

# Anomalous Precession Frequency Analysis at Fermilab Muon g-2 Experiment

DPF-PHENO 2024

May 15 2024

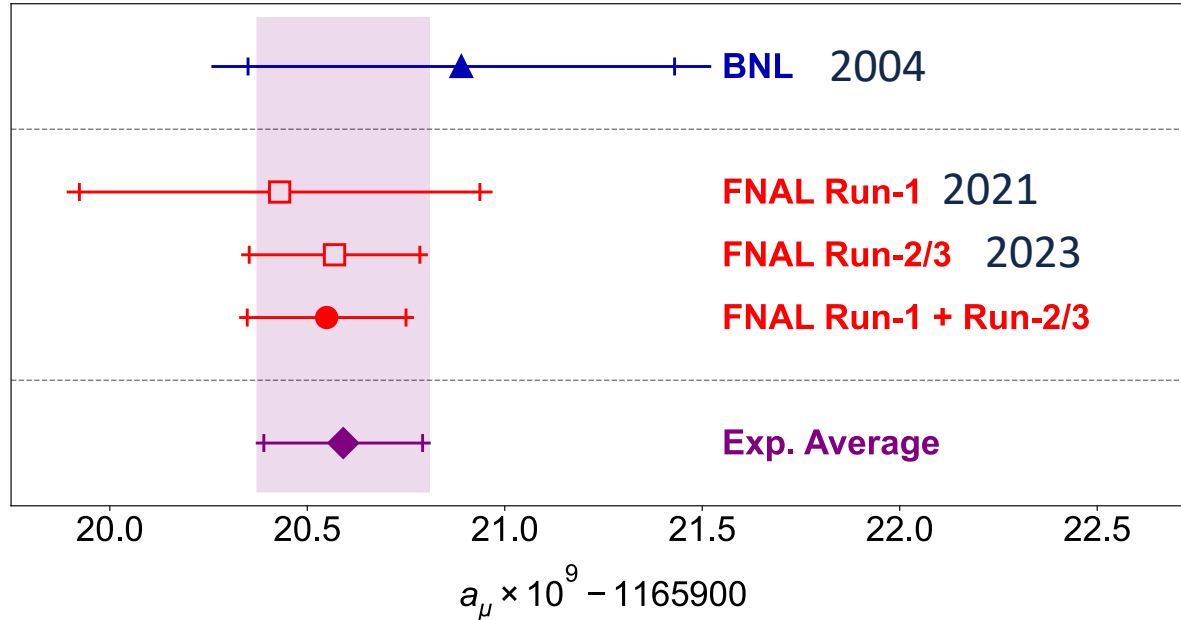
Murong Cheng

On behalf of the Muon g-2 collaboration



UNIVERSITY OF  
**ILLINOIS**  
URBANA-CHAMPAIGN

# Muon $g-2$ Measurement Result

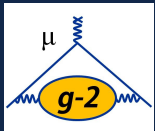


$$a_\mu(\text{FNAL; Run-2/3}) = 0.00116592057(25) \text{ [215 ppb]}$$

$$a_\mu(\text{FNAL}) = 0.00116592055(24) \text{ [203 ppb]}$$

$$a_\mu(\text{Exp}) = 0.00116592059(22) \text{ [190 ppb]}$$

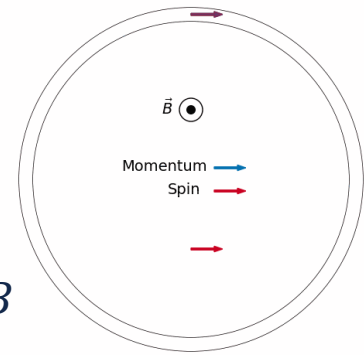
- Run-2/3 result is consistent with Run-1 and BNL
- Uncertainty reduced by more than x2 compared to Run-1



# Muon g-2 Experiment at Fermilab

To measure the magnetic anomaly:  $\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{m}$

Spin frequency:  $\omega_s = -\frac{e}{\gamma m} B (1 + \gamma a_\mu)$       Cyclotron frequency:  $\omega_c = -\frac{e}{\gamma m} B$



Highly polarized muon from  $\pi^+ \rightarrow \mu^+ + \nu_\mu$

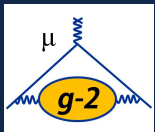
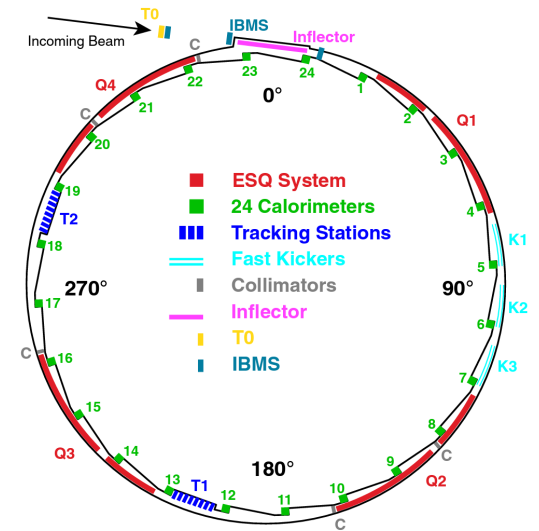
Inflector: creates an almost field-free region in the storage ring avoid muon deflected by magnetic field

Kicker: kick the muon beam to the central orbit by producing another magnetic field

Electrostatic quadrupole system: vertically focus muon beam

Tracker: measure beam oscillation

Calorimeter: measure time and energy of decayed positrons

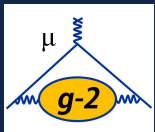


# Measuring $a_\mu$

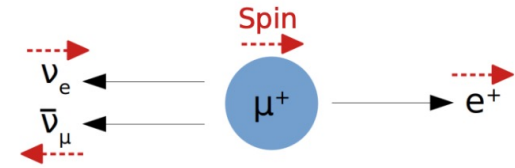
$$\omega_a = a_\mu \frac{eB}{m} \quad \xrightarrow{B = \frac{\omega'_p}{2\mu_p}} \quad a_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \frac{m_\mu}{m_e} \frac{\mu_p}{\mu_e} \frac{g_e}{2}$$

Real world equation:

$$\frac{\omega_a}{\omega_p} = \frac{\omega_a^m}{\omega_p^m} \times \frac{\overbrace{1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}}^{\text{Corrections from Beam Dynamics}}}{\underbrace{1 + B_k + B_q}_{\text{Corrections from Magnetic Field Transient}}}$$

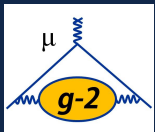
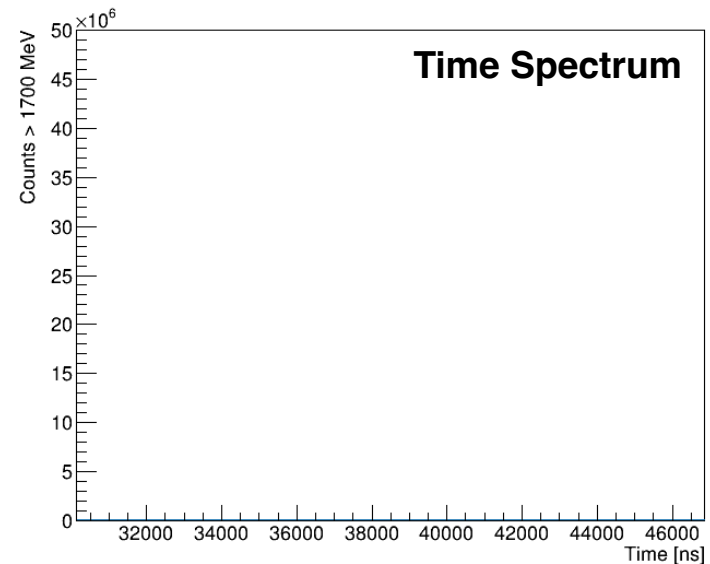
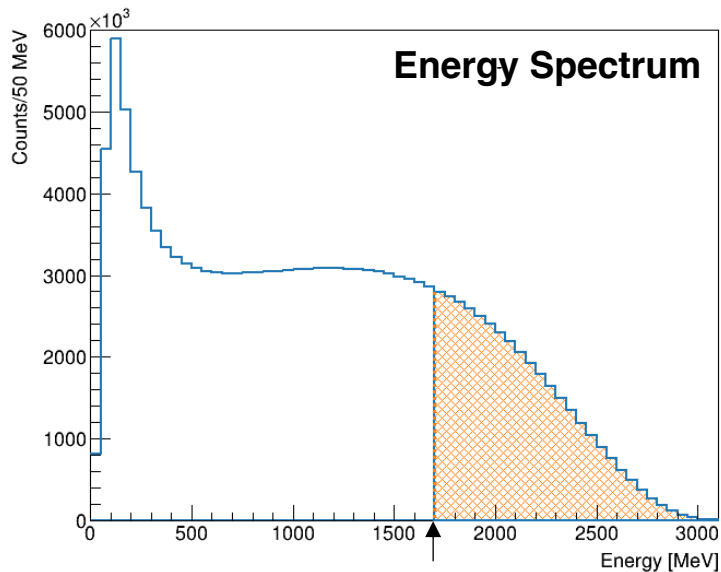


# To measure $\omega_a$

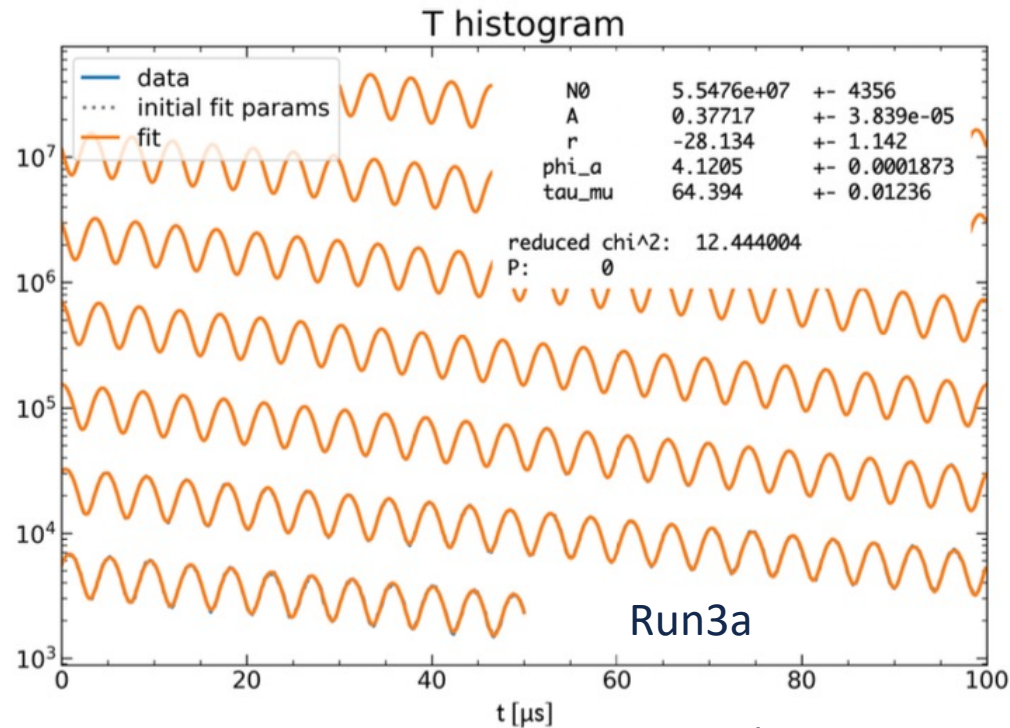


Muon decay

- Parity violation
- Positron tends to be emitted along the changing spin direction
- As the muon spin points towards & away from calorimeters, the number of high energy positrons oscillates

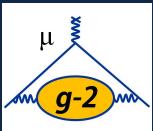


# $\omega_a$ analysis

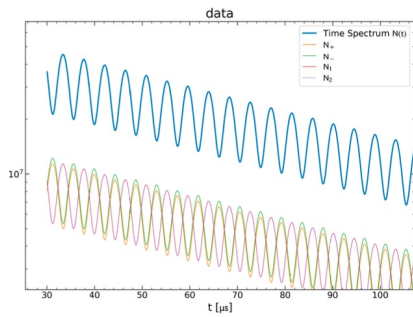


$$N(t) = N_0 \cdot e^{-\frac{t}{\tau}} \cdot [1 + A \cos(\omega_a - \phi_a)]$$

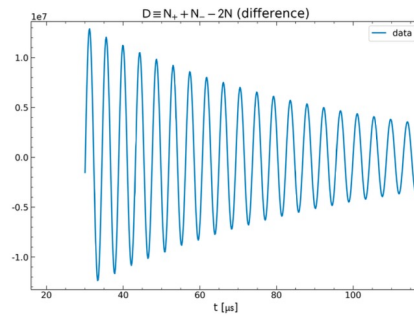
- Threshold method:
  - Highest energy positrons contain highest signals
  - Apply an energy threshold cut
  - Fit all positrons with energies above the threshold



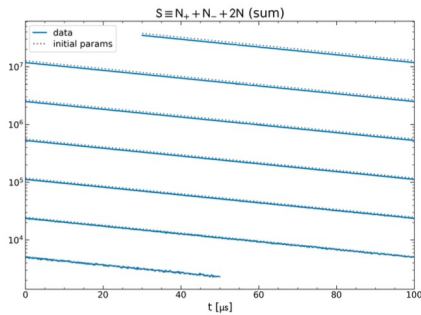
# $\omega_a$ analysis



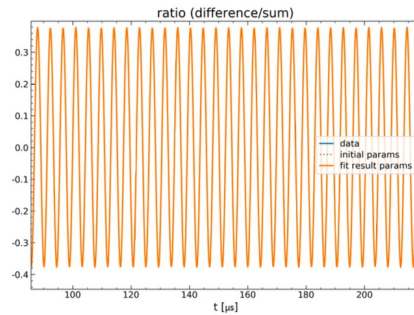
(a) R-method subsets



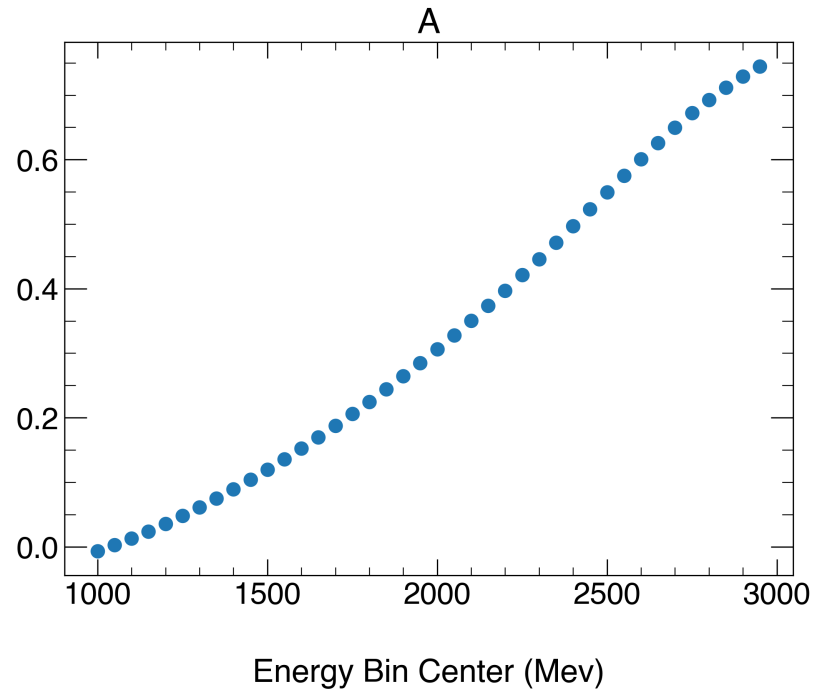
(b)  $D = N_+ + N_- - 2N$



(c)  $S = N_+ + N_- + 2N$

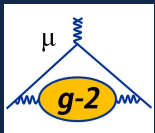


(d)  $R = D/S$



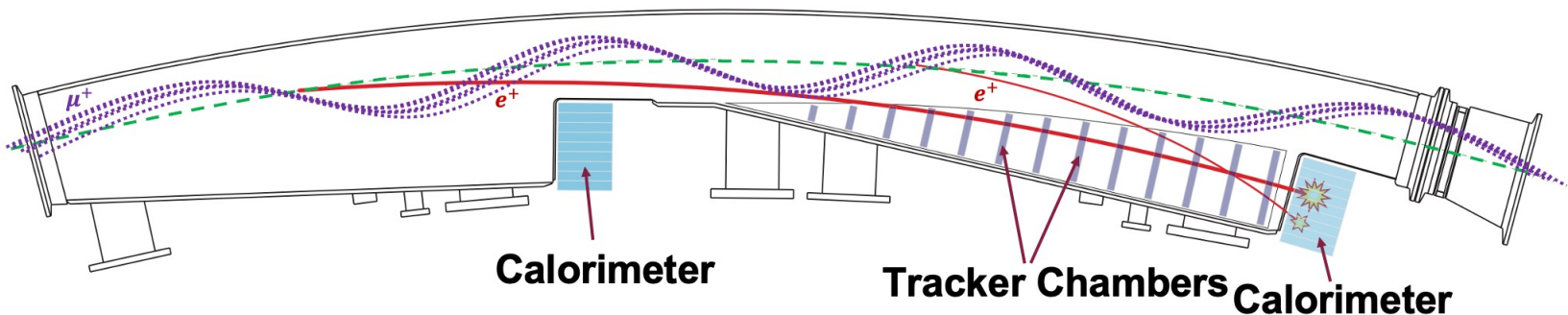
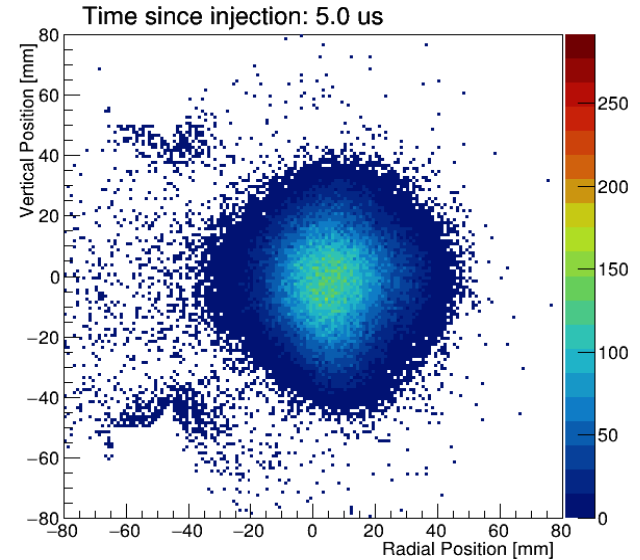
- **Ratio method:**
  - Enhance the g-2 oscillation in the numerator and remove it in the denominator.

- **Asymmetry-weighted method:**
  - Energy-weighted version of Threshold method



# Coherent Betatron Oscillation (CBO)

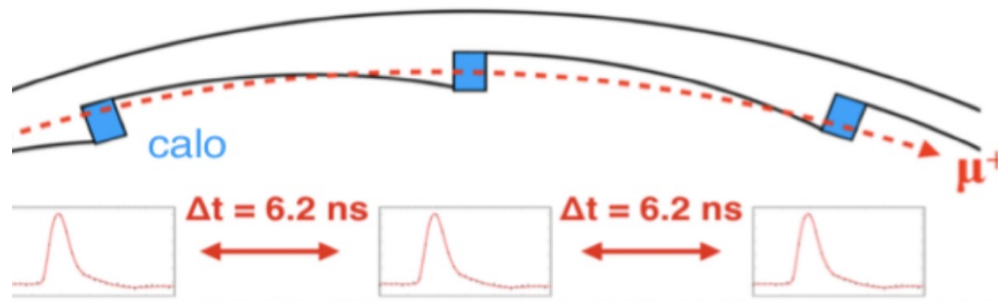
- Muons fail to stay in the perfect circular orbit, oscillate in radial direction
- $\omega_{CBO} = \omega_c - \omega_x$  cyclotron frequency – horizontal beam oscillation frequency
- $N_{CBO}(t) = 1 + A_{CBO} \cdot e^{-t/\tau_{CBO}} \cdot \cos(\omega_{CBO}t + \phi_{CBO})$





# Muon loss

- Some muons are lost from the storage ring before they decay during the measurement window
- When passing through consecutive calorimeters, it has a characteristic energy of around 170 MeV and a characteristic time of flight of around 6.15 ns.



Combining CBO and ML, got 9-par fit function:

$$N(t)_{9\text{-par}} = N_0 \cdot e^{-\frac{t}{\tau_\mu}} [1 + A \cos(\omega_a + \phi_0)] \cdot \Lambda(t) \cdot N_{CBO}(t)$$

Muon loss



# $\omega_a$ analysis - Fit function

$$N(t) = N_0 e^{-t/\tau_\mu} [1 + A \cdot A_{cbo}(t) \cos(\omega_a t) - (\phi_0 + \phi_{cbo}(t))] \\ \cdot \Lambda(t) \cdot N_{cbo}(t) \cdot N_{2cbo}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_n(t)$$

$$\phi_{cbo}(t) = A_{cbo-\phi} (e^{-t/\tau_{cbo}} + C_{cbo}) \cos(\omega_{cbo}(t)t - \phi_{cbo-\phi})$$

$$A_{cbo}(t) = 1 + A_{cbo-A} (e^{-t/\tau_{cbo}} + C_{cbo}) \cos(\omega_{cbo}(t)t - \phi_{cbo-A})$$

$$N_{cbo}(t) = 1 + A_{cbo} (e^{-t/\tau_{cbo}} + C_{cbo}) \cos(\omega_{cbo}(t)t - \phi_{cbo})$$

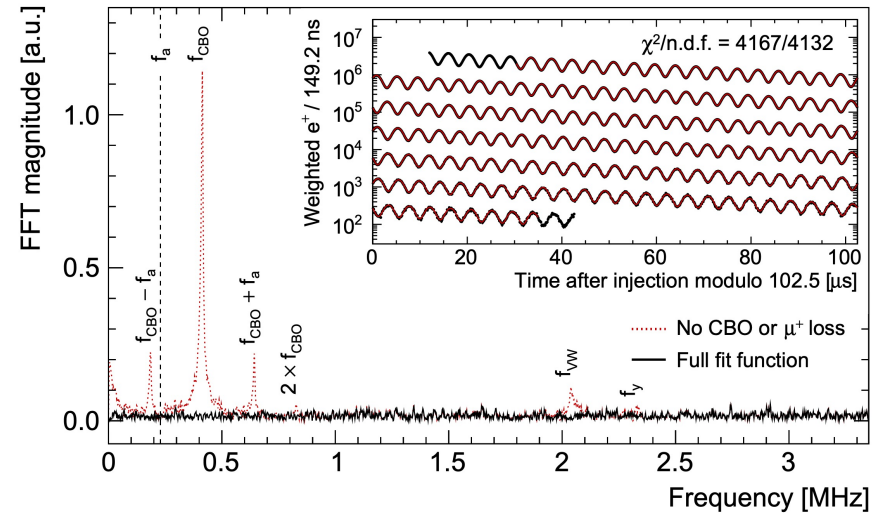
$$N_{2cbo}(t) = 1 + A_{2cbo-N} (e^{-2t/\tau_{cbo}} + C_{cbo}) \cos(2\omega_{cbo}(t)t - \phi_{2cbo-N})$$

$$\omega_{cbo}(t)t = \omega_{cbo}t + A_d e^{-t/\tau_d}$$

$$N_{VW}(t) = 1 + A_{VW} e^{-t/\tau_{VW}} \cos(\omega_{VW}t - \phi_{VW})$$

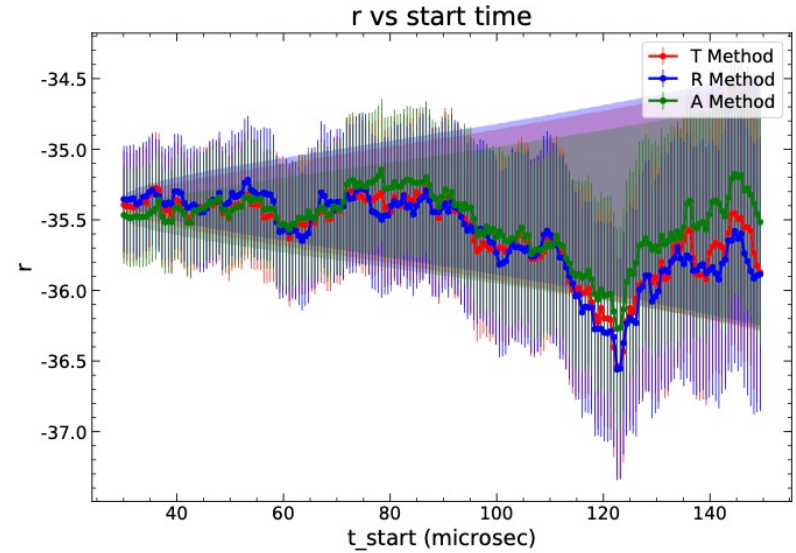
$$N_y(t) = 1 + A_y e^{-t/\tau_y} \cos(\omega_y t - \phi_y)$$

$$N_n(t) = 1 + A_n e^{-t/\tau_n} \cos(\omega_n t - \phi_n)$$

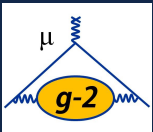
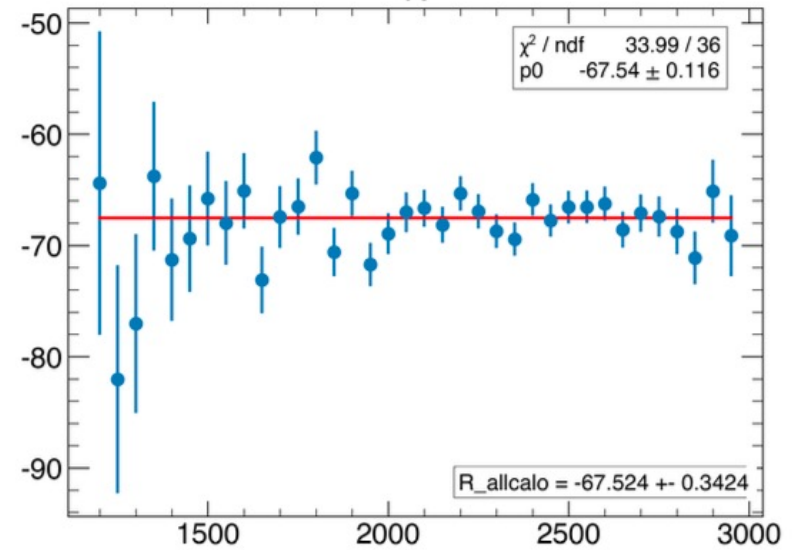
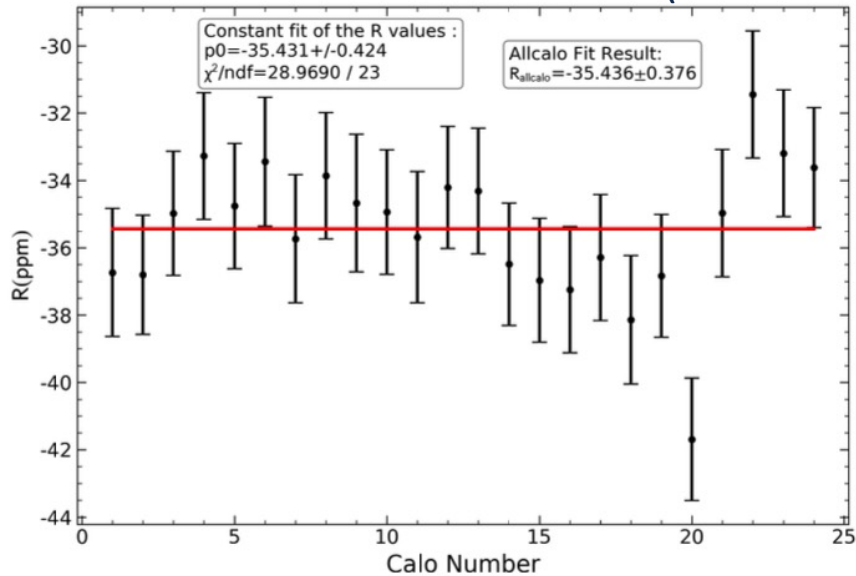


# Consistency checks

- Fit start time scans
- Positron energy binned scans
- Per calorimeters scans

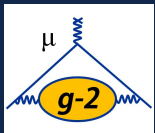
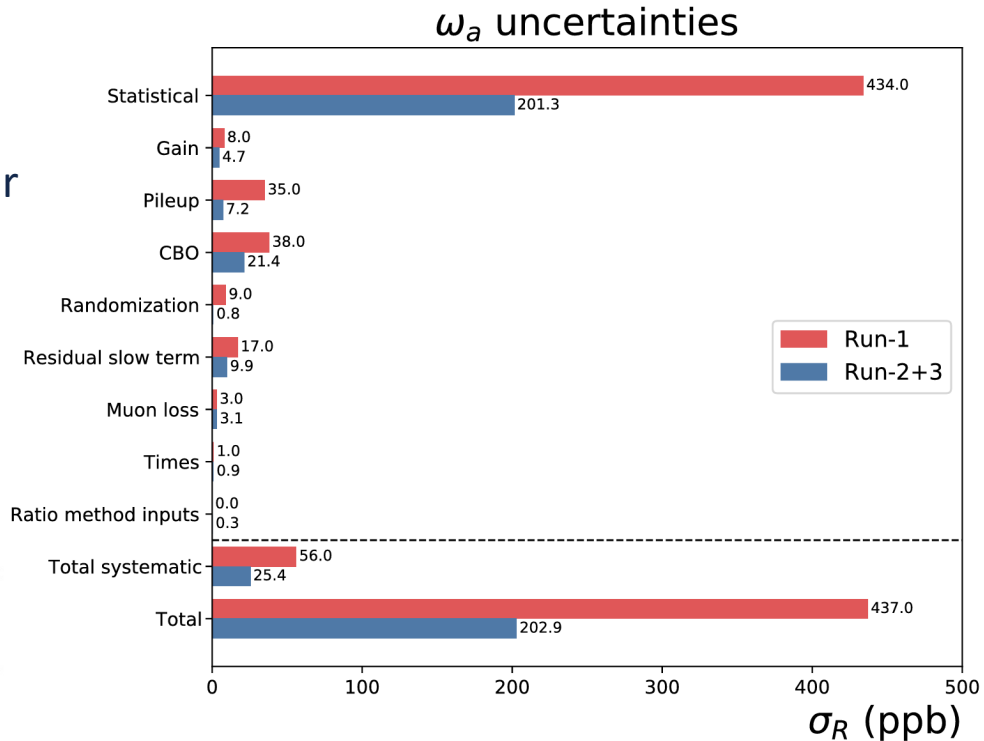
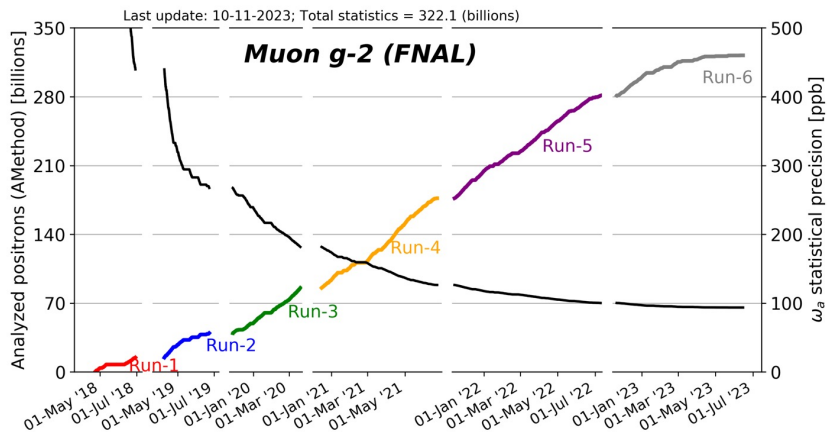


Run-2 R value (blinded  $\omega_a$ ) consistency checks

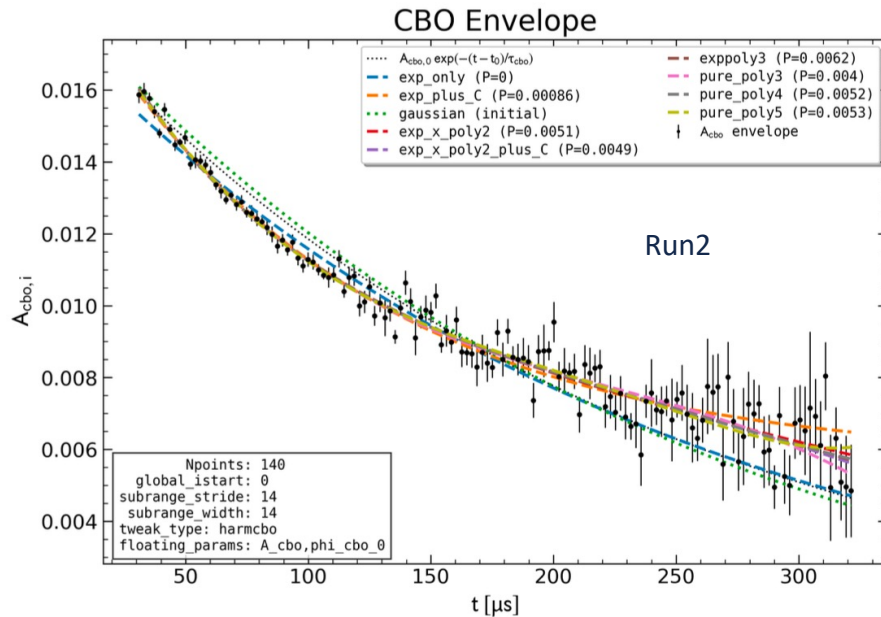


# Statistical and systematic uncertainties

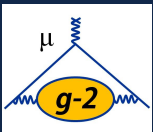
- Comparing to Run-1, Run-2/3 statistical and systematic uncertainties are both reduced by factor of 2.2
- Statistical uncertainty dominates the error
- Run-4/5/6 aims to lead to another factor of 2 improvement in statistical precision and reduce the systematic uncertainties



# CBO decoherence envelope study

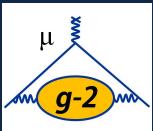
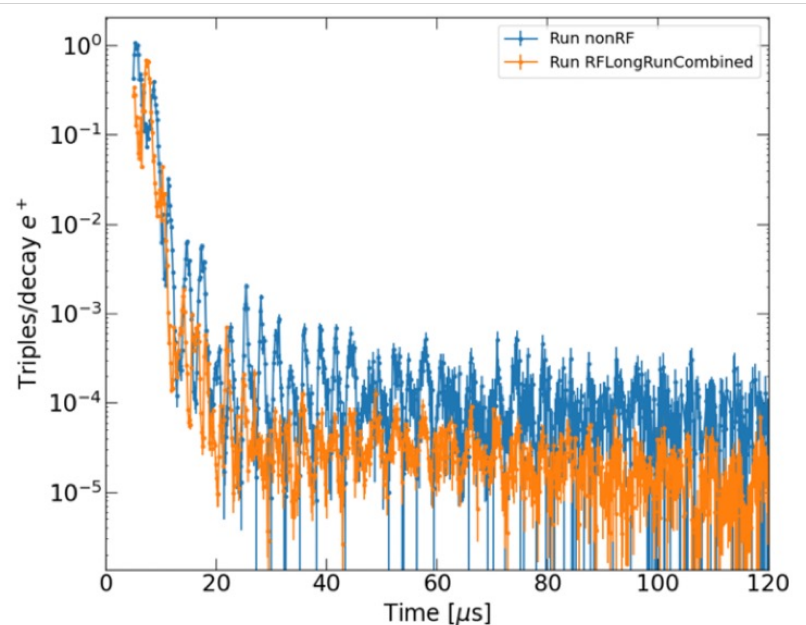
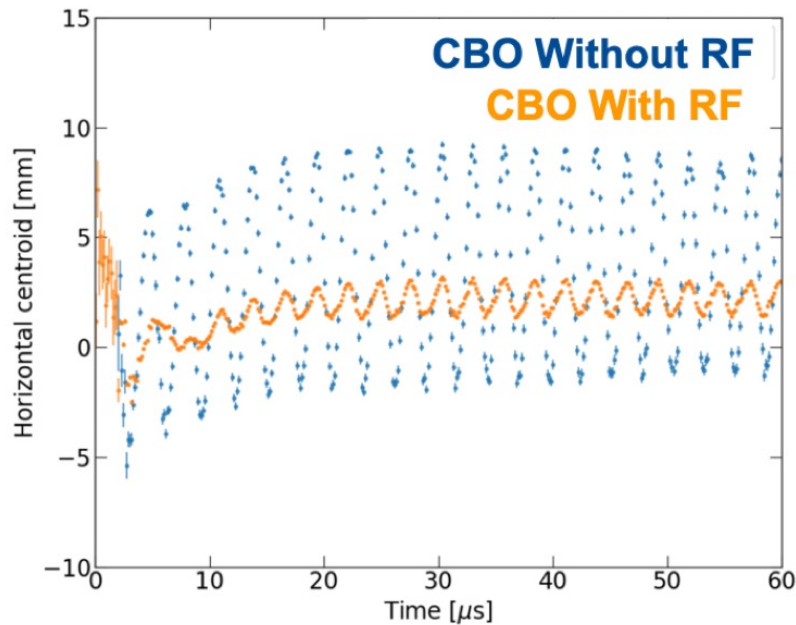


- Perform sliding window fit and show how different models describe the CBO envelope
- Run-2/3: Estimate the R value shift to be used as uncertainty on the CBO decoherence envelope

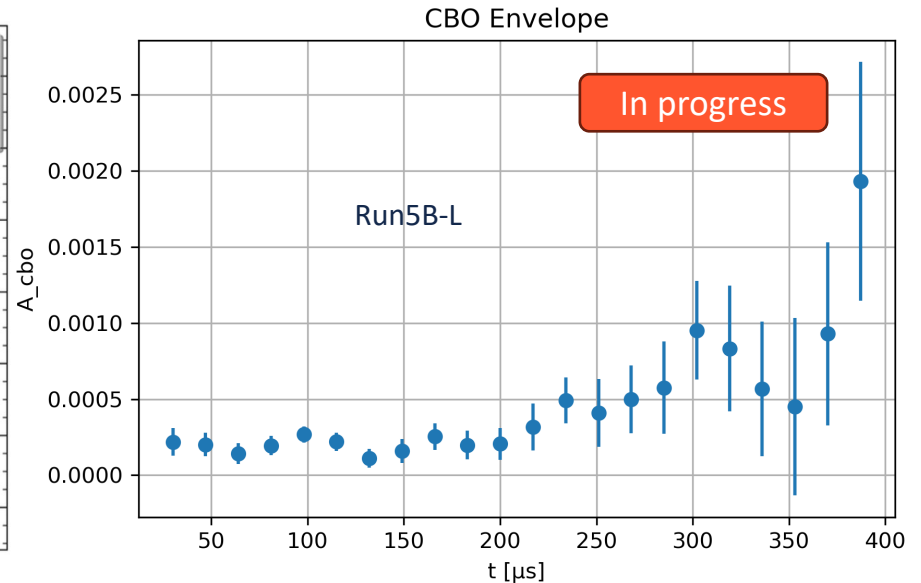
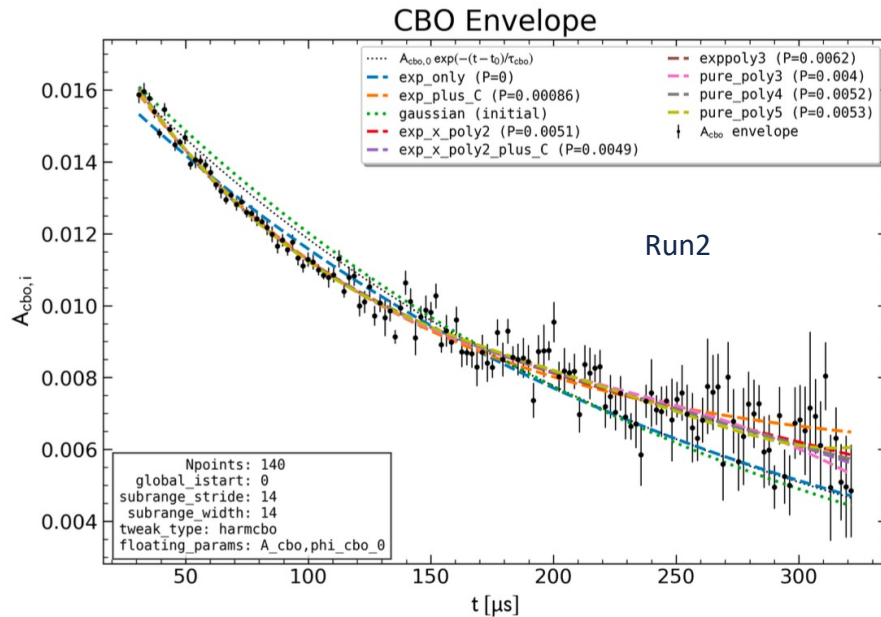


# Radio Frequency (RF) system

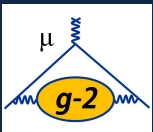
- Starting from Run-5, the RF system is employed
  - A harmonically varying horizontal dipole electric field is applied to the beam out of phase with the CBO.
  - Reduce CBO amplitude by factor of 5.
  - Also reduce muon loss by factor of 4.



# CBO decoherence envelope study



- Perform sliding window fit and show how different models describe the CBO envelope
- Run-2/3: Estimate the R shift to be used as uncertainty on the CBO decoherence envelope
- CBO amplitude is much reduced in Run-5/6, the similar studies are being performed. Beam dynamics changed due to the RF system, so different CBO envelopes are under investigation.



# Thanks for listening! Questions?

## Acknowledgements

- Department of Energy (USA),
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- Science and Technology Facilities Council (UK),
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- Strong 2020 (EU),
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- National Natural Science Foundation of China,
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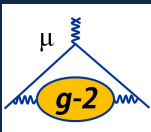


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MSIP

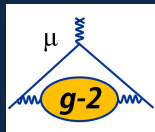


National Research  
Foundation of Korea





# Back-up

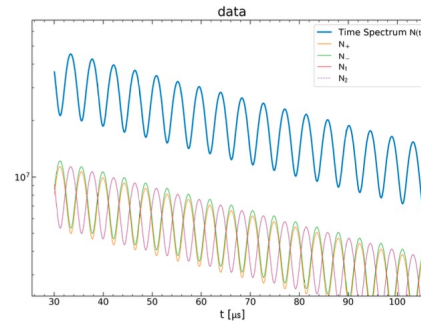


# $\omega_a$ analysis – R-method

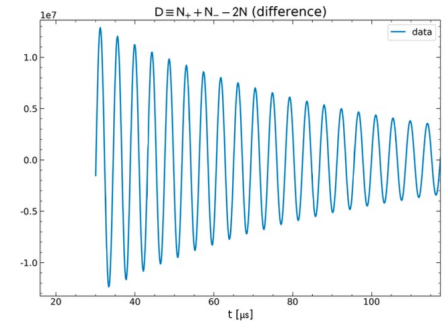
- Ratio method:
  - Decayed positron counts randomly divided into 4 histograms
  - Enhance the  $g-2$  oscillation in the numerator and remove it in the denominator.

$$R(t) = \frac{N_+(t + \frac{Ta}{2}) + N_-(t - \frac{Ta}{2}) - 2N(t)}{N_+(t + \frac{Ta}{2}) + N_-(t - \frac{Ta}{2}) + 2N(t)}$$

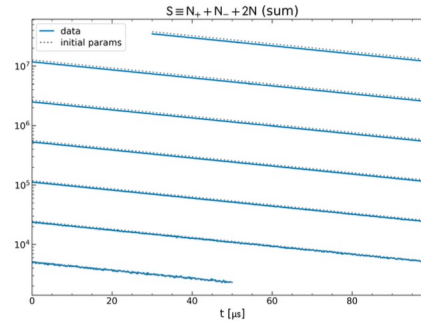
- The ratio divides out the slow effects



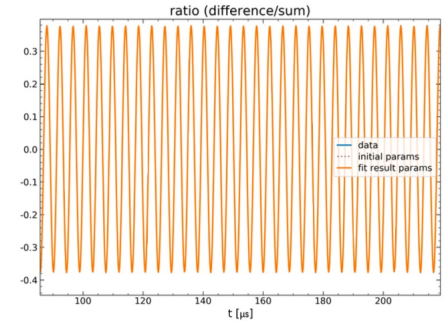
(a) R-method subsets



(b)  $D = N_+ + N_- - 2N$



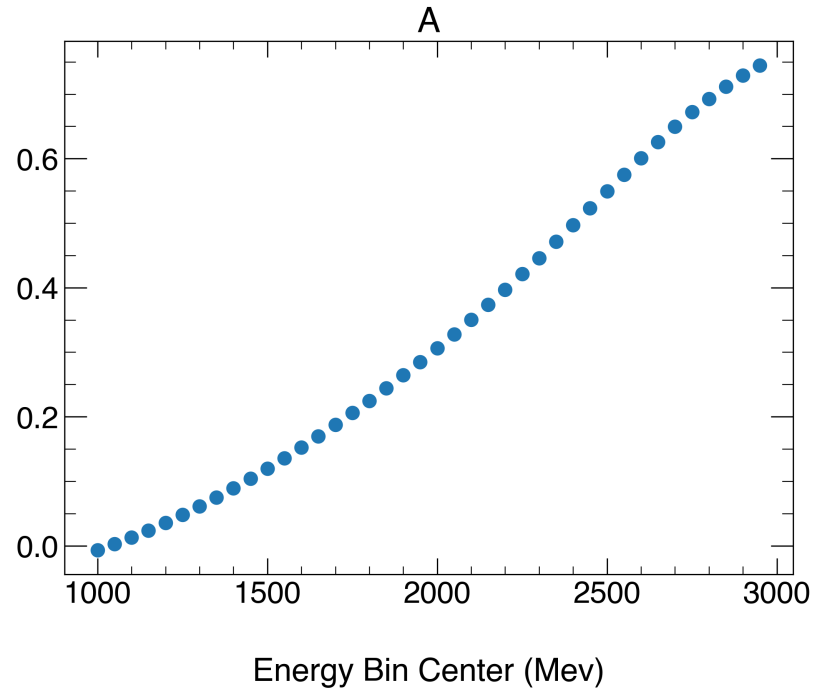
(c)  $S = N_+ + N_- + 2N$



(d)  $R = D/S$

# $\omega_a$ analysis – A-method

- Asymmetry-weighted method:
  - Energy-weighted version of Threshold method
  - T-method  $w(E) = 1, E > 1750$  MeV
  - A-method  $w(E) = A(E), 1050$  MeV  $< E < 3050$  MeV
  - Reduce the statistical uncertainty
- Asymmetry-weighted **Ratio** method:
  - Energy-weighted version of Ratio method

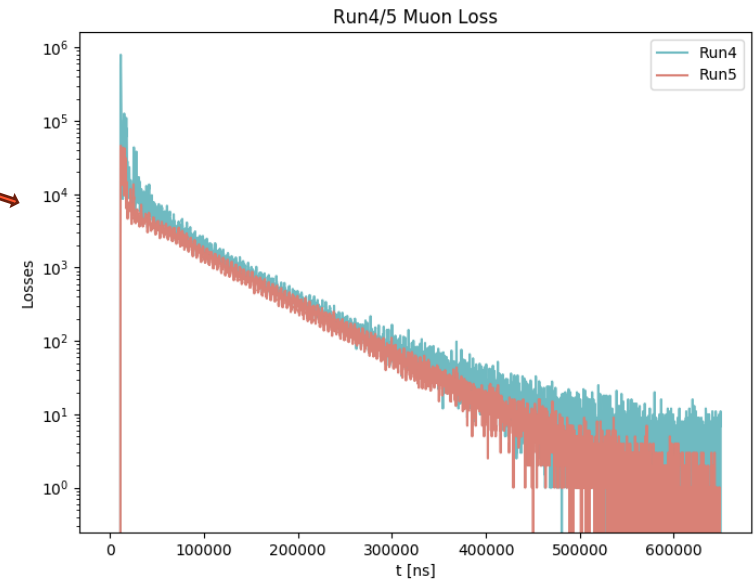
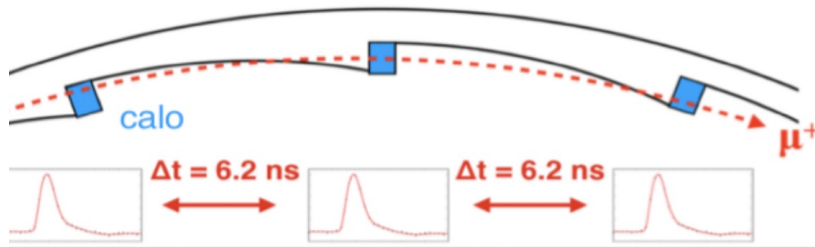


# Muon loss

- Some muons are lost from the storage ring before they decay during the measurement window
- When passing through consecutive calorimeters, it has a characteristic energy of around 170 MeV and a characteristic time of flight of around 6.15 ns.

$$\Lambda(t) = 1 - \kappa_{loss} \int_0^t dt' L(t') e^{t'/\tau_\mu}$$

detection efficiency of lost muons



Combining CBO and ML, got 9-par fit function:

$$N(t)_{9-par} = N_0 \cdot e^{-\frac{t}{\tau_\mu}} [1 + A \cos(\omega_a + \phi_0)] \cdot \Lambda(t) \cdot N_{CBO}(t)$$

# Muon loss correction

True rate of muon loss

$$\frac{dN}{dt} = -\frac{N}{\tau} - \frac{L(t)}{\epsilon}$$

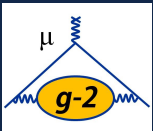
Proportionality constant,  $\epsilon$ , characterizes the above method's efficiency of detecting the lost muons

$$N(t) = N(t_0) \cdot e^{-(t-t_0)/\tau} \cdot \left[ 1 - \frac{1}{N(t_0)\epsilon} \int_{t_0}^t L(t') \cdot e^{(t'-t_0)/\tau} dt' \right]$$

**Kloss**

depends only on the initial number of stored muons and the detection efficiency of lost muons

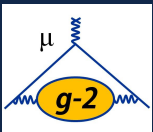
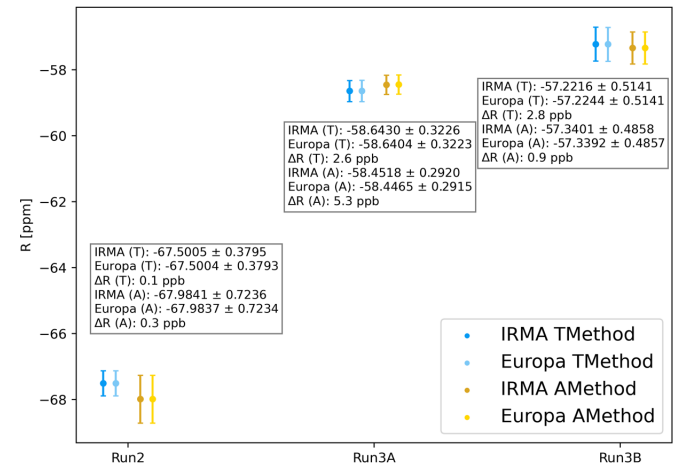
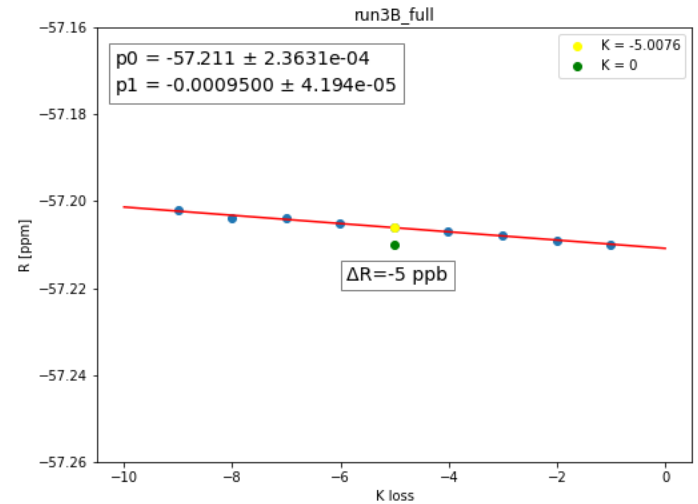
Cut criteria	Description
Coincidence level = 1.	Exclude the singlets.
Time < 15 $\mu$ s.	Exclude the first 15 $\mu$ s after the injection.
The number of hits in crystals $\geq 3$ .	Exclude non-MIP-like events.
The maximum crystal energy / cluster energy < 0.8.	Exclude non-MIP-like events.
$\Delta t_{12,23} < 5$ ns or $\Delta t_{12,23} > 7.5$ ns	Coincidence time interval cuts.
$\Delta t_{13} > 14.4$ ns	Deuteron subtraction cut.
Energy < 100 MeV or Energy > 250 MeV	MIP energy cuts.



# Muon loss systematic study

$$\Lambda(t) = 1 - \kappa_{loss} \int_0^t dt' L(t') e^{t'/\tau_\mu}$$

- Fixed  $\kappa_{loss}$ 
  - R-method fit:  $\tau_\mu$  and  $\kappa_{loss}$  fixed to the values extracted from the T-method fit. This introduces uncertainty on the R-value for the R-method  $\omega_a$  analysis.
  - Extract the R sensitivity for various  $\kappa_{loss}$  values and then multiply this sensitivity with the uncertainty on  $\kappa_{loss}$  from T-method fit results.
- Different muon loss models
  - Use the different Run-2/3 muon loss spectra from the Europa group, which have quadruple and quintuple coincidences besides triples.
  - Nominal fits were performed by using both IRMA and Europa's muon loss spectra

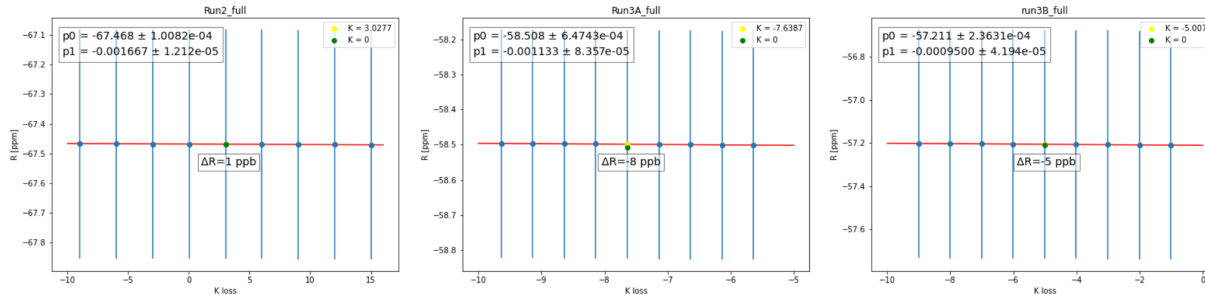


# 1-)Fixed Kloss Systematic

- Fix Kloss in R method (from T method)
- Scan over different Kloss value

Systematic Error due to fixed Kloss (seed 0)

	dR/dK	$\sigma_k$	$\delta R$ [ppb]
<b>Run2</b>	-1.667	0.318	0.530
<b>Run3A</b>	-1.113	1.785	1.987
<b>Run3B</b>	-0.950	1.744	1.657



## The sensitivity of R to the fixed Kloss value for Run2, Run3A, Run3B. (Seed 0)

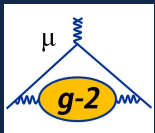
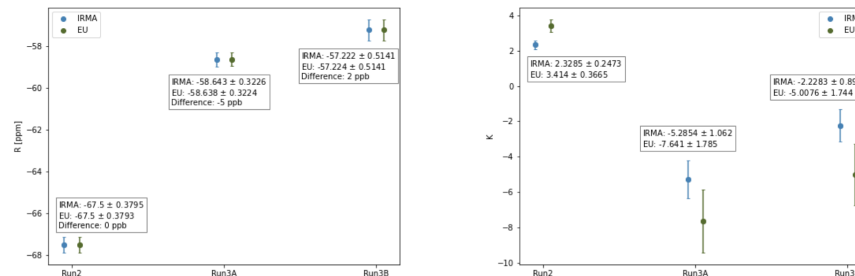
Run2: Best-fit value of Kloss from T method is 3.0277, the corresponding R method fit result R is -67.469 ppm. Turn off Kloss value, R is -67.468 ppm. Difference is 1 ppb.

Run3A: Best-fit value of Kloss from T method is -7.6387, the corresponding R method fit result R is -58.499 ppm. Turn off Kloss value, R is -58.507 ppm. Difference is -8 ppb.

Run3B: Best-fit value of Kloss from T method is -5.0076, the corresponding R method fit result R is -57.206 ppm. Turn off Kloss value, R is -57.21 ppm. Difference is -5 ppb.

# 2-)Different Models Systematic

- Kloss values are different by using two muon loss models
- R values are the same in Run2 dataset
- Small differences of R values in Run3A and Run3B



Results are from T method fit