THERMAL EFFECTS IN FREEZE-IN NEUTRINO DARK MATER PRODUCTION

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Based on:
A. Abada, G. Arcadi, ML, G. Piazza and S. Rosauro-Alcaraz, **JHEP 11 (2023) 180**
ML, **Phys.Lett.B 846 (2023) 138206**

ALMA MATER STUDIORUM
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Observational problems of the SM

Two seemingly unrelated observations cannot be accounted for in the Standard Model

NuFIT 5.0 (2020)

Neutrinos are massive and leptons mix

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.233 \to 0.507 & 0.461 \to 0.694 & 0.631 \to 0.778 \\ 0.261 \to 0.526 & 0.471 \to 0.701 & 0.611 \to 0.761 \end{pmatrix}$$

I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou, arXiv:2007.14792 [hep-ph]

The Universe has a dark matter component

$$\Omega_{\mathrm{m}} h^2 = 0.1430 \pm 0.0011$$

$$\Omega_{\mathrm{c}} h^2 = 0.1200 \pm 0.0012$$

$$\Omega_{\mathrm{b}} h^2 = 0.02237 \pm 0.00015$$

N. Aghanim *et al.* [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO]

The natural (simple) way

Complete the SM field pattern with **right-handed neutrinos**

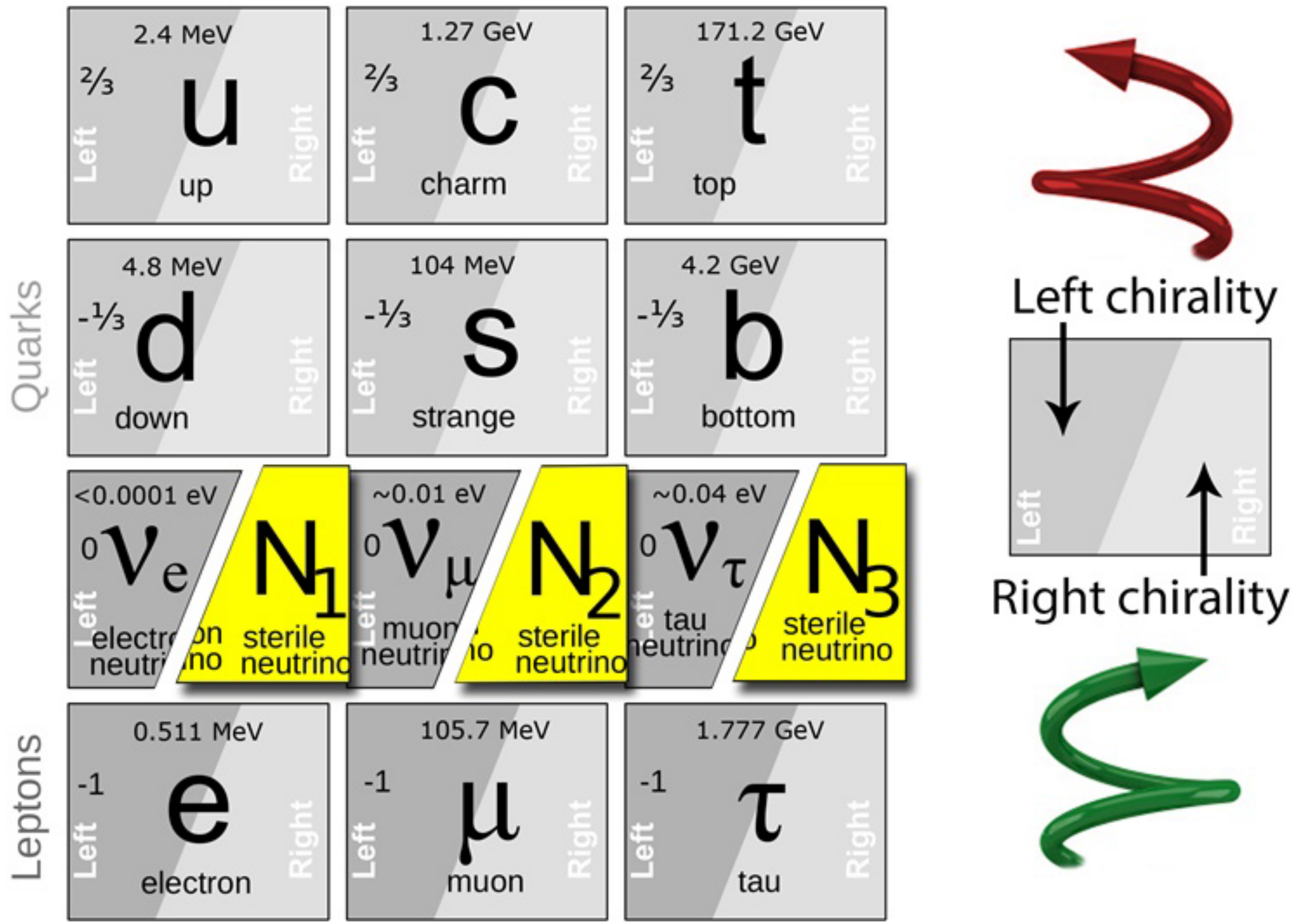

Figure from S. Alekhin *et al.*, arXiv:1504.04855 [hep-ph]

Neutrino masses and dark matter

Type-I seesaw mechanism: SM + gauge singlet fermions N_I

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N_I}\partial\!\!\!/ N_I - \left(F_{\alpha I}\overline{\ell_L^\alpha}\widetilde{\phi}N_I + \frac{M_{IJ}}{2}\overline{N_I^c}N_J + h.c.\right)$$

After electroweak phase transition < Φ > = v ≃ 174 GeV

$$m_\nu = -v^2 F\frac{1}{M}F^T$$

The new fields N_I can be viable DM candidates:

- **No electromagnetic interactions**
- **Potentially long-lived**
- Produced in the early Universe

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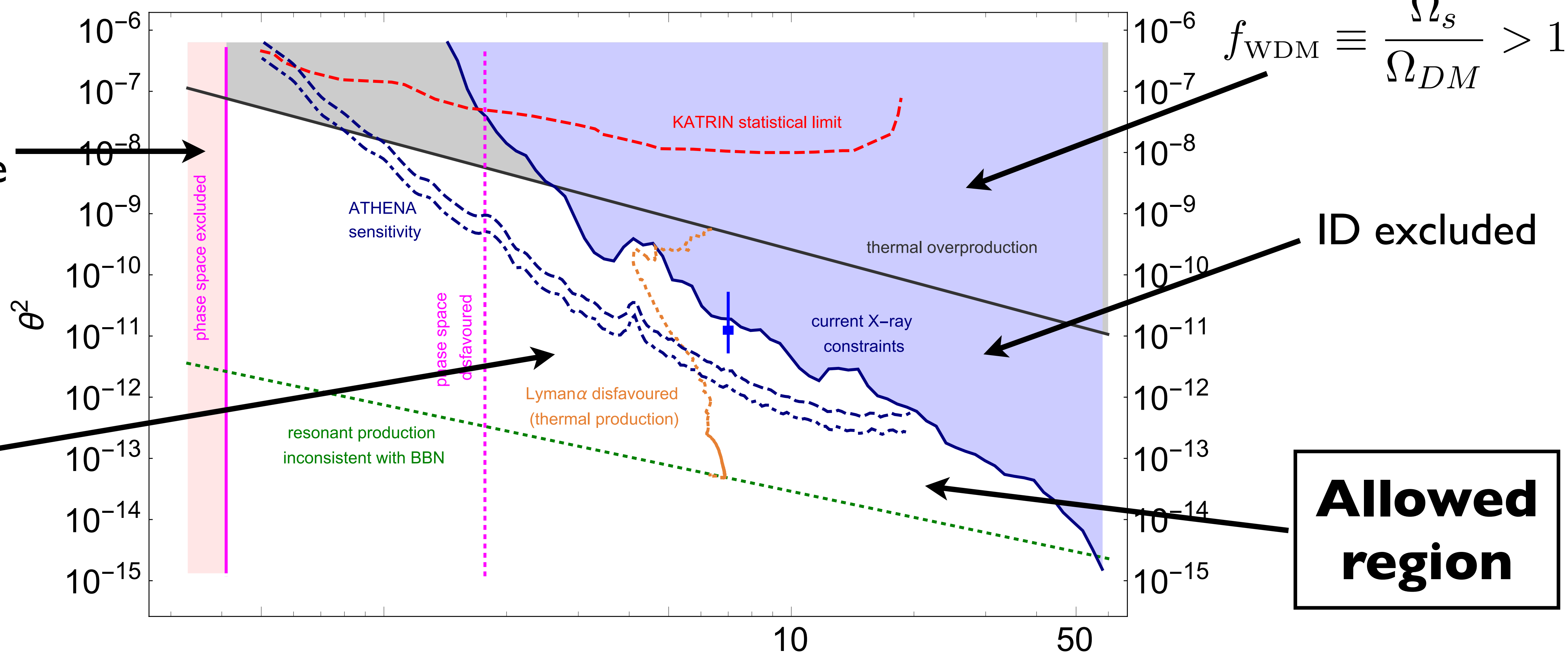
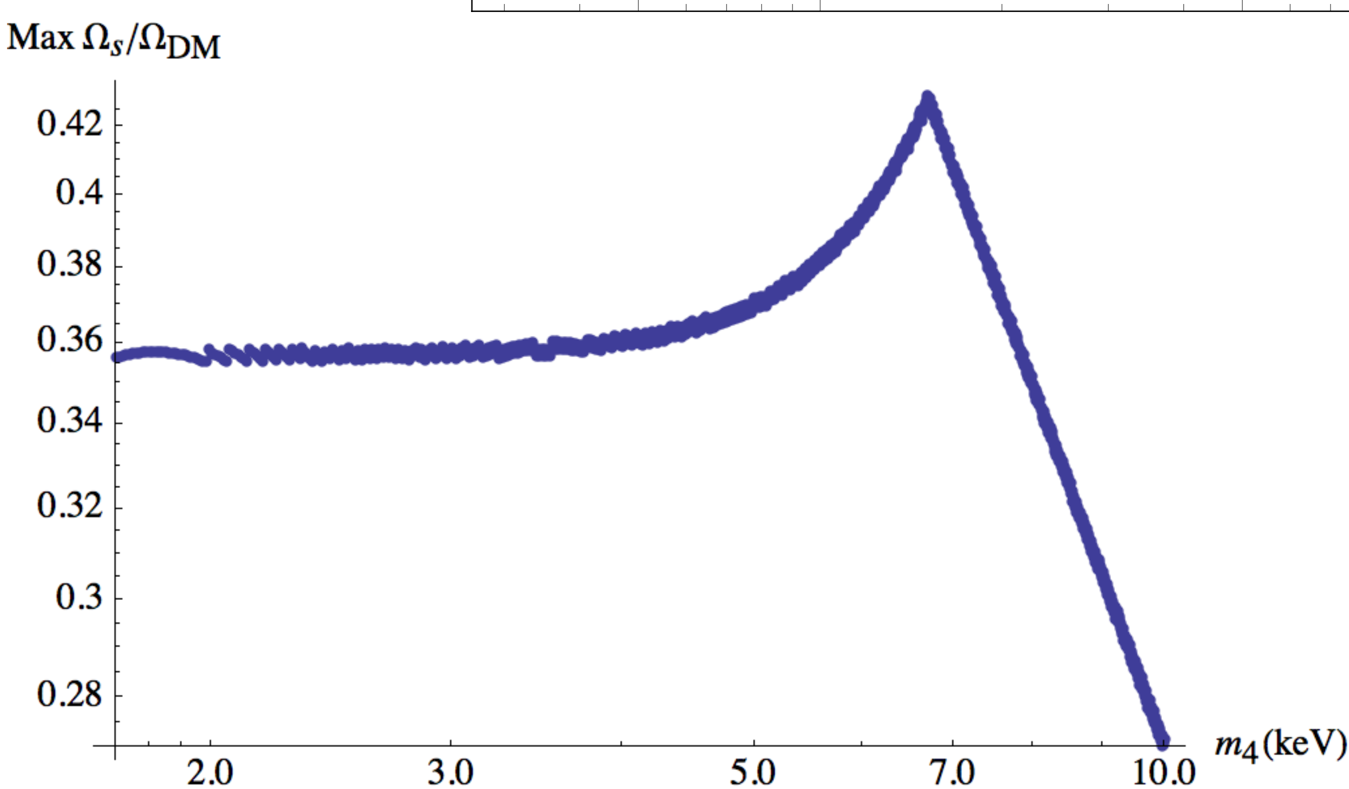
WDM constraints

DW produced sterile ν are warm dark matter





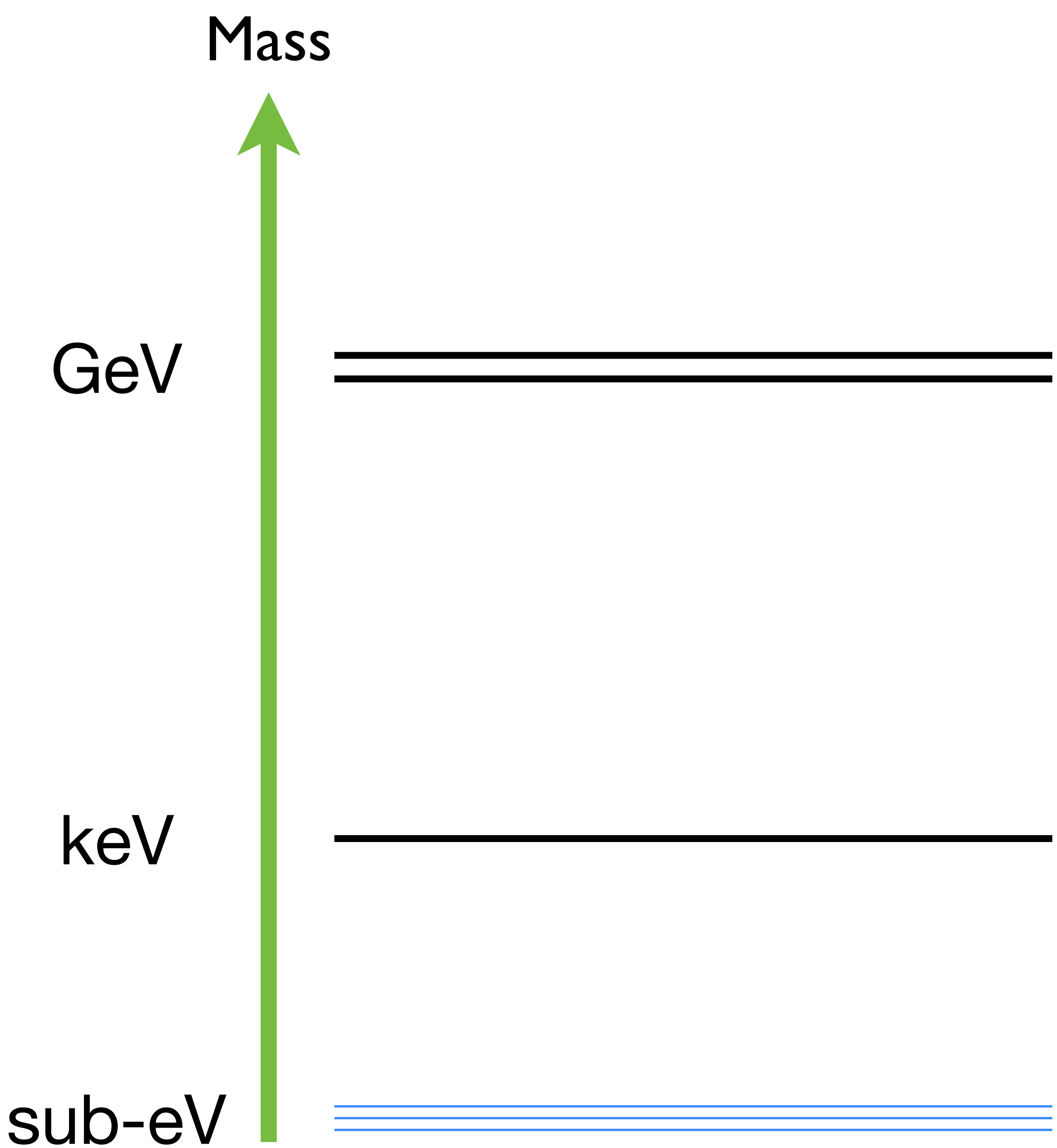
Sterile ν produced via **DW cannot** account for the 100% of the **observed DM abundance**

Figure from
A. Abada, G. Arcadi and M.L., arXiv:1406.6556 [hep-ph]

Known solution: The νMSM

T. Asaka, S. Blanchet and M. Shaposhnikov, hep-ph/0503065
T. Asaka and M. Shaposhnikov, hep-ph/0505013
M. Shaposhnikov and I. Tkachev, hep-ph/0604236

Type-I Seesaw with a phenomenologically motivated mass spectrum



At the origin of the lepton **asymmetries** of the Universe and of **neutrino masses**

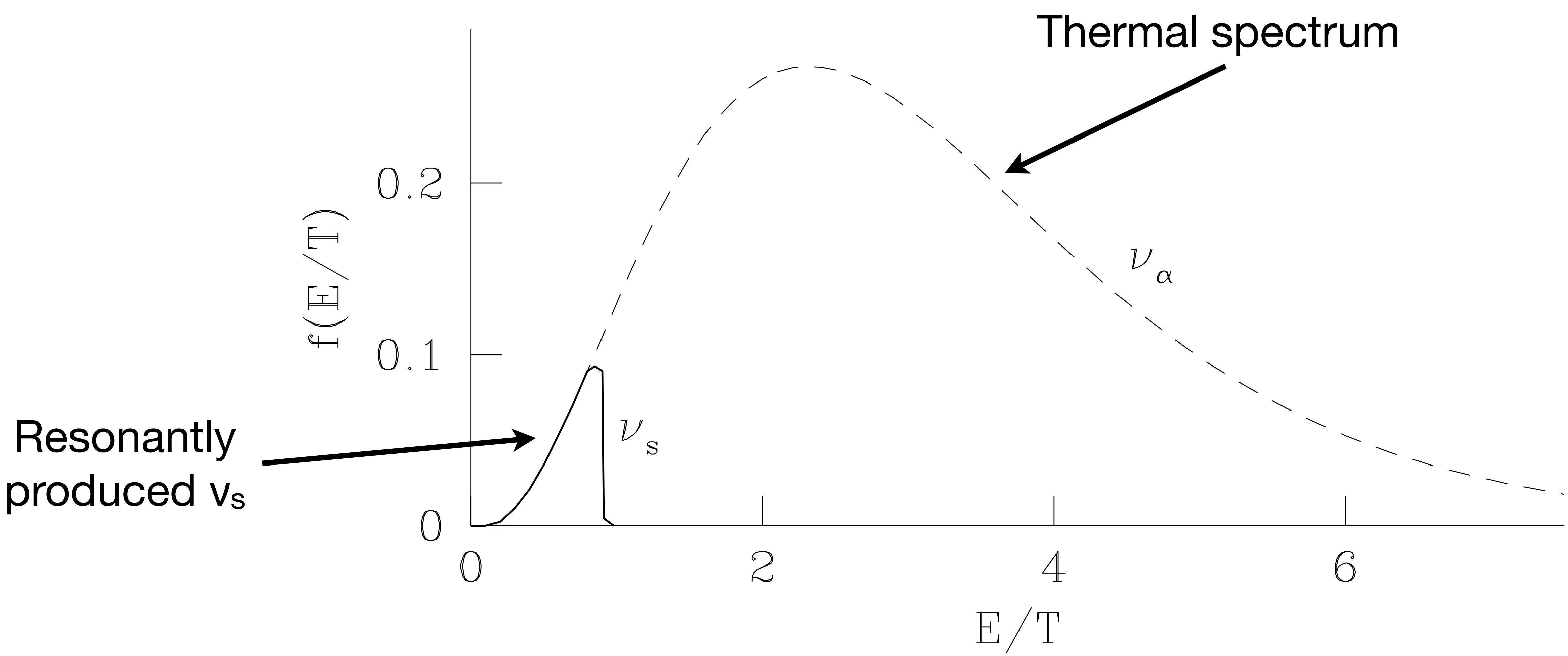
Dark matter candidate
(does not significantly contribute to ν masses)

Active neutrinos

νMSM dark matter solution

Shi-Fuller mechanism: **lepton number-driven resonant** MSW conversion of active neutrinos

X. D. Shi and G. M. Fuller, astro-ph/9810076



Required lepton asymmetry

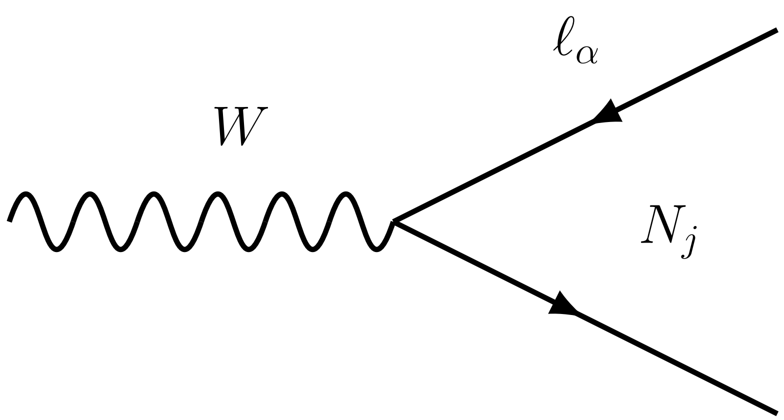
$$\mu_\alpha = \frac{n_\alpha}{s} \gtrsim 8 \cdot 10^{-6}$$

Required mass degeneracy

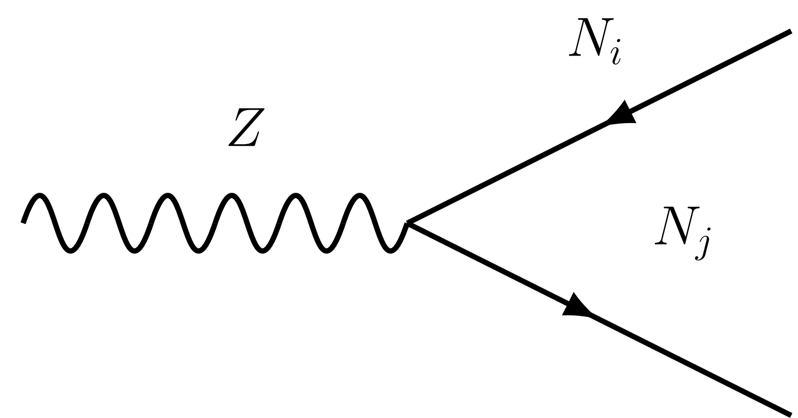
$$\frac{\delta M}{M} \lesssim 10^{-14}$$

Can we think of other production mechanisms?

In experimental searches, HNL are looked for in W, Z mediated channels



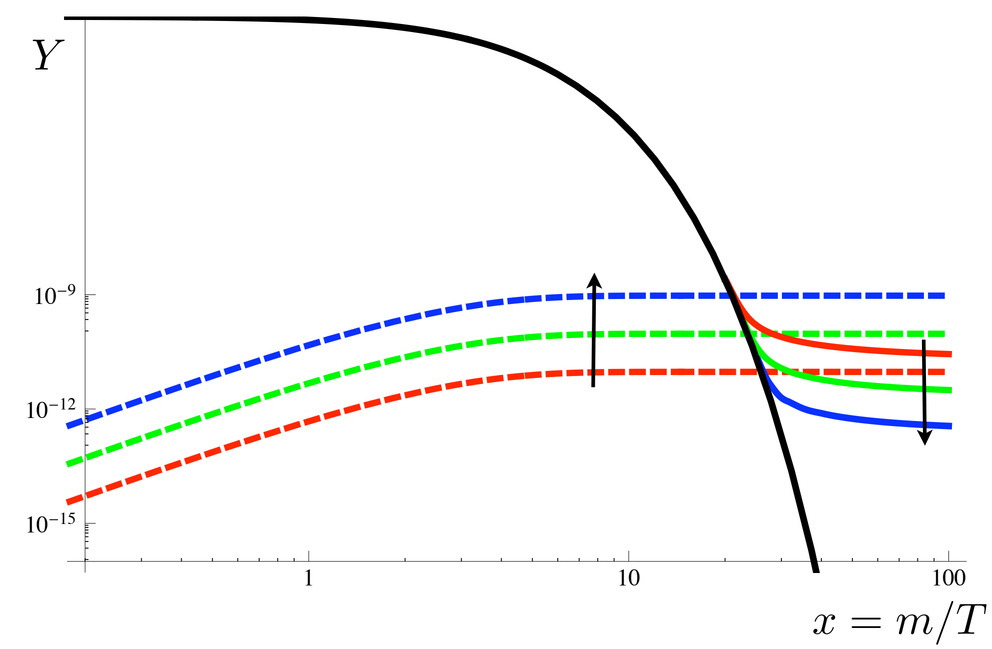
$\propto U_{\alpha i}$



$\propto U^*_{\alpha i} U_{\alpha j}$

In principle we have the necessary ingredients for successful production of DM

- Parent particle in thermal equilibrium by EW interactions
- DM coupling suppressed by small active-sterile mixing



L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, arXiv:0911.1120 [hep-ph]

Does it work?

Thermal effects suppress the rate

Simplified scenario - Standard Model with only one leptonic generation: one active neutrino and its charged lepton partner and one SU(2) singlet Dirac sterile neutrino

$$\mathcal{L} = \mathcal{L}_{SM} + \overline{\nu}_s\, i \not{\partial} \nu_s - \overline{\nu}_\alpha\, \mathbb{M}_{\alpha\beta}\, \nu_\beta + \mathrm{h.c}\ ;\ \alpha, \beta = a, s\ ,$$

L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

$$\mathbb{M} = \begin{pmatrix} 0 & m \\ m & M_s \end{pmatrix}$$

$$\begin{pmatrix} \nu_a \\ \nu_s \end{pmatrix} = U(\theta) \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \ ;\ U(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

In the seesaw limit: m ≪ M_s

$$M_1 \simeq -\frac{m^2}{M_s} \ ;\ M_2 \simeq M_s \ ;\ \sin(2\theta) \simeq 2\theta \simeq \frac{2m}{M_s} \ll 1$$

active ν **DM**

One-generation seesaw

Introducing the flavour doublet $\Psi = \begin{pmatrix} \nu_a \\ \nu_s \end{pmatrix}$

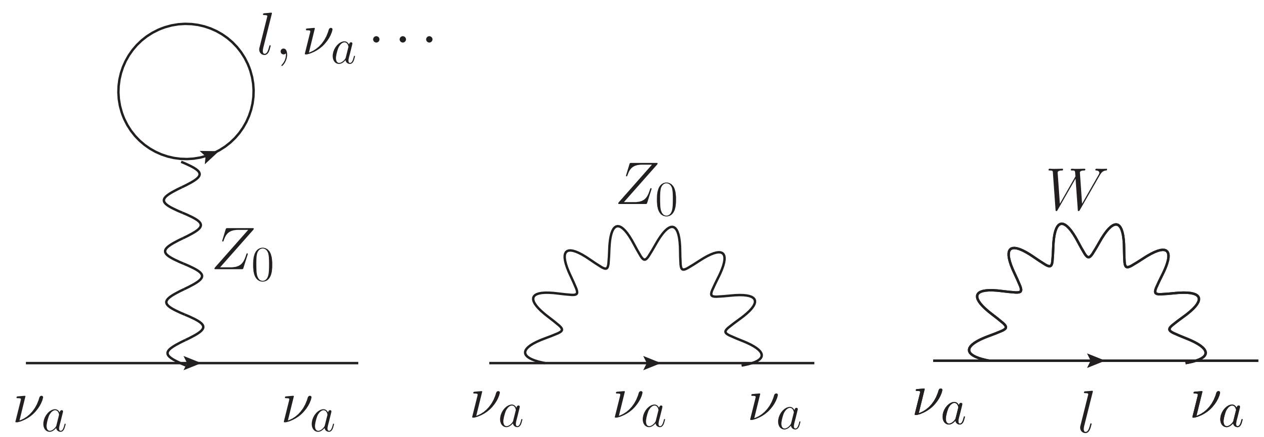
The equations of motion in flavour basis are

$$\left[\left(\gamma_0 \omega - \vec{\gamma} \cdot \vec{q}\right) \mathbb{1} - \mathbb{M} + \left(\boldsymbol{\Sigma}^t + \boldsymbol{\Sigma}(\omega, \vec{q}) \right) \mathbb{L} \right] \tilde{\Psi}(\omega, \vec{q}) = 0$$

tadpole

Σ are the self-energy corrections, which are diagonal in flavour basis

$$\boldsymbol{\Sigma} \equiv \Sigma \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$



The presence of Σ generally changes the mixing angle with respect to zero temperature

L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

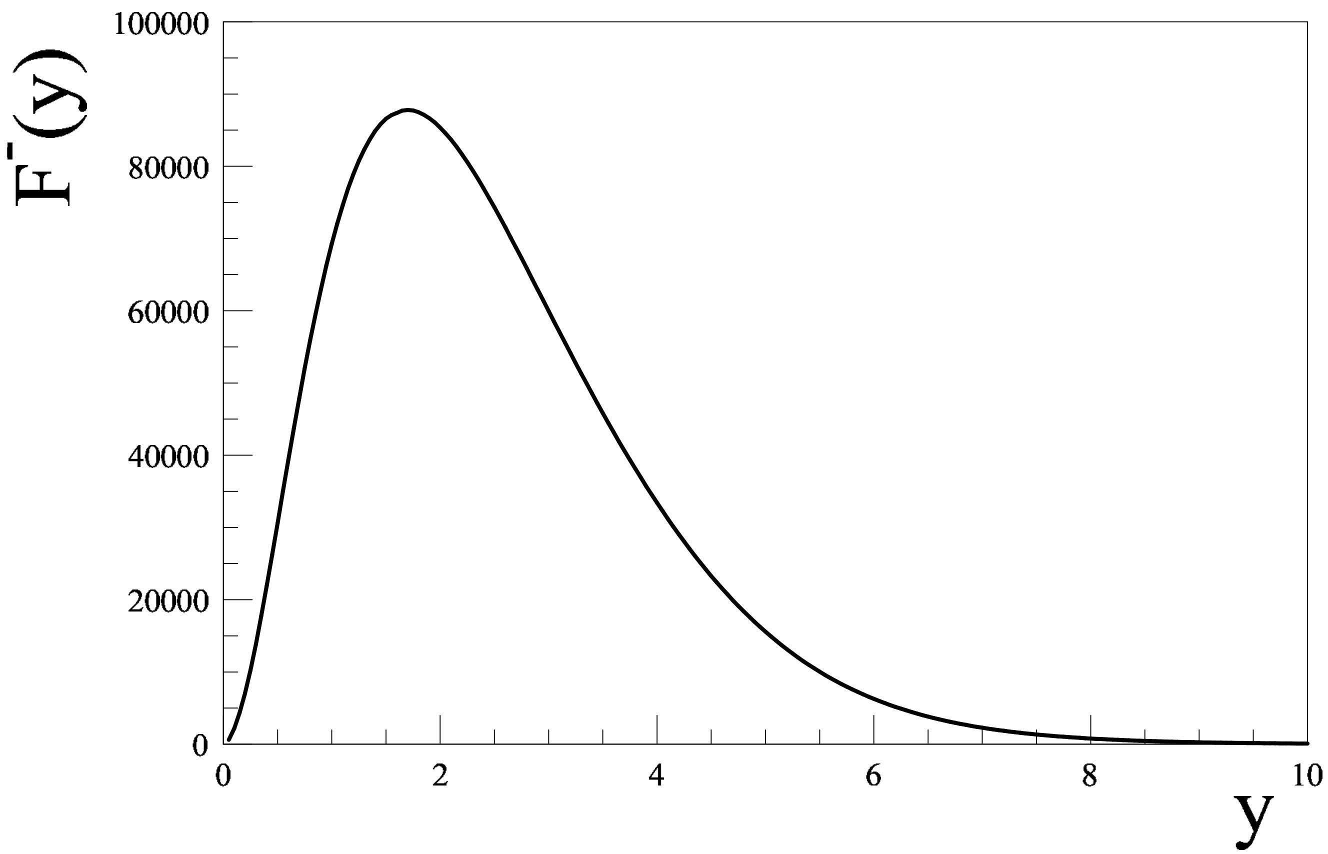
Single generation production rates

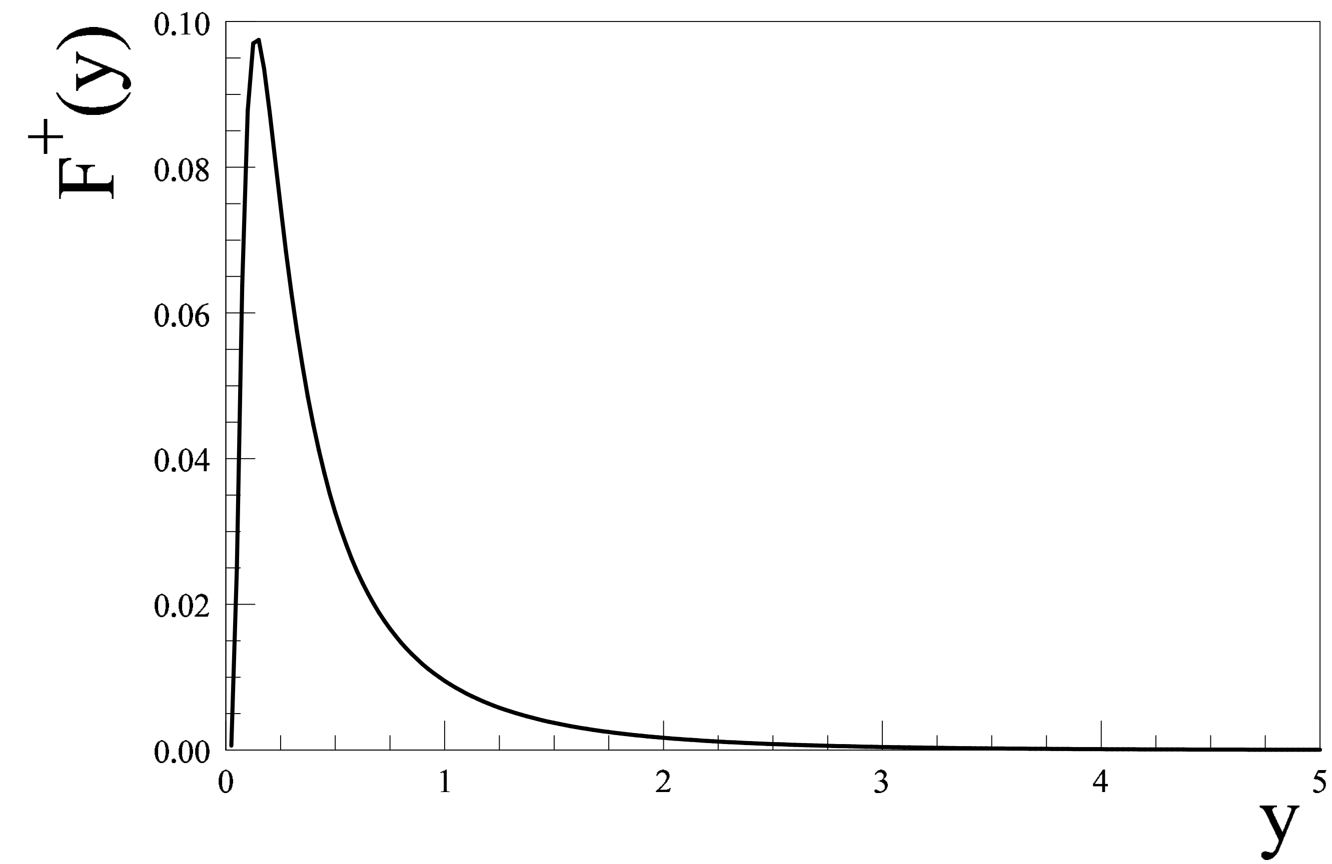
By computing the finite-temperature self energy one finally obtains the production rates

$$y = \frac{q}{T} \qquad \tau \equiv \frac{M_W}{T}$$

$$n_2^-(y) = \int_1^\infty \frac{dn_2^-(\tau, y)}{d\tau}\, d\tau = 0.92 \times 10^{16}\, \theta^2 \left(\frac{M_s}{M_W}\right)^4 F^-(y)$$

$$n_2^+(y) = 0.92 \times 10^{16}\, \theta^2 \left(\frac{M_s}{M_W}\right)^2 F^+(y)$$





By integrating these expressions the fraction of DM results

$$\mathcal{F}_2 = 0.97 \left(\frac{\theta^2}{10^{-8}}\right) \left(\frac{M_s}{\mathrm{MeV}}\right)^3 \left[1 + \left(\frac{M_s}{8.35\,\mathrm{MeV}}\right)^2\right]$$

These masses/mixings result in a too short-living sterile neutrino

L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

End of the story?

Not really! This conclusion was derived in a simple toy model

If we want to explain neutrino masses, more ingredients are necessary

$$\mathcal{L} \;=\; \mathcal{L}_{\rm SM} + i\overline{N_I}\partial\!\!\!/ N_I - \left(F_{\alpha I}\overline{\ell_L^\alpha}\widetilde{\phi}N_I + \frac{M_{IJ}}{2}\overline{N_I^c}N_J + h.c.\right)$$

- **Include multiple right-handed neutrinos:** at least 3, two to generate neutrino masses and the other as DM candidate
- **Include the Higgs sector!**

Key differences

- **RHN masses and Yukawa couplings relatively unconstrained** (while gauge couplings and boson masses are fixed);
- **RHN have direct couplings with the Higgs** (additional production channels)
- **Neutrinos are Majorana particles**

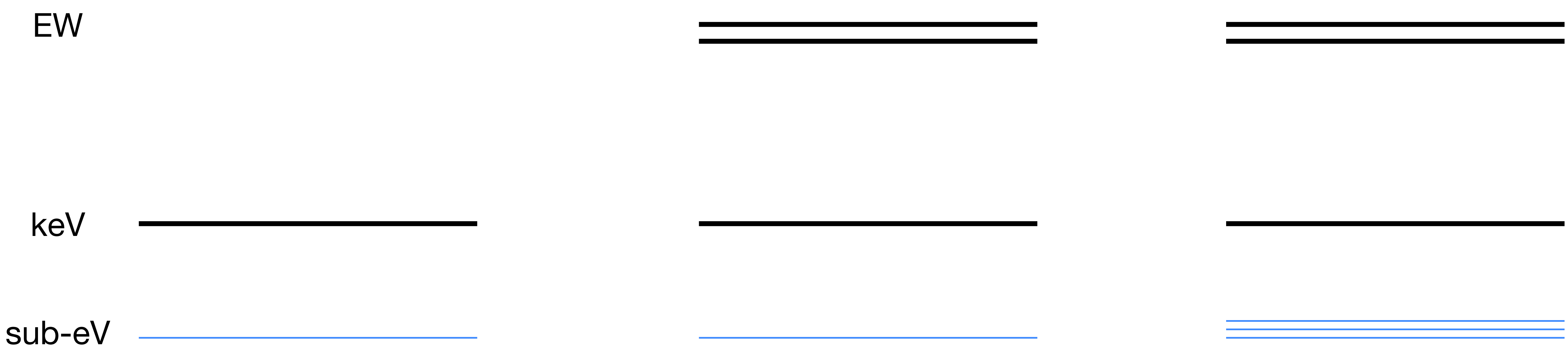
The model(s)

We consider increasing levels of complexity to single-out the relevant dynamics

Single flavour + DM candidate (toy 2 × 2)

Single flavour + 3 RHN (ISS-4×4)

Type-I seesaw



Only production via gauge bosons is relevant (Higgs coupling proportional to mass)

HNL decay can be relevant, mixings constrained to a vanilla scenario

Fully reproduces oscillation data

Scenario	Decay channels
Toy 2×2	$W(Z) \to \ell_\alpha(n_l) + n_{DM}$
ISS 4×4	$W(Z) \to \ell_\alpha(n_l) + n_{DM}$, $H \to n_l + n_{DM}$, $n_h \to H(Z) + n_{DM}$
Type-I seesaw w/o Higgs	$W(Z) \to \ell_\alpha(n_l) + n_{DM}$, $n_h \to Z + n_{DM}$
Type-I seesaw	$W(Z) \to \ell_\alpha(n_l) + n_{DM}$, $H \to n_l + n_{DM}$, $n_h \to H(Z) + n_{DM}$

Real-time formalism of thermal QFT

Four types of propagators

$$\begin{cases} \tilde{D}^{(++)}_{\alpha\beta}(k) = \left[\frac{i}{k^2-m^2+i\epsilon} + 2\pi\eta f(|k_0|)\delta(k^2-m^2)\right] d_{\alpha\beta}, \\ \tilde{D}^{(--)}_{\alpha\beta}(k) = [\tilde{D}^{(++)}(k)]^* d_{\alpha\beta}, \\ \tilde{D}^{(+-)}_{\alpha\beta}(k) = e^{\beta k_0/2} f(k_0) 2\pi\varepsilon(k_0)\delta(k^2-m^2) d_{\alpha\beta}, \\ \tilde{D}^{(-+)}_{\alpha\beta}(k) = \eta \tilde{D}^{(+-)}_{\alpha\beta}(k), \end{cases}$$

$$f(k_0) = \frac{1}{e^{\beta k_0} - \eta}$$

$$d_{\alpha\beta} = \begin{cases} 1, & \text{for scalars,} \\ \not{k} \pm m\mathbb{1}, & \text{for fermions,} \\ -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}, & \text{for vector bosons} \end{cases}$$

Some relations exist that can simplify computations

$$\mathrm{Im}\sigma^R = -\cosh\left(\frac{\beta|p_0|}{2}\right)\left[\Theta(p_0) - \Theta(-p_0)\right] i\sigma^{(+-)}$$

$$\mathrm{Re}\sigma^R = \frac{1}{\pi}\int_{-\infty}^{\infty} dq_0 \mathcal{P}\left[\frac{\mathrm{Im}\sigma^R(q_0)}{p_0 - q_0}\right]$$

Finite temperature dispersion relations

$$\Sigma(\omega, \vec{q}) = \mathrm{Re}\Sigma(\omega, \vec{q}) + i\,\mathrm{Im}\Sigma(\omega, \vec{q})$$

"Index of refraction" in the medium

"Damping rate" of (quasi) particle excitations

They feature a dispersive representation

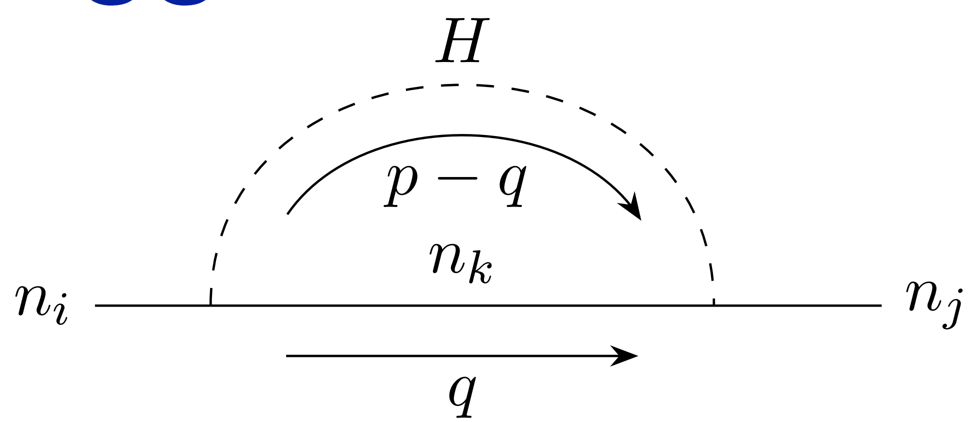
$$\mathrm{Re}\Sigma(\omega, \vec{q}) = \frac{1}{\pi}\int_{-\infty}^{\infty} dq_0\, \mathcal{P}\left[\frac{\mathrm{Im}\Sigma(q_0, \vec{q})}{q_0 - \omega}\right]$$

We can generally write

$$\Sigma_{ij} \propto \sum_k \mathcal{A}_{ik}\mathcal{A}_{kj}\sigma_k^R$$

$$\Sigma = \gamma_0\Sigma^{(0)} - \vec{\gamma}\cdot\hat{p}\Sigma^{(1)} + \Sigma^{(2)}$$

Higgs contribution



$$C_{ij} \equiv \sum_{\alpha=e,\mu,\tau} \mathcal{U}^\dagger_{i\alpha}\mathcal{U}_{\alpha j}$$

$$\mathcal{L}_{H-\nu} \supset -\frac{H}{v_H}\sum_{i,j} C_{ij}\bar{n}_i\left(m_i P_L + m_j P_R\right)n_j = -\frac{H}{2v_H}\sum_{i,j}\bar{n}_i\left[C_{ij}\left(m_i P_L + m_j P_R\right) + C^*_{ij}\left(m_i P_R + m_j P_L\right)\right]n_j$$

$$i\Sigma^{(+-)}_{ij} = -\frac{2\pi}{v_H^2}e^{\frac{\beta p_0}{2}}f_F(p_0)\sum_k\int\frac{d^4q}{(2\pi)^3}\left[1+f_B(r_0)-f_F(q_0)\right]\varepsilon(q_0)\varepsilon(r_0)$$
$$\left[\left(\mathcal{A}^R_{kj}\not{q}+\mathcal{A}^L_{kj}m_k\right)\mathcal{A}^L_{ik}P_L + \left(\mathcal{A}^L_{kj}\not{q}+\mathcal{A}^R_{kj}m_k\right)\mathcal{A}^R_{ik}P_R\right]\delta(q^2-m_k^2)\delta(r^2-M_H^2)$$

$$r_\mu \equiv (p-q)_\mu \qquad \mathcal{A}^L_{ij} \equiv \tfrac{1}{2}\left(C_{ij}m_i + C^*_{ij}m_j\right) \qquad \mathcal{A}^R_{ij} = \left(\mathcal{A}^L_{ij}\right)^*$$

We define $\mathcal{R}\left(\Sigma_{ij}\right) + i\mathcal{I}\left(\Sigma_{ij}\right) = \mathrm{Re}\Sigma_{ij} + i\mathrm{Im}\Sigma_{ij}$

$\mathcal{A}_{ik}\mathcal{A}_{kj}\mathrm{Re}\sigma^R_k$ $\qquad$ $\mathcal{A}_{ik}\mathcal{A}_{kj}\mathrm{Im}\sigma^R_k$

Higgs-mediated self energy

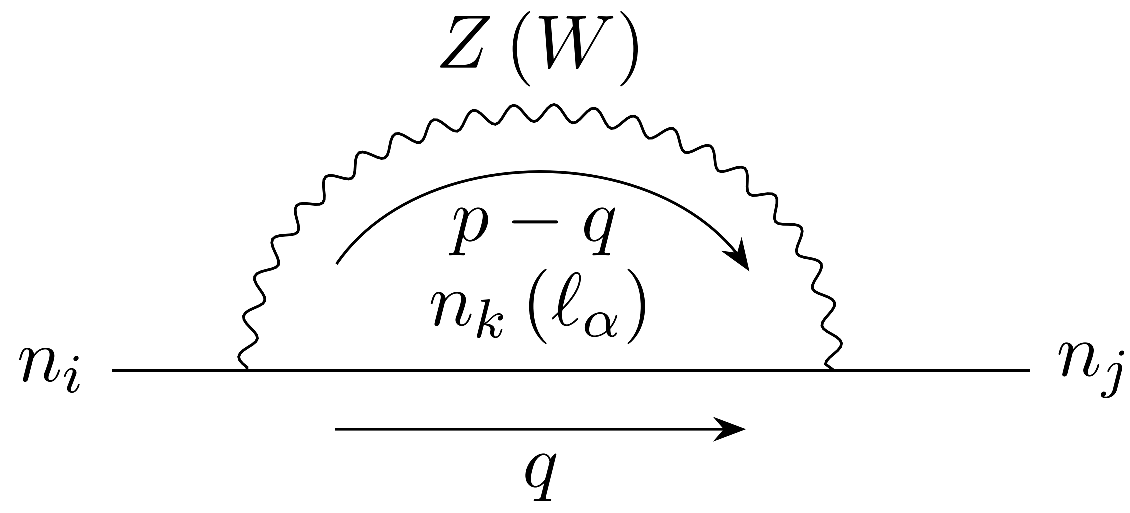
$$\omega_H^{\pm} = \sqrt{(p \pm q)^2 + M_H^2}$$

$$I^{(0)} \equiv \frac{1}{p} \int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_H^-}^{\omega_H^+} d\omega_H \frac{q q_0}{(2\pi)^2 4\omega_k} \left[1 + f_B(r_0) - f_F(q_0)\right]$$
$$\left[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k)\right]\left[\delta(r_0 - \omega_H) - \delta(r_0 + \omega_H)\right],$$
$$I^{(1)} \equiv \frac{1}{p^2} \int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_H^-}^{\omega_H^+} d\omega_H \frac{q}{(2\pi)^2 4\omega_k} \left(p_0 q_0 - p\tilde{\mu}\right) \left[1 + f_B(r_0) - f_F(q_0)\right]$$
$$\left[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k)\right]\left[\delta(r_0 - \omega_H) - \delta(r_0 + \omega_H)\right],$$
$$I^{(2)} \equiv \frac{1}{p} \int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_H^-}^{\omega_H^+} d\omega_H \frac{q}{(2\pi)^2 4\omega_k} \left[1 + f_B(r_0) - f_F(q_0)\right]$$
$$\left[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k)\right]\left[\delta(r_0 - \omega_H) - \delta(r_0 + \omega_H)\right],$$

with prefactors

$$\xi_L^{(I)} = \begin{cases} A_{kj}^R \mathcal{A}_{ik}^L q_0, & \text{for } I = 0, \\ -A_{kj}^R \mathcal{A}_{ik}^L \frac{\vec{p}\cdot\vec{q}}{|\vec{p}|}, & \text{for } I = 1, \\ A_{kj}^L \mathcal{A}_{ik}^L, & \text{for } I = 2, \end{cases}$$

Z contribution



$$\mathcal{L}_{Z-\nu} \supset \frac{g}{2c_W} \sum_{i,j} C_{ij} \bar{n}_i \gamma^\mu P_L n_j Z_\mu = \frac{g}{4c_W} \sum_{i,j} \bar{n}_i \left[C_{ij} \gamma^\mu P_L - C_{ij}^* \gamma^\mu P_R \right] n_j Z_\mu$$

$$\begin{aligned} \mathcal{I}\left(\Sigma_{ij}\right) = & \left(\frac{g}{2c_W}\right)^2 \pi \sum_k \int_{-\infty}^{\infty} dq_0 \int \frac{d^3q}{(2\pi)^3} \frac{1}{4\omega_k \omega_Z} \left[1 + f_B(r_0) - f_F(q_0)\right] \\ & \frac{1}{4}\gamma^\mu \left\{ \left(C_{kj} \not{q} - C_{kj}^* m_k\right) C_{ik} \gamma^\nu P_L + \left(C_{kj}^* \not{q} - C_{kj} m_k\right) C_{ik}^* \gamma^\nu P_R \right\} \left(-\eta_{\mu\nu} + \frac{r_\mu r_\nu}{M_Z^2}\right) \\ & \left[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k)\right] \left[\delta(r_0 - \omega_Z) - \delta(r_0 + \omega_Z)\right], \end{aligned}$$

Z mediated self-energy

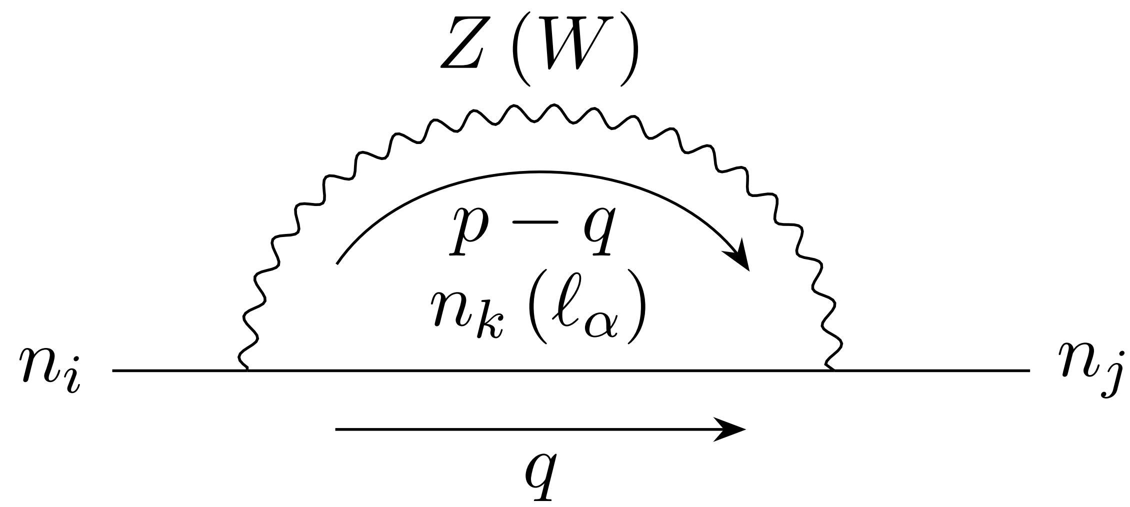
$$I_Z^{(0)} \equiv \frac{1}{p}\int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_Z^-}^{\omega_Z^+} d\omega_Z \frac{q}{(2\pi)^2 4\omega_k}\left[A_0(p_0)p_0 + B(p_0)q_0\right]\left[1 + f_B(r_0) - f_F(q_0)\right]$$
$$\left[\delta(q_0-\omega_k) - \delta(q_0+\omega_k)\right]\left[\delta(r_0-\omega_Z) - \delta(r_0+\omega_Z)\right],$$
$$I_Z^{(1)} \equiv \frac{1}{p^2}\int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_Z^-}^{\omega_Z^+} d\omega_Z \frac{q}{2(2\pi)^2 4\omega_k}\left[A_1(p_0) + 2p_0 B(p_0)q_0\right]\left[1 + f_B(r_0) - f_F(q_0)\right]$$
$$\left[\delta(q_0-\omega_k) - \delta(q_0+\omega_k)\right]\left[\delta(r_0-\omega_Z) - \delta(r_0+\omega_Z)\right],$$
$$I_Z^{(2)} = I^{(2)}\left(\omega_H \to \omega_Z\right).$$

with prefactors $$\mathscr{P}_L^{(I)} = \begin{cases} \frac{1}{4}C_{ik}C_{kj}\left[A_0(p_0)p_0 + B(p_0)q_0\right], & \text{for } I=0, \\ -\frac{1}{8p}C_{ik}C_{kj}\left[A_1(p_0) + 2p_0B(p_0)q_0\right], & \text{for } I=1, \\ \frac{3}{2}C_{ik}C_{kj}^* m_k, & \text{for } I=2. \end{cases}$$

$$A_0(p_0) \equiv \frac{p_0^2 - p^2 - M_Z^2 - m_k^2}{M_Z^2},$$
$$B(p_0) \equiv 1 - A_0(p_0),$$
$$A_1(p_0) \equiv p^2 + M_Z^2 - p_0^2 - m_k^2 + A_0(p_0)\left[p^2 + m_k^2 - M_Z^2 + p_0^2\right]$$

For $\mathscr{P}_R^{(I)}$
$C_{ij} \to C_{ij}^*$

W contribution



$$\mathcal{L}_{W-\nu} \supset \frac{g}{\sqrt{2}} \sum_{i,\alpha} \mathcal{U}_{\alpha i} \bar{\ell}_\alpha \gamma^\mu P_L n_i W^-_\mu + h.c.$$

Can be derived from Z contribution

$$g/(2c_W) \to g/\sqrt{2}$$

$$\sum_k C_{ik} C_{kj} \to 2 \sum_\alpha \mathcal{U}^\dagger_{i\alpha} \mathcal{U}_{\alpha j} = 2C_{ij}$$

$$M_Z \to M_W$$

$$\mathscr{P}^{(I)}_R = 0$$

$$\mathscr{P}^{(2)}_L = 0$$

$$m_k \to m_\ell \sim 0$$

Propagating states in the medium

Once the self-energy correction to the propagator is known

$$\not{\Sigma}(p_0) = \left(\gamma_0 \Sigma_L^{(0)} - \vec{\gamma}\cdot\hat{p}\Sigma_L^{(1)} + \Sigma_L^{(2)}\right) P_L + \left(\gamma_0 \Sigma_R^{(0)} - \vec{\gamma}\cdot\hat{p}\Sigma_R^{(1)} + \Sigma_R^{(2)}\right) P_R$$

we can insert this in the Dirac equation

$$\left(\not{p} - \mathcal{M} + \not{\Sigma}(p_0)\right)\tilde{\Psi} = 0$$

and project into helicity eigenstates

$$\tilde{\Psi} = \sum_{h=\pm 1} v^h \otimes \begin{pmatrix} \varphi^h \\ \zeta^h \end{pmatrix}, \quad \vec{\sigma}\cdot\hat{p}v^h = hv^h$$

$$\begin{aligned} &\left[p_0 + \Sigma_R^{(0)} - hp - h\Sigma_R^{(1)}\right]\zeta^h - \left(\mathcal{M} + \Sigma_L^{(2)}\right)\varphi^h = 0 \\ &\left[p_0 + \Sigma_L^{(0)} + hp + h\Sigma_L^{(1)}\right]\varphi^h - \left(\mathcal{M} + \Sigma_R^{(2)}\right)\zeta^h = 0 \end{aligned} \quad \Rightarrow \quad \zeta^h = \left[p_0 + \Sigma_R^{(0)} - hp - h\Sigma_R^{(1)}\right]^{-1}\left(\mathcal{M} + \Sigma_L^{(2)}\right)\varphi^h$$

The dispersion relations are given by the zeroes of the inverse propagator

$$\boxed{\left[p_0^2 - p^2 + (p_0 - hp)\left(\Sigma_L^{(0)} + h\Sigma_L^{(1)}\right) - (p_0 - hp)\left(\mathcal{M} + \Sigma_R^{(2)}\right)\left[p_0 + \Sigma_R^{(0)} - hp - h\Sigma_R^{(1)}\right]^{-1}\left(\mathcal{M} + \Sigma_L^{(2)}\right)\right]\varphi^h = 0}$$

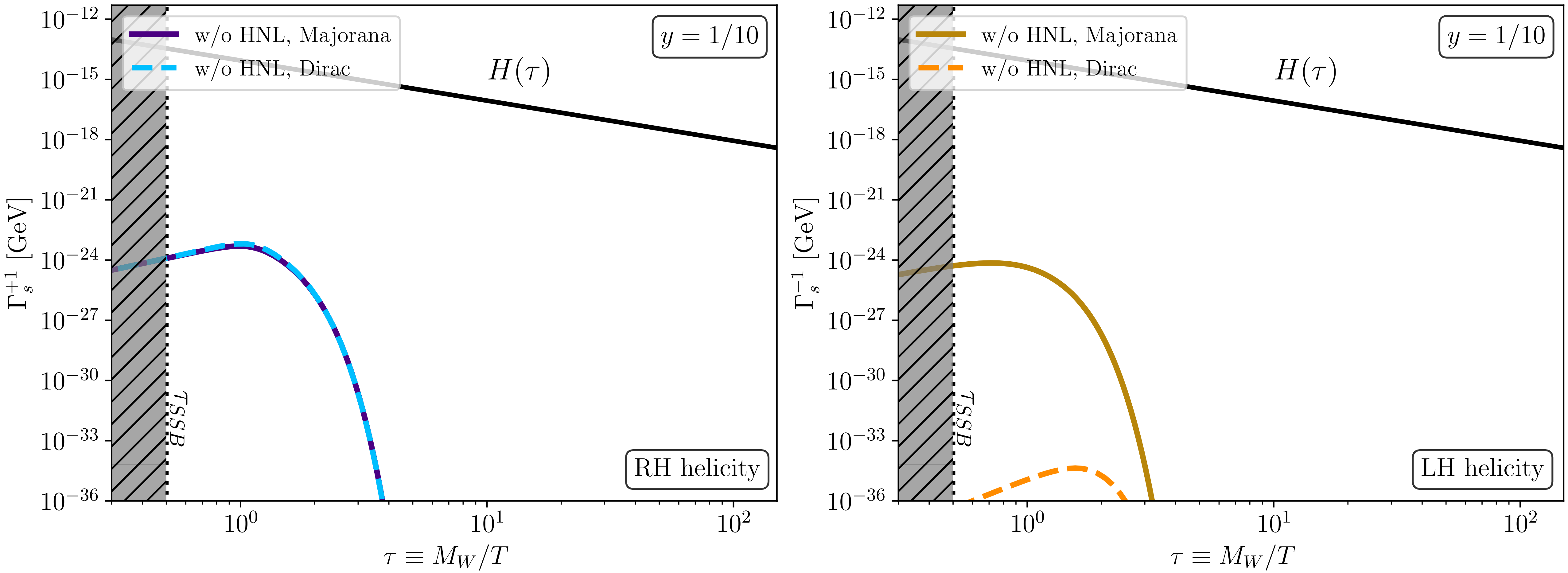
Results: toy 2x2

Single flavour without HNL

$$\Gamma(p_0) = -\left(\frac{\mathrm{Im}\,\bar{\Pi}_{\mathrm{F}}(p_0)}{p_0}\right)$$

H. A. Weldon, Phys. Rev. D 28 (1983), 2007

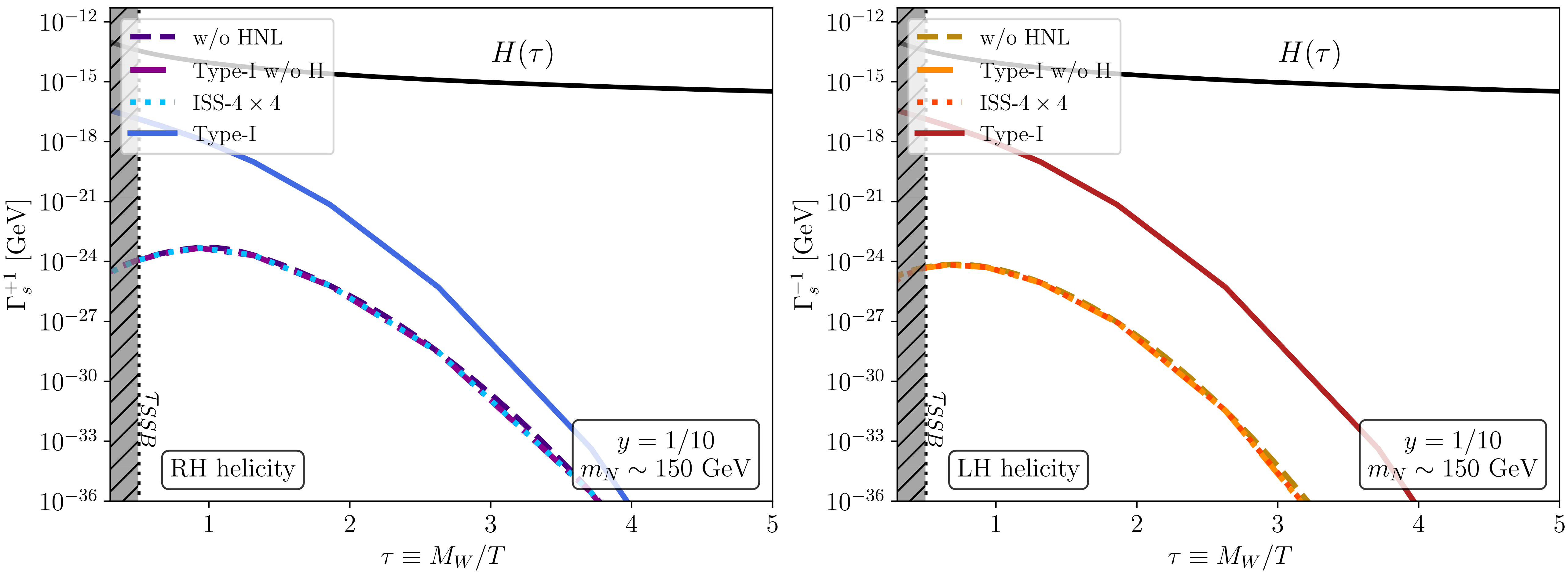
$$y = \frac{p}{T} \qquad \tau = \frac{M_W}{T}$$



$$m_{DM} = 10 \text{ keV and } |\mathcal{U}_{\alpha 4}| \sim 10^{-6}$$

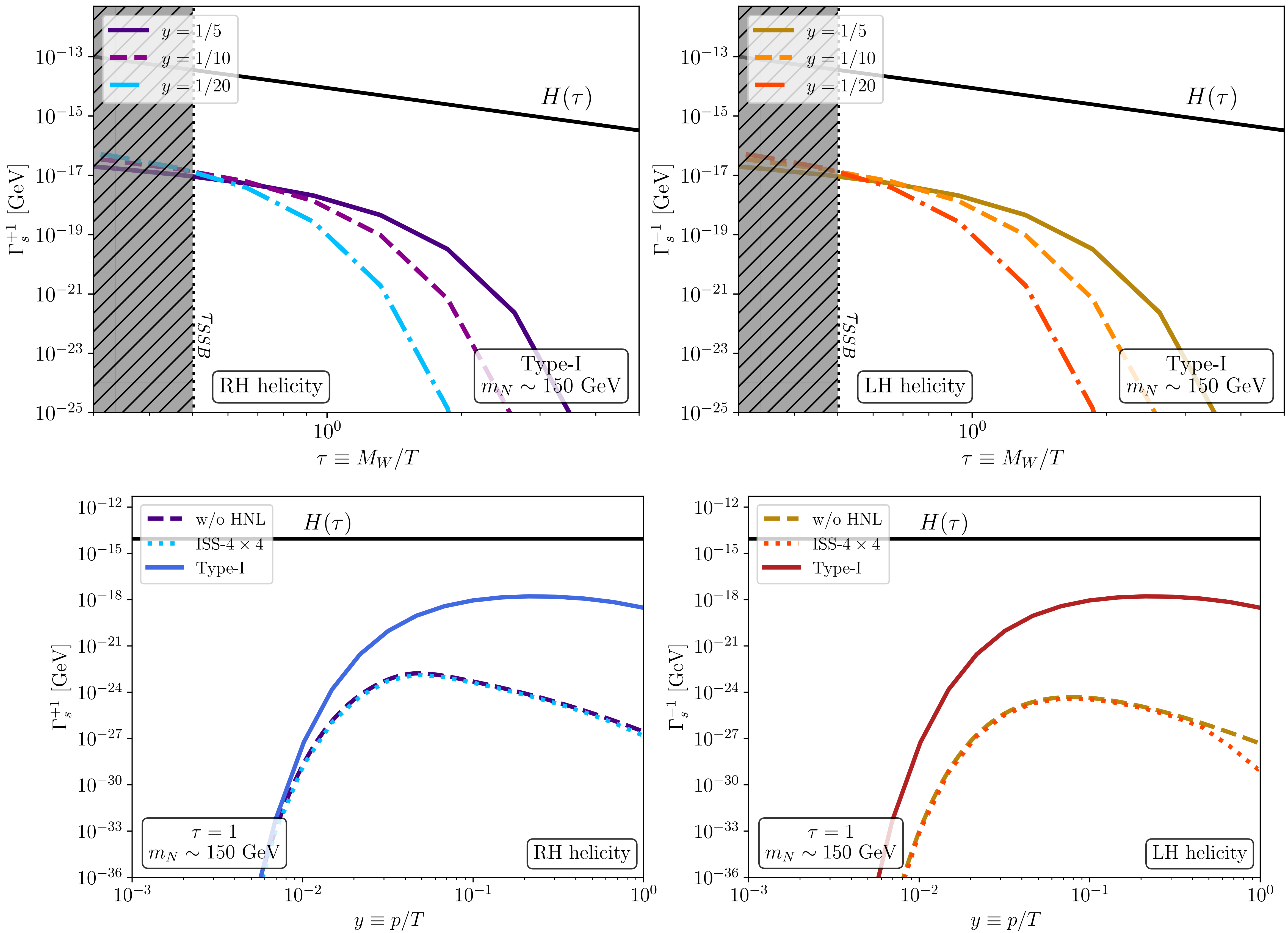
$\Gamma \sim \mathcal{O}(10^{-16})$ GeV is needed to generate the correct DM abundance

Results: 4x4 ISS and Type-I Seesaw



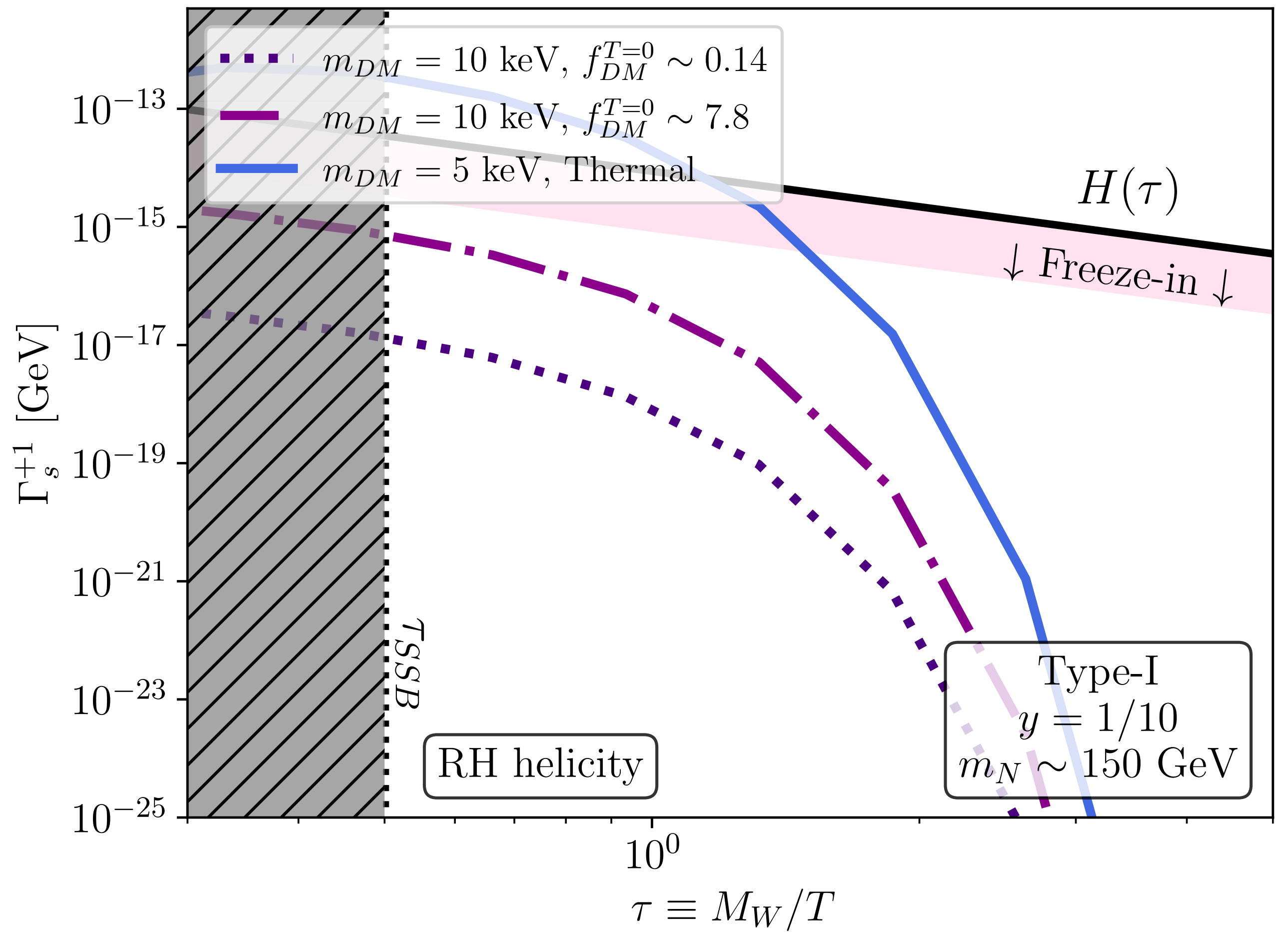
- **The 4x4 ISS is not qualitatively different from the 2x2 toy-model;**
- **Type-I seesaw rates are many orders of magnitude larger;**
- **If we neglect Higgs contributions in Type-I we recover the same rates as toy models**

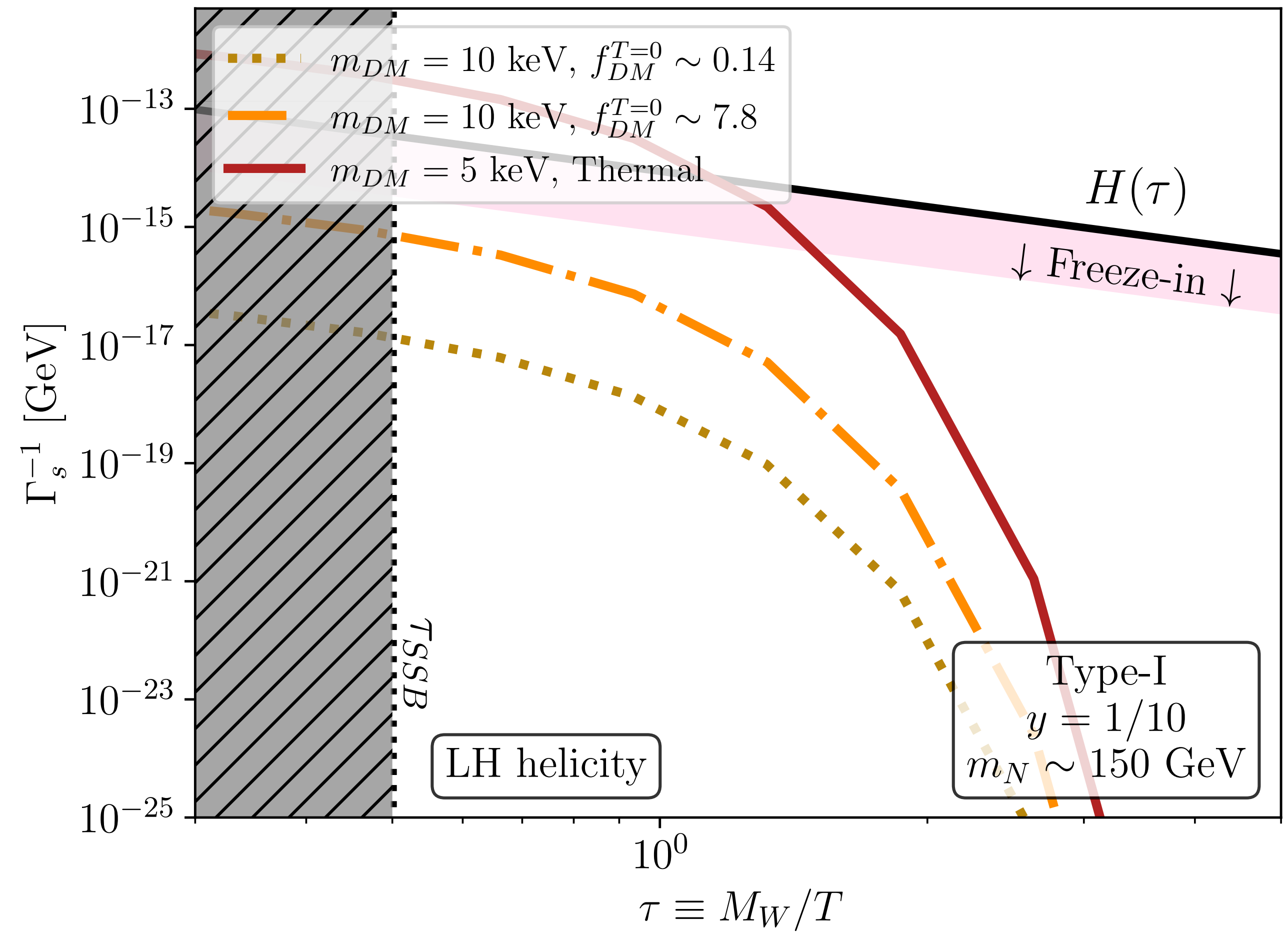
Temperature and momenta dependence



Type-I realisations

Type-I seesaw rates can go from fully freeze-in regime to thermalised scenarios





	$f_{DM}^{T=0} \sim 0.14$	$f_{DM}^{T=0} \sim 7.8$	Thermal
m_{DM} (keV)	10	10	5
ω_{12}	10^{-7}	10^{-7}	$(0.089 + 2i) \times 10^{-8}$
ω_{13}	0	0	$(71 + 3.7i) \times 10^{-9}$
ω_{23}	$9i$	$10i$	$5.5 \times 10^{-10} + 10.5i$

Numerical integration over T and p is currently challenging, due to the computation of the real part being very demanding

Conclusion

Sterile neutrinos are viable DM candidates, and are motivated by massive neutrinos

Many phenomenological constraints: the only known solution in the minimal framework of SM + HNL is the νMSM

Can sterile neutrinos be produced by the decay of heavier particles? Diagrammatically yes

However thermal effects make gauge boson production negligible

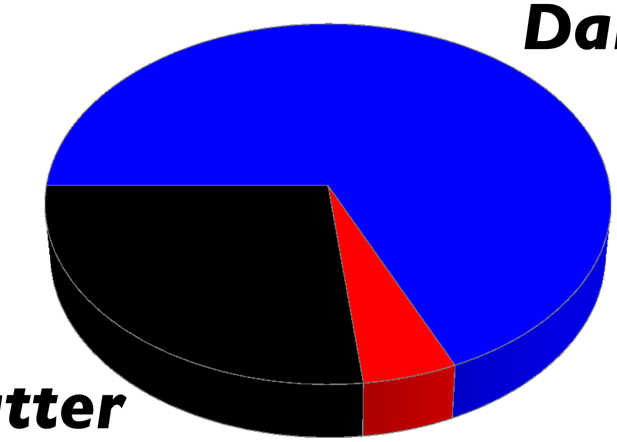
Interestingly, production from HNL decay does not appear to be suppressed!

Need of a realistic full model to accomodate sizeable Yukawas, otherwise neutrinos + DM constraints suppress the production rate

Solutions exist in the zero temperature approximation. Currently working on an effective numerical integration of Boltzmann equations

Backup

Neutrinos as Dark Matter?



$$\begin{aligned} \Omega_{\rm B} h^2 &= 0.02237 \pm 0.00015 \\ \Omega_{\rm DM} h^2 &= 0.1200 \pm 0.0012 \\ \Omega_\Lambda &= 0.6847 \pm 0.0073 \end{aligned}$$

$$h = 0.6736 \pm 0.0054$$

N. Aghanim *et al.* [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO]

Sterile neutrinos can be viable DM candidates: they are produced by oscillations of active ones as long as an active-sterile mixing is present

S. Dodelson and L. M. Widrow, hep-ph/9303287

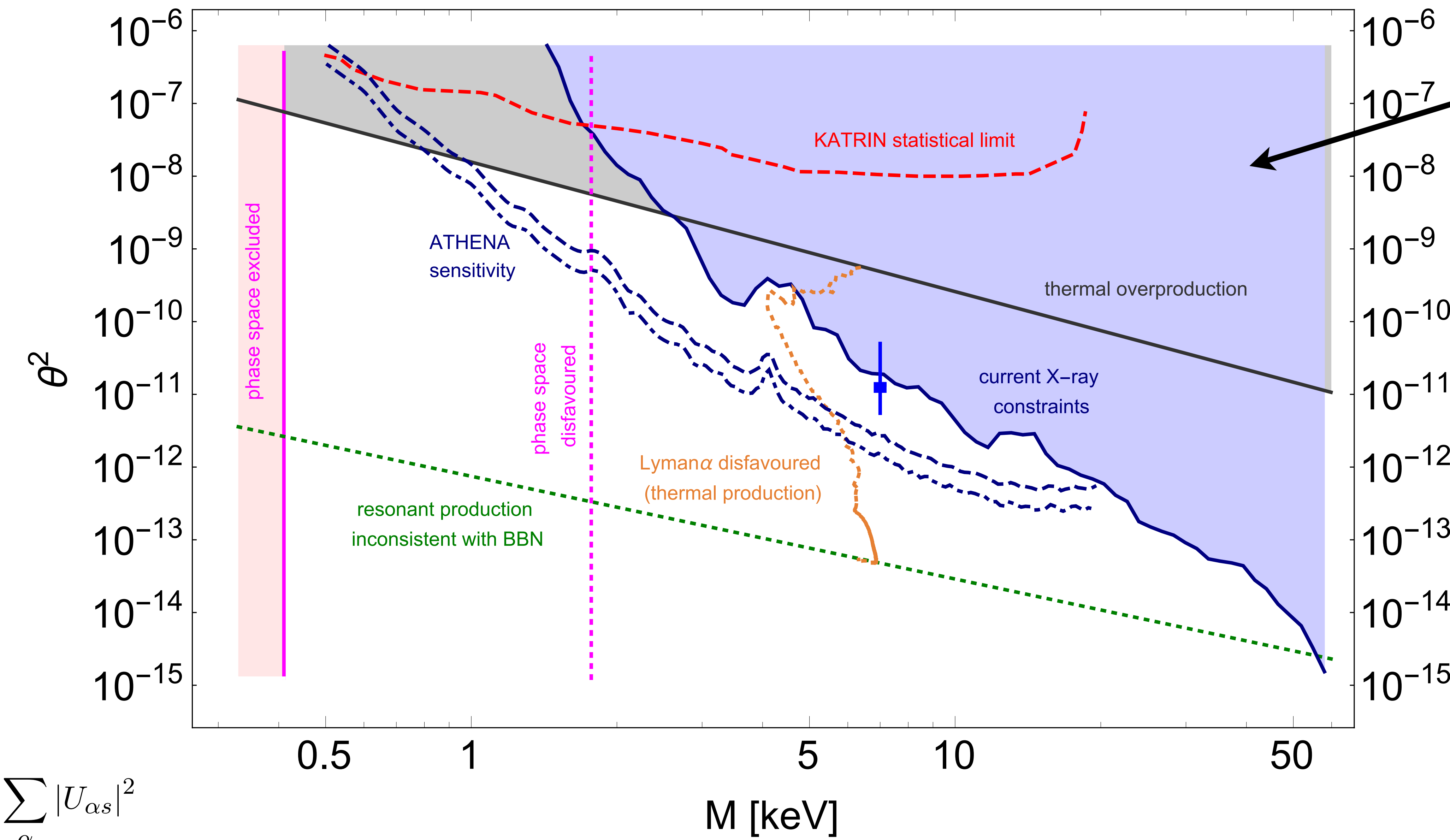
Constraints: abundance

DW: as long as an active-sterile mixing is present, a population of sterile ν is produced by oscillations in the primordial plasma

S. Dodelson and L. M. Widrow, hep-ph/9303287

$$\Omega_s h^2 = 1.1 \cdot 10^7 \sum_\alpha C_\alpha(m_s)\,|U_{\alpha s}|^2 \left(\frac{m_s}{\mathrm{keV}}\right)^2, \quad \alpha = e, \mu, \tau$$

T. Asaka, M. Laine and M. Shaposhnikov, hep-ph/0612182



$$f_{\mathrm{WDM}} \equiv \frac{\Omega_s}{\Omega_{DM}} > 1$$

$$\theta^2 = \sum_\alpha |U_{\alpha s}|^2$$

Figure from A. Boyarsky, M. Drewes, T. Lasserre, S. Mertens and O. Ruchayskiy, arXiv:1807.07938 [hep-ph]

Constraints: phase-space density

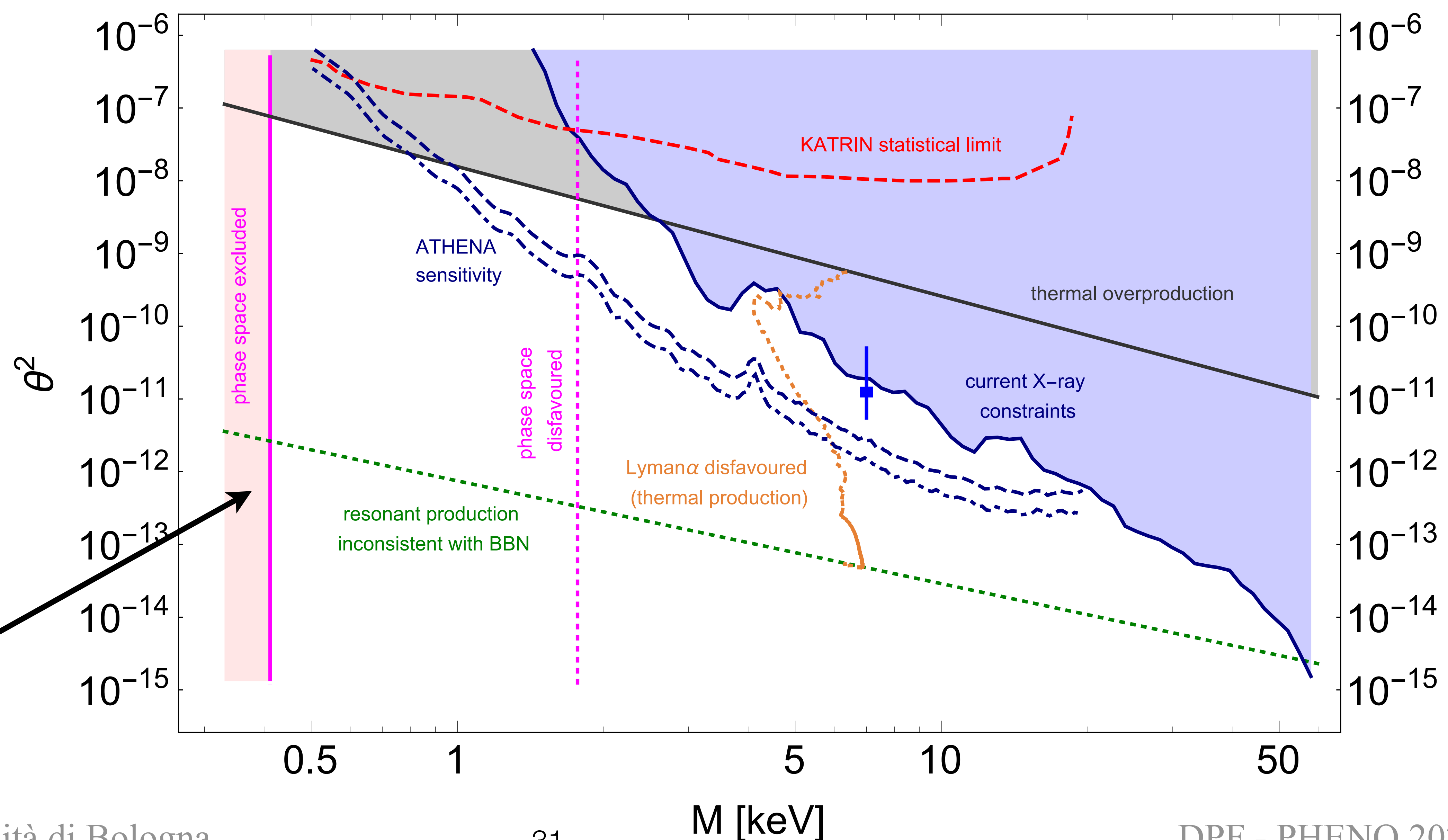
For fermionic DM, Pauli exclusion principle impose a maximum on its distribution function (degenerate Fermi gas). Imposing that inferred phase-space density does not excess this bound, it is possible to extract a lower bound on the DM mass

S. Tremaine and J. E. Gunn, Phys. Rev. Lett. 42 (1979) 407

$$f_{max,\mathrm{NRP}} = \frac{94\ \omega_{\mathrm{DM}}}{2\,(2\pi\hbar)^3}\frac{m_{\mathrm{NRP}}^3}{\mathrm{eV}^3} \quad \Longrightarrow \quad m_{\mathrm{NRP}} > 1.77\,\mathrm{keV}$$

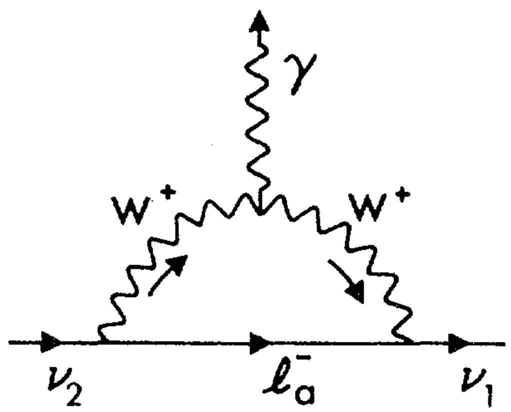
from dSphs observations

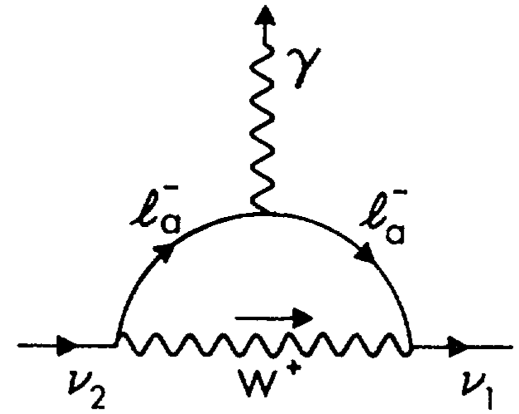
A. Boyarsky, O. Ruchayskiy and D. Iakubovskyi, 0808.3902 [hep-ph]



Too dense phase-space

Constraints: stability and indirect detection (ID)



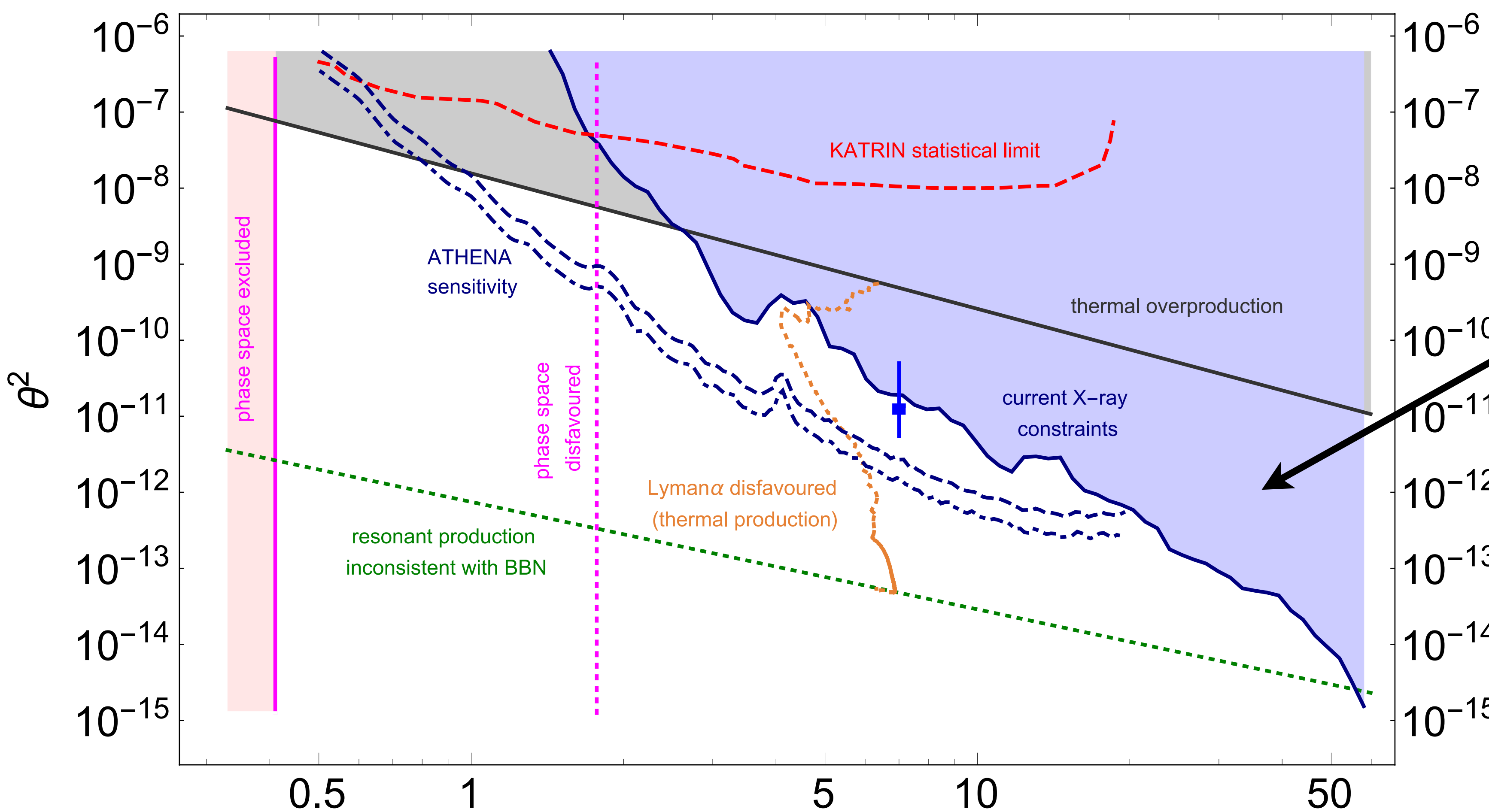


Massive ν can decay radiatively producing monochromatic γ

P. B. Pal and L. Wolfenstein, Phys. Rev. D 25 (1982) 766

Due to the lack of signature (e.g. CHANDRA, XMN)

$$f_{\rm WDM} \sin^2 2\theta \lesssim 1.5 \times 10^{-4} \left(\frac{m_s}{1\rm keV}\right)^{-5}$$



ID excluded

Constraints: Lyman-α

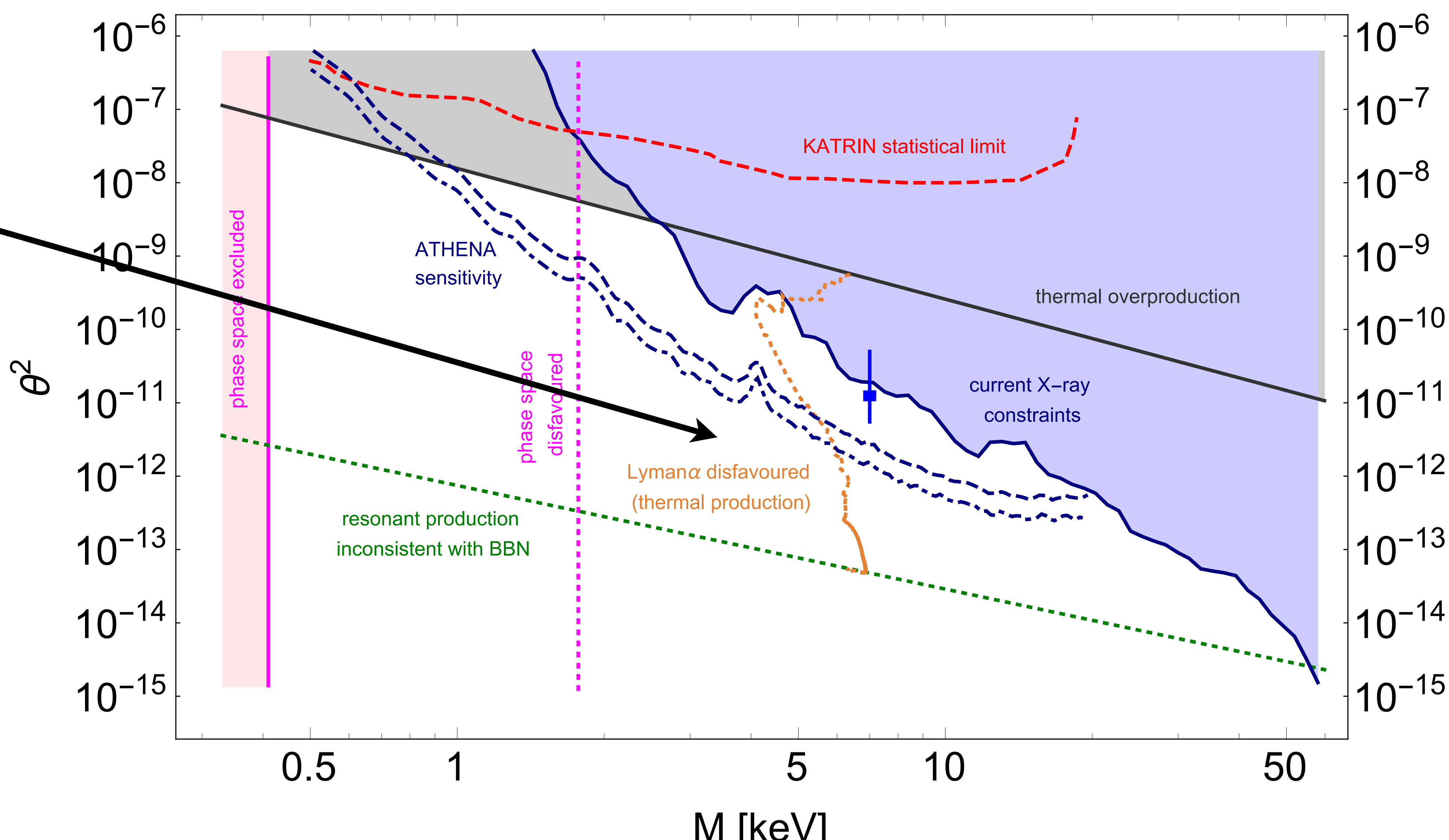
The absorption in the spectra of QSOs by the H (Ly-α: 1s → 2p) in IGM can trace matter distribution at scales: 1-80 h^{-1} Mpc

Narayanan, Vijay K.; Spergel, David N.; Davé, Romeel; Ma, Chung-Pei, Astrophys. J. 543, 103 (2000)

Ly-α constraints highly model dependent → limits for DW produced sterile ***ν***

J. Baur, N. Palanque-Delabrouille, C. Yeche, A. Boyarsky, O. Ruchayskiy, E. Armengaud and J. Lesgourgues, arXiv:1706.03118 [astro-ph.CO]

Ly-α disfavoured



Helicity decomposition

Expand the **chiral** fields in terms of **helicity** eigenstates

$$\Psi_L = \sum_{h=\pm1} v^h \otimes \Psi_L^h \quad ; \quad \Psi_L^h = \begin{pmatrix} \nu_a^h \\ \nu_s^h \end{pmatrix}_L \qquad \Psi_R = \sum_{h=\pm1} v^h \otimes \Psi_R^h \quad ; \quad \Psi_R^h = \begin{pmatrix} \nu_a^h \\ \nu_s^h \end{pmatrix}_R$$

$$\mathbb{L} = (1-\gamma^5)/2 \qquad \mathbb{R} = (1+\gamma^5)/2$$

where $\mathbf{v^h}$ are **eigenstates** of the **helicity** operator

$$\widehat{h}(\widehat{\mathbf{q}}) = \gamma^0 \vec{\gamma} \cdot \widehat{\mathbf{q}}\, \gamma^5 = \vec{\sigma} \cdot \widehat{\mathbf{q}} \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} \qquad \vec{\sigma} \cdot \widehat{\mathbf{q}}\, v^h = h\ v^h \quad ; \quad h = \pm 1$$

The equation of motion are then

$$\boxed{\Big[(\omega^2 - q^2)\mathbb{1} + \big(\omega - hq\big)\big(\mathbb{A} + h\mathbb{B}\big) - \mathbb{M}^2\Big]\Psi_L^h = 0} \qquad \boxed{\Big[\omega - h\,q\Big]\Psi_R^h = \mathbb{M}\,\gamma^0\,\Psi_L^h}$$

with $$\mathbf{\Sigma}^t + \mathbf{\Sigma}(\omega, \vec{q}) = \gamma^0\,\mathbb{A}(\omega, \vec{q}) - \vec{\gamma} \cdot \widehat{\mathbf{q}}\,\mathbb{B}(\omega, \vec{q})$$

$$\mathbb{A}(\omega, \vec{q}) = \begin{pmatrix} A(\omega, \vec{q}) & 0 \\ 0 & 0 \end{pmatrix} \qquad \mathbb{B}(\omega, \vec{q}) = \begin{pmatrix} B(\omega, \vec{q}) & 0 \\ 0 & 0 \end{pmatrix}$$

L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

Propagating states

The dispersion relations for the (quasi) particle states in the medium are given by the complex poles of the propagator (or by the zeroes of the inverse propagator)

$$\left[\mathbb{S}_L^h(\omega,q)\right]^{-1} = \left[(\omega^2-q^2)\mathbb{1} + \big(\omega - hq\big)\big(\mathbb{A}+h\mathbb{B}\big) - \mathbb{M}^2\right]$$

To leading order in θ and assuming $M_1 \ll M_2$ the corrections are given by

$$\boxed{\delta\omega_1^h = -\Big(\Delta_1^h(q)+i\gamma_1^h(q)\Big) - \frac{\theta^2\ \Big(\xi+\Delta_1^h(q)-i\gamma_1^h(q)\Big)}{\left[\left(1+\frac{\Delta_1^h(q)}{\xi}\right)^2+\left(\frac{\gamma_1^h(q)}{\xi}\right)^2\right]} + \theta^2\xi}$$

active-like

$$\boxed{\delta\omega_2^h = \frac{\theta^2\ \Big(\xi+\Delta_2^h(q)-i\gamma_2^h(q)\Big)}{\left[\left(1+\frac{\Delta_2^h(q)}{\xi}\right)^2+\left(\frac{\gamma_2^h(q)}{\xi}\right)^2\right]} - \theta^2\xi}$$

sterile-like

$$\omega = \omega_j(q) + \delta\omega_j^h$$

$$\omega_j(q) = \sqrt{q^2+M_j^2}$$

$$\Delta_j^h(q) + i\gamma_j^h(q) = \frac{\Omega^h(\omega_j,q)}{2q}$$

$$\Omega^h \equiv (\omega - h\,q)\,\big(A(\omega,q)+h\,B(\omega,q)\big)$$

$$\xi = \frac{M_s^2}{2q}$$

L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

Kinetic equations

From the dispersion relations is it possible to define an "effective" mixing angle in medium

$$\theta^h_{eff}(q) = \frac{\theta}{\left[\left(1+\frac{\Delta^h_j(q)}{\xi}\right)^2 + \left(\frac{\gamma^h_j(q)}{\xi}\right)^2\right]^{1/2}}$$

This quantity enters kinetic equation for sterile-like production

$$\frac{dn^h_2(q;t)}{dt} = \Gamma^h_2(q)\Big[n_{eq}(q) - n^h_2(q;t)\Big] \qquad \Gamma^h_{prod}(q) = \Gamma^h_2(q)\, n_{eq}(q)$$

$$\begin{aligned}
\Gamma^-_{prod}(q) &= 2\Big(\theta^-_{eff}(q)\Big)^2 \,\mathrm{Im}\Sigma^-(q)\, n_{eq}(q) \\
\Gamma^+_{prod}(q) &= 2\Big(\theta^+_{eff}(q)\Big)^2 \left[\frac{M_s}{2q}\right]^2 \mathrm{Im}\Sigma^+(q)\, n_{eq}(q)
\end{aligned}$$

L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

Thermal Field Theory in a nutshell

The partition function at finite temperature (β = 1/T) can be formally expressed as a path integral with imaginary time evolution

$$Z = \int \mathcal{D}\phi \langle \phi | \mathrm{e}^{-\beta H} | \phi \rangle = \int \mathcal{D}\phi \exp\left[- \int_0^\beta d\tau \mathcal{L}(\tau) \right]$$

The fields at finite temperature are periodic in the imaginary time component

$$\langle \phi(\mathbf{x},t)\phi(\mathbf{y},0)\rangle_\beta = \frac{1}{Z}\mathrm{Tr}\left[\mathrm{e}^{-\beta H}\phi(\mathbf{x},t)\phi(\mathbf{y},0)\right] = \frac{1}{Z}\mathrm{Tr}\left[\phi(\mathbf{x},t)\mathrm{e}^{-\beta H}\mathrm{e}^{\beta H}\phi(\mathbf{y},0)\mathrm{e}^{-\beta H}\right] = \langle \phi(\mathbf{y},-i\beta)\phi(\mathbf{x},t)\rangle_\beta$$

Kubo-Martin-Schwinger (KMS) condition $\phi(\mathbf{x},0) = \pm\, \phi(\mathbf{x}, i\beta)$

R. Kubo, J. Phys. Soc. Jap. 12 (1957), 570-586; P. C. Martin and J. S. Schwinger, Phys. Rev. 115 (1959), 1342-1373

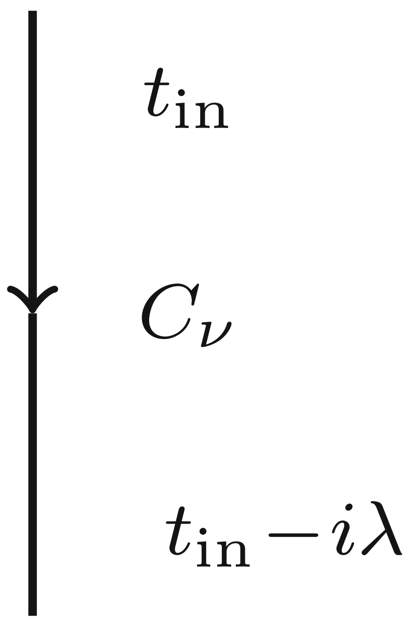
A finite-temperature path-integral formalism can be derived in analogy to the standard QFT treatment: thermal Wick theorem, thermal diagrammatic expansion, thermal propagators, etc…

See e.g. T. Lundberg and R. Pasechnik,arXiv:2007.01224 [hep-th] for a modern review and collection of references

Imaginary vs Real time formalism

It is necessary to specify a contour of time integration in the complex plane

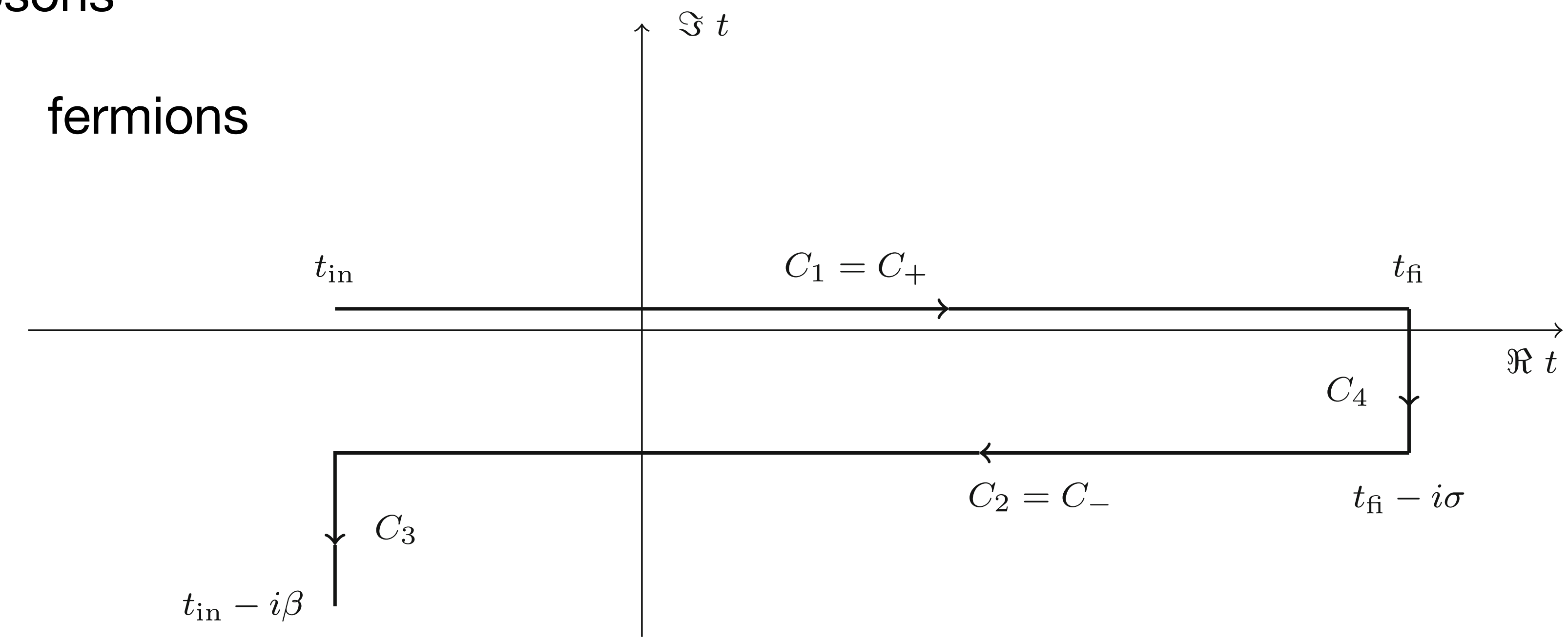
Imaginary time



$\omega_n = \frac{2\pi n}{\beta}$ bosons

$\omega_n = \frac{2\pi(n+1)}{\beta}$ fermions

Real time



\+ Same diagrammatic structure as zero temperature QFT

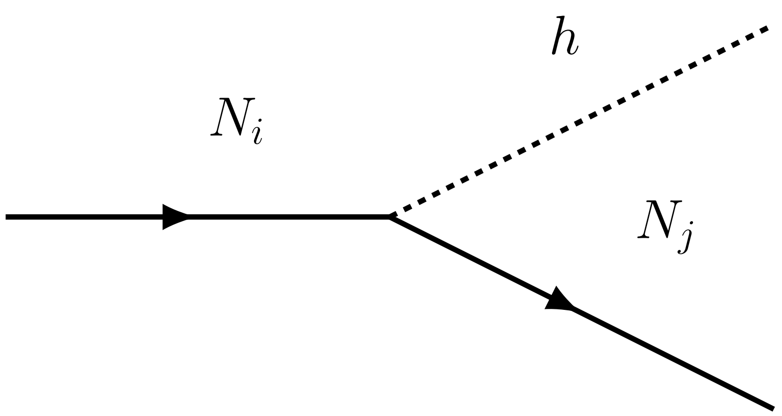
\- Need to perform non-trivial sum on Matsubara frequencies ω_n

\+ Contribution from vertical paths factorises, no discrete sums are needed

\- Propagators are matrices

We make use of the real-time formalism of thermal QFT

Solutions in the zero T approximation



$$\propto \left(\frac{i}{2v}\right)\left[C_{ji}(M_j P_L + M_i P_R) + C^*_{ji}(M_j P_R + M_i P_L)\right]$$

$$C_{ij} = \sum_{\alpha} U^*_{\alpha i} U_{\alpha j}$$

This operator is only present after electroweak symmetry breaking (v≠0)

If $M_i > M_h$ and $M_j \approx$ keV, this decay can produce keV sterile neutrinos (DM) while N_i are in thermal equilibrium

$$\Omega_{\rm DM} h^2 \simeq \frac{1.07 \times 10^{27}}{g_*^{3/2}} \sum_I g_I \frac{m_{\rm s} \Gamma\left(N_I \to {\rm DM} + \text{ anything}\right)}{m_I^2} \varepsilon(m_I)$$

A. Abada, G. Arcadi and M.L., arXiv:1406.6556 [hep-ph]

For general freeze-in studies, see e.g. L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, arXiv:0911.1120 [hep-ph]; X. Chu, T. Hambye and M. H. G. Tytgat, arXiv:1112.0493 [hep-ph]; X. Chu, Y. Mambrini, J. Quevillon and B. Zaldivar, arXiv:1306.4677 [hep-ph]; M. Klasen and C. E. Yaguna, arXiv:1309.2777 [hep-ph]; M. Blennow, E. Fernandez-Martinez and B. Zaldivar, arXiv:1309.7348 [hep-ph]

The ε suppression function

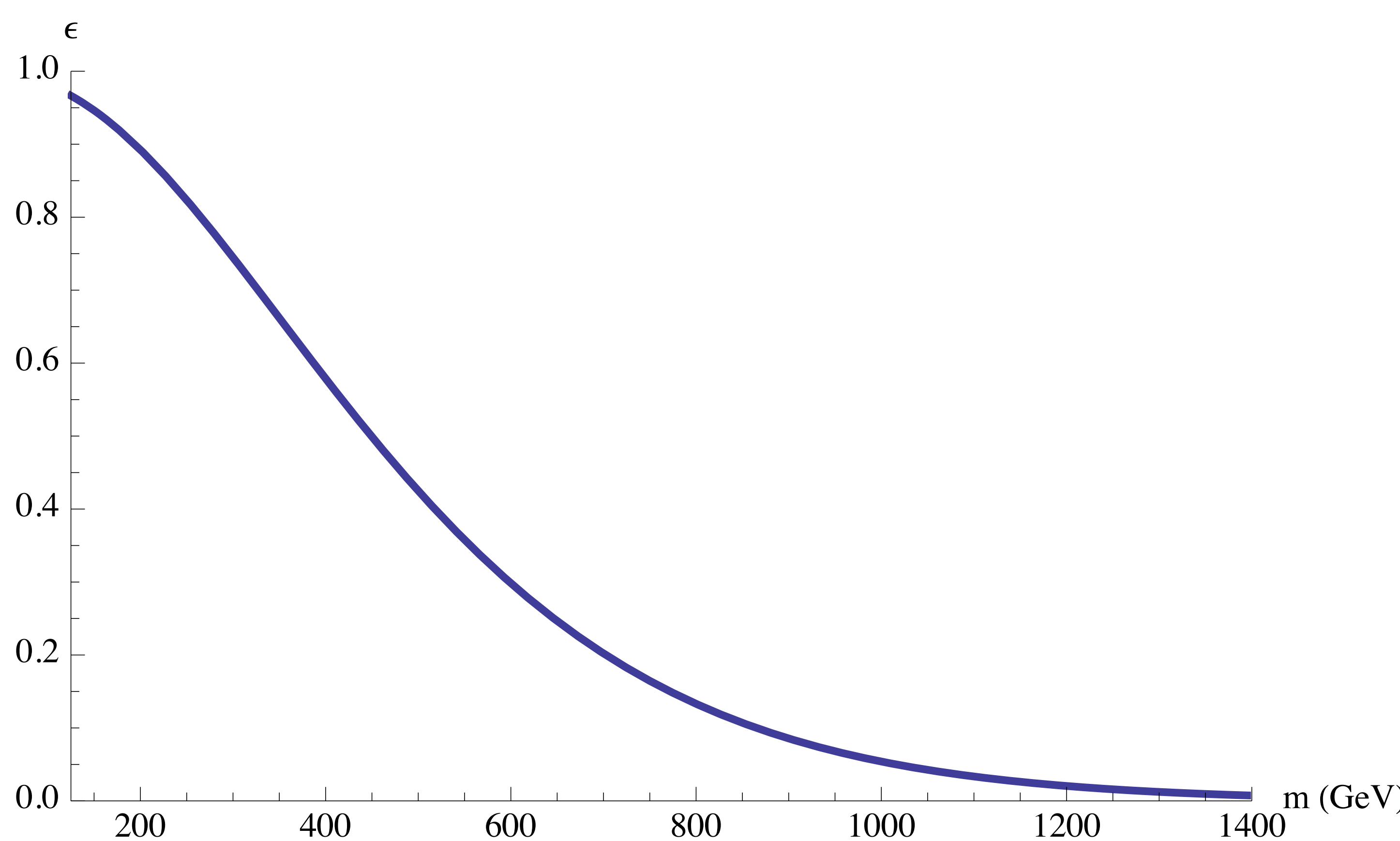
The function ε takes into account the fraction of decays taking place **after** electroweak symmetry breaking

$$\varepsilon(m_I) = \frac{2}{3\pi}\int_0^\infty f(x_I)^2 x_I^3 K_1(x_I) dx_I, \quad x_I = \frac{m_I}{T}$$

with *f(x_I)* tracking the evolution of Higgs vev with temperature

M. D'Onofrio, K. Rummukainen and A. Tranberg, arXiv:1404.3565 [hep-ph]

$$\frac{v(T)}{v(T=0)} = \begin{cases} 1 & T < T_{\rm EW} \\ 8 - \frac{m_I}{20 x_I} & T_{\rm EW} \leq T \leq 160 \text{ GeV} \\ 0 & T > 160 \text{ GeV} \end{cases} \qquad T_{\rm EW} \approx 140 \text{ GeV}$$



Reproducing neutrino data

To conveniently reproduce neutrino oscillation data we employ the Casas-Ibarra parametrisation

J. A. Casas and A. Ibarra, arXiv:hep-ph/0103065 [hep-ph]

$$m_D \equiv vF = -i\, \underbrace{U^*_{\mathrm{PMNS}}}_{\text{Mixing matrix}} \sqrt{\hat{m}_\nu}\, R \sqrt{M}$$

Yukawa couplings (F)

Mixing matrix (U^*_{PMNS})

Light neutrino masses ($\hat{m}_\nu$)

Heavy neutrino masses (M)

where R is an orthonormal matrix parametrised by 3 complex angles ω_{ij}

$$R = V_{23}V_{13}V_{12}, \qquad \text{with} \qquad V_{12} = \begin{pmatrix} \cos\omega_{12} & \sin\omega_{12} & 0 \\ -\sin\omega_{12} & \cos\omega_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

Two dark matter populations

Dodelson-Widrow production

$$\Omega_{\rm DM}^{DW} h^2 = 0.11 \cdot 10^5 \frac{M_1}{\rm keV} \sum_\alpha C_\alpha(M_1) \left| \left(U^*_{\rm PMNS} \sqrt{\frac{\hat{m}}{\rm eV}} R \right)_{\alpha 1} \right|^2$$

$\Omega_{DM} < 0.12$ sets strong upper bounds on R_{i1}

Freeze-in production

$$\Omega_{\rm DM}^{FI} h^2 = 2.16 \cdot 10^{22} \sum_{J=2,3} \left| \left(R^\dagger \frac{\hat{m}}{v} R \right)_{1J} \right|^2 g_J \left(1 - \frac{m_h^2}{M_J^2} \right)^2 \varepsilon\left(M_J\right),$$

For sub-eV neutrinos $(\hat{m}/v)^2 \lesssim 10^{-23}$ giving the correct abundance for R ≈ 1

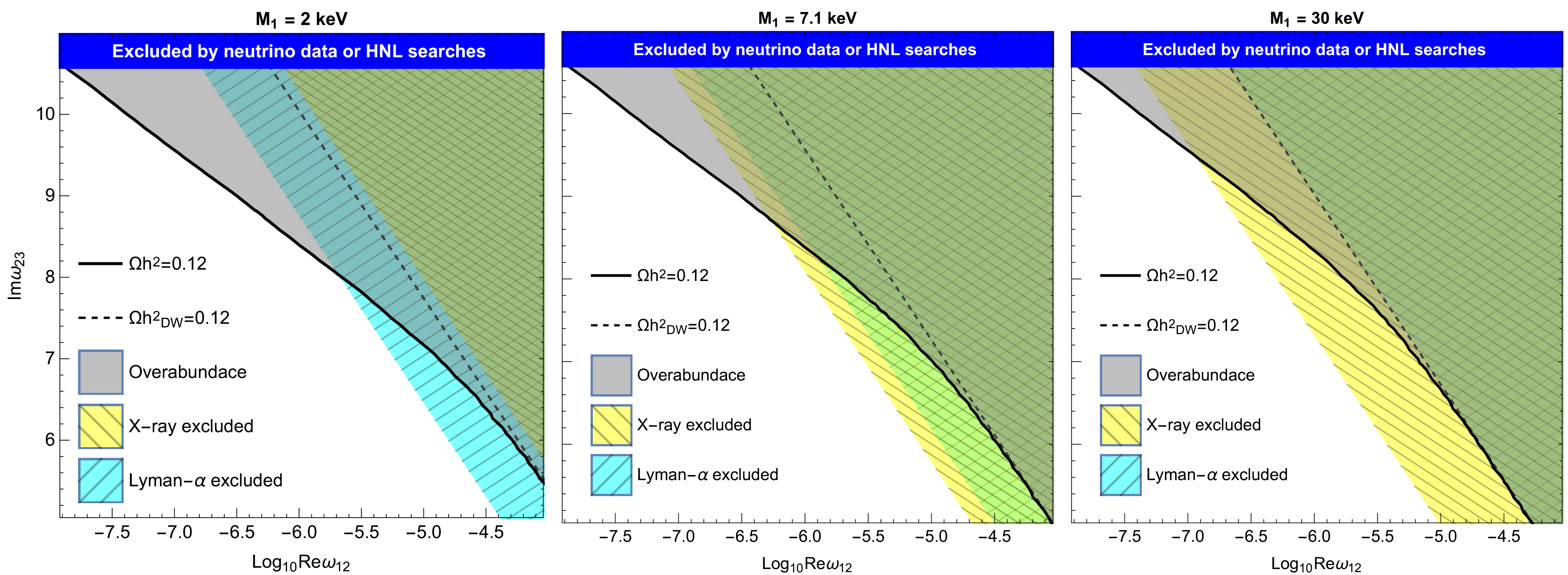
Freeze-in produced DM has a colder spectrum than DW

A. Boyarsky, M. Drewes, T. Lasserre, S. Mertens and O. Ruchayskiy, arXiv:1807.07938 [hep-ph]; A. Abada, G. Arcadi and M. Lucente, arXiv:1406.6556 [hep-ph]; F. Bezrukov and D. Gorbunov, arXiv:1403.4638 [hep-ph]; M. Shaposhnikov and I. Tkachev, arXiv:hep-ph/0604236 [hep-ph]; A. Merle and M. Totzauer, arXiv:1502.01011 [hep-ph]; J. Heeck and D. Teresi, arXiv:1706.09909 [hep-ph]

Example of solutions

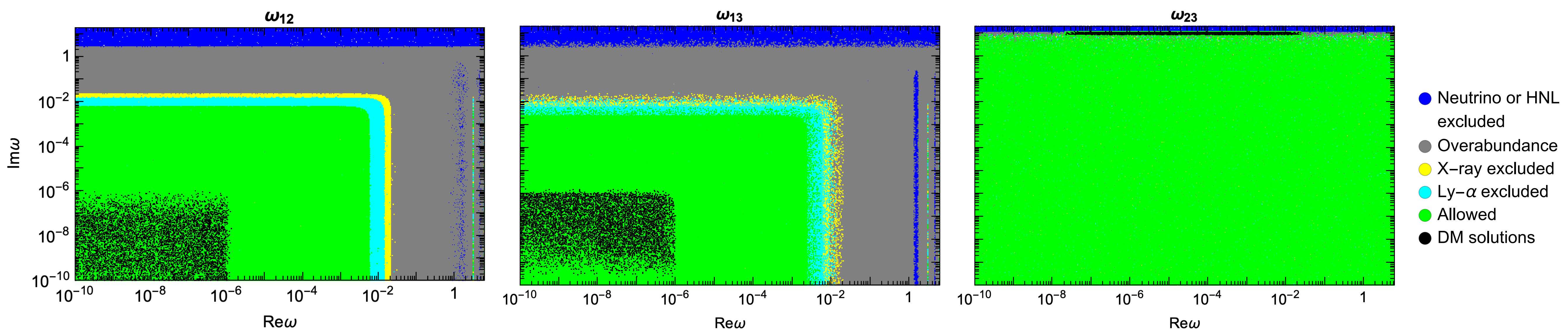
We fix for definitiveness the neutrino oscillation parameters to their best-fit values in the NO case

and $m_1 = 0$, $M_2 = M_3 = 300$ GeV, $\omega_{13} = 0$, $\mathrm{Im}\omega_{12} = 0$, $\mathrm{Re}\omega_{23} = 0$



No solution for $M_1 > 60$ keV ($M_1 > 49$ keV for IO)

General structure of solutions



$M_1 \approx$ keV
$M_{23} > 125$ GeV

Hierarchy of masses

$|\omega_{1j}| \ll 1$
$\mathrm{Im}\,\omega_{23} \simeq 10$

Hierarchy in the CI complex angles

<u>Fine-tuned solutions?</u>

Lepton number symmetry

In the limit

$$\begin{aligned} M_1 &\to 0 & \qquad \omega_{1i} &\to 0 \\ M_2 &\to M_3 & \qquad \omega_{23} &\to \pm i\infty \end{aligned}$$

the Lagrangian acquires a global lepton number symmetry. The mass spectrum becomes

- 3 massless active neutrinos
- 1 massless decoupled state N_1
- 1 Dirac heavy neutrino (linear combination of N_2 and N_3)

Approximate lepton number symmetry

Because neutrinos have (tiny) masses, the symmetry must be broken at some level. In the scenario:

$$\begin{aligned} M_1 &\ll M_{2,3} \\ M_2 &\simeq M_3 \end{aligned} \qquad\qquad \begin{aligned} |\omega_{1i}| &\ll 1 \\ |\mathrm{Im}\,\omega_{23}| &\gg 1 \end{aligned}$$

the mass spectrum features

- 3 light (massive) active neutrinos
- 1 light sterile neutrino with mass M_1 (e.g. keV)
- 2 heavy Majorana neutrinos with almost degenerate masses forming a pseudo-Dirac pair (e.g. EW scale)

Moreover, the approximate symmetry protects light neutrino masses from large loop corrections even if sizeable Yukawas are present