Dimension-8 operators in SMEFT

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Effective Field Theory in Multiboson Production

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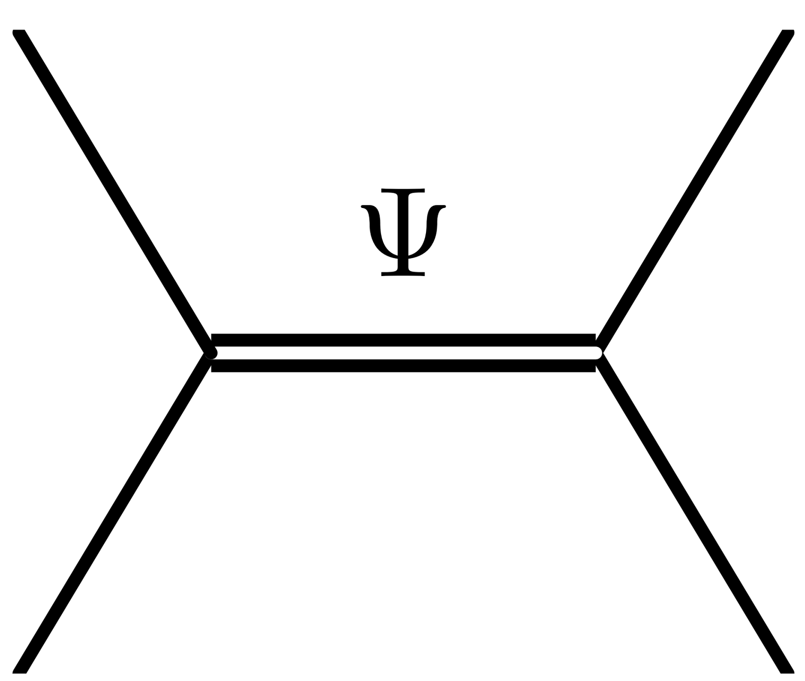
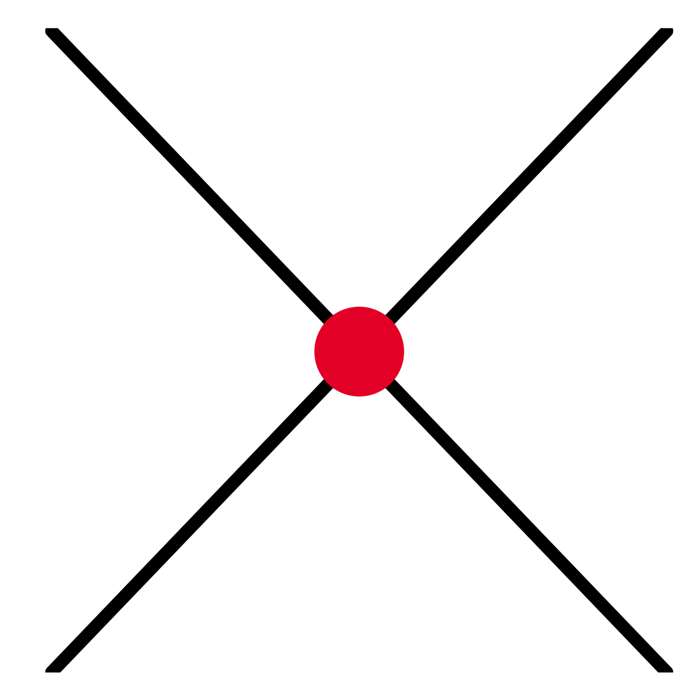
EFT expansion

$$\mathcal{A}^{n}_{\mathrm{BSM}}(E, M) \sim E^{4-n}\left(a_0 + a_1\frac{E}{M} + a_2\frac{E^2}{M^2} + \cdots\right), \qquad E \ll M$$



$$\frac{g^2}{p^2 - M^2} \approx -\frac{g^2}{M^2}\left[1 + \frac{p^2}{M^2} + \frac{p^4}{M^4} + \cdots\right]$$



$$\mathscr{L} \sim g\Psi\left(\phi^\dagger\phi\right) \approx \frac{g^2}{M^2}(\phi^\dagger\phi)^2 + \frac{g^2}{M^4}\phi^\dagger\phi\,\partial^\mu\phi^\dagger\partial_\mu\phi + \frac{g^2}{M^6}\partial^\mu\phi^\dagger\partial_\mu\phi\partial^\nu\phi^\dagger\partial_\nu\phi + \cdots$$

$s = p^2 < M^2$: EFT validity criterion

g not too large: perturbative UV completion

Beyond dim-6

$$\mathcal{L}_{\text{eff}} = \sum_i \frac{c_i \mathcal{O}_i^D}{\Lambda^{D-4}}$$

Is the EFT interpretation valid?

(also, what EFT? See Dave's talk)

- Does my truncated amplitude faithfully reproduce full new physics effect?
- Or, are higher order terms in Λ^{-1} relevant?

Theory ⇔ Amplitudes

$$\mathcal{A}_{\text{SMEFT}} = \mathcal{A}_{\text{SM}} + \sum_i \mathcal{A}_i^{(6)} \frac{C_i^{(6)}}{\Lambda^2} + \sum_j \mathcal{A}_j^{(8)} \frac{C_j^{(8)}}{\Lambda^4} + \mathcal{O}\left(\frac{1}{\Lambda^5}\right)$$

We measure cross-sections

$$\sigma_{\text{SMEFT}} \propto |\mathcal{A}_{\text{SMEFT}}|^2 = |\mathcal{A}_{\text{SM}}|^2 + \sum_i 2\text{Re}\left[\mathcal{A}_{\text{SM}}^* \mathcal{A}_i^{(6)}\right] \frac{C_i^{(6)}}{\Lambda^2}$$

$$+ \sum_{i \leq j} 2\text{Re}\left[\mathcal{A}_i^{(6)*} \mathcal{A}_j^{(6)}\right] \frac{C_i^{(6)} C_j^{(6)}}{\Lambda^4} + \sum_i 2\text{Re}\left[\mathcal{A}_{\text{SM}}^* \mathcal{A}_i^{(8)}\right] \frac{C_i^{(8)}}{\Lambda^4} + \cdots$$

Ideally we would sensitive enough to neglect Λ^{-4}

- In practice, not always the case…

$O(\Lambda^{-4})$

$$\sigma_{\text{SMEFT}} \propto |\mathcal{A}_{\text{SMEFT}}|^2 = |\mathcal{A}_{\text{SM}}|^2 + \sum_i 2\text{Re}\left[\mathcal{A}^*_{\text{SM}}\mathcal{A}^{(6)}_i\right]\frac{C^{(6)}_i}{\Lambda^2} + \sum_{i\leq j} 2\text{Re}\left[\mathcal{A}^{(6)*}_i\mathcal{A}^{(6)}_j\right]\frac{C^{(6)}_i C^{(6)}_j}{\Lambda^4} + \cdots$$

When might it matter?

- Data are not very sensitive - C_i/Λ^2 poorly constrained: *e.g.* 4 top production
- Dim-6 interference term is suppressed at high energy: helicity selection
  [Azatov et al.; PRD 95 (2017) no. 6, 065014]
- Sensitivity from energy growing effects $\mathcal{A} = \mathcal{A}_{\text{SM}}\left(1 + \frac{v^2}{\Lambda^2} + \frac{vE}{\Lambda^2} + \boxed{\frac{E^2}{\Lambda^2} + \cdots}\right)$
- $\mathcal{A}^{(6)*}_i\mathcal{A}^{(6)}_j$ will dominate over $\mathcal{A}^*_{\text{SM}}\mathcal{A}^{(6)}_i$ at high energy
- Some effects only arise at dim-8: *e.g.* neutral triple gauge interaction (ZZZ,…)

(See Fabian's talk)

Dim-8 is interesting from a phenomenological perspective

- Global fits, UV model interpretations, validity, unique dim-8 effects,…

Also interesting from a theoretical perspective

- Positivity bounds on dim-$(n \geq 8)$ scattering amplitudes

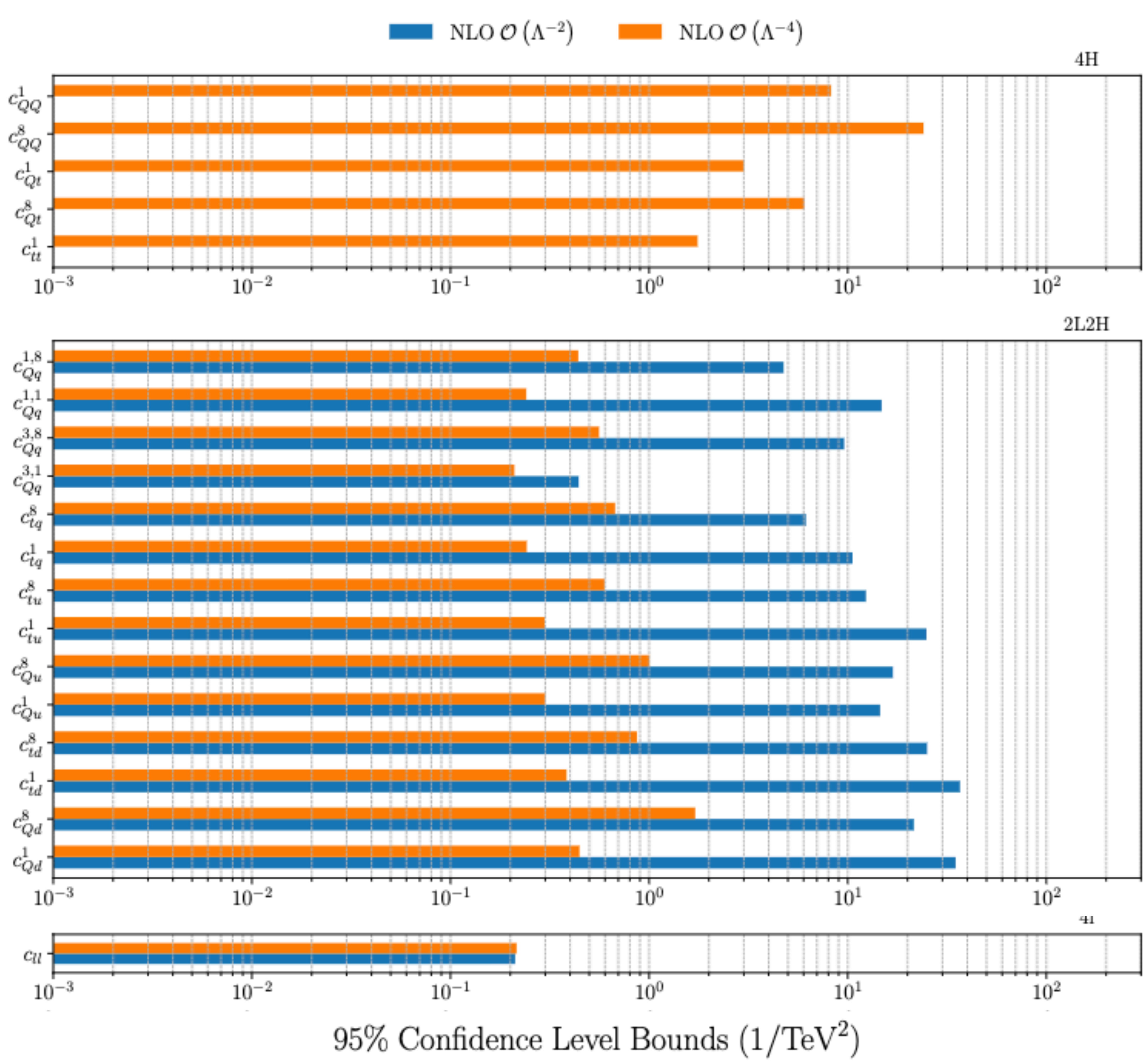
Dim-6: Λ^{-2} vs Λ^{-4}

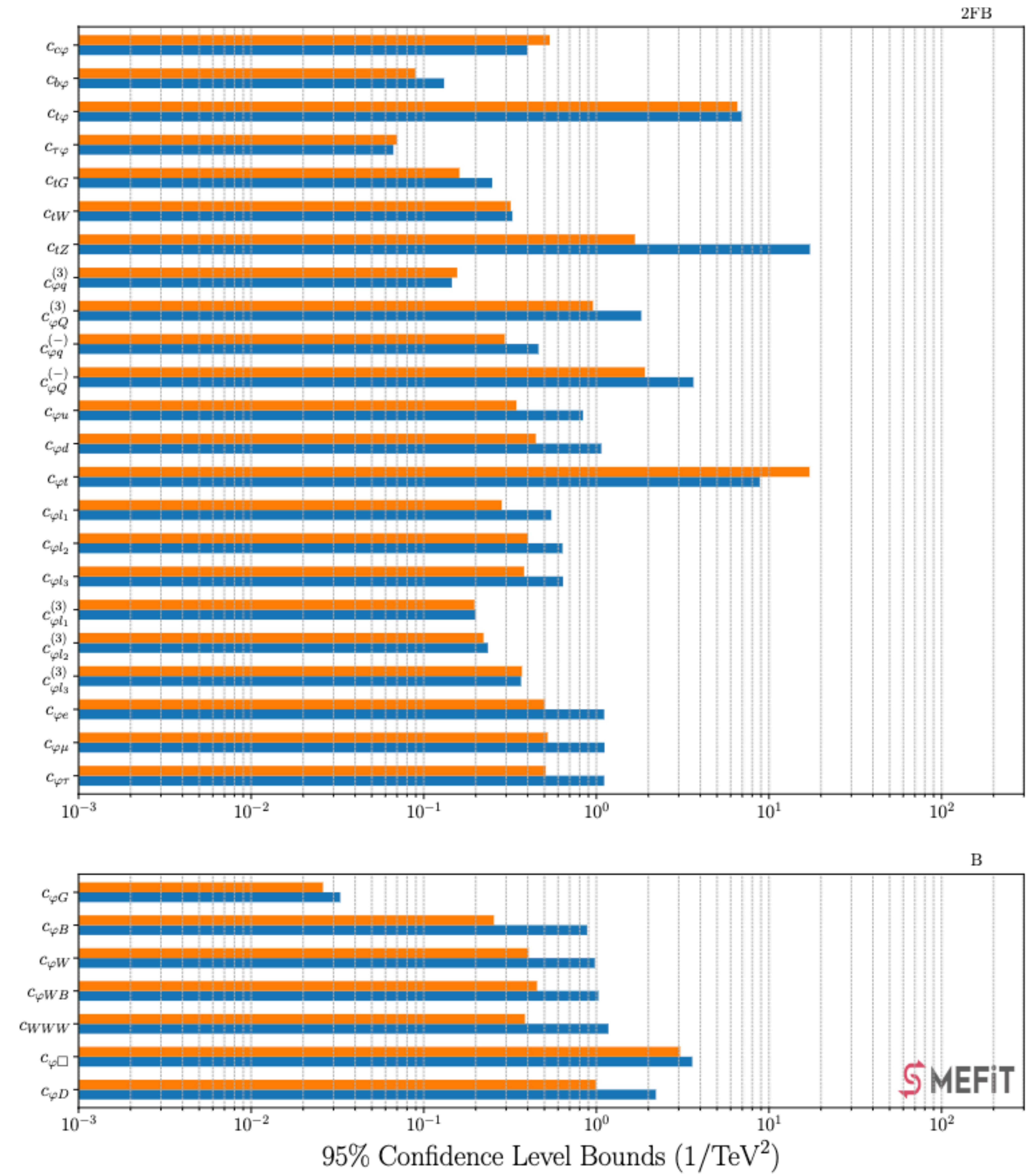
[Celada et al.; 2404.12809]

SMEFiT3.0

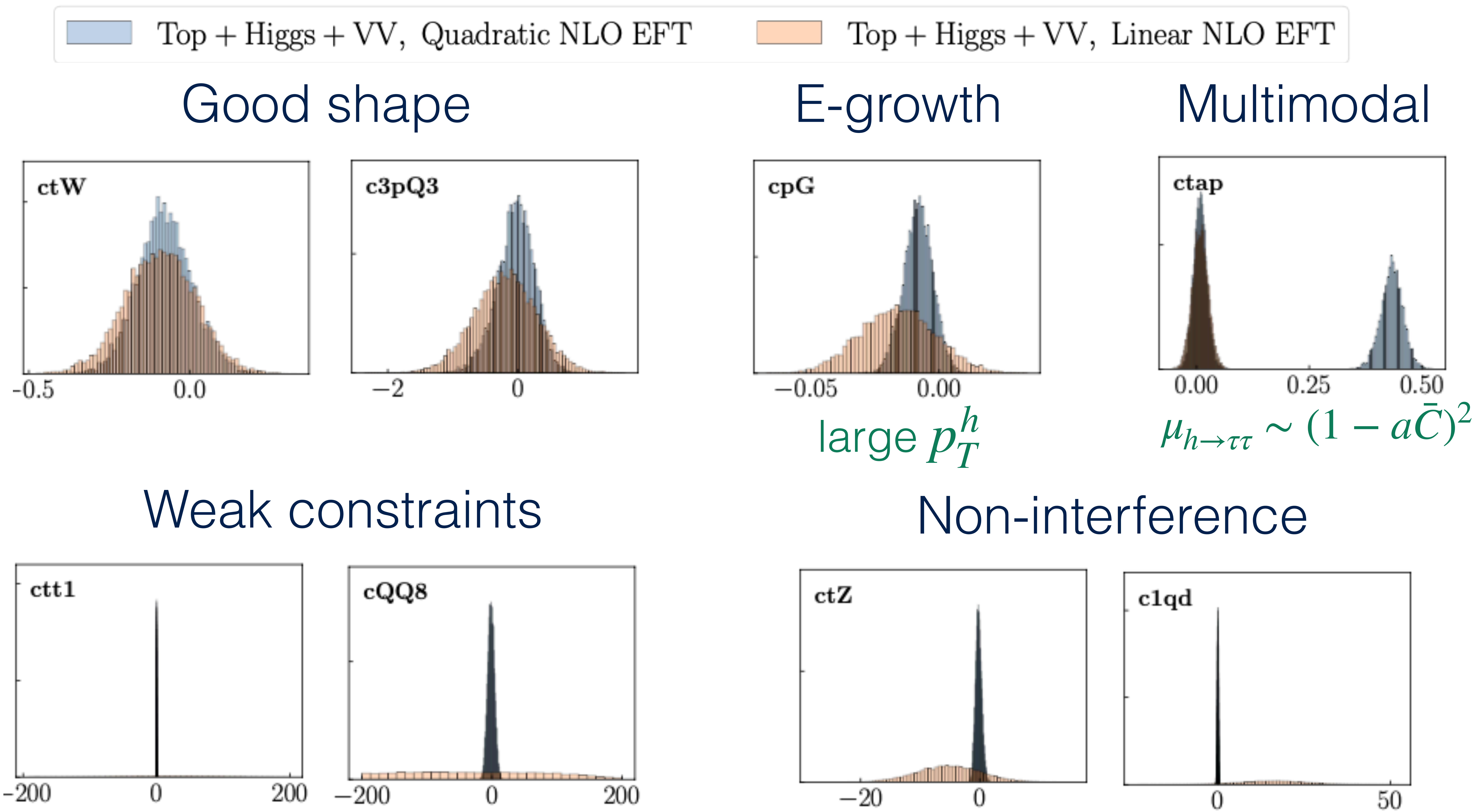
(See Eleni's talk)

The only thing we can do at dimension-6:
linear vs. quadratic





Dim-6: Λ^{-2} vs Λ^{-4}



$\sigma(t\bar{t}t\bar{t})$ $\sigma(t\bar{t}b\bar{b})$ $t\bar{t}Z$ helicity $t\bar{t}$ colour

New effects at dimension-8

Higher canonical dimension ⇒ new Lorentz structures

- More derivatives = higher energy growth

'Decorrelation' effects

- Anomalous QGC independent from TGC: beyond $\mathcal{O}_W = \epsilon_{IJK} W^{I,\nu}_{\mu} W^{J,\rho}_{\nu} W^{K,\mu}_{\rho}$
- h^4 independent from h^3: $(H^\dagger H)^3$ & $(H^\dagger H)^4$

New gauge self-interactions

$F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$ — *Light-by-light*

$G^A_{\mu\nu}G^{\mu\nu}_A V_{\rho\sigma}V^{\rho\sigma}, V = Z, \gamma, W^\pm$ — $gg \to VV$

$iH^\dagger \tilde{B}_{\mu\nu} W^{\mu\rho} \{D_\rho, D^\nu\} H$ — *Neutral TGC*

Much left to study

- Non-redundant basis is known *[Murphy; JHEP 10 (2020) 174]* *[Li et al.; PRD 104 (2020) 015026]*

Dim-8 & EFT validity

a) Model independent: study effects of dim-8 operators

- Understand where these could be relevant
- Global analyses up to dimension-8 (not yet…)
- Identify processes/observables that are uniquely sensitive
- Positivity connection: additional theory priors, inverse problem,…

[Boughezal, Mereghetti & Petriello; PRD 104 (2021) 9, 095022]
[Boughezal, Petriello & Wiegand; PRD104 (2021) 1, 016005]

b) Model dependent: *predict* effects of dim-8 operators

- Apply results of a) by making some assumptions about $C_i^{(8)}$
- Study classes of explicit UV models up to dimension-8

[Hays et al; JHEP 02 (2019) 123]
[Hays et al.; JHEP 11 (2020) 087]

Triplet scalar & dark photon [Corbett et al; JHEP 06 (2021) 076]
Vector-like quarks [Dawson, Homiller & Sullivan; PRD 104 (2021) 11, 115013]
2HDM [Dawson, et al; PRD 106 (2022) 5, 055012]
Z' [Dawson, Forslund & Schnubel; 2404.01375]
Various… [Corbett; 2405.04570]

Singlet scalar to dim-8

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu S \partial^\mu S - m_S S^2 - \kappa_S H^\dagger H S - \lambda_S H^\dagger H S^2 - \kappa_{S^3} S^3 - \kappa_{S^4} S^4$$

SM + real, singlet scalar, S

- S obtains a vev, v_S, in general & mixes with h after EWSB
- Potential parameters subject to theoretical constraints

$V(S,H)$ bounded from below $\quad \lambda_h, \kappa_{S^4} > 0, \quad \lambda_S > -2\sqrt{\lambda_h \kappa_{S^4}}$

Perturbative unitarity in h, S scatterings $\quad |\lambda_S| \leq 4\pi, \quad |\lambda_h| \leq \frac{8\pi}{3}, \quad |\kappa_{S^4}| \leq \frac{2\pi}{3}$

Global EW minimum $\quad V(v, v_s) \leq V(S,H)$

[Dawson, Giardino, Homiller; PRD 103 (2021) 7, 075016]
[Dawson et al.; PRD 106 (2022) 5, 055012]

[Jiang et al.; JHEP 02 (2019) 031]
[Haisch et al.; JHEP 04 (2020) 164]

Matching

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu S \partial^\mu S - m_S S^2 - \boxed{\kappa_S H^\dagger H S - \lambda_S H^\dagger H S^2 - \kappa_{S^3} S^3} - \kappa_{S^4} S^4$$

Tree-level: solve EoM for S and plug back into $\mathscr{L}$ $(m_S, \kappa_S, \kappa_{S^3} \gg v)$

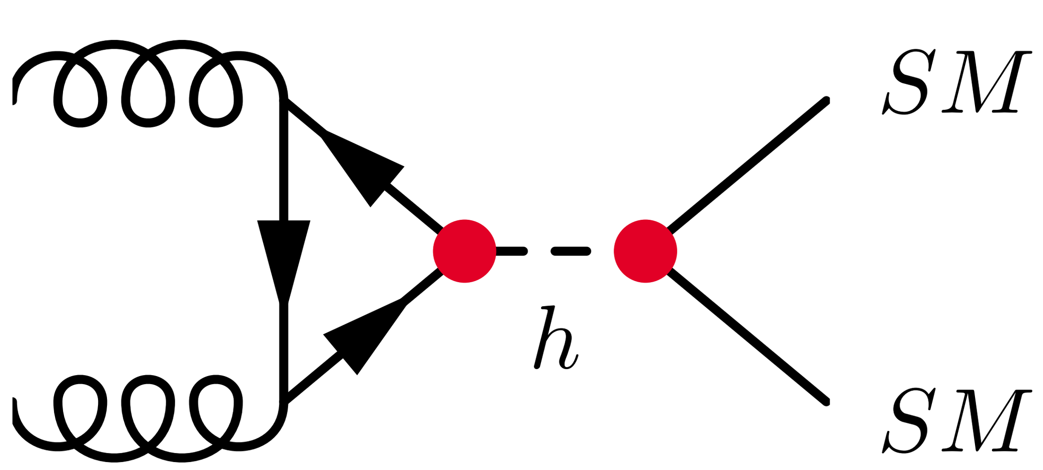
- Only 2 operators at dimension-6

e.g. *[de Blas et al.; JHEP 03 (2018) 109]*

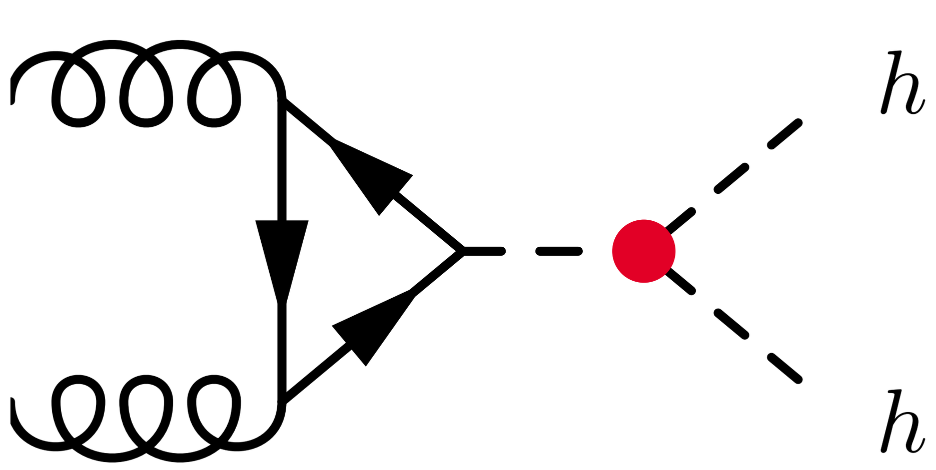
$$(H^\dagger H)\Box(H^\dagger H) : C_{H\Box} = -\frac{1}{2}\frac{\kappa_S^2}{m_S^2}$$

$$(H^\dagger H)^3 : C_H = -\frac{\kappa_S^2}{m_S^2}\left(\lambda_S - \frac{\kappa_S \kappa_{S^3}}{m_S^2}\right)$$

$\Rightarrow$ ***All Higgs couplings***
Single Higgs production & decay



$\Rightarrow$ ***Higgs self-coupling***
Higgs pair production



To dim-8…

[Neglecting y_d, y_e]

Dim - 4	C_{H4}	$\frac{(\kappa_s)^2}{2m_S^2}\left(1-\frac{4\mu^4}{m_S^4}\right)$	$(H^\dagger H)^2$
Dim - 6	C_H	$-\frac{(\kappa_s)^2}{m_S^2}\left(\lambda_s\left(1-\frac{4\mu^2}{m_S^2}\right)-\frac{\kappa_s\kappa_{s3}}{m_S^2}\left(1-\frac{6\mu^2}{m_S^2}\right)\right)$	$(H^\dagger H)^3$
	$C_{H\Box}$	$-\frac{(\kappa_s)^2}{2m_S^2}\left(1-\frac{4\mu^2}{m_S^2}\right)$	$(H^\dagger H)\Box(H^\dagger H)$
Dim - 8	C_{H^8}	$\frac{(\kappa_s)^2}{m_S^2}\left(2(2\lambda_h-\lambda_S)^2+6\frac{\kappa_S\kappa_{S3}}{m_S^2}(2\lambda_h-\lambda_S)+\frac{(\kappa_s)^2}{m_S^2}\left(\frac{9}{2}\frac{\kappa_{S3}^2}{m_S^2}-\kappa_{S^4}\right)\right)$	$(H^\dagger H)^4$
	$C_{H^6}^{(1)}$	$-\frac{4(\kappa_s)^2}{m_S^2}\left(2\lambda_h-\lambda_S+\frac{3}{2}\frac{\kappa_{S3}^2}{m_S^2}\right)$	$(H^\dagger H)^2\lvert D_\mu H\rvert^2$
	$C_{H^4}^{(3)}$	$\frac{2(\kappa_s)^2}{m_S^2}$	$\lvert D_\mu H\rvert^4$
	C_{quH^5}	$-\frac{2(\kappa_s)^2}{m_S^2}\left(2\lambda_h-\lambda_S+\frac{3}{2}\frac{\kappa_{S3}^2}{m_S^2}\right)y_u$	$(H^\dagger H)^2\bar{Q}u\tilde{H}$
	$C_{q^2u^2H^2}^{(1)}$	$-\frac{(\kappa_s)^2}{2m_S^2}\lvert y_u\rvert^2$	$(H^\dagger H)(\bar{Q}\gamma^\mu Q)(\bar{u}\gamma^\mu u)$
	$C_{q^2u^2H^2}^{(5)}$	$\frac{(\kappa_s)^2}{2m_S^2}(y_u)^2$	$(\bar{Q}u\tilde{H})^2$
	$C_{quH^3D^2}$	$-\frac{2(\kappa_s)^2}{m_S^2}y_u$	$\lvert D_\mu H\rvert^2\bar{Q}u\tilde{H}$

Sensitivity from dim-8

C_{H^8}	$\frac{(\kappa_s)^2}{m_S^2}\Big(2(2\lambda_h-\lambda_S)^2+6\frac{\kappa_S\kappa_{S3}}{m_S^2}(2\lambda_h-\lambda_S)+\frac{(\kappa_s)^2}{m_S^2}\Big(\frac{9}{2}\frac{\kappa_{S3}^2}{m_S^2}-\kappa_{S^4}\Big)\Big)$
$C_{H^6}^{(1)}$	$-\frac{4(\kappa_s)^2}{m_S^2}\Big(2\lambda_h-\lambda_S+\frac{3}{2}\frac{\kappa_{S3}^2}{m_S^2}\Big)$
C_{quH^5}	$-\frac{2(\kappa_s)^2}{m_S^2}\Big(2\lambda_h-\lambda_S+\frac{3}{2}\frac{\kappa_{S3}^2}{m_S^2}\Big)y_u$

+ others…

$O_{H^8} : (H^\dagger H)^4$ ⇒ ***Higgs trilinear & quartic-couplings*** *Higgs pair & triple Higgs production*

$O_{H^6}^{(1)} : (H^\dagger H)^2 |D_\mu H|^2$ ⇒ ***All Higgs couplings & Higgs ZZ/WW*** *Single Higgs production & decay*

$O_{quH^5} : (H^\dagger H)^2 \bar{Q} u \tilde{H}$ ⇒ ***Top quark Yukawa coupling*** *Gluon fusion & $t\bar{t}H$*

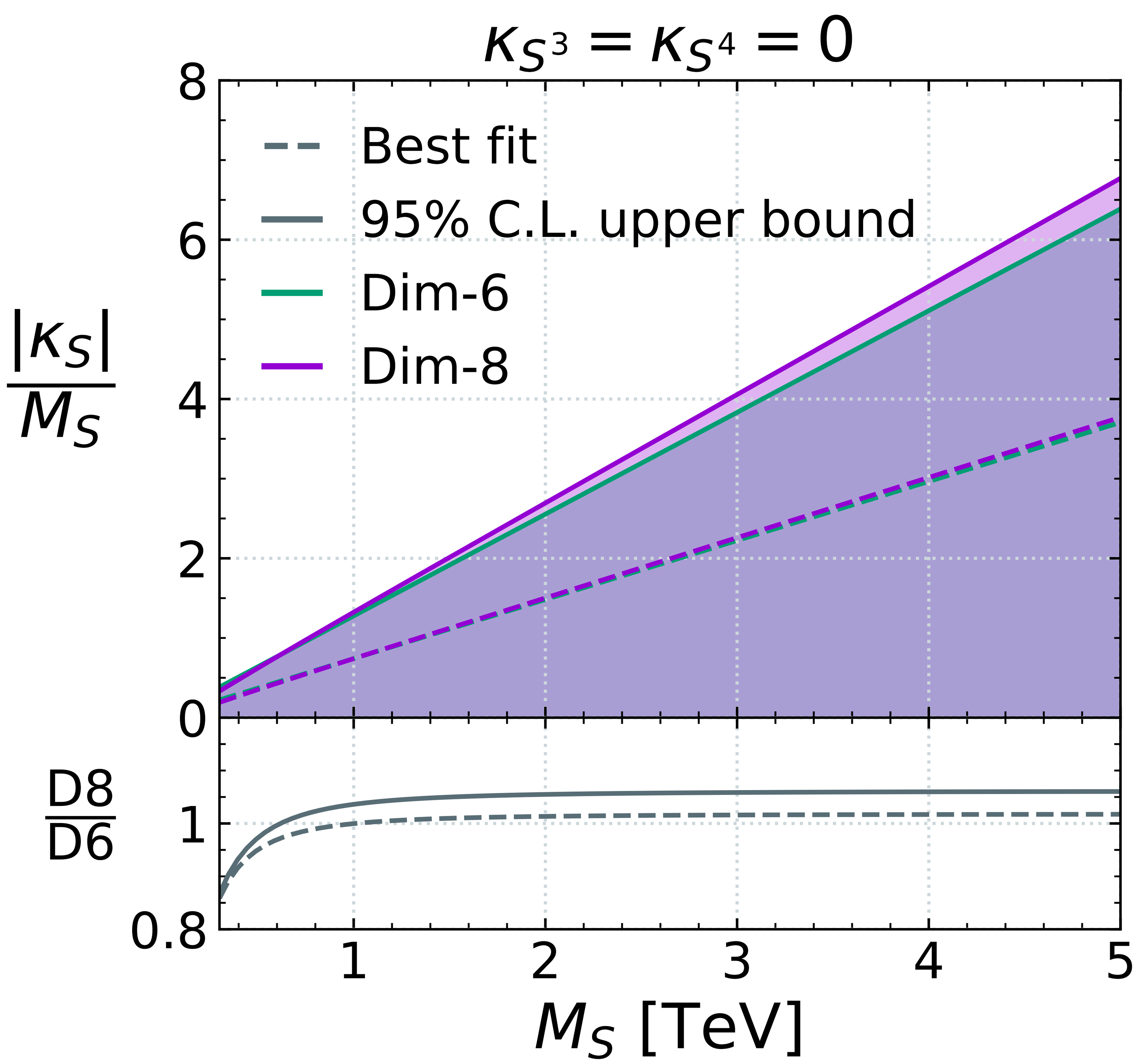
- New sensitivity to λ_S, κ_{S^3} in single Higgs production & decay
- New sensitivity to κ_{S^4} in Higgs pair production

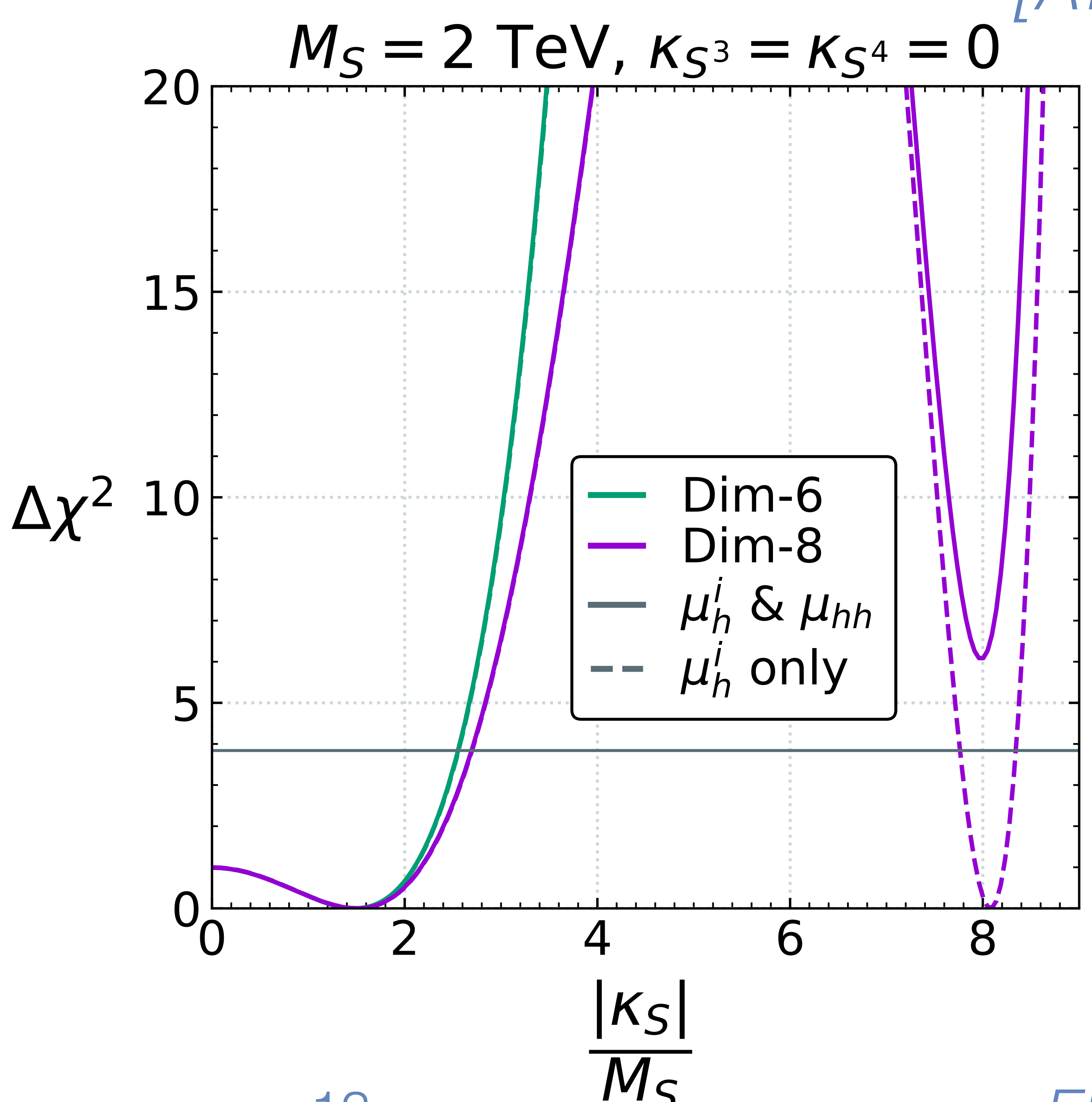
Constraints from h & hh

Global SMEFT fit of Higgs data to dimension-8

- Using **fitmaker** code *[Ellis, Madigan, KM, Sanz & You; JHEP 04 (2021) 279]*
- Latest Higgs signal strengths + di-Higgs cross section
- Mapped to singlet parameter space

[ATLAS; Nature 607 (2022) 52-59]
[CMS; Nature 607 (2022) 60-68]
[ATLAS-CONF-2022-050]

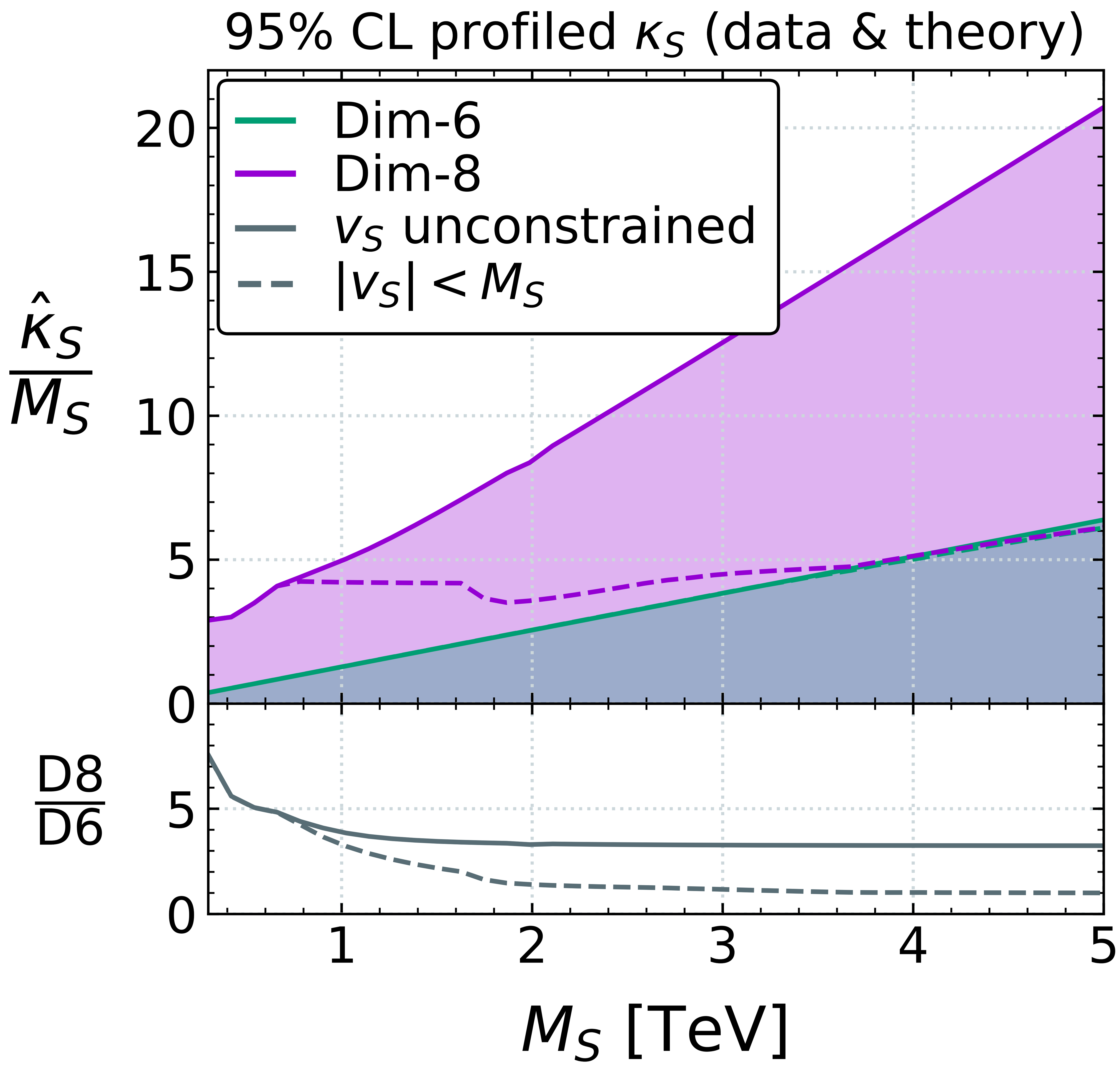




$\kappa_S H^\dagger H S$

$M_S = 1$ TeV, $\kappa_{S^3} = \kappa_{S^4} = 0$

Profiled κ_S bound



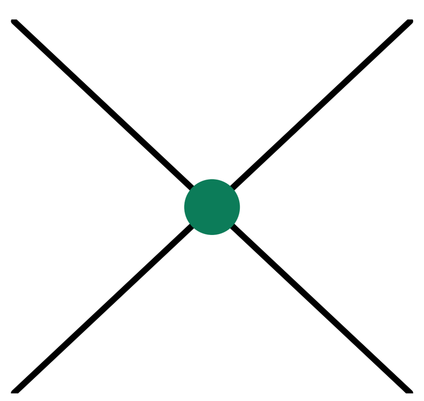
- Data + theory still allow large κ_S
- Much of allowed parameter space in regions where $v_S \gg M_S$
- Cannot be described by EFT, which assumes small v_S… validity?
- Repeat analysis requiring $v_S < M_S$
- Smaller region allowed & D8 result converges to D6
- Still some significant differences for $M_s \lesssim 3$ TeV

Positivity

[Pham & Troung; PRD 31 (1985 3027)]
[Anathanarayan et al.; PRD 51 (1995) 1093-1100]
[Adams et al.; JHEP 10 (2006) 014]

Can we bound EFT coefficients for theory alone?

- **Yes**, *e.g.*, partial wave unitarity in the EFT

$c_i \frac{s}{\Lambda^2}$ 

$$S^\dagger S = 1 \rightarrow i(T^\dagger - T) = T^\dagger T$$

$$M(s,\theta) = 16\pi \sum_\ell (2\ell + 1)\, a_\ell(s)\, P_\ell(\cos\theta)$$

$\Rightarrow$ $\mathrm{Im}(a_i) < |a_i|^2$, $c_i \frac{s}{\Lambda^2} < 16\pi$

We can do more with some reasonable assumptions

- Causal, unitarity & local (yet unknown) UV
- Analytic properties of scattering amplitudes in complex s-plane

EFT (IR) $\frac{1}{2}\frac{d^2 M(0)}{ds^2} = \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi i \mu^3} \mathrm{Im}\left(M(\mu)\right) > 0$ UV $\Rightarrow$ $\sum_i b_i C_i^{(8)} > 0$

"theory prior" on EFT space **OR** testing QFT axioms of UV

Positivity cone

$$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi\mu^3} \left(m_{ij} m_{kl}^* + m_{i\tilde{l}} m_{k\tilde{j}}^* \right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

Simplest case: elastic scattering $\mathscr{A}_{ij\to ij} \Rightarrow \frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} \geq 0$

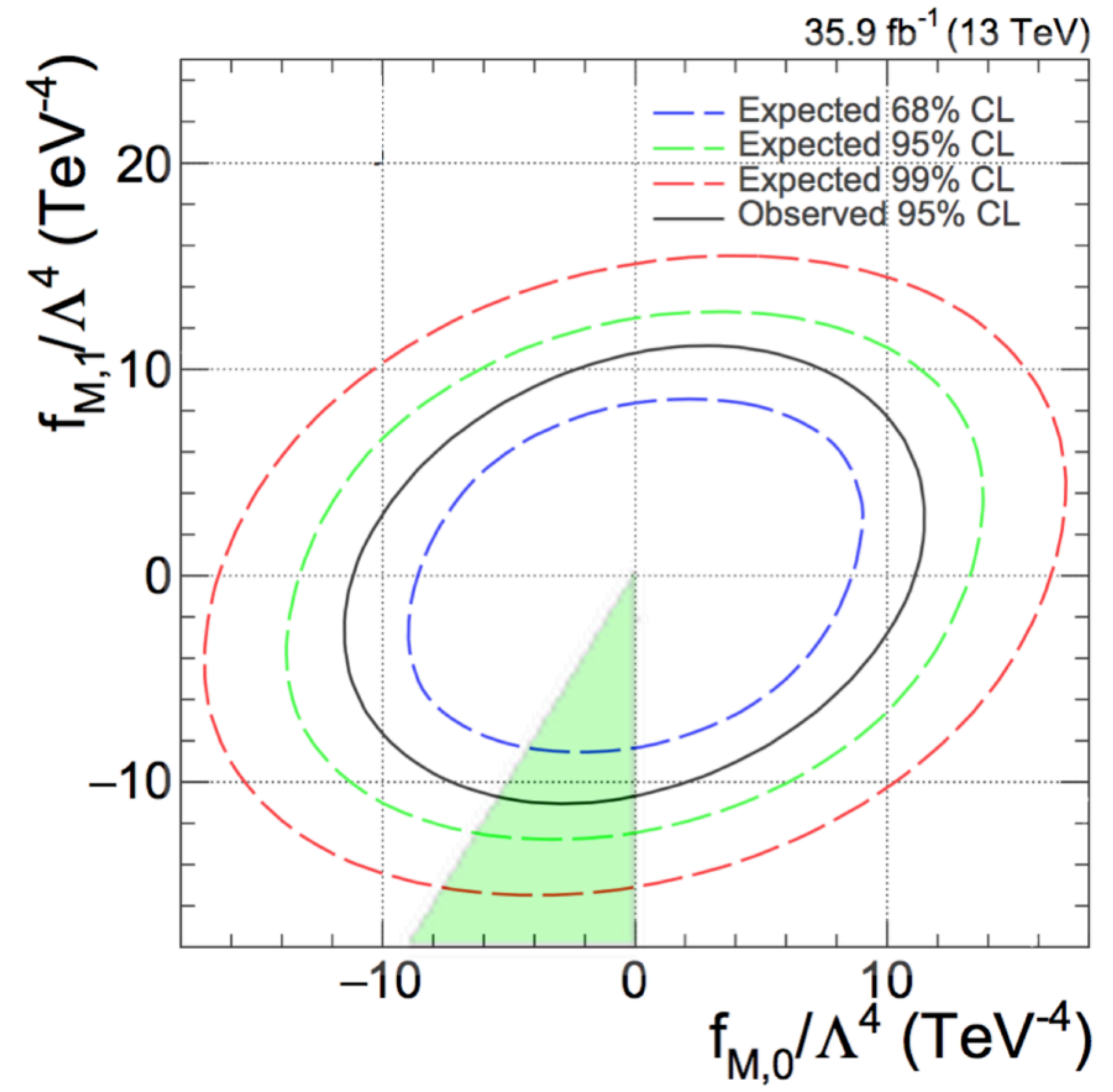
Beyond elastic scattering: S-matrix forms a cone

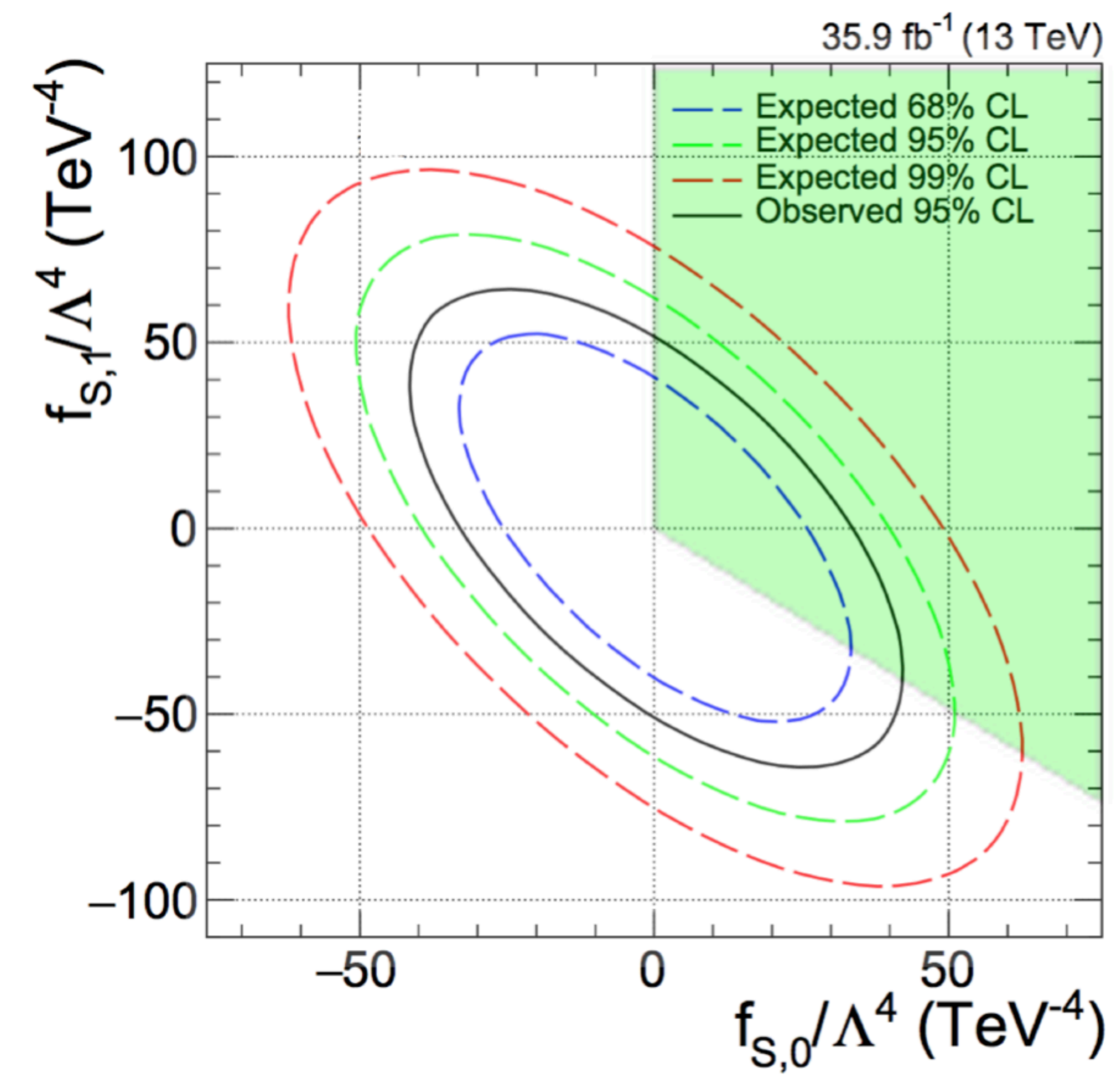
Finding optimal bounds is a solved (numerical) problem

Vector boson scattering

[Bi, Zhang, Zhou; JHEP 06 (2019) 137]

$$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,2} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi]$$
$$O_{M,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times [(D_\beta\Phi)^\dagger D^\beta\Phi]$$
$$O_{M,1} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times [(D_\beta\Phi)^\dagger D^\mu\Phi]$$





Positivity ⇔ experiment

Assume positivity: UV completion is a normal QFT

- Use bounds as a theory prior ⇒ enhanced global sensitivity

Test positivity: is the UV completion a normal QFT?

- "Test the fundamental axioms of QFT"
- Find dedicated processes/observables

Example: Drell-Yan angular distributions

$$\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}d\Omega_\ell} = \frac{3}{16\pi}\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}}\Bigg[(1+c_\theta^2) + \frac{\tilde{A}_0}{2}(1-3c_\theta^2) + \tilde{A}_1 s_{2\theta}c_\phi + \frac{\tilde{A}_2}{2}s_\theta^2 c_{2\phi} + \tilde{A}_3 s_\theta c_\phi + \tilde{A}_4 c_\theta + \tilde{A}_5 s_\theta^2 s_{2\phi} + \tilde{A}_6 s_{2\theta}s_\phi + \tilde{A}_7 s_\theta s_\phi\Bigg]$$

- SM: Spin-1 photon & Z-boson $\to l \leq 2$ angular dependence
- LO is ϕ symmetric: $\tilde{A}_{1,4} \neq 0$, NLO: $\tilde{A}_{1-7} \neq 0$

Higher moments

[Alioli et al.; PLB 809 (2020) 135703]
See also:
[Banerjee et al.; JHEP 09 (2020) 170
& JHEP 06 (2021) 013]

$$\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}d\Omega_\ell} = \frac{3}{16\pi}\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}}\Bigg[(1+c_\theta^2) + \frac{\tilde{A}_0}{2}(1-3c_\theta^2) + \tilde{A}_1 s_{2\theta}c_\phi$$

$l \leq 2$

$$+\frac{\tilde{A}_2}{2}s_\theta^2 c_{2\phi} + \tilde{A}_3 s_\theta c_\phi + \tilde{A}_4 c_\theta + \tilde{A}_5 s_\theta^2 s_{2\phi} + \tilde{A}_6 s_{2\theta} s_\phi + \tilde{A}_7 s_\theta s_\phi$$

$l = 3$

$$+\frac{\tilde{B}_1^e}{2}s_\theta\left(5c_\theta^2-1\right)c_\phi + \frac{\tilde{B}_1^o}{2}s_\theta\left(5c_\theta^2-1\right)s_\phi + \boxed{\frac{\tilde{B}_0}{2}\left(5c_\theta^3-3c_\theta\right)}$$

$$+\tilde{B}_3^e s_\theta^3 c_{3\phi} + \tilde{B}_3^o s_\theta^3 s_{3\phi} + \tilde{B}_2^e s_\theta^2 c_\theta c_{2\phi} + \tilde{B}_2^o s_\theta^2 c_\theta s_{2\phi}\Bigg]$$

$\tilde{B}_i$ coefficients: $Y_{3,m}$ spherical harmonics

- Only populated by certain class of dim-8 4F operators $\mathscr{A}_{SM}\mathscr{A}_{EFT} \sim \cos^3\theta$
- At LO, no SM or dim-6 contribution
- Dominant moment $\tilde{B}_0$: $Y_{3,0}$ spherical harmonic (no ϕ dependence)
- Clean probe of dim-8 effects in Drell Yan

Which operators

Relevant dim-8 operators: $\mathcal{A}(q\bar{q} \to \ell^+\ell^-) \sim t^2$

- Two-derivative 4F operators, $\psi^4 D^2$
- No additional Higgs fields (powers of E, not v)

$$O_{8,lq\partial 3} = (\bar{\ell}\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{q}\gamma^\mu \overleftrightarrow{D}^\nu q) \qquad O_{8,lq\partial 4} = (\bar{\ell}\tau^I\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{q}\tau^I\gamma^\mu \overleftrightarrow{D}^\nu q)$$
$$O_{8,ed\partial 2} = (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{d}\gamma^\mu \overleftrightarrow{D}^\nu d) \qquad O_{8,eu\partial 2} = (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)$$
$$O_{8,ld\partial 2} = (\bar{\ell}\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{d}\gamma^\mu \overleftrightarrow{D}^\nu d) \qquad O_{8,lu\partial 2} = (\bar{\ell}\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)$$
$$O_{8,qe\partial 2} = (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{q}\gamma^\mu \overleftrightarrow{D}^\nu q)$$

- Other class of $\psi^4 D^2$: $(\bar{\ell}\gamma_\mu \ell)\partial^2(\bar{q}\gamma^\mu q) \Rightarrow \mathcal{A}(q\bar{q} \to \ell^+\ell^-) \sim s^2$

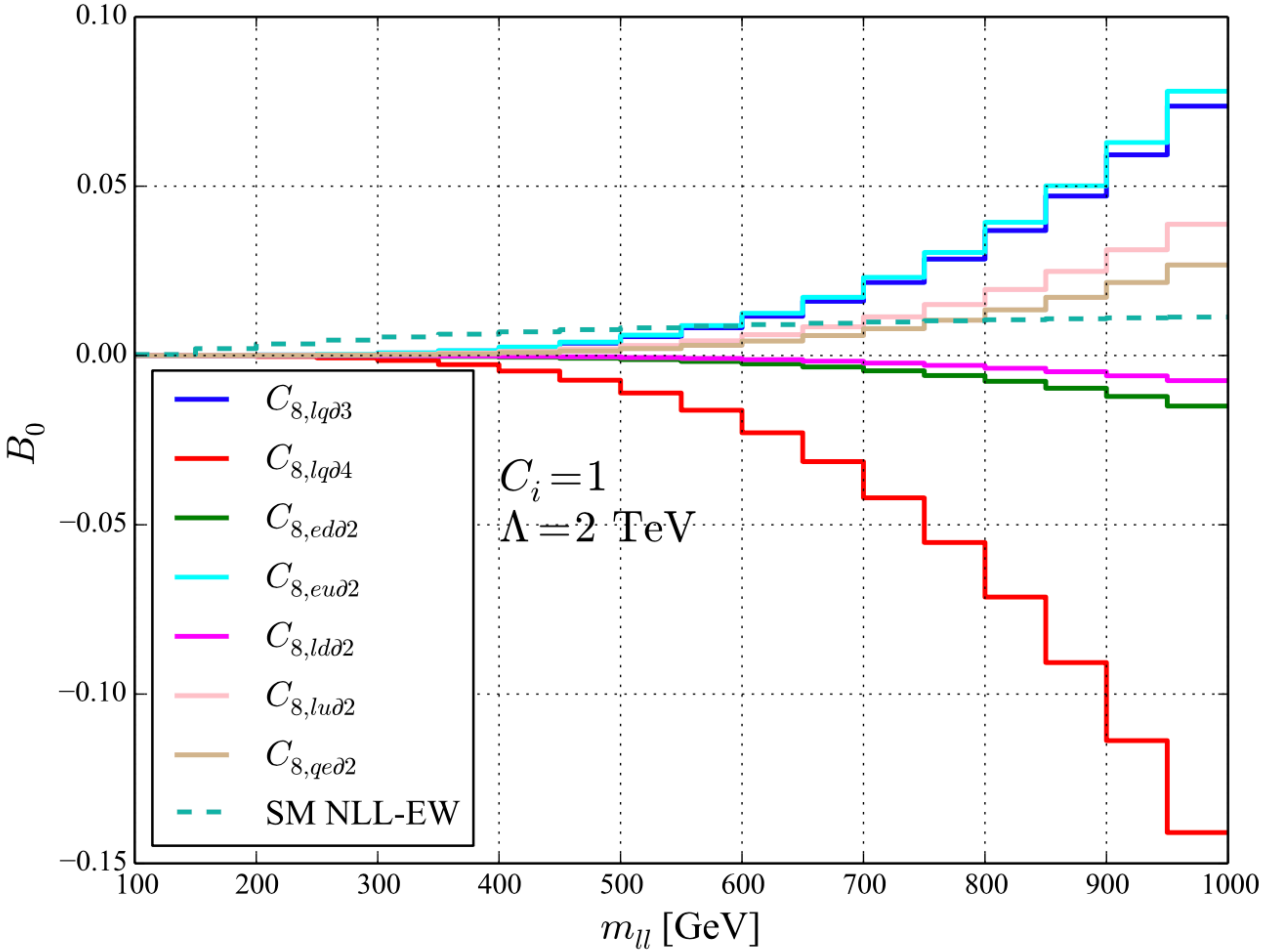
No new angular dependence

Crossing symmetry: $\mathcal{A}(q\ell \to q\ell) \sim s^2!$

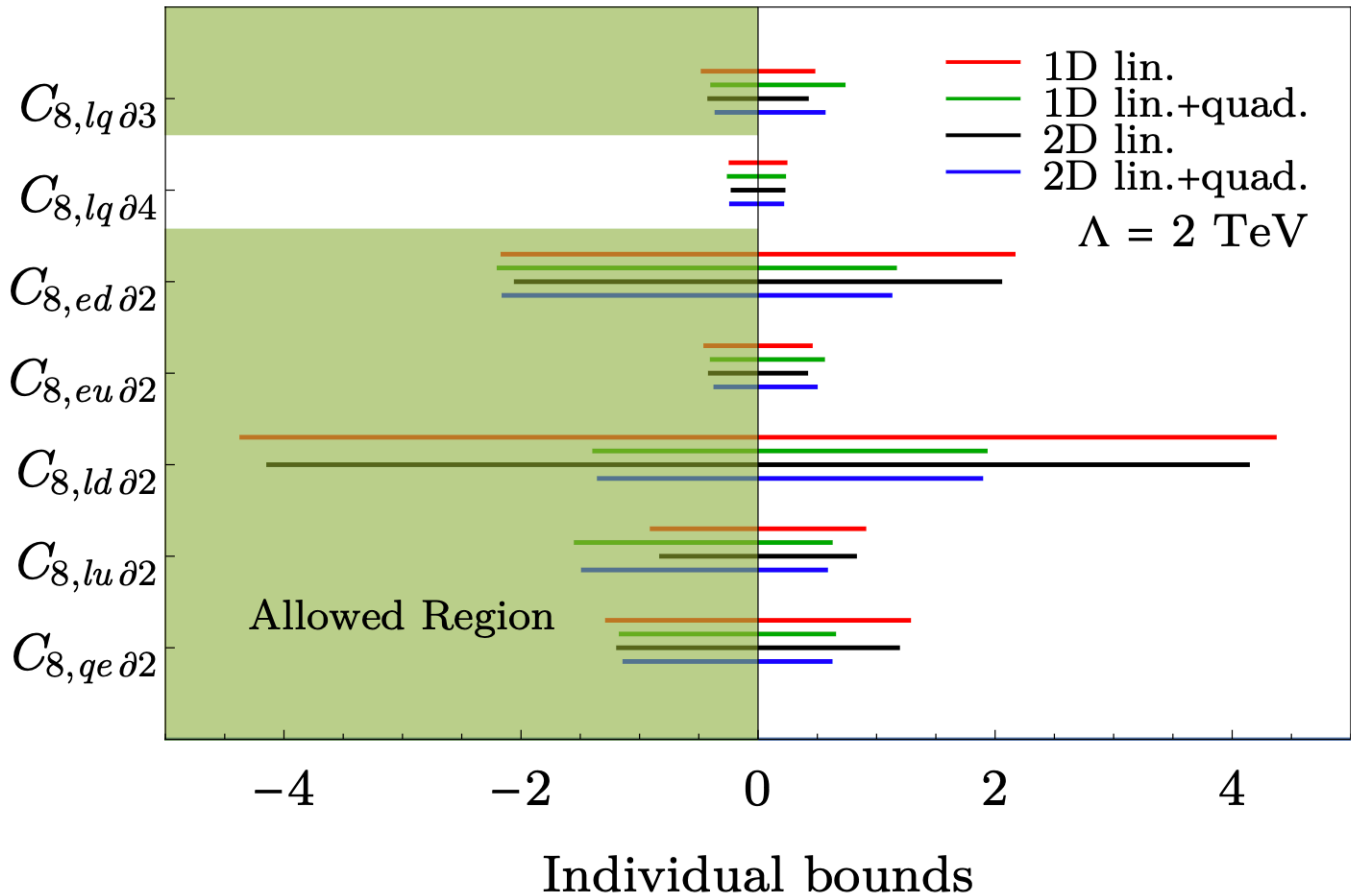
Higher moments in $q\bar{q} \to \ell^+\ell^- \Leftrightarrow$ Positivity bounds

[Li, KM, Yamashita, Yang, Zhang, Zhou; JHEP 10 (2022) 107]

$\tilde{B}_0(m_{\ell^+\ell^-})$



Individual bounds on C_i



A priori restricted parameter space to consider

Can also be used to search for violations of positivity

Connection to the "inverse problem"

More information?

$$\frac{1}{2}\frac{d^2M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi\mu^3}\left(m_{ij}m_{kl}^* + m_{i\tilde{l}}m_{k\tilde{j}}^*\right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

Positivity cone uses "half" of UV amplitude information

- Partial wave coefficients, $a_{ijkl}(\mu)$, are also bounded from above
- In addition to $s \leftrightarrow u$ crossing symmetry, we have $s \leftrightarrow t$

$$0 < \rho_\ell^{iiii} \leq 2$$

$$\rho_\ell^{ijkl} \equiv \mathrm{Im}[a_\ell^{ijkl}]$$

$$\rho_\ell^{ijkl} = (-1)^\ell \rho_\ell^{jikl} = (-1)^\ell \rho^{ijlk}$$

$s \leftrightarrow t$ crossing leads to a series of null constraints

$$0 = \sum_\ell 16(2\ell+1)\int_{\Lambda^2}^{\infty}\frac{d\mu}{\mu^{r+4}}\Big[C_{r,i_r}(\ell)\rho_\ell^{ijkl}(\mu) + D_{r,i_r}(\ell)\rho_\ell^{ijlk}(\mu) + E_{r,i_r}(\ell)\rho_\ell^{ikjl}(\mu) + F_{r,i_r}(\ell)\rho_\ell^{iklj}(\mu) + G_{r,i_r}(\ell)\rho_\ell^{iljk}(\mu) + H_{r,i_r}(\ell)\rho_\ell^{ilkj}(\mu)\Big]$$

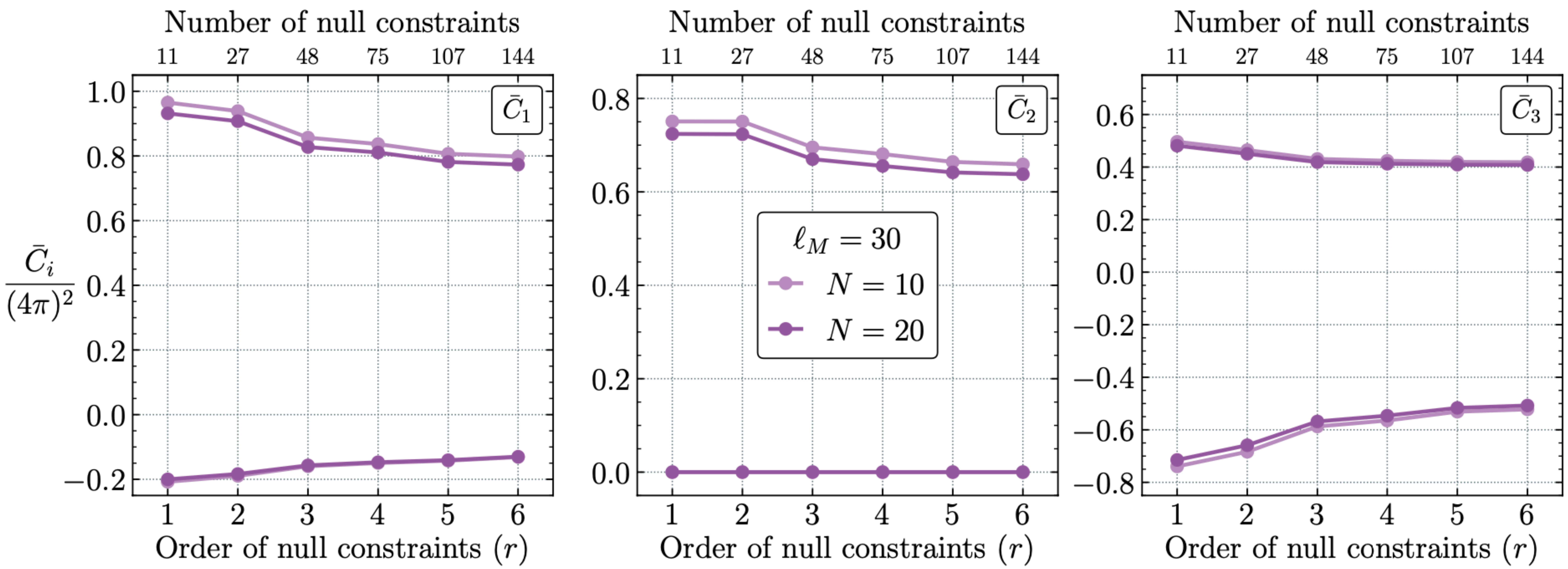
Capping the cone

Implement as a linear programming problem

- Discretising μ and summing over large, finite number of ℓ
- Numerically maximise $c^{m,n}_{ijkl}$ coefficients of EFT expansion of $\mathscr{A}_{ijkl}(s,t)$

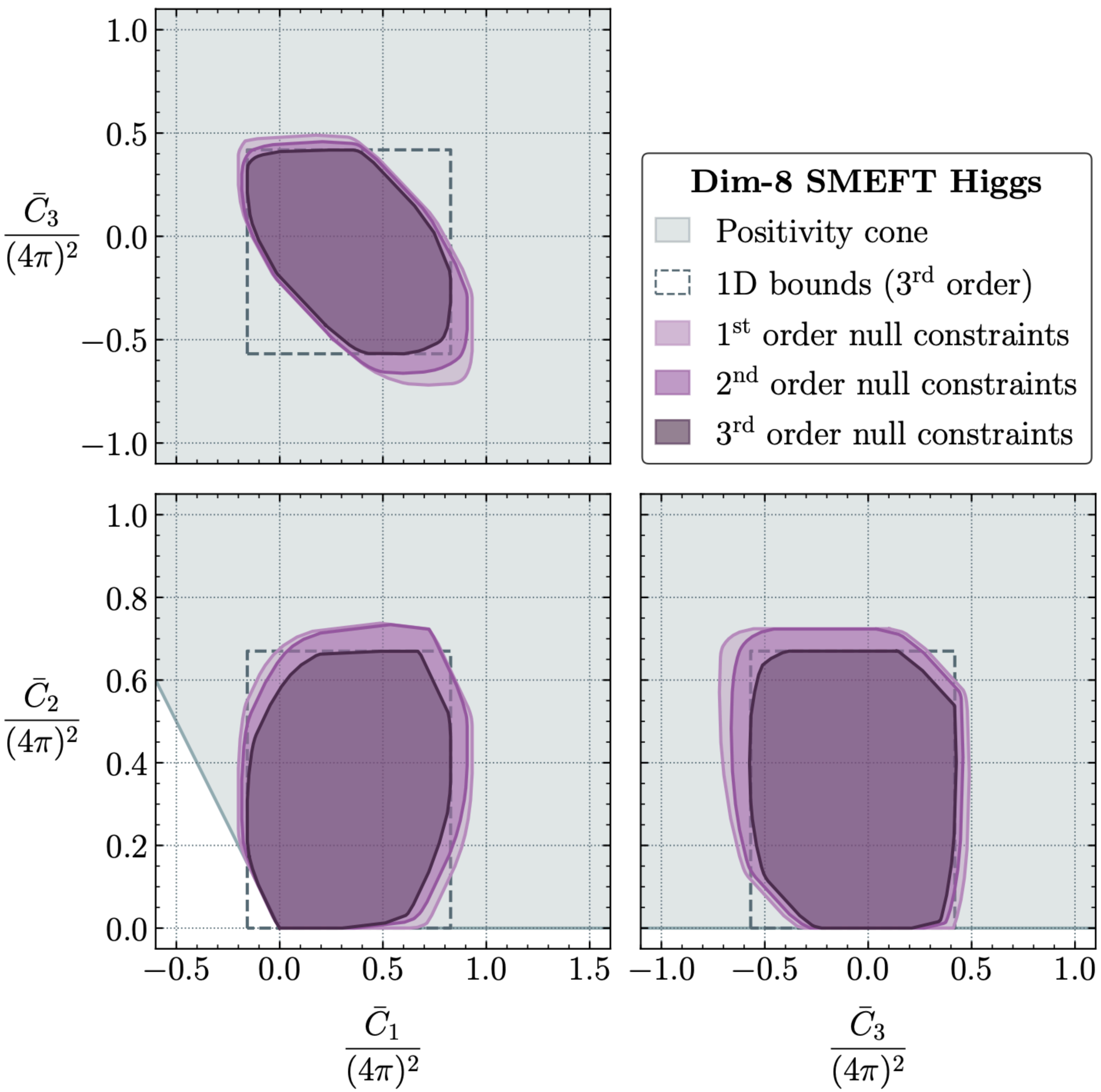
$c^{2,0}_{ijkl}$: Higgs operators at dimension 8:

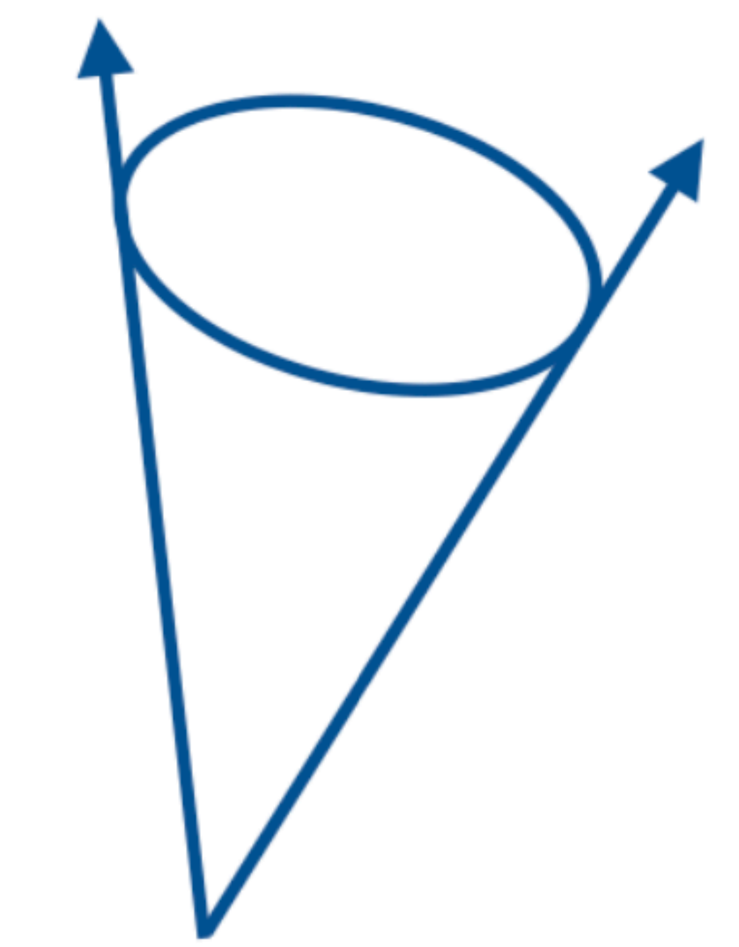
$$\mathcal{O}^{(1)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H) \quad \mathcal{O}^{(2)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H) \quad \mathcal{O}^{(3)}_{H^4} = (D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$$



$$-\frac{0.130}{\Lambda^4} \le \frac{C_1}{(4\pi)^2} \le \frac{0.774}{\Lambda^4}$$
$$0 \le \frac{C_2}{(4\pi)^2} \le \frac{0.638}{\Lambda^4}$$
$$-\frac{0.508}{\Lambda^4} \le \frac{C_3}{(4\pi)^2} \le \frac{0.408}{\Lambda^4}$$

Capping the cone

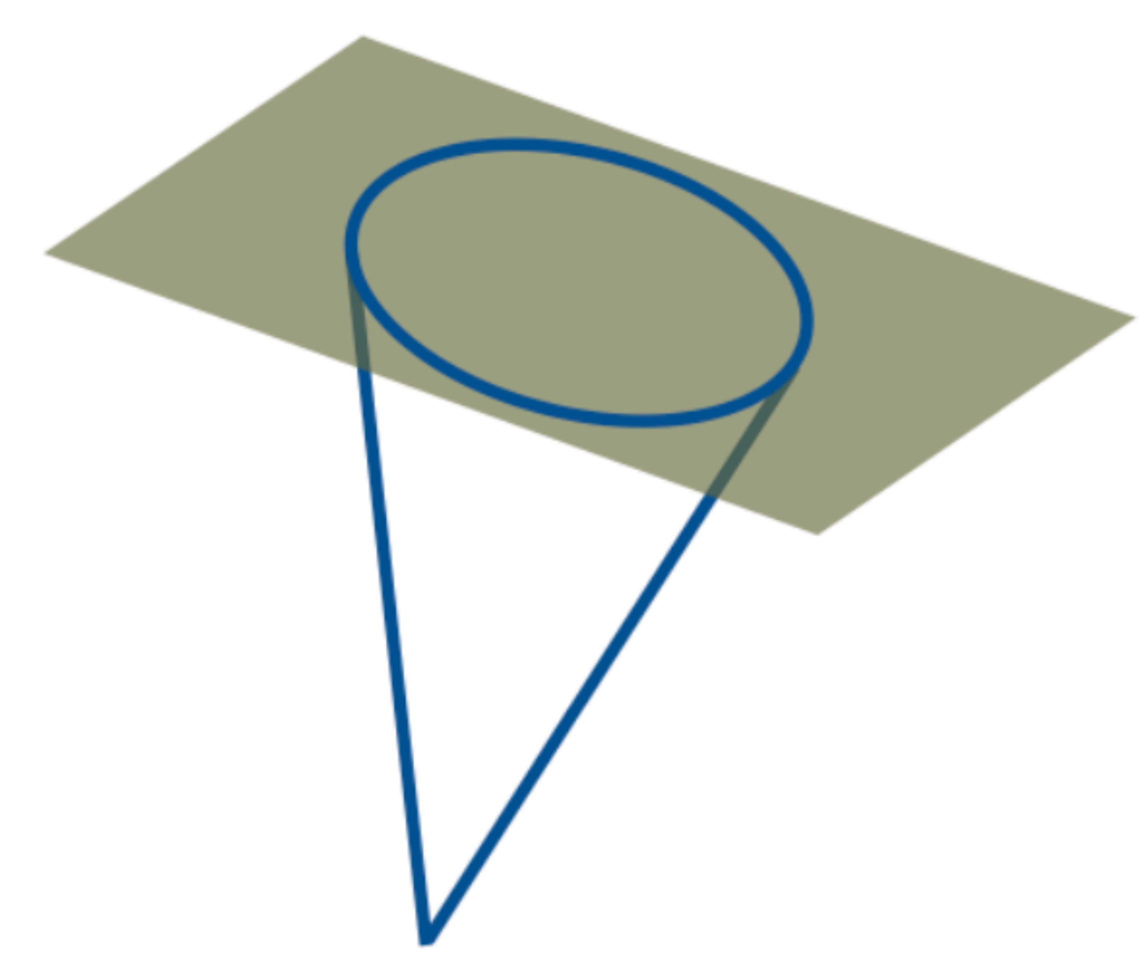




positivity part of UV unitarity

$(\mathrm{Im}\, a_\ell^{ijkl} \succeq 0)$

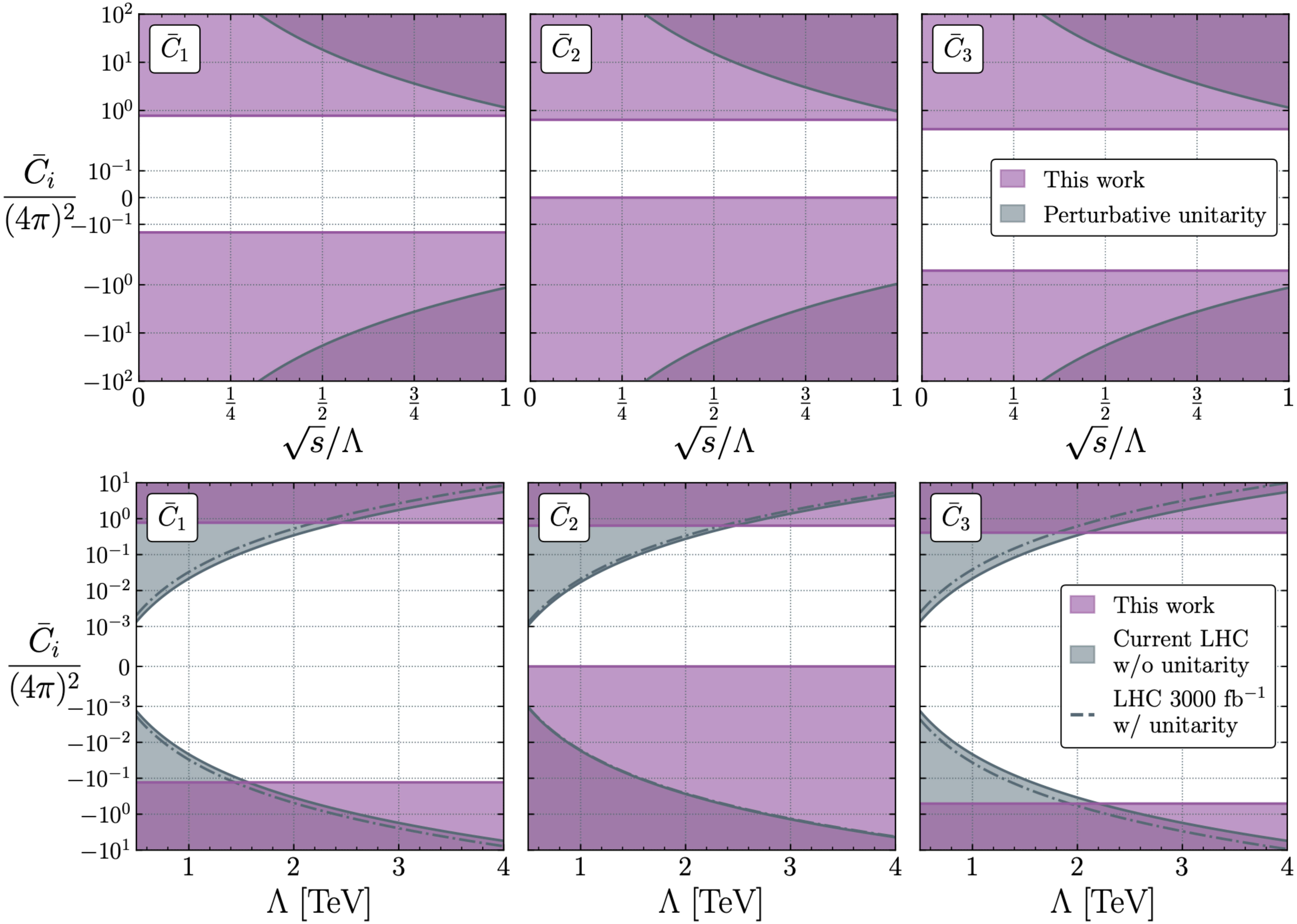
$\Downarrow$



fuller use of UV unitarity

+ null constraints

Comparisons



Perturbative unitarity in the EFT

[Almeida, Eboli & Gonzelez-Garcia; PRD 101 (2020) 11, 113003]

HL-LHC projections from VBS

[Capati et al.; JHEP 09 (2022) 038]

(See R. Covarelli's talk)

Conclusions

Dimension-8 effects are phenomenologically interesting

- EFT validity question & unique BSM signatures
- Connection to positivity & inverse problem *[Zhang; JHEP 12 (2022) 096]*

Instructive to study explicit UV completions

- Dim-8 brings richer sensitivity to underlying model parameters
- Identify regions where the EFT expansion may be breaking down

Use the information offered by positivity

- Lower & upper bounds $\Rightarrow$ theory prior for statistical analyses
- Test the fundamental axioms of QFT

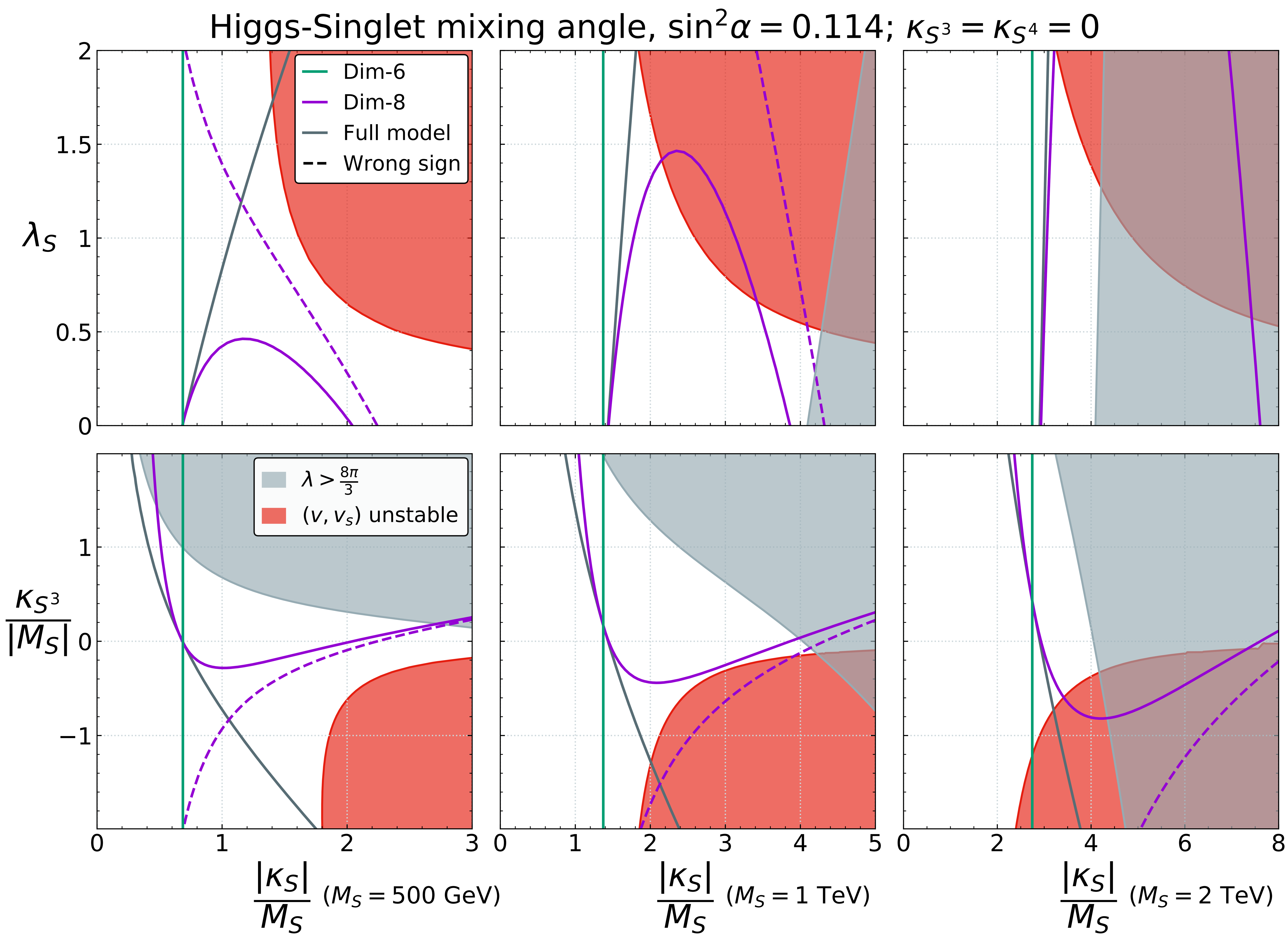
Unique effects: higher angular moments in $\mathscr{A}(2 \to 2)$

- I expect that there are multi boson analogues

Backup

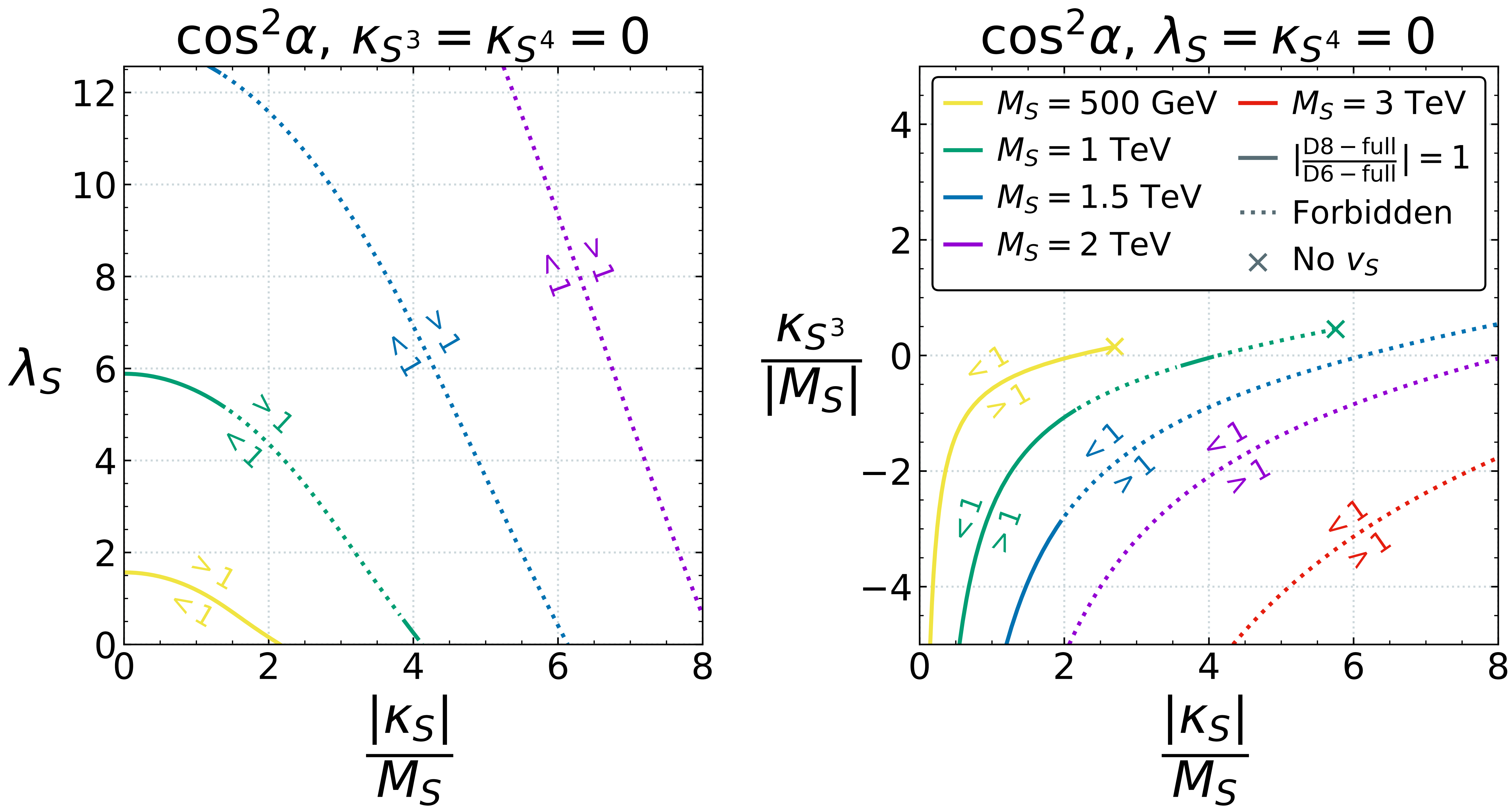
Validity

Current sensitivity from data



Validity

$$\delta = \left| \frac{(\cos^2\alpha)_{\text{D8}} - (\cos^2\alpha)_{\text{full}}}{(\cos^2\alpha)_{\text{D6}} - (\cos^2\alpha)_{\text{full}}} \right|$$



Some regions where dimension-8 improves predictions

Others where SMEFT expansion breaks down

- Large couplings, λ_S & κ_{S^3}, non-perturbative region
- Depends on M_S

Positivity for pedestrians

Set of theoretical constraints on scattering amplitudes

- Apply to a subset of $D \geq 8$ Wilson coefficients

Result from basic assumptions about UV QFT/S-matrix

- Lorentz invariance, unitarity, causality & locality

  [Pham & Troung; PRD 31 (1985 3027)]
  [Anathanarayan et al.; PRD 51 (1995) 1093-1100]
  [Adams et al.; JHEP 10 (2006) 014]

  + many more in recent years…

Unitarity ⇔ conservation of probability in full theory

- Generalised optical theorem: scattering amplitude $\mathcal{M}_{ij\to kl}$ satisfies

$$\frac{1}{2i}\left(\mathcal{M}_{ij\to kl} - \mathcal{M}^*_{kl\to ij}\right) = \frac{1}{2}\sum_X \int d\Pi_X\, \mathcal{M}_{ij\to X}\mathcal{M}^*_{kl\to X}$$

Elastic case: $\mathrm{Im}\mathcal{M}_{ij\to ij} = \frac{1}{2}\sum_X \int d\Pi_X\, |\mathcal{M}_{ij\to X}|^2 > 0$ ⇒ Elastic positivity bounds

Analyticity

Tree-level dimension 8 $\Rightarrow$ highest growth: s^2, st, t^2

- We will be taking 2 derivatives of w.r.t. s $\Rightarrow$ set $t = 0$ w.l.o.g.

Causality $\Rightarrow$ $\mathscr{M}(s, t = 0)$ analytic in the complex s plane

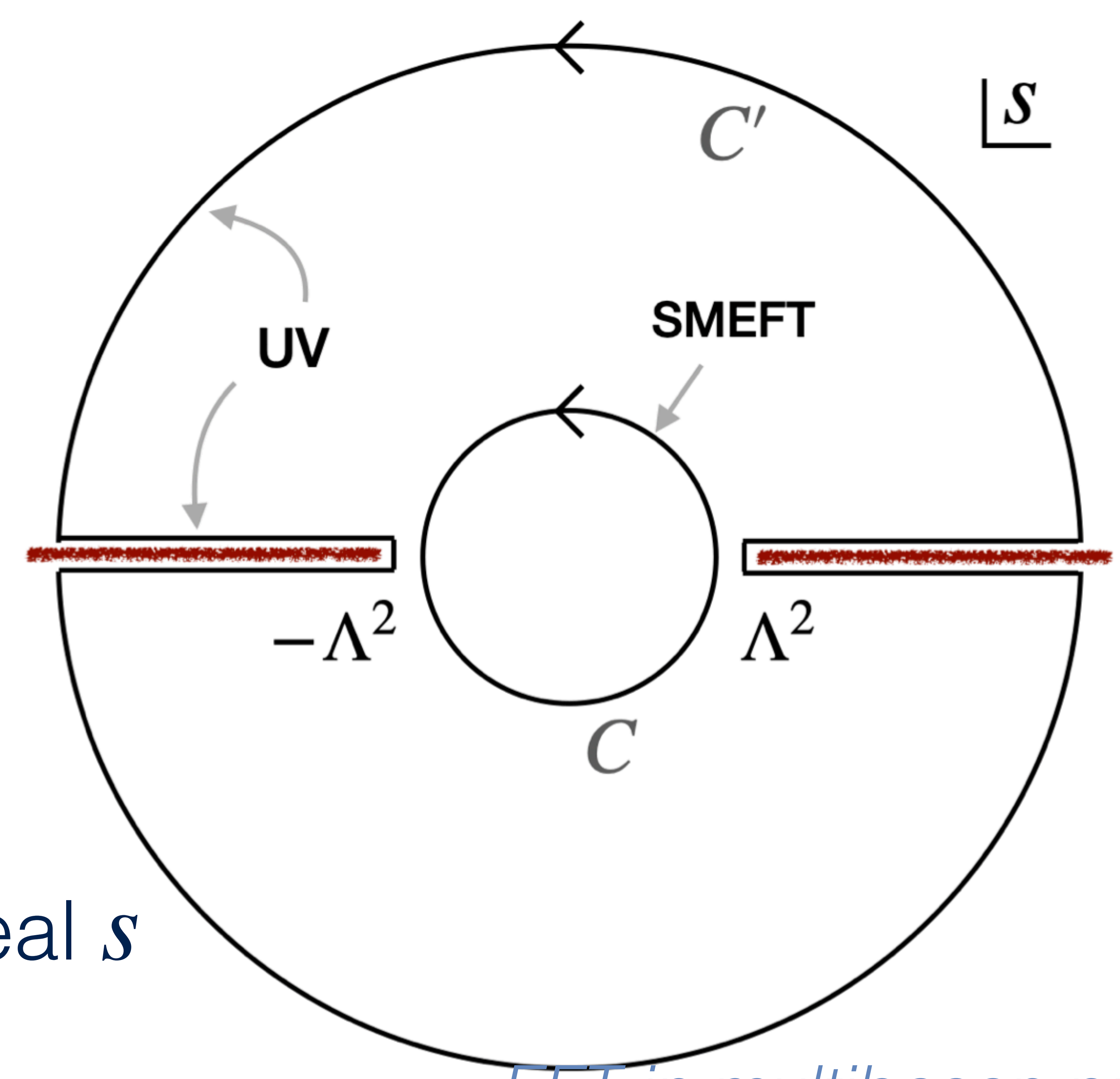
- Only poles & branch cuts on the real axis
- Define a pole subtracted amplitude: $M_{ijkl} \equiv \mathscr{M}_{ij\to kl} - (\text{low energy poles})$

Cauchy's intergral formula

$$\frac{1}{2}\frac{d^2 M_{ijkl}(s)}{ds^2} = \oint_C \frac{d\mu}{2\pi i}\frac{M_{ijkl}(\mu)}{(\mu - s)^3}$$

Avoiding UV branch cuts

- Deform contour to infinity $C \Rightarrow C'$
- C' = 2 semi-circles + discontinuities along real s



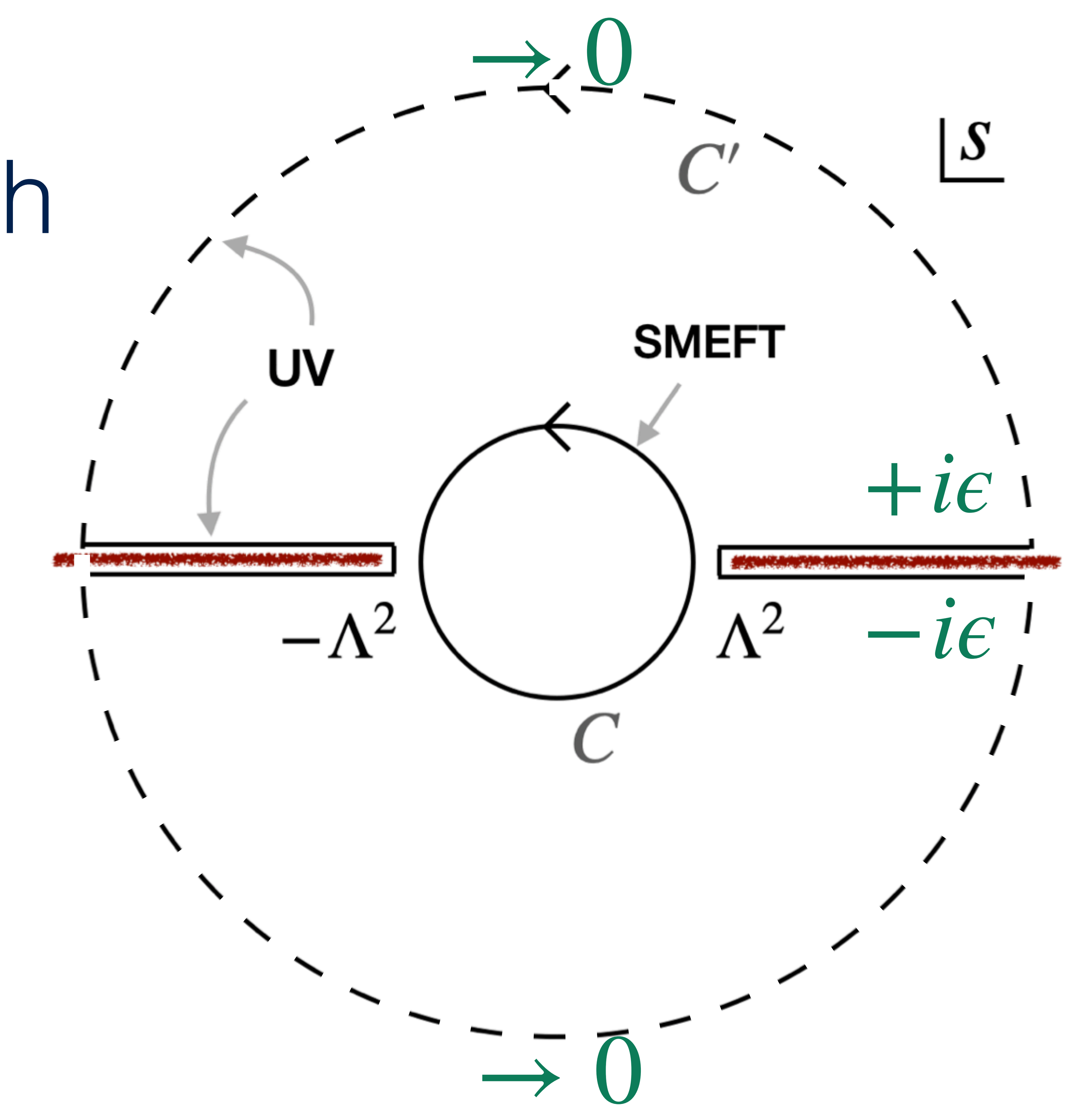
Dispersion relation

Infinite semi-circle contributions vanish

- Froissart bound for unitary & local amplitude

$$M \lesssim s \log^2 s, \;\; s \to \infty$$

[Froissart; Phys. Rev. 123 (1961) 1053-1057]



$$\Rightarrow \;\; \frac{1}{2}\frac{d^2 M_{ijkl}(s)}{ds^2} = \int_{-\infty}^{\infty} \frac{d\mu}{2\pi i} \frac{\text{Disc}[M_{ijkl}(\mu)]}{(\mu - s)^3}$$

$$\text{Disc}[f(s)] \equiv f(s + i\epsilon) - f(s - i\epsilon)$$

"Twice subtracted" dispersion relation

For small s:

LHS: approximated by EFT amplitude (IR)

RHS: integral to infinity: full amplitude (UV)

Analyticity + Unitarity

$\Lambda \gg v$: take SM particles massless $\qquad s \leftrightarrow u$

- Crossing symmetry in massless, forward limit: $M_{ijkl}(\mu) = M_{i\tilde{l}k\tilde{j}}(-\mu)$
- No discontinuities in M below $s = \Lambda^2$ (we subtracted them)

$$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} = \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi i \mu^3}\left(\text{Disc}[M_{ijkl}(\mu)] + \text{Disc}[M_{i\tilde{l}k\tilde{j}}(\mu)]\right)$$

Recall:

$$\frac{1}{2i}\left(\mathcal{M}_{ij\to kl} - \mathcal{M}^*_{kl\to ij}\right) = \frac{1}{2}\sum_X \int d\Pi_X \, \mathcal{M}_{ij\to X}\mathcal{M}^*_{kl\to X}$$

Hermitian analyticity

$$\mathcal{M}^*_{kl\to ij}(s,t) = \mathcal{M}_{ij\to kl}(s^*,t) \quad \Rightarrow \quad \frac{1}{2i}\text{Disc}[\mathcal{M}_{ij\to kl}] = \frac{1}{2}\sum_X \int d\Pi_X \, \mathcal{M}_{ij\to X}\mathcal{M}^*_{kl\to X}$$

- Generalised optical theorem + twice subtracted dispersion relation:

$$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi\mu^3}\left(m_{ij}m^*_{kl} + m_{i\tilde{l}}m^*_{k\tilde{j}}\right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

Positivity cone

$$\frac{1}{2}\frac{d^2M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi\mu^3}\left(m_{ij}m^*_{kl} + m_{i\tilde{l}}m^*_{k\tilde{j}}\right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

$2 \to n$ amplitudes m_{ij} are unknown complex functions of μ

- Encode masses of new states at & above Λ^2

Elastic ($ij = kl$): $$\frac{1}{2}\frac{d^2M_{ijij}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi\mu^3}\left(|m_{ij}|^2 + |m_{i\tilde{j}}|^2\right) \geq 0$$

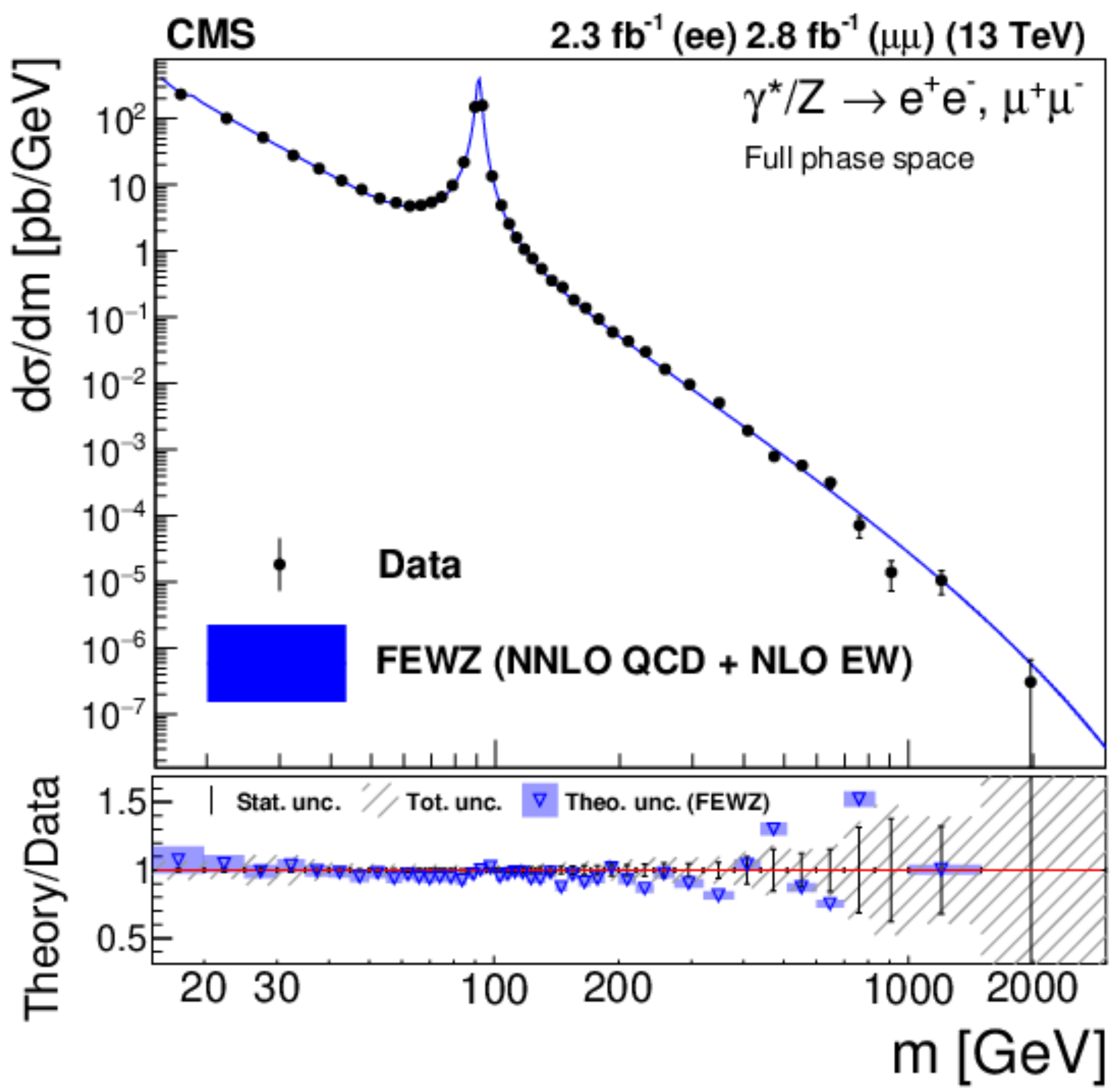
Inelastic ($ij \neq kl$): more information

- $m_{ij}m^*_{kl}$ are not positive-definite. However, RHS is a **positive sum** (integral)

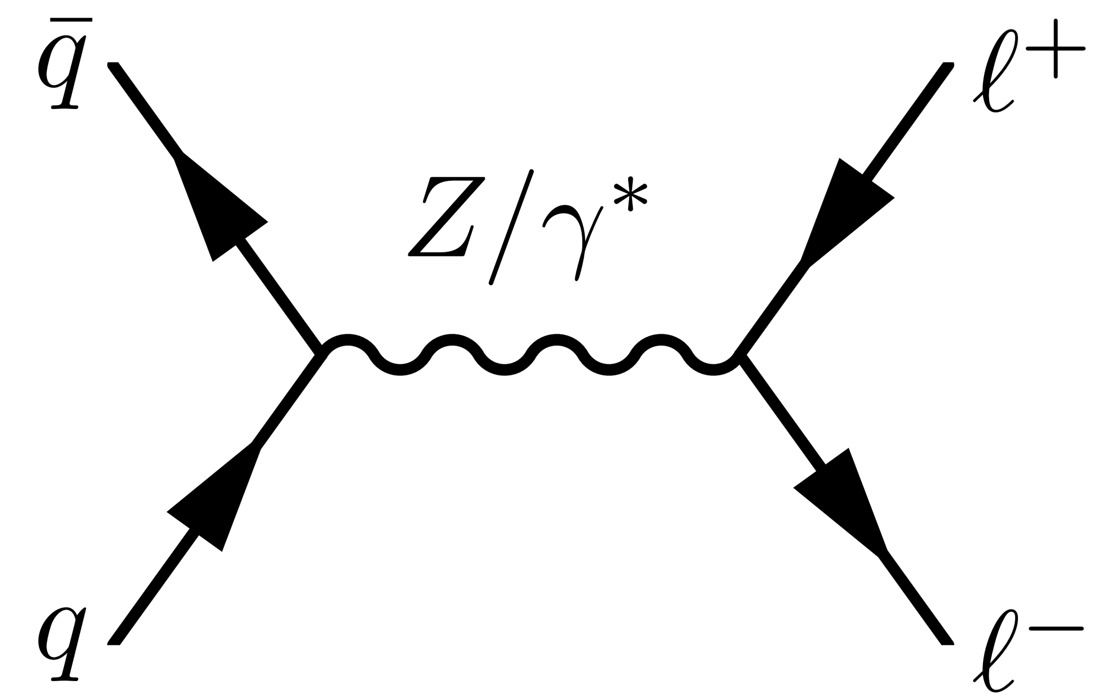
$$\frac{1}{2}\frac{d^2M_{ijkl}(0)}{ds^2}$$ forms a **convex cone**

- Constraints the Wilson coefficients to a non-trivial, conical subspace

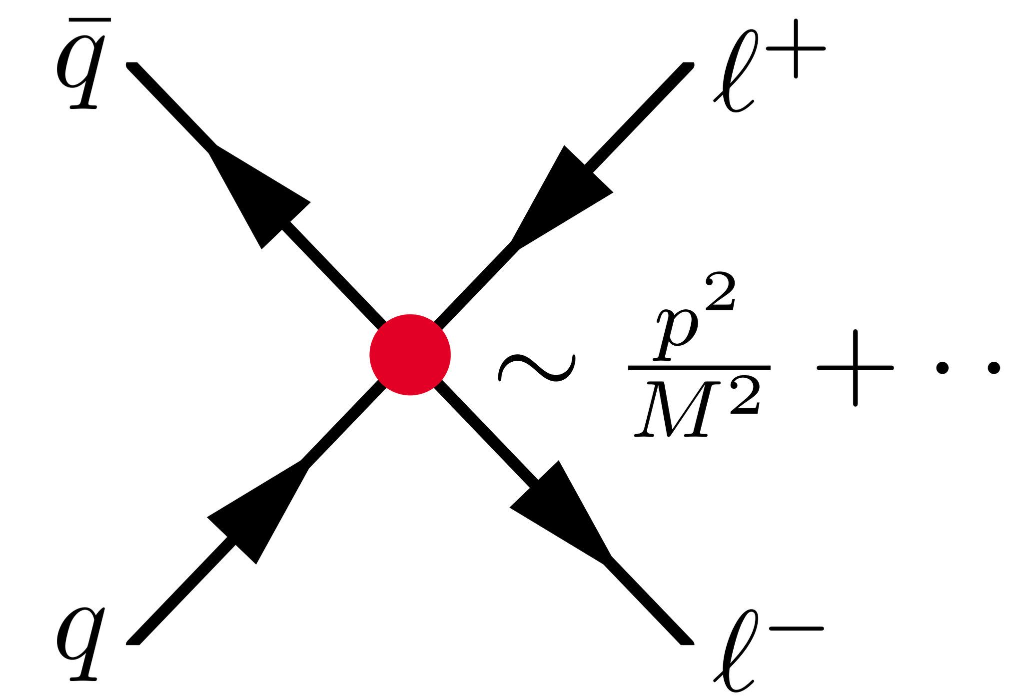
Drell Yan at the LHC



$pp \to \ell^+\ell^-$ at high mass



SM

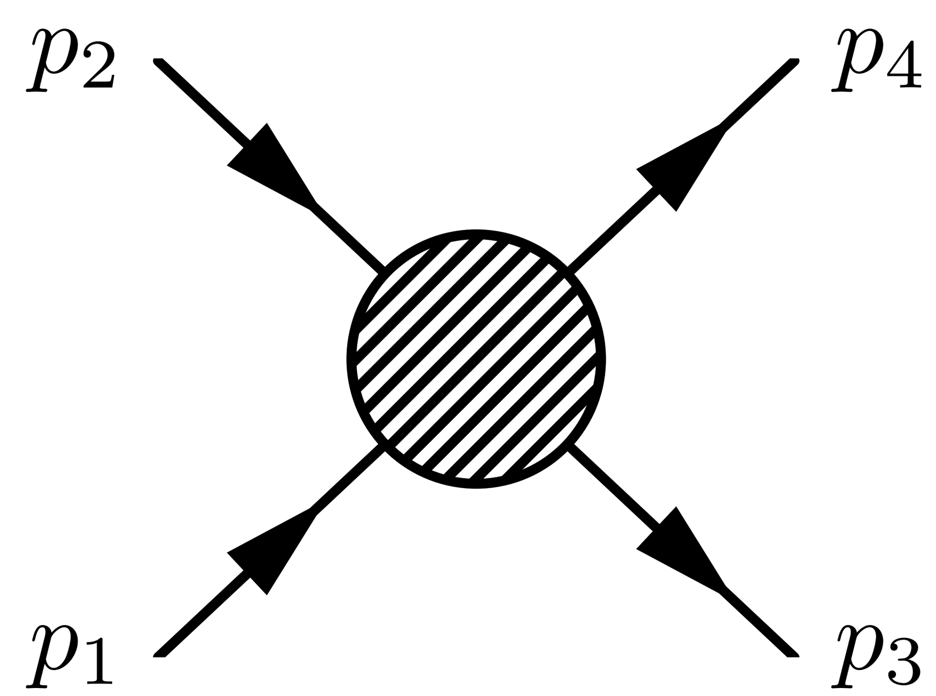


EFT

Clean, high-energy probe of 2 → 2 scattering

- Strong bounds on new resonances
- Sensitive to energy-growing contact interactions: 4F operators
- Complete reconstruction of final state: fully differential cross section

New angular dependence



$$\rightarrow \mathscr{A}(s,t) \qquad s = (p_1 + p_2)^2 \quad t = (p_1 - p_3)^2 \qquad \cos\theta \sim 1 - \frac{2t}{s}$$

$\mathscr{A}_{SM}$: spin-1 $\rightarrow \propto \cos\theta \sim \dfrac{t}{s}$

- Differential cross section $|\mathscr{A}|^2 \sim t, t^2$: $Y_{l\leq 2,m}$
- QCD corrections factorise, $l \leq 2$ unchanged
- Leading higher l contribution: NLL EW Sudakov $\sim \dfrac{\alpha^2}{16\pi^2} \log \dfrac{t}{m_W^2}$

$\mathscr{A}_{BSM}$: new Lorentz structures

- Higher spin states or contact interactions (4F operators)

Dim 6 (E^2)

$$\mathscr{A} \sim s, t \Rightarrow |\mathscr{A}|^2 : \; l \leq 2$$

Dim 8 (E^4)

$$\mathscr{A} \sim s^2, t^2 \Rightarrow \mathscr{A}_{SM}\mathscr{A}_{EFT} : \; l \leq 3$$

Angular dependence

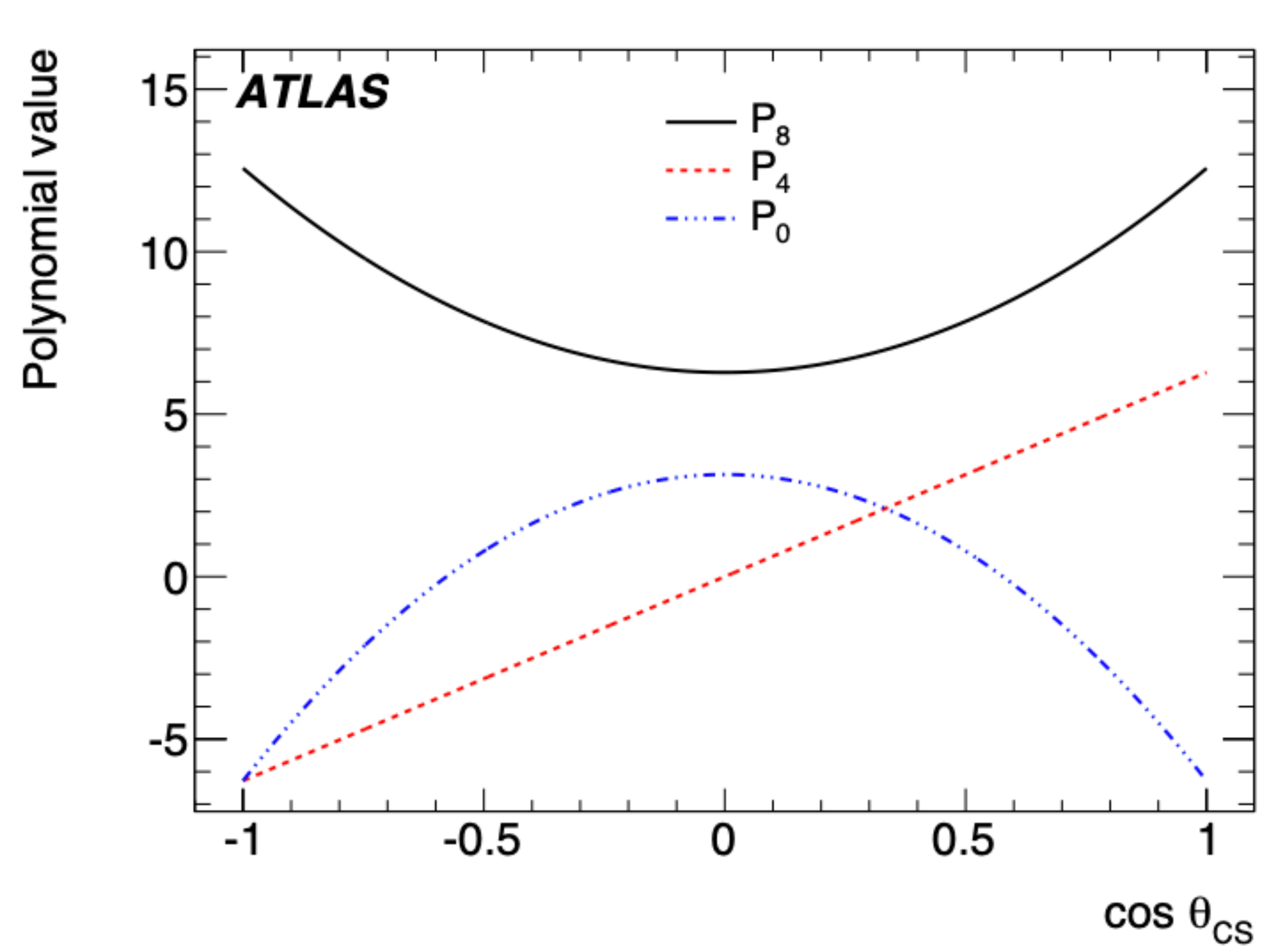
Extracting the $\tilde{A}_i$: moments of spherical harmonics *

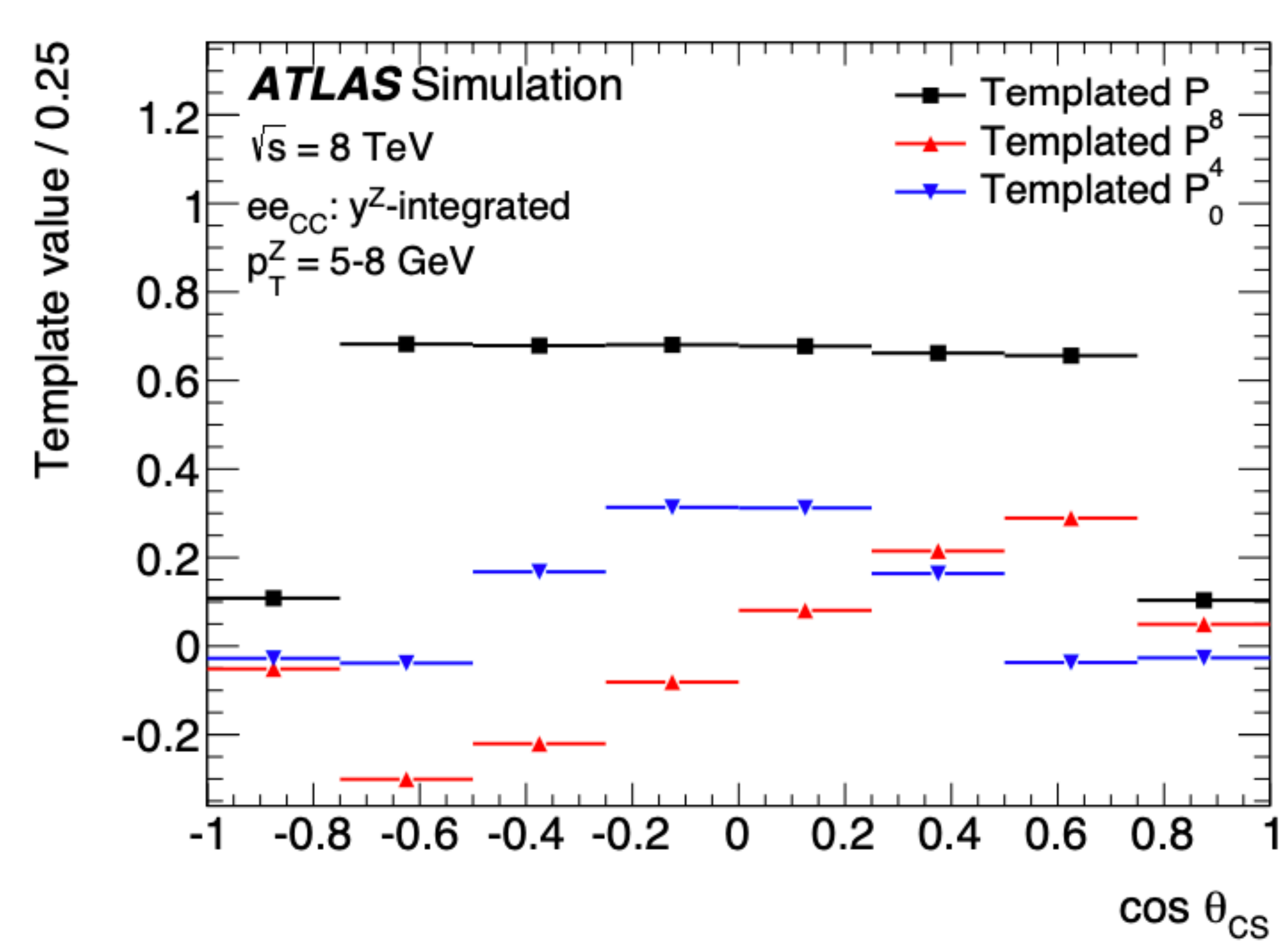
$$\langle f(\theta,\phi)\rangle \equiv \left(\frac{d\sigma}{dm\,d\eta\,d\Omega}\right)^{-1}\int d\Omega_\ell \,\frac{d\sigma}{dm\,d\eta\,d\Omega}\, f(\theta,\phi) \qquad f(\theta,\phi) \propto \left\{Y_{0,0}, Y_{1,0}, Y_{1,\pm1}, Y_{2,0}, Y_{2,\pm1}, Y_{2,\pm2}\right\}$$

- A.K.A. weighted sum of the basis functions over event sample
- $\tilde{A}_i$'s are linear functions of the $\langle Y_{l,m}\rangle$

* In practice, finite experimental acceptance

- Spoils the orthonormality of spherical harmonics





Extracted by fit to signal templates

[CMS; PLB 750 (2015) 154-175]
[ATLAS; JHEP 08 (2016) 159]

Even higher moments

Exploit the full information of D8 amplitude: $l = 4$ moments

$$\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}d\Omega_\ell} = \frac{3}{16\pi}\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}}\Bigg[(1+c_\theta^2) + \frac{\tilde{A}_0}{2}(1-3c_\theta^2) + \tilde{A}_1 s_{2\theta}c_\phi$$

$l \leq 2$

$$+\frac{\tilde{A}_2}{2}s_\theta^2 c_{2\phi} + \tilde{A}_3 s_\theta c_\phi + \tilde{A}_4 c_\theta + \tilde{A}_5 s_\theta^2 s_{2\phi} + \tilde{A}_6 s_{2\theta}s_\phi + \tilde{A}_7 s_\theta s_\phi$$

$l = 3$

$$+\frac{\tilde{B}_1^e}{2}s_\theta\left(5c_\theta^2-1\right)c_\phi + \frac{\tilde{B}_1^o}{2}s_\theta\left(5c_\theta^2-1\right)s_\phi + \boxed{\frac{\tilde{B}_0}{2}\left(5c_\theta^3-3c_\theta\right)}$$

$$+\tilde{B}_3^e s_\theta^3 c_{3\phi} + \tilde{B}_3^o s_\theta^3 s_{3\phi} + \tilde{B}_2^e s_\theta^2 c_\theta c_{2\phi} + \tilde{B}_2^o s_\theta^2 c_\theta s_{2\phi}$$

$l = 4$

$$+\tilde{D}_4^e s_\theta^4 c_{4\phi} + \tilde{D}_4^o s_\theta^4 s_{4\phi} + \tilde{D}_3^e s_\theta^3 c_\theta c_{3\phi} + \tilde{D}_3^o s_\theta^3 c_\theta s_{3\phi}$$

$$+\tilde{D}_2^e s_\theta^2(7c_\theta^2-1)c_{2\phi} + \tilde{D}_2^o s_\theta^2(7c_\theta^2-1)s_{2\phi} + \tilde{D}_1^e s_\theta(7c_\theta^3-3c_\theta)c_\phi$$

$$+\tilde{D}_1^o s_\theta(7c_\theta^3-3c_\theta)s_\phi + \boxed{\frac{\tilde{D}_0}{2}(35c_\theta^4-30c_\theta^2+3)}\Bigg]$$

Use $(\tilde{B}_0, \tilde{D}_0)$ to constrain the space of dim-8 WCs

- Quantify the ability of the LHC to test positivity in $q\ell$ scattering

LHC sensitivity

1 TeV cut to mitigate impact of quadratics

Consider 10 × 10 square $\{m_{\ell\ell}, \eta_{\ell\ell}\}$ binning:

$$m_{\ell\ell}\colon \{100,\ 190,\ 280,\ 370,\ 460,\ 550,\ 640,\ 730,\ 820,\ 910,\ 1000\}\,\text{GeV},$$
$$\eta_{\ell\ell}\colon \{-5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5\},$$

Binned $\Delta\chi^2$, combining (B_0, D_0), for $L_{\text{int.}} = 3000\,\text{fb}^{-1}$

$$\chi^2(C_i) \equiv \Delta\chi^2(C_i) = \sum_i \left(B_0^i(\vec{C}), D_0^i(\vec{C})\right) \cdot \mathbf{V}^{-1} \cdot \left(B_0^i(\vec{C}), D_0^i(\vec{C})\right) \leq 3.84,$$

- B_0 & D_0 are correlated: statistical covariance matrix $\mathbf{V}$

$$V_{ij} = \frac{1}{L}\int_{m_{\text{min.}}}^{m_{\text{max.}}} dm_{\ell\ell} \int_{\eta_{\text{min.}}}^{\eta_{\text{max.}}} d\eta_{\ell\ell} \int_{-1}^{1} dc_\theta \frac{d\sigma_{pp\to\ell^-\ell^+}}{d\eta_{\ell\ell}\, dm_{\ell\ell}\, dc_\theta} \cdot F_{ij}(c_\theta),$$

(co)variance of weighted average(s)

$$F_{11} = \frac{448\pi}{9}\left(Y_3^0(c_\theta)\right)^2; \quad F_{22} = \frac{36\pi^3}{49}\left(Y_4^0(c_\theta)\right)^2; \quad F_{12} = F_{21} = \sqrt{\frac{16}{7}}4\pi^2 Y_3^0(c_\theta) Y_4^0(c_\theta)$$

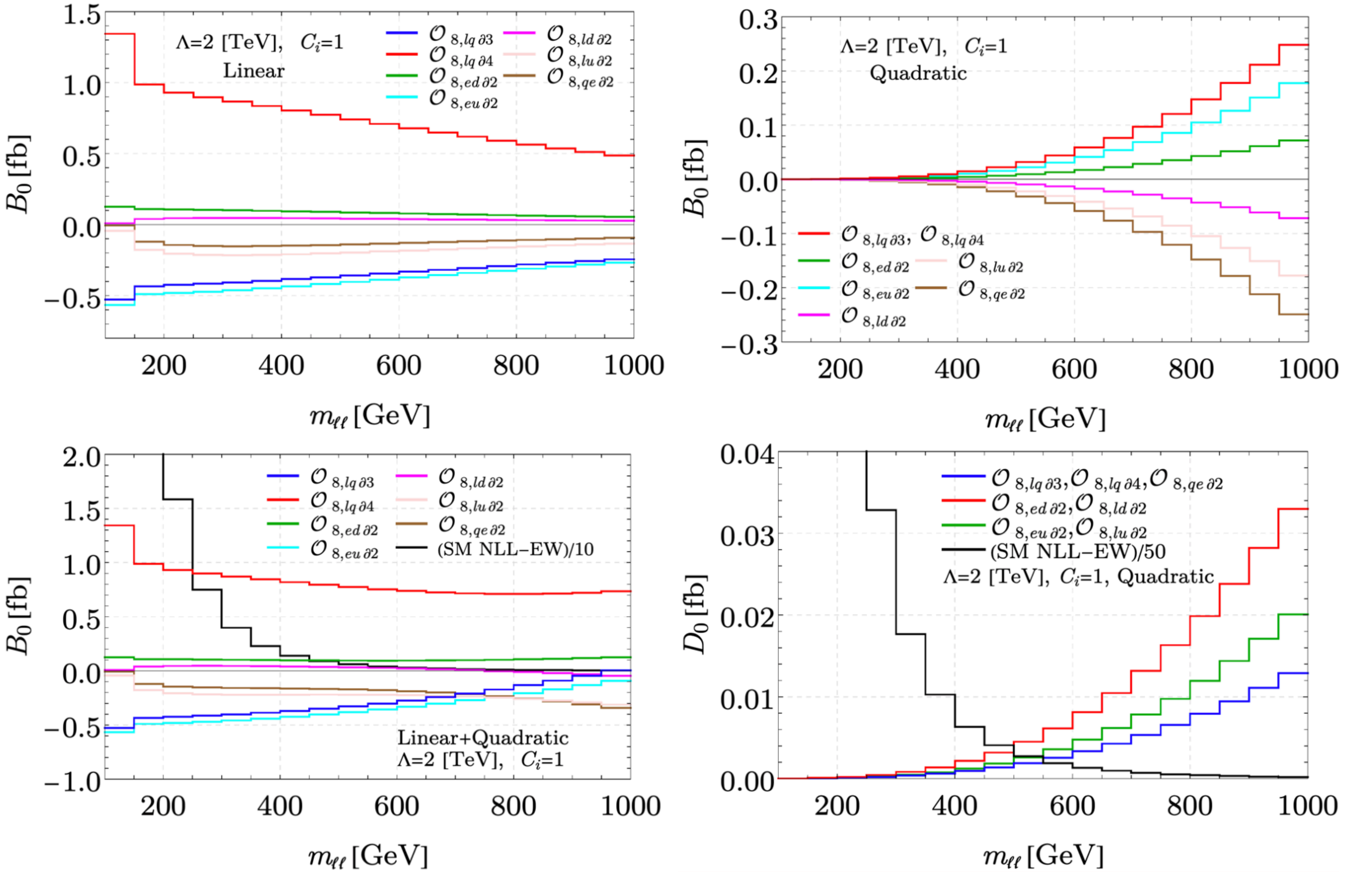
- Variances dominated by SM, computed @ NLO QCD with `mg5`

[Li, KM, Yamashita, Yang, Zhang, Zhou; JHEP 10 (2022) 107]

PDF: `NNPDF_nlo_as_0118_luxqed`

LHC predictions

$\sqrt{s} = 14\,\text{TeV}$



Connecting to positivity

Relevant dim-8 operators: $\mathscr{A}(q\bar{q} \to \ell^+\ell^-) \sim t^2$

- By crossing symmetry, elastic amplitude: $\mathscr{A}(q\ell \to q\ell) \sim s^2!$
- Novel angular dependence in Drell Yan $\Leftrightarrow$ positivity bounds from $ql \to ql$

Positivity bound	channel: $\lvert 1\rangle + \lvert 2\rangle \to \lvert 1\rangle + \lvert 2\rangle$
$-4C_{8,lq\partial 3} + 4C_{8,lq\partial 4} \geq 0$	$\lvert 1\rangle = \lvert e^-_L\rangle\,,\quad \lvert 2\rangle = \lvert u_L\rangle$
$-4C_{8,lq\partial 3} - 4C_{8,lq\partial 4} \geq 0$	$\lvert 1\rangle = \lvert e^-_L\rangle\,,\quad \lvert 2\rangle = \lvert d_L\rangle$
$-4C_{8,ed\partial 2} \geq 0$	$\lvert 1\rangle = \lvert e^-_R\rangle\,,\quad \lvert 2\rangle = \lvert d_R\rangle$
$-4C_{8,eu\partial 2} \geq 0$	$\lvert 1\rangle = \lvert e^-_R\rangle\,,\quad \lvert 2\rangle = \lvert u_R\rangle$
$-4C_{8,ld\partial 2} \geq 0$	$\lvert 1\rangle = \lvert e^-_L\rangle\,,\quad \lvert 2\rangle = \lvert d_R\rangle$
$-4C_{8,lu\partial 2} \geq 0$	$\lvert 1\rangle = \lvert e^-_L\rangle\,,\quad \lvert 2\rangle = \lvert u_R\rangle$
$-4C_{8,qe\partial 2} \geq 0$	$\lvert 1\rangle = \lvert e^-_R\rangle\,,\quad \lvert 2\rangle = \lvert u_L\rangle$

- Use higher angular moments in DY to test positivity $\Rightarrow$ Fundamental properties of QFT in the UV

Higher orders

$l \leq n, O(\Lambda^{-m})$ A_i B_i D_i

D=	n,m	1,0	1,2	2,4	3,6	…
4	1,0	2,0				
6	1,2	2,2	2,4			
8	2,4	3,4	3,6	4,8		
10	3,6	4,6	4,8	5,10	6,12	
…	…					

$M_{SM,D=6} \Leftrightarrow l = 1$ & $M_{D=8} \Leftrightarrow l = 2, \ldots$

- $M_{D=6} \times M_{D=8}$ populates $l = 3$ at $O(\Lambda^{-6})$
- $M_{SM} \times M_{D=10}$ populates $l = 4$ at $O(\Lambda^{-8})$
- $M_{D=6} \times M_{D=10}$ populates $l = 4$ at $O(\Lambda^{-10})$
- …

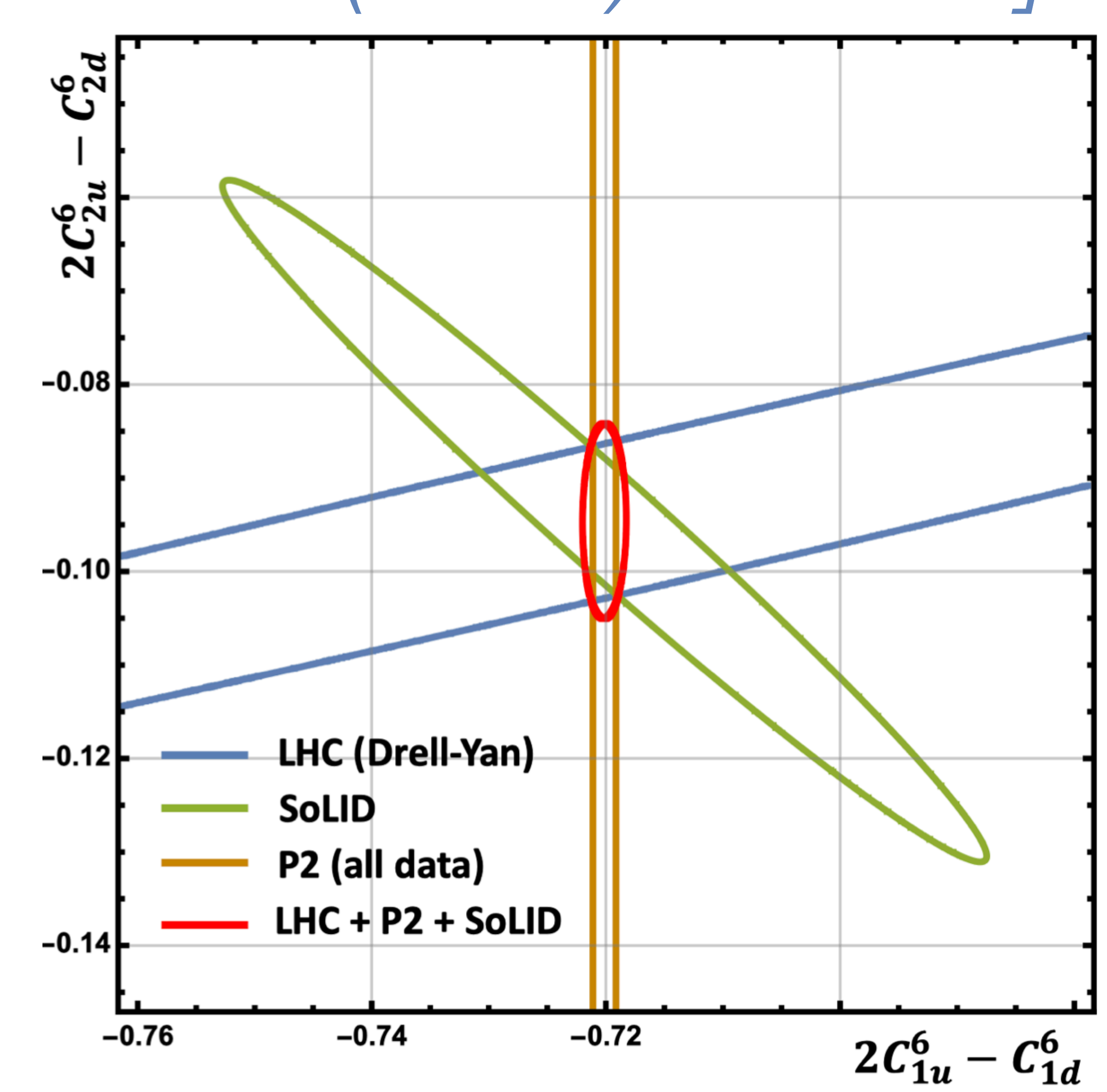
Our assumption: neglect possible $D = 6$

- Constrained elsewhere: A_0 moments, APV, β-decays,…

$l = 4$ have other possible contributions

- We currently neglect them, try to mitigate impact

[Boughezal et al.; PRD 104 (2021) 016005]



Ultimately, more complete (higher l) & global ($D = 6,8,\ldots$) analysis needed

Testing positivity

No concrete reason to expect violation of positivity

- Nature (data) should have the last word
- Probe the violation of positivity to test the axiomatic principles of QFT

Define "distance" from region allowed by elastic positivity

$$-\Delta^{-4} \equiv \min\left[\min_{\text{processes}} \frac{1}{2}\frac{d^2M(0)}{ds^2}, 0\right] = \frac{\delta(\vec{C}_0)}{\Lambda^4},$$

$$\delta(\vec{C}_0) \equiv \min\Big[-4C_{8,lq\partial 3} + 4C_{8,lq\partial 4}, -4C_{8,lq\partial 3} - 4C_{8,lq\partial 4}, -4C_{8,ed\partial 2}, -4C_{8,eu\partial 2}, -4C_{8,ld\partial 2}, -4C_{8,lu\partial 2}, -4C_{8,qe\partial 2}, 0\Big]$$

elastic ql scatterings

"most non-positive" direction

Associates a scale, Δ, to positivity violation

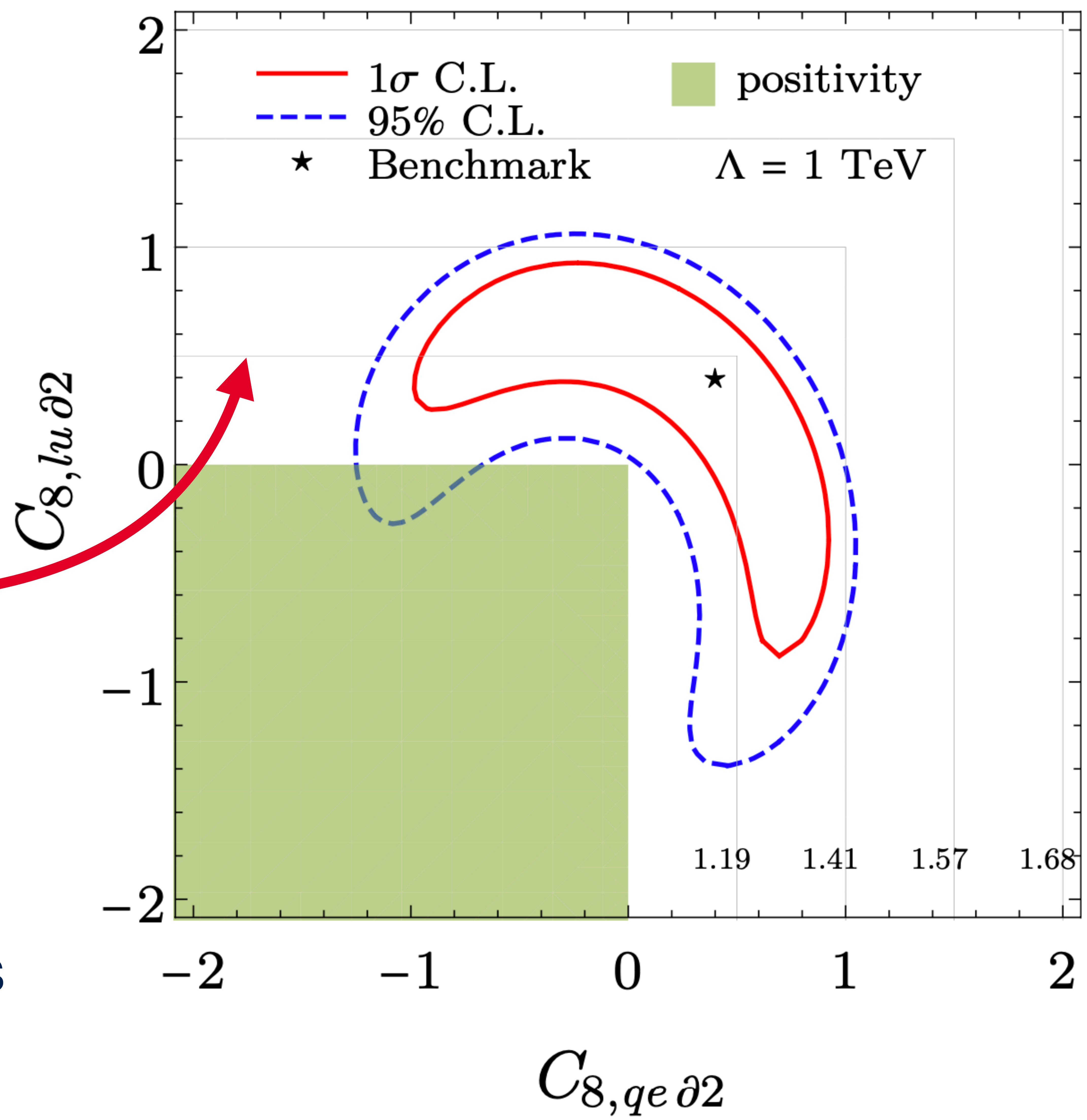
Satisfied: $\Delta = \infty$

Violated: $\Delta = \dfrac{\Lambda}{\sqrt[4]{\delta C_{\text{min.}}}}$

Testing positivity

Suppose we observe some non-zero WCs, $\overrightarrow{C}_0$

- Uncertainty crucial to determine whether we claim evidence for positivity violation: $\Delta\chi^2$
- If 95% confidence region overlaps with positivity allowed region, cannot rule out positivity
- Δ values shown in TeV
- Cannot exclude positivity at 95% C.L in this case: $\overrightarrow{C}_0 = (0.4, 0.4)$
- $\Delta^{-1} = [\Delta^{-1}_{\text{low}}, \Delta^{-1}_{\text{high}}]$, Δ_{low} gives conservative estimate

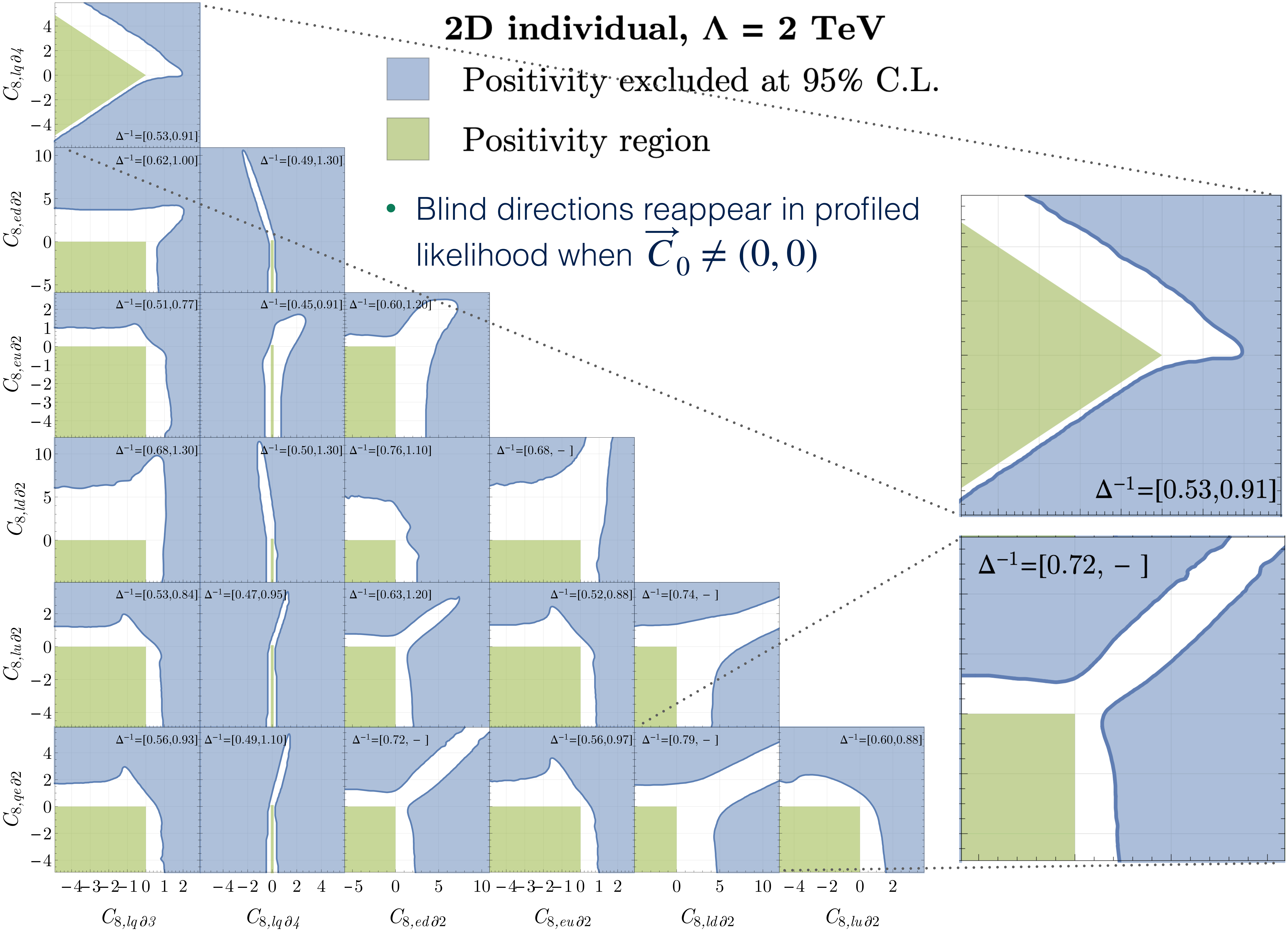


2D individual, Λ = 2 TeV

Positivity excluded at 95% C.L.

Positivity region

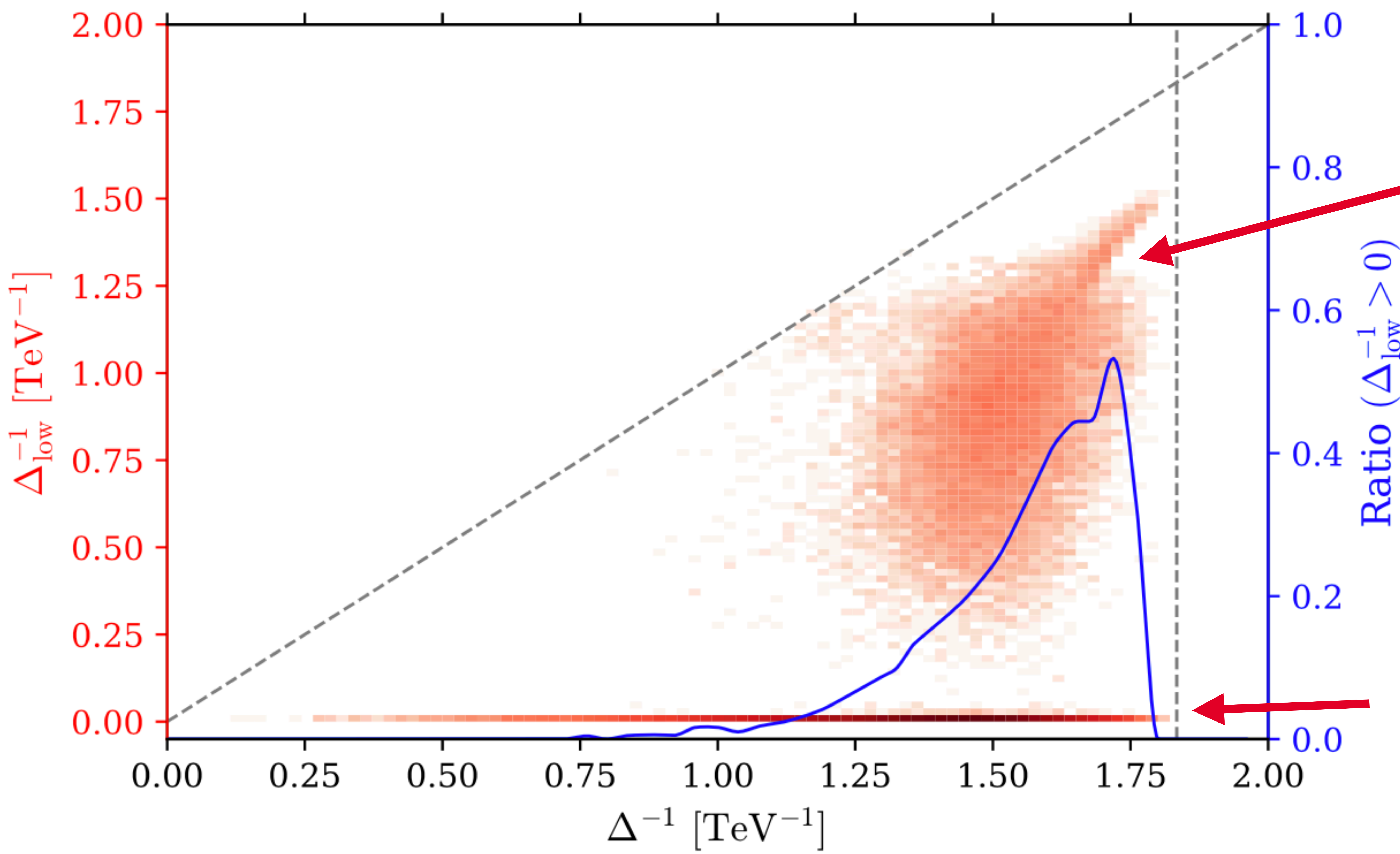
- Blind directions reappear in profiled likelihood when $\overrightarrow{C}_0 \neq (0,0)$



Testing positivity

7D case: does the allowed region intersect positivity region?

- $\Delta^{-1} = [\Delta^{-1}_{\rm low}, \Delta^{-1}_{\rm high}]$, $\Delta_{\rm low}$ gives conservative estimate (highest scale)
- Uniformly sample a ball of radius 2, with $\Lambda = 1\,\mathrm{TeV}$



Pos. excluded

Pos. not excluded