Multi Higgs boson signals of a modified muon Yukawa coupling

Taegyu Lee

Phys. Rev. D 109, 095003

Collaboration with R. Dermisek, K. Hermanek, N. McGinnis, and S. Yoon

INDIANA UNIVERSITY BLOOMINGTON

Summary of the talk

Multi-Higgs boson signals at muon collider

- Precise measurement of the magnitude of Muon Yukawa coupling

- Obtain information about the *complex phase* of Muon Yukawa coupling

Deviation in muon Yukawa coupling

Deviation in muon Yukawa coupling

$$\mathscr{L} \supset -y_\mu \bar{l}_L \mu_R H + \text{h.c.}$$

$$\supset -m_\mu \bar{\mu}_L \mu_R - \frac{1}{\sqrt{2}} \lambda^{h,\text{SM}}_{\mu\mu} \bar{\mu}_L \mu_R h + \text{h.c.}$$

$$l_L = (\nu_\mu, \mu_L)^T \qquad H = (G^+, v + (h + iG^0)/\sqrt{2})^T$$

$$m_\mu = y_\mu v \qquad \lambda^{h,\text{SM}}_{\mu\mu} = \frac{m_\mu}{v}$$

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$$m_\mu = y_\mu v \qquad \lambda_{\mu\mu}^{h,\mathrm{SM}} = \frac{m_\mu}{v}$$

Using the general Kappa Framework, we can express deviation in Yukawa coupling

Kappa framework arXiv:1209.0040. arXiv:1307.1347.

$$\lambda_{\mu\mu}^{h} = \frac{m_\mu}{v} \kappa_\mu$$

For SM, $\kappa_\mu = 1.$

$$\Delta\kappa_\mu = \kappa_\mu - 1$$

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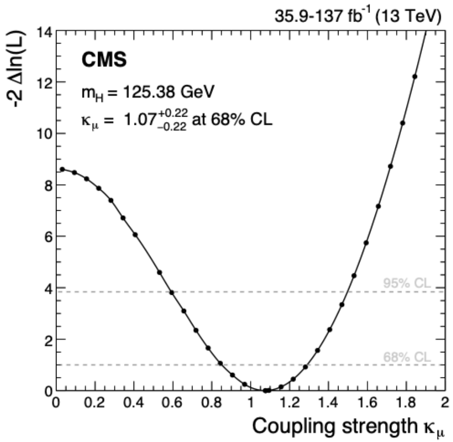

CMS : arXiv:2009.04363

Currently, $0.59 < \kappa_\mu < 1.50$ at 95 % confidence level

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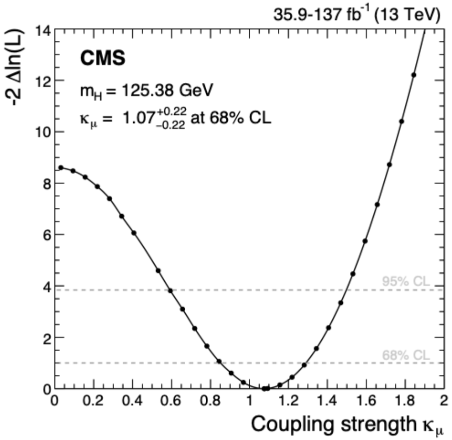

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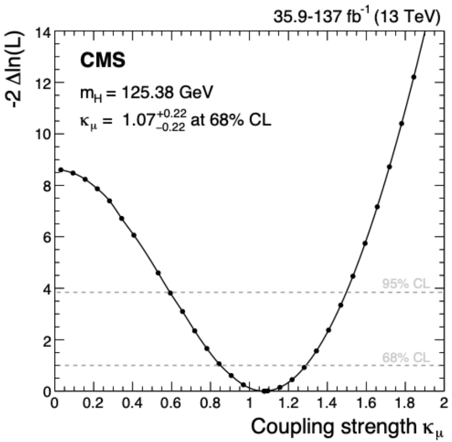

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Limits : Can’t measure complex phase

For complex κ_μ, $\Delta\kappa_\mu < 2.50$

Standard Model Effective Field Theory (SMEFT)

If SM is considered as low energy effective theory, the effect of UV theory at some high scale shows up as a form of higher dimensional operator.

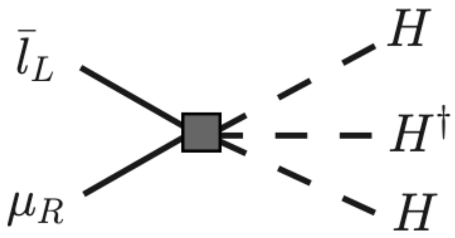
Warsaw Basis : arXiv:1008.4884

The complete set of dimension 6 operators are given in Warsaw basis.
(59 independent operators)

There is only one dimension 6 SMEFT operator responsible for deviation Yukawa coupling

$$\mathscr{L} \supset -C_{\mu H}\,\bar{l}_L \mu_R H \left(H^\dagger H\right) + \mathrm{h.c.}$$

Dimension 6 mass operator



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$$\lambda^h_{\mu\mu} = \frac{m_\mu}{v} + 2\, C_{\mu H} v^2$$

$$\Delta\kappa_\mu = 2C_{\mu H} v^3 / m_\mu$$

$$\lambda^h_{\mu\mu} = \frac{m_\mu}{v} \kappa_\mu \qquad \Delta\kappa_\mu = \kappa_\mu - 1$$

Thus we can parametrize the processes from dim 6 mass operator in terms of $\Delta\kappa_\mu$.

Dimension 6 mass operator

Di-Higgs, tri-Higgs : arXiv:2108.10950

	in terms of $\Delta\kappa_\mu$	in units of $\sigma_{\mu^+\mu^-\to hh}$
$\sigma_{\mu^+\mu^-\to hh}$	$\frac{9}{256\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2$	1
$\sigma_{\mu^+\mu^-\to hZ_L}$	$\frac{1}{128\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2$	$\frac{2}{9}$
$\sigma_{\mu^+\mu^-\to Z_L Z_L}$	$\frac{1}{256\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2$	$\frac{1}{9}$
$\sigma_{\mu^+\mu^-\to W_L^+ W_L^-}$	$\frac{1}{128\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2$	$\frac{2}{9}$

Assuming muon and Higgs mass zero

	in terms of $\Delta\kappa_\mu$	in units of $\sigma_{\mu^+\mu^-\to hhh}$
$\sigma_{\mu^+\mu^-\to hhh}$	$\frac{3s}{2^{14}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2$	1
$\sigma_{\mu^+\mu^-\to hhZ_L}$	$\frac{s}{2^{14}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2$	$\frac{1}{3}$
$\sigma_{\mu^+\mu^-\to hZ_L Z_L}$	$\frac{s}{2^{14}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2$	$\frac{1}{3}$
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$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
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Derivative operators

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Derivative operators

$$\mu^+\mu^- \to W^+W^-, hZ, W^+W^-h, W^+W^-Z, Zhh, ZZZ$$

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Derivative operators

$$\mu^+\mu^- \to W^+W^-, hZ, W^+W^-h, W^+W^-Z, Zhh, ZZZ$$

Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$

Dipole operators

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Dipole operators

$$\mu^+\mu^- \to hZ, ZZ, W^+W^-, W^+W^-h$$

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- Is the Standard Model background small?

Dimension 6 mass operator

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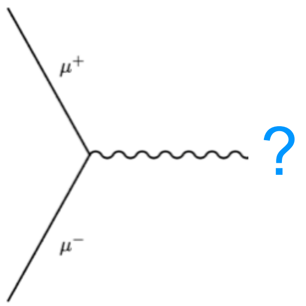
- Is the Standard Model background small?

Large SM background comes from gauge boson interaction.



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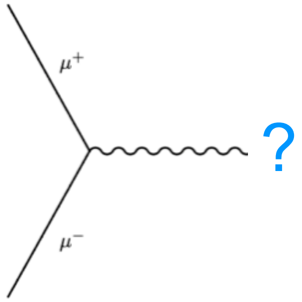

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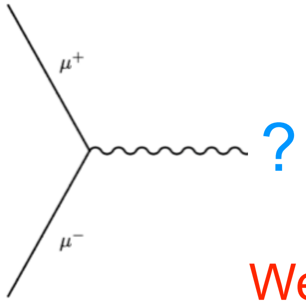
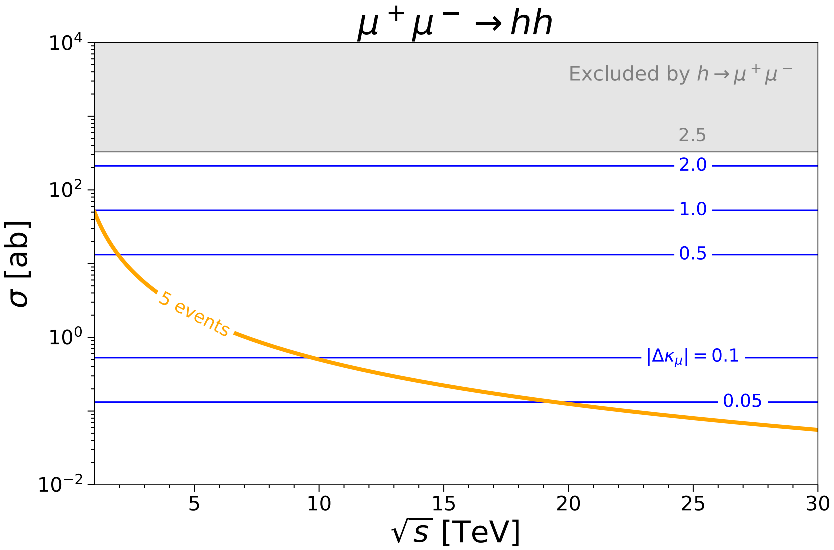
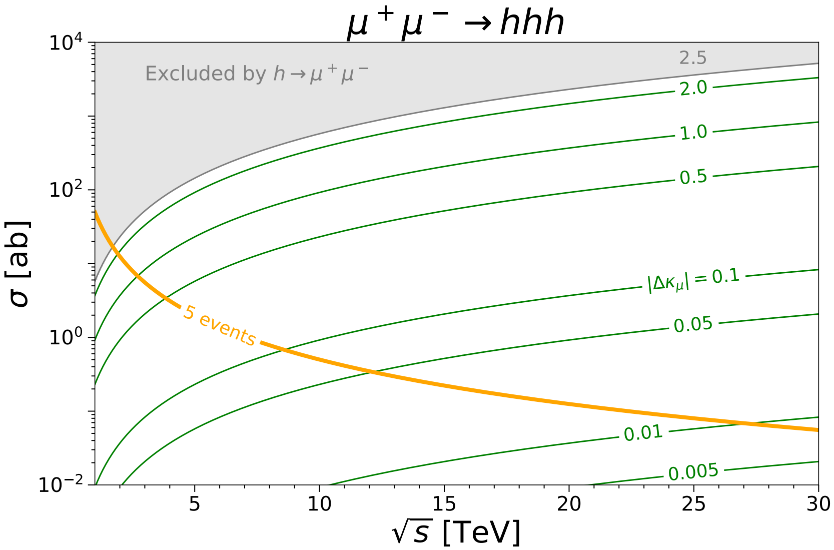


$\mu^+\mu^- \to hh, hhh$ survived.

We identify $\mu^+\mu^- \to hh, hhh$ as a golden channel for measuring deviation in muon Yukawa coupling

Dimension 6 mass operator





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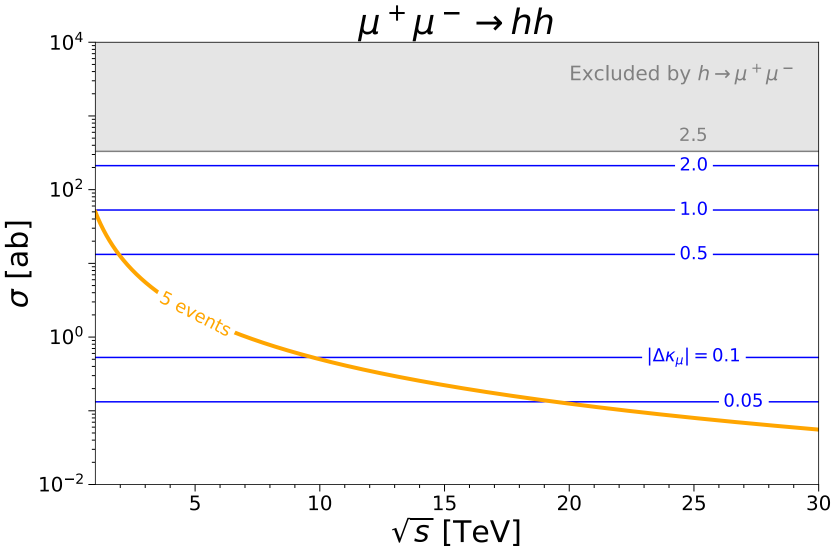
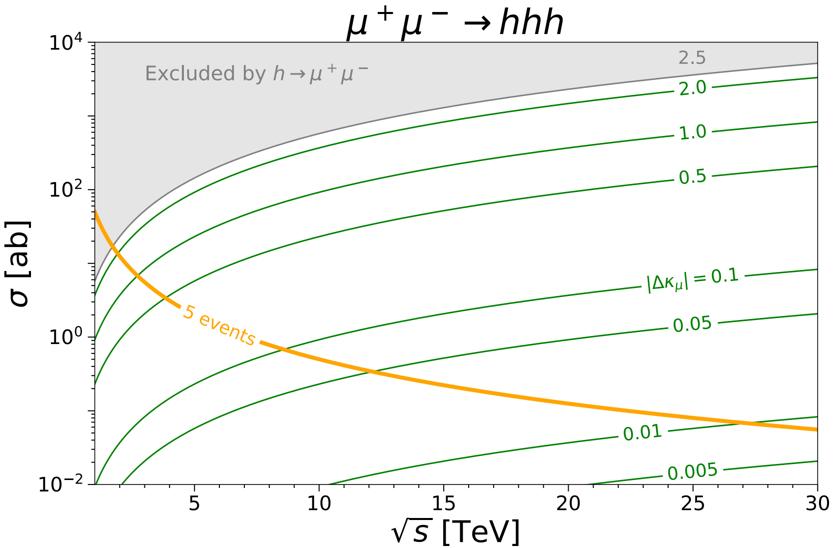


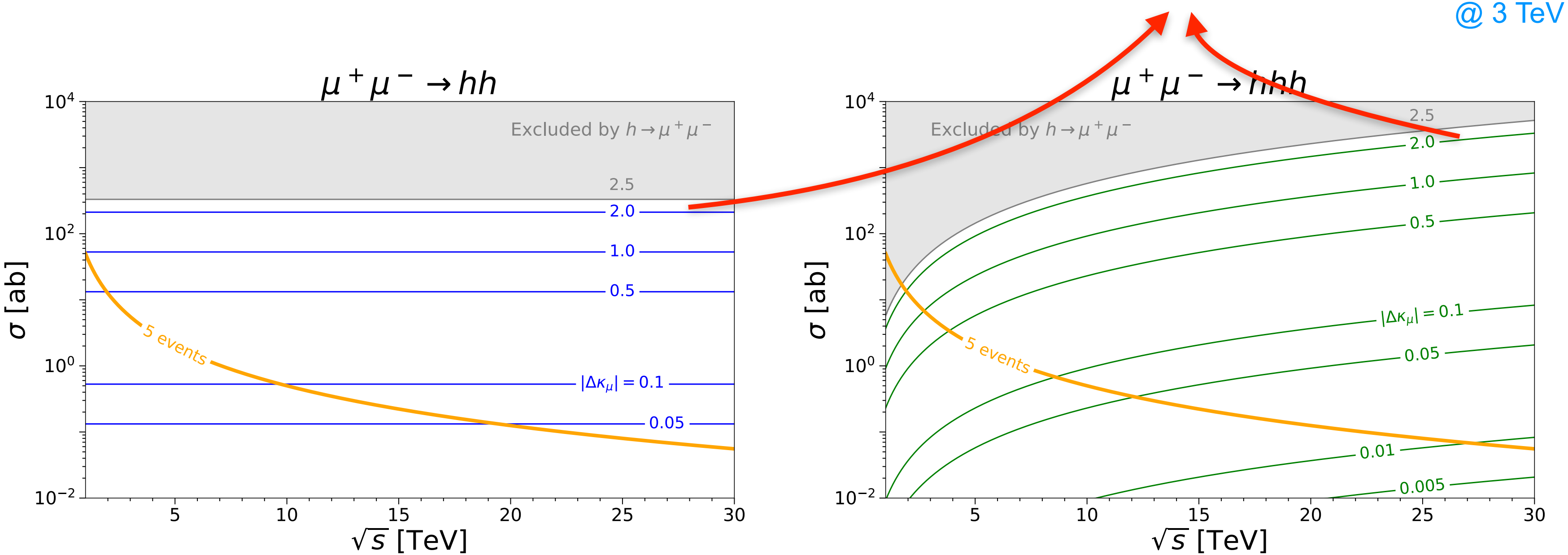


$\sigma(\mu^+\mu^- \to hh)_{SM} = 1.6 \times 10^{-4}$ ab and $\sigma(\mu^+\mu^- \to hhh)_{SM} = 2.9 \times 10^{-5}$ ab at $\sqrt{s} = 3$ TeV, and falling as $1/s^2$and $1/s$ at larger energies.

Dimension 6 mass operator

Opposite sign Yukawa

191 di-Higgs
30 tri-Higgs
@ 3 TeV



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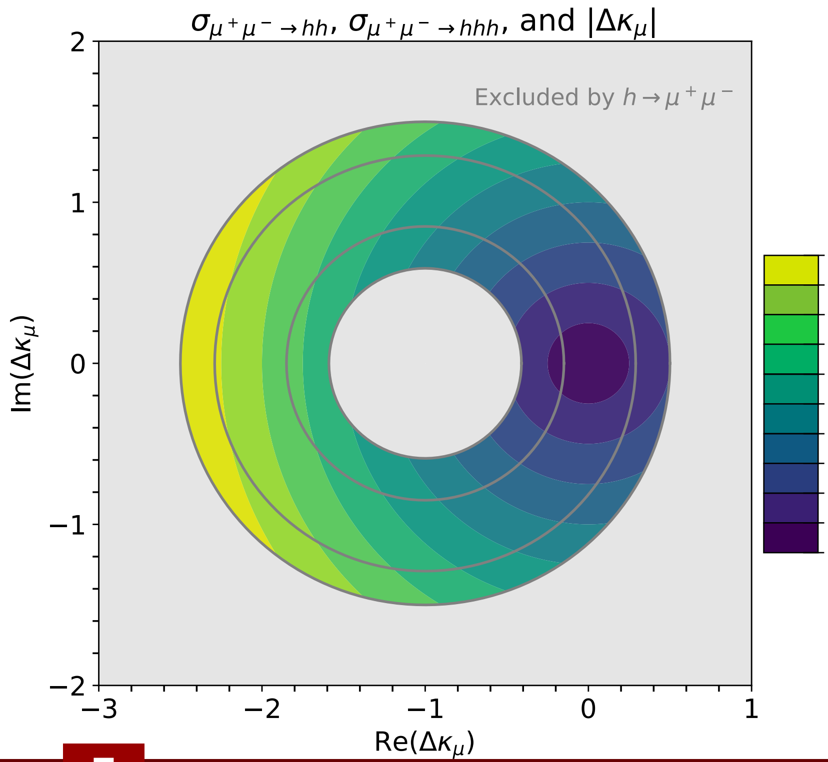
Complex $\Delta\kappa_\mu$ ($C_{\mu H}$) plane



$$\lambda^h_{\mu\mu} = \frac{m_\mu}{v}\left(1 + \Delta\kappa_\mu\right)$$

$$\sigma_{\mu\mu\to hh,hhh} \propto |\Delta\kappa_\mu|^2$$

$\lvert\Delta\kappa_\mu\rvert$	$\sigma_{\mu^+\mu^-\to hh}$ [ab]	$\sigma_{\mu^+\mu^-\to hhh}$ $[(\frac{\sqrt{s}}{10\text{ TeV}})^2$ ab]
2.5	332	578
2.25	269	468
2.0	212	370
1.75	163	283
1.5	119	208
1.25	83	145
1.0	53	93
0.75	30	52
0.5	13	23
0.25	3	6
0	0	0

Extension to 2HDM type II

$$\mathscr{L} \supset -y_\mu \bar{l}_L \mu_R H_d - C_{\mu H_d} \bar{l}_L \mu_R H_d \left(H_d^\dagger H_d\right)$$
$$-C_{\mu H_u}^{(1)} \bar{l}_L \mu_R H_d \left(H_u^\dagger H_u\right) - C_{\mu H_u}^{(2)} \bar{l}_L \mu_R \cdot H_u^\dagger \left(H_d \cdot H_u\right) - C_{\mu H_u}^{(3)} \bar{l}_L \mu_R \cdot H_u^\dagger \left(H_d^\dagger \cdot H_u^\dagger\right) + \text{h.c.}$$

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$$-C^{(1)}_{\mu H_u} \bar{l}_L \mu_R H_d \left(H_u^\dagger H_u\right) - C^{(2)}_{\mu H_u} \bar{l}_L \mu_R \cdot H_u^\dagger \left(H_d \cdot H_u\right) - C^{(3)}_{\mu H_u} \bar{l}_L \mu_R \cdot H_u^\dagger \left(H_d^\dagger \cdot H_u^\dagger\right) + \mathrm{h.c.}$$

1. Comes from new leptons with same quantum number as SM leptons
2. Leads to largest $\tan\beta$ enhancement to the processes.

Extension to 2HDM type II

$$\mathscr{L} \supset -y_\mu \bar{l}_L \mu_R H_d \; - \; C_{\mu H_d} \bar{l}_L \mu_R H_d \left(H_d^\dagger H_d \right)$$

$$\Delta\kappa_\mu = 2C_{\mu H_d} v_d^3 / m_\mu \qquad\qquad v_d = v\cos\beta$$

For each heavy Higgs in the process leads to $\tan^2\beta$ enhancement to the cross section

$$H_d^\pm = \cos\beta G^\pm - \sin\beta H^\pm$$

$$H_d = (H_d^+, H_d^0)^T$$

$$H_d^0 = v\cos\beta + \frac{1}{\sqrt{2}}(h\cos\beta + H\sin\beta) + \frac{i}{\sqrt{2}}(G\cos\beta - A\sin\beta)$$

Extension to 2HDM type II

	in terms of $\Delta\kappa_\mu$
$\sigma_{\mu^+\mu^-\to hh}$	$\frac{9}{256\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2$
$\sigma_{\mu^+\mu^-\to AA}$	$\frac{1}{256\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^4\beta$
$\sigma_{\mu^+\mu^-\to HH}$	$\frac{9}{256\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^4\beta$
$\sigma_{\mu^+\mu^-\to hH}$	$\frac{9}{128\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^2\beta$
$\sigma_{\mu^+\mu^-\to hA}$	$\frac{1}{128\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^2\beta$
$\sigma_{\mu^+\mu^-\to HA}$	$\frac{1}{128\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^4\beta$
$\sigma_{\mu^+\mu^-\to H^+H^-}$	$\frac{1}{128\pi}\left(\frac{m_\mu}{v^2}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^4\beta$

	in terms of $\Delta\kappa_\mu$
$\sigma_{\mu^+\mu^-\to hhh}$	$\frac{3s}{2^{14}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2$
$\sigma_{\mu^+\mu^-\to AAA}$	$\frac{3s}{2^{14}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^6\beta$
$\sigma_{\mu^+\mu^-\to HHH}$	$\frac{3s}{2^{14}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^6\beta$
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$\sigma_{\mu^+\mu^-\to HAA}$	$\frac{s}{2^{14}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^6\beta$
$\sigma_{\mu^+\mu^-\to hH^+H^-}$	$\frac{s}{2^{13}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^4\beta$
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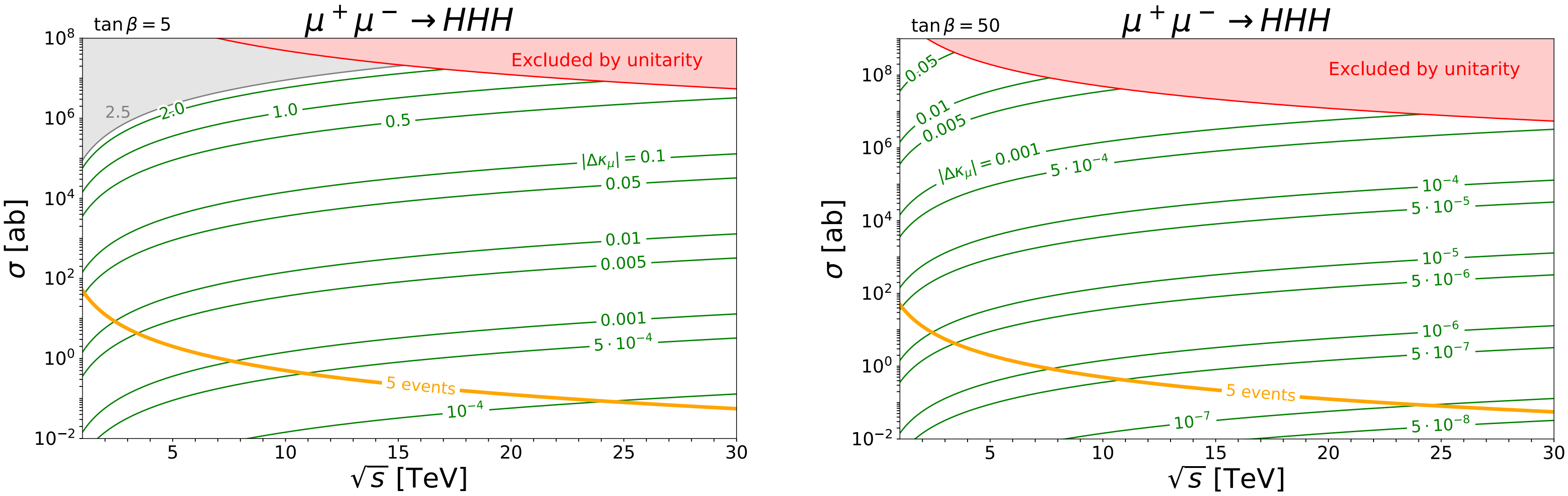
$\sigma_{\mu^+\mu^-\to AH^+H^-}$	$\frac{s}{2^{13}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^6\beta$
$\sigma_{\mu^+\mu^-\to hHA}$	$\frac{s}{2^{13}\pi^3}\left(\frac{m_\mu}{v^3}\right)^2 \lvert\Delta\kappa_\mu\rvert^2 \tan^4\beta$

Extension to 2HDM type II



Summary

- Future muon collider is expected to probe deviation to muon Yukawa coupling up to 7% (0.8 %) at 10 TeV (30 TeV) from $\mu^+\mu^- \to hhh$
- Expected to measure opposite sign Yukawa even at low energy muon collider.
- 2HDM type II can be tested and if there are 2HDM type II present, we can probe deviation to muon Yukawa coupling up to the $\sim 10^{-6}$ (10^{-8}) level at 10 TeV (30 TeV) from $\mu^+\mu^- \to HHH$

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Thanks for listening

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$$\mu^+\mu^- \to hhh, HHH, hhH, hAA, hHH, HAA$$

$$\mu^+\mu^- \to hAZ, HZZ$$

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$$\mu^+\mu^- \to hAZ,\, HZZ$$

Thus, $\mu^+\mu^- \to HH$ and $\mu^+\mu^- \to HHH$ could be a golden channel for large $\tan\beta$