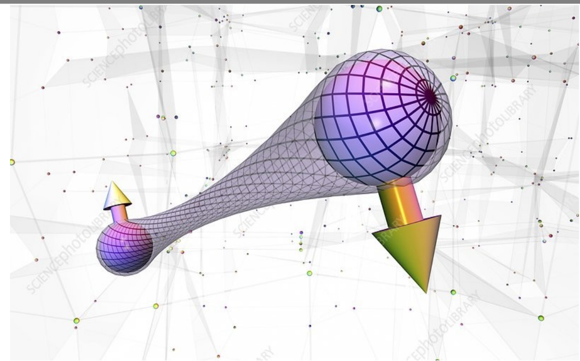
# Quantum Entanglement in Diboson production



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**Effective Field Theory in Multiboson production**

Padova, June 10 – 11, 2024

in collaboration with: Marco Fabbrichesi, Roberto Floreanini, and Luca Marzola

based on: **JHEP 09** (2023) 195; arXiv:2304.02403; **EPJC 83** (2023) 9, 823; arXiv:2302.00683;
**PRD 109** (2024) 3, L031104; arXiv:2305.04982

- "Entanglement" between two systems is a pure quantum phenomena

- Expected to violates Bell inequalities → set of (classical) probability correlations

- Violations incompatible with classical physics based on causality and local realism (locality) (EPR paradox, hidden variables theories)

- Violation of Bell inequality observed at **low energy**

  pair of photons Freedman-Clauser PRL 28 (1972), Aspect-Dalibard-Roger PRL 49 (1982), Zelinger et al. , Nature 433, 230 (2005)

**At high energy (B physics):**

- **first observation** of **Bell violation** at high energy in B meson decays and in the presence of WEAK and STRONG interactions ! (large significance >> 5$\sigma$)

- observation of **Entanglement** (as ATLAS [arXiv:2311.07288] and CMS [CMS-PAS-TOP-23001] results for tops)

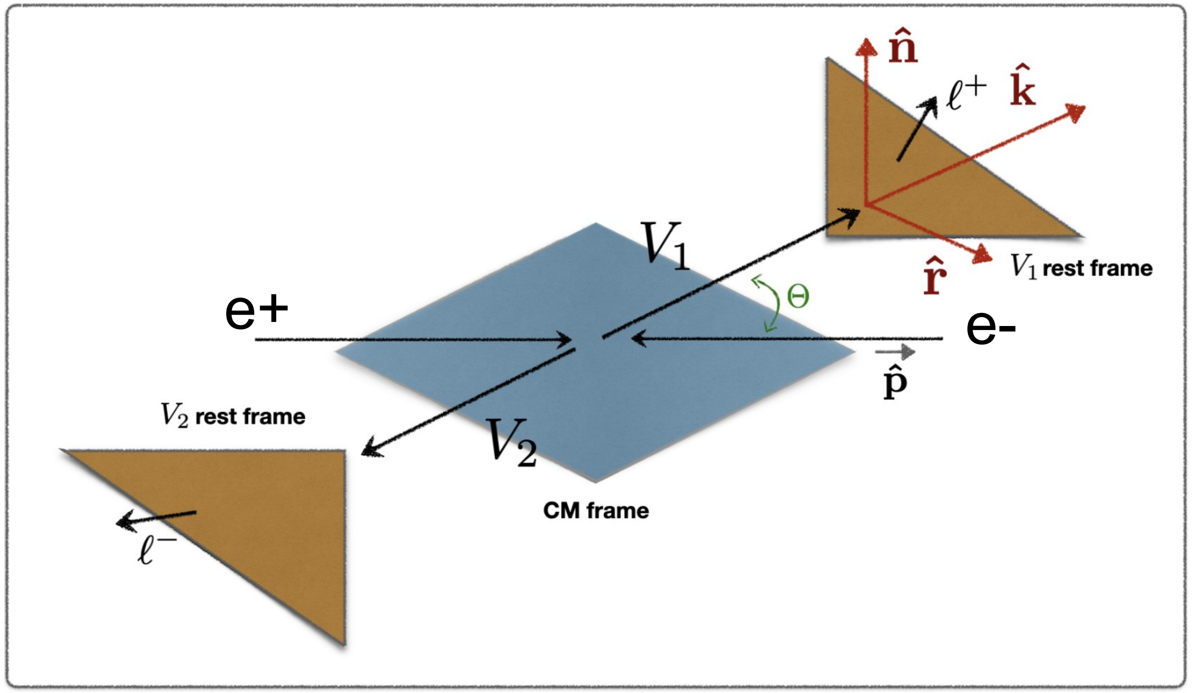
Fabbrichesi, Floreanini, EG, Marzola, PRD 109 (2024) 3, L031104 ; arXiv: 2305.04982

# Entanglement & Bell inequalities in spin-1 systems (qutrits)

- Requires the knowledge of the **Polarization Density Matrix (PDM)** of spin-1 $V_1\,V_2$ production

- PDM can be fully reconstructed from the angular distributions of the single $V_1\,V_2$ **decay products**

PDM depends only on invariant mass of $V_1V_2$ system and on scattering angle $\theta$ in the $V_1V_2$ c.m. frame



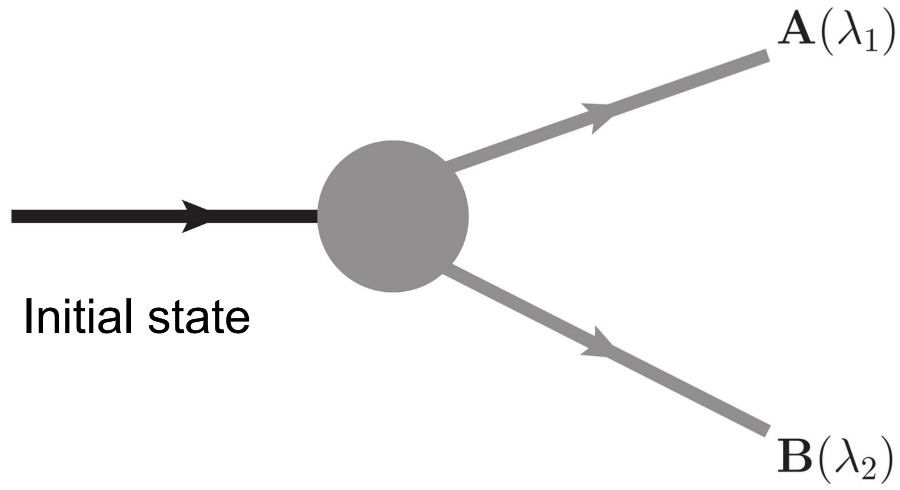

- **but it can also be computed analytically**

- knowledge of full PDM allows to quantify (where possible) entanglement and Bell inequality violation

# Polarization density matrix: two final spin-1 particles

- Consider an initial quantum state going into two final **A,B** spin-1 particles

Quantum polarized amplitude $\mathcal{M}(\lambda_1, \lambda_2) \quad =$




helicities of A,B

- polarization density matrix

$$\rho(\lambda_1, \lambda_1', \lambda_2, \lambda_2') \quad = \quad \frac{\mathcal{M}(\lambda_1, \lambda_2)\mathcal{M}^\dagger(\lambda_1', \lambda_2')}{|\overline{\mathcal{M}}|^2}$$

9x9 matrix

unpolarized

- Two methods available for computing or reconstructing PDM of two qutrits
  - Gell-Mann matrices decomposition (or spherical harmonics basis) → more general
  - using **helicity amplitudes** → **suitable for pure states**

# Decomposition of Polarization Density Matrix for qutrits

PDM $\rho$ can be decomposed on the basis of tensor products of **Gell-Mann matrices** $T^a$

9x9 matrix

$$[A \otimes B]_{ii'jj'} = A_{ii'} B_{jj'}$$

Unpolarized

$$\rho(\lambda_1, \lambda_1', \lambda_2, \lambda_2') = \left( \frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [\mathbb{1} \otimes T^a] + \sum_a g_a [T^a \otimes \mathbb{1}] + \sum_{ab} h_{ab} \left[ T^a \otimes T^b \right] \right)_{\lambda_1 \lambda_1', \lambda_2 \lambda_2'}$$

helicities

$$f_a = \frac{1}{6} \mathrm{Tr} \left[ \rho \left( \mathbb{1} \otimes T^a \right) \right], \quad g_a = \frac{1}{6} \mathrm{Tr} \left[ \rho \left( T^a \otimes \mathbb{1} \right) \right], \quad h_{ab} = \frac{1}{4} \mathrm{Tr} \left[ \rho \left( T^a \otimes T^b \right) \right]$$

Spin polarization of particle 1

Spin polarization of particle 2

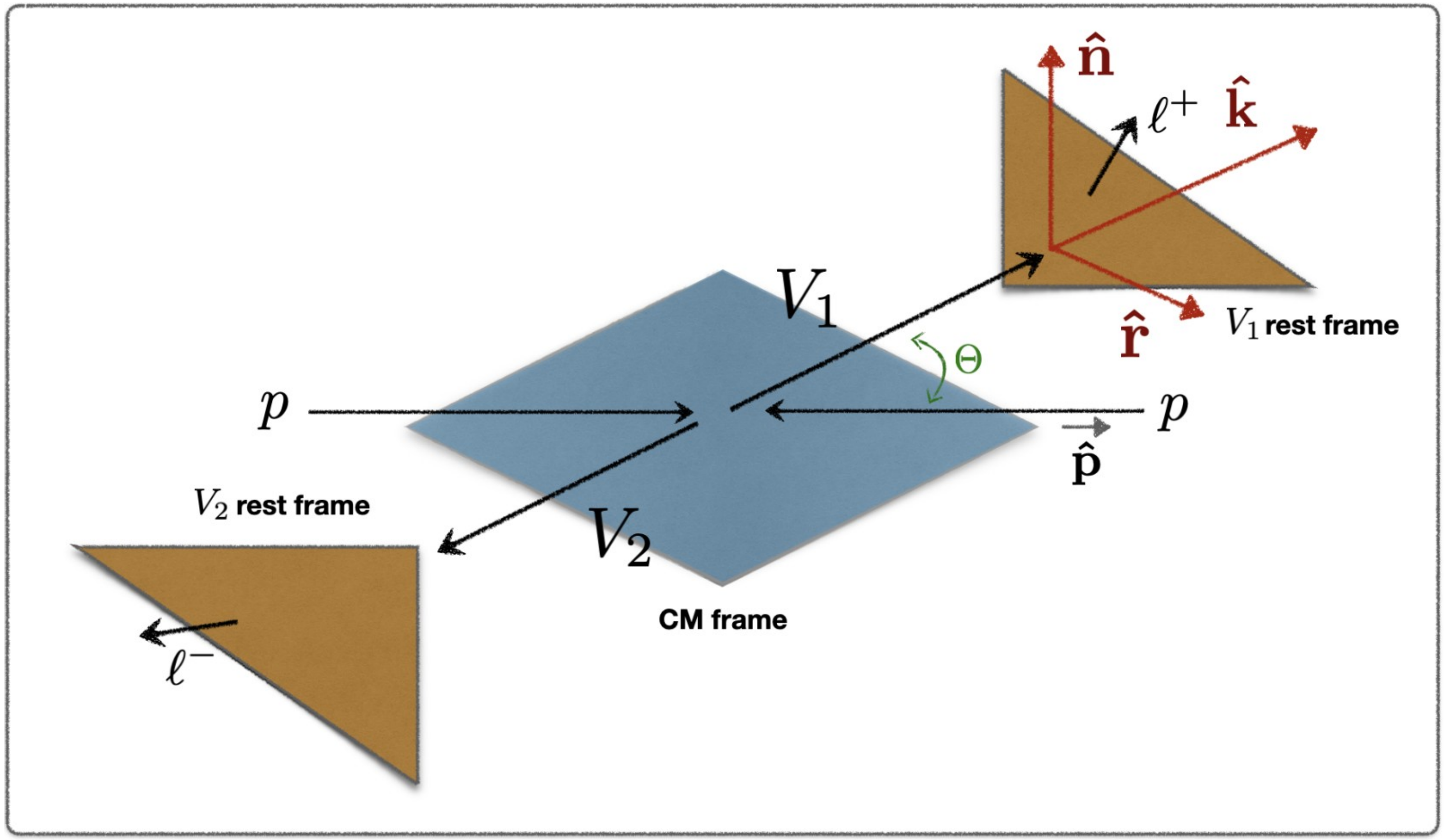
Spin correlations of particles 1 and 2

- extract ***f,g,h*** coefficients from data using the angular distributions of their decay products

Right-handed basis $\{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\}$:

$$\hat{\mathbf{p}} \cdot \hat{\mathbf{k}} = \cos\Theta.$$

$$\hat{\mathbf{n}} = \frac{1}{\sin\Theta}\left(\hat{\mathbf{p}} \times \hat{\mathbf{k}}\right), \qquad \hat{\mathbf{r}} = \frac{1}{\sin\Theta}\left(\hat{\mathbf{p}} - \cos\Theta\hat{\mathbf{k}}\right)$$




$$d\Omega^{\pm} = \sin\theta^{\pm} d\theta^{\pm}\, d\phi^{\pm}$$

phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

# Extracting PDM for Two-Qutrits from data

The following example is for W+ and W- final states, but the method is general

Rahaman, Singh, NPB 984 (2022), arXiv:2109.09345

## Differential cross section

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^{+}\,d\Omega^{-}} = \left(\frac{3}{4\pi}\right)^{2}\mathrm{Tr}\left[\rho_{V_1 V_2}\left(\Gamma_{+}\otimes\Gamma_{-}\right)\right]$$

Kronecker product

$$d\Omega^{\pm} = \sin\theta^{\pm} d\theta^{\pm}\, d\phi^{\pm} \longrightarrow$$

phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

$\rho_{V_1 V_2}$ = density matrix of $V_1 V_2$

$\Gamma_{\pm} \longrightarrow$ Density matrices that describe the polarization of the two spin-1 massive vectors into final leptons (the charged ones assumed to be massless)

(in the case of the W-bosons these are projectors because of their chiral couplings to leptons ) can be computed by rotating to an arbitrary polar axis the spin $\pm 1$ states of gauge bosons from the ones given in the **k-direction** quantization axis

$$\Gamma_{\pm} = \frac{1}{3}\,\mathbb{1} + \sum_{i=1}^{8} \mathfrak{q}_{\pm}^{a}\, T^{a}$$

Example for the W+ W- final states

→ Density matrices for W-bosons

$\mathfrak{q}_{\pm}^{a}$ → Wigner q-symbols (see backup slides) are functions of the solid angles $\Omega^{\pm}$

set of polynomials of spherical coordinates (see backup slide)

$$h_{ab} = \frac{1}{\sigma}\int\int \frac{d\sigma}{d\Omega^{+}\, d\Omega^{-}}\, \mathfrak{p}_{+}^{a}\, \mathfrak{p}_{-}^{b}\, d\Omega^{+} d\Omega^{-}$$

$$f_{a} = \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^{+}}\, \mathfrak{p}_{+}^{a}\, d\Omega^{+}$$

$$g_{a} = \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^{-}}\, \mathfrak{p}_{-}^{a}\, d\Omega^{-}$$

$\mathfrak{p}_{\pm}^{n}$ a particular set of orthogonal functions (see backup slides)

$$\left(\frac{3}{4\,\pi}\right)\int \mathfrak{p}_{\pm}^{n}\, \mathfrak{q}_{\pm}^{m}\, d\Omega^{\pm} = \delta^{nm}$$

For **ZZ** case, the set of functions are linear combinations of $\mathfrak{q}_{\pm}^{a}$ (see backup slides)

# Quantify entanglement

**For <u>pure states</u>** $\rho = |\psi\rangle\langle\psi| \Rightarrow \rho^2 = \rho \Rightarrow \mathrm{Tr}[\rho^2] = 1$

- CONCURRENCE

$$\mathcal{C}[|\Psi\rangle] = \sqrt{2\left(1 - \mathrm{Tr}\left[(\rho_r)^2\right]\right)}$$

$r = A$ or $B$

Hill, Wootters, PRL 78 (1997)
P. Rungta et al, PRA 64 (2001)

→ vanish for separable (not entangled) states

Trace performed in the subsystems *r = A or B*

**For <u>mixed states</u>** $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$ , $p_i \geq 0$ , $\sum_i p_i = 1$

$$\mathcal{C}[\rho] = \inf_{\{|\Psi\rangle\}} \sum_i p_i \, \mathcal{C}[|\Psi_i\rangle]$$

→ optimization problem (analytical solution only for qubit)

- analytical solution exists for the lower bound $\left(\mathcal{C}[\rho]\right)^2 \geq \mathscr{C}_2[\rho]$

Mintert, Buchleitner, PRL 98 (2007)

(witness of entanglement)

$$\mathscr{C}_2[\rho] = 2\max\left(0,\ \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_A)^2\right],\ \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_B)^2\right]\right)$$

- If non-vanishing unequivocally signal the presence of entanglement
- used as observable for entanglement in WW, ZZ and WZ productions

# Quantify entanglement for pure states

**Fabbrichesi,Floreanini, EG, Marzola**

EPJC 83 (2023) 9, 823 ; arXiv: 2302.00683]

- If the bipartite (A,B) system is a pure state it is possible to **quantify** its entanglement via

## Entropy of entanglement

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

Log is in natural basis

equality holds if and only
if the bipartite is separable

$$0 \leq \mathcal{E}[\rho] \leq \log d \qquad d = 3 \text{ for qutrits}$$

(Von Neumann Entropy)

- Very useful in the case of Higgs decay into H → WW* or ZZ* (pure state)

# Bell inequality for Two-QUTRITS

- take a bi-partite system with components A,B
- perform two measurements $\longrightarrow (A_1 \;,\; A_2) \quad (B_1 \;,\; B_2)$ each can take values $\rightarrow \{0,1,2\}$
- consider the following correlator $\mathcal{I}_3$ for probability measurements

$$\begin{aligned}\mathcal{I}_3 \;=\;& P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)\end{aligned}$$

can be expressed as

$$\mathcal{I}_3 = \mathrm{Tr}\left[\rho\, \mathcal{B}\right]$$

with $\mathcal{B}$ a suitable Bell operator

**Collins, Gisin, Linden, Massar, Popescu,** PRL 88 (2002)

**Bell inequalities for two-qutrits**

For deterministic local models $\mathcal{I}_3 \leq 2$

**QM for qutrits can violate this inequality with upper bound = 4**

in order to **maximize Bell's inequality**

$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

U,V unitary 3x3 matrices

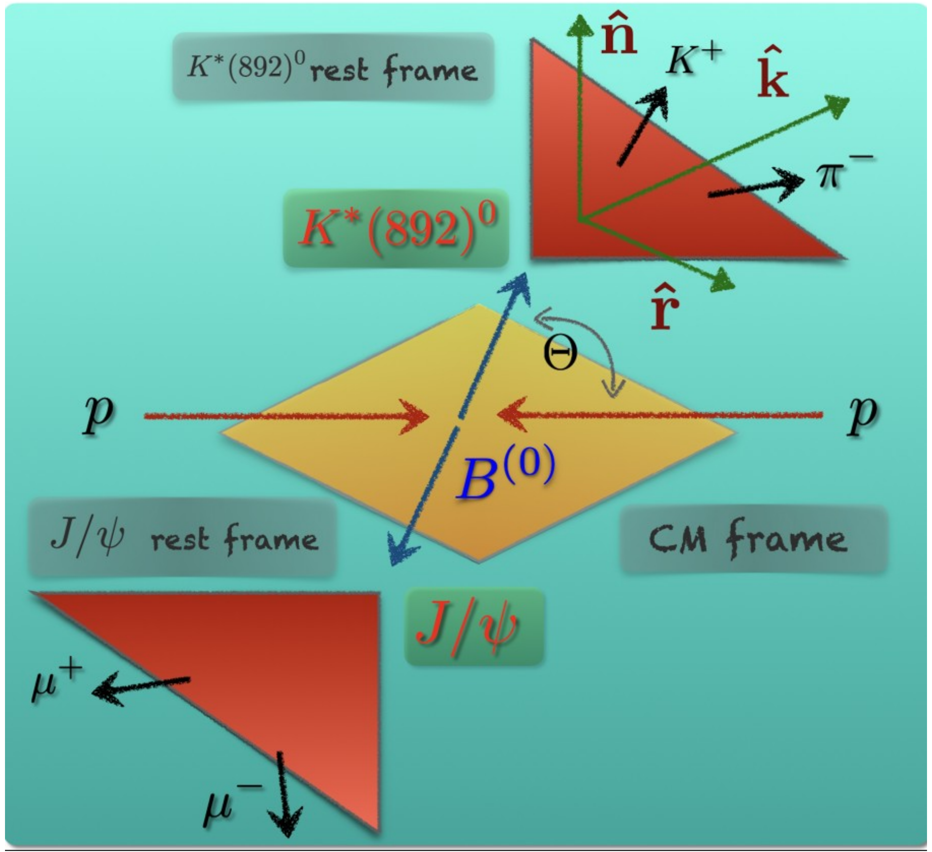
- **first observation** of **Bell violation** at high energy in B meson decays and in the presence of WEAK and STRONG interactions and for qutrit systems (spin-1) !

- work based on data from LHCb experiment at CERN [Phys. Rev. D 88, 052002 (2013)]

$$B^0 \to J/\psi K^*(892)^0$$

| | $\mathscr{E}$ | $\mathcal{I}_3$ |
|---|---|---|
| • $B^0 \to J/\psi K^*(892)^0$ [5] | $0.756 \pm 0.009$ | $2.548 \pm 0.015$ |
| • $B^0 \to \phi K^*(892)^0$ [18] | $0.707 \pm 0.133^*$ | $2.417 \pm 0.368^*$ |
| • $B^0 \to \rho K^*(892)^0$ [19] | $0.450 \pm 0.077^*$ | $2.208 \pm 0.151^*$ |
| • $B_s \to \phi\phi$ [20] | $0.734 \pm 0.037$ | $2.525 \pm 0.064$ |
| • $B_s \to J/\psi\phi$ [21] | $0.731 \pm 0.032$ | $2.462 \pm 0.080$ |

PHYSICAL REVIEW D **109**, L031104 (2024)




M. Fabbrichesi, EG, L. Marzola, R. Floreanini

CGLMP inequality $\quad \mathcal{I}_3 < 2$

(for qutrits)

$$\mathcal{I}_3 = 2.548 \pm 0.015,$$

Significance (to reject the null hypothesis)

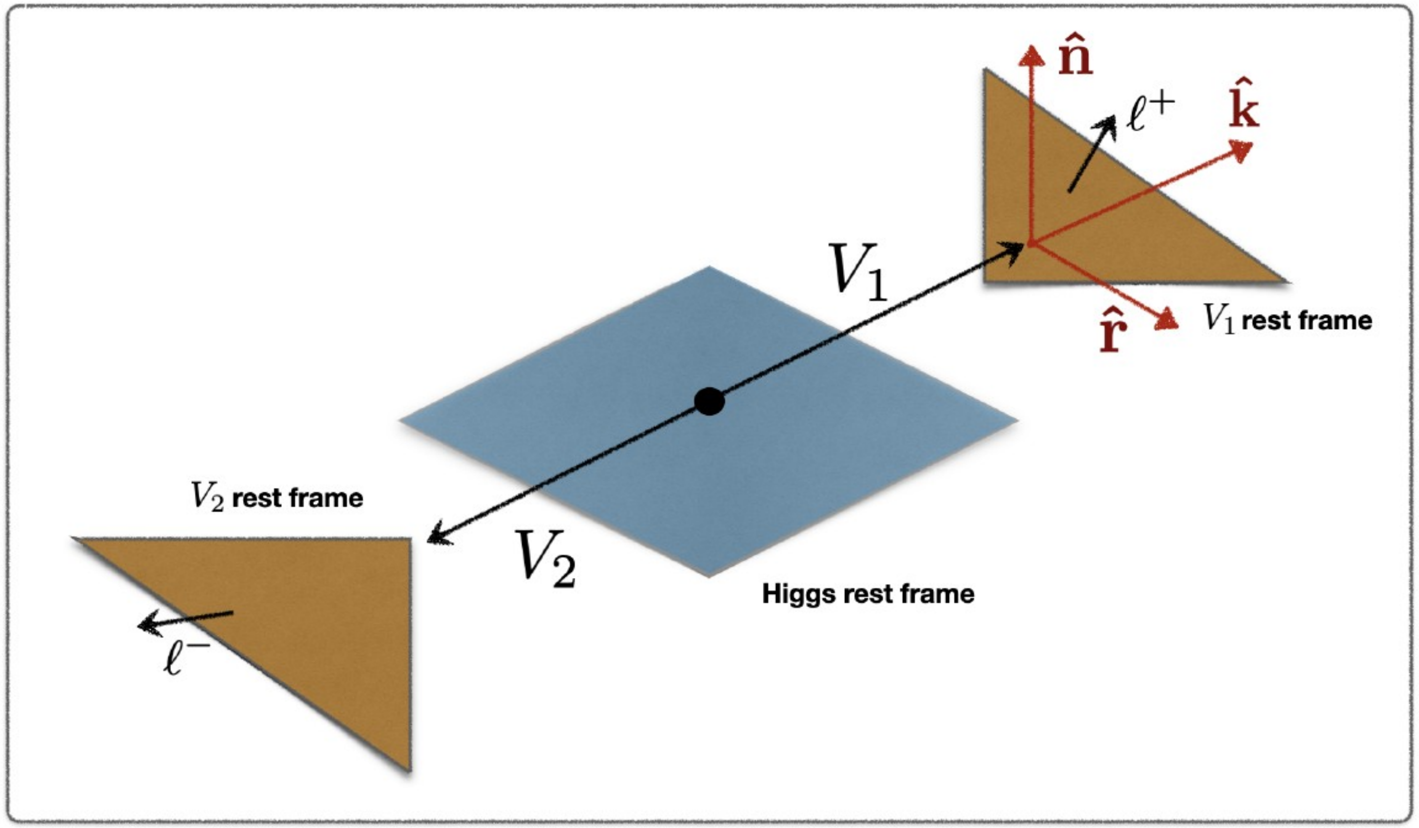
Entanglement (Entropy): **86 σ**

Bell inequality violation: **36 σ**

# Di-boson production in Higgs boson decays

EPJC 83 (2023) 9, 823 ; arXiv: 2302.00683

$$H \to V(k_1, \lambda_1)\, V^*(k_2, \lambda_2)$$




In the Higgs boson rest frame the density matrix of the bipartite VV* system does not depend on the scattering angle, but only by the Higgs mass, the V mass and the off-shell V* mass

# Di-boson production in Higgs boson decays

**A. Barr**, PLB 825 (2022) ; 2106.01377

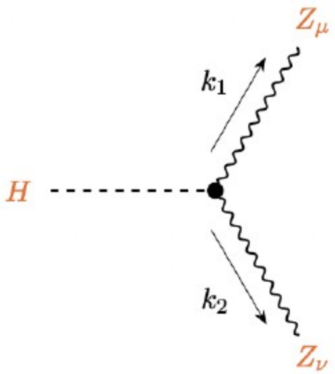
**Aguilar-Saavedra**, **Bernal, Casas, Moreno** PRD 107 (2023) 1; 2209.13441

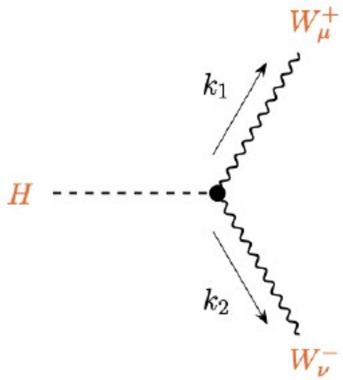
$$H \to V(k_1, \lambda_1)\, V^*(k_2, \lambda_2)$$

**Results based on**
EPJC 83 (2023) 9, 823 ; 2302.00683

analytic computation of PDM

PDM extracted from simulation at parton level




$$\begin{cases} \xi_W = 1, \text{ and } \xi_Z = 1/(2c_W), \text{ with } c_W = \cos\theta_W \\ g \text{ is the weak coupling} \end{cases}$$

Weinberg angle

$V^*$ regarded as an off-shell vector boson with mass $M^*_V = f M_V \qquad 0 < f < 1$

mass of V boson $M_V$

Quantum Amplitude

$$\mathcal{M}_H(\lambda_1, \lambda_2) = g\, M_V\, \xi_V\, g_{\mu\nu} \varepsilon^{\mu\star}(k_1, \lambda_1) \varepsilon^{\nu\star}(k_2, \lambda_2)$$

$$\mathcal{M}_H(\lambda_1, \lambda_2) \mathcal{M}_H(\lambda'_1, \lambda'_2)^\dagger = g^2\, M_V^2\, \xi_V^2\, g_{\mu\nu} g_{\mu'\nu'} \mathscr{P}^{\mu\mu'}_{\lambda_1\lambda'_1}(k_1)\, \mathscr{P}^{\nu\nu'}_{\lambda_2\lambda'_2}(k_2)$$

By projecting into the Gell-Mann matrix basis for 2-qutrits one can extract the **f,g** and **h** coefficients

# Inserting the f,g and h coefficients into the Gell-Mann basis for 2-qutrits

$$\rho_H = 2\begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&h_{44}&0&h_{16}&0&h_{44}&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&h_{16}&0&2\,h_{33}&0&h_{16}&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&h_{44}&0&h_{16}&0&h_{44}&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix},$$

$$\mathrm{Tr}\left[\rho_H\right] = 1$$

density matrix is idempotent

$$\boxed{\rho_H^2 = \rho_H}$$

Signaling that H → VV* is a **pure state**

$$\boxed{\rho_H = |\Psi_H\rangle\langle\Psi_H|}$$

**Aguilar-Saavedra, Bernal, Casas, Moreno**
PRD 107 (2023) 1; 2209.13441

using the basis

$$|\lambda\,\lambda'\rangle = |\lambda\rangle \otimes |\lambda'\rangle \text{ with } \lambda, \lambda' \in \{+, 0, -\}$$

where the pure state is

EPJC 83 (2023) 9, 823 ; 2302.00683

$$\boxed{|\Psi_H\rangle = \frac{1}{\sqrt{2+\varkappa^2}}\left[|+-\rangle - \varkappa\,|0\,0\rangle + |-+\rangle\right]}$$

$$\varkappa = 1 + \frac{m_H^2 - (1+f)^2 M_V^2}{2 f M_V^2}$$

EPJC 83 (2023) 9, 823 ;
2302.00683

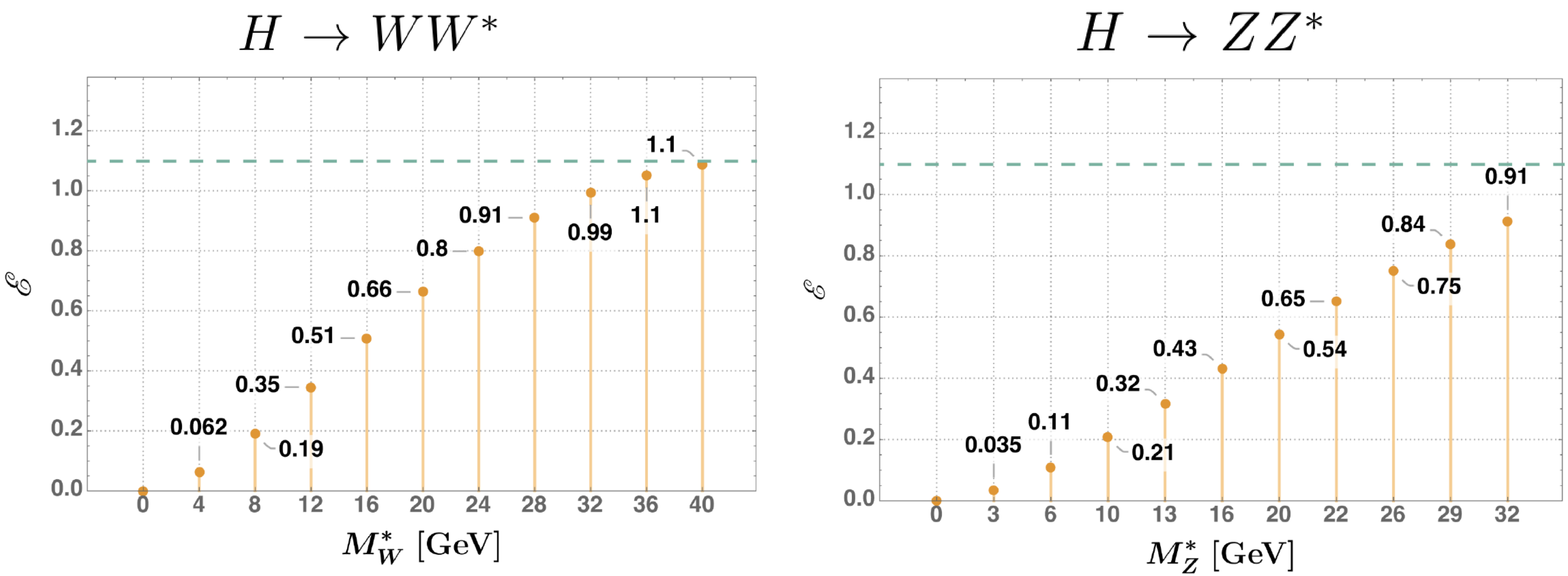
# Entropy of entanglement for H → VV*

- Being a pure state we can **quantify** entanglement via **Entropy of entanglement**
(Von Neumann Entropy)

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

$$0 \leq \mathcal{E}[\rho] \leq \log 3 \quad \sim 1.01$$

equality holds if and only
if the bipartite is separable

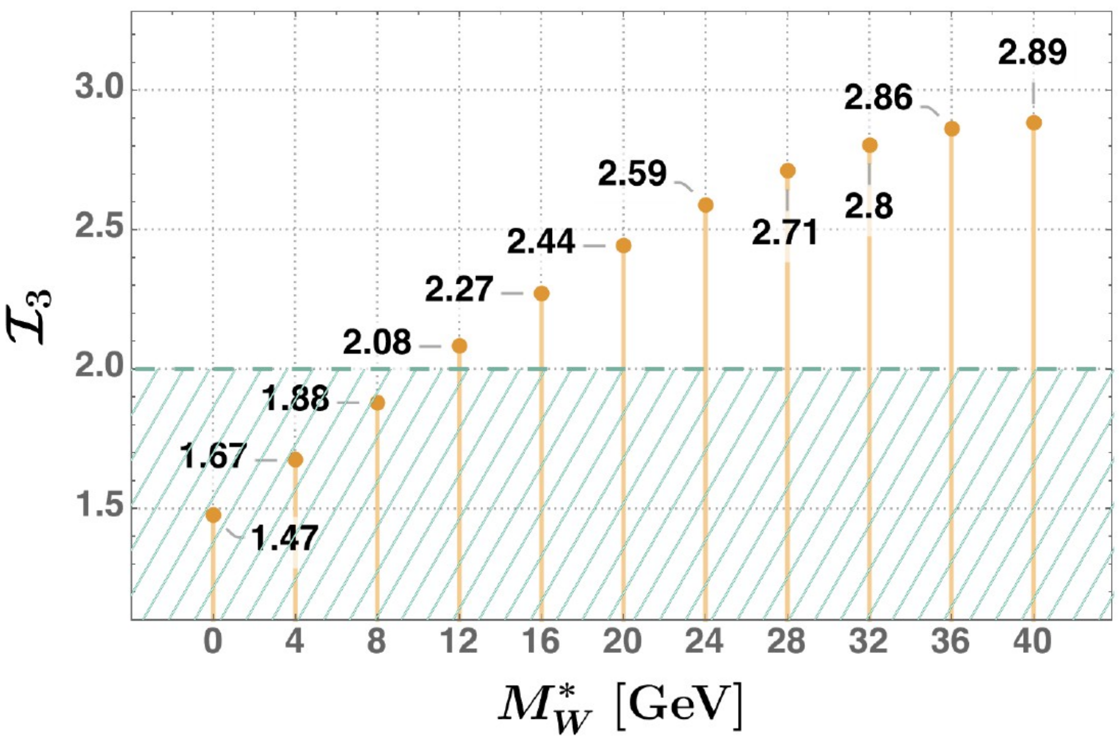


EPJC 83 (2023) 9, 823 ;
2302.00683

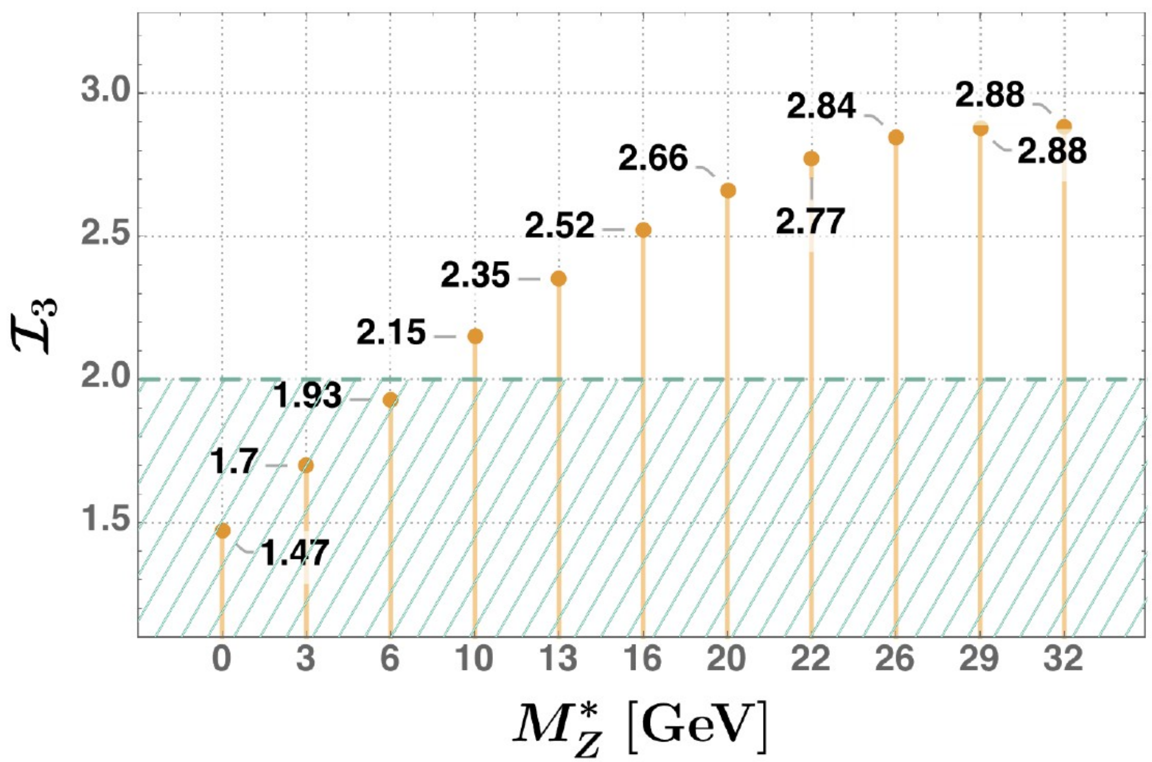
# Bell inequality violation for H → VV*

$$\mathcal{I}_3 > 2$$

$H \to WW^*$




$H \to ZZ^*$




Maximization of $\mathcal{I}_3$ performed point by point as function of $M_{V^*}$

# Summary of results for $H \to WW^*$ and $H \to ZZ^*$

significance ***S*** for rejecting the null hypothesis $\mathcal{I}_3 \leq 2$

**1000 pseudo experiments**

| | ***S*** (WW*) | ***S*** (ZZ*) |
|---|---|---|
| **LHC Run 2** | **1** | **1.3** |
| **LHC Hi-Lumi** | **1** | **5.6** |

***S*** (WW*) → uncertainty dominated by the systematic error

***S*** (ZZ*) → uncertainty dominated by the statistical error

5.6 → larger due to smaller uncertainty and higher mean value (2.9)

## ZZ* promising channel at LHC Hi-Lumi

- In real simulation a reduced significance is expected (systematic uncertainties due to unfolding, background, and detector were neglected or only partially modeled here)

- Results are also consistent with analysis of → **A. Barr,** PLB 825 (2022) ; 2106.01377 **Aguilar-Saavedra, Bernal, Casas, Moreno** PRD 107 (2023) 1; 2209.13441 [hep-ph]
where simulations of events have been adopted for the reconstruction of the density matrix

# Constraining HWW and HZZ anomalous couplings with Quantum Tomography

Fabbrichesi, Floreanini, EG, Marzola,
JHEP 09 (2023) 195; 2304.02403

CP-even

CP-odd

We use polarization density matrix of the processes

$$H \to WW^* \qquad H \to ZZ^*$$

to constrain anomalous Higgs couplings to WW and ZZ

$V^{\mu\nu} \longrightarrow$ Field strength , V=W,Z

$$\tilde{V}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} V_{\alpha\beta}$$

## Effective Higgs-VV Lagrangian (including SM)

$$\begin{aligned}
\mathcal{L}_{HVV} \;\; = \;\; & g\, m_W W^+_\mu W^{-\mu} H + \frac{g}{2\cos\theta_W} m_Z Z_\mu Z^\mu H \\
& - \frac{g}{m_W}\Bigg[ \frac{\lambda_1^W}{2} W^+_{\mu\nu} W^{-\mu\nu} + \lambda_2^W \Big( W^{+\nu} \partial^\mu W^-_{\mu\nu} + \text{H.c.} \Big) + \frac{\widetilde{\lambda}^W_{CP}}{4} W^+_{\mu\nu} \widetilde{W}^{-\mu\nu} \\
& + \frac{\lambda_1^Z}{2} Z_{\mu\nu} Z^{\mu\nu} + \lambda_2^Z Z^\nu \partial^\mu Z_{\mu\nu} + \frac{\widetilde{\lambda}^Z_{CP}}{4} Z_{\mu\nu} \widetilde{Z}^{\mu\nu} \Bigg] H \,,
\end{aligned}$$

# This state can be described in terms of helicity amplitudes $h_i$

$$|\Psi\rangle = \frac{1}{|\overline{\mathcal{M}}|}\left[h_+\,|V(+)V^*(-)\rangle + h_0\,|V(0)V^*(0)\rangle + h_-\,|V(-)V^*(+)\rangle\right]$$

$$h_\lambda = \langle V(\lambda)V^*(-\lambda)| - \mathcal{L}|H\rangle \quad \text{with} \quad \lambda = 0, \pm$$

$$|\overline{\mathcal{M}}|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2 \,,$$

$$\rho_H = \frac{1}{|\overline{\mathcal{M}}|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ see backup slide for same density matrix in Gell-Mann basis

$$\boxed{\rho_H^2 = \rho_H}$$

$\Rightarrow$ again a pure state!

# observables to constrain anomalous couplings in H → VV*

**Entropy of entanglement**

$$\mathscr{E}_{ent} = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

CP-even

**Lower bound on concurrence (entanglement)**

$$\begin{aligned} \mathscr{C}_2 \quad = \quad & 2\max\Big[0,\, -\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,, \\ & -\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2\Big]\,. \end{aligned}$$

CP-even

$$\mathscr{C}_{odd} = \frac{1}{2}\sum_{\substack{a,b\\ a<b}} \left| h_{ab} - h_{ba} \right|$$

CP-odd

# Constraining the anomalous couplings in H $\rightarrow$ VV*

$$\chi^2 = \sum_i \left[\frac{O_i(a_V, \widetilde{a}_V) - O_i(0,0)}{\sigma_i}\right]^2 \leq 5.991$$

95% CL

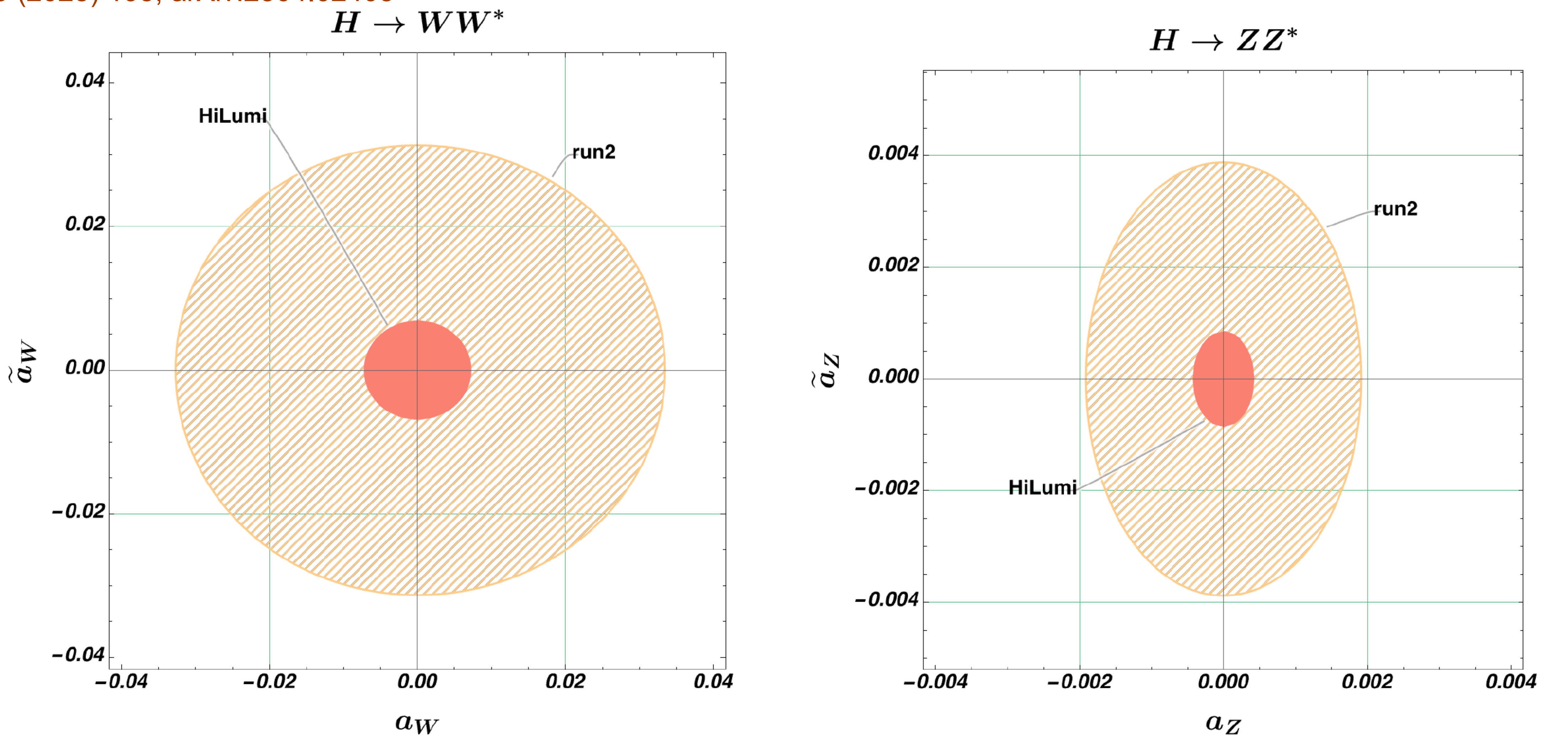
i=1,2

Combinations of two $O_i$

$$m_H = 124 \pm 0.18 \pm 0.04$$

- Main uncertainty comes from the measurement of polarizations $\rightarrow$ originates in the reconstruction of the rest frame of the decaying Higgs boson $\rightarrow$ related to VV* invariant mass
- We propagate uncertainty in the Higgs mass to the operators by means of a Montecarlo simul. $\rightarrow$ giving variances $\sigma_i^2$ associated to the operators $O_i$
- Works well for ZZ*
- For WW we can determine only transverse mass $\rightarrow$ comes with an error of 5 GeV for fully leptonic decays, and we take half for semileptonic decays

JHEP 09 (2023) 195; arXiv:2304.02403




HiLumi → rescaling run2 uncertainties by

→ ratio of Luminosities $\sigma_i \to \sigma_i \sqrt{\frac{14}{300}}$

including bckg effects weakens a bit the bounds (see case for ZZ in backup slides)

95% CL limits

No background

| LHC | run2 | HiLumi |
|---|---|---|
| | $\|a_W\| \leq 0.033$ | $\|a_W\| \leq 0.0070$ |
| | $\|\tilde{a}_W\| \leq 0.031$ | $\|\tilde{a}_W\| \leq 0.0068$ |
| | $\|a_Z\| \leq 0.0019$ | $\|a_Z\| \leq 0.00040$ |
| | $\|\tilde{a}_Z\| \leq 0.0039$ | $\|\tilde{a}_Z\| \leq 0.00086$ |

- Comparison with other theoretical analysis based on polarizations (not entanglement)

1σ limits and for $L$=1ab$^{-1}$ ➡ $a_Z = 6.88 \times 10^{-3}\,, \quad \tilde{a}_Z = 9.53 \times 10^{-3}$

**K. Rao, S.D. Rindani, P. Sarmah**
NPB 964 (2021) 115317; 2009.00980

- Comparison with CMS $\quad f_{g2} = \frac{\sigma_2}{\sigma}\,|a_V|^2\,, \quad \text{and} \quad f_{g3} = \frac{\sigma_3}{\sigma}\,|\tilde{a}_V|^2$

Our 95% CL limits $\quad f^Z_{g2} < 7.8 \times 10^{-6}\,, \quad f^Z_{g3} < 1.5 \times 10^{-5}$

CMS 95% CL limits $\quad f^V_{g2} < 3.4 \times 10^{-3}\,, \quad f^V_{g3} < 1.4 \times 10^{-2}$

# Di-boson production in pp collisions

$$p\,p \to V_1\,V_2$$

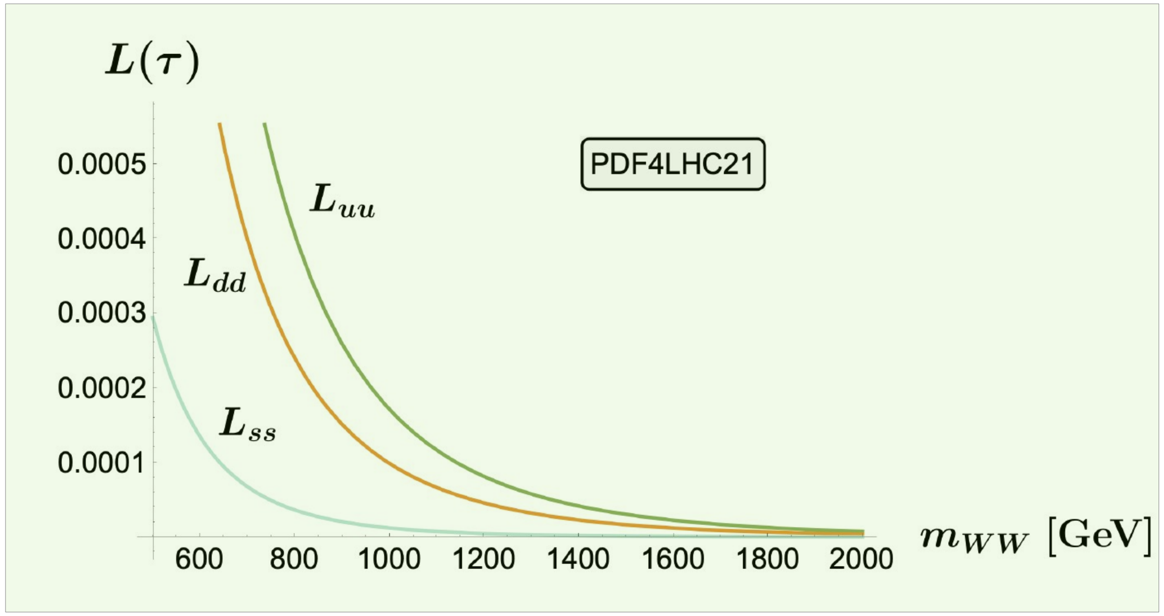
$$V_1, V_2 \in \{W, Z\}$$

(away from the Higgs resonance)

Proceeds via the partonic quark-antiquark process

$$q_1\,\bar{q}_2 \to V_1 V_2$$

weighted by the parton luminosity $L^{q_1\bar{q}_1}(\tau)$




parton luminosity of the initial $q_1\bar{q}_2$ state

$$L^{q_1\bar{q}_1}(\tau) = \frac{4\tau}{\sqrt{s}} \int_{\tau}^{1/\tau} \frac{\mathrm{d}z}{z}\, q_{q_1}(\tau\, z)\, q_{\bar{q}_2}\left(\frac{\tau}{z}\right)$$

$\tau = m_{VV}/\sqrt{s}$

pp c.o.m. energy

invariant mass of $V_1V_2$ system

## Decomposing the polarization matrix density into the Gell-Mann matrix basis

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \left(\frac{1}{9}\left[\mathbb{1}\otimes\mathbb{1}\right] + \sum_a f_a \left[\mathbb{1}\otimes T^a\right] + \sum_a g_a \left[T^a\otimes\mathbb{1}\right] + \sum_{ab} h_{ab}\left[T^a\otimes T^b\right]\right)_{\lambda_1\lambda_1',\lambda_2\lambda_2'}$$

we obtain for the **h** correlations coefficients in VV production → depend also on scattering angle $\Theta$

$$h_{ab}[m_{VV},\Theta] = \frac{\sum_{q=u,d,s} L^{q\bar{q}}(\tau)\left(\tilde{h}^{q\bar{q}}_{ab}[m_{VV},\Theta] + \tilde{h}^{q\bar{q}}_{ab}[m_{VV},\Theta+\pi]\right)}{\sum_{q=u,d,s} L^{q\bar{q}}(\tau)\left(A^{q\bar{q}}[m_{VV},\Theta] + A^{q\bar{q}}[m_{VV},\Theta+\pi]\right)}$$

and analogously for the $f_a$ and $g_a$ correlation coefficients, where

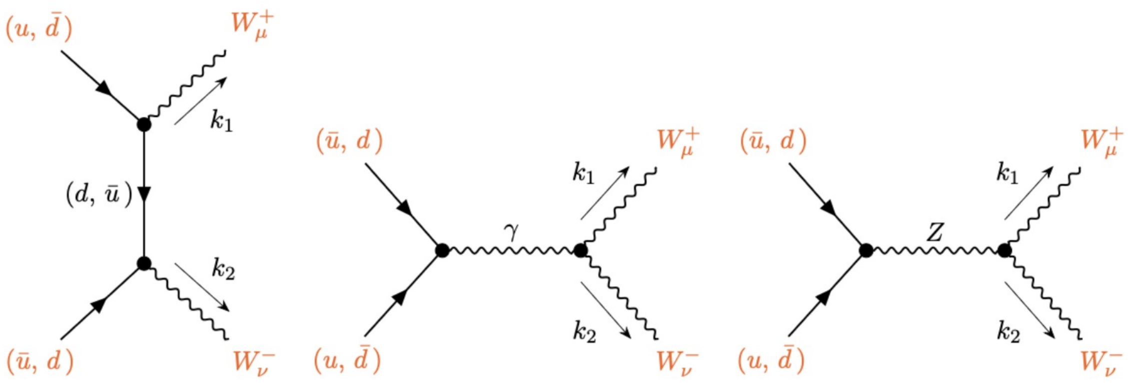
$$\tilde{h}_{ab} = A^{q\bar{q}} h_{ab}$$

$A^{q\bar{q}}$ = unpolarized square amplitude of the partonic process

- main uncertainty on the correlation coefficients comes from the missing higher order QCD corrections
- giving approx a 10% uncertainty on the main entanglement observables
- other theoretical uncertainties, mainly from PDF and other SM inputs, is negligible → of the order of permille effect

Fabbrichesi,Floreanini, EG, Marzola
EPJC 83 (2023) 9, 823 ; arXiv: 2302.00683]

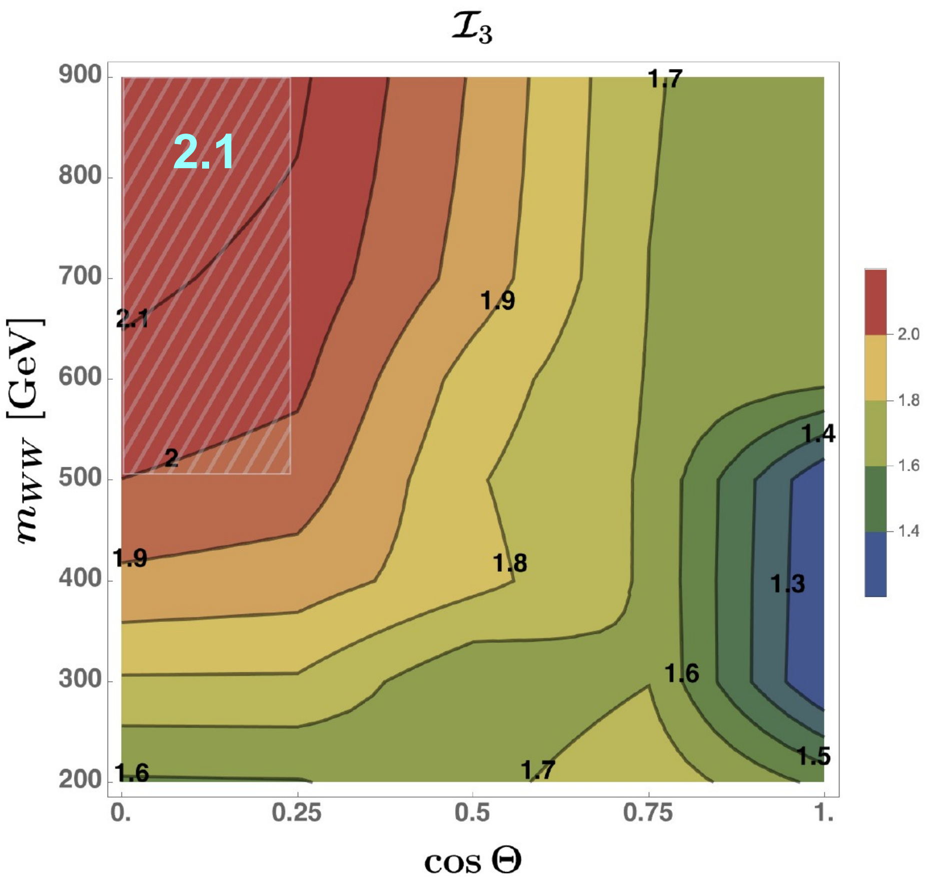
$$p\,p \to W^+W^-$$

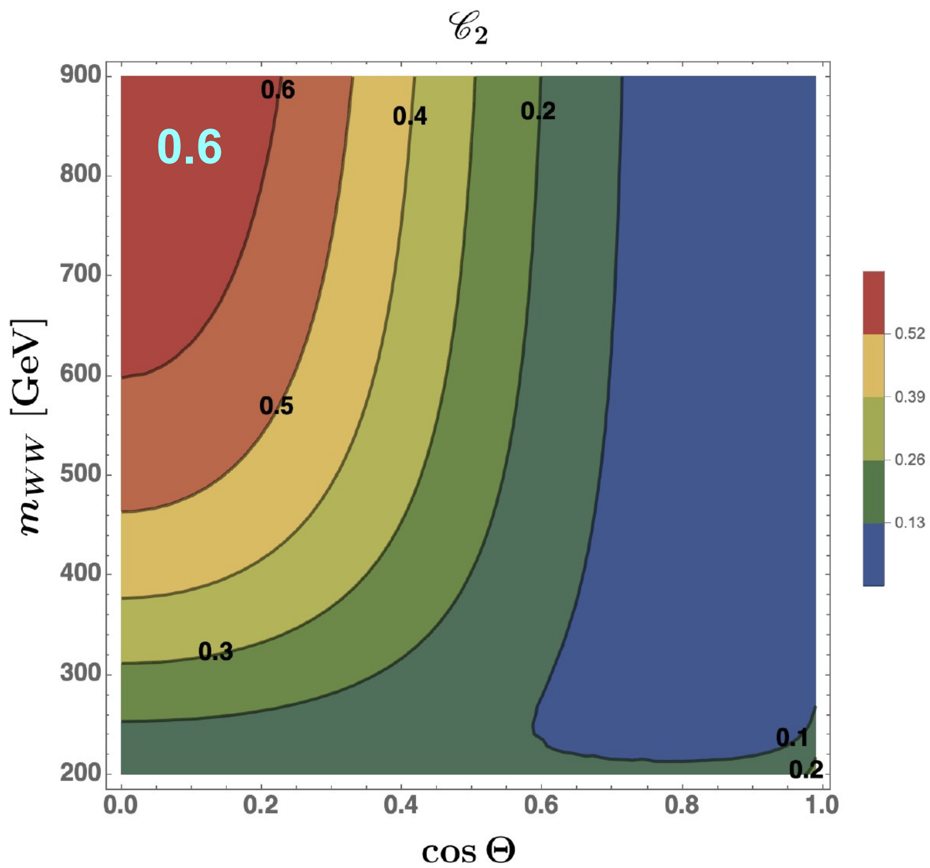



optimization for maximum value of $\mathcal{I}_3$ of Bell's inequality violation is employed point by point in the $\Theta$ $m_{WW}$ space

$$\mathcal{B} \to (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

(see backup slides for their expressions in hatched area)

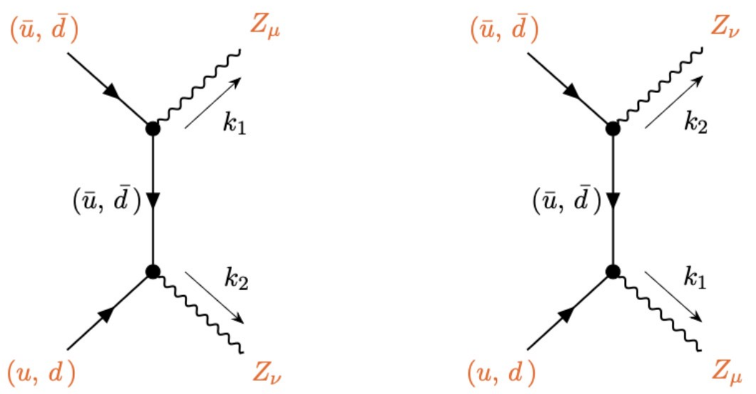


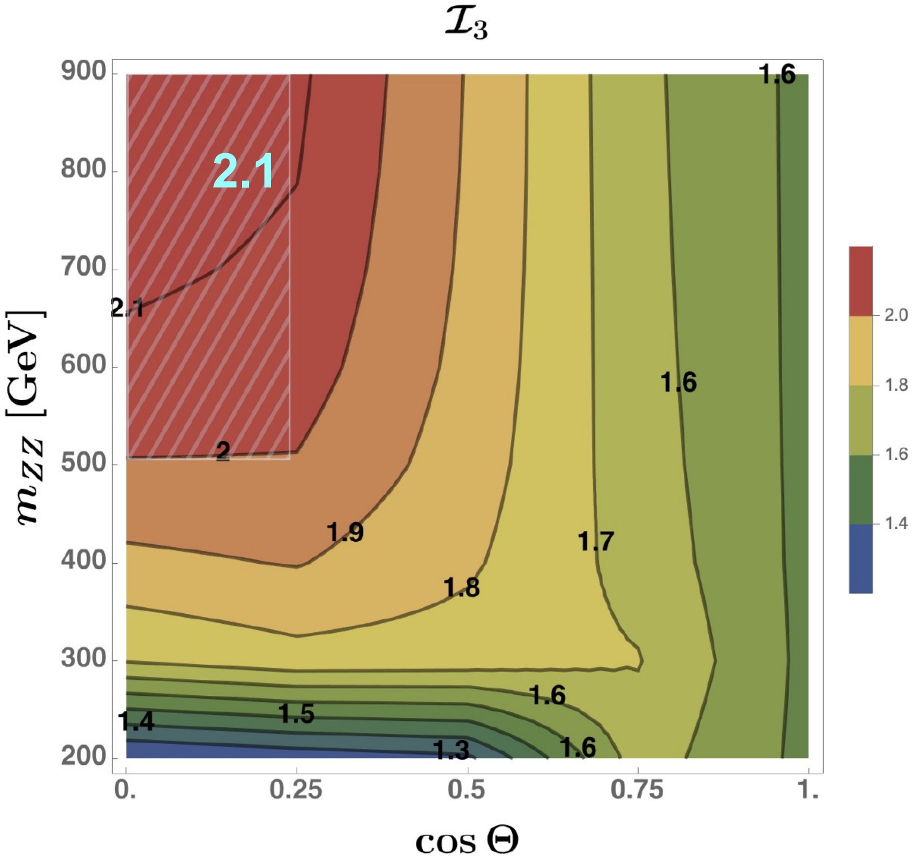

hatched area in the left-plot for $\mathcal{I}_3$ >2 indicates bin used as reference for our estimation of the significance (see next slides)

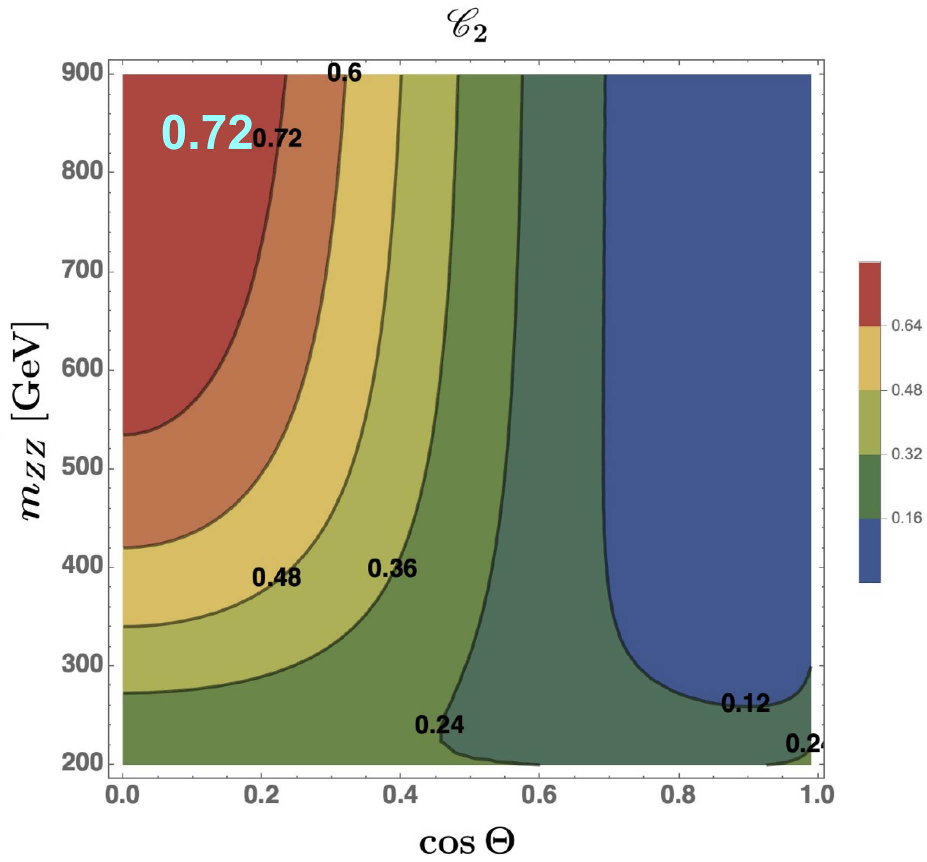
EPJC 83 (2023) 9, 823 ; arXiv: 2302.00683

# $pp \to ZZ$




- same strategy as in WW for optimization of maximum value of of Bell's inequality violation
- reduced effect for Bell violations with respect to WW, but error is smaller being dominated by statistics




hatched area in the left-plot for $\mathcal{I}_3$ >2 indicates bins used as reference for our estimation of the significance (see next slides)

# Bell's inequalities in Di-boson production at LHC

EPJC 83 (2023) 9, 823 ; arXiv: 2302.00683

$$p\,p \to W^+W^- \quad p\,p \to ZZ \qquad p\,p \to WZ$$

(backup slides for details)

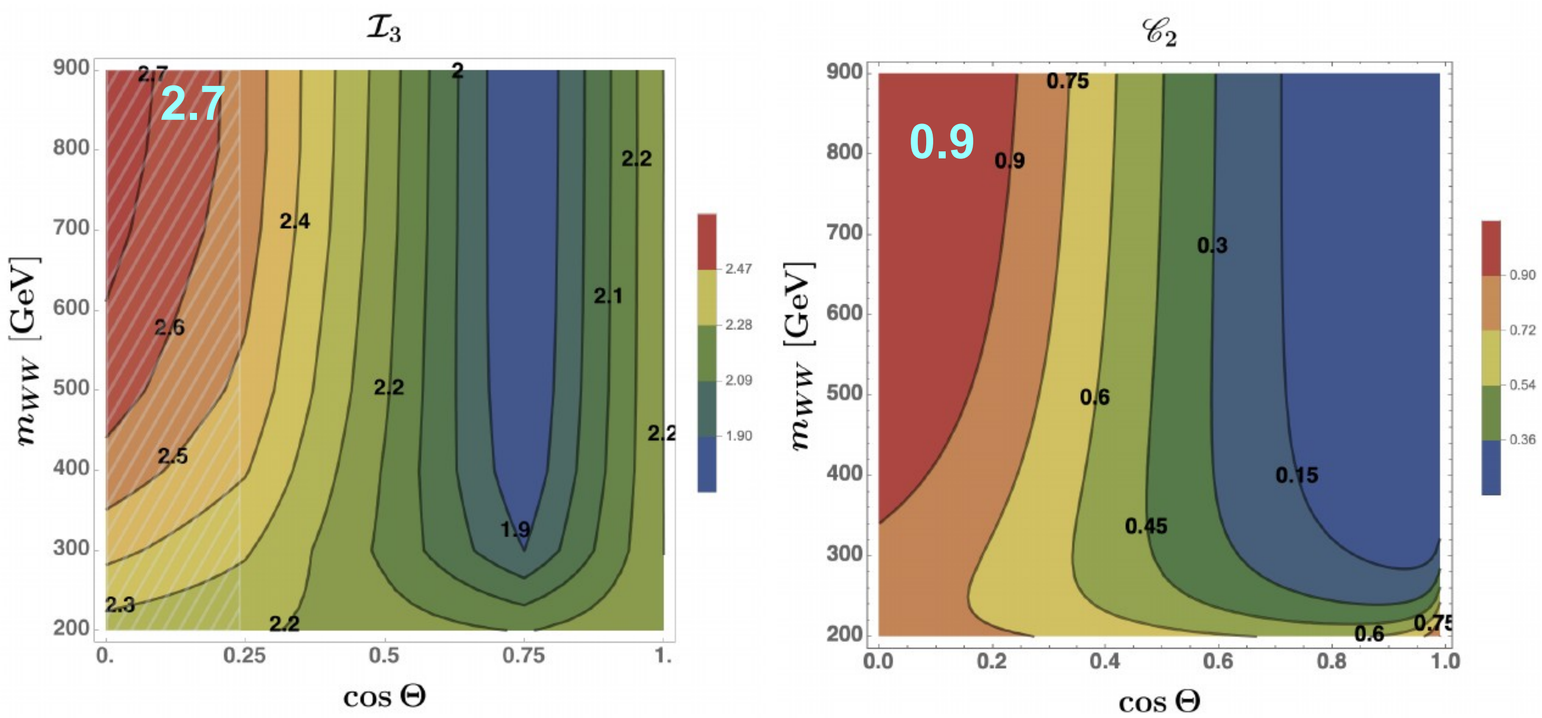
- not sufficient N. of events to test Bell's inequalities with large significance in the relevant kinematic region
- pp → WW largely affected by systematic error induced by the missing neutrino momentum reconstruction
- pp → ZZ is the most promising channel

At Hi-lumi significance **more than 2** to reject the null hypothesis

- estimates of significance expected to be reduced by a full simulation analysis

# Lepton colliders

$$e^+ \; e^- \to W^+ \; W^-$$

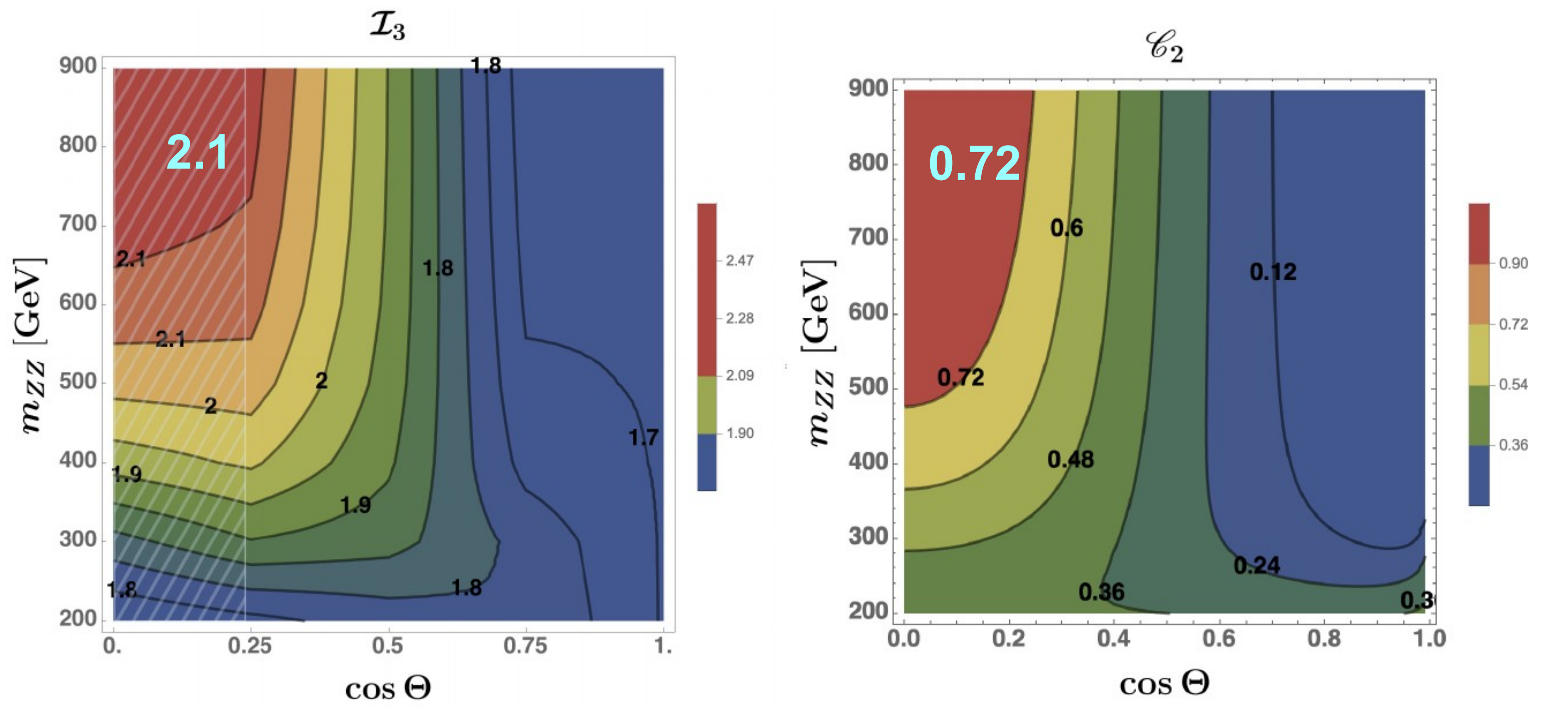



$$h_{ab}[m_{WW},\Theta] = \frac{\tilde{h}^{\ell\bar{\ell}}_{ab}[m_{WW},\Theta]}{A^{\ell\bar{\ell}}[m_{WW},\Theta]},$$

$$f_a[m_{WW},\Theta] = \frac{\tilde{f}^{\ell\bar{\ell}}_{a}[m_{WW},\Theta]}{A^{\ell\bar{\ell}}[m_{WW},\Theta]},$$

$$g_a[m_{WW},\Theta] = \frac{\tilde{g}^{\ell\bar{\ell}}_{a}[m_{WW},\Theta]}{A^{\ell\bar{\ell}}[m_{WW},\Theta]},$$

absence of PDF increases entanglement and Bell violations in WW




$$e^+ \; e^- \to Z \; Z$$

ZZ channel not much different than LHC → due to the fact that PDF almost factorize (t,u channels)

# Lepton colliders : events and sensitivity

$$e^+ \, e^- \to Z \, Z$$

**promising channel**

| | $\sqrt{s} = 1$ TeV | $\sqrt{s} = 368$ GeV |
|---|---|---|
| | (muon) $\mathcal{L} = 1$ ab$^{-1}$ | (FCC) $\mathcal{L} = 1.5$ ab$^{-1}$ |
| events | 44 | 748 |

$$m_{WW} > 200 \text{ GeV and } \cos\Theta < 0.25$$

- Events computed with MADGRAPH@LO
- Efficiency 70% in each lepton identification
  → n. of events reduced accordingly
- only statistic error assumed
- no background considered

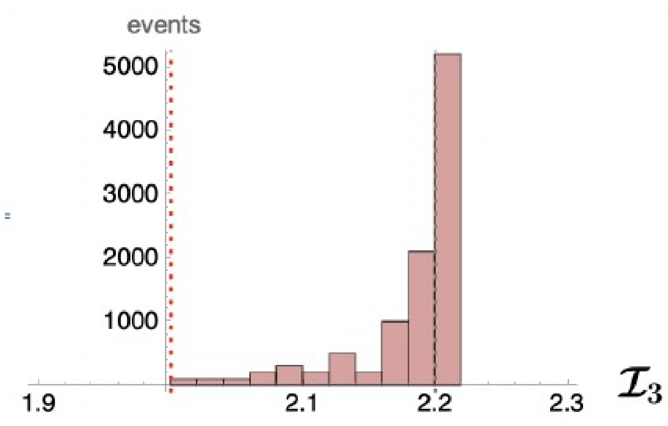
FCC significance is larger than muon collider due to larger n. of events at FCC (statistical error reduced)

Significance to reject null hypothesis:
**2** @ muon collider and > **4** @ FCC

**$10^4$ pseudo experiments**

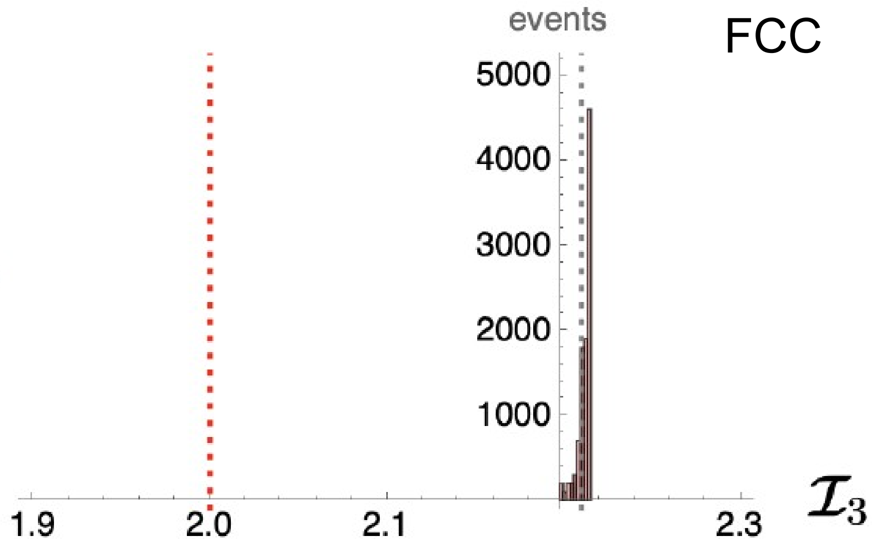
$\mathcal{I}_3 = 2.2$

Muon collider




$\mathcal{I}_3 = 2.2$

FCC



# Summary

- Quantum tomography in diboson production is a very powerful tool
- require the knowledge of polarization density matrix
- that can be fully reconstructed from decay products of W and Z
- and allow to investigate quantum entanglement and Bell inequality violation

- Entanglement in diboson production difficult to quantify in general case (mixtures) (but we can use lower bound on concurrence as witness of entanglement)
- It is possible to quantify entanglement in pure states **H** $\rightarrow$ **VV*** (using Entropy)

- Large significance (> $5\sigma$) to observe entanglement and Bell inequality violation in **H** $\rightarrow$ **ZZ*** expected at LHC Hi-Lumi

- Entanglement as powerful tool to constrain anomalous couplings in H $\rightarrow$ VV*

# Backup slides

- Matrix element of **two-Vector Boson** production $\bar{q}(p_1)\, q(p_2) \to V_1(k_1,\lambda_1)\, V_2(k_2,\lambda_2)$

$$\mathcal{M}(\lambda_1,\lambda_2) \quad = \quad \mathcal{M}_{\mu\nu}\varepsilon^{\mu\star}(k_1,\lambda_1)\varepsilon^{\nu\star}(k_2,\lambda_2)$$

**Density matrix**

covariant expression

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \frac{\mathcal{M}_{\mu\nu}\mathcal{M}^{\dagger}_{\mu'\nu'}\,\mathscr{P}^{\mu\mu'}_{\lambda_1\lambda_1'}(k_1)\,\mathscr{P}^{\nu\nu'}_{\lambda_2\lambda_2'}(k_2)}{|\overline{\mathcal{M}}|^2}$$

**9 x 9 matrix**

unpolarized matrix element square

$$\mathscr{P}^{\mu\nu}_{\lambda\lambda'}(p) \quad = \quad \varepsilon^{\mu}(p,\lambda)^{\star}\varepsilon^{\nu}(p,\lambda')$$

(spin-1 eigenstates at rest)

**k**=axis of spin-quantization ⇒ $\psi_{\pm} = -\frac{1}{\sqrt{2}}(\pm\hat{\mathbf{n}} + i\,\hat{\mathbf{r}}) \quad \text{and} \quad \psi_0 = \hat{\mathbf{k}}$

- How to related spin-1 eigenstates at rest to the boosted frame ? (see next slide)

## covariant polarization vector of spin-1

$$\varepsilon^\mu(p,\lambda) = -\frac{1}{\sqrt{2}}|\lambda|\,(\lambda\, n_1^\mu + i\, n_2^\mu) + \Big(1-|\lambda|\Big) n_3^\mu$$

helicity $\lambda = \pm 1, 0$

rest frame limit

$$\varepsilon^\mu(p,\lambda) \xrightarrow[(\beta\to 0)]{} \psi_\pm \,,\ \psi_0$$

$\beta$ is the velocity

boosted base

$$n_1^\mu = (0,\ \hat{\mathbf{n}})\,,\ \ n_2^\mu = (0,\ \hat{\mathbf{r}})\,,\ \ n_3^\mu = \frac{E}{M}(\beta,\ \hat{\mathbf{k}})$$

## Covariant Projector

$$\mathscr{P}^{\mu\nu}_{\lambda\lambda'}(p) \quad = \quad \varepsilon^\mu(p,\lambda)^\star \varepsilon^\nu(p,\lambda')$$

$\varepsilon^{0123} = 1$

**master formula**

$$= \frac{1}{3}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}\right)\delta_{\lambda\lambda'} - \frac{i}{2M}\epsilon^{\mu\nu\alpha\beta} p_\alpha n^i_\beta\,(S_i)_{\lambda\lambda'} - \frac{1}{2} n^\mu_i n^\nu_j\,(S_{ij})_{\lambda\lambda'}$$

$S_i,\ i \in \{1,2,3\} \longrightarrow$ rotation matrices for spin-1 particle

H.S. Song, Lett. Nuovo Cim. 25 (1979)
S.Y. Choi, T. Lee, H.S. Song, PRD 40 (1989)
Fabbrichesi, Floreanini, EG, Marzola, 2302.00683 [hep-ph]

(see backup slides)

$$S_{ij} = S_i S_j + S_j S_i - \frac{4}{3}\mathbb{1}\,\delta_{ij}$$

basis correspondence for $(S_i)_{\lambda\lambda'}$

$$|+\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix},\quad |0\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix},\quad |-\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

corresponding to eigenevalues $\lambda = \pm 1, 0$

# Sensitivity to the Bell violation

## $H \rightarrow WW^* \quad ZZ^*$

| | | $\ell^+\nu_\ell\,\ell^-\bar{\nu}_\ell$ | $\ell^-\ell^+\ell^-\ell^+$ |
|---|---|---|---|
| LHC run2 | $(\mathcal{L} = 140\ \text{fb}^{-1})$ | 4571 | 28 |
| Hi-Lumi | $(\mathcal{L} = 3\ \text{ab}^{-1})$ | $9.8 \times 10^3$ | 589 |

cuts in V* invariant mass: 40 GeV for WW* and 30 GeV for ZZ*

- we used MADGRAPH5 @ LO corrected by k factors at N3LO + N3LL
  Bonvini, Marzani, Muselli, Rottoli, JHEP 08, 105 (2016) 1603.08000 [hep-ph]
- take into account efficiency (70%) in the identification of final leptons
- events reduced of 25% for ZZ* and 50% for WW*

- backgrounds can be efficiently reduced after considering topology of final states and use of suitable cuts
- we neglect all background effects in our assessment of significance

- We model uncertainty as a Gaussian dispersion in the determination of the mass of the off-shell weak boson controlled by the number of events

- For the WW* we add a systematic error taking into account uncertainty in the reconstruction of W* mass, taking as benchmark value = 5 GeV for the systematic error in reconstruction of neutrino momenta

# Relations between helicity amplitudes $h_i$ and $h_{ab}$ , $f_a$ , $g_a$ , of Gell-Mann basis

$$
\begin{aligned}
\hat{h}_-\hat{h}_-^* &= \frac{1}{9}\Big[1 + 3\sqrt{3}\,\left(f_8 - 2g_8 - 2h_{38}\right) + 9f_3 - 6h_{88}\Big]\,,\\
\hat{h}_0\hat{h}_-^* &= h_{16} + i\,\left(h_{17} - h_{26}\right) + h_{27}\,,\\
\hat{h}_+\hat{h}_-^* &= h_{44} + i\,\left(h_{45} - h_{54}\right) + h_{55}\,,\\
\hat{h}_0\hat{h}_0^* &= \frac{1}{9}\Big[1 - 9\left(f_3 + g_3 - h_{33}\right) + 3\sqrt{3}\,\left(f_8 + g_8 - h_{38} - h_{83}\right) + 3h_{88}\Big]\,,\\
\hat{h}_+\hat{h}_0^* &= h_{61} + i\,\left(h_{62} - h_{71}\right) + h_{72}\,,\\
\hat{h}_+\hat{h}_+^* &= \frac{1}{9}\Big[1 + 3\sqrt{3}\,\left(g_8 - 2f_8 - 2h_{83}\right) + 9g_3 - 6h_{88}\Big]\,,
\end{aligned}
$$

# Including background for $H \to Z\ell^+\ell^-$

$$\rho_{\rm ZZ} = \alpha\rho_{\rm H\,\to\,ZZ} + (1-\alpha)\rho_{\rm BCKG}$$

$$0 \leq \alpha \leq 1$$

$$S/(S+B)$$

Main EW background

$$pp \to ZZ/Z\gamma \to 4\ell$$

We take $\alpha = 0.8$ corresponding to bckg 4 times smaller than signal

| LHC | run2 | HiLumi |
|---|---|---|
| | $\lvert a_Z\rvert \leq 0.0028$ | $\lvert a_Z\rvert \leq 0.00062$ |
| | $\lvert\tilde{a}_Z\rvert \leq 0.0039$ | $\lvert\tilde{a}_Z\rvert \leq 0.00086$ |

➡ bckg effects as expected weakens a bit the bounds

# Lepton colliders : events and sensitivity

$$e^+ \, e^- \rightarrow W^+ \, W^-$$

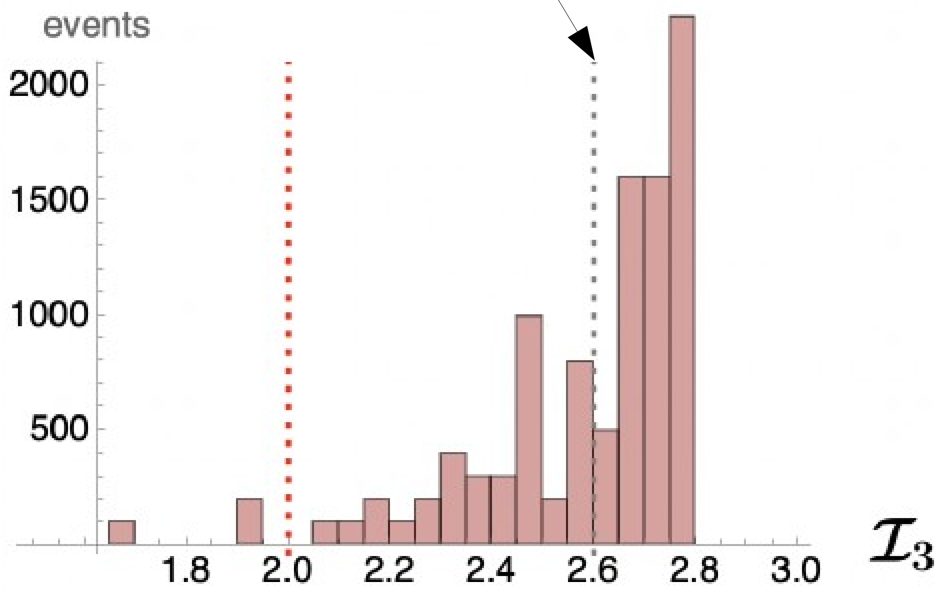
| | $\sqrt{s} = 1$ TeV | $\sqrt{s} = 368$ GeV |
|---|---|---|
| | (muon) $\mathcal{L} = 1 \text{ ab}^{-1}$ | (FCC) $\mathcal{L} = 1.5 \text{ ab}^{-1}$ |
| events | $3.6 \times 10^3$ | $5.8 \times 10^4$ |

$$m_{WW} > 200 \text{ GeV and } \cos\Theta < 0.25$$

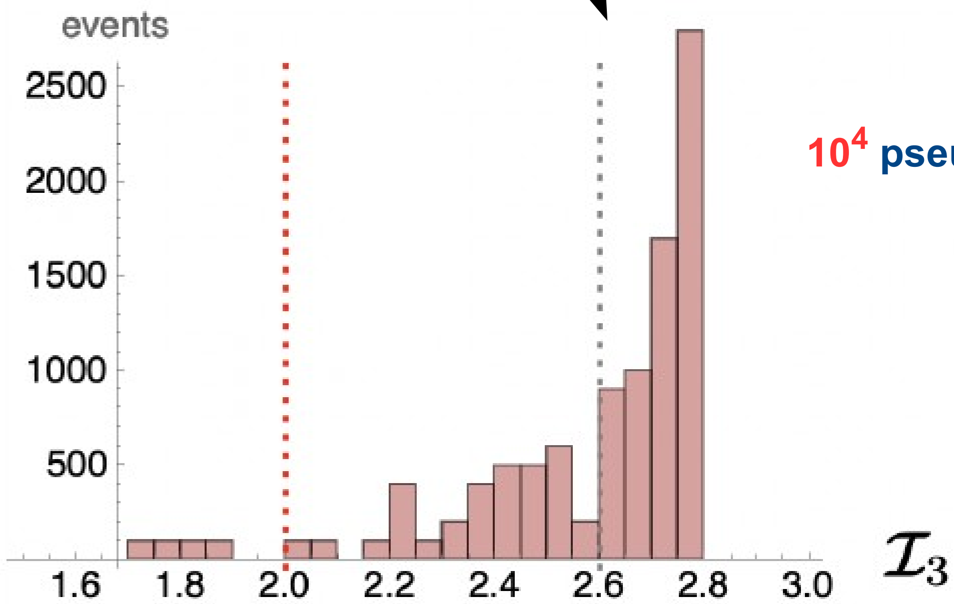
- Events computed with MADGRAPH@LO
- Efficiency 70% in each lepton identification
  → n. of events reduced accordingly
- systematic error due to missing neutrino in WW mass reconstruction largely dominates against statistical one
- no background considered

Significance = **2** to reject the null hypothesis both at muon collider and FCC

mean value = **2.6**




mean value = **2.6**




$10^4$ **pseudo experiments**