A Common Origin for the QCD Axion and Sterile Neutrinos from $SU(5)$ Strong Dynamics

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Outline

Strong CP problem

- In principle, the QCD Lagrangian can contain the following term:

$$\mathcal{L} \supset \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\,\mu\nu}\,, \tag{1}$$

where, $\widetilde{G}^{a\,\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta}$.

- This term - (i) is Lorentz-invariant (ii) gauge-invariant
  (ii) has mass dimension-4 (hence, renormalizable)

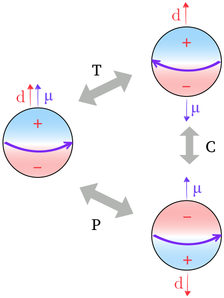
- The θ-term breaks CP (charge conjugation + parity) symmetry.

$$G\widetilde{G} \longrightarrow -G\widetilde{G} \tag{2}$$

Strong CP problem

- A non-zero neutron electric dipole moment (EDM) also breaks CP :

Figure 1: C, P and T transformations of the neutron EDM (d) and magnetic moment (μ).



- The neutron EDM is proportional to the θ parameter:

$$d_n = |\theta| \times 2.4 \cdot 10^{-16}\, e \cdot \text{cm}\,. \tag{3}$$

- The current experimental bound on the neutron EDM $d_n < 0.18 \times 10^{-25}\, e \cdot \text{cm}$.
- This sets an upper bound: $\boxed{\theta < 10^{-10}\,.}$

Strong CP Problem

- $\boxed{\theta < 10^{-10}\,.}$
- Why is θ so small?
- This is known as the strong CP problem.

PQ Solution - Overview

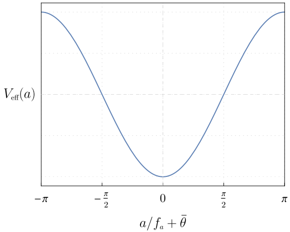
- Add a dynamical real scalar field field $a(x)$:

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \frac{g_s^2}{32\pi^2}\left(\theta + \frac{a(x)}{f_a}\right) G^a_{\mu\nu}\widetilde{G}^{a\,\mu\nu}\,. \tag{4}$$

- The effective axion potential induced by QCD is obtained -

$$V_{\text{eff}}(a) \sim -m_a^2 f_a^2 \cos\left(\theta + \frac{a(x)}{f_a}\right), \tag{5}$$

Figure 2: QCD induced effective potential $V_{\text{eff}}(a)$ of the axion as a function of $\frac{a(x)}{f_a} + \theta$.



- $V_{\text{eff}}(a)$ has a minimum at $\frac{a(x)}{f_a} + \theta = 0$, hence dynamically fixing the CP-violating term to zero.

PQ Mechanism - few important points

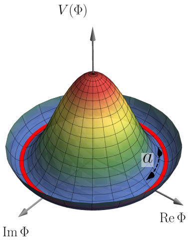
- Introduce a **global**, **chiral** $U(1)_{\mathrm{PQ}}$ symmetry.
- $U(1)_{\mathrm{PQ}}$ transformations of a PQ-charged complex scalar Φ :

$$\Phi \to e^{i\alpha}\,\Phi\,. \tag{6}$$

- Spontaneous breaking of $U(1)_{\mathrm{PQ}}$ with the VEV of the potential:

$$V(\Phi) = \lambda\left(\Phi^\dagger\Phi - f_a^2\right)^2$$

$$\Rightarrow \langle\Phi\rangle = f_a$$



- Reparametrize Φ:

$$\Phi(x) = f_a\, e^{i\, a(x)/f_a} \tag{7}$$

- PQ symmetry of the NG boson $a(x)$:

$$\frac{a(x)}{f_a} \longrightarrow \frac{a(x)}{f_a} + \alpha\,. \tag{8}$$

Axion Quality Problem

- Recall that $U(1)_{\mathrm{PQ}}$ is a **global** symmetry.
- $U(1)_{\mathrm{PQ}}$ needs to be *anomalous* under $SU(3)_c$ to generate the QCD induced effective potential, hence cannot be *gauged*.
- Quantum gravity should explicitly break the **global** $U(1)_{\mathrm{PQ}}$ symmetry. Generically,

$$\mathcal{L} \supset |c| e^{i\delta} \cdot \frac{\Phi^n}{M_{\mathrm{Pl}}^{n-4}} + \mathrm{h.c.}\,, \tag{9}$$

- The above Planck suppressed operator modifies the axion potential:

$$V_{\mathrm{eff}}(a) \simeq -m_a^2 f_a^2 \cos\left(\theta + \frac{a}{f_a}\right) + |c| \frac{f_a^n}{M_{\mathrm{Pl}}^{n-4}} \cos\left(\frac{a}{f_a} + \delta\right) \tag{10}$$

Axion Quality Problem

- The potential minimum is shifted by,

$$\Delta\theta \sim \delta \left(\frac{f_a}{M_{\rm Pl}}\right)^{n-4} \left(\frac{f_a}{m_a}\right)^2 \tag{11}$$

- $\theta < 10^{-10} \quad \Rightarrow \boxed{n \gtrsim 8}$ (assuming $f_a \gtrsim 10^8\ GeV$)

- Why is the global symmetry of $U(1)_{\rm PQ}$ protected from quantum gravity corrections up to $n \gtrsim 8$?

Origin of Neutrino Mass

- In the Standard Model, gauge invariance does not allow a mass term for the left-handed (active) neutrino.
- However, neutrino oscillation experiments set a bound on the neutrino mass $m_\nu \gtrsim 0.05\,\text{eV}$.
- Introduce right-handed sterile neutrinos with a heavy Majorana mass, m_R.
- This allows a Dirac mass (m_D) term in the Lagrangian,

$$\mathcal{L} \supset m_D \nu_L^\dagger \nu_R + \frac{1}{2} m_R \nu_R \nu_R + \text{h.c.}\,, \tag{12}$$

$$= \frac{1}{2} \begin{pmatrix} \nu_L^\dagger & \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L^\dagger \\ \nu_R \end{pmatrix} + \text{h.c.} \tag{13}$$

- Eigenvalues: m_D^2/m_R , m_R (assume: $m_R \gg m_D$)
- Assuming $m_D \simeq m_e = .51\,MeV$, we obtain $m_R \lesssim 10^{12}\,GeV$.

Common Origin for QCD axion and neutrino mass

- The axion decay constant, $f_a \gtrsim 10^8\ GeV$ (SN1987A observations)

- Majorana mass of neutrino, $m_R \lesssim 10^{12}\ GeV$.

- Consider an $SU(5)$ strong dynamics to naturally relate f_a and m_R.

- The $SU(5)$ gauge invariance will protect PQ symmetry from quantum gravity up to a higher dimensional operator.

SU(5) Chiral Gauge Theory [2]

Table 1: Representations of the $SU(5)$ chiral fermions under gauge symmetry

	Gauge		Global
	$SU(5)$	$SU(3)_c$	$U(1)_{\mathrm{PQ}}$
$\psi_{\bar{5}}$	$\overline{\square}$	$\mathbf{R}_\psi$	-3/5
ψ_{10}	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{R}_\psi$	$+1/5$

- This PQ charge assignment sets $U(1)_{\mathrm{PQ}}$ symmetry $SU(5)$-anomaly free.

[2] M. B. Gavela, M. Ibe, P. Quilez, and T. T. Yanagida, Automatic Peccei–Quinn symmetry, Eur. Phys. J. C 79, 542 (2019), arXiv:1812.08174 [hep-ph].

Composite Axion

- The $SU(5)$ confinement scale $\Lambda_5 \sim \mathcal{N} f_a$. ($\mathcal{N}$ is the QCD anomaly coefficient - depends on the QCD rep $\mathbf{R}_\psi$)

- The lowest dimension $SU(5)$-singlet operators that carry PQ charge contain **six fermions** and are **dimension-9**:

$$\begin{aligned}
\Phi_{\mathrm{PQ},1} &\equiv \psi_{\bar{5}}\,\psi_{\bar{5}}\,\psi_{10}\,\psi_{\bar{5}}\,\psi_{\bar{5}}\,\psi_{10}\,,\\
\Phi_{\mathrm{PQ},2} &\equiv \psi_{\bar{5}}\,\psi_{10}^{\dagger}\,\psi_{10}^{\dagger}\,\psi_{\bar{5}}\,\psi_{10}^{\dagger}\,\psi_{10}^{\dagger}\,,\\
\Phi_{\mathrm{PQ},3} &\equiv \psi_{\bar{5}}\,\psi_{\bar{5}}\,\psi_{10}\,\psi_{\bar{5}}\,\psi_{10}^{\dagger}\,\psi_{10}^{\dagger}\,,\\
\Phi_{\mathrm{PQ},4} &\equiv \psi_{\bar{5}}\,\psi_{\bar{5}}\,\psi_{\bar{5}}\,\psi_{\bar{5}}\,\psi_{\bar{5}}^{\dagger}\,\psi_{10}^{\dagger}\,.
\end{aligned} \tag{14}$$

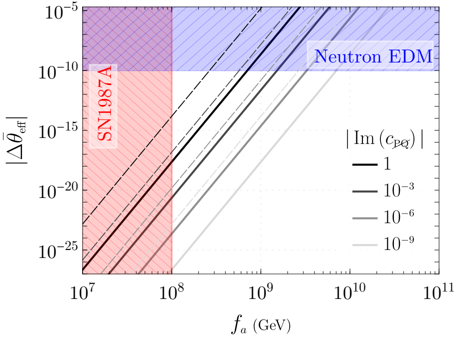
- The explicit PQ-breaking term at Planck scale:

$$\mathcal{L}_{\not{\mathrm{PQ}}} = \frac{c_{\not{\mathrm{PQ}}}}{4\pi}\frac{1}{M_{\mathrm{Pl}}^5}\;\psi_{\bar{5}}\,\psi_{\bar{5}}\,\psi_{10}\,\psi_{\bar{5}}\,\psi_{\bar{5}}\,\psi_{10} + \text{h.c.}\,, \tag{15}$$

Composite Axion

- The displacement of axion potential minimum:

$$|\Delta\theta| \sim |\mathrm{Im}\,(c_{\not{\mathrm{PQ}}})|\,\mathcal{N}^8 \left(\frac{f_a}{M_{\mathrm{Pl}}}\right)^5 \left(\frac{f_a}{m_a}\right)^2 . \qquad (16)$$



The solid (dashed) lines correspond to QCD rep: $\mathbf{R}_\psi = \mathbf{3} \oplus \bar{\mathbf{3}}$ ($\mathbf{R}_\psi = \mathbf{8}$).

- A solution to the axion quality problem requires $10^8 \lesssim f_a/\mathrm{GeV} \lesssim 10^9$. [assuming $|\mathrm{Im}\,(c_{\not{\mathrm{PQ}}})| \in (0.001, 1)$]

Composite Neutrino

- **QCD singlet 3-fermion** bound states which will be realized as RH neutrino:

$$N_1 \equiv \psi^\dagger_{\bar{5}}\psi^\dagger_{\bar{5}}\psi^\dagger_{10}\,, \tag{17}$$

$$N_2 \equiv \psi^\dagger_{\bar{5}}\psi_{10}\psi_{10}\,, \tag{18}$$

- The EFT operators to generate Dirac mass at EFT scale $\Lambda_L > \Lambda_5$:

$$\mathcal{L}_{\rm EFT} = \frac{\widetilde{\xi}_{ij}}{\Lambda_L^3} L_i H \big(\psi_{\bar{5}}\psi_{\bar{5}}\psi_{10}\big)_j + \frac{\widetilde{\xi}'_{ij}}{\Lambda_L^3} L_i H \big(\psi_{\bar{5}}\psi^\dagger_{10}\psi^\dagger_{10}\big)_j + \text{h.c.}\,, \tag{19}$$

- Below the $SU(5)$ resonance scale Λ_5:

$$\mathcal{L} = \xi_{ik}\xi_{jk}\left(\frac{\Lambda_5^3}{\Lambda_L^3}\right)^2 \frac{1}{\Lambda_5}(L_i H)(L_j H)e^{-2ia/f_{\rm PQ}} + \text{h.c.}\,, \tag{20}$$

Composite Neutrino

- After electroweak symmetry breaking, this generates Majorana masses for the neutrinos,

$$m_{\nu,i}^{\text{active}} \sim \mathcal{N}^5 \left(\frac{f_a}{\Lambda_L}\right)^6 \frac{v^2}{f_a}\,, \tag{21}$$

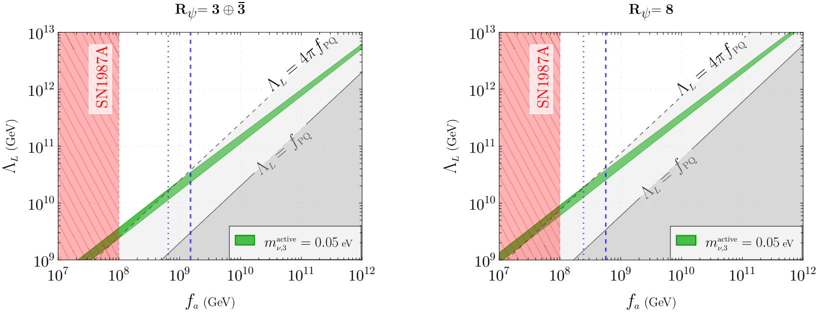


Figure 3: EFT scale Λ_L versus f_a in the heavy sterile neutrino model with an active neutrino mass $m_{\nu,3}^{\text{active}} = 0.05\,\mathrm{eV}$.

UV completion of operator $LH\psi\psi\psi$

- Introduce two massive scalar fields ϕ and ϕ_2 with $m_\phi, m_{\phi_2} \gg \Lambda_5$

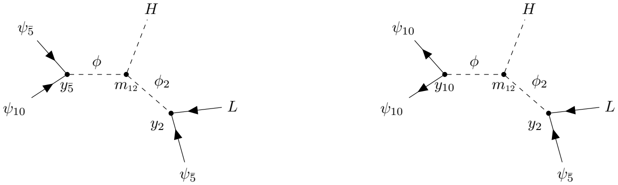
Table 2: Representations of the fields in the UV completion of the heavy sterile neutrino scenario.

	$SU(5)$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{\text{PQ}}$
$\psi_{\bar{5}}$	$\overline{\square}$	$\mathbf{R}_\psi$	$\mathbf{1}$	0	$-3/5$
ψ_{10}	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{R}_\psi$	$\mathbf{1}$	0	$1/5$
ϕ	$\overline{\square}$	$\mathbf{R}_\psi$	$\mathbf{1}$	0	$2/5$
ϕ_2	$\overline{\square}$	$\mathbf{R}_\psi$	$\square$	$-1/2$	$2/5$
L	$\mathbf{1}$	$\mathbf{1}$	$\square$	$-1/2$	1
H	$\mathbf{1}$	$\mathbf{1}$	$\square$	$1/2$	0

UV completion of operator $LH\psi\psi\psi$

- The interaction Lagrangian in UV

$$\mathcal{L} = y_{\bar{5}}\psi_{\bar{5}}\psi_{10}\phi + \frac{1}{2}y_{10}\psi_{10}\psi_{10}\phi^\dagger + y_2 L\psi_{\bar{5}}\phi_2^\dagger + m_{12}\phi\phi_2^\dagger H^\dagger + \text{h.c.}\,, \tag{22}$$



(a) $\psi_{\bar{5}}\psi_{\bar{5}}\psi_{10}LH$ (b) $\psi_{\bar{5}}\psi_{10}^\dagger\psi_{10}^\dagger LH$

- Integrating out ϕ and ϕ_2, the effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{y_{\bar{5}}y_2 m_{12}}{m_\phi^2 m_{\phi_2}^2} H(L\psi_{\bar{5}})(\psi_{\bar{5}}\psi_{10}) + \frac{1}{2}\frac{y_{10}y_2 m_{12}}{m_\phi^2 m_{\phi_2}^2} H(L\psi_{\bar{5}})(\psi_{10}^\dagger\psi_{10}^\dagger) + \text{h.c.}\,. \tag{23}$$

Alternative: Light Sterile Neutrino

- Introduce elementary, massless, right-handed neutrinos ν_R with PQ charge -1.
- No Majorana mass term for ν_R in UV (due to PQ symmetry)
- Below EFT scale Λ_R (assuming $\Lambda_5 < \Lambda_R < M_{\rm Pl}$):

$$\mathcal{L}_{\rm EFT} = \frac{\tilde{\zeta}_{ij}}{\Lambda_R^2}\nu_{R,i}\left(\psi_{\bar{5}}^\dagger\psi_{\bar{5}}^\dagger\psi_{10}^\dagger\right)_j + \frac{\tilde{\zeta}'_{ij}}{\Lambda_R^2}\nu_{R,i}\left(\psi_{\bar{5}}^\dagger\psi_{10}\psi_{10}\right)_j + \text{h.c.}\,, \tag{24}$$

- Below scale Λ_5, the neutrino mass term becomes:

$$\mathcal{L} = y_\nu^{ij}\frac{v}{\sqrt{2}}\,\nu_{L,i}^\dagger\nu_{R,j} + \frac{1}{2}m_R^{ij}\nu_{R,i}\nu_{R,j}e^{2ia/f_{\rm PQ}} + \text{h.c.}\,. \tag{25}$$

with

$$m_R^{ij} \sim \zeta_{ik}\zeta_{jk}\left(\frac{\Lambda_5}{\Lambda_R}\right)^4\Lambda_5\,, \tag{26}$$

Alternative: Light Sterile Neutrino

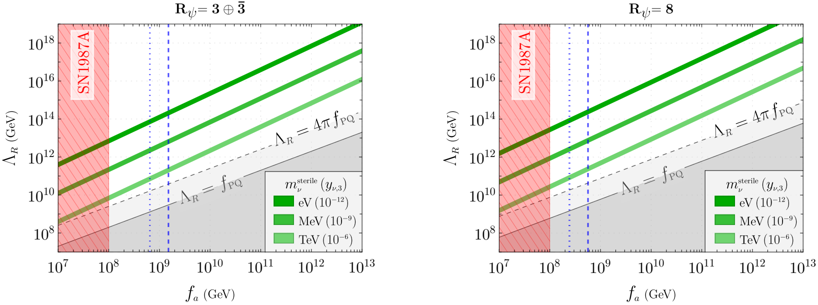


Figure 4: EFT scale Λ_R versus f_a in the light sterile neutrino model with an active neutrino mass $m_{\nu,3}^{\text{active}} = 0.05\,\text{eV}$.

Conclusion

- In the light sterile neutrino model, sterile neutrinos of mass scale ranging from **eV to TeV** can be generated.
- This model naturally relates axion decay constant f_a and sterile neutrino mass m_R.
- Solves axion quality problem since the Planck-scale violation appears at **mass dimension 9**.
- A first order SU(5) phase transition could give rise to GW signal associated with the PQ scale (i.e, $\sim 10^8$ GeV) [Von Harling et al., 2020].
- Predicts a coupling between axion and neutrino $\Rightarrow$ effects in neutrino oscillations within local DM axion halo [Gherghetta and Shkerin, 2023].

Thank you!

References

Cox, P., Gherghetta, T., and Paul, A. (2023).
A common origin for the QCD axion and sterile neutrinos from SU(5) strong dynamics.
JHEP, 12:180.

Gavela, M. B., Ibe, M., Quilez, P., and Yanagida, T. T. (2019).
Automatic Peccei–Quinn symmetry.
Eur. Phys. J. C, 79(6):542.

Gherghetta, T. and Shkerin, A. (2023).
Probing the Local Dark Matter Halo with Neutrino Oscillations.
arXiv:2305.06441.

Von Harling, B., Pomarol, A., Pujolàs, O., and Rompineve, F. (2020).
Peccei-Quinn Phase Transition at LIGO.
JHEP, 04(2020):195.

Backup slide: UV completion of $\nu_R\psi\psi\psi$

	$SU(5)$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathrm{PQ}}$
$\psi_{\bar{5}}$	$\overline{\square}$	$\mathbf{R}_\psi$	$\mathbf{1}$	0	$-3/5$
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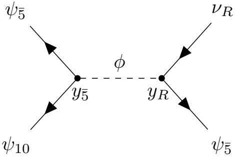
ϕ	$\overline{\square}$	$\mathbf{R}_\psi$	$\mathbf{1}$	0	$2/5$
ν_R	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	-1
L	$\mathbf{1}$	$\mathbf{1}$	$\square$	$-1/2$	-1
H	$\mathbf{1}$	$\mathbf{1}$	$\square$	$1/2$	0

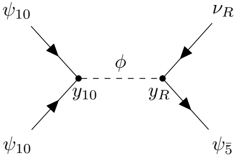
Backup slide: UV completion of $\nu_R\psi\psi\psi$

- Interaction term in the UV Lagrangian:

$$\mathcal{L} = y_\nu L^\dagger H^\dagger \nu_R + y_{\bar{5}}\psi_{\bar{5}}\psi_{10}\phi + \frac{1}{2}y_{10}\psi_{10}\psi_{10}\phi^\dagger + y_R\nu_R\psi_{\bar{5}}^\dagger\phi + \mathrm{h.c.}\,, \tag{27}$$



(a) $\psi_{\bar{5}}^\dagger\psi_{\bar{5}}^\dagger\psi_{10}^\dagger\nu_R$



(b) $\psi_{\bar{5}}^\dagger\psi_{10}\psi_{10}\nu_R$

- Integrating out the massive scalar ϕ,

$$\mathcal{L}_{\text{eff}} = \frac{y_{\bar{5}}y_R}{m_\phi^2}(\nu_R\psi_{\bar{5}}^\dagger)(\psi_{\bar{5}}^\dagger\psi_{10}^\dagger) + \frac{1}{2}\frac{y_{10}y_R}{m_\phi^2}(\nu_R\psi_{\bar{5}}^\dagger)(\psi_{10}\psi_{10}) + \mathrm{h.c.}\,, \tag{28}$$