Thermodynamics of “Continuous Spin” Photons

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DPF – Pheno - 16th May 2024

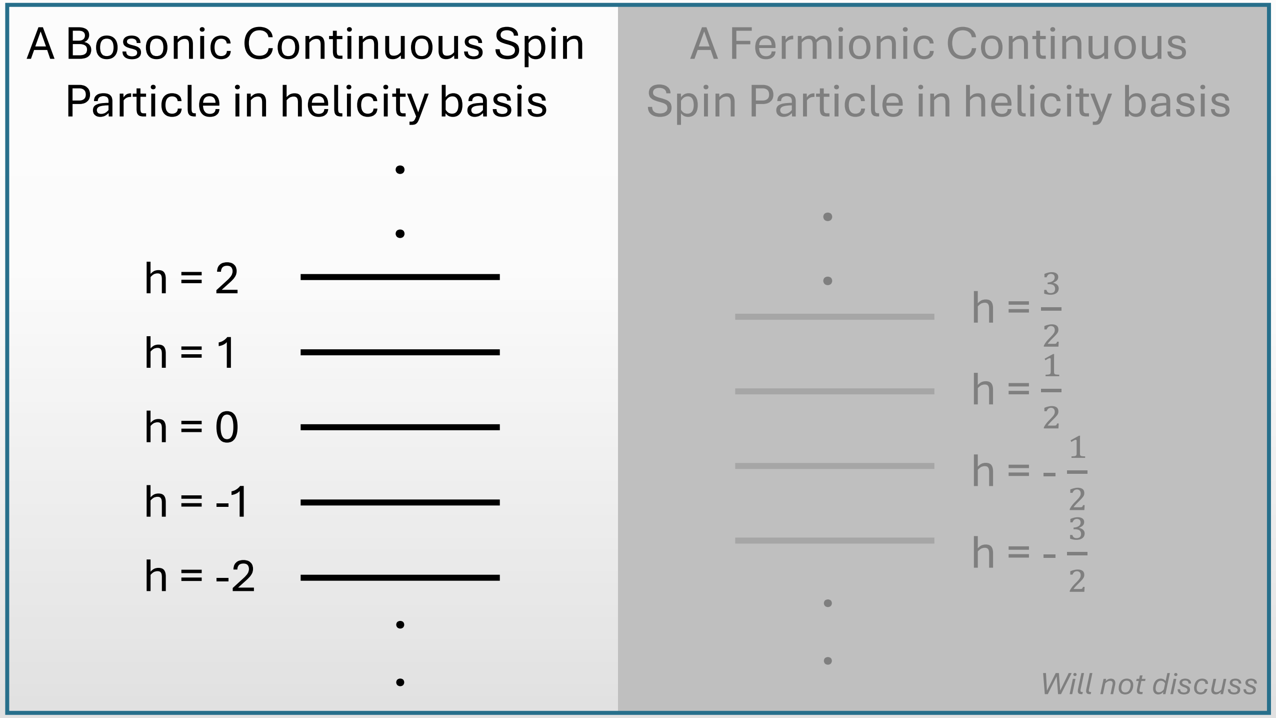
arXiV: 24xx.xxxxx, with Philip Schuster & Natalia Toro

Outline

1. What are Continuous Spin particles (CSPs)?
2. Why are we talking about CSP thermodynamics?
3. The CSP photon and the structure of its interactions
4. Key properties of CSP photon thermodynamics
5. Conclusion

1 What are "Continuous spin" particles (CSPs)?

First things first – "Continuous spin" is a bad name we seem to be stuck with – spin is quantized
In helicity basis, a CSP is characterized by an infinite tower of integer-spaced eigen states



CSPs are the general massless particles allowed by relativity + QM

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- **Definition** of particle in relativity + QM – Unitary irreducible representation of Poincaré group
- Let $| \; k^{\mu} \; , \;\; \sigma \;\; \rangle$ be a momentum eigen-state of a particle. Consider its Poincaré transformations

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- Dictates properties under space-time translations P^{μ}

- Under transformations generated by Lorentz group $J^{\mu\nu}$
  - Some change k^{μ} …
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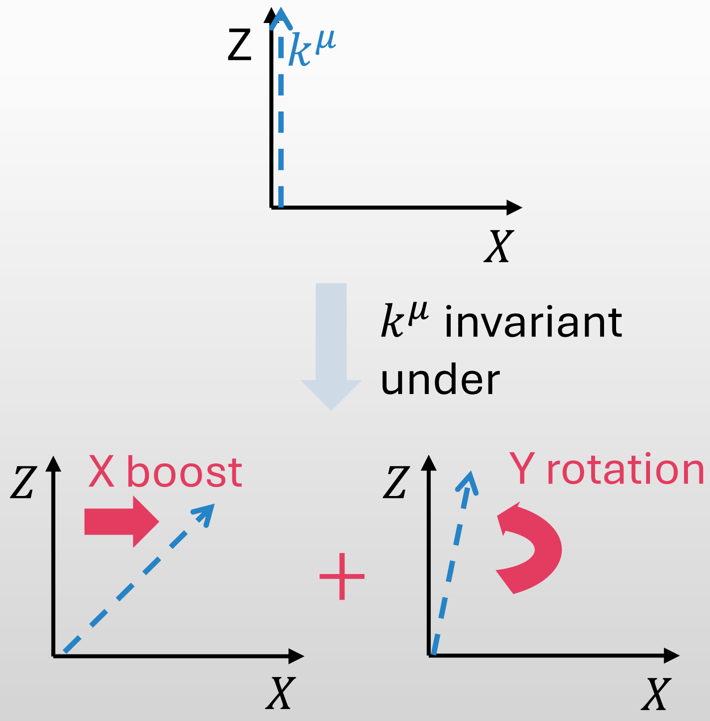
- The sub-group of Lorentz group that leaves 4-momentum invariant encodes "internal space-time symmetries" of a particle: Little Group

- Little Group generators W^μ uniquely specify how a particle's internal state σ (spin degrees of freedom) transforms

Little Group transformation example:



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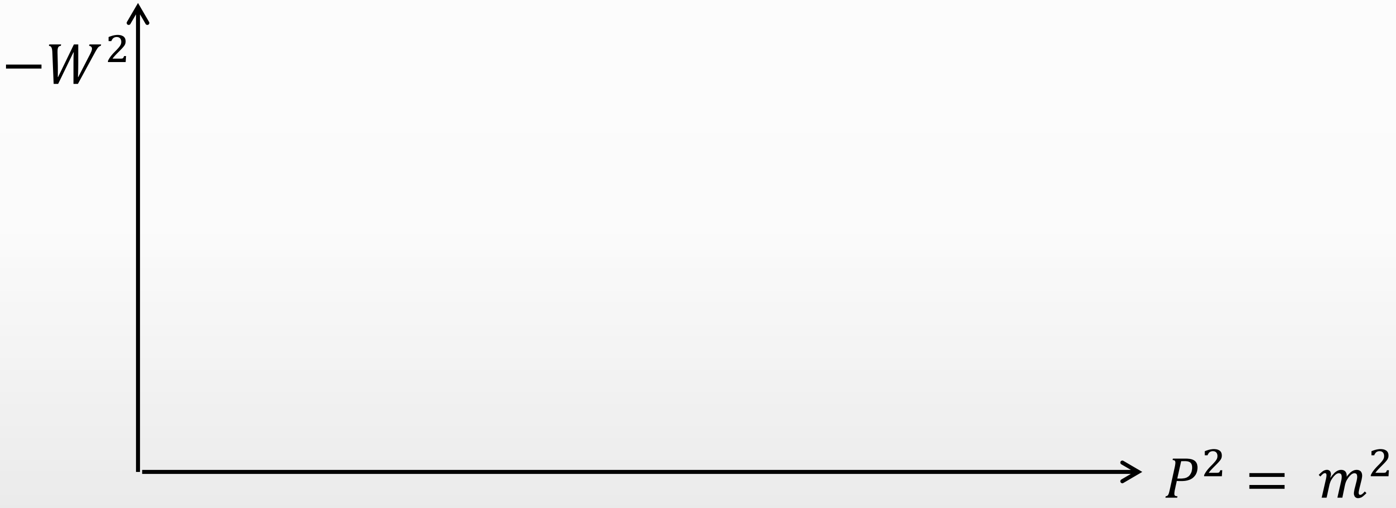
- Wigner (1939) classified all unitary irreps of the Poincaré group using group invariants: P^2 and W^2 ➡ Gives us all particle types allowed by relativity + QM



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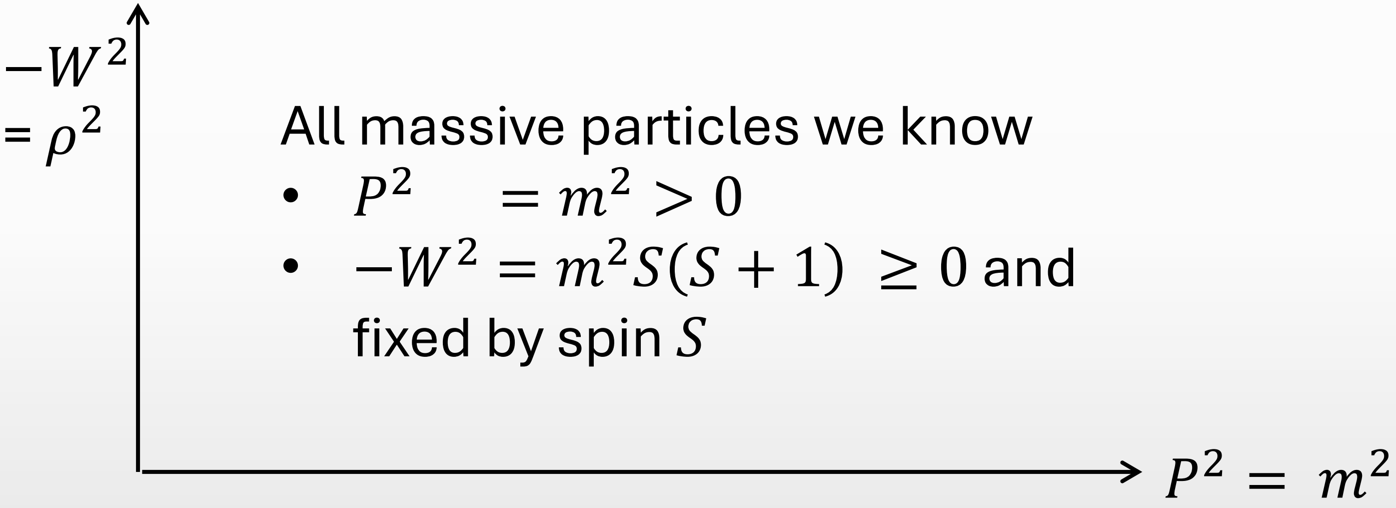

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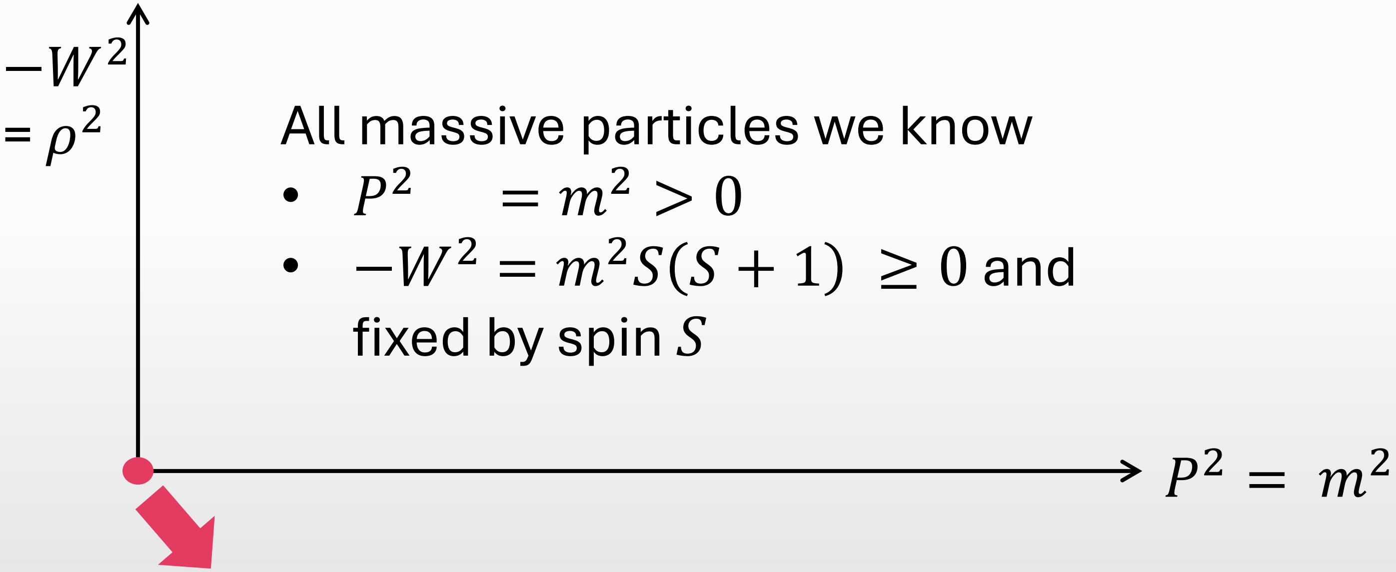

All massive particles we know

- $P^2 \quad = m^2 > 0$
- $-W^2 = m^2 S(S+1) \ \geq 0$ and fixed by spin S

All massless particles we typically work with

- $P^2 = m^2 \ = 0$
- And we **assume** $W^2 = 0$
  i.e., $\rho = 0$ → Clearly not the most general case!

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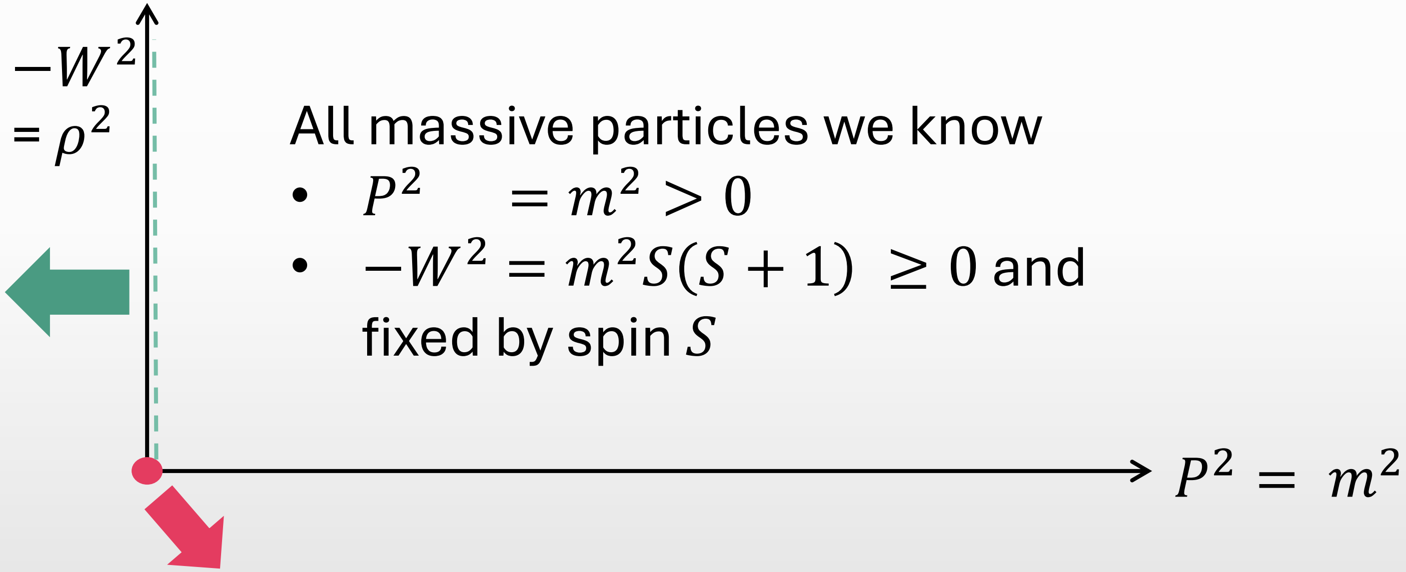
CSP is the **general massless particle**
- $P^2 \quad = m^2 \ = 0$
- $-W^2 = \rho^2 \neq 0$, make contact with known theory when $\rho \to 0$

"Spin scale" ρ is a fundamental property of massless particles
- Units of energy
- Gives the infinite tower of eigen states in helicity basis & controls their "mixing" under Little Group



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Why are we talking about CSP thermodynamics?

- One of the (several) reasons CSPs have been relatively unstudied despite being known since 1930’s

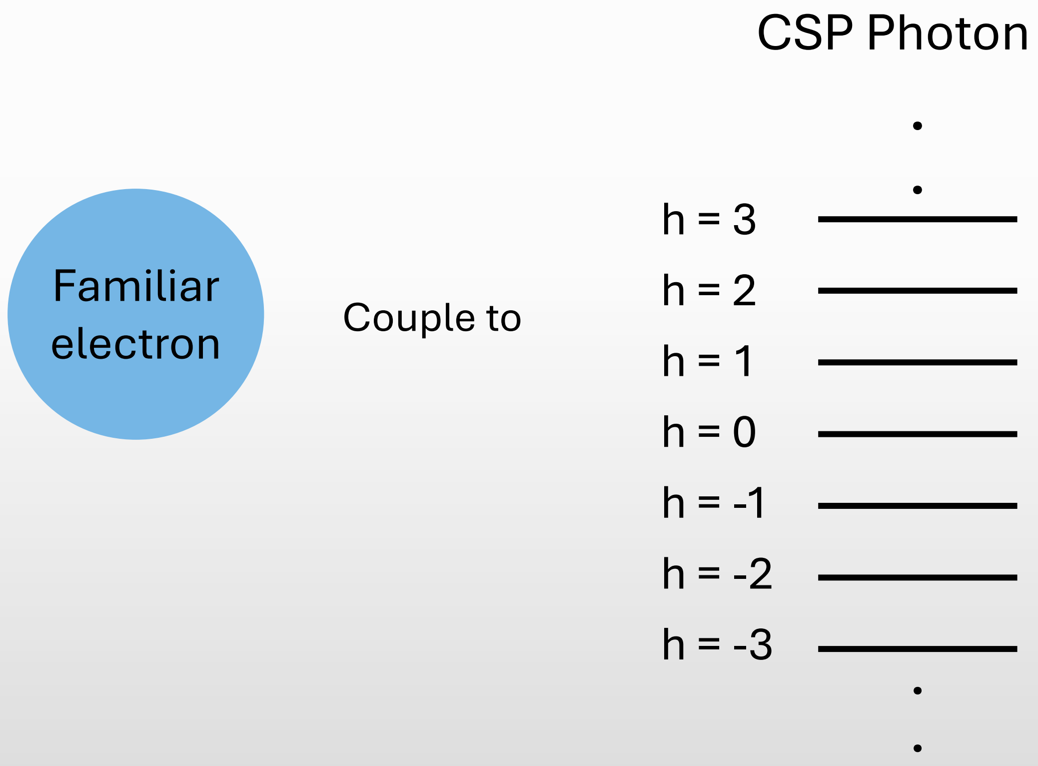
“ The mere possibility of the existence of these particles would give an infinite heat capacity to vacuum ”

- Wigner

CSPs were thought to be thermodynamically unphysical due to their infinity of polarizations, i.e., infinite internal degrees of freedom

The structure of CSP interactions allows us to re-evaluate this

- CSP interactions follow a "helicity correspondence" : Covariant interactions single out a **single** helicity, whose couplings to matter is unsuppressed by ρ at energies > spin scale



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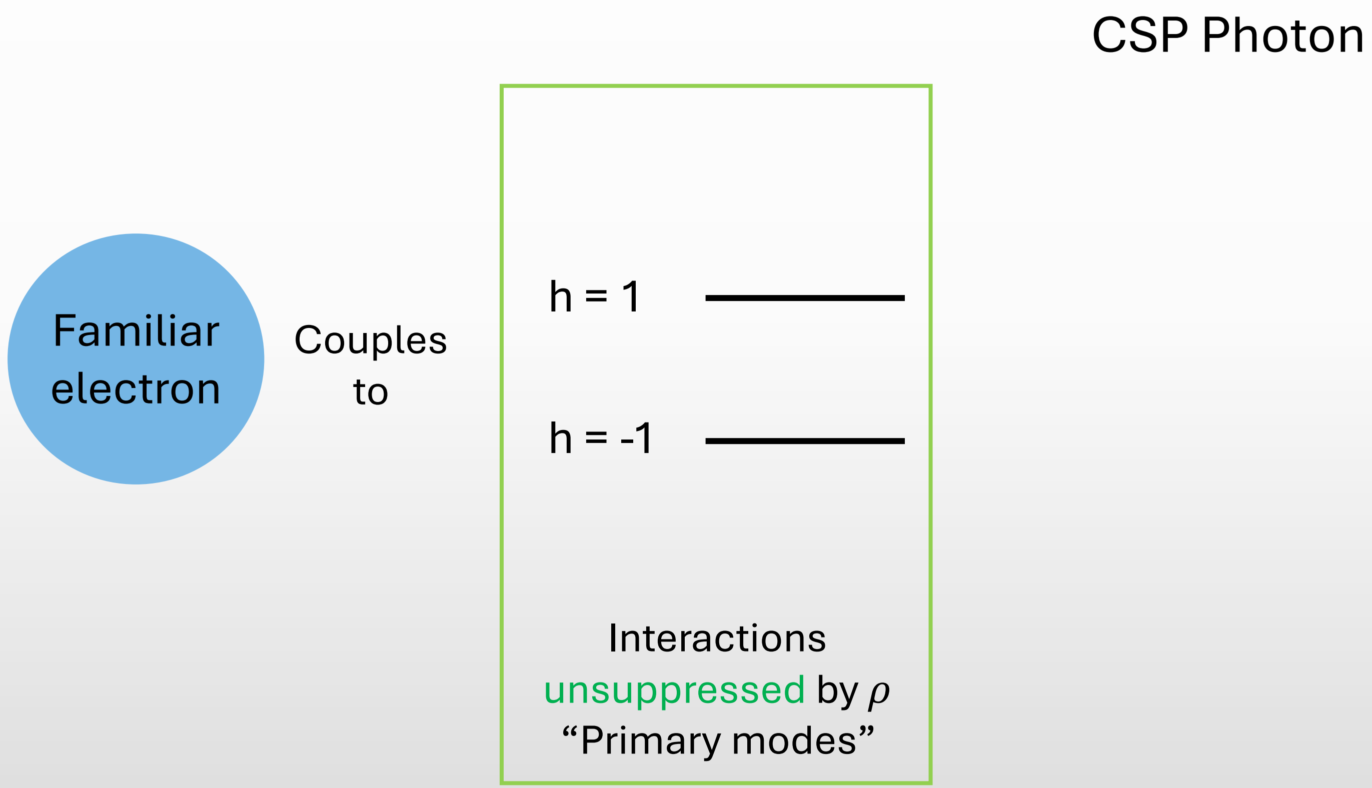

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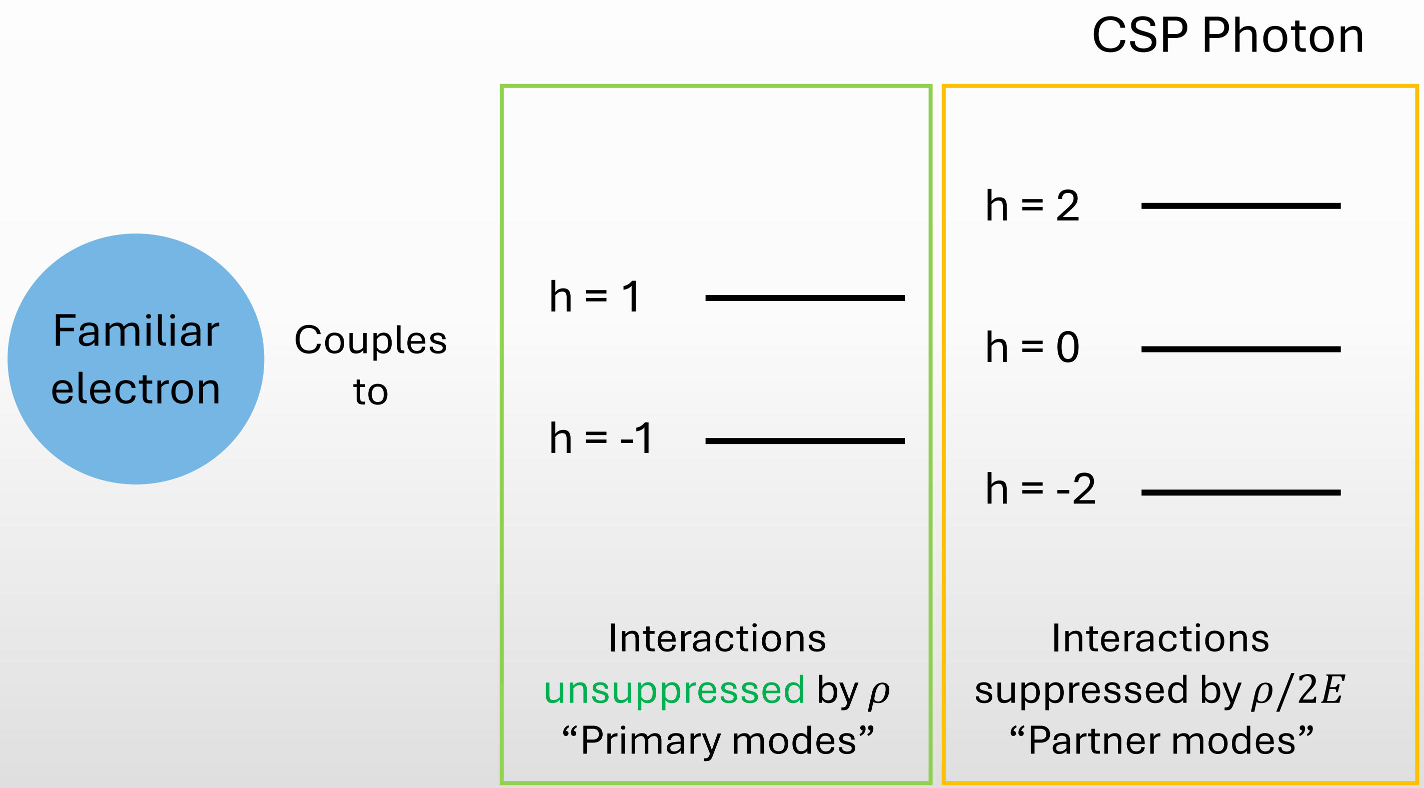

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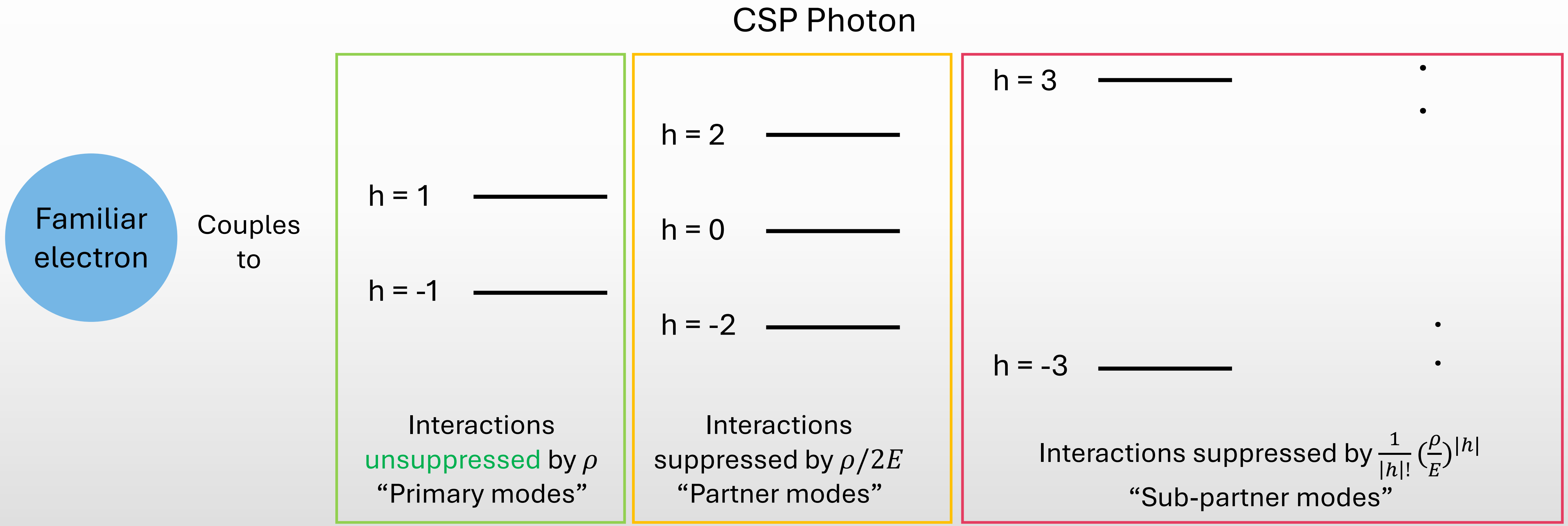

- Interaction amplitudes in the soft limit look like QED amplitudes x $\frac{1}{\rho}$ Bessel J $[h, \frac{\rho}{E}]$

h=0 sees modifications to the Bessel form

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Could our Standard Model photon be a CSP photon?

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Need to address Wigner's thermodynamic argument

Next – Going to use the structure of CSP photon interactions to calculate thermodynamic properties that arise from $\rho \neq 0$,
and we will contrast that with
familiar QED photon thermodynamics (that uses $\rho = 0$)

CSP Photon thermodynamics: Set up

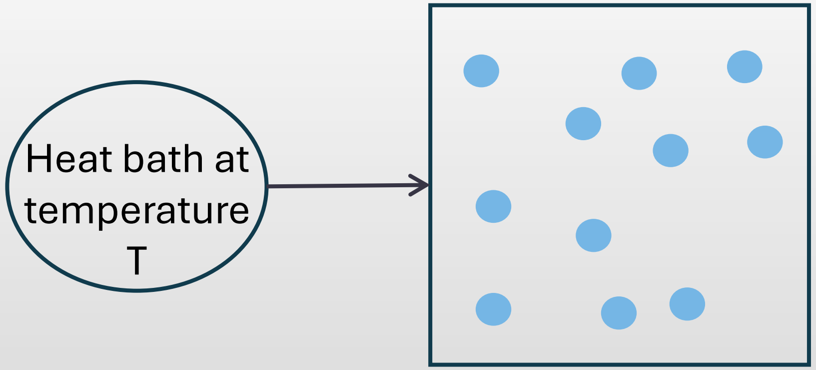
2 fundamental energy scales that control thermalization now: Temperature & Photon spin scale

We will consider an isothermal system with $T \gg \rho$
(Photon spin scale $\rho \leq$ neV, so most familiar thermal systems will have $T \gg \rho$)

Isothermal thermodynamics - System set up:

At t < 0

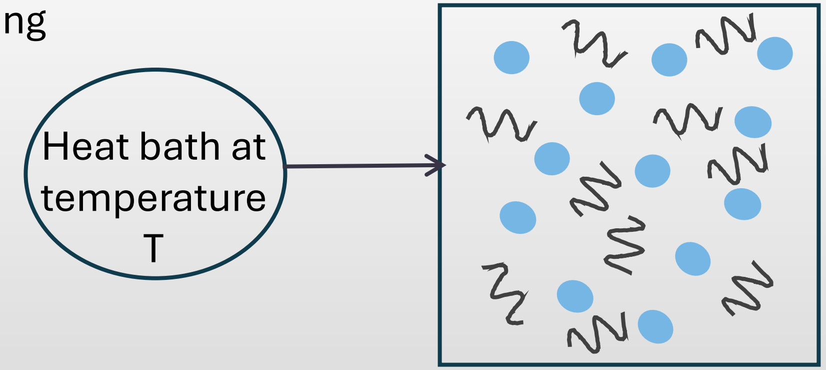
Non-interacting charged particles held in equilibrium with heat bath at temperature T



t = 0
"Turn on" charged matter-CSP photon coupling

t > 0

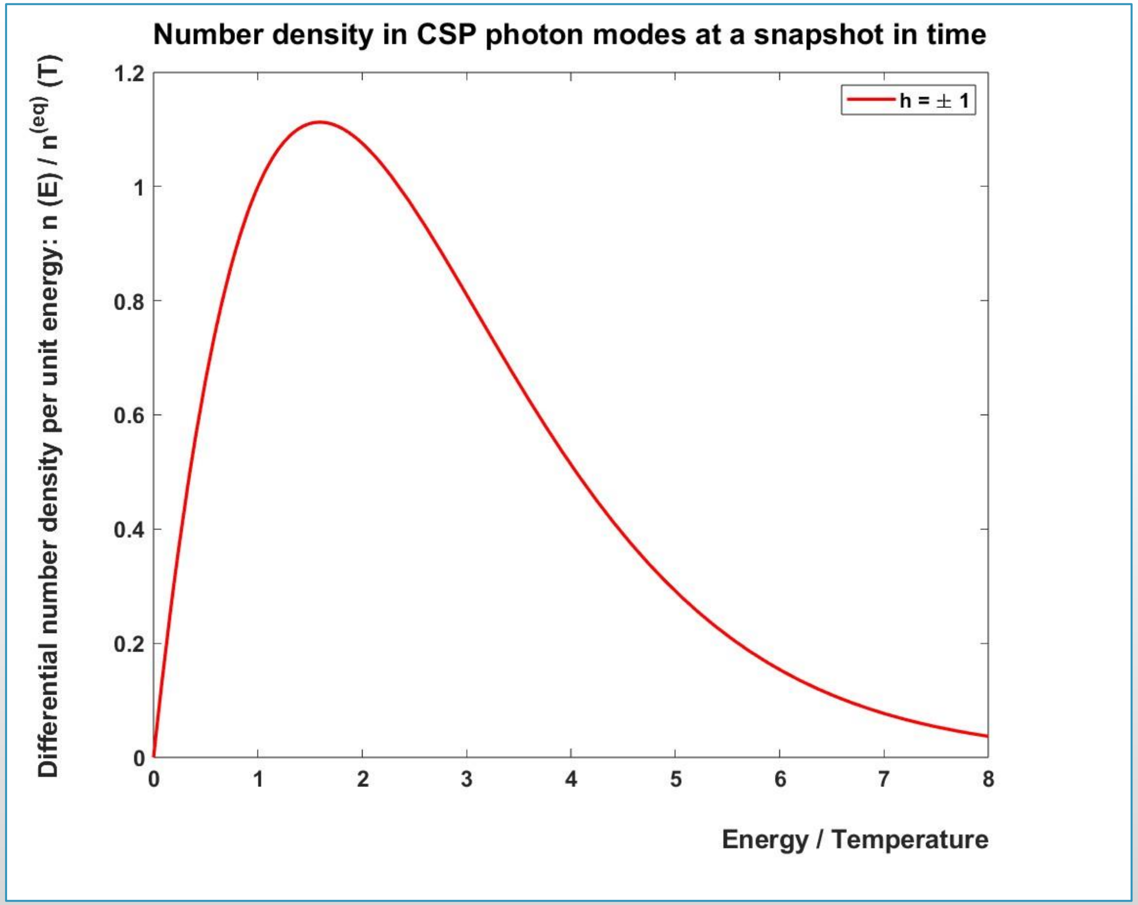
Charged particle interactions start populating the phase space of CSP photon modes



Run the clock and track CSP gas evolution
QED gas thermalization time in a similar set up is our benchmark

Key results #1: CSP photon gas (mostly) looks like the QED gas

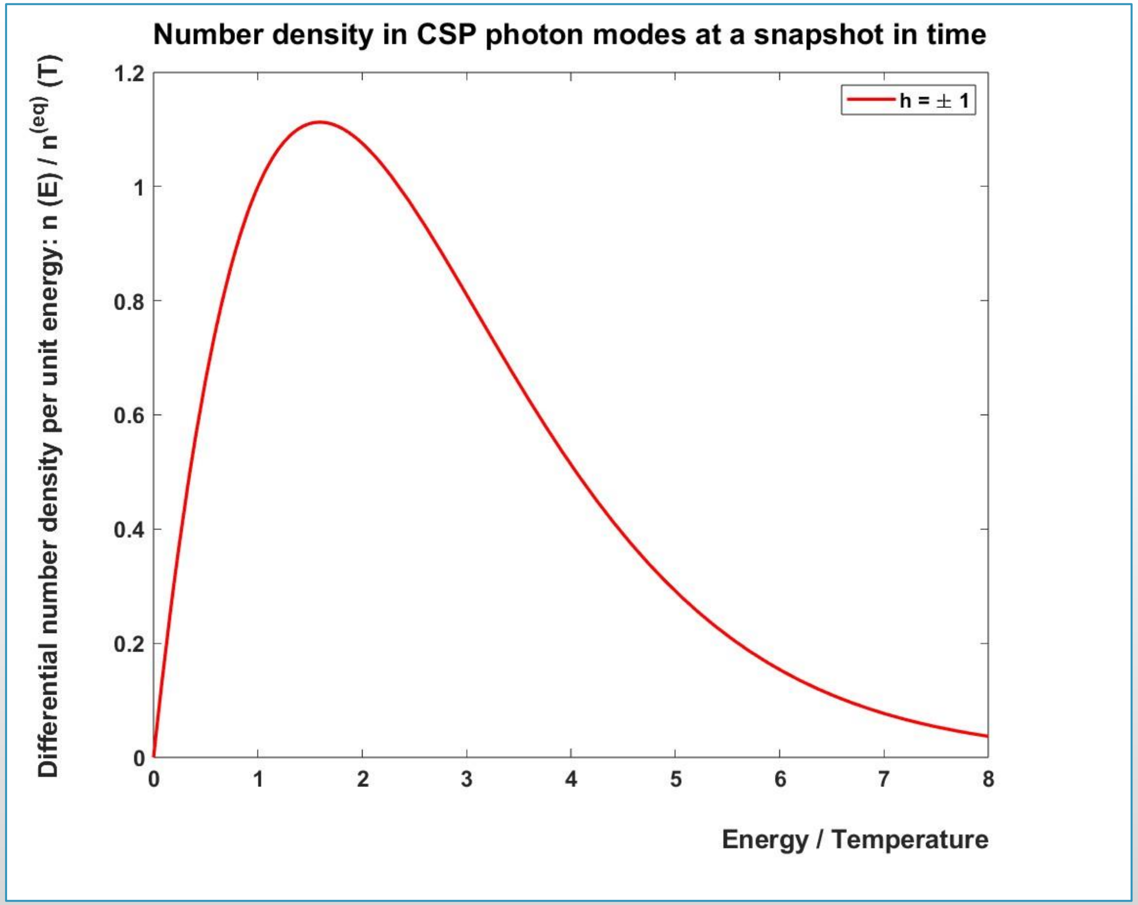
CSP photon mode number density at time = QED gas thermalization time



Plot uses $T = 10^4 \rho$ and electrons Boltzmann distributed with $\langle v_\perp \rangle = 0.1$
"QED gas thermalization time" taken to be time taken for QED gas to fully populate its phase space up to E = 10T

Key results #1: CSP photon gas (mostly) looks like the QED gas

CSP photon mode number density at time = 100 x QED gas thermalization time



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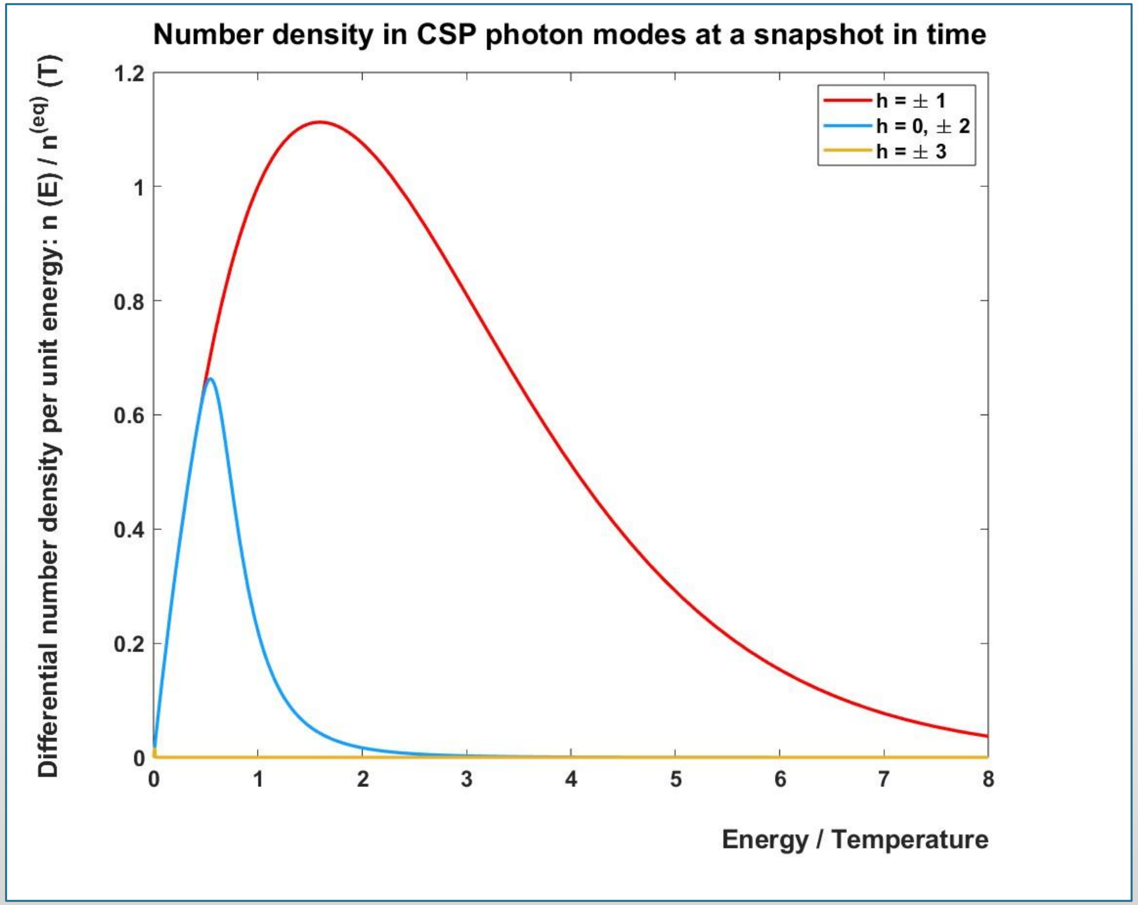
Key results #1: CSP photon gas (mostly) looks like the QED gas

CSP photon mode number density at time = 10^8 x QED gas thermalization time



- CSP gas as a whole is always partially thermal
  - Different modes thermalize on different timescales – primary modes first, partner modes next, and so on
  - "Doubly hierarchical" thermalization of phase space at $E > \rho$ → high energy phase space of higher helicities is super-exponentially suppressed

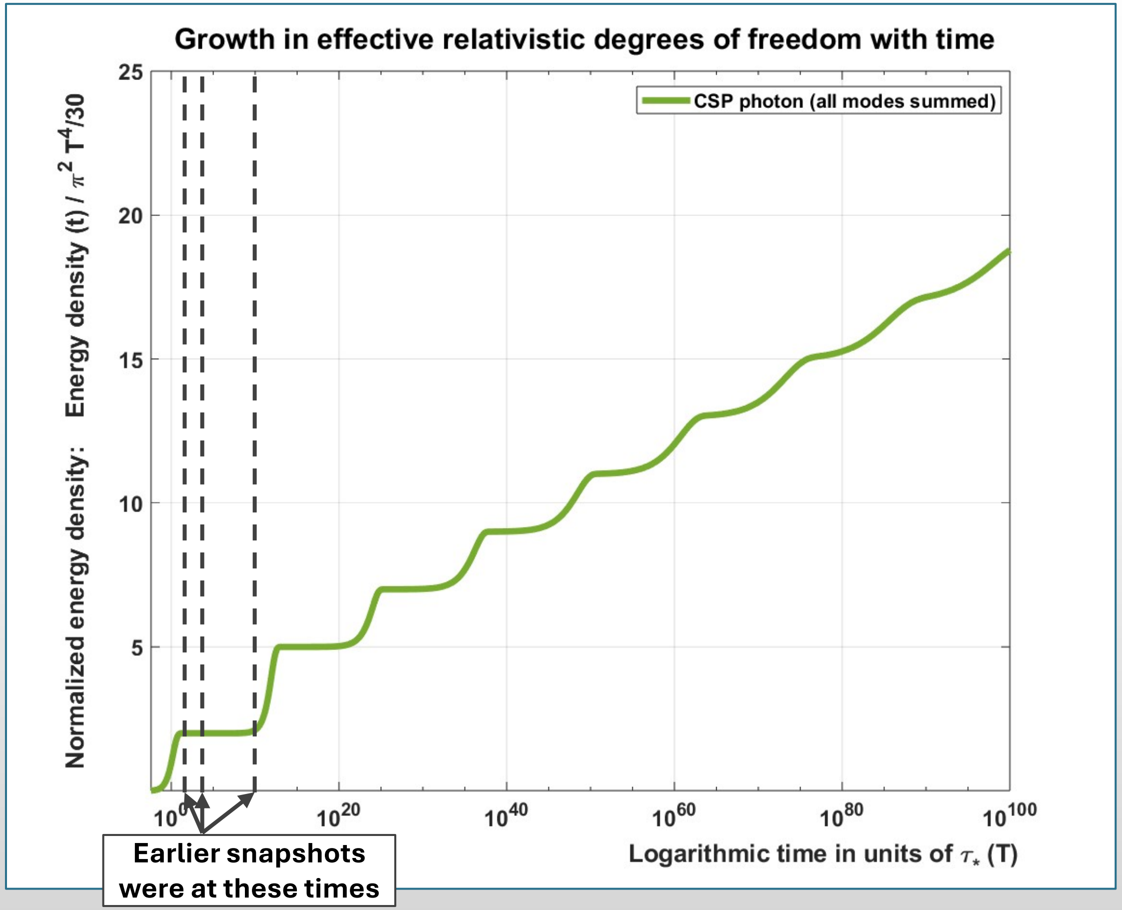
- Have to wait a really long time to see even 10% deviations from familiar QED!

Plot uses $T = 10^4 \rho$ and electrons Boltzmann distributed with $\langle v_\perp \rangle = 0.1$
"QED gas thermalization time" taken to be time taken for QED gas to fully populate its phase space up to E = 10T

Key results #2: The infinite degrees of freedom never* unlock

*requires infinite time

CSP Photon Effective relativistic degrees of freedom vs. time



Plot uses $\mathrm{T} = 10^4 \rho$ and electrons Boltzmann distributed with $\langle v_\perp \rangle = 0.1$

$\tau_*(T)$ is the time taken for QED gas to fully populate its phase space up to E = T

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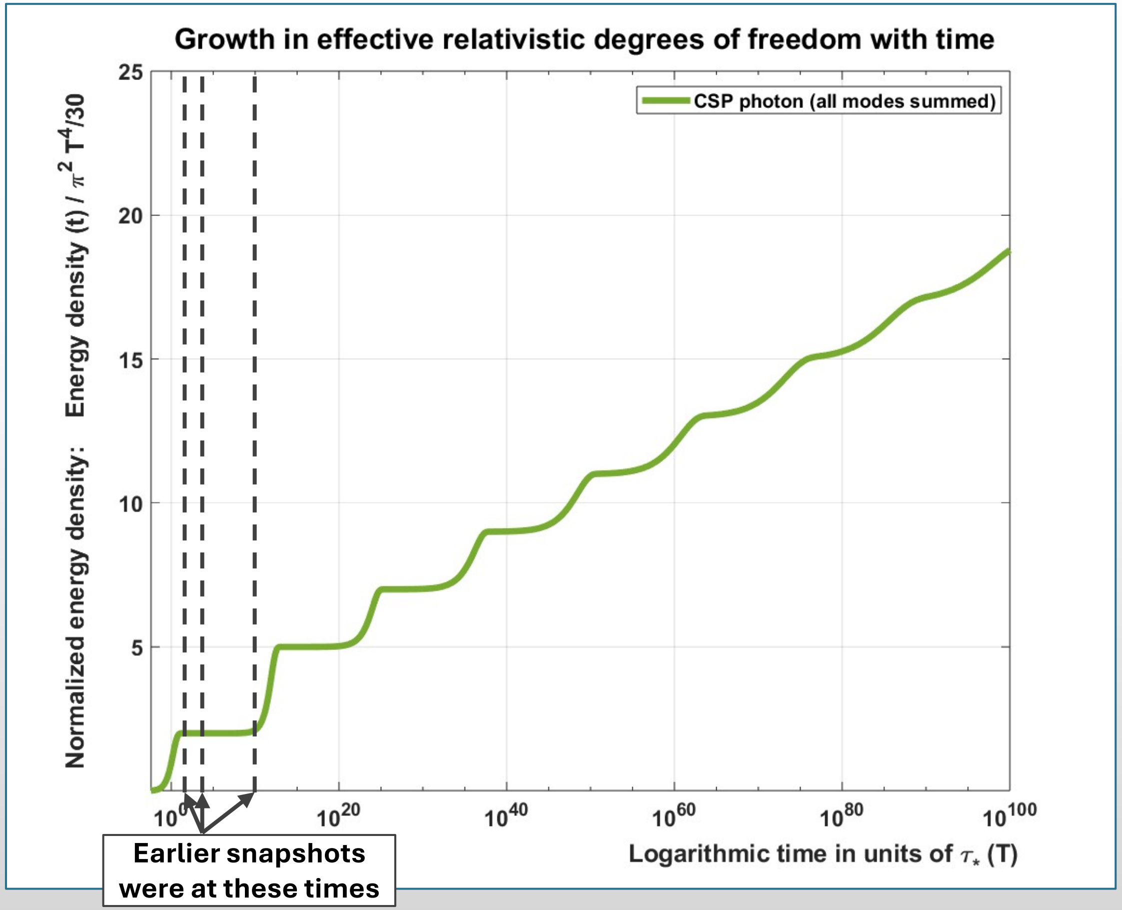
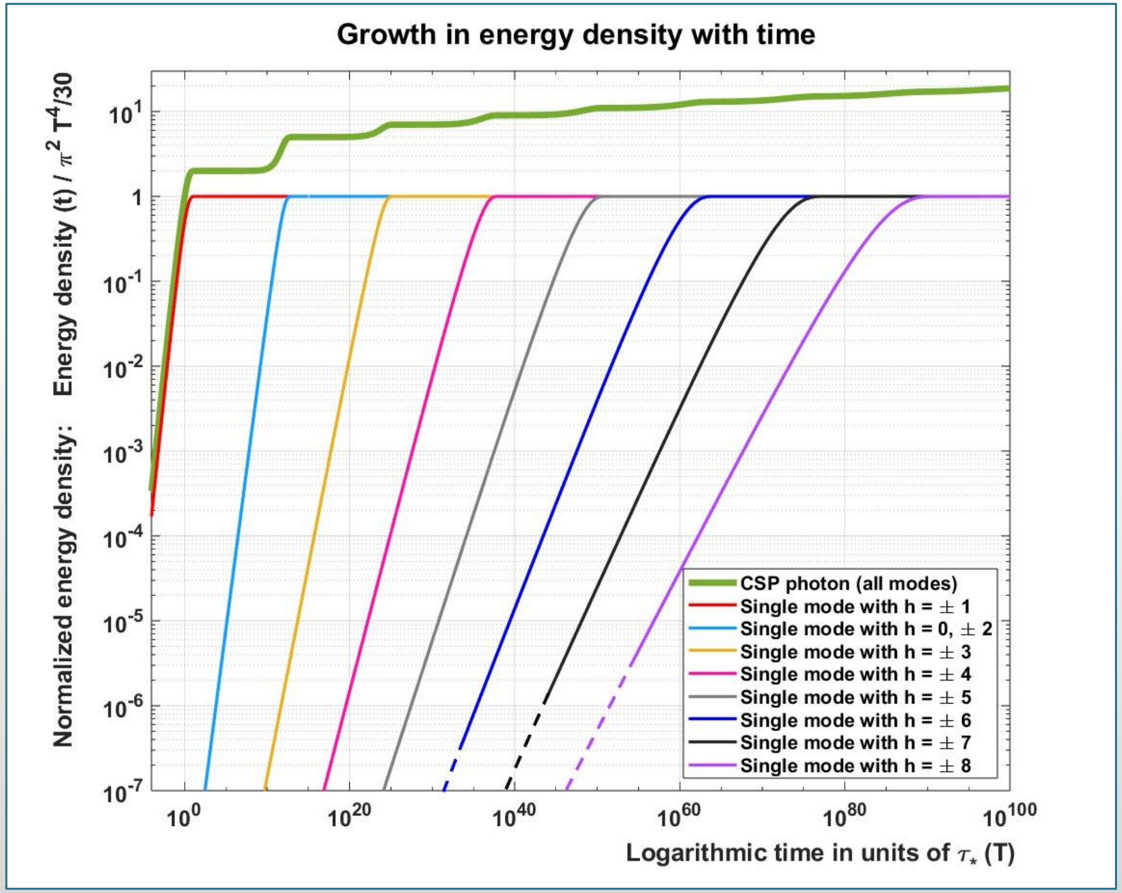


Mode energy density growth is slower than $t^{\frac{3}{2|h|}}$ when thermalizing - "frozen" as $h \to \infty$

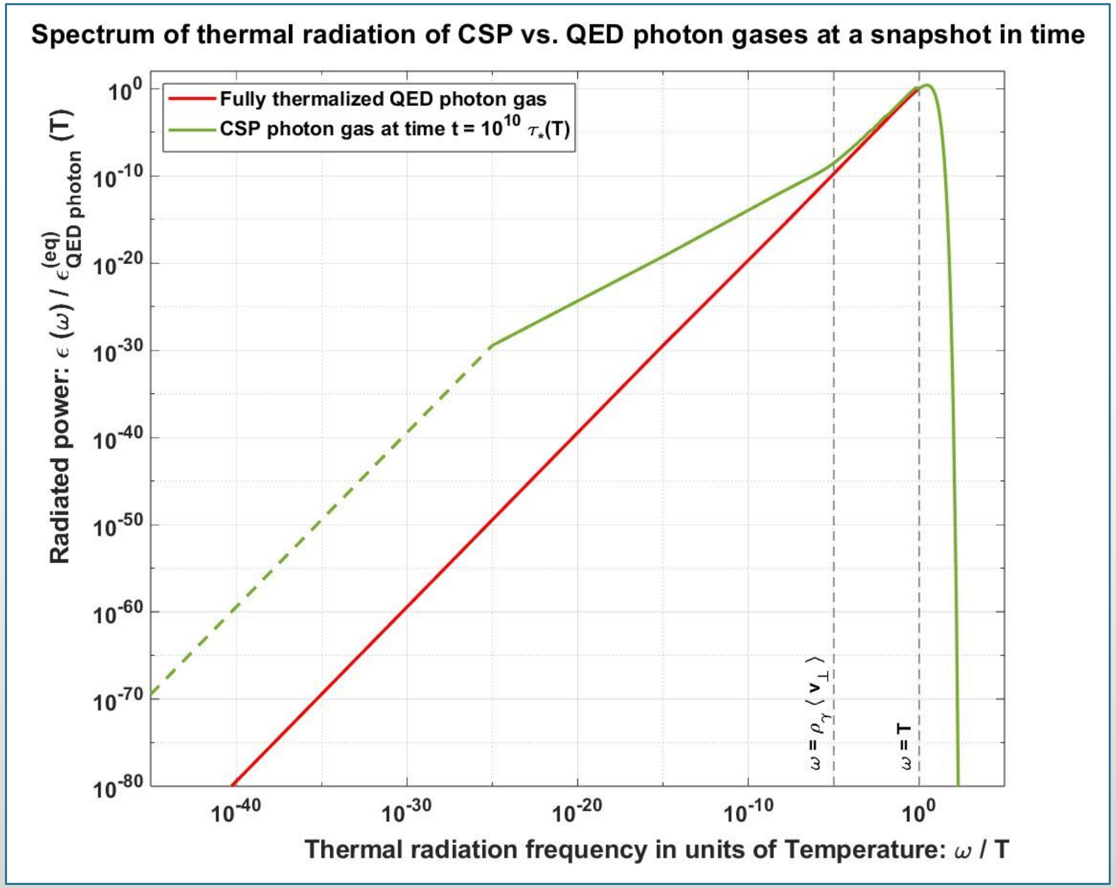


Except h=0 whose energy density grows as $t^{3/4}$ when populating phase space at E << T

Plot uses $\mathrm{T} = 10^4 \rho$ and electrons Boltzmann distributed with $\langle v_\perp \rangle = 0.1$
$\tau_*(T)$ is the time taken for QED gas to fully populate its phase space up to E = T

Key results #3: Things get interesting in the IR, at energies less than the photon spin scale

Thermal radiation spectrum at time = 10^8 x QED gas thermalization time



- All modes including primary modes behave differently vs. standard QED at energies < spin scale
- All thermodynamic quantities (summed over modes) remain finite and sub-dominant
- In thermal spectrum, orders of magnitude stronger radiated power (vs. QED) at Frequencies < Spin scale
  - Deep IR spectrum is calculable
  - Despite large fractional deviations from QED, carries very small fraction of the total power

Plot uses $T = 10^4 \rho$ and electrons Boltzmann distributed with $\langle v_\perp \rangle = 0.1$
"QED gas thermalization time" taken to be time taken for QED gas to fully populate its phase space up to E = 10T

Conclusion

- CSPs are the general massless particles allowed by relativity + QM
  - Bosonic CSPs have an infinite tower of integer eigen-valued polarization states that mix under Lorentz symmetry
  - Spin-scale ρ is a fundamental particle property
  - CSP photon's interactions with charged matter are hierarchical, with $h = \pm 1$ modes dominating at energies >> spin scale ("Helicity correspondence")

Could our Standard Model bosons be CSPs?

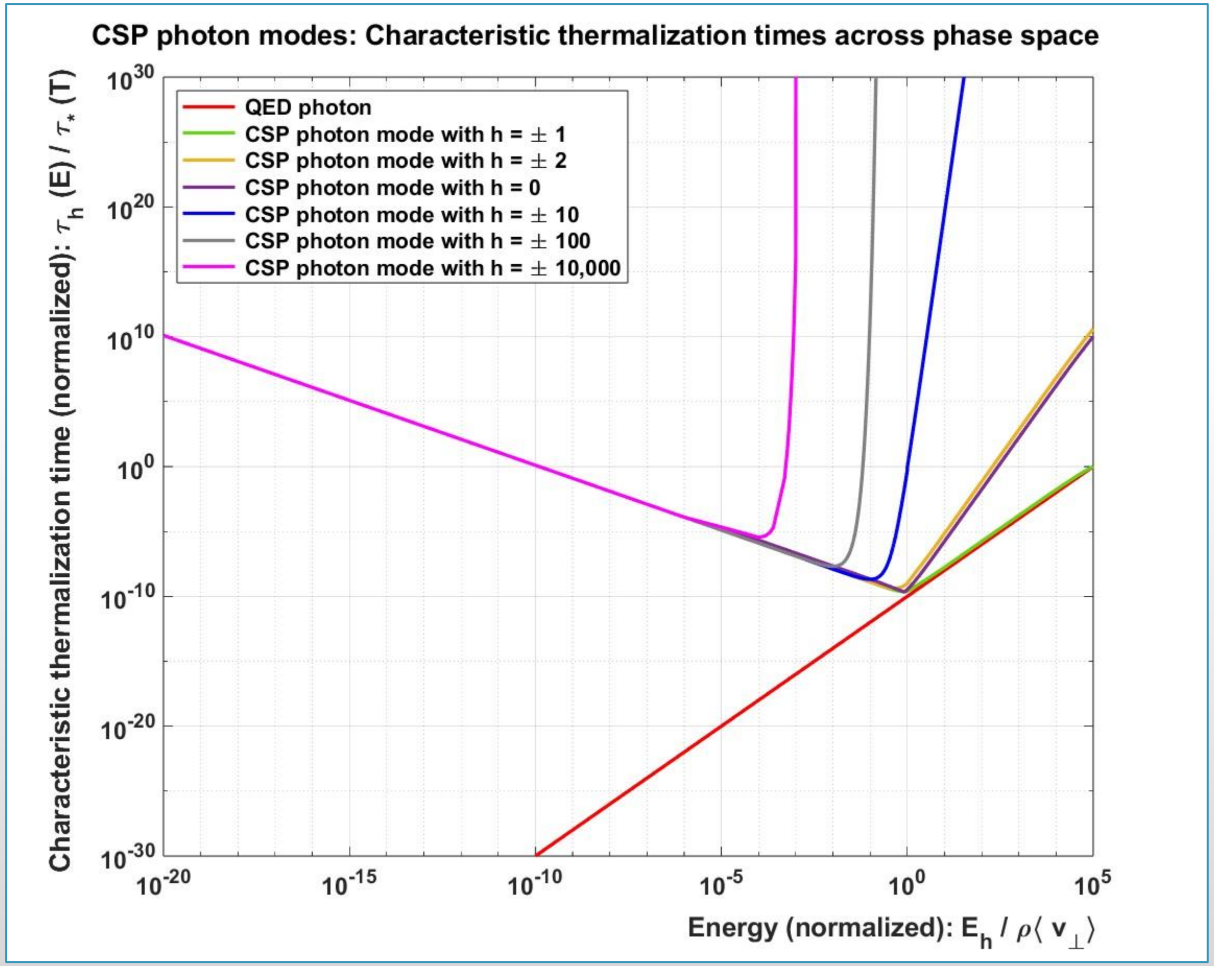
Exciting possibility that deserves further theoretical and phenomenological investigation!

- CSP photon thermodynamics looks like QED thermodynamics, unless we wait very long and/ or look in the deep IR
  - In familiar thermal systems, negligible deviations from standard QED, up to exponentially late times
  - Precision measurements of deep IR phase space of thermal systems can lead to discovery
  - All thermodynamic properties in all regions of phase space calculable and finite – the infinite degrees of freedom never physically manifest!
- So, was Wigner wrong in his thermodynamic argument precluding the existence of CSPs?
  - Strictly technically, no (it is correct in infinite time)
  - But physically, yes. Our universe is a "finite time system" – so physically, Wigner's argument was an incorrect statement about our universe

Back up

Mode thermalization behavior across phase space – fastest at a “characteristic energy” that is > 0 and less than spin scale

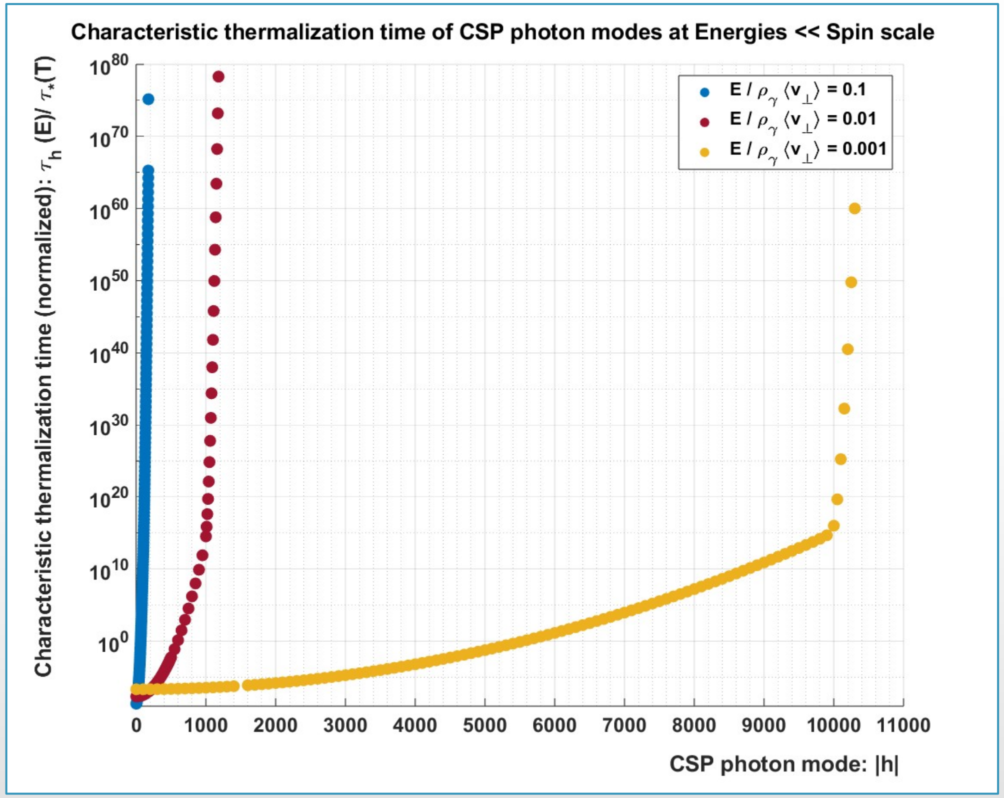
Mode thermalization time across phase space



Plot uses $T = 10^4\rho$ and electrons Boltzmann distributed with $\langle v_\perp \rangle = 0.1$
Characteristic energy → 0 and h → ∞

Mode thermalization hierarchies at energies << spin scale and at energies >> spin scale

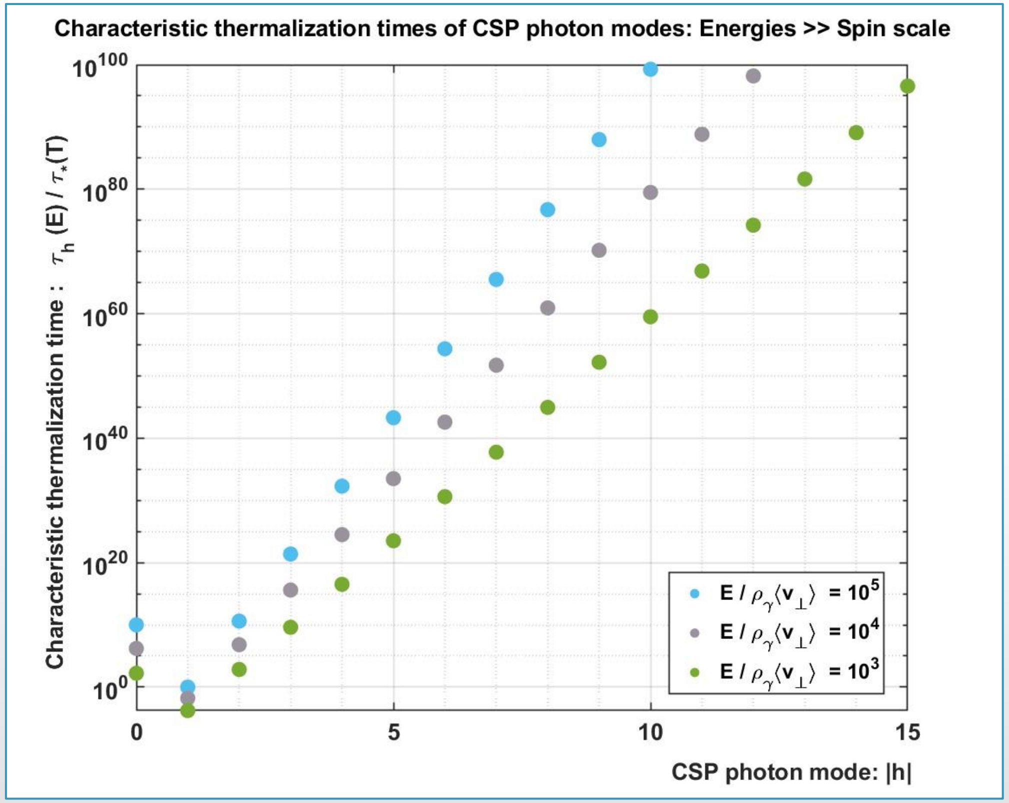
"Weak hierarchy at $E \ll \rho$"



$$\frac{\tau_h(E)}{\tau_*(E)} \sim \begin{cases} \left(\frac{\rho_\gamma\langle v_\perp\rangle}{E}\right)^3 & |h| \lesssim \left(\frac{\rho_\gamma\langle v_\perp\rangle}{E}\right)^{1+\varepsilon} \\ 2^{\tilde{h}}(\tilde{h}+1)!^2\left(\frac{E}{\rho_\gamma\langle v_\perp\rangle}\right)^{2\tilde{h}} & |h| > \frac{\rho_\gamma}{E} \\ & \text{and } > \frac{1}{2\langle v_\perp\rangle^2} \end{cases}$$

Characteristic thermalization time of a CSP photon mode h / Characteristic thermalization time of QED photon at same energy

"Strong double hierarchy at $E \gg \rho$"



$$\frac{\tau_h(E)}{\tau_*(E)} \sim \frac{\langle |z|^2\rangle}{\langle |zF_h(\rho_\gamma z)|^2\rangle} \approx 2^{\tilde{h}}(\tilde{h}+1)!^2\left(\frac{E}{\rho_\gamma\langle v_\perp\rangle}\right)^{2\tilde{h}}$$

Plots use $T = 10^4\rho$ and electrons Boltzmann distributed with $\langle v_\perp\rangle = 0.1$