Theoretical Prediction for Double Higgs Production via Photon Fusion at Muon Colliders

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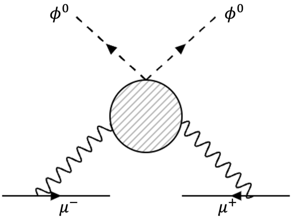
Introduction

1. Double Higgs production is important for testing the Higgs self-coupling (λ_{hhh}) which is responsible for giving mass to elementary particles and the shape of the Higgs potential.
2. λ_{hhh} and λ_{hhhh} are challenging to measure directly, as it requires the production of two or more Higgs bosons simultaneously.
3. To measure the tri-linear/ quartic Higgs coupling at the LHC requires high luminosity.
4. Muon colliders can reach higher center of mass energies than proton colliders ($\sqrt{s} \sim \mathcal{O}(10\,TeV)$) , which can increase the production rate of triple Higgs events.

(E. Asakawa, CO 2008)



Higgs Triplet Model

1. The motivation for the Higgs Triplet Model (Type II Seesaw Model) stems from the observation that two doublets can be decomposed into a triplet and a singlet representation ($2 \otimes 2 = 3 \oplus 1$).

$$\Delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}$$

2. The general scalar potential term , $V(\Phi, \Delta)$,

$$\begin{aligned} V(\Phi, \Delta) &= -\mu_\Phi^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2 + \mu_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta) \\ &+ [\mu(\Phi^T i\sigma^2 \Delta^\dagger \Phi) + \mathrm{h.c}] + \lambda_1 (\Phi^\dagger \Phi) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr \Delta^\dagger \Delta)^2 \\ &+ \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 \Phi^\dagger \Delta \Delta \Phi \end{aligned}$$

3. There are 7 physical Higgs states: $h^0, H^0, A^0, H^\pm, H^{\pm\pm}$

(A. Arhrib, CO 2011)

Constraints-ρ parameter

1. We can establish two distinct parameter spaces as follows:

$$\mathcal{P}_1 = \{\mu, \lambda, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \tan\beta, \cos\alpha\},$$

$$\mathcal{P}_2 = \{M_{h^0} = 125\mathrm{GeV}, M_{H^0}, M_{A^0}, M_{H^\pm}, M_{H^{\pm\pm}}, v_\Delta, v_\Phi, \cos\alpha\}.$$

   μ - LFV parameter.
   α, β denote the rotation angles respectively in the CP-even and CP-odd sectors.

2. The ρ parameter

$$\rho = \frac{1 + \frac{1}{2}\tan^2\beta}{1 + \tan^2\beta}.$$

   Where $\tan\beta = 2v_\Delta/v_\Phi$. The experimental value of the rho parameter, $\rho^{exp} = 1.0008^{+0.0017}_{-0.0007}$, being close to unity, leads to the bound $\tan\beta \lesssim 0.0633$.

Theoretical Constraints

Vacuum stability

$$\lambda \geq 0; \lambda_2 + \lambda_3 \geq 0; \lambda_2 + \frac{\lambda_3}{2} \geq 0$$

$$\lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0; \lambda_1 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0$$

$$\lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0$$

In the Higgs Triplet model, vacuum stability is required to ensure that the electroweak symmetry breaking minimum of the Higgs potential is stable.

Tachyonic modes

For the neutral pseudoscalar field (A^0),

$$\mu > 0 = \mu_0,$$

For the singly charged field ($H^{\pm}$),

$$\mu > \frac{\lambda_4 M_W S_W}{4\sqrt{2\pi\alpha_e}} \frac{\tan\beta}{\sqrt{1 + \frac{\tan^2\beta}{2}}} = \mu_1,$$

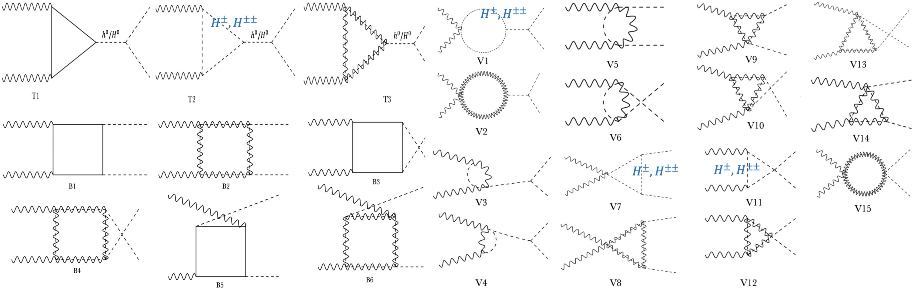
For the doubly charged field ($H^{\pm\pm}$)

$$\mu > \frac{M_W S_W}{4\sqrt{2\pi\alpha_e}} \frac{(2\lambda_4 \tan\beta + \lambda_3 \tan^3\beta)}{\sqrt{1 + \frac{\tan^2\beta}{2}}} = \mu_2.$$

For the Heavy Higgs state (H^0), $\mu \in [\mu_L = max\{\mu_0, \mu_1, \mu_2, \mu_-\}, \mu_+]$.

$$\mu_{\pm} = \frac{M_W S_W}{8\tan\beta\sqrt{\pi\alpha_e}(1 + \frac{\tan^2\beta}{2})}\Big[A_1(\lambda, \lambda_{1,4}, \tan\beta) \pm A_2(\lambda, \lambda_{1,2,3,4}, \tan\beta)\Big]$$

$A_1 = \lambda + 2(\lambda_1 + \lambda_4)\tan^2\beta$, $A_2 = \sqrt{2\lambda^2 + 8\lambda\tan^2\beta(\lambda_1 + \lambda_4 + \lambda_2\tan^2\beta + \lambda_3\tan^2\beta)} \in \mathbb{R}$ under the vacuum stability.



$\gamma\gamma \to h^0 h^0 \Rightarrow$ 430 diagrams
$\gamma\gamma \to A^0 A^0 \Rightarrow$ 262 diagrams

EPA and Iterative Solutions for QED

The case of collinear photon emission from an muon at leading order (LO) can be described using the equivalent photon approximation (EPA) or Weizsacker-Williams Approximation,

$$f_{\gamma,l}(x) \approx \frac{\alpha_e}{2\pi} P_{\gamma,l}(x) \ln \frac{E^2}{m_l^2}$$

Where the splitting function are, $P_{\gamma,l} = (1 + (1 - x^2))/x$ for $l \to \gamma$.
By solving iteratively the DGLAP equations, the approximate solutions for the PDF, $(LO + \mathcal{O}(\alpha_e^2 t^2))$,

$$f_\gamma(x,t) = \frac{\alpha_e}{2\pi} t P_{vf}^f + \frac{1}{2}\left(\frac{\alpha_e t}{2\pi}\right)^2 [(P_\gamma^v + P_f^v) P_{vf}^f + I_{vfff}].$$

$$t = log\left(\frac{E^2}{M_\mu}\right), P_f^v = 3/2, P_f^v = -40/9, P_{vf}^f = \frac{1 + (1 - x)^2}{x}$$

$$I_{vfff} = \left(\frac{3}{2} + 2log(1 - x)\right) P_{vf}^f + \frac{(1 - x)(2x - 3)}{x} + (2 - x) log(x)$$

(F Garosi, CO 2023)

Total cross sections $\mu^+\mu^- \to \gamma\gamma \to \phi\phi$

Since the Higgs pair production via photon-photon collisions is a subprocess of $\mu^+\mu^-$ collisions at the muon collider, the total cross section can be obtained by,

$$\sigma(s, \mu^+\mu^- \to \gamma\gamma \to \phi\phi) = \int_{\frac{2M_\phi^2}{\sqrt{s}}}^{1} d\tau \frac{d\mathcal{L}_{\gamma\gamma}}{d\tau} \hat{\sigma}(\hat{s} = \tau s, \gamma\gamma \to \phi\phi),$$

along with the photon luminosity

$$\frac{d\mathcal{L}_{\gamma\gamma}}{d\tau} = \int_{\tau}^{1} \frac{dx}{x} f_{\gamma/\mu}(x) f_{\gamma/\mu}(\frac{\tau}{x}),$$

In our analysis, we explore various hierarchies between $M_{H^\pm}$ and $M_{H^{\pm\pm}}$, primarily determined by the sign of λ_4 at $\tan\beta << 1$ values.

$$M_{H^\pm}^2 - M_{H^{\pm\pm}}^2 = \frac{M_W^2 S_W^2}{4\alpha_e \pi}\lambda_4 + \frac{\mu M_W S_W}{\sqrt{2\pi\alpha_e}} \tan\beta + \mathcal{O}(\tan^2\beta)$$

$\lambda_4 > 0$	$M_{H^{\pm\pm}} < M_{H^\pm} < M_H \approx M_{A^0}$
$\lambda_4 = 0$	$M_{H^{\pm\pm}} \approx M_{H^\pm} \approx M_H \approx M_{A^0}$
$\lambda_4 < 0$	$M_H \approx M_{A^0} < M_{H^\pm} < M_{H^{\pm\pm}}$

Numerical Results

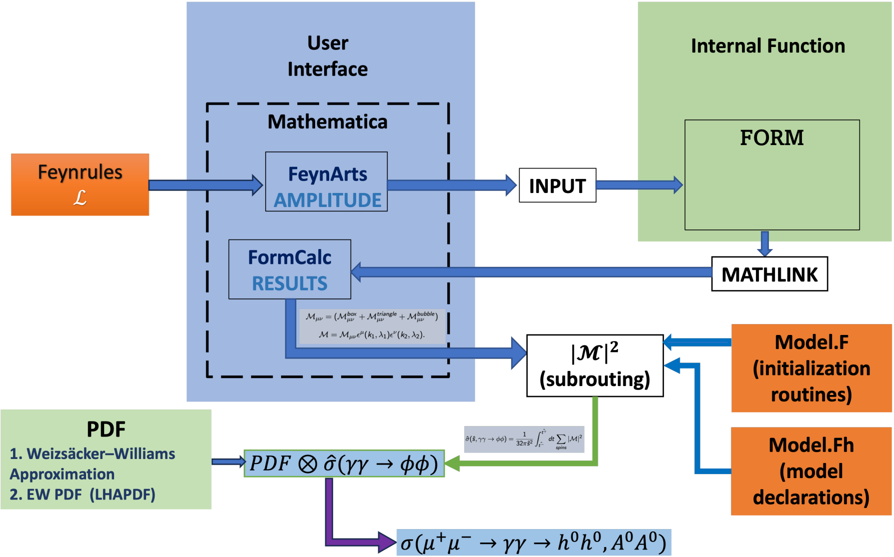
We have used the following input values: $\tan\beta = 0.001$, $\lambda = 0.51$, $\lambda_1 = 10$, $\lambda_2 = 1$, $\lambda_3 = -1$, and $\mu = [0.2, 2]$. The table below displays the mass ranges corresponding to these input values.

	$M_{H^\pm}$ GeV	$M_{H^{\pm\pm}}$ GeV	M_{H^0} GeV	M_{A^0} GeV
$\lambda_4 = 1.82$	201-809.6	114-729.5	261-826.5	261-826.5
$\lambda_4 = 0$	261-827	261-827	261-827	261-827
$\lambda_4 = -1.82$	309-843	351-859	261-826.5	261-826.5

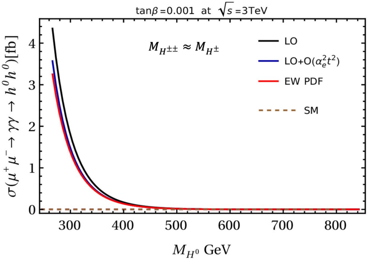
According to the ATLAS collaboration, searches for di-W bosons are excluded from the mass region where $M_{H^{\pm\pm}}$ lies between 200 GeV and 220 GeV for $Br(H^{\pm\pm} \to W^\pm W^\pm) \approx 1$.

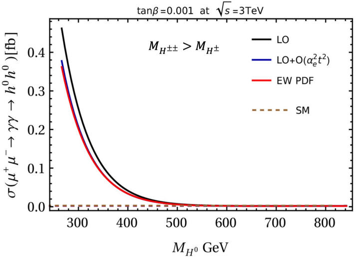
(P Siddharth, CO 2023)

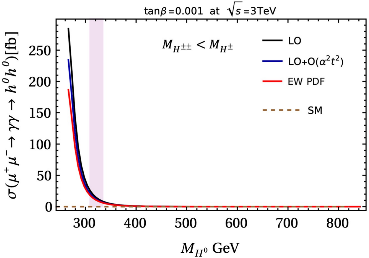
The calculations



$\sigma(\mu^+\mu^- \to \gamma\gamma \to h^0 h^0)$



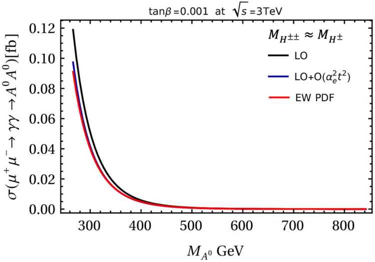


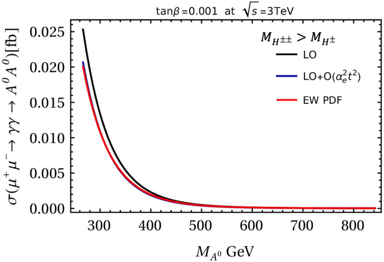


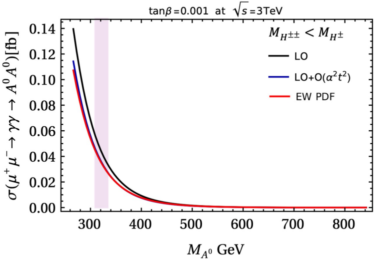
pink region in the plot: In this scenario, $H^{\pm\pm}$ meets the exclusion limits from 200 GeV to 220 GeV. The corresponding H^0 mass values range from 315 GeV to 335 GeV.

$$\{h^0H^+H^-, h^0h^0H^+H^-\}|_{(\lambda_4>0)} > \{h^0H^+H^-, h^0h^0H^+H^-\}|_{(\lambda_4<0)}$$

$\sigma(\mu^+\mu^- \to \gamma\gamma \to A^0 A^0)$







pink region in the plot: In this scenario, $H^{\pm\pm}$ meets the exclusion limits from 200 GeV to 220 GeV. The corresponding A^0 mass values range from 315 GeV to 335 GeV.

$h^0 A^0 A^0|_{(\lambda_4>0)} > h^0 A^0 A^0|_{(\lambda_4<0)}$ and $H^0 A^0 A^0 \to 0$ for small μ (weak-coupling decoupling limit)

$h^0 h^0 h^0$ and $H^0 h^0 h^0$ Couplings

1. Trilinear SM-like Higgs Coupling at $\alpha \approx 0$ and,

$$\lambda^{'} = -\frac{3\sqrt{\pi\alpha_e}}{vM_W S_W}\left(\frac{M_h^2}{-\frac{3}{2} + \frac{3}{8}\tan^2\beta + \mathcal{O}(\tan^4\beta)}\right)$$

behaves as follows:

$$\lambda_{hhh}^{HTM}\Big|_{\lambda=\lambda'} \rightarrow \lambda_{hhh}^{SM}$$

Numerically, $\lambda \approx 0.51$, $\left|\frac{\Delta\lambda_{hhh}}{\lambda_{hhh}^{SM}}\right| \times 100 \lesssim 1\%$

2. For $\tan\beta << 1$, if we define

$$\mu^d = \frac{M_W S_W(\lambda_1 + \lambda_4)}{\sqrt{8\pi\alpha_e}}\tan\beta,$$

the $\lambda_{Hhh}^{HTM}\Big|_{\mu=\mu^d} \rightarrow 0$.

λ_4	μ^d	$M_{H^0}(\mathrm{GeV})$	λ_{Hhh}^{HTM}
+1.82	1.047	597.98	10^{-6}
0	0.869	544.78	10^{-7}
-1.82	0.725	497.60	10^{-7}

Summary

1. The cross sections of $\mu^+\mu^- \to \gamma\gamma \to \phi\phi$ can be enhanced for $M_{H^\pm} > M_{H^{\pm\pm}}$. The lowest cross sections can be observed for $M_{H^{\pm\pm}} > M_{H^\pm}$.
2. We showed how the cross sections for $\phi\phi$ production increase noticeably because of the significant contributions from interactions like $h^0 H^+ H^-$, $h^0 h^0 H^+ H^-$, and $h^0 A^0 A^0$.
3. We employed the Weizsäcker-Williams approximation at leading order (LO) and its second-order correction $(\mathcal{O}(\alpha_e^2 t^2))$, alongside the EW PDF set. Comparing the results, we observed that the cross sections obtained from the second-order corrections closely matched those from the EW PDF set.

Thank you!

Appendix A

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi \\ \delta \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta' & \sin\beta' \\ -\sin\beta' & \cos\beta' \end{pmatrix} \begin{pmatrix} \phi^\pm \\ \delta^\pm \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi \\ \eta \end{pmatrix}$$

Appendix B

$$v_\phi^2 = \frac{1}{1 + \frac{1}{2}\tan^2\beta}\frac{S_W^2 M_W^2}{\pi\alpha}$$

$$v_\Delta^2 = \frac{\tan^2\beta}{1 + \frac{1}{2}\tan^2\beta}\frac{S_W^2 M_W^2}{4\pi\alpha}$$

$$\mu = \frac{\sqrt{2}v_\Phi}{v_\Phi^2 + 4v_\Delta^2}M_A^2$$

$$\lambda = -\frac{2}{v_\Phi^2}(c_\alpha^2 M_h^2 + s_\alpha^2 M_H^2)$$

$$\lambda_1 = -\frac{2}{v_\Phi^2 + 4v_\Delta^2} M_A^2 + \frac{2}{v_\Phi^2 + 2v_\Delta^2} M_{H^\pm}^2 + \frac{\sin 2\alpha}{2v_\Delta v_\Phi}(M_h^2 - M_H^2)$$

$$\lambda_2 = \frac{1}{2v_\Delta^2}\left(c_\alpha^2 M_h^2 + s_\alpha^2 M_H^2 + \frac{v_\Delta^2}{v_\Phi^2 + 4v_\Delta^2} M_A^2 - \frac{4v_\Delta^2}{v_\Phi^2 + 2v_\Delta^2} M_A^2 + 2M_{H^{\pm\pm}}^2\right)$$

$$\lambda_3 = \frac{1}{v_\Delta^2}\left(-\frac{v_\Phi^2}{v_\Phi^2 + 4v_\Delta^2} M_A^2 + \frac{2v_\Phi^2}{v_\Phi^2 + 2v_\Delta^2} M_{H^\pm}^2 - M_{H^{\pm\pm}}^2\right)$$

$$\lambda_4 = \frac{4}{v_\Phi^2 + 4v_\Delta^2} M_A^2 - \frac{4}{v_\Phi^2 + 2v_\Delta^2} M_{H^\pm}^2$$