THERMAL EFFECTS IN FREEZE-IN NEUTRINO DARK MATER PRODUCTION

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Based on:

A. Abada, G. Arcadi, ML, G. Piazza and S. Rosauro-Alcaraz, **JHEP 11 (2023) 180** ML, **Phys.Lett.B 846 (2023) 138206**

ALMA MATER STUDIORUM UNIVERSITÀ DI BOLOGNA

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Observational problems of the SM Section 7.2 and Planck Collaboration X (2020) discuss the implems of the SM

Two seemingly unrelated observations cannot be accounted for in the Standard Model dependent scalar spectral index. *3.5. Matter densities* matter density reduces the small-scale CMB power. The matter duservations cannot be e Standard Model strained at the percent level: ⌦c*h*² ⁼ ⁰.¹²⁰⁰ [±] ⁰.⁰⁰¹² (68 %, *Planck* TT,TE,EE

The Universe has a dark matter

 0.12 0.1420 0.0011 **component**

N. Aghanim *et al.* [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO] $\Omega_{\rm m}h^2 = 0.1430 \pm 0.0011$ st al. p failed conaboration, always tool tooloo $\Omega_c h^2 = 0.1200 \pm 0.0012$ density parameter is measured at sub-percent level at sub-percent level at sub-percent level accuracy with the $\Omega_{\rm m}h^2=$ $\Omega_b h^2 = 0.02237 \pm 0.00015$ $\sqrt{2}$ astro-ph COI

The matter density can be measured from the \sim

The natural (simple) way

Complete the SM field pattern with **right-handed neutrinos**

Figure from S. Alekhin *et al.*, arXiv:1504.04855 [hep-ph]

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Left chirality

Neutrino masses and dark matter

Type-I seesaw mechanism: SM + gauge singlet fermions *NI* \mathcal{L} = $\mathcal{L}_{\text{SM}} + i\overline{N_I}\partial N_I \sqrt{ }$ $F_{\alpha I} \overline{\ell_L^{\alpha}} \phi N_I +$ M_{IJ} 2 $\overline{N_I^c} N_J + h.c.$

After electroweak phase transition < Φ > = $v \approx 174$ GeV α *R* α *P* α = α = 174 GeV
 Extra α

$$
m_{\nu}=-v^2F\frac{1}{M}F^T
$$

The new fields N_I can be viable DM candidates:

- **No** *L* \blacksquare \blacksquare **nagnetic int**
 S .eractior *MIJ*
	- **Potentially long-lived**
	- Produced in the early Universe

Neutrino masses and dark matter

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The new fields *N_I* can be viable DM candidates:

- **No electromagnetic interactions** *L* \blacksquare \blacksquare ! **nagnetic int** .eractior
. *MIJ*
	- **Potentially long-lived**
	- **• Produced in the early Universe** *Rj*%

Michele Lucente - Università di Bologna DPF - PHENO 2024 $\frac{2}{3}$

WDM constraints

DW produced sterile v are warm dark matter

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Known solution: The νMSM

T. Asaka, S. Blanchet and M. Shaposhnikov, hep-ph/0503065 T. Asaka and M. Shaposhnikov, hep-ph/0505013 M. Shaposhnikov and I. Tkachev, hep-ph/0604236

Type-I Seesaw with a phenomenologically motivated mass spectrum

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νMSM dark matter solution

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Can we think of other production *{* (3) α **mechanisms?** \blacksquare *\bl <u>folleti</u>* keV)−⁵

In experimental searches, HNL are looked for in W, Z mediated channels *Nj*

Z 1 **In principle we have the necessary ingredients for successful production of DM**

- Parent particle in thermal equilibrium by EW interactions
- DM coupling suppressed by small active-sterile mixing

L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, arXiv:0911.1120 [hep-ph]

out (solid coloured) and freeze-in via a Yukawa interaction (dashed coloured) as a **Does** it work?

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Thermal effects suppress the rate spontaneus symmetry symmetry and in unitary gauge the Yukawa coupling yields yields and in unitary coupling yields and the Yukawa coupling yields and the Yukawa coupling yields and the Yukawa coupling yields and the Vulkar we are the that mixes of the Higgs and the Higgs and the Higgs and the Higgs and the mixes of production on pro
Active-sterile coupling, since with the production of production on production on production on production on s_{max} symmetry breaking and in unitary gauge the \sim -vector mass terms ι nermal eπects suppress the rate Thermal effects suppress the rate

Simplified scenario - Standard Model with only one leptonic generation: one active neutrino and its charged lepton partner and one SU(2) singlet Dirac sterile neutrino Simplified scenario - Standard Model with only Simplified scenario - Standard Mode # ν^a ιeι
κα o - Stan .
1 ν1 ν1 **ια**
Σ $\ddot{}$, US SUPP
del with only a cos(θ) sin(θ) **1e reptonic gener**a
∪(2) singlet Dirac s **n**: one active

$$
\mathcal{L} = \mathcal{L}_{SM} + \overline{\nu}_s \, i \partial \nu_s - \overline{\nu}_{\alpha} \, \mathbb{M}_{\alpha\beta} \, \nu_{\beta} + \text{h.c} \, ; \, \alpha, \beta = a, s \, ,
$$

where a, s refer to active and sterile respectively and L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph] where a, s refers to active and sterile respectively and sterile respec Pisarski, ar<mark>)</mark> 09.07647 [hep-ph] L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-porcess)

$$
\mathbb{M} = \left(\begin{array}{cc} 0 & m \\ m & M_s \end{array}\right)
$$

$$
\begin{pmatrix} \nu_a \\ \nu_s \end{pmatrix} = U(\theta) \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} ; U(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}
$$

In the seesaw limit: m $\ll M$.

 \sim th ϵ $\mathsf{In the \; seesaw \; limit: \; m \, \ll \, M_s}$

$$
M_1 \simeq -\frac{m^2}{M_s} \quad ; \quad M_2 \simeq M_s \quad ; \quad \sin(2\theta) \simeq 2\theta \simeq \frac{2m}{M_s} \ll 1
$$
\nactive v

\nDM

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10

One-generation seesaw \mathbf{R} √ e era eigen ein ein ein ein der Stadt $\overline{(\nu)}$ Γ = (νa, νs) the flavor doublet ΨT $_{\rm eff}$ = (va, \sim 92–94](for details see the flavor basis is \sim 0.94](for details see the flavor basis is \sim appendix in ref.[93]).

Introducing the flavour doublet $\qquad \Psi = \begin{pmatrix} \nu_a \ \nu_{_S} \end{pmatrix}$

$$
\Psi = \begin{pmatrix} \nu_a \\ \nu_s \end{pmatrix}
$$

The equations of motion in flavour basis are
Where **1** one-particle irreducible self-energy includes local tadpole (Σt) and non-local dispersive (Σr(⃗x−⃗x′

$$
\left[\left(\gamma_0 \omega - \vec{\gamma} \cdot \vec{q} \right) \mathbb{1} - \mathbb{M} + \left(\sum_{\mathbf{t}}^t + \Sigma(\omega, \vec{q}) \right) \mathbb{L} \right] \tilde{\Psi}(\omega, \vec{q}) = 0
$$

tadpole

Σ are the self-energy corrections, which are diagonal in flavour basis Σ are the self-energy corrections, which are diagonal in flavour basis It are the contently corrections, minon are all gonal in havear basis **z** are the se

The presence of Σ generally changes the mixing angle with respect to zero temperature The presence of Σ generally changes the mixing angle with respect to zero temperature

L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

Michele Lucente - Università di Bologna 11 DPF - PHENO 2024 $\sum_{i=1}^{n}$

Single generation production rates distribution as a function of τ, as the rate vanishes for large τ, larger values of y freeze-out at larger τ but with much single generation production rations and vector bosons are suppressed at \sim and are further suppressed by the detailed by the detailed by the \sim the study with cosmological expansion in the next sections we take as a reference mass that of the W vector boson Single generation production rates respectively and fig. (12) shows R+(t, y) vs. t for y = 1, 3, 5 respectively. Together the "filling" show the " α and α take longer to be populated and freeze out later, but the populated and freeze out is strongly in α which is shown in fig. (10). Although we have set the lower limit to the lower limit to consistency in (VII.11 Single generation production rates

By computing the finite-temperature self $y = \frac{q}{T}$ $\tau \equiv \frac{M}{T}$ \overline{q} M_W By computing the finite-temperature self q $y = \frac{y}{T}$ $\tau \equiv \frac{y}{T}$ $\frac{q}{T}$ $\tau \equiv \frac{M}{T}$ $\tau \equiv$ re inite-temperature seil by computing the imite-temperature seil
energy one finally obtains the production rates energy one finally obtains the production rates $y = T$ and $y = T$ energy one finally obtains the production rates M_{\odot} 1 $\int_0^\infty dr^{-1} (\tau, r)$ and defining $\int_0^\infty d\tau$ $\frac{1}{2}(y) = \int_{1}^{\infty}$ $dn_2^+(\tau, y)$ M_s \setminus^2 $\bigwedge M_s$ \setminus^4 $n_2^+(y)=0.92\times 10^{16}\,\theta^2$ $F^{\pm}(y)$ $n_2^ \frac{(1, y)}{d\tau} d\tau = 0.92 \times 10^{16} \theta^2$ $F^-(y)$ $n_2^+(y)=0.92\times 10^1$ $\begin{array}{ccccc} \hline L^{(1)} & & \hline L^{(2)} & & \hline \ \hline \end{array}$ = 0.10 $\begin{array}{ccccc} \hline L^{(2)} & & \hline L^{(1)} & & \hline \end{array}$ M_W ' M_W $\mathcal{F}(\mathcal{Y})$ $d\tau$ (M_W) = (3) \sim 2 (3) e−n 2/4y
e−n 2/4y = n 2/4y = $\overline{}$ <u>)</u> e−n y − (−1)n y − (−1 \sim $\frac{100000}{\sqrt{100000}}$ 1 − e−t 2/4y e−t 2/4y e−t 2/4y e−t 2/4y e−t 2/4y e− \sum \Box ² (y) is shown in fig. (13), it is dominated by the region 0 < y ! 8 with \setminus $\begin{array}{|c|c|c|c|c|}\n\hline\n & \text{I} & 0.08 & \text{+} \\
\hline\n & & & & & \\
\hline\n & & & & & & \\
\hline\n\end{array}$ 80000 $\begin{bmatrix} 1 & 0.08 \\ 1 & 0.08 \end{bmatrix}$ the sum of the sum of the contributions $\mathbb{F}(\mathcal{X})$ $\frac{1}{3}$. \overline{a} 60000 0.06 , \mathbb{R} $\begin{array}{|c|c|c|c|c|}\n\hline\n&0.04 &\mbox{\LARGE\end{array}$ 0.04 40000 We note that the distribution function function function function function $\mathbb H$. Only peaked at small momenta $\mathbb H$ $\begin{array}{c|c} 0.02 & \text{if} & \$ 20000 the positive helicity component \mathbb{R}^n . The reason for this compare figures (10 and 13) \downarrow \mathcal{L} ($\frac{1}{10}$ 1 4
4
4 year - Ann 1 M^W 8 * |
|
|-* discrepancy is the fact that the production rate for the negative helicity component features a competition between $\mathbf{0}$ $\begin{array}{c|c|c|c|c} \hline \rule{0pt}{1ex} & \rule{0pt}{1ex} & \rule{0pt}{1ex} \hline \rule{0pt}{1ex} & \rule{0pt}{1ex} & \rule{0pt}{1ex} \end{array}$ 0.00 $\begin{array}{|c|c|c|}\n\hline\n2 & 3 \\
\hline\n\end{array}$ \overline{z} $\frac{1}{5}$ Ω $\overline{4}$ $\mathbf y$ damping rate, but an increasing $\mathbf w$ increases (temperature decreases). This increases (temperature decreases). This is the set of $\mathbf w$ increases (temperature decreases). This is the set of $\mathbf w$ increases (te $\frac{1}{\sqrt{2}}$ $\mathcal T$

By integrating these expressions the fraction of DM results number of ultrarelativistic degrees of freedom at decoupling (freeze-out) which occurs at T^f ≃ 5 − 8 GeV, yielding

$$
\mathcal{F}_2 = 0.97 \left(\frac{\theta^2}{10^{-8}}\right) \left(\frac{M_s}{\text{MeV}}\right)^3 \left[1 + \left(\frac{M_s}{8.35 \text{ MeV}}\right)^2\right]
$$

These masses/mixings result in a too short-living sterile neutrino

dominates for M^s ≫ 8.35 MeV. L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

Michele Lucente - Università di Bologna (12) 12 The vanishing of F − Children of the vanishing of the vanishing of the imaginary part of ≃ KeV warm dark matter component, since the X-ray data constrains such component to the mass range ≃ few KeV

End of the story?

Not really! This conclusion was derived in a simple toy model

If we want to explain neutrino masses, more ingredients are necessary

$$
\mathcal{L} = \mathcal{L}_{\rm SM} + i \overline{N_I} \partial N_I - \left(F_{\alpha I} \overline{\ell_L^{\alpha}} \widetilde{\phi} N_I + \frac{M_{IJ}}{2} \overline{N_I^c} N_J + h.c. \right)
$$

- **•** Include multiple right-handed neutrinos: at least 3, two to generate neutrino masses and the other as DM candidate generate neutrino masses and the other as DM candidate $\overline{2}$ *k* right-handed neutrinos: at least 3, two
o masses and the other as DM candidate *^L*φν" *Ri* − *F*[∗]
	- **• Include the Higgs sector!**

Key differences

- **• RHN masses and Yukawa couplings relatively unconstrained** (while gauge couplings and boson masses are fixed);
- **• RHN have direct couplings with the Higgs** (additional production channels)
- **• Neutrinos are Majorana particles**

The model(s) Dirac neutrinos, here we will instead consider them to be Majorana. When referring to this

We consider increasing levels of complexity to single-out the relevant dynamics sider inorecaing levele of complexity to cinele out the relevent dynam sider increasing levels of com

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Real-time formalism of thermal QFT becomes a 2 matrix. For a particle of mass α mass α mass α mass α in the symmetric and in the symmetric a **In the** *real-time formalism* of the rmal C becomes a 2 ⇥ 2 matrix. For a particle of mass *m*, taking the symmetric-Keldysh contour and in the >>>>< *D*˜() ↵ (*k*)=[*D*˜(++)(*k*)]⇤*d*↵*, D*˜(+) ↵ (*k*) = *^ek*0*/*2*f*(*k*0)2⇡"(*k*0)(*k*² *^m*2)*d*↵*, FREAL* is the function of the function $\boldsymbol{\theta}$ *ek*⁰ ⌘ with ⌘ = *±*1 for bosons and fermions, respectively, and is the inverse of the temperature of the

Four types of propagators , (16) **, (16)** , (16) , (

$$
\begin{cases}\n\tilde{D}_{\alpha\beta}^{(++)}(k) = \left[\frac{i}{k^2 - m^2 + i\epsilon} + 2\pi\eta f(|k_0|) \delta(k^2 - m^2)\right] d_{\alpha\beta}, \\
\tilde{D}_{\alpha\beta}^{(--)}(k) = \left[\tilde{D}^{(++)}(k)\right]^* d_{\alpha\beta}, \\
\tilde{D}_{\alpha\beta}^{(+-)}(k) = e^{\beta k_0/2} f(k_0) 2\pi \varepsilon(k_0) \delta(k^2 - m^2) d_{\alpha\beta}, \\
\tilde{D}_{\alpha\beta}^{(--)}(k) = \eta \tilde{D}_{\alpha\beta}^{(+-)}(k),\n\end{cases}
$$

$$
f(k_0) = \frac{1}{e^{\beta k_0} - \eta} \qquad d_{\alpha\beta} = \begin{cases} 1, & \text{for scalars,} \\ k \pm m\mathbb{1}, & \text{for fermions,} \\ -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}, & \text{for vector bosons} \end{cases}
$$

Sonie relations exist that can simplify computations with ⌘ = *±*1 for bosons and fermions, respectively, and is the inverse of the temperature of the **Some relations exist that can simplity comp** Some relations exist that can simplify computations **Uttile retarders chist friat care simplify computations**

$$
\text{Im}\sigma^{R} = -\cosh\left(\frac{\beta|p_{0}|}{2}\right) \left[\Theta(p_{0}) - \Theta(-p_{0})\right] i\sigma^{(+)}\n\qquad
$$
\n
$$
\text{Re}\sigma^{R} = \frac{1}{\pi} \int_{-\infty}^{\infty} dq_{0} \mathcal{P}\left[\frac{\text{Im}\sigma^{R}(q_{0})}{p_{0} - q_{0}}\right]
$$
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15 - Universita di Bologna di Retarde that independent self-energy that will even the relationship of the

Finite temperature dispersion relations Finition in Finiti

$$
\Sigma(\omega, \vec{q}) = \text{Re}\Sigma(\omega, \vec{q}) + i \text{Im}\Sigma(\omega, \vec{q})
$$

⃗) = ¹ **in the medium "Index of refraction"** ealum n = Re

particle excitations # **"Damping rate" of (quasi)** index *k* runs over mass eigenstates. Thus, we can still make use of the dispersion relation for *^k* in the medium and the neutrino service of the neutrino service self-energy self-energy particle excitations with singletial in the medium
and imaginary particle excitations **A**
*i*n the couplings **A**ⁱj do not depend on the couplings α

ture a dispersive representation $\frac{d}{dt}$ They feature a dispersive representation *Ney reature a dispersive representation*
momenta, in the following we can use the relation from Eq. (18) in The relation Γ

$$
\text{Re}\Sigma(\omega,\vec{q}) = \frac{1}{\pi} \int_{-\infty}^{\infty} dq_0 \ \mathcal{P}\left[\frac{\text{Im}\Sigma(q_0,\vec{q}\,)}{q_0-\omega}\right]
$$

of the single (quasi) particle excitations. The tadpole term must be calculated separately and does not feature a **We can generally write** A s will be apparent in the next section, in general we will be able to write the self-energy contributions of \mathbb{R}^n On general grounds, in the following, we will decompose the self-energy as (suppressing generation

$$
\Sigma_{ij} \propto \sum_{k} A_{ik} A_{kj} \sigma_k^R \qquad \qquad \Sigma = \gamma_0 \Sigma^{(0)} - \vec{\gamma} \cdot \hat{p} \Sigma^{(1)} + \Sigma^{(2)}
$$

$$
\Sigma_{ij} \propto \sum_{k} \mathcal{A}_{ik} \mathcal{A}_{kj} \sigma_k^R \qquad \qquad \Sigma = \gamma_0 \Sigma^{(0)} - \vec{\gamma} \cdot \hat{p} \Sigma^{(1)} + \Sigma^{(2)}
$$

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16 the tadpoone) is generally valid in principle to all orders in standard model in the V orders which all orders in the V orders in the V orders which are of the V orders in the V orders which are of the V orders which are loop propagators for neutrinos. The propagators correspond to mass eigenstates, the perturbe the perturbe the p Wichele Lucente - Università di Bologna 16
16 able to prove the scalar work with the scalar functions \sim

Higgs contribution n_i *n*_i *n*_i *n*_j *q* n_k $p - q$ *H nⁱ n^j* $\alpha = e, \mu, \tau$ \mathcal{U}_{α} $\frac{1}{2}$ $\sum C_{ij}\bar{n}_i(m_iP_L+m_jP_R)\,n_j=-\frac{1}{2n_H}\sum\bar{n}_i\left[C_{ij}(m_iP_L+m_jP_R)+C^*_{ij}(m_iP_R+m_jP_L)\right]n_j$ $C_{ij}\equiv\sum_{\alpha=e,\mu,\tau}\mathcal{U}^{\dagger}_{i\alpha}$ $j^!_{i\alpha}\mathcal{U}_{\alpha j}$ ω ² $\frac{1}{2}$ reaches equilibrium will be given by the imaginary part of the self-energy. This is precisely the quantity we are interested in within the freeze-in framework. The advantage of using the one-loop ϵ is that it directed all possible interactions with the plasma, as well as well as ϵ $\frac{d\mathbf{d}}{dt}$ $m_i \longrightarrow \infty$ $\mathcal{L}_{H-\nu} \supset -$ *H* v_H \sum $C_{ij}\bar{n}_i\left(m_iP_L + m_jP_R\right)n_j =$ dependence with temperature and momenta for the production rates. 3.2 Higgs contribution \mathcal{L} study contribution from the theorem the Higgs boson contribution from the Higgs boson coupling, which after SSB $\mathcal{L}_{\mathcal{D}}$ *LH*⌫ *H* X *Cijn*¯*ⁱ* (*miP^L* + *mjPR*) *n^j* = $-\frac{H}{2v}$ $2v_H$ \sum $\bar{n}_i \left[C_{ij} (m_i P_L + m_j P_R) + C_{ij}^* (m_i P_R + m_j P_L) \right] n_j$ We study the thermal self-energy contribution from the Higgs boson coupling, which after SSBB boson coupling, which after S p' $p - q$), *Cij* (*miP^L* + *mjPR*) + *C*⇤ *ij* (*miP^R* + *mjPL*) $v_H \frac{v_j}{i,j}$, v_f , v_H , v *nⁱ n^j nk p q* $C_{ij} \equiv \sum_{\alpha=e,\mu,\tau} \mathcal{U}^{\dagger}_{i\alpha} \mathcal{U}_{\alpha j}$ $n_i \longrightarrow \frac{\kappa}{2}$ and $n_j \longrightarrow \frac{\kappa}{2}$ bosons) contribution from the gauge n_j q and \tilde{q} each species would reach $H \sum_{n=1}^{\infty} a_n$ *nⁱ n^j* $\frac{1}{\sqrt{2}}$ *nk* n_k *H* $\alpha = e, \mu$ $\epsilon_{H-\nu} \geq \nu_H \sum_{i,j} \nu_{ij} \nu_i$ (iii) μ_I is the singlet of the $2\nu_H \sum_{i,j} \nu_i$ (iii) μ_I is the SM extension with singlet $2\nu_H \sum_{i,j} \nu_i$ *nⁱ n^j* n_i

 i,j

fermions, there is the Higgs contribution (left) an the gauge (*W* and *Z* bosons) contribution from the

$$
i\Sigma_{ij}^{(+-)} = -\frac{2\pi}{v_H^2} e^{\frac{\beta p_0}{2}} f_F(p_0) \sum_k \int \frac{d^4 q}{(2\pi)^3} \left[1 + f_B(r_0) - f_F(q_0)\right] \varepsilon(q_0) \varepsilon(r_0)
$$

$$
\left[\left(\mathcal{A}_{kj}^R \phi + \mathcal{A}_{kj}^L m_k\right) \mathcal{A}_{ik}^L P_L + \left(\mathcal{A}_{kj}^L \phi + \mathcal{A}_{kj}^R m_k\right) \mathcal{A}_{ik}^R P_R\right] \delta(q^2 - m_k^2) \delta(r^2 - M_H^2)
$$

$$
r_{\mu} \equiv (p - q)_{\mu} \qquad A_{ij}^{L} \equiv \frac{1}{2} \left(C_{ij} m_{i} + C_{ij}^{*} m_{j} \right) \qquad A_{ij}^{R} = \left(A_{ij}^{L} \right)^{*}
$$

We define
$$
R(\Sigma_{ij}) + i\mathcal{I}(\Sigma_{ij}) = \text{Re}\Sigma_{ij} + i\text{Im}\Sigma_{ij}
$$

 $A_{ik}A_{kj}\text{Re}\sigma_k^R$ $A_{ik}A_{kj}\text{Im}\sigma_k^R$

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2024 to note that *R* (⌃*ij*) + *iI* (⌃*ij*) = Re⌃*ij* + *i*Im⌃*ij* , which is the final quantity we are interested in.

 i,j

fermions, there is the Higgs contribution (left) an the gauge (*W* and *Z* bosons) contribution from the

Higgs-mediated self energy (*q*² *^m*² *^k*) = ¹ 2!*^k* [(*q*⁰ !*k*) + (*q*⁰ + !*k*)] *,* (26) q

$$
\omega_H^\pm\,=\,\sqrt{(p\pm q)^2+M_H^2}
$$

$$
I^{(0)} = \frac{1}{p} \int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_H^-}^{\omega_H^+} d\omega_H \frac{qq_0}{(2\pi)^2 4\omega_k} [1 + f_B(r_0) - f_F(q_0)]
$$

\n
$$
[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k)] [\delta(r_0 - \omega_H) - \delta(r_0 + \omega_H)],
$$

\n
$$
I^{(1)} = \frac{1}{p^2} \int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_H^-}^{\omega_H^+} d\omega_H \frac{q}{(2\pi)^2 4\omega_k} (p_0 q_0 - p\tilde{\mu}) [1 + f_B(r_0) - f_F(q_0)]
$$

\n
$$
[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k)] [\delta(r_0 - \omega_H) - \delta(r_0 + \omega_H)],
$$

\n
$$
I^{(2)} = \frac{1}{p} \int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_H^-}^{\omega_H^+} d\omega_H \frac{q}{(2\pi)^2 4\omega_k} [1 + f_B(r_0) - f_F(q_0)]
$$

\n
$$
[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k)] [\delta(r_0 - \omega_H) - \delta(r_0 + \omega_H)],
$$

with prefactors
$$
\xi_L^{(I)} = \begin{cases} A_{kj}^R A_{ik}^L q_0, & \text{for } I = 0, \\ -A_{kj}^R A_{ik}^L \frac{\vec{p} \cdot \vec{q}}{|\vec{p}|}, & \text{for } I = 1, \\ A_{kj}^L A_{ik}^L, & \text{for } I = 2, \end{cases}
$$

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⌘⇤

$$
\mathcal{L}_{Z-\nu} \supseteq \frac{g}{2c_W} \sum_{i,j} C_{ij} \bar{n}_i \gamma^\mu P_L n_j Z_\mu = \frac{g}{4c_W} \sum_{i,j} \bar{n}_i \left[C_{ij} \gamma^\mu P_L - C_{ij}^* \gamma^\mu P_R \right] n_j Z_\mu
$$

$$
\mathcal{I}(\Sigma_{ij}) = \left(\frac{g}{2c_W}\right)^2 \pi \sum_{k} \int_{-\infty}^{\infty} dq_0 \int \frac{d^3q}{(2\pi)^3} \frac{1}{4\omega_k \omega_Z} \left[1 + f_B(r_0) - f_F(q_0)\right]
$$

$$
\frac{1}{4} \gamma^{\mu} \left\{ \left(C_{kj} \not q - C_{kj}^* m_k\right) C_{ik} \gamma^{\nu} P_L + \left(C_{kj}^* \not q - C_{kj} m_k\right) C_{ik}^* \gamma^{\nu} P_R \right\} \left(-\eta_{\mu\nu} + \frac{r_{\mu} r_{\nu}}{M_Z^2}\right)
$$

$$
\left[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k)\right] \left[\delta(r_0 - \omega_Z) - \delta(r_0 + \omega_Z)\right],
$$

Michele Lucente - Università di Bologna 19
19 Michele Lucente - Università di Bologna 19 inclusion from Eq. (20). In Eq. (20). When Eq. (20). When $DPF - P$ *ij* ⌘ ✓ *g*

Z mediated self-energy *^µ Ckj/q ^C*⇤ *kjm^k Cik*⌫*P^L* + *C*⇤ *kj/q Ckjm^k* [(*q*⁰ !*k*) (*q*⁰ + !*k*)] [(*r*⁰ !*Z*) (*r*⁰ + !*Z*)] *, kj/q Ckjm^k* ⌘*µ*⌫ + *M*² [(*q*⁰ !*k*) (*q*⁰ + !*k*)] [(*r*⁰ !*Z*) (*r*⁰ + !*Z*)] *,*

$$
I_Z^{(0)} \equiv \frac{1}{p} \int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_Z^-}^{\omega_Z^+} d\omega_Z \frac{q}{(2\pi)^2 4\omega_k} \left[A_0(p_0) p_0 + B(p_0) q_0 \right] \left[1 + f_B(r_0) - f_F(q_0) \right]
$$

$$
\left[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k) \right] \left[\delta(r_0 - \omega_Z) - \delta(r_0 + \omega_Z) \right],
$$

$$
I_Z^{(1)} \equiv \frac{1}{p^2} \int_{-\infty}^{\infty} dq_0 \int_0^{\infty} dq \int_{\omega_Z^-}^{\omega_Z^+} d\omega_Z \frac{q}{2(2\pi)^2 4\omega_k} \left[A_1(p_0) + 2p_0 B(p_0) q_0 \right] \left[1 + f_B(r_0) - f_F(q_0) \right]
$$

$$
\left[\delta(q_0 - \omega_k) - \delta(q_0 + \omega_k) \right] \left[\delta(r_0 - \omega_Z) - \delta(r_0 + \omega_Z) \right],
$$

$$
I_Z^{(2)} = I^{(2)} \left(\omega_H \to \omega_Z \right).
$$

with prefactors
$$
\mathscr{P}_L^{(I)} = \begin{cases} \frac{1}{4} C_{ik} C_{kj} \left[A_0(p_0) p_0 + B(p_0) q_0 \right], & \text{for } I = 0, \\ -\frac{1}{8p} C_{ik} C_{kj} \left[A_1(p_0) + 2p_0 B(p_0) q_0 \right], & \text{for } I = 1, \\ \frac{3}{2} C_{ik} C_{kj}^* m_k, & \text{for } I = 2. \end{cases}
$$

$$
A_0(p_0) \equiv \frac{p_0^2 - p^2 - M_Z^2 - m_k^2}{M_Z^2},
$$
 For $\mathcal{P}_R^{(I)}$
\n
$$
B(p_0) \equiv 1 - A_0(p_0),
$$

\n
$$
A_1(p_0) \equiv p^2 + M_Z^2 - p_0^2 - m_k^2 + A_0(p_0) [p^2 + m_k^2 - M_Z^2 + p_0^2]
$$

\n
$$
B(p_0) \equiv 1 - A_0(p_0),
$$

\n
$$
C_{ij} \rightarrow C_{ij}^*
$$

\n
$$
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$$

Michele Lucente - Università di Bologna DPF - PHENO 2024 case.
Case de la case.
Case de la case.

 DPF - PHENO 2024

Can be derived from Z contribution Can be derived from Z contribution *^LW*⌫ *^g* uti *i,*↵ **The interact interact interacts contribution** can be derived from Z contribution

2*Cij* , and noticing that in the *W* case we get *P*(*I*) $\sum C C$ $\sum P$ $g/(2c_W) \rightarrow g/\sqrt{2}$ $\sum_k C_{ik} C_{kj} \rightarrow 2 \sum_{\alpha} U_{i\alpha}^{\dagger} U_{\alpha j} = 2C_{ij}$ $E_y(\omega_{\text{W}})$ if $y/\sqrt{2}$ and ω_{k} contributes to the structure of the structure $q/(\Omega_{\text{CUT}}) \rightarrow q/2$ $\sqrt{2}$ $\sum_{l} C_{lk} C_{lk} \rightarrow 2 \sum_{l} U_{l}^{\dagger} U_{lk} = 2C_{lk}$ grangian is such that these terms can be readily taken into account from the *Z*-boson ones. Indeed, by $g/(2c_W) \rightarrow g/\sqrt{2}$ \sum $\int_{k} C_{ik} C_{kj} \rightarrow 2 \sum_{\alpha} \mathcal{U}^{\dagger}_{i\alpha} \mathcal{U}_{\alpha j} = 0$ $2C_{ij}$ \Rightarrow $a/\sqrt{2}$ $\qquad \qquad \sum_{i} C_{ik} C_{ki} \rightarrow 2 \sum_{i} U^{\dagger}_{i} U_{\alpha i} = 2C_{ii}$ \sum_{k} one is \sum_{k} that the *indeed*, \sum_{k} and \sum_{k} into a *z*-boson ones.

making the changes *g/*(2*c^W*) ! *g/*p2, *^M^Z* ! *^M^W* , *^m^k* ! *^m*` ⇠ ⁰¹³ and ^P

Even if the *W*-boson also contributes to the neutrino self-energy corrections, the structure of the La-

 $\frac{3}{2}$ $\frac{3}{2}$ $\frac{3}{2}$ $M_z \rightarrow M$ $M_Z \rightarrow M_W$
 $\mathscr{P}_R^{(I)} = 0$ $\mathscr{P}_R^{(I)}=0$ $M \rightarrow M$ $M_Z \rightarrow M_W$ $\mathscr{P}_R^{(I)} = 0$ $M_Z\rightarrow M_W$ and $\mathscr{P}_R^{(I)}=0$ $\Omega(I)$ $\partial \phi_R^{(I)}$ neutrino self-energy corrections from *W*-boson exchange using the same master integrals as for the *Z* making the changes *g/*(2*c^W*) ! *g/*p2, *^M^Z* ! *^M^W* , *^m^k* ! *^m*` ⇠ ⁰¹³ and ^P

$$
m_k \to m_\ell \sim 0 \qquad \qquad \mathscr{P}_L^{(2)} = 0
$$

grangian is such that these terms can be readily taken into account from the *Z*-boson ones. Indeed, by

^R = 0 and *^P*(2)

Michele Lucente - Università di Bologna 21 can use the following presentation from Eq. (18) in the following present p we are interested in finding the propagation \mathcal{L} states in the medium, which follow Diraction Di Given the self-energy corrections to the propagator, written on the property assumed to the property a 2*Cij* , and noticing that in the *W* case we get *P*(*I*) neutrino self-energy corrections from *W*-boson exchange using the same master integrals as for the *Z* Michele Lucente - Università di Bologna

 $\mathcal{C}^{(2)}$

2*Cij* , and noticing that in the *W* case we get *P*(*I*)

ⁱ↵*U*↵*^j* =

 $\mathscr{P}_L^{(2)}=0$

 $\mathscr{P}^{(2)}=0$

 $\mathscr{P}_L^{(-)}$ =

Propagating states in the medium 3.5 Propagating states in the medium **1.5 Propagating states in the medium of the m** *^L* ~ *· ^p*ˆ⌃(1) we are the propagation of the property states in the medium of medium, which in the medium \mathbf{u}

Once the self-energy correction to the propagator is known Given the self-energy corrections to the propagator, written on the propagator, written on σ $logator$ is knowr ˜ = 0*.* (41)

$$
\Sigma(p_0) = \left(\gamma_0 \Sigma_L^{(0)} - \vec{\gamma} \cdot \hat{p} \Sigma_L^{(1)} + \Sigma_L^{(2)}\right) P_L + \left(\gamma_0 \Sigma_R^{(0)} - \vec{\gamma} \cdot \hat{p} \Sigma_R^{(1)} + \Sigma_R^{(2)}\right) P_R
$$

 can incort this in the Dirac equation we can insert this in the Dirac equation we can insert this in the Dirac equation \overline{z} *h*=*±*1 *^v^h* ⌦

$$
\left(\rlap{\hspace{0.02cm}/}{p}-\mathcal{M}+\rlap{\hspace{0.02cm}/}{\mathfrak{X}}(p_0)\right)\tilde{\Psi}=0
$$

and project into helicity eigenstates and project into belicity aigenstates and project into helicity eigenstates
 $\langle \rangle$ ^{*h*</sub>}

$$
\tilde{\Psi} = \sum_{h=\pm 1} v^h \otimes {\phi^h \choose \zeta^h}, \quad \vec{\sigma} \cdot \hat{p} v^h = h v^h
$$

 $\begin{bmatrix} -1 \end{bmatrix}$ is the second term in Eq. (42) is the possible eigenvalues $\begin{bmatrix} 1 \end{bmatrix}$ is the possible eigenvalues $\begin{bmatrix} 1 \end{bmatrix}$ $\left|p_0 + \Sigma_R^{(0)} - hp - h\Sigma_R^{(1)}\right|\zeta^h - \left(\mathcal{M} + \Sigma_L^{(2)}\right)\varphi^h = 0$ $\lceil p_0 \rceil$ $\binom{2}{1}$ $\zeta^h = \left[p_0 + 2R^{-1} \mu - \frac{1}{2}R\right] \zeta^h = \left[p_0 + \Sigma_R^{(0)} - hp - h\Sigma_R^{(1)}\right]^{-1} \left(\mathcal{M} + \Sigma_L^{(2)}\right) \varphi^h$ $\left[p_0 + \Sigma_L^{(0)} + hp + h\Sigma_L^{(1)}\right]\varphi^h - \left(\mathcal{M} + \Sigma_R^{(2)}\right)\zeta^h = 0$ where the second term in Eq. (42) is the helicity operator, with the possible eigenvalues *h* = *±*1. $\left[p_0 + \Sigma_R^{(0)} - hp - h\Sigma_R^{(1)}\right]\zeta^h - \left(\mathcal{M} + \Sigma_L^{(2)}\right)\varphi^h = 0$ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ *p ^R hp ^h*⌃(1) *M* + ² $\left[p_0 + \Sigma_R^{(0)} - hp - h \Sigma_R^{(1)} \right]$ $\int \zeta^h - \left(\mathcal{M} + \Sigma^{(2)}_L \right)$ $\Big)\,\varphi^h=0\,,$ $\begin{bmatrix} 0 & h(n+1) \end{bmatrix}$ (a) $\begin{bmatrix} h & h(n+1) \end{bmatrix}$ (a) $\begin{bmatrix} 1 & h(n+1) \end{bmatrix}$ $p_0 + \Sigma_L^{(0)} + hp + h\Sigma_L^{(1)}\Big\vert\ \varphi^h - \left(\mathcal{M} + \Sigma_R^{(2)}\right)\zeta^h = 0\ .$ $\Sigma_R^{(3)}$ $\zeta^h =$ $\left[p_0 + \Sigma_R^{(0)} - hp - h\Sigma_R^{(1)} \right]$ $\left[\begin{matrix} -1\ \mathcal{M} + \Sigma^{(2)}_L \end{matrix} \right]$ $\Bigr)$ φ^h ¹²Performing a similar change of variables as for the Higgs case we find !*[±] Z* . 13Given that we are interested in temperatures at the EW scale, we need the following charged lepton masses,
The following charged lepton masses, we need the following charged lepton masses, we are interested in the and substitute in the second line in order to find the equation of motion for '*^h* as

*p*⁰ + ⌃(0) *^R hp ^h*⌃(1) *R* ⇣*^h ^M* ⁺ ⌃(2) *L* '*^h* = 0*,* ⇣*^h* relations are given by
———————————————————— '*^h* eroes or un
———————————————————— ⇣*^h* = 0*.* From the first line in Eq. (43) we can solve for ⇣*^h* as h <u>i</u> Ì. *m*`. $\overline{ }$ p is are given by the zero ϵ ⌘ **The dispersion relations are given by the zeroes of the inverse propagator**

$$
\left[p_0^2 - p^2 + (p_0 - hp) \left(\Sigma_L^{(0)} + h \Sigma_L^{(1)} \right) - (p_0 - hp) \left(\mathcal{M} + \Sigma_R^{(2)} \right) \left[p_0 + \Sigma_R^{(0)} - hp - h \Sigma_R^{(1)} \right]^{-1} \left(\mathcal{M} + \Sigma_L^{(2)} \right) \right] \varphi^h = 0
$$
\nMichele Lucente - Università di Boloona

\n22

Michele Lucente - Università di Bologna DPF - PHENO 2024 ⇣*h* = h
a 14IN GENERAL OF PROPERTY OF *POSSIC* (*p*⁰ *hp*) *^M* ⁺ ⌃(2) *R* ⌘ h*p*⁰ ⁺ ⌃(0) *^R hp ^h*⌃(1)

and substitute in the second line in order to find the equation of motion for '*^h* as

¹⁴In general ⌃ is a function of *T, p*0, and *p*, but we are interested here in its dependence with *p*⁰ and thus omit the

 $m_{DM} = 10 \text{ keV} \text{ and } |\mathcal{U}_{\alpha 4}| \sim 10^{-6}$ $m_{DM} = 10$ keV and m_{α} $\frac{1}{\sqrt{2}}$ comes with internal or $\frac{1}{\sqrt{2}}$ comes with internal or Lorentz indices, $\frac{1}{\sqrt{2}}$ contrary in the set of $\frac{1}{\sqrt{2}}$ contrary in the set of $\frac{1}{\sqrt{2}}$ contrary in the set of $\frac{1}{\sqrt{2}}$ contrary in

 $\Gamma \sim \mathcal{O}(10^{-16})$ GeV is needed to generate the correct DM abundance expansion *rate as a function of a function of* $\frac{1}{\sqrt{10}}$ $\Gamma \sim \mathcal{O}(10^{-16})\;$ GeV is needed to generate the correct DM abundance $T_{\rm eff}$ medium is filled with particles that couple to Φ

Michele Lucente - Università di Bologna DPF - PHENO 2024 a_{label} The Eurente extending the Results di Release constant a_{label} and a_{label} and b_{label} b_{label} b_{label} b_{label} b_{label} b_{label} b_{label} c_{label} c_{label} c_{label} c_{label} c_{label} c_{label} *m*_D = 1000 km² component of Dologna cente - Uni

Results: 4x4 ISS and Type-I Seesaw

- The 4x4 ISS is not qualitatively different from the 2x2 toy-model; represent the results from the type-I seesaw without the contribution from the Higgs. The vertical
- Type-I seesaw rates are many orders of magnitude larger;
- If we neglect Higgs contributions in Type-I we recover the same rates as toy models

Type-I realisations

Type-I seesaw rates can go from fully freeze-in regime to thermalised scenarios

Numerical integration over T and p is currently challenging, due to the computation of the real part being very demanding

Michele Lucente - Università di Bologna 26
29 a definition of a definition of $\omega = \frac{1}{2}$, when $\omega = \frac{1}{2}$ phases in the PMNS mixing matrix, ↵¹ ⇠ 1*.*06⇡ and ↵² ⇠ 0*.*38⇡.

Conclusion

Sterile neutrinos are viable DM candidates, and are motivated by massive neutrinos

Many phenomenological constraints: the only known solution in the minimal framework of SM + HNL is the νMSM

Can sterile neutrinos be produced by the decay of heavier particles? Diagrammatically yes

However thermal effects make gauge boson production negligible

Interestingly, production from HNL decay does not appear to be suppressed!

Need of a realistic full model to accomodate sizeable Yukawas, otherwise neutrinos + DM constraints suppress the production rate

Solutions exist in the zero temperature approximation. Currently working on an effective numerical integration of Boltzmann equations

Backup

Neutrinos as Dark Matter?

$$
\Omega_{\rm B}h^2 = 0.02237 \pm 0.00015
$$

\n
$$
\Omega_{\rm DM}h^2 = 0.1200 \pm 0.0012 \qquad h = 0.6736 \pm 0.0054
$$

\n
$$
\Omega_{\Lambda} = 0.6847 \pm 0.0073
$$

N. Aghanim *et al.* [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO]

Sterile neutrinos can be viable DM candidates: they are produced by oscillations of active ones as long as an active-sterile mixing is present

S. Dodelson and L. M. Widrow, hep-ph/9303287

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Constraints: abundance

DW: as long as an active-sterile mixing is present, a population of sterile V is produced by oscillations in the primordial plasma

S. Dodelson and L. M. Widrow, hep-ph/9303287

$$
\Omega_s h^2 = 1.1 \cdot 10^7 \sum_{\alpha} C_{\alpha}(m_s) \left| U_{\alpha s} \right|^2 \left(\frac{m_s}{\text{keV}} \right)^2, \quad \alpha = e, \mu, \tau
$$

T. Asaka, M. Laine and M. Shaposhnikov, hep-ph/0612182

Constraints: phase-space density *m*¹c: *nh* 1 *Y ^T* ⌃ *M*⌃ *Y*⌃ (21) 0 1

For fermionic DM, Pauli exclusion principle impose a maximum on its distribution function (degenerate Fermi gas). Imposing that inferred phase-space density does not excess this bound, it is possible to extract a lower bound on the DM mass **M** Deuli ate Fermi gas). Imposing that inferred phase

Constraints: stability and indirect detection (ID) limit of the order of the integration can be obtained from the μ

monochromatic γ

extending the limits of the limits of the May 1982 of the M-ray searches of the M-ray S-ray S-ray S-ray S-ray S-ray S-ray searches are ported and L. Wolfenstein, Phys. Rev. D 25 (1982) 766

 g_{max} is the leak of signature (σ σ CLIANIDDA, VMNI) Due to the lack of signature (e.g. CHANDRA, XMN)

Constraints: Lyman-α

The absorption in the spectra of QSOs by the H (Ly-a: $1s \rightarrow 2p$) in IGM can trace matter distribution at scales: I-80 h⁻¹ Mpc

Narayanan, Vijay K.; Spergel, David N.; Davé, Romeel; Ma, Chung-Pei, Astrophys. J. 543, 103 (2000)

J. Baur, N. Palanque-Delabrouille, C. Yeche, A. Boyarsky, O. Ruchayskiy, E. Armengaud and J. Lesgourgues, arXiv:1706.03118 [astro-ph.CO]

The equation simplify by projection simplify decomposition eigenstates. Following the steps of ref. ϵ ⃗) − ⃗γ · q s Bioity decempeoitien , prima → que establecen en la construction de la construction de la construction de la construction de la const
De la construction de la constr L ; Ψh, Ψh, Ψh, Ψ ^L = **Example 19 Helicity decomposition** Ψ^R = ' and neutral current interactions, the form of the equations of motion and the dispersive form of the self-energy (not A subtle but important conceptual issue arises in the neutral current contribution to the self-energy with internal A subtle but important conceptual issue arises in the neutral current contribution to the self-energy with internal

Expand the **chiral** fields in terms of **helicity** eigenstates Expand the chiral fields in terms of helicity eigenstates and neutral current interactions, the form of the equations of motion and the dispersive form of the self-energy (notations of $\frac{1}{2}$

$$
\Psi_L = \sum_{h=\pm 1} v^h \otimes \Psi_L^h \; ; \; \Psi_L^h = \begin{pmatrix} \nu_a^h \\ \nu_s^h \end{pmatrix}_L \qquad \Psi_R = \sum_{h=\pm 1} v^h \otimes \Psi_R^h \; ; \; \Psi_R^h = \begin{pmatrix} \nu_a^h \\ \nu_s^h \end{pmatrix}_R
$$

$$
\mathbb{L} = (1 - \gamma^5)/2 \qquad \mathbb{R} = (1 + \gamma^5)/2
$$

eio ler & where **vh** are **eigenstates** of the **helicity** operator \ddotsc where where the and right and right the flavor operator Where **v**ⁿ are eigenstates of the nelicity operator where **vhalu equilibrium. Where with an example is dependent** mixture. We note that the **helicity** operator micity and digenstates of the neitury operator where v^h are eigenstates of the helicity operator assumed a later will be considered the internal loop of the internal local loc

$$
\hat{h}(\hat{\mathbf{q}}) = \gamma^0 \vec{\gamma} \cdot \hat{\mathbf{q}} \gamma^5 = \vec{\sigma} \cdot \hat{\mathbf{q}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \vec{\sigma} \cdot \hat{\mathbf{q}} \ v^h = h \ v^h \quad ; \quad h = \pm 1
$$

 \geq ϵ quai
— The equation of motion are then
According the equation of motion are then The equation of motion are then

————————————————————

$$
\left[\left(\omega^2 - q^2 \right) \mathbb{1} + \left(\omega - hq \right) \left(\mathbb{A} + h \mathbb{B} \right) - \mathbb{M}^2 \right] \Psi_L^h = 0 \qquad \qquad \left[\left[\omega - h q \right] \Psi_R^h = \mathbb{M} \gamma^0 \Psi_L^h \right]
$$

s

vvici i t
Vite
Vite where $\sum^{\nu} + \sum(\omega, q) = \gamma^{\nu} \mathbb{A}(\omega, q) - \gamma \cdot \mathbf{q} \mathbb{B}(\omega, q)$ $\mathcal{L} = \mathcal{L}(\mathbf{w}, \mathbf{y})$ in the flavor basis, and vertex \mathbf{y} $\frac{1}{\sqrt{2}}$ \mathbf{w} it by \mathbf{w} is and \mathbf{w} is an order by \mathbf{w} . $\boldsymbol{\Sigma}^t + \boldsymbol{\Sigma}(\omega, \vec q) = \gamma^0 \, \mathbb{A}(\omega, \vec q) - \vec \gamma \, \cdot \widehat{\textbf{q}} \,\, \mathbb{B}(\omega, \vec q)$ with where in the flavor basis

 \mathbb{R}^n

$$
\mathcal{A}(\omega, \vec{q}) = \begin{pmatrix} A(\omega, \vec{q}) & 0 \\ 0 & 0 \end{pmatrix} \qquad \qquad \mathcal{B}(\omega, \vec{q}) = \begin{pmatrix} B(\omega, \vec{q}) & 0 \\ 0 & 0 \end{pmatrix}
$$

Michele Lucente - Università di Bologna 34 DPF - PHENO 2024 e Lucente - Universit Leno
di Bol D_{on} 1 34 L. Lello, D. Boyanovsky and R. D. Pisarski,
^{cente} - Università di Bologna The end, D. Boyanovsky and R. D. Fisarski, arxiv.1009.07647 [nep-prij
DPF - PHENO 2024
DPF - PHENO 2024 L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

 \vec{q})

 $=$ $\frac{1}{2}$. (II.21) $\frac{1}{2}$. (II.2

Propagating states and M1,2 are given by equality (II.6). The results (II.23) are general for standard model couplings of the activ Propagating states ! " and M1,2 are given by equation of the results (II.22, II.23) are general for standard model couplings of the a (flavor) neutrinos and sterile neutrinos that only interact with active ones via a see-saw type mass matrix. Before

The dispersion relations for the (quasi) particle states in the medium are given by the complex poles of the propagator (or by the zeroes of the inverse propagator) $\overline{}$ The dispersion relations for the (quest) pertials states in the medium are given by the $\overline{}$ fire dispersion relations for the (quasi) particle states in the medium are given by the media of the propagator
Complex poles of the propagator (or by the zeroes of the inverse propagator) given by the The (quasi) particle states in the medium are given by the
Expansion for by the zeroes of the inverse preparator) e dispersion relations for the (quasi) particle states in the medium are given by the
complex poles of the propagator (or by the zeroes of the inverse propagator) e propagator (or by the Z o areportion relations for the ₍quadi) particle states in the medium are given by the side of the prencess of the operator on the side of the operator of the operator of the side of the side of the side of the inverse pr

$$
\left[\mathbb{S}_{L}^{h}(\omega, q)\right]^{-1} = \left[(\omega^2 - q^2)\mathbb{1} + (\omega - hq)(\mathbb{A} + h\mathbb{B}) - \mathbb{M}^2\right]
$$

To leading order in θ and assuming $\mathsf{M}_1 \ll \mathsf{M}_2$ the corrections are given by To leading order in θ and assuming M₁ ≪ M₂ the corrections are given by To leading order in θ and assuming M₁ \ll M₂ the corrections are given by

$$
\delta\omega_1^h = -\left(\Delta_1^h(q) + i\gamma_1^h(q)\right) - \frac{\theta^2\left(\xi + \Delta_1^h(q) - i\gamma_1^h(q)\right)}{\left[\left(1 + \frac{\Delta_1^h(q)}{\xi}\right)^2 + \left(\frac{\gamma_1^h(q)}{\xi}\right)^2\right]} + \theta^2\xi
$$
active-like

$$
\delta\omega_2^h = \frac{\theta^2 \left(\xi + \Delta_2^h(q) - i\gamma_2^h(q)\right)}{\left[\left(1 + \frac{\Delta_2^h(q)}{\xi}\right)^2 + \left(\frac{\gamma_2^h(q)}{\xi}\right)^2\right]} - \theta^2 \xi \quad \text{sterile-like} \quad \omega = \omega_j(q) + \delta\omega_j^h
$$
\n
$$
\omega_j(q) = \sqrt{q^2 + M_j^2}
$$

$$
\Delta_j^h(q) + i\gamma_j^h(q) = \frac{\Omega^h(\omega_j, q)}{2q} \qquad \qquad \Omega^h = (\omega - hq) \left(A(\omega, q) + h B(\omega, q) \right) \qquad \qquad \xi = \frac{M_s^2}{2q}
$$

rski, arXiv:1609.07647 [hep-ph] 0. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph] 35 ! Lello, D. Boyano $\overline{}$ L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph] and using (II.24) we obtain the contract of the

Michele Lucente - Università di Bologna 35 5 2024 $\overline{\text{nc}}$ \mathbf{d} −1. $\overline{31}$

 \overline{a} \mathbf{p} \overline{P} – \overline{P} $\text{I EIVU } 2024$)2 \overline{O} 2024

Example 18 Security 19 Security 19 Sequations \mathbf{r} the plasma degrees of freedom are in the damping rate of \mathbb{R}^n and \mathbb{R}^n are in the damping rate of single \mathbb{R}^n Fundamentally the heart of the argument is simply detailed balance, a consequence of the main assumption that

From the dispersion relations is it possible to define an "effective" mixing angle in medium From the disne From the dispersion relations is it possible to define an "effective" mixing angle in
medium is linear in the population n2 to leading order in each control term in the gain term in the gain term in the
In the gain term in the gain term in the quantum kinetic equation is simply the gain term in the gain term in Γ2(q) determines the approach to equilibrium in linear response[95–98] and for θ ≪ 1 the quantum kinetic equation is it possible to define an "effective" mixing angle in the gain term in the quantum term in the quantum kinet
Exercises $r = \frac{1}{2}$

$$
\theta_{eff}^{h}(q) = \frac{\theta}{\left[\left(1 + \frac{\Delta_{j}^{h}(q)}{\xi}\right)^{2} + \left(\frac{\gamma_{j}^{h}(q)}{\xi}\right)^{2}\right]^{1/2}}
$$

admny omoro mnono oquanon ior olonio i '!θh **This quantity enters kinetic equation for sterile-like production** on the mixing angle in the quantum kinetic equation are incorporated by the simple replacement sin(θ) → θef f (q) in .
die **2 (2) are given by Given by City of Stead by City (III.55). Hence, the production**

$$
\frac{dn_2^h(q;t)}{dt} = \Gamma_2^h(q) \left[n_{eq}(q) - n_2^h(q;t) \right] \qquad \Gamma_{prod}(q) = \Gamma_2^h(q) n_{eq}(q)
$$

$$
\Gamma_{prod}(q) = 2 \left(\theta_{eff}^-(q) \right)^2 \text{Im}\Sigma^-(q) n_{eq}(q)
$$

$$
\Gamma_{prod}^+(q) = 2 \left(\theta_{eff}^+(q) \right)^2 \left[\frac{M_s}{2q} \right]^2 \text{Im}\Sigma^+(q) n_{eq}(q)
$$

L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]
DPI behele Lucente - Università di Bologna \overline{D} . B L. Lello, D. Boyanovsky and R. D. Pisarski, arXiv:1609.07647 [hep-ph]

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Michele Lucente - Università di Bologna DPF - PHENO 2024 L
Michele Lucente - Università (III.19,III.20) Σ(q) is the standard model self-energy for flavor neutrinos evaluated on the relativistic mass shell, Michele Lucente - Università di Bologna and 36 anno 1992, ISBN 0-2024

Thermal Field Theory in a nutshell case, where we use the canonical ensemble. The use of grand canonical ensemble field theory textbooks [6], we have the field theory textbooks [6], we have 0. *y* ... a rice

The partition function at finite temperature ($\beta = 1/T$) can be path integral with imaginary time evolu i iuncuon at imite temperature $\mathfrak{p} = \mathfrak{p}$ is a can be formally expressed as \mathfrak{p} paan megraf wan miaginaly anno ovolution ϵ \bm{x} pressed as a **path** integral with imaginary time evolution **The partition function at finite temperature (β = 1/T) can be formally expressed as a**

$$
Z=\int {\cal D}\phi\langle\phi|{\rm e}^{-\beta H}|\phi\rangle =\int {\cal D}\phi \exp\left[-\int_0^\beta d\tau {\cal L}(\tau)\right]
$$

The fields at finite temperature are $\langle \phi(\mathbf{x}, t) \phi(\mathbf{y}, 0) \rangle_{\beta} =$ 1 *Z* $\text{Tr}\left[e^{-\beta H}\phi(\mathbf{x},t)\phi(\mathbf{y},0)\right] =$ $\overline{\mathsf{C}}$ 1 *Z* Ω -Martin-Schwinger (KMS) **z eriodic in the imaginary time** 1 *Z* $\text{Tr} \left[\phi(\mathbf{x}, t) e^{-\beta H} e^{\beta H} \phi(\mathbf{y}, 0) e^{-\beta H} \right] = \langle \phi(\mathbf{y}, -i\beta) \phi(\mathbf{x}, t) \rangle_{\beta}$ 1 **z** $\phi(\mathbf{x},0) = \pm \phi(\mathbf{x}, i\beta)$ *Z* The fields at finite temperature are periodic in the imaginary time component Kubo-Martin-Schwinger (KMS) condition $\phi(\mathbf{x},0) = \pm \phi(\mathbf{x},i\beta)$ part in the path of the control of the con \overline{A}

s. 1 $\overline{\mathcal{C}}$ r. Jap. 12 (1957), 570-586; P. C. Martin and J. S. 1 *Z* Aartin and J. S. Schwinger, Phys surprisingly see that imaginary temperature plays the role as a time variable. If we $\frac{d}{dx}$ define the imaginary time $\frac{d}{dx}$ R. Kubo, J. Phys. Soc. Jap. 12 (1957), 570-586; P. C. Martin and J. S. Schwinger, Phys. Rev. 115 (1959), 1342-1373

$\overline{\mathsf{m}}$ 1 *Z* A finite-temperature path-integral formalism can be derived in analogy to the red. Productional and the cyclic process of the cyclic permutation propagators, etc... ⌧ = *it t* = *i*⌧ (2.11) standard QFT treatment: thermal Wick theorem, thermal diagrammatic expansion,

where we used the cyclic permutation property of a trace of operator products. We surprisingly see that imaginary temperature plays the role as a time variable. If we have variable the role as See e.g. T. Lundberg and R. Pasechnik,arXiv:2007.01224 [hep-th] for a modern review and collection of references

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Imaginary vs Real time formalism

It is necessary to specify a contour of time integration in the complex plane necessary to specity a contour of time integration in the comp

presented by Daniel Band
Same diagrammat **T** as zero temperatu Same diagrammatic structure as zero temperature QFT as zero temperature QFT as and in imaginary time with the series of $\mathbf{F}_{\mathbf{a}}$ and $\mathbf{F}_{\mathbf{a}}$ and $\mathbf{F}_{\mathbf{a}}$ and $\mathbf{F}_{\mathbf{a}}$ are the series of $\mathbf{F}_{\mathbf{a}}$ and $\mathbf{F}_{\mathbf{a}}$ are th **+**

above is valid for any system that is defined by any system is defined by a system in the interaction of the i
Separate is defined by a distribution of the interaction of the interaction of the interaction of the interact
 \blacksquare on Matsubara frequencies ω_η ial
مد ˆ *(t)* = field theory textbooks [6], we have Need to perform non-trivial sum on Matsubara frequencies ω_η

- in the limit of the contract of the contractions that the con- Ω and Ω ¹ Ω ¹ Ω ¹ Ω ¹ Contribution from vertical paths factorises, no discrete sums are needed **+**
- *(t) t* ∈ R on *C*¹ ∪ *C*2*,* contour shown in Fig. 5, the time coordinates of *x*, *x*′ may be $-$ Propagators are

matrices um **Propagators are**
- **functions** arises. Let us label them as $\frac{1}{2}$ so that if $\frac{1}{2}$ so *^s* ∈ *Cs* for *r,s* = {1*,* 2*,* 3*,* 4}. Then, the different atr *dt*⁰⁰*L*(*t*) matrices **-**

We make use of the real-time formalism of thermal QFT point of the formalism has been shown as a shown \mathbf{W} to \mathbf{W} the formalism \mathbf{W} $\mathbf{g} = \mathbf{g} = \mathbf{g} = \mathbf{g}$ We make use of the real-time for

tr ∈ *Cr*, *t*′

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-

Solutions in the zero T approximation θ² = ! *|U*α*s|* ² (5) ⌦DM*h*² '

This operator is only present after electroweak symmetry breaking (v≠0) \overline{a} DM. On the above analytical expression is not strictly applicable for \overline{a} pseudwear synnicu y preaking (v⊬v)

If $M_i > M_h$ and $M_j \approx$ keV, this decay can produce keV sterile neutrinos (DM) while *Ni* are in thermal equilibrium expressed as: *Y*eff sin ✓ *<* 107. T in $\frac{1}{N}$ ivih and $\frac{1}{N}$ Nev, this decay can produce Nev steme neutrinos (DIVI) $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ boson and is thus zero above the EW phase transition temperature. To a good transition temperature. dina dia memberikan berju

$$
\Omega_{\rm DM} h^2 \simeq \frac{1.07 \times 10^{27}}{g_*^{3/2}} \sum_I g_I \frac{m_s \Gamma(N_I \to \rm DM + \text{ anything})}{m_I^2} \varepsilon(m_I)
$$

A. Abada, G. Arcadi and M.L., arXiv:1406.6556 [hep-ph] A. Abdud, G. Arcaul dirac with, dixiv. 1400.0000 [Hep-ph] L., arXiv:1406.6556 [hep-ph] with the temperature and which can be temperature and which can be temperature and w
 $\frac{1}{2}$

For general freeze-in studies, see e.g. L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, arXiv:0911.1120 [hep-ph]; X. Chu, T. Hambye and M. H. G. Tytgat, arXiv:1112.0493 [hep-ph]; X. Chu, Y.
Mambrini ، L Quevillon and B. Zaldivar, arXiv:1306 4677 [hen-ph]: M. Klasen and C. E. Yaquna Mambrini, J. Quevillon and B. Zaldivar, arXiv:1306.4677 [hep-ph]; M. Klasen and C. E. Yaguna, arXiv:1309.2777 [hep-ph]; M. Blennow, E. Fernandez-Martinez and B. Zaldivar, arXiv:1309.7348 [hep-ph] *v*(*T*)

Michele Lucente - Università di Bologna 39 *Samma DPF - PHENO 2024* of the Higgs boson and is thus zero above the EW phase transition temperature. To a good transition temperature. To a g

The ε suppression function \mathbf{F} **FILE & Supplession initially a** pseudo-Dirac neutrinos since the mixing angle \mathbf{r} of the Higgs boson and is thus zero above the EW phase transition temperature. To a good

The function ε takes into account the fraction of decays taking place after electroweak symmetry breaking pseudo-Dirac neutrinos since the mixing angle ✓ depends on the vacuum expectation value (vev) The function ε takes into account the Iraction of decays taking a of decouse to consider \mathbf{a} place of the \mathbf{r} "(*m^I*) given by:

$$
\varepsilon(m_I) = \frac{2}{3\pi} \int_0^\infty f(x_I)^2 x_I^3 K_1(x_I) dx_I, \quad x_I = \frac{m_I}{T}
$$

with $f(x_i)$ tracking the evolution of Higgs vev with temperature *v*(*T*)

in turn approximated, according the results presented in [54], by: M. D'Onofrio, K. Rummukainen and A. Tranberg, arXiv:1404.3565 [hep-ph]

MICHELE LUCENTE - ONIVERSITA OI DOIOGNA

Reproducing neutrino data the heavier mass eigenstates have masses coinciding with the eigenvalues of the matrix *M*. Without loss of generality, it is always possible to chose a basis in which the matrix *M* is

To conveniently reproduce neutrino oscillation data we employ the Casas-Ibarra parametrisation chtry reproduce neutrino oscination data we employ *II* α *Darra* parametrisation Io conveniently reproduce neutrino oscillation data we employ the Casas-**DALIUI I**
And *A L M <i>O L O O C C L* agreement the agreement to the agreement

J. A. Casas and A. Ibarra, arXiv:hep-ph/0103065 [hep-ph]

parametrised by 3 complex angles !*ij* , where R is an orthonormal matrix parametrised by 3 complex angles ω_{ij}

$$
R = V_{23}V_{13}V_{12}, \t\t\t\t\t\twith \t\t\t V_{12} = \begin{pmatrix} \cos \omega_{12} & \sin \omega_{12} & 0 \\ -\sin \omega_{12} & \cos \omega_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},
$$

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Two dark matter populations

Dodelson-Widrow production

$$
\Omega_{\rm DM}^{DW}h^2=0.11\cdot 10^5\frac{M_1}{\rm keV}\sum_\alpha C_\alpha(M_1)\left|\left(U_{\rm PMNS}^*\sqrt{\frac{\hat{m}}{\rm eV}}R\right)_{\alpha 1}\right|^2
$$

 $\Omega_{\rm DM} < 0.12$ sets strong upper bounds on $\rm H_{11}$ Q_{out} of 12 sets strong unner bounds on R_{in} Ω_{DM} < 0.12 sets strong upper bounds on R_{i1}

DM *h*² 0*.*12 imposes a strong upper bound on the elements of the first column of *R*, and/or **Freeze-in production**

$$
\Omega^{FI}_{\rm DM}h^2 = 2.16 \cdot 10^{22} \sum_{J=2,3} \left| \left(R^\dagger \frac{\hat{m}}{v} R \right)_{1J} \right|^2 g_J \left(1 - \frac{m_h^2}{M_J^2} \right)^2 \varepsilon \left(M_J \right),
$$

For sub-eV neutrinos $(\hat{m}/v)^2 \gtrsim 10^{-23} \,$ giving the correct abundance for R \approx 1 For sub-eV neutrinos $(\hat{m}/v)^2 \lesssim 10^{-23}\;$ giving the correct abundance for R \approx 1

Freeze-in produced DM has a colder spectrum than DW Freeze-in produced DM has a colder spectrum than DW

A. Boyarsky, M. Drewes, T. Lasserre, S. Mertens and O. Ruchayskiy, arXiv:1807.07938 [hep-ph]; A. Abada, G. Jucents and in a reliction of the Regulator and D. Certures: and the Regulator and the results in a reliction of th DM*h*² = 2*.*¹⁶ *·* ¹⁰²² ^X arXiv:hep-ph/0604236 [hep-ph]; A. Merle and M. Totzauer, arXiv:1502.01011 [hep-ph]; J. Heeck and D. Teresi,
arXiv:1706.09909 [hep-ph]
arXiv:1706.09909 [hep-ph] $\sqrt{2}$ 06.09909 arXiv:1706.09909 [hep-ph]Lucente, arXiv:1406.6556 [hep-ph]; F. Bezrukov and D. Gorbunov, arXiv:1403.4638 [hep-ph]; M. Shaposhnikov and I. Tkachev, kev sterile neutrinos can independ radiatively decay independent radiative neural and a photon and a light activ
The photon and a light active neural and a light active neural active neural and a light active neural and a A. Boyarsky, M. Drewes, T. Lasserre, S. Mertens and O. Ruchayskiy, arXiv:1807.07938 [hep-ph]; A. Abada, G. Arcadi and M.

Example of solutions

We fix for definitiveness the neutrino oscillation parameters to their best-fit values in the NO case

and $m_1 = 0$, $M_2=M_3=300$ GeV, $\omega_{13}=0$, $\lim \omega_{12}=0$, $\text{Re}\omega_{23}=0$

 $F(X \cup S)$ is parameter $F(Y \cap T)$ in the choice of $F(Y \cap T)$ is described with the choice of parameters as $F(Y \cap T)$ No solution for $M_1 > 60$ keV (M₁ > 49 keV for IO)

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General structure of solutions

 $M_1 \approx keV$
≈ Hierarchy of masses $M_1 \approx k$ eV **M23 > 125 GeV**

the criteria reported in the Figure. Finally, black points reproduce the observed DM relic density at the 3 precision. $|\omega_{1j}| \ll 1$ **Im** $\omega_{23} \simeq 10$ Hierarchy in the CI complex angles

fire and the Home of the Higgs boson, the H Fine-tuned solutions?

Michele Lucente - Università di Bologna $\frac{44}{4}$ DPF - PHENO 2024 $\frac{1}{2}$ mass degeneracy between *M*2*,*3, this has no phenomenological motivation; however, relaxing

Lepton number symmetry Lepton number symmetry

In the limit

$$
M_1 \rightarrow 0 \qquad \omega_{1i} \rightarrow 0
$$

\n
$$
M_2 \rightarrow M_3 \qquad \omega_{23} \rightarrow \pm i\infty
$$

the Lagrangian acquires a global lepton number symmetry. The mass spectrum becomes

- 3 massless active neutrinos
- \bullet 1 massless decoupled state N₁
- 1 Dirac heavy neutrino (linear combination of N_2 and N_3)

Approximate lepton number symmetry !1*ⁱ* ! 0 (13)

Because neutrinos have (tiny) masses, the symmetry must be broken at some level. In the scenario:

$$
M_1 \ll M_{2,3}
$$
\n
$$
M_2 \sim M_3
$$
\n
$$
| \omega_{1i} | \ll 1
$$
\n
$$
M_2 \sim M_3
$$
\n
$$
| \text{Im } \omega_{23} | \gg 1
$$

the mass spectrum features

- 3 light (massive) active neutrinos
- 1 light sterile neutrino with mass M_1 (e.g. keV)
- 2 heavy Majorana neutrinos with almost degenerate masses forming a pseudo-Dirac pair (e.g. EW scale)

Moreover, the approximate symmetry protects light neutrino masses from large loop corrections even if sizeable Yukawas are present