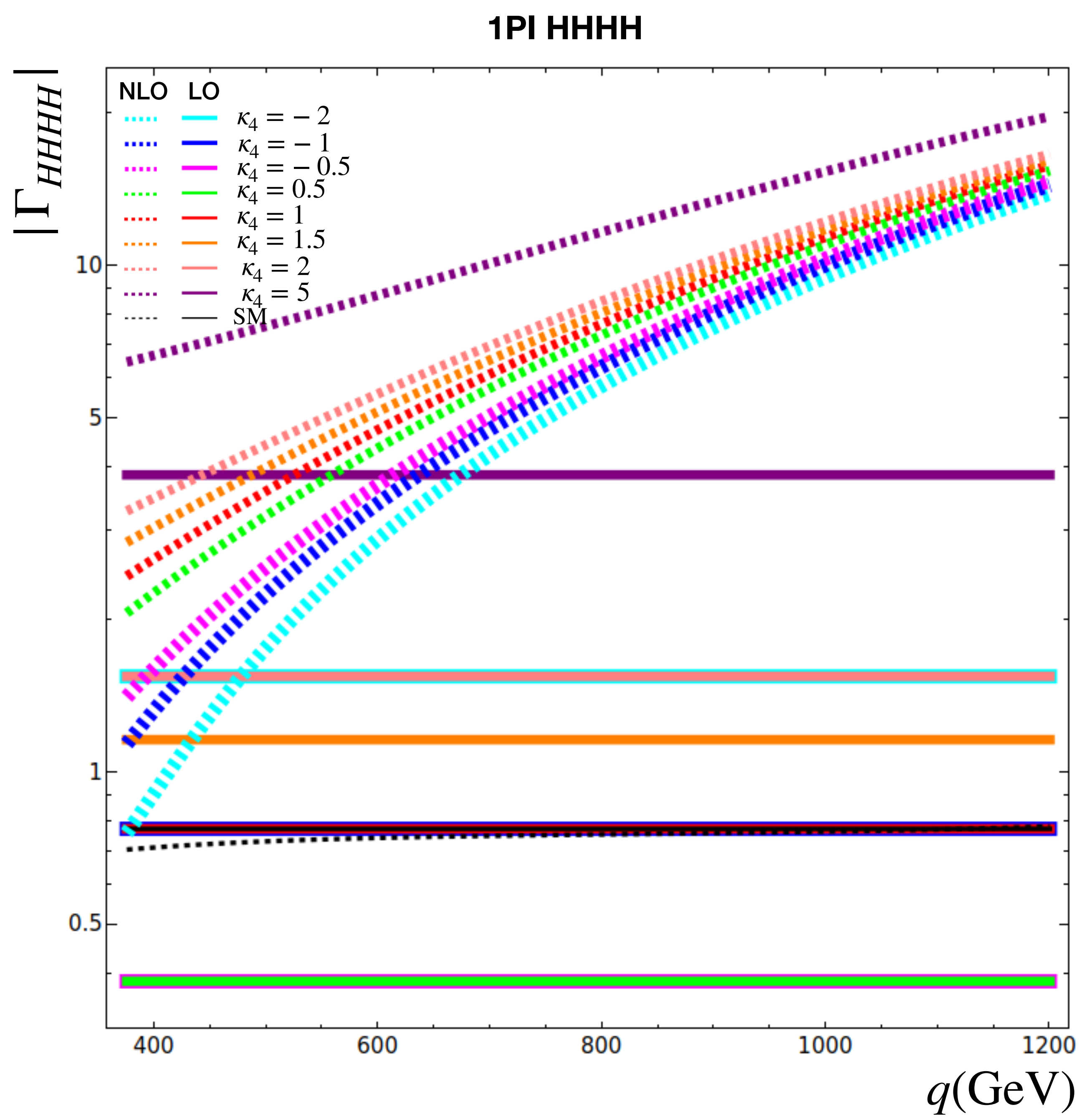
Γ^{Loop}: From loop diagrams computed with $\mathscr{L}^{\mathrm{LO}}$ FRs

Γ^{CT}: From $\mathscr{L}^{\mathrm{LO}}$ counterterms $\delta Z_{W,Z,H..}\, \delta g, \delta g', \delta a, \delta b, \delta\kappa_3, \delta\kappa_4 \ldots$

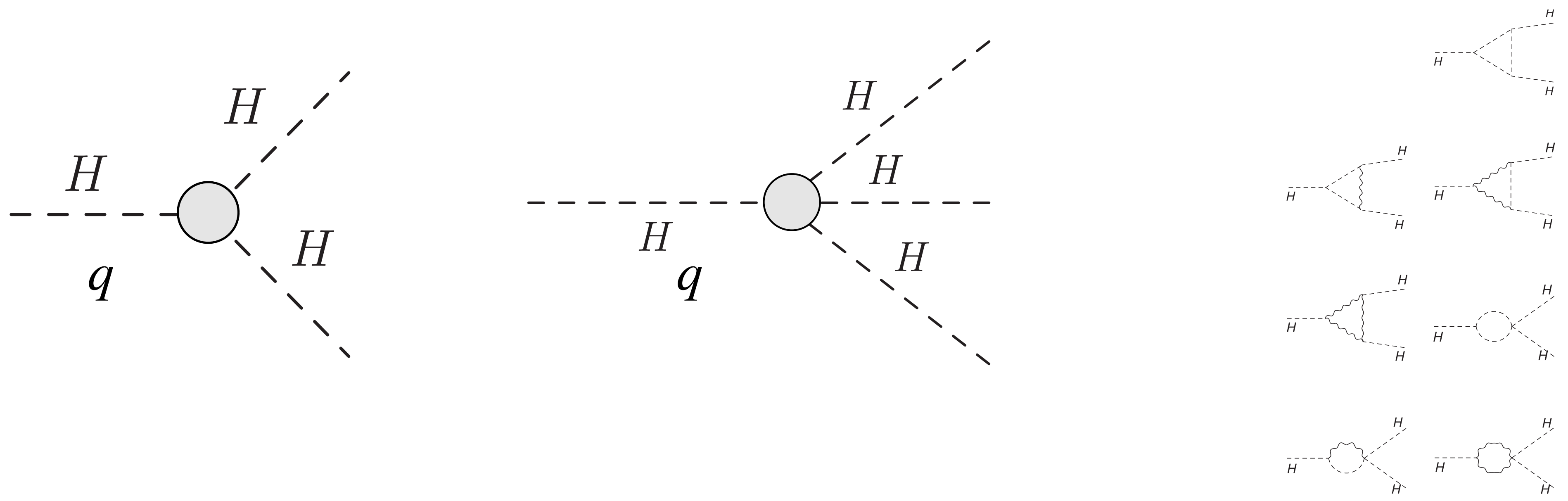
Several examples: 2005.03537 and 2208.09334 (H decays), 2107.07890 (WZ to WZ), 2208.09334 (WW to HH) 2405.05385 (gg to HH, gg to HHH)

Radiative corrections in 1PIs: the case of Γ_{HHH} and Γ_{HHHH}





In general, departures respect to the SM grow with offshellness q



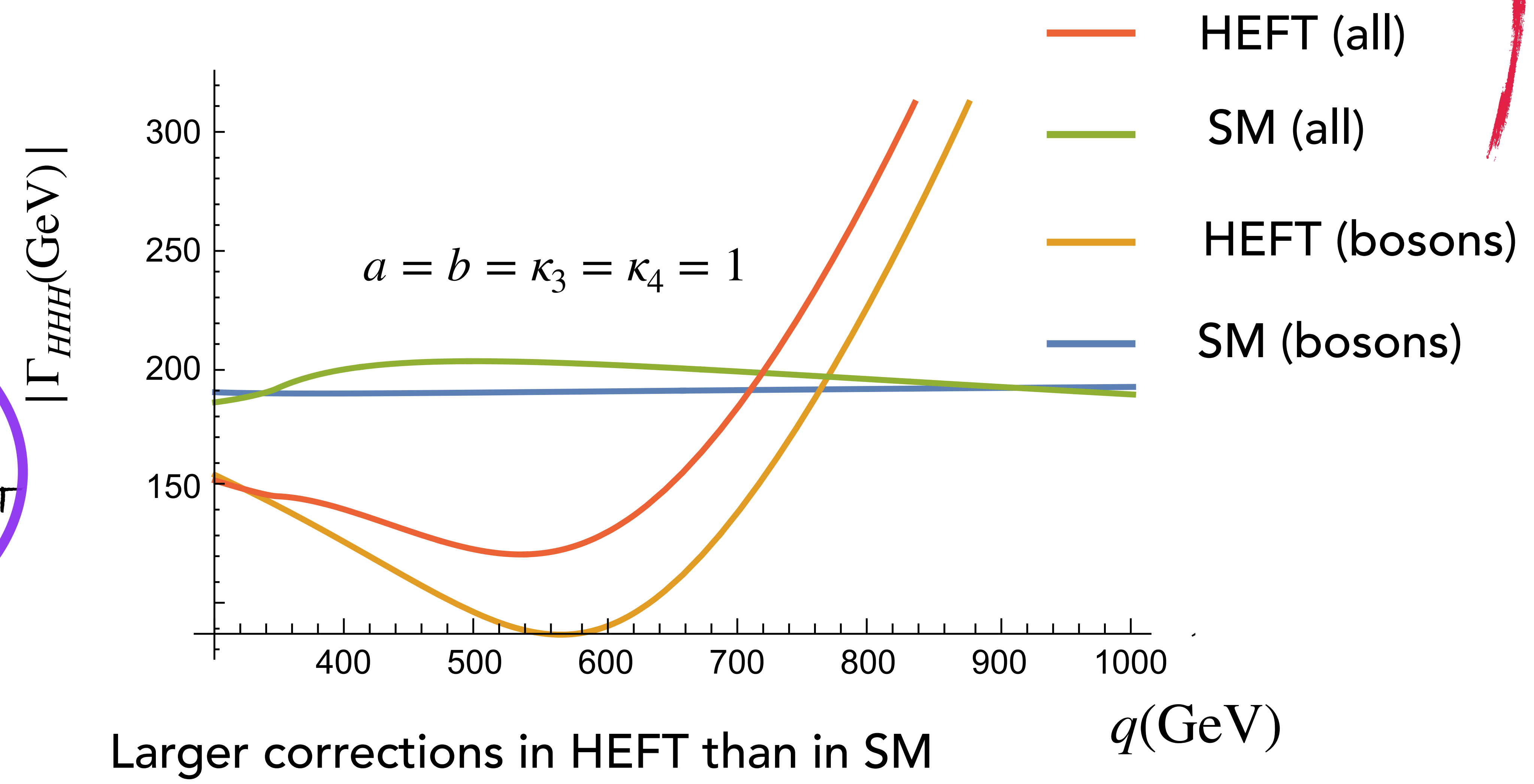
Loops of bosons and tops included (Feynman gauge)

Loops of bosons clearly dominate if fermions assumed SM like

The size of the corrections can be large at large q

$|\Gamma^{NLO}_{HHH}|/|\Gamma^{LO}_{HHH}|$			$|\Gamma^{NLO}_{HHHH}|/|\Gamma^{LO}_{HHHH}|$		
κ_3	$q = 251$ GeV	$q = 1000$ GeV	κ_4	$q = 376$ GeV	$q = 1000$ GeV
-1	1.1	4.4	-2	0.49	6.2
-0.5	1.2	7.9	-1	1.5	13
0.5	0.77	6.3	-0.5	3.7	27
1	0.84	2.8	0.5	5.4	29
1.5	0.82	1.7	1	3.2	15
2	0.76	1.3	1.5	2.5	10
			2	2.1	7.9
			5	1.7	4.0
SM	0.97	1.0	SM	0.91	0.99

Non-linearity
H-singlet
growing with energy
of interactions within HEFT
are the resons for this



Larger corrections in HEFT than in SM

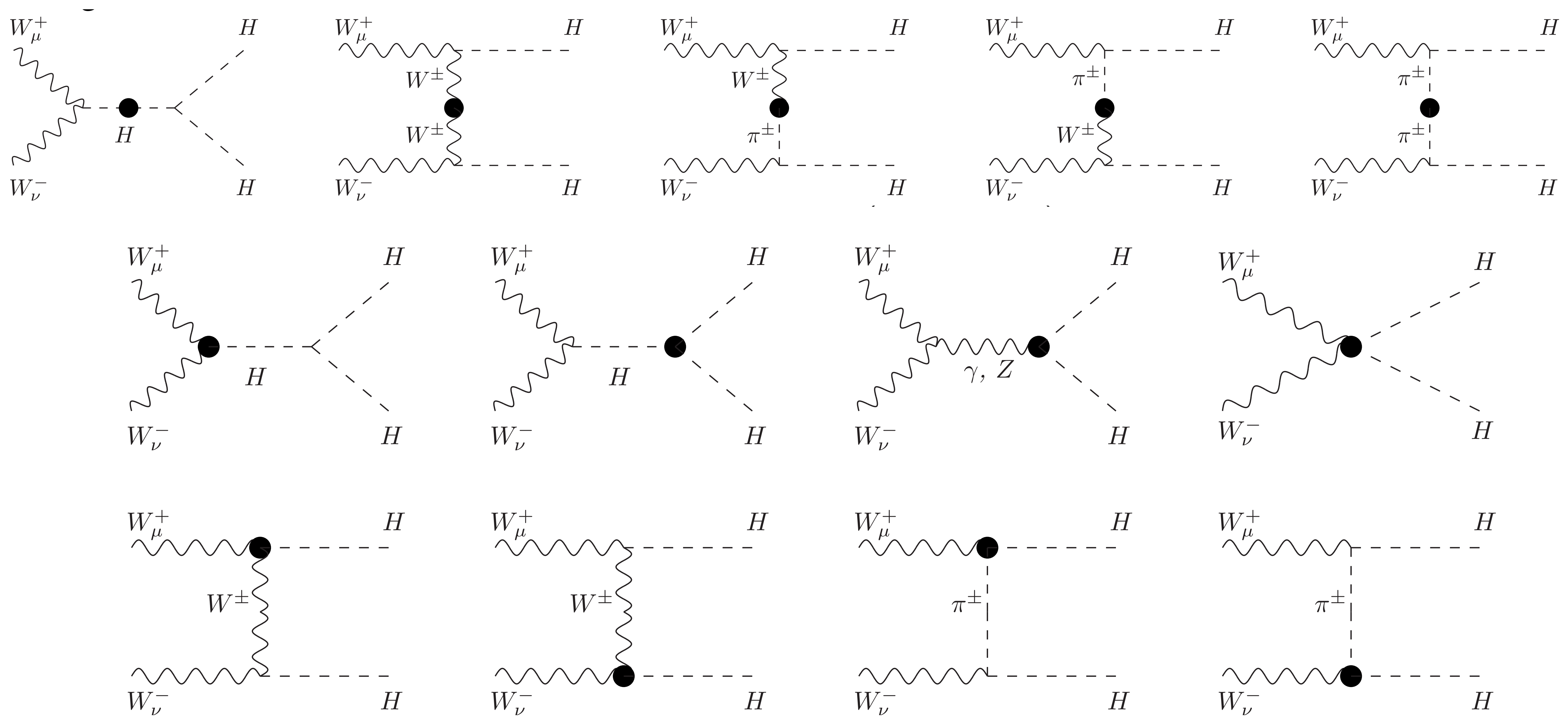
SM corrections almost flat with virtuality q

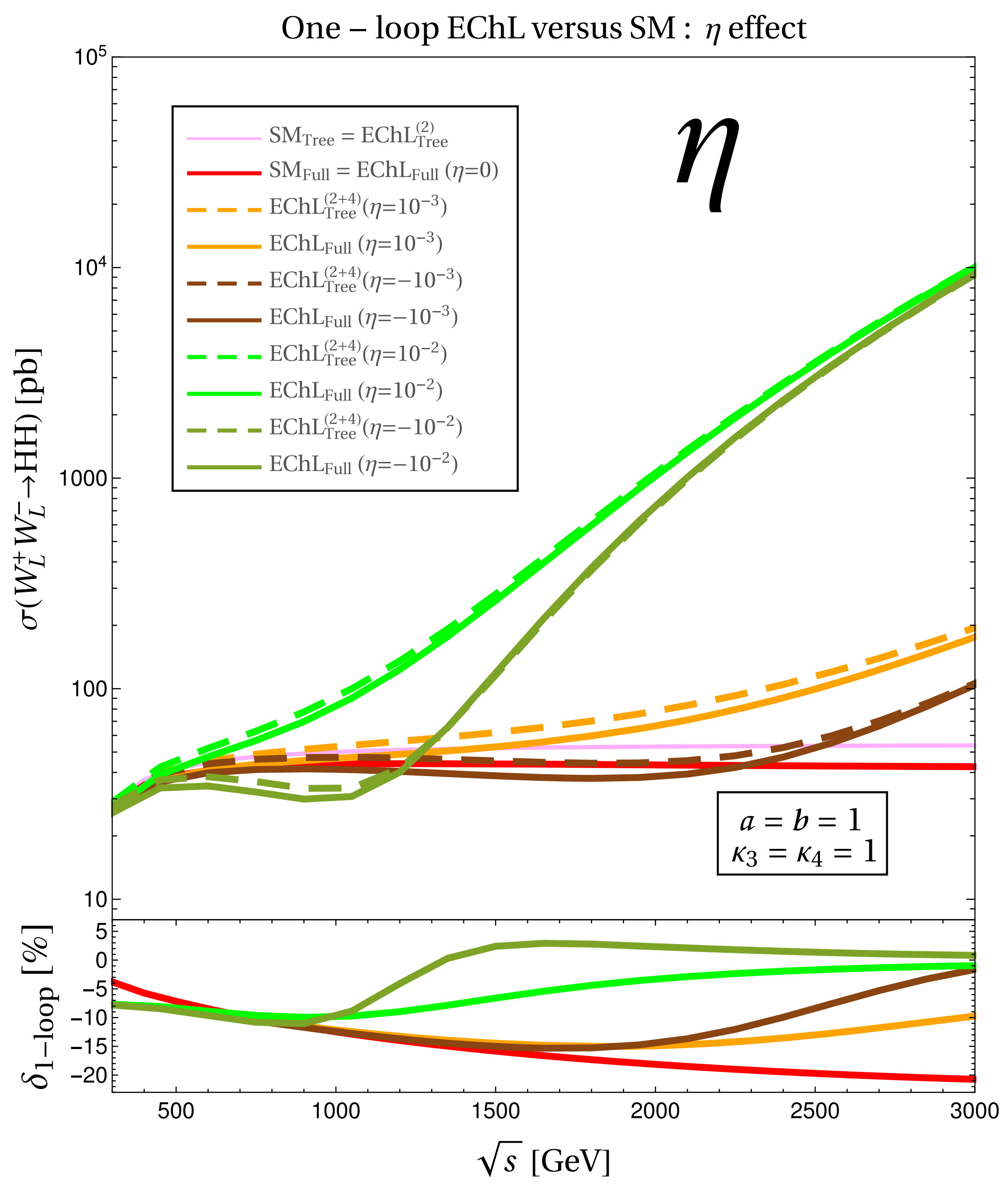
HEFT corrections highly sensitive to virtuality q

Radiative corrections in $WW \to HH$

M.J. Herrero and R.A Morales , PRD106,073008(2022) 2208.05900

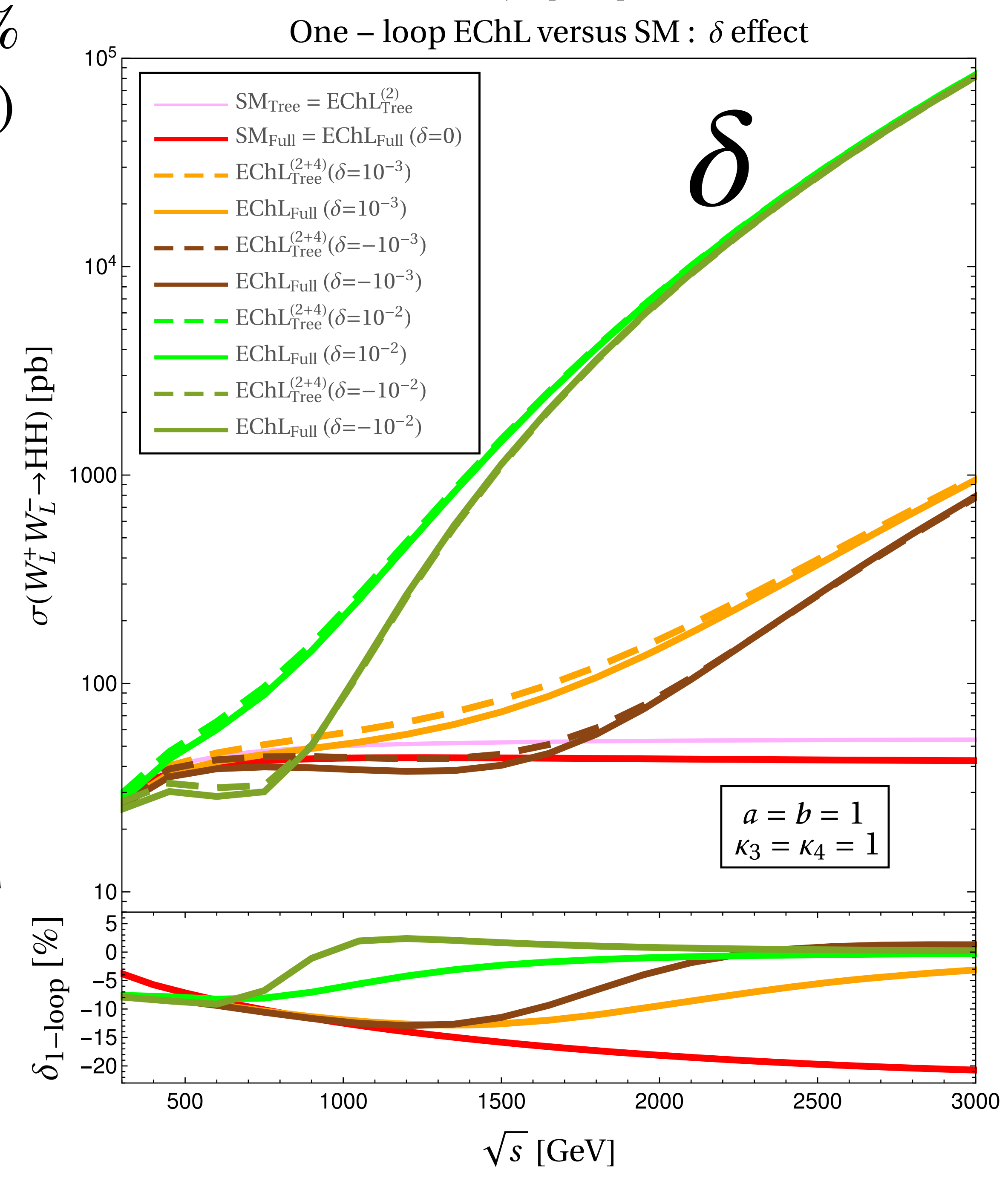
Renormalized one-loop 1PIs $\hat{\Gamma}^{\rm NLO}_{\rm HEFT}$ computed in the R_ξ gauges = black balls inserted in the FDs





Size of $\delta_{1-\rm loop} \in (5, -15)\,\%$ comparable to SM $(-20\,\%)$

But different behaviour with energy



We have extracted all the needed CTs

In particular, all the involved $\delta a_i's$ ξ independence checked

RGEs for all the involved HEFT coefficients derived

We checked some $\delta a_i's$ with previous results in specific limits (pure scalar, isospin limit $m_W = m_Z$) Others were unknown before our work (see paper)

Interesting RGE invariants for $(a^2 = b)$

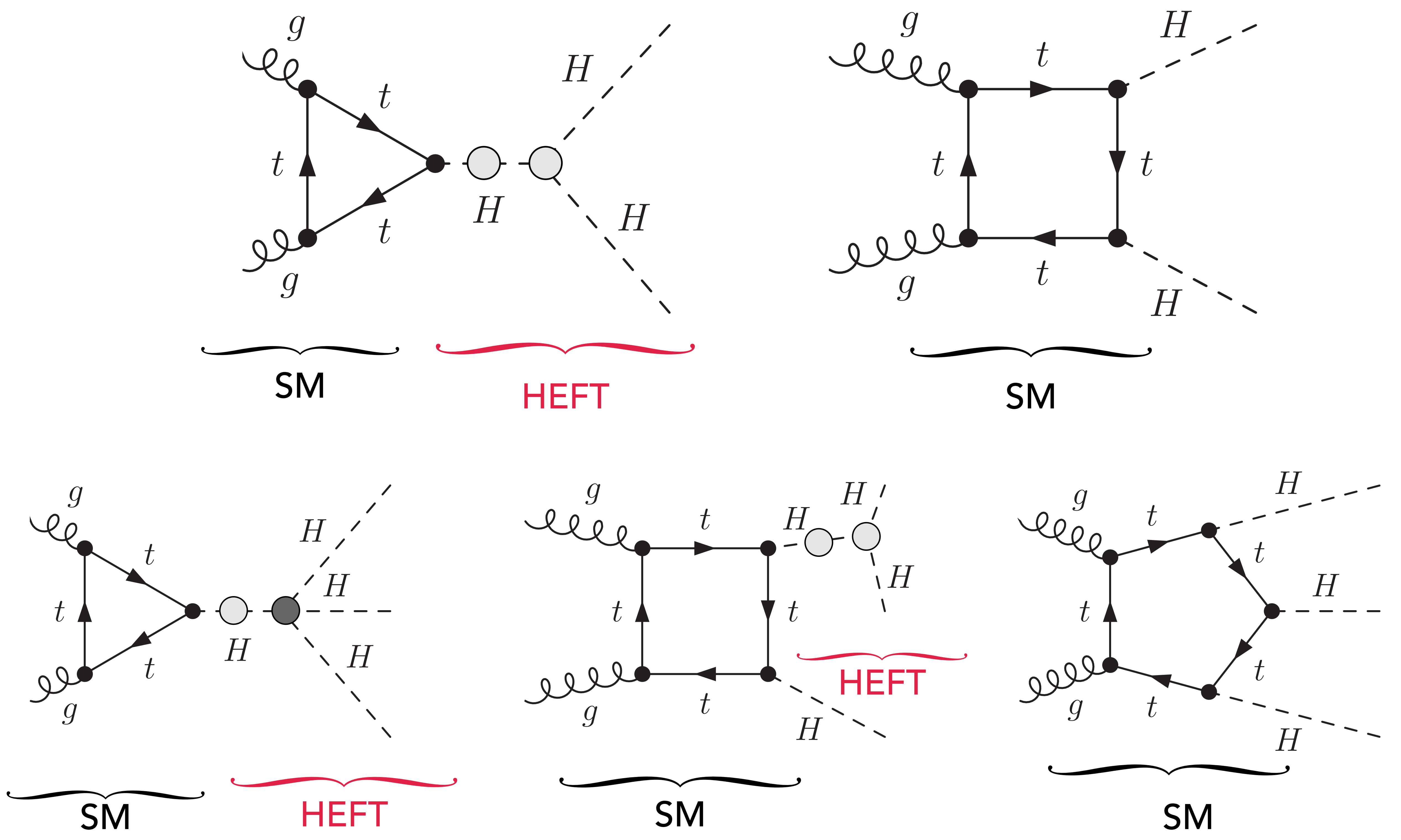
$$\eta(\mu) = \eta(\mu') - \frac{1}{16\pi^2}\frac{1}{3}(a^2-b)^2 \log\left(\frac{\mu^2}{\mu'^2}\right),$$

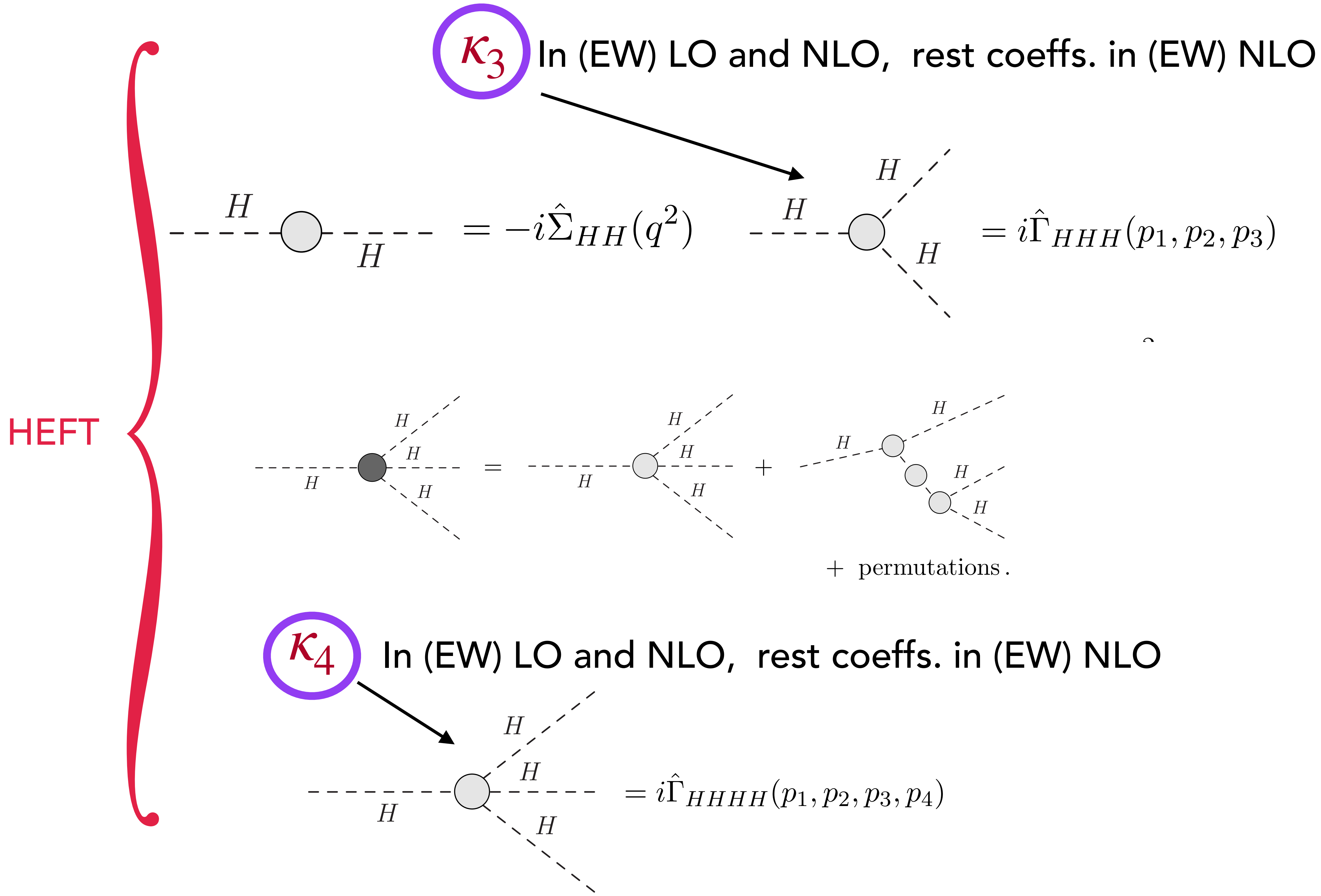
$$\delta(\mu) = \delta(\mu') + \frac{1}{16\pi^2}\frac{1}{12}(a^2-b)(7a^2-b-6)\log\left(\frac{\mu^2}{\mu'^2}\right)$$

(EW) Radiative corrections in $gg \to HH$ and in $gg \to HHH$

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385 (numerical estimates with VBFNLO)

Renormalized one-loop 1PIs $\hat{\Gamma}^{\rm NLO}_{\rm HEFT}$ computed in Feynman 'tHooft gauge = shaded balls inserted in the FDs





The loops in HHH and HHHH vertices and the non-trivial off-shell momenta dependencies produce relevant changes respect to LO

Renormalization of κ_3, κ_4 and of new $a_i's$ also set, RGEs etc

1311.5993,14091571 (pure scalar)
2109.02673 ($m_W = m_Z$)

OK with others in simplified limits

$$\delta_\epsilon \kappa_3 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_{\rm H}^2 v^2}\Big(\kappa_3(a^2 - b + 9\kappa_3^2 - 6\kappa_4)m_{\rm H}^4 - 3(1-a^2)\kappa_3 m_{\rm H}^2(m_{\rm W}^2 + m_{\rm Z}^2) + 6(-2ab + 2a^2\kappa_3 + b\kappa_3)(2m_{\rm W}^4 + m_{\rm Z}^4)\Big)\ ,$$

$$\delta_\epsilon \kappa_4 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_{\rm H}^2 v^2}\Big(\kappa_4(2a^2 - 2b + 9\kappa_3^2 - 6\kappa_4)m_{\rm H}^4 - 6(1-a^2)\kappa_4 m_{\rm H}^2(m_{\rm W}^2 + m_{\rm Z}^2) + 6(-2b^2 + 2a^2\kappa_4 + b\kappa_4)(2m_{\rm W}^4 + m_{\rm Z}^4)\Big)\ ,$$

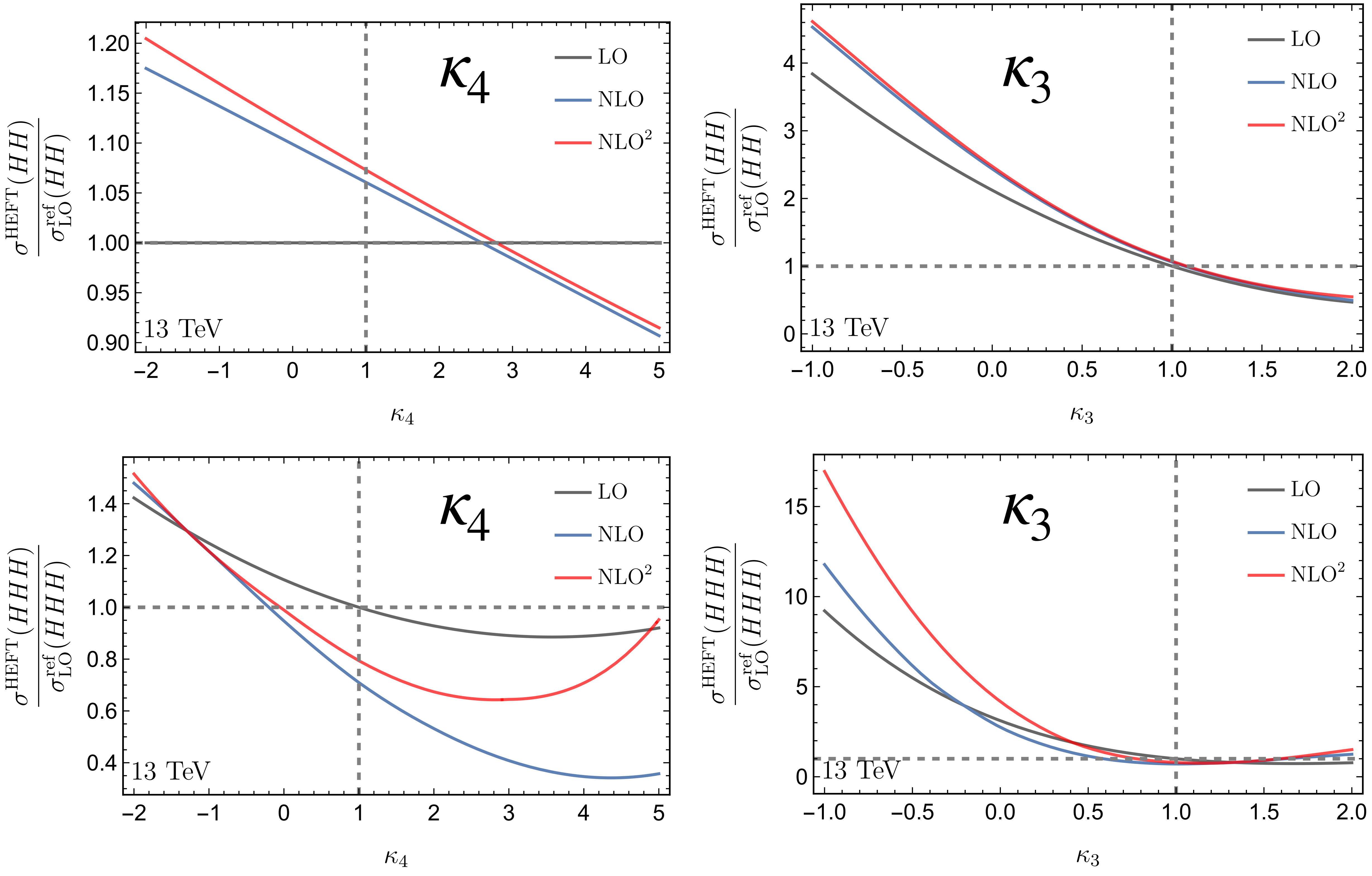
Size of the corrections in $gg \to HH$ and in $gg \to HHH$

Corrections at LHC (13 TeV) cross sections

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385

$gg \to HH$

$gg \to HHH$



$$\sigma^{\mathrm{SM}}_{\mathrm{LO}}(HH) = \sigma^{\mathrm{ref}}_{\mathrm{LO}}(HH) = 17.40\,\mathrm{fb}\,;\, \sigma^{\mathrm{SM}}_{\mathrm{LO}}(HHH) = \sigma^{\mathrm{ref}}_{\mathrm{LO}}(HHH) = 0.041\,\mathrm{fb}$$

All simulations done with BVFNLO

Most important message:
(EW) radiative corrections within NLO-HEFT change
the sensitivity to κ_3 and κ_4 in HH and HHH production at LHC

The most relevant change is in κ_3
For κ_3<0, we find relevant
enhancements in the NLO/LO prediction
$\sigma(HH)$ of $\sim 10\,\%$
and in
$\sigma(HHH)$ of $\sim 30\,\%$ ($\sim 80\,\%$ if NLO^2)

Also large changes in κ_4
For κ_4>0, we find relevant
reductions in the NLO/LO prediction
$\sigma(HHH)$ of $\sim 50\,\%$

Large effects from NLO coefficients

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385

$\mathcal{O}_{\Box\Box}$	$a_{\Box\Box} \frac{\Box H \Box H}{v^2}$	$\mathcal{O}_{H\Box\Box}$	$a_{H\Box\Box} \left(\frac{H}{v}\right) \frac{\Box H \Box H}{v^2}$
$\mathcal{O}_{Hdd}$	$a_{Hdd} \frac{m_H^2}{v^2} \left(\frac{H}{v}\right) \partial^\mu H \partial_\mu H$	$\mathcal{O}_{HHdd}$	$a_{HHdd} \frac{m_H^2}{v^2} \left(\frac{H^2}{v^2}\right) \partial^\mu H \partial_\mu H$
$\mathcal{O}_{ddW}$	$a_{ddW} \frac{m_W^2}{v^2} \left(\frac{H}{v}\right) \partial^\mu H \partial_\mu H$	$\mathcal{O}_{HddW}$	$a_{HddW} \frac{m_W^2}{v^2} \left(\frac{H^2}{v^2}\right) \partial^\mu H \partial_\mu H$
$\mathcal{O}_{ddZ}$	$a_{ddZ} \frac{m_Z^2}{v^2} \left(\frac{H}{v}\right) \partial^\mu H \partial_\mu H$	$\mathcal{O}_{HddZ}$	$a_{HddZ} \frac{m_Z^2}{v^2} \left(\frac{H^2}{v^2}\right) \partial^\mu H \partial_\mu H$
$\mathcal{O}_{dd\Box}$	$a_{dd\Box} \frac{1}{v^3} \partial^\mu H \, \partial_\mu H \, \Box H$	$\mathcal{O}_{Hdd\Box}$	$a_{Hdd\Box} \frac{1}{v^3} \left(\frac{H}{v}\right) \partial^\mu H \, \partial_\mu H \, \Box H$
$\mathcal{O}_{HH\Box\Box}$	$a_{HH\Box\Box} \left(\frac{H^2}{v^2}\right) \frac{\Box H \Box H}{v^2}$	$\mathcal{O}_{dddd}$	$a_{dddd} \frac{1}{v^4} \partial^\mu H \, \partial_\mu H \, \partial^\nu H \, \partial_\nu H$

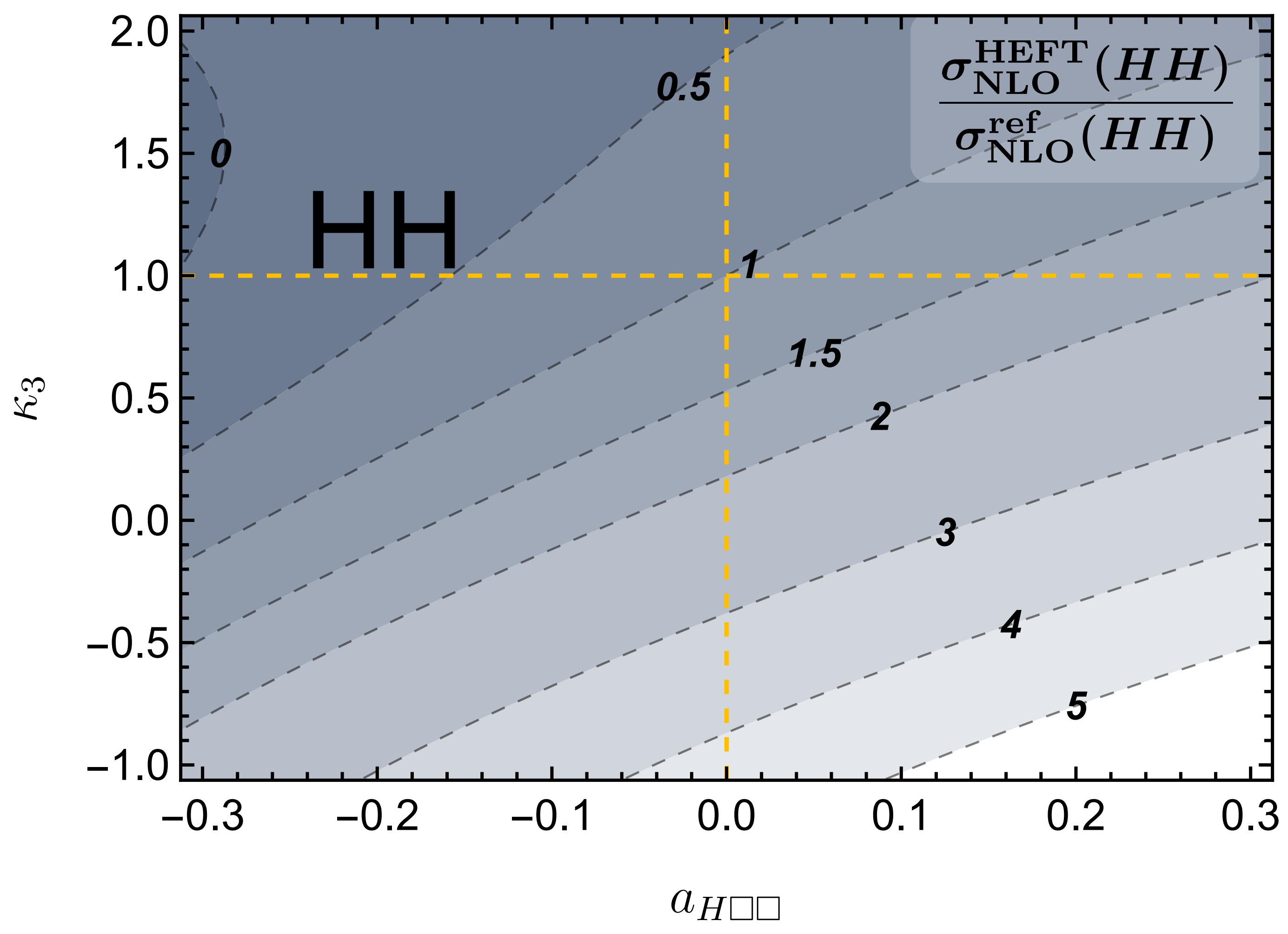
The largest effects are from operators with higher number of derivatives: $a_{dd\Box}, a_{H\Box\Box}, a_{dddd} \ldots$

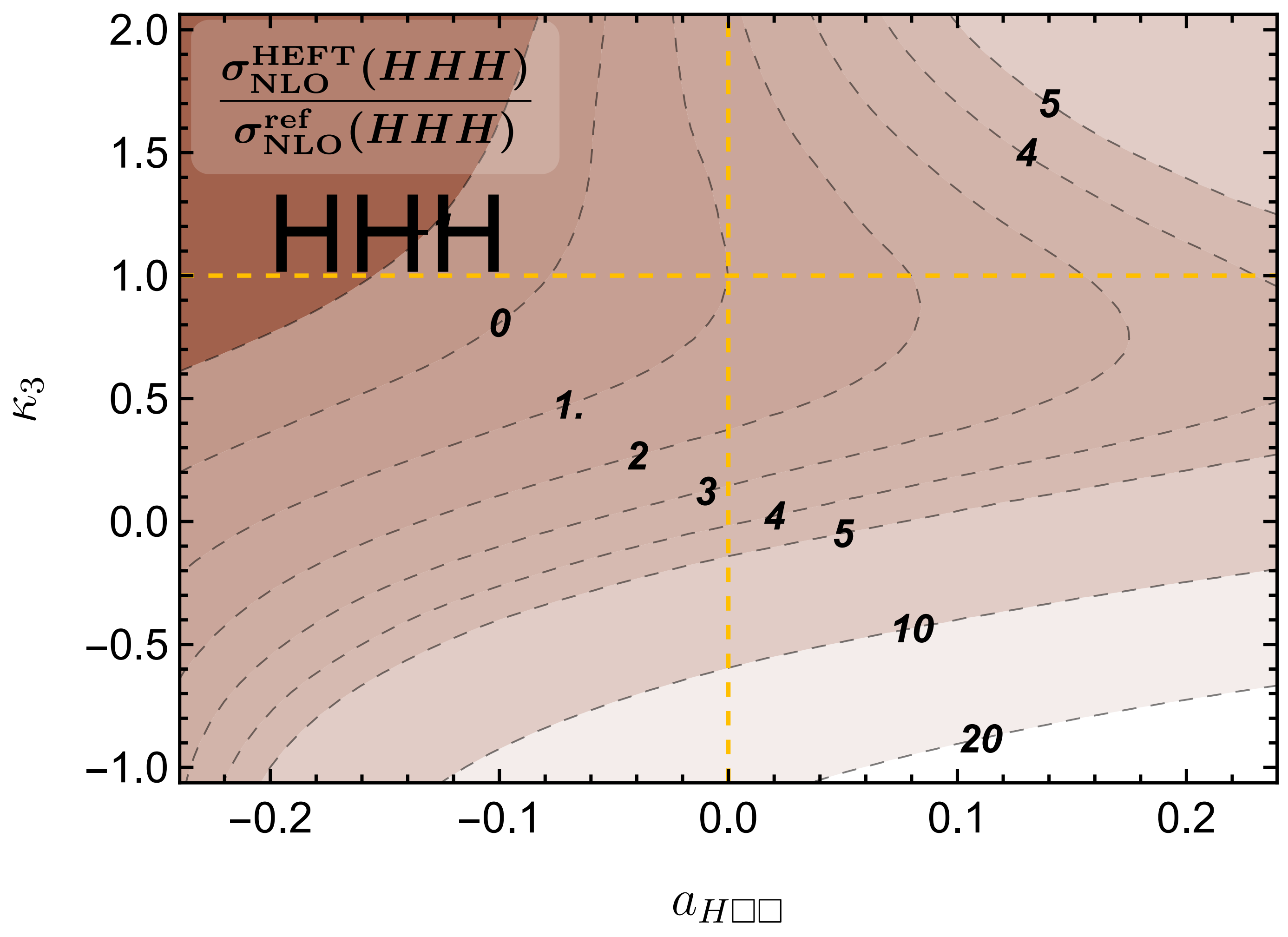
For instance, for $a_{H\Box\Box} = 0.1$ and $\kappa_3 = 1$

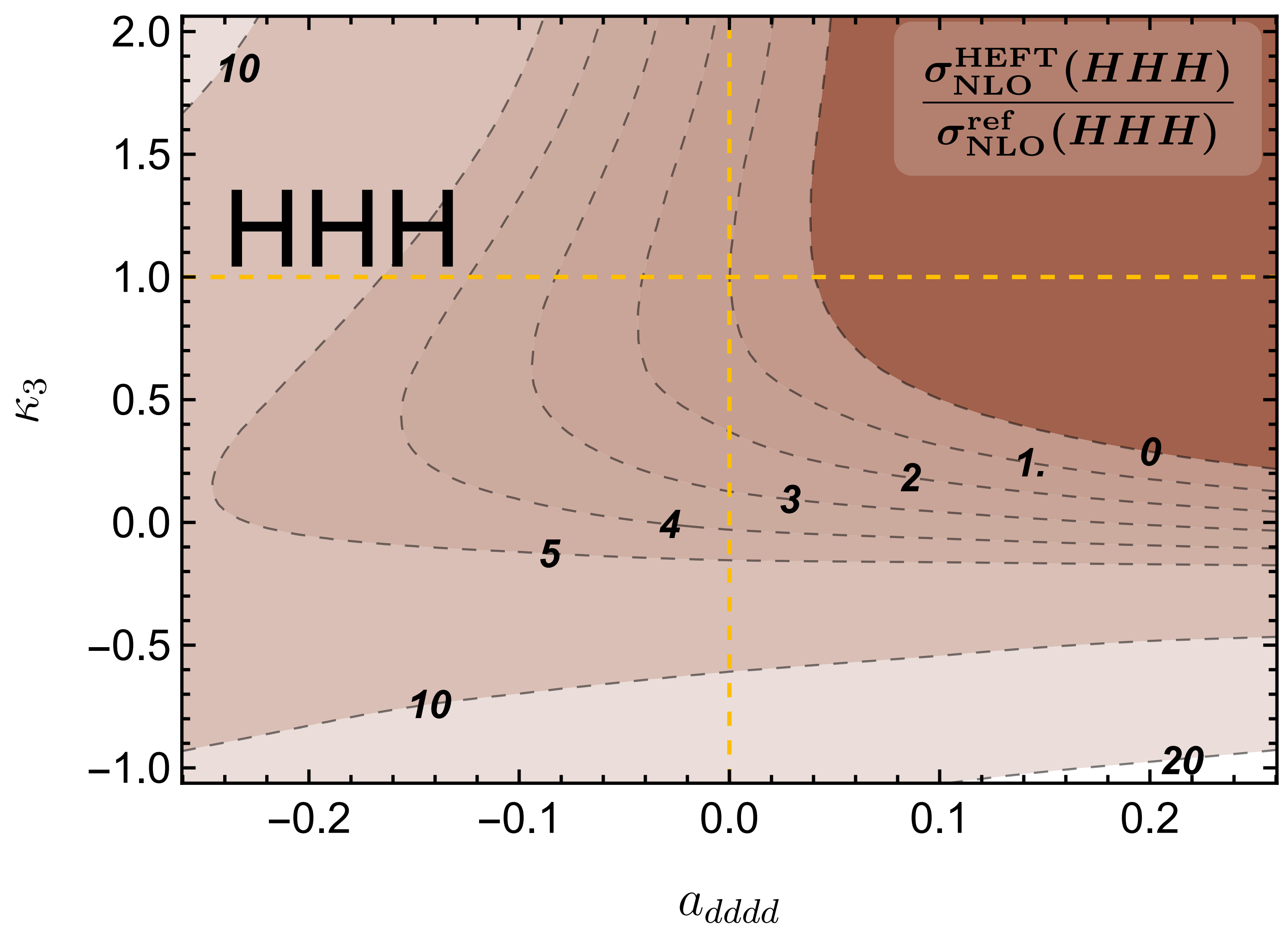
$$\sigma^{\rm HEFT}(HH) \sim 1.5\, \sigma^{\rm SM}(HH) \quad (50\%)$$

$$\sigma^{\rm HEFT}(HHH) \sim 1.8\, \sigma^{\rm SM}(HHH) \; (80\%)$$

Other 2D correlation plots in 2405.05385







Conclusions

Multiple Higgs production at colliders (HH, HHH,..) will test the Higgs potential and BSM Higgs couplings to gauge bosons. Some correlations could also be tested:

$\mathbf{V_{HWW}}$ / $\mathbf{V_{HHWW}}$, $\lambda_{\mathbf{HHH}}$ / $\lambda_{\mathbf{HHHH}}$,…uncorrelated in HEFT because H is a singlet but correlated in other specific scenarios.
In particular: 2HDM, SMEFT, …. where H is part of a doublet

Both HL-LHC (14 TeV) and CLIC (3TeV) will give access to LO and NLO HEFT coefficients. Studying specific difxsections will help in exploring potential correlations: Ex. $d\sigma/d\eta_H$ for $\kappa_V^2 \leftrightarrow \kappa_{2V}$

Including radiative corrections within HEFT predictions is important

Back up slides

Best prospects for κ_3 are at future e^+e^- colliders

- *Proposed* high-energy linear e^+e^- colliders: **ILC** and **CLIC**
- Projected sensitivity to κ_3 from hhZ and $hh\nu\bar{\nu}$ *(better than HL-LHC!)*:

At **ILC**:

500 GeV (4 ab^{-1}):	±27%
500 GeV (4 ab^{-1}) + 1 TeV (5* ab^{-1}):	±10%

[Dürig, 16] [Fujii *et al*., 15]

At **CLIC**: (at 68% CL)

1.4 TeV (2.5 ab^{-1}):	-29%, +67%
1.4 TeV (2.5 ab^{-1}) + 3 TeV (5 ab^{-1}):	-8%, +11%

[CLICdp Collab., 15]

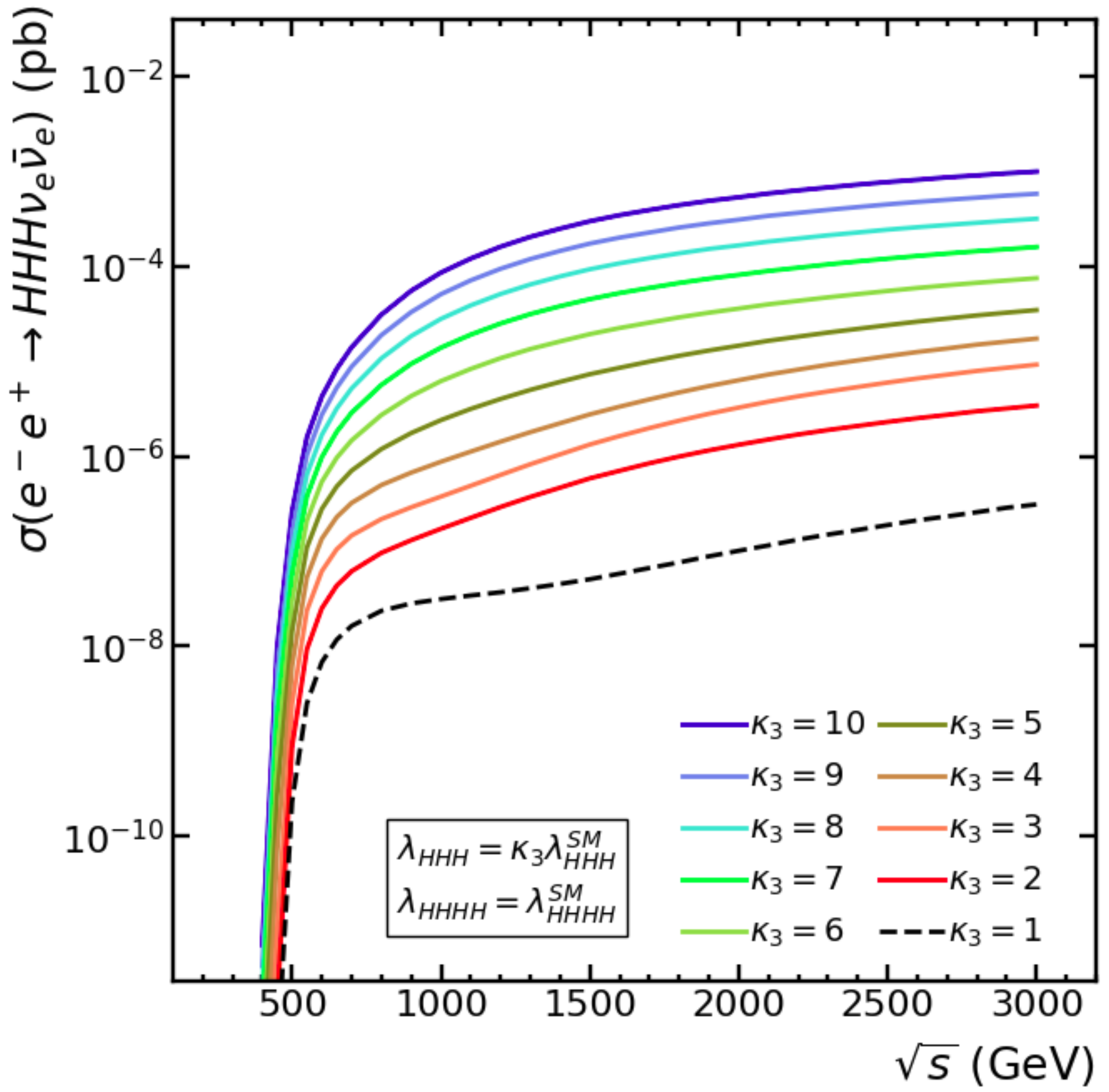
Best expected sensitivities $\Delta\kappa_3 \sim 0.1$

Cortesy by Francisco Arco

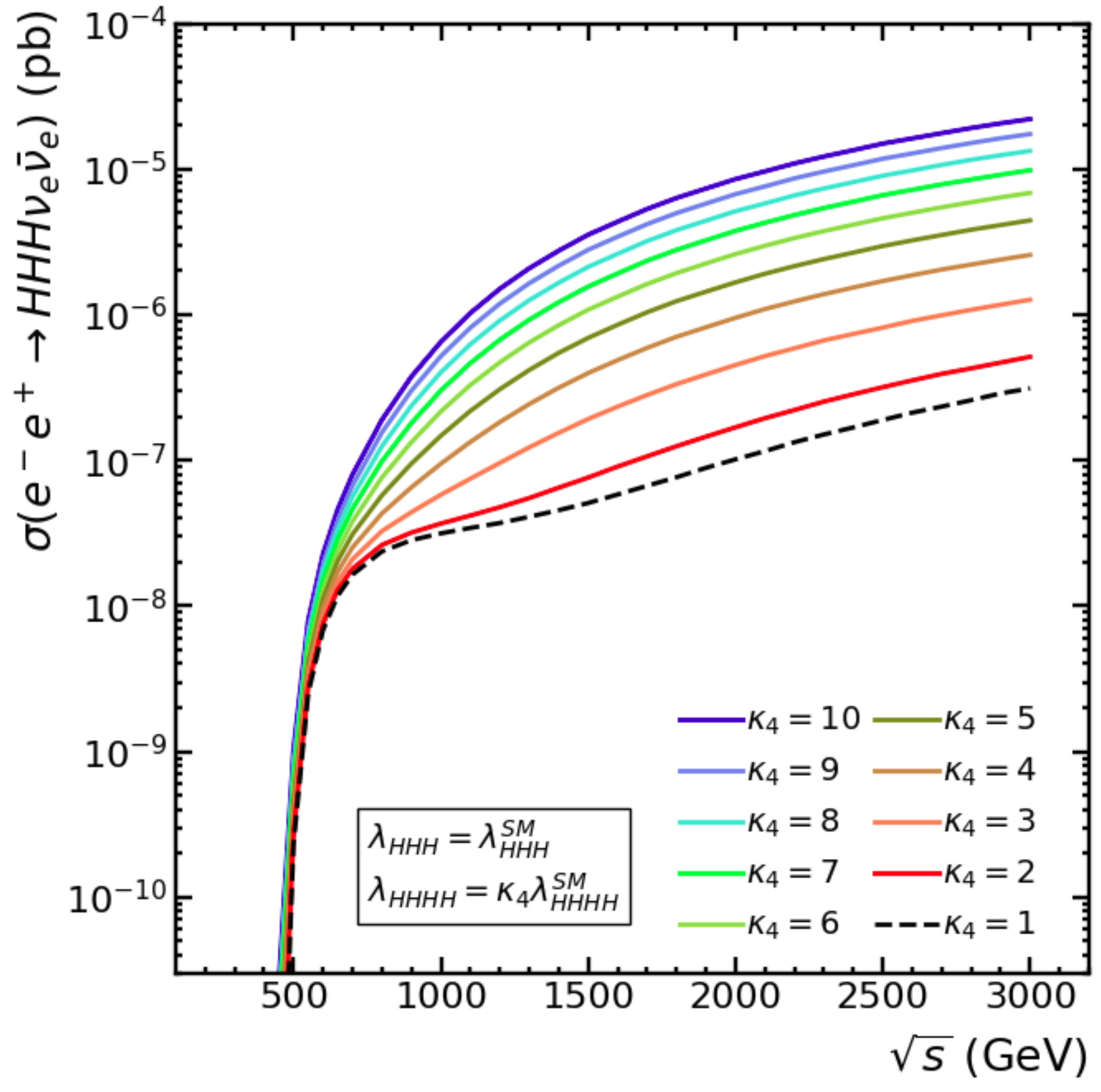
Sensitivity to κ_3 and κ_4 in $e^+e^- \to HHH\nu\bar{\nu}$

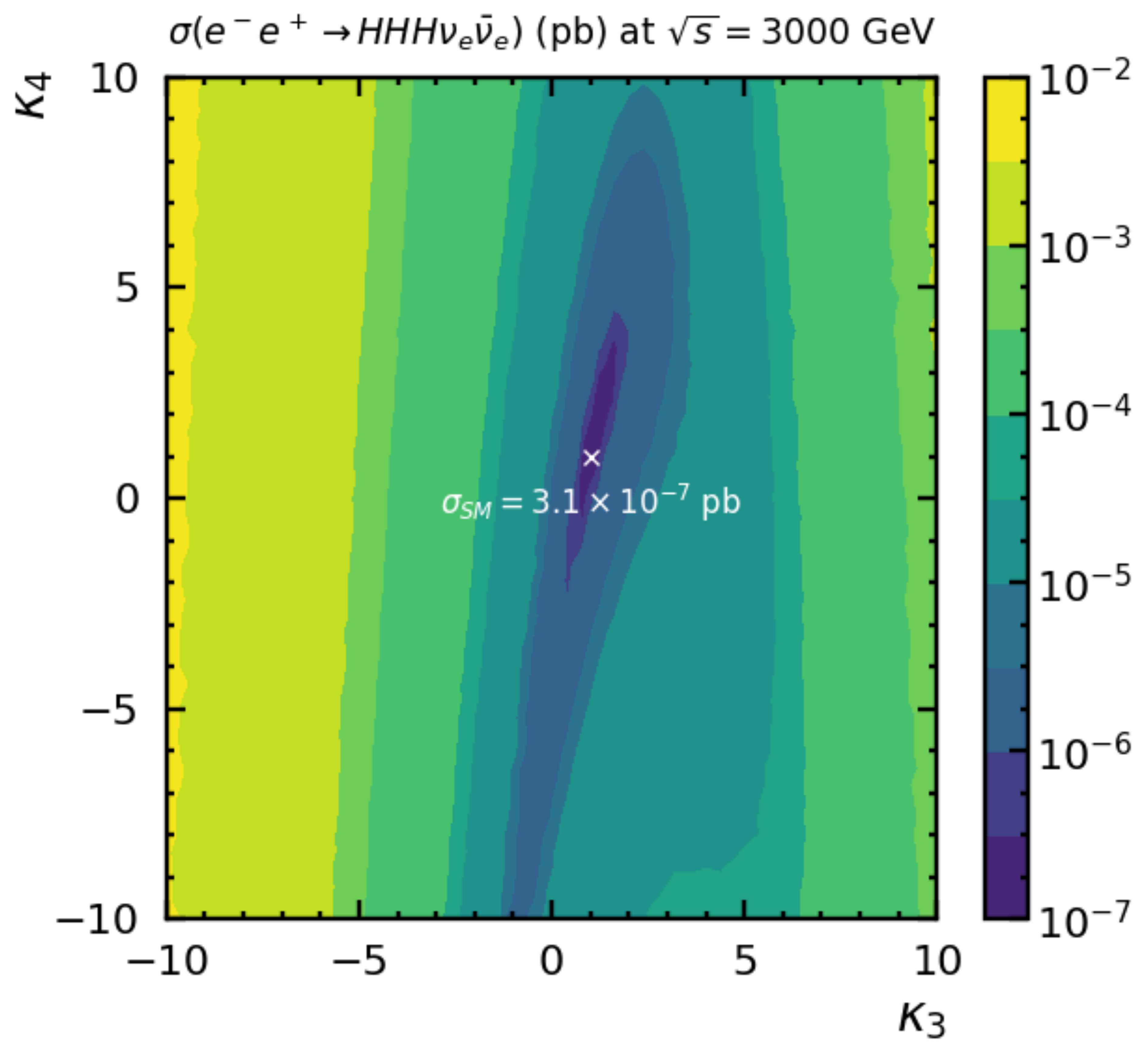
2011.13195, EPJC 81 (2021)3, 260, González-López, Herrero, Martínez-Suárez

κ_3



κ_4





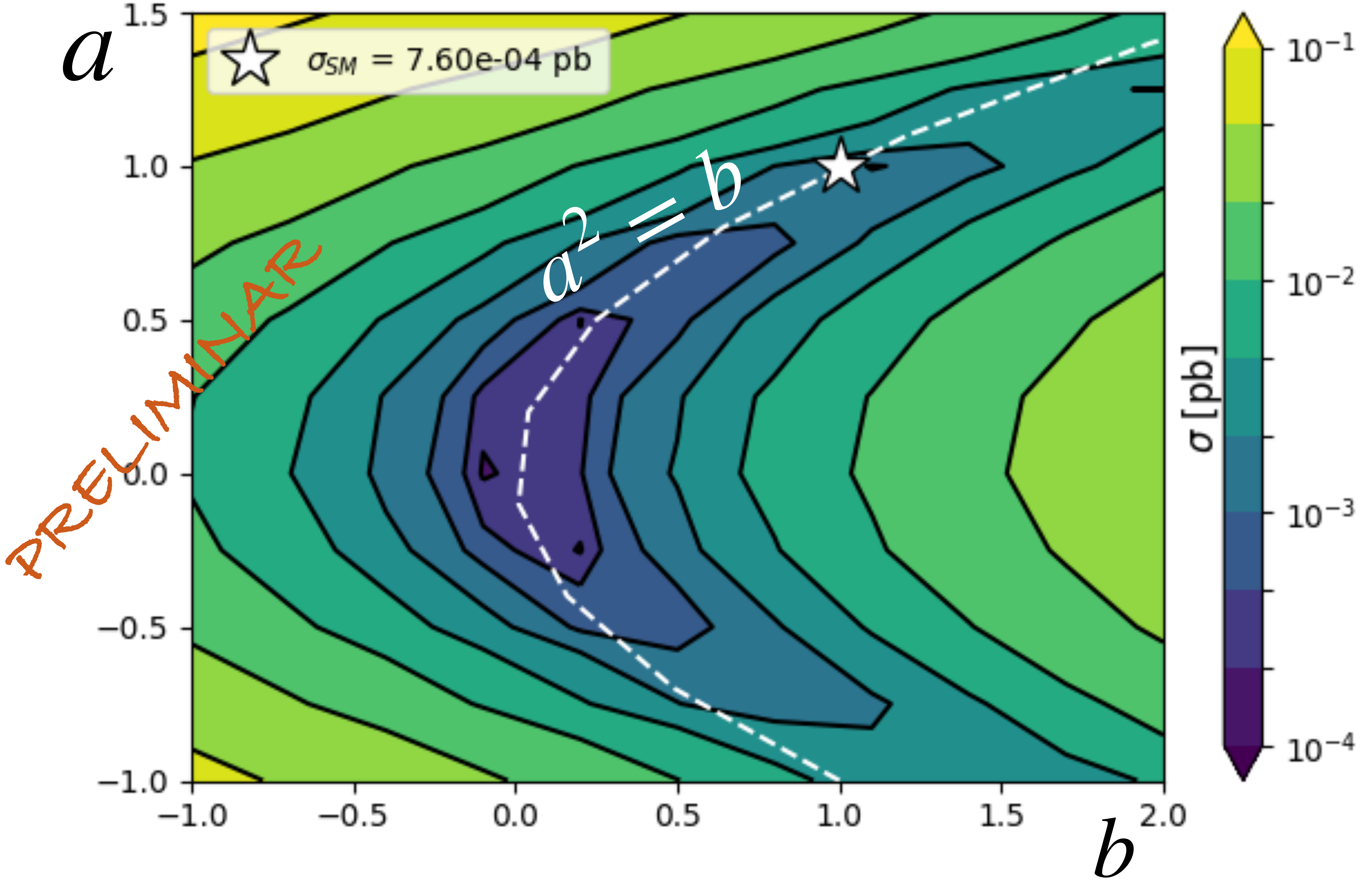
The best expectations are for CLIC (3 TeV) where

BSM/SM $\gtrsim 10$ for $\kappa_3 \gtrsim 2$ $(\kappa_4 = 1)$

BSM/SM $\gtrsim 10$ for $\kappa_4 \gtrsim 4$ $(\kappa_3 = 1)$

Higher sensitivity to κ_3 than to κ_4 !!

Sensitivity to a=κ_V, b=κ_{2V} in $pp \to HHj_1j_2$ (LHC, 14TeV)



Largest sensitivity expected if

$$a^2 \neq b \quad \text{(again)}$$

producing the largest deviations compared to SM predictions

$$\sigma^{BSM}(a=1, b=-0.5)/\sigma^{SM} \simeq 33 \;!!$$

Stringent exp. constraints on b
For 13TeV, in 4b channel:

$$b^{exp} \in [-0.1, 2.2]^{(1)}, [-0.03, 2.11]^{(2)}$$

(1) CMS, PRL129, 081802(2022) [2202.09617]

(2) ATLAS, PRD108, 052003(2023) [2301.03212]

Applied cuts: $P_{Tj}^{min} = 20\,GeV$, $\eta_{j1}\eta_{j2} < 0$, $2 \leq |\eta_j| \leq 5$, $M_{jj}^{min} = 500 GeV$, $\Delta R_{jj}^{min} = 0.4$

Cepeda, Domenech, Garcia-Mir, Herrero (Work in progress)

Models with $a^2 = b$ very difficult to test

Exploring correlations at LHC more difficult than at e^+e^- but yet possible by studying differential xsections with specific variables (angular, rapidity etc..see next)

Matching amplitudes

We do matching at amplitude level (more useful to compare with data).
In contrast to other approaches: matching Lagrangians, matching Effective Actions …etc

Matching amplitudes requires:

$$\mathscr{A}^{\mathrm{HEFT}} = \mathscr{A}^{\mathrm{UV}}(\mathrm{m_{heavy}} \gg \mathrm{m_{light}})$$

- Setting the HEFT order (LO, NLO,..)
- Setting the n-loop order $\mathcal{O}(\hbar^n)$, same in both sides
- Setting the input parameters, in both sides
- Setting the proper large mass expansion in the UV theory

2307.15693, Phys.Rev.D 108 (2023)9, 095013, Arco, Domenech, Herrero, Morales

Matching HEFT and 2HDM Amplitudes

Matching several amplitudes:

LO	$h \to WW^* \to W f \bar{f}'$		tree
LO	$h \to ZZ^* \to Z f \bar{f}$		tree
LO	$W^+W^- \to hh$		tree
LO	$ZZ \to hh$		tree
LO	$hh \to hh$		tree
NLO	$h \to \gamma\gamma$	R_ξ	1-loop
NLO	$h \to \gamma Z$	R_ξ	1-loop

Choose input parameters:

(HEFT) $m_h, m_W, m_Z, c_i's$

(2HDM)
- m_h, m_W, m_Z, m_{12} (light)
- $m_H, m_A, m_{H^\pm}$ (heavy)
- $\tan\beta,\ \cos(\beta-\alpha)$ (free)

Proper large mass expansion is in $\left(\frac{m_{\mathrm{light}}}{m_{\mathrm{heavy}}}\right)^n$. Other parameters are derived $(\lambda_{h_i h_j h_k}, \ldots.)$

Solution to the matching equations: HEFT versus 2HDM

2307.15693, Phys.Rev.D 108 (2023)9, 095013, Arco, Domenech, Herrero, Morales

Solving the matching equations implies identifying all momenta and Lorentz structures involved and extracting the corresponding HEFT $c_i's$ coeffs

$$(a = 1 - \Delta a\,, b = 1 - \Delta b\,, \kappa_3 = 1 - \Delta\kappa_3\,, \kappa_4 = 1 - \Delta\kappa_4)$$

$$\begin{aligned}
\Delta a|_{\rm 2HDM} &= 1 - s_{\beta-\alpha}\,,\\
\Delta b|_{\rm 2HDM} &= -c^2_{\beta-\alpha}(1 - 2c^2_{\beta-\alpha} + 2c_{\beta-\alpha}s_{\beta-\alpha}\cot 2\beta)\,,\\
\Delta\kappa_3|_{\rm 2HDM} &= 1 - s_{\beta-\alpha}(1 + 2c^2_{\beta-\alpha}) - c^2_{\beta-\alpha}\left(-2s_{\beta-\alpha}\frac{m^2_{12}}{m^2_h s_\beta c_\beta} + 2c_{\beta-\alpha}\cot 2\beta\left(1 - \frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right)\right)\,,\\
\Delta\kappa_4|_{\rm 2HDM} &= -\frac{c^2_{\beta-\alpha}}{3}\left(-7 + 64c^2_{\beta-\alpha} - 76c^4_{\beta-\alpha} + 12\left(1 - 6c^2_{\beta-\alpha} + 6c^4_{\beta-\alpha}\right)\frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right.\\
&\quad +4c_{\beta-\alpha}s_{\beta-\alpha}\cot 2\beta\left(-13 + 38c^2_{\beta-\alpha} - 3(-5 + 12c_{\beta-\alpha})\frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right)\\
&\quad \left.+4c^2_{\beta-\alpha}\cot^2 2\beta\left(3c^2_{\beta-\alpha} - 16s^2_{\beta-\alpha} + 3(-1 + 6s^2_{\beta-\alpha})\frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right)\right)\,,\\
a_{h\gamma\gamma}|_{\rm 2HDM} &= -\frac{s_{\beta-\alpha}}{48\pi^2}\,,\\
a_{h\gamma Z}|_{\rm 2HDM} &= -\frac{(2c^2_{\rm w} - 1)s_{\beta-\alpha}}{96c^2_{\rm w}\pi^2}\,.
\end{aligned}$$

These contributions are $\mathcal{O}\left(\frac{m_{\rm light}}{m_{\rm heavy}}\right)^0$

Leading terms in the large $m_{\rm heavy}$ expansion

Summarize the Non-Decoupling effects of the heavy Higgs bosons at low energies

They are valid for arbitrary $t_\beta\,, c_{\beta-\alpha}$

when $c_{\beta-\alpha} \ll 1$ is required (quasi-alignement)

These non-decoupling effects from the heavy bosons are not obtained in the SMEFT where all effects are decoupling $\left(\frac{m_{\rm light}}{m_{\rm heavy}}\right)^n, n \geq 2$

Interesting correlations found

$$\Delta a|^{\rm qal}_{\rm 2HDM} = -\frac{1}{2}\Delta b|^{\rm qal}_{\rm 2HDM}$$

$$2\Delta\kappa_3|^{\rm qal}_{\rm 2HDM} + \Delta\kappa_4|^{\rm qal}_{\rm 2HDM} = -\frac{2}{3}c^2_{\beta-\alpha}$$

Matching amplitudes: HEFT versus SMEFT

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos

Requiring matching of the amplitudes for WW—>HH (similar for ZZ —>HH) and identifying all momenta and Lorentz structures involved

$$\mathcal{A}(WW \to HH)|_{\rm HEFT} = \mathcal{A}^{(2)} + \mathcal{A}^{(4)} \quad \longleftrightarrow \quad \mathcal{A}(WW \to HH)_{\rm SMEFT} = \mathcal{A}_{\rm SM} + \mathcal{A}^{[6]} + \mathcal{A}^{[8]}$$

$$\mathcal{A}^{(2)}|_S = \frac{g^2}{2} 3a\kappa_3 \frac{m_{\rm H}^2}{S - m_{\rm H}^2} \epsilon_+ \cdot \epsilon_-$$

$$\mathcal{A}^{(2)}|_T = g^2 a^2 \frac{m_{\rm W}^2 \epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_1 \epsilon_- \cdot k_2}{T - m_{\rm W}^2}$$

$$\mathcal{A}^{(2)}|_U = g^2 a^2 \frac{m_{\rm W}^2 \epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_2 \epsilon_- \cdot k_1}{U - m_{\rm W}^2}$$

$$\mathcal{A}^{(2)}|_C = \frac{g^2}{2} b \epsilon_+ \cdot \epsilon_-$$

$$\begin{aligned}\mathcal{A}^{(4)}|_S = \frac{g^2}{2v^2} \frac{1}{S - m_{\rm H}^2} & (3\kappa_3 a_{d2} m_{\rm H}^2 (S\epsilon_+ \cdot \epsilon_- - 2\epsilon_+ \cdot p_- \epsilon_- \cdot p_+) \\ & + 6\kappa_3 a_{HWW} m_{\rm H}^2 ((S - 2m_{\rm W}^2)\epsilon_+ \cdot \epsilon_- - 2\epsilon_+ \cdot p_- \epsilon_- \cdot p_+) - (3\kappa_3 a_{H\mathcal{V}\mathcal{V}} m_{\rm H}^4 + a a_{Hdd} m_{\rm H}^2 (S + 2m_{\rm H}^2))\epsilon_+ \cdot \epsilon_-)\end{aligned}$$

$$\begin{aligned}\mathcal{A}^{(4)}|_T = \frac{g^2}{2v^2} \frac{a}{T - m_{\rm W}^2} & (a_{d2}(4m_{\rm W}^2 m_{\rm H}^2 \epsilon_+ \cdot \epsilon_- + 2(T + 3m_{\rm W}^2 - m_{\rm H}^2)\epsilon_+ \cdot k_1 \epsilon_- \cdot k_2 \\ & - 4m_{\rm W}^2(\epsilon_+ \cdot k_1 \epsilon_- \cdot p_+ + \epsilon_+ \cdot p_- \epsilon_- \cdot k_2)) \\ & - 8a_{HWW} m_{\rm W}^2 ((T + m_{\rm W}^2 - m_{\rm H}^2)\epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_1 \epsilon_- \cdot p_+ + \epsilon_+ \cdot p_- \epsilon_- \cdot k_2) \\ & - 4a_{H\mathcal{V}\mathcal{V}} m_{\rm H}^2 (m_{\rm W}^2 \epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_1 \epsilon_- \cdot k_2))\end{aligned}$$

$$\mathcal{A}^{(4)}|_U = \mathcal{A}^{(4)}|_T \quad \text{with} \quad T \to U \quad \text{and} \quad k_1 \leftrightarrow k_2$$

$$\begin{aligned}\mathcal{A}^{(4)}|_C = \frac{g^2}{2v^2} & (-2a_{dd\mathcal{V}\mathcal{V}1}(\epsilon_+ \cdot k_2 \epsilon_- \cdot k_1 + \epsilon_+ \cdot k_1 \epsilon_- \cdot k_2) \\ & + (-2a_{dd\mathcal{V}\mathcal{V}2}(S - 2m_{\rm H}^2) + 4a_{HHWW}(S - 2m_{\rm W}^2) + a_{Hd2} S - a_{HH\mathcal{V}\mathcal{V}} m_{\rm H}^2)\epsilon_+ \cdot \epsilon_- \\ & - 2(a_{Hd2} + 4a_{HHWW})\epsilon_+ \cdot p_- \epsilon_- \cdot p_+)\end{aligned}$$

$$\mathcal{L}_6 = \frac{a_{\phi\Box}}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \frac{a_{\phi D}}{\Lambda^2}(\phi^\dagger D_\mu \phi)((D^\mu\phi)^\dagger\phi) + \frac{a_{\phi W}}{\Lambda^2}(\phi^\dagger\phi) W^a_{\mu\nu} W^{a\mu\nu} + \ldots$$

$$\mathcal{L}_8 = \frac{a^{(1)}_{\phi^6}}{\Lambda^4}(\phi^\dagger\phi)^2 (D_\mu\phi^\dagger D^\mu\phi) + \frac{a^{(2)}_{\phi^6}}{\Lambda^4}(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)(D_\mu\phi^\dagger\sigma^I D^\mu\phi) + \frac{a^{(1)}_{\phi^4}}{\Lambda^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi) + \frac{a^{(2)}_{\phi^4}}{\Lambda^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi) + \frac{a^{(3)}_{\phi^4}}{\Lambda^4}(D_\mu\phi^\dagger D_\mu\phi)(D^\nu\phi^\dagger D^\nu\phi) + \ldots$$

$$\begin{aligned}\mathscr{A}_{\rm SM} &= \frac{g^2}{2} 3 \frac{m_{\rm H}^2}{S - m_{\rm H}^2}\epsilon_+ \cdot \epsilon_- + g^2 \frac{m_{\rm W}^2 \epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_1 \epsilon_- \cdot k_2}{T - m_{\rm W}^2} \\ & + g^2 \frac{m_{\rm W}^2 \epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_2 \epsilon_- \cdot k_1}{U - m_{\rm W}^2} + \frac{g^2}{2}\epsilon_+ \cdot \epsilon_- \qquad (2.17)\end{aligned}$$

$$\mathcal{A}^{[6]}|_S = \frac{g^2}{4}\frac{v^2}{\Lambda^2}\delta a_{\phi D}\frac{S + 8m_{\rm H}^2}{S - m_{\rm H}^2}\epsilon_+ \cdot \epsilon_- + 6\frac{v^2}{\Lambda^2} a_{\phi W}\frac{m_{\rm H}^2}{v^2}\frac{2\epsilon_- \cdot p_+ \epsilon_+ \cdots - (S - 2m_{\rm W}^2)\epsilon_+ \cdot \epsilon_-}{S - m_{\rm H}^2}; \qquad \delta a_{\phi D} \equiv 4a_{\phi\Box} - a_{\phi D}$$

$$\begin{aligned}\mathcal{A}^{[6]}|_T &= \frac{g^2}{2}\frac{v^2}{\Lambda^2}\delta a_{\phi D}\frac{m_{\rm W}^2 \epsilon_+ \cdot \epsilon_- + (\epsilon_- \cdot p_+ - \epsilon_- \cdot k_1)\epsilon_+ \cdot k_1}{T - m_{\rm W}^2} \\ & + 2g^2 \frac{v^2}{\Lambda^2} a_{\phi W}\frac{\epsilon_+ \cdot \epsilon_-(-m_{\rm H}^2 + m_{\rm W}^2 + T) - \epsilon_- \cdot k_1 \epsilon_+ \cdot p_- + \epsilon_- \cdot p_+(\epsilon_+ \cdot p_- + \epsilon_+ \cdot k_1)}{T - m_{\rm W}^2};\end{aligned}$$

$$\mathcal{A}^{[6]}|_U = \mathcal{A}^{(6)}|_T \quad \text{with} \quad T \to U \quad \text{and} \quad k_1 \leftrightarrow k_2$$

$$\mathcal{A}^{[6]}|_C = \frac{g^2}{4}\frac{v^2}{\Lambda^2}\delta a_{\phi D}\epsilon_+ \cdot \epsilon_- + \frac{v^2}{\Lambda^2} a_{\phi W}\frac{1}{v^2}(-2(S - 2m_{\rm W}^2)\epsilon_+ \cdot \epsilon_- + 4\epsilon_- \cdot p_+ \epsilon_+ \cdot p_-);$$

$$\mathcal{A}^{[8]}|_C = -\frac{g^2}{4}\frac{v^2}{\Lambda^4}((a^{(1)}_{\phi^4} + a^{(2)}_{\phi^4})(\epsilon_- \cdot p_+ \epsilon_+ \cdot k_1 + \epsilon_- \cdot k_1(\epsilon_+ \cdot p_- - 2\epsilon_+ \cdot k_1)) + a^{(3)}_{\phi^4}\epsilon_+ \cdot \epsilon_-(S - 2m_H^2))$$

Interesting correlations found

$$\Delta b|_{\rm SMEFT} = 4\Delta a|_{\rm SMEFT}$$

$$a_{HWW}|_{\rm SMEFT} = 2a_{HHWW}|_{\rm SMEFT}$$

Solutions to the matching:

$$a - 1 = \frac{1}{4}\frac{v^2}{\Lambda^2}\delta a_{\phi D} \qquad \kappa_3 - 1 = \frac{5}{4}\frac{v^2}{\Lambda^2}\delta a_{\phi D} \qquad a_{HWW} = -\frac{v^2}{2m_{\rm W}^2}\frac{v^2}{\Lambda^2}a_{\phi W} \qquad a_{dd\mathcal{V}\mathcal{V}1} = \frac{v^4}{4\Lambda^4}[a^{(1)}_{\phi^4} + a^{(2)}_{\phi^4}]$$

$$b - 1 = \frac{v^2}{\Lambda^2}\delta a_{\phi D} \qquad a_{HHWW} = -\frac{v^2}{4m_{\rm W}^2}\frac{v^2}{\Lambda^2}a_{\phi W} \qquad a_{dd\mathcal{V}\mathcal{V}2} = \frac{v^4}{4\Lambda^4}a^{(3)}_{\phi^4}$$

CTs in NLO $WW \to HH$ and derived RGEs

M.J. Herrero and R.A Morales , PRD106,073008(2022) 2208.05900

$$\delta_\epsilon a = \frac{\Delta_\epsilon}{16\pi^2}\frac{3}{2v^2}\left((a^2-b)(a-\kappa_3)m_{\rm H}^2 + a((1-3a^2+2b)m_{\rm W}^2+(1-a^2)m_{\rm Z}^2)\right),$$

$$\boldsymbol{\delta_\epsilon b} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2v^2}\big((a^2-b)(8a^2-2b-12a\kappa_3+3\kappa_4)m_{\rm H}^2$$
$$+6a^2b(2m_{\rm W}^2+m_{\rm Z}^2)-6b(m_{\rm W}^2+m_{\rm Z}^2)-6b^2m_{\rm W}^2\big),$$

$$\delta_\epsilon\kappa_3 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_{\rm H}^2v^2}\big(\kappa_3(a^2-b+9\kappa_3^2-6\kappa_4)m_{\rm H}^4 - 3(1-a^2)\kappa_3 m_{\rm H}^2(m_{\rm W}^2+m_{\rm Z}^2)$$
$$+6(-2ab+2a^2\kappa_3+b\kappa_3)(2m_{\rm W}^4+m_{\rm Z}^4)\big)\ ,$$

$$\boldsymbol{\delta_\epsilon a_{dd\mathcal{V}\mathcal{V}1}} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a^4+a^2b+b^2}{3}, \qquad \boldsymbol{\delta_\epsilon a_{dd\mathcal{V}\mathcal{V}2}} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{(a^2-b)(2a^2+b+6)}{12},$$

$$\delta_\epsilon a_{11} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a^2}{4}, \qquad \delta_\epsilon a_{H11} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{2}, \qquad \boldsymbol{\delta_\epsilon a_{HH11}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4-5a^2b+b^2}{4},$$

$$\delta_\epsilon a_{HWW} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{12}, \qquad \boldsymbol{\delta_\epsilon a_{HHWW}} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4-5a^2b+b^2}{24},$$

$$\delta_\epsilon a_{d2} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{6}, \qquad \boldsymbol{\delta_\epsilon a_{Hd2}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4-5a^2b+b^2}{6},$$

$$\delta_\epsilon a_{\Box\mathcal{V}\mathcal{V}} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a(2+a^2)}{4}, \qquad \boldsymbol{\delta_\epsilon a_{H\Box\mathcal{V}\mathcal{V}}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4+a^2(4-3b)-2b}{4},$$

$$\delta_\epsilon a_{d3} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2+b)}{2}, \qquad \boldsymbol{\delta_\epsilon a_{Hd3}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{-4a^4+a^2b+b^2}{2},$$

$$\delta_\epsilon a_{\Box\Box} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3a^2}{4}, \qquad \boldsymbol{\delta_\epsilon a_{H\Box\Box}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3a(2a^2-b)}{2},$$

$$\boldsymbol{\delta_\epsilon a_{dd\Box}} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3(a^2-b)}{2}, \qquad \boldsymbol{\delta_\epsilon a_{Hdd}} = 0, \qquad \boldsymbol{\delta_\epsilon a_{ddW}}/2 = \boldsymbol{\delta_\epsilon a_{ddZ}} = -\frac{\Delta_\epsilon}{16\pi^2}3a(a^2-b),$$

$$\boldsymbol{\delta_\epsilon a_{H\mathcal{V}\mathcal{V}}} = \boldsymbol{\delta_\epsilon a_{HH\mathcal{V}\mathcal{V}}} = 0, \qquad \Delta_\epsilon = \frac{2}{\epsilon} - \gamma_E + \log(4\pi).$$

Comment: $\delta_\epsilon\kappa_4$ and others $\delta_\epsilon a_i's$ are fixed in NLO $gg \to HH$ and $gg \to HHH$ (see next)

Combinations appearing in scattering amplitude :
(=use of e.o.m)

$$\delta_\epsilon\eta = \delta_\epsilon\tilde{a}_{dd\mathcal{V}\mathcal{V}1} = \delta_\epsilon(a_{dd\mathcal{V}\mathcal{V}1} - 4a^2a_{11} + 2aa_{d3}) = -\frac{\Delta_\epsilon}{16\pi^2}\frac{(a^2-b)^2}{3},$$

$$\delta_\epsilon\delta = \delta_\epsilon\tilde{a}_{dd\mathcal{V}\mathcal{V}2} = \delta_\epsilon\left(a_{dd\mathcal{V}\mathcal{V}2} + \frac{a}{2}a_{dd\Box}\right) = \frac{\Delta_\epsilon}{16\pi^2}\frac{(a^2-b)(7a^2-b-6)}{12},$$

$$\delta_\epsilon(a_{H\mathcal{V}\mathcal{V}} - 2a_{\Box\mathcal{V}\mathcal{V}} + 2aa_{\Box\Box}) = \frac{\Delta_\epsilon}{16\pi^2}a(1-a^2),$$

$$\delta_\epsilon(a_{HH\mathcal{V}\mathcal{V}} - 6\kappa_3a_{\Box\mathcal{V}\mathcal{V}} - 4a_{H\Box\mathcal{V}\mathcal{V}} + 4ba_{\Box\Box} + 6\kappa_3aa_{\Box\Box} + 4aa_{H\Box\Box}) = \frac{\Delta_\epsilon}{16\pi^2}(3\kappa_3a(1-a^2)+2b-2a^2(2+3b)+8a^4),$$

$$\delta_\epsilon(a_{Hdd} - a_{dd\Box}) = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3a(a^2-b)}{2}.$$

RGE easily derived for all these $c_i's$ HEFT coefficients

$$c_i(\mu) = c_i(\mu') + \frac{1}{16\pi^2}\gamma_{c_i}\log\left(\frac{\mu^2}{\mu'^2}\right)\ , \quad \delta_\epsilon c_i = \frac{\Delta_\epsilon}{16\pi^2}\gamma_{c_i}$$

We checked some $\delta c_i's$ with previous
results in specific limits :
pure scalar (1311.5993,14091571)
isospin limit $m_W = m_Z$ (2109.02673)
Others were unknown
before our work (see paper)

Comparing SMEFT and HEFT : LO and NLO

$WW \to HH$ subprocess

Some preliminary results (D. Domenech, M. Herrero, R. Morales, M. Ramos, 2022)

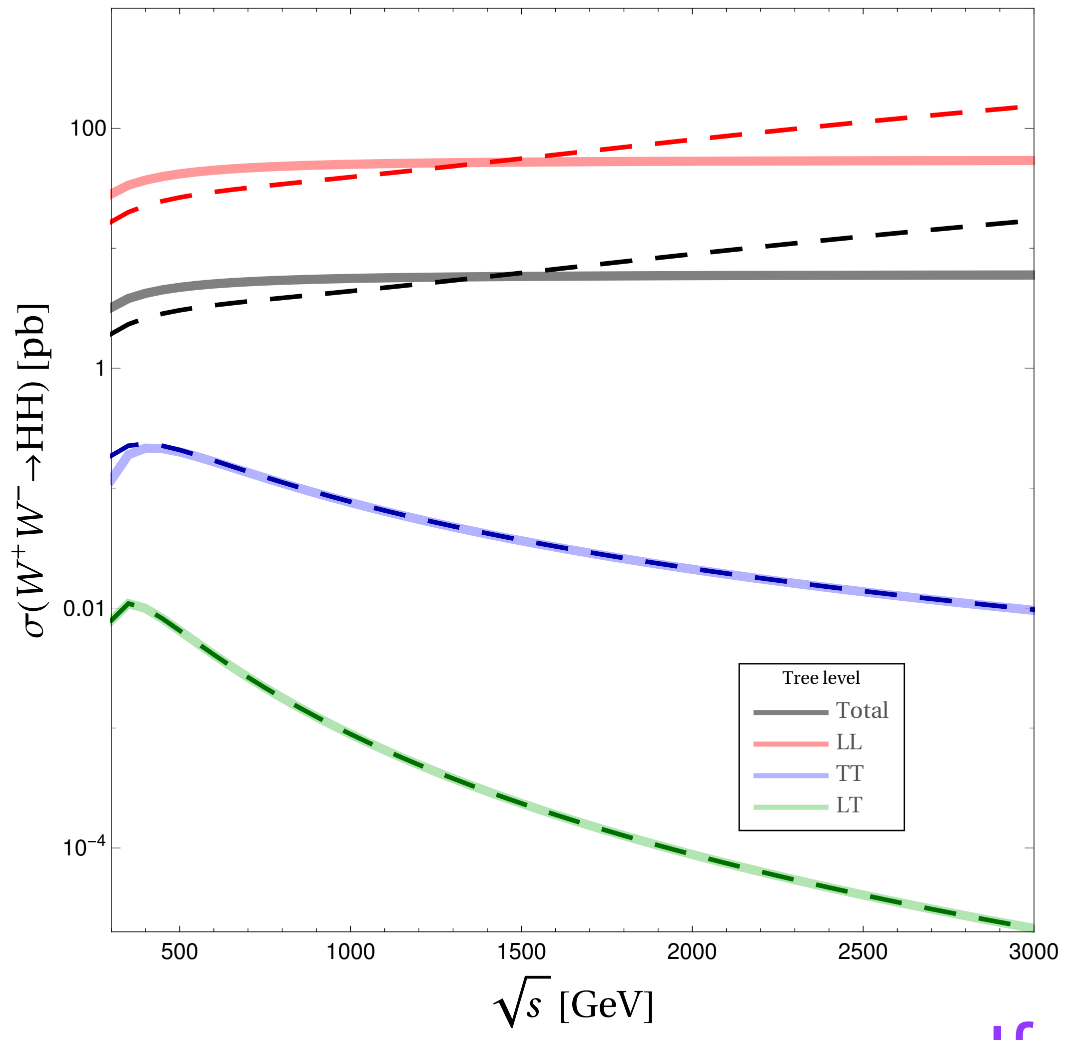
$$\mathcal{L}_6 \supset c_{\phi^6}(\phi^\dagger\phi)^3 + c_{\phi\Box}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + c_{\phi D}(\phi^\dagger D_\mu\phi)((D^\mu\phi)^\dagger\phi) + c_{\phi W}(\phi^\dagger\phi)W^a_{\mu\nu}W^{a\mu\nu} \qquad c_i \equiv a_i/\Lambda^2$$

$$\mathcal{L}_8 \supset c^{(1)}_{\phi^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi) + c^{(2)}_{\phi^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi) + c^{(3)}_{\phi^4}(D_\mu\phi^\dagger D_\mu\phi)(D^\nu\phi^\dagger D^\nu\phi) + \ldots \qquad c_i \equiv a_i/\Lambda^4$$

Again: the largest BSM deviations in Longitudinal modes $W_L W_L \to HH$ Transverse modes are less affected. At TeV: dim8 compete with dim6 !!

$\Lambda = 1\,\mathrm{TeV}$
In these plots

$a_{\phi\Box} = 1.0$



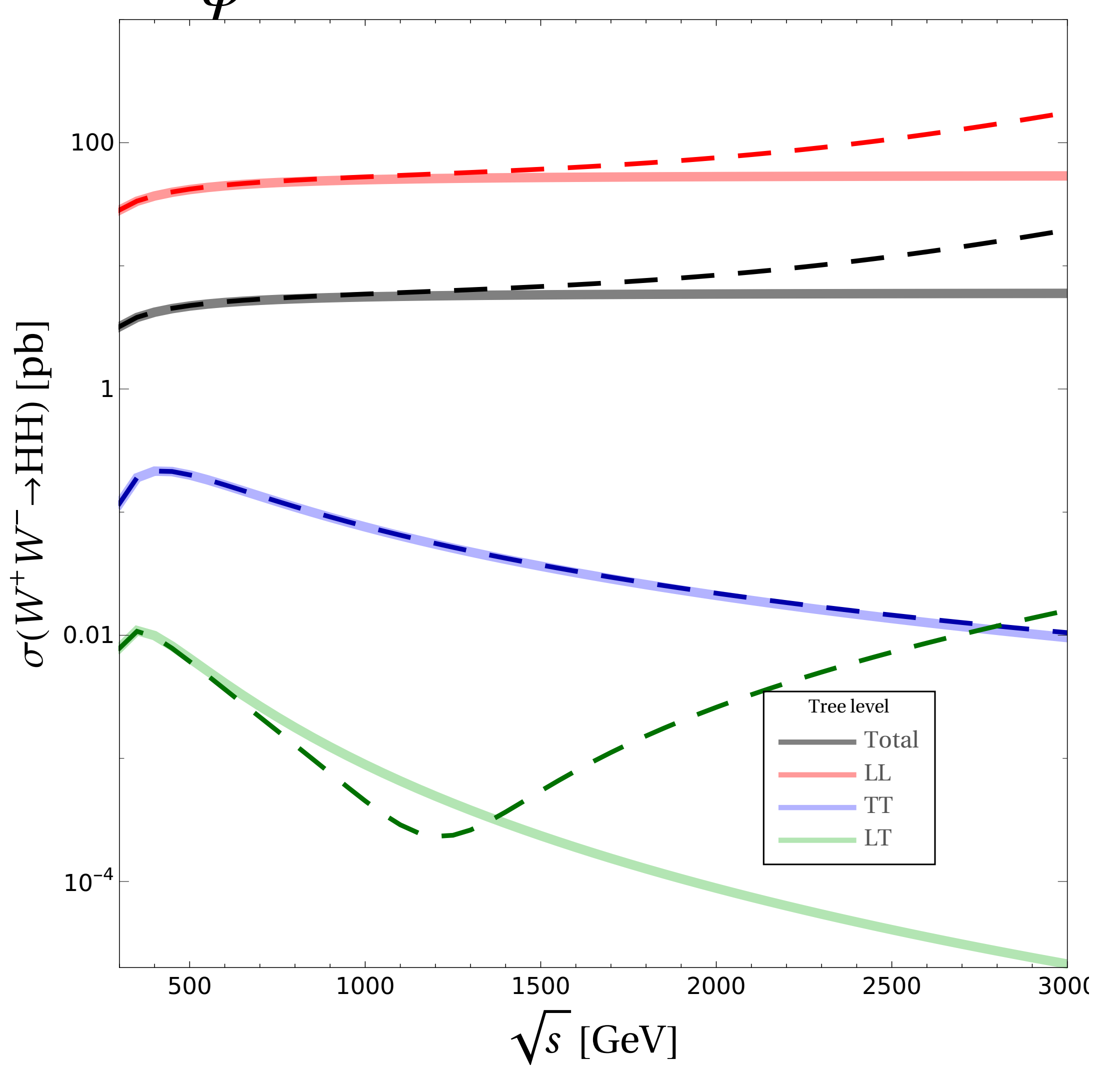
dim 6

$a_{\phi\Box}$

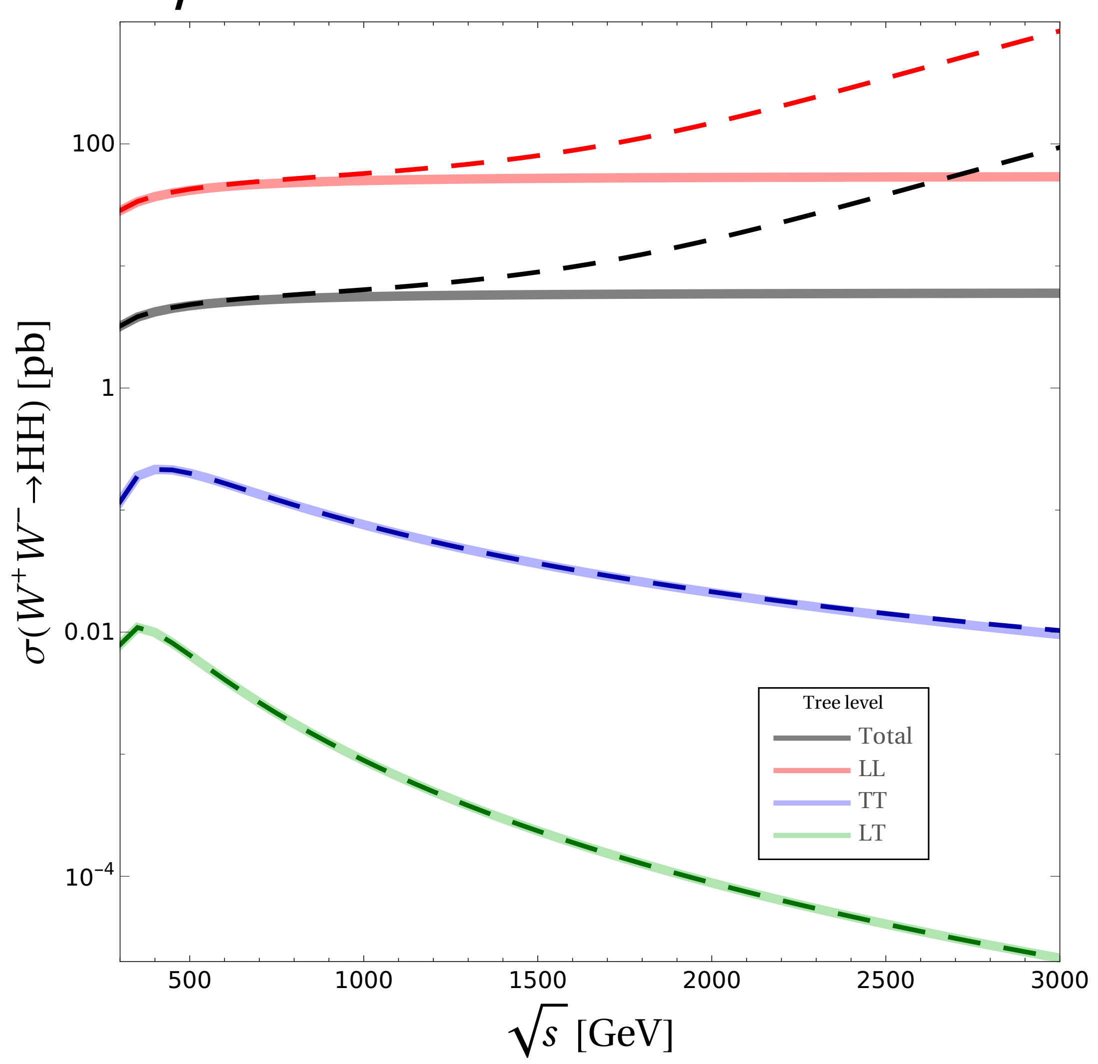
compares to

$a\,(\kappa_V)$ **and** $b\,(\kappa_{2V})$

$a^{(1)}_{\phi^4} = 1.0$



$a^{(3)}_{\phi^4} = 1.0$



dim 8

$a^{(1)}_{\phi^4}, a^{(3)}_{\phi^4}$

compare to

a_{ddVV1}, a_{ddVV2}

If matching in amplitudes according to behavior with energy: SMEFT dim 8 (6) <—> HEFT chi-dim 4 (2)