N^3LO soft-gluon corrections to H^+H^- production

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- **N^3LO in single-particle-inclusive kinematics**
- **H^+H^- cross sections**
- **Charged-Higgs rapidity distributions**

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Soft-gluon corrections in 1PI kinematics

partonic processes $q(p_a) + \bar{q}(p_b) \to H^+(p_1) + H^-(p_2)$

define $s = (p_a + p_b)^2$**,** $t_1 = (p_a - p_1)^2 - m^2$**,** $u_1 = (p_b - p_1)^2 - m^2$

we define the threshold variable
$s_4 = s + t_1 + u_1 = (p_2 + p_g)^2 - m^2$
where extra gluon with p_g **emitted**

At partonic threshold $p_g \to 0$ **and thus** $s_4 \to 0$

Soft corrections $\left[\frac{\ln^k(s_4/m^2)}{s_4}\right]_+$ **with** $k \leq 2n-1$ **for the order** α_s^n **corrections**

Resum these soft corrections for the double-differential cross section

Finite-order expansions $\to$ **no prescription needed or used**
(this avoids underestimating the size of the corrections)

Approximate NNLO (aNNLO) and approximate N^{3}LO (aN^{3}LO) predictions for cross sections and differential distributions

Analytical results are applicable to other processes with colorless final states

Soft-gluon Resummation

$$d\sigma_{pp\to H^+H^-} = \sum_{q,\bar{q}} \int dx_a\, dx_b\, \phi_{q/p}(x_a,\mu_F)\, \phi_{\bar{q}/p}(x_b,\mu_F)\, d\hat{\sigma}_{q\bar{q}\to H^+H^-}(s_4,\mu_F)$$

take Laplace transforms $d\tilde{\hat{\sigma}}_{q\bar{q}\to H^+H^-}(N) = \int (ds_4/s)\, e^{-Ns_4/s}\, d\hat{\sigma}_{q\bar{q}\to H^+H^-}(s_4)$

Then $d\tilde{\sigma}_{q\bar{q}\to H^+H^-}(N) = \tilde{\phi}_{q/q}(N_q,\mu_F)\, \tilde{\phi}_{\bar{q}/\bar{q}}(N_{\bar{q}},\mu_F)\, d\tilde{\hat{\sigma}}_{q\bar{q}\to H^+H^-}(N,\mu_F)$

Refactorization of the cross section

$$d\sigma_{q\bar{q}\to H^+H^-}(N) = \tilde{\psi}_q(N_q,\mu_F)\, \tilde{\psi}_{\bar{q}}(N_{\bar{q}},\mu_F)\, H_{q\bar{q}\to H^+H^-}\, \tilde{S}_{q\bar{q}\to H^+H^-}\left(\frac{\sqrt{s}}{N\mu_F}\right)$$

where $H_{q\bar{q}\to H^+H^-}$ **is hard function and** $S_{q\bar{q}\to H^+H^-}$ **is soft function**

Thus $d\tilde{\hat{\sigma}}_{q\bar{q}\to H^+H^-}(N) = \dfrac{\tilde{\psi}_{q/q}(N_q,\mu_F)\, \tilde{\psi}_{\bar{q}/\bar{q}}(N_{\bar{q}},\mu_F)}{\tilde{\phi}_{q/q}(N_q,\mu_F)\, \tilde{\phi}_{\bar{q}/\bar{q}}(N_{\bar{q}},\mu_F)}\, H_{q\bar{q}\to H^+H^-}\, \tilde{S}_{q\bar{q}\to H^+H^-}\left(\dfrac{\sqrt{s}}{N\mu_F}\right)$

Renormalization group evolution → resummation

$$d\tilde{\hat{\sigma}}^{\text{resum}}_{q\bar{q}\to H^+H^-}(N) = \exp\left[E_q(N_q) + E_{\bar{q}}(N_{\bar{q}})\right] \exp\left[2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\left(\gamma_{q/q}(N_q,\alpha_s(\mu)) + \gamma_{\bar{q}/\bar{q}}(N_{\bar{q}},\alpha_s(\mu)\right)\right]$$
$$\times H_{q\bar{q}\to H^+H^-}\left(\alpha_s(\sqrt{s})\right)\; \tilde{S}_{q\bar{q}\to H^+H^-}\left(\alpha_s\left(\frac{\sqrt{s}}{N}\right)\right)$$

where $E_q(N_q) = \int_0^1 dz \frac{z^{N_q-1}-1}{1-z} \left\{ \int_1^{(1-z)^2} \frac{d\lambda}{\lambda} A_q\left(\alpha_s(\lambda s)\right) + D_q\left[\alpha_s((1-z)^2 s)\right] \right\}$

NLO soft+virtual corrections

$$\frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}\to H^+H^-}}{dt_1\,du_1} = F^B_{q\bar{q}\to H^+H^-} \frac{\alpha_s(\mu_R)}{\pi} \{c_3\,\mathcal{D}_1(s_4) + c_2\,\mathcal{D}_0(s_4) + c_1\,\delta(s_4)\}$$

where

$$\mathcal{D}_k(s_4) = \left[\frac{\ln^k(s_4/m^2)}{s_4}\right]_+ ,$$

$$c_3 = 4C_F \qquad \text{and} \qquad c_2 = -2\,C_F\,\ln\left(\frac{t_1u_1}{m^4}\right) - 2\,C_F\,\ln\left(\frac{\mu_F^2}{s}\right) ,$$

$$c_1 = C_F\,\ln^2\left(\frac{-t_1}{m^2}\right) + C_F\,\ln^2\left(\frac{-u_1}{m^2}\right) + \left[C_F\,\ln\left(\frac{t_1u_1}{m^4}\right) - 2\gamma_q^{(1)}\right]\ln\left(\frac{\mu_F^2}{s}\right) + V_1 ,$$

$$\text{and} \qquad V_1 = 2C_F(-2+\zeta_2)$$

NNLO soft+virtual corrections

$$
\begin{aligned}
\frac{d^2\hat{\sigma}^{(2)}_{q\bar{q}\to H^+H^-}}{dt_1\,du_1} = {} & F^B_{q\bar{q}\to H^+H^-}\frac{\alpha_s^2(\mu_R)}{\pi^2}\left\{\frac{1}{2}c_3^2\,\mathcal{D}_3(s_4)+\left[\frac{3}{2}c_3c_2-\frac{\beta_0}{4}c_3\right]\mathcal{D}_2(s_4)\right.\\
& +\left[c_3c_1+c_2^2-\zeta_2c_3^2-\frac{\beta_0}{2}c_2-\beta_0C_F\ln\left(\frac{\mu_F^2}{\mu_R^2}\right)+4A_q^{(2)}\right]\mathcal{D}_1(s_4)\\
& +\left[c_2c_1-\zeta_2c_3c_2+\zeta_3c_3^2+\frac{\beta_0}{4}c_2\ln\left(\frac{\mu_R^2}{s}\right)\right.\\
& \quad -\frac{\beta_0}{2}C_F\ln^2\left(\frac{-t_1}{m^2}\right)-\frac{\beta_0}{2}C_F\ln^2\left(\frac{-u_1}{m^2}\right)-2A_q^{(2)}\ln\left(\frac{t_1u_1}{m^4}\right)+2D_q^{(2)}\\
& \quad \left.+\frac{\beta_0}{4}C_F\ln^2\left(\frac{\mu_F^2}{s}\right)-2A_q^{(2)}\ln\left(\frac{\mu_F^2}{s}\right)\right]\mathcal{D}_0(s_4)\\
& +\left[V_2+\frac{1}{2}\left(c_1^2-V_1^2\right)-\frac{\zeta_2}{2}c_2^2+\zeta_3c_3c_2+\frac{\beta_0}{6}C_F\left(\ln^3\left(\frac{-t_1}{m^2}\right)+\ln^3\left(\frac{-u_1}{m^2}\right)\right)\right.\\
& \quad +\left(\frac{\beta_0}{4}C_F+A_q^{(2)}\right)\left(\ln^2\left(\frac{-t_1}{m^2}\right)+\ln^2\left(\frac{-u_1}{m^2}\right)\right)+\frac{\beta_0}{4}c_1\ln\left(\frac{\mu_R^2}{s}\right)-2\gamma_q^{(2)}\ln\left(\frac{\mu_F^2}{s}\right)\\
& \quad \left.\left.+A_q^{(2)}\ln\left(\frac{t_1u_1}{m^4}\right)\ln\left(\frac{\mu_F^2}{s}\right)+\frac{\beta_0}{8}\left(2\gamma_q^{(1)}-C_F\ln\left(\frac{t_1u_1}{m^4}\right)\right)\ln^2\left(\frac{\mu_F^2}{s}\right)\right]\delta(s_4)\right\}
\end{aligned}
$$

where $V_2=C_F^2\left(\frac{511}{64}-\frac{35}{8}\zeta_2-\frac{15}{4}\zeta_3+\frac{\zeta_2^2}{10}\right)+C_FC_A\left(-\frac{1535}{192}+\frac{37}{9}\zeta_2+\frac{7}{4}\zeta_3-\frac{3}{20}\zeta_2^2\right)+C_Fn_f\left(\frac{127}{96}-\frac{7}{9}\zeta_2+\frac{\zeta_3}{2}\right)$

$\mathbf{N^3LO}$ soft-gluon corrections

$$
\begin{aligned}
\frac{d^2\hat{\sigma}^{(3)}_{q\bar{q}\to H^+H^-}}{dt_1\,du_1} = {} & F^B_{q\bar{q}\to H^+H^-}\,\frac{\alpha_s^3(\mu_R)}{\pi^3}\left\{\frac{1}{8}c_3^3\,\mathcal{D}_5(s_4)+\left[\frac{5}{8}c_3^2c_2-\frac{5}{24}c_3^2\beta_0\right]\mathcal{D}_4(s_4)\right. \\
& +\left[c_3c_2^2+\frac{1}{2}c_3^2c_1-\zeta_2c_3^3+\frac{\beta_0^2}{12}c_3-\frac{5}{6}\beta_0c_3c_2-\beta_0C_Fc_3\ln\left(\frac{\mu_F^2}{\mu_R^2}\right)+4c_3A_q^{(2)}\right]\mathcal{D}_3(s_4) \\
& +\left[\frac{3}{2}c_3c_2c_1+\frac{1}{2}c_2^3-3\zeta_2c_3^2c_2+\frac{5}{2}\zeta_3c_3^3-\frac{\beta_0}{4}c_3c_1+\frac{9}{8}\beta_0\zeta_2c_3^2\right. \\
& \quad\left.+(3c_2-\beta_0)\left(-\frac{\beta_0}{4}c_2-\frac{\beta_0}{2}C_F\ln\left(\frac{\mu_F^2}{\mu_R^2}\right)+2A_q^{(2)}\right)-C_F\frac{\beta_1}{4}-\frac{3}{2}c_3X_1\right]\mathcal{D}_2(s_4) \\
& +\left[\frac{1}{2}c_3c_1^2+c_2^2c_1-\zeta_2c_3^2c_1-\frac{5}{2}\zeta_2c_3c_2^2+5\zeta_3c_3^2c_2+\frac{5}{4}\zeta_2^2c_3^3-\frac{15}{4}\zeta_4c_3^3\right. \\
& \quad-\frac{\beta_0^2}{4}\zeta_2c_3-\frac{5}{3}\beta_0\zeta_3c_3^2+\beta_0\zeta_2c_3c_2+(2c_1-5\zeta_2c_3)\left(-\frac{\beta_0}{4}c_2-\frac{\beta_0}{2}C_F\ln\left(\frac{\mu_F^2}{\mu_R^2}\right)+2A_q^{(2)}\right) \\
& \quad+(\beta_0-2c_2)X_1+c_3X_0+4A_q^{(3)}+C_F\frac{\beta_0^2}{4}\ln^2\left(\frac{\mu_F^2}{\mu_R^2}\right)-2\beta_0A_q^{(2)}\ln\left(\frac{\mu_F^2}{\mu_R^2}\right) \\
& \quad\left.+C_F\frac{\beta_1}{4}\ln\left(\frac{\mu_R^2}{s}\right)+C_F\frac{\beta_1}{4}\ln\left(\frac{t_1u_1}{m^4}\right)\right]\mathcal{D}_1(s_4)
\end{aligned}
$$

$$
+\left[\frac{1}{2}c_2 c_1^2+3\zeta_5 c_3^3-\frac{15}{4}\zeta_4 c_3^2 c_2-2\zeta_2\zeta_3 c_3^3+\zeta_3 c_3^2 c_1+2\zeta_3 c_3 c_2^2+\frac{5}{4}\zeta_2^2 c_3^2 c_2-\zeta_2 c_3 c_2 c_1-\frac{\zeta_2}{2}c_2^3\right.
$$

$$
+\frac{\beta_0}{12}c_3\left(15\zeta_4 c_3-8\zeta_3 c_2-6\zeta_2^2 c_3+3\zeta_2 c_1\right)
$$

$$
+(4\zeta_3 c_3-3\zeta_2 c_2)\left(-\frac{\beta_0}{4}c_2-\frac{\beta_0}{2}C_F\ln\left(\frac{\mu_F^2}{\mu_R^2}\right)+2A_q^{(2)}\right)
$$

$$
+(\zeta_2 c_3-c_1)\,X_1+c_2 X_0-\frac{\beta_0^2}{4}C_F\left(\ln^2\left(\frac{-t_1}{m^2}\right)+\ln^2\left(\frac{-u_1}{m^2}\right)\right)\ln\left(\frac{\mu_R^2}{s}\right)
$$

$$
+\frac{\beta_0^2}{16}c_2\ln^2\left(\frac{\mu_R^2}{s}\right)+\frac{\beta_0^2}{8}C_F\ln\left(\frac{\mu_F^2}{s}\right)\ln^2\left(\frac{\mu_R^2}{s}\right)
$$

$$
\left.-\frac{\beta_1}{8}C_F\left(\ln^2\left(\frac{-t_1}{m^2}\right)+\ln^2\left(\frac{-u_1}{m^2}\right)\right)+Y_0\right\}\ \mathcal{D}_0(s_4)
$$

where

$$
X_1=\frac{\beta_0}{4}\zeta_2 c_3-\frac{\beta_0}{4}c_2\ln\left(\frac{\mu_R^2}{s}\right)+\frac{\beta_0}{2}C_F\ln^2\left(\frac{-t_1}{m^2}\right)+\frac{\beta_0}{2}C_F\ln^2\left(\frac{-u_1}{m^2}\right)
$$

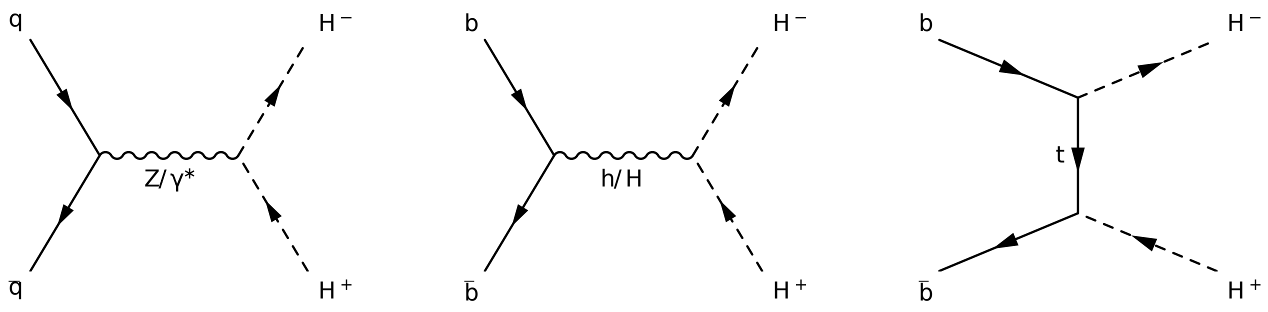
$$
+2A_q^{(2)}\ln\left(\frac{t_1 u_1}{m^4}\right)-2D_q^{(2)}-\frac{\beta_0}{4}C_F\ln^2\left(\frac{\mu_F^2}{s}\right)+2A_q^{(2)}\ln\left(\frac{\mu_F^2}{s}\right),
$$

$$
\begin{aligned}
X_0 \quad = \quad & V_2 - \frac{1}{2} V_1^2 - \frac{1}{4}\zeta_2^2 c_3^2 + \frac{3}{4}\zeta_4 c_3^2 + \frac{\beta_0}{4} c_1 \ln\left(\frac{\mu_R^2}{s}\right) + \frac{\beta_0}{6}\zeta_3 c_3 - \frac{\beta_0}{4}\zeta_2 c_2 - \frac{\beta_0}{2}\zeta_2 C_F \ln\left(\frac{\mu_F^2}{\mu_R^2}\right) \\
& + 2A_q^{(2)}\zeta_2 - 2\gamma_q^{(2)} \ln\left(\frac{\mu_F^2}{s}\right) + \frac{\beta_0}{8}\left[2\gamma_q^{(1)} - C_F \ln\left(\frac{t_1 u_1}{m^4}\right)\right] \ln^2\left(\frac{\mu_F^2}{s}\right) \\
& + A_q^{(2)} \ln\left(\frac{t_1 u_1}{m^4}\right) \ln\left(\frac{\mu_F^2}{s}\right) + \frac{\beta_0}{6} C_F \ln^3\left(\frac{-t_1}{m^2}\right) + \frac{\beta_0}{6} C_F \ln^3\left(\frac{-u_1}{m^2}\right) \\
& + \left(C_F \frac{\beta_0}{4} + A_q^{(2)}\right)\left[\ln^2\left(\frac{-t_1}{m^2}\right) + \ln^2\left(\frac{-u_1}{m^2}\right)\right] ,
\end{aligned}
$$

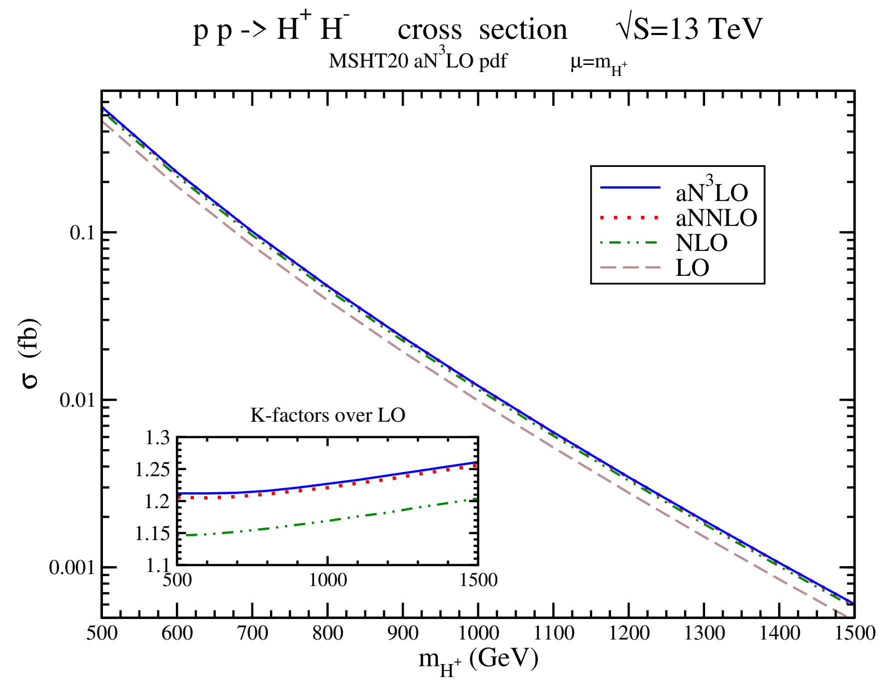
and

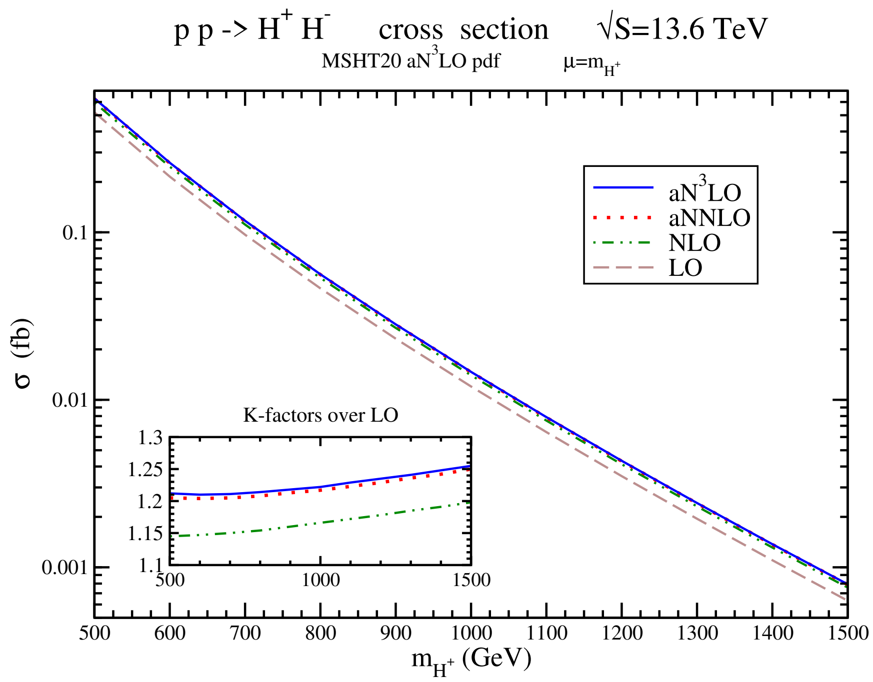
$$
\begin{aligned}
Y_0 \quad = \quad & -C_F \frac{\beta_0^2}{24}\left[\ln^3\left(\frac{\mu_F^2}{\mu_R^2}\right) + \ln^3\left(\frac{\mu_R^2}{s}\right)\right] + \frac{1}{16}\left(C_F \beta_1 + 8\beta_0 A_q^{(2)}\right)\left[\ln^2\left(\frac{\mu_F^2}{\mu_R^2}\right) - \ln^2\left(\frac{\mu_R^2}{s}\right)\right] \\
& - \left(A_q^{(2)}\beta_0 + C_F \frac{\beta_1}{8}\right) \ln\left(\frac{t_1 u_1}{m^4}\right) \ln\left(\frac{\mu_R^2}{s}\right) + D_q^{(2)}\beta_0 \ln\left(\frac{\mu_R^2}{s}\right) - 2A_q^{(3)} \ln\left(\frac{\mu_F^2}{s}\right) + 2D_q^{(3)} \\
& - \frac{C_F}{6}\beta_0^2\left[\ln^3\left(\frac{-t_1}{m^2}\right) + \ln^3\left(\frac{-u_1}{m^2}\right)\right] - \frac{\beta_0}{4}\left(C_F \beta_0 + 4A_q^{(2)}\right)\left[\ln^2\left(\frac{-t_1}{m^2}\right) + \ln^2\left(\frac{-u_1}{m^2}\right)\right] \\
& + \left(\beta_0 D_q^{(2)} - 2A_q^{(3)}\right) \ln\left(\frac{t_1 u_1}{m^4}\right)
\end{aligned}
$$

H^+H^- cross sections

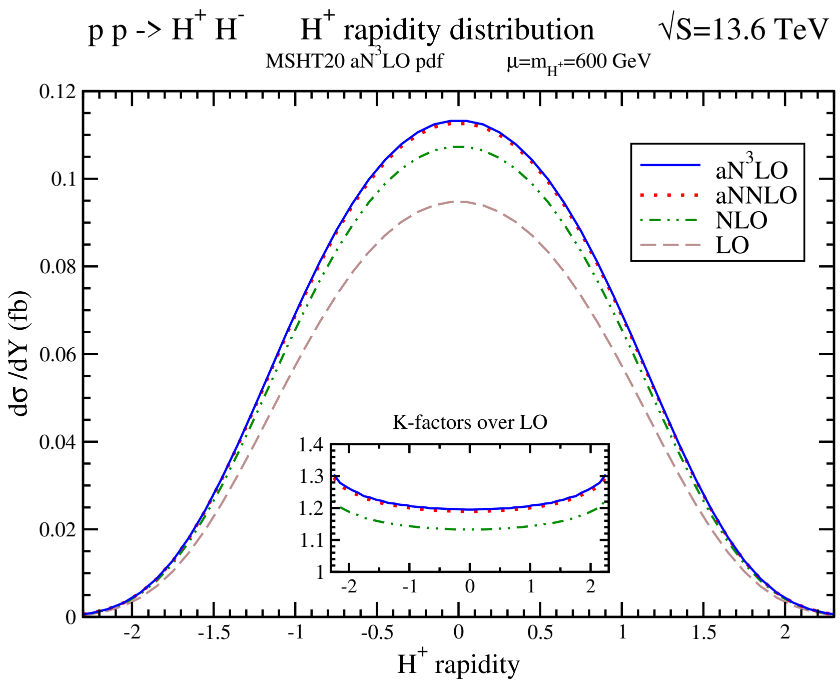


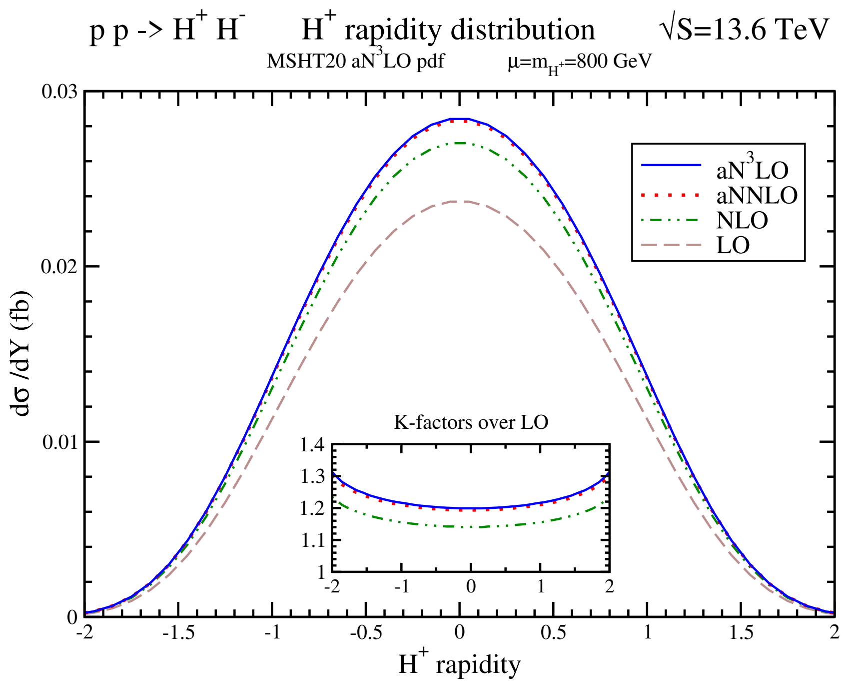
we consider Drell-Yan type production of H^+H^- (left diagram); the other contributions are negligible





H^+ rapidity distributions at 13.6 TeV





K-factors increase at larger rapidities

Summary

- H^+H^- **production**

- **soft-gluon corrections through aN^{3}LO in single-particle-inclusive kinematics**

- **results for total cross sections and rapidity distributions**

- **higher-order corrections further enhance and improve the theoretical predictions**

- **analytical results for resummation and N^{3}LO expansions applicable to other processes with colorless final states**