Quasiclassical Solutions For Static Quantum Black Holes

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II. **Importance**: Unlike standard EFT our formulation is sensitive to potentially new phenomenology by *not assuming they are of higher curvature form*.

III. **Unique and New**: We systematically derive quantum effects in canonical gravity, capturing quantum fluctuations using a *Scalar Field modification to the Hamiltonian.*

IV. **Implications**: Our analysis reveals features of underlying *non-local quantum effects* and shows that the quasiclassical methods we used are promising for future applications to inhomogeneous models of quantum gravity.

Background Independence and General Covariance

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- General Relativity (GR) vs. Special Relativity (SR):
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- Dynamical nature of spacetime geometry:
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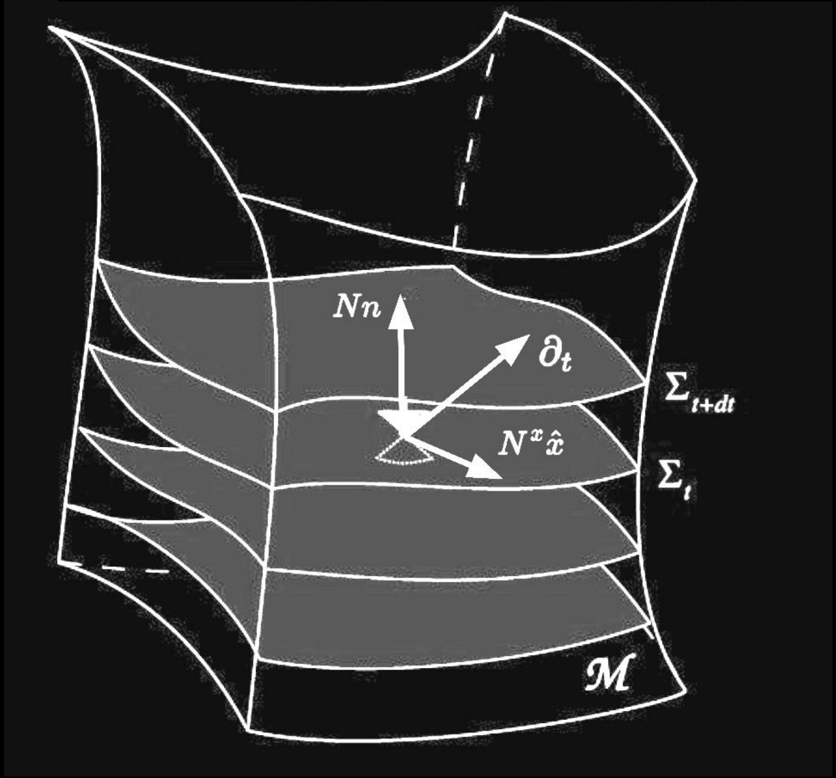
- General Covariance:
  - The lack of fixed background geometry in GR is what Einstein referred to as general covariance.

Spherically Symmetric Space-Time

$$ds^2 = -N(t,x)^2 dt^2 + q_{xx}(t,x)(dx + N^x(t,x)dt)^2 + q_{\varphi\varphi}(t,x)d\Omega^2$$



Space-Time Foliation

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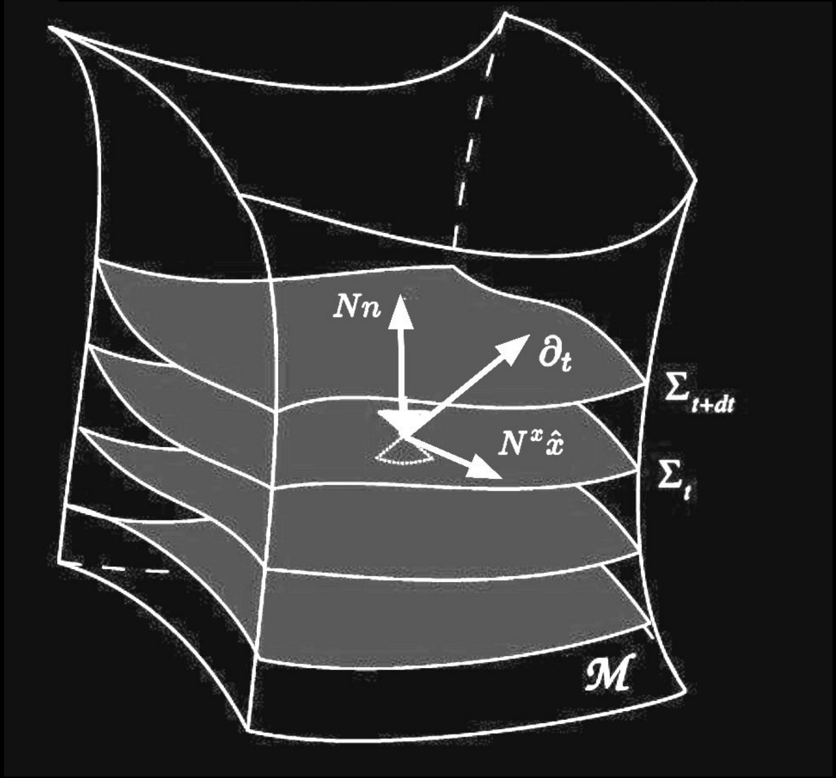

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$$q_{xx}(t,x) = \frac{(E^\varphi)^2}{|E^x|}$$

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$$\{K_x(x), E^x(y)\} = 2G\delta(x,y)$$

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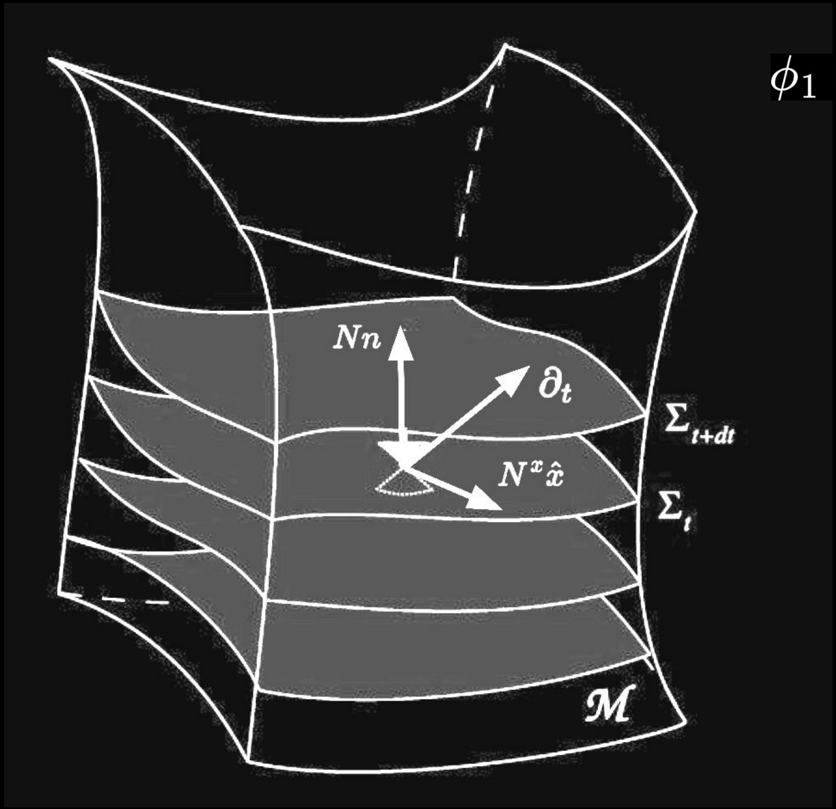
Canonical Fields



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$$\phi_1 = E^x,\ p_1 = -K_x,\ \phi_2 = 2E^\varphi,\ p_2 = -K_x$$

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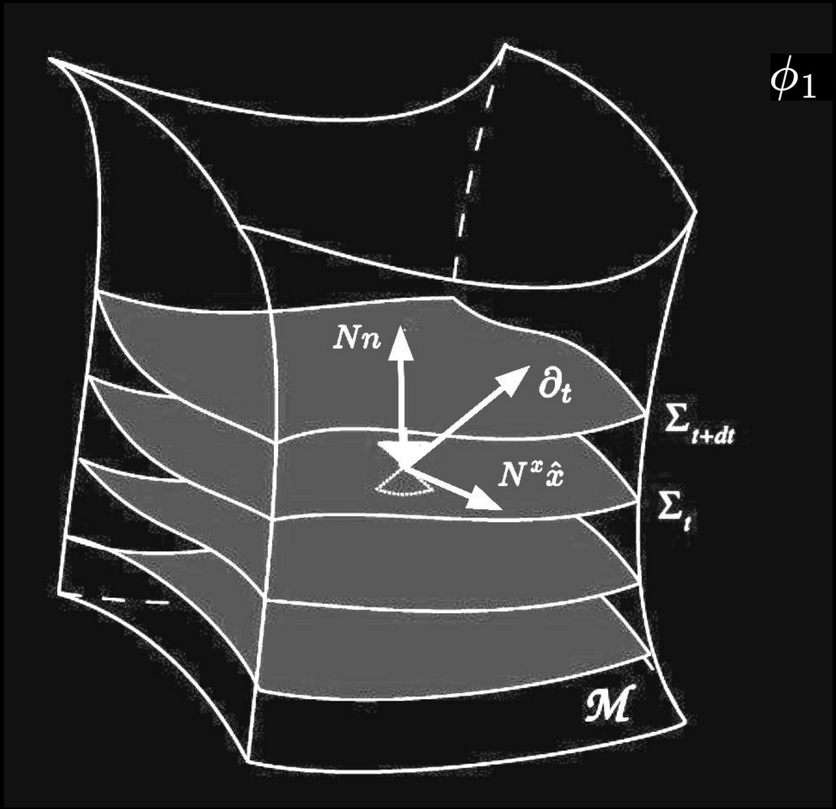

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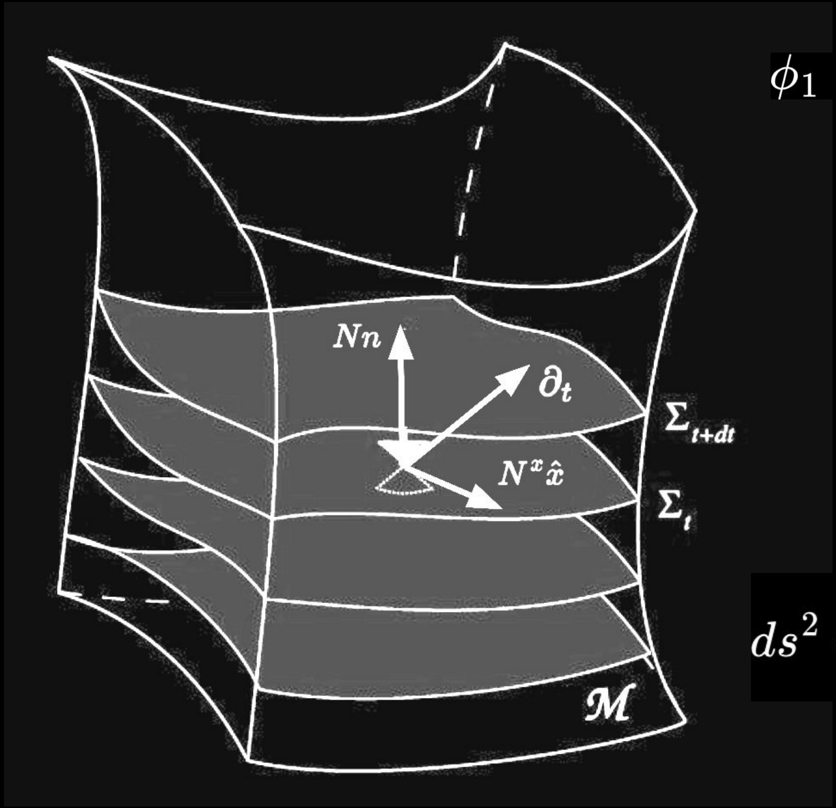

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Canonical Formulation

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General Relativity is a constrained system $\Longrightarrow H[N] = 0\ ,\ D[N^x] = 0$

Hypersurface Deformation Algebroid (HDA)

$$\{D[N^x], D[M^x]\} = D\left[N^x M^{x\prime} - M^x N^{x\prime}\right]$$

$$\{H[N], D[N^x]\} = -H\left[MN'\right]$$

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⇒ General Covariance

Hypersurface Deformation Algebroid (HDA)

$D_{\delta N^a}$ Is the generator of stretchings and $H_{\delta N}$ is the generator of translations or deformations.

$$[D_{\delta N^a}, D_{\delta M^a}] = D_{\delta N^b \partial_b \delta M^a - \delta M^b \partial_b \delta N^a}$$

$$[H_{\delta N}, D_{\delta N^a}] = -H_{N^a \delta N} = -H_{\delta M}$$

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$\Rightarrow$ General Covariance

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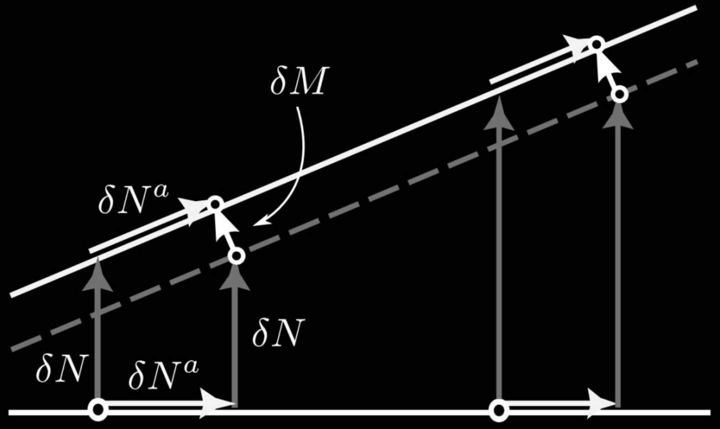

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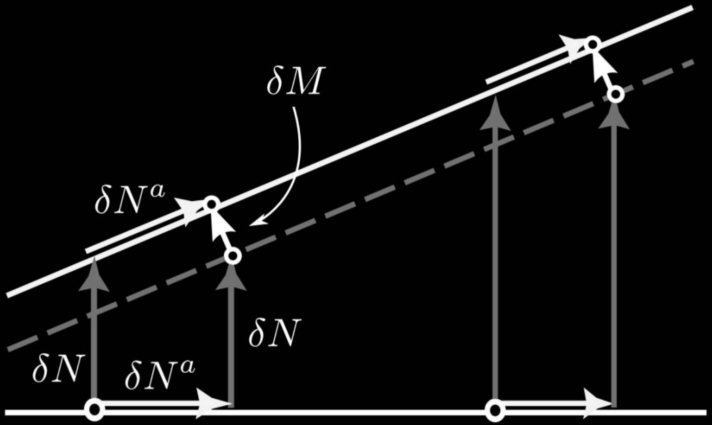
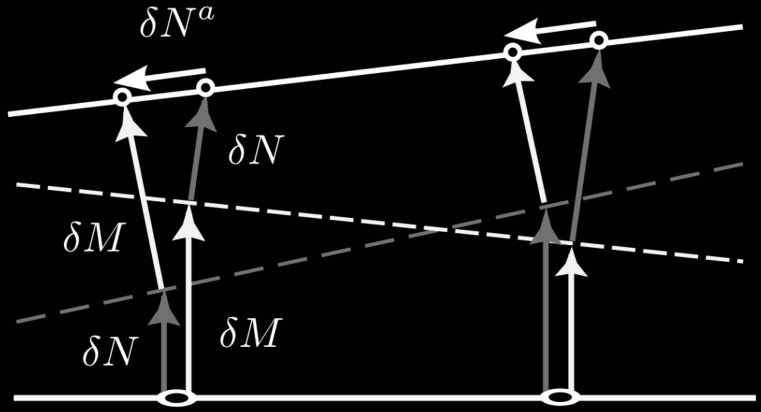
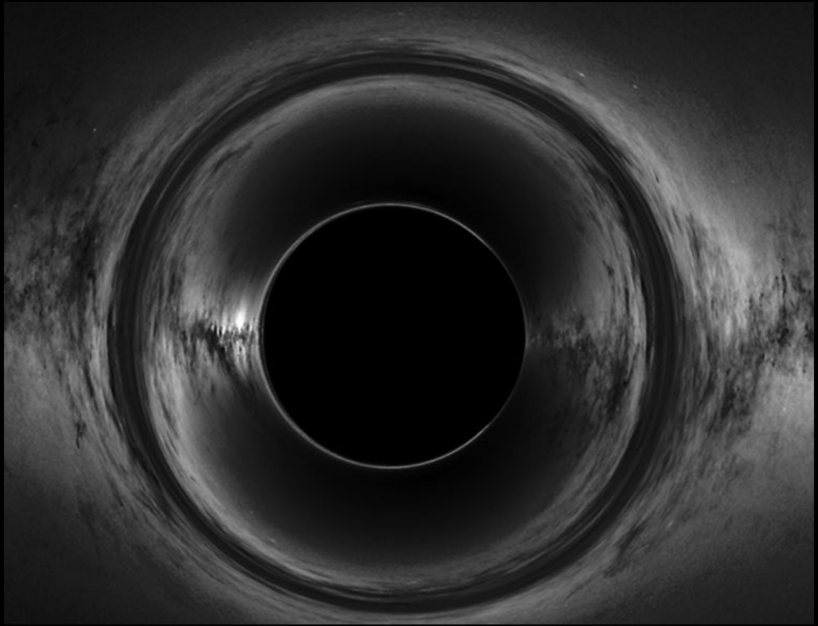




Partial Gauge Fixing in a static configuration

Letting ϕ_1 take on it's classical value: $\phi_1 = x^2$



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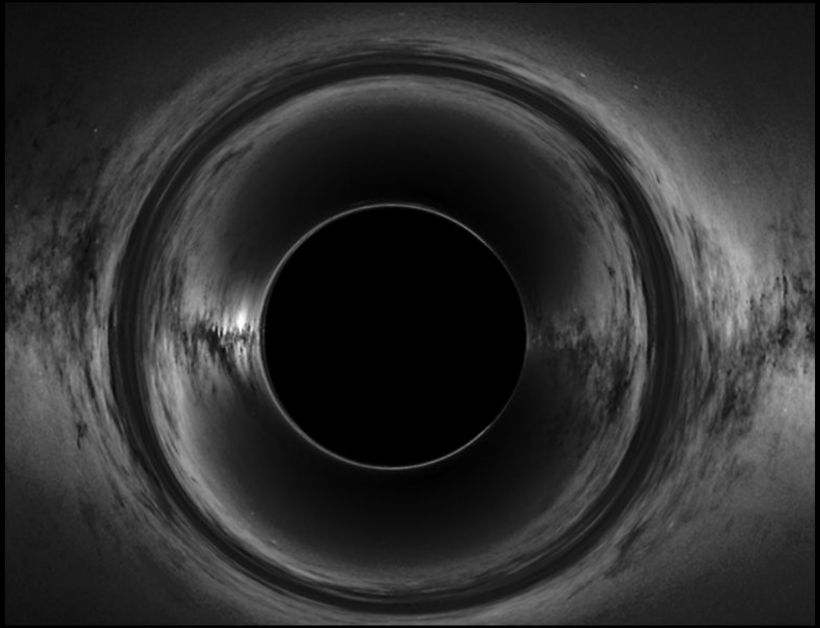
For the Static case we get the Schwarzschild line element: $ds^2 = -N^2 dt^2 + \frac{1}{4}\frac{(\phi_2)^2}{x^2}dx^2 + x^2 d\Omega^2$



Nonadiabatic Quantum Dynamics

Canonical Variables and moments

$$
\begin{aligned}
q &= \langle \hat{q} \rangle \\
\Delta q &= \langle \hat{q} - q \rangle = \langle \hat{q} \rangle - q = 0 \\
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Classical Harmonic Oscillator

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H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2
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Quasiclassical Hamiltonian with truncation s = 2

$$H_{\text{eff},2} = H + \frac{1}{2}\frac{\partial^2 H}{\partial p^2}\Delta\left(p^2\right) + \frac{1}{2}\frac{\partial^2 H}{\partial q^2}\Delta\left(q^2\right)$$
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Hamiltonian dynamics

$$\left\{\left\langle \hat{A} \right\rangle, \left\langle \hat{B} \right\rangle\right\} = \frac{\left\langle \left[\hat{A}, \hat{B}\right] \right\rangle}{i\hbar}$$

$$\left\{\left\langle \hat{A} \right\rangle, H_{\text{eff}}\right\} \qquad \frac{\left\langle \left[\hat{A}, \hat{H}\right] \right\rangle}{i\hbar} = \frac{d\left\langle \hat{A} \right\rangle}{dt}$$

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$$\Delta\left(q^2\right) = s^2$$
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$$U = \Delta\left(q^2\right)\Delta\left(p^2\right) - \Delta(qp)^2 \geq \frac{\hbar^2}{4}$$

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Adiabatic approximation

$$V_{\text{low-energy}}(q) = V(q) + \sqrt{\frac{UV''(q)}{m}}$$

$$U_{\text{min}} = \frac{\hbar^2}{4}$$

$$V''(q) = m\omega^2$$

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$$U_{\min} = \frac{\hbar^2}{4}$$
$$V''\left(q\right) = m\omega^2$$

Zero-Point Energy

$$E_{\min} = \frac{1}{2}\hbar\omega$$

Scalar Field modification to the Hamiltonian

$$\Delta(\phi_2^2) = \phi_3^2, \quad \Delta(\phi_2 p_2) = \phi_3 p_3, \quad \Delta(p_2^2) = p_3^2 + \frac{U(x)}{\phi_3^2}.$$

$$H_{\text{eff}} = H[N] + H_2[N]$$

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$$H_2[N] = \int \mathrm{d}x N(x) \left(\frac{1}{2} \frac{\partial^2 H}{\partial p_2^2} \left(p_3^2 + \frac{U}{\phi_3^2} \right) + \frac{\partial^2 H}{\partial \phi_2 \partial p_2} \phi_3 p_3 + \frac{1}{2} \frac{\partial^2 H}{\partial \phi_2^2} \phi_3^2 + \frac{\partial^2 H}{\partial \phi_2 \partial \phi_2'} \phi_3 \phi_3' \right)$$

$$= -\int \mathrm{d}x N(x) \left(\frac{\phi_2 p_3^2}{2\sqrt{\phi_1}} + \frac{\phi_3 p_2 p_3}{\sqrt{\phi_1}} + \left(6 \frac{\sqrt{\phi_1} \phi_1' \phi_2'}{\phi_2^4} - \frac{1}{2} \frac{(\phi_1')^2}{\sqrt{\phi_1} \phi_2^3} - 2 \frac{\phi_1'' \sqrt{\phi_1}}{\phi_2^3} \right) \phi_3^2 - 4 \frac{\sqrt{\phi_1} \phi_1' \phi_3 \phi_3'}{\phi_2^3} + \frac{U(x) \phi_2}{2\sqrt{\phi_1} \phi_3^2} \right).$$

Scalar Field modification to the Hamiltonian

$$\Delta(\phi_2^2)=\phi_3^2, \quad \Delta(\phi_2 p_2)=\phi_3 p_3, \quad \Delta(p_2^2)=p_3^2+\frac{U(x)}{\phi_3^2}.$$

$$H_{\text{eff}} = H[N] + H_2[N]$$

$$H_2[N]=\int \mathrm{d}x N(x)\left(\frac{1}{2}\frac{\partial^2 H}{\partial p_2^2}\left(p_3^2+\frac{U}{\phi_3^2}\right)+\frac{\partial^2 H}{\partial\phi_2\partial p_2}\phi_3 p_3+\frac{1}{2}\frac{\partial^2 H}{\partial\phi_2^2}\phi_3^2+\frac{\partial^2 H}{\partial\phi_2\partial\phi_2'}\phi_3\phi_3'\right)$$

$$=-\int \mathrm{d}x N(x)\left(\frac{\phi_2 p_3^2}{2\sqrt{\phi_1}}+\frac{\phi_3 p_2 p_3}{\sqrt{\phi_1}}+\left(6\frac{\sqrt{\phi_1}\phi_1'\phi_2'}{\phi_2^4}-\frac{1}{2}\frac{(\phi_1')^2}{\sqrt{\phi_1}\phi_2^3}-2\frac{\phi_1''\sqrt{\phi_1}}{\phi_2^3}\right)\phi_3^2-4\frac{\sqrt{\phi_1}\phi_1'\phi_3\phi_3'}{\phi_2^3}+\frac{U(x)\phi_2}{2\sqrt{\phi_1}\phi_3^2}\right).$$

$$H_{\phi_2}[L]=\int \mathrm{d}x L(x)\left(\frac{\partial H}{\partial\phi_2}\phi_3^2+\frac{\partial H}{\partial\phi_2'}\phi_3\phi_3'+\frac{\partial H}{\partial p_2}\phi_3 p_3\right)$$

$$=-\int \mathrm{d}x L(x)\left(\left(\frac{p_2^2}{2\sqrt{\phi_1}}+\frac{1}{2\sqrt{\phi_1}}+\frac{(\phi_1')^2+4\phi_1\phi_1''}{2\sqrt{\phi_1}\phi_2^2}-4\frac{\sqrt{\phi_1}\phi_1'\phi_2'}{\phi_2^3}\right)\phi_3^2\right.$$

$$\left.+\frac{2\sqrt{\phi_1}\phi_1'}{\phi_2^2}\phi_3\phi_3'+\left(\frac{\phi_2 p_2}{\sqrt{\phi_1}}+2\sqrt{\phi_1}p_1\right)\phi_3 p_3\right).$$

Scalar Field modification to the Hamiltonian

$$\Delta(\phi_2^2) = \phi_3^2, \quad \Delta(\phi_2 p_2) = \phi_3 p_3, \quad \Delta(p_2^2) = p_3^2 + \frac{U(x)}{\phi_3^2}.$$

$$H_{\text{eff}} = H[N] + H_2[N]$$

$$H_2[N] = \int \mathrm{d}x N(x) \left(\frac{1}{2} \frac{\partial^2 H}{\partial p_2^2} \left(p_3^2 + \frac{U}{\phi_3^2} \right) + \frac{\partial^2 H}{\partial \phi_2 \partial p_2} \phi_3 p_3 + \frac{1}{2} \frac{\partial^2 H}{\partial \phi_2^2} \phi_3^2 + \frac{\partial^2 H}{\partial \phi_2 \partial \phi_2'} \phi_3 \phi_3' \right)$$

$$= -\int \mathrm{d}x N(x) \left(\frac{\phi_2 p_3^2}{2\sqrt{\phi_1}} + \frac{\phi_3 p_2 p_3}{\sqrt{\phi_1}} + \left(6 \frac{\sqrt{\phi_1} \phi_1' \phi_2'}{\phi_2^4} - \frac{1}{2} \frac{(\phi_1')^2}{\sqrt{\phi_1} \phi_2^3} - 2 \frac{\phi_1'' \sqrt{\phi_1}}{\phi_2^3} \right) \phi_3^2 - 4 \frac{\sqrt{\phi_1} \phi_1' \phi_3 \phi_3'}{\phi_2^3} + \frac{U(x) \phi_2}{2\sqrt{\phi_1} \phi_3^2} \right).$$

$$H_{\phi_2}[L] = \int \mathrm{d}x L(x) \left(\frac{\partial H}{\partial \phi_2} \phi_3^2 + \frac{\partial H}{\partial \phi_2'} \phi_3 \phi_3' + \frac{\partial H}{\partial p_2} \phi_3 p_3 \right)$$

$$= -\int \mathrm{d}x L(x) \left(\left(\frac{p_2^2}{2\sqrt{\phi_1}} + \frac{1}{2\sqrt{\phi_1}} + \frac{(\phi_1')^2 + 4\phi_1 \phi_1''}{2\sqrt{\phi_1} \phi_2^2} - 4 \frac{\sqrt{\phi_1} \phi_1' \phi_2'}{\phi_2^3} \right) \phi_3^2 + \frac{2\sqrt{\phi_1} \phi_1'}{\phi_2^2} \phi_3 \phi_3' + \left(\frac{\phi_2 p_2}{\sqrt{\phi_1}} + 2\sqrt{\phi_1} p_1 \right) \phi_3 p_3 \right).$$

$$\phi_3(x) = \frac{C}{(1 - \mu/x)^{3/2}}$$

Quantum-Corrected Space-Time Geometry

$$ds^2 = -(N + \delta N)^2 dt^2 + \frac{(\phi_2 + \delta\phi_2)^2}{4x^2} dx^2 + x^2 d\Omega^2$$

$$= -\left(N^2 + 2N\delta N\right) dt^2 + \frac{\phi_2^2 + 2\phi_2 \delta\phi_2}{4x^2} dx^2 + x^2 d\Omega^2$$

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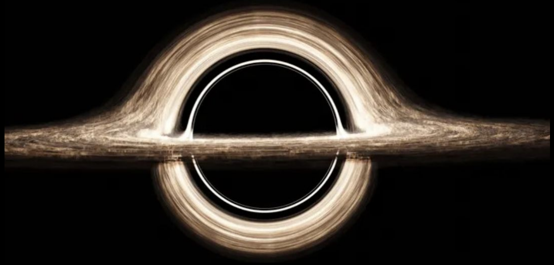
Schwarzschild Plus Small Corrections

$$\phi_2 + \delta\phi_2 = \frac{2x}{\sqrt{1 - \frac{2GM}{x}}} + \delta\phi_2$$

$$N + \delta N = \sqrt{1 - \frac{2GM}{x}} + \delta N$$



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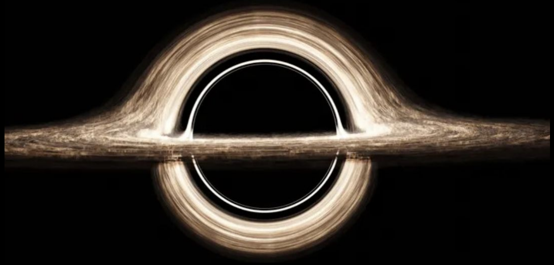

For Asymptotically Constant U(x)

$$\delta\phi_2 \sim \sqrt{U}x$$

$$\delta N \sim \sqrt{U}\log\left(\frac{2GM}{x}\right)$$

Quantum-Corrected Space-Time Geometry

$$ds^2 = -(N+\delta N)^2 dt^2 + \frac{(\phi_2+\delta\phi_2)^2}{4x^2}dx^2 + x^2 d\Omega^2$$

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$$\phi_2 + \delta\phi_2 = \frac{2x}{\sqrt{1-\frac{2GM}{x}}} + \delta\phi_2$$

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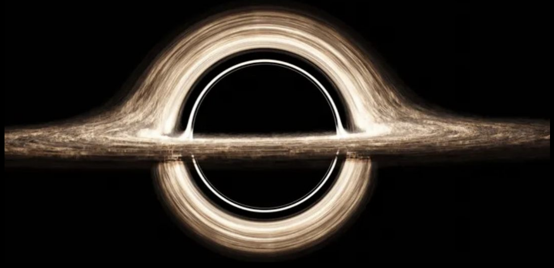

For Asymptotically Constant U(x)

$$\delta\phi_2 \sim \sqrt{U}x$$

$$\delta N \sim \sqrt{U}\log\left(\frac{2GM}{x}\right)$$

In a QM settings U(x) can be interpreted as Classical Remnant of Zero-Point Fluctuations.

Thank You For Your Attention!

Amherst Center for Fundamental Interactions

Physics at the interface: Energy, Intensity, and Cosmic frontiers

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