CP violation in tau decay into Kaons

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CP violation in $\tau^{\pm} \to \nu K_S \pi^{\pm}$

- CP violation in tau decay in one kaon was firstly predicted by Bigi and Sanda (*arXiv:hep-ph/0506037*);

- The origin of CP asymmetry is the $K^0 - \bar{K}^0$ mixing;

$$|K_{S,L}\rangle = p|K^0\rangle \pm q|\overline{K}^0\rangle \qquad \tau^+(\tau^-) \to \nu K^0(\bar{K}^0)\pi$$

- K-short in final state is identified by a **two pion system** $m_{\pi\pi} \approx m_K$;

$$A_1 = \frac{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu_\tau}) - \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu_\tau}) + \Gamma(\tau^- \to \pi^- K_S \nu_\tau)} = |p|^2 - |q|^2 \approx 2\mathrm{Re}[\epsilon] \approx 3.3 \cdot 10^{-3}$$

Experimental search

- Bigi and Sanda (*arXiv:hep-ph/0506037*):

$$A_1 = \frac{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu_\tau}) - \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu_\tau}) + \Gamma(\tau^- \to \pi^- K_S \nu_\tau)} = |p|^2 - |q|^2 \approx 3.3 \cdot 10^{-3}$$

- BaBar collaboration (*Phys. Rev. D 85, 099904*):

$$A_1 = (-3.6 \pm 2.3 \pm 1.1) \cdot 10^{-3}$$

- BELLE collaboration (*PhysRevLett.107.131801*): compatible with zero with precision $O(10^{-3})$

Measurement are **3-sigma away** from SM prediction

Experimental search

(*arXiv:1110.3790*)

Theoretical prediction is wrong! Missing KS-KL interference

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$$A_1 = \frac{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu_\tau}) - \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu_\tau}) + \Gamma(\tau^- \to \pi^- K_S \nu_\tau)} = |p|^2 - |q|^2 \approx 3.3 \cdot 10^{-3}$$

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$K_S - K_L$ interference

$$\tau^\pm \to \nu K_S \pi^\pm$$

- The K-short in final state is identified by a **two pion system** $m_{\pi\pi} \approx m_K$

$$A_1(E) = \frac{\int_0^\infty dt\ F(t,E)[\Gamma(t) - \bar{\Gamma}(t)]}{\int_0^\infty dt\ F(t,E)[\Gamma(t) + \bar{\Gamma}(t)]}$$

$$\Gamma(t) = \Gamma(K^0(t) \to 2\pi), \qquad \bar{\Gamma}(t) = \Gamma(\bar{K^0}(t) \to 2\pi)$$

- The interference amplitude $K_S - K_L$ is as important as the pure K-short

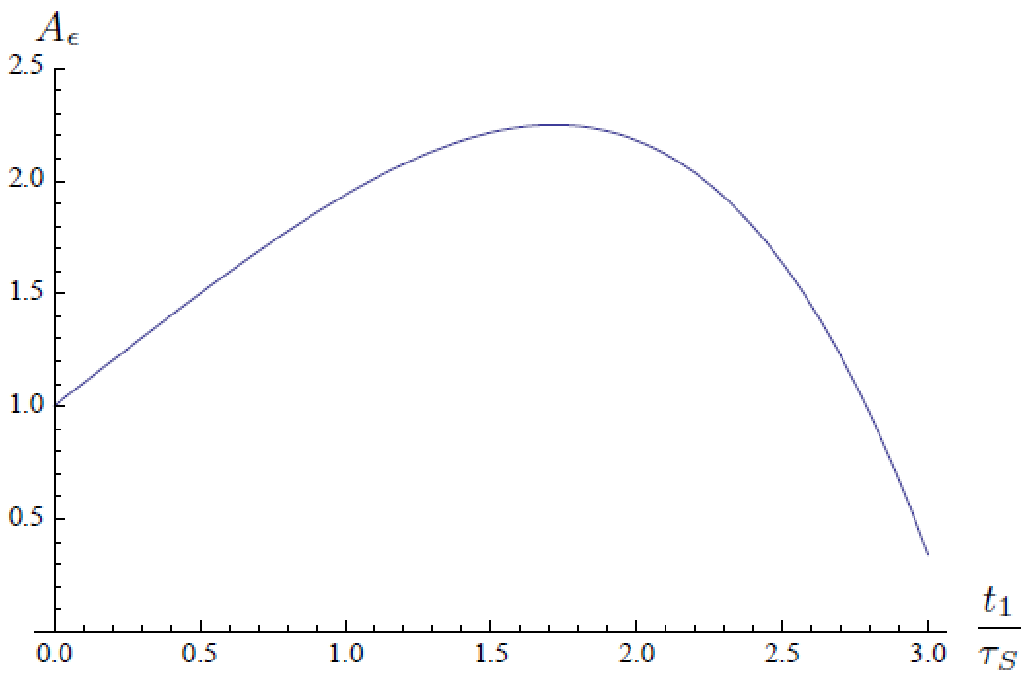
$$\Gamma(t) = \Gamma(K^0(t) \to \pi\pi) = \frac{|A_S|^2}{4|p|^2}(e^{-\Gamma_S t} + |\epsilon|^2 e^{\Gamma_L t} + 2\mathrm{Re}[\epsilon^* e^{i\Delta m - \Gamma t}])$$

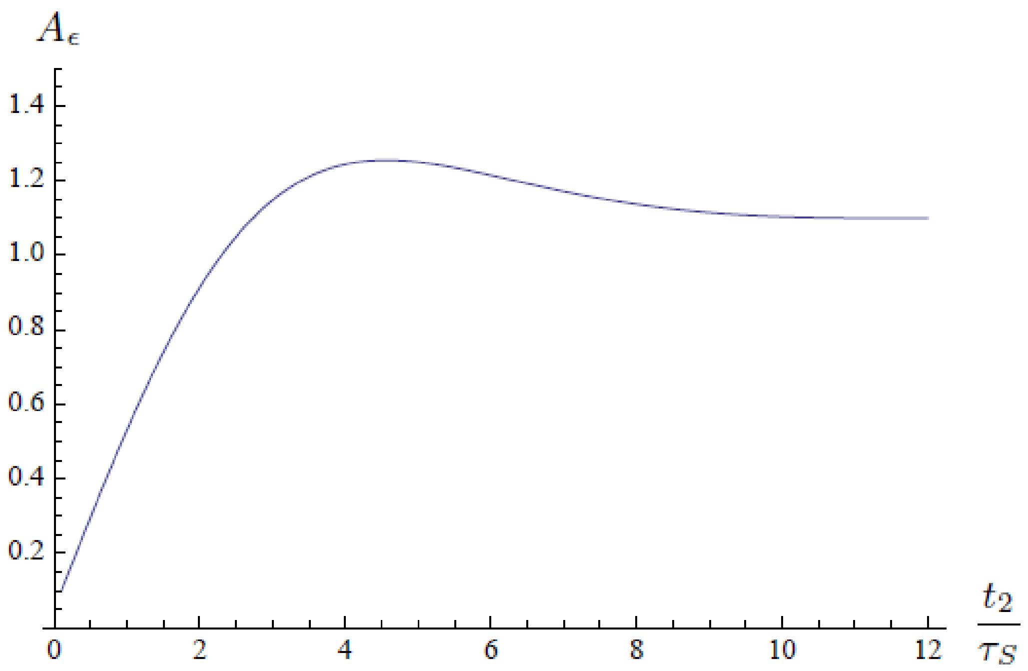
⇨ Asymmetry depends on the **experimental reconstruction efficiency** $F(t, E)$

Fixed experimental cuts in
Kaon's reference frame: $F(t) = \theta(t-t_1)\theta(-t+t_2)$

$$A_\epsilon(t_1,t_2) = \frac{\int_{t_1}^{t_2} dt[\Gamma(K^0(t)\to 2\pi) - \Gamma(\bar{K}^0\to 2\pi)]}{\int_{t_1}^{t_2} dt[\Gamma(K^0(t)\to 2\pi) + \Gamma(\bar{K}^0\to 2\pi)]}$$

$$= -2Re[\epsilon]\left(1 - \frac{\int_{t_1}^{t_2} dt\; e^{-\Gamma t(\cos(\Delta m t)+\frac{Im(\epsilon)}{Re[\epsilon]}\sin(\Delta m t))}}{\int_{t_1}^{t_2}\; e^{-\Gamma t}}\right)$$





$$t_1 \ll \tau_s \ll t_2 \ll \tau_L \quad \Longrightarrow \quad A_\epsilon(t_1,t_2) \approx +2Re[\epsilon] \approx 3.3\times 10^{-3}$$

$$A_1(E) = \frac{\int_0^\infty dt\ F(t,E)[\Gamma(t) - \bar{\Gamma}(t)]}{\int_0^\infty dt\ F(t,E)[\Gamma(t) + \bar{\Gamma}(t)]}$$

Relevance for BELLE II

- The **efficiency function** $F(t, E)$ must be determined as part of the experimental analysis and is part of the theoretical prediction;

- The dependence on the **decay time** and t and **energy** makes the CP asymmetry for other decay modes non trivial:

$\tau^- \to \pi^- K_S^0 (\geq 0\pi^0)\nu_\tau$,	$\tau^- \to K^- K_S^0 (\geq 0\pi^0)\nu_\tau$,	$\tau^- \to \pi^- K^0 \bar{K}^0 (K_S^0 K_L^0)\nu_\tau$
A_1	A_2	A_3

$$\mathcal{A} = \frac{f_1 A_1 + f_2 A_2 + f_3 A_3}{f_1 + f_2 + f_3} = \frac{f_1 - f_2}{f_1 + f_2 + f_3} A_1$$

$$A_2 = -A_1$$
$$A_3 = 0$$

$$A_1(E) = \frac{\int_0^\infty dt\ F(t,E)[\Gamma(t) - \bar{\Gamma}(t)]}{\int_0^\infty dt\ F(t,E)[\Gamma(t) + \bar{\Gamma}(t)]}$$

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A_1	A_2	A_3

$$\mathcal{A} = \frac{f_1 A_1 + f_2 A_2 + f_3 A_3}{f_1 + f_2 + f_3} = \frac{f_1 - f_2}{f_1 + f_2 + f_3} A_1$$

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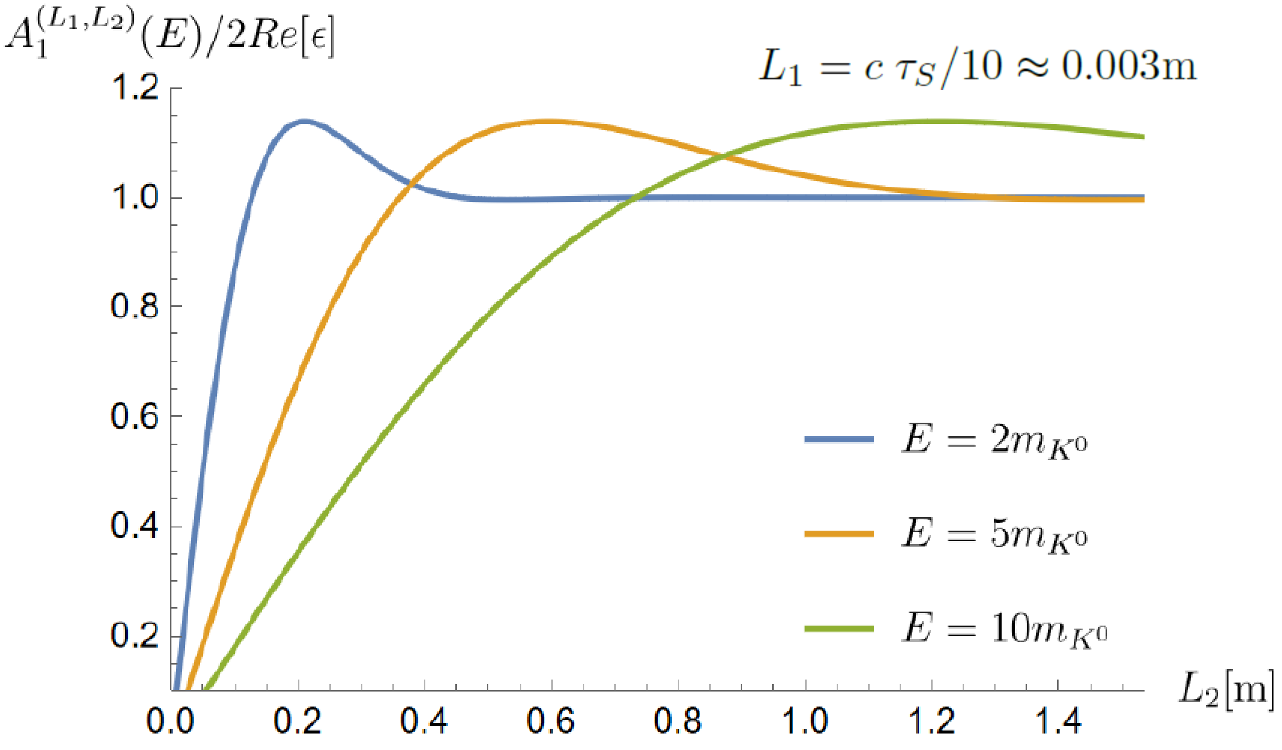
Experimental efficiency

Realistic example: fixed length cuts in LAB frame:

$$t_{1,2}^{LAB} = \frac{L_{1,2}}{\beta c} \quad \rightarrow \quad F(t,E) = \theta(t - \frac{L_1}{\beta\gamma c})\theta(-t + \frac{L_2}{\beta\gamma c})$$

The experimental efficiency depends of the **decay tim**e t and the **energy of the kaon**.

$$A_1 = \int f(E) A_1(E) dE$$



CP asymmetry in tau decay in 2 kaons

$$A_3 = \frac{\Gamma(\tau^+ \to \bar{\nu_\tau} K_L K_S \pi^+) - \Gamma(\tau^- \to \nu_\tau K_L K_S \pi^-)}{\Gamma(\tau^+ \to \bar{\nu_\tau} K_L K_S \pi^+) + \Gamma(\tau^- \to \nu_\tau K_L K_S \pi^-)}$$

- We are looking at the process $\tau \to \pi K^0 \bar{K}^0 (K_S K_L) \nu$ and would expect A3=0 because the asymmetry of K and K-bar cancel each other;

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- We are looking at the process $\tau \to \pi K^0 \bar{K}^0 (K_S K_L) \nu$ and would expect A3=0 because the asymmetry of K and K-bar cancel each other;

- However, the **energy dependence of the efficiency function** makes the asymmetry non vanishing when K^0 and $\bar{K}^0$ are produces with different energies:

$$A_3 = \frac{1}{2}\left[\int f_{K^0}(E)dE \; A_1(E) - \int f_{\bar{K}^0}(E)dE \; A_1(E)\right]$$

CP violation in $\tau \to \pi K^0 \bar{K}^0 (K_S K_L) \nu$

$$A_3 = \frac{1}{2}\left[\int f_{K^0}(E) dE\ A_1(E) - \int f_{\bar{K}^0}(E) dE\ A_1(E)\right]$$

$$A_1(E) = \frac{\int_0^\infty dt\ F(t,E)[\Gamma(t) - \bar{\Gamma}(t)]}{\int_0^\infty dt\ F(t,E)[\Gamma(t) + \bar{\Gamma}(t)]}$$

- The A3 asymmetry is only a consequence of the **different energy (different time dilatation)** of K and Kbar;
- The asymmetry is a consequence of the energy dependence of F=F(t,E): **finite experimental efficiency**. We would have A3=0 in a perfect experiment;
- Need of **Kaons spectrum** for theoretical predictions;

Relevance for BELLE II

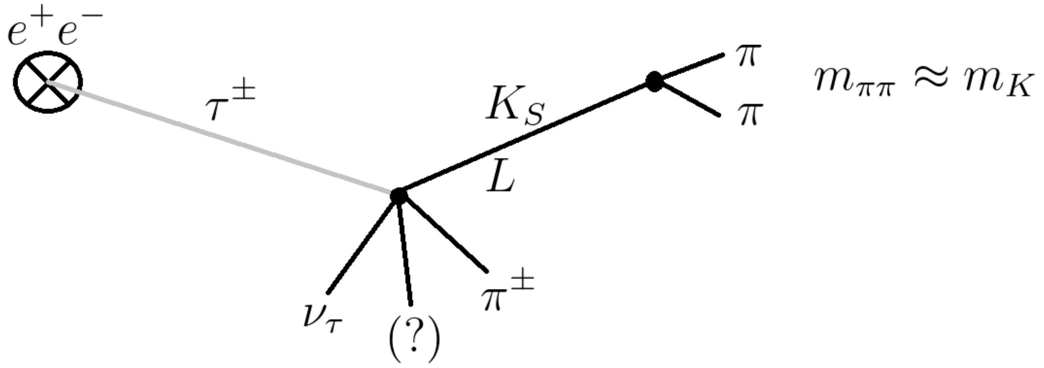
- We need the reconstruction efficiency of the experiment parametrized by **kaon decay time** and **energy** in the LAB frame

$$A_3 = \frac{1}{2}\sum_{i=1}^{10}\left(\int_{\Delta E} f_{K^0}(E)A_\epsilon(E_i)dE \; - \int_{\Delta E} f_{\bar{K}^0}(E)A_\epsilon(E_i)dE\right)$$

$$F(t, E_i) \quad \text{for} \quad i = 1, \ldots 10$$

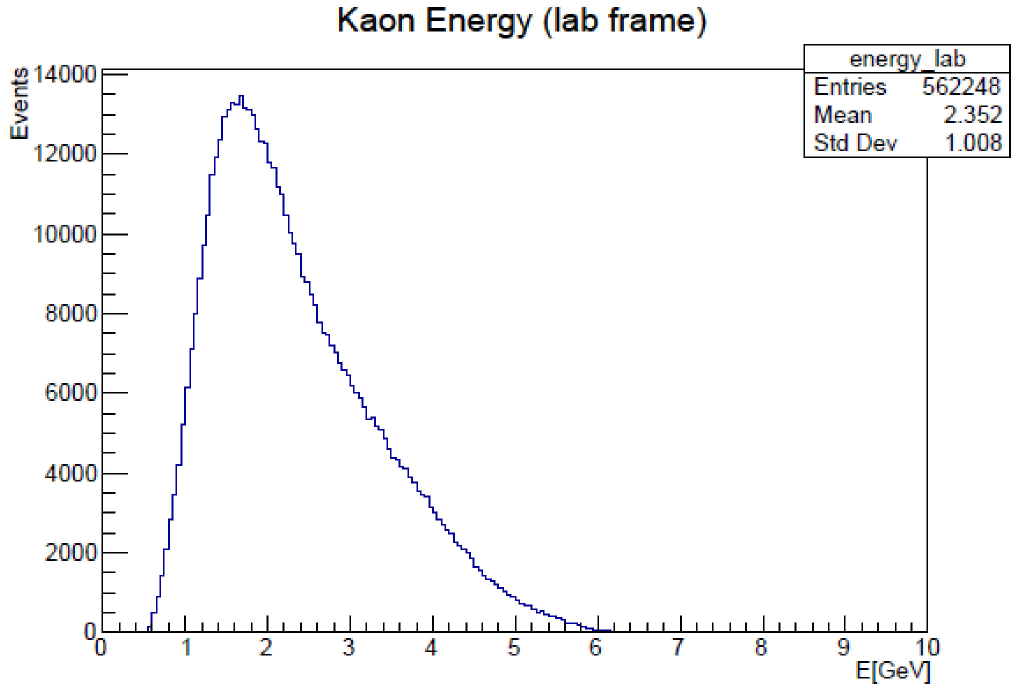
For each kaon event we measure energy E and decay length:

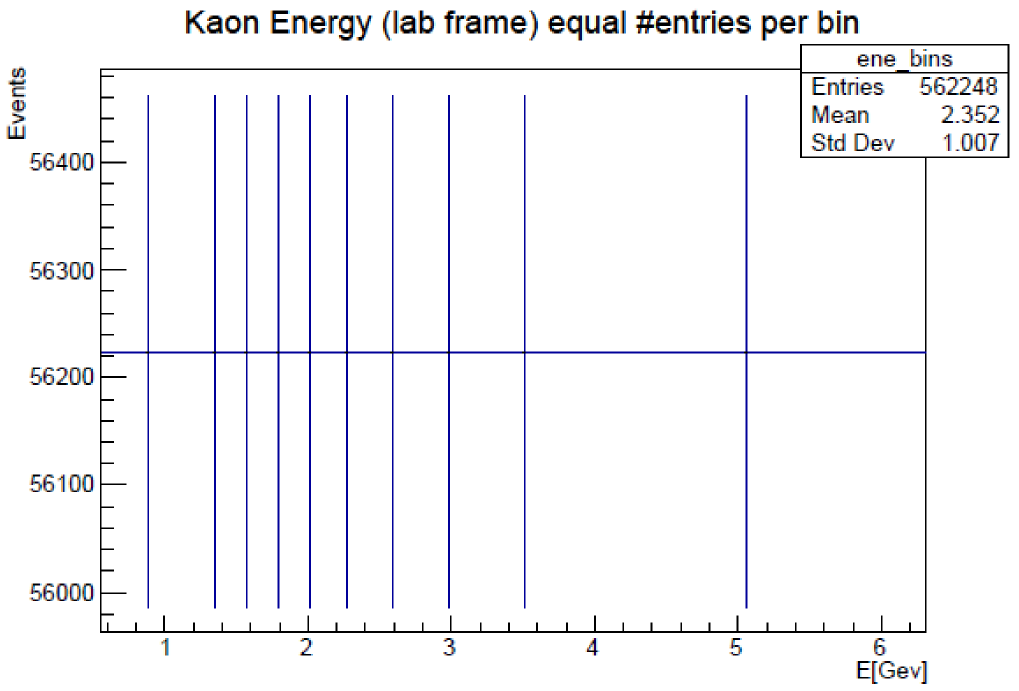
$$t = \frac{L}{c\beta\gamma}$$



$$A_3 = \frac{1}{2}\sum_{i=1}^{10}\left(\int_{\Delta E} f_{K^0}(E)A_\epsilon(E_i)dE \; - \; \int_{\Delta E} f_{\bar{K}^0}(E)A_\epsilon(E_i)dE\right) \qquad F(t, E_i) \quad \text{for} \;\; i = 1, ...10$$

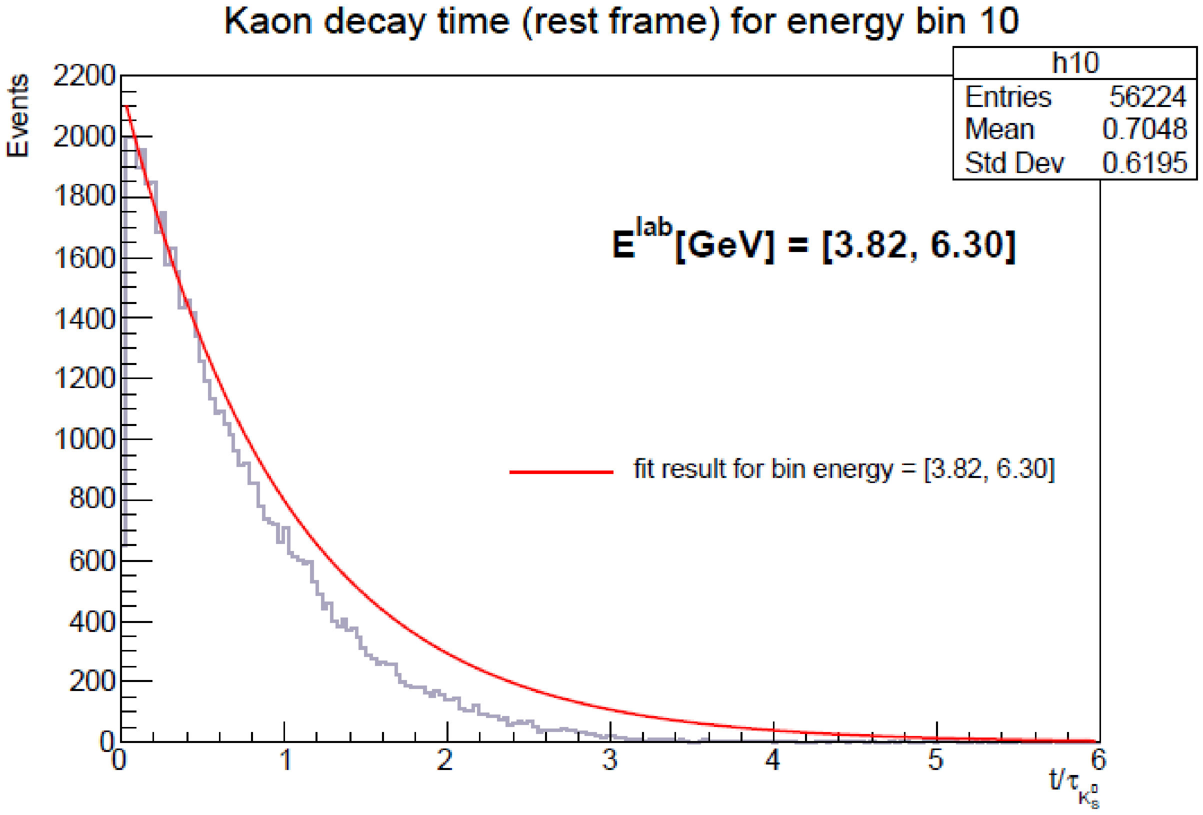
We divide the energy distribution of the kaon in 10 bins with the same number of entries.

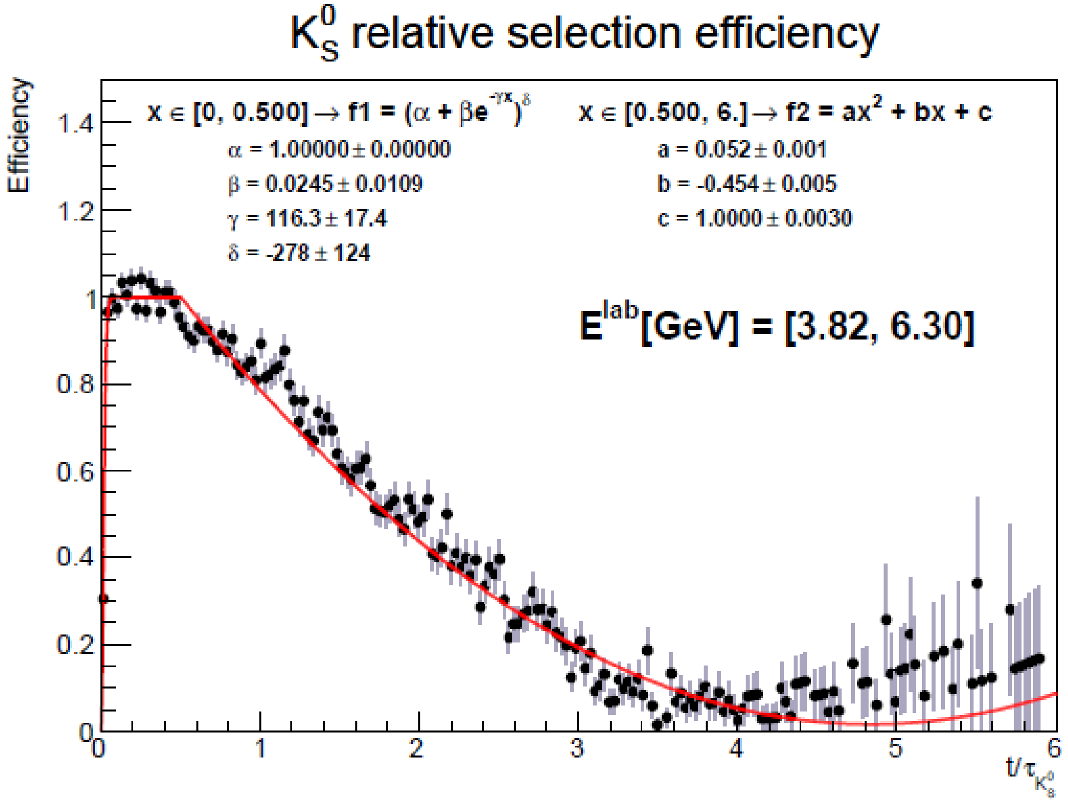


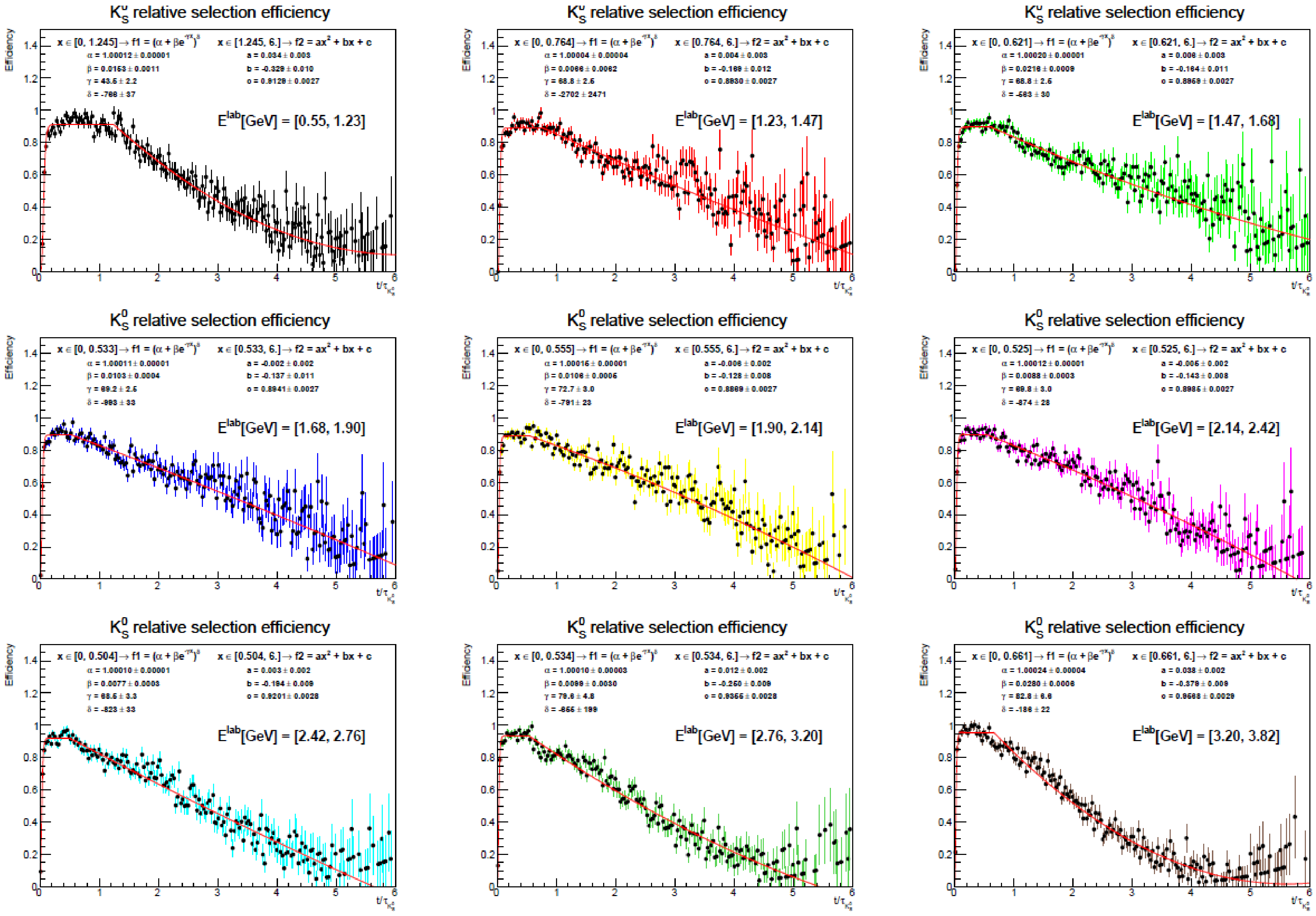


$$f(t) = N_0 e^{-t/\tau_{K_S^0}}$$

The **relative selection efficiency** is the ratio between the decay time distribution and the reference one.





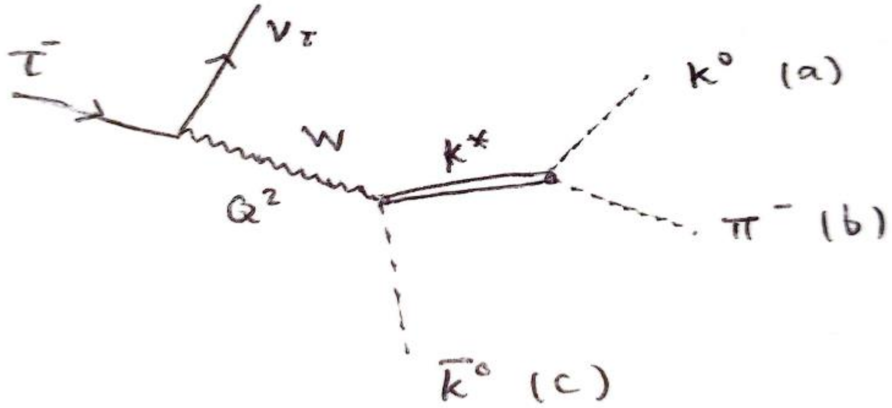


K_S^0 relative selection efficiency
$x \in [0, 1.245] \rightarrow f1 = (\alpha + \beta e^{-\gamma x})^{\delta}$
$x \in [1.245, 6.] \rightarrow f2 = ax^2 + bx + c$
E^{lab}[GeV] = [0.55, 1.23]
K_S^0 relative selection efficiency
$x \in [0, 0.764] \rightarrow f1 = (\alpha + \beta e^{-\gamma x})^{\delta}$
$x \in [0.764, 6.] \rightarrow f2 = ax^2 + bx + c$
E^{lab}[GeV] = [1.23, 1.47]
K_S^0 relative selection efficiency
$x \in [0, 0.621] \rightarrow f1 = (\alpha + \beta e^{-\gamma x})^{\delta}$
$x \in [0.621, 6.] \rightarrow f2 = ax^2 + bx + c$
E^{lab}[GeV] = [1.47, 1.68]
K_S^0 relative selection efficiency
$x \in [0, 0.533] \rightarrow f1 = (\alpha + \beta e^{-\gamma x})^{\delta}$
$x \in [0.533, 6.] \rightarrow f2 = ax^2 + bx + c$
E^{lab}[GeV] = [1.68, 1.90]
K_S^0 relative selection efficiency
$x \in [0, 0.555] \rightarrow f1 = (\alpha + \beta e^{-\gamma x})^{\delta}$
$x \in [0.555, 6.] \rightarrow f2 = ax^2 + bx + c$
E^{lab}[GeV] = [1.90, 2.14]
K_S^0 relative selection efficiency
$x \in [0, 0.525] \rightarrow f1 = (\alpha + \beta e^{-\gamma x})^{\delta}$
$x \in [0.525, 6.] \rightarrow f2 = ax^2 + bx + c$
E^{lab}[GeV] = [2.14, 2.42]
K_S^0 relative selection efficiency
$x \in [0, 0.504] \rightarrow f1 = (\alpha + \beta e^{-\gamma x})^{\delta}$
$x \in [0.504, 6.] \rightarrow f2 = ax^2 + bx + c$
E^{lab}[GeV] = [2.42, 2.76]
K_S^0 relative selection efficiency
$x \in [0, 0.534] \rightarrow f1 = (\alpha + \beta e^{-\gamma x})^{\delta}$
$x \in [0.534, 6.] \rightarrow f2 = ax^2 + bx + c$
E^{lab}[GeV] = [2.76, 3.20]
K_S^0 relative selection efficiency
$x \in [0, 0.661] \rightarrow f1 = (\alpha + \beta e^{-\gamma x})^{\delta}$
$x \in [0.661, 6.] \rightarrow f2 = ax^2 + bx + c$
E^{lab}[GeV] = [3.20, 3.82]
Efficiency
$t/\tau_{K_S^0}$

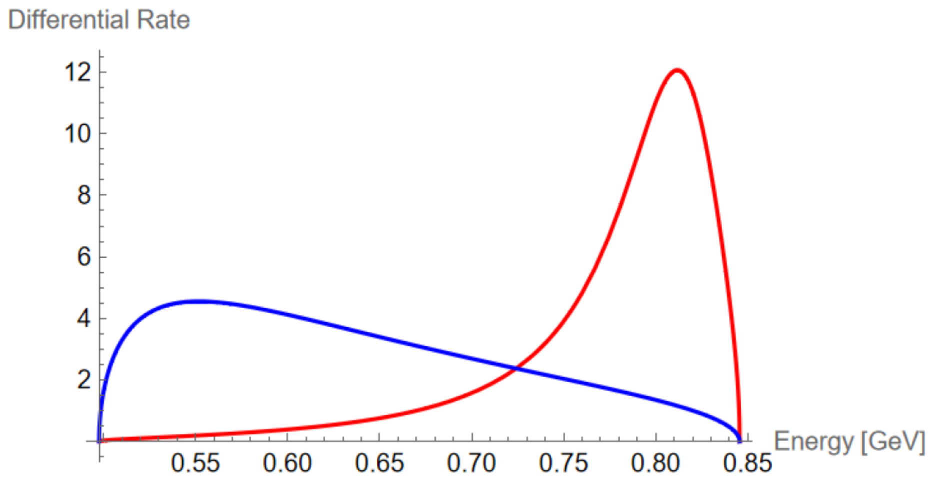
Predictions for BELLE II

$$\sqrt{s}|_{e^+e^-} = 10.58 \text{ GeV}$$

- Need of energy distribution for K and Kbar, simplified toy model with K* resonance:



$$A_3 = \frac{1}{2}\left[\int f_{K^0}(E)dE\ A_1(E) - \int f_{\bar{K}^0}(E)dE\ A_1(E)\right]$$



- This sets an upper bound on A3:

$$A_3 \approx 9.2 \cdot 10^{-5} \approx 0.03 \cdot 2Re[\epsilon]$$

Conclusions

- CP asymmetry has been studied in $\tau^{\pm} \rightarrow \nu K_S \pi^{\pm}$ with measurement that at 3 sigma are not consistent with the SM;
- BELLE II aims to study CP violation in tau decay into kaons: we need theoretical predictions for all the decay modes involved:

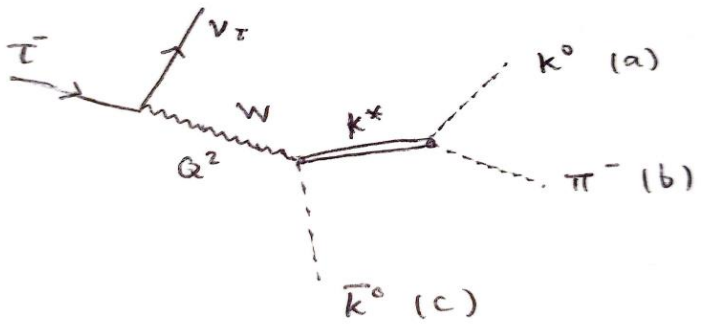
$$\tau^- \rightarrow \pi^- K_S^0 (\geq 0\pi^0)\nu_\tau , \qquad \tau^- \rightarrow K^- K_S^0 (\geq 0\pi^0)\nu_\tau , \qquad \tau^- \rightarrow \pi^- K^0 \bar{K}^0 (K_S^0 K_L^0)\nu_\tau$$

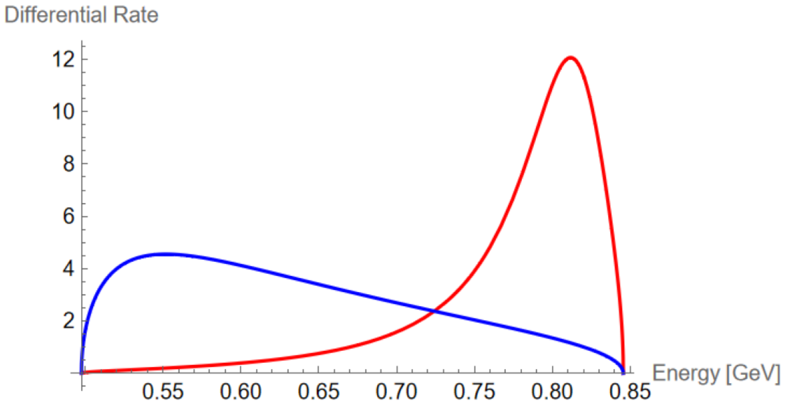
$$A_1 \qquad\qquad\qquad A_2 \qquad\qquad\qquad A_3$$

- The asymmetry depends on the **reconstruction efficiency F(t,E)** parametrized by the decay time and the energy of the kaon;
- The energy dependence makes the **asymmetry A3 non vanishing**;

Backup Slides

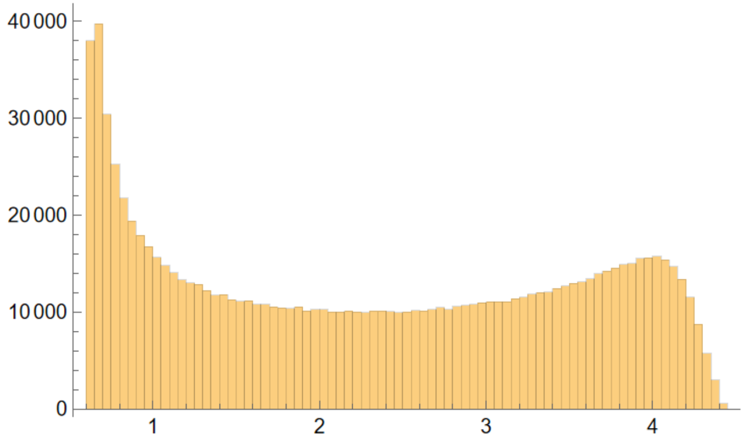
Toy model for Kaon and anti-Kaon energy spectrum to set an upper bound for the A3 asymmetry.

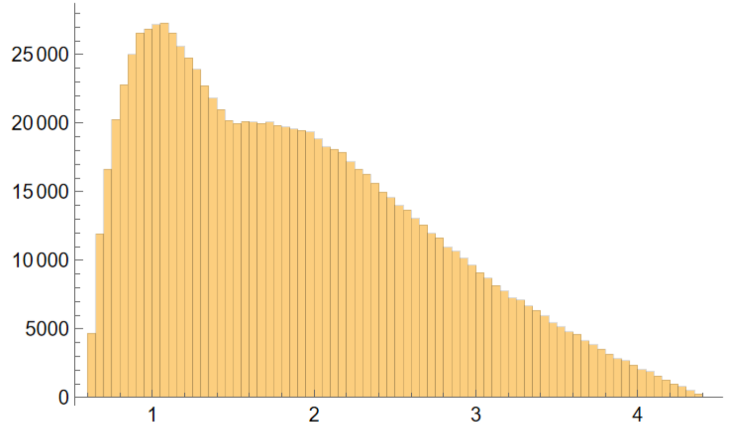




Boost in LAB frame

$$\sqrt{s}|_{e^+e^-} = 10.58 \text{ GeV}$$





Extreme case: Kaon at max energy and anti-Kaon at min energy

$$A_3 \approx 0.19(2Re[\epsilon])$$

Toy model case: $A_3 \approx 9.2 \cdot 10^{-5} \approx 0.03 \cdot 2Re[\epsilon]$

CP-violation asymmetry has been observed in the decay of the τ into a Kaon:

$$\tau \to \nu K_S \pi$$

$$A_\epsilon = \frac{\int_0^\infty F(t)[\Gamma(\tau^+ \to \bar{\nu_\tau} K_S \pi^+) - \Gamma(\tau^- \to \nu_\tau K_S \pi^-)]dt}{\int_0^\infty F(t)[\Gamma(\tau^+ \to \bar{\nu_\tau} K_S \pi^+) + \Gamma(\tau^- \to \nu_\tau K_S \pi^-)]dt}$$
$$= \frac{\int_0^\infty dt\, F(t)[\Gamma(K^0(t) \to 2\pi) - \Gamma(\bar{K^0}(t) \to 2\pi)]}{\int_0^\infty dt\, F(t)[\Gamma(K^0(t) \to 2\pi) + \Gamma(\bar{K^0}(t) \to 2\pi)]}$$

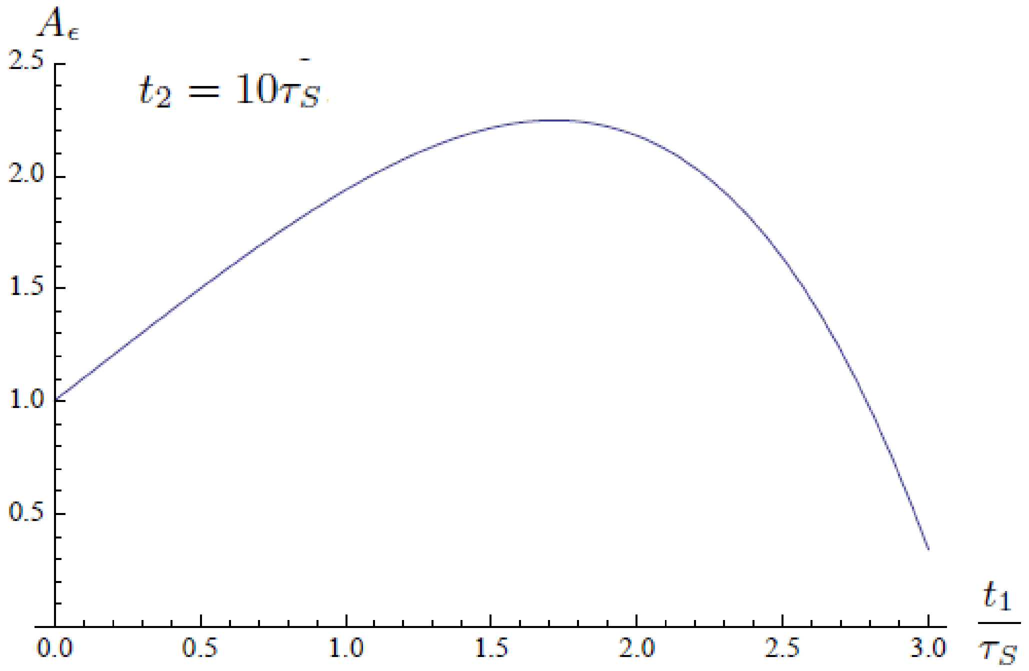
$t \to$ Kaon proper time

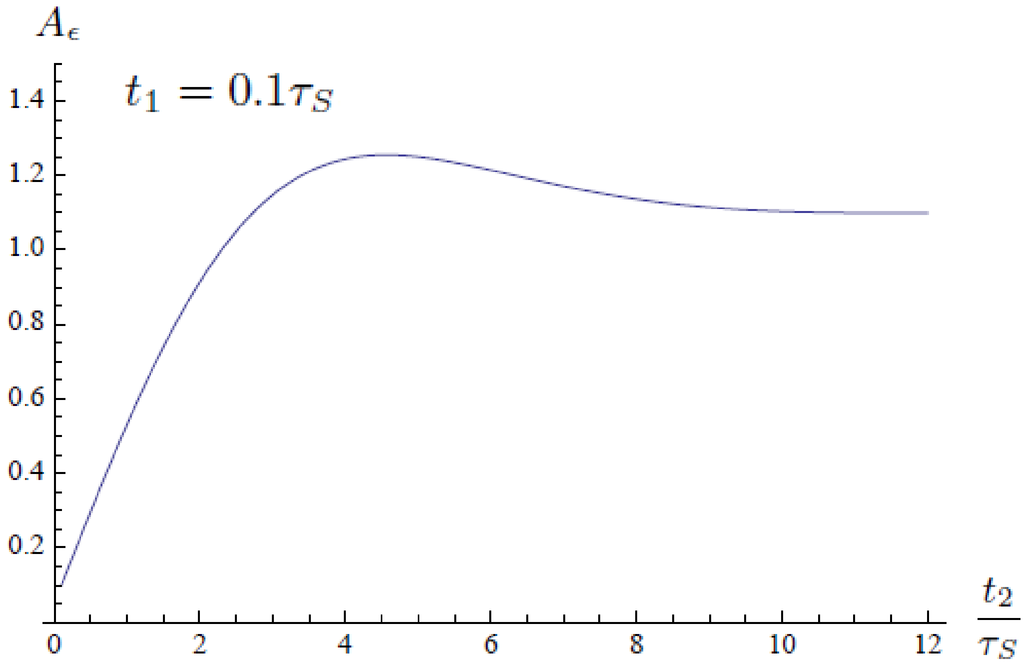
Fixed experimental cuts in the the Kaon's frame: $F(t) = \theta(t - t_1)\theta(-t + t_2)$

$$t_1 \ll \tau_s \ll t_2 \ll \tau_L \implies A_\epsilon(t_1, t_2) \approx +2Re[\epsilon] \approx 3.3 \times 10^{-3}$$

$$A_\epsilon(t_1,t_2) = \frac{\int_{t_1}^{t_2} dt[\Gamma(K^0(t)\to 2\pi) - \Gamma(\bar{K^0}\to 2\pi)]}{\int_{t_1}^{t_2} dt[\Gamma(K^0(t)\to 2\pi) + \Gamma(\bar{K^0}\to 2\pi)]}$$

$$= -2Re[\epsilon]\left(1 - \frac{\int_{t_1}^{t_2} dt\ e^{-\Gamma t(\cos(\Delta m t) + \frac{Im(\epsilon)}{Re[\epsilon]}\sin(\Delta m t))}}{\int_{t_1}^{t_2}\ e^{-\Gamma t}}\right)$$





$$t_1 \ll \tau_s \ll t_2 \ll \tau_L \quad \Longrightarrow \quad A_\epsilon(t_1,t_2) \approx +2Re[\epsilon] \approx 3.3\times 10^{-3}$$

We are interested in determining the asymmetry coefficient related to the time integrated asymmetry of the decay of the τ into three pseudoscalar mesons:

$$A_3(t_1,t_2) = \frac{\int_{t_1}^{t_2}[\Gamma(\tau^+ \to \bar{\nu_\tau} K_L K_S \pi^+) - \Gamma(\tau^- \to \nu_\tau K_L K_S \pi^-)]dt}{\int_{t_1}^{t_2}[\Gamma(\tau^+ \to \bar{\nu_\tau} K_L K_S \pi^+) + \Gamma(\tau^- \to \nu_\tau K_L K_S \pi^-)]dt}$$

$$\begin{aligned}
A_3(t_1,t_2) &= \frac{\int_{t_1}^{t_2} dt[\Gamma(t)_{\tau^+} + \bar{\Gamma}(t)_{\tau^+}] - \int_{t_1}^{t_2} dt[\Gamma(t)_{\tau^-} + \bar{\Gamma}(t)_{\tau^-}]}{\int_{t_1}^{t_2} dt[\Gamma(t)_{\tau^+} + \bar{\Gamma}(t)_{\tau^+}] + \int_{t_1}^{t_2} dt[\Gamma(t)_{\tau^-} + \bar{\Gamma}(t)_{\tau^-}]} \\
&= \frac{\int_{t_1}^{t_2} dt[\Gamma(t)_{\tau^+} - \bar{\Gamma}(t)_{\tau^-}] - \int_{t_1}^{t_2} dt[\Gamma(t)_{\tau^-} - \bar{\Gamma}(t)_{\tau^+}]}{\int_{t_1}^{t_2} dt[\Gamma(t)_{\tau^+} + \bar{\Gamma}(t)_{\tau^+}] + \int_{t_1}^{t_2} dt[\Gamma(t)_{\tau^-} + \bar{\Gamma}(t)_{\tau^-}]} \\
&\approx \frac{A_\epsilon(t_1,t_2)^{K^0(\tau^+)} - A_\epsilon(t_1,t_2)^{K^0(\tau^-)}}{2}
\end{aligned}$$

$$\Gamma(t) = \Gamma(K^0 \to 2\pi)$$

$$\bar{\Gamma}(t) = \Gamma(\bar{K^0} \to 2\pi)$$

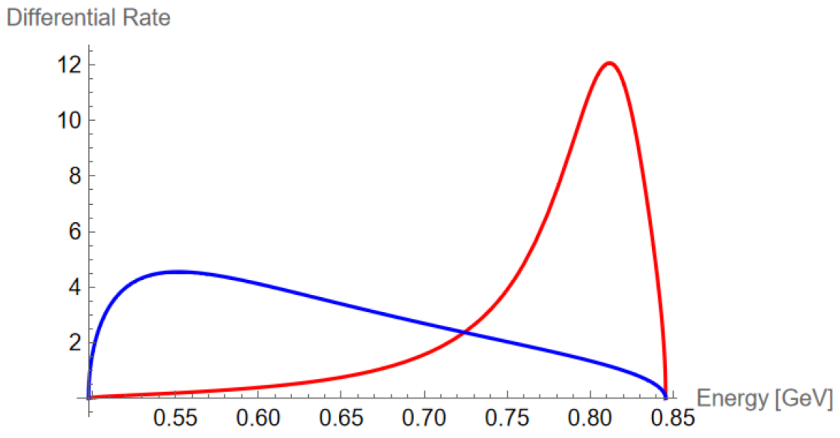
It does not make sense to take fixed experimental cuts in the Kaon frame: we **fix time cuts in LAB frame**

$$A_3(t_1^{LAB}, t_2^{LAB}) = \frac{A_\epsilon(t_1^{LAB}/\gamma, t_2^{LAB}/\gamma)^{K^0(\tau^+)} - A_\epsilon(t_1^{LAB}/\gamma, t_2^{LAB}/\gamma)^{K^0(\tau^-)}}{2}$$

$$A_3(t_1^{LAB}, t_2^{LAB}) = \frac{1}{2}\left[\int f_{K^0}(E)dE\, A_\epsilon\left(\frac{t_1^{LAB}}{\gamma(E)}, \frac{t_2^{LAB}}{\gamma(E)}\right) - \int f_{\bar{K}^0}(E)dE\, A_\epsilon\left(\frac{t_1^{LAB}}{\gamma(E)}, \frac{t_1^{LAB}}{\gamma(E)}\right)\right]$$

Fixed time cutes in the LAB frame:

$$A_3(t_1^{LAB}, t_2^{LAB}) = \frac{1}{2}\left[\int f_{K^0}(E)dE\ A_\epsilon\left(\frac{t_1^{LAB}}{\gamma(E)}, \frac{t_2^{LAB}}{\gamma(E)}\right) - \int f_{\bar{K}^0}(E)dE\ A_\epsilon\left(\frac{t_1^{LAB}}{\gamma(E)}, \frac{t_1^{LAB}}{\gamma(E)}\right)\right]$$



- Kaons energy spectrum in the hadronic reference frame (CM of K, Kbar and pion);
- The asymmetry A3 is only a consequence of the **different energy** of K and Kbar: more precisely is a consequence of **different time dilatation** effect for the two kaons;
- **This is not a realistic model!** But makes us understand that the origin of the asymmetry A3 is **the energy dependence of the efficiency function**

$$F(t,E) = \theta(\gamma(E)t - t_1^{LAB})\theta(-\gamma(E)t + t_2^{LAB})$$

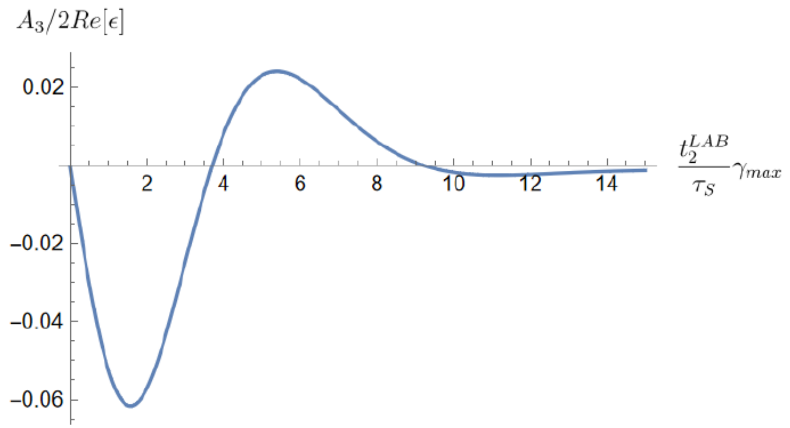


Figure 5: Asymmetry A_3 assuming a fixed value for $t_1^{LAB} = \tau_S/100$. The value $\gamma_{max} = E_K^{max}/m_K \approx 1.7$ is used assuming the case in which the kaon is emitted with maximum energy.

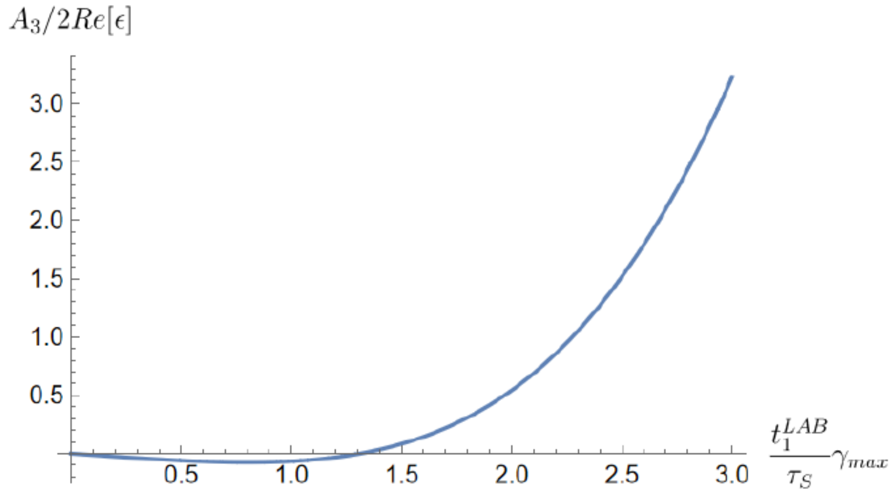


Figure 4: Asymmetry A_3 assuming a fixed value for $t_2^{LAB} = 100\tau_S$. The value $\gamma_{max} = E_K^{max}/m_K \approx 1.7$ is used assuming the case in which the kaon is emitted with maximum energy.

$$A_3 = \frac{1}{2}\left[\int f_{K^0}(E)dE\ A_\epsilon(E) - \int f_{\bar{K}^0}(E)dE\ A_\epsilon(E)\right]$$

$$A_\epsilon(E) = \frac{\int_0^\infty dt\ F(t,E)\ [\Gamma(K^0(t)\to 2\pi) - \Gamma(\bar{K}^0(t)\to 2\pi)]}{\int_0^\infty dt\ F(t,E)\ [\Gamma(K^0(t)\to 2\pi) + \Gamma(\bar{K}^0(t)\to 2\pi)]}$$

The asymmetry A3 is a consequence of **the energy dependence of the efficiency function** **F=F(t,E)**:

- In the case of fixed time cuts in Kaon frame (Grossman paper): $F(t) = \theta(t-t_1)\theta(-t+t_2) \rightarrow A_3 = 0$
- In the case of fixed time cuts in LAB frame: $F(t,E) = \theta(\gamma(E)t - t_1^{LAB})\theta(-\gamma(E)t + t_2^{LAB})$

The asymmetry A3 is only a consequence of the **limit of experimental efficiency**: if we were integrating the all the times then **A3=0**; On the other hand **A1 is a physical CP symmetry**;

Fixing time cuts in the LAB frame is **not a realistic model**. A more realistic model is to **fix length cuts** in the experimental apparatus:

$$t_{1,2}^{LAB} = \frac{L_{1,2}}{\beta c} \quad \rightarrow \quad F(t,E) = \theta(t - \frac{L_1}{\beta\gamma c})\theta(-t + \frac{L_2}{\beta\gamma c})$$

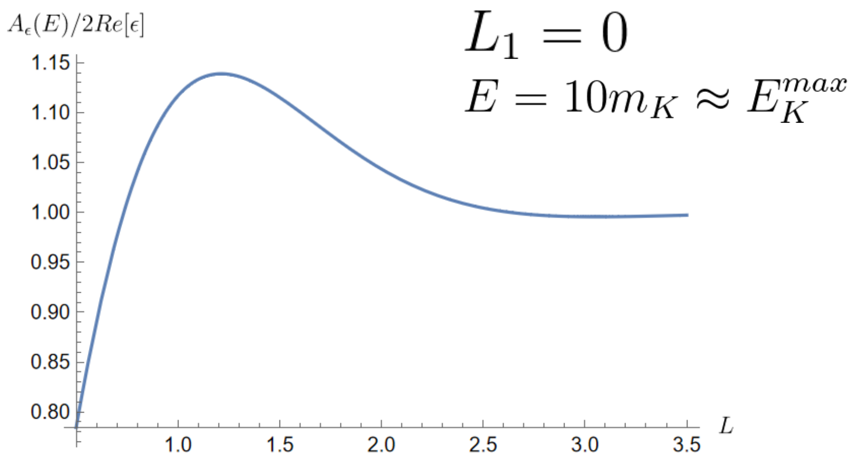
Fixed length cuts of the experimental apparatus: we detect all the Ks that decay within the length L1 and L2:

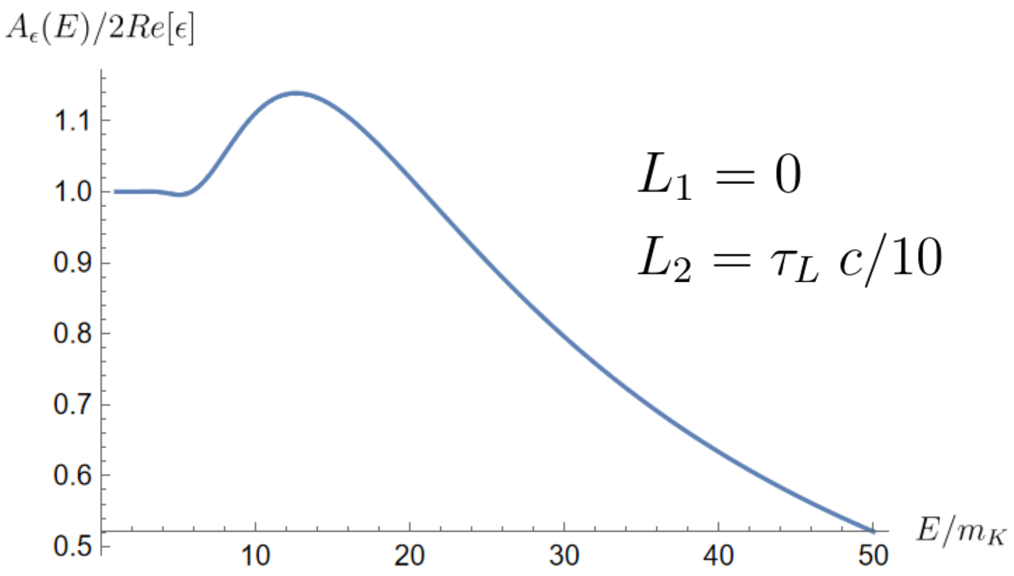
$$t_{1,2}^{LAB} = \frac{L_{1,2}}{\beta c} \quad \rightarrow \quad F(t,E) = \theta\left(t - \frac{L_1}{\beta\gamma c}\right)\theta\left(-t + \frac{L_2}{\beta\gamma c}\right)$$

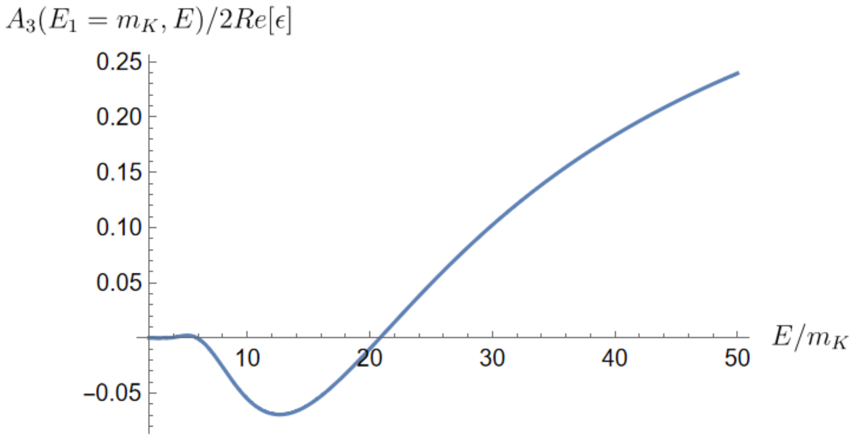
→ Fix lengths cuts in LAB frame means energy dependent time cuts in Kaon frame

$$A_\epsilon(E) = \frac{\int_0^\infty dt\ F(t,E)\ [\Gamma(K^0(t) \to 2\pi) - \Gamma(\bar{K}^0(t) \to 2\pi)]}{\int_0^\infty dt\ F(t,E)\ [\Gamma(K^0(t) \to 2\pi) + \Gamma(\bar{K}^0(t) \to 2\pi)]}$$

$$A_3 = \frac{1}{2}\left[\int f_{K^0}(E)dE\ A_\epsilon(E) - \int f_{\bar{K}^0}(E)dE\ A_\epsilon(E)\right]$$







The **origin of asymmetry A3** is
to fix cut-offs in the LAB frame (fix time cuts, fix length cuts)

⇩

This creates energy dependent cuts in the Kaon rest frame because of **time dilatation** that make the A1 asymmetry energy dependent:

$$t_{1,2}^{LAB} = \frac{L_{1,2}}{\beta c} \quad \rightarrow \quad F(t,E) = \theta(t - \frac{L_1}{\beta\gamma c})\theta(-t + \frac{L_2}{\beta\gamma c})$$

$$F(t,E) = \theta(\gamma(E)t - t_1^{LAB})\theta(-\gamma(E)t + t_2^{LAB})$$

$$A_\epsilon(E) = \frac{\int_0^\infty dt\ F(t,E)\ [\Gamma(K^0(t)\to 2\pi) - \Gamma(\bar{K}^0(t)\to 2\pi)]}{\int_0^\infty dt\ F(t,E)\ [\Gamma(K^0(t)\to 2\pi) + \Gamma(\bar{K}^0(t)\to 2\pi)]}$$

⇩

Since **Kaon and anti Kaon are emitted with different energy** they will have different time dilatation and asymmetry A1 that will not cancel out:

$$A_3 = \frac{1}{2}\left[\int f_{K^0}(E)dE\ A_\epsilon(E) - \int f_{\bar{K}^0}(E)dE\ A_\epsilon(E)\right]$$

Real experiment case

$$A_\epsilon(E) = \frac{\int_0^\infty dt\, F(t,E)\, [\Gamma(K^0(t) \to 2\pi) - \Gamma(\bar{K^0}(t) \to 2\pi)]}{\int_0^\infty dt\, F(t,E)\, [\Gamma(K^0(t) \to 2\pi) + \Gamma(\bar{K^0}(t) \to 2\pi)]}$$

$$A_3 = \frac{1}{2}\left[\int f_{K^0}(E) dE\ A_\epsilon(E) - \int f_{\bar{K}^0}(E) dE\ A_\epsilon(E)\right]$$

- We need the **experimental efficiency function** F(t,E) parametrized by **kaon decay time** and **energy** in the LAB frame;

Dividing the whole experimental energy spectrum into 10 bins
and building the efficiency function for each bin **(Thank you Paolo!)** $F(t, E_i) \quad \text{for} \quad i = 1, \ldots 10$

$$A_3 = \frac{1}{2}\sum_{i=1}^{10}\left(\int_{\Delta E} f_{K^0}(E) A_\epsilon(E_i) dE\ - \int_{\Delta E} f_{\bar{K}^0}(E) A_\epsilon(E_i) dE\right)$$

- We need the **energy distribution** for the Kaon and Kaon-bar;