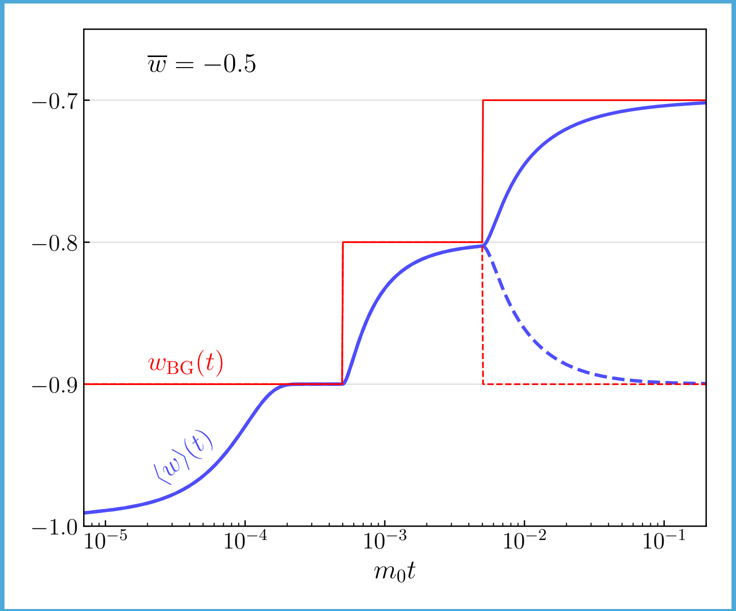
Cosmological Stasis from Dynamical Scalars



Brooks Thomas

LAFAYETTE COLLEGE

Work supported in part by NSF

Based on work done in collaboration with:

Keith R. Dienes, Fei Huang, Lucien Heurtier, and Timothy M. P. Tait [arXiv:2405.xxxxx]

DPF/PHENO, University of Pittsburgh, May 15th, 2024

Cosmological Stasis

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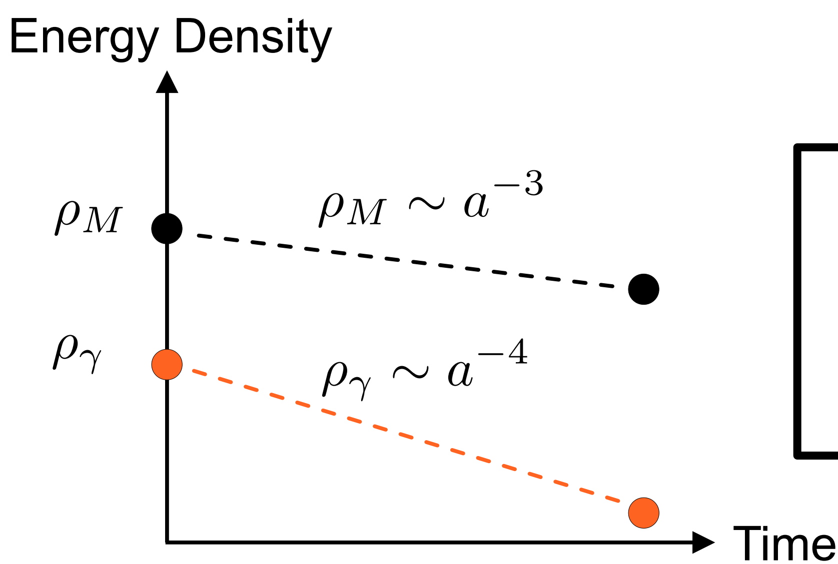

Boltzmann Equations

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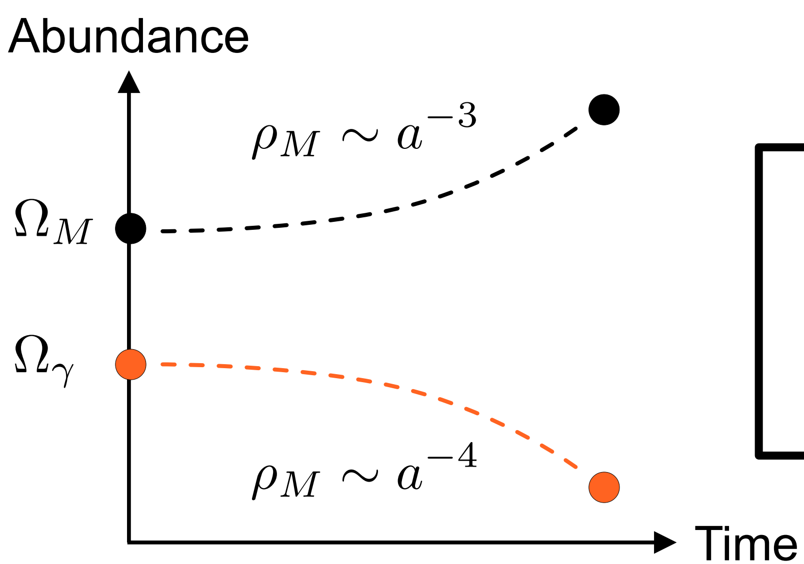

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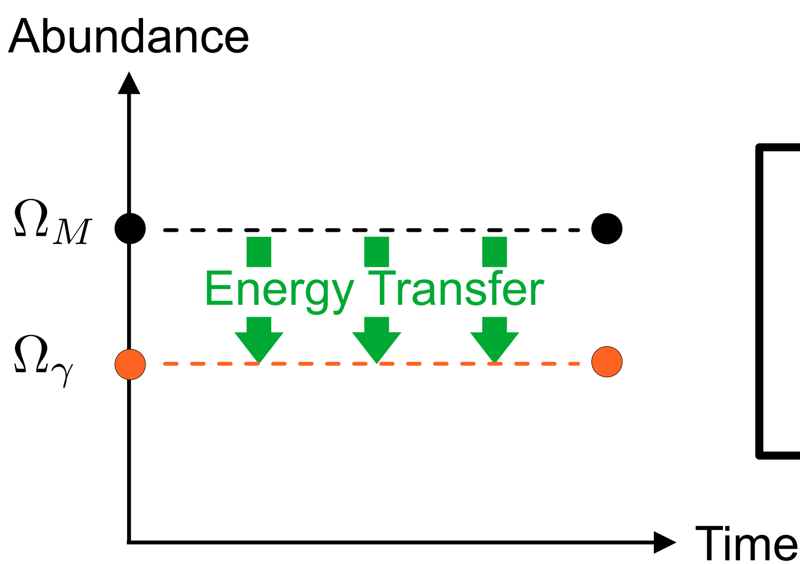

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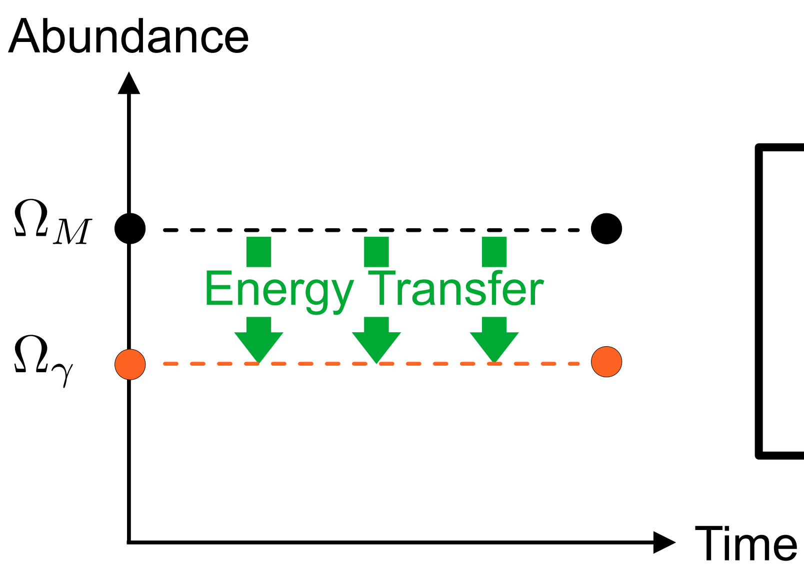

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$\propto t^{-1}$, so $P(t)$ should be as well.

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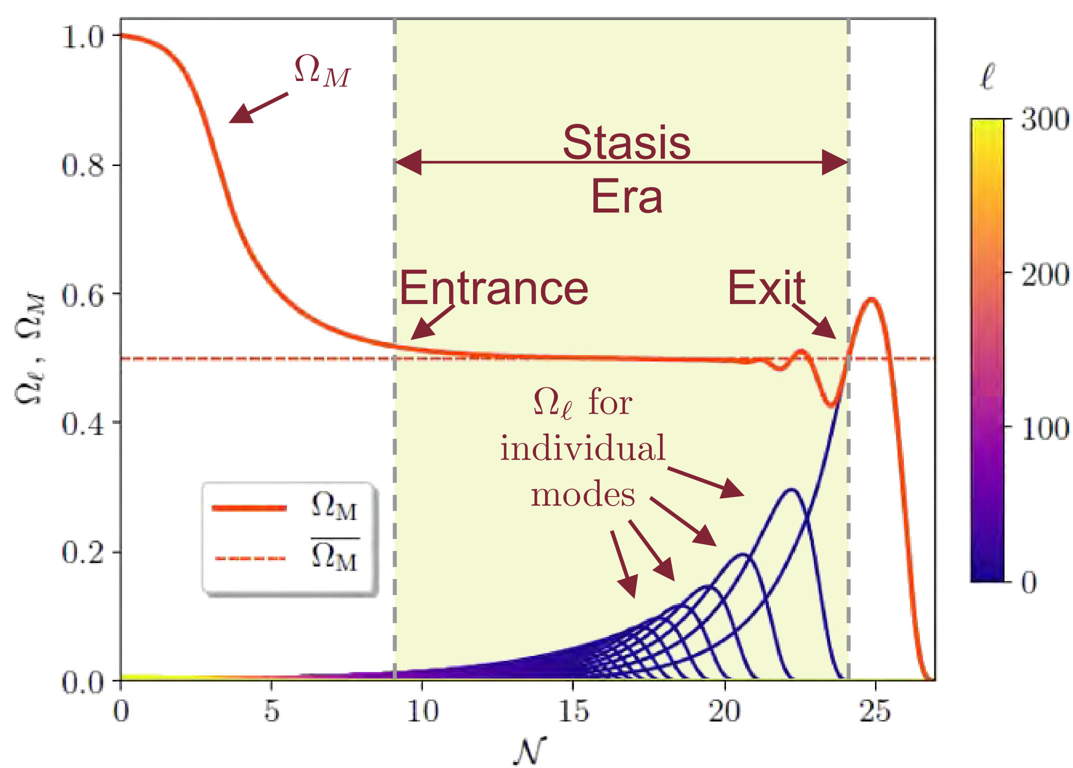

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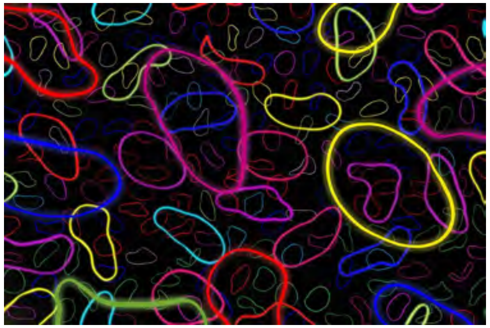
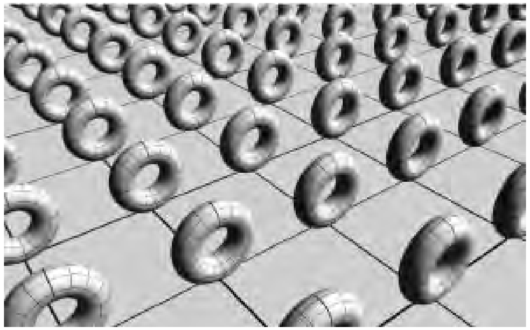
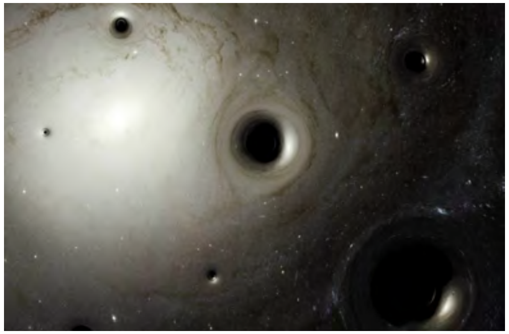

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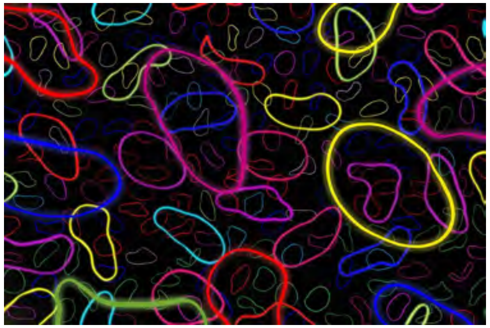

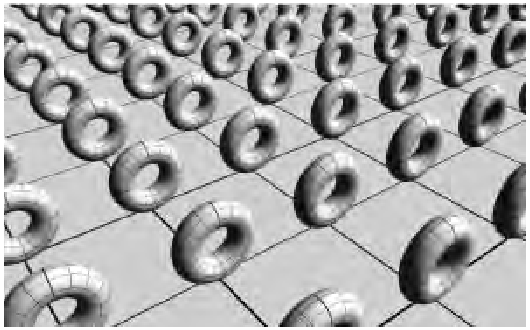

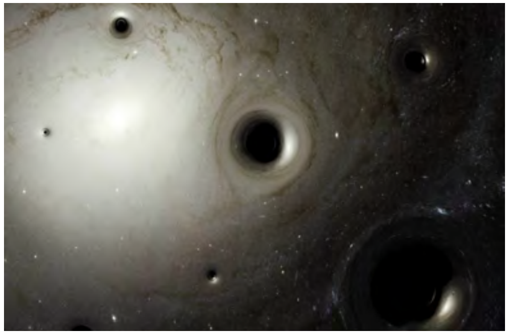

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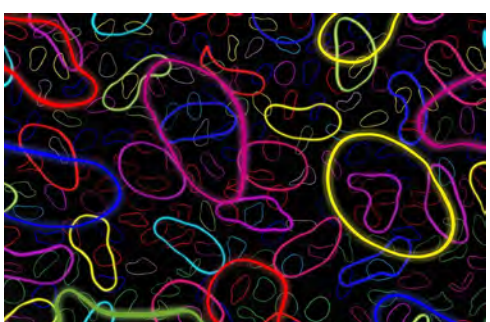
- When they do emerge, stasis is typically a **global attractor**: the universe will evolve toward stasis regardless of initial conditions.
- The modified cosmological histories associated with stasis can affect the evolution of **scalar and tensor perturbations**.

See Anna Paulsen's talk (directly after this one)



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- Such a stasis, as we'll see, would be characterized by an effective equation-of-state parameter **between that of vacuum energy** ($w_\Lambda = -1$) **and matter** ($w_\Lambda = 0$)
- Moreover, stases involving dynamical scalars give rise to some **phenomena not seen in other realizations of stasis** which could potentially useful for addressing fundamenal questions in cosmology.

Warm-Up: A Single Scalar

- To set the stage, let's recall how the homogeneous zero-mode of a **single scalar field** ϕ of mass m with a quadratic potential $V(\phi)$ evolves in a flat FRW universe.

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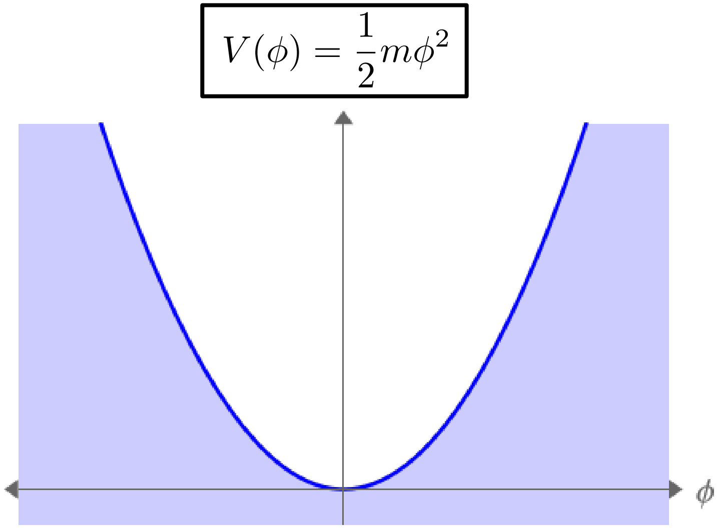

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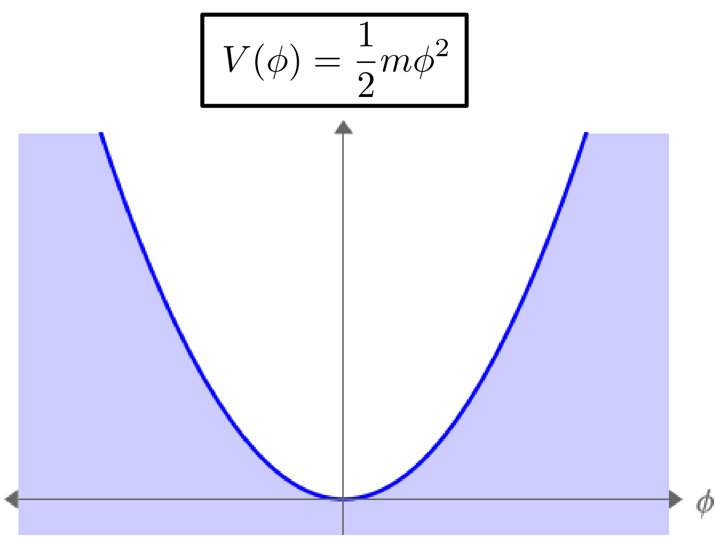

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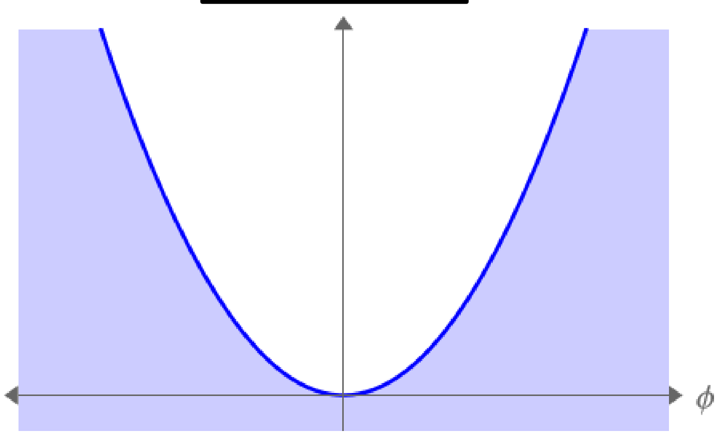

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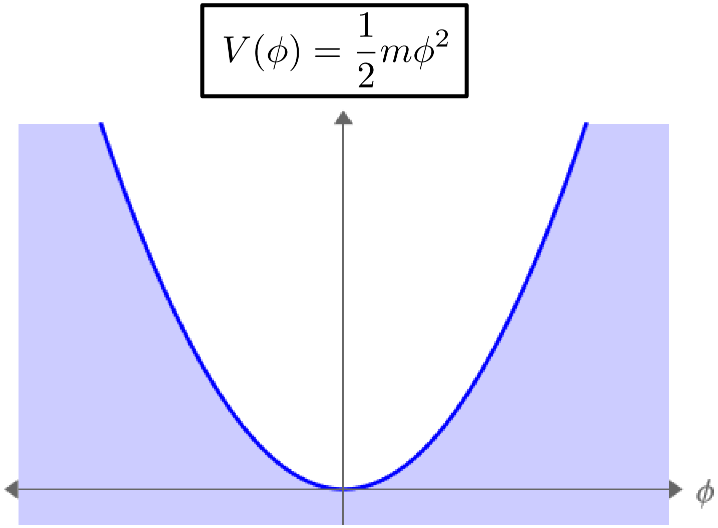

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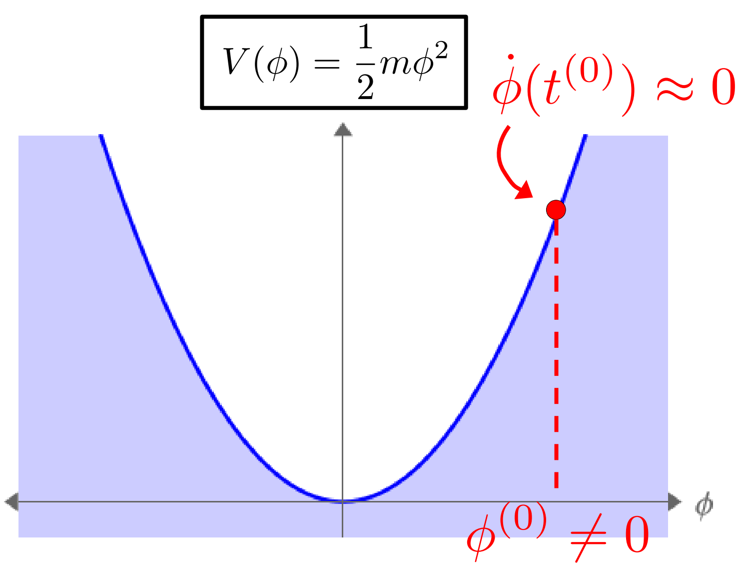

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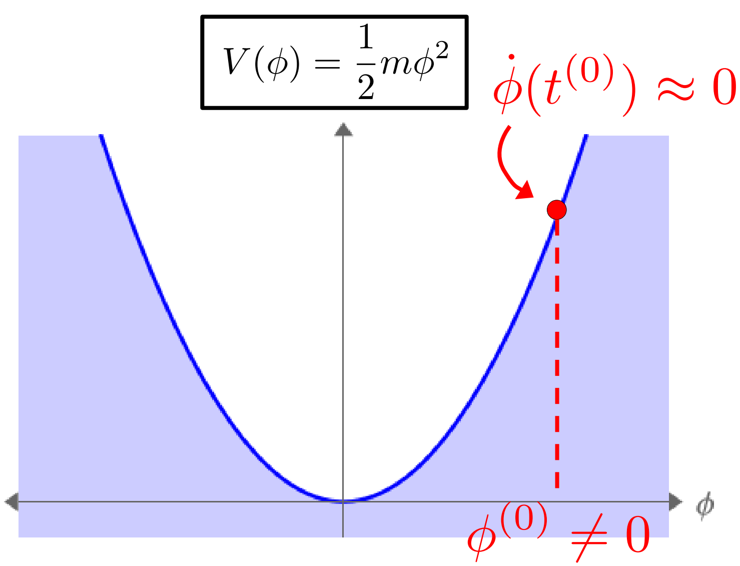

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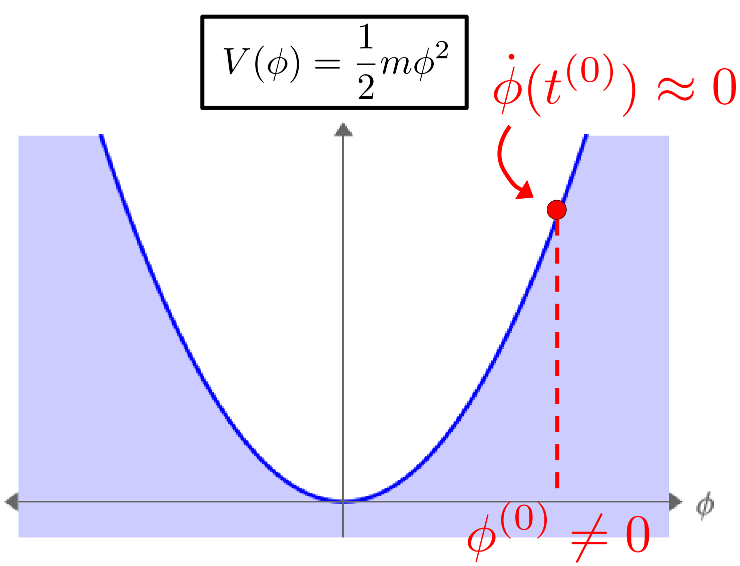

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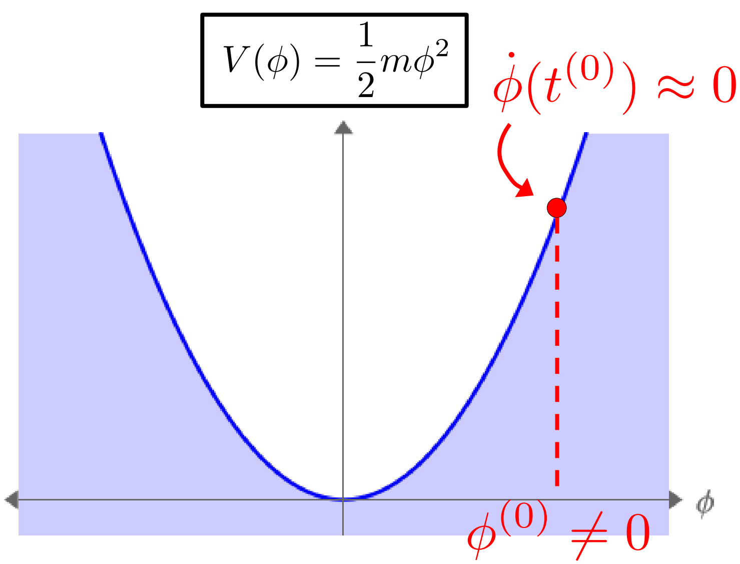
$$H \approx \frac{\kappa}{3t}\,, \quad \text{where} \quad \kappa \equiv \frac{2}{1+w}$$



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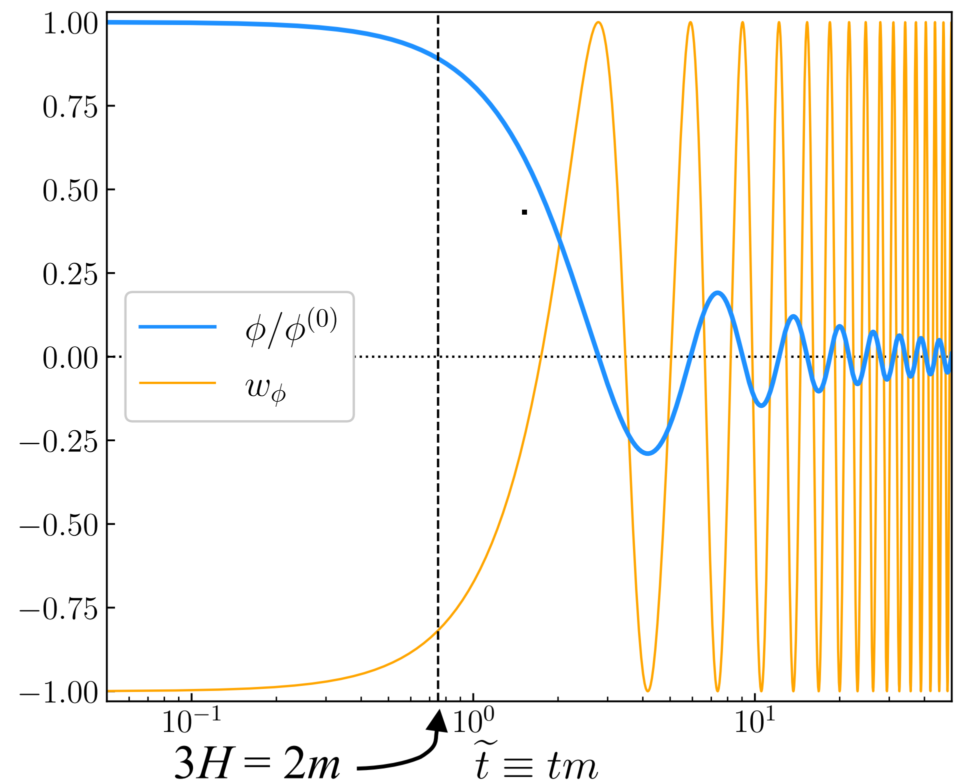

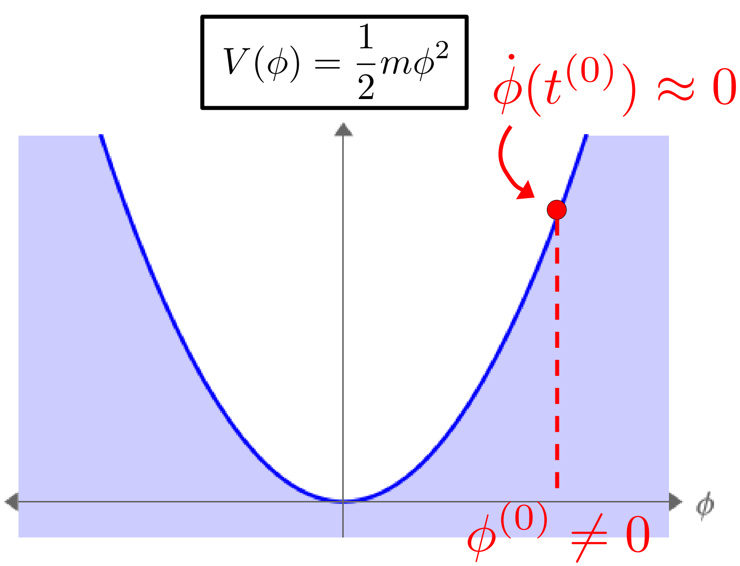

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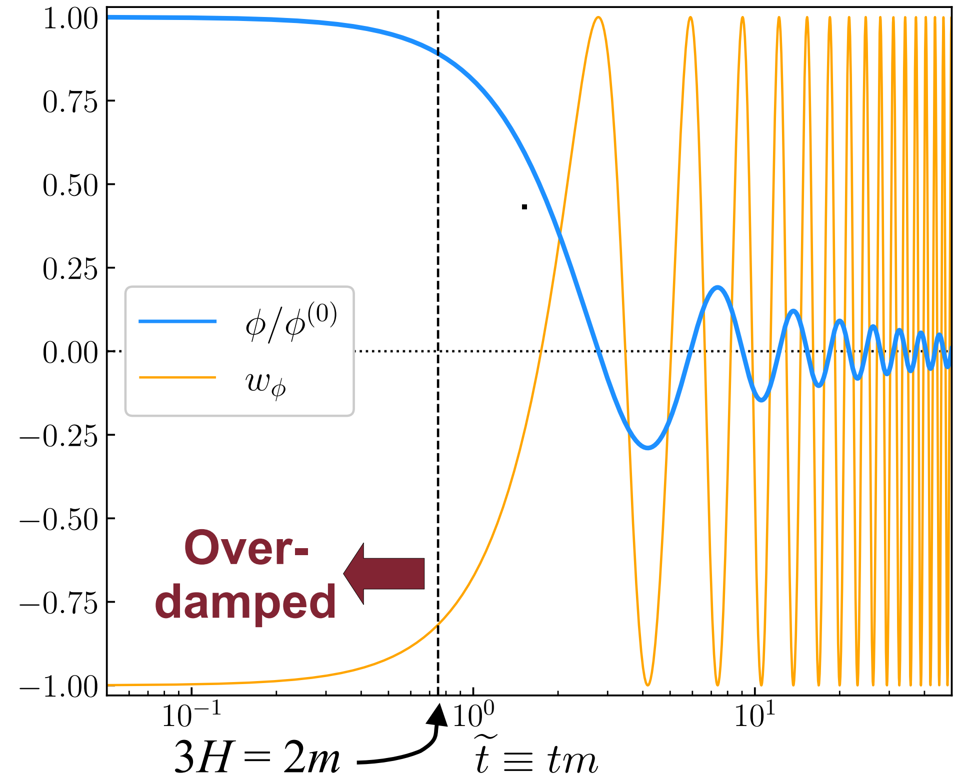

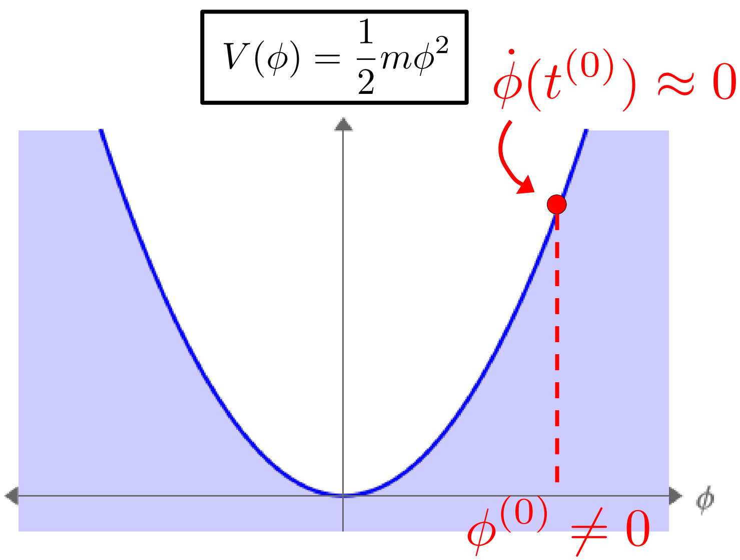

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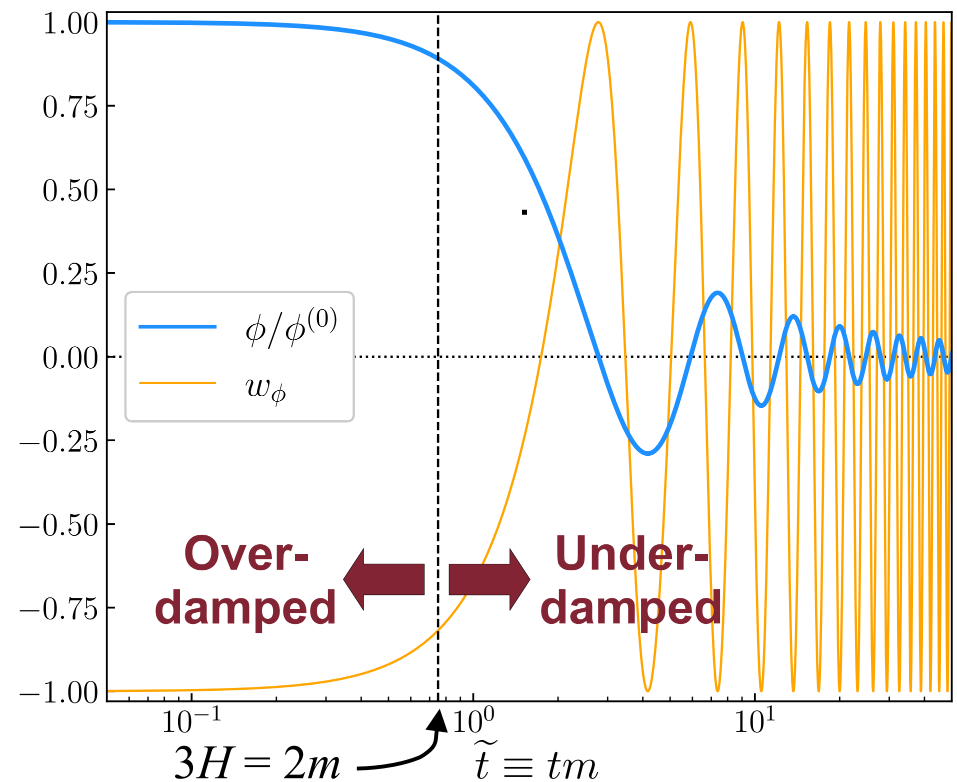

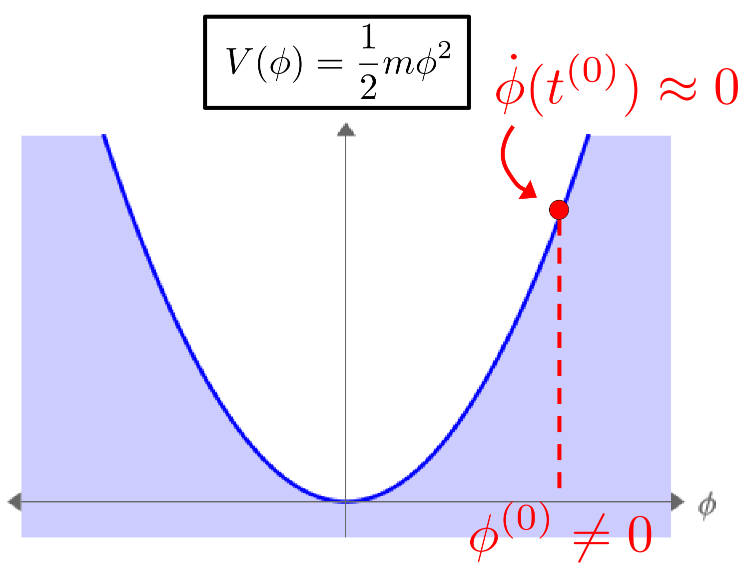

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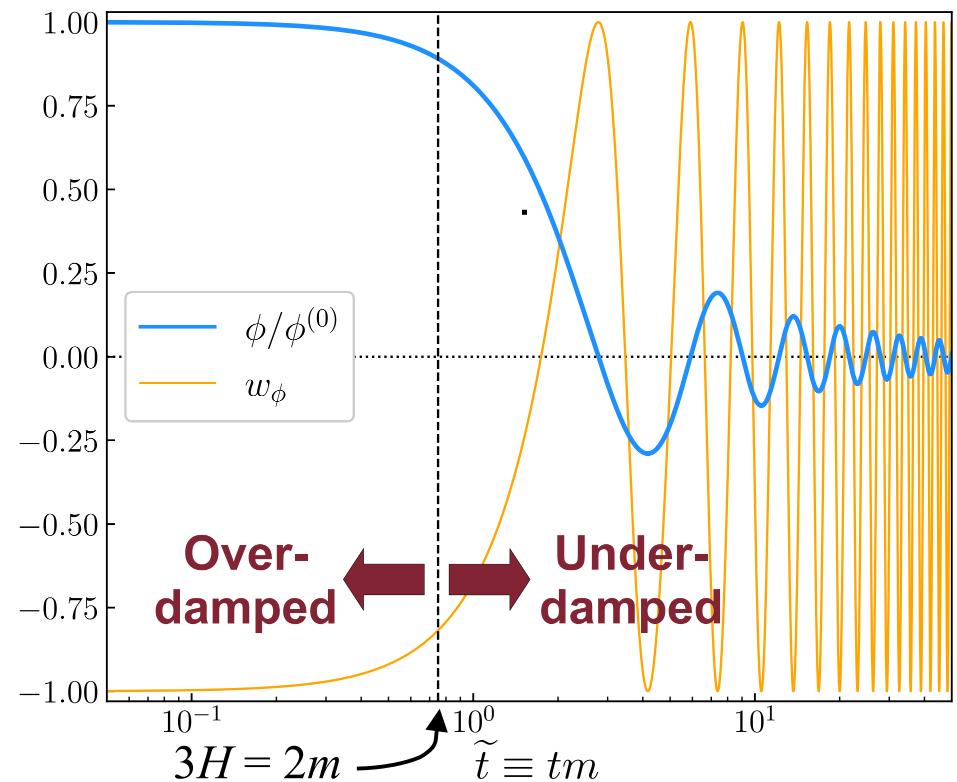

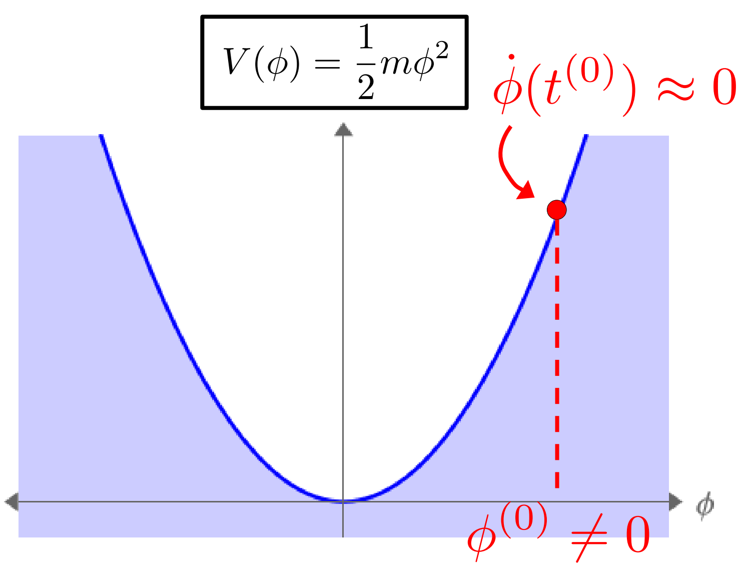

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➡ **Behaves like massive matter**

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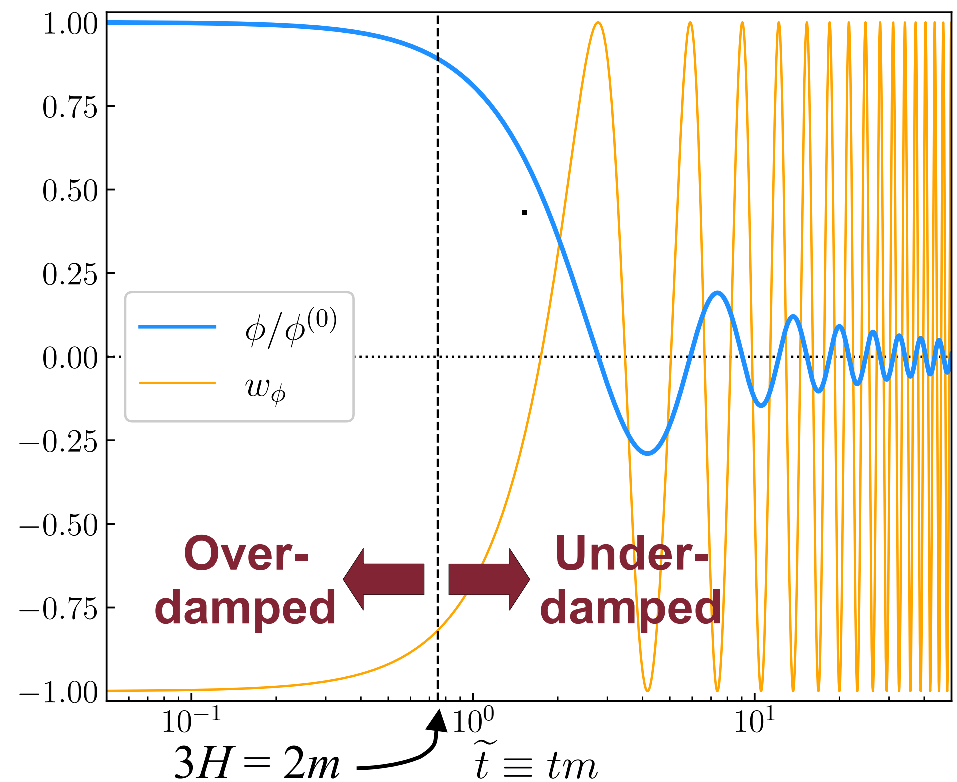

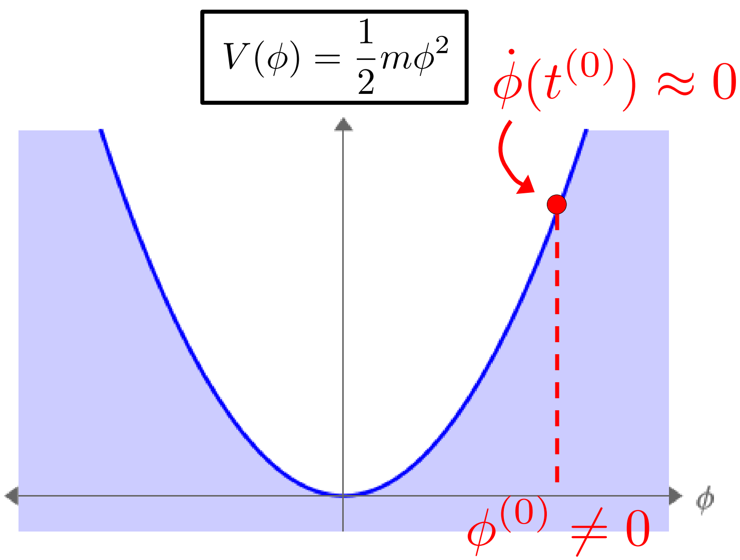

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- Consider a **mass spectrum** (motivated by KK towers):

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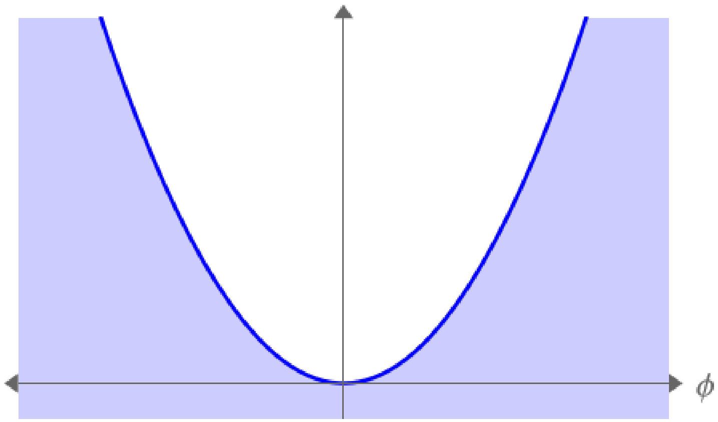

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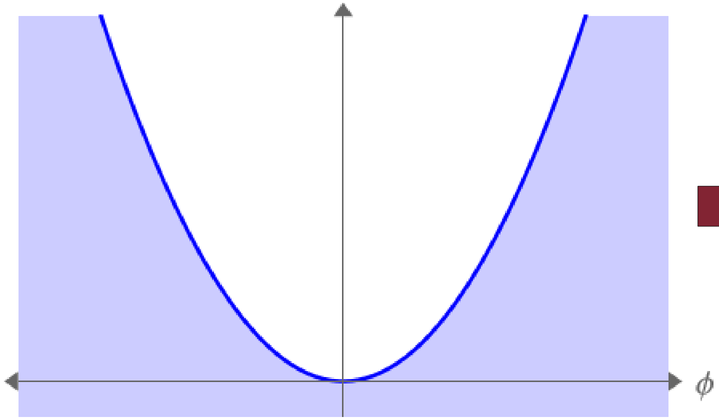

For Each Field

$$\rho_\ell = \frac{1}{2}\dot{\phi}_\ell^2 + \frac{1}{2}m_\ell^2\phi_\ell^2 \qquad w_\ell = \frac{\dot{\phi}_\ell^2 - m_\ell^2\phi_\ell^2}{\dot{\phi}_\ell^2 + m_\ell^2\phi_\ell^2}$$

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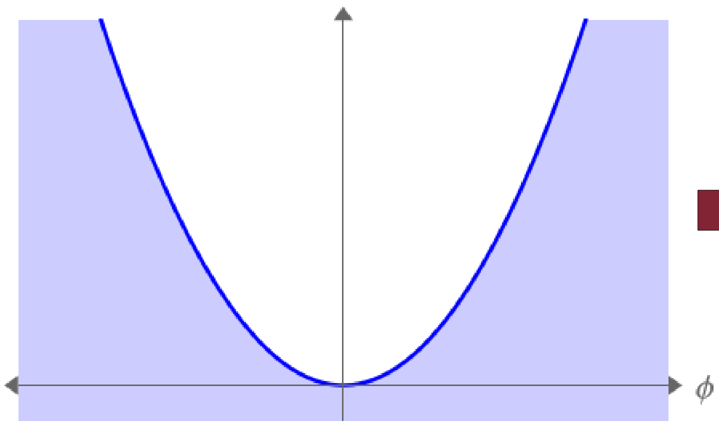

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- We'll also assume (for the moment) that there's **no background energy component**: the collective energy density of the ϕ_ℓ dominates the universe.

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N "copies" of this

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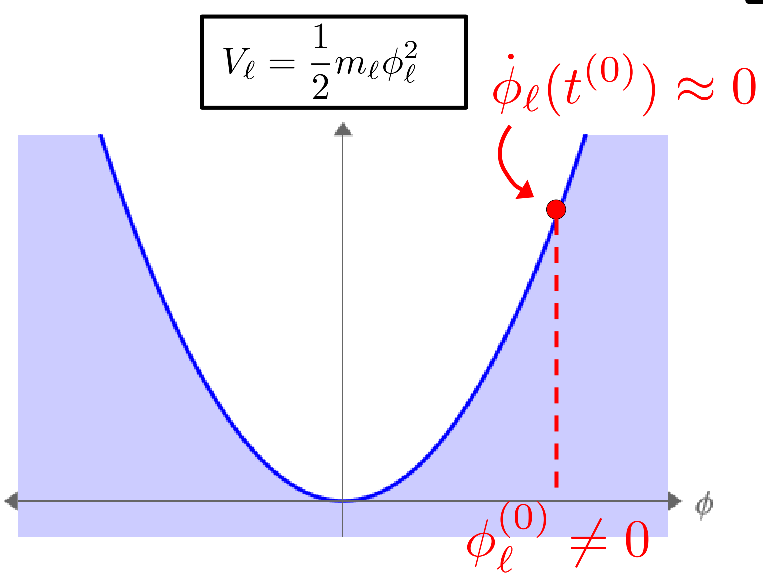
Let's see what the cosmology of such a tower of scalar-field zero modes looks like!

Initial Conditions

- All that now remains is to specify the initial conditions for our scalars.
- For simplicity (and because it's consistent with many standard abundance-generation mechanisms for fields of this sort – e.g., vacuum misalignment), we once again take $\dot{\phi}_\ell(t^{(0)}) \approx 0$ for all of the ϕ_ℓ.
- However, we still need both an **overall mass scale** for the displacements and to know **how they scale** with ℓ across the tower.
- We assume a power-law scaling for the initial abundances of the form

$$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0} \right)^\alpha$$



- For a given mass spectrum, the overall scale of the abundances can be parameterized by the ratio $\phi_0^{(0)}/M_P$, or, equivalently, by the ratio $H^{(0)}/m_{N\text{-}1}$.

Dynamical Evolution

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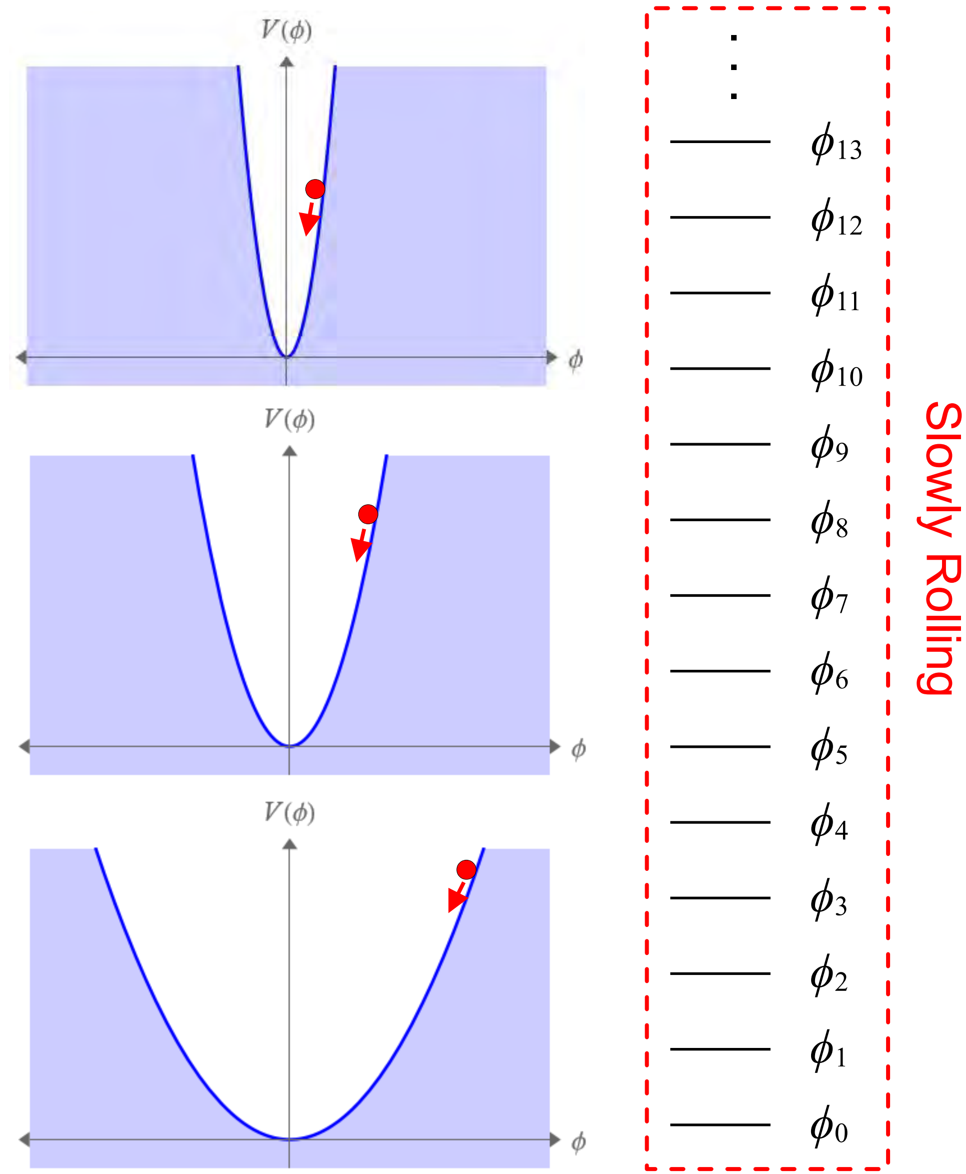

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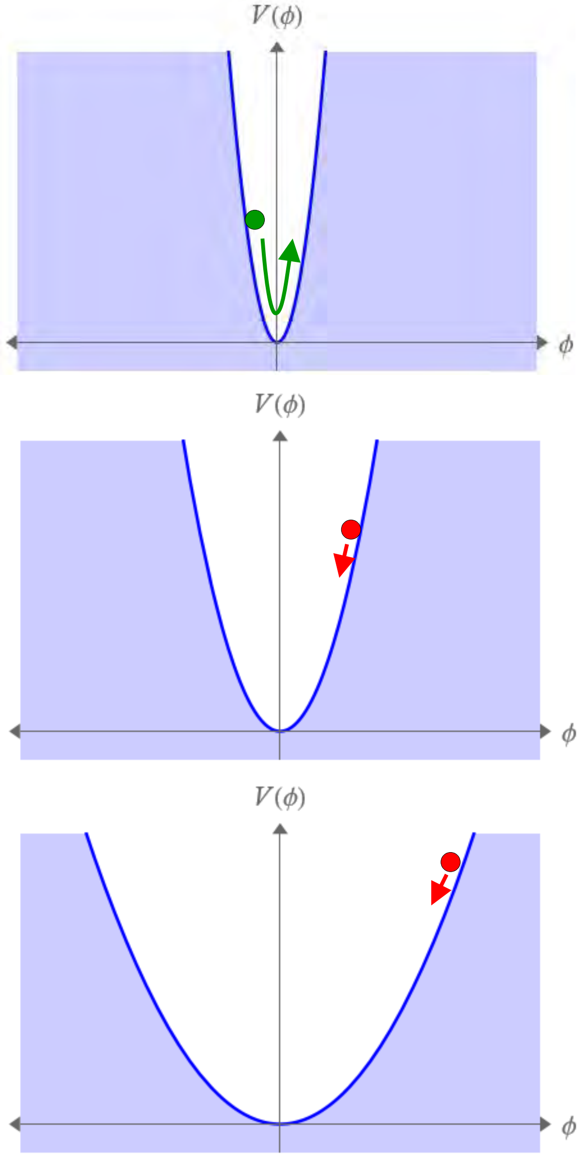

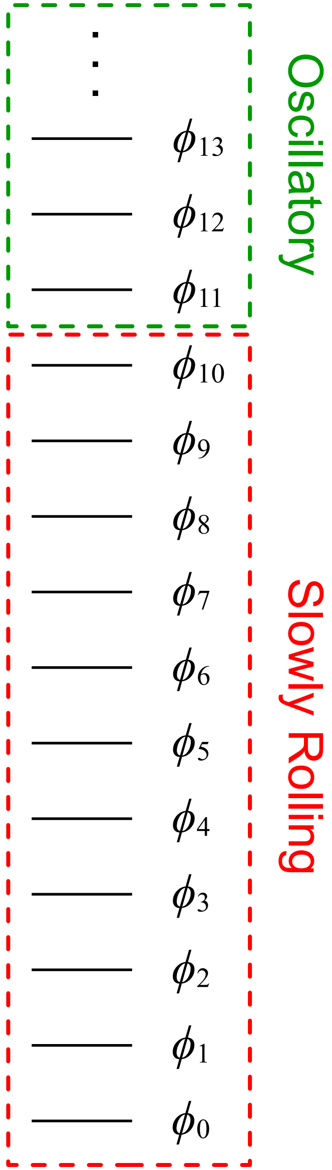

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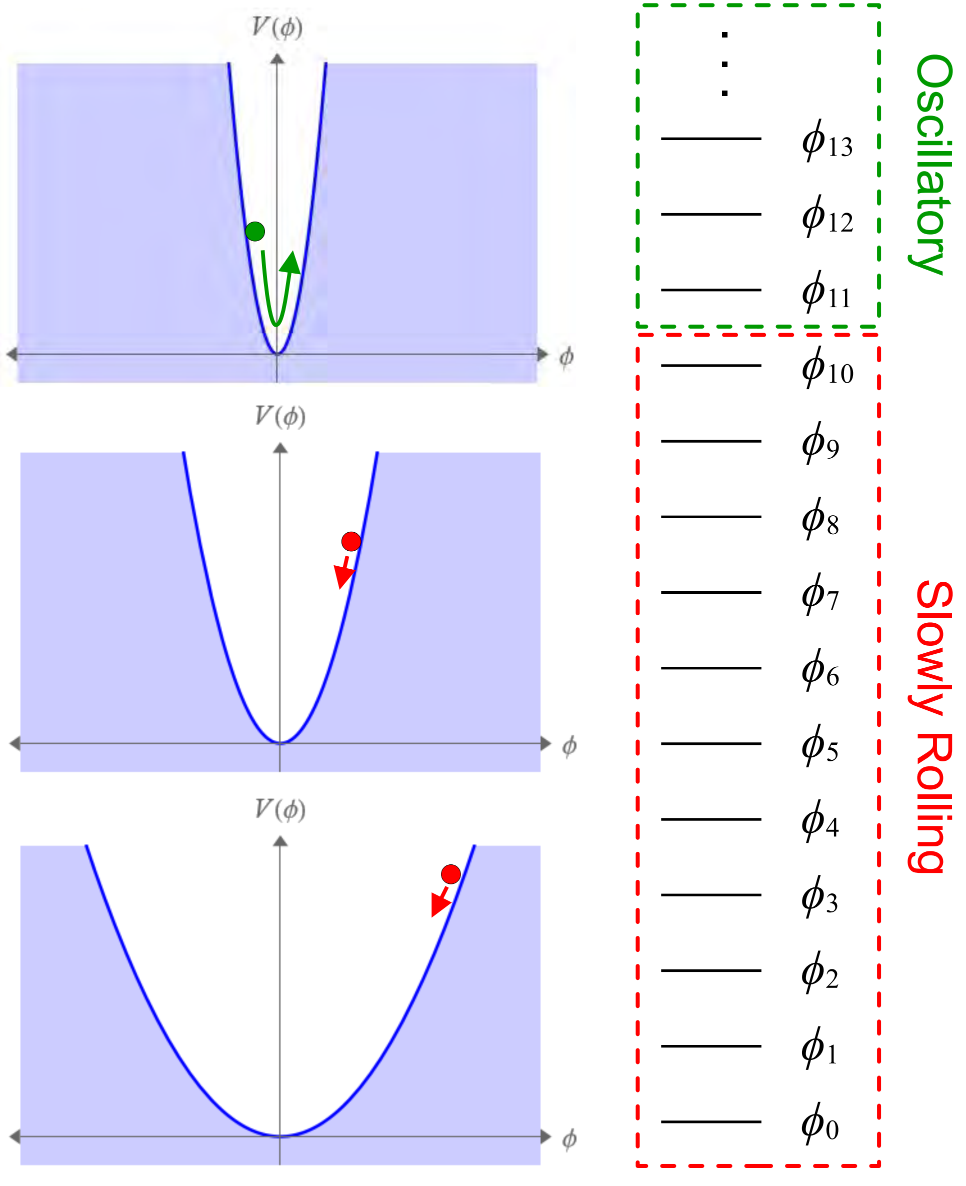

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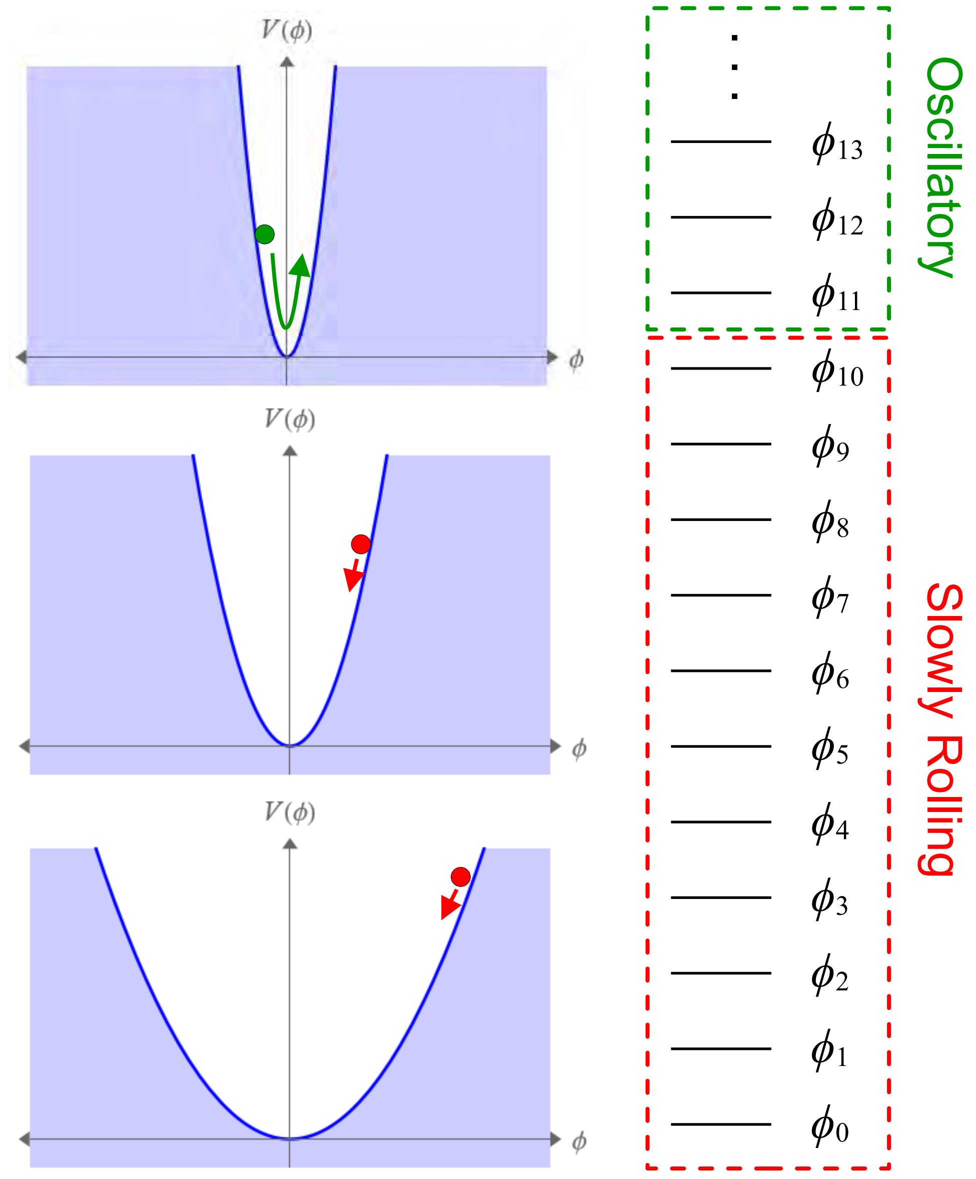

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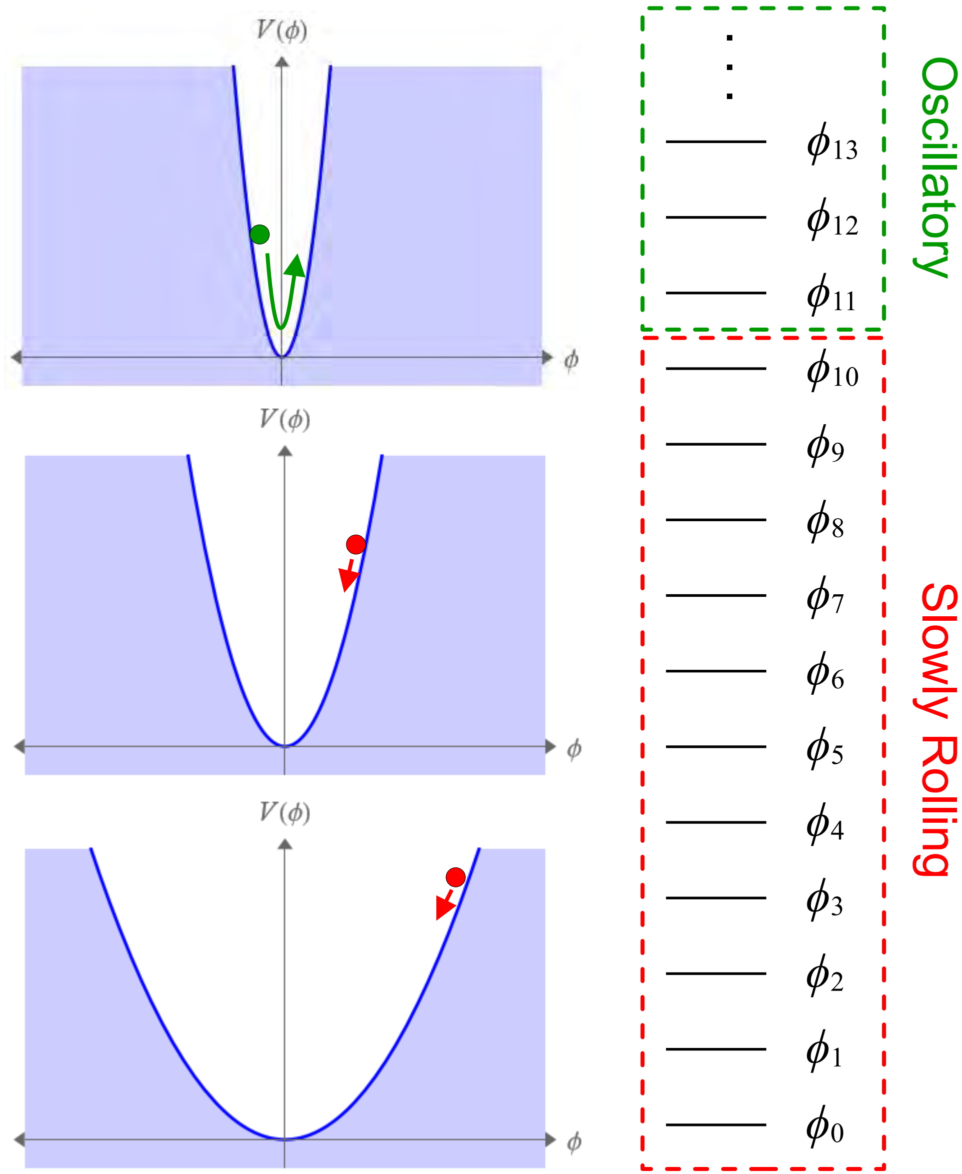

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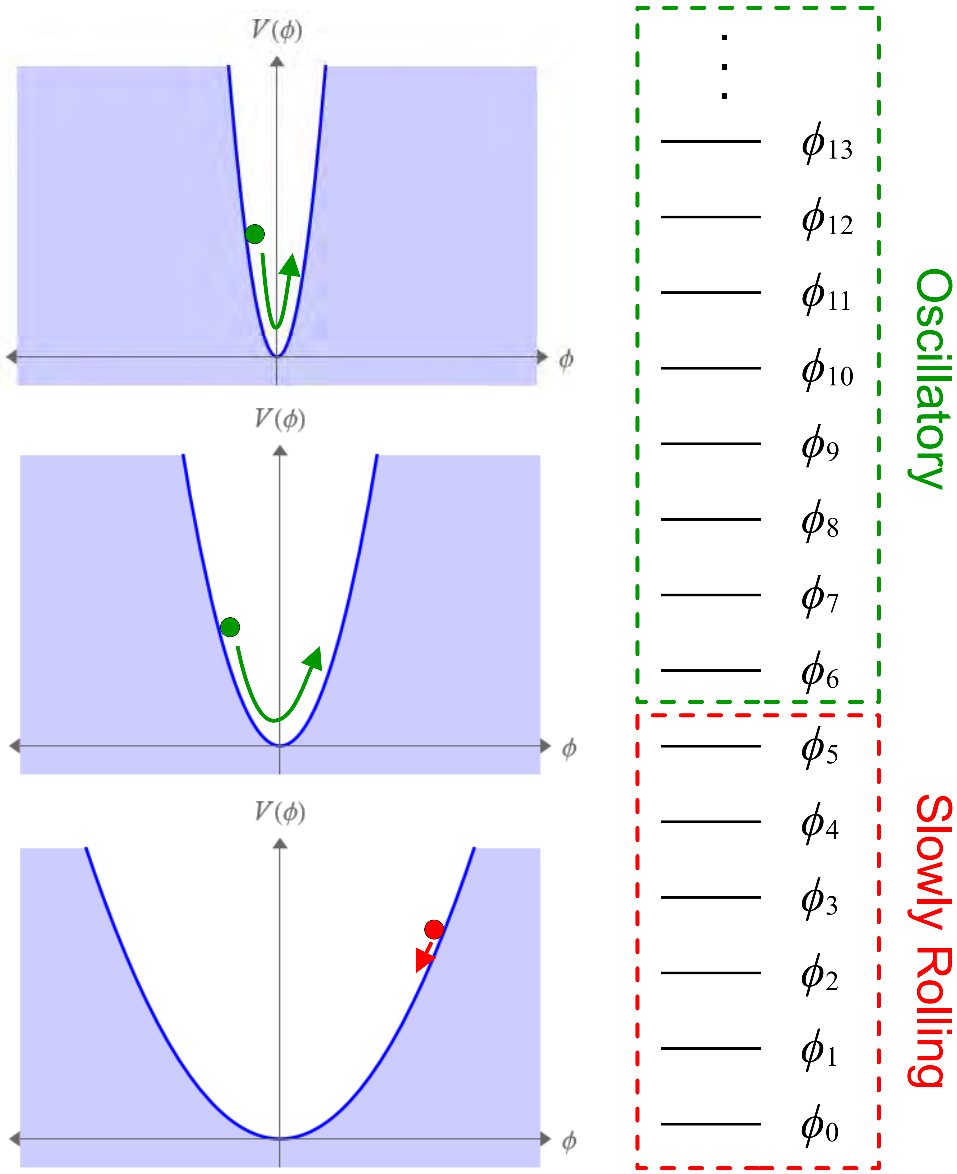

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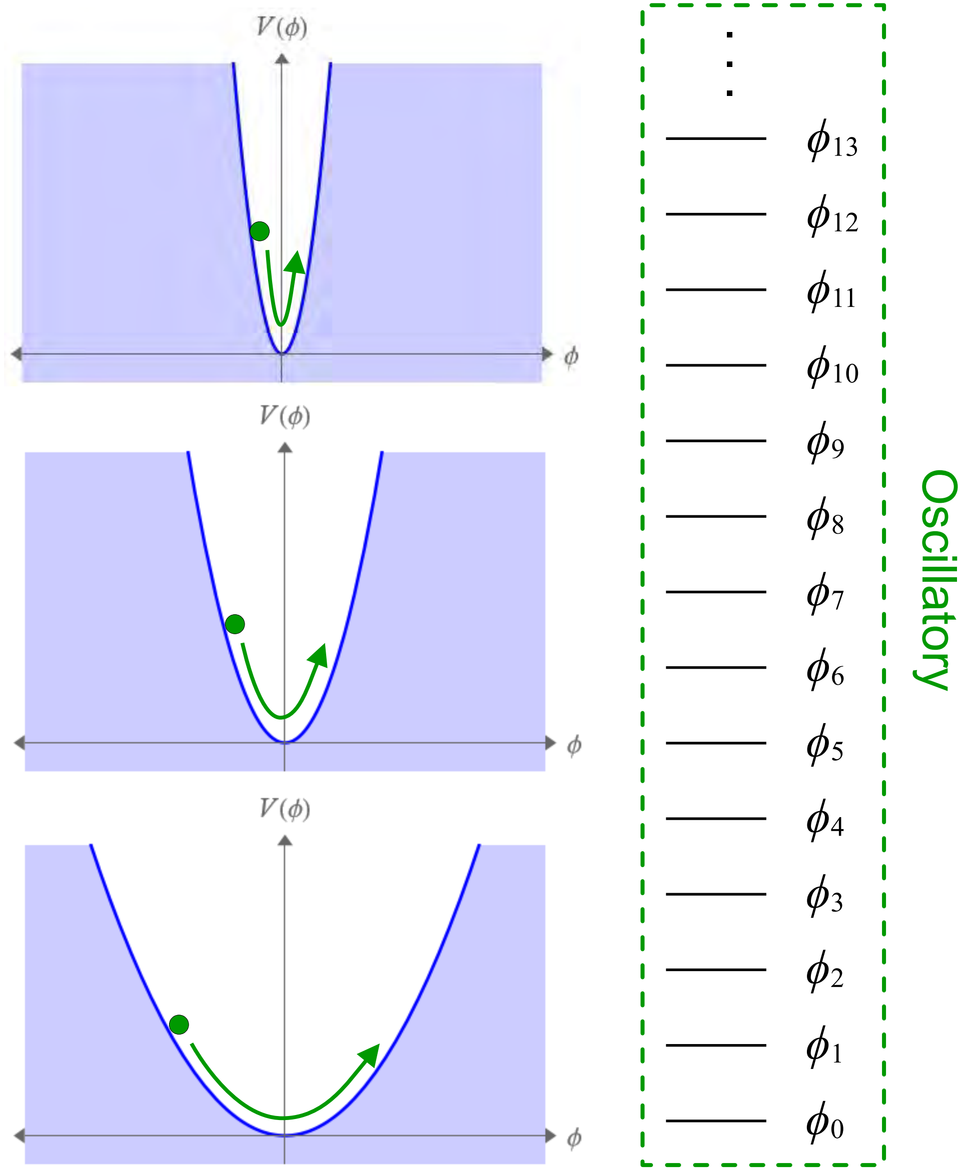

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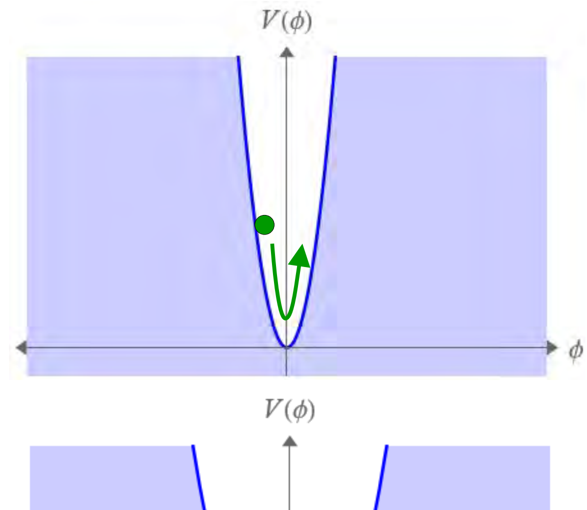

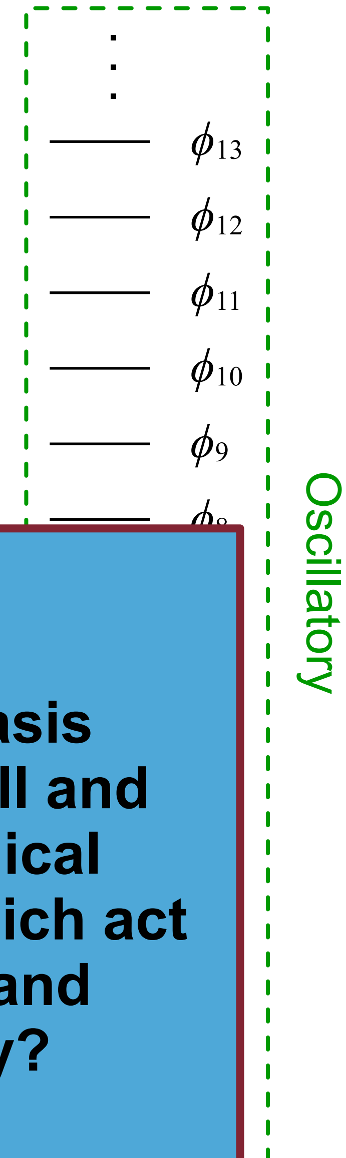

The Question:

Can we achieve a stasis between these slow-roll and oscillatory cosmological energy components, which act like vacuum energy and matter, respectively?

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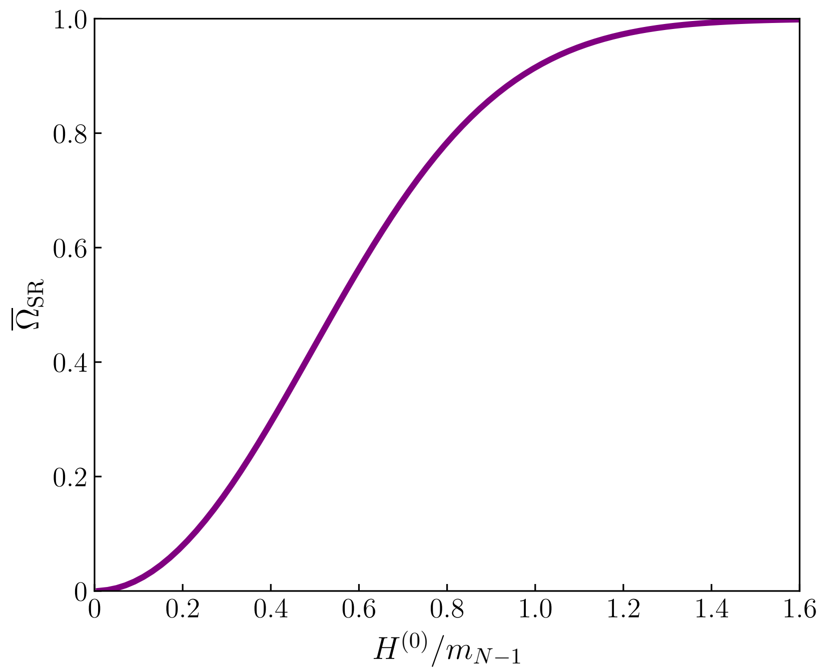
Towers which satisfy this relation give rise to stasis. For $\delta = 1$, this corresponds to $\phi_\ell^{(0)} \sim \ell^{-1/2}$

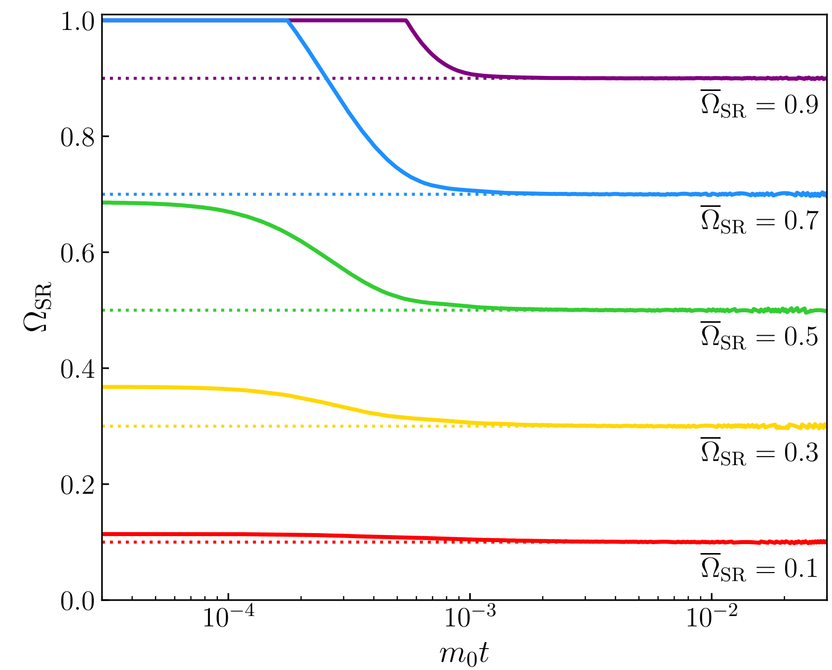
Effect of Initial Conditions

- Unlike in previous realizations of stasis, the stasis abundances $\overline{\Omega}_{SR}$ and $\overline{\Omega}_{SR}$ depend on the **initial conditions** for the scalar tower.
- In particular, $\overline{\Omega}_{SR}$ and $\overline{\Omega}_{SR}$ are sensitive to the ratio $\phi_0^{(0)}/M_P$ which parametrizes the overall scale of the initial zero-mode displacements.

Impact of Initial Conditions



Evolution Toward Stasis



$\alpha = 1$ $\delta = 1$ $N = 5000$

Duration of Stasis

$$\mathcal{N}_s \approx \frac{\overline{\kappa}}{3}\left[\delta \log N + \log\left(\frac{\Delta m}{m_0}\right) + \log\left(\frac{3H^{(0)}}{2m_{N-1}}\right)\right]$$

Background Components and Tracking

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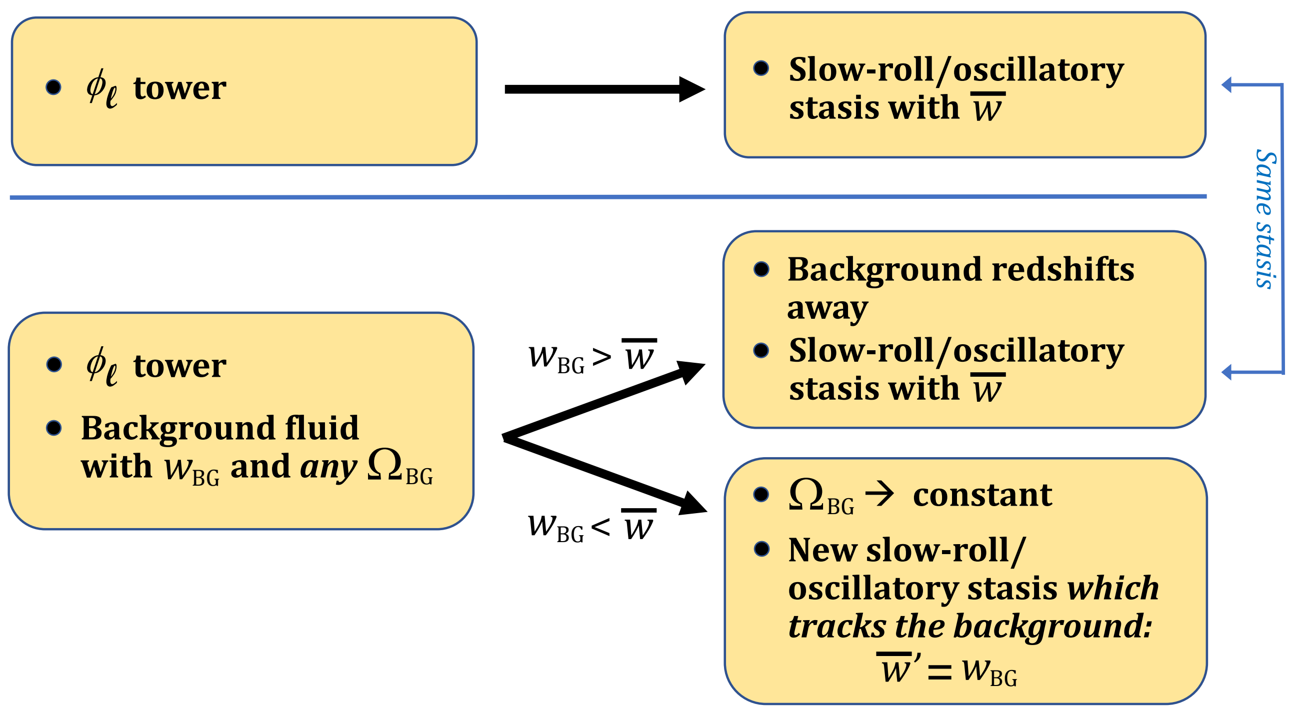
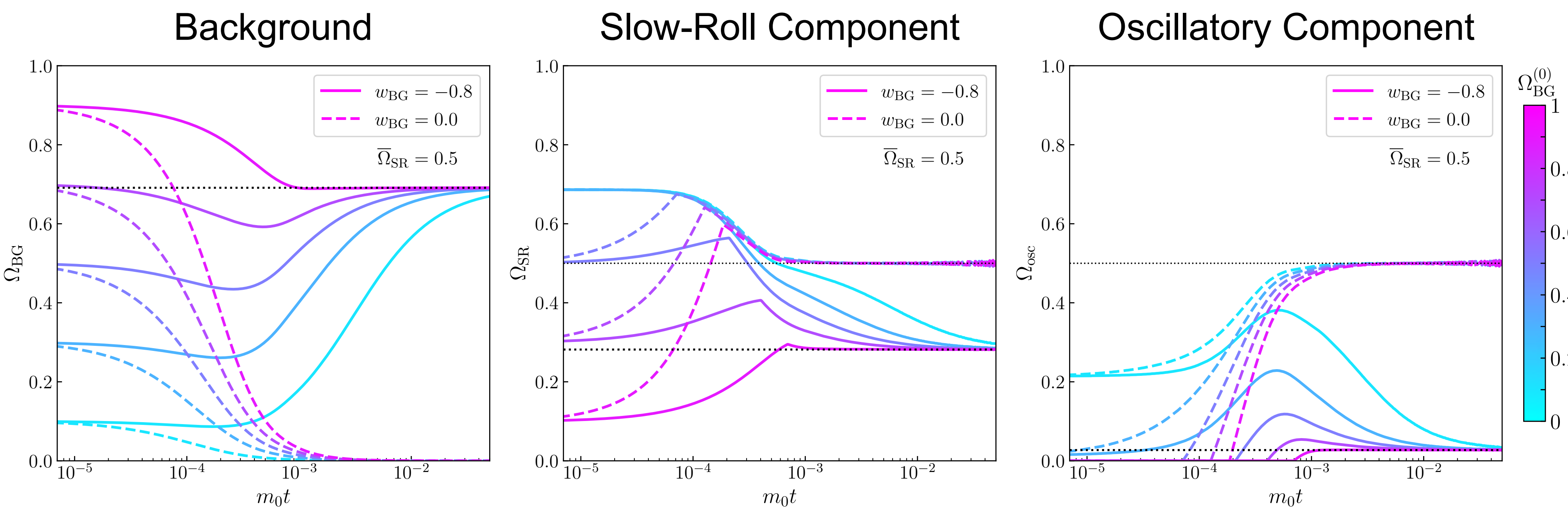
- Let's now consider how the cosmological dynamics is modified if we include a **background energy component** with a constant equation-of-state parameter $w_{\rm BG}$ in addition to the tower.
- It turns out that in the presence of such an energy component, the universe still evolves toward stasis (or something like it).
- However, the outcome depends on the relationship between $w_{\rm BG}$ and the equation-of-state parameter $\overline{w}$ the tower *would* have had during stasis if the background component weren't present.



Background Components and Tracking

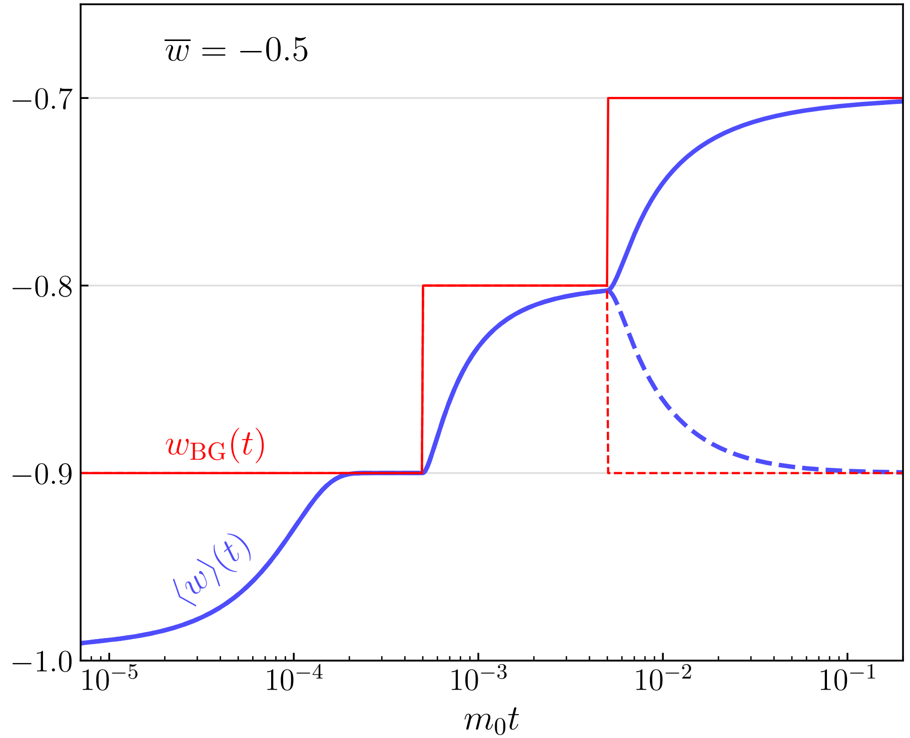
- The **tracking phenomenon** which arises in wBG < w has not been observed in other realizations of stasis.



- These results provide insight about how th universe might enter into – or exit from – an stasis epoch involving dynamical scalars.

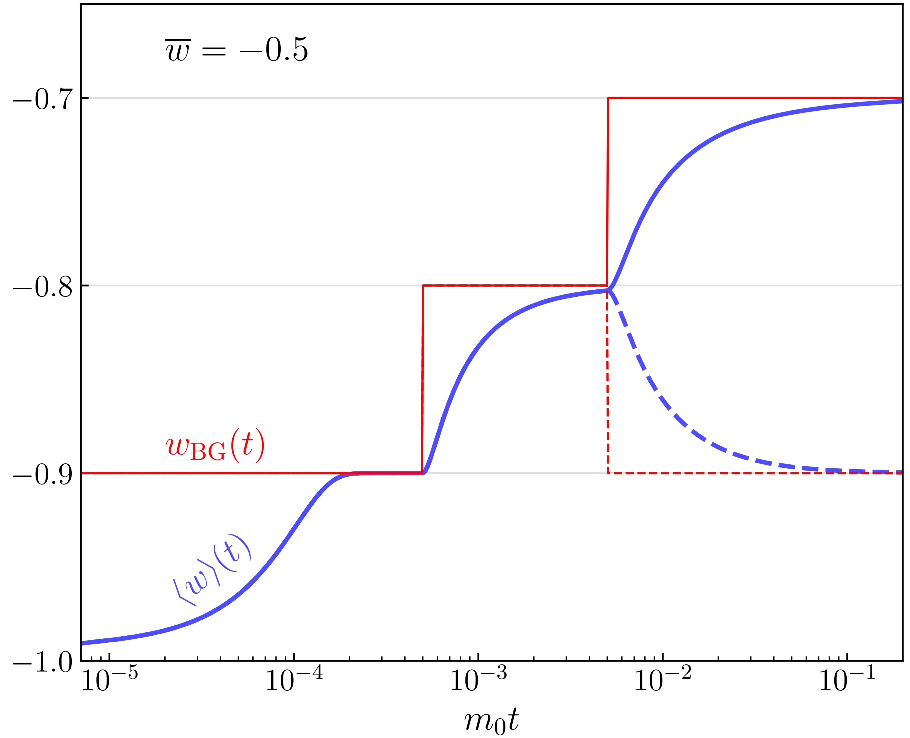
Background Components and Tracking

- It's also noteworthy that this tracking behavior is quite robust and persists even when $w_{\rm BG}$ **experiences an abrupt shift** (as might occur, for example, as the result of a phase transition).



Background Components and Tracking

- It's also noteworthy that this tracking behavior is quite robust and persists even when $w_{\rm BG}$ **experiences an abrupt shift** (as might occur, for example, as the result of a phase transition).
- Indeed, as long as $w_{\rm BG}$ remains below $\overline{w}$, the tower's equation-of-state parameter $\langle w \rangle$ continues to evolve toward the new value of $w_{\rm BG}$ after the shift, regardless of whether this shift is positive or negative.



Summary

- ***Stable, mixed-component cosmological eras*** – *i.e.* ***stasis eras*** – are indeed a viable cosmological possibility – and one that can arise naturally in many extensions of the Standard Model.
- A **tower of scalar fields** which undergo a transition from overdamped to underdamped evolution can give rise to stasis.
- Stasis itself is an **attractor** in these systems, but several fundamental characteristics of the stasis epoch toward which the universe evolves depend on the initial conditions.
- In the presence of an additional background component with equation-of-state parameter w_{BG}, the tower exhibits a **tracking behavior** in which its own equation-of-state parameter evolves toward w_{BG}.