(image source: Muon (g-2) Collaboration)

Muon (g-2): current theoretical status

Ethan T. Neil (Colorado)
DPF-PHENO 2024
05/17/24

Outline

<u>Goal:</u> clarify the current SM theory prediction for muon (g-2), focusing on the QCD contributions.

1. Muon (g-2): experiment and theory

2. Light-by-light scattering

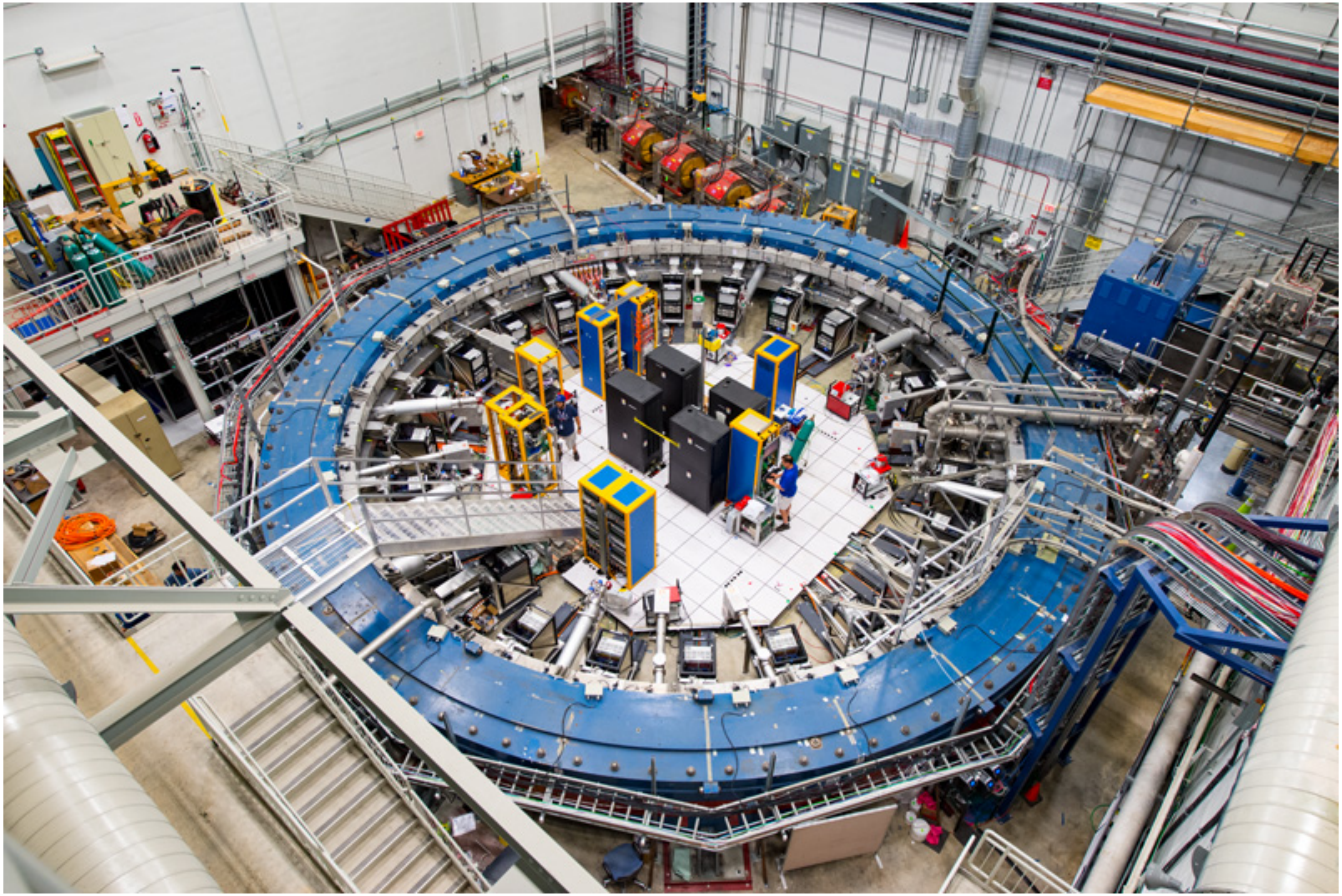
3. Hadronic vacuum polarization

Useful refs:

- Muon (g-2) Theory Initiative whitepaper: T. Aoyama et al, arXiv:2006.04822
- Snowmass update for Theory Initiative whitepaper: G. Colangelo et al., arXiv:2203.15810
- 6th plenary workshop of the Theory Initiative, University of Bern, 09/23: https://indico.cern.ch/event/1258310/

Fine print: I am involved with the Muon (g-2) Theory Initiative, but I am not giving this talk on behalf of the Initiative; **opinions expressed are my own**.

1. Muon (g-2): experiment and theory



(image credit: Fermilab Muon (g-2) Experiment)

- <u>Above:</u> the Fermilab muon (g-2) experiment - basically, a huge *magnetic storage ring*. Precession of muons' spin vs. momentum as they go around the ring is used to measure magnetic moment —> (g-2) directly!

The Big Move!

(image credits: Fermilab Muon (g-2) Experiment - except the beer photo)

Experiment uses a 50-ft diameter, 1.45T superconducting electromagnet, originally at Brookhaven on Long Island!

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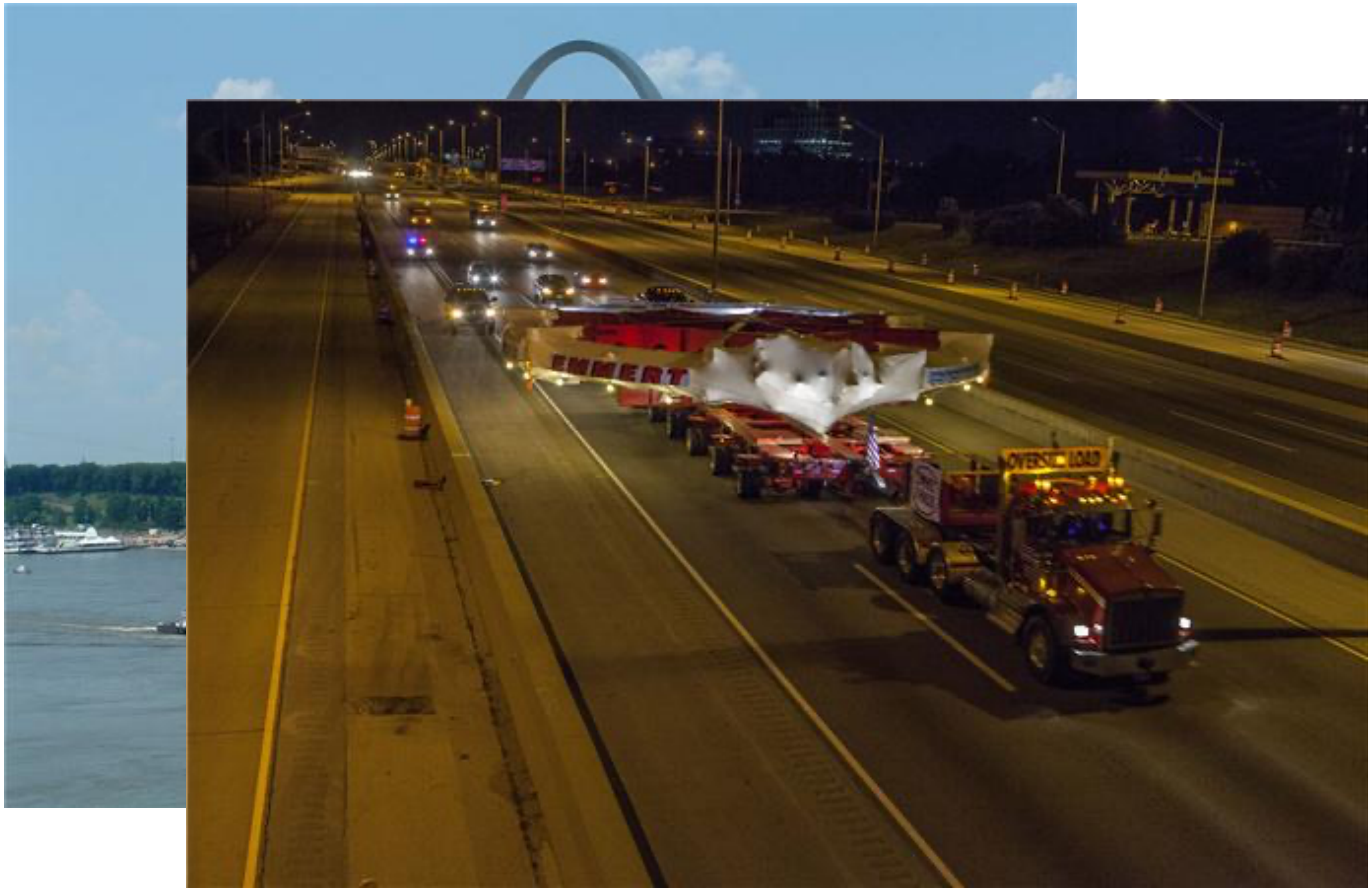

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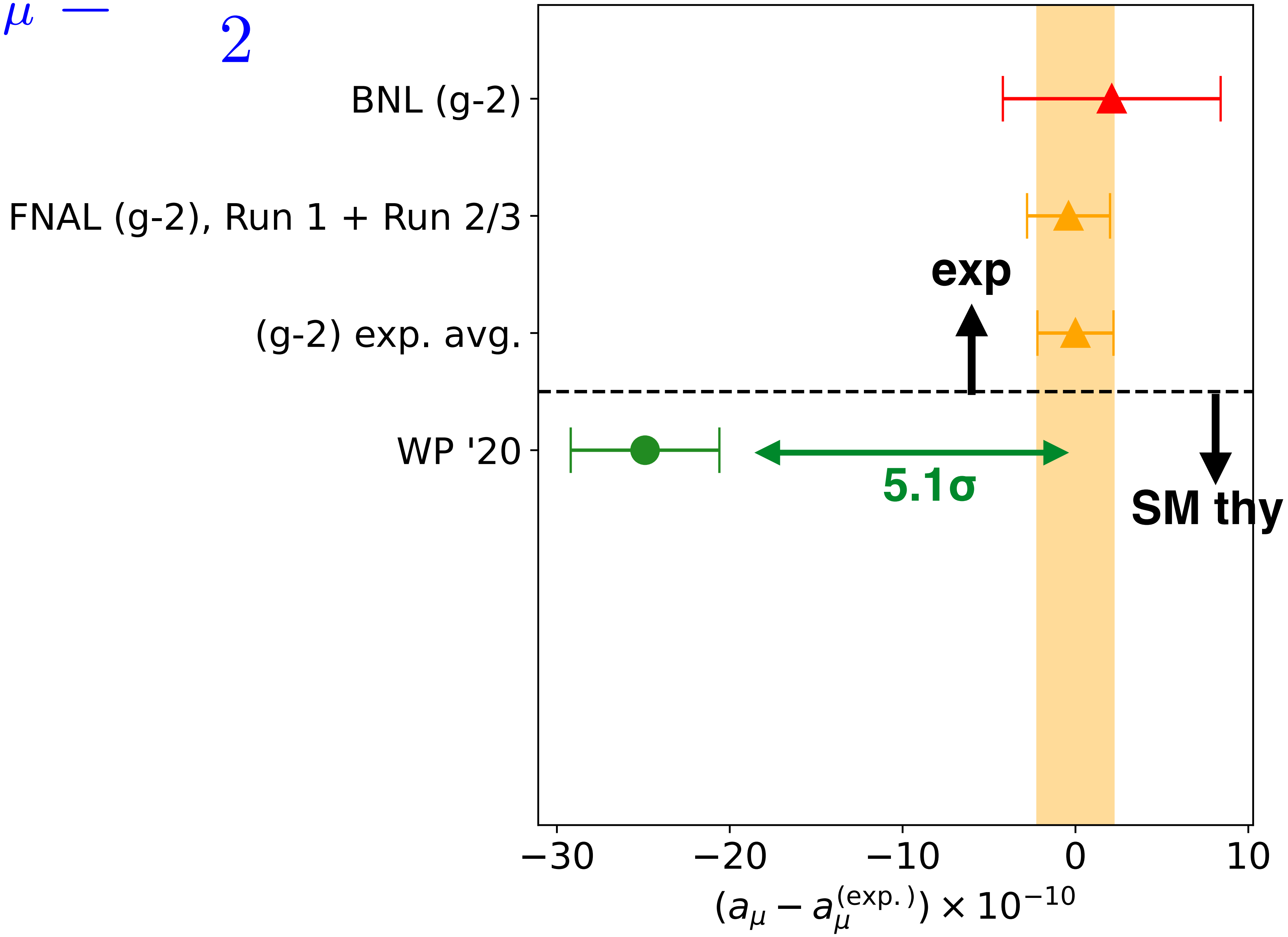
Summary of current results

$$a_\mu \equiv \frac{g_\mu - 2}{2}$$

(Latest experiment: FNAL Muon (g-2) Collaboration, PRL **131**, 161802 (2023); arXiv:2308.06230)
(WP '20: T. Aoyama et al (Muon (g-2) Theory Initiative), arXiv:2006.04822)



- Latest FNAL (g-2) results in ~5σ tension with "SM theory" prediction from Theory Initiative whitepaper!

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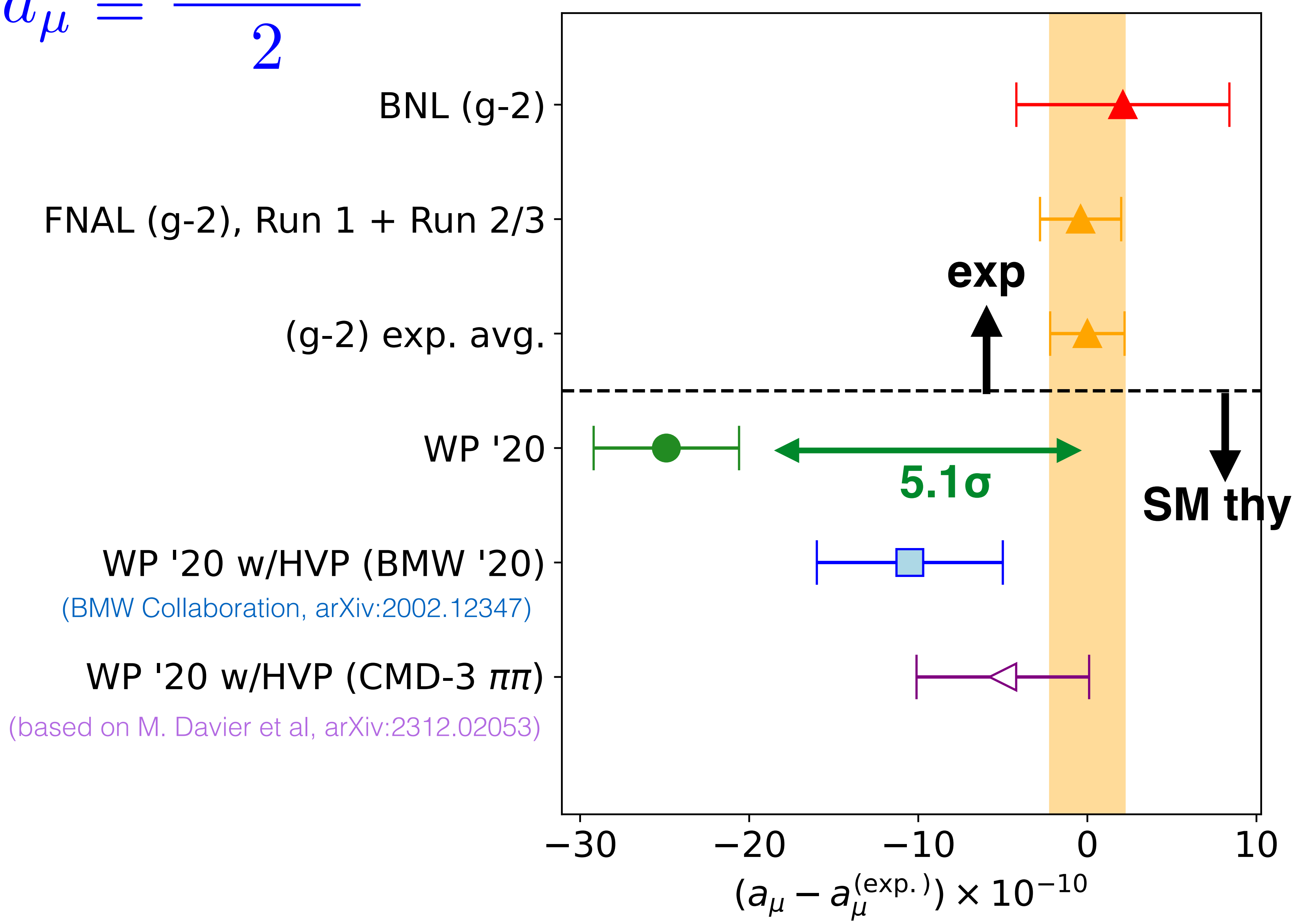
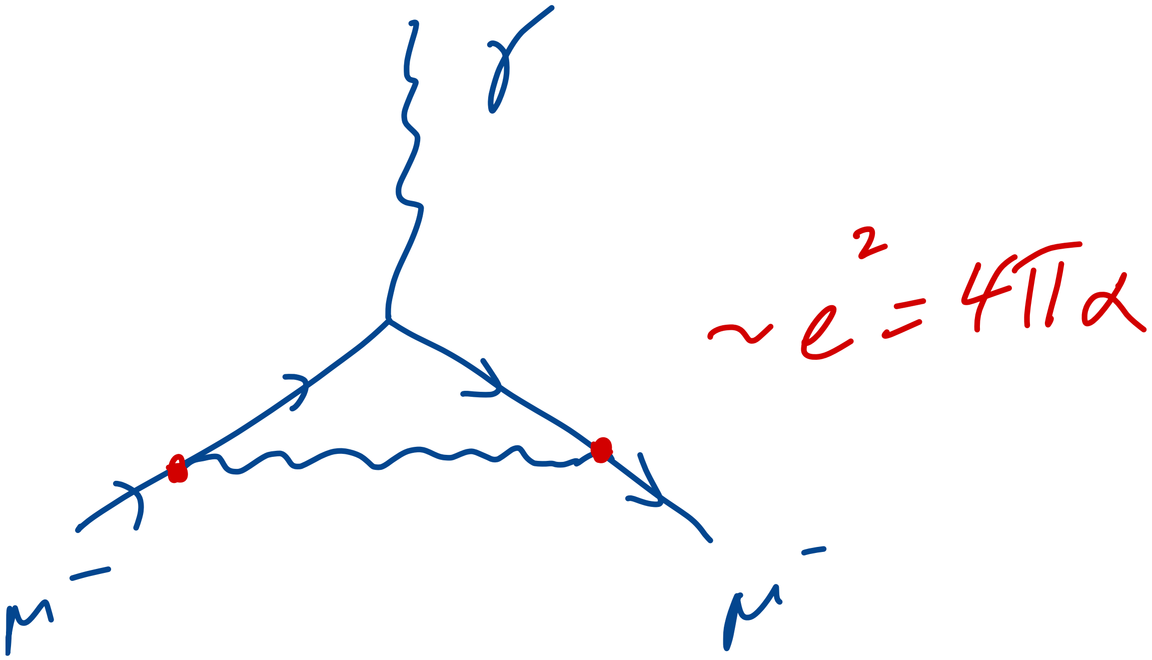


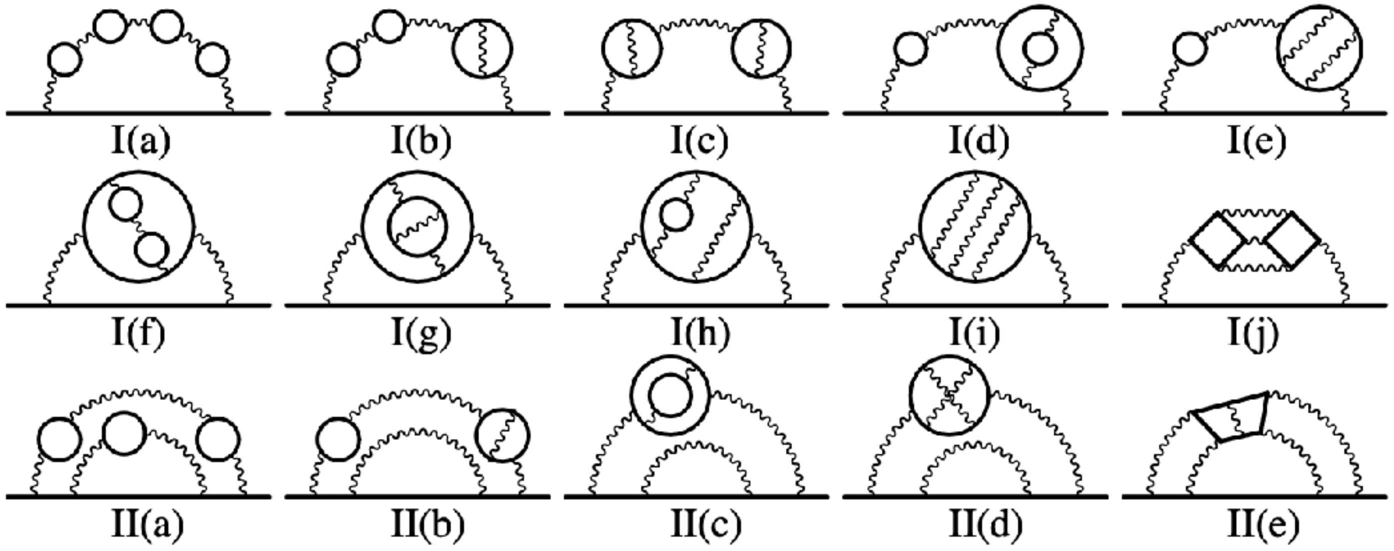
- Latest FNAL (g-2) results in ~5σ tension with "SM theory" prediction from Theory Initiative whitepaper!

- But, not so fast…*(sometimes crude, unofficial)* updates of WP number with newer results indicate a value closer to experiment; there are some internal tensions.

- "Anomalous" muon (g-2) starts with simple one-loop QED:



$$a_\mu = \frac{(g_\mu - 2)}{2} \approx \frac{\alpha}{2\pi} = 0.00116$$



(from T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, arXiv:1205.5368)

(image credit: Wikimedia Foundation)

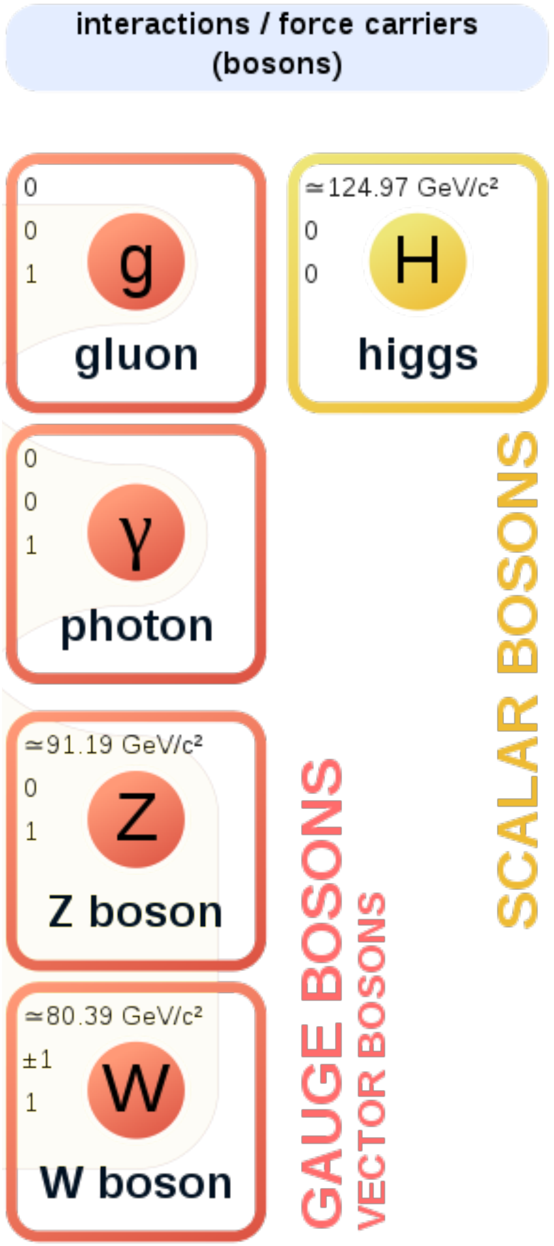
- Focus today on QCD, but QCD contributions mean nothing without the *heroic* QED calculations that dominate the SM prediction!
- **<u>Left:</u>** "Tenth order" corrections, <u>12,672</u> unique diagrams! Same calculation gives electron (g-2), where there are also some tensions (a story for another talk…)

(WP '20: T. Aoyama et al (Muon (g-2) Theory Initiative), arXiv:2006.04822)

- Full result: *(from (g-2) Theory Initiative '20)*

$$a_\mu(\text{SM}) \equiv \frac{(g_\mu - 2)}{2} = 1165918.10(43) \times 10^{-9}$$

$$a_\mu(\text{exp.}) = 1165920.59(22) \times 10^{-9}$$



- Break apart the SM contribution into pure QED, EW (involving W/Z/h) and QCD (involving gluons.)

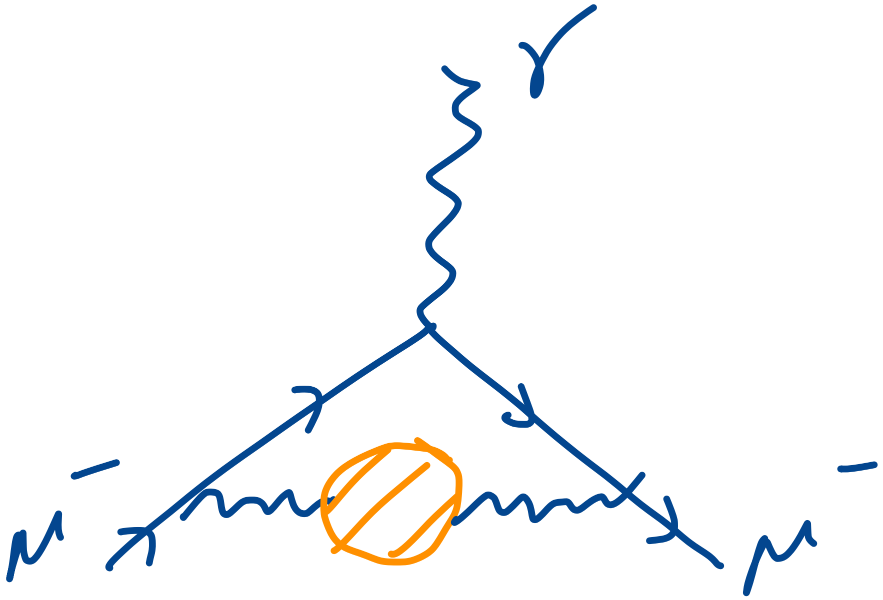
$$a_\mu(\text{QED}) = 1165847.189(1) \times 10^{-9}$$

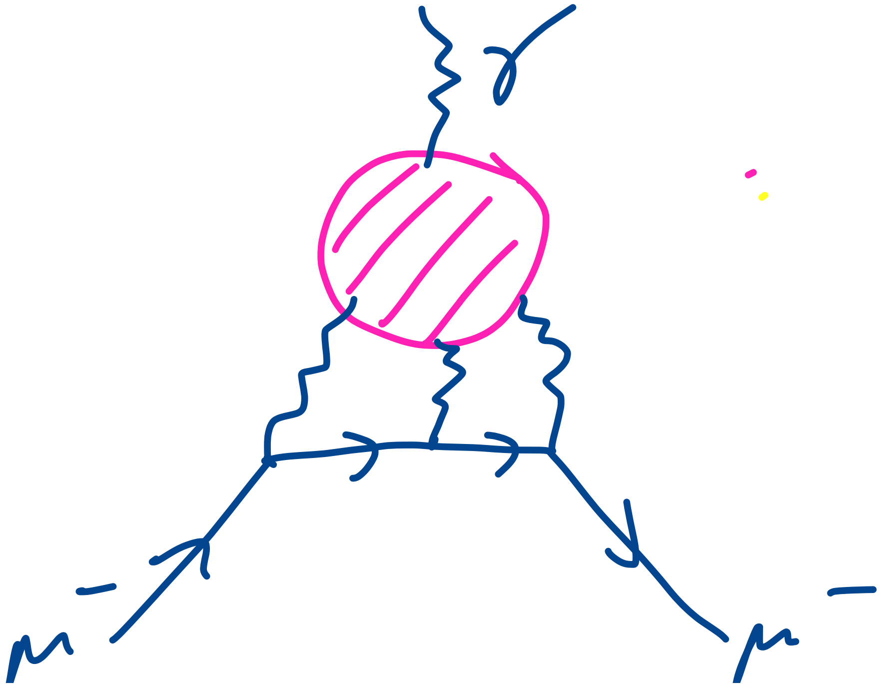
$$a_\mu(\text{EW}) = 1.536(10) \times 10^{-9}$$

$$a_\mu(\text{QCD}) = 69.370(430) \times 10^{-9}$$

  - QCD contribution is relatively small, but *by far* dominates the uncertainty! Precision goal of experiment is (16). Commensurate precision from QCD would be 0.16/69.37 ~ 0.2%. (Currently, 0.43/69.37 ~ 0.6%, although some internal conflicts…)

- Describe QCD contribution using "QCD blobs". Two meaningful diagrams: **hadronic vacuum polarization** (HVP) and **hadronic light-by-light scattering** (HLbL).





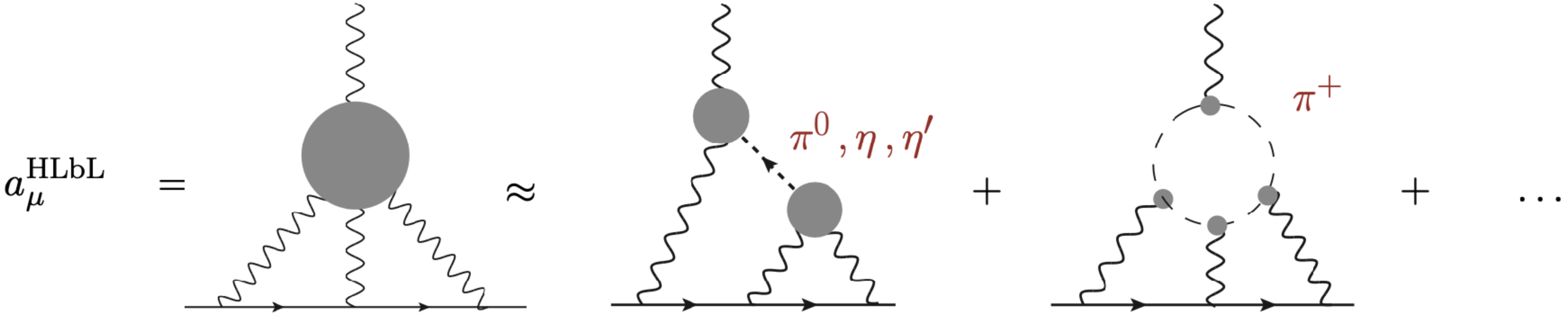
- Relative size of contributions: $a_\mu^{HVP} \sim 70 \times 10^9$; $a_\mu^{HLbL} \sim 1 \times 10^9$.

- Overall precision goal of 0.2% for QCD overall —> 0.2% precision for HVP, and 10% precision for HLbL.

- Both terms can be estimated both by leveraging experimental input (dispersive analysis), or *ab initio* by lattice QCD calculation. Improvements are in the works for both approaches; the more cross-checks, the better!

2. Light-by-light scattering

- Dispersive analysis uses experimental input; pseudoscalar meson terms (e.g. form factors) are most significant.



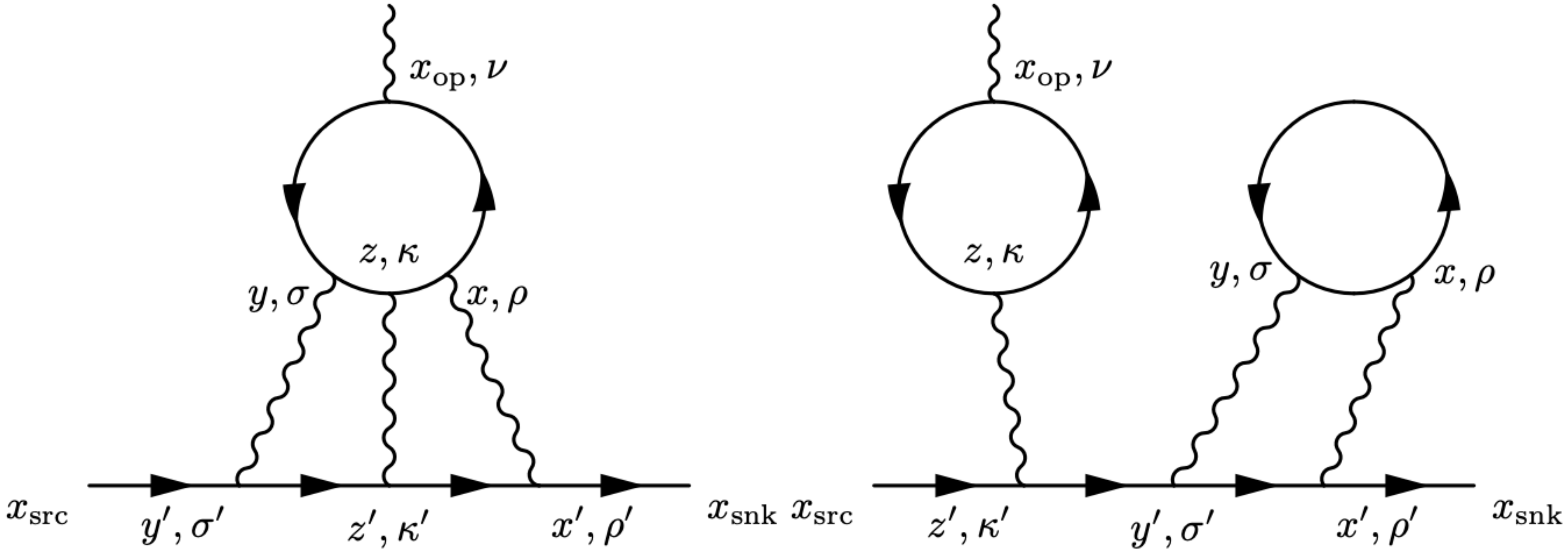
- Dominant contributions from π^0/η/η’ well-understood.

  Larger uncertainty from higher states and “short distance” contributions.

- 19/92 ~ 21% rel. error (remember, 10% target.)

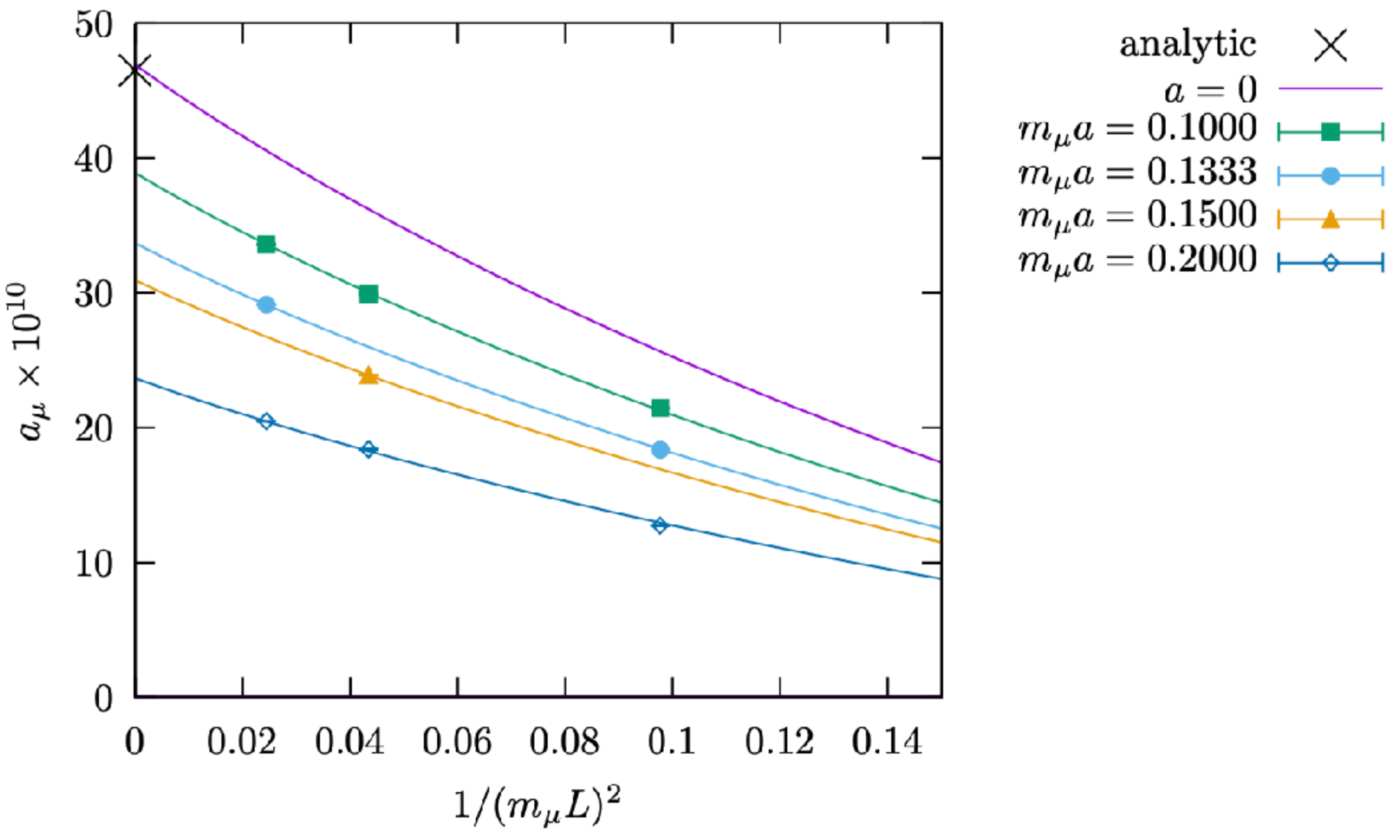
Dispersive framework ('21)	$a_\mu \times 10^{11}$
π^0, η, η'	93.8 ± 4
pion/kaon loops	-16.4 ± 0.2
S-wave $\pi\pi$	-8 ± 1
axial vector	6 ± 6
scalar + tensor	-1 ± 3
q-loops / short. dist. cstr	15 ± 10
charm + heavy q	3 ± 1
HLbL (dispersive)	92 ± 19

(from talk by A. Gerardin, 5th Muon g-2 Theory Initiative workshop)



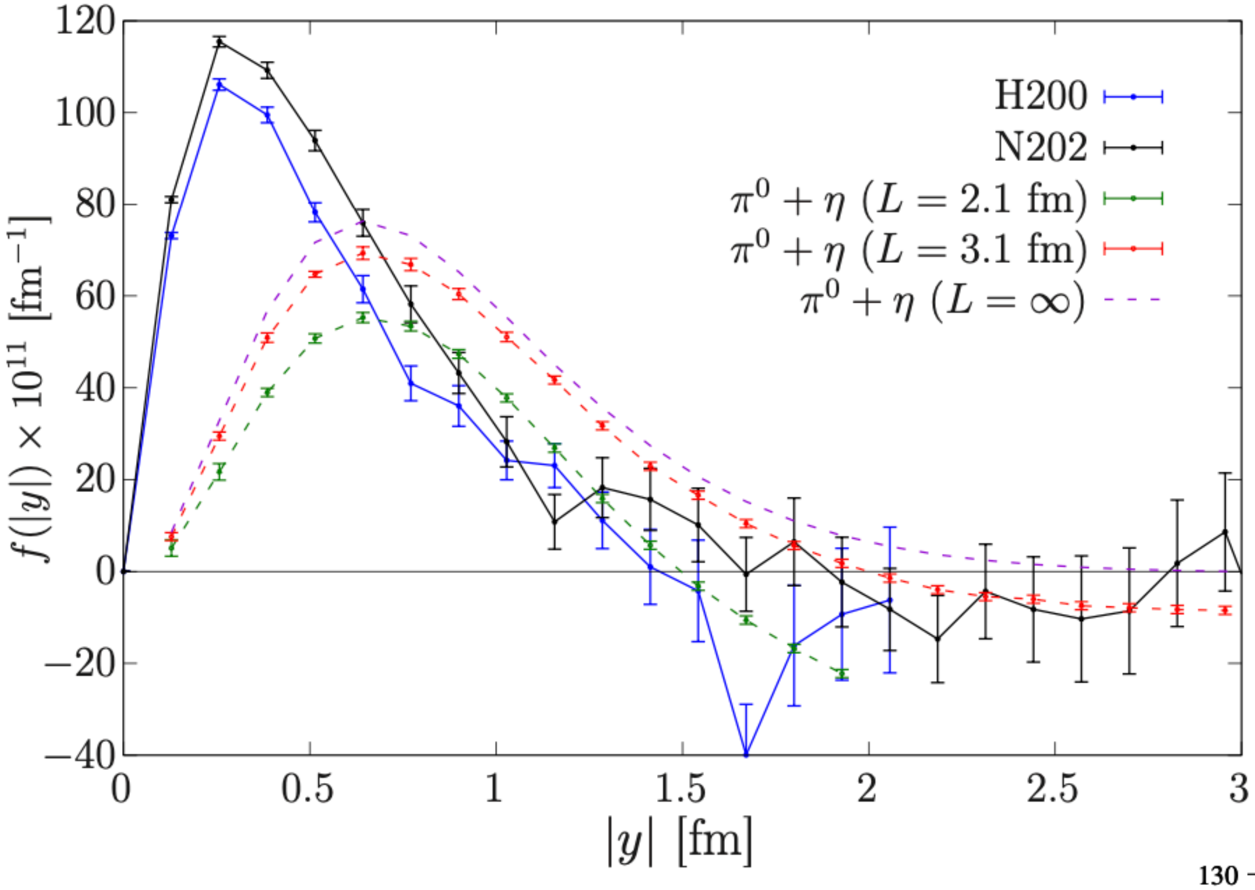
(from talk by T. Blum, RBC/UKQCD collaborations, 5th Muon g-2 Theory Initiative workshop)

- Lattice QCD can calculate HLbL **directly**; correlation between four insertions of EM current.

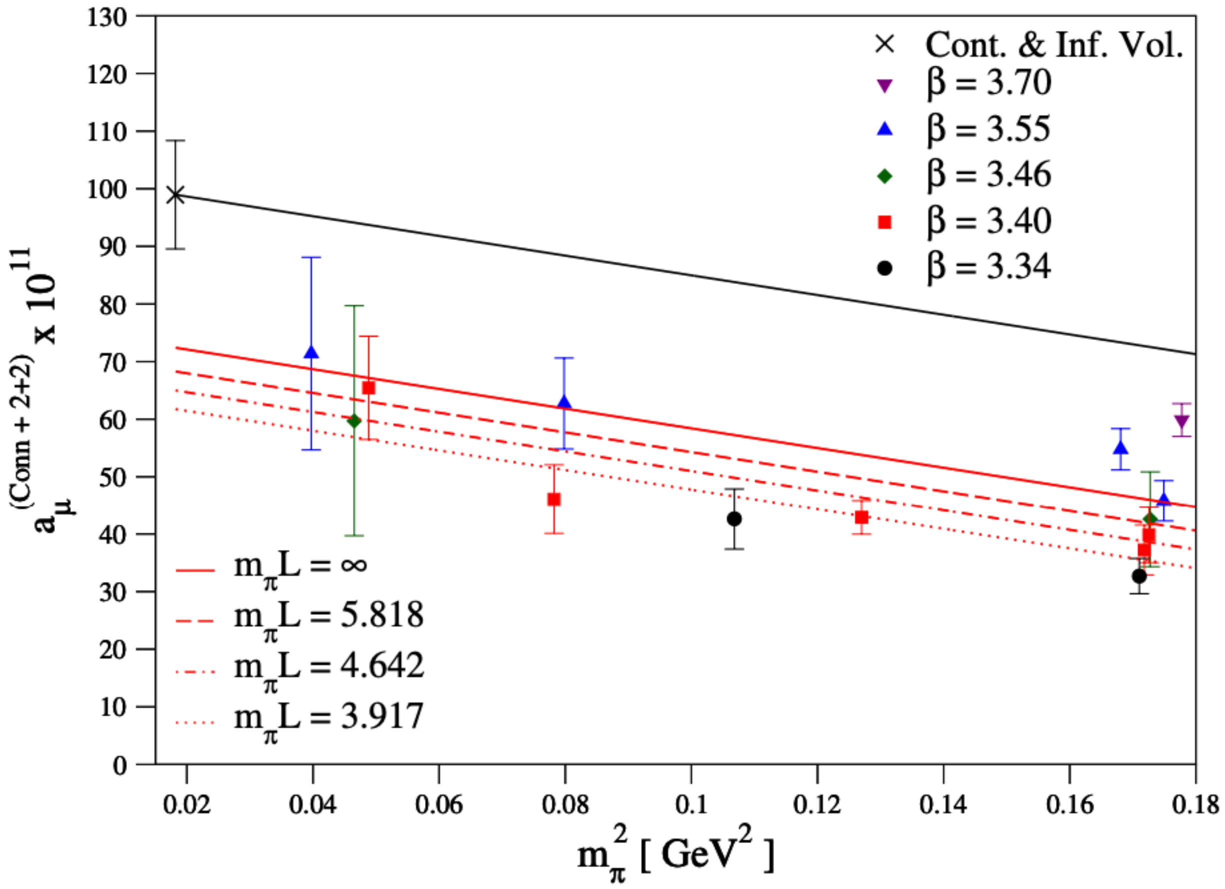


- "QED in a box" is hard! Work directly in finite box, or separate out QED and treat in infinite-volume perturbation theory.
- **Left:** testing QED FV method with QCD turned off, against exactly calculable "muon loop".

(from talk by A. Gerardin, Mainz group, 5th Muon g-2 Theory Initiative workshop)

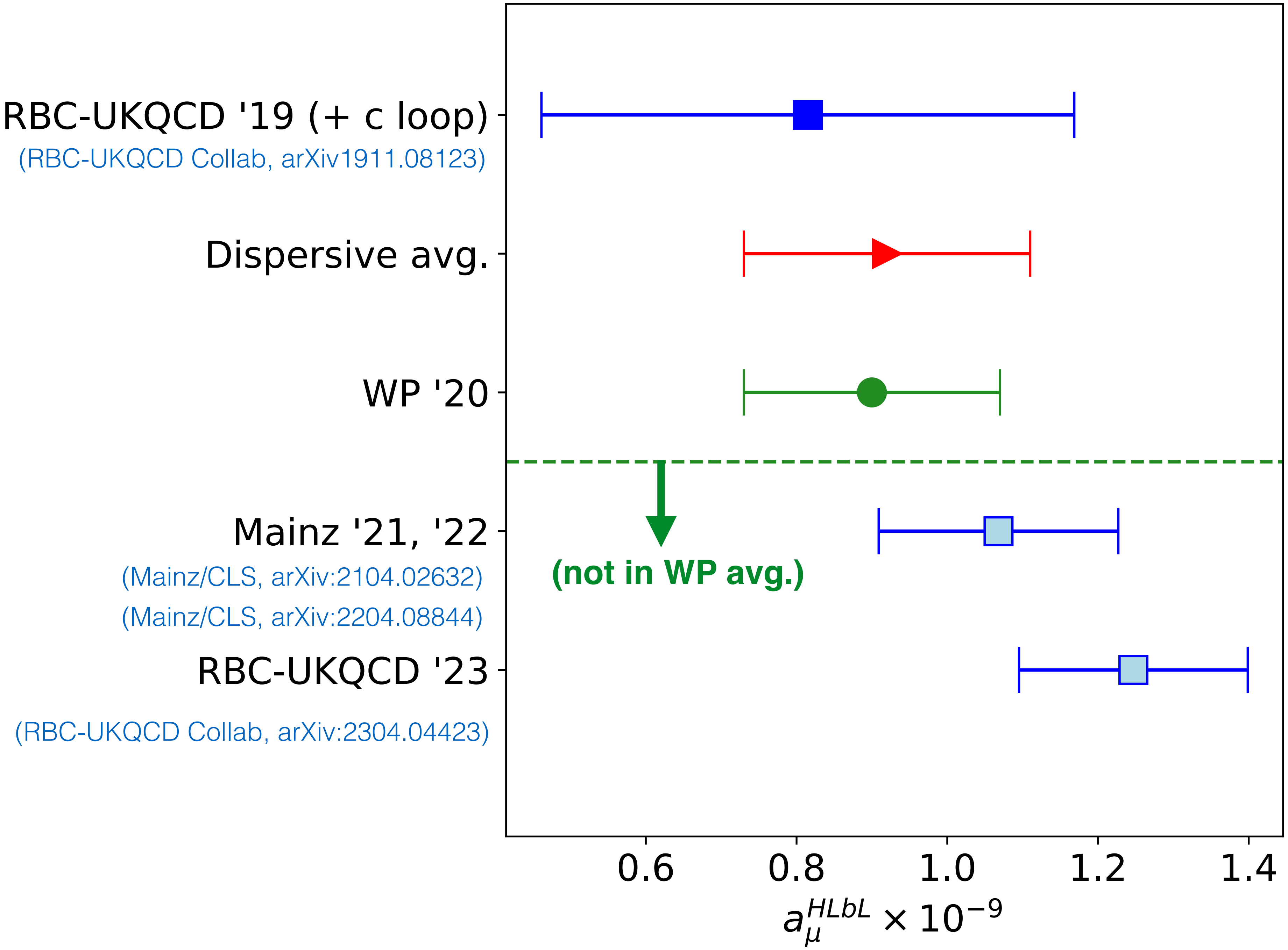


- **Left:** Integrate over one current position, show remaining integrand f(y) to get a sense for energy/distance dependence.
- (Solid lines: lattice complete; dashed lines: π^0/η form factor only. "SU(3) point".)



- **Right:** Continuum and infinite-volume extrapolation of (physical, connected) light-quark HLbL. Other terms (heavy quarks, disconnected) also calculated on lattice.

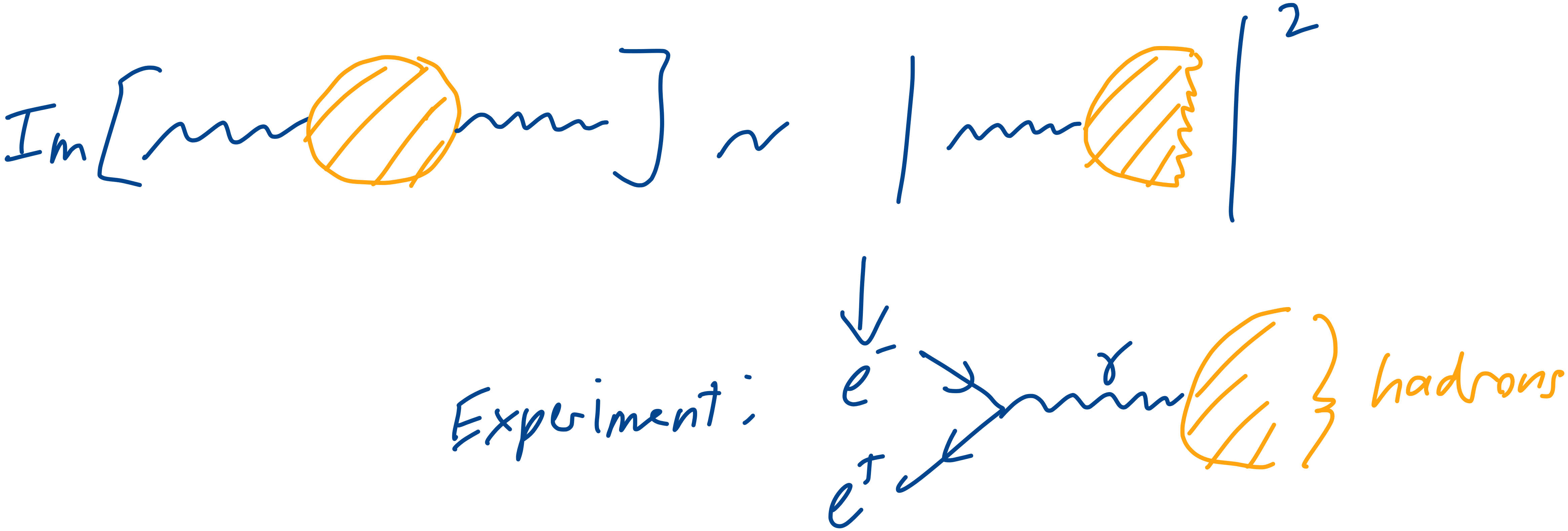
(from talk by A. Gerardin, Mainz group, 5th Muon g-2 Theory Initiative workshop)



- <u>Summary:</u> direct lattice and dispersive approaches agree well; lattice input on form factors —> dispersive gives further checks. Precision is getting close to 10% goal already!

3. Hadronic vacuum polarization

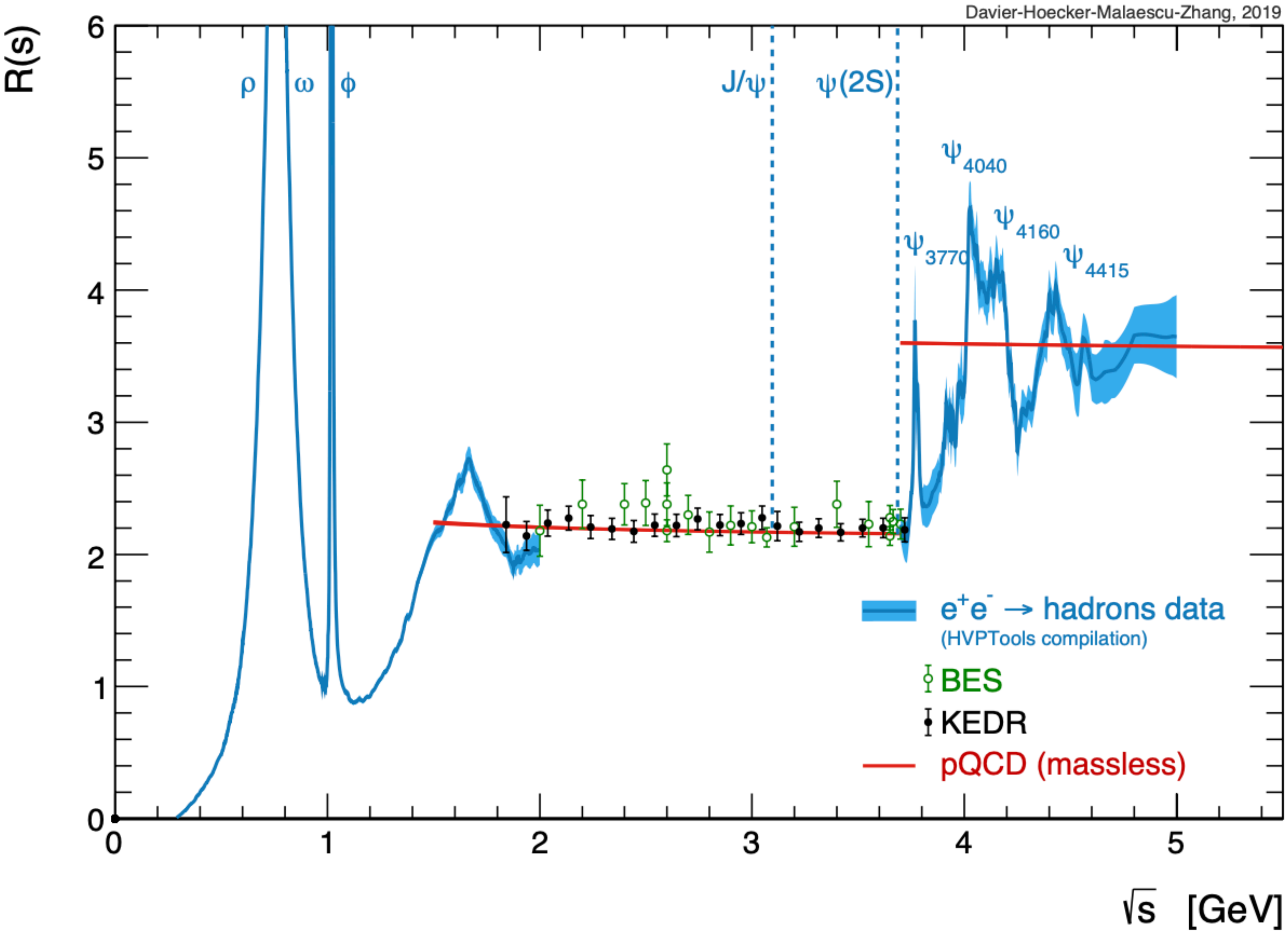
- HVP can be estimated from experiment using a dispersion relation:



- In principle, this is a simple integral over the ratio:

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

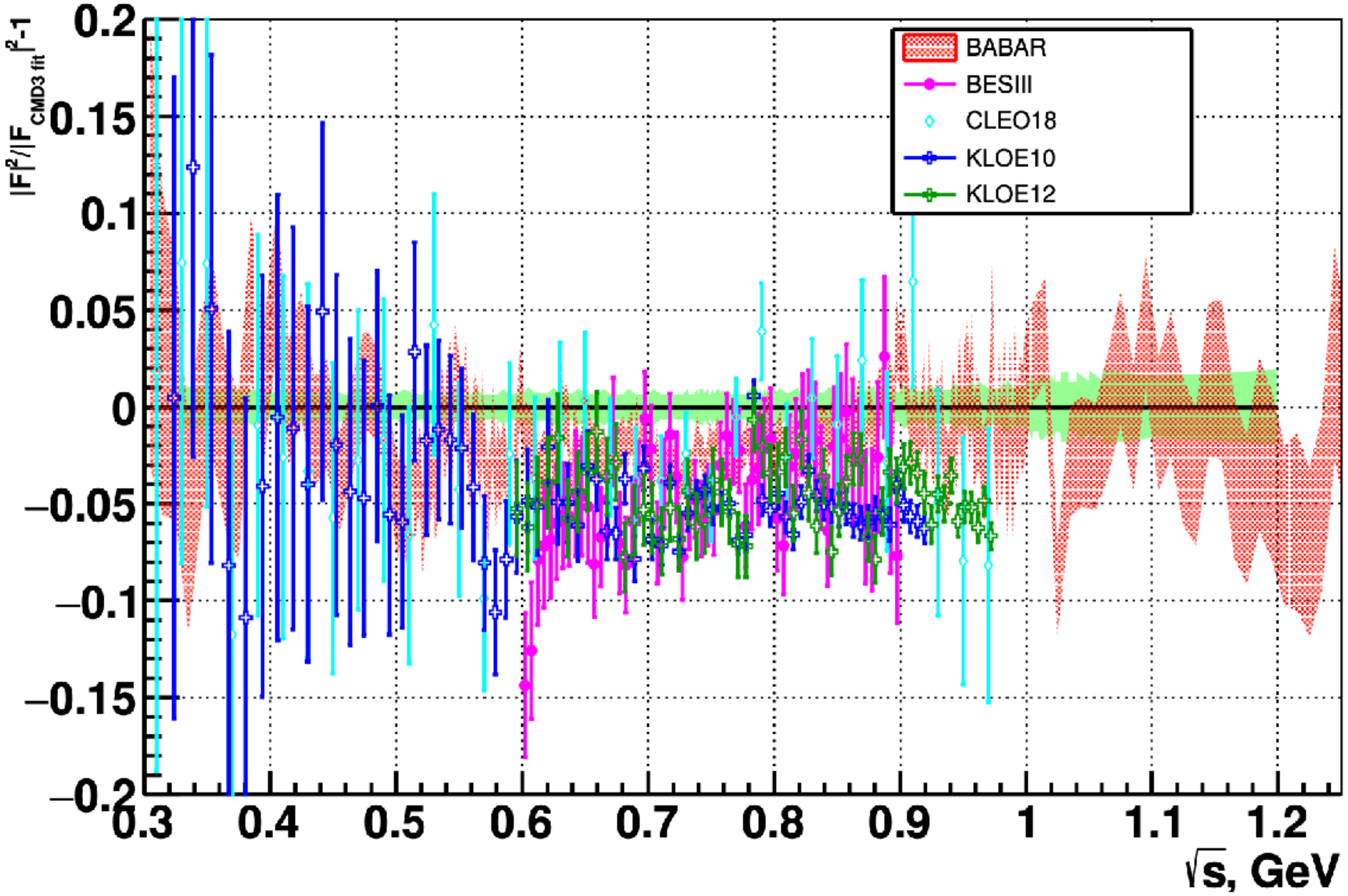
(from M. Davier, A. Hoecker, B. Malaescu, and Z. Zhang, 1908.00921)

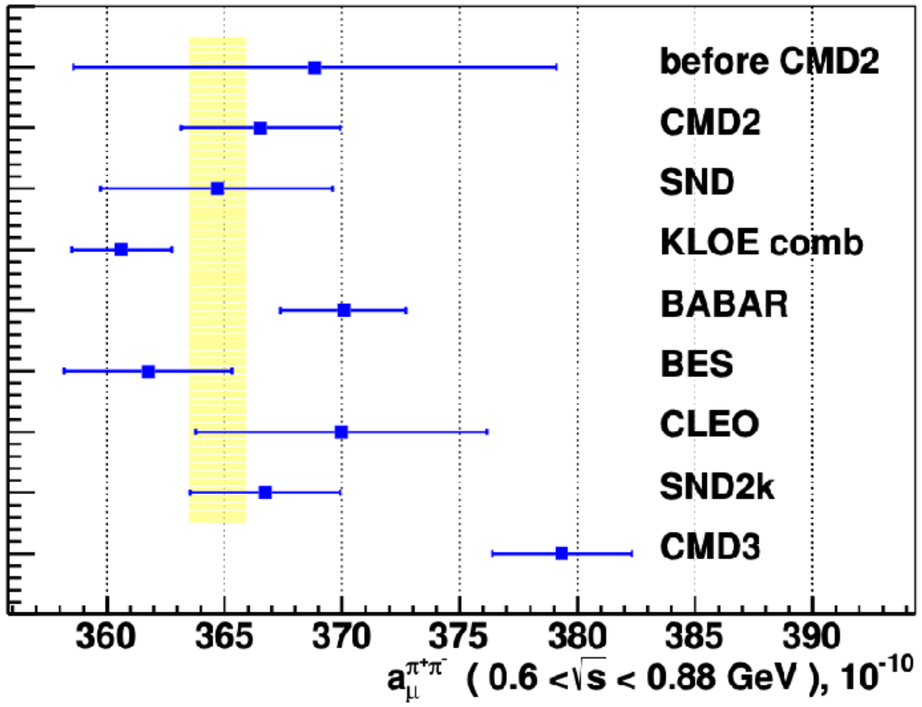


- In *practice*, combination of results from multiple experiments is intricate; multiple groups do the extraction of HVP independently; systematic errors assigned to cover disagreements between experiments.

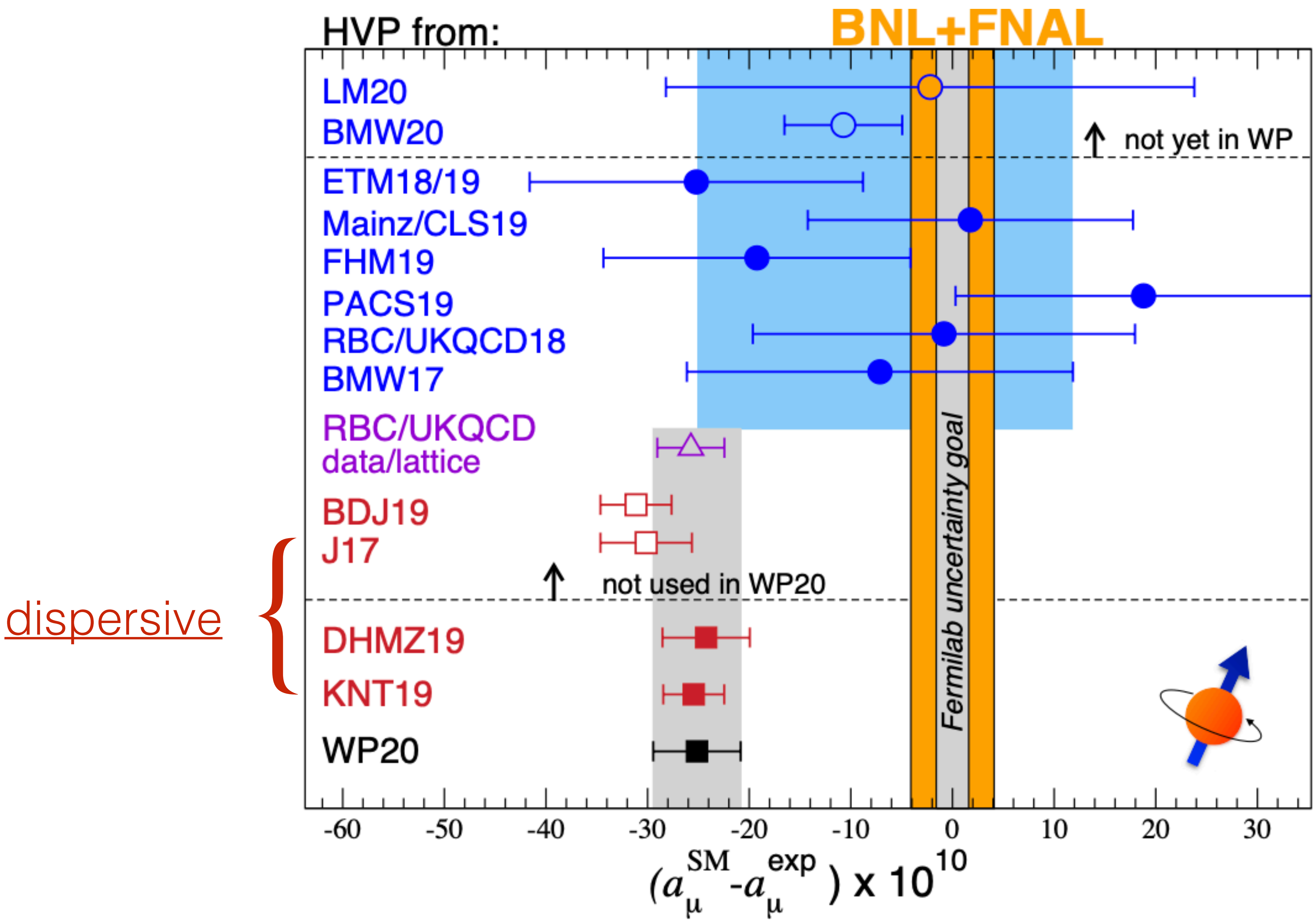
(F.V. Ignatov *et al*, arXiv:2302.08834)

Measurement of the $e^+e^- \to \pi^+\pi^-$ cross section from threshold to 1.2 GeV with the CMD-3 detector





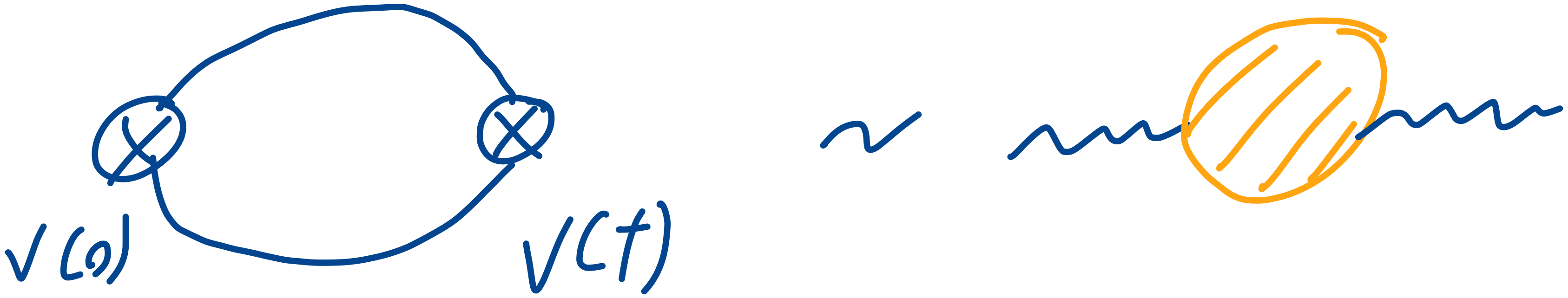
- Some tension between experiments in places, particularly two-pion data from 600-900 MeV; new CMD-3 result, arXiv:2302.08834, shows larger discrepancies.



(from the Muon g-2 Theory Initiative, arXiv:2203.15810)

- Current WP ’20 theory value for HVP **dominated by dispersive results**; lattice determinations not precise enough at the time! **BMW ’20** (most precise full lattice calculation) not in the WP ’20 average, in particular.

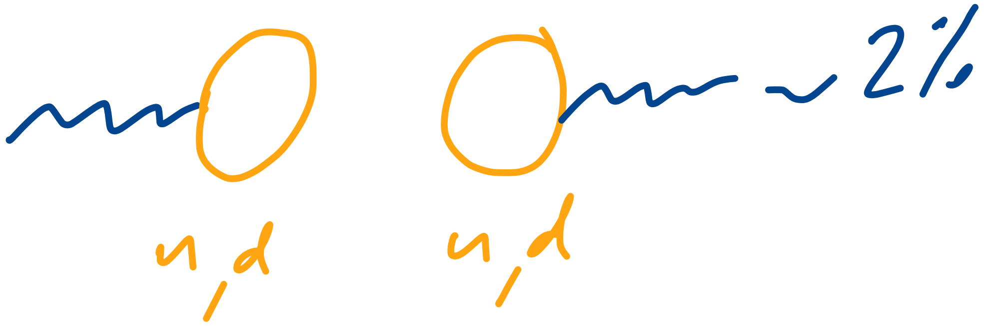
- To compute HVP on lattice, we compute *correlation* between two vector currents:

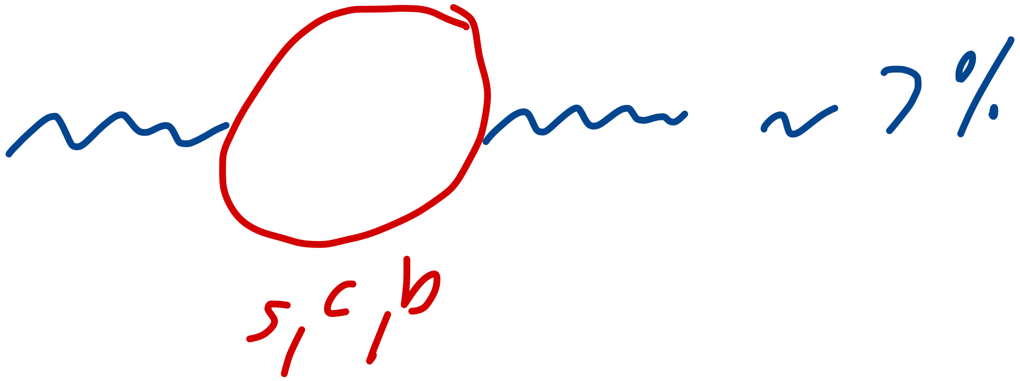


  - Path integration automatically “sums over” all gluon configurations, but fermions treated more explicitly:

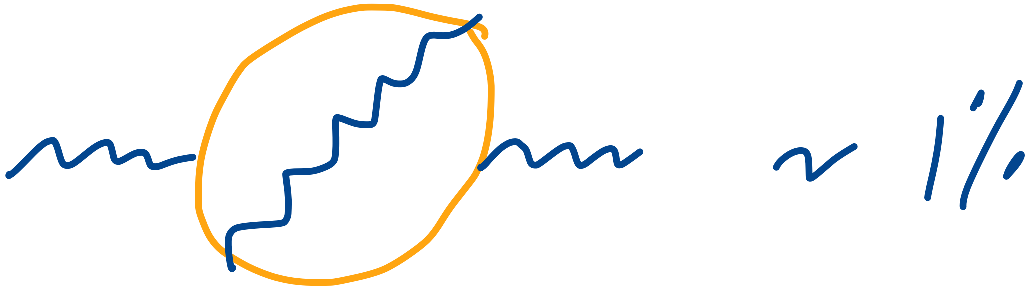
“light-quark connected”

“light-quark disconnected”





“heavy-quark connected”

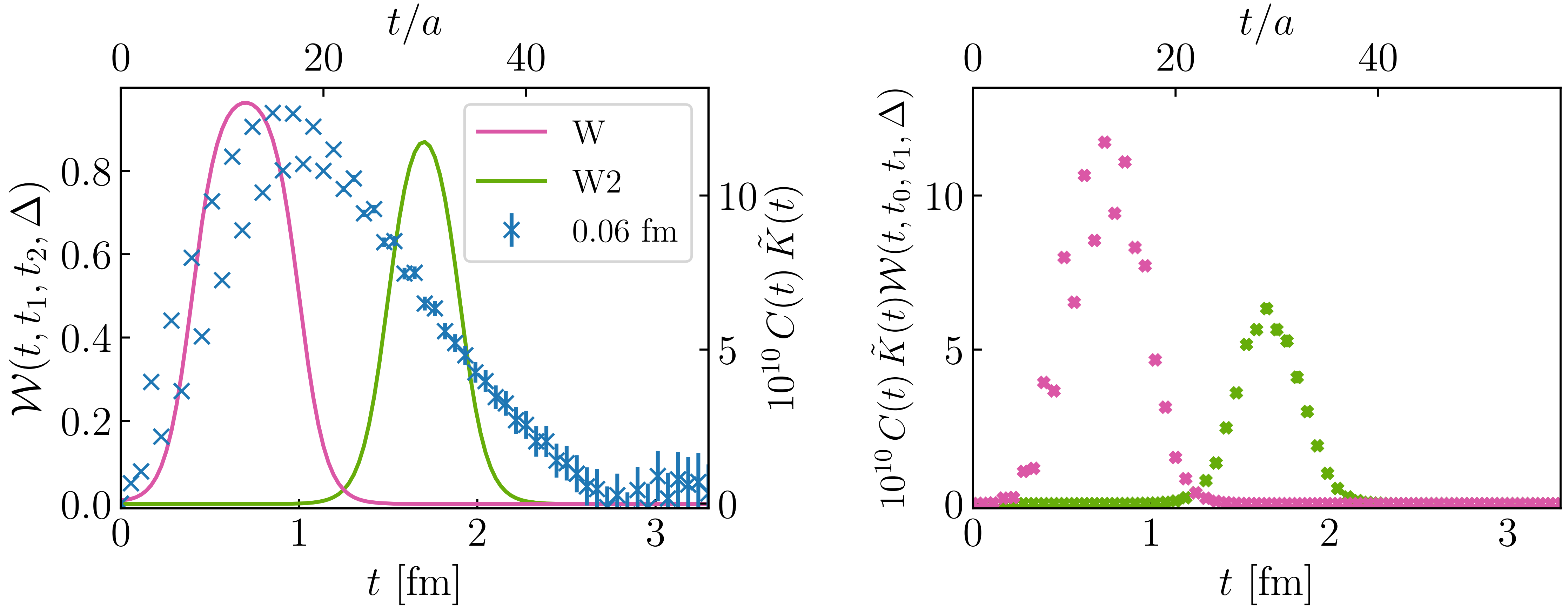


“isospin-breaking corrections”

*Bernecker and Meyer, Eur.Phys.J. A47 (2011) 148

*Blum et al. (RBC/UKQCD), Phys. Rev. Lett 121 (2018) 2, 022003

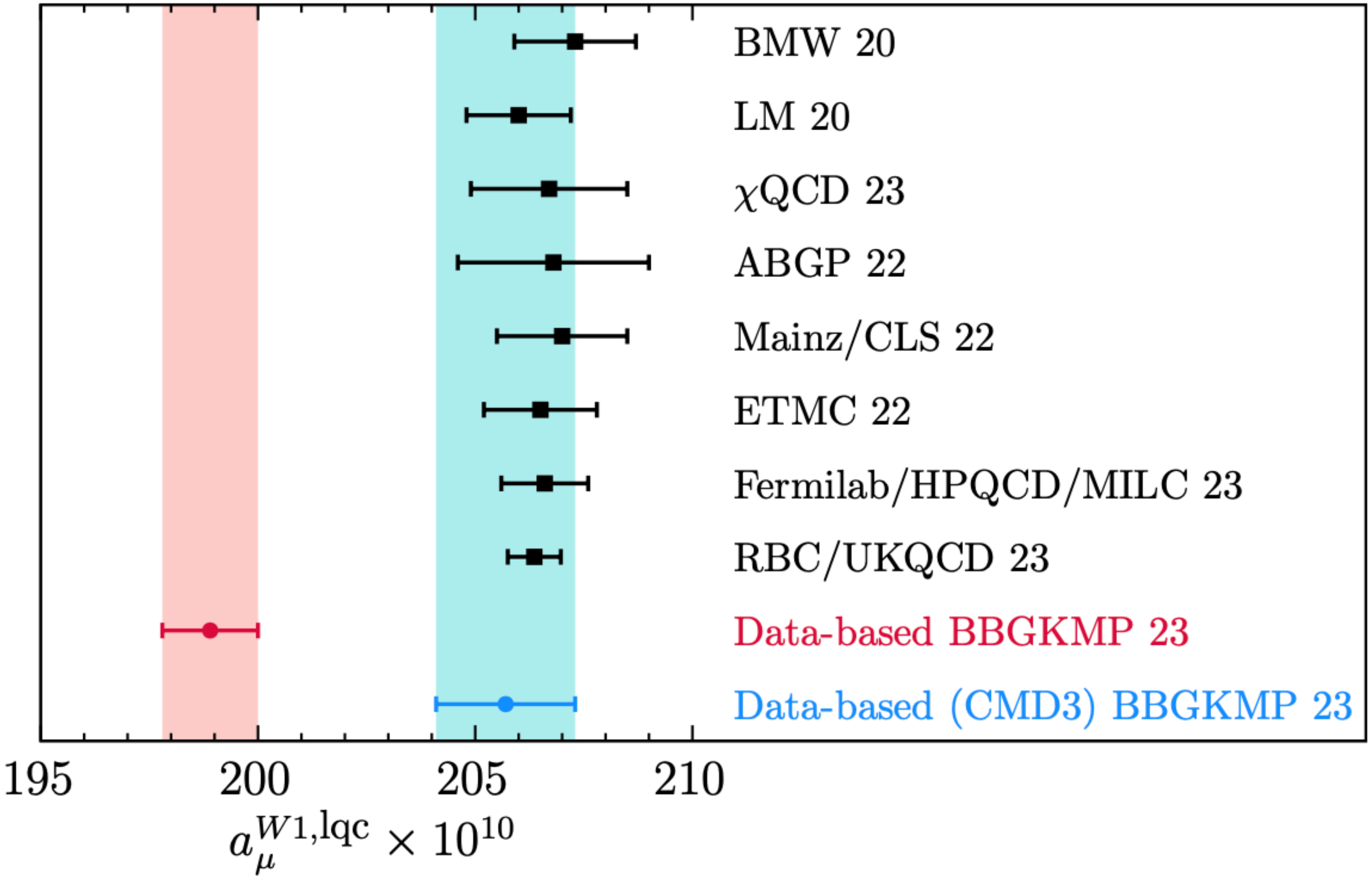
(Fermilab/HPQCD/MILC collaborations, arXiv:2301.08274)



- To validate and cross-check lattice results, calculation can be restricted to an “**intermediate window**”, **W**; contributions in [0.4, 1.0] fm. (Other windows are considered, but this is the most common choice.)

- No long-distance or short-distance contributions makes this piece of HVP **relatively clean** and **highly precise**. On the dispersive side, this window corresponds to a specific energy range

(from G. Benton, D. Boito, M. Golterman, A. Keshavarzi, K. Maltman and S. Peris, arXiv:2311.09523)

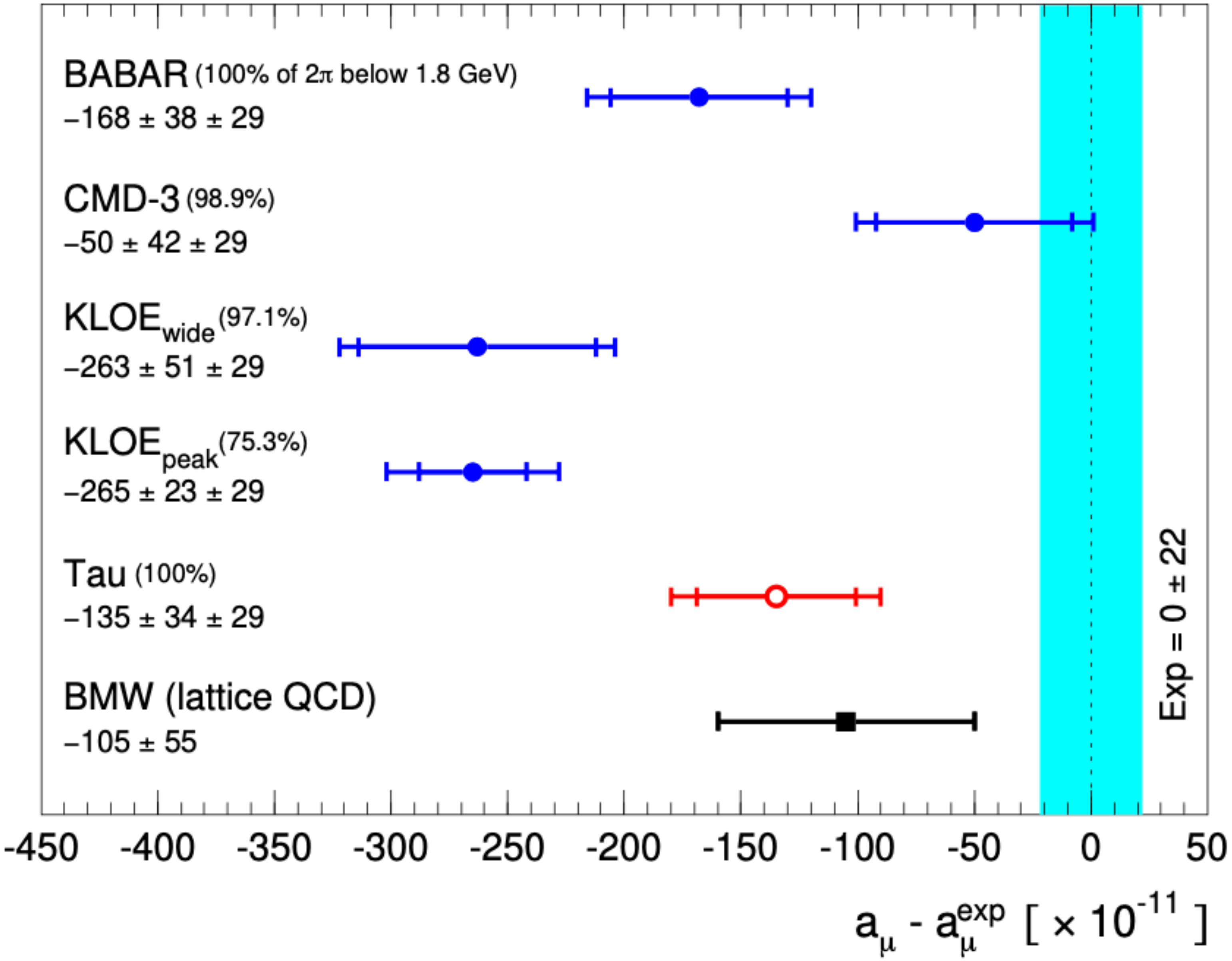


8 lattice results!

- Results for "intermediate window", "light-quark connected" from **several lattice groups** all show excellent agreement, increasing precision.

- Significant tension with **dispersive result** persists here (although, some effort/modeling is needed to translate real-world experiment into "light-quark connected".) Likewise, using ***only*** **new CMD3 dataset** over [0.33, 1.2] GeV in LQC window is much closer to lattice…but discarding all other experimental results is premature, this is just a hint!

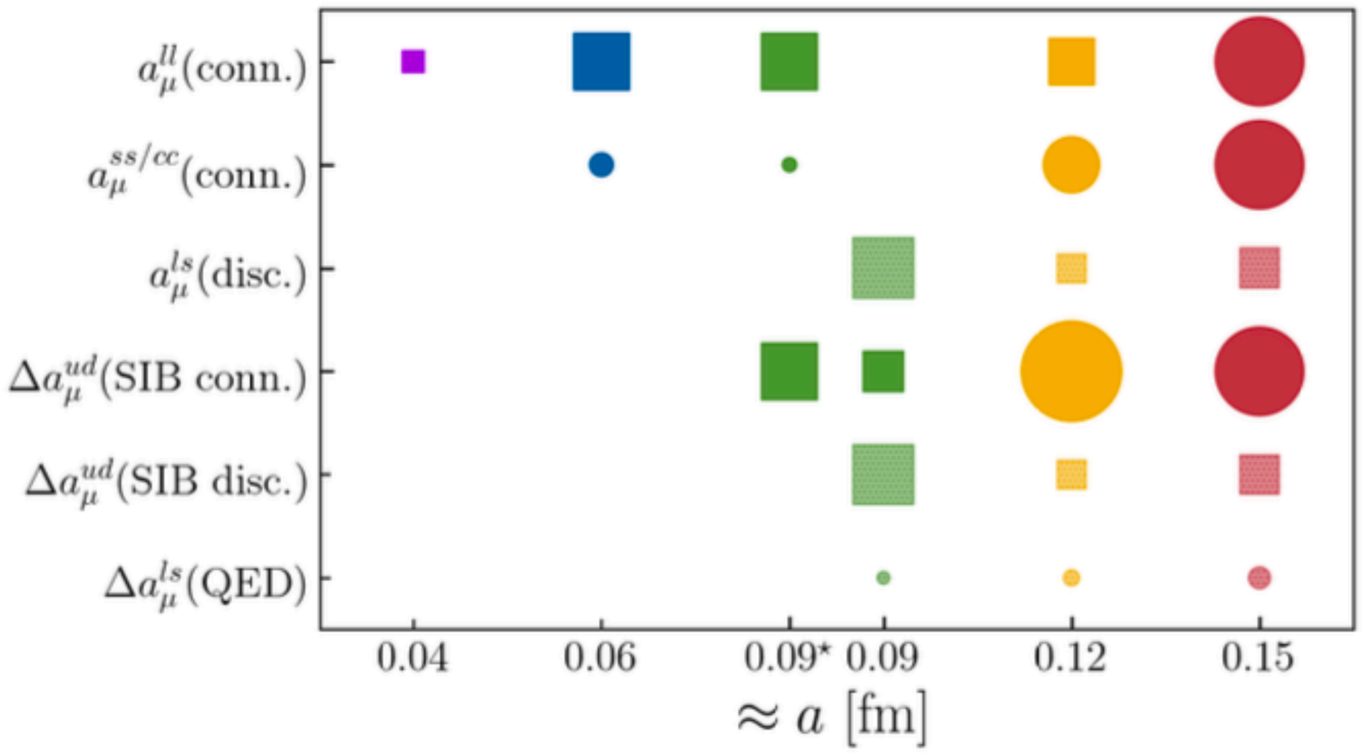
(from M. Davier, A. Hoecker, A.M. Lutz, B. Malaescu, and Z. Zhang, arXiv:2312.02053)



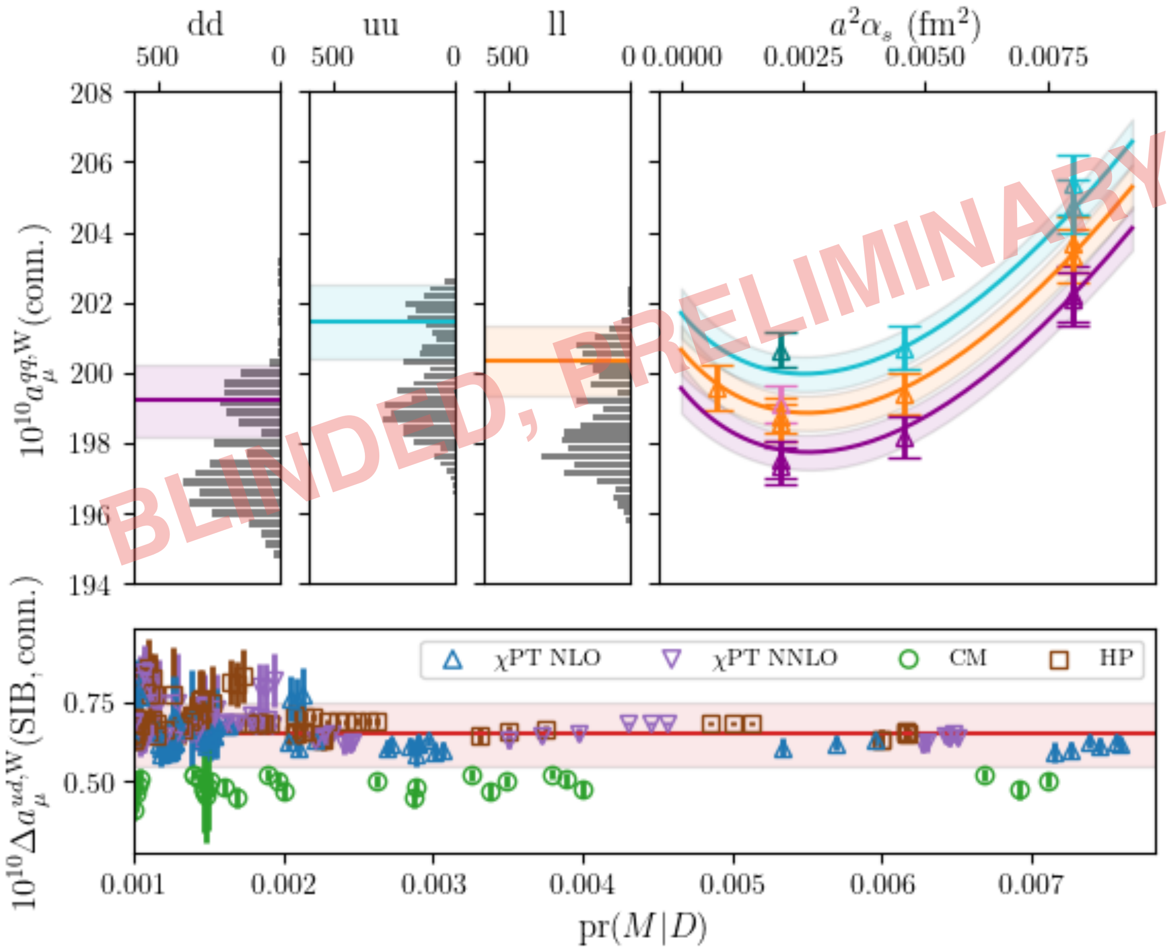
- Breaking down into individual results for “two-pion” energy region of R(s) total a_μ (or for the window) shows *internal discrepancy* between results from different experiments.

- More lattice calculations at high precision will help, as will further scrutiny on the experimental/dispersive side. Work is in progress!

- On the lattice QCD side, many groups are working hard on updated calculations with better statistics, and a close look at all individual contributions.

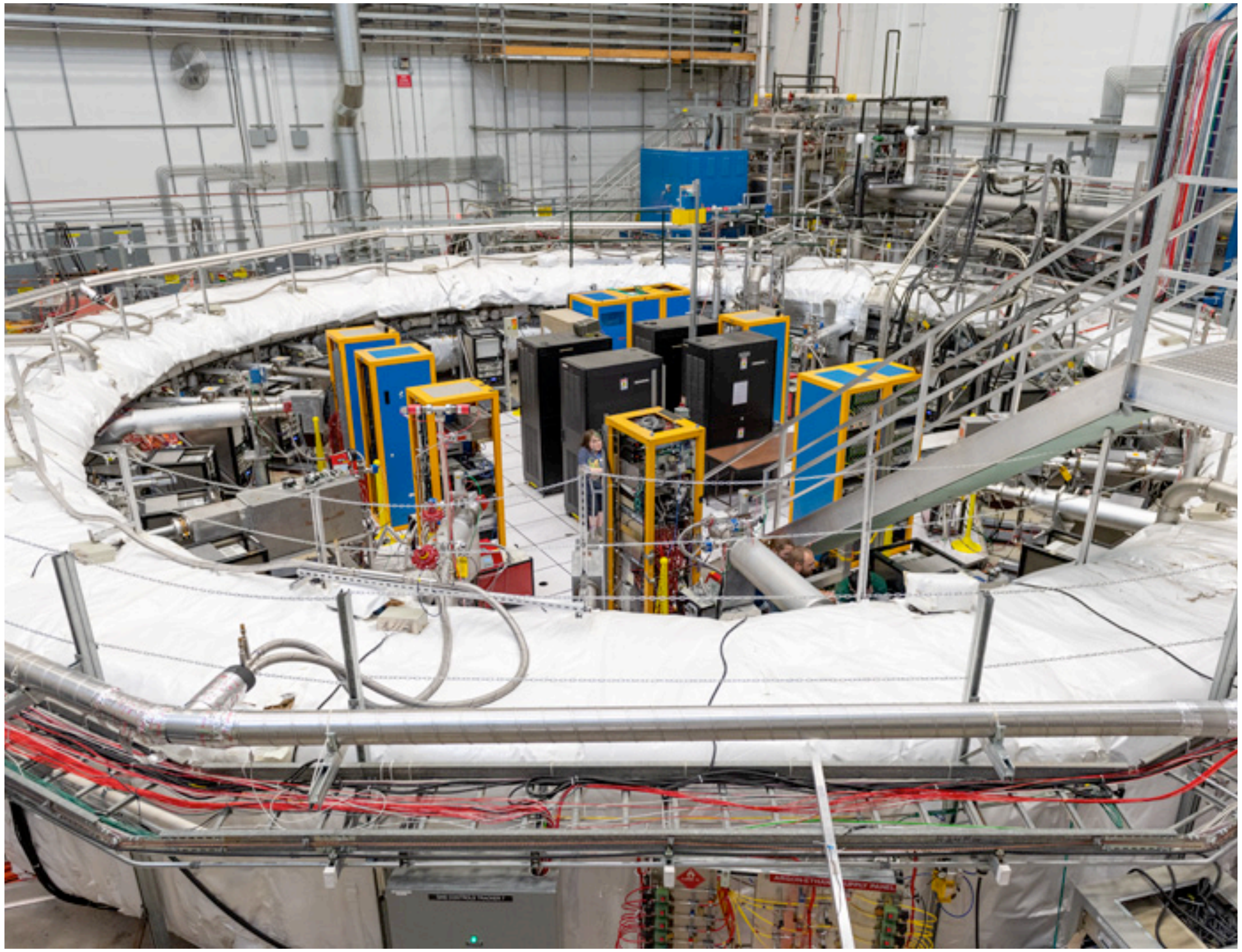


(Fermilab/HPQCD/MILC collaborations, **preliminary**)



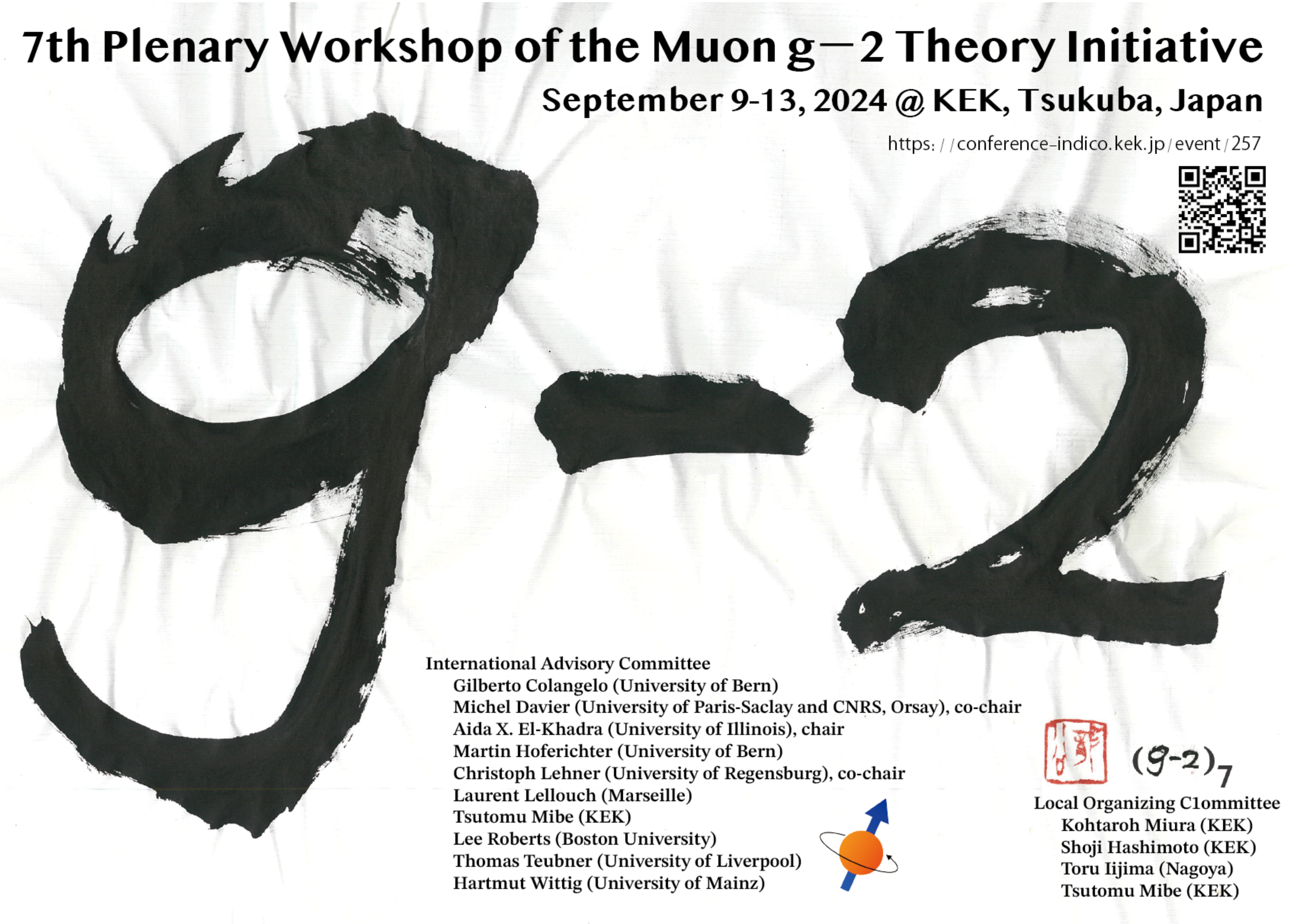
- My collaboration (**Fermilab/MILC/HPQCD**) is currently working on full a_μ in W and short-distance; all contributions for a “complete” determination. Improved statistics, multiple lattice spacings (above.)
- New paper soon! Preliminary (and **blind**!) results (e.g. for strong-isospin breaking, left) show good control of systematics and better statistics.

Summary



- Muon (g-2) is an incredibly precise test of the SM, and discrepancies could be a sign of new physics! (But we need to understand QCD better.)
- Hadronic light-by-light seems under control, uncertainty is near the precision goal
- Better HVP precision in progress; understanding current discrepancies will be critical (group vs. group, lattice vs. dispersive, "windows")
- Stay tuned for new theory results and an update from the Theory Initiative in the next year!

Thank you!



7th Plenary Workshop of the Muon g−2 Theory Initiative

September 9-13, 2024 @ KEK, Tsukuba, Japan

https://conference-indico.kek.jp/event/257

International Advisory Committee

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- Shoji Hashimoto (KEK)
- Toru Iijima (Nagoya)
- Tsutomu Mibe (KEK)

Backup

(summary from P.P. Giardino and H. Davoudiasl, arXiv:2311.12112)

Electron (g-2)

(T. Aoyama, T. Kinoshita, and M. Nio, arXiv:1712.06060)

(X. Fan, T.G. Myers, B.A.D. Sukra, and G. Gabrielse, arXiv:2209.13804)

- Latest $(g-2)_e$ measurement gives:

$$a_e^{(\text{exp.})} = 1159652.18059(13) \times 10^{-9}$$

- SM theory prediction depends strongly on input value of fine-structure constant. Two recent measurements with large discrepancy:

$$\alpha^{-1}(\text{Rb}) = 137.035999206(11)$$

$$\Rightarrow a_e^{(\text{exp.})} - a_e^{(\text{SM})} = (34 \pm 16) \times 10^{-14} \quad [+2.2\sigma]$$

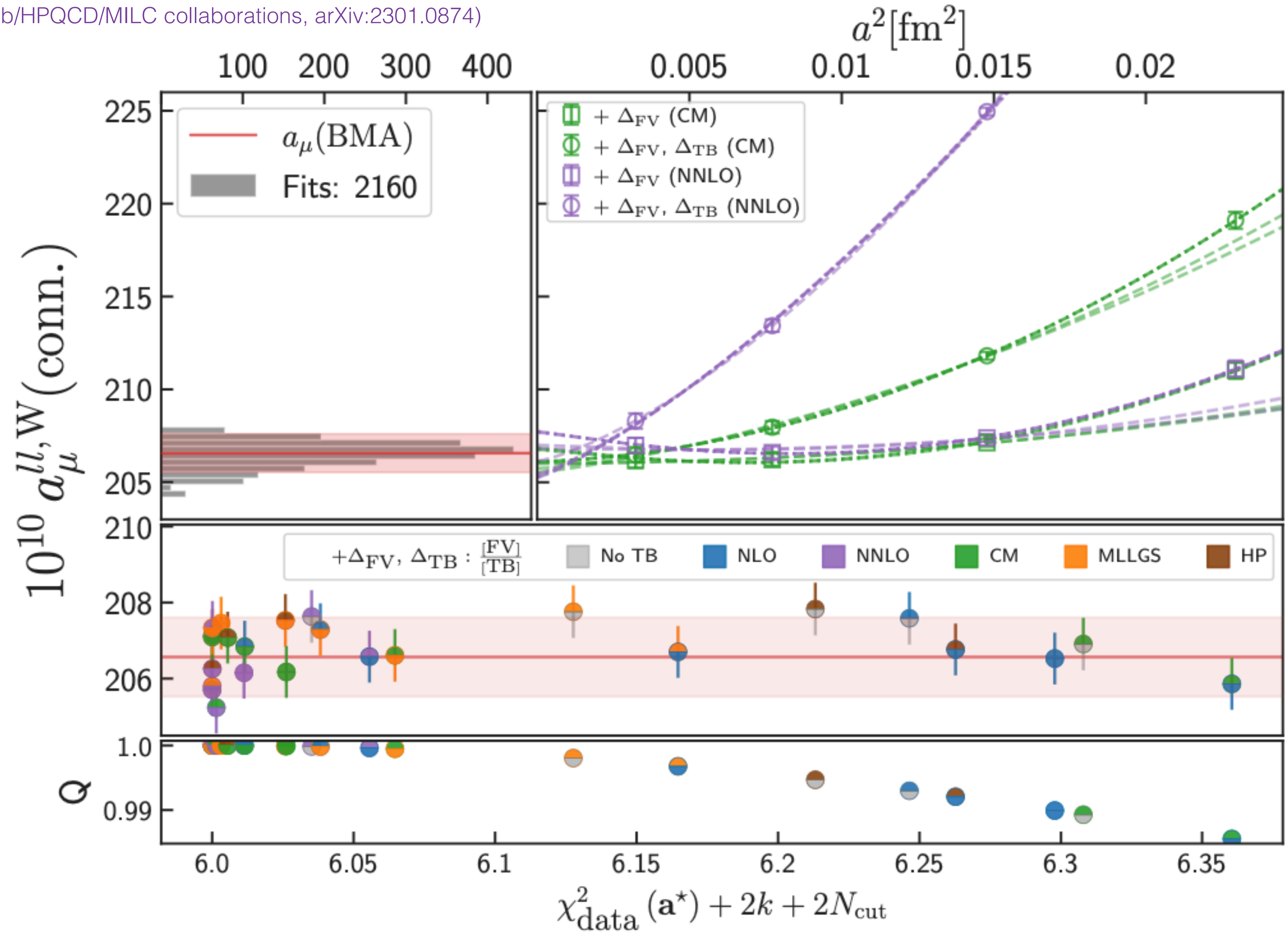
$$\alpha^{-1}(\text{Cs}) = 137.035999046(27)$$

$$\Rightarrow a_e^{(\text{exp.})} - a_e^{(\text{SM})} = (-101 \pm 27) \times 10^{-14} \quad [-3.7\sigma]$$

- The lattice QCD "world" in which HVP/HLbL are calculated is necessarily distorted: various effects must be removed to get the "real world" answer.
- Discretization (grid size) and finite volume (box size) must be *extrapolated* or otherwise removed
- Physical scales (quark masses and overall energy) must be *tuned*. Also QED effects and $(m_u - m_d)$ corrections.
- Important to have **analyses from different lattice groups**: independent statistics, but also variety of methods for dealing with all of the above (fermion discretizations, finite-volume extrapolations, etc.)

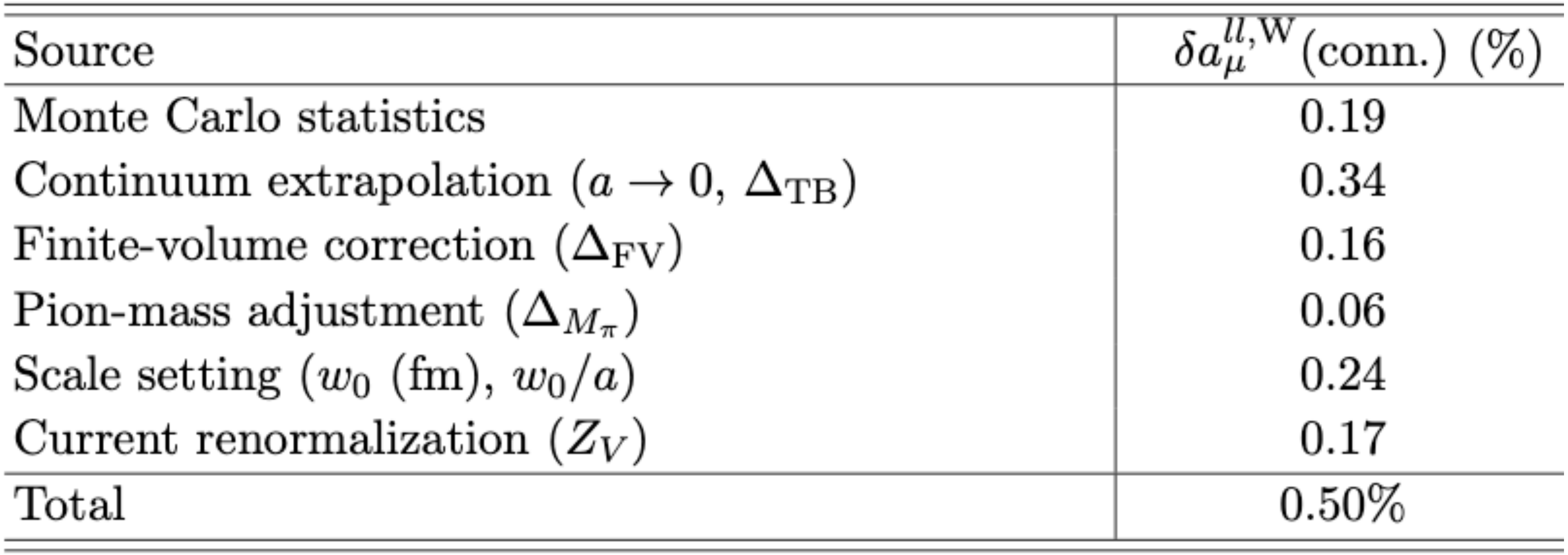
(image credits: amazon.com)

(Fermilab/HPQCD/MILC collaborations, arXiv:2301.0874)



- Example of analysis details from recent paper on light-quark connected W HVP. **2160** fit variations - dealing with discretization, finite volume, mass corrections, and other effects! Variations combined using **Bayesian model averaging**.

(Fermilab/HPQCD/MILC collaborations, arXiv:2301.08274)

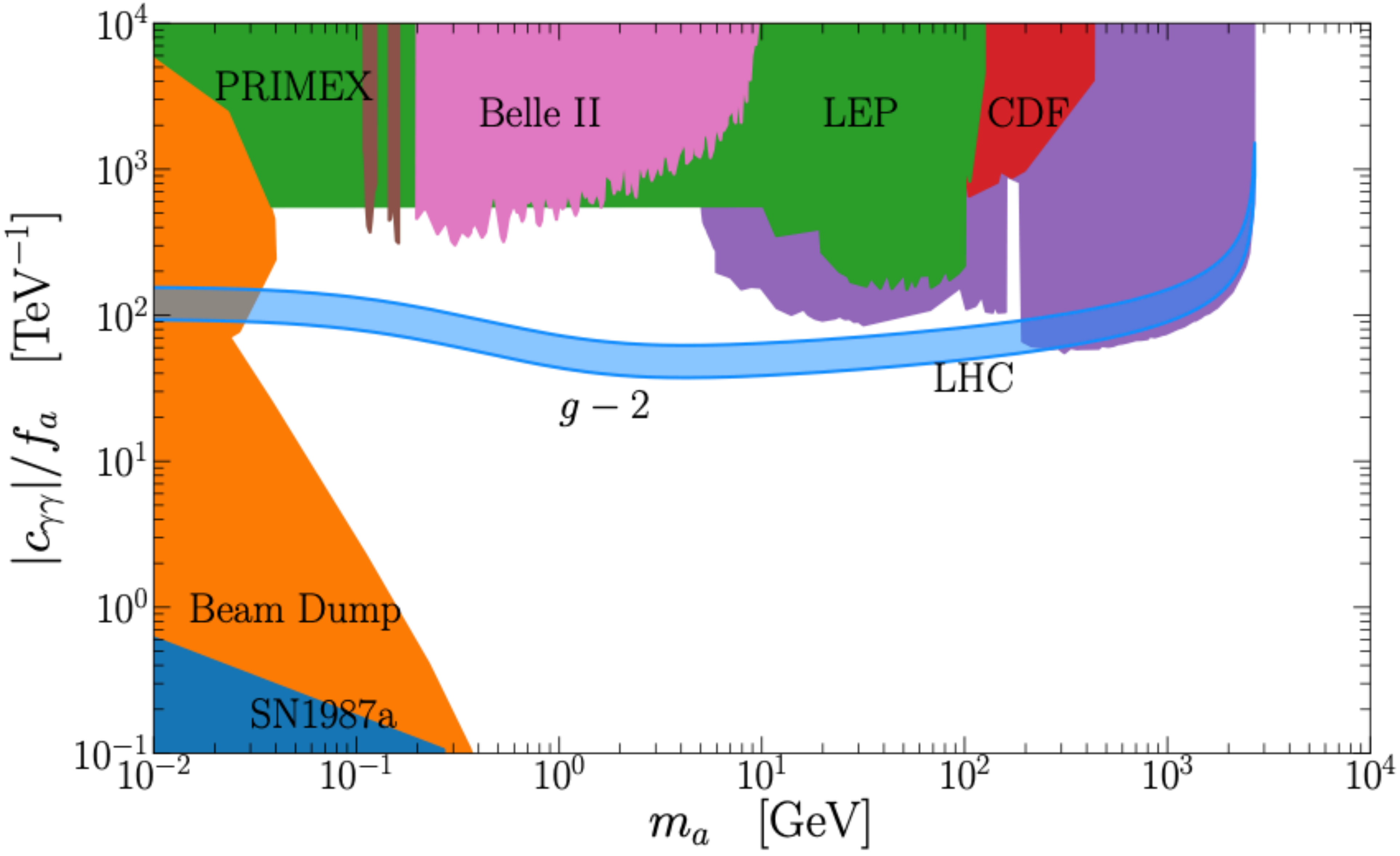


Source	$\delta a_\mu^{ll,\mathrm{W}}$(conn.) (%)
Monte Carlo statistics	0.19
Continuum extrapolation ($a \to 0$, Δ_{TB})	0.34
Finite-volume correction (Δ_{FV})	0.16
Pion-mass adjustment (Δ_{M_π})	0.06
Scale setting (w_0 (fm), w_0/a)	0.24
Current renormalization (Z_V)	0.17
Total	0.50%

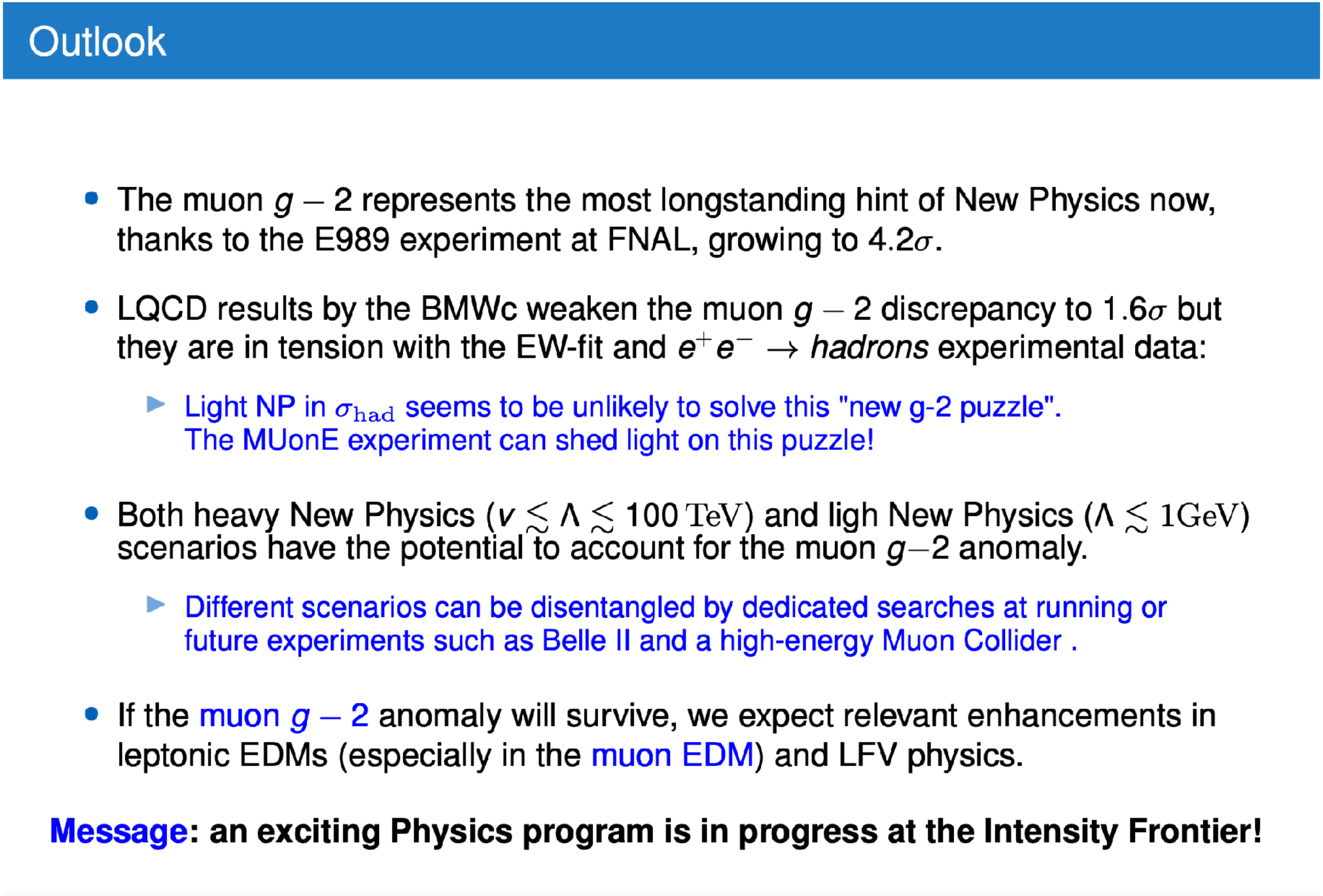
- “Error budget” shows the contribution to uncertainty from various sources

- Continuum extrapolation is currently dominant; scale setting error is larger for total HVP, but sub-leading for the window.

- An example new-physics model that can give rise to muon (g-2) is an axion-like particle (ALP) (although the authors note some model-building challenges:)



(Buen-Abad, Fan, Reece, and Sunk, https://arxiv.org/pdf/2104.03267) (summary slide taken from talk by P. Paradisi, sixth plenary meeting of (g-2) Theory Initiative)

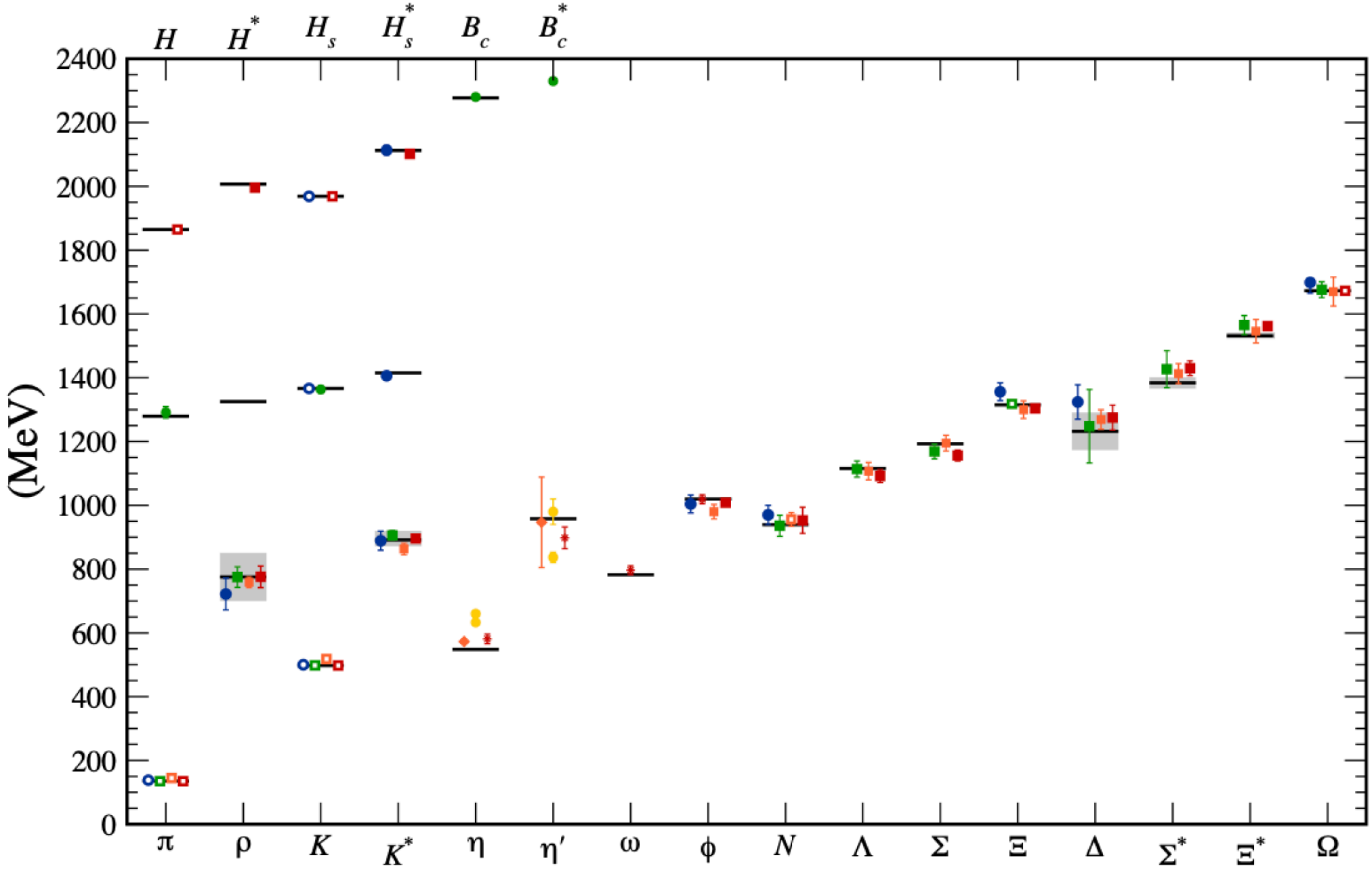


Outlook

- The muon $g-2$ represents the most longstanding hint of New Physics now, thanks to the E989 experiment at FNAL, growing to 4.2σ.
- LQCD results by the BMWc weaken the muon $g-2$ discrepancy to 1.6σ but they are in tension with the EW-fit and $e^+e^- \to \textit{hadrons}$ experimental data:
  - Light NP in $\sigma_{\rm had}$ seems to be unlikely to solve this "new g-2 puzzle". The MUonE experiment can shed light on this puzzle!
- Both heavy New Physics ($v \lesssim \Lambda \lesssim 100\,\mathrm{TeV}$) and ligh New Physics ($\Lambda \lesssim 1\mathrm{GeV}$) scenarios have the potential to account for the muon $g-2$ anomaly.
  - Different scenarios can be disentangled by dedicated searches at running or future experiments such as Belle II and a high-energy Muon Collider .
- If the muon $g-2$ anomaly will survive, we expect relevant enhancements in leptonic EDMs (especially in the muon EDM) and LFV physics.

Message: an exciting Physics program is in progress at the Intensity Frontier!

(from A. Kronfeld, *Ann.Rev.Nucl.Part.Sci. 62* (2012), arXiv:1203.1204)



- Modern lattice QCD is well-tested and reliable, for a wide variety of *ab initio* predictions - like the hadron spectrum above!