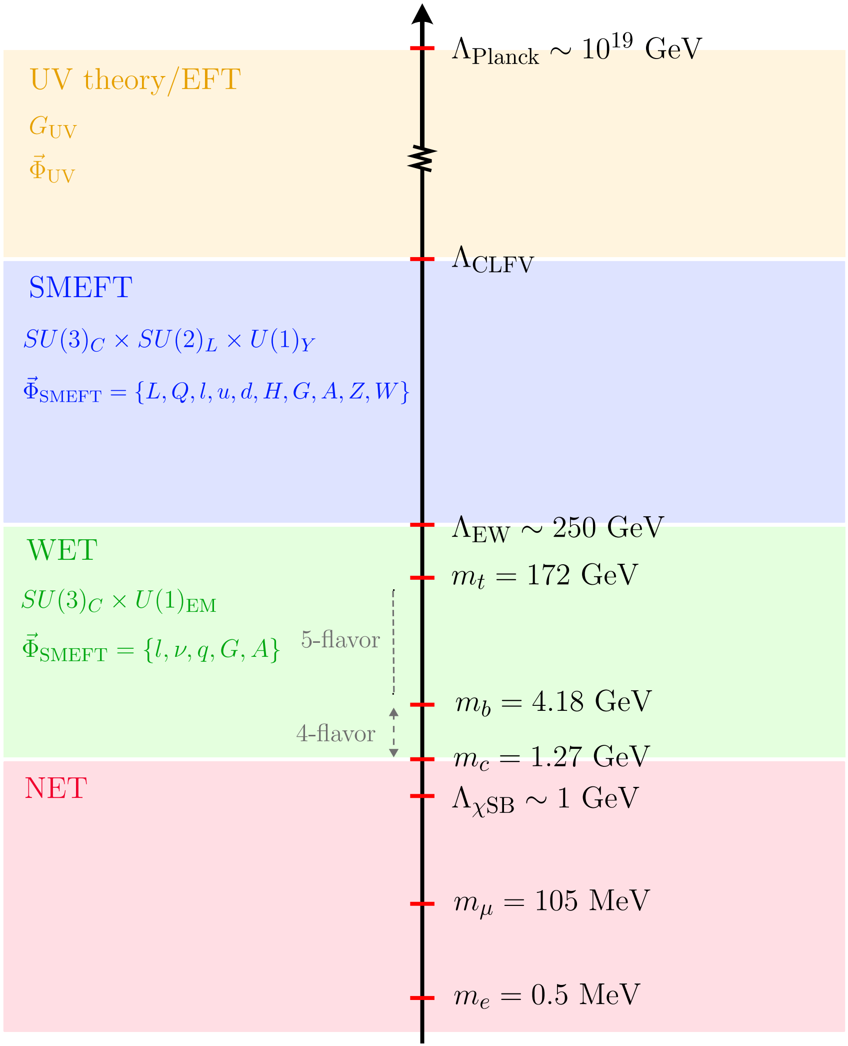
UV imprints on muon-to-electron conversion

PHENO24
May 13th, 2024

Tony Menzo
PhD candidate, University of Cincinnati



Based on 2405.xxxxx with Wick Haxton, Evan Rule, Ken McElvain, and Jure Zupan

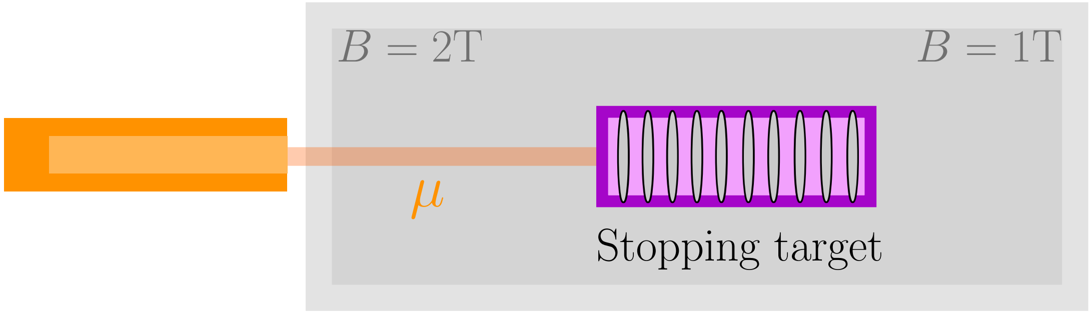
$\mu \to e$

- Limits on $\mu \to e$ conversion have improved by roughly 12 orders of magnitude over the last 75 years

Upcoming experiments:

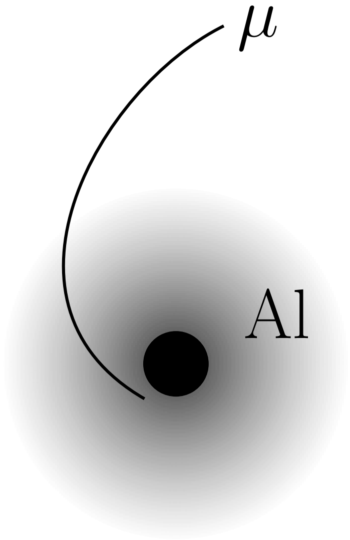
Mu2e @ Fermilab, COMET @ J-PARC - $N\,\mu^- \to N\,e^-$

Projected sensitivity: $\mathrm{CR}(\mu^- \mathrm{Al} \to e^- \mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$



~10^{10} stopped μ/sec

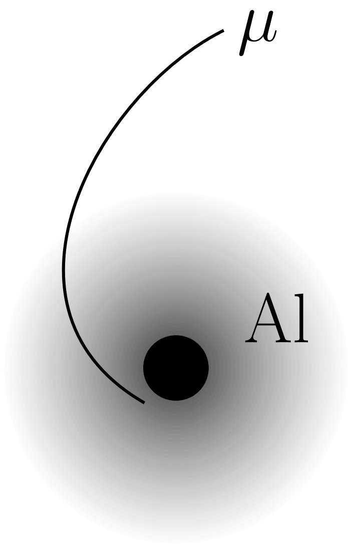
Trapped muon



Trapped muons can:

1. Decay in orbit
2. Be captured by the nucleus
3. Convert to an electron
   a) Mono-energetic electron signal ($E_e = m_\mu$)

Trapped muon



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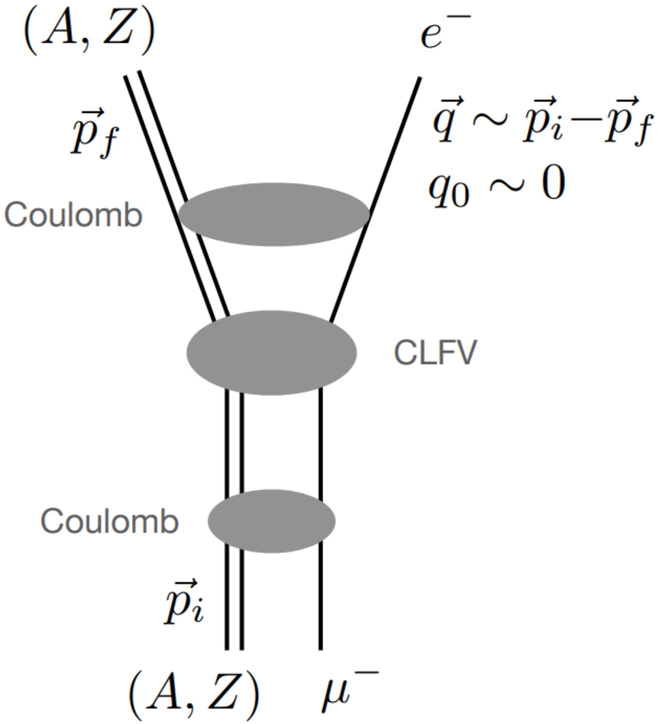
N μ⁻ → N e⁻

- **Natural hierarchy of dimensionless scales**

$$y \equiv \left(\frac{qb}{2}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$$



N μ⁻ → N e⁻

- **Natural hierarchy of dimensionless scales**

$$y \equiv \left(\frac{qb}{2}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T| \quad {}^{27}\mathrm{Al}$$

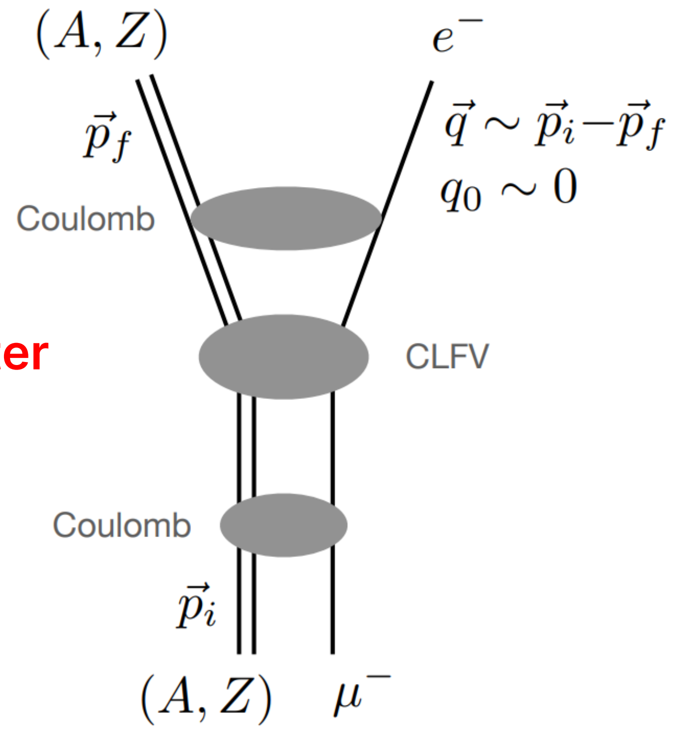
$\left(\frac{qb}{2}\right)^2 \approx 0.25$, $|\vec{v}_N| \approx 0.2$, $|\vec{v}_\mu| \approx 0.05$

Leptonic momentum transfer

$$q \approx m_\mu = 1/1.86 \text{ fm}$$

Harmonic oscillator parameter

$$b = 1.85 \text{ fm}$$



Nuclear-level effective theory

- Inconsistent truncations plague previous treatments (see 2208.07945 for more details)

- General treatment results in 11 allowed nuclear response functions

  * CP constraints on elastic scattering reduce this to 8

2208.07945

arXiv:2208.07945v3 [nucl-th] 1 Apr 2023

Nuclear-level Effective Theory of $\mu \to e$ Conversion: Formalism and Applications

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(Dated: April 4, 2023)

Over the next decade new $\mu \to e$ conversion searches at Fermilab (Mu2e) and J-PARC (COMET, DeeMe) are expected to advance limits on charged lepton flavor violation (CLFV) by more than four orders of magnitude. By considering the consequence of P and CP on elastic $\mu \to e$ conversion and the structure of possible charge and current densities, we show that rates are governed by six nuclear responses and a single scale, q/m_N, where $q \approx m_\mu$ is the momentum transferred from the leptons to the nucleus. To relate this result to microscopic formulations of CLFV, we construct in nonrelativistic effective theory (NRET) the CLFV nucleon-level interaction, pointing out the relevance of the dimensionless scales $y = (\frac{qb}{2})^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$, where b is the nuclear size, $\vec{v}_N$ and $\vec{v}_\mu$ are the nucleon and muon intrinsic velocities, and $\vec{v}_T$ is the target recoil velocity. We discuss previous work, noting the lack of a systematic treatment of the various small parameters. Because the parameter y is not small, a proper calculation of $\mu \to e$ conversion requires a full multipole expansion of the nuclear response functions, an apparently daunting task with Coulomb-distorted electron partial waves. We demonstrate that the multipole expansion can be carried out to high precision by introducing a simplifying local momentum $q_{\rm eff}$ for the electron. Previous work has been limited to simple charge or spin interactions, thereby treating the nucleus effectively as a point particle. We show that such formulations are not compatible with the general form of the $\mu \to e$ conversion rate, failing to generate three of the six allowed nuclear response functions. The inclusion of the nucleon velocity $\vec{v}_N$ yields an NRET with 16 operators and a rate of the general form. Consequently, in the current discovery era for CLFV, it provides the most sensible starting point for experimental analysis, defining what can and cannot be determined about CLFV from the highly exclusive process of $\mu \to e$ conversion. Finally, we expand the NRET operator basis to account for the effects of $\vec{v}_\mu$, associated with the muon's lower component, generating corrections to the CLFV coefficients of the point-nucleus response functions. Using advanced shell-model methods, we compute $\mu \to e$ conversion rates for a series of experimental targets, deriving bounds on the coefficients of the CLFV operators. These calculations are the first to include a general basis of CLFV operators, full evaluation of the associated nuclear response functions, and an accurate treatment of electron and muon Coulomb effects. We discuss target selection as an experimental "knob" that can be turned to probe the microscopic origins of CLFV. We describe two types of coherence that enhance certain CLFV operators and selection rules that blind elastic $\mu \to e$ conversion to others. We discuss the matching of the NRET onto higher level effective field theories, such as those constructed at the light quark level, noting opportunities to build on existing work in direct detection of dark matter. We discuss the relation of $\mu \to e$ conversion to $\mu \to e + \gamma$ and $\mu \to 3e$, showing how MEG II and Mu3e results will complement those of Mu2e and COMET. Finally we describe a accompanying script – in Mathematica and Python versions – that can be used to compute $\mu \to e$ conversion rates in various nuclear targets for the full set of NRET operators.

I. INTRODUCTION

Muon-to-electron conversion, in which a muon bound to a nucleus converts to a mono-energetic outgoing electron, occurs at an observable level only if there are new sources of flavor violation, beyond those responsible for neutrino mixing [1–4]. It has yond the standard model [5–7]. This has motivated a series of experimental advances that, in sum, have improved limits on $\mu \to e$ conversion rates by ≈12 orders of magnitude over the past 75 years [8].

The experimental attributes of $\mu \to e$ conversion are quite attractive. Intense muon beams exist, with rates on target of $\approx 10^{11}$/s expected in the

Nuclear-level effective theory

- **CLFV'ing single-nucleon operators**

$$i\hat{q} = \frac{i\vec{q}}{|\vec{q}|}, \qquad \vec{\sigma}_L, \qquad \vec{\sigma}_N, \qquad \vec{v}_N, \qquad \vec{v}_\mu.$$

$$\begin{aligned}
\mathcal{O}_1 &= 1_L\ 1_N, & \mathcal{O}'_2 &= 1_L\ i\hat{q}\cdot\vec{v}_N, \\
\mathcal{O}_3 &= 1_L\ i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], & \mathcal{O}_4 &= \vec{\sigma}_L\cdot\vec{\sigma}_N, \\
\mathcal{O}_5 &= \vec{\sigma}_L\cdot(i\hat{q}\times\vec{v}_N), & \mathcal{O}_6 &= i\hat{q}\cdot\vec{\sigma}_L\ i\hat{q}\cdot\vec{\sigma}_N, \\
\mathcal{O}_7 &= 1_L\ \vec{v}_N\cdot\vec{\sigma}_N, & \mathcal{O}_8 &= \vec{\sigma}_L\cdot\vec{v}_N, \\
\mathcal{O}_9 &= \vec{\sigma}_L\cdot(i\hat{q}\times\vec{\sigma}_N), & \mathcal{O}_{10} &= 1_L\ i\hat{q}\cdot\vec{\sigma}_N, \\
\mathcal{O}_{11} &= i\hat{q}\cdot\vec{\sigma}_L\ 1_N, & \mathcal{O}_{12} &= \vec{\sigma}_L\cdot[\vec{v}_N\times\vec{\sigma}_N], \\
\mathcal{O}'_{13} &= \vec{\sigma}_L\cdot(i\hat{q}\times[\vec{v}_N\times\vec{\sigma}_N]), & \mathcal{O}_{14} &= i\hat{q}\cdot\vec{\sigma}_L\ \vec{v}_N\cdot\vec{\sigma}_N, \\
\mathcal{O}_{15} &= i\hat{q}\cdot\vec{\sigma}_L\ i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], & \mathcal{O}'_{16} &= i\hat{q}\cdot\vec{\sigma}_L\ i\hat{q}\cdot\vec{v}_N.
\end{aligned}$$

Non-relativistic effective theory:

$$\mathcal{L}_{\text{eff}}^{\text{NRET}} = \sum_{\tau=0,1}\sum_{i=1}^{16} c_i^\tau \mathcal{O}_i t^\tau + \cdots ,$$

The rate

- **The conversion rate:** $\mathcal{O}(y)$

$$\Gamma(\mu \to e) = \frac{1}{2\pi} \frac{q_{\text{eff}}^2}{1 + q/M_T} \left|\phi_{1s}^{Z_{\text{eff}}}(\vec{0})\right|^2 \sum_{\tau,\tau'} \Big[R_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\text{eff}}) + R_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\text{eff}}) + R_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\text{eff}}) \Big],$$

$$\begin{aligned} R_{MM}^{\tau\tau'} &= c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*}, \\ R_{\Sigma'\Sigma'}^{\tau\tau'} &= c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*}, \\ R_{\Sigma''\Sigma''}^{\tau\tau'} &= \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*}, \end{aligned}$$

The rate

- The conversion rate: $\mathcal{O}(\vec{v}_N)$

*All nuclear responses allowed by symmetries are generated

$$\omega = \frac{G_F^2}{\pi} \frac{q_{\rm eff}^2}{1+\frac{q}{M_T}} |\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ \Big[\tilde{R}_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff})\Big]$$
$$+ \frac{q_{\rm eff}^2}{m_N^2} \Big[\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} W_{\Phi''\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} W_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Delta}^{\tau\tau'} W_{\Delta\Delta}^{\tau\tau'}(q_{\rm eff})\Big]$$
$$- \frac{2q_{\rm eff}}{m_N} \Big[\tilde{R}_{\Phi''M}^{\tau\tau'} W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'} W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff})\Big] \Bigg\} \quad (59)$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$
$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$
$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

$$\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} = \tilde{c}_3^\tau \tilde{c}_3^{\tau'*} + (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)(\tilde{c}_{12}^{\tau'*} - \tilde{c}_{15}^{\tau'*})$$
$$\tilde{R}_{\Phi''M}^{\tau\tau'} = \mathrm{Re}[\tilde{c}_3^\tau \tilde{c}_1^{\tau'*} - (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\, \tilde{c}_{11}^{\tau'*}]$$
$$\tilde{R}_{\Delta\Delta}^{\tau\tau'} = \tilde{c}_5^\tau \tilde{c}_5^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_8^{\tau'*}$$
$$\tilde{R}_{\Delta\Sigma'}^{\tau\tau'} = \mathrm{Re}[\tilde{c}_5^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_9^{\tau'*}]$$
$$\tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} = \tilde{c}_{12}^\tau \tilde{c}_{12}^{\tau'*} + \tilde{c}_{13}^\tau \tilde{c}_{13}^{\tau'*}$$

Nuclear response hierarchy

$$W_{MM}^{00} \sim \mathcal{O}(A^2) \gg \left\{ W_{\Sigma'\Sigma'}^{00}, W_{\Sigma''\Sigma''}^{00}, \frac{q_{\text{eff}}^2}{m_N^2} W_{\Phi''\Phi''}^{00} \right\} \gg \left\{ \frac{q_{\text{eff}}^2}{m_N^2} W_{\Delta\Delta}^{00}, \frac{q_{\text{eff}}^2}{m_N^2} W_{\tilde{\Phi}'\tilde{\Phi}'}^{00} \right\}.$$

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*Can become semi-coherent in some nuclei with half-filled shells (e.g. Al, Cu)

$$W_M^{00} \gg q/m_N \; W_{\Phi'' M}^{00} \gg q^2/m_N^2 \; W_{\Phi''}^{00}$$

Nuclear response hierarchy

$$W^{00}_{MM} \sim \mathcal{O}(A^2) \gg \left\{ W^{00}_{\Sigma'\Sigma'}, W^{00}_{\Sigma''\Sigma''}, \frac{q^2_{\text{eff}}}{m_N^2} W^{00}_{\Phi''\Phi''} \right\} \gg \left\{ \frac{q^2_{\text{eff}}}{m_N^2} W^{00}_{\Delta\Delta}, \frac{q^2_{\text{eff}}}{m_N^2} W^{00}_{\tilde{\Phi}'\tilde{\Phi}'} \right\}.$$

*Can become semi-coherent in some nuclei with half-filled shells (e.g. Al, Cu)

$$W^{00}_{M} \gg q/m_N \; W^{00}_{\Phi'' M} \gg q^2/m_N^2 \; W^{00}_{\Phi''}$$

Where does the UV physics sit?

$$\Gamma \propto \sum R \times W, \qquad R(c_i)$$

EFT tower

- WET

$$\mathcal{L}_{\text{eff}}^{\text{WET}} = \sum_{a,d} \hat{\mathcal{C}}_a^{(d)} \mathcal{Q}_a^{(d)},$$

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha q)\,,$$

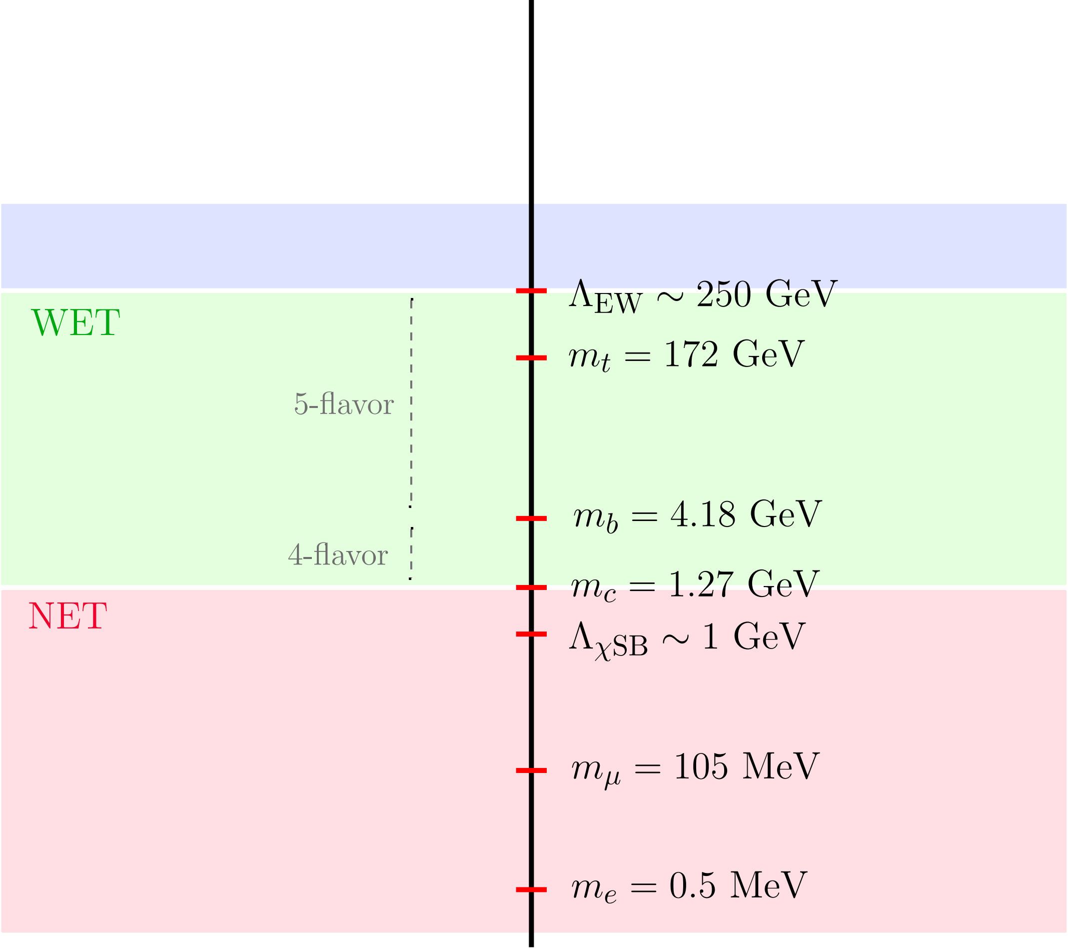
$$\mathcal{Q}_{3,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,.$$

$$\mathcal{Q}_{5,q}^{(6)} = (\bar{e}\mu)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}q)\,,$$

$$\mathcal{Q}_{7,q}^{(6)} = (\bar{e}\mu)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}i\gamma_5 q)\,,$$

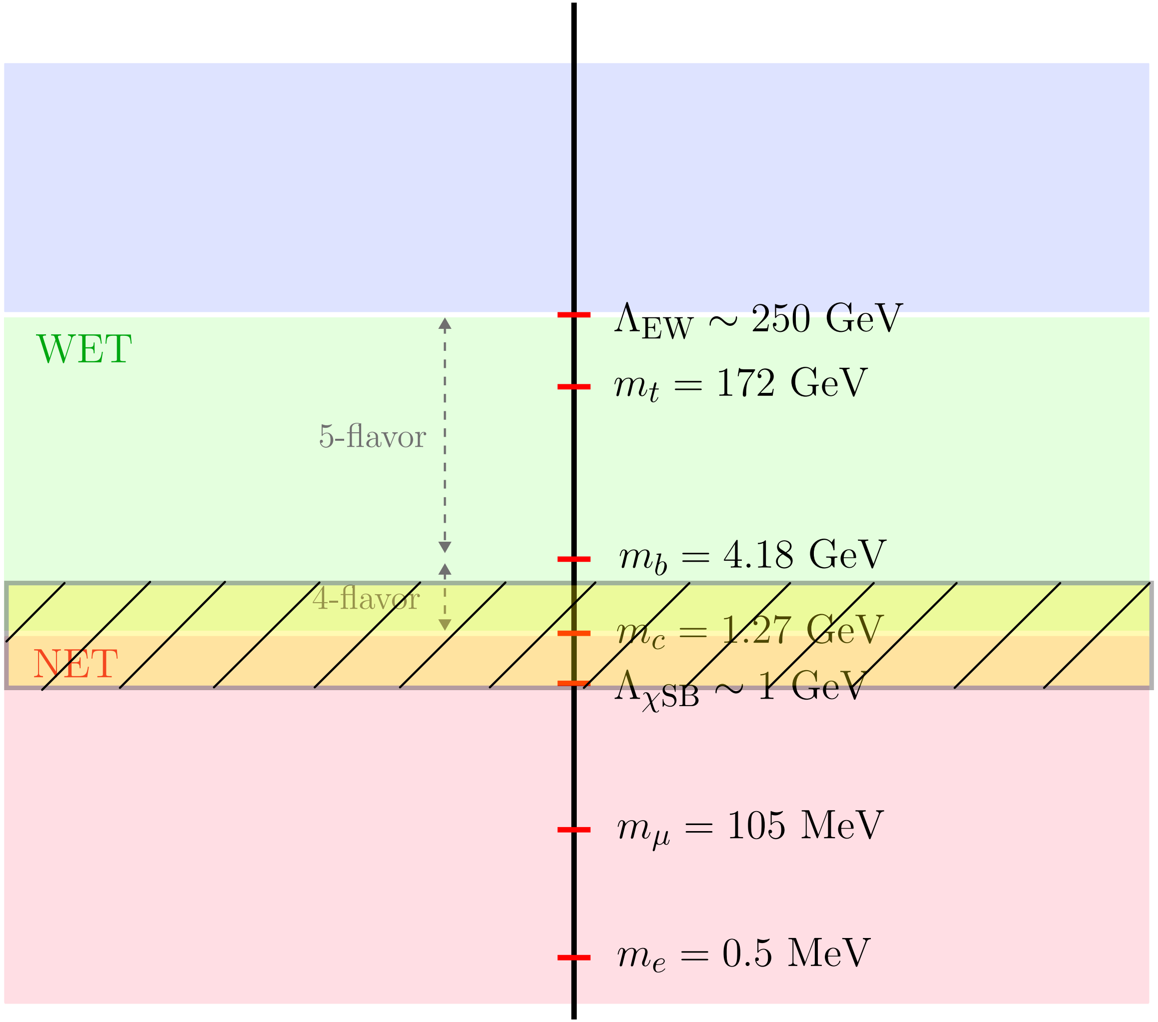
$$\mathcal{Q}_{9,q}^{(6)} = (\bar{e}\sigma^{\alpha\beta}\mu)(\bar{q}\sigma_{\alpha\beta}q)\,, \qquad \mathcal{Q}_{10,q}^{(6)} = (\bar{e}i\sigma^{\alpha\beta}\gamma_5\mu)(\bar{q}\sigma_{\alpha\beta}q)\,.$$

* We work to dim-7



EFT tower

- **Matching between WET and NRET (hadronization)**



EFT tower

- **Matching between WET and NRET (hadronization)**
- **Parameterize with nuclear form factors**

$$\langle N'|\bar{q}\gamma^\mu q|N\rangle = \bar{u}'_N\Big[F_1^{q/N}(q^2)\gamma^\mu - \frac{i}{2m_N}F_2^{q/N}(q^2)\sigma^{\mu\nu}q_\nu\Big]u_N\,,$$

$$\langle N'|\bar{q}\gamma^\mu\gamma_5 q|N\rangle = \bar{u}'_N\Big[F_A^{q/N}(q^2)\gamma^\mu\gamma_5 - \frac{1}{2m_N}F_{P'}^{q/N}(q^2)\gamma_5 q^\mu\Big]u_N\,,$$

$$\langle N'|m_q\bar{q}q|N\rangle = F_S^{q/N}(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|m_q\bar{q}i\gamma_5 q|N\rangle = F_P^{q/N}(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\frac{\alpha_s}{12\pi}G^{a\mu\nu}G^a_{\mu\nu}|N\rangle = F_G^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha_s}{8\pi}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}|N\rangle = F_{\tilde{G}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\bar{q}\sigma^{\mu\nu}q|N\rangle = \bar{u}'_N\Big[\hat{F}_{T,0}^{q/N}(q^2)\,\sigma^{\mu\nu} - \frac{i}{2m_N}\gamma^{[\mu}q^{\nu]}\hat{F}_{T,1}^{q/N}(q^2) - \frac{i}{m_N^2}q^{[\mu}k_{12}^{\nu]}\hat{F}_{T,2}^{q/N}(q^2)\Big]u_N\,,$$

$$\langle N'|\frac{\alpha}{12\pi}F^{\mu\nu}F_{\mu\nu}|N\rangle = F_\gamma^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha}{8\pi}F^{\mu\nu}\tilde{F}_{\mu\nu}|N\rangle = F_{\tilde{\gamma}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,.$$

EFT tower

- **Matching between WET and NRET (hadronization)**
- **Matching expressions →**

$$
\begin{aligned}
c_1^N = &-\frac{\alpha}{\pi q}\hat{\mathcal{C}}_1^{(5)}\sum_q Q_q F_1^{q/N} + \sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + \sum_q \frac{1}{m_q}\hat{\mathcal{C}}_{5,q}^{(6)} F_S^{q/N} \\
&- \frac{q}{m_N}\sum_q \hat{\mathcal{C}}_{9,q}^{(6)}\left(\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + 4\hat{F}_{T,2}^{q/N}\right) \\
&+ \hat{\mathcal{C}}_1^{(7)} F_G^N + \hat{\mathcal{C}}_5^{(7)} F_\gamma^N + (q + m_+)\sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N} \\
&- \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left[\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + \left(4 + \frac{q^2}{m_N^2}\right)\hat{F}_{T,2}^{q/N}\right], \\
c_2^N = &\, i\Big[\sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + m_+ \sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N} + \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left(\hat{F}_{T,1}^{q/N} - 4\hat{F}_{T,2}^{q/N}\right)\Big],
\end{aligned}
$$

$$\vdots$$

EFT tower

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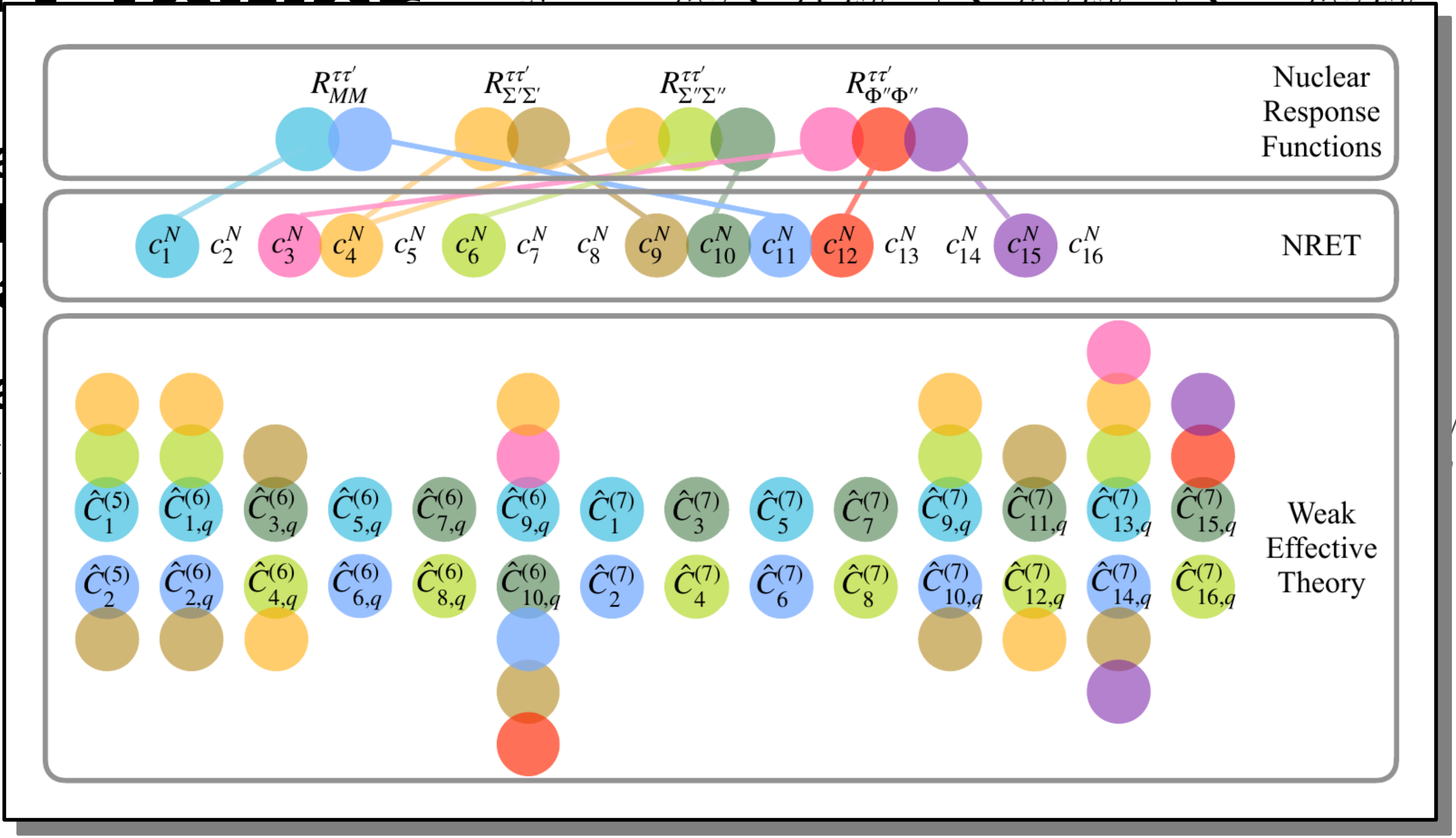
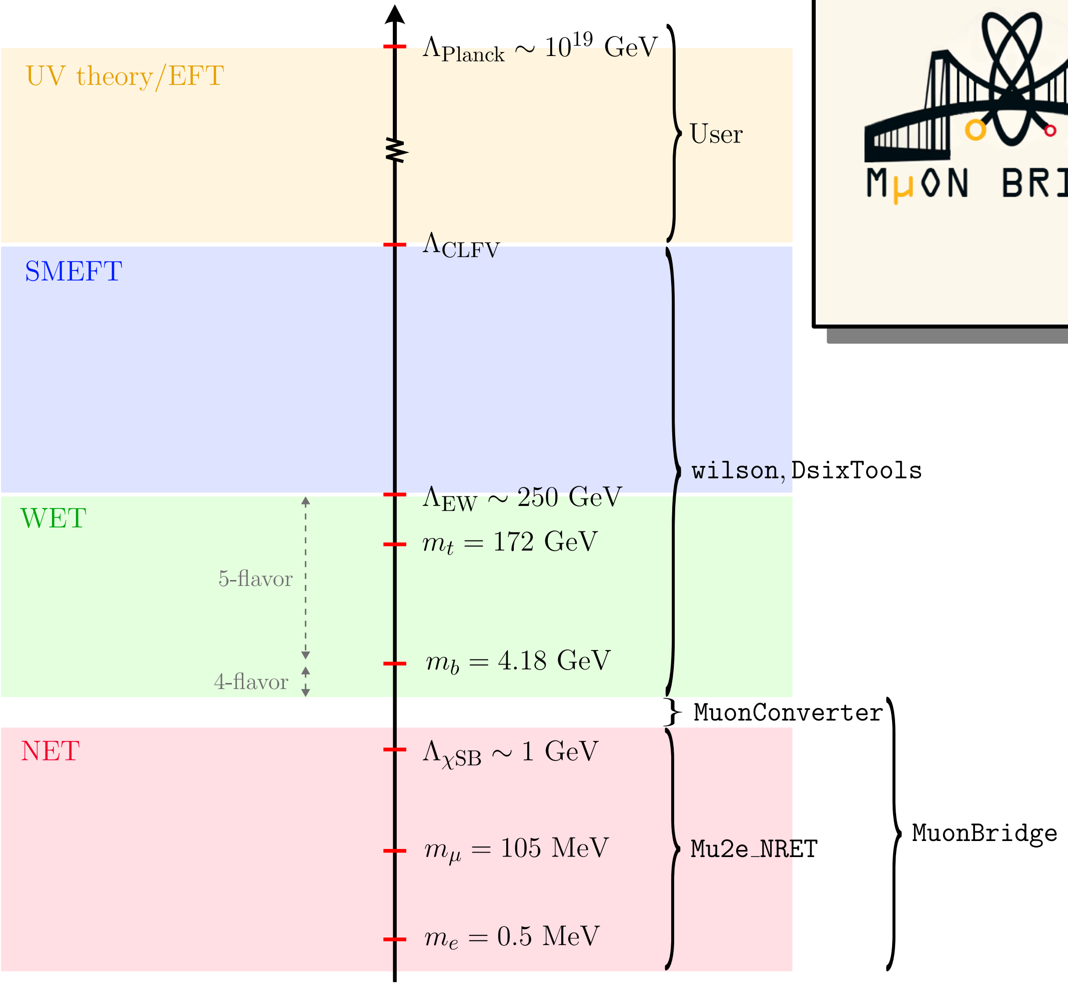


MuonBridge

Three components:

1. Elastic - one body density matrices
2. Mu2e_NRET – Computes μ⟶e conversion rate
3. MuonConverter – matches WET to NRET and facilitates interface with existing EFT software



Mu

Three

1. Elas
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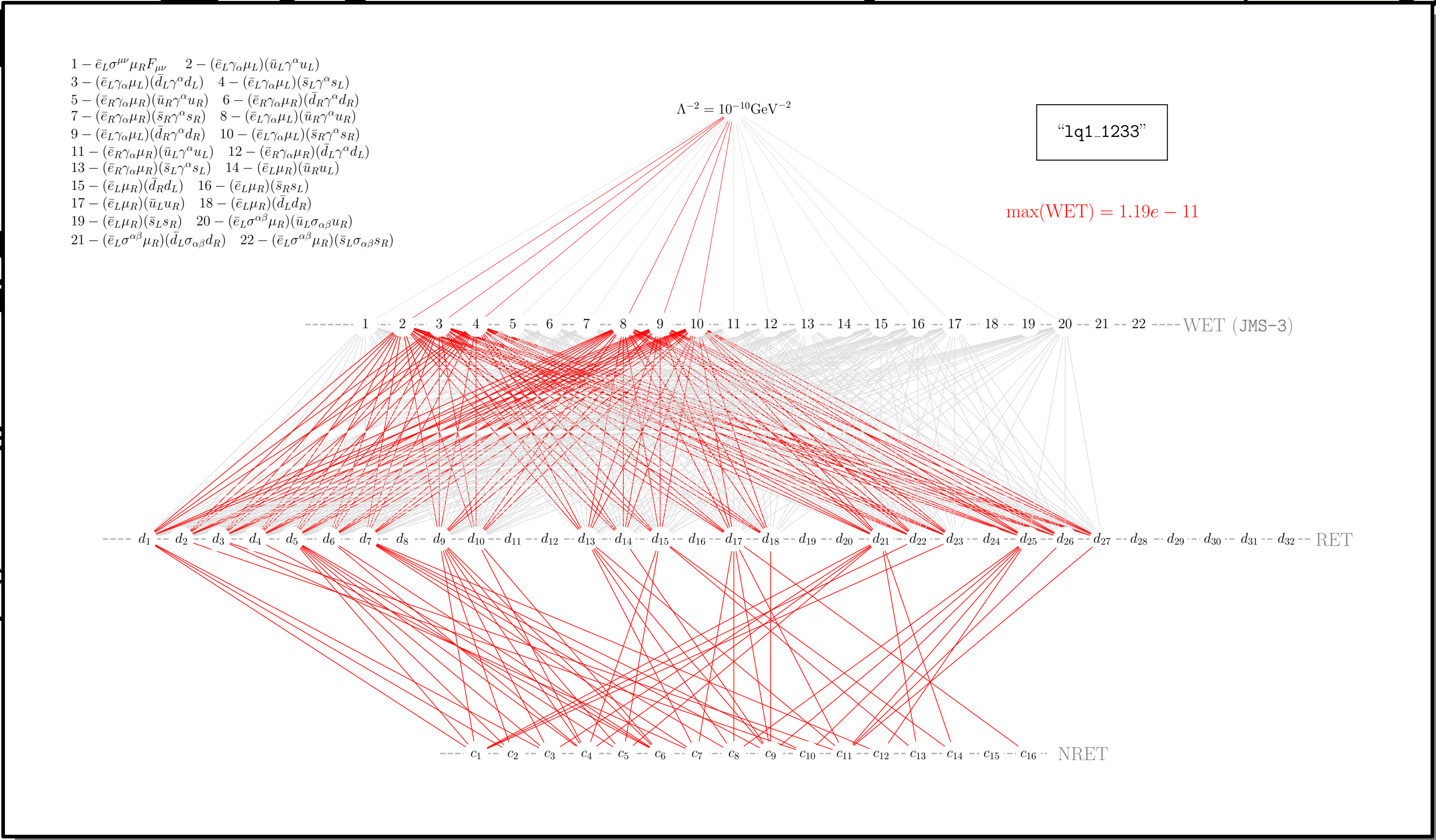
2. Mu2
   μ→e
3. Muo
   matc
   facili
   exist



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Usage

In practice the code can be used in ~2 ways

1. Bottom-up analyses where μ → e is investigated in the context of an EFT

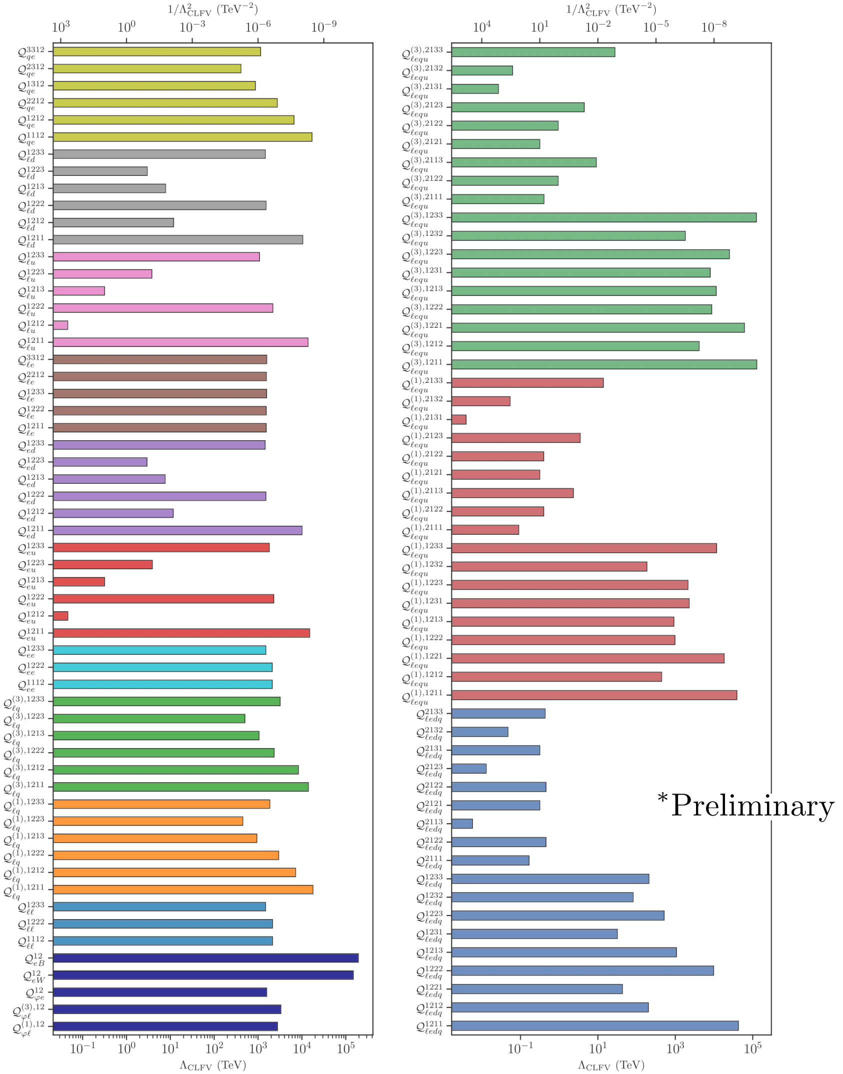
$$\{\mathcal{O}_1, \mathcal{O}_2, \cdots\} \to \Gamma(\mu N \to eN)$$

2. Top-down analyses where an explicit UV model is matched onto the SMEFT and run down to the nuclear scale

$$M_{\rm UV}(\Lambda_{\rm UV}, \vec{\Phi}, \mathcal{L}_{\rm UV}) \to \Gamma(\mu N \to eN)$$

Bottom-up

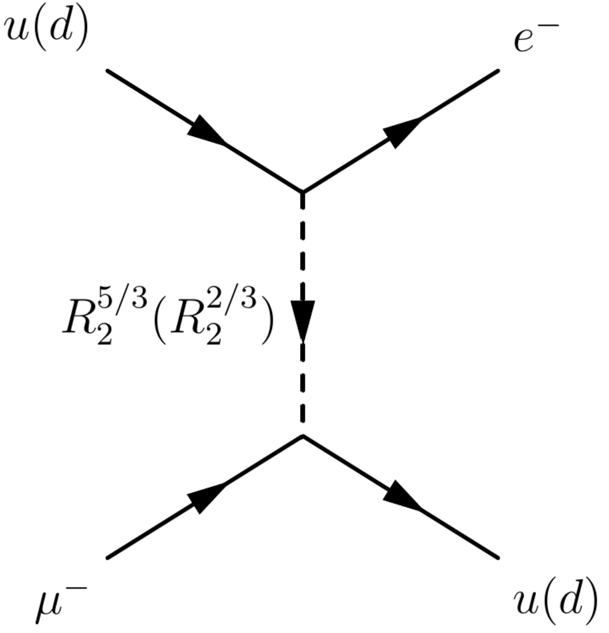
- Single dim-6 SMEFT operator bounds (with one-loop running down to 2 GeV)



Top-down

- **Consider the following leptoquark model**

$$\mathcal{L} \supset -y_{2\,ij}^{RL}\,\bar{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} + y_{2\,ij}^{LR}\,\bar{e}_R^i R_2^{a\,*} Q_L^{j,a} + \text{h.c.},$$



Match onto SMEFT @ $\Lambda = m_{LQ}$:

$$C_{\substack{\ell u\\12ii}} = -\frac{1}{2m_{LQ}^2} y_{2\,i2}^{RL} y_{2\,i1}^{RL*},$$

$$C_{\substack{qe\\ii12}} = -\frac{1}{2m_{LQ}^2} y_{2\,2i}^{LR*} y_{2\,1i}^{LR},$$

$$C^{(1)}_{\substack{\ell equ\\12ii}} = 2C^{(3)}_{\substack{\ell equ\\12ii}} = -\frac{1}{2m_{LQ}^2} y_{2\,2i}^{LR*} y_{2\,i1}^{RL*},$$

$$C^{(1)*}_{\substack{\ell equ\\21ii}} = 2C^{(3)*}_{\substack{\ell equ\\21ii}} = -\frac{1}{2m_{LQ}^2} y_{2\,i2}^{LR} y_{2\,1i}^{RL},$$

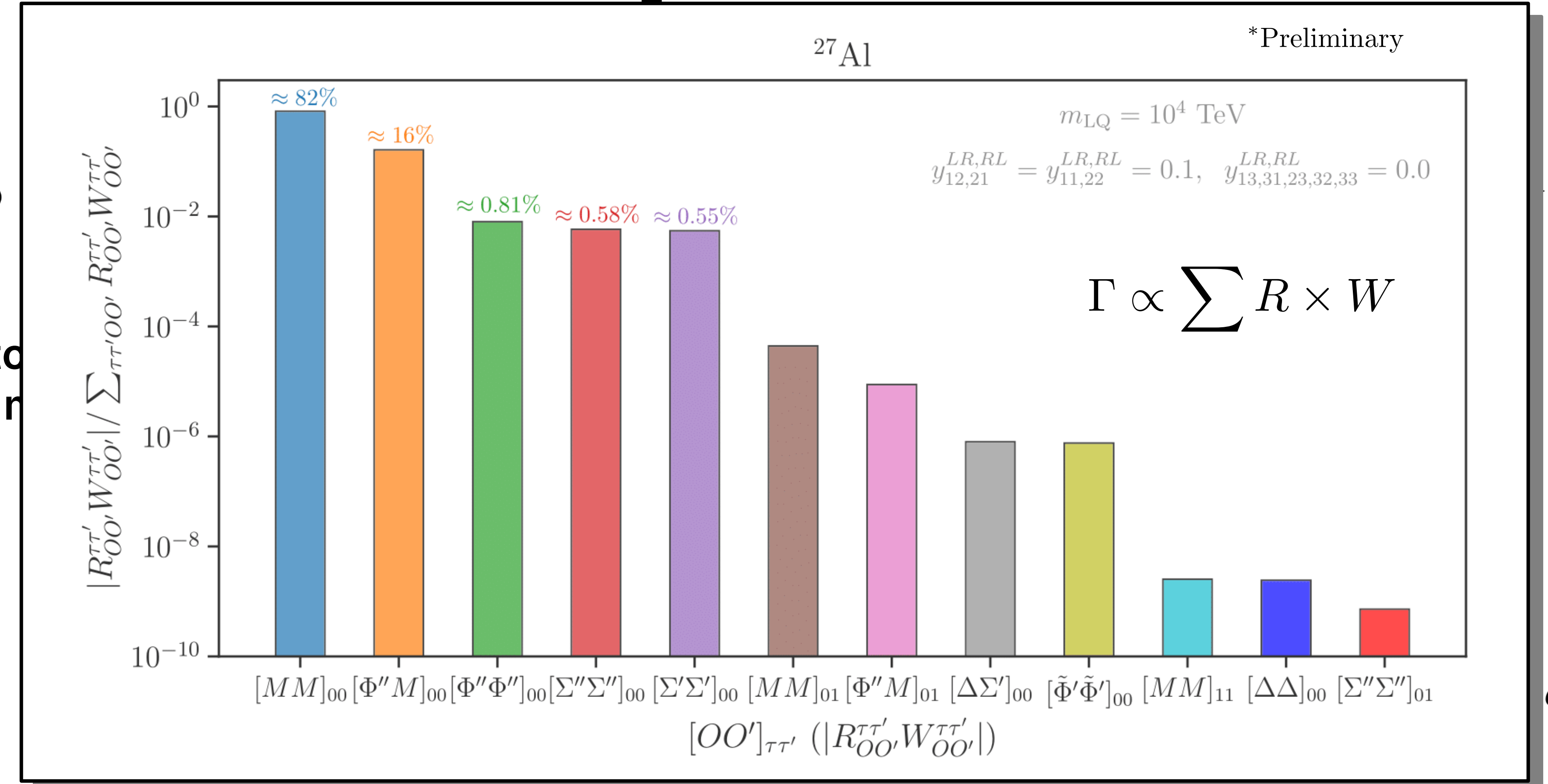
Top-down

•

$\mathcal{L} \supset$

Matc

Λ = r



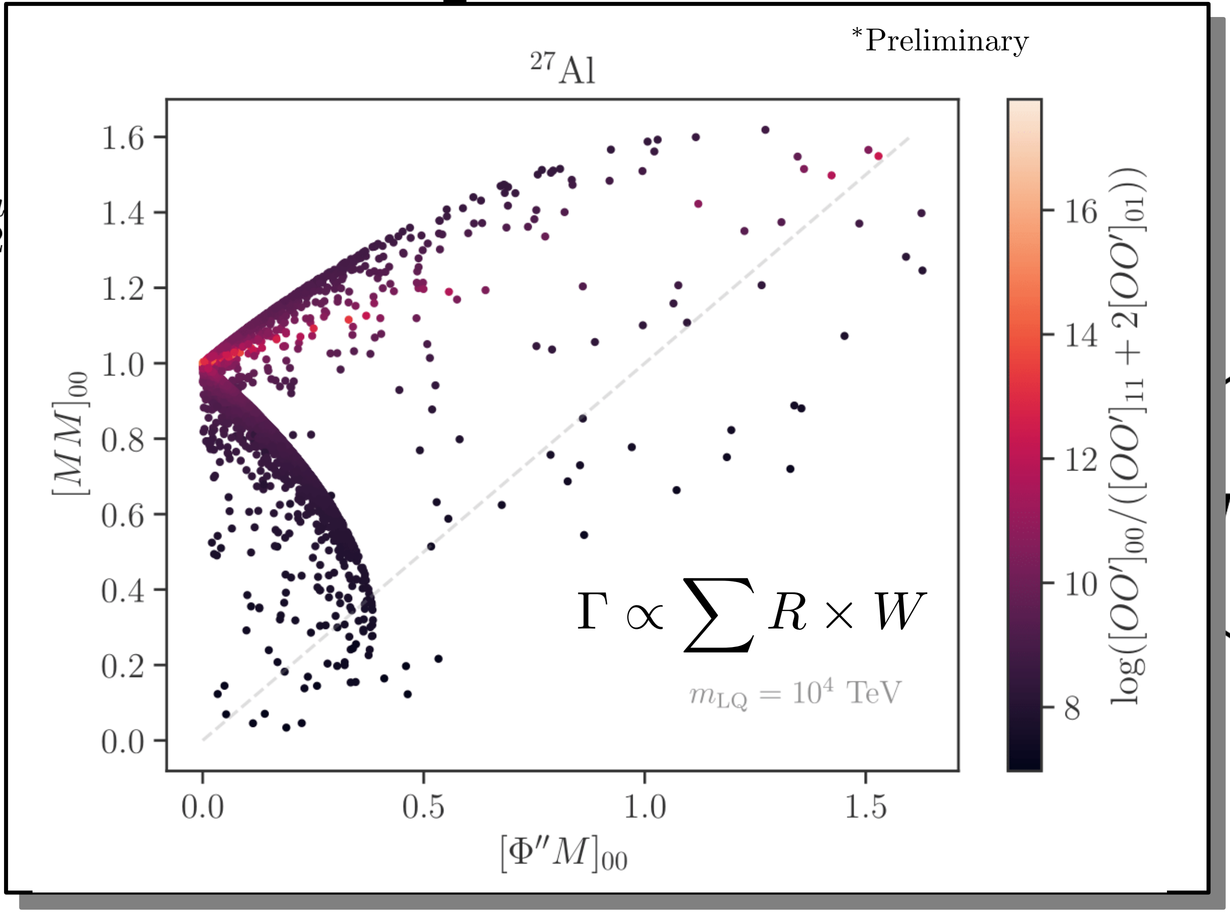
$d)$

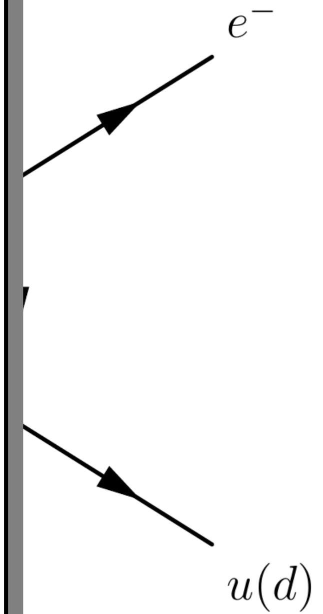
Top-down

- **Consider**

$$\mathcal{L} \supset -y^{RL}_{2\,ij}\,\bar{u}^i_R R^a_2$$

Match onto SME
Λ = m_{LQ}:





Conclusions

- Presented a very flexible EFT framework for computing μ→e conversion rates within UV theories.
  - Consistent nuclear physics truncation with a complete treatment of the nucleon and muon velocity operators (tensor-current interactions are important)
- Facilitated by a soon-to-be open-source software MuonBridge (available in Python and Mathematica)

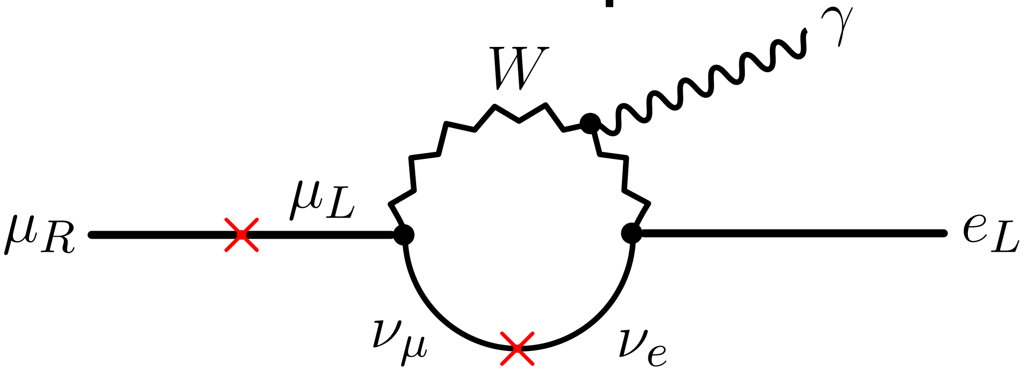
BACK-UP

μ → e

- **The Standard Model (SM) has an accidental global flavor symmetry**

$$U(1)_e \times U(1)_\mu \times U(1)_\tau$$

- **Because $m_\nu \neq 0$ charged-lepton-flavor violation (CLFV) can occur at one-loop**



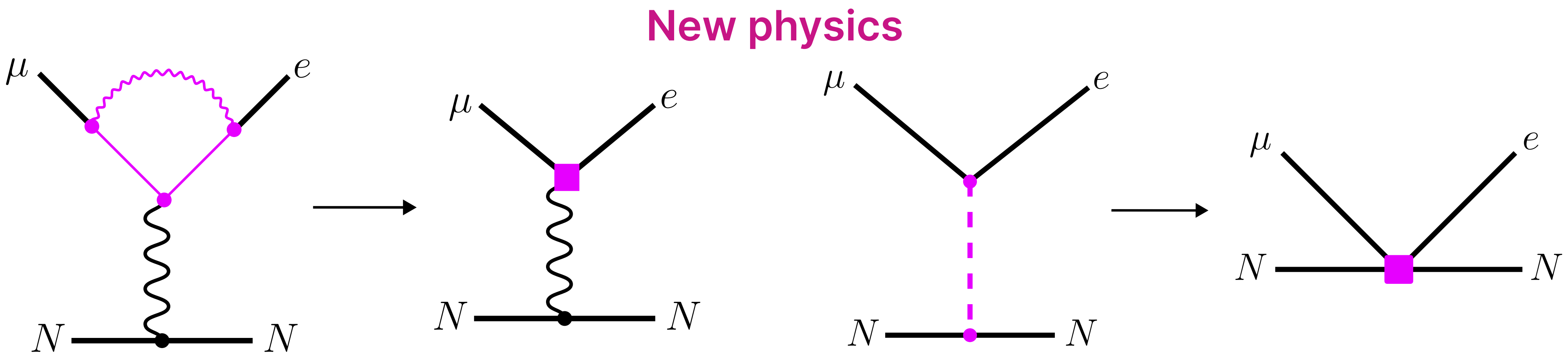
$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{3\alpha}{32\pi M_W^4} |U_{\mu 3}U_{e3}^* \Delta m_{31}^2 + U_{\mu 2}U_{e2}^* \Delta m_{21}^2|^2$$
$$\simeq 10^{-54}$$

Bottom line: Observing CLFV = new physics

Exotic μ → e

In the space of all UV models, CLFV is common. For heavy new physics:

- “Photonic” – e.g. SUSY, heavy steriles, ...
- “Contact” – e.g. Z’, leptoquarks, ...



Upcoming experiments

- **Mu → E Gamma (MEG) @ PSI - μ → eɣ**

  Projected: $\mathrm{BR}(\mu^+ \to e^+\gamma) \lesssim 6 \times 10^{-14}$

- **Mu3e @ PSI - μ → eee**

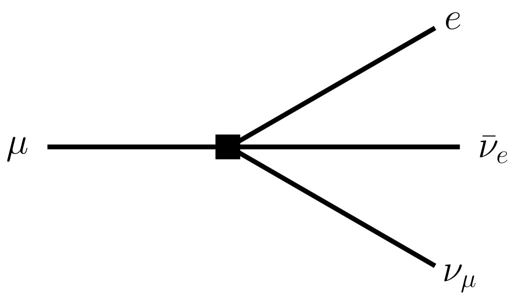
  Projected: $\mathrm{BR}(\mu^+ \to e^+e^-e^+) \lesssim 10^{-16}$

- **Mu2e @ Fermilab, COMET @ J-PARC - Nμ → Ne**

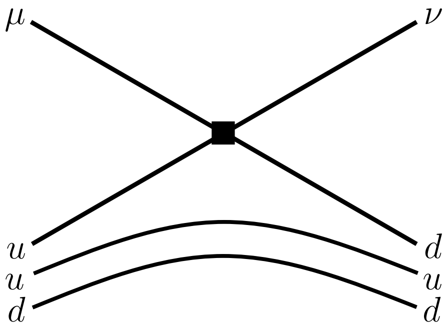
  Projected: $\mathrm{CR}(\mu^- \mathrm{Al} \to e^- \mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$

Upcoming experiments

$$\mathrm{CR} \equiv \frac{\Gamma(\mu \to e)}{\Gamma(\mu p \to e\nu)}, \mathrm{BR} \simeq \frac{\Gamma(\mu \to e)}{\Gamma(\mu \to e\nu\nu)}, \quad \Gamma(\mu \to e\nu\nu) = \frac{G_F^2 m_\mu^5}{192\pi^3} \simeq 10^{-16}\ \mathrm{MeV}$$



$^{27}\mathrm{Al}$ $\approx$



Projected: $\mathrm{CR}(\mu^- \mathrm{Al} \to e^- \mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$

Upcoming experiments

- **Mu → E Gamma (MEG) @ PSI - μ → eγ**

Projected: $\mathrm{BR}(\mu^+ \to e^+\gamma) \lesssim 6 \times 10^{-14}$ $(\Gamma(\mu \to e) \lesssim 10^{-33}\ \mathrm{GeV})$

- **Mu3e @ PSI - μ → eee**

Projected: $\mathrm{BR}(\mu^+ \to e^+e^-e^+) \lesssim 10^{-16}$ $(\Gamma(\mu \to e) \lesssim 10^{-35}\ \mathrm{GeV})$

- **Mu2e @ Fermilab, COMET @ J-PARC - Nμ → Ne**

Projected: $\mathrm{CR}(\mu^-\mathrm{Al} \to e^-\mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$ $(\Gamma(\mu \to e) \lesssim 10^{-36}\ \mathrm{GeV})$

Upcoming experiments

- **Mu → E Gamma (MEG) @ PSI - μ → eɣ**

Projected: $\mathrm{BR}(\mu^+ \to e^+\gamma) \lesssim 6 \times 10^{-14}$ $(\Gamma(\mu \to e) \lesssim 10^{-10}\ \mathrm{Hz})$

- **Mu3e @ PSI - μ → eee**

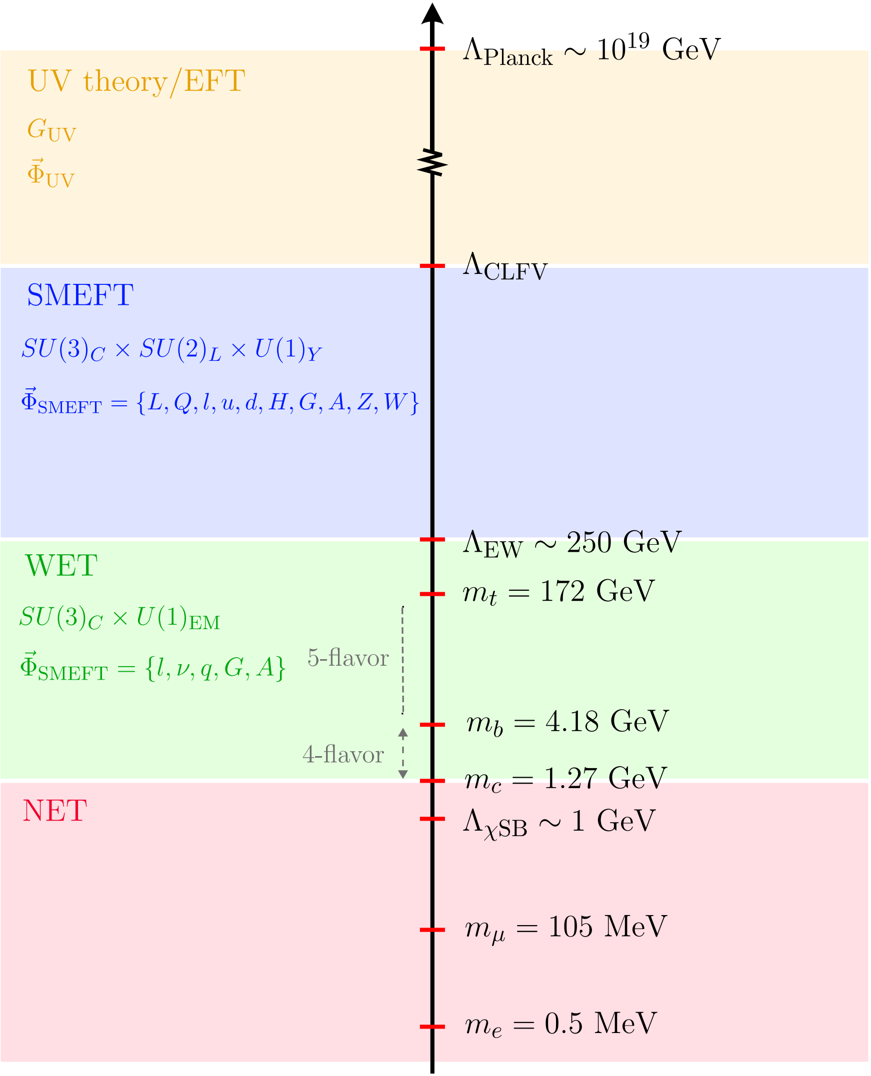
Projected: $\mathrm{BR}(\mu^+ \to e^+e^-e^+) \lesssim 10^{-16}$ $(\Gamma(\mu \to e) \lesssim 10^{-12}\ \mathrm{Hz})$

- **Mu2e @ Fermilab, COMET @ J-PARC - Nμ → Ne**

Projected: $\mathrm{CR}(\mu^-\mathrm{Al} \to e^-\mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$ $(\Gamma(\mu \to e) \lesssim 10^{-13}\ \mathrm{Hz})$

EFT tower

- Consider a series of EFTs
- EFT tower ingredients
  1. Scales (Λ_i)
  2. Fields ($\vec{\Phi}_i$)
  3. Interactions ($\{\mathcal{O}\}_i$)
  4. Matching conditions $c_I^{\mathrm{IR}}(\{\vec{c}^{\,\mathrm{UV}}\})$



EFT tower

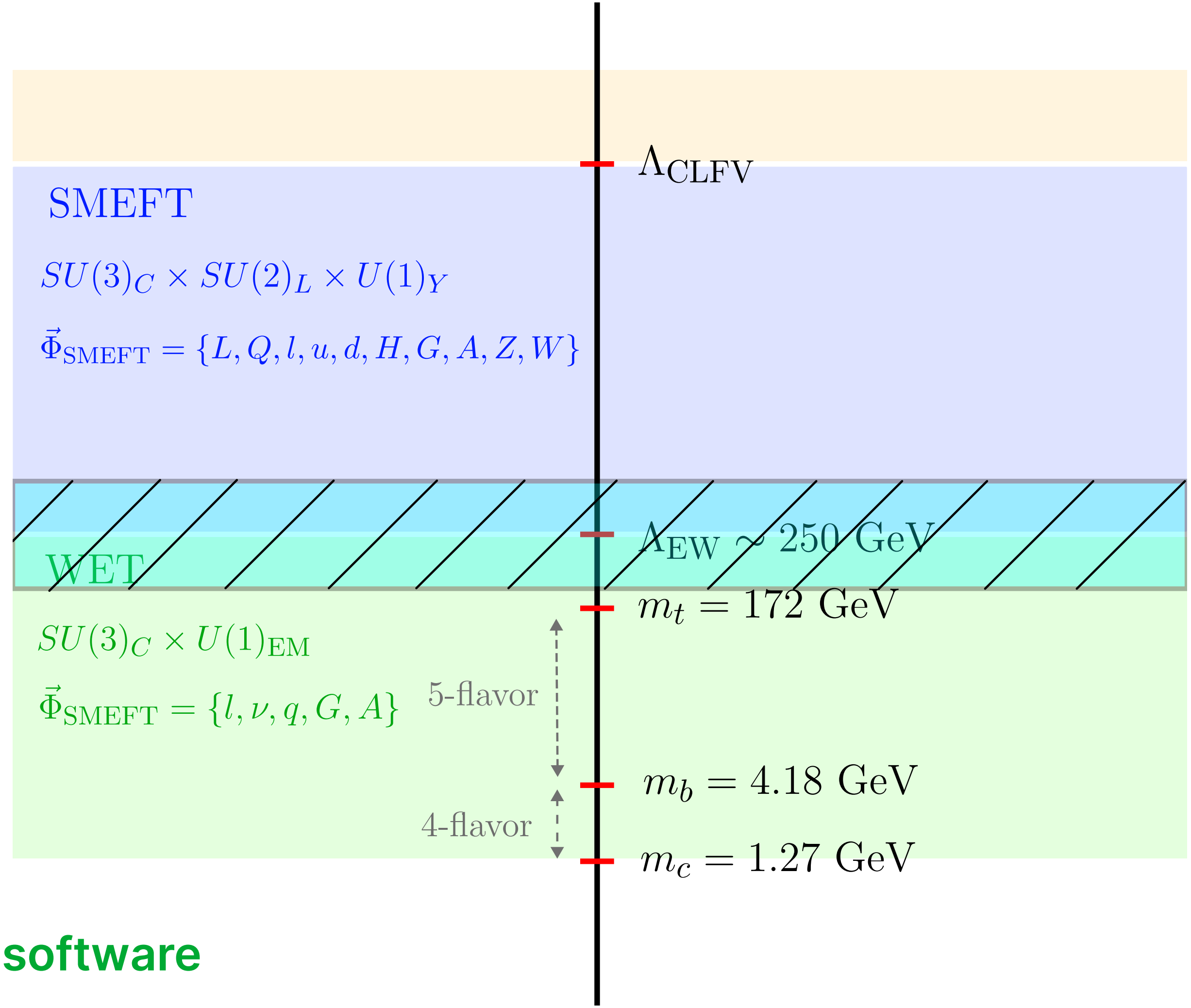
- SMEFT operators

$$
\begin{aligned}
Q_{e\varphi}: &\quad (\varphi^\dagger\varphi)(\bar{L}_2 e_R\varphi), \quad (\varphi^\dagger\varphi)(\bar{\ell}_1\mu_R\varphi),\\
Q_{eW}: &\quad (\bar{\ell}_2\sigma^{\mu\nu}e_R)\tau^I\varphi W^I_{\mu\nu}, \quad (\bar{\ell}_1\sigma^{\mu\nu}\mu_R)\tau^I\varphi W^I_{\mu\nu},\\
Q_{eB}: &\quad (\bar{\ell}_2\sigma^{\mu\nu}e_R)\varphi B_{\mu\nu}, \quad (\bar{\ell}_1\sigma^{\mu\nu}\mu_R)\varphi B_{\mu\nu},\\
Q^{(1)}_{\varphi\ell}: &\quad (\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{\ell}_2\gamma^\mu\ell_1),\\
Q^{(3)}_{\varphi\ell}: &\quad (\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{\ell}_2\gamma^\mu\tau^I\ell_1),\\
Q_{\varphi e}: &\quad (\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{\mu}_R\gamma^\mu\tau^I e_R),\\
Q^{(1)}_{\ell q}: &\quad (\bar{\ell}_2\gamma^\mu\ell_1)(\bar{q}\gamma^\mu q),\\
Q^{(3)}_{\ell q}: &\quad (\bar{\ell}_2\gamma^\mu\tau^I\ell_1)(\bar{q}\gamma^\mu\tau^I q),\\
Q_{eu}: &\quad (\bar{\mu}_R\gamma^\mu e_R)(\bar{u}_R\gamma^\mu u_R),\\
Q_{ed}: &\quad (\bar{\mu}_R\gamma^\mu e_R)(\bar{d}_R\gamma^\mu d_R),\\
Q_{\ell u}: &\quad (\bar{\ell}_2\gamma^\mu\ell_1)(\bar{u}_R\gamma^\mu u_R),\\
Q_{\ell d}: &\quad (\bar{\ell}_2\gamma^\mu\ell_1)(\bar{d}_R\gamma^\mu d_R),\\
Q_{qe}: &\quad (\bar{q}\gamma^\mu q)(\bar{\mu}_R\gamma^\mu e_R),\\
Q_{\ell edq}: &\quad (\bar{\ell}_2 e_R)(\bar{d}_R q), \quad (\bar{\ell}_1\mu_R)(\bar{d}_R q),\\
Q^{(1)}_{\ell equ}: &\quad (\bar{\ell}^j_2 e_R)\varepsilon_{jk}(\bar{q}^k u), \quad (\bar{\ell}^j_1\mu_R)\varepsilon_{jk}(\bar{q}^k u),\\
Q^{(3)}_{\ell equ}: &\quad (\bar{\ell}^j_2\sigma_{\mu\nu}e_R)\varepsilon_{jk}(\bar{q}^k\sigma^{\mu\nu}u), \quad (\bar{\ell}^j_1\sigma_{\mu\nu}\mu_R)\varepsilon_{jk}(\bar{q}^k\sigma^{\mu\nu}u),
\end{aligned}
$$

EFT tower

- SMEFT-WET matching

*Very well-developed literature related to running and matching for both EFTs, e.g. 1308.2627, 1310.4838, 1312.2014, 1709.04486, 1711.05270



Implemented in efficient public software