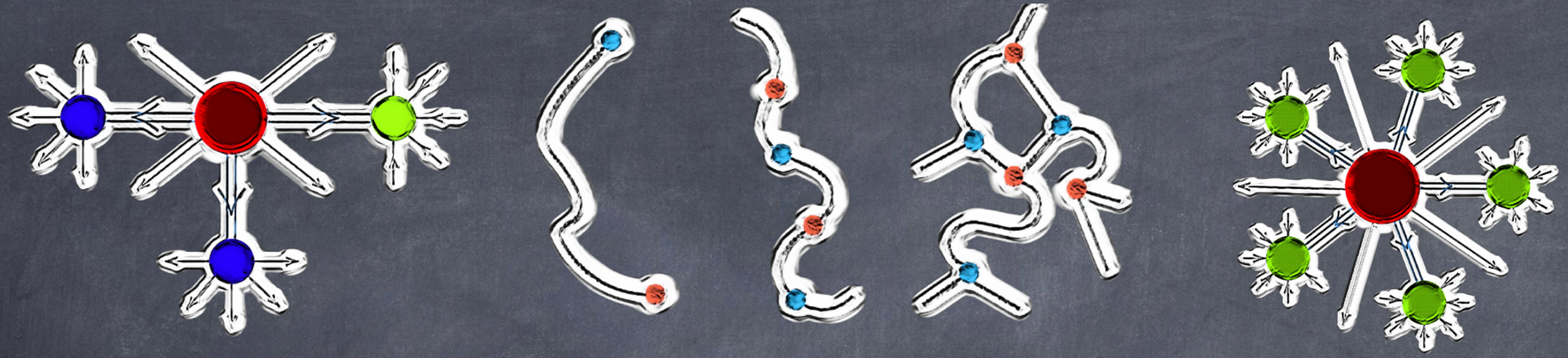

Composite Topological Structures in SO(10)

Published in: JHEP 05 (2023), 119

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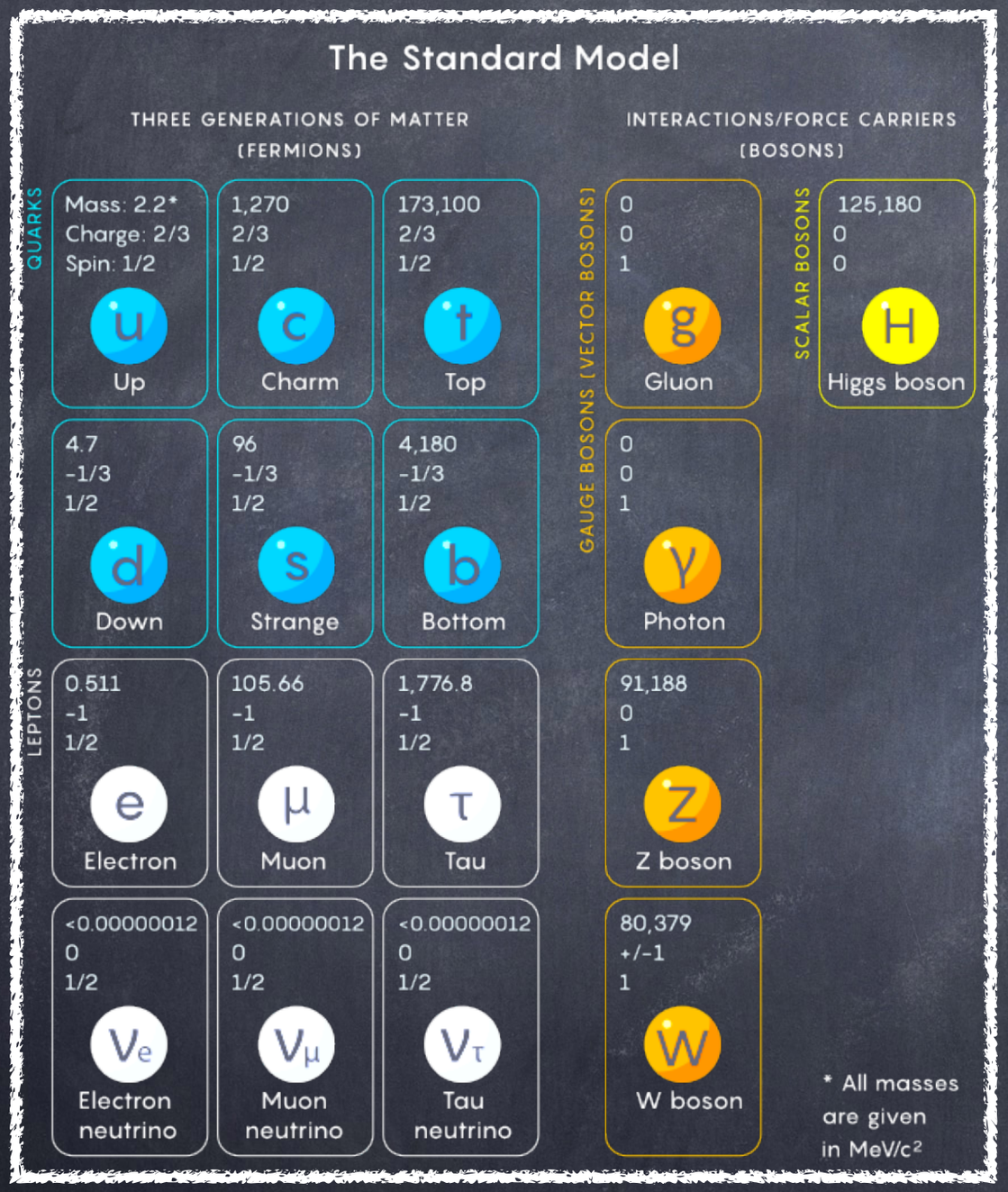
Talk prepared for DPF-PHENO-2024

Motivation

- $SO(10)$ is a prominent example of grand unified theories (GUT) that provide unification at high scales ($M_{GUT} = 10^{16}\ GeV$) of the Standard Model 'forces' (strong, weak and electromagnetic) and 'elementary matter' (quarks and leptons).
- It's spontaneously broken at M_{GUT} to the SM, which is hard to test in collider physics.
- This paper studies the existence of new topological structures which can be produced during the phase transitions that occurred in the early universe.
- Large detectors such as PTA, LIGO-VIRGO, LISA, Einstein telescope will test.

Standard Model



Gauge group of the Standard Model (SM)

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

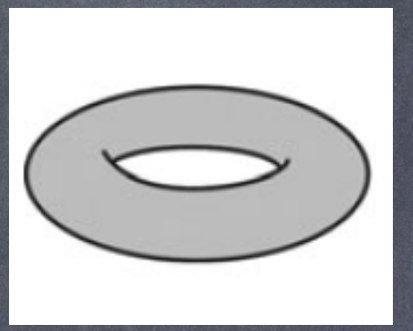
QCD - strong interactions involving 'colored' quarks & gluons

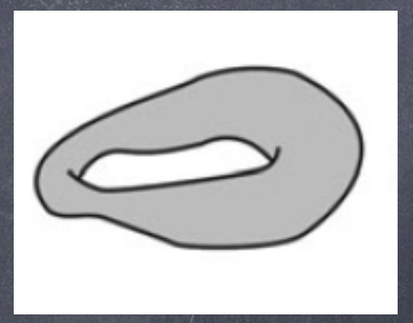
Electromagnetic and weak interactions mediated by $W^{\pm}$, Z^0 bosons and γ, which have been found

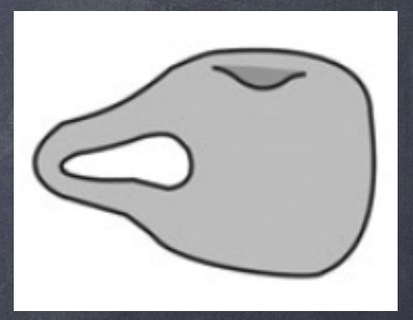
Note: there are no stable topological structures in SM.

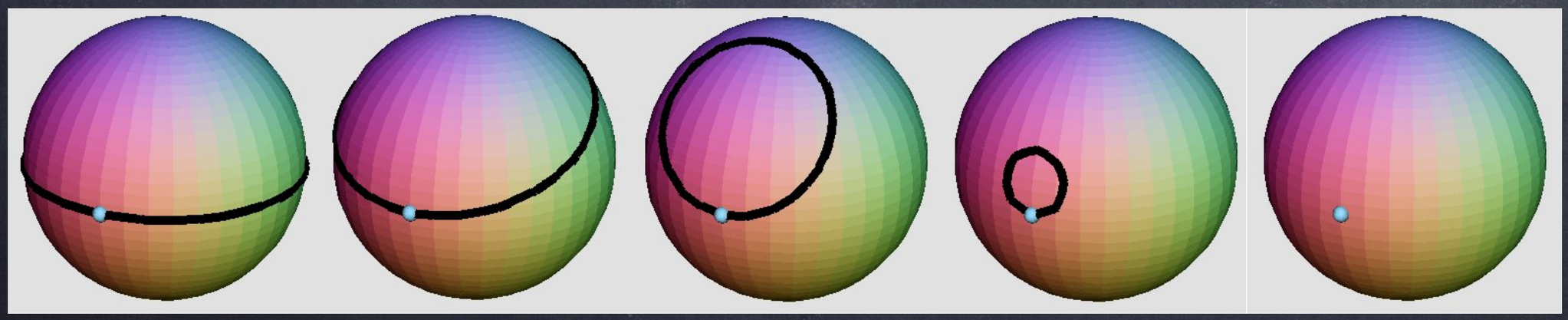
Lie group as Topological space

- Topology is the mathematical study of the properties of spaces that are preserved under continuous deformations. Eg: Donut and Cup are topology equivalent.
- $SU(2)$ is the Lie group of 2×2 unitary matrices with determinant of $+1$.
- $SU(2)$ has the topology of three sphere. Its parameters span this space (live in this space).
- If I draw a circle on this sphere, I can collapse it to a point, $SU(2)$ is simply connected.





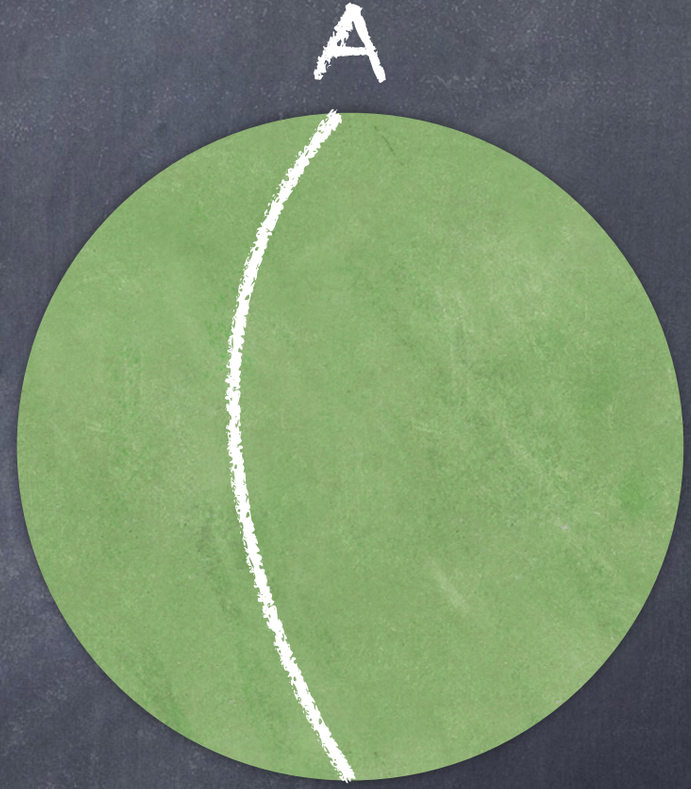




Attribution: Salix alba at English Wikipedia

Lie group as Topological space

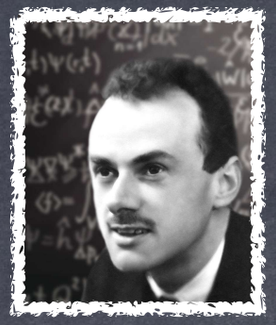
- $SO(3)$ is a special orthogonal group in dimension 3 (also called group of all rotations). $SO(3)$ is same as 3−sphere with antipodal points identified. A closed loop on it cannot be contracted to a point.
- $SO(10)$ is a special orthogonal group in dimension 10.
- $SO(10)$ is not simply connected, but has the simply connected covering group $Spin(10)$.



$A = A'$

Topological structures

- Quarks, leptons, gauge bosons are point particles in $SO(10)$ and SM.
- Spontaneous $SO(10)$ breaking predicts extended topological structures.
- Such structures expected from phase transitions during cooling of the universe, similar to phase transitions and structure production in condensed matter physics.
- The energy scale (mass, mass per unit length or area) is related to the symmetry breaking scale at which the structure arises.



Magnetic Monopole

- $A_\varphi^U = \frac{g}{4\pi}(1 - cos(\theta))$, upper hemisphere $0 \leq \theta \leq \pi/2$
- $A_\varphi^L = -\frac{g}{4\pi}(1 + cos(\theta))$, lower hemisphere $\pi/2 \leq \theta \leq \pi$
- A^U and A^L are connected by gauge transformation at $\theta = \pi/2$

$$A_\varphi^U(\theta = \pi/2) - A_\varphi^L(\theta = \pi/2) = \frac{2g}{4\pi} = \frac{1}{ie}(\partial\Omega)\Omega^{-1},$$

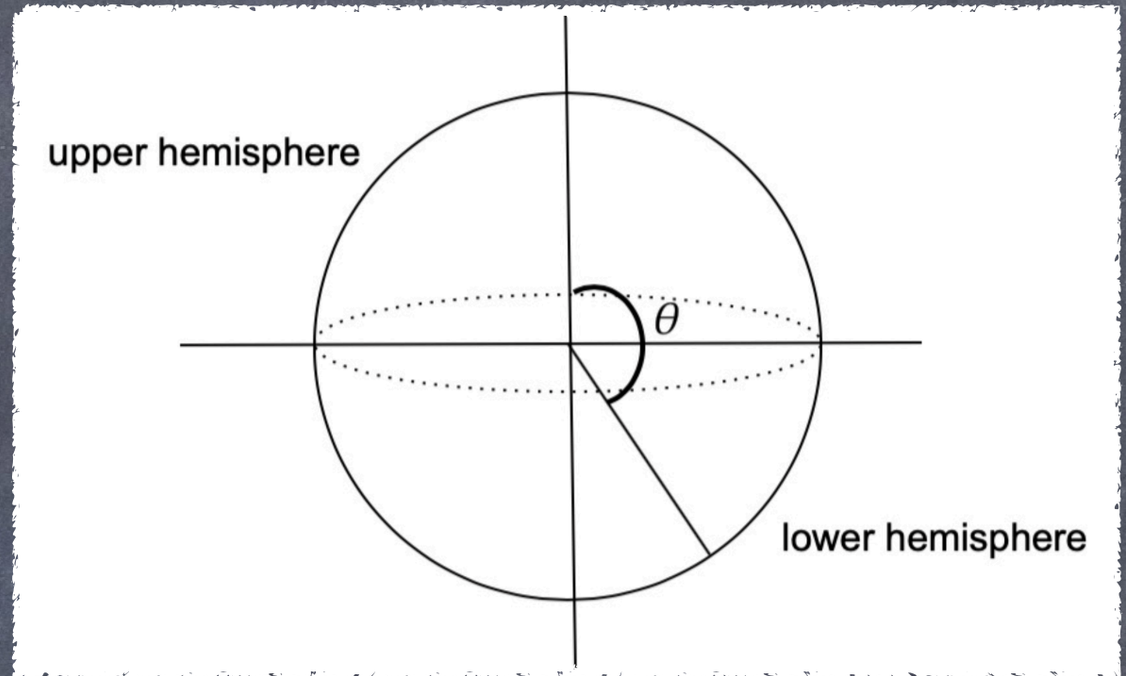
where $\Omega(\varphi) = exp[i2eg\varphi/4\pi]$

- For $\Omega(\varphi)$ to be single-valued it follows,

$$\frac{eg}{4\pi} = \frac{n}{2}$$

Dirac quantization condition

$$g = \frac{2\pi n}{e}$$

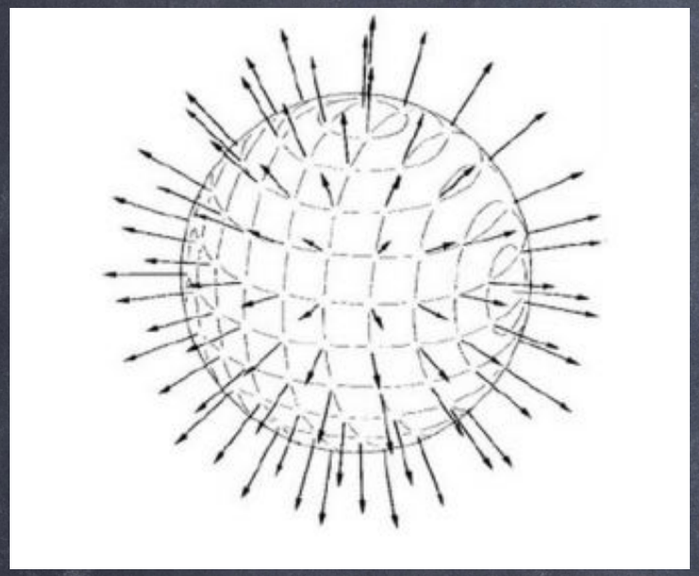


Mapping of equatorial circle to the $U(1)$ gauge transformation is denoted by homotopy group $\pi_1(U(1)) = Z$ (set of integers).

This mapping is more subtle when we have SM unbroken symmetry group $SU(3)_c \times U(1)_{em}$

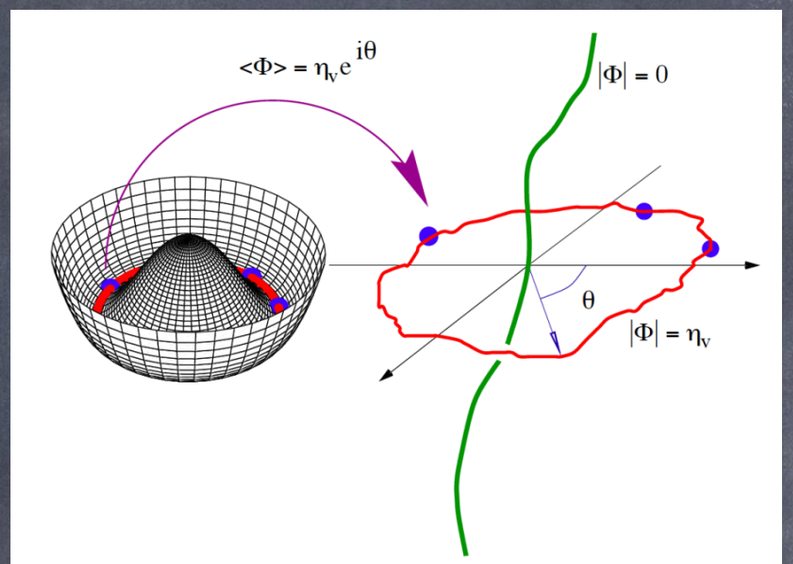
Symmetry breaking $G \to H$

Second homotopy group $\pi_2(G/H)$



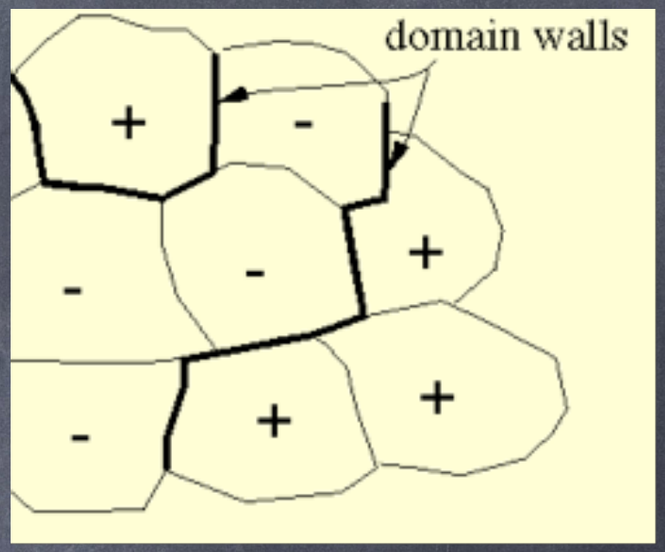
Mapping from the 2–sphere at spatial infinity to the vacuum manifold. Eg: Monopole

First homotopy group $\pi_1(G/H)$



Mapping from the circle at spatial infinity to the vacuum manifold. Eg: cosmic string; vortex in superfluid

Zeroth homotopy group $\pi_0(G/H)$

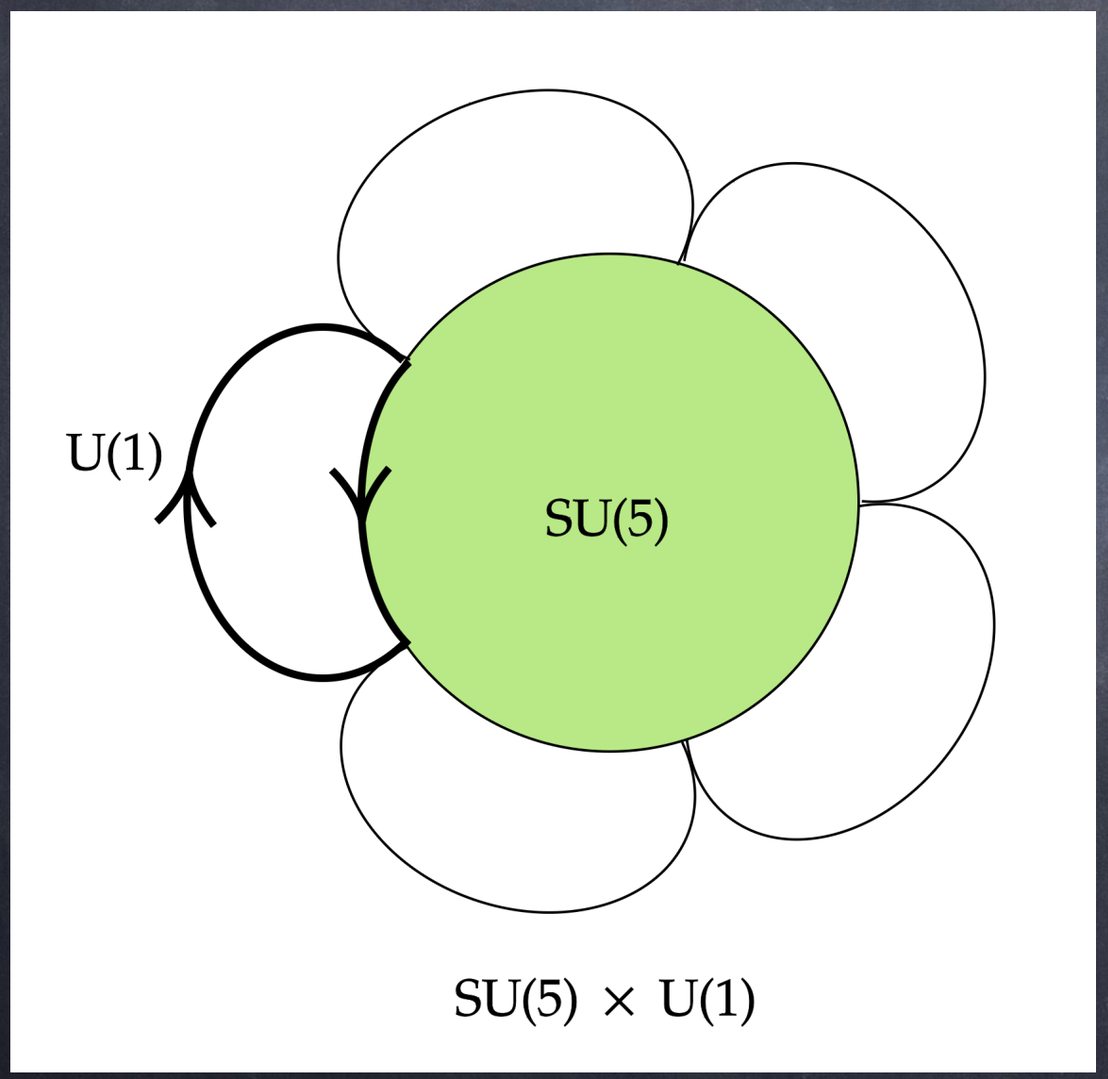


If a discrete symmetry is spontaneously broken we produce domain walls that separate the distinct vacua.

In our $SO(10)$ paper, we showed a large number of composite structures are produced from these basic building blocks.

Symmetry breaking
$G \to H$

$$SO(10) \longrightarrow SU(5) \times U(1)_\chi$$



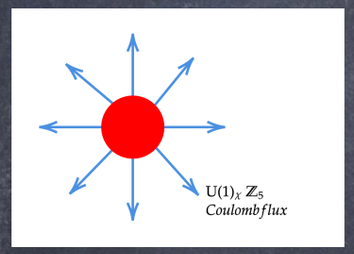
Theorem: $\pi_n(G/H) = \pi_{n-1}(H)$ if G is simply connected. We use covering group $G = Spin(10)$ for $SO(10)$.

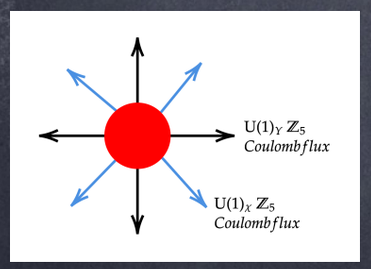
We study
$\pi_2(G/H) = \pi_1(SU(5) \times U(1)_\chi)$

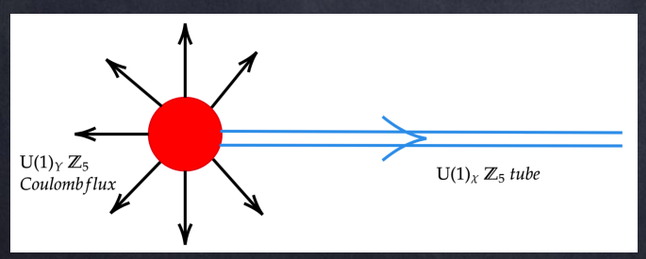
The monopole with minimal $U(1)$ magnetic charge is associated with a loop that winds only one-fifth around $U(1)$.

Metastable strings

$$SO(10) \longrightarrow SU(5) \times U(1)_\chi$$
$$\longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_\chi$$
$$\longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$

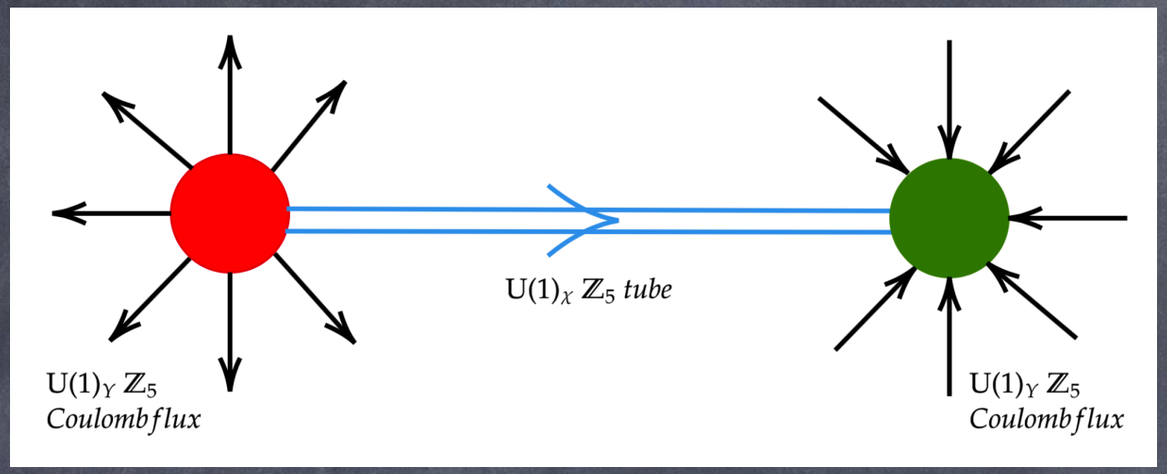


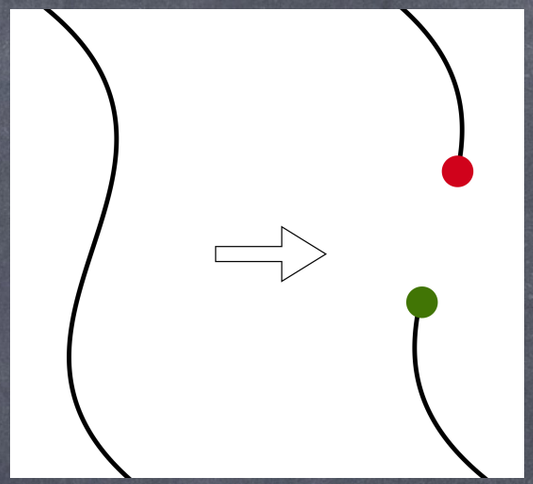




- First breaking: $SO(10)$ monopole appears carrying $SU(5)$ and $U(1)_\chi$ charges.
- Second breaking: $SO(10)$ monopole produced at the first step ends up with a combination of $U(1)_\chi$ and $U(1)_Y$ Coulomb magnetic fields.
- Third step spontaneously breaks $U(1)_\chi$, which means the flux is confined in a tube (analog to Meissner effect in superconductor).

Metastable strings



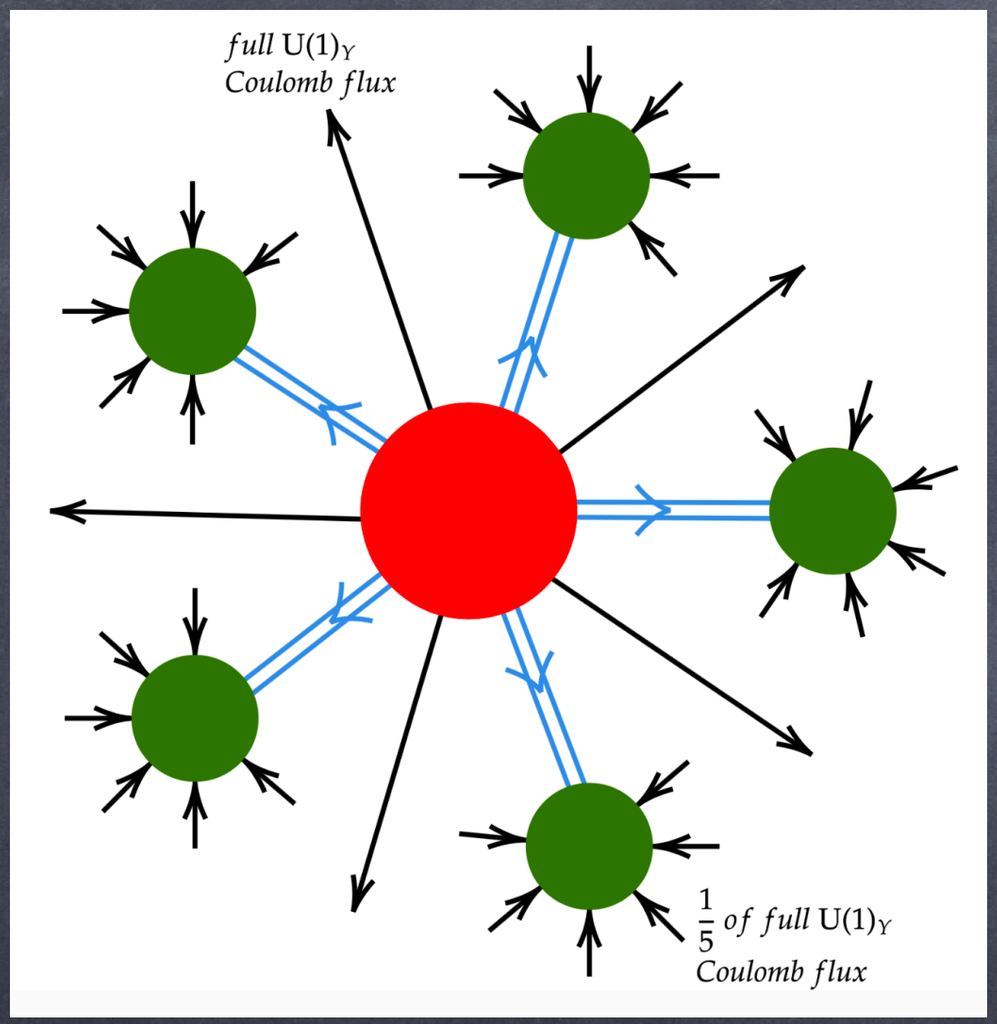


- We finally end up with a dumbbell configuration as shown here. To get a finite energy configuration it should end on antimonopole (magnetic analog of QCD quark-antiquark).

- Long tube could break through quantum mechanical tunneling of monopole-antimonopole pair.

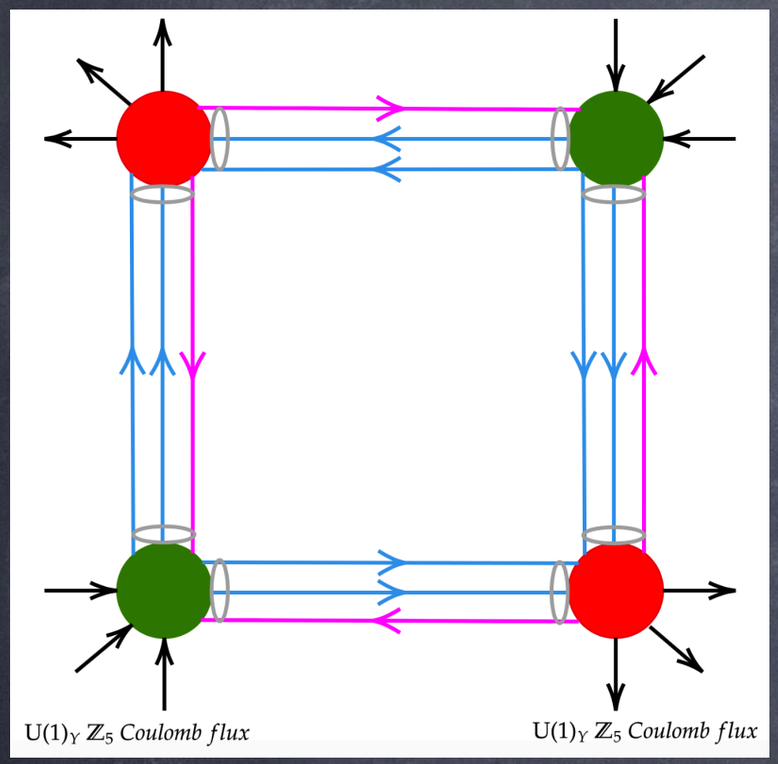
- Emit gravitational radiation over a wide frequency range which can be tested in a variety of experiments including pulsar timing array and LIGO-VIRGO.

Starfish

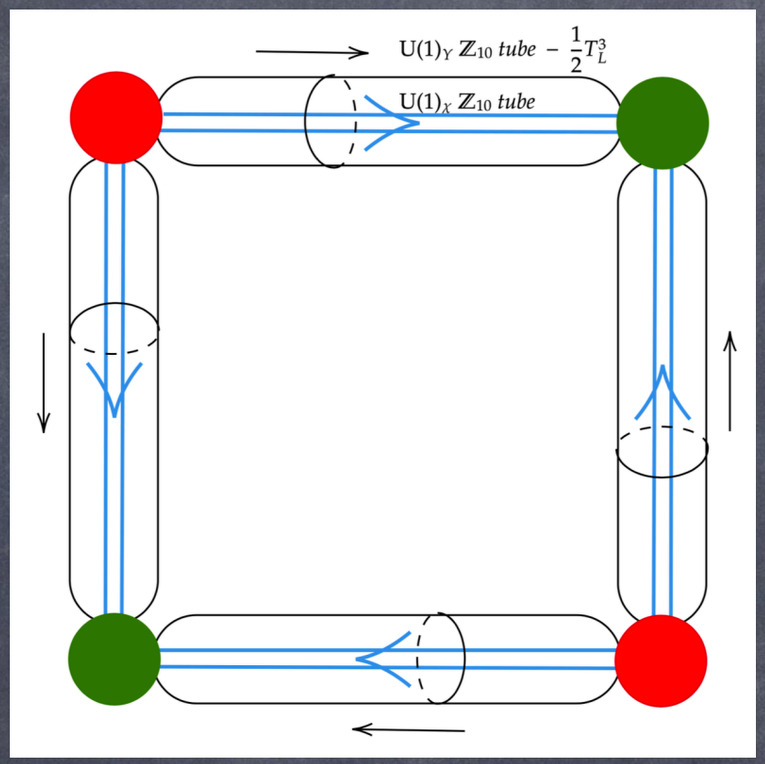


Taking five $SO(10)$ monopoles together we get a multimonopole configuration with full $U(1)_\chi$ and $U(1)_Y$ Coulomb fluxes corresponding to 2π rotations about these two abelian groups. We named this new configuration 'Starfish' .

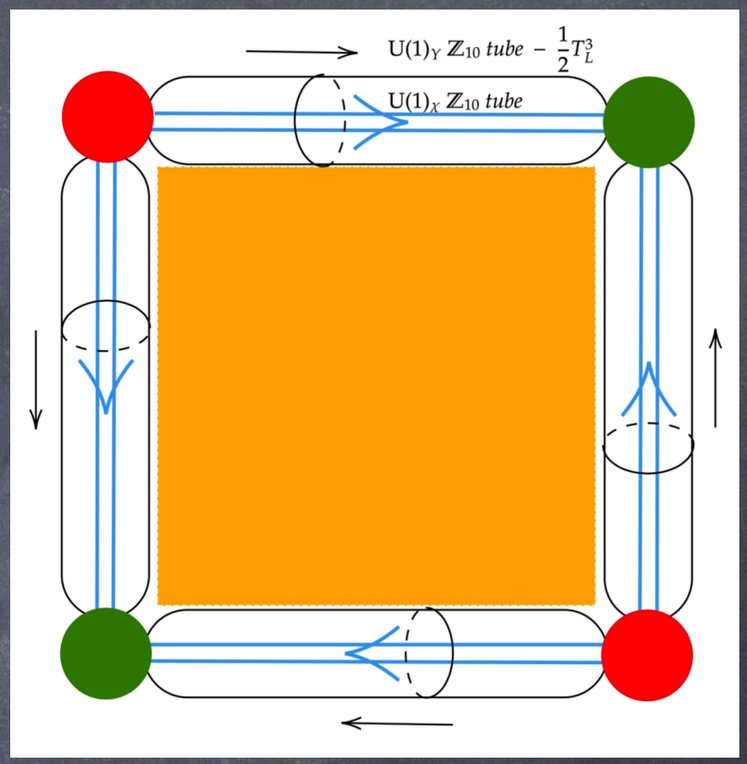
Domain walls bounded by Necklace



The group $U(1)_\chi$ is broken to its $Z_{10} = Z_5 \times Z_2$ subgroup. Necklace of monopoles (red) and antimonopoles (green) connected by $U(1)_\chi Z_{10}$ tubes.

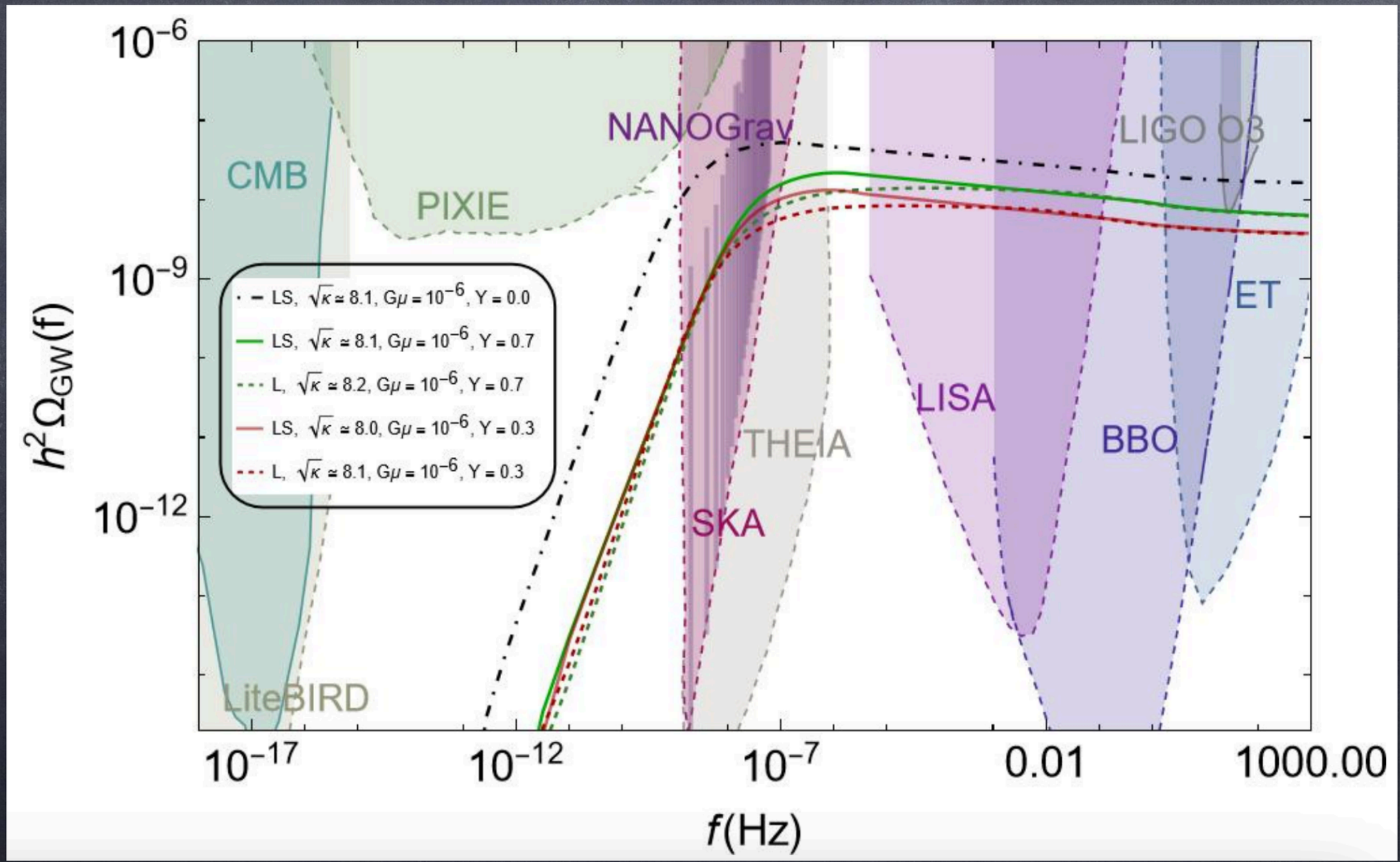


Necklace configuration after Electroweak breaking.



Emergence of a domain wall (orange) from each segment of the necklace due to the breaking of the Z_2 subgroup of $U(1)_\chi$, with the necklace ultimately becoming the boundary of the wall.

GW spectrum for metastable current carrying strings



Adeela, Qaisar, Amit (arxiv: 2311.05564)

Summary

- For the first time we have found structures like starfish, polypole and walls bounded by necklaces.
- Discovery of primordial monopoles has implications for particle physics and cosmology. Measuring magnetic charge would reveal underlying unified theory. (Recall that SM does not contain monopoles.)
- Cosmic strings (stable or metastable) radiate gravitational waves, will be tested by current/proposed experiments.
- Domain walls bounded by strings leave gravitational wave traces. They were first discovered in $SO(10)$, recently in superfluid $He-3B$.
- Study of more complex structures in the early universe such as the necklace configuration is just beginning.
- Overlap with similar composite structures recently reported in the condensed matter literature is quite exciting.

Thank you!

Backup slides

SO(10): Predictions

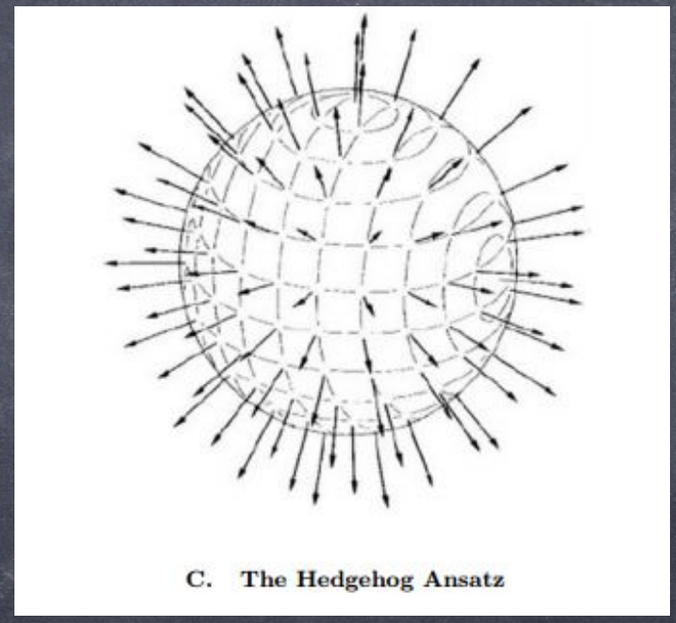
- Gauge coupling unification.
- Quark and lepton unification.
- Three observed SM neutrino are majorana particles with non-zero masses.
- Existence of right-handed neutrinos (RHNs) - key for explaining baryon asymmetry via leptogenesis.
- Proton decay - though not yet observed, ongoing experiments continue to test.
- Electric charge quantization.
- Topological structures - main focus of this talk.

t'Hooft-Polyakov Monopole (1974)

- Scalar triplet ϕ^a in the adjoint representation of $SU(2)$ breaks $SU(2) \longrightarrow U(1)_{em}$.

- We want to find a mapping between the sphere at potential minima to the spatial boundary sphere at infinity.

- We choose "hedgehog" configuration such that at large r, $\phi^a(\hat{r}) = v\hat{r}^a$.



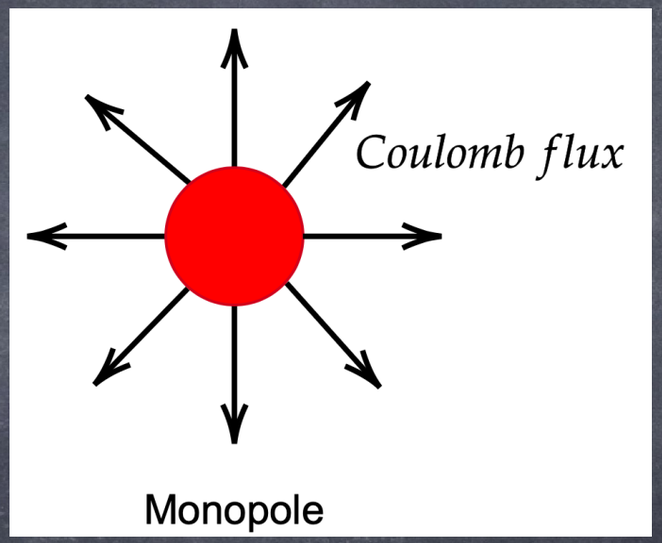
C. The Hedgehog Ansatz

Source: http://bartholomewandrews.com/projects/Skyrmions.pdf

t'Hooft-Polyakov Monopole (1974)

- Energy is finite only if $D_i\phi^a \sim 0$ for large r.

$$W_i^a \sim \frac{\epsilon_{iak}\hat{r}_k}{er} \text{ and } B_i^a \sim \frac{\hat{r}_i\hat{r}^a}{er^2}$$



- It carries two units of Dirac charge $g = 1/e$.
- Monopole mass $M \sim (4\pi/e)M_W$, core size M_W^{-1}.

Homotopy

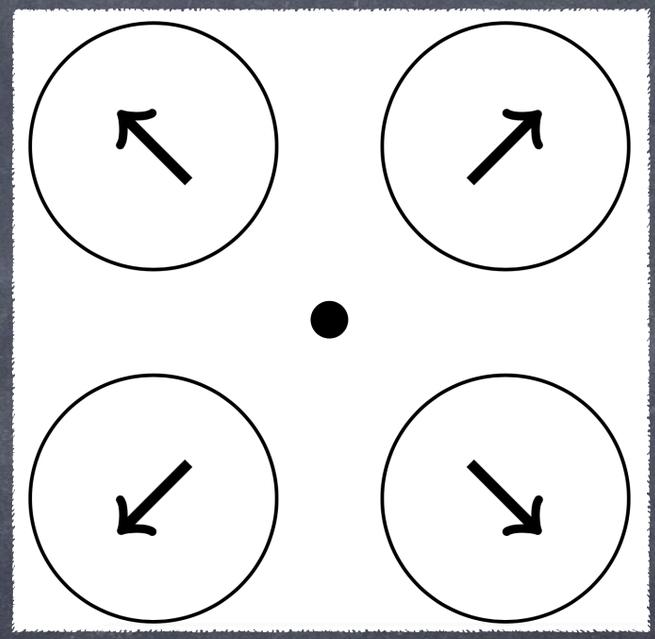
- $\pi_0(G/H)$ is non-trivial $\Longrightarrow$ Domain Walls
- $\pi_1(G/H)$ is non-trivial $\Longrightarrow$ Strings
- $\pi_2(G/H)$ is non-trivial $\Longrightarrow$ Monopoles
- $\pi_3(G/H)$ is non-trivial $\Longrightarrow$ Textures
- Theorem: $\pi_n(G/H) = \pi_{n-1}(H)$ if $\pi_n(G) = 1 = \pi_{n-1}(G)$.

Monopoles in GUTs

- In particular, if H contains any $U(1)$ factors, $\pi_1(H)$ is non-trivial and the model has monopoles. In GUTs, the GUT group is usually taken to be simply connected and then, since H necessarily contains the electromagnetic $U(1)$ symmetry group, $\pi_2(G/H)$ is non-trivial and hence, (magnetic) monopoles are necessarily predicted.

Primordial Monopoles

- They are produced via the Kibble Mechanism as $G \longrightarrow H$
- Higgs field in each region takes different direction.
- At the center unbroken phase gets trapped (a defect).
- Center of monopole has G symmetry $< \phi > = 0$.
- Monopole problem (too many). Need Inflation.



SU(5) monopole

- $SU(5) \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_{em} \longrightarrow SU(3)_c \times U(1)_{em}$

Lightest monopole carries one unit of Dirac magnetic charge even though there exist fractionally charged quarks.

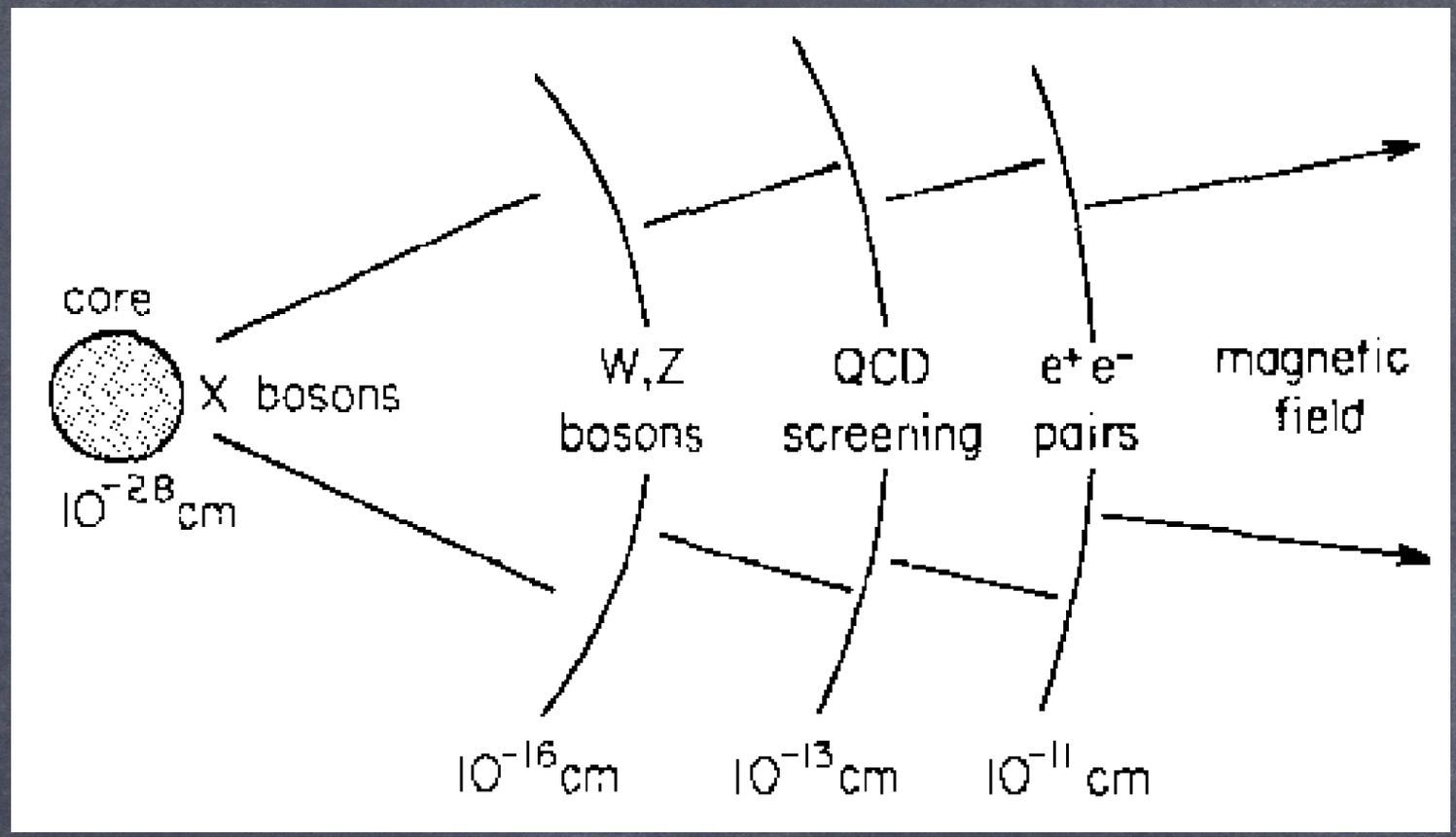
- $Q_{em} = diag\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, -1\right)$ (5 representation of $SU(5)$: antidown quarks, in three colors, the neutrino and the electron)

- A 2π rotation with Q_{em} yields: $diag\left(\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}, 1, 1\right)$

SU(5) monopole

- $Q_{color} = diag\left(\frac{-1}{3}, \frac{-1}{3}, \frac{2}{3}, 0, 0\right)$ (traceless 3 x 3 matrices acting on the quarks only)
- A 2π rotation with Q_{color} yields: $diag\left(\frac{-2\pi}{3}, \frac{-2\pi}{3}, \frac{4\pi}{3}, 1, 1\right)$
- Net effect: we return to identity element.
- The monopole carries one unit of Dirac magnetic charge and color magnetic charge.
- Color magnetic field is screened at distances greater than $\sim \Lambda_{QCD}^{-1}$

Structure of a grand unified monopole



Refer review by John Preskill on Magnetic Monopoles

- Concentrated mass within a tiny core (radius: M_X^{-1}).
- Virtual W and Z bosons influence interactions within distances of $M_Z^{-1}(\sim 10^{-16}cm)$.
- Exhibits a color magnetic field, extending up to 10^{-13} cm, screened by strong-interaction effects.
- Because of large magnetic charge monopole is strongly coupled to cloud of virtual electron-positron pairs, extends up to $m_e^{-1}(\sim 10^{-11}cm)$.

News from Condense Matter physics

- Walls bounded by strings have recently been found in superfluid helium ^3He-B.

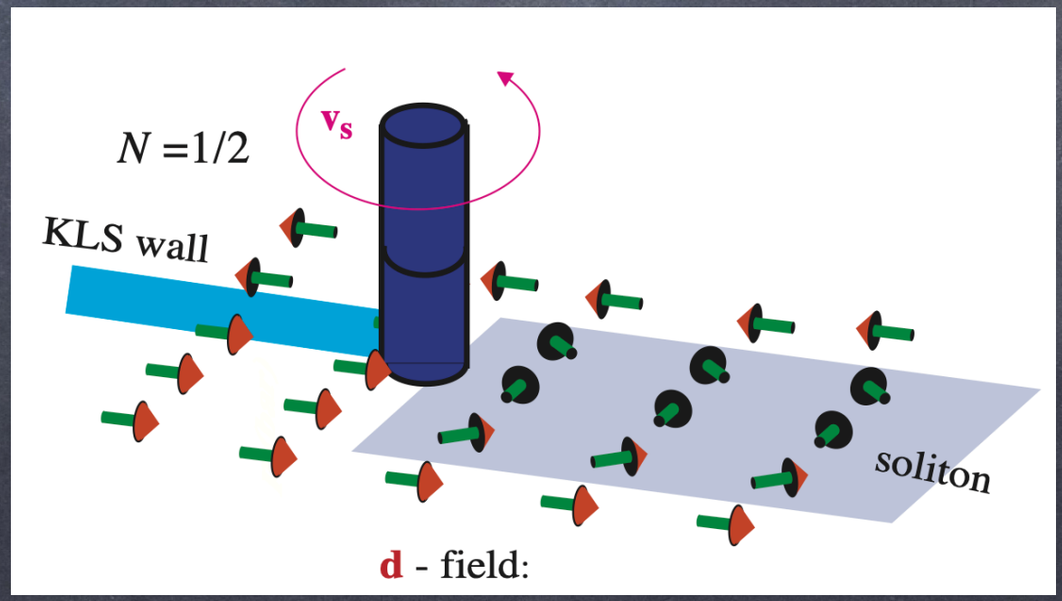
nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-018-08204-8 OPEN

Half-quantum vortices and walls bounded by strings in the polar-distorted phases of topological superfluid ^{3}He

J. T. Mäkinen [1], V. V. Dmitriev[2], J. Nissinen[1], J. Rysti[1], G. E. Volovik[1,3], A. N. Yudin[2], K. Zhang[1,4] & V. B. Eltsov[1]



Composite Objects to be Observed

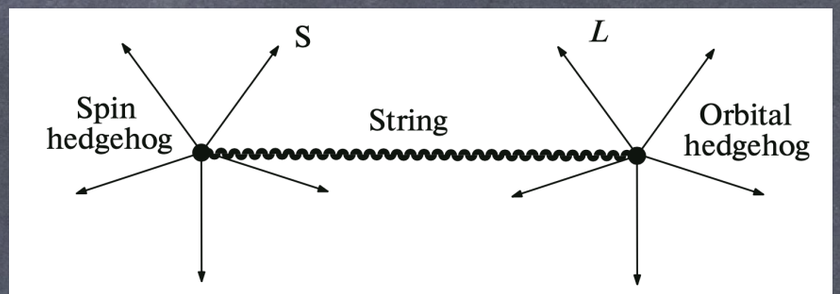


Fig. 9. Spin and orbital hedgehogs connected by string in magnon BEC (HPD), from [51].

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Composite Topological Objects in Topological Superfluids

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Abstract—The spontaneous phase coherent precession of magnetization, discovered in 1984 by Borovik-Romanov, Bunkov, Dmitriev, and Mukharskiy [1] in collaboration with Fomin [2], became now an important experimental tool for study complicated topological objects in superfluid ^{3}He.

DOI: 10.1134/S1063776120070146

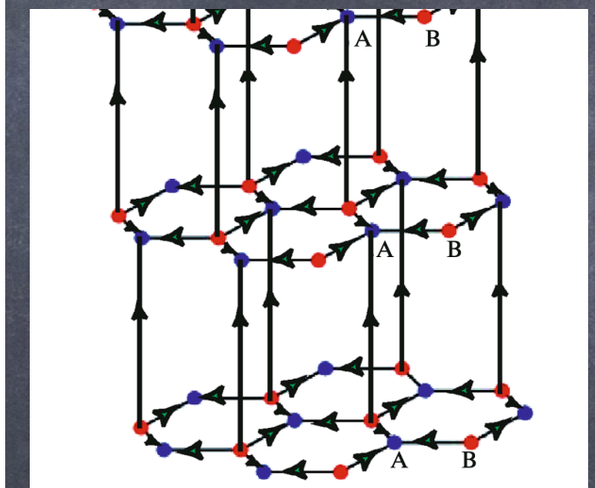


Fig. 11. (Color online) Three dimensional lattice of monopoles (on sites A) and anti-monopoles (on sites B), which are joined together by Alice strings (half-quantum vortices).

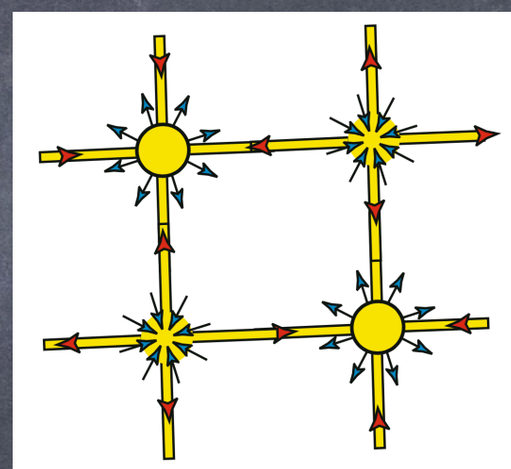


Fig. 10. (Color online) Two dimensional lattice of monopoles (hedgehogs in the $\hat{I}$-field) joined together by Alice strings (half-quantum vortices). Each monopole is the source or sink of 4 strings.

Bounds on monopoles

- Let's consider how much dilution of the monopoles is necessary. $M_I \sim 10^{13}$ GeV corresponds to monopole masses of order $M_M \sim 10^{14}$ GeV. For these intermediate mass monopoles the MACRO experiment has put an upper bound on the flux of 2.8×10^{-16} cm^{-2} s^{-1} sr^{-1}. For monopole mass $\sim 10^{14}$ GeV, this bound corresponds to a monopole number per comoving volume of $Y_M \equiv n_M/s \lesssim 10^{-27}$. There is also a stronger but indirect bound on the flux of $(M_M/10^{17}$ GeV$)10^{-16}$cm^{-2} s^{-1} sr^{-1} obtained by considering the evolution of the seed Galactic magnetic field.

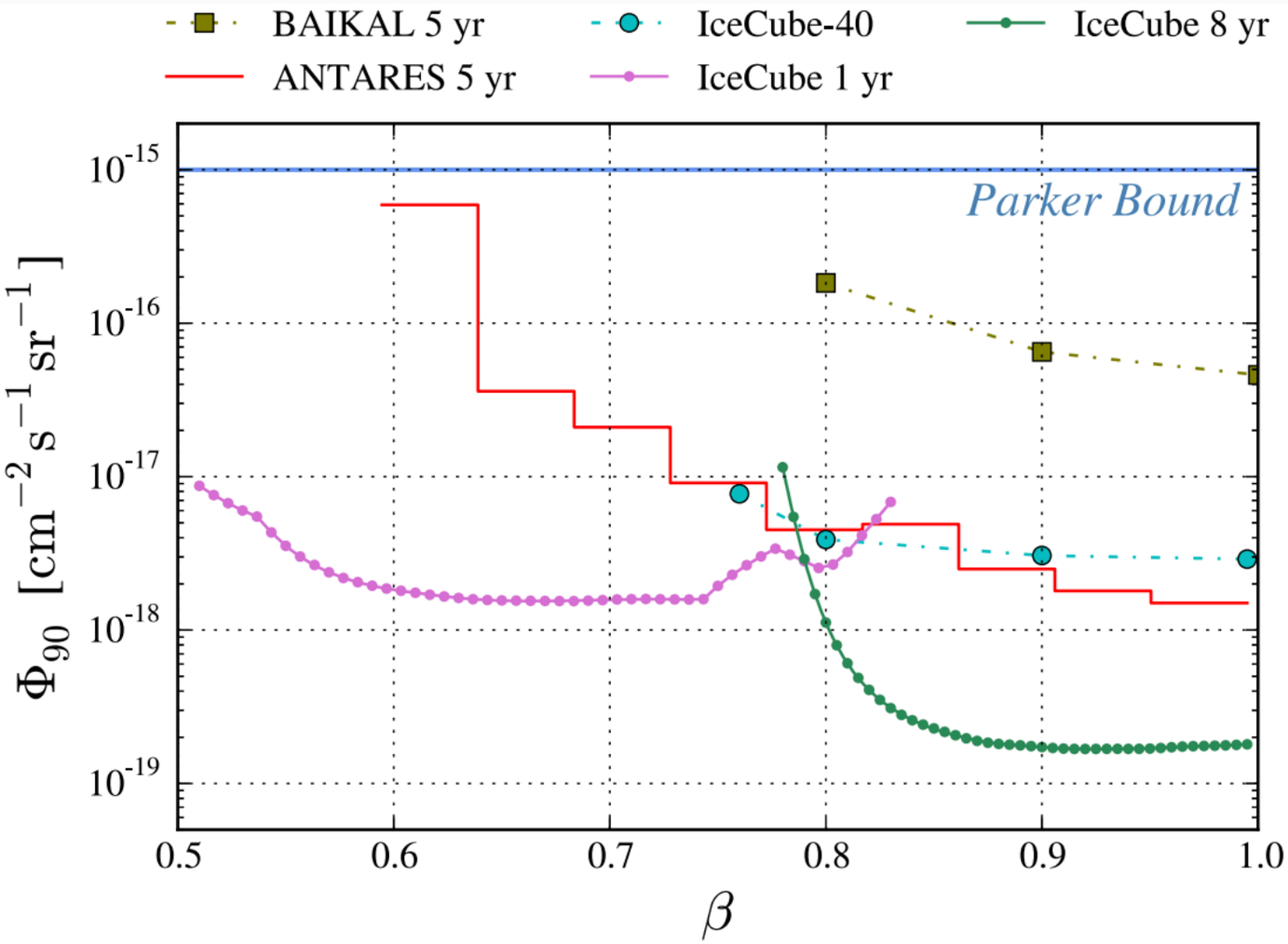
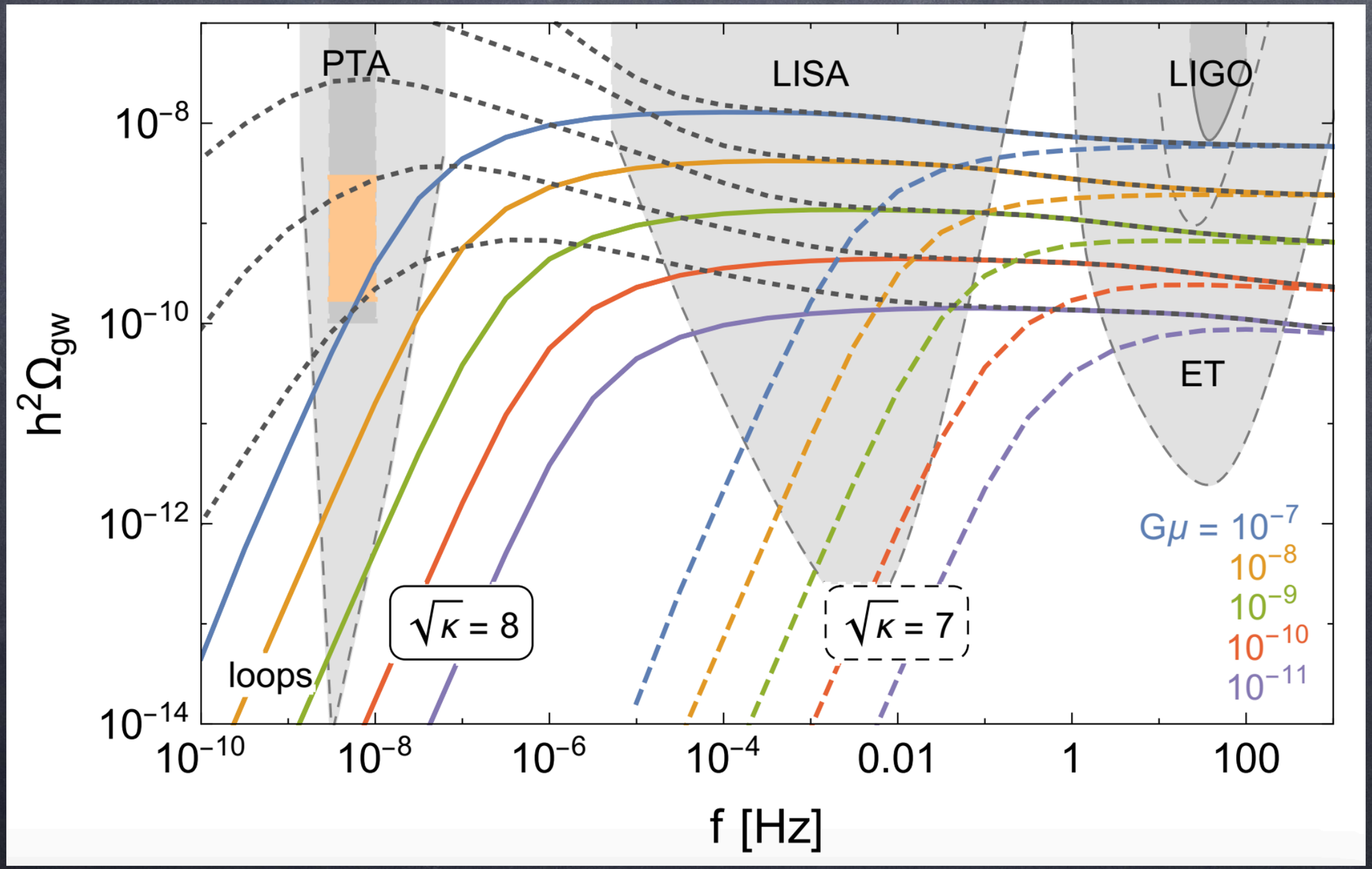


FIG. 4. 90% confidence level upper limit on the cosmic flux of relativistic monopoles as a function of true particle β obtained in the present analysis assuming zero background (dark green curve). Also included are previous results of IceCube [14,15], ANTARES [16], and Baikal [17]. The limits are valid for monopoles with the given β at the detector. The Parker bound [37,38] is shown as reference.

GW spectrum for metastable strings



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