

Extraction of the ^{12}C Longitudinal (R_L) and (R_T) Nuclear Electromagnetic Response Functions from all Electron Scattering Measurements on Carbon

1. Testing first principle nuclear theory predictions
2. Provide a platform for verification of electron and neutrino MC generators over the entire kinematic range of interest

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Presented by **Zihao Lin**

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12 min talk + 3 min Q&A

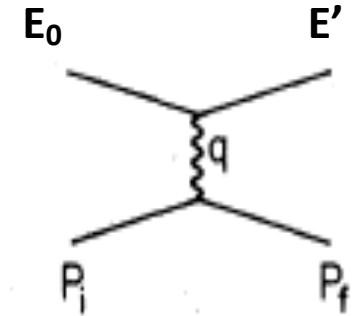
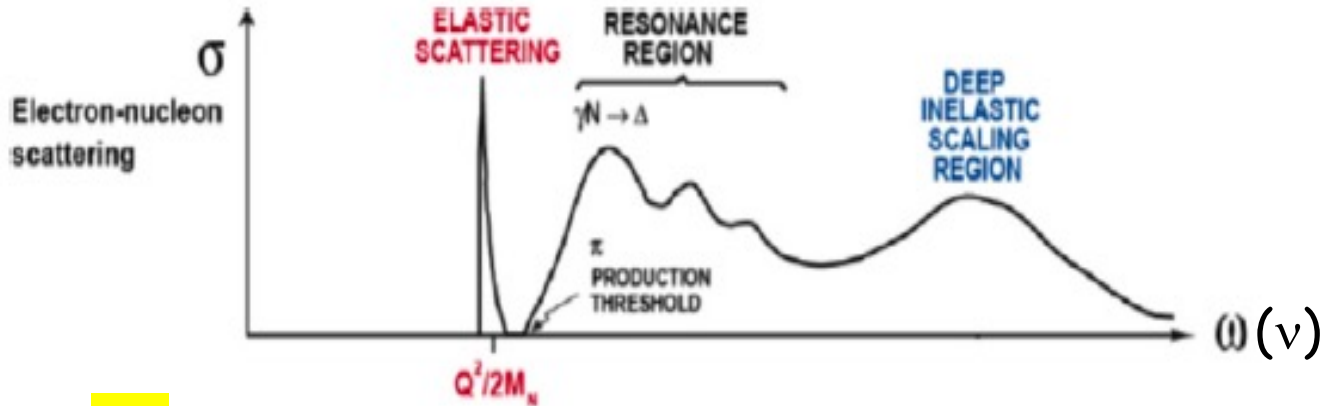


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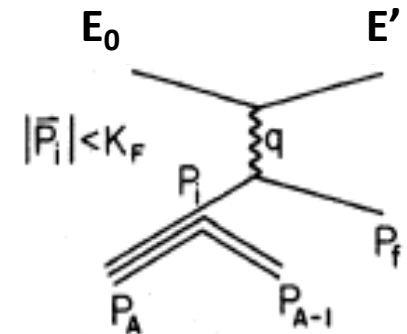
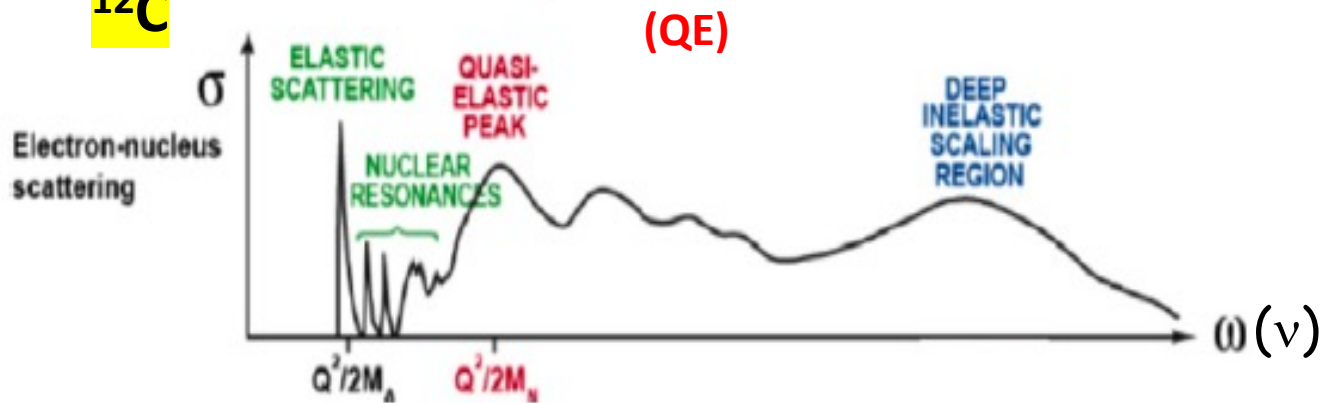
Introduction

P, N

G.T. Garvey et al. / Physics Reports 580 (2015) 1–45



^{12}C



Nuclear corrections effect **quench the Longitudinal Response (R_L)** and **enhance the Transverse Response (R_T)** in QE scattering. There're 3 formalisms:

1. R_L and R_T
2. F_1 and F_L
3. σ_L and σ_T .

We use R_L and R_T .

Physical Quantities

Nuclear Physics:

This description is primarily used in the nuclear excitation and QE regions. The electron scattering differential cross section is written in terms of longitudinal ($\mathcal{R}_L(Q^2, \nu)$) and transverse ($\mathcal{R}_T(Q^2, \nu)$) nuclear response functions [24]

$$\frac{d\sigma}{d\nu d\Omega} = \sigma_M [A \mathcal{R}_L(Q^2, \nu) + B \mathcal{R}_T(Q^2, \nu)] \quad (20)$$

where σ_M is the Mott cross section, $A = (Q^2/\mathbf{q}^2)^2$ and $B = \tan^2(\theta/2) + Q^2/2\mathbf{q}^2$.

Particle Physics:

This description is primarily used in the inelastic continuum region. In the one-photon-exchange approximation, the spin-averaged cross section for inclusive electron-proton scattering can be expressed in terms of two structure functions as follows

$$\frac{d\sigma}{d\Omega dE'} = \sigma_M [\mathcal{W}_2(W^2, Q^2) + 2 \tan^2(\theta/2) \mathcal{W}_1(W^2, Q^2)]$$

$$\sigma_M = \frac{\alpha^2 \cos^2(\theta/2)}{[2E_0 \sin^2(\theta/2)]^2} = \frac{4\alpha^2 E'^2}{Q^4} \cos^2(\theta/2) \quad (10)$$

energy transfer = ν (or ω)

$$\nu = E_0 - E'$$

Q^2 = 4-momentum transfer squared

$$Q^2 = (-q)^2 = 4E_0 E' \sin^2 \frac{\theta}{2},$$

W^2 = final state invariant mass squared

$$W^2 = M^2 + 2M\nu - Q^2$$

\mathbf{q} = 3-momentum transfer

$$\mathbf{q}^2 = Q^2 + \nu^2$$

E_x = Excitation energy

$$E_x = \nu - \nu_{elastic}$$

$$x = Q^2 / (2M\nu).$$

$$\mathcal{F}_1 = M\mathcal{W}_1 \text{ and } \mathcal{F}_2 = \nu\mathcal{W}_2.$$

$$\mathcal{F}_L(x, Q^2) = \mathcal{F}_2 \left(1 + \frac{4M^2 x^2}{Q^2} \right) - 2x\mathcal{F}_1,$$

$$\mathcal{R}_L(\mathbf{q}, \nu) = \frac{\mathbf{q}^2}{Q^2} \frac{\mathcal{F}_L(\mathbf{q}, \nu)}{2Mx}$$

$$\mathcal{R}_T(\mathbf{q}, \nu) = \frac{2\mathcal{F}_1(\mathbf{q}, \nu)}{M}$$

Project Overview

The RL RT extraction Project:

- **Extract RL and RT values on various nuclei using all available data.**
Today: Carbon (16k electron scattering and photoproduction cross-section measurements.)
- **Cover all kinematic regions:** nuclear elastic, nuclear excitations, quasi-elastic (QE), resonance region and inelastic scattering.
- For Carbon, we use **Rosenbluth linear fit** to extract at 18 fixed Q^2 values: $0 < Q^2 < 3.45 \text{ GeV}^2$,
and at 18 fixed q values: $0.1 < |q| < 2.78 \text{ GeV}$,
both as functions of ν .
- ν ranges from $\nu=0 \text{ GeV}$ to the end of the resonance region (where $W=2.0 \text{ GeV}$).

Project Overview

Goals:

- To test first-principle nuclear theories.
- To validate MC generators.

Advantage: covers all kinematic regions, more comprehensive than comparison with a few cross-section measurements in limited kinematic regions.

Note: Where there is no data, we provide our universal fit.

Part 1: updated Christy-Bodek Universal Fit

- Includes **all the world's electron scattering data** on H, D and nuclear targets.
- Fits for all kinematic regions:
 - **QE cross section** (including Transverse Enhancement/MEC and longitudinal low q Quenching), **Resonance and pion production**, **deep inelastic scattering**, **nuclear excitations**, elastic scattering.

Since the cross sections span a large range of energies and scattering angles, we can extract both the **longitudinal R_L** and **transverse R_T** contributions.
- Parameterizes both the **Enhancement of the Transverse QE** cross section and the **Quenching of the Longitudinal QE** cross section.
 - We also extract the most precise Coulomb Sum rule as a function of q and compare to theoretical calculations.
- The fit alone can be used to evaluate Monte Carlo predictions for electron-nucleus scattering.

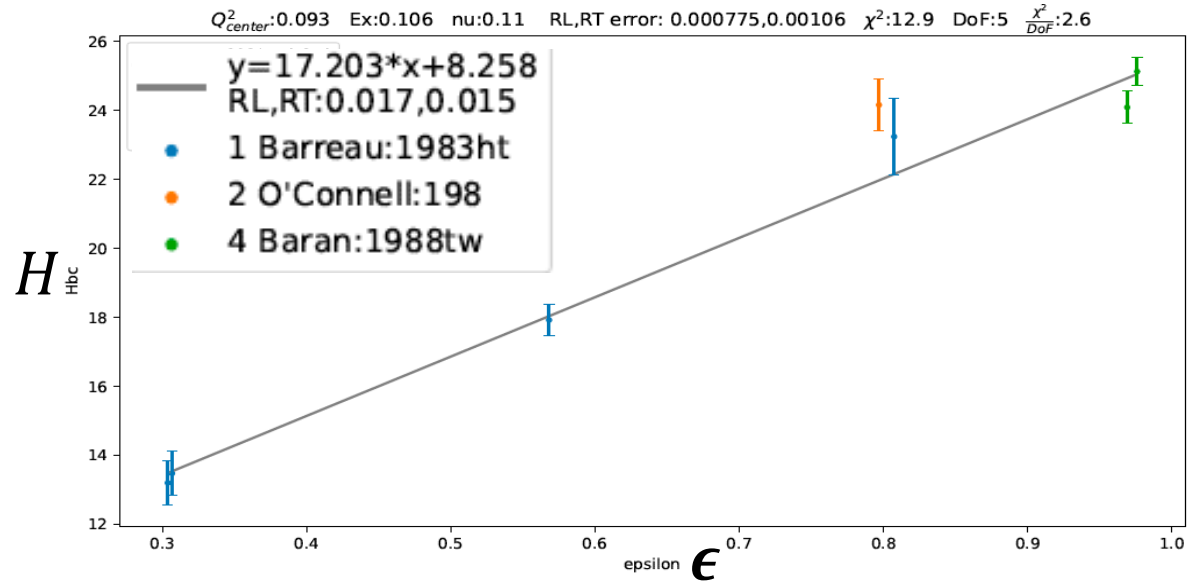
Part 2: Individual R_L R_T extractions (Rosenbluth Linear Fits)

Rosenbluth linear fit with
Coulomb corrections,
bin-centering corrections
to extract R_L , R_T :

Slope $\propto R_L$;

Intercept $\propto R_T$.

H : the “Rosenbluth quantity”
 ϵ : virtual photon polarization



$$H = \left(\frac{E_0}{E_0 + V_{eff}} \right)^2 \frac{\mathbf{q}_{eff}^4}{4\alpha^2 E'_{eff}{}^2} \frac{1}{\cos^2\left(\frac{\theta}{2}\right) + 2\left(\frac{\mathbf{q}_{eff}}{Q_{eff}}\right)^2 \sin^2\left(\frac{\theta}{2}\right)}$$

$$\epsilon = \left[1 + 2 \left(1 + \frac{\nu^2}{Q^2} \right) \tan^2\left(\frac{\theta}{2}\right) \right]^{-1}$$

Part 2: Individual R_L R_T extractions (Rosenbluth Linear Fits)

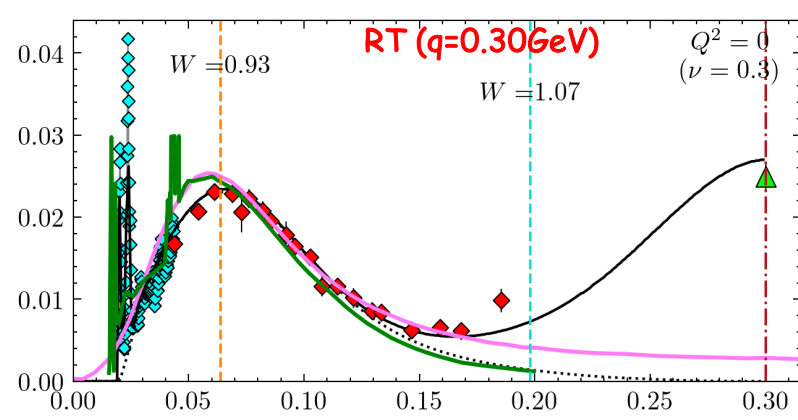
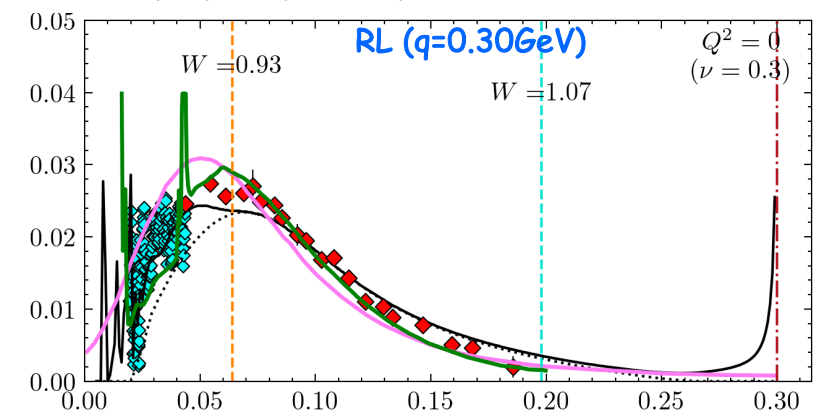
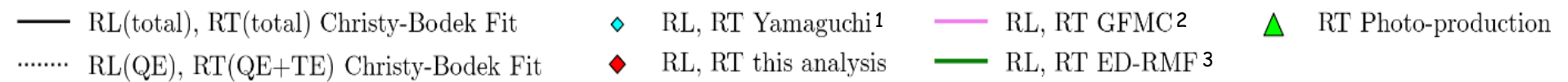
Analysis in fixed q (or in fixed Q^2) bin:

1. Bin all cross-section data in q ;
2. Apply **Coulomb corrections**; apply **bin-centering corrections**.
For $\nu < 50\text{MeV}$: bin-centered in E_x (excitation energy);
For $\nu > 50\text{MeV}$: bin-centered in W^2 (final state invariant mass squared);
Later convert E_x and W^2 to ν .
3. Bin again in ν .
4. Finally, perform Rosenbluth fit to subdivisions of data to extract R_L and R_T .

Our fixed **q bin-centers**: 0.100, 0.148, 0.167, 0.205, 0.240, 0.300, 0.380, 0.475, 0.570, 0.649, 0.756, 0.991, 1.659, 1.921, 2.213, 2.500, 2.783, 3.500 GeV

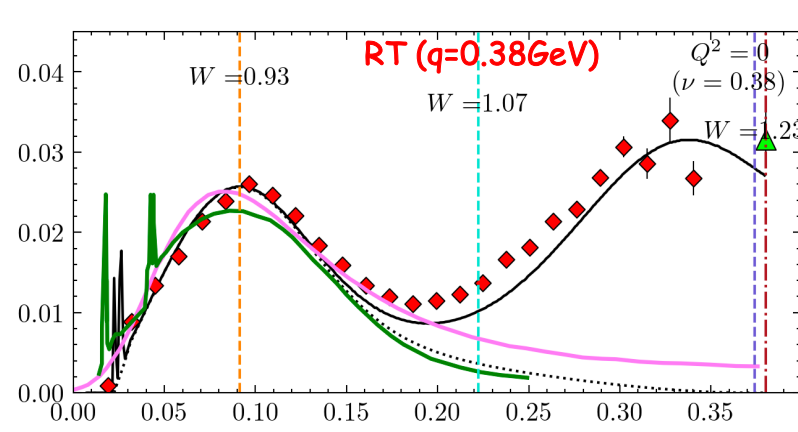
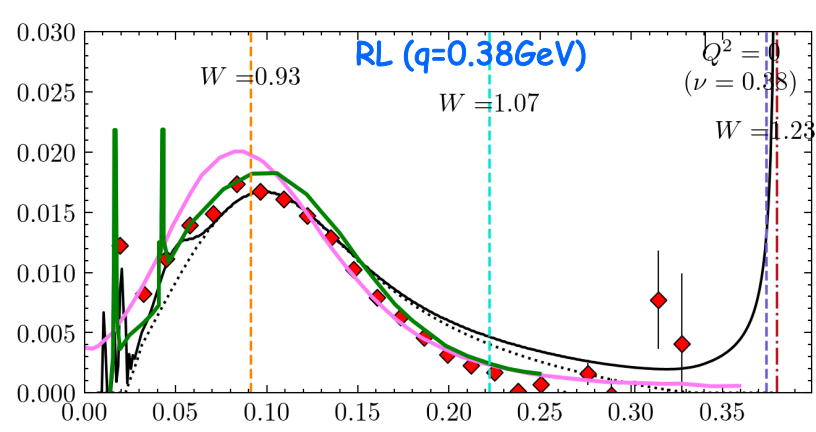
Our fixed **Q^2 bin-centers**: 0.00 (photoproduction), 0.010, 0.020, 0.026, 0.040, 0.056, 0.093, 0.120, 0.160, 0.265, 0.38, 0.50, 0.80, 1.25, 1.75, 2.25, 2.75, 3.25, 3.75 GeV^2

Note: Christy-Bodek fit is universal, while Rosenbluth fit uses only a small subset.

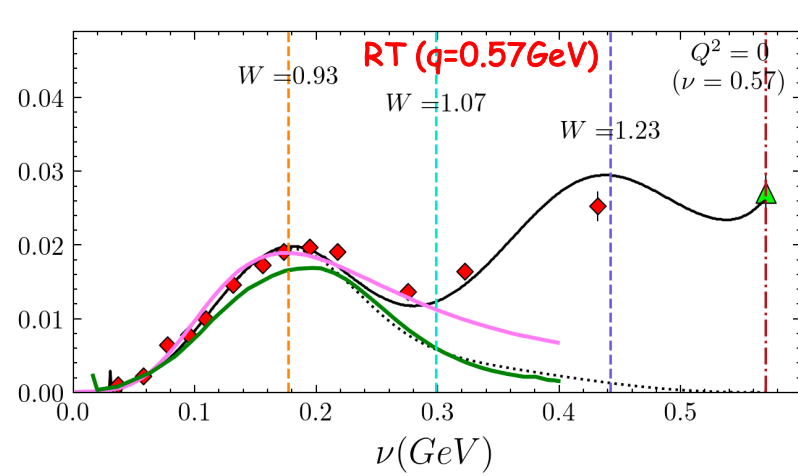
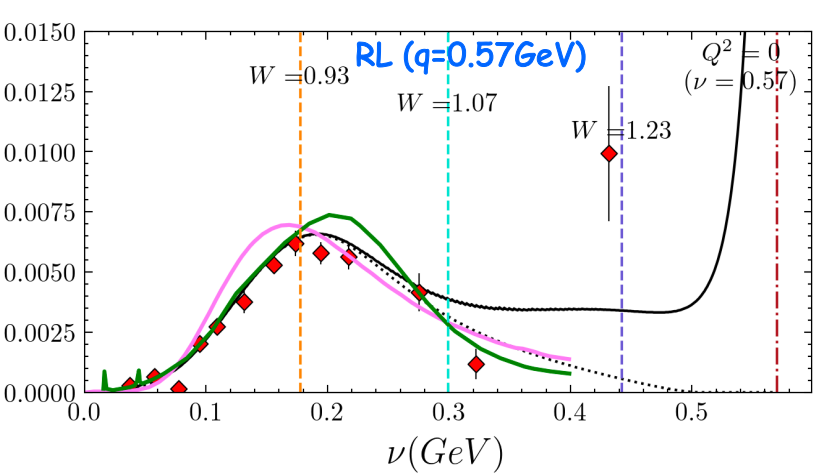


Comparison to Nuclear Theory (for QE 1p1h process only)

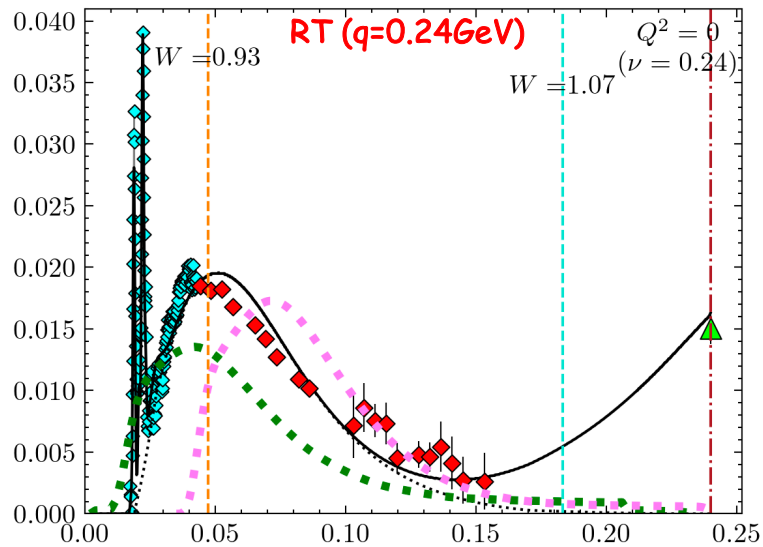
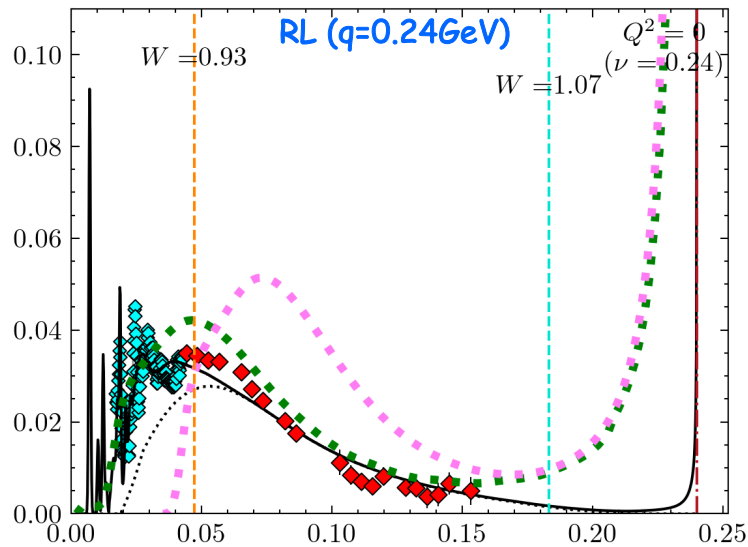
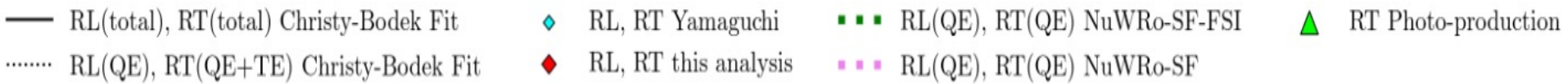
GFMC:
Green's Function Monte-Carlo



ED-RMF:
Energy Dependent Relativistic Mean Field



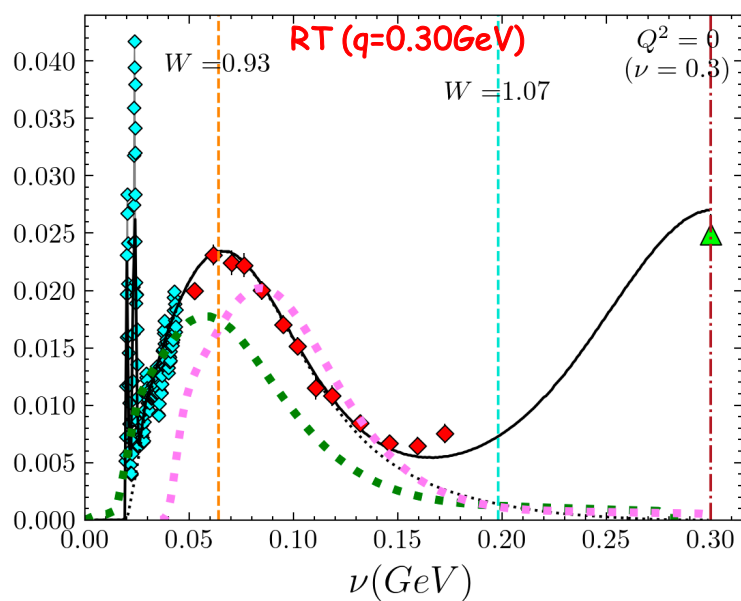
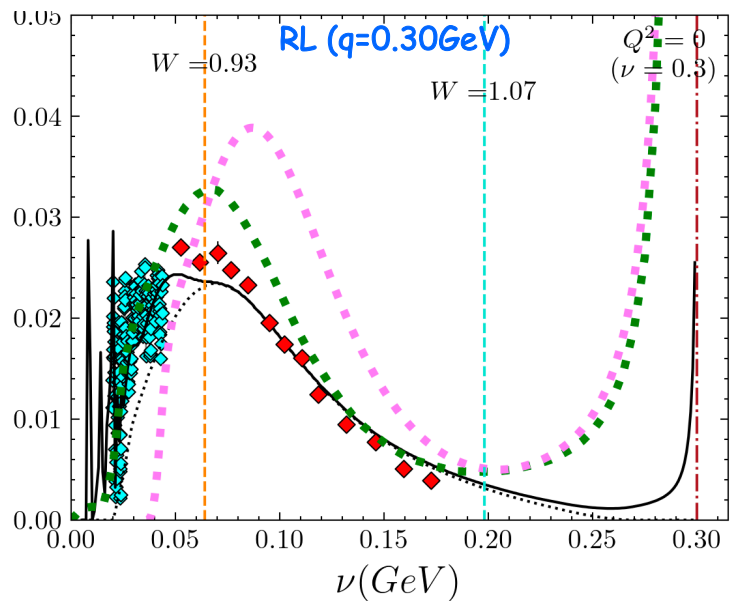
1: A. Yamaguchi et al, Phys. Rev. C 3, 1750 (1971).
 2: A. Lovato et al, Phys. Rev. Lett. 117, 082501 (2016),
 3: T. Franco-Munoz et al., (2022) arXiv:2203.09996

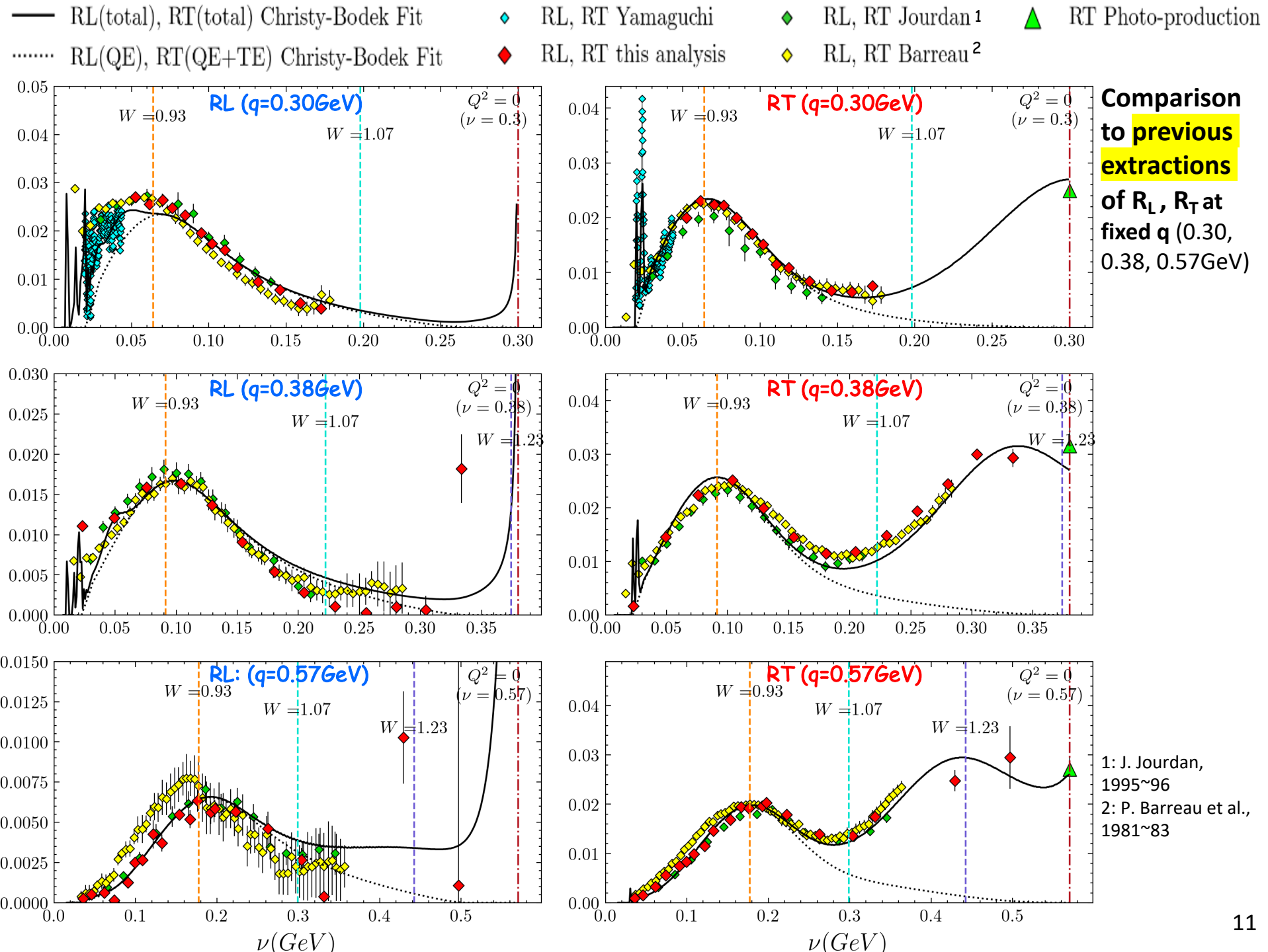


Comparison to NuWRo Neutrino MC generator for QE (in electron scattering mode)

SF:
Spectral function

FSI:
Final state interaction

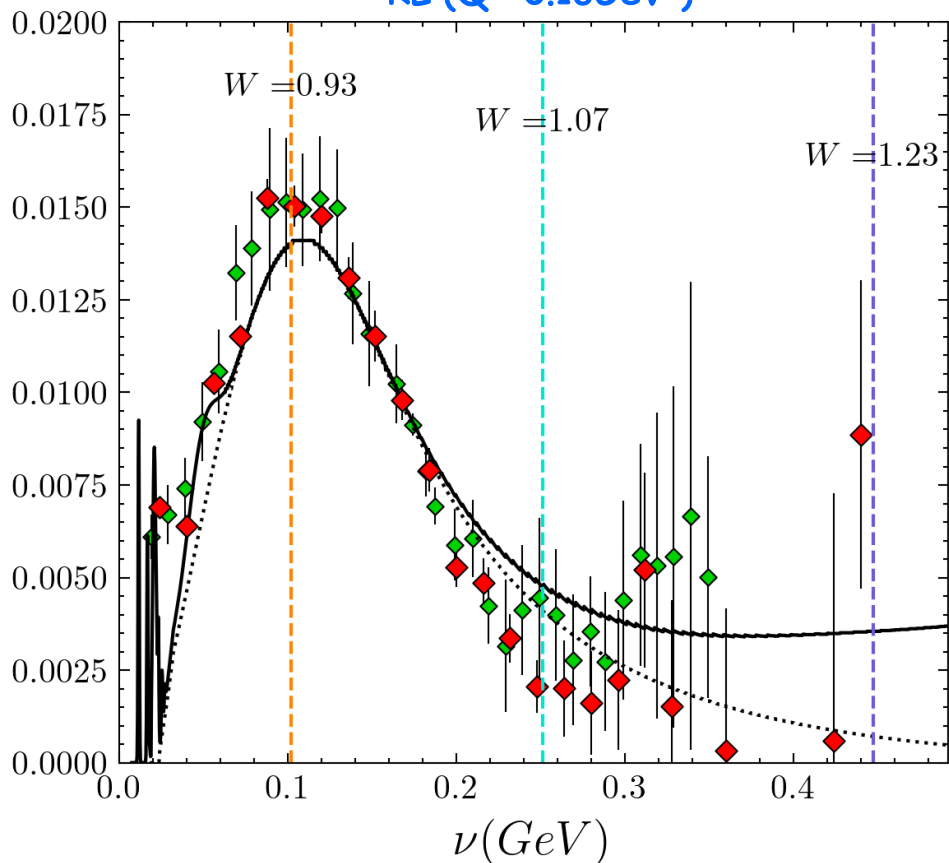




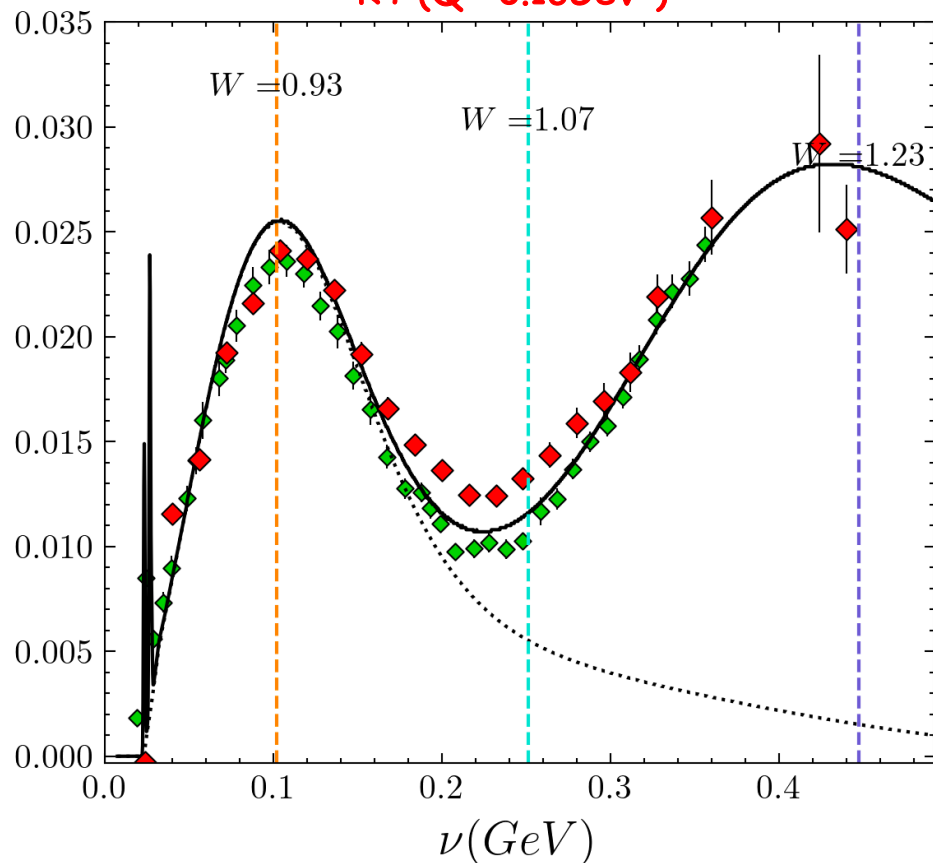
Comparison to previous extractions of $R_L R_T$ at fixed Q^2 (0.16 GeV^2)

- $R_L(\text{total}), R_T(\text{total})$ Christy-Bodek Fit
- ⋯ $R_L(\text{QE}), R_T(\text{QE+TE})$ Christy-Bodek Fit
- ◆ R_L, R_T this analysis
- ◆ R_L, R_T Barreau

$R_L (Q^2=0.16\text{GeV}^2)$



$R_T (Q^2=0.16\text{GeV}^2)$



Conclusion

- The 18 R_L and R_T extractions cover a large kinematic range. The values are in excellent agreement with the Christy-Bodek Universal fit to all cross-section values. The universal fit covers an even larger kinematic range.
- The R_L and R_T measurements as well as the universal fit provide a simple way to validate electron and neutrino MC generators over the entire kinematic range of interest.
- Good agreement in the QE region with nuclear theory for 3 values of q . Predictions for all other values of q not yet available.

In Supplemental Materials we will provide:

- Tables of the extracted R_L and R_T .
- Tables of the Christy-Bodek Universal fit values for R_L and R_T . The contributions of nuclear excitations, QE, transverse enhancement and inelastic scattering will be listed separately.
- Code for the Christy-Bodek Universal fit.

Thank you!

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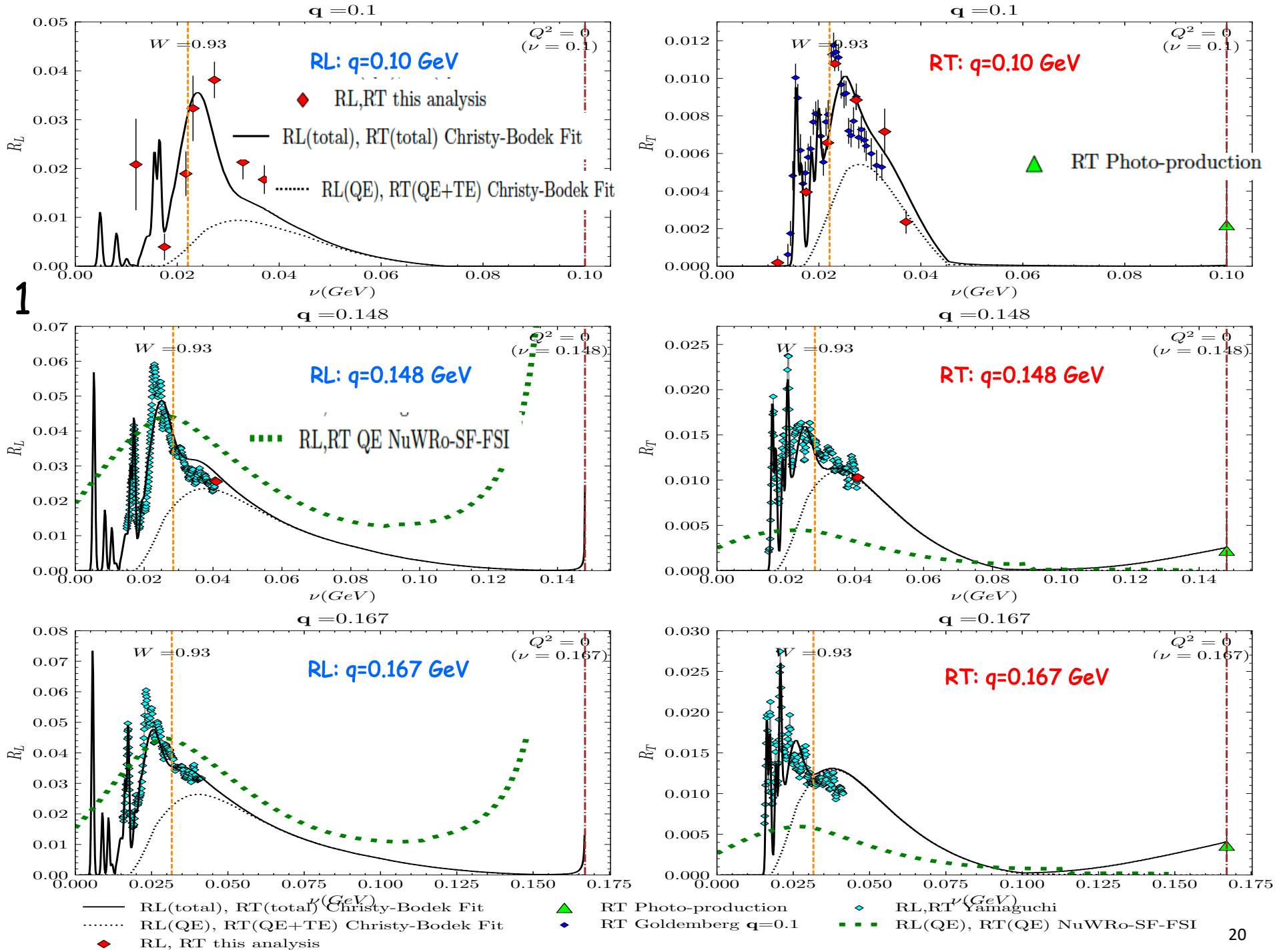
Backup slides

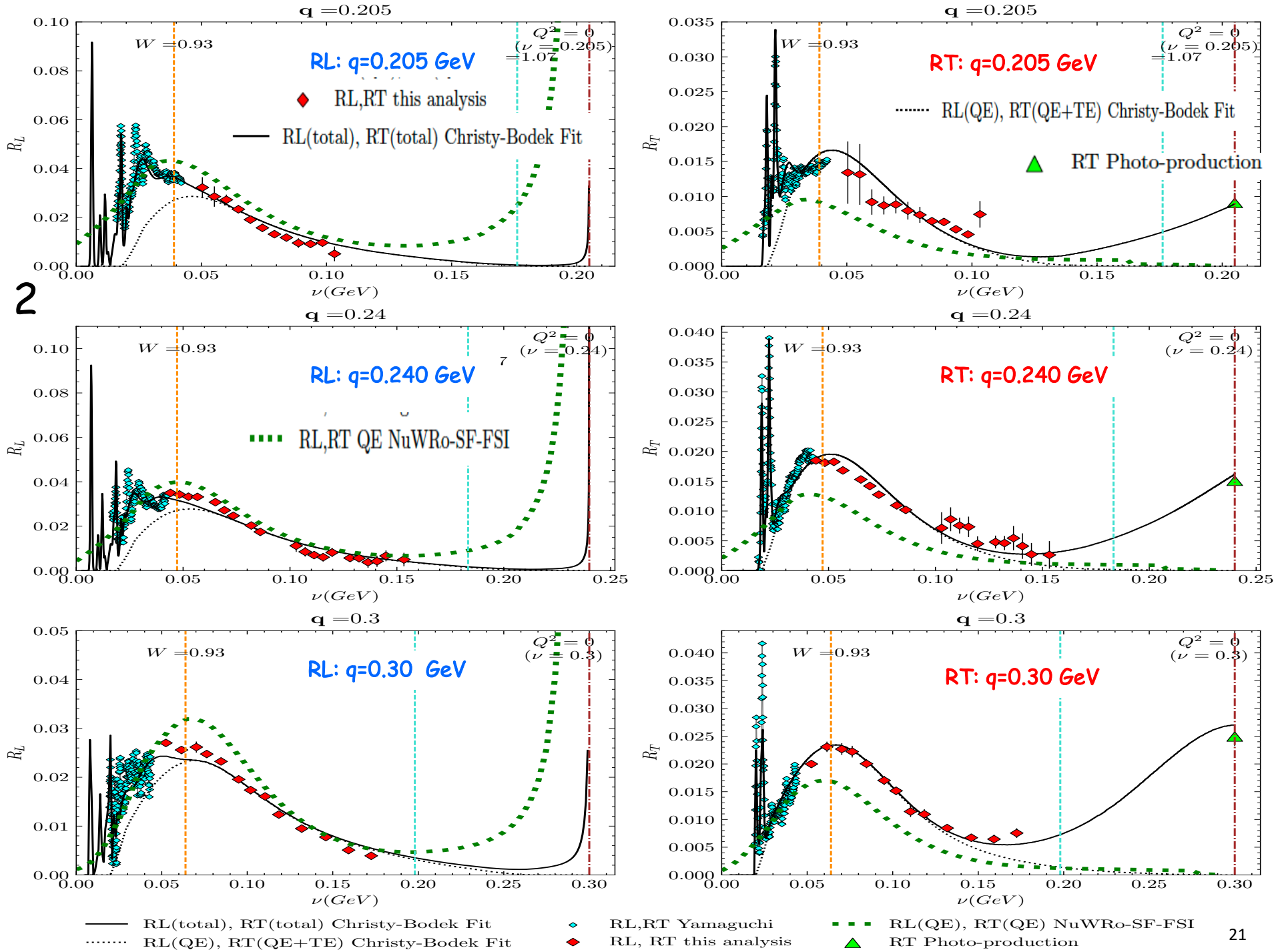
Data sets and normalizations:

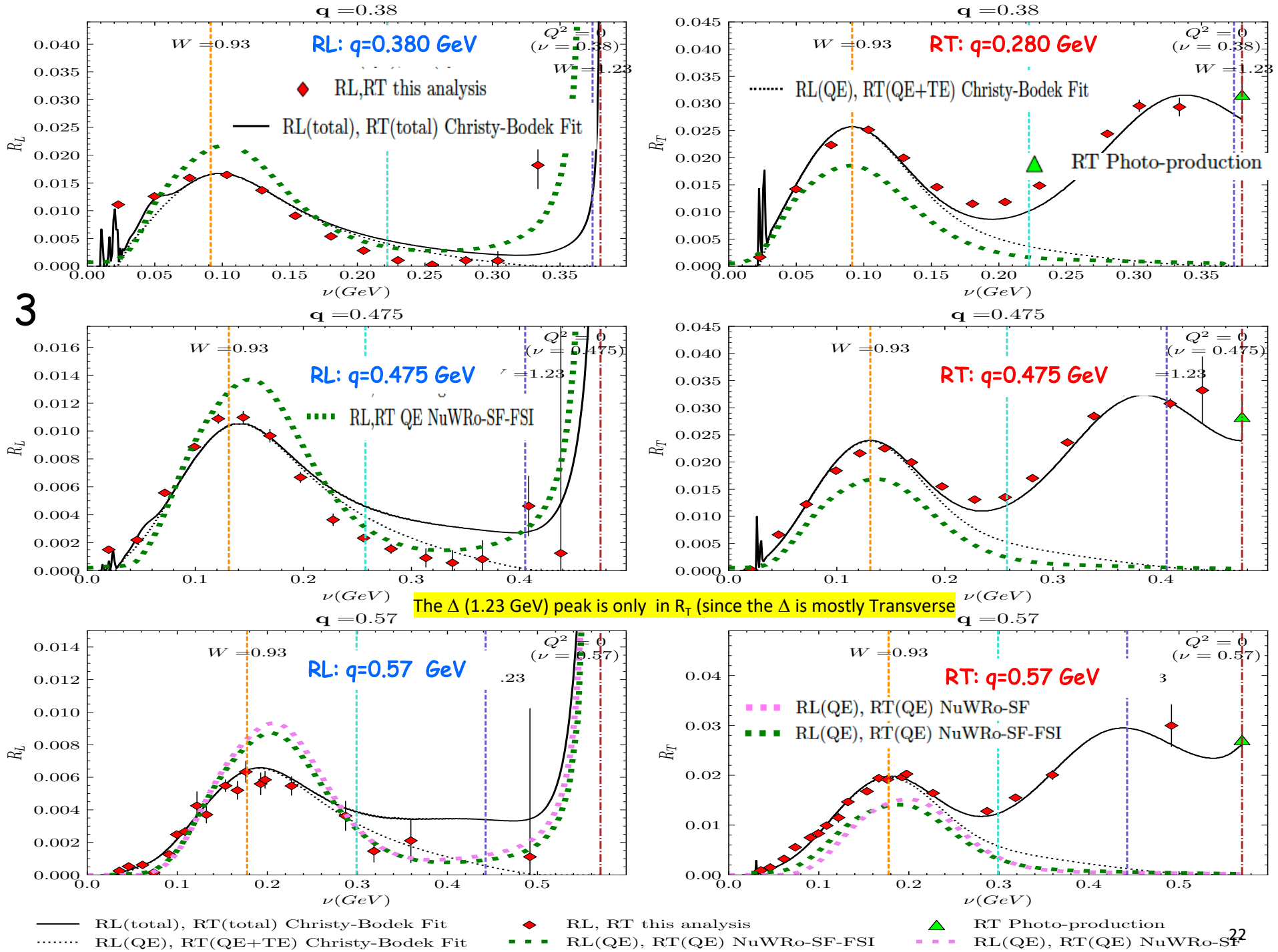
	Data Set	Normalization	Error
1	Barreau83 [11–13]	0.9919	0.0024
2	O’Connell87 [26]	0.9787	0.0086
3	Sealock89 [27]	1.0315	0.0048
4	Baran88 [28]	0.9924	0.0046
5	Bagdasaryan88 [29]	0.9878	0.0083
6	Dai19 [30]	1.0108	0.0053
7	Arrington96 [31]	0.9743	0.0133
8	Day93 [32]	1.0071	0.0033
9	Arrington99 [33]	0.9888	0.0034
10	Gaskell21 [34, 35]	0.9934	0.0051
11	Whitney74 [36, 37]	1.0149	0.0153
12	E04-001-2005 [14–16]	0.9981	0.0067
13	E04-001-2007 [14–16]	1.0029	0.0070
14	Gomez74 [38, 39]	1.0125	0.0149
15	Fomin10 [40, 41]	1.0046	0.0031
16	Yamaguchi71 [7]	1.0019	0.0029
17	Ryan84 [8] (180 ⁰)	1.0517	0.0130
18	Czyk63 [42]	1.0	0.1
19	Bounin63 [43, 44]	1.15	0.23
21	Spamer70 [7, 45]	1.2	0.1
22	Goldemberg64 [24] (180 ⁰)	1.1	0.1
23	Deforest65 [25] (180 ⁰)	0.85	0.1
	Donnelly68 [46, 47] (not used)	-	-
	Zeller73 [48] (not used)	-	-

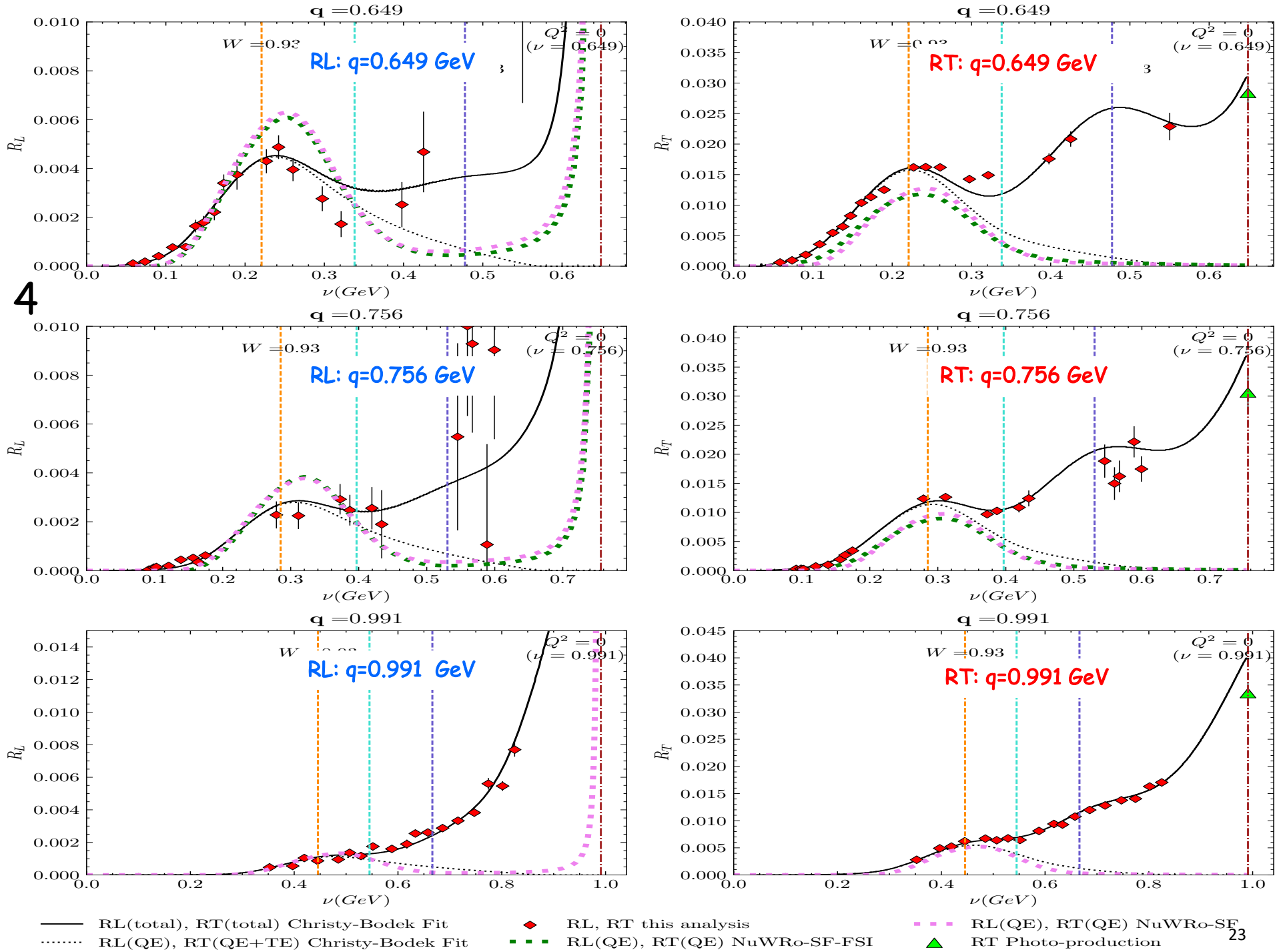
Comments

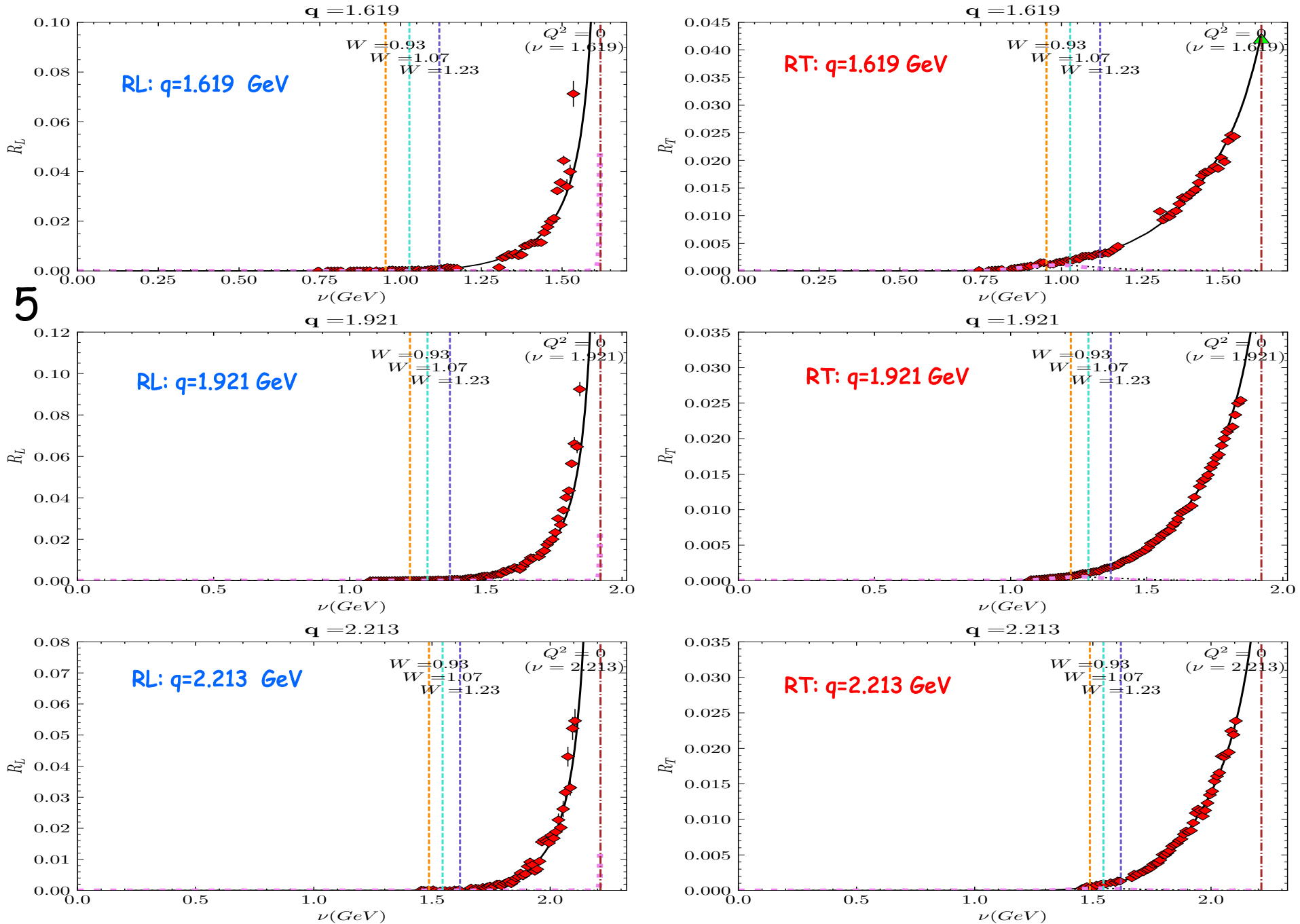
- The extracted R_L and R_T are in good agreement with the Christy-Bodek Universal Fit. The fit can be used for validation of MC generators over a more extended range of q and ν .
- The Christy-Bodek fit includes **Longitudinal Quenching** at low q and **Transverse Enhancement** at intermediate q . It also includes nuclear excitations
- The Δ (1.23 GeV) peak is only seen in R_T (since the Δ is mostly Transverse).
- For fixed q the maximum value of ν is $\nu=q$ (where R_T can also be extracted from photoproduction data).
- Validation of Neutrino Generator NuWro in electron model: NuWro only models QE with a spectral function, and adds Final State Interaction (FSI)
- NuWro (spectral function) requires FSI for better agreement with the data.
- However, even with FSI for $q < 0.3$ GeV NuWro overestimates R_L (**requires Longitudinal Quenching**)
- NuWro underestimates R_T (in neutrino mode it has Transverse Enhancement/MEC, but not in electron mode).
- Nuclear excitations are not included
- Comparisons with GENIE (in electron mode) will be available shortly.











— RL(total), RT(total) Christy-Bodek Fit ◆ RL, RT this analysis ▲ RT Photo-production
 - - - RL(QE), RT(QE+TE) Christy-Bodek Fit - - - RL(QE), RT(QE) NuWRo-SF

